The Casualty Actuarial Society *Forum*
Fall 2002 Edition
Including the Reserves Discussion Papers

To CAS Members:

This is the Fall 2002 Edition of the Casualty Actuarial Society *Forum*. It contains nine Reserves Discussion Papers, one committee report, and six additional papers.

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The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted. The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Dennis L. Lange

Dennis L. Lange, CAS *Forum* Chairperson

The 2002 CAS Reserves Discussion Papers
Presented at the
2002 Casualty Loss Reserve Seminar
September 23-25, 2002
Crystal Gateway Marriott
Arlington, Virginia

The Fall 2002 Edition of the CAS *Forum* is a cooperative effort between the CAS *Forum* Committee and the CAS Committee on Reserves.

The CAS Committee on Reserves presents for discussion nine papers prepared in response to its Call for 2002 Reserves Discussion Papers.

This *Forum* includes papers that will be discussed by the authors at the 2002 CAS Casualty Loss Reserve Seminar, September 23-25, 2002, in Arlington, Virginia.

Reserves Discussion Papers

Committee Reports

Additional Papers

Correlation and the Aggregation of Unpaid Loss Distributions

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CORRELATION AND THE AGGREGATION OF UNPAID LOSS DISTRIBUTIONS

PAUL BREHM, FCAS, MAAA

Abstract

Significant progress has been made in the last decade in developing models to describe the distribution of unpaid losses for a line of insurance. Line-by-line distributions must be aggregated, however, in order to address company-wide issues such as enterprise risk, capital requirements, fair value, and more.

Using U.S. industry commercial lines data, this paper uses Zehnwirth's method to produce distributions of unpaid losses by line of insurance and Wang's standard normal copula method of aggregating correlated risk portfolios to create aggregate distributions of unpaid losses. In doing so, a methodology for direct estimation of correlations between lines is proposed.

1. MODEL OF UNPAID LOSSES[1]

Means. Actuarial loss reserving training is focused on getting a decent estimate of the mean of the unpaid loss and loss expense liability. Witness the CAS syllabus does not contain one reading concerning models for unpaid losses that reflect the stochastic nature of unpaid losses, producing distributions, or confidence intervals. Perhaps that's fair: it is certainly necessary to get the mean right. It's necessary, but not sufficient.

ASOP 36 speaks of "materiality," "ranges of reasonable estimates," "adverse deviations," and "risk margins." Statutory codification refers to "best estimates" and "risk based capital." The IASB speaks of "fair value liabilities." All of these concepts have a common element: they require a view of the higher moments of an unpaid loss distribution. We are, after all, in the business of risk, and to truly understand the risk associated with a portfolio of loss reserves the actuary must have a view about the distribution of unpaid losses, not just the mean.

Higher moments. A number of methodologies have been proposed to produce confidence intervals about a mean reserve estimate, or alternatively, produce an estimated distribution of unpaid losses. In particular the 1994 call paper program on the variability of loss reserves and subsequent papers published some truly landmark methodologies: Halliwell [9], Holmberg [10], Mack [12], Murphy [15], Verall [20], and Zehnwirth [24] just to name a few. While each model has its strengths and weaknesses, this paper

[1] Throughout this paper the term "loss" is used to refer to all loss and defense costs and adjusters fees (the old allocated loss adjustment expenses) for simplicity.

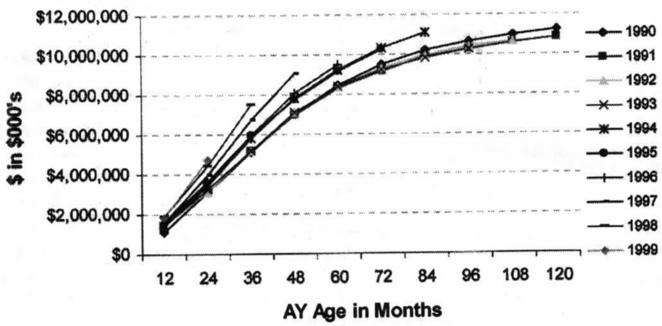
applies Zehnwirth's model to industry Schedule P data to illustrate the estimation of a distribution of unpaid losses.

Zehnwirth example. The data used in the following example was taken from the U.S. total industry 2000 Schedule P as compiled by A.M. Best[2]. Commercial lines, excluding excess of loss reinsurance lines, were analyzed. Data was grouped into six lines (commercial auto, workers' compensation, commercial multi-peril, medical malpractice, all other liability, and all other[3]) in an attempt to minimize detail.

The industry (non-medical) liability triangle will be used to illustrate Zehnwirth's modeling framework. Graph 1.1 shows the cumulative development by accident year for these data.

Graph 1.1

Industry Liability Cumulative Paid Losses



Zehnwirth's model can be characterized by the following algorithm.

1. Start with a cumulative paid loss data triangle (illustrated above).
2. Calculate an incremental paid triangle.
3. Inflation adjust the incremental paids, if desired.
4. Preferably divide each incremental paid by exposure, or in the absence of exposure as with industry data, by earned premium.
5. Calculate the natural logs of the incremental ratios in step #4.

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[2] The 1990 accident year was included, too, at least through year-end 1999, from the 1999 report. Current reserves for the 1999 accident year were estimated by running off a portion of those carried at year-end 1999.

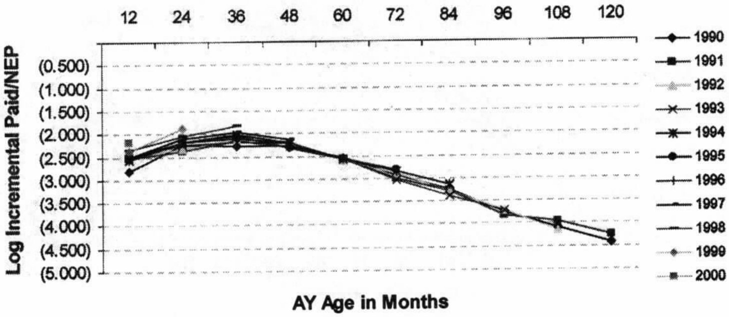
[3] The 'all-other' -- principally short tailed -- triangle was created by subtracting each reported line triangle (including homeowners and private passenger auto liability) from the total, all-lines triangle. This does not yield a pure short tailed commercial lines triangle. The triangle will still include personal auto physical damage and inland marine floaters. This triangle was included anyway, for illustrative purposes.

6. Model the log incremental paid ratios with a regression model, incorporating dummy variables for the accident year, development year, and calendar year dimensions of the triangle.

Focus is on paid data because logs are used on the incremental ratios. Since the increments cannot be negative, downward development is problematic. In fact, even paid data can present a problem in lines where negative incremental paids are common (e.g., surety).

Graph 1.2 shows the transformed triangle for the cumulative data in Graph 1.1: the log of incremental paid losses to net earned premium. The data was not inflation adjusted, but could be without loss of generality.

Graph 1.2
Log Incremental Paid/NEP



Zehnwirth's model describes the above data patterns with parameters for the accident year ("i") dimension (α_i, which are essentially the vertical leveling of the accident year), the development year ("j") dimension (γ_j, which actuaries would call the incremental payout pattern), and the calendar year (i+j) dimension (ι_{ij}, which would represent some sort of calendar period distortion not otherwise picked up by the parameters in the other two dimensions, e.g., a shift in inflation or a change in the claims department). The parameterization is such that tail development is already calculated as part of the assumed exponential decay by the last fitted γ. The general formula, then, looks as follows. For the log incremental paid ratio for accident year i and development period j:

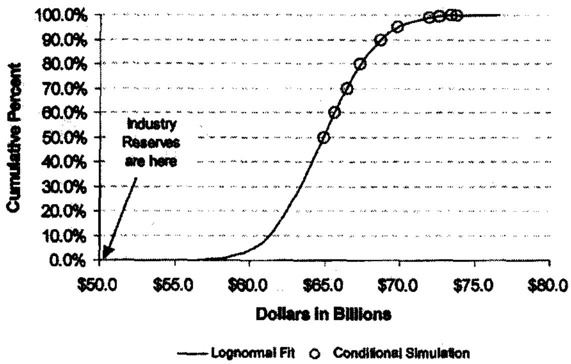
$$y_{ij} = \alpha_i + \Sigma^{j}_{k=1}\gamma_k + \Sigma^{i+j}_{t=1}\iota_t + \varepsilon_{ij} \qquad 1.1$$

In essence, Zehnwirth's model describes the log of each incremental payment ratio by the combination of three vectors, one each for the accident year, the development period, and the calendar year.

The regression model (1.1) assumes that the ε will be normally distributed. Thus, each estimated incremental payment, when transformed back into dollars, will be distributed logormally. Since the distribution of the sum of lognormals is not a simple closed form distribution, the aggregate unpaid loss must be simulated in a conditional simulation using formula 1.1 above.

Graph 1.3 shows the simulated CDF for the industry liability unpaid losses. A lognormal curve was fit to the simulated distribution using a simple method-of-moments fit. Despite the fact that the sum of lognormals theoretically isn't lognormal, a lognormal curve fits the data well.

Graph 1.3
Industry Commercial Liability Unpaid Loss CDF



Loss triangles for each commercial line were modeled as above. Distributions were estimated using conditional simulations with formula 1.1. As with the liability example, lognormal models were fit to each distribution with good results. Table 1.1 below summarizes the results.

Table 1.1

U.S. Industry Commercial Lines Reserves 1990-2000

	Model Unpaid	Industry Carried	Redundancy /(Deficiency)	% tile	Lognormal Parameters μ	σ
Commercial Auto [C]	23,008	18,911	(4,097)	0.0%	3.135	0.032
Work Comp. [D]	66,535	60,597	(5,938)	15.8%	4.194	0.089
CMP [E]	27,734	24,753	(2,981)	0.0%	3.322	0.018
Med Mal [F]	26,098	19,478	(6,620)	0.0%	3.261	0.032
Liability [H,R]	64,950	50,148	(14,802)	0.0%	4.173	0.045
All Other	26,254	24,578	(1,676)	27.0%	3.263	0.099
	234,579	198,465	(36,114)	?	?	?

Table 1.1 highlights the central issue addressed in this paper. While estimating the distributions of unpaid losses by line is an important advance in actuarial science, the actuary is still ill equipped to estimate the distribution of unpaid losses for the entire portfolio.

We could apply the Zehnwirth model to an industry total, all lines triangle and get both a mean result and an associated distribution. As actuaries, we bristle at the thought: the lack of homogeneity in such a triangle would surely distort our ability to get the mean unpaid loss estimate correct. So, we model unpaid losses in more homogenous, hopefully still credible, segments. Line-by-line we can calculate the mean unpaid loss and its distribution, but now we have lost the ability to calculate a simple closed form distribution for the aggregate unpaid loss reserve requirement. We cannot fill in the empty boxes in Table 1.1 without a methodology for aggregating correlated distributions.

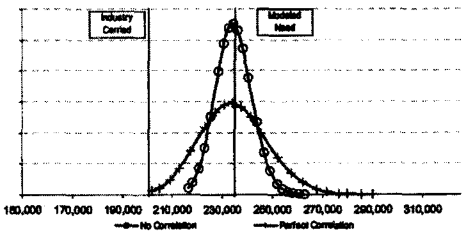
2. AGGREGATE DISTRIBUTIONS

Correlations. Methods exist for aggregating distributions, but the correlations between the distributions are always the critical component. Actuaries seem to appreciate the potentially profound impact correlations have on required economic capital, capital allocations or risk loads, reinsurance buying, etc., but very little is written about it[4].

Polar cases of correlations are simple and illustrative. For example, given marginal distributions of unpaid losses for two lines of business, an aggregate distribution can be easily created if one assumes the correlation is –1, 0, or 1 between the lines. If correlation is 0, one could simulate an amount from each of the distributions and simply add the two numbers together. If the correlation is 1, amounts are simulated for each distribution, sorted, and matched up from smallest to largest, and then added together (for –1 match opposite rank orders). Alternatively, an aggregate distribution can be computed in closed form with a simple variance-covariance matrix and an assumption regarding functional form.

As a basis, the marginal lognormal distributions shown in Table 1.1 were aggregated in just such manners for the polar cases of no correlation and perfect correlation. The results are shown in graph 2.1 below.

Graph 2.1
Aggregate Unpaid Loss Distributions
with Polar Correlation Assumptions



[4] Wang [22] mentions a constant correlating factor for unpaid losses in an appendix. Myers [23] discussion of Wang illustrates the incorporation of correlation into the calculation of aggregate loss distributions in a collective risk theory framework. The correlation parameter behaves much like the contagion parameter of the collective risk model. Frequency is the correlating factor between lines.

In conversations with Todd Bault, he has hypothesized a model of correlation wherein correlation behaves as a scaling factor. His argument goes something like this: if you believe that everything is correlated to the whole, then the larger the line, the higher the correlation. Chris Gross has offered a similar model, one where there exists a correlation within a line of subsets of that line (e.g., a new account). While currently unpublished, I think these views have some merit and should be explored.

The public access DFA model [5] addresses correlations in unpaid losses by essentially injecting them into future loss payments. Simple linear models are used to describe line of business loss cost inflation as a function of the CPI. Thus all lines are correlated with each other since they are based on the CPI. This correlating inflation is applied in the accident year dimension.

Graph 2.1 clearly shows the importance of correlation assumptions in computing an aggregate distribution – not for the mean – but for the risk component. If the industry were required to hold capital to, say, the 99.9th percentile over carried reserves, required economic capital would be roughly $60 billion for the uncorrelated case and over $80 billion for the perfectly correlated case.

Correlations are likely not zero or one. Nor are we typically dealing with only two lines. For correlations other than the polar values, or for more than two lines, mixing distributions is no longer so simple.

Measuring correlations. The single biggest source of risk in an unpaid loss portfolio is arguably the potential distortions that can affect all open accident years, i.e., changes in calendar year trends. Such a distortion could be a social inflation, say in court judgments, that affect all open claims (subsequent incremental payments) at once. In Zehnwirth's model, this is saying that the future ι_{ij}'s turn out to be much different than predicted and reserved for.

Our line-by-line distributions reflect the calendar year inflation risk, since ι_{ij} is a statistical estimate and has an associated variability that is incorporated into the conditional simulations. But what happens if a calendar year distortion affects more than one triangle simultaneously? This is the sort of thing that keeps chief actuaries awake at nights: the fear that a systematic distortion will affect multiple lines, in the same (bad) direction all at once.

Having modeled a number of lines of business, we have a vector of calendar period trend parameters, ι_{ij}, for each line. By measuring the line-by-line correlations between these historic parameters, we can estimate a correlation matrix for calendar period movements. The estimated calendar year parameters and associated correlation matrix from this exercise are shown below[5].

[5] It's worth mentioning correlation measures that don't work for the purposes of unpaid loss distributions. Correlations in future unpaid losses cannot be calculated by looking at time series of loss ratios. Historic loss ratios are typically highly correlated because of cyclical pricing impacts in the denominator of the loss ratio. For loss reserving purposes pricing induced correlations do not matter. Furthermore, correlations in the numerator of a loss ratio, if measured at an ultimate value are suspect depending on methodology. If, for example, two lines are developed to ultimate using the Bornhuetter-Ferguson methodology with the same or similar seed loss ratios, correlation is actually injected into the "data."

Figure 2.1
Calendar Period Inflation Parameter Estimates (ι_{ij})

	Commercial Auto [C]	Work Comp. [D]	CMP [E]	Med Mal [F]	Liability [H, R]	Other
1991-1992	9.18%	3.70%	-5.96%	0.00%	0.00%	0.00%
1992-1993	5.09%	3.70%	2.67%	0.00%	0.00%	0.00%
1993-1994	5.09%	3.70%	2.67%	0.00%	0.00%	0.00%
1994-1995	5.09%	3.70%	-3.29%	0.00%	0.00%	0.00%
1995-1996	5.09%	3.70%	-3.29%	0.00%	0.00%	0.00%
1996-1997	2.70%	3.70%	-3.29%	-5.64%	0.00%	0.00%
1997-1998	2.70%	10.24%	-3.29%	6.06%	-3.25%	0.00%
1998-1999	5.74%	0.00%	-3.29%	6.06%	5.34%	0.00%
1999-2000	5.74%	15.05%	-3.29%	6.06%	5.34%	5.01%

Correlation in the historical calendar year inflation trends is evident by inspection of the above table. While not uniformly true, it is apparent that inflation is low and remarkably stable through out the middle-90's. Inflation even inexplicably declines in some lines in or near 1996. Finally, there seems to be evidence that inflation accelerates some time in 1998 or 1999.

To compute the correlation matrix from the above table, simply calculate all pair-wise correlations between the lines of business. In Excel, for the correlation between auto and work comp for example, this would be the formula: =correl(AUTO, COMP). These calculations were made and are shown below in Figure 2.2.

Figure 2.2
Correlation Matrix

	Commercial Auto [C]	Work Comp. [D]	CMP [E]	Med Mal [F]	Liability [H, R]	Other	average
Commercial Auto [C]	1.000	(0.169)	(0.259)	0.100	0.337	0.115	0.025
Work Comp. [D]	(0.169)	1.000	(0.138)	0.465	0.079	0.812	0.210
CMP [E]	(0.259)	(0.138)	1.000	(0.139)	(0.118)	(0.132)	(0.157)
Med Mal [F]	0.100	0.465	(0.139)	1.000	0.396	0.444	0.253
Liability [H,R]	0.337	0.079	(0.118)	0.396	1.000	0.611	0.261
Other	0.115	0.812	(0.132)	0.444	0.611	1.000	0.370
							0.160

The average correlation across all values is 0.16 (ignoring the diagonal of the correlation matrix). While fairly small, it is obviously the result of offsetting negative and positive values. The above matrix was accepted as-is in further examples below, but could be judgmentally amended to cap correlations at a certain level, set small (statistically insignificant) correlations to zero, or to erase correlations that lack an intuitive reason for being. No consideration was given to statistical significance, but this may be a fruitful area for future research.

This methodology provides a simple and practical method to measure correlations of unpaid losses between lines of business given typical actuarial data arrays. It is dependent on a loss reserving model capable of estimating calendar year trends, in this case the Zehnwirth model.

The method proposed above is not without its shortcomings.

1. Such a framework is that is inherently a slave to the data at hand. The decade of the 90's, used here for illustration, was a decade marked by low, stable inflation in a prosperous economy. Correlations measured in such an environment will reflect that environment and perhaps nothing more. Actuarial judgment should play a role in adjusting assumptions in changing or different environments.

2. The calendar year inflation parameters are themselves estimates from a model. There is often no one 'true' model, and different model parameters will yield different correlation results.

3. This model measures correlations in unpaid loss distributions by asserting that the principal correlating factor is severity. There is no consideration of frequency, i.e., correlations between lines in pure IBNR claims.

There are undoubtedly additional critiques that could be raised. But, as my father used to say, "it's way better than what comes in second place" – which, from a review of available literature, appears to be nothing.

Between line correlations could be estimated directly if we could model all lines simultaneously with regression model 1.1. By estimating parameters for all lines at once, the variance-covariance matrix from the regression would reflect the correlations between lines. But models and computing power aren't quite there yet. In the mean time the method proposed here serves as a proxy.

Aggregation of Line Distributions – Model 1. Given distributions by line and a correlation matrix, an aggregate distribution can be created numerically using a standard normal copula[6]. Shaun Wang [21] proposed this method in his paper, "Aggregation of Correlated Risk Portfolios: Models and Algorithms," PCAS LXXXV, 1998 (pp. 887-891)[7].

[6] The standard normal copula has been criticized recently for producing uncorrelated tail values, which would clearly defeat the purpose at hand. Cf, Mango [14].
[7] Wang's method is not the only such method. Nakada [16] uses a numerical integration routine that is mathematically similar and does not require simulation.

Wang's standard normal copula algorithm is as follows.

1. Measure the correlation matrix Σ with elements ρ_{ij} (above).

2. From Σ construct the lower triangular matrix **B** via the Cholesky decomposition such that $\Sigma = \mathbf{BB'}$. Each element of B can be defined by:

$$b_{ij} = \frac{\rho_{ij} - \sum_{s=1}^{j-1} b_{is} b_{js}}{\sqrt{1 - \sum_{s=1}^{j-1} b_{js}^2}} \qquad 1 \le j \le i \le n \ \ \& \ \Sigma^0 = 0$$

3. Generate a vector $\mathbf{Y} = (y_1, \ldots, y_k)'$ of standard normals , where k=# of lines,. This is just the Excel function NORMINV(RAND()).

4. Define $\mathbf{Z} = \mathbf{BY}$. $\mathbf{Z} = (z_1, \ldots, z_k)'$ has the appropriate joint pdf defined by the correlation matrix. In Excel this is the array function MMULT(**B**,**Y**).

5. Set $u_i = \Phi(z_i)$, where Φ denotes the standard normal cdf. In Excel, NORMDIST(z_i,0,1,TRUE).

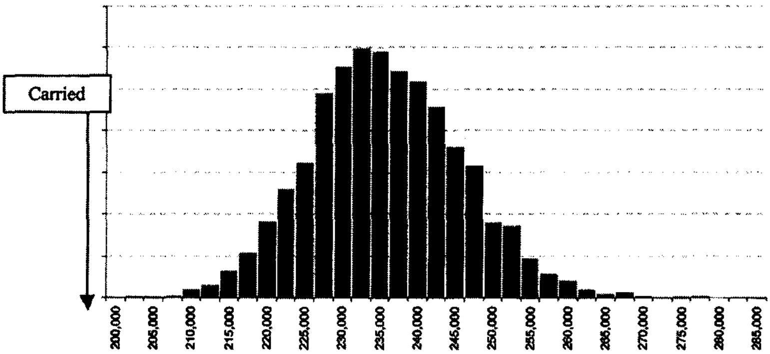
6. Set $x_i = F_i^{-1}(u_i)$, where F_i is the marginal distribution function for the modeling line i. In our case the F_i's are the assumed lognormal distributions produced by ICRFS. However, there is no restriction on the marginals. In fact, F need not be parameterized. An empirical distribution can be used. If a lognormal is used, the Excel formula is LOGINV(u_i,μ_i,σ_i)

7. Iterate steps 3-6 as many times as desired.

Wang shows that the standard normal copula methodology has the nice properties of creating a distribution of values with the desired correlations and still retaining the original marginal distributions. Furthermore, the required calculations are easily accomplished in a spreadsheet. If there is a drawback to this numerical methodology, it lies in the requirement to simulate to calculate the distribution. Inherent to all such methodologies, the subsequent calculations will not typically replicate the original estimated distribution.

For the industry example, the empirical pdf, based on 3,000 simulations, of the aggregate unpaid loss distribution looks as follows.

Graph 2.2
Distribution of Unpaid Losses
U.S. Commercial Lines Total



The minimum value in the above distribution is \$200 billion. The maximum value is \$276 billion. (Recall that the mean is \$234.5 billion.) The observed aggregate standard deviation is \$10 billion. The industry carried reserves for commercial lines loss and allocated loss expense from 1990 to 2000, at \$198.4 billion, are below the scale based on the above aggregate distribution.

Aggregation of Line Distributions – Model 2. Model 2 is a quick-and-dirty alternative. The mean of the aggregate distribution is known. It is simply the sum of the line means. The variance of the aggregate distribution can be calculated from the estimated variance-covariance matrix (VCM). The aggregate variance is the sum of the elements in the matrix. Given the aggregate mean and variance, a distribution can be estimated by assuming an appropriate functional form, e.g., lognormal. Following are the computations for the running example to compute a method of moments lognormal from the observed data.

Given the standard deviations calculated by line, create a matrix, σ =diagonal (σ_1, σ_2, …,σ_6). With the correlation matrix , Σ, the VCM will take the form:

$$VCM = \sigma'\Sigma\sigma$$

The VCM from the running example is this paper is shown below.

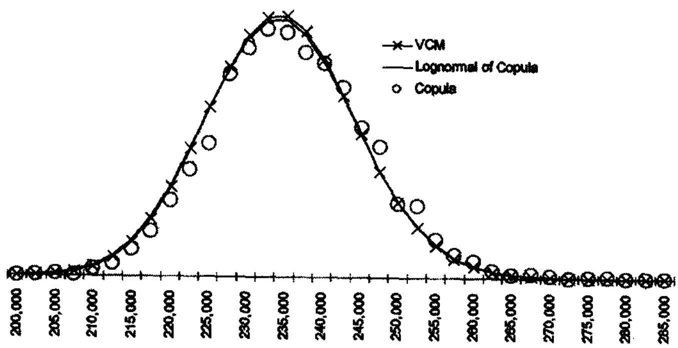
Figure 2.3

Variance-Covariance Matrix

	Auto [C]	Comp. [D]	CMP [E]	Med Mal [F]	Liability [H,R]	Other
Auto [C]		(0.75)	(0.10)	0.06	0.73	0.22
Comp. [D]	(0.75)		(0.41)	2.31	1.36	12.66
CMP [E]	(0.10)	(0.41)		(0.06)	(0.17)	(0.17)
Med Mal [F]	0.06	2.31	(0.06)		0.96	0.97
Liability [H,R]	0.73	1.36	(0.17)	0.96		4.63
Other	0.22	12.66	(0.17)	0.97	4.63	

The aggregate mean, M, is $234.5 billion. The aggregate standard deviation, S, is $9.8 billion. The method of moments estimates for lognormal parameters μ and σ of a distribution defined by observed moments M and S are 12.364 and 0.043 respectively. This lognormal distribution from method 2 is graphed below along with the results from the copula and the lognormal fit to the copula results.

Graph 2.3



3. FUN WITH DISTRIBUTIONS

Given distributions of unpaid losses by line and in total, there are a number of interesting uses and implications. This section covers some topics where distributions are valuable in actuarial practice.

Capital requirements. Economic capital requirements line-by-line could be established using a ruin theory or value-at-risk construct. How much capital is needed to be sure that there are enough funds for claimants at some extreme probability? For example, at a 3 in 10,000 ruin probability (the equivalent of a AA credit rating default value) the required risk based capital, by line is the difference between the carried reserve and the 99.97th percentile ($F^{-1}(0.9997)$-reserve):

Table 3.1

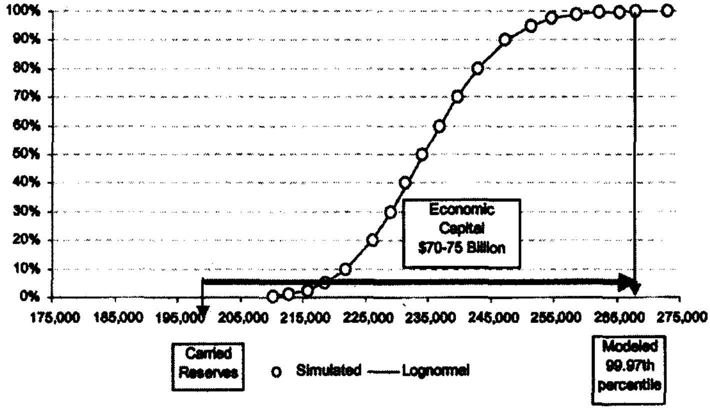
Capital Requirements

	Carried Provision	$F^{-1}(0.9997)$	Stand Alone Capital	Reserves-to-Capital
Commercial Auto [C]	18,911	25,694	6,783	2.79
Work Comp. [D]	60,597	90,057	29,460	2.06
CMP [E]	24,753	29,496	4,743	5.22
Med Mal Claims made [F2]	19,478	29,107	9,629	2.02
General Liability Occurrence [H1]	50,148	75,605	25,457	1.97
Short Tailed Lines	24,578	36,745	12,167	2.02
Sum Total	198,465	286,704	88,239	2.25
Modeled Aggregate	198,465	271,161	72,696	2.73

The stand alone and aggregate capital needs shown above include a provision to cover the reserve inadequacies. If booked reserves were increased, supporting capital would decline, and the reserves-to-capital ratios shown above would increase.

The sum of the stand-alone capital amounts shown above, $88 billion, is a meaningless number: it is more economic capital than is required. To calculate the required economic capital to support the industry reserves, the total industry distribution must be used. This distribution reflected the portfolio aggregation and diversification effects for the industry. Required aggregate economic capital is shown in the table above. Its derivation is shown graphically below.

Graph 3.1
Commercial Reserve Cumulative Distribution



Graph 3.1 implies a required economic capital of roughly $73 billion. The sum of the line-by-line, stand-alone capital was $88 billion. Thus, there is a $15 billion natural diversification effect[8].

This example highlights a flaw in standard rules of thumb, such as reserves-to-surplus or premium-to-surplus. Though widely accepted, they mismatch numerator (partial economic risk factor) and denominator (accounting based total surplus).

Capital allocation. It was not the intent of this paper to dive into the capital allocation debate. However, given an aggregate risk distribution as described in the previous section, and given the variance-covariance matrix that necessarily underlies it, the source information exists to allocate capital to an unpaid loss portfolio in a defensible fashion.

A few years ago there was a terrific discussion thread in the CAS Proceedings, starting with Shalom Feldblum [7] and ending with Todd Bault [8] thrashing about whether one should allocate capital (risk load) in proportion to standard deviation or variance. Bault proved that the two (and actually others as well) were part of a broader, unified theory of risk based on correlations. When correlation approaches zero, the allocation is in

[8] This capital requirement is not the same as the total capital required for the commercial lines industry. The analysis would have to incorporate additional risks: volatility in new business (UPR), cats, investment risk, operational risks, etc. This could be easily accomplished. We need only to measure risk distributions for each risk type and use the same integration routine to get a total aggregate risk distribution (see [17]).

proportion to variance. When correlation approaches one, the allocation basis tends toward relative standard deviation.

Bault's conclusions immediately present a plausible capital allocation mechanism, incorporating standard deviation/variance and all of the measured covariances.
Zero and one are obviously just the polar cases. The more general result can be seen with a simple two-by-two VCM for risks 1 and 2:

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

Given the above variance-covariance matrix, the capital allocation to a line is simply the sum of a given row in the variance-covariance matrix divided by the sum of the entire matrix. This generalized construct accounts for those cases where lines have perfect correlation, no correlation, or anything in between. And it is computationally tractable with any number of lines.

In the simplified VCM above for risk 1 and risk 2, the allocation formula for, say risk 1, would be

$$\frac{\sigma_1^2 + \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2}$$

It is easy to see from this formula that, if ρ equals 0, the allocation basis is variance, and, if ρ equals 1, the allocation basis is standard deviation.

In the industry example, the resulting capital allocation looks as follows.

Figure 3.2
Capital Allocation from the VCM

	Commercial Auto [C]	Work Comp. [D]	CMP [E]	Med Mal [F]	Liability [H, R]	Other	Row Sum	Capital Allocation %
Commercial Auto [C]	0.6	(0.7)	(0.1)	0.1	0.7	0.2	0.7	0.7%
Work Comp. [D]	(0.7)	35.5	(0.4)	2.3	1.4	12.7	50.7	52.4%
CMP [E]	(0.1)	(0.4)	0.2	(0.1)	(0.2)	(0.2)	(0.7)	-0.7%
Med Mal [F]	0.1	2.3	(0.1)	0.7	1.0	1.0	4.9	5.1%
Liability [H,R]	0.7	1.4	(0.2)	1.0	8.4	4.6	15.9	16.4%
Other	0.2	12.7	(0.2)	1.0	4.6	6.8	25.1	26.0%
							96.7	100.0%

Our numerical example yields a negative capital allocation, which, while problematic, is not unique. Myers and Read [16] suffer the same problem. For the purposes of this paper, the observation is noted and accepted, in the spirit of presenting methodology first, an illustration second, and specific parameters a distant third.

The allocated capital and associated reserves-to-capital ratio are shown below, along side the stand alone capital estimates. In this exhibit, the total required capital and the allocated amounts are predicated on reserves being booked at an adequate level according to the analysis presented in this paper.

Figure 3.3

Capital Allocations for Unpaid Losses

	Required Reserves	Capital Allocation %	Allocated Capital	Reserves-to-Capital	Stand Alone Capital	Reserves-to-Capital
Auto [C]	$23,008	0.7%	$286	80.55	$2,686	8.57
Comp. [D]	$66,535	52.4%	$20,146	3.30	$23,522	2.83
CMP [E]	$27,734	-0.7%	-$261	(106.22)	$1,762	15.74
Med Mal [F]	$26,098	5.1%	$1,962	13.30	$3,009	8.67
Liability [H, R]	$64,950	16.4%	$6,314	10.29	$10,655	6.10
Short	$26,254	26.0%	$9,999	2.63	$10,491	2.50
	$234,579	100.0%	$38,446	6.10	$52,125	4.50

Valuation methodology. When placing an intrinsic or actuarial value on a prospective acquisition target, actuaries look at pricing 1) the true economic value imbedded in the balance sheet, and 2) the present value of the ongoing business. The method essentially turns everything past, present, and future into tangible (but stochastic) cash flows, capitalizes them appropriately, and then calculates a present value[9].

In light of the diversification benefit observed in the industry example above, it is apparent that valuation methodologies cannot attempt to measure the 'true value' of a target company in a vacuum, i.e., on a stand-alone basis. There exists a portfolio effect that will allow the merging companies to free up some amounts of capital for other investments or return to shareholders.

This free capital flow should be accounted for in the valuation, presuming that the theoretical diversification effect can be harvested given regulatory restrictions. Or should it? There is an open and active debate on this very subject in actuarial circles today. The opposing view would say that mergers between companies cannot create capital efficiencies that the market would not have already anticipated or that market instruments could not replicate.

[9] As an alternative, one can discount the cash flows at a risk-adjusted rate. As another, perhaps best, alternative, one can compute the risk adjusted distribution of cash and then discount at the risk free rate.

Fair Value Liabilities. The IASB has proposed that all liabilities, like assets, should be marked to market for financial reporting purposes. Lacking an active market of unpaid loss liabilities, a "fair value" must be estimated based on cash flows and risk. In the absence of an aggregate distribution as presented here, the total reserve portfolio risk cannot be quantified. Further, it should be apparent that it is insufficient to conduct a fair-value-ing exercise on a line-by-line basis, as it misses the diversification effect.

In the absence of a liquid market, think of fair value as the amount it would cost to pay a reinsurer to take the unpaid loss liability off your balance sheet. Two calculations are shown below for the running liability example. For these calculations, it is assumed that the immunized, risk free discount rate is 4.25% and the weighted average life of the payments is 4.5 years. The implied risk free discount factor is 0.829; discounted reserves equal $54 billion.

Fair value could be calculated using risk adjusted discount rates from Butsic [1]. Butsic shows that the appropriate risk adjusted rate, given the immunized risk free rate and the capital requirement is a liability analog to the CAPM formula.

$$r_{riskadjusted} = \left(\frac{capital}{reserves}\right)\left(ROE - r_{riskfree}\right)$$

so,

$$r_{riskadjusted} = \left(\frac{\$6}{\$65}\right)\left(20\% - 4.25\%\right) = 1.5\%$$

The above formula assumes a 20% pre-tax ROE target. Using the average life of about 4.5 years, the risk adjusted discount factor is 0.935 ($=1.015^{-4.5}$). Total value would be $65*0.935 = $61 billion. The implied loading for risk is $7 billion (= $61 - $54).

Alternatively, the value of the unpaid loss liabilities could be assessed using risk neutral distributions via PH-transforms as discussed by Wang [21] and Butsic [2]. The simple lognormal assumptions applied through out this paper come in handy here. A transformed lognormal entails shifting only the location parameter. If the underlying distribution has parameters μ and σ and the appropriate risk load is $\lambda\%$, the risk neutral distribution will be lognormal with parameters $\mu' = \ln(1+\lambda) + \mu, \sigma$. A starting point for the risk load percentage could be the ratio of the risk charge, above, to required reserves shown in 3.3. Again using the industry liability data:

$$\lambda = (7.0/65) = 0.108$$

$$\mu' = \ln(1+\lambda) + \mu = \ln(1+0.108) + 4.173 = 4.275$$

$$\sigma = 0.045$$

Transforming back to dollars, the expected value of the risk neutral distribution is

$$E[X] = e^{\mu' + \frac{1}{2}\sigma^2} = \$72 \text{ billion}$$

Allowing for time value, the risk neutral value is on the order of $60 billion (= 72*.829). The results from the risk adjusted discounting methodology and the PH-transform are very similar.

One of the nice features of the PH-transform is the ability to use the risk neutral distribution to price layers of the distribution by simply taking the difference of the respective limited expected values. Perhaps this could facilitate a more active, liquid market in unpaid loss liabilities. Ironically, though, if we had a liquid market, none of the above calculations of fair value would be necessary.

Statutory risk based capital. The treatment of aggregate distributions also highlights some of the flaws with the mechanical formulas for statutory risk based capital:

1. Supporting economic capital makes sense only in the aggregate and only then when correlations have been appropriately reflected. Correlations between lines of business are imperative and cannot be ignored.

2. RBC, like economic capital, should be the difference between the aggregate value at risk (F^{-1}(1-ruin)) and the carried provision for the unpaid loss liabilities. If reserves are strengthened, required supporting capital should decrease. RBC, however, inappropriately assumes companies are currently adequately, *and only adequately,* reserved, *charging* for *any* additions to reserves. This penalizes well-reserved companies and those wishing to become better reserved.

3. Using techniques illustrated here, true economic capital requirements *can* be calculated with accepted actuarial techniques at an individual company level. Industry norms would have little use.

The above remarks refer only to RBC charges for unpaid losses. Of course other risk factors need to be integrated into a total economic capital figure. But this is only a simple extension of Wang's standard normal copula method shown above. Marginal risk distributions can be created for investment portfolios, catastrophe exposures, etc., and integrated into a total distribution for use in the calculation of required economic capital.

Best estimates. Statutory accounting requires that we establish a best estimate reserve. Further, we must establish that reserve by line of business. Further still, if management has a number of alternative estimates of unpaid loss, all equally likely, the best estimate is presumably the average of all estimates.

If regulation was principally focused on solvency, all that matters is the aggregate distribution of unpaid losses, the actual provision for unpaid losses, and the amount of capital available. In this construct, best estimates of line-by-line reserves are less relevant.

Materiality. ASOP 36 places great emphasis on materiality but does not define the term. Materiality is best defined with a distribution like that shown in Graph 2.1 (pdf) as the basis. Whether or not an issue is material depends on the answer to questions such as, "where could this issue move my estimate of unpaid losses in the *a priori* distribution of possible unpaid losses?" "Could it change the shape of the distribution?" In the end, if a company is reserved at the 51^{st} percentile (the mean of the industry aggregate distribution in our example), a material movement is one that would drop you to the N^{th} percentile in the posterior distribution.

Unfortunately, now the definition of 'material' hinges on the definition of N. Having a definition of N, however, yields an interesting implication. If a company is already just at or below the N^{th} percentile of the distribution of unpaid losses, materiality disclosures are almost a moot point. Regulatory emphasis should be placed on reserve adequacy.

Materiality is clearly a function of a company's size. A large company could conceivably have an issue looming that could move their best estimate of unpaid losses by millions of dollars, but this might be the difference of being at the 51^{st} percentile and the 50^{th}. This should not be considered material. On the other hand there are clearly companies where materiality would be measured in the thousands of dollars.

Furthermore, portfolio diversification is again key. Any actuary can think of a handful of nasty things that could cause adverse development in unpaid losses. Perhaps these nasty things could even be characterized with a mean and distribution of possible results. If we went further and thought in terms of the distributions of nasty things *and* the inter-nasty-things-correlations, we could aggregate using the same technology presented here. Would the aggregate distribution of potentially 'material' things be material? Perhaps not. Why? Because it is precisely the highly skewed, generally independent distributions that get heavily diversified away.

4. CONCLUSIONS

This paper used published and readily available data and techniques, along with a simple proposal for measuring correlations amongst unpaid losses, to produce a sample aggregate loss distribution for the U.S. commercial lines unpaid losses (1990-2000) as of year-end 2000. By the time the end of this paper mercifully came, the author had whined about a great many things. In conclusion:

1. Point estimates for unpaid loss and loss expense are insufficient. Methods to produce distributions exist and are reasonably approachable.
2. Methods exist to aggregate risk distributions, given correlations.
3. Correlations can be measured directly from the data normally employed for loss reserve analysis.
4. Aggregate distributions of unpaid losses are useful analytical tools, with implications for required economic capital, capital allocation, pricing and valuation, and various issues associated with accounting such as "best estimate," "materiality," and RBC.

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Reserving for Catastrophes

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RESERVING FOR CATASTROPHES

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Abstract
The National Association of Insurance Commissioners (NAIC) has worked since the mid-1990s to facilitate pre-event tax-deferred catastrophe reserves. The proposal was adopted by the NAIC in 2001.

The catastrophe reserve will not be a traditional liability reserve, nor will it specifically be included in the scope of the NAIC Statement of Actuarial Opinion on loss reserves. However, actuarial expertise will be needed to evaluate additions and drawdowns of the reserve. The reserve should be evaluated, along with additional information in the Annual Statement and from other sources, as part of a program to protect a company's financial integrity.

This paper provides background information on the problem and constraints currently faced by the insurance industry with respect to accumulating funds for catastrophic events. It also presents a description of the catastrophe reserve. This explanation includes definitions pertinent to this reserve, recommended statutory accounting treatment of the reserve and step-by-step instructions on annual calculations needed. Example calculations are provided and described in detail. The evolution of this proposal is discussed, along with changes to the current design that might be made over time. Finally, the paper outlines future activities needed to enable and fully implement the catastrophe reserve.

Reserving for Catastrophes

Introduction
The NAIC has worked since the mid-1990s to facilitate pre-event tax-deferred catastrophe reserves. The proposal, which includes recommended accounting treatment, was adopted by the Property & Casualty (C) Committee at the June 12, 2001 meeting in New Orleans and by the entire NAIC membership (Plenary) at the October 24, 2001 meeting in Washington, D.C. The Internal Revenue Code changes that are needed are being closely monitored.

The catastrophe reserve will not be a traditional liability reserve, nor will it specifically be included in the scope of the NAIC Statement of Actuarial Opinion on loss reserves. However, actuarial expertise will be needed to evaluate additions, accumulations and drawdowns of the reserve. The reserve should be evaluated, along with additional information in the Annual Statement and from other sources, as part of a program to protect a company's financial integrity.

The purpose of this paper is to summarize the development of the reserve proposal, provide an explanation of how it might work, and discuss changes to the current design that might be made over time.

Problem and Constraints
Catastrophes, whether natural or man-made, pose a significant challenge to the U.S. economy and to the U.S. insurance industry. As the 2001 terrorist attacks demonstrated, unexpected extreme events can, and do, occur. The insurance industry's successful response to such events relies on insurance companies' ability to manage their risk exposures prior to the occurrence of a catastrophic event.

Current tax laws and accounting principles discourage U.S. property and casualty insurers from accumulating funds specifically to pay for catastrophe losses for which the probability of occurrence in any given year is very low. Presently, loss reserves are established upon the occurrence of an insured event. As a result, payments for losses from a future catastrophic event must come from unrestricted policyholders' surplus. Existing U.S. tax law does not permit deduction of reserves for losses that have not yet been incurred.

Solution
Companies can currently use a variety of methods to manage their risk exposure, such as deductibles, co-payments, coverage limits, loss mitigation, geographic spread of risk and reinsurance. Recent additions to this repertoire include capital market securitizations to reduce the net amount of risk held by the company.

Insurance regulators in several foreign countries allow, and some even require, some form of pre-event reserves. These reserves are set aside prior to a loss occurring and are used by insurance companies to manage earnings during years in which significant losses are sustained. The NAIC Tax-Deferred Pre-Event Catastrophe Reserve has some similarities to, and some significant differences from, these reserves. Tax advantages are included in the NAIC proposal, but earnings management is limited to mitigating the effects of extreme catastrophes. More details regarding pre-event reserves for other countries can be found in the Other Countries' Approaches section and in Appendix G.

The catastrophe reserve is a pre-event reserve to be used solely for catastrophic losses. The reserve is designed to remove disincentives for long-term management of catastrophe exposure. When an insurance company experiences significant losses from a catastrophe, the company can draw upon these segregated funds to maintain financial strength. The reserve does not provide a measurement or quantification of the maximum amount a catastrophic event may cost an insurance company or the insurance industry.

Description of NAIC proposal

The NAIC Catastrophe Reserve Subgroup was appointed to design a mechanism that would address the issues mentioned above. After much discussion, the following general conclusions were reached:

1. The best solution is a private market mechanism that encourages prudent management of risk.
2. Only mega-catastrophes should be allowed a tax deferral.
3. Capital should be used efficiently; trapped capital is not desirable.
4. The mechanism should be as simple as possible and should be auditable.

Traditional loss reserves are only established upon the occurrence of a covered event and are estimated to be the amount of the expected ultimate payout. In contrast, the catastrophe reserve is established to cover future losses for which the company's ultimate payout is not yet known. The intent of the catastrophe reserve is to provide segregated funds for use upon the future occurrence of a catastrophic event. It is recommended that it be reported as a liability for statutory accounting purposes, separate and distinct from loss, loss adjustment expense and unearned premium reserves. As such, it is easily identifiable for tax purposes, solvency regulation, and stockholder and policyholder scrutiny.

An insurance company may elect to establish such a reserve in order to have funds readily available when catastrophes occur. The maximum amount a company can have in the reserve is determined by multiplying its net written premium for each covered line of business by a factor. The resulting amounts for each line of business are summed to obtain a single aggregate reserve cap for the company. The factors have been determined by line of business and are designed to address that portion of the premium that could be associated with low-frequency mega-catastrophes. Appendix C provides a description of the methodology used to derive the reserve cap factors.

The proposed federal tax treatment provides for a phase in of tax-deferral for reserve contributions over a period of 20 years. The deferral allowed equals the full amount at the end of that period. In this proposal, the targeted accumulation amount for the entire property/casualty insurance industry is $40 billion. The NAIC proposal is designed to work in conjunction with federal legislation, but it does not contain the phase-in language.

The annual statement lines of business covered include fire, allied lines, farm owners, homeowners, commercial multi-peril (non-liability portion), earthquake, inland marine, auto physical damage and non-proportional reinsurance for the qualifying lines.

Losses from the following catastrophic events are covered by the reserve: wind, hail, earthquake and fire following, winter catastrophes, fire, tsunami, flood, volcanic eruption and lahar.

A company can access its catastrophe reserve in three situations. Primarily, the catastrophe reserve would be used for losses incurred as a result of a catastrophic event in a covered line of business. In this case, the reserve is drawn down according to criteria designed to protect solvency and to limit use of the reserve to the payment of truly catastrophic losses.

A second situation in which the company may need to draw down its catastrophe reserve is when the reserve balance exceeds the reserve cap.

Finally, the company's domiciliary Commissioner may require an insurer to release its catastrophe reserve as a rehabilitation, conservation or liquidation measure or to forestall insolvency of the insurer.

Annual Calculations
A company would need to go through the following steps annually to determine the amount of its catastrophe reserve.

1. Calculate the reserve cap for the year. The reserve cap is the sum of the amounts determined by multiplying the catastrophe reserve cap factor for each qualifying line of business by the insurer's net written premium for the corresponding line.

2. Determine qualifying losses. Qualifying losses for the catastrophe year are those that arise from a covered peril, are attributable to a covered line of business and are due to a catastrophe. Appendix B provides an outline of the conditions an event must meet to be considered a catastrophe for the purposes of the proposal.

   Qualifying losses must be evaluated by catastrophe year, the year in which the catastrophe began. The development of losses from a covered catastrophic event must be monitored since such losses may not reach one of the thresholds for withdrawal from the reserve until several years after the occurrence. Therefore, each company must maintain losses by catastrophe year along with the qualifying loss limitation thresholds as a permanent benchmark.

Although the drawdown triggers are not complicated, the proposal is somewhat confusing in its descriptions. Item 7 in the Summary and Definitions section of the proposal (Appendix B) defines gross qualifying losses, which do not recognize any reduction for salvage, subrogation, or recoveries from risk transfer mechanisms. This amount is used for calculating what amounts can be released from the reserve and provides a benchmark of how large the catastrophic event is for the company. Item 11a of the Summary and Definitions section also refers to qualifying losses. This is discussed in more detail below.

3. <u>Calculate thresholds and determine allowable drawdown.</u> "The Catastrophe Reserve shall be drawn down in an amount not to exceed the lesser of the amount determined under subparagraph (1) or (2):

   (1) Qualifying Losses for the Catastrophe Year net of contractual payments from catastrophe management resources and net of salvage and subrogation, or

   (2) Qualifying Losses for the Catastrophe Year to the extent that such Qualifying Losses exceed the lesser of:

      i. 100% of the insurer's prior year Reserve Cap, or

      ii. 30% of the insurer's surplus at December 31 of the prior year."[4]

   It is helpful to look at each subparagraph individually. The drawdown can never be more than the amount described in 3(1), the company's losses net of salvage, subrogation, and other recoveries. Losses include loss adjustment expenses and non-recoverable assessments, surcharges, etc. Catastrophe management resources are defined in item 7 of the Summary and Definitions section.

   Drawdowns are further limited by comparison to items that reflect, but do not necessarily measure, a company's catastrophe exposure and financial resources. Paragraph 3(2) above selects the lesser of an insurer's prior year reserve cap, which is related to catastrophic loss exposure, and 30% of its prior year surplus, which is related to ability to absorb the impact of catastrophic losses. Since the reserve cap amount is based on net written premium in lines of business that tend to be affected by catastrophes, the amount of the cap is reflective of how much catastrophe exposure the company has on its books.

   If the insurer's prior year reserve cap is more than 30% of prior year surplus (which means the amount in 3(2)(ii) is the lesser), the company can be considered more vulnerable to the financial impact of catastrophic events than a company whose percent of surplus exceeds the reserve cap. Note that this is independent of how much the company has actually set aside in the reserve. The trigger is designed to recognize this company variation by allowing the larger of the two amounts in 3(2) to be compared to the net qualifying losses in 3(1). Losses in excess of the smaller of two amounts will lead to the larger resulting loss amount.

The qualifying losses used in the 3(2) calculation are gross qualifying losses.

An example may be helpful. Suppose a company has these characteristics.

Gross qualifying losses	1,350
Salvage, subrogation & other recoveries	800
Net qualifying losses	550
Prior year-end surplus	6,175
30% of prior year-end surplus	1,853
Prior year reserve cap	584

The maximum amount the company could withdraw according to paragraph 3(1) above is the net qualifying losses of $550. However, the drawdown is further limited according to the calculations involving its prior year reserve cap and prior year surplus, as described in paragraph 3(2). The prior year reserve cap of $584 is less that 30% of the prior year surplus of $1,853. So, the calculation specified by paragraph 3(2) is the gross qualifying losses minus the prior year reserve cap or $1,350 – $584 = $766. This is more than the net qualifying losses of $550. Therefore, the maximum allowable drawdown amount is $550.

4. Reduce the previous year-end reserve balance by the current year drawdown amount calculated in step 3.

5. Determine the maximum amount of the current year reserve addition by subtracting the reserve balance after drawdowns from the reserve cap calculated in step 1. Contributions to the reserve are tax-deferred to the extent of additions that result in a balance that does not exceed the reserve cap, with a 20 year phase in.

6. Calculate the PMLE and decide the best balance of reinsurance, other risk transfer mechanisms, and pre-event reserving that will suit applicable regulatory safeguards and the company's own goals and management philosophies. Many companies may choose to contribute to a reserve based on its net PMLE, after the effects of any risk transfer have been evaluated.

7. Add the selected amount to the reserve.

8. Verify that the resulting reserve balance does not exceed the reserve cap. The company must draw down its reserve if the reserve amount exceeds the allowable cap, as calculated in step 1.

Financial Statement Disclosures
The amount, if any, to be added to the reserve each year is up to each company. However, if a reserve is established, specified accounting procedures must be followed.

Calculation of amount to be added to the reserve
The reserve balance at the end of the year cannot exceed the cap. Each company calculates its own cap. As an example, consider a company that writes only 2 lines of business: Homeowners and Private Passenger Auto. The reserve cap is determined as shown below.

Line of Business	(A) Net Premiums Written	(B) Reserve Cap Cap Factor	(A x B) Reserve Cap Amount
Homeowners	100,000	.60	60,000
PPA liability	150,000	na	0
Auto Physical Damage	80,000	.01	800
Total	**330,000**		**60,800**

If we assume that this company is setting up its reserve in the first year after the tax change has been enacted, 1/20th of this $60,800 cap, $3,040, can be tax-deferred if set aside in a catastrophe reserve.

Note that the reserve cap amount does not have a contribution from the liability portion of the Private Passenger Auto business. Only property lines that have been identified as being catastrophe-prone have factors to allow contribution to the reserve cap calculation. The total amount in the reserve is available in response to a catastrophe in any of these property lines. For example, the entire reserve amount for the company shown here could be used to pay for homeowners losses sustained by this company in any part of the country.

A more detailed exhibit showing how a company would calculate its reserve cap is shown in Appendix H.

Notes to Financial Statements
The following entries would be made in the Notes to the Financial Statements for the first year's accumulation of the reserve, based on the information in our example above and assuming that no qualifying losses occurred:

Catastrophe Reserve Balance at the Beginning of Year	$	0
Current Annual Reserve Addition to the Catastrophe Reserve	$	3,040
Current Drawdowns Due to Qualifying Losses	$	0
Current Drawdowns for Amounts in Excess of the Reserve Cap	$	0
Catastrophe Reserve Balance at the End of the Year	$	3,040

Balance Sheet - Liabilities, Surplus And Other Funds
A new line, Line 16 - Catastrophe Reserve, will be added to the Balance Sheet –

Liabilities, Surplus and Other Funds. This amount will change over time, and should reflect the amount in the reserve at the end of the year for which the statement is being filed. For our example it should show $3,040. A copy of the proposed page 3 is shown in Appendix F.

Underwriting and Investment Exhibit – Statement of Income
Line 4 - Change in Catastrophe Reserve is to be added to the Underwriting Income section of the Underwriting and Investment Exhibit. Additions to and drawdowns from the catastrophe reserve are addressed in the proposed Statement of Statutory Accounting Principles, which is shown in Appendix E. Since our example is for the first year of accumulation, Line 4, Column 1, Change in Catastrophe Reserve, Current Year would show $3,040 ($3,040 – $0). The federal tax change will allow this amount as a deduction from taxable income for the year the contribution is made, similar to current tax treatment for setting up a traditional loss reserve. If we use a tax rate of 35% and assume that all else in the company's tax situation remains the same, the tax savings to the company for this year would be $1,064 ($3,040 x .35).

When any amounts from the reserve are drawn down to pay for losses, the amounts withdrawn from the reserve are added back into taxable income. A reserve withdrawal is most likely to happen due to an extremely large catastrophe, rather than due to an overpayment or financial difficulties. Large underwriting losses in the catastrophe year are expected to offset this increase in reported income. It is likely that much of the catastrophe reserve will be, in effect, tax-free.

Company Considerations
Since the NAIC proposal allows voluntary participation, each company must decide if a catastrophe reserve is appropriate, given its unique exposure to catastrophe risk. There is not a documented method to determine when the accrual of a catastrophe reserve is appropriate. Generally speaking, the costs and benefits of establishing a catastrophe reserve should be compared to the costs and benefits of other catastrophe management tools available to the company.

It is not expected that every company will accumulate reserves to the maximum amount allowed. There will be situations where prudent financial planning dictates other uses of funds. Nevertheless, it is instructive to look at how various companies could use this mechanism to pre-fund catastrophic loss payments.

Exhibit 1, below, shows data for 12 companies of various sizes, as well as countrywide information. The data was extracted from the 2000 NAIC Annual Statements, with the reserve cap factors applied to each company's net written premium. Appendix H gives more detail on how the reserve cap for each company was calculated and contains detailed information on Company B in Exhibit 2.

Columns (3), (4) and (5) compare maximum reserve amounts that could be set aside for each company. Column (3) shows the percentage of premium that is in catastrophe-prone property lines. Columns (4) and (5) show how each company's reserve compares to its net written premium for all lines and for the catastrophe-prone lines, respectively. As can be seen, the reserve cap varies widely by company when measured relative to the net written premium. This is due, of course, to the mix of lines that each company writes. The range in percentages narrows somewhat if the reserve cap is compared to the net written premium for covered lines only, Column (5). A wide range is still expected, since the reserve cap factors for covered lines range from .01 to over 15.

Column (6) applies to catastrophic losses that might have occurred during 2001. The reserve cap and surplus used are both prior year-end, based on 2000 Annual Statement data. Since the reserve amount that can be drawn down is dependent on qualifying losses in excess of either the reserve cap or of surplus, a comparison of these two items is in order. Column (6) indicates which of these two will be the amount entering into a drawdown calculation. If the amount in Column (6) is less than 100%, then the qualifying losses in excess of the prior year reserve cap is used in the drawdown calculation. If it is greater than 100%, then the qualifying losses in excess of 30% of the prior year's surplus is used. As stated above, this latter situation might mean that a company is considered more vulnerable to the financial impact of catastrophic events.

As additional information becomes available, amounts actually accumulated, as well as amounts paid, for catastrophic events can be evaluated and analyzed.

EXHIBIT 1

(1) Company	(2) 2000 Net Written Premium (000s)	(3) Covered Lines Net Written Premium As % Of All Lines Net Written Premium	(4) Reserve Cap As % Of All Lines Net Written Premium	(5) Reserve Cap As % Of Covered Lines Net Written Premium	(6) Reserve Cap As % Of (Surplus x .30)
Countrywide	318,739,936	42.0%	15.3%	36.4%	
A	7,821,650	46.8%	16.8%	36.0%	66.1%
B	3,415,196	60.3%	17.1%	28.4%	31.5%
C	3,051,503	39.6%	0.7%	1.7%	7.0%
D	2,146,285	26.6%	10.1%	37.9%	39.9%
E	1,058,715	16.5%	6.0%	36.4%	19.9%
F	959,759	46.3%	29.4%	63.4%	109.4%
G	870,624	59.6%	37.6%	63.1%	223.4%
H	207,302	53.4%	15.5%	28.9%	66.3%
I	119,993	0.3%	0.0%	1.0%	0.0%
J	41,394	64.2%	10.5%	16.3%	48.6%
K	22,863	11.7%	3.5%	30.0%	2.5%
L	22,008	100.0%	61.6%	61.6%	437.4%

Companies will also be interested in how rating agencies view the accrual of a catastrophe reserve. The reserve does not transfer risk, but does assure that specified funds will be immediately available to pay for catastrophic losses. Will accruing a catastrophe reserve have a positive or negative impact on a company's rating? How will it be viewed in comparison with the use of other catastrophe management tools? To date, rating agencies have not commented on this.

The reserve should not have any effect on rates charged by companies to their policyholders. However, consumers may be more accepting of the need for long-term catastrophe charges in their premiums if they know there are funds specifically set aside for this purpose.

Evolution of Proposal
In December 1995 the NAIC established the charge to "Develop a proposal for a reserve for future catastrophes that will include consideration of how and where the reserve should be shown in the financial statements, calculation of contributions to the reserve, calculation of withdrawals from the reserve, calculation of a maximum reserve, treatment of reinsurance ceded and reinsurance assumed and how to obtain federal tax deferment." [3] The Catastrophe Reserve Subgroup of the Catastrophe Insurance Working Group was appointed to carry out this charge.

The Reserve Subgroup was composed of state regulators with expertise in actuarial science, accounting, economics, financial examination and law. Industry representatives were invited to participate in the discussions and development of the proposal; they provided invaluable feedback and input. These interested parties were also instrumental in the analysis leading to the development of the reserve factors. The proposal was exposed for public comment and discussion at several points throughout the development.

Tax Deferral
As discussions began regarding the design of the proposal, it became apparent that certain characteristics of the reserve would generate significant discussion. One major item that insurance regulators and insurance industry representatives readily agreed upon is the tax-deferral. Regulators agree that the reserve should only be established to the extent that the federal government allows tax-deferred status. As of this writing, the tax code has not yet been changed to allow the categorization of these pre-event funds as tax-deferred.

Mandatory vs. Voluntary
The proposal began as a mandatory reserve requirement. All insurance companies writing the covered lines would be required to establish a catastrophe reserve based on the factors developed. As work continued, concerns were voiced about a mandatory reserve. Each company has a unique mix of business and risk profile, which make it difficult to specify an optimal tax-deferred reserve for prudent catastrophe risk management for all companies to use. There was also the possibility that requiring a specific level could lead to capital trapped in the reserve. As a result, the proposal became a voluntary, pre-event tax-deferred reserve within a regulatory framework.

Lines of Business
Line of business and peril were selected as identifiers that would be effective, simple and auditable. Determining lines of business generated significant discussion.

At the time the proposal was conceived, it was assumed that natural catastrophes would be most likely to cause market disruptions, as evidenced by Hurricane Andrew and the Northridge Earthquake. Property lines of business were targeted as being the most exposed. Discussion was given to the inclusion of workers compensation risk, specifically due to the earthquake-related exposure. The decision was reached to leave workers' compensation out of the proposal at that time.

Boiler and machinery was also discussed, and was incorporated into early designs. Because its inclusion contributed so little to the reserve and because it appeared to lend itself to the possibility of trapped capital for some writers, it was eliminated. Auto physical damage, not normally considered a line prone to catastrophic disruption, was evaluated carefully, and finally included since significant payouts were observed after both Hurricane Andrew and the Northridge Earthquake. Inland marine was initially excluded from the proposal because losses for this line are generally not included with the data used, but was added in the final proposal with an appropriate factor.

Excess reinsurance was not originally included, at the request of the professional reinsurers. They did, however, ask for these lines to be included in the final proposal, and sufficient changes were made to incorporate their request into the design.

While the reserve cap factors (and hence reserve amounts) vary by line, the reserve does not "belong" to any one line of business. Reserves can be used to pay for qualifying losses for any of the lines or perils defined in the proposal.

Perils

The perils to be included also generated some discussion. In its current state, the proposal allows fund drawdowns for losses resulting from wind, hail, earthquake and fire following, winter catastrophes, fire, tsunami, flood, volcanic eruption and lahar. These events must be designated a catastrophe by the Property Claims Service or any successor organization or declared a catastrophe by the President of the United States pursuant to the Stafford Act. Discussions on specifically including terrorism are underway.

Net Written Premium as Base

Net written premiums were decided upon as a surrogate for a company's catastrophe exposure. Premiums were selected since they should vary with coverage, geographic location and other classification factors that measure relative exposure. In addition, net premiums should reflect the risk a company is comfortable retaining. To the degree that lines are cross-subsidized, that premiums are excessive or inadequate, or that data is inaccurately reported, the usefulness of premium as a surrogate for exposure is reduced.

Factors by Line of Business

The factors are intended to indicate the portion of premium that can be associated with low-frequency catastrophic events. The analysis and development included looking at industry events everyone agreed were truly catastrophic. These appeared to be relatively well defined by line, by state, and by peril. Since there was data available in this amount of detail, the most useful factors originally appeared to be those that would take all of these categorizations into account. The details of the methodology to develop the first set of factors, which varied by state as well as by line, can be found in the NAIC Proceedings.

A simpler structure for the factors was later selected based only on line of business since catastrophic exposure is assumed to vary with geographic location, which should already be incorporated in the premium. The factors were developed by determining the relative contribution of each line of business to the target accumulation amount, $40 billion, with specified assumptions and adjustments. This was selected as an amount that would provide stability following a catastrophic event in developing for both the by-state, by-line factors and the current by-line factors.

Reserve Cap Factors

As mentioned above, the original proposal was for a mandatory annual reserve accumulation, calculated with annual accumulation factors that varied by line and by state. Caps on the total amount that could be accrued were also part of the calculations. Although annual accumulation factors are no longer needed, a cap on the total amount that can be accumulated is necessary since the IRS is not likely to allow an infinite tax-deferral.

A voluntary reserve provides each company with greater flexibility than a mandatory reserve. A single pre-event fund with one cap for all lines, regions and perils also provides some diversification benefits. The American Academy of Actuaries has expressed the belief that countrywide cap factors should be set at a level that will allow all companies to receive full benefit from the reserve. [1]

The reserve cap factors are in Appendix B and the methodology used to derive them is in Appendix C. The NAIC will periodically review the factors and the reserve mechanism itself. The factors are not designed to be either a correct or accurate measure of a company's catastrophic exposure. As a set, they help define a mechanism for individual companies to defer federal tax payments on monies that will be used to pay catastrophic claims.

State Catastrophe Programs

Many states have programs to assist their voluntary markets in providing catastrophe insurance. These programs are targeted towards the perils, lines of business and demographic characteristics that are of concern to a given state. A discussion of the structure, coverage, administration and specific characteristics of each program are beyond the scope of this paper. However, consideration must be given to the interaction of state catastrophe programs with both the financial responsibility framework discussed below and the catastrophe reserve.

The Florida Hurricane Catastrophe Fund (FHCF) acts as a reinsurer for the hurricane peril for all companies writing residential property coverages in the state. Participation in the program is mandatory. Like any other reinsurer meeting certain standards, credit for this risk transfer is allowed in the ceding company's financial statements. Since the premiums are considered "ceded to a non-affiliate," the premium paid to the FHCF each hurricane season does not enter into the reserve amount calculation. Any losses reimbursed by the FHCF are netted out of net qualifying losses. Since each company can select its participation percentage in the FHCF (45%, 75% or 90%), a company has some discretion as to how much FHCF coverage it purchases. Each company could then fund as much of its retained hurricane risk as it desires, up to the net reserve cap.

Membership in the California Earthquake Authority (CEA) is voluntary. The CEA offers basic earthquake insurance for California homeowners, renters, condominium owners and mobile home owners. The CEA participating insurers write about two-thirds of the homeowners premium in California. The earthquake coverage for participating insurer homeowners policyholders is provided directly by a CEA policy, although the companies market and service the policy.

The CEA has approximately $7 billion in capacity, from participating insurer capital contributions, retained earnings, letters of credit, reinsurance or bonds. About half of the capacity comes from two layers of potential assessments of the participating insurers. Although the participating insurers have a potential liability that must be backed by surplus or reserves, they do not have any net premium and thus they would not be able to set aside tax-exempt catastrophe reserves under the proposal.

Regulatory Framework

Regulators agreed to make the reserve voluntary as long as a company demonstrates to regulators that it has adequately managed its catastrophe exposures. The regulatory mechanism for evaluating this exposure management is currently known as the Framework for Demonstrating Insurer Financial Responsibility for Catastrophe Loss Exposures and has yet to be finalized and adopted. The framework provides a level of assurance to regulators that an insurance company has adequate financial resources to deal with its catastrophe exposures. The reserve is an integral part of the framework since it can be a useful tool for a company to fund losses that might threaten its solvency.

The framework requires an insurance company to determine its probable future losses using generally accepted actuarial methods. The analysis produces an estimate of the insurance company's Probable Maximum Loss Exposure (PMLE).

The insurance company must also look at the resources it has to manage its exposure. Besides the catastrophe reserve, resources currently accepted by regulators include reinsurance, securitizations, unrestricted surplus, capital notes, surplus notes, commissioner-approved support agreements from parent or affiliated entities, and other risk transfer arrangements as permitted by state law or regulation or approved by the commissioner of the state of domicile.

Once the insurance company has determined its PMLE and has identified all of its resources for managing that exposure, it is to provide a report (Net PMLE Report) to the regulator demonstrating that it has adequate resources to handle its exposures. If the insurer's net probable maximum loss exposures are unacceptable to the regulatory authority, then the insurance company must provide a plan that demonstrates how the company will resolve the deficiency.

As currently proposed, a qualified actuary must evaluate the Net PMLE Report. The American Academy of Actuaries, in response to a request from the NAIC, has commented that actuaries are qualified to provide an opinion of this nature. [2] The AAA continues to monitor the work of the NAIC on the reserve proposal and related topics.

There are many issues that still need to be addressed relative to the currently proposed framework. These items are more fully addressed in the section titled Enabling and Implementation Activities.

Other Countries' Approaches

Other countries allow, and some even mandate, pre-event reserves. We conducted a survey to learn more about the types of reserves that are established on a mandatory and voluntary basis in other countries. Based upon this research it is apparent that the following countries have some kind of pre-event reserve: Australia, Barbados, Canada, Finland, Germany, Italy, Japan, Puerto Rico, Switzerland and the United Kingdom. It is also evident that in different countries the needs vary because the reserves are designed to cover different exposures and release of the reserve is triggered by different events.

There are two important considerations to keep in mind when comparing the NAIC proposal to what other countries have done. One is that required disclosure, accounting practices and terminology vary from country to country. Second, equalization reserves do not serve exactly the same purposes as the proposed catastrophe reserve. As the name implies, equalization reserves may be used as a management tool to even out uneven performance from year to year, while catastrophe reserves are specifically designed only to pay losses and related expenses from catastrophic events. Appendix G contains more information about the specific reserves of some of the countries mentioned above.

Enabling and Implementation Activities

Although the full body of the NAIC has adopted the NAIC Catastrophe Reserve Proposal, before it is implemented and used by insurance companies several things must happen.

Federal Tax Law

Federal tax law must be changed to allow for tax deferral of funds held for future catastrophe losses. HR 785 was referred to the House Ways and Means Committee on February 28, 2001 as the "Policyholder Disaster Protection Act of 2001." No further progress has been made on this bill.

HR 785 proposes changes to Section 832 of the Internal Revenue Code that decrease a company's taxable income for the amount of contributions to the reserve in the year the contributions are made and add withdrawals from the reserve back into taxable income in the year they are drawn down. In addition, the bill phases in the reserve tax-deductibility over 20 years at the rate of 5% (1/20th) per year. Assets are "required to be invested in a manner consistent with the investment requirements applicable to the qualified insurance company under the laws of its jurisdiction of domicile," and "the net income for the taxable year derived from the assets in the fund is required to be distributed no less frequently than annually."[6]

NAIC Actions

Possible Additions

Due to the significant losses experienced by insurance companies from the tragic events of September 11, 2001, discussion has begun with respect to updating the catastrophe reserve proposal to specifically include terrorism as a covered peril or catastrophic event. Inclusion of workers' compensation as a covered line of business is also being considered. The Catastrophe Insurance Working Group is analyzing these issues and may recommend changes.

Update and Review

Upon passage of the federal tax legislation, the NAIC will have the task of periodic review and update of the Catastrophe Reserve Factors. Analysis of the expected losses to pre-fund will need to be reviewed as additional years of experience are accumulated and as existing modeling techniques are refined and new ones are developed. How much companies use the reserve for contributions and for drawdowns will need to be evaluated, as well as the tax revenue losses. The mix of business written by the U.S. admitted voluntary primary market, reinsurance and other risk transfer mechanisms, and changes in accounting may all require adjustments to this initial design.

NAIC Annual Statement Blanks

NAIC working groups must address additional specifics of the catastrophe reserve proposal. The NAIC Blanks Task Force must approve a proposal that would add disclosure of the reserve amount and disclosure of annual changes in the reserve amount to the NAIC Property and Casualty Annual Statement. They must also add a description of the process to the NAIC Annual Statement Instructions. These items are shown in detail in Appendix F. However, the catastrophe reserve can be used before these items are finalized since both the Balance Sheet and Underwriting and Investment Exhibit contain lines for write-in items.

Statutory Accounting Principles

The NAIC Statutory Accounting Principles Working Group needs to add guidance to the Statutory Accounting Principles. A proposed Statutory Issue Paper and Statement of Statutory Accounting Principles have been drafted. They can be found in Appendix D and Appendix E, respectively. These two items provide detailed discussion of the issue that the catastrophe reserve is meant to address and specifics on how a company would calculate its catastrophe reserve additions and draw downs. They also indicate the appropriate disclosures needed by a company that has established a catastrophe reserve.

Risk-Based Capital and IRIS Ratios
The NAIC Property and Casualty Risk-Based Capital Working Group must address how the catastrophe reserve will be viewed in light of the current minimum capital requirements. The current formula is being evaluated to ascertain how to best incorporate the reserve in the required calculations. The NAIC Financial Analysis Research and Development Working Group must undertake similar analysis with respect to the IRIS Ratios.

Regulatory Framework
The NAIC Framework for Demonstrating Insurer Financial Responsibility for Catastrophe Loss Exposures that accompanies the Catastrophe Reserve Proposal is still in proposal form. However, the catastrophe reserve can be adopted and used by companies before the framework is finalized or implemented. There are many issues that still need to be addressed relative to the framework.

- Should demonstrating financial responsibility be limited to catastrophic exposures?
- Should gross exposure rather than, or in addition to, net exposure be required?
- Are there parts of risk transfer arrangements that need to be kept confidential?
- Should reviewing the framework be part of the more traditional financial exam?
- How should review of the framework fit into target exams?
- Should the reserve become mandatory in the event a proposed plan is not acceptable to a domiciliary commissioner?
- What will be required as part of an actuarial review?
- Is it appropriate for regulators to be concerned with the mix of catastrophe management tools and if so, how should this be evaluated?

Conclusion
It is the authors' hope that this document provides guidance for both insurance regulators and actuaries. Insurance regulators will be responsible for reviewing insurance companies' catastrophe reserves. Their review is likely to encompass the appropriateness and adequacy of the reserve as well as compliance with statutory accounting requirements. Actuaries are expected to play an integral part in the analysis needed to establish and fund the reserve.

Even more importantly, regulators must verify that each company has selected a prudent mix of catastrophe management resources in light of its unique book of business. Actuaries may be asked to evaluate and comment on the specifics of a company's overall catastrophe management program. We hope this paper contributes to the understanding of how the catastrophe reserve will work as part of a company's catastrophe risk management program.

Appendices

Explanation of Appendices

Appendices A through F comprise the NAIC Pre-Event Catastrophe Reserve Proposal as adopted by the NAIC Plenary and as it has been referred to in this paper. They are NAIC copyrighted materials. Along with or subsequent to its adoption by the NAIC, the following changes to these appendices should be noted:

1. The discussion in Appendix B about the potential treatments of assumed excess reinsurance resulted in the acceptance of the Model 1 factors shown in Appendix F. Details of the discussion may be found in the NAIC Proceedings for the June 2001 meetings.
2. All references to the reserve factors will be maintained by the NAIC and updated annually. The factors will be available via the NAIC website. Factors will not be part of the NAIC Annual Statement Instructions or any other instructions. This is to facilitate and expedite any changes to the factors as they are made.
3. The Underwriting and Investment Exhibit, Statement of Income, found in Appendix F contains a typographical error. Line 4, Change in Catastrophe Reserve should say Page 3, Line 16, Column 1 minus Column 2.

Appendix A: Problem Statement

PROBLEM STATEMENT

NAIC TAX-DEFERRED PRE-EVENT CATASTROPHE RESERVE

(Original Adopted July 28, 1997 – Changes proposed below)

Catastrophes present a significant challenge to the U.S. economy and to the U.S. property-casualty insurance industry, posing financial solvency, capital accumulation and insurance availability issues.

Insurers should be encouraged to engage in prudent risk management behavior. Existing methods to manage insurer risk include loss mitigation, geographic spreading of risk, reinsurance, capital market instruments, derivatives products and short and long-term pre and post-event financing. One additional method proposed to encourage prudent risk behavior - which should augment, not supplant those existing risk management methods - is to permit insurers to establish tax-deferred reserves for future catastrophes.

Current tax laws and accounting principles discourage U.S. property and casualty insurers from accumulating assets specifically to pay for future catastrophe losses. Instead, payments for catastrophe losses are made from unrestricted policyholder surplus after the losses are incurred.

Current U.S. tax law does not permit deduction of reserves for future catastrophe losses.

Current U.S. accounting principles (both Generally Accepted Accounting Principles and Statutory Accounting Principles) applicable to property and casualty insurers: 1) limit the recording of loss reserves to losses which have already occurred, and 2) require the recognition of catastrophe premiums in periods prior to the periods in which catastrophe losses are incurred.

Some non-U.S. insurers, because of their domestic tax laws and accounting principles, are able to deduct reserves for future catastrophe losses free of tax. That ability gives those non-U.S. insurers a competitive advantage over U.S. insurers enabling them to attract insurance and reinsurance business, which would otherwise be written, by U.S. insurers.

U.S. State insurance regulators believe that existing tax treatment of catastrophe risk fails to maximize the ability of insurers to appropriately respond to catastrophe coverage related solvency and availability issues. State insurance regulators recognize that the portion of approved rates earmarked for catastrophes is intended to finance catastrophe losses that are not expected to occur on an annual basis. If these funds are not set aside in a dedicated reserve for these losses, funds may not be available to meet policyholder obligations. Accordingly, regulators may be more inclined to approve catastrophe-related rates if they were assured that the resultant premiums would be set aside for their intended purpose until the indicated catastrophe exposure has had adequate opportunity to materialize.

Some state insurance regulators recognize that U.S. property and casualty insurers, in the absence of assurance that approved catastrophe insurance rates will reflect past experience and projected exposures, are reluctant to maintain or expand their catastrophe insurance writings in regions with significant historical or projected catastrophe exposures.

In the absence of tax deductibility and in consideration of domestic accounting principles, a required or permitted catastrophe reserve would provide no additional assets to finance insured catastrophe claims. Requiring or permitting such a reserve without tax deductibility would diminish insurer's capital and would likely restrict availability of insurance coverages to consumers.

It is in the public interest to consider whether current U.S. accounting principles and tax law should be revised to permit tax-deferred reserves for future catastrophe losses.

While considering those steps, it is in the public interest to determine whether a tax-deferred reserve for future catastrophes can be structured in such a way as to provide incentives for:

- property and casualty insurers to appropriately manage their catastrophic loss exposures by making specific provision for future catastrophe losses while continuing to provide and expand catastrophe insurance coverage in regions with significant catastrophe exposure, and

- state insurance regulators to approve catastrophe insurance rates which, among other appropriate factors, reflect past experience and projected exposures,

so that the U.S. federal government and taxpayers benefit, in the form of possible reduced demand for catastrophe relief, as a result of these changes in the insurance regulatory and tax systems.

PROPOSAL - SUMMARY AND DEFINITIONS

NAIC VOLUNTARY TAX-DEFERRED PRE-EVENT CATASTROPHE RESERVES

For regulatory purposes, certain insurers that write qualifying property lines of business will be allowed to establish voluntary tax-deferred pre-event catastrophe reserves within the parameters set forth in the definitions and related materials set forth below. These parameters contemplate that such voluntary reserves will be accorded tax-deferred status under federal tax law and that implementation of such reserves will not be effected until and unless such tax deferral is granted. Within this voluntary program, insurers may, but shall not be required to, establish and maintain a pre-event catastrophe reserve in excess of amounts that could be accorded tax deferred status under federal tax law subject to the limits set forth below. However, those insurers that choose to establish a tax-deferred disaster fund under federal tax law must establish and maintain a pre-event catastrophe reserve for statutory reporting purposes at a level not less than the outstanding balance of such fund. The following definitions assume that a tax-deferred, pre-event catastrophe reserve has been elected.

This voluntary program is based on the regulatory approach that insurers will be allowed to use voluntary tax deferred pre event catastrophe reserves as one available tool among many to be financially ready to effectively deal with their catastrophe exposures.

1. **Subject Entities** – Insurers shall be allowed to establish a statutory Catastrophe Reserve if they: (1) write property/casualty contracts for the Qualifying Lines of Business; (2) file an NAIC annual statement for property casualty companies and (3) are subject to federal tax on net income.
2. **Catastrophe Reserve** – A voluntary pre-event provision for future Qualifying Losses within specified Thresholds attributable to Qualifying Catastrophe Events that are insured under Qualifying Lines of Business. An aggregate reserve for all Qualifying Lines of Business shall be established.
3. **Statutory Reporting** A Catastrophe Reserve shall be reported in statutory financial statements as a separate liability; distinct from loss, loss adjustment expense and unearned premium liabilities. Additions to, and reductions from, the Catastrophe Reserve shall be reported through a Change in Catastrophe Reserve in the Underwriting Income section of the Statutory Statement of Income. The Catastrophe Reserve shall be included in the scope of the statutory financial statement audit performed by a certified public accountant.
4. **Geographic Scope** – A Catastrophe Reserve shall be established for Qualifying Losses for Qualifying Catastrophes to which the United States, its territories and possessions are exposed in the Qualifying Lines of Business. Insurers domiciled in Puerto Rico shall not establish a Catastrophe Reserve for catastrophes to which Puerto Rico is exposed as a catastrophe reserve is already included in the Puerto Rican insurance code.
5. **Qualifying Lines of Business** - Fire, allied lines, farm-owners multiple-peril, homeowners multiple-peril, commercial multiple-peril (non-liability portion), earthquake, private

passenger auto physical damage, commercial auto physical damage, inland marine and non-proportional reinsurance for the qualified lines of business.

6. **Qualifying Catastrophe Events** – Wind; hail; earthquake and fire following; winter catastrophes such as snow, ice, freezing; fire; tsunami; flood; and volcanic eruption (including lahar). These events shall be 1) designated a catastrophe by Property Claims Service or any successor organization or 2) declared an emergency or disaster by the President of the United States pursuant to the Stafford Act or 3) declared an emergency or disaster in a similar declaration by the chief executive official of a State, territory or possession of the United States or the District of Columbia.

7. **Qualifying Losses** – Direct and assumed losses and loss adjustment expenses incurred and any non-recoverable assessments, surcharges or other liabilities attributable to Qualifying Catastrophe Events borne by such insurer in the qualifying lines of business as reported in such insurer's annual statement that are attributable to one or more qualifying events, plus the amounts by which such losses and loss adjustment expenses have been reduced for contractual payments from catastrophe management resources and for salvage and subrogation. Qualifying Losses shall be determined and maintained by Catastrophe Year for purposes of determining how they apply to the Catastrophe Reserve drawdown criteria.

   Note: Catastrophe management resources may include reinsurance that meets the risk transfer definition contained in Statement of Statutory Accounting Principle No. 62, Property and Casualty Reinsurance, of the *NAIC Accounting Practices and Procedures Manual* and qualifies for credit for reinsurance per the domiciliary state's credit for reinsurance law; insurance risk securitizations that constitute risk transfer or reinsurance per the NAIC or domiciliary state rules; unrestricted surplus, capital notes and surplus notes to the extent that, in combination, they exceed the insurer's Company Action Level Risk-Based Capital of the NAIC Risk-Based Capital Model Act; commissioner-approved support agreements from parent or affiliated entities such as intragroup reinsurance, stop loss, guarantees, net worth maintenance or other similar arrangements; voluntary tax-deferred pre-event catastrophe reserves as defined herein; and other risk transfer arrangements (including contingent equity or contingent surplus notes or contingent capital notes that, when issued, would meet risk-based capital credit, surplus enhancement, credit for reinsurance, or risk transfer requirements) as permitted by state law or regulation or approved by the Commissioner of the state of domicile. Catastrophe management resources are subject to regulatory review.

8. **Catastrophe Year** - Catastrophe Year in concept and application is similar to the insurance accounting concept of accident year since it represents the identification and development of losses occurring within a particular calendar year. Catastrophe Year is somewhat different from accident year, however, since individual Qualifying Losses, which are attributable to a Qualifying Catastrophe Event, are included in the year in which the Qualifying Catastrophe Event started, disregarding the possibility that the Qualifying Catastrophe Event spans more than one calendar year. For purposes of determining Qualifying Losses, Catastrophe Year shall be determined from the time that a Qualifying

Catastrophe Event first occurs and shall include all Qualifying Losses attributable to that Qualifying Catastrophe Event regardless of the year incurred.

9. **Reserve Cap** – The Reserve Cap establishes a maximum limit for the insurer's Voluntary Tax-deferred Pre-event Catastrophe Reserve. The Reserve Cap is calculated using a formula that multiplies Catastrophe Reserve Cap Factors by an insurer's net written premiums (including net written premiums under intercompany pooling arrangements) for each corresponding Qualifying Line of Business. The resulting amounts are added together to obtain a single aggregate Catastrophe Reserve Cap for the insurer. The Catastrophe Reserve shall be established and maintained at a level not to exceed the Reserve Cap. Additions to the Catastrophe Reserve shall be limited to an amount equal to the difference between the Reserve Cap and the reserve balance at the time of the addition.

10. **Catastrophe Reserve Cap Factors** - The following Catastrophe Reserve Cap Factors are to be used to calculate the insurer's Reserve Cap.

Fire	0.25
Allied lines	0.85
Farmowners Multi-peril	0.10
Homeowners Multi-peril	0.60
Commercial Multi-peril	0.30
Earthquake	16.30
Private Passenger Auto Physical Damage	0.01
Commercial Auto Physical Damage	0.01
Inland Marine	0.20
Non-proportional Reinsurance for Other Qualifying Lines	0.45

[Note: The above factors are subject to revision to accommodate non-proportional reinsurance for the qualifying lines of business. Alternative factors (Models 2 & 3) are currently being considered for non-proportional reinsurance and whether Reinsurance A and B should be combined or split. The factors for Model 1 are included above. Model 2 & 3 factors are attached to the Exposure Draft.]

The Catastrophe Reserve Cap Factors will be the same as the factors promulgated under federal tax law to allow tax deferral of such reserves. Given the unique prerogative of the U.S. Congress over the determination of the basis for taxation, these factors may be updated periodically by federal law with advice and counsel from other parties, to include the NAIC. Any changes to such factors for use in the federal tax law shall also be made to the Catastrophe Reserve Cap Factors, herein. The methodology used to determine these factors is included for reference in the Appendix.

11. **Reserve Drawdown/Thresholds** – Drawdowns from the Catastrophe Reserve shall be made in accordance with the following criteria:

3. **Drawdown for Qualifying Losses in Excess of Threshold** –The Catastrophe Reserve shall be drawn down in an amount not to exceed the lesser of the amount determined under subparagraph (1) or (2):

   1) Qualifying Losses for the Catastrophe Year net of contractual payments from catastrophe management resources and net of salvage and subrogation, or

   2) Qualifying Losses for the Catastrophe Year to the extent that such Qualifying Losses exceed the lesser of:

      i. 100% of the insurer's prior year Reserve Cap, or

      ii. 30% of the insurer's surplus at December 31 of the prior year.

Note: Federal enabling tax legislation includes a second event trigger that is not included here subject to further discussion.

b. **Drawdown for Amounts in Excess of Cap** - The catastrophe reserve balance shall be drawn down to the extent it exceeds the Reserve Cap.

c. **Drawdown for Insolvency** - The domiciliary Commissioner may cause an insurer to release the Catastrophe Reserve as a rehabilitation, conservation or liquidation measure or to forestall insolvency of the insurer.

12. **Adjustment for Affiliate Risk Sharing Arrangements** - Many insurers share risk among affiliates through excess of loss or stop loss reinsurance agreements. For such insurers, the reserve cap shall be computed on a consolidated basis for the participating affiliates and then allocated to each participating affiliate on a basis that reasonably reflects the relative retained exposure of each entity to Qualifying Losses.

13. **Effective Date and Transition** - No reporting or calculation of a catastrophe reserve shall be required until enabling federal tax legislation is in effect.

Derivation of Line of Business Catastrophe Reserve Cap Factors
For Voluntary Tax-Deferred Pre-Event Catastrophe Reserves

Objective - Reserve cap factors by line of business were derived to produce a maximum insurance industry reserve of $40 billion when applied to 1999 net written premiums. The reserve cap factors are designed to reflect the historic variability in industry loss ratios for the relevant lines of business, as well as the expected catastrophe losses implied by a catastrophe model.

Reserve Calculation Methodology

The proposed reserve cap factors are based on the following methodology:

1. Adjust the reinsurance A and B premiums and losses to account for the portion attributable to the qualified lines only (qualified lines in reinsurance A are fire, allied, inland marine, earthquake, private passenger auto physical damage, and commercial auto physical damage; qualified lines in reinsurance B are farmowners, homeowners, and CMP non-liability) by multiplying the reinsurance A and B premiums and losses by the following ratio: (reinsurance X qualified lines premiums ceded to non-affiliates)/(total reinsurance X premiums ceded to non-affiliates), where X = A or B.
2. Combine the reinsurance A and B lines into one reinsurance line by summing the respective reinsurance A and B premiums and losses.
3. Calculate industry direct incurred loss ratios (direct incurred losses and loss adjustment expenses divided by direct earned premiums) from 1967 through 1999 for each of the subject lines of business based on data published by A.M. Best. (Farmowners data was only available starting in 1973; commercial multiple peril data was only available in total from 1967 through 1991 [the non-liability portion of CMP was available starting in 1992]; earthquake data was not available in 1971 and 1972; reinsurance A and B data was only available starting in 1976 [only total reinsurance data was available from 1976-1987; beginning in 1988, the data was split into the separate reinsurance lines].)
4. Calculate the mean and standard deviation of the annual direct incurred loss ratios for each line.
5. Adjust the loss ratios from step 3 such that there is no loss ratio greater than the mean plus one standard deviation as calculated in step 4. In other words, if an annual loss ratio is greater than the mean plus one standard deviation, set the loss ratio equal to the mean plus one standard deviation. If an annual loss ratio is less than the mean plus one standard deviation, do not adjust it.
6. Calculate the adjusted mean and standard deviation of the direct incurred loss ratios from step 5 for each line.

7. Calculate a threshold loss ratio for each line, equal to the adjusted mean loss ratio plus two adjusted standard deviations from step 6.
8. Calculate the excess loss ratio for each line and year equal to the excess of the actual loss ratio over the threshold loss ratio. (It was assumed that non-liability represented 50% of CMP premiums and losses during the period 1967-1991. To reflect this assumption, any excess loss ratios during this period were doubled. In addition, CMP excess loss ratios for 1983-1985 were set to zero because the large loss ratios in those years were primarily due to inadequate casualty pricing rather than property catastrophes. For farmowners, the excess loss ratios for 1967-1972 are the homeowners excess loss ratios. For earthquake, the excess loss ratios for 1971 and 1972 are allied lines excess loss ratios.)
9. Calculate losses in excess of the threshold loss ratio for each line by applying the all years sum of the excess loss ratios for each line to the 1999 direct written premium.
10. Prorate the excess losses from step 9 to a total of two thirds of the projected cap (two-thirds of $40 billion based on 1999 premiums).
11. Estimate expected catastrophe losses by line of business, based on an analysis of data published by a prominent catastrophe modeling firm.
12. Prorate the expected catastrophe losses from step 11 to a total of one-third of the projected cap (one-third of $40 billion based on 1999 premiums).
13. Add the amounts from steps 10 and 12 to produce an initial reserve cap by line of business.
14. Calculate an initial reserve cap factor by dividing the reserve caps for each line (from step 13) by 1999 net written premium by line.
15. Cap the factor in step 14 for the private passenger auto physical damage line of business equal to 0.01
16. Cap the factor in step 14 for the commercial auto physical damage line of business equal to 0.01.
17. Using the capped factors in steps 15 and 16, calculate the reserve caps for private passenger auto physical damage and commercial auto physical damage.
18. Prorate the difference between the original private passenger auto physical damage and the commercial auto physical damage reserve caps from step 13 and the reserve caps from step 17 back to the other lines and recalculate the reserve cap factors.
19. Select a reserve cap factor for each line by rounding the ratios calculated in step 18 to the nearest 0.05, with a minimum factor of 0.05 (except the auto physical damage lines).

Statutory Issue Paper No. ___

Title:
CATASTROPHE RESERVES

Status:
Revised Draft of the Catastrophe Insurance Working Group February 2001

Type of Issue:
Property-Casualty

SUMMARY OF ISSUE:

1. Catastrophes present a significant challenge to the U.S. economy and to the U.S. property-casualty insurance industry, posing financial solvency, capital accumulation and insurance availability issues. Insurers should be encouraged to engage in prudent risk management behavior. Existing methods to manage insurer risk include loss mitigation, geographic spreading of risk, reinsurance, capital market instruments, derivatives products and short and long-term pre and post-event financing. One additional method proposed to encourage prudent risk behavior - which should augment, not supplant those existing risk management methods - is to permit insurers to establish a reserve for future catastrophes.

2. Current statutory and GAAP accounting limits the recognition of loss reserves to losses which have been incurred. A reserve for future catastrophes is intended to enhance property and casualty insurers' capability to manage their catastrophic loss exposures by allowing specific provision for future catastrophe losses. Existing statutory accounting and tax treatment of catastrophe exposure fails to maximize the ability of insurers to appropriately respond to catastrophe related and availability issues. With respect to solvency issues, the current statutory accounting and tax treatment does not encourage the prudent accumulation of capital to absorb catastrophe losses.

3. It is in the public interest to determine whether a reserve for future catastrophes can be structured in such a way so as to provide incentives for:

    - property and casualty insurers to appropriately manage their catastrophic loss exposures by making specific provision for future catastrophe losses while continuing to provide and expand catastrophe insurance coverage in regions with significant catastrophe exposure, and

    - state insurance regulators to approve catastrophe insurance rates which, among other appropriate factors, reflect past experience and projected exposures, so that the U.S. federal government and taxpayers benefit, in the form of possible reduced demand for catastrophe relief, as a result of these changes in the insurance regulatory and tax systems.

4. This paper establishes statutory accounting criteria for recording a pre-event catastrophe reserve.

SUMMARY CONCLUSION:

5. Insurers writing property and casualty contracts as defined in SSAP 50 - Classifications and Definitions of Insurance or Managed Care Contracts In Force, writing the lines of business described in paragraph 7 below and required under state law to file a NAIC annual statement shall establish a catastrophe reserve for the purpose of paying for future catastrophe losses occurring in the United States and its territories and possessions. Insurers domiciled in Puerto Rico shall not establish the reserve for Puerto Rico risks as a catastrophe reserve is already included in the Puerto Rico insurance code.

6. The catastrophe reserve shall be a separate liability on the balance sheet distinct from loss and loss adjustment expense reserves and unearned premium reserves. Additions to, and deductions from, the catastrophe reserve shall be reported through a change in catastrophe reserve in the underwriting income section of the statutory statement of income.

7. The catastrophe reserve shall be established in the aggregate for the following lines: fire, allied lines, farm-owners multiple-peril, homeowners multiple-peril, commercial multiple-peril (non-liability portion), earthquake, private passenger auto physical damage commercial auto physical damage, inland marine, and non-proportional reinsurance for the Qualifying Lines of Business.

8. The types of events that must occur for an insurer to be allowed to deduct Qualifying Losses (as defined in paragraph 9 below) from the catastrophe reserve are as follows:

   a. Wind
   b. Hail
   c. Earthquake/fire following
   d. Winter catastrophes (snow, ice, freezing)
   e. Fire
   f. Tsunami
   g. Flood
   h. Volcanic eruption (including lahar)

These events shall be 1) designated a catastrophe by Property Claims Service or any successor organization, 2) declared an emergency or disaster by the President of the United States pursuant to the Stafford Act, or 3) declared to be an emergency or disaster in a similar declaration by the chief executive official of a State, possession, or territory of the United States, or the District of Columbia.

9. Qualifying Losses are: direct and assumed losses and loss adjustment expenses incurred and any non-recoverable assessments, surcharges or other liabilities attributable to Qualifying Catastrophe Events borne by such insurer in the qualifying lines of business as reported in such insurer's annual statement that are attributable to one or more qualifying events, plus the amounts by which such losses and loss adjustment expenses have been reduced for contractual payments from catastrophe management resources and for salvage and subrogation.

Calculation and Basis for Reserve Additions

10. Permissible annual reserve additions to the catastrophe reserve will be determined by applying catastrophe reserve cap factors to the insurer's net premiums written. Although the maximum permissible reserve is to be derived from premiums multiplied by an exposure factor, the design of the reserve is not intended to affect rates.

11. Catastrophe reserve cap factors, promulgated under federal law and developed and updated periodically with input from the NAIC, will be established for each line of business written (as specified in paragraph 7). These catastrophe reserve cap factors shall be applied to net written premiums (including net written premium under inter-company pooling arrangements) by qualifying line of business in order to determine the maximum permissible amount of the catastrophe reserve (or "catastrophe reserve cap"). The catastrophe reserve cap factors are included in federal legislation and may be revised periodically. They will be maintained as updated for any subsequent changes, in the *NAIC Annual Statement Instructions – Property & Casualty*.

Criteria for Catastrophe Reserve Drawdowns

12. The following criteria shall be followed for the drawdown of the reserve.

    a. For purposes of the catastrophe reserve drawdown criteria, catastrophe year shall be determined from the time that a qualifying event's first loss occurs and shall include all qualifying losses (as defined in paragraph 9) related to that qualifying event regardless of the year incurred. Catastrophe year in concept and application is similar to the insurance accounting concept of accident year since it represents the identification and development of losses occurring within a particular calendar year. Catastrophe year is somewhat different from accident year, however, since individual losses, which are part of a catastrophic event, are included in the year in which the catastrophic event started, disregarding the possibility that the event spans more than one calendar year. Qualifying losses shall be maintained by catastrophe year.

13. **Reserve Drawdown/Thresholds** – Drawdowns from the Catastrophe Reserve shall be made in accordance with the following criteria:

a. **Drawdown for Qualifying Losses in Excess of Threshold** – The Catastrophe Reserve shall be drawn down in an amount not to exceed the lesser of the amount determined under subparagraph 1) or 2):

1) Qualifying Losses for the Catastrophe Year net of contractual payments from catastrophe management resources and net of related subrogation and salvage, or

2) Qualifying Losses for the Catastrophe Year to the extent that such Qualifying Losses exceed the lesser of:

i. 100% of the insurer's prior year Reserve Cap, or

ii. 30% of the insurer's surplus at December 31 of the prior year.

Note: Federal enabling tax legislation includes a second event trigger that is not included here subject to further discussion.

b. **Drawdown for Amounts in Excess of Reserve Cap** - The catastrophe reserve balance shall not exceed the reserve cap (the sum of the amounts obtained by multiplying each line of business reserve cap factor by its corresponding net written premium for the insurer). The reserve shall be drawn down to the extent the balance of the reserve exceeds the reserve cap.

c. **Drawdown for Insolvency** - The insurer's domiciliary Commissioner may cause an insurer to release the reserve as a rehabilitation, conservation or liquidation measure or to forestall insolvency.

14. Many insurers share risk among affiliates through excess of loss or stop loss reinsurance agreements. For such insurers, the reserve cap should be computed on a consolidated basis for the participating affiliates and then allocated to each participating affiliate on a basis that reasonably reflects the relative retained exposure of each entity to Qualifying Losses.

Disclosures

15. The annual financial statements shall disclose a reconciliation of the catastrophe reserve between years, including:

a. The balance of the catastrophe reserve at the beginning of the year;
b. The annual reserve addition;
c. Drawdowns of the reserve, including the nature of the drawdown; (i.e. qualifying losses, amounts in excess of the reserve cap, etc.)
d. The balance of the catastrophe reserve at the end of the year.

Effective Date and Transition

16. No reporting or calculation of a catastrophe reserve shall be required until enabling federal tax legislation is in effect.

DISCUSSION:

17. Under the preamble to the NAIC Accounting Practices and Procedures Manual effective January 1, 2001, it is contemplated that special reserves may be established for regulatory solvency. That provision of the preamble provides the basis for the establishment of this reserve.

18. Consistent with the solvency and conservatism concepts in the Statutory Accounting Principles Statements of Concepts and Statutory Hierarchy, the statutory accounting model uses numerous accounting methods to accomplish the objective of reporting a company's statutory financial position to demonstrate solvency. Notwithstanding the accounting guidance in SSAP 5 and SSAP 55 recording of a catastrophe reserve is consistent with the solvency and conservatism concepts.

19. The catastrophe reserve does not meet the definition of a liability, which is set forth in SSAP 5 – Liabilities, Contingencies and Impairments of Assets. Nor does the catastrophe reserve meet the definition and characteristics of a liability as defined in FASB Statement of Concepts No. 6 – Elements of Financial Statements. However, it is consistent with the "*ultimate objective of solvency regulation*" as stated in the Statement of Concepts. This states:

    The ultimate objective of solvency regulation is to ensure that policyholder, contract holder and other legal obligations are met when they come due and that companies maintain capital and surplus at all times and in such forms as required by statute to provide an adequate margin of safety.

    Additionally, recording the catastrophe reserve, as a liability is consistent with the Statement of Concepts, which states:

    Liabilities require recognition as they are incurred. Certain statutorily mandated liabilities may also be required to arrive at conservative estimates of liabilities and probable loss contingencies.

20. This issue paper is consistent with certain principles discussed in SSAP 60 – Financial Guaranty Insurance regarding a contingency reserve. Similarities can be draw between the calculation of the contingency reserve as specified in SSAP 60 and the calculation of the catastrophe reserve as specified in this issue paper. Both reserves are on a pre-event basis and reference premiums written. The purpose of both is to protect policyholders.

21. The statutory accounting principles outlined in the conclusion above are consistent with the conservatism and recognition concepts in the Statement of Concepts. Pertinent excerpts follow:

Conservatism

Financial reporting by insurance enterprises requires the use of substantial judgments and estimates by management. Such estimates may vary from the actual amounts for numerous reasons. To the extent that factors or events result in adverse variation from management's accounting estimates, the ability to meet policyholder obligations may be lessened. In order to provide a margin of protection for policyholders, the concept of conservatism should be followed when developing estimates as well as establishing accounting principles for statutory reporting.

22. A catastrophe reserve without tax-deductibility would diminish an insurer's capital and would likely have the effect of restricting availability of insurance coverage to consumers in catastrophe prone areas. It is anticipated that tax-deductibility of the catastrophe reserve will provide incentive for insurers to write in catastrophe prone areas and should result in greater availability of insurance and increase risk-bearing capacity to cover catastrophic exposure.

23. The specified treatment of the annual reserve additions and qualifying losses do not meet the defined recording and treatment of claims, losses, loss/claim adjustment expense, unpaid claims, unpaid losses and unpaid loss/claim adjustment expenses as defined in SSAP 55 - Unpaid Claims, Losses, and Loss Adjustment Expenses or FASB Statement No. 60, Accounting and Reporting by Insurance Enterprises (FAS 60). These pronouncements specify that the event must have occurred for a liability to be recorded and an estimate of the ultimate cost must be determined. However the treatment is consistent with the *"ultimate objective of solvency regulations"* as stated in the Statement of Concepts, please refer to paragraph 18 above.

RELEVANT LITERATURE:

Statutory Accounting Practices and Procedures

- Preamble to the NAIC Accounting Practices and Procedures Manual
- Statutory Accounting Principles Statement of Concepts and Statutory Hierarchy
- SSAP 5 – Definition of Liabilities, Loss Contingencies and Impairments of Assets
- SSAP 55 – Unpaid Claims, Losses, and Loss Adjustment Expenses
- SSAP 60 – Financial Guaranty Insurance

Generally Accepted Accounting Principles

- FASB Statement of Concepts No. 6 – Elements of Financial Statements
- FASB Statement No. 60 – Accounting and Reporting by Insurance Enterprises

State Regulations

- Puerto Rico Statues, Title 26, Ins. Code §25.010, Reserve for Catastrophic Insurance Losses
- Florida Statues, Title XXXVII, Chapter 627, Insurance Rates and Contracts §067.062

Statement of Statutory Accounting Principles No. __

Catastrophe Reserves

Scope of Statement

1. This statement establishes statutory accounting principles for a pre-event catastrophe reserve for property and casualty companies.

Summary Conclusion

2. Insurers writing property and casualty contracts as defined in SSAP 50 – Classifications and Definitions of Insurance or Managed Care Contracts In Force, writing the lines of business described in paragraph 4 below and required under state law to file a NAIC annual statement shall establish a catastrophe reserve for the purpose of paying for future qualifying catastrophe losses occurring in the United States and its territories and possessions. Insurers domiciled in Puerto Rico shall not establish the reserve for Puerto Rico risks as a catastrophe reserve is already included in the Puerto Rico insurance code.

3. The catastrophe reserve shall be a separate statutory liability on the balance sheet distinct from loss and loss adjustment expense reserves and unearned premium reserves. Additions to, and deductions from, the catastrophe reserve shall be reported through a change in catastrophe reserve in the underwriting income section of the statutory statement of income.

4. The catastrophe reserve shall be established in the aggregate for the following lines: fire, allied lines, farm-owners multiple-peril, homeowners multiple peril, commercial multiple-peril (non-liability portion), earthquake, private passenger auto physical damage, commercial auto physical damage, inland marine, and non-proportional reinsurance for Qualifying Lines of Business.

5. The types of events that must occur for an insurer to be allowed to deduct qualifying losses (defined in paragraph 6) from the catastrophe reserve are as follows:

   a) Wind
   b) Hail
   c) Earthquake/fire following
   d) Winter catastrophe (snow, ice, freezing)
   e) Fire
   f) Tsunami
   g) Flood
   h) Volcanic eruption (including lahar)

   These events shall be 1) designated a catastrophe by Property Claims Service or any successor organization, 2) declared an emergency or disaster by the President of the United States pursuant to the Stafford Act, or 3) declared to be an emergency or disaster in a similar declaration by the chief executive official of a State, possession, or territory of the United States, or the District of Columbia.

6. Qualifying losses are: Direct and assumed losses and loss adjustment expenses incurred and any non-recoverable assessments, surcharges or other liabilities attributable to Qualifying Catastrophe Events borne by such insurer in the qualifying lines of business as reported in such insurer's annual statement that are attributable to one or more qualifying events, plus the amounts by which such losses and loss adjustment expenses have been reduced for contractual payments from catastrophe management resources and for salvage and subrogation.

7. Calculation and Basis for Reserve Additions

a) Annual reserve additions for each insurer shall not exceed the catastrophe reserve cap. The catastrophe reserve cap is the sum of the amounts determined by multiplying the catastrophe reserve cap factor for each qualifying line of business by the insurer's net written premium for the corresponding qualifying line of business.

b) Catastrophe reserve cap factors, promulgated under federal law and developed and updated periodically with input from the NAIC will be established for each line of business written (as specified in paragraph 4). These catastrophe reserve cap factors shall be applied to net written premiums (including net written premiums under inter-company pooling arrangements) by qualifying line of business in order to determine the maximum permissible amount (or "reserve cap") of the catastrophe reserve. The catastrophe reserve cap factors are included in federal tax legislation and may be revised periodically and will be maintained in *the NAIC Annual Statement Instructions – Property & Casualty.*

8. Criteria for Drawdown of the Catastrophe Reserve

The following criteria shall be followed for the drawdown of the reserve:

a. For purposes of the catastrophe reserve drawdown criteria, catastrophe year shall be determined from the time that a qualifying event's first loss occurs and shall include all qualifying losses (as defined in paragraph 6) related to that qualifying event regardless of the year incurred. Catastrophe year in concept and application is similar to the insurance accounting concept of accident year since it represents the identification and development of losses occurring within a particular calendar year. Catastrophe year is somewhat different from accident year, however, since individual losses, which are part of a catastrophic event, are included in the year in which the catastrophic event started, disregarding the possibility that the event spans more than one calendar year. Qualifying losses shall be maintained by catastrophe year.

b. The Catastrophe Reserve shall be drawn down in an amount not to exceed the lesser of the amount determined under subparagraph 1) or 2):

1) Qualifying Losses for the Catastrophe Year net of contractual payments from catastrophe management resources and net of subrogation and salvage, or

2) Qualifying Losses for the Catastrophe Year to the extent that such Qualifying Losses exceed the lesser of:

   i. 100% of the insurer's prior year initial reserve cap, or

   ii. 30% of the insurer's surplus at December 31 of the prior year

Note: Federal enabling tax legislation includes a second event trigger that is not included here subject to further discussion.

c. The catastrophe reserve shall be drawn down to the extent it exceeds the reserve cap.

d. The insurer's domiciliary commissioner may cause an insurer to release the reserve as a rehabilitation, conservation or liquidation measure or to forestall insolvency.

9. Many insurers share risk among affiliates through excess of loss or stop loss reinsurance agreements. For such insurers, the reserve cap should be computed on a consolidated basis for the participating affiliates and then allocated to each participating affiliate on a basis that reasonably reflects the relative retained exposure of each entity to Qualifying Losses.

Disclosures

10. The annual financial statements shall disclose a reconciliation of the catastrophe reserve between years, including:

   a) The balance of the catastrophe reserve at the beginning of the year;
   b) The annual reserve addition;
   c) Drawdowns of the reserve, including the nature of the drawdown (i.e. qualifying losses, amounts in excess of the reserve cap, etc.);
   d) The balance of the catastrophe reserve at the end of they year.

Effective Date

11. No reporting or calculation of a catastrophe reserve shall be required until enabling federal tax legislation is in effect. A change resulting from the adoption of this statement shall be accounted for as a change in accounting principle in accordance with *SSAP No. 3 – Accounting Changes and Corrections of Errors.*

Authoritative Literature

Statutory Accounting

- Preamble to the NAIC Accounting Practices and Procedures Manual
- Statutory Accounting Principles Statement of Concepts and Statutory Hierarchy
- SSAP 5 – Definition of Liabilities, Contingencies, and Impairments of Assets

- SSAP 55 – Unpaid Claims, Losses and Loss Adjustment Expenses
- SSAP 60 – Financial Guaranty Insurance

Relevant Issues Papers

- Issue Paper No. ___, Catastrophe Reserves
- Issue Paper No. 5 – Liabilities, Contingencies, and Impairments of Assets
- Issue Paper No. 55 – Unpaid Claims, Losses and Loss Adjustment Expenses
- Issue Paper No. 69 – Financial Guaranty Insurance

SSAP No. __ - Exhibit A Implementation Guidance

Voluntary tax-deferred pre-event catastrophe reserve calculation rules

For insurer's that choose to use the reserve, the reserve balance calculation should be completed on an **annual basis** in the order as listed below.

1. *Calculate the annual reserve cap and thresholds.*

2. *Determine the allowable drawdown for qualifying losses.*

3. *Reduce the reserve balance for the amount of qualifying losses to be drawn down from the reserve subject to the limit in 2 above.*

4. *Determine the maximum amount of the annual reserve addition allowed by subtracting the remaining reserve balance from the reserve cap.*

5. *Add the desired reserve addition to the remaining balance of the reserve subject to the limitation in 4 above*

6. *Determine if the resulting balance exceeds the reserve cap, if so, reduce the reserve for any balance in excess of the reserve cap.*

The determination of qualifying losses is a complicated procedure since all losses shall be maintained by catastrophe year. Catastrophe year in concept and application is similar to the commonly understood insurance accounting concept of accident year since it represents the identification and development of losses occurring within a particular calendar year. Catastrophe year is somewhat different from accident year, however, since individual losses that are part of a catastrophic event are included in the year in which the catastrophic event started, disregarding the possibility that the event spans more than one calendar year.

The development of losses from a qualifying event shall be continuously maintained. Losses from a qualifying event might not reach the qualifying loss threshold for several years after the year of occurrence. Accordingly, each company shall maintain losses by catastrophe year and, their qualifying loss limitation thresholds for each year as a permanent benchmark. Once the annual determination of qualifying losses is completed for the current year the qualifying losses are deducted from the reserve. Favorable loss development should not be used to re-establish reserves previously drawdown.

The reserve cap is the sum of the amounts determined by multiplying the catastrophe reserve cap factor for each qualifying line of business by the insurer's net written premium for the corresponding qualifying line of business.

Proposed NAIC Annual Statement Instructions for Property and Casualty Insurance Companies

LIABILITIES, SURPLUS AND OTHER FUNDS

Line 16 - Catastrophe Reserve

Include: Amounts for catastrophe reserves. Refer to SSAP #__, Voluntary Tax-deferred Pre-event Catastrophe Reserves for guidance regarding these amounts.

The following represents the catastrophe reserve cap factors as specified in SSAP #__, paragraph 7. These factors are used in the calculation of catastrophe reserve cap, which limits annual reserve additions.

Fire	0.25
Allied lines	0.85
Farmowners Multi-peril	0.10
Homeowners Multi-peril	0.60
Commercial Multi-peril	0.30
Earthquake	16.30
Private Passenger Auto Physical Damage	0.01
Commercial Auto Physical Damage	0.01
Inland Marine	0.20
Non-proportional Reinsurance for Qualifying Lines	0.45

[Note: The above factors are subject to revision to accommodate non-proportional reinsurance for the qualifying lines of business. Alternative factors (Models 2 & 3) are currently being considered for non-proportional reinsurance and whether Reinsurance A and B should be combined or split. The factors for Model 1 are included above. Model 2 & 3 factors are attached to the Exposure Draft.]

LIABILITIES, SURPLUS AND OTHER FUNDS	1 Current Year	2 Prior Year
1. Losses (Part 3A, Line 32, Column 5)		
2. Reinsurance payable on paid loss and loss adjustment expenses (Schedule F, Part 1, Column 6)		
3. Loss adjustment expenses (Part 3A, Line 34, Column 9)		
4. Commissions payable, contingent commissions and other similar charges		
5. Other expenses (excluding taxes, licenses and fees)		
6. Taxes, licenses and fees (including $........net deferred tax liability)		
7. Federal and foreign income taxes (excluding deferred taxes)		
8. Borrowed money $........and interest thereon $		
9. Unearned premiums (Part 2A, Line 36, Column 5) (after deducting unearned premiums for ceded reinsurance of $............and including warranty reserves of $..............)		
10. Dividends declared and unpaid:		
10.1 Stockholders		
10.2 Policyholders		
10.3 Ceded reinsurance premiums payable (net of ceding commissions)		
11. Funds held by company under reinsurance treaties (Schedule F, Part 3, Column 14)		
12. Amounts withheld or retained by company for account of others		
13. Remittances and items not allocated		
14. Provision for reinsurance (Schedule F, Part 7)		
15. Net adjustments in assets and liabilities due to foreign exchange rates		
16. Catastrophe Reserve		
17. Drafts outstanding		
18. Payable to parent, subsidiaries and affiliates		
19. Payable for securities		
20. Liability for amounts held under uninsured accident and health plans		
21. Capital notes $....................and interest thereon $		
22. Aggregate write-ins for liabilities		
23. Total liabilities (Lines 1 through 22)		
24. Aggregate write-ins for special surplus funds		
25. Common capital stock		
26. Preferred capital stock		
27. Aggregate write-ins for other than special surplus funds		
28. Surplus notes		
29. Gross paid in and contributed surplus		
30. Unassigned funds (surplus)		
31. Less treasury stock, at cost:		
31.1 ....shares common (value included in Line 25 $..........)		
31.2 ....shares preferred (value included in Line 26 $..........)		
32. Surplus as regards policyholders (Lines 24 to 30, less 31) (Page 4, Line 38)		
33. TOTALS (Page 2, Line 24, Col. 4)		
DETAILS OF WRITE-INS		
2201.		
2202.		
2203.		
2298. Summary of remaining write-ins for Line 22 from overflow page		
2299. Totals (Lines 2201 through 2203 plus 2298) (Line 22 above)		
2401.		
2402.		
2403.		
2498. Summary of remaining write-ins for Line 24 from overflow page		
2499. Totals (Lines 2301 through 2403 plus 2498) (Line 24 above)		
2701.		
2702.		
2703.		
2798. Summary of remaining write-ins for Line 27 from overflow page		
2799. Totals (Lines 2701 through 2703 plus 2798) (Line 27 above)		

UNDERWRITING AND INVESTMENT EXHIBIT
STATEMENT OF INCOME

Line 4 - Change in Catastrophe Reserve

Include: Additions to and drawdowns from the catastrophe reserve. Refer to SSAP #__, Voluntary Tax-deferred Pre-event Catastrophe Reserves for guidance regarding these amounts.

UNDERWRITING AND INVESTMENT EXHIBIT STATEMENT OF INCOME	1 Current Year	2 Prior Year
UNDERWRITING INCOME		
1. Premiums earned (Part 2, Line 34, Column 4)		
DEDUCTIONS		
2. Losses incurred (Part 3, Line 34, Column 7)		
3. Loss expenses incurred (Part 4, Line 25, Column 1)		
4. Change in Catastrophe Reserve (Page 3, Line 16, Column 2 minus Column 1)		
5. Other underwriting expenses incurred (Part 4, Line 25, Column 2)		
6. Aggregate write-ins for underwriting deductions		
7. Total underwriting deductions (Lines 2 through 6)		
8. Net underwriting gain or (loss) (Line 1 minus Line 7)		
INVESTMENT INCOME		
9. Net investment income earned (Part 1, Line 15)		
10. Net realized capital gains or (losses) (Part 1A, Line 10)		
11. Net investment gain or (loss) (Lines 9 + 10)		
OTHER INCOME		
12. Net gain or (loss) from agents' or premium balances charged off (amount recovered $ .... amount charged off $ ....)		
13. Finance and service charges not included in premiums		
14. Aggregate write-ins for miscellaneous income		
15. Total other income (Lines 12 through 14)		
16. Net income before dividends to policyholders and before federal and foreign income taxes (Lines 8+11+15)		
17. Dividends to policyholders (Exhibit 2, Line 16, Column 1 plus Page 3, Line 10.2, Column 1 minus Column 2)		
18. Net income, after dividends to policyholders but before federal and foreign income taxes (Line 16 minus Line 17)		
19. Federal and foreign income taxes incurred		
20. Net income (Line 18 minus Line 19) (to Line 22)		
CAPITAL AND SURPLUS ACCOUNT		
21. Surplus as regards policyholders, December 31 prior year (Page 4, Line 37, Column 2)		
GAINS AND (LOSSES) IN SURPLUS		
22. Net income (from Line 19)		
23. Net unrealized capital gains or (losses) (Part 1A, Line 11)		
24. Change in net deferred income tax		
25. Change in nonadmitted assets (Exhibit 1, Line 6, Col. 3)		
26. Change in provision for reinsurance (Page 3, Line 14, Column 2 minus Column 1)		
27. Change in net unrealized foreign exchange capital gain or (loss)		
28. Change in surplus notes		
29. Cumulative effect of changes in accounting principles		
30. Capital changes:		
30.1 Paid in		
30.2 Transferred from surplus (Stock Dividend)		
30.3 Transferred to surplus		
31. Surplus adjustments:		
31.1 Paid in		
31.2 Transferred to capital (Stock Dividend)		
31.3 Transferred from capital		
32. Net remittances from or (to) Home Office		
33. Dividends to stockholders		
34. Change in treasury stock (Page 3, Line 31.1 and (31.2), Column 2 minus Column 1)		
35. Extraordinary amounts of taxes for prior years		
36. Aggregate write-ins for gains and losses in surplus		
37. Change in surplus as regards policyholders for the year (Lines 22 through 36)		
38. Surplus as regards policyholders, December 31 current year (Line 21 plus Line 37) (Page 3, Line 32)		
DETAILS OF WRITE-INS		
0601.		
0602.		
0603.		
0698. Summary of remaining write-ins for Line 6 from overflow page		
0699. Totals (Lines 0601 through 0603 plus 0698) (Line 6 above)		
1401.		
1402.		
1403.		
1498. Summary of remaining write-ins for Line 14 from overflow page		
1499. Totals (Lines 1401 through 1403 plus 1498) (Line 14 above)		
3601.		
3602.		
3603.		
3698. Summary of remaining write-ins for Line 36 from overflow page		
36599. Totals (Lines 3601 through 3603 plus 3698) (Line 36 above)		

NOTES TO FINANCIAL STATEMENTS

The financial statements shall disclose the following:

a. The balance of the catastrophe reserve at the beginning of the year;
b. The annual reserve addition;
c. Drawdowns of the reserve, including the nature of the drawdown (i.e. qualifying losses, amounts in excess of the reserve cap, etc.); and
d. The balance of the catastrophe reserve at the end of the year.

Note: Refer to SSAP #__, Voluntary Tax-deferred Pre-event Catastrophe Reserves for the guidance on calculating and maintaining catastrophe reserves.

Illustration:

i.	Catastrophe Reserve Balance at the Beginning of Year	$__________
ii.	Current Annual Reserve Addition to the Catastrophe Reserve	$__________
iii	Current Drawdowns from Qualifying Losses	$__________
iv.	Current Drawdowns for Amounts in Excess of the Reserve Cap	$__________
v.	Catastrophe Reserve Balance at the End of the Year	$__________

Appendix G: Other Countries' Approaches

As mentioned in the text, we gathered information from other countries with respect to insurance companies' ability to establish pre-event reserves. A brief description of each is provided here. We are aware that France, Mexico and Puerto Rico all have reserves of this nature also, but the details were not available to us.

Barbados

In Barbados, non-life insurance companies are allowed to establish a Catastrophe Reserve Fund to cover natural catastrophes only. This fund is tax deductible and is accounted for as an appropriation of retained earnings. Annually, each company may deduct 20% of its net premium income up to a limit of 100% of its shareholder's equity, from its property insurance business, to be added to its Catastrophe Reserve Fund. The occurrence of any catastrophic event, as defined in the statute, can trigger the release of the reserve.

Canada

Canadian insurers are required to establish pre-event reserves to cover nuclear liability exposures and, at certain levels, to cover earthquake exposures. Each reserve is established differently based upon the exposure it is meant to cover. In the case of the nuclear liability, companies are required to establish an appropriation of surplus equal to 100% of the net written premiums, less commission, for the nuclear liability policies. To cover earthquake exposures, property and casualty companies must demonstrate that they have sufficient resources to absorb losses of a certain level. Otherwise, there is a requirement to carry a reserve, again an appropriation of surplus, up to a certain level. Upon meeting established financial requirements, a company can set aside a reserve on a voluntary basis. Both mandatory and voluntary reserves are tax deductible.

Finland

In Finland, non-life insurance companies are required to establish what we would call pre-event reserves. These reserves serve as an equalization provision[1] for years in which the loss ratio of the accounting year exceeds the average loss ratio from previous years. All classes of business are covered by the equalization provision. Companies domiciled in Finland are also required to establish a pre-event reserve to cover a collective guarantee item in statutory motor vehicle liability and in statutory workers' compensation insurance. In Finland there is a joint liability among insurance companies writing these lines of business in case of an insolvency of one of the companies. This collective guarantee item is meant to cover part of the expenses paid by the company due to an insolvency of another company. These liabilities are tax-exempt.

Germany

In Germany, property/casualty insurers and reinsurers are required to establish an equalization provision and a large risk provision. These provisions are tax deductible. The equalization provision applies to all insurance lines and is concerned with normal

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[1] In Finland, Germany and Switzerland a provision is a liability, used to pay claims, while a reserve is equity.

claims fluctuations occurring at random. It is based upon the assumption that claims vary from year to year and ensures that the yearly fluctuations are spread out evenly. The large risk provision is required for nuclear installation insurance risk and pharmaceutical product risk insurance. It is concerned with fluctuations during the year, which occur because of individually exceptional claims, by providing a further layer of reserve over the general equalization reserve.

Italy
In Italy, companies are required to supplement unearned premium reserves for hail and other natural catastrophes. Companies are also required to establish equalization reserves, which cover nuclear energy and credit risks as well as natural catastrophes. All reserves of this type are tax deductible. Italian law also requires an auditing company to opine on all annual accounts. An actuary must support the auditor and provide a certification of the sufficiency of all technical reserves, including pre-event reserves.

Japan
In Japan, it is mandatory for non-life insurance companies to establish a catastrophe provision. This provision is partially tax-deductible, depending upon the class of business. The main objective is to cover natural catastrophes, however it is not limited to them. The release of the fund is determined by the written paid loss ratio regardless of the event.

Switzerland
Equalization reserves are required in Switzerland for credit insurance and are allowed for other classes of business. Normally, the taxing authority allows the provision to be tax deductible if it is stipulated by law or by the supervisory authority and financed by policyholders. These provisions are established not only to cover natural catastrophes, but also to equalize fluctuations in long-tail lines of business. Companies are also allowed to establish a liability that serves as equity, financed by shareholders and is therefore not tax deductible.

United Kingdom
In the United Kingdom, companies are required to maintain tax-deductible equalization reserves. They are established based upon a formula and a set of rules for the transfer of funds in and out and for the maximum permitted reserve. This requirement covers the following lines of business: property and proportional and non-proportional property reinsurance, business interruption and proportional and non-proportional business interruption reinsurance, ocean marine, aviation, nuclear risks and credit insurance. The company's auditors should check the amount carried as part of the audit. Companies are allowed to release the reserve if their loss ratio calculation for the year for the relevant lines of business is greater than the trigger ratio, or if the company's business in the relevant lines decreases so that the carried reserve is greater than the permitted maximum.

Appendix H: Detailed Example

Annual statements must show specified information about catastrophe reserve additions, balances and drawdowns. The calculation of the maximum dollar amount allowed in the reserve at the end of a year could easily be based on the company's Underwriting and Investment Exhibit, Part 2B and the reserve cap factors mentioned above and described in Appendix C. This calculation is shown in Exhibit 2, below.

The calculation illustrated in Exhibit 2 makes some assumptions about how a company would calculate its reserve cap.

1. The proposal does not specify the data source for the company's net written premium. In this example, the company's 2000 Annual Statement, Underwriting and Investment Exhibit, Part 2B was used.
2. The proposal does not define how a company calculates its split between commercial multiple peril liability and non-liability portions, although the reserve cap factor is to be applied to only the non-liability portion. The proportion can be estimated by looking at lines 5.1 and 5.2 on the company's state pages, which show direct written premium but not net written premium. Company B does not write any CMP, so this was not needed in Exhibit 2. The countrywide percentage of non-liability CMP was used for companies F and G, which did not report any direct written premium. The calculations for companies F and G are found in Exhibit 1.
3. For excess reinsurance, some companies may report certain lines of insurance in Reinsurance A while others may report the same lines in Reinsurance B. For this reason, a single factor, adjusted to reflect the catastrophe-prone property lines only, is used for both of these net written premium amounts.
4. The data is on an individual company basis. Many companies are in a group, and have risk sharing arrangements among affiliated members. "For such insurers, the reserve cap should be computed on a consolidated basis for the participating affiliates and then allocated to each participating affiliate on a basis that reasonably reflects the relative retained exposure of each entity to Qualifying Losses." [5]
5. While the proposal does not differentiate between reinsurance agreements with affiliates and non-affiliates, the annual statement page does. This might be helpful information in analyzing net PMLE, as discussed in the paper.

Exhibit 2
Company B
2000 UNDERWRITING INVESTMENT EXHIBIT PART 2B - 009

		(1)	(2a)	(2b)	(3a)	(3b)	(4)	(5)	(6)
			Reinsurance Assumed		Reinsurance Ceded				(4) x (5)
	LINE	Direct Business	From Affiliates	From Non Affiliates	To Affiliates	To Non Affiliates	Net Premiums Written	Reserve Cap Factor	Reserve Cap Amount
1	Fire	44,391,571	13,235	493,695	1,779,501	664,830	42,454,170	0.25	10,613,543
2	Allied lines	43,444,005	0	1,074,589	750,038	3,708,546	40,060,010	0.85	34,051,009
3	Farmowners multiple peril	0	0	0	0	0	0	0.10	0
4	Homeowners multiple peril	837,211,018	556	364,183	334,787	47,881,199	789,359,772	0.60	473,615,863
5	Commercial multiple peril	0	0	0	0	0	0	0.30	0
6	Mortgage guaranty	0	0	0	0	0	0		0
8	Ocean marine	15,629,728	0	0	0	400,931	15,228,797		0
9	Inland marine	63,633,958	0	0	610	1,887,021	61,746,327	0.20	12,349,265
10	Financial guaranty	0	0	0	0	0	0		0
11.1	Medical malpractice - occurrence	0	0	0	0	0	0		0
11.2	Medical malpractice - claims-made	0	0	0	0	0	0		0
12	Earthquake	16,443,167	0	0	189,001	13,765,654	2,488,512	16.30	40,562,746
13	Group accident and health	0	0	0	0	0	0		0
14	Credit accident and health (group and individual)	0	0	0	0	0	0		0
15	Other accident and health	0	0	0	0	0	0		0
16	Workers' compensation	104,351	13,254	0	0	0	117,605		0
17.1	Other liability - occurrence	67,706,541	315,623	0	0	6,641,381	61,380,783		0
17.2	Other liability - claims-made	0	0	0	0	0	0		0
18.1	Products liability - occurrence	0	962,935	0	0	0	962,935		0
18.2	Products liability - claims-made	0	0	0	0	0	0		0
19.1	Private passenger auto liability	1,245,407,092	6,282,755	32,993,020	9,976,505	9,188,499	1,265,517,863		0
19.3	Commercial auto liability	31,083	9,240	151,955	71,297	0	120,981		0
21	Auto physical damage	1,109,886,048	4,332,464	11,219,223	2,532,723	4,266,129	1,118,638,883	0.01	11,186,389
22	Aircraft (all perils)	0	14,028,239	0	0	0	14,028,239		0
23	Fidelity	0	0	0	0	0	0		0
24	Surety	0	0	0	0	0	0		0
26	Burglary and theft	0	0	0	0	0	0		0
27	Boiler and machinery	0	0	0	0	0	0		0
28	Credit	0	0	0	0	0	0		0
29	International	0	0	0	0	0	0		0
30A	Reinsurance - nonproportional assumed property		1,200,184	0	0	0	1,200,184	0.45	540,083
30B	Reinsurance - nonproportional assumed liability		1,890,810	0	0	0	1,890,810	0.45	850,865
30C	Reinsurance - nonproportional assumed financial lines		0	0	0	0	0		0
31	Aggregate write-ins for other lines of business	0	0	0	0	0	0		0
32	Totals	3,443,888,563	29,049,295	46,296,666	15,634,462	88,404,190	3,415,195,872		583,769,761

References

[1] American Academy of Actuaries, "Actuarial Issues Related to Catastrophe Reserves," presented by Wayne Fisher, July 12,1999.

[2] American Academy of Actuaries, letter to Elise Liebers, July 2, 1999.

[3] National Association of Insurance Commissioners. NAIC Proceedings 1995 4th Quarter. Kansas City, MO. page 925.

[4] NAIC Voluntary Tax-Deferred Pre-Event Catastrophe Reserve Proposal, Appendix B: Proposal – Summary and Definitions.

[5] NAIC Voluntary Tax-Deferred Pre-Event Catastrophe Reserve Proposal, Appendix E: Statement of Statutory Accounting Principles.

[6] Representative Foley, 107th Congress, H.R. 785 "Policyholder Disaster Protection Act of 2001." February 28, 2001. House of Representatives, January 21, 2001.

Annotated Bibliography

American Academy of Actuaries, "Actuarial Issues Related to Catastrophe Reserves," presented by Wayne Fisher. Presentation to Subcommittee on Natural Disaster Insurance Legislation, National Conference of Insurance Legislators. July 12, 2001.

A brief background on catastrophe risk management is given, along with some comments on actuarial expertise relevant to such a reserve. The voluntary nature of the reserve, regional exposure to catastrophic events and return periods are also touched on.

American Academy of Actuaries. Actuarial Update. "Academy Report Card, Casualty Practice Council." December 2001.

The Newsmonthly of the American Academy of Actuaries can also be found at www.actuary.org/

American Academy of Actuaries, Catastrophe Management Work Group, "Catastrophe Exposures and Insurance Industry Catastrophe Management Practices," June 10, 2001

The paper addresses how property and casualty insurers manage catastrophe risks. A number of observations are made, including a 5-step description of how catastrophic risk can be managed. In this paper, a pre-event reserve is treated as "ring fenced" capital.

American Academy of Actuaries, letter to Bill Archer commenting on the Policyholder Disaster Protection Act of 1999. September 20, 2000.

The Academy offers comment on the determination of the proposed fund cap.

American Academy of Actuaries, letter to Elise Liebers commenting on the Probably Maximum Loss Cap Proposal. July 2, 1999.

The Academy offers comments on three aspects of the Proposal: 1. Can an individual insurer's Net Probably Maximum Loss Exposure be determined with reasonable accuracy? 2. Are actuaries professionally qualified to render an Opinion on the Net Probable Maximum Loss Exposure? 3. What issues or parameters do the NAIC need to specify to ensure consistency in determining PML Caps?

American Academy of Actuaries, letter to Kevin McCarty commenting on actuarial issues in the NAIC Proposal. February 5, 2001.

The Academy reiterates its comments on relevant aspects of the voluntary proposal.

American Academy of Actuaries, "Preparing for Catastrophe: A Better Way?" Actuarial Update, November, 2000.

The Academy points out the desirability of having a financial responsibility framework and of including workers' compensation in the reserve design.

Davidson, Jr., Ross J. "Tax-Deductible, Pre-Event Catastrophe Reserves." Journal of Insurance Regulation. Winter, 1996: 175-190.

Mr. Davidson covers the status of the NAIC proposal at the time of the article. He also provides background information on the desirability of such a mechanism.

Eley, David. "Creating Catastrophe Reserves: A Balancing Act." Journal of Insurance Regulation. Winter, 1996: 191-193.

Mr. Eley's paper is a companion to the one by Mr. Davidson. It provides another perspective on Mr. Davidson's report.

Musulin, Rade, "Would a Federal Role in Disaster Protection Be a Catastrophe?" Contingencies, November/December 2000.

Mr. Musulin gives his perspective on the weaknesses and strengths of bills before the U.S. Congress at that time. HR2749 is the precursor to HR785.

National Association of Insurance Commissioners. NAIC Proceedings, 1995 – 2001 inclusive. Kansas City, MO.

The NAIC Proceedings provide minutes from its meetings. The interested reader is directed to the minutes of the Property and Casualty (C) Committee during the times listed above. The Catastrophe Reserve Subgroup and the Catastrophe Insurance Working Group also have minutes that may give additional detail.

Representative Foley. 107th Congress. H.R. 785 "Policyholder Disaster Protection Act of 2001." 2/28/2001. House of Representatives. 1/21/2002. http://www.theorator.com/bills107/hr785.html/

The text of the bill as it was introduced to the House Ways and Means Committee can be found at the above website as well as in the Congressional Record.

Rep. Foley, Mark. HR785 "Bill Summary and Status for the 107th Congress." 1/21/2002. http://thomas.loc.gov/cgi~bin/bdquery/z?d107:h.r.00785:/

We have found this website to be a useful tool in checking the status of various bills.

United States Government. Internal Revenue Code of 1986. Title 26, Subtitle A, Chapter 1, Subchapter L, Part II, Sections 831 and 832.

Other sections of the Internal Revenue Code are referenced in these Sections; however, we did not analyze them. The interested reader may wish to do so.

Asbestos and Environmental Reserves Increases and Shareholder Wealth

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Asbestos and Environmental Reserves Increases and Shareholder Wealth

Abstract

Between 1992 and 2000, significant reserves increase announcements were made by several major property/liability insurers. These reserves increases were for the purpose of funding expected asbestos and environmental liability. Although most analysts agree that U.S. insurers are underreserved for asbestos and environmental liability, how the market reacts to an insurer's announcement of an increase in these reserves has not been analyzed. An insurer that is significantly underreserved is likely to be viewed by the market as lacking financial stability for the long term. However, when a company increases its reserves there is a charge to income and a reduction in capital. If surplus is diminished sufficiently as a result of the increased reserving, regulatory attention and eroding shareholder and market confidence could result as well. The goal of this study is twofold. First, by calculating the sample insurers' cumulative abnormal returns surrounding the largest asbestos and environmental reserves increase announcements made between 1992 and 2000, the study estimates and documents the market's reaction to these reserves increase announcements. Second, by considering the market reaction for both announcing and non-announcing firms, the study seeks to evaluate the reasons for this reaction. Using market data for both announcing and non-announcing insurers with potential environmental exposure provides a useful paradigm for identifying the most likely explanation for the market's reaction to the announcements of environmental reserves increases.

When looking at reserves increase announcements from 1992 to 1995, we find that most insurers announcing large increases in asbestos and environmental reserves experience a significant reduction in stock price in the days surrounding the announcement. During the period 1996 to 2000, a period during which additional information disclosures of A & E payments and reserves were required of insurers, many of the announcing insurers saw little impact on their stock prices. We find some evidence that the additional required accounting disclosures provided valuable valuation information to the market. We also find that with the exception of the largest announcement, the stock price reaction is isolated to the announcing firm, indicating that the announcement by one firm does not impact the market's assessment of the entire industry.

Asbestos and Environmental Reserves Increases and Shareholder Wealth

Introduction

During 1992 and 1993, four insurers (ITT Hartford, Aetna, CNA and Travelers) announced significant reserves increases for the purpose of funding expected asbestos and environmental (A&E) liability. In addition, over a period of four months during 1995, four other reserves increase announcements were made by Swiss Re America, Fireman's Fund, Aetna, and Cigna. Also, during this same period, CNA acquired Continental Corporation and Zurich Insurance Group acquired Home Holdings, both acquisitions prompted at least in part by A & E liability problems of the targeted firms. Alan Levin, managing director of Standard and Poor's described the events of 1995 collectively as "perhaps the most significant event in the property/casualty industry in decades."[1] Later in 1995, Nationwide followed with the announcement of an increase in its A & E liability reserves as well.[2]

Although most analysts agree that U.S. insurers are underreserved for A & E liability,[3] how the market reacts to an insurer's announcement of an increase in A & E reserves remains unclear. An insurer that is significantly underreserved is likely to be viewed by the market as lacking financial stability for the long term. However, when a company increases its A & E liability reserves there is a charge to income and a reduction in reported capital. If surplus is diminished

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[1] Aetna's reserving turns up pressure, *Business Insurance*, July 17, 1995, p. 1.

[2] An extensive search for the reserves increase announcements made through 1995 has been conducted. For the period 1996 through 2000 a preliminary investigation into A & E reserves announcements suggests that there were eighteen significant announcements made during this period that also are relevant and will be included in the study.

[3] In 1995, Standard and Poor's (S&P) and Tillinghast estimated that U.S. insurers' environmental liability - including amounts already paid or reserved - was somewhere between $40 and $60 billion. S&P also estimated that the total amount reserved by U.S. insurers for this exposure was around $12 billion (Lenckus, 1995). A.M. Best analysts later reported in 1998 that net asbestos and environmental reserves were deficient by approximately $41 billion (Sclafane, 1998). While by 2000, Best has not had an opportunity to revisit ultimate loss estimates since 1997, they state that "intuitively, [they] do not think insurers are that close to being fully funded [for asbestos and environmental liability]" (Sclafane, 2000).

sufficiently as a result of the reserving, regulatory attention and eroding shareholder and market confidence could result as well. Yet, Sean Mooney, former senior vice president at the Insurance Information Institute, stated that "the reduction in capital [as a result of increasing environmental reserves] can be viewed as positive as it removes some of the excess 'paper' capital from the industry, and thus can lead to firmer pricing."[4]

The goal of this study is twofold. First, the study estimates and documents the market's reaction to the reserves increase announcements made between 1992 and 2000.[5] The announcement of a reserves increase could have one of three effects on the market price of insurers with A & E liability exposure: 1) a statistically significant positive effect; 2) a statistically significant negative effect; or 3) no significant effect. Each outcome contains specific information about the market's ability to detect understated reserves and the value assigned to changes in loss reserves. Although some anecdotal evidence currently exists, the market reaction to changes in A & E reserves has not been statistically assessed. Second, by considering the market reaction for both announcing and non-announcing firms, the study seeks to evaluate the reasons for this reaction. Using market data for both announcing and non-announcing insurers with potential A & E exposure provides a useful paradigm for identifying the most likely explanation for the reaction to the announcements of A & E reserves increases. This provides a unique opportunity to evaluate the information effect associated with these dramatic announcements.

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[4] Insurer Glasses Are Both Half Full and Half Empty, *National Underwriter (Property/Casualty Edition)*, October 24, 1994, p. 43.

[5] Market price data are not available for mutual insurers. However, announcements made by mutual insurers will be used in the portion of the analysis where the price reaction of non-announcing firms is evaluated.

The impact of A & E reserves increases on the market value of insurers has become an increasingly important issue for regulators with the introduction of Footnote 24 by the NAIC (see Simpson, 1996)[6]. Starting with 1995 filings (which were made public in 1996), insurers are required to provide a five-year history of environmental reserves and claim payments. Given that our study includes announcements made over the period 1992 to 2000, the results will allow us to determine if the information provided as a result of Footnote 24 has changed the way in which the market interprets the reserves announcements made by insurers. This enables us to construct a meaningful test of the value of this additional information disclosure that has been mandated by the regulators.

The remainder of the study is divided into five sections. The next section of the paper provides a brief review of the prior research related to this study. This is followed by discussions related to the data, as well as the methodology and hypotheses used in the paper. The results of the paper are presented and the conclusions and implications of the study are outlined in the final two sections.

Prior Literature

Event study methodologies similar to the one used in this paper have been used to assess the overall impact of a variety of issues on shareholder wealth. Examples of these issues include: regulatory changes (Chen and D'Arcy, 1986; Moore and Schmit, 1987; and Horton and Macve, 1998), changes in business strategies (VanDerhei, 1987; Alahegbe, Borde, and Madura, 1993; and McNamara, 1997), as well as the reporting of increases in liabilities or large losses (Sprecher and Pertel, 1983; Davidson, Chandy, and Cross, 1987; Baginski, Carbeth, and Oleega, 1991; Lamb,

[6] Although when the new reporting requirements were introduced the A&E reserves exhibit was referred to as Footnote 24, subsequent additions have resulted in the actual footnote number varying from year to year in the annual statement blank. Our reference here to Footnote 24 is to the NAIC's requirement that the reporting insurer provide

1995; and Cagle, 1996). This framework allows us to determine whether the announcement contains new, valuable information for the shareholders. If the announcement conveys new information that changes investors' views of the firm's value, then there will be a significant change in the stock price around the announcement. If investors had already built the information into their assessment of the firm's value, or if it will not have a meaningful impact on the firm's value, then there will not be a significant change in the stock price. As stated earlier, industry analysts have publicly stated that insurers are viewed as being dramatically underreserved in the area of A & E liability (Sclafane, 1998; Sclafane, 2000).[7] Based on this fact, it is questionable as to whether new information will be conveyed in these announcements. Event study methodologies also create a means to quantify how the market perceives the announcement. As mentioned earlier, investors could view the announcement of the increase in A & E reserves as an event that either increases or decreases the firm's value.

In addition to assessing the impact on the announcing firm, other authors have used event study methodologies to assess the impact of announcements on other firms in the industry. For example, Fenn and Cole (1994) investigate the impact that the announcements related to the investment problems at First Executive and Travelers had on the stock prices of other life insurers.

The decision to manage loss reserves in an effort to achieve financial goals also has been documented in several studies. For example, Grace (1990) and Gaver and Paterson (1999) find that insurers have incentives to overstate loss reserves in an effort to reduce tax liabilities. On the other

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information on its five-year historical pattern of payments and reserves for environmental exposures.

[7] In a 2001 study, A. M. Best estimates property-liability insurers will ultimately incur more than $121 billion in net A&E losses. It projects that the industry's unfunded asbestos position is $33 billion and environmental exposures are underfunded by $24 billion.

hand, financially troubled firms often understate loss reserves in an effort to reduce the level of regulatory scrutiny (see Petroni, 1992 and Gaver and Paterson, 1999). Incentives to smooth earnings through managing loss reserves are documented by Weiss (1985) and Grace (1990). Additionally, Nelson (2000) comments that insurers implicitly discount loss reserves to reflect the time value of money. In this case, insurers making large changes in A & E loss reserves face these same financial implications from their adjustments. Given insurance companies' incentives to manipulate loss reserves to achieve a variety of financial goals, there is some question as to the degree of credibility that a reserves increase announcement will have with investors. Christensen, Hoyt, and Paterson (1999) comment that the credibility placed on insurers' earnings announcements is tempered by the level of ability and the incentives managers have to manage earnings.

Also, prior researchers have argued that due to high monitoring costs investors holding the equity of financial firms (i.e., banks and insurance companies) are "rationally uninformed" about the quality of their assets and liabilities. Polonchek and Miller (1996) empirically demonstrate that the level of these information asymmetries is even higher for insurance companies than for commercial banks. Fenn and Cole (1994) and Avila and Eastman (1995) provide additional evidence that investors are relatively uninformed regarding insurer asset quality and, hence, new information is likely to have valuation relevance. For property-liability insurers the heterogeneity and complexity of the risk assumed, as well as the considerable managerial discretion available in setting reserves, arguably contributes to even greater information asymmetries with respect to the firm's liabilities. These factors make the insurance industry an especially interesting environment in which to evaluate the relevance of public information releases such as reserves increases. We anticipate increased relevance relative to firms in non-financial industries and would expect reserves adjustment announcements during periods of increased uncertainty to be especially relevant to the valuation of security prices.

Data

For our analysis we calculate the sample insurers' cumulative abnormal returns surrounding the A & E reserves increase announcements made between 1992 and 2000. The sample of insurers includes those firms that in 1985 were writing "other liability" insurance, which includes environmental liability and comprehensive general liability (CGL) policies.[8] This information is available on the *National Association of Insurance Commissioners (NAIC)* data tapes. This group of insurers is compared to a listing of insurers whose stock is currently traded on one of the open exchanges. Then the list is compared with information from *Best's Review* regarding the "50 Largest Writers of Other Liability Insurance" for the years 1985, 1975 and 1967. Additionally, the list was compared to the set of insurers with A & E exposures that were analyzed in a study by Standard & Poor's (1995). The final sample of insurers includes 25 publicly-traded insurers that had potential exposure to A & E liability. We believe that insurers entering the A & E liability lines of insurance after 1985 are not likely to announce significant revisions in their A & E loss reserves during the 1990's. This expectation is based on the fact that pollution exclusions substantially limited A & E exposures arising from policies written after 1985.

[8] 1985 was chosen since most commercial liability policies were modified to contain so-called "absolute" pollution exclusions after 1985. The name of the CGL was changed effective in 1986 from "comprehensive" general liability to the "commercial" general liability policy.

The environmental liability exposure information (volume of "other liability" insurance written) is obtained from the 1985 *National Associations of Insurance Companies (NAIC) Database*. Data regarding the 1992 through 2000 event dates and other announcements related to the sample insurers have been identified through a *Lexis/Nexis* search of the *Wall Street Journal, Business Insurance* and the *National Underwriter (Property/Casualty Edition).* Finally, preliminary stock price data have been collected from the *Center for Research in Security Prices (CRSP)* data tapes.

Table 1 provides a complete listing of firms included in the sample and Table 2 provides a listing of the announcement dates and the amount of the reserves changes for the announcing firms in our sample. In our study we identified 16 announcements made by 12 firms during the years 1992 through 2000. The reserves increases ranged from $134 million to $1.5 billion. Of the total number of firms in the sample, 40 percent (10 out of 25 – the other two announcing firms are not publicly traded) announced an increase in A & E reserves during the time period.

Methodology

In the empirical analysis, firms are designated as announcing firms for event dates for which they announced a change in A & E loss reserves. All other firms are designated as non-announcing firms for that event data. In order to determine if an announcement had a significant impact on the shareholders wealth, the following methodology is utilized.

First, the expected returns for each insurer are estimated by fitting the market model given in equation (1) to the insurer's historical returns.

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \quad t = 1, \ldots, T, \qquad (1)$$

where

R_{it}	=	return on shares of insurer i at time t, $(Price_{t+1} - Price_t + Dividend)/Price_t$,
R_{mt}	=	the CRSP equally-weighted market return at time t,
α_i	=	a coefficient representing the return of insurer i that is independent of the market,
β_i	=	a constant representing the market sensitivity of insurer i,
T	=	number of time periods,
ε_{it}	=	an error term.

We estimate all market model parameters using OLS regression analysis over a one-year estimation period up to day –5 (e.g., five days before the announcement), relative to the reserves adjustment announcement date and employ the CRSP equally weighted market index in market model regressions. This is done to develop an estimate of what the stock's return relative to the market would have been in absence of a major event.

Once the expected return is estimated, abnormal returns for each insurer are calculated by taking the difference between the insurer's actual returns and its expected returns, as shown in equation (2).

$$AR_{it} = R_{it} - (\alpha_i + \beta_i R_{mt}) \qquad (2)$$

Cumulative abnormal returns for each insurer are computed by summing the abnormal returns over an event window from t_1 to t_2, as shown in equation (3).

$$CAR_i(t_1,t_2) = \sum_{t=t_1}^{t_2} AR_{it} \qquad (3)$$

The cumulative abnormal returns for each firm are calculated for the two-day period from the day prior to the event date through the day of the event (days –1 and 0). The procedure is followed for both the announcing firm and all of the non-announcing firms. T-tests (and non-parametric tests) are conducted to determine if the cumulative abnormal returns for each firm are statistically different from zero. Significant changes in the firm's cumulative abnormal returns indicate that the market has revised its estimation of firm value based on the announcement.

Hypotheses Development

As stated previously, the announcement of an increase in reserves could have any one of the following effects on the market price of an insurer with A & E liability exposure: 1) a statistically significant positive effect; 2) a statistically significant negative effect; or 3) no significant effect. Each outcome provides important information concerning the market's ability to detect the misstatement of loss reserves prior to the announcement, as well as the impact that the announcement has on firm value.

Impact on Shareholder Wealth for the Announcing Firms

Two hypotheses support a positive stock price reaction for an announcing firm. First, the market may reflect a prior overestimation of the expected A & E liability of an insurer and the reserves increase announcement indicates a lower expected assessment of liability by the insurer. Second, the market already has adjusted for the extent of potential environmental liability, but it expects the increase in reserves to result in a decrease in expected taxes and a resulting increase in firm value.[9]

The hypothesis supporting a negative stock price reaction of the announcing firm states that the market underestimated the A & E liability of the insurer and the announcement verifies a higher expected assessment of liability by the insurer. No significant stock price reaction suggests that the

[9] Merritt Insurance Report (Jan 8, 1996, p. 12) states that "the IRS will announce its intent to investigate several insurers concerning "questionable" A & E loss reserve increases" and "will focus on whether or not these reserve increases were tax deferral or avoidance mechanisms."

market already has properly assessed the insurer's environmental liability and the insurer is simply recognizing a liability that was already fully discounted in the market price.

Impact on Shareholder Wealth for Non-announcing Firms

Interestingly, the effect of one firm's reserves adjustment announcement on the stock prices of other firms is likely to provide the most insight into the explanation for a stock price movement of the announcing firm. A positive or negative stock price movement around the event date of a firm that is not increasing its reserves suggests that the market misestimated the A & E liability for the entire industry and is adjusting stock prices of all insurers exposed to A & E liability to account for this misestimation. An increase of the stock price of the announcing firm coupled with no significant stock price movement for the non-announcing firms supports the previously stated tax deferral hypothesis. A decrease of the stock price of the announcing firm coupled with no significant stock price movement for the non-announcing firms would suggest that the announcement is providing new information to investors regarding the announcing firm, but is not affecting the market's overall assessment of A & E liability exposures for the industry.

The likelihood of a statistically significant price reaction, positive or negative, will be greater the more uncertainty investors face. In those situations additional information released to the market will be more likely to affect the market's assessment of firm values.

Table 3 provides a summary of the hypotheses related to the potential impact of the announcement of increases in A & E liability reserves on the market price of announcing and non-announcing firms.

Results

Due to the possible impact of the NAIC's increased disclosure requirements (Footnote 24), we review our results over the two periods 1992 to 1995 and 1996 to 2000. The NAIC's disclosure requirements were effective for 1995 annual statements which would have been available in early 1996. Cumulative abnormal returns for the announcing insurers are presented in Table 4. Four of the six announcements in 1992 to 1995 resulted in negative and significant CARs for the announcing insurers. These results suggest that the market determined that new and negative information on the announcing firms was introduced in these announcements. The largest of these CARs was associated with the announcement by CNA which represented the largest reserves increase announcement during the period of our study ($1.5 billion).

During the period 1996 to 2000 three of the seven announcements resulted in negative CARs for the announcing insurers. However, one announcement, by Allstate, resulted in a positive CAR. Also, it should be noted that the second announcement by Reliance Group came when the insurer was already plunging into serious financial difficulties that ultimately resulted in its insolvency. Of particular interest in these results for the 1996 to 2000 period is the fact that the CARs for the first three announcements in 1996 were not negative and significant. These are the first announcements that occur after the Footnote 24 requirements were established. Given the statistically significant negative results for the announcing firms during the 1992 to 1995 period, the statistically insignificant results for these first three post-Footnote 24 announcements suggest that the information provided as a result of Footnote 24 is sufficient for the market to adequately assess the insurer's reserves position. However, while we would require more data to reach a solid conclusion, the statistically significant negative reaction to the AIG announcement in 2000 may signal that these statutory statement disclosures are no longer sufficient for the market to formulate complete assessments of an insurer's A & E exposures.

While the shareholders of the announcing firms experienced a decrease in shareholder wealth, in most cases, the shareholders of non-announcing firms were not impacted. With the exception of the largest environmental liability announcement, non-announcing firms do not experience significant changes in stock price around the event dates. This suggests that the market viewed most of the announcements as providing information on the individual insurer and not on the market as a whole.

It is worth noting that the CNA announcement is the one that produced a statistically significant negative result for the non-announcing firms. Results associated with the CNA announcement are presented in Table 5. Not only was this announcement the largest of any of the sample announcements, it also occurred fairly early in the time period, only four months after the initial ITT Hartford announcement. Also, the CNA reserves announcement was reportedly tied to the company's decision to reach a settlement with an insured (Fibreboard) that was facing "tens of thousands of [asbestos-related] claims" (see Greenwald, 1993). The decision to reach a settlement with Fibreboard was prompted by a San Francisco Superior Court judge's ruling that each claim constituted a separate occurrence and that Chubb, the parent of another of Fibreboard's insurers, owed Fibreboard the duty to defend and indemnify them for each claim. Also contributing to the decision to settle was the "trend toward mass tort actions and consolidated litigation, which do not give the defendant the opportunity to adequately defend themselves because they are grouped with others" (Greenwald, 1993). It appears that it was not only the amount of the increase that prompted the statistically significant negative result for the other insurers, but also the circumstances that reportedly led to the announcement.

We also employ cross-sectional regressions in which we measure the incremental effects of various characteristics of the firm (size, volume of A & E exposed insurance written, etc.) and of the announcements (size of adjustment, prior adjustments, etc.) on the magnitude of beta-adjusted returns. In this analysis we follow the approach of several prior event studies.[10] As an example of the construction of these cross-sectional regressions, one variable that is included on the right-hand side of the regressions of all announcements is a variable reflecting the A & E liability exposure of each of the sample insurers. Each insurer's exposure to A & E liability is measured by the percent of "other liability" insurance to total insurance written in 1985. This information is contained in Part 2 of the Underwriting and Investment Exhibit of the statutory annual statement. The insurers' cumulative abnormal returns are then regressed against the degree of environmental liability exposure in 1985 to determine the effect of the reserves increase announcements on the stock price of the firm. This was done in a multivariate framework so that a number of relevant factors can be assessed simultaneously. Thus far, the results of this additional analysis have been inconclusive.

[10] See, for example, in an insurance context Baginski, Corbett, and Ortega (1991) and Marlett and Pacini (1999); and in a general context Schwert (1981) and Binder (1985).

Conclusions and Implications

While many industry experts have stated that the insurance industry is underreserved with respect to asbestos and environmental liability exposures, it appears that the market has not fully discounted firm value to reflect the potential understatement of liabilities. For the period 1992 to 1995, the majority of announcements of increased asbestos and environmental reserves were associated with a decrease in firm value for the announcing firm.

In reaction to concerns over the magnitude of A & E exposures and the adequacy of insurers' reserves for these exposures, the NAIC required with the filing of 1995 annual statement blanks that insurers provide additional information on A & E claims and reserves (the five-year exhibit that was initially referred to as Footnote 24). We find some empirical evidence that is consistent with the notion that the information made public by the Footnote 24 requirements is meaningful to the market.

However, with one notable exception, the A & E reserves increases did not result in a change in the market's assessment of firm values for the non-announcing insurers. Our results are consistent with the idea that public information releases will be most influential on the market's assessment of firm values when financial statement disclosures are inadequate (lack of transparency in accounting information) and when uncertainty is greatest.

Our findings have important implications for valuation actuaries. First, they show that the market is not consistently able to adequately detect large misstatements of loss reserves. This suggests that accurate reserves statements by actuaries are critical and that adjustments to erroneous ones do have an effect on the market's assessment of firm value. Second, faced with the potential reduction in firm value and possible increase in regulatory costs, insurers have the incentive to understate or further delay the announcement of increased asbestos and environmental liability reserves. In reviewing reserves actuaries must be cognizant of this potential conflict in incentives.

After several years of relative quiet in the A & E area, 2001 saw a dramatic jump in the number of reserves increase announcements by insurers and concerns over the magnitude of insurers' A & E exposures (American Academy of Actuaries, 2001; Banham, 2001; Lemke, 2002). Due to data limitations we are not yet able to evaluate the 2001 announcements. Given the increased attention to and uncertainty associated with A & E liabilities, at some point it will be especially informative to extend the analysis in our study to the announcements made during 2001.

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Table 1
Sample Firms
1992-2000

Announcing Firms	Non-announcing Firms
Aetna Allstate Insurance Group American International Group American Reinsurance Cigna CNA Fireman's Fund (Allianz) ITT Hartford Reliance Group Travelers	American Financial Group American General Group Berkshire Hathaway Chubb Group Fremont Insurance Group General Reinsurance Group Home Insurance Group (Zurich Re) Kemper National Group Lincoln National Group Ohio Casualty Group Old Republic Group Orion Group St. Paul TIG Insurance Group United States Fidelity & Guaranty

Table 2
Event Dates for Years 1992 – 2000

Event	Date	Amount of Increase
ITT Hartford	10/1/92	$582M
Aetna	2/3/93	$180M
CNA	2/8/93	$1.5B
Travelers	10/14/93	$325M
Swiss Re America	4/12/95	$700M
Fireman's Fund	6/23/95	$800M
Aetna	7/13/95	$750M
Cigna	10/2/95	$1.2B
Nationwide	12/12/95	$1.1B
American Re	1/30/96	$587M
Aetna	2/8/96	amount unknown
Reliance Group	6/27/96	$134M
Allstate	10/9/96	$245M
ITT Hartford	10/21/96	$543M
Reliance Group	6/15/99	amount unknown
AIG	10/27/00	amount unknown

Table 3
Summary of Hypotheses

Impact on Stock Price	Announcing Firms	Non-announcing Firms
Positive	-The market over estimated the expected A&E liability for the insurer -The market had adjusted for the expected A&E liability but it expects that the announcement will decrease taxes	-The market misestimated the A&E liability for the entire industry
Negative	-The market underestimated the A&E liability	-The market misestimated the A&E liability for the entire industry
No Effect	-The market has already properly assessed the insurers A&E liability	(If found with a positive move in stock price for the announcing firms this is supportive of the tax hypothesis)

Table 4
Cumulative Abnormal Returns for Announcing Insurers
(Insurer's own announcement)

Announcing Insurer	CAR (-1,0)	Event Date
Aetna	-.0285**	2/3/93
CNA	-.0352**	2/8/93
Travelers	.0012	10/14/93
Fireman's Fund (Allianz)	-.0056	6/23/95
Aetna	-.0254**	7/13/95
CIGNA	-.0108*	10/2/95
American Re	.0097	1/30/96
Aetna	-.0074	2/8/96
Reliance Group	.0017	6/27/96
Allstate	.0224**	10/9/96
ITT Hartford	-.0118*	10/21/96
Reliance Group	-.2222**	6/15/99
AIG	-.0532**	10/27/00

* CARs are significant at p=.05

** CARs are significant at p=.01

Table 5
Cumulative Abnormal Returns for the CNA Announcement

CNA Announcement (2/8/93)	
Insurer	**CAR (-1,0)**
Aetna	-.0109*
CNA	-.0352**
Travelers	-.0331**
CIGNA	-.0253**
Average CARs for other insurers	-.0111*

* CARs for these insurers are significant at p=.05
** CARs for these insurers are significant at p=.01

For the other 15 announcements the CARs for the announcing and non-announcing insurers are not statistically significant

Materiality and Statements of Actuarial Opinion

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Materiality and Statements of Actuarial Opinion

ABSTRACT

How should practicing actuaries consider materiality in the context of formal Statements of Actuarial Opinion? The specific issue of materiality has come to the forefront for casualty actuaries recently with the requirements of Actuarial Standard of Practice (ASOP) 36.

The Actuarial Standards Board Casualty Committee's Subcommittee on Reserves was involved in drafting ASOP 36. After its third draft, the Subcommittee held a hearing on the proposed standard. There were many controversial issues expressed at the hearing, especially those involving materiality. While the Subcommittee admitted that a standard of practice on the topic of materiality itself was perhaps a good idea, the implementation of ASOP 36 went forward, despite pointed opposition by many actuaries.

This paper will address materiality from external points of view (i.e., U.S. Supreme Court, Securities and Exchange Commission, Financial Accounting Standards Board), then present findings from research on materiality standards commonly used by both the actuarial and regulatory communities. Next, we present a framework for determining materiality thresholds in the context of the Statement of Actuarial Opinion for practicing actuaries ranging from the very simple (rules of thumb) to the more complex (stochastic modeling).

This paper presumes the reader is well versed in the requirements of ASOP 36 and has a good working knowledge of the requirements for Statements of Actuarial Opinion promulgated by the National Association of Insurance Commissioners (NAIC).

Materiality and Statements of Actuarial Opinion

Introduction

Casualty actuaries have not had to deal with the issue of materiality explicitly until ASOP 36 became effective for Statements of Actuarial Opinion prepared subsequent to October 15, 2000. For many actuaries, the issue of materiality is nebulous, falling under the general banner of "actuarial judgment." While this may be technically true, it's fair to say that most practicing actuaries' sense of materiality may be very different from that of the users of the actuarial work products.

This paper is an attempt to begin a serious dialogue within the actuarial community on materiality. It is not an issue easily dismissed as being "in the eyes of the beholder." Critical issues face the actuary in making determinations of materiality and become readily apparent when discussing the results of a work product with outside third parties, such as regulators, auditors or rating agencies.

Of course, there are many ways to look at materiality. The focus of this paper will relate to materiality considerations associated with formal Statements of Actuarial Opinion, but the general discussion will have more far-ranging implications elsewhere.

According to a recent draft of a document by the American Academy of Actuaries regarding the "Actuaries' Responsibilities to Users of Their Work Products", regulators have suggested that some actuaries may be meeting the letter of regulatory requirements without satisfying their underlying intent, perhaps due to the actuary's efforts to mitigate the costs of regulatory compliance. With some of the perspectives provided in this paper, the author attempts to heighten the sensitivities of practicing actuaries as to the perspectives of the regulatory community and other users of the actuarial work products.

Where do we look for guidance in addressing materiality? There are several sources within the literature and in case law that provide perspectives important to any discussion of materiality. With such historical perspectives, we will then discuss issues unique to the property/casualty insurance market. We will supplement the general discussion with findings gleaned from a survey of regulators and feedback from practitioners subsequent to the 2000 reserve opinion season.

We then suggest a multiple-trigger threshold for determining materiality and try to put that in context, given the perspectives of several audiences to the actuarial work product. The process considers both quantitative and qualitative factors. We finish with commentary on the use of judgment by the actuary, not as a panacea, but as an affirmative obligation that should not be taken lightly. Reconciling differing views of materiality from our various audiences will perhaps be the biggest challenge for actuaries. Lastly, we provide four brief case studies with relevant commentary on the approach used for determining materiality.

This paper will focus on statutory Statements of Actuarial Opinion that relate to requirements promulgated by the NAIC. We recognize there are other Statements of Actuarial Opinion that must adhere to the professional guidelines of ASOP 36 (such as for self-insured entities or non-U.S. domiciled companies). Nevertheless, much of the discussion in this paper will be relevant for Statements of Opinion that are not specifically prepared under the auspices of the NAIC requirements.

If the comments herein provoke controversy and discussion within the actuarial community, the author will deem the paper a success.

ASOP 36 and Materiality

ASOP 36 requires the actuary to consider materiality from a variety of perspectives. The issue itself is of such importance that the Valuation, Finance and Investment Committee (VFIC) of the Casualty Actuarial Society (CAS) prepared a special document discussing materiality considerations for the practicing actuary. In particular, ASOP 36 requires materiality to be considered in at least the following ways:

1. Determining whether to issue a qualified opinion;
2. Determining the need for disclosure of significant risks of material adverse deviation;
3. Consideration of factors likely to affect the actuary's reserve analysis; and,
4. Determining the need for a number of other disclosures.

The VFIC document is attached as an Appendix to the Property and Casualty Practice Note, prepared each year by the Committee on Property and Liability Financial Reporting (COPLFR) of the American Academy of Actuaries. We recommend all readers of this paper first be familiar with the VFIC document, as it presents many of the issues in general terms.

The requirements of ASOP 36 indicate that when evaluating materiality within the context of a reserve opinion, the actuary should consider the purposes and intended uses for which the actuary prepared the Statement of Actuarial Opinion. The actuary is instructed to evaluate materiality based on professional judgment, materiality guidelines or standards applicable to the Statement of Actuarial Opinion.

When the ASB Casualty Committee' Subcommittee on Reserves discussed questions regarding preliminary drafts of ASOP 36, there was a general feeling that a separate standard of practice on materiality would probably be a good idea, but that the lack of such a standard was not critical to the use of ASOP 36 by practicing actuaries. So, we had to look elsewhere for guidance on materiality.

Search For Guidance

Where should we look for guidance on materiality? Let us start with guidance from outside the insurance market generally, then move to guidance from sources specific to the property/casualty insurance market. We will begin with pronouncements of the Financial Accounting Standards Board (FASB), the Securities and Exchange Commission (SEC) and the U.S. Supreme Court. We then follow with a discussion from the guidance from VFIC, the NAIC Financial Condition Examiners Handbook and the Accounting Practices and Procedures Manual.

According to the FASB Statement of Accounting Standard Number 5, the omission or misstatement of an item in a financial report is material if, in the light of surrounding circumstances, the magnitude of the item is such that it is probable that the judgment of a reasonable person relying upon the report would have been changed or influenced by the inclusion or correction of that item. From this author's reading of that standard, the operative phrases in determining materiality include "probable," "reasonable person" and "changed or influenced." The standard further states that management must consider both quantitative and qualitative factors in assessing an item's materiality.

According to the SEC Staff Accounting Bulletin Number 99, the exclusive reliance on certain quantitative benchmarks to assess materiality in preparing financial statements is inappropriate; misstatements are not immaterial simply because they fall beneath a numerical threshold. However, the SEC did state that it had no objection to using such rules of thumb as an ***initial step*** in assessing materiality.

The most authoritative pronouncement on the topic of materiality comes from the U.S. Supreme Court in its 1976 decision in the TSC Industries v Northway, Inc., 426 U.S. 438,449. The Court stated that an omitted fact is material if there is a substantial likelihood that its disclosure would have been viewed by the reasonable investor as having significantly altered the "total mix" of information made available. Determinations of materiality require "delicate assessments of inferences a reasonable shareholder would draw from a given set of facts and the significance of those inferences to him."

In sum, these three authoritative sources indicate that materiality must be judged:

1. Using a "reasonable person" test;

2. In both quantitative and qualitative terms;

3. Within the context of probability (the author is substituting this phrase for the Supreme Court's "substantial likelihood" phraseology); and,

4. In context of changing or significantly altering someone's judgment about a matter

At least three other sources exist providing guidance on materiality issues. Earlier, we mentioned the document prepared by the VFIC. We do not reproduce the elements of that document herein, but encourage the reader to be familiar with its content. A fifth source is the Accounting Practices and Procedures Manual (i.e., statutory accounting). In Section VI of the Manual, it states that materiality judgments are primarily quantitative in nature. The question of materiality is posed as follows: Is this item large enough for users of the information to be influenced by it? Generally, an item is deemed material if the magnitude of the item is such that it is probable that the judgment of a reasonable person relying upon the statutory financial statement would have been changed or influenced by the inclusion or correction of the item.

A sixth source of guidance on the issue of materiality is the Financial Examiners Handbook prepared by the National Association of Insurance Commissioners (NAIC). Section 4 of the Handbook is titled "Understanding Materiality and Risk." More details of the regulatory perspective on materiality is provided later in this paper.

For practicing actuaries, we must consider the viewpoints of the users of our work product in assessing materiality, which can include a broad and diverse audience. Section 3.4 of ASOP 36 says very clearly that the actuary should consider the purposes and intended uses for which the actuary prepared the Statement of Actuarial Opinion when evaluating materiality. Those "intended" users likely include regulators, company management (including the Board of Directors), the company auditors, and perhaps even rating agencies.

Unfortunately, the Statement of Actuarial Opinion is a publicly available document. Hence, it may be used by a number of other "unintended" parties, such as reinsurers, financial analysts and investors (both current and potential). It isn't beyond the realm of comprehension that other third parties, such as policyholders and claimants, may also be interested in such public documents.

Since the Statement of Actuarial Opinion is a public document, the opining actuary may face a real dilemma. The materiality standard to which the actuary must abide relative to ASOP 36 relates to those "intended" users of the work product. However, another (perhaps very different) materiality standard may apply in those instances where an "unintended" user is reviewing that work product. In the former case, the actuary may take some comfort in Precept 8 of the American Academy of Actuaries Code of Professional Conduct, Annotation 8-1 which says "The Actuary should recognize the risks of misquotation, misinterpretation, or other misuse of the Actuarial Communication and should therefore take reasonable steps to ... include ... limitations on the distribution and utilization of the Actuarial Communication." As a practical matter though, the actuary usually includes a phrase such as "the statement of opinion is solely for the use of, and only to be relied upon by, the Company and the various state departments with which it files its Annual Statement." Despite such language, other audiences will be reviewing the document. It is virtually impossible for the actuary to limit the distribution and utilization of such a public document.

Another audience that must be considered by actuaries is the Actuarial Board for Counseling and Discipline (ABCD). While the ABCD does not routinely review an actuary's work product, issues of materiality may ultimately be judged by the Board if a matter involving allegations of unprofessional conduct were to arise. The ABCD will be the judge of whether the actuary's work product is in compliance with the standards of practice and a de facto judge of whether considerations about materiality meet the intent of ASOP 36.

A significant difficulty is determining which audience to consider when assessing materiality. The level of discussion and documentation required in the Statement of Actuarial Opinion may vary depending on the particular audience being considered. For example, if we presume the audience is another actuary, the level of documentation and disclosure may be less than if the user were a member of the Company's Board of Directors. This is because each potential user has a different level of knowledge about the significance of loss and loss adjustment expense reserves, and the nuances associated with evaluating the adequacy of such reserves.

Therefore, does this not imply that the actuary should consider the materiality standard for the potential audience with the least knowledge and experience with loss reserving? Or, should the actuary focus solely on the materiality standards for the primary users of the Statement of Actuarial Opinion (i.e., company management and regulators)? The author suggests that the "reasonable person" standard should apply in any case, regardless of whether the individual practitioner considers each potential user a "reasonable person."

The crux of the problem for the actuary may be stated succinctly:

How do I determine what a reasonable person will view as material?

Materiality in hindsight can be far different from what one views as material prospectively. It is hoped that the standard to which actuaries will be held will relate only to the facts and evidence available at the time of rendering the Statement of Actuarial Opinion.

Regulatory View of Materiality

One of the primary responsibilities of individual state regulators is to monitor the solvency of the companies licensed to do business in the state. As a result, regulators are a primary user of the Statement of Actuarial Opinion. Often, the Statement of Opinion is used as a tool to separate those companies that demonstrate potential financial problems from those that do not.

Financial examiners routinely conduct detailed assessments of an individual company's financial condition. In most instances, the financial examination process involves a review of the actuarial report supporting the findings in the Statement of Actuarial Opinion. Therefore, it is important for actuaries to have a good understanding of the materiality thresholds used by regulators in the process of reviewing the financial condition of companies.

According to the NAIC Financial Condition Examiners Handbook, materiality is defined as the dollar amount above which the examiner's perspective of the company's financial position will be influenced. The amount is determined at two levels during the examination's planning stage: (1) an overall level as it relates to the Annual Statement taken as a whole; (2) an individual balance sheet (Annual Statement line item) level. Risk and materiality are addressed at an overall level to help develop a strategy that will provide sufficient evidence to enable the examiner to reasonably evaluate whether the Annual Statement is materially misstated, or whether the company has a high likelihood of becoming insolvent.

Planning Materiality (PM) is the examiner's preliminary judgment of materiality made during initial planning. It is used in developing the overall scope of the examination procedures. At the examination's conclusion, the examiner evaluates whether the total effect of differences identified is material to the Annual Statement. The estimate of PM requires judgment based on the examiner's understanding of the company's operations. The examiner is instructed to consider the (1) nature of the business, (2) operating results (e.g., stable earnings, consistently near break-even, volatile results), and (3) financial position. Consideration should also be given to how close the company's surplus is to levels that would trigger regulatory action.

According to the Handbook, an appropriate starting point for PM is 1% to 5% of surplus. The actual percentage used depends on the circumstances of the examination. This author found this to be somewhat startling, as practicing actuaries have typically used a much wider materiality threshold.

Subsequent to the passage of ASOP 36 and prior to the preparation of Statements of Actuarial Opinion for year-end 2000, the author conducted an informal survey of insurance regulators inquiring as to the materiality threshold commonly used in testing the adequacy of a company's held loss and loss adjustment expenses reserves. Responses were received from sixteen individual jurisdictions, and the results were enlightening. The responses fall into four categories. Since some responses were "unofficial," the results are reproduced below without identifying specific regulators' responses:

TABLE 1

Number of Jurisdictions	Materiality Threshold
7	"It all depends"
6	1% - 5% of surplus, per the guidelines in NAIC Financial Examiners Handbook
2	"It is up to you"
1	10% of surplus

Despite receiving responses from only 16 of the 51 state regulatory authorities, we expect that responses from other jurisdictions would be similar to those indicated in Table 1. Virtually every response mentioned that the actuary must use judgment in assessing materiality and that the actuary should be guided by ASOP 36. Most responses indicated that percentages of surplus would generally be used as the first measure of determining materiality, but depending on the circumstances of the individual company involved, Risk-Based Capital (RBC) may be used instead.

One regulator indicated that "tolerable" error, the materiality for a particular account balance, is generally set at 50% of the planning materiality. Perhaps the most instructive comment came from a couple of regulators that encouraged the practicing actuary to put himself in the position of the regulator. The viewpoint of some regulators is that the Statement of Actuarial Opinion is intended (among other things) to assure the regulator that the Company's reserve position will be adequate for the next 12 months until a new Opinion is issued. So, those regulators feel the actuary should disclose any reason for concern that the reserves could be materially understated. In effect, those regulators want to know whether they can "set aside" the Company or whether there is a need for close monitoring during the course of the upcoming year.

Viewpoint of Practicing Actuaries

For many practicing actuaries, these survey results may be the first perspective available on the materiality threshold for the users of the Statement of Actuarial Opinion. Furthermore, it should be illuminating for many practitioners whose sense of materiality is much different from that of regulators.

In fact, subsequent to the year-end 2000 reserve opinion season, this author was privy to a discussion regarding materiality thresholds among several leading practitioners representing their firms. Again, without naming names, the results of that informal survey revealed materiality thresholds significantly different from those of the regulatory community:

TABLE 2

# Firms	Materiality Threshold - 2000
3	10% of reserves/20% of surplus
2	15% of surplus
1	15% of reserves/25% of surplus
1	1% - 5% of surplus

While each firm represented that these were the typical guidelines used for assessing materiality in the context of ASOP 36, many other factors were also considered when making a determination as to whether disclosures were required for the risk of material adverse deviation. Furthermore, the justification many advanced for the recommended thresholds related to the thresholds used for the Insurance Regulatory Information System (IRIS) Test ratios 10-12 (one-year development to surplus at 20%, two-year development to surplus at 20% and current reserve deficiency to surplus at 25%).

It is important to note that the results of the regulatory survey were not commonly known at the time the year-end 2000 Statements of Actuarial Opinions were issued. From anecdotal evidence, this author can state that the materiality thresholds used by many practitioners for year-end 2001 Statements of Actuarial Opinion were much more narrow than those used previously.

The author suggests that materiality be considered using a multiple-trigger approach. The first trigger would include quantifying the difference between the Company's held reserves and the high end of the actuary's reasonable range of reserves. This approach may cause practitioners who formerly relied strictly upon "best estimates" for rendering reserve opinions to consider supplementing such an analysis with a reasonable range of reserves. While development of such a range is generally not required, such a range provides a direct application for assessing materiality. For example, an actuary could measure the difference between a Company's held reserves with those indicated by the high end of the actuary's reasonable range. If that difference is deemed material, a disclosure of the risk of material adverse deviation may be considered.

A second trigger would involve a determination whether the actuary's range of indicated reserves may cause exceptional values for IRIS Tests 10-12.

A third consideration could be whether the actuary's indicated reserves may trigger an RBC value at or below Company Action Level RBC. For a Company in precarious financial condition, virtually every potential risk factor facing the Company may be deemed material in the context of potential adverse deviation. The actuary must consider the materiality threshold in terms of the unique characteristics of the particular company. If the company being examined wrote long tail liability lines of business, it may be highly leveraged in terms of reserves (i.e., a high reserves to surplus ratio), but have an acceptable premium-to-surplus ratio. In such a situation, perhaps the materiality standard used by the actuary shouldn't relate strictly to surplus, but to reserves or some combination of reserves and surplus.

We recognize that the computations involving RBC are not trivial. Many considerations are involved in any computation of a change in surplus. The impact of reserve adjustments may also involve other balance sheet items, such as contingent commissions, retrospective premiums due, taxes and others.

Often, discussion of materiality revolves around the adjectives "remote," "reasonably possible" and "probable." According to Statement of Statutory Accounting Principles (SSAP) 5, these terms are defined as follows:

TABLE 3

Threshold	**SSAP #5 Definition**
Remote	The chance of the future event occurring is slight
Reasonably Possible	The chance of the future event occurring is more than remote but less than probable
Probable	The future event is likely to occur

Items with only a remote chance of happening will generally be viewed as immaterial by the actuary. Matters that are reasonably possible fall into a gray area depending on circumstances. Matters that are probable should be considered material. The author suggests these thresholds for discussion purposes within the actuarial community.

An Approach to Evaluating Materiality

What follows is a suggested approach for evaluating materiality. In all cases, we begin with a simple rule of thumb as the starting point. Then, we examine the relevant financial facts for each Company, postulate the current reserve position relative to actuarially indicated reserves and discuss the various considerations an actuary should make in assessing materiality. Lastly, we focus on a more quantitative methodology for assessing materiality for one specific example.

The remainder of this paper will present four case studies providing relevant commentary on issues involving materiality for illustrative purposes. One can imagine many other scenarios; however, the purpose of this paper is to generate future discussion, not to provide an exhaustive discussion of any and all materiality issues.

In any loss reserving exercise, materiality should be judged based on the totality of the facts and circumstances facing the Company. We will assume several facts in each instance, including the supposition that the actuary has completed a thorough analysis of required reserves, has interviewed

Company management regarding all the operational characteristics of the Company that may impact reserves, is familiar with external factors that may be relevant to reserve adequacy and that the actuary's opinion on reserve adequacy is a reasonable representation of true required reserves.

Furthermore, we assume the actuary will have computed a reasonable range of reserves, and that this range is also an accurate representation of the true underlying variability in required reserves. This paper is not intended as a treatise on the need for computing a range of required reserves, nor is it designed to provide guidance for the actuary on how to compute such a range. On the contrary, it is up to the individual actuary's judgment to develop a reasonable range of reserves and/or a best estimate of required reserves, depending on the needs of Company management and the circumstances peculiar to each situation.

We also assume that there are no disputes between the Company and its reinsurers regarding collectibility. This issue can be a very important materiality consideration if the Company in question has significant amounts of reinsurance recoverables relative to surplus. And, we assume that unearned premium reserves are adequate to fund the future run-off of liabilities and expenses for in-force business.

The four case studies presented will involve (a) a mutual company licensed in all states writing personal lines coverages (b) a commercial multiple line carrier (c) a writer of lawyers professional liability in a single jurisdiction and (d) a reinsurance company.

Case Study #1

The first case study we present is for a mutual insurance company that writes private passenger automobile insurance coverages and minor amounts of other business. As outlined in Exhibit 1, some of the pertinent facts for this Company are as follows:

1. Increasing surplus each year and a declining premium-to-surplus ratio;
2. Strong financial strength indicated by A++ rating by A. M. Best & Company each of the past 5 years;
3. The ratio of reserves to surplus has remained fairly steady over 5-year period;
4. Favorable (i.e., negative) loss development in each of the past 4 years;
5. Held surplus far in excess of indicated RBC; and,
6. The Company is a member of a group of insurers

For the purpose of this first case study, we postulate that the Company's held reserves are above the midpoint of the actuarially indicated range of reasonable reserves, but within the high end of the range. Specifically, held reserves are 4% higher than the actuary's "best estimate" of required reserves. Furthermore, we assume that since it is a mutual insurance Company, it has no readily available access to the capital markets.

It should be apparent to the reader that this Company's financial picture is very strong. Downward loss development that has emerged consistently over the past several years is an indicator that future adverse loss development is unlikely. Furthermore, a 5% upward deviation in reserves would amount to less than 2% of Company surplus. While a conservative actuary might consider that the Company faces a material risk of adverse deviation, the author submits that this Company's reserve position is very solid. The risk of material adverse deviation may come from one of many potential sources.

For example, since the Company writes mostly private passenger auto coverages, it should be worried about broader issues facing all carriers, such as the uninsured/underinsured motor vehicle coverage extension from commercial vehicles to private passenger types in the state of Ohio. Or, the ultimate impact of the so-called "Broadnax" matter in West Virginia, whereby all exclusions written into the auto insurance policy were deemed unenforceable if there was no justification for such exclusions in the rating plan. Private passenger auto insurers are also concerned about the ultimate impact of recent court cases involving diminution of value; other mass tort actions are also of concern. Such potential future loss development is foreseeable, but does the actuary consider it material for the purpose of making a disclosure in the Statement of Actuarial Opinion, consistent with the requirements of ASOP 36?

For the purpose of this example, we would suggest using a materiality threshold of 10% - 15% of reserves, which is approximately 3.2% - 4.8% of surplus. Is this materiality standard too narrow, or too broad? We must consider the Company has a history of reserve redundancies, and the current held reserves also indicate a redundancy. For such a conservatively run company, perhaps a wider threshold is warranted. Individual opinions will vary.

Clearly, if the Company in question were not as adequately protected from a surplus point of view, or if its history of reserve development were different, the practicing actuary would likely consider many such conditions to be material. From a regulatory point of view, the practicing actuary should try to make a determination of disclosing relevant factors material to the Company's operations, and not to focus on such broad, all-encompassing statements regarding future loss development.

The actuary should consider potential downward loss development in the future as an offset to potential adverse development. The writers of ASOP 36 were very purposeful in focusing the disclosure requirements in the Statement of Actuarial Opinion only for material ***adverse*** deviation. But there are examples of circumstances that occur that have resulted in systematic downward loss development. The one most obvious example relates to the impact of managed care initiatives and benefits reforms to the workers compensation system in the early 1990's. The systematic reduction in prior years' loss reserves of more than 9% of premium in each of calendar years 1994 - 1998 was so significant (and unexpected by many) that it masked any adverse development. If the actuary believes held reserves are redundant, the materiality threshold for determining whether a disclosure is required by ASOP 36 becomes even broader.

In making a final decision as to the materiality threshold, the author suggests the minimum measurement point be from the current held reserves to the top end of the actuary's range of reasonable reserves. In this case, that amounts to only 1% of reserves. According to our two other thresholds, we find little chance that the IRIS Test results would change significantly due to our indicated reserves. And we find the company's current surplus levels are so high relative to RBC that the likelihood of a significant drop in surplus is remote. Hence, the overall likelihood of material adverse deviation is deemed remote and no disclosure is required per ASOP 36.

Case Study #2

The second case study presented is for a multiple line casualty stock Company that writes primarily commercial lines coverages (but no workers compensation). As outlined in Exhibit 2, some of the pertinent facts for this Company are as follows:

1. History of significant reserve deficiencies over past 4 years;
2. Volatile ratios of premiums to surplus and reserves to surplus;
3. Downgrade in A. M. Best Rating from A- to B++ in latest year;
4. Net income losses in 4 of past 5 years;
5. A significant decline in surplus of more than 31% in the latest year, resulting in a deterioration in the ratio of surplus to RBC of 3.5; and,
6. Significant reductions in total admitted assets in 1998 and 2000

For the sake of this case study, we postulate that held reserves are at the low end of the actuary's indicated range of reasonable reserves (4% below the "best estimate" of reserves). Furthermore, we assume there are no significant retrospective reserving issues associated with the indicated reserve deficiency.

We postulate that the Company's domiciliary regulator uses the 1% to 5% materiality threshold for determining materiality, consistent with the provisions in the Financial Condition Examiners Handbook. From a materiality point of view, the projected 4% deficiency in held reserves is more than 5% of held surplus. Hence, the issue of materiality may be easier to ascertain. Since we're starting with a deficiency of more than 5% of surplus and we know the regulatory threshold is between 1% and 5% of surplus, there appears to be a de facto requirement to make disclosures of risks of adverse deviation.

The fact the Company has a history of reserve deficiencies would tend to support the need for disclosures, regardless. That is, if the Company's held reserves were virtually identical to the actuary's "best estimate," disclosures would likely still be necessary given the Company's history. If, on the other hand, there was a significant reserve correction made in the prior year so that management thinks it had caught up with all prior year's deficiencies, the actuary's job is perhaps a bit more difficult. The actuarial report's findings and diagnostics regarding reserve adequacy would need to be factored into any determination of possible future adverse deviation.

Since the Company appears to have significant prior loss development problems, we stipulate the IRIS test ratios are already outside the acceptable range. Hence we know the Company is likely being given regulatory scrutiny, given that it lost more than 30% of its surplus last year. In such a case, it would appear reasonable that the Appointed Actuary would tend to be more conservative in any assessment of materiality, given the declining financial condition of the Company.

Case Study #3

The third case study we present is for a mutual insurance Company licensed in only one state, writing lawyers professional liability on a claims-made basis. Again, some of the pertinent facts for this Company (refer also to Exhibit 3) are as follows:

1. Positive growth in premiums and surplus each of past 5 years;
2. Favorable reserve development each of past 2 years;
3. Stable B++ rating by A. M. Best Company over past 5 years;
4. Per occurrence retention of $250,000 (no change over past 10 years);
5. Growth in reserves roughly in tandem with growth in surplus; and,
6. Positive net income in each of past 5 years

For the sake of this example, we assume the mutual Company books the Appointed Actuary's best estimate of required reserves, hence reserves are considered reasonable for the purpose of the Statement of Actuarial Opinion. Furthermore, we assume there are no significant reserves indicated for tail policies or for extended reporting endorsements. We also assume the types of law practices insured are small 1- to 2-person firms.

Given the nature of the business written (i.e., claims-made) and the Company's $250,000 per occurrence retention, the ultimate resolution of a single claim may be considered material. That is, the $250,000 represents 2.6% of the Company's held surplus. A potential 5% adverse deviation in reserves would amount to approximately 6% of surplus. Hence, even though the Company books the Appointed Actuary's best estimate of reserves, the potential for material adverse deviation is readily apparent. Or, is it?

During the past two years, however, the Company has realized significant reserve redundancies. Let us postulate the actuary's reasonable range of reserves is +/- 5% of the so-called "best estimate" of required reserves. Hence, the top end of the range is a 5% deviation from the best estimate, which corresponds with the materiality threshold suggested above.

Materiality is somewhat more difficult to ascertain in this case, because the Company's premium volume is fairly small, but it has grown its surplus steadily over five years. Reserves have historically been conservatively stated, but the nature of the Company's business is such that there is a potential for a surplus impairment of more than 5% if two specific claims were to exceed the Company net retention.

The second trigger (IRIS test results) doesn't appear likely to be affected, since prior year reserves have been conservatively stated. The Company would need to lose more than half its surplus before an RBC event would be triggered. When we consider the company writes only one line of business in a single jurisdiction, it doesn't have the same diversification of risk that a multiple line company would realize. Hence, the author would suggest a materiality threshold in such an instance that is somewhat more conservative than for a similar size company operating in multiple jurisdictions writing numerous lines of business.

This would argue for a materiality threshold of 5% of surplus.

Case Study #4

The fourth case study presented is for a reinsurance company, writing no direct business, but assuming more than $650 million in premium annually, mostly commercial. Pertinent facts from Exhibit 4 for this Company are as follows:

1. Reserve deficiencies in each of the past 4 years;
2. Increasing ratios of reserves to surplus;
3. A+ rating by A. M. Best in each of past 4 years;
4. Member of a larger group (multiple line casualty companies); and,
5. Premium is in "Reinsurance" category constituting 26% of the total

This reinsurance Company has reserves almost three times its surplus. The impact of this leverage is that a relatively small change in reserves may result in a material change in surplus levels. In this example, we postulate that the Company's held reserves are near the midpoint of the actuary's range of reasonable reserves. We also postulate that the Company's parent has demonstrated its commitment to add capital to the Company when necessary.

Specifics are not provided with regard to the Company's share of asbestos and environmental reserves, nor do we have disclosures as to exposure from other mass torts including terrorism. However, it's safe to say that with such a leveraged position, the actuary should disclose several risk factors facing reinsurers that could result in material adverse deviation. In this instance, the author suggests a materiality threshold of approximately 2% of reserves, which is roughly equivalent to 5% of surplus.

Quantitative Approaches to Materiality

The use of modeling to assess materiality is a natural outgrowth of loss reserving and financial risk management software. Such modeling provides a perspective on variability not otherwise reflected in static loss reserve analyses. The interaction of internal company factors with those external to the company can have a significant impact on the adequacy of reserves. In particular, future inflationary trends that may be significantly different from those in the underlying database could render held reserves deficient. Likewise, a strategy implemented by the company to control its legal costs via in-house counsel could result in reserve redundancies. One can imagine a variety of other factors that may influence the level of required reserves, some of which may already be embedded in the actuary's analysis:

1. Formation of special investigative units (SIU) to combat fraud;
2. Implementation of managed care initiatives;
3. General changes in economy (inflation, interest rates, unemployment);
4. Regulatory/legislative/judicial changes;
5. Potential bad faith claims; and,
6. Reinsurance collectibility problems

The use of modeling enables the actuary to not only assess a reasonable range of reserves, but also to assess the pertinent risk factors that may lead to material adverse deviation. The real value of this process is to determine which "levers" are most significant to the situation at hand.

For example, suppose the company recently changed its claims handling practices to offer more generous settlements earlier in the life of claims than before. The stated purpose of this new strategy by management is to reduce the costs of defending claims as more will be settled early and fewer claims will end up in litigation. The actuary should somehow reflect such changes in the estimate of required reserves, but must also consider potential adverse effects that may ensue. That is, if the company becomes viewed by claimants as an easy target for claims, there may be an increase in the number of claims filed and the ultimate result may be higher costs. This fact alone may be enough of a concern to the actuary to cause a disclosure of this material risk of adverse deviation.

Of course, the opposite may be true as well. A company that changes its strategy of claims handling to be tougher on settlements may be subjecting themselves to a material risk of potential bad faith claims in subsequent years.

Stochastic techniques can be used in the loss reserving process to model the potential for such circumstances (and others), providing the actuary useful information as to which risk factors may be the most material in terms of potential future adverse deviation. The results of any such modeling must be reviewed carefully, not only for what the numbers indicate, but also for what elements the model may not be taking into consideration. There is always a danger of specifying a model that produces "elegant" results, but may not stand up to scrutiny in light of empirical data. As with any tool, the modeling should be used to glean information that may not otherwise be readily apparent.

There are a number of statistical techniques developed in the actuarial literature in recent years to quantify the variability underlying traditional "chain ladder" loss reserving data and the resulting estimates of indicated loss and loss adjustment expense reserves. Three examples of these include:

1. "Murphy" method which uses a regression techniques

2. "Mack" method which uses a distribution free statistical approach

3. "Renshaw & Vernall" which uses generalized linear models

Each of these techniques has strengths and weaknesses, however, their goals are comparable. They seek to provide estimates of the variability underlying the estimates of future claims development. These variance estimates can be used for a number of applications, including estimating reserves at higher levels of statistical confidence.

We do not mean to suggest there is a direct linkage between variability and materiality. However, the tail value at risk (TVAR) applications of such models can be used to assess probability levels that a Company's reserves may develop adversely, or the probability a company's surplus may drop below RBC thresholds. Given such information, the actuary can make a more thoughtful determination about potential future loss development and whether it is deemed material.

Conclusion

Materiality may be in the eye of the beholder, but the practicing actuary preparing a Statement of Actuarial Opinion must consider the intended uses of that opinion when assessing materiality. Even though the intended users of the Statement of Opinion are specific audiences (regulators, company management, auditor and rating agencies), the document itself is in the public realm. This means that investment analysts, reinsurers, policyholders, claimants and possibly even the ABCD may be reviewing the document. Because of these many audiences, the actuary must consider the points of view of a "reasonable person" when assessing materiality in the context of ASOP 36.

For the purpose of determining the materiality for regulators, we have provided some evidence as to their materiality thresholds. Likewise, we have provided some background on the materiality thresholds commonly in use up through year-end 2000 in the actuarial community. We expect the latter to approach the former rather than vice versa. Materiality must be considered from a reasonable person point of view. It must be considered given the totality of information available about a company's financial and operational circumstances. And, it must be given thoughtful consideration by the actuary. Those are the standards by which we must abide, and those are the standards by which we will be judged.

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Mutual Auto Insurance Company EXHIBIT 1
all figures in (000's)

		Calendar Year				
		1996	1997	1998	1999	2000
(1)	Surplus	25,120	30,054	37,608	41,766	45,792
(2)	% Change	18.8%	19.6%	25.1%	11.1%	9.6%
(3)	DWP	22,634	23,483	23,675	23,391	22,692
(4)	% Change		3.8%	0.8%	-1.2%	-3.0%
(5)	NWP	24,283	25,060	25,223	24,733	23,994
(6)	% Change		3.2%	0.6%	-1.9%	-3.0%
(7)	Loss+LAE Reserves	8,810	10,433	12,501	14,525	14,555
(8)	Net Income	972	2,343	2,450	1,013	842
(9)	Admitted Assets	54,756	60,892	69,442	74,579	80,114
(10)	Combined Ratio	105.0	97.4	97.2	106.7	108.3
(11)	Pretax Op. Income	1,216	3,144	3,225	918	434
(12)	Total Inv. Gains	4,311	3,795	7,077	4,923	5,110
(13)	Pre-Tax ROR	22.0%	23.1%	27.4%	14.0%	12.1%
(14)	NWP to Surplus	0.97	0.83	0.67	0.59	0.52
(15)	Reserves to Surplus	0.35	0.35	0.33	0.35	0.32
	Loss Reserve Dev.					
(16)	% of Original	-12.6%	-9.5%	-5.2%	-3.9%	
(17)	% of Surplus	-10.3%	-6.3%	-2.6%	-1.7%	
(18)	RBC	2,502	2,688	3,211	3,643	3,900
(19)	Best's Rating	A++	A++	A++	A++	A++
	5% Reserve Deviation as % of					
(20)	Surplus	1.8%	1.7%	1.7%	1.7%	1.6%
(21)	Net Income	45.3%	22.3%	25.5%	71.7%	86.4%

Member of Group?	YES
Lines of Business	Dist. Of NWP
Private Passenger Auto Liability	53%
Private Passenger Auto Physical Damage	40%
A&H	2%
Other	4%

Multi-line Casualty Co. EXHIBIT 2
all figures in (000's)

		Calendar Year				
		1996	1997	1998	1999	2000
(1)	Surplus	121,337	115,728	128,811	123,289	84,851
(2)	% Change	53.9%	-4.6%	11.3%	-4.3%	-31.2%
(3)	DWP	199,115	211,445	193,224	183,940	163,665
(4)	% Change		6.2%	-8.6%	-4.8%	-11.0%
(5)	NWP	122,945	158,182	93,341	99,390	75,892
(6)	% Change		28.7%	-41.0%	6.5%	-23.6%
(7)	Loss+LAE Reserves	97,070	97,212	113,354	113,867	111,829
(8)	Net Income	(6,401)	(2,893)	10,733	(5,231)	(3,857)
(9)	Admitted Assets	294,805	319,920	282,994	282,415	244,291
(10)	Combined Ratio	114.7	110.4	111.4	118.9	138.8
(11)	Pretax Op. Income	(10,437)	(7,928)	4,072	(9,905)	(23,268)
(12)	Total Inv. Gains	7,438	1,080	10,539	(225)	1,725
(13)	Pre-Tax ROR	-2.5%	-5.9%	11.3%	-8.2%	-25.4%
(14)	NWP to Surplus	1.01	1.37	0.72	0.81	0.89
(15)	Reserves to Surplus	0.80	0.84	0.88	0.92	1.32
	Loss Reserve Dev.					
(16)	% of Original	22.0%	21.9%	20.2%	17.0%	
(17)	% of Surplus	23.7%	25.4%	19.9%	18.2%	
(18)	RBC	17,334	19,288	25,762	30,822	24,243
(19)	Best's Rating	A-	A-	A-	A-	B++
	5% Reserve Deviation as % of					
(20)	Surplus	4.0%	4.2%	4.4%	4.6%	6.6%
(21)	Net Income	-75.8%	-168.0%	52.8%	-108.8%	-145.0%

Member of Group? YES

Lines of Business

Private Passenger Auto Liability	5%
Private Passenger Auto Physical Damage	0%
Commercial Auto Liability	11%
Commercial Auto Physical Damage	6%
Homeowners Multiple Peril	6%
Commercial Multiple Peril	22%
Fire	5%
Allied Lines	2%
Inland Marine	9%
General Liability	30%
A&H	1%
Other	3%

Lawyers Prof. Liab. Ins Co
all figures in (000's)

EXHIBIT 3

		Calendar Year				
		1996	1997	1998	1999	2000
(1)	Surplus	6,305	7,329	7,334	8,154	9,641
(2)	% Change	19.3%	16.2%	0.1%	11.2%	18.2%
(3)	DWP	7,036	7,197	7,298	7,664	7,815
(4)	% Change		2.3%	1.4%	5.0%	2.0%
(5)	NWP	3,347	4,593	4,864	5,678	5,979
(6)	% Change		37.2%	5.9%	16.7%	5.3%
(7)	Loss+LAE Reserves	6,401	8,178	8,536	10,141	11,291
(8)	Net Income	742	703	278	721	1,079
(9)	Admitted Assets	19,605	22,256	23,261	26,531	29,599
(10)	Combined Ratio	119.4	128.2	130.3	107.3	101.7
(11)	Pretax Op. Income	926	1,134	(254)	859	1,278
(12)	Total Inv. Gains	47	204	358	91	364
(13)	Pre-Tax ROR	15.4%	18.3%	1.4%	11.7%	17.0%
(14)	NWP to Surplus	0.53	0.63	0.66	0.70	0.62
(15)	Reserves to Surplus	1.02	1.12	1.16	1.24	1.17
	Loss Reserve Dev.					
(16)	% of Original	4.2%	-0.5%	-18.4%	-13.4%	
(17)	% of Surplus	4.3%	-0.5%	-21.4%	-16.7%	
(18)	RBC	1,235	1,358	1,676	1,833	2,338
(19)	Best's Rating	B++	B++	B++	B++	B++
	5% Reserve Deviation as % of					
(20)	Surplus	5.1%	5.6%	5.8%	6.2%	5.9%
(21)	Net Income	43.1%	58.2%	153.5%	70.3%	52.3%

Member of Group? NO

Lines of Business

Lawyers Professional Liability 100%

Reinsurance Company
all figures in (000's)

EXHIBIT 4

		Calendar Year				
		1995	1996	1997	1998	1999
(1)	Surplus	272,374	396,677	423,616	402,652	401,392
(2)	% Change	11.9%	45.6%	6.8%	-4.9%	-0.3%
(3)	DWP	0	0	0	0	0
(4)	% Change					
(5)	NWP	205,065	429,870	645,832	698,440	653,984
(6)	% Change		109.6%	50.2%	8.1%	-6.4%
(7)	Loss+LAE Reserves	659,145	971,859	1,059,040	1,038,460	1,179,181
(8)	Net Income	22,980	(17,164)	35,794	1,264	8,375
(9)	Admitted Assets	733,225	1,491,776	1,608,026	1,745,156	1,877,779
(10)	Combined Ratio	101.0	99.8	104.5	112.4	113.8
(11)	Pretax Op. Income	25,022	(18,977)	58,304	10,226	(6,092)
(12)	Total Inv. Gains	7,729	20,066	4,710	10,883	24,968
(13)	Pre-Tax ROR	12.0%	0.3%	14.9%	5.2%	4.7%
(14)	NWP to Surplus	0.75	1.08	1.52	1.73	1.63
(15)	Reserves to Surplus	2.42	2.45	2.50	2.58	2.94
	Loss Reserve Dev.					
(16)	% of Original	1.8%	7.9%	7.5%	6.7%	
(17)	% of Surplus	2.7%	16.4%	15.8%	17.3%	
(18)	RBC					
(19)	Best's Rating	A	A	A+	A+	A+
	5% Reserve Deviation as % of					
(20)	Surplus	12.1%	12.3%	12.5%	12.9%	14.7%
(21)	Net Income	143.4%	-283.1%	147.9%	4107.8%	704.0%

Member of Group?	YES
Lines of Business	
Commercial Auto Liability	17%
Commercial Auto Physical Damage	10%
Homeowners Multiple Peril	3%
Commercial Multiple Peril	12%
Allied Lines	2%
Workers Compensation	4%
General Liability	7%
Reinsurance	26%
Other	20%

Reserving for Runoff Operations—A Real Life Claims Specific Methodology for Reserving a Workers Compensation Runoff Entity

James B. Kahn, FCAS, MAAA

Reserving for Runoff Operations - A Real Life Claims Specific Methodology for Reserving a Workers Compensation Runoff Entity

James B. Kahn, FCAS, MAAA

ABSTRACT:

The paper takes the reader through a real life example of an entity in runoff. In some instances, certain calculations and data examples have been amended from their original forms for the purposes of simplicity and demonstration.

The runoff operation's reserves are predominately those of Florida Workers Compensation (WC) self-insured funds. WC has its own unique properties, which need to be considered when reserving in a runoff environment.

Two observations in particular have been seen within the data: (1) occasional spikes in the Workers Compensation data as a result of settlement activity, (2) extraordinary ALAE costs incurred during the years following the 1994 Tort Reforms. The changes to both the type of remaining claimants as well as the Workers Compensation environment may produce distortions to loss development triangles using so called "traditional" reserving methodologies.

APPROACH:

When a limited number of open claims remain, a claims specific model could be set up whereby scenario testing can be performed on data segments to reserve to an "ultimate" loss reserve amount.

Interaction with a company's claims department will be essential in both the setup and application of the Workers Compensation model. The claims unit can give input as to the state of the market for the lines of business, law changes, and perception of future settlement activity. Eventually, the individual claim model will need to be audited, with integral help from a company's claims department.

Model scenarios here could include differing medical escalation percentages, longevity of claimants, or inuring reinsurance arrangements (and tracking exposure for recovery likelihood of carriers rated below A-).

ALAE

The ALAE distortion in the data caused by the 1994 Florida Tort Reforms can be overcome with modifications to the same claims specific model. Solutions may involve applying ALAE caps or taking average yearly payments for typical years to apply to future periods.

BIOGRAPHY:

James Kahn is an Assistant Vice President and Actuary with Folksamerica Reinsurance in New York, NY. He is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He received his B.A. degree in Mathematics (with an application in Probability and Statistics) from the Johns Hopkins University in Baltimore, MD. Mr. Kahn has over 12 years of actuarial experience, having held various positions at a primary insurance company and consulting firms prior to coming to Folksamerica Reinsurance.

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* * * *

To my friends across the street who I wasn't given the chance to say goodbye to on September 11, 2001, thank you for putting my day-to-day struggles into a well-needed perspective.

I. INTRODUCTION

In this example, a run-off company (no business written since 1997) has reserves, which are predominately those of primary Workers Compensation in the State of Florida (for self-insured funds). The majority of the data shown is actual data for an entity hereafter referred to under the fictional name "ABC Insurance Company" or "ABC" or "the Company" (not to be confused with any potential entity bearing the letters "ABC" in any part of their name). Whereas it should be understood that many of the conclusions reached and methodologies used herein have been derived using our anecdotal evidence, the readers should feel more than free to amend any or all of the methodologies contained herein for their own particular situations. In some instances, certain calculations and data examples have been amended from their original forms for the purposes of simplicity and demonstration.

We will look to address some observations seen within this data: (1) occasional spikes in the Workers Compensation data as a result of settlement activity, (2) extraordinary ALAE costs following the tort reforms of 1994.

In many cases, the most difficult projection work for an actuary occurs when business for a current segment is different from historical data for the same segment. For a runoff workers compensation writer, over time, we should be left with only claims which will not take an offered settlement; and as such, will be subject to parameter risks such as tort reforms and inflation. The fact that a body of claims should "migrate" over time into a more severely injured population does not necessarily mean that historical data as of the same maturity is different than the current data. It is possible that changes in injury mix of open claims as an accident year matures could be consistent from accident year to accident year.

This paper takes the approach of setting ultimate reserves for both the "likely to settle" and "unlikely to settle" groups of claimants as determined by the company claims department. The differences in these bodies of claims will need to be reflected in determining final IBNR amounts.

Additionally, this paper will show why exceptions may occur to many normally sound schools of thought and potentially lead to counter-intuitive conclusions if one does not consider the particulars of the given runoff situation.

This paper will project ultimate IBNR reserves using a "non-traditional" claims specific methodology, and attempt to explain the conditions under which this method is an appropriate approach. Hopefully, the understanding of these techniques will have merit for all Workers Compensation states as well as for ongoing situations (with appropriate modifications).

II. WORKERS COMPENSATION LOSS RESERVING

A. Background

1. Workers Compensation Characteristics

Workers Compensation losses have their own unique properties:

- Most of the eligible payments are set by State law, which could vary significantly from state to state for such items as maximum and minimum payments, rulings for manifestation of claims, statute of limitation requirements, and integration of benefits. All contain some form of payments for indemnity (representing payment for lost wages) and medical costs associated with the injuries themselves.
- After the first couple of years, there should be very little, if any, smaller "nuisance" type claims remaining open. Occasionally, claimants could attempt to reopen earlier claims, even those with very little merit.
- Because of stringent reporting statutes in most states, very few, if any, IBNR claims will be reported going forward. A company in runoff will very likely be in a position to fight newly reported claims on the basis of Statute of Limitations filing requirements. As ABC's data is mostly construction related risks, there should be virtually no exposure to asbestosis, latent injuries or occupational disease, which may have late reporting patterns and claims could potentially be accepted as a result of a late manifestation of injury.
- Depending on a company's case reserving practices, loss reserves may be carried at implicit or explicit discounted values.
- Settlement "spikes" may be seen periodically throughout the paid loss data, particularly with new chief claims officers or the assignment of a new third party administrator. These settlements distort ultimate loss estimates using the traditional paid loss development methods, especially when claims are settled at an amount below the reported loss amount on the company's financial ledgers. It is easy to see how additional volatility could be added to the latest diagonal in such situations.
- Occasionally, claims handling operations may reach discount arrangements with medical providers, whereby these providers will perform services associated with these claims for a reduced cost.
- The possibility exists that claims, which were previously closed, may still have remaining exposure. A company should consider the possibility of "reopening" of closed claims.
- Outside sources of recovery (reinsurance, second injury fund, etc.) need to be considered in determining what the final ultimate reserve would be.
- Workers Compensation losses have substantial early payments (our data shows about four years beyond the initial accident date) as claimants have initial hospitalizations, surgeries and treatments. Over the near term, yearly costs and utilization may very well decrease after the initial injury or surgery date until such time when follow-up surgeries or additional therapy may become necessary due to the aging process.

A runoff situation will have its own particular nuances:

- Over time many of the remaining claimants will have suffered permanent total injuries, leading to claims oftentimes being reserved at a lower "settlement" value, not necessarily at a 'true' ultimate value. This would include those claimants, who will never end up accepting a settlement and will have their ultimate values increase over time from the held settlement value. Factors influencing settlement are discussed later.
- A runoff company will most likely be at a point in the timeline beyond the initial high cost of hospitalization and surgeries (about four years after the initial accident date in this case).
- Claimants **may** pursue claims involving a runoff company less aggressively than they would with an ongoing active writer (the "deep pockets" theory).

Workers Compensation statutes are under a constant state of change. It is therefore critical for recent developments and trends to be understood prior to any projections of ultimate losses. To arrive at sensible projections, both actuaries and claims personnel should communicate their knowledge of relevant state changes and trends to each other as often as possible.

On a related note, there have been recent discussions concerning Medicare efforts to take credit for portions of a medical settlement paid to claimants when the bills submitted are for medical services provided after the Workers Compensation settlement was struck. Put differently, if an injured worker has already accepted a medical settlement, and subsequently bills Medicare for treatment, it is quite possible that Medicare will take the viewpoint that this is "double dipping" of benefits, and should not be provided for. It is unclear at this point how this should be handled from a company standpoint and several questions still remain as to logistics of any additional company exposure (Who would pay if the claimant has died? Can they recover attorney fees? In Florida, will the SDTF pay for closed cases? Will it be up to an insurance carrier to contribute in such cases? What do the settlement documents provide? etc.). There is a Medicare signoff process prior to the settlement that will enable a company to confirm upfront the amount of a Medicare credit. At this point, we have not reflected any potential for Medicare in projections of ultimate, but settlement language can provide adequate protection from possible reopening potential.

2. Workers Compensation Industry Reserving Practices and Philosophies

a. General Overview

There are probably almost as many company Workers Compensation reserving philosophies and styles as there are companies. For an actuary to arrive at proper assessments of reserve or reserve adequacy trends, it is most crucial to understand the internal philosophies and definitions (not everyone will define closed or reopened claims the same way especially in regard to closed without payment claims).

Many companies consider forms of discounting (explicit or implicit) in setting case reserves; some don't consider discounting at all, and some may only discount the so-called "catastrophic" type claims. It is a widely held belief that most Workers Compensation carriers apply some form of discounting, and the unwinding of this discount over time is, in fact, reflected in Industry loss development patterns.

Some companies set reserves to consider the expected lifetime of a claimant using a standard mortality table, others consider an impaired worker mortality table, and still others treat all claimants as if they'll live to a common age such as 90 or 85 years of age (sometimes referred to as the "Rule of 90" or "Rule of 85" philosophies).

For accounting purposes, some entities implicitly reflect subrogation recoverables whereas others may explicitly state the recoverable amounts separately. Florida companies may or may not handle recoveries from the Florida Special Disability Trust Fund (or a Second Injury Fund in other states) in a similar fashion. Companies may have different philosophies for reserving to the reinsurance retentions, or to various reinsurance layers.

b. Settlement Provision Background

This is probably a good time to introduce the concept of "settlement value" reserving as it may pertain to a given company's case reserving philosophy. It is an important idea in our development of the claims-specific model. Per the ABC claims department, it is Industry practice to employ some form of "settlement value" reserving in Workers Compensation (as well as in liability lines of insurance). As such, Industry loss development patterns should already reflect this practice to some degree. As Workers Compensation payments may take place over several years, claims departments could easily establish initial case reserve amounts at "settlement value" and subsequently refine these estimates over the course of many years. The term "ultimate value" reserve will hereby be defined to mean the claims department's estimate of ultimate reserve using expected payments and mortality assumptions, prior to any adjustments including discounting.

ABC case reserves, for the remaining claimants, are held at a specified percentage below the discounted "ultimate value" reserve. In negotiating with claimants or their outside attorneys, it has been the historical practice for ABC to get settlements below the "settlement value" reserves. Historically, ABC has been successful at settling about 70% of the held discounted reserve, though as of late, this figure has begun to approach 85-90% of the held discounted amount.

Whether or not a claimant accepts a settlement appears to be more a function of the claimants themselves than a function of the severity of injury types. Analogies can be drawn to utility theory (there are probably high enough offers whereby almost anyone would accept a settlement) or whether or not a lottery winner would be willing to accept a lump sum payment or a steady stream of future payments (a big difference discussed later would be the Workers Compensation loss of future payments upon early death). It is largely up to a company's internal claims department or Third Party Administrator to understand what external or internal factors may lead to whether or not claimants ultimately accept a settlement.

Why would someone with a minor injury not be willing to accept a settlement? Some may view having an open Workers Compensation case as an "insurance policy" against potential unforeseen circumstances (latent disease) or injury re-aggravation. As a result, there are claimants, some of which have small or zero reserves outstanding, that the ABC claims department has deemed "unlikely to settle" following discussions with the claimant and/or a claimant's attorney.

On the other hand, some of the more seriously injured are willing to consider settlement offers. A claimant who has dependant children may be willing to take a settlement below projected medical costs rather than take a chance of leaving the lower fatality benefits to their dependants upon an untimely death. Other very seriously injured workers, those without a spouse or dependant children, may not have a reason for taking a settlement (no one to inherit future streams of payments or death benefit).

Occasionally, a claimant who had previously rejected settlement offers may suddenly want to change course. A few reasons are sometimes cited: (1) the claimant may have recently been diagnosed with a limited lifetime where they'll have to reassess the differences of fatality benefits versus settlement offers, (2) outside debt obligations for non-medical related costs, (3) general state of the economy, which may lead to more cash settlements when the economy is not performing well, and (4) possible influence of outside sources, which could include, among others, claimant's attorneys.

In Florida prior to 10/1/2001, a judge needed to approve all outstanding settlements, and would occasionally not allow some if they thought the seriously injured claimant was being taken advantage of. Effective 10/1/2001, a judge's approval is no longer needed in the State of Florida for settlement of newly tendered settlement offers, if the claimant is represented by an attorney.

Since claimants 62 years of age or older have a potential Medicare offset to their benefits through the current Social Security plan, agreeing to settle their Workers Compensation claim would void the additional qualifying benefits. Since older claimants have fewer remaining years to live, setting a settlement figure much more than an additional couple of years of current payments could be economically risky to an insurance company. As a result, ABC does not focus on settlements with older claimants, preferring to work toward settling those claimants between the ages of 30 and 60 years of age, who are more likely to settle where the present value of the settlement is a much more attractive savings.

Under most Workers Compensation statutes, settlement is not allowed until a condition of Maximum Medical Improvement (MMI) has been reached as determined by a physician. (Note: As of 10/1/2001, in Florida, the claimant does not need to have a condition of MMI to settle. The examples shown within the exhibits are prior to the 10/1/2001 change in statute).

It may certainly be worthwhile for ABC to settle claims already beyond the reinsurance retention. This is required for ABC to eliminate uncertainty, such as the risk either insolvency of reinsurance carriers or the claimants exceeding reinsurance maximums. Discussions would most likely involve all applicable reinsurers in such a case, including the SDTF or 2nd Injury Fund.

3. 1994 Florida Tort Reforms

a. Background

More information concerning the 1994 Florida tort reforms and the Special Disability Trust Fund is given in Appendix A. A couple points should be understood to see why various assumptions are made within the claims specific model.

ALAE Impact - Beginning with the 1994 Reforms, *Temporary Total* (TT) benefits were reduced from a maximum duration of 260 months to 104 weeks. When the 104 weeks of time expired for new claims on 1/1/1996, a great number of filings were made to extend benefits, or push for more reclassifications into the greater benefits of Permanent Total categories. The initial push resulted in large legal costs generated from calendar years 1996 and 1997. The costs subsequently subsided, but not after seeing a two-year spike in paid allocated loss adjustment expenses.

Allowance of Indemnity Settlements - Prior to 1994, entities were allowed to settle indemnity (lost wages) portions of a claim, but not medical loss or loss adjustment expense. Beginning 1/1/1994, companies were allowed to settle the entire Workers Compensation claim, or any portion thereof (medical or lost wage) including claims open at 1/1/1994. As a result, there was a big Industry push to settle claims (or the medical portion only of claims) beginning in 1994.

As a company practice, ABC will no longer settle an indemnity portion of a Workers Compensation claim only. Claims which ABC currently classifies as "medical maintenance" claims are those which had the indemnity portion settled prior to the 1994 Tort Reforms. ABC still reserves the right to settle the indemnity portion of claims without settling the medical if it would so desire.

b. Special Disability Trust Fund (SDTF)

At this point, some background on the Florida Special Disability Trust Fund (SDTF) is necessary to better understand the Non Traditional Workers Compensation model for ABC (Florida's version of other states' "Second

Injury Funds"). The SDTF reimburses the employer when an employee suffers an injury, directly or indirectly, as a result of a previous disability or coupled with a previous medical condition, which has worsened due to the new injury. SDTF recoveries inure to the benefit of all additional reinsurance coverage.

The Florida SDTF operates on a pay-as-you-go basis whereby each Florida Workers Compensation writer contributes a percentage of current premium writings (~4.5% currently). In recent years, ABC has received more than $10 Million of SDTF recoveries per annum (net of reinsurance cessions).

The SDTF will pay for paid claims on a first come, first served basis. As additional payments are made on case reserves or development on known cases, additional papers (called SDF-2 forms) will need to be filed with the State. Currently, the time from request for reimbursement until recovery from the Fund is received is a little more than three years as recouping assessments have lagged behind payment of benefits.

B. Non-Traditional Methodology (The Introduction of the "Claims Specific" Model)

1. Categories of Claims and Definitions:

The following list applies to the various segments of the "Claims Specific" Workers Compensation model:

- ***As Reported*** – Loss reserves as displayed in the company's financial statements. These may be held at "settlement value" in the cases where it would apply.
- ***Ceded*** – Loss reserves, which will be ceded to reinsurance companies after recovery from SDTF.
- ***Closed Claims*** – Claims, which are no longer open and active. As a result, reserves will need to be taken down, or in the cases where they're open for an SDTF recovery only, the remaining recovery will need to be booked when received.
- ***Coverage B/Coverage Issues*** – Claims pertaining to Employers Liability (Section 1B) or other miscellaneous items where required payment may be in question based on policy language.
- ***Direct*** – Liability reserve amount to the Company before any additional recoveries
- ***Likely Exposure @X%*** - Percentage of "As Reported" reserves, which the likely to settle claims will eventually settle for (approximately X% of currently held direct reserves on this segment).
- ***Likely to Settle*** – Claims deemed by management as being *likely* to accept a tendered settlement offer within the timeframe being considered.
- ***Maximum Exposure*** – Scenario under which no settlements occur. Closed claims, resolved claims, Coverage B/Coverage Issues, and Unlikely to Settle scenarios are at maximum exposure amount under all percentages of the likely to settle scenarios.
- ***Maximum Reinsurer Reserve Claim*** – Claims, which are already being carried at an amount greater than the specific reinsurer's retention amount after SDTF recoveries. These claims may or may not be greater than the reinsurer's coverage limit.

- ***Medical Maintenance*** – Claims where the indemnity is settled and the medical portion remains open. For ABC, this will pertain to claims Prior to the 1994 Florida Tort Reforms.

- ***Net*** – Amount retained by the company after recoveries from SDTF and reinsurance companies.

- ***Not at MMI*** – Claimant hasn't reached maximum medical impairment as determined by a doctor and is therefore unable to settle (as of 10/1/2001, in Florida, the claimant does not need to have a condition of MMI to settle).

- ***Other*** – Claims which are not classified as Closed, Coverage B/Coverage Issues, Maximum Reinsurer Reserve, Medical Maintenance, Not MMI, PT, PT Pending, Resolved, or Special Disability Accepted. The majority of these claimants are those who have gone back to work. These would include those falling under Permanent Partial (PP) and Temporary Total (TT) classifications.

- ***PT*** – Claims, which have been accepted under the Workers Compensation statute as being Permanent Total.

- ***PT Pending*** – Claims where a petition for Benefits has been filed for Permanent Total classification.

- ***Resolved Claims*** – Claims, which have not been formally settled, but which are in the process of settling. These can be further categorized into (1) claims which remain open for recovery only, (2) claims where a settlement has been reached, but the check has not been issued, (3) claims where a settlement order has been prepared and is being presented to a Judge for approval, (4) claims where a judge has approved a settlement, and the claim is open to pay final medical and legal bills and will close or transfer to the SDTF. The majority of claims in this category are those described in item (4). Changes to settlements involving judges were noted in a previous section.

- ***SDF-Reqstd*** – Paid loss amounts which have been filed with the SDTF, approved, and thus due the company. These are not reported in company financial statements as reported losses recoveries.

- ***SDF-on Res*** – Reserve and IBNR amounts pertaining to claims, which have already had paid losses accepted by the SDTF. Recovery for these reserves will need to be filed with the State following additional payments on these claims. These are not reported in company financial statements as reported loss recoveries.

- ***Special Disability Accepted Claims*** – Claims which have been accepted by the Special Disability Trust Fund as being eligible for recovery because of a condition suffered by the claimant prior to his work related injury.

- ***Unlikely to Settle*** - Claims deemed by management as being *unlikely* to accept a tendered settlement offer within the timeframe being considered.

2. Background and Methodology

a. Need for Non-Traditional Method

The internal need for a non-traditional method arose after observing great calendar year differences between actual and expected loss emergence using the so-called traditional loss reserving methodologies (chain ladder, Bornhuetter-

Ferguson, etc.). Several factors, not previously seen in the historical data, have contributed to these differences: (1) the impact of reinsurance leveraging once active claims began to pierce the applicable retention and render historically observed cession percentages too low for given accident years, (2) the receipt of higher than historical SDTF recoveries now that enough "lag time" has passed on older claims, (3) a concerted effort by the Third Party Administrator (TPA) to settle more claims quicker than the historical pattern. The effort cited in item (3) will result in fewer outstanding cases per claims handler going forward, assuming an identical number of claims handlers.

Item (3) can lead to big spikes in paid loss development methods for some years, while having relatively little impact on incurred loss development methods in many cases. These spikes will create more of a distortion as the number of open claims begins to dwindle when settlements occur, and subsequently lead to reduced case reserves. A runoff entity, unlike an ongoing operation, will not reach a steady state of reserves where new claims enter the reserve base as older claims settle. This leads to a shift in the type of claims remaining open.

The following table shows the recent trend in claim closures for ABC:

ABC INSURANCE COMPANY SUMMARY OF CLOSED CLAIMS FROM 12/1/1996-12/1/2000

Evaluation Date	Total Open Claims	Closed from 12/1-12/1
12/1/1996	18,341	N/A
12/1/1997	8,719	9,622
12/1/1998	5,370	3,349
12/1/1999	4,274	1,096
12/1/2000	3,068	1,206

Open claims shown above include those claimants who have agreed to settle in principle. On the summary examples in Appendix C, these types of claimants are classified in the "resolved claims" category.

The ABC claims department estimates that by the end of 2002, there will be somewhere between 500 and 750 remaining claims, all of which can be considered "Unlikely to Settle". There will still be additional claims that have either agreed to settle, or which have open files while awaiting recovery from either a reinsurance company or the SDTF (resolved claims noted above).

The more difficult challenges for actuaries exist in cases where future patterns will differ from historical ones. Given that the future population of remaining claimants will be those who will not settle, we would expect future development to look different from the previous population (which was more similar to the "steady state" situation mentioned previously). The reserves for this group of claimants should ultimately exceed settlement value, whereas many of historical claimants accepted payments for reserves at some percentage of the settlement value.

Looking at data on a claim-by-claim basis could pose logistical difficulties for a large entity or an ongoing entity, which will still have significant "true" IBNR claims. However, as a runoff company handles fewer and fewer claims, this procedure should become less cumbersome. At the end of 2002, this may become very little more than applying a maximum exposure procedure with appropriate modifications.

Traditional actuarial methods may be slow to recognize the change in claimant population, especially with a rapidly dwindling book such as ABC's. As mentioned earlier, this population is not necessarily different than those claimants historically seen at the same maturity. Additionally, the Industry development tail **may** be overstated for a runoff entity if claimants are less aggressive in their filing of claims than they would be for a "deep pockets" ongoing company.

Traditional aggregate grouping methodologies may also be slow to reflect leveraging of reinsurance cessions once a retention level is reached. It may also be slow to reflect future SDTF recovery now that the last two calendar years have produced significant increases in the amount of recovery received. As the claim database for a runoff entity becomes more manageable, the ability to look at these parameters on a case-by-case basis becomes a lot more appealing. With appropriate judgment, many of these difficulties can be overcome with a claims specific reserve model.

b. Development of Model

1) General Background

Because of reserves being held at "settlement value", claims that will not settle will need a lengthy procedure whereby payments will continually be made to a claimant until either death (for the claimant) or remarriage (widow's benefits). If no claimants ever accept a settlement, the exercise will reduce to projecting future payments multiplied by the number of remaining months to live. We will refer to this going forward as the "maximum exposure" example.

Because some claimants will, in fact, accept settlement offers, we will need to divide our data into the "likely to settle" and "unlikely to settle" categories as determined by the claims unit. For likely to settle claims, we can set reserves at a specified percentage of the carried reserves – perhaps somewhere around the targeted settlement amount (currently 80% as mentioned previously). Unlikely to settle claims will essentially need to be held at the "maximum exposure" reserve figures. Details as to the assumptions for each settlement possibility will be provided later. An adjustment is later performed outside the scope of the model to adjust for the possibilities that claimants deemed as "likely to settle" will in fact never settle and vice versa. In many cases, it is not implied that a claimant will never accept a settlement, just that they will not accept the current offer – an offer currently targeted at a specified percentage of the held reserve amount within the specified time frame.

In its most basic form, the "unlikely to settle" category of the claims projection model applies an average monthly payment amount (average yearly divided by 12) to the number of remaining months. This procedure is similar to that proposed by Teng in his 2001 Call Paper. Any standard mortality table (in this case, the most recent State of Florida, "non-impaired" table) can be used to estimate the number of remaining months in a claimant's lifetime. Not using the "impaired" table could provide an element of conservatism if one is to believe that ABC's seriously injured claimants have a lower expected lifetime than the general population. Also, for simplicity, this model projects the claimant's lifetime as the number of remaining months without considering a likelihood distribution. If a claimant lives longer than the expected lifetime, the burden of additional cost may be the responsibility of reinsurance carriers, whereas a shorter lifetime may lead to lower ultimate values.

Projected payments are made separately for medical losses, indemnity (lost wage) losses, and allocated loss adjustment expense. Assumptions regarding these projections will be discussed in separate sections. Final ultimate reserve amounts based on these payments are then "netted down" to consider the impact of reinsurance cessions and SDTF recoveries.

Looking at data on a claim-by-claim basis also provides an excellent opportunity to audit the data, especially when performed in conjunction with a company's claims department. In the case of ABC's held reserves, a handful of adjustments were made to the data following the audit of claims within this model. It was discovered that a few of the medical maintenance claim files still had reserves being held for future indemnity payments. Additionally, there were still some reserves being held on a number of claim files that were already closed. The final scenarios have reflected what should be the true reserves on these claims. These corrections can best be observed on any of the attached "100% of reserves, likely to settle" scenarios (since the amounts are not equal to held amounts).

Summaries of the claims specific model are shown in Appendix C. Exhibits 1-5 show three effective groups of figures. The information at the far left of the page shows the ultimate reserve amounts as would be projected using the given assumptions for each scenario. The middle section shows the case reserves being reported in ABC's financial statements. The section on the far right of the page takes the difference between the two other sections. This would be the final IBNR. It should be understood that the "Direct" minus "SDF" minus "ceded" would be the amount "net" of recoveries from reinsurers or the SDTF. "SDF-Reqstd" represents SDTF recoverable amounts for paid losses already approved by the State of Florida, and due ABC. This differs from "SDF-on Res", which are reserves (case or IBNR) for these approved claims, which would be eligible for SDTF recovery once payment is made and SDF-2 forms are filed. The Descriptions of the claim categories seen in the rows on the far left were defined earlier.

2) Model Scenarios

As the number of open claims for ABC begins to dwindle, management will need to understand how to handle the ongoing 'parameter' risk associated with a loss sensitive model like the one produced. Rather than view one scenario as the "best" estimate of ultimate loss, it may make sense to run several different versions of the model, and consider applying likelihood weights to each. This is not too much different than the ultimate loss reserving process of applying various weights to different actuarial methods.

For likely to settle claims, ABC has chosen to show these claims settling at both 80% of the current reserve value (the current target and an amount closer to the historical figure) and 100% (an amount which would make sense if one believes that the population will have fewer claimants willing to accept 80% going forward).

Particulars of the escalating loss payments will be discussed in a later section. However, it should be noted that the model is able to handle the concept of escalating loss payments with the help of common reserving software. With minor adjustments, the model should be able to even handle escalation payments with different inflation/trend factors for different time periods.

3) Reinsurance Recovery Calculations

It is not at all difficult to calculate reinsurance recovery for each claimant after determination of the ultimate loss. In the case of ABC, ALAE cession arrangements are on a pro rata basis except for two Fund years pertaining to one fund. Such adjustments for the differences in ALAE arrangement can be easily handled for different reinsurance treaties.

Should any of ABC's reinsurers look to commute ongoing treaties, it would not be difficult to project a reasonable settlement using ultimate losses as determined by the claims specific model. Additionally, handling of previously commuted reinsurance arrangements on a claims basis may prove to be much easier to calculate than would a "traditional method" based on aggregate loss information. Any traditional cession analysis using aggregate data runs the risk of being too conservative in their projection of ultimates if there is a difficulty quantifying the leverage effect of reserves already at the maximum retention. Additionally, one may want to consider the likelihood of a claimant accepting a settlement (perhaps at an amount below the reinsurance retention) prior to any commutation discussions.

During the years 1995-1997, ABC had a Quota Share arrangement, whereby 75% of loss and ALAE was ceded after an inuring $1M maximum per claim. When losses exceed the $1M retention, expenses will be ceded on a pro rata basis. On the Appendix C summary exhibits, any cessions seen on losses, which are not at a "maximum reinsurance reserve" classification, pertain to this 1995-1997 Quota Share.

A handful of ABC's reinsurers have been declared insolvent. Cessions to these carriers have been removed from the cession calculations. If management would want to test cession scenarios should certain other reinsurance carriers become insolvent (even at future points in time), we could remove these carriers from ceded recovery calculations. For example, you could remove those carriers rated as "A-" or below by A.M.Best.

4) SDTF Recovery Calculations

SDTF recovery is calculated for each eligible claimant, inuring to the benefit of all other recoveries except for subrogation. For the 1995-1997 years, the phase-out by the State of Florida resulted in only a 50% recovery for SDTF claims. This arrangement has been handled for each applicable claimant. For a handful of other claimants, the State approved recovery amounts other than 50% or 100% for SDTF claims. Modifications for these different percentages are trivial. Future recoveries from the SDTF may need to be ceded back to the reinsurers (or occasionally even the SDTF itself in a subrogation situation). SDTF recoveries shown in the calculations are figures 'net' of reinsurance cessions.

5) Social Security Offsets

A 20% offset for Social Security has been applied to the indemnity reserves for the PT Pending and Not at MMI Unlikely to Settle Categories, since these types of claims would most likely qualify for Social Security recovery (if the claimant has worked enough quarters). Injured workers under the age of 62 may qualify for disability benefits under Social Security.

As current PT claimants should already qualify for Social Security Offset, historical indemnity losses should already be reflected net of Social Security offsets. The average calendar year payment calculations, used in the unlikely to settle situations, are based on historical paid information. As such, Permanent Total historical payments should already implicitly reflect payments net of Social Security offsets. As a result, only PT Pending claims and claims not at MMI would need adjustments (since they wouldn't be reflected in the historical data)

Although hard to quantify, other segments could have claimants qualify for Social Security, based mostly on age of claimant (62 and older). The omission of such an offset applies a little bit of conservatism to the model.

6) Medical Reserve Overrides

Sometimes the claims department can identify "problem" or unique claims where applying an average payment based on the general population would not give optimal results. In such cases, these claims have been entered using a claim "override" instead of the usual methodology (ultimate loss based on a maximum exposure method for "unlikely to settle" claimants).

The two overrides on "likely to settle claims" (one at reinsurance maximum and the other in SDTF) have not been entered at the claims department estimates, but instead at the likely to settle reserve percentage (either 80% or 100% depending upon scenario). These claims should be monitored in the event that they switch from likely to settle to unlikely to settle categories.

c. "Likely to Settle" Ultimate Reserve

The model applies a selected percentage of the current reserve amount to likely to settle claims (in our examples, we've shown both 80% and 100%).

Appendix C, Exhibit 9 shows historical closed reserve percentages based upon reserve values at both 12 and 24 months prior to closing. Since settlement value reserves may very well be substantially below where ultimate reserves should be booked if the claimant never accepts a settlement (and even well below where ABC would expect a claim to realistically settle for), it is occasionally ABC's practice to raise the reserve figures to an amount equal to or above the final agreed upon value just prior to the final agreement. As such, comparing where the percentage of the final payment amount will be to where the reserve values were at points 12 and 24 months prior to closing will prove to be a more useful relationship than would be comparing the relationship of final agreed upon settlement amount to the held reserves carried one day prior to settlement.

Historically, claimants have accepted settlements, which were approximately 70% of the held reserve value (the "settlement" value) at 12 and 24 months before closing. As of late, claimants are starting to require settlements that are 85-90% of the settlement value reserve. The underlying thought expectation is that over time there will be fewer and fewer claimants who would accept amounts as low as 70% of the settlement value as their final claim settlement – meaning that since there is no new business coming on, only the "harder to settle" claims remain at any point in time. In some ways, this pattern parallels the same logic behind the classic actuarial Salzmann principle that states, "The average size of unpaid claims generally increases with the age of development." We would expect the average percentage of settlement value, which would entice a claimant to settle their claim, to also increase over time. However, certain outside influences (such as state of the economy mentioned previously) could impact how these percentages apply in a given calendar year.

While ABC may consider using higher percentages of held reserve amounts to attract more settlements (hopefully, at a lower value than an "ultimate" reserve figure), an immediate increase would alert outsiders that higher percentage values are now being offered. Such a situation could very well result in an upward bias of future claim amounts after the claimants' bar figures out this change in handling philosophy. As such, ABC raises the settlement value amounts in a slow fashion.

Of note, losses already at the reinsurance retention may have a "reverse leveraging" effect if reduced by a likely to settle target percentage. Supposing the target settlement rate in our model uses 80% of reported direct reserves as the ultimate losses for likely to settle claims. If paid losses for a particular claim already exceed the retention, then the net reserve reduction would be 0%. On the other hand, some reserves could be reduced by the entire 20% amount (an example would be a case where no losses have been paid, but the reserve amount is equal to the retention). As a result, the selected **net** reserve percentage will end up being somewhere between 80% and 100% of held **net** reserves for the body of claims already at the maximum reinsurance retention. SDTF claims could have distortions for a couple reasons: (1) reverse leveraging for claims at maximum reinsurance retentions, and (2) the replacement of ceded losses by SDTF recoveries once amounts are received and thus placed on company financials.

d. "Unlikely to Settle" Ultimate Reserves

The unlikely to settle claims take the ALL year ***indemnity*** paid average monthly average and multiply this factor by the number of remaining months. Florida statute specifies set wage amounts to be applied to a claimant's lifetime, and thus should have very little fluctuation in these amounts. A supplemental benefit (to be discussed later and calculated on Appendix C, Exhibit 8) is applied in some cases to handle a cost of living increase, subject to certain caps. If one wanted to apply this procedure to states with unique Cost of Living Adjustment (COLA) procedures (like those used in Massachusetts for instance), appropriate modifications would need to be made.

The unlikely to settle claims take the latest four-year ***ALAE*** paid average monthly average and multiply this factor by the number of remaining months. The logic used in selecting a four-year average is given in the ALAE section.

For unlikely to settle claims, we've calculated the ***medical*** yearly payment amounts (differing by accident year) as shown on Appendix C, Exhibit 7. The model then takes these yearly calculations, and applies them to the remaining number of months (after dividing by twelve). The derivation of the yearly payments is detailed in the next section.

1) Medical Escalation

Unlike the indemnity portion of Workers Compensation claims, where payments should remain relatively steady for a claimant as payments are set by statute (usually as a percentage of the claimant's salary or statewide average weekly wage), medical payments have many outside factors, which add volatility to the reserving process. Significant volatility exists for medical payments depending on number of years from accident date (whereas indemnity payments should remain relatively steady). As such, it may prove to be too cumbersome, and more importantly not as accurate, to attempt to take a straight average of historical medical payments by claimant and apply these to future remaining months using mortality assumptions. The model will use a combination of individual claim history as well as aggregate claim data by accident year.

Appendix C, Exhibit 7 shows the historical calendar year payments (by accident year) for the open claims **not** at the reinsurance retention. A calculation was then performed whereby payments for each accident year were trended by a 6% inflation factor to put all years at the 1997 accident year cost level. A weighted average for the 1984-1997 years was then determined using the number of open claims as weights.

Of importance is the apparent "U-Shaped" payment pattern seen on the graph associated with this group of open claims (Appendix C, Exhibit 7, Page 1). Because this is a forever changing body of claims, the information should be reviewed periodically. The spike in early year payment is consistent with claimants needing the initial costly surgeries following an injury, and the intensive physical therapy associated with treatment. Over time, there is a decrease in utilization, which may counteract the effects of medical inflation. The decrease appears to end about 11-12 years following the initial accident date where we begin to see a gentle rise in payments once again. It is very likely that as the population ages, they begin to need follow-up surgeries and additional treatments. As ABC has been in runoff for four years, we will look at payments beginning at year 5. After appearing to be at a low point at around year 11-12, there begins to be an increase in average payment.

The rise in payments beginning at year 11 can be looked at as a combination of two pieces: (1) those who need therapy only – this should go up annually by about the amount of people's wages or hourly costs~4% at this writing, and (2) those who need additional surgeries. Anecdotal data would suggest that surgeries and hospitalizations increase at a higher trend. We can think of a 6% escalation in loss trend as a combination of therapy cost increase with a virtual "cat load" of 2% for those needing surgeries or other additional costs.

To handle the "u-shaped" payment behavior, the model has observed that, for the case where all accident years are trended to the same point in time, the year 5 and year 16 payment average is approximately equal to the average of the years 5 through 16 payment average. In other words, historical medical payments show a relatively level average payment for years 5 through 16. Since ABC has been in runoff for four years, payments for the first four years would not be applicable to the current book of claims. The model will assume payments will be flat for calendar years 5-16, and then escalate payments beginning at calendar year 16 (in those scenarios where we apply escalation percentages). This procedure takes a summation of all projected calendar year payments for each claimant, considering each particular accident year.

The by accident year averages show very little difference among payments for accident years 1979-1991. As such, the procedure is simplified by taking a weighted average for calendar years 5-16 payments to apply to these accident years. For the years, which do not have 16 calendar years of payments yet, we have taken the average of years 5 through N, where N is the most recent calendar year. For accident years 1992 and 1993, we have determined the average of the payments for years 5 through N. Because there have been very few historical payments for accident years 1994-1997, we have trended the 1993 selected 5 through N average by the selected escalation percentages - at

6% for the 6% escalation scenario, and at 10% for both the 10% escalation scenario as well as the "10% year 17-21, 6% thereafter" scenario. These values will be used in the model during the escalation procedures.

The model takes the average monthly paid amount, by claim, through the first eight years following the date of the accident, and applies these amounts to the remaining months up to the first eight years. As our data shows that the first four years are among the highest payments because of the initial surgeries and comprehensive treatments, we would expect the first eight year average to be especially high for the claims whose accident dates are less than eight years ago. In ABC's case, since a 75% quota share applies to Accident Years 1995-1997, the impact of this conservatism in somewhat counteracted.

For the remaining months for years 9-16, the model takes the lesser of the monthly average of years 1-8 and the selected 5 through N average as determined previously – a weighted average for years 5-16 (or 5-N) for accident years 1979-1991, the actual 5-N average for accident years 1992 and 1993, and the trended average for accident years 1994-1997. It is possible that for the **second** eight years, there may be a shortfall in applying a lower payment for a particular claimant (first 8 year average) instead of the 5-16 all claim average. This should be offset in the cases where the individual claim average for the **first** eight years is higher than the 5-16 all claim average.

Beginning at age 17, we escalate the age 16 payments by the appropriate escalation scenario (assuming there is one) by accident year, and apply this procedure to each claimant. In the case where 10% escalation is assumed for years 17-21, and 6% thereafter, the payment for year 17 is 10% higher than the payment for year 16, the payment for year 18 is 10% higher than the payment for year 17, etc. The payment for year 22 should be only 6% higher than the payment for year 21, etc. In the scenario assuming no medical escalation, identical payments will be made for all years beginning with year 9.

2) Supplemental Indemnity Escalation

Permanent Total claims qualify for a supplemental indemnity (SI) benefit under Florida statute. The model adjusts reserves for Unlikely to Settle PT claims for the additional benefit to claimants.

The benefit allows a 5% yearly increase (additive, not compounded) to the base weekly wage. For instance, in year one, the increase would be 5% of the weekly wage; in year two it would be 10% and so forth. The benefit is eligible for all applicable recoveries including SDTF. For each accident year, the benefit would be limited to certain Statutory maximums. The model has performed a separate adjustment whereby eligible claims are looked at separately, and the monthly SI benefit is applied to the number of remaining months (subject to the maximums). Necessary adjustments are then applied for cessions where necessary. Appendix C, Exhibit 8 includes sample SI calculations.

Over time, the model can perform an adjustment to project claims, which will become PT claims, and then determine an SI adjustment for these. The same body of claims, however, may be eligible for a Social Security offset, and so there may be a negligible impact after offset. When only unlikely to settle claims remain, it may be a future exercise of the claims department to determine, which remaining claims have a realistic impact of reclassification. Following this determination, both SI and Social Security adjustments could be determined.

C. ABC Specific Adjustments (Outside the Claims Specific Model)

1. Background

Now that we have established Workers Compensation IBNR using the payment specific claims model, we will need to adjust the model's IBNR figure for items, which make sense to be considered outside the claims specific model. The items listed in the next section were not considered in the payment model proposed by Teng in his 2001 Reserve Call paper, but the resulting additional IBNR or valid recovery amounts would be necessary adjustments to consider prior to booking a final net IBNR provision.

2. Specific Adjustments

a. Subrogation

The model doesn't adjust for recoveries for subrogation. In the Workers Compensation line of business, salvage recoveries are rare or non-existent. Subrogation recoveries could result in the case of carrier dispute (for instance, in the case of disease manifestation or injury triggers), or line of business dispute (for instance, an automobile accident in the course of employment) just to give a couple of examples.

To arrive at anticipated subrogation recoveries, we have applied a traditional paid subrogation to paid loss and ALAE development approach (using annual link ratios) to derive an overall percentage of losses and ALAE, which will be subject to subrogation recoveries. We then applied the ultimate percentage to the ultimate IBNR (including ALAE). Based upon our scenario using 100% of carried reserves as the settlement amount for likely to settle claims, and 6% escalation after year 17 (Appendix C, Exhibit 4), the ultimate subrogation ratio of 1.9% is multiplied by ultimate IBNR of $1,590,341 to arrive at IBNR recovery of $30,216 (Appendix D).

An enhancement for companies with credible enough information would be to apply ultimate subrogation recovery ratios to IBNR for each accident year. Also, if specifics knowledge of recoveries are known or anticipated, adjustments could be made to each of the specific claimants (as well as seeing impact of cessions to reinsurers or the SDTF).

With reinsurance recoveries already being contemplated in the model, determining anticipated subrogation recoveries may not be quite as simple as reducing IBNR by the subrogation amount. Since subrogation recoveries inure to the benefit of reinsurers, taking both subrogation and anticipated reinsurance recoveries could in effect double count the anticipated recoveries. However, in the case where claim payments have exceeded the reinsurance limits, subrogation recoveries would serve to effectively reinstate a portion of the reinsurance limits. In its most basic case, if all subrogation recoveries pertained only to claims below the reinsurance attachment point, 100% of the anticipated subrogation recoveries could be applied to IBNR. If all subrogation recoveries effected claims already at the reinsurance attachment point or greater, 0% of the subrogation recoveries should be applied to IBNR. Should a particular company's practice be to not pursue subrogation recoveries as aggressively when a claimant is already in the reinsurance layer, they could consider applying a factor (such as 75%) to the anticipated subrogation recoveries to consider the possibility that some anticipated recoveries may potentially inure to the benefit of reinsurance recoveries.

b. New/Reopened Claims

A provision for new and reopened claims has been handled as an outside-the-model adjustment. As not all carriers are consistent in their definitions of new, reopened, or even closed claims, some entities may be more subject to additional claims going forward. For example, some entities may deem a claim closed if there is a lack of payment activity for a set number of months. Others may only treat a claim as closed if a complete release has been signed. Obviously the former entity would be expected to have more future activity from reopened claims than the latter.

New/IBNR Claims - As mentioned earlier, Workers Compensation claims have specific statute of limitations regarding reporting requirements for new claims. In most cases, ABC (in runoff for four years) will be able to deny payment for new claims on the grounds that they are being filed after the required statute of limitations for Florida. At this point in time, there can still be some exposure under Employers Liability – Section B as a result of third party over actions (for instance, if an employee gets hurt while using a product, sues the product manufacturer, who then countersues the employee's supervisor for negligence – this would be considered a "third party over action"). Exposure for Employers Liability is fairly remote with the passage of time. Although no specific adjustment has been made for IBNR claims (under Section B or otherwise), a provision could be put in as a conservative adjustment to the model.

Reopened Claims – Recent legal contract language and Florida Statutes have made reopening of closed claims a virtual impossibility, especially after a settlement offer has already been accepted by a claimant. Occasionally, courts may allow a reopening of a claim based on alleged fraud in settlement (not disclosing immediate necessary medical procedures upon settlement would be an example), but such an instance is viewed as the very rare exception rather than the rule.

More commonly, ABC occasionally receives a late medical procedure bill for services rendered prior to settlement. As such, a claim file is temporarily opened, a payment is rendered, and then the claim file will immediately be "re-closed". The provision for this type of adjustment has been added to the IBNR calculated on the model. Based on the most recent statistics, ABC has received a monthly average of $108,361 for reopened claims, with the yearly median decreasing by 15.68% annually. As seen on Exhibit E, this will result in another $6.991 Million of IBNR, calculated using a geometrical decay procedure.

c. Amounts in Excess of Reinsurance Limits

Given the mortality assumptions in the table used, and the average payments calculated per claimant, it was determined that no claims would pierce the reinsurance limit, and result in additional liability to ABC. However, the contingency that claimants may ultimately incur costs in excess of the reinsurance limit could exist under a few scenarios: (1) individual claimants with significant payments outliving the expected number of years, (2) modest deterioration of the large "override" claims, and (3) a current sound reinsurer may become impaired or insolvent at some point in the future.

To calculate the amount of liability for claims exceeding the reinsurance maximum, we assumed that each claimant would live to be 95 years old, and multiplied the amounts in excess of the reinsurance limit by the probability that a claimant would live to the age of 95. We have used this method as a shortcut approximation of an approach, which would multiply the payment for a given age of claimant by the probability that this claimant survives each passing year (up to age 95). Using our scenario of 100% of carried reserves as the settlement amount for likely to settle claims, and 6% escalation after year 17, we have discovered that three claimants would exceed the applicable maximum reinsurance thresholds. An additional $665,497 (calculated in Appendix F) has been added to the model's ultimate net IBNR.

Mathematically, a company with credible enough data could consider estimating these loss amounts by performing a stochastic simulation. However, parameter risk would most likely increase as the number of open claims begins to decline.

d. Migration of Claimants from 'Likely to Settle' to 'Unlikely to Settle'

Claimants have been assigned into the classifications of likely to settle and not likely to settle based on judgment of ABC's claims operations. As would be expected, as time passes, not all "likely" claimants settle, nor do all "unlikely" claimants continue to refuse settlement offers. We will also assume that our calculation will cover claimants who switch categories i.e. going from the "other" category (which would include permanent partial or temporary total classifications) into the "permanent total" category. Companies with more credible information may consider further studying this switch as an enhancement to the ABC model.

Based upon a calendar year analysis, discussions with the claims department, and judgment, we have determined that roughly 10% of claimants deemed as likely to settle will eventually be in the unlikely to settle category (this is a net figure with an assignment of +1 given to a claimant who switches from likely to unlikely and –1 given to a claimant who accepts a settlement after being classified as unlikely). Based upon our scenario using 100% of carried reserves as the settlement amount for likely to settle claims, and 6% escalation after year 17, we have determined that the average amount of net payment for likely to settle claims is $41,699 (equals $28,787,584 divided by 690). The similar calculation for the unlikely to settle claims gives an average net amount of $160,329 (equals $62,688,761 divided by 391). We have selected the approximate difference in these averages of $125,000 as the increase in severity due to the changing of classification. The final adjustment will multiply 10% of the number of likely claimants (690) by $125,000 equals 690*10%*$125,000=$8,625,000.

In this example, we have conservatively not considered the additional impact that reinsurance cessions may have on the classification switch. Companies with more credible data may want to consider switching from likely to unlikely by explicit category.

D. Possible Enhancements

While the assumptions used for the ABC non-traditional model are appropriate for an entity with such few open claims, additional endless possibilities could be used depending on data availability, credibility considerations, and management's tolerance for change.

1. BF Test A Priori to be used for Traditional Methods

If vast differences exist between the selected ultimates determined by traditional methods and those of the claim specific method, a company could aggregate the claim specific ultimates by accident year, and use these selections as the a priori in a traditional method Bornhuetter-Ferguson test.

The effect of this would be to smooth in the claim specific ultimates to the comfort level of management (or shareholders) while still being in a position to make refinements to ultimates for differences from expected results going forward.

2. Restatement of Loss Development Triangles to Reflect Experience of Open Claims Only

Historical loss development triangles can be developed whereby the experience shown is that of only the remaining open claimants. Loss development and BF methods can be applied to this triangle to determine ultimate loss amounts. The results of this method can be compared with those already determined by the claims specific model for reasonableness. Some reserving actuaries already employ similar procedures to remove commuted treaties and treaties at a loss "maximum cap" from their loss development history.

As mentioned previously, an appropriate provision should be considered here to reflect payments associated with the reopening of claims that were previously closed.

3. Accident Year by Calendar Year Average for Indemnity and ALAE

ABC has taken a monthly average, claim by claim, of average indemnity payments (monthly four year average for ALAE payments) to project the ultimate paid reserve. Companies with a greater number of unlikely to settle claimants may find this procedure too cumbersome, and think about taking an average by accident year for indemnity and ALAE, identical or similar to the procedure used by ABC for medical payments. States with more stringent statutory escalation for Cost of Living Adjustment (COLA) such as Massachusetts would need appropriate modifications.

4. By All Type splits

ABC, with a non-increasing number of claimants, looks to group all claimants into either the "likely to settle" or "unlikely to settle" category. Companies with a credible enough body of open claimants could consider refining the data set (PT Pending vs. Medical Maintenance vs. Other, etc.) to see if significantly different conclusions may be reached as to ultimate reserve amounts. These companies can also further study migration among classification types.

5. Hospital vs. Home Healthcare splits

Companies with credible enough data could look to see if different trends and development are visible when medical payment data is split between Hospital/Surgeon costs and Home Healthcare costs.

6. Actual vs. Expected Calculations

To test the assumptions of a claims specific model, the previous year's model can be run to assume each claimant will live only 12 more months. The expected and actual payments and reported losses can then be compared and refinements to assumptions made accordingly.

III. WORKERS COMPENSATION ALLOCATED LOSS ADJUSTMENT EXPENSE (ALAE) ISSUES

A. Background

As mentioned in an earlier section, the implementation of the Florida Tort Reforms of 1994 has impacted the ALAE development of Florida Workers Compensation carriers in general, and ABC in particular. Most companies, including ABC, saw large increases in their ALAE costs in Calendar Years 1996 and 1997; a direct result of the first push to reclassify claimants' injury types to the Permanent Total category (recall that Temporary Total and Temporary Partial benefits were to expire after 104 weeks of coverage - 1/1/1996 if classification began at 1/1/1994). It is our experience, that once a claim is considered a "PT Pending" claim, it would be a rare exception for it not to ultimately become a PT claim.

Although the initial push to file for additional benefits subsided following the 1996 and 1997 calendar years, there remains no statute of limitations for attempted reclassifications. However, one would expect that over time, a runoff entity like ABC will have fewer and fewer older non-PT claims that would have enough PT characteristics to eventually qualify for inclusion.

Additionally, a conscious effort was made by the ABC claims department beginning in 1999 to put more borderline claims into the PT category immediately, rather than pay additional ALAE costs to fight the classification. This manner is similar to the way some claims departments may never fight any claim they can settle for under a given amount, say $5,000.

Several factors may, in fact, lead to an exception to the actuarial Salzmann principle, which states, "The ratio of paid allocated loss expense to paid loss generally increases with the age of development." One obvious reason is that a lot of ALAE may be expended initially to reclassify a claim as PT. Once the reclass has taken place, there should be less ALAE expended unless a settlement is rendered. Also, over time, there will be fewer remaining non-PT claimants, and therefore, there will be fewer claims to pursue reclassification with.

Finally, as mentioned earlier, effective 10/1/2001 in Florida, a judge no longer needs to approve an agreed upon settlement for new claims. Additionally, judges would also be allowed to review legal expense for reasonableness. These changes should serve to cut down on ALAE going forward. It should also lead to ABC having fewer resolved claims at any given point in time, all things being equal.

B. ALAE in Claims Specific Model

Given the information in the previous section, it should be apparent that the paid ALAE pattern was somewhat erratic for the 1996 and 1997 calendar years, and should probably be less so going forward.

Many companies don't even establish case reserves for ALAE, booking only a bulk IBNR figure for their financial statements. Traditional ALAE reserving methodologies usually involve variations of Paid Loss development methods (Paid ALAE Development, Paid ALAE Bornhuetter-Ferguson, Paid ALAE/Paid Loss Development). In our Florida Workers Compensation case, we would need to adjust the development factors and expected ALAE ratios accordingly in some sort of judgmental fashion. Settlements of claims would create yet another distortion. The possibility of overstating the needed reserve would be very realistic. In fact, the ALAE case reserves for ABC have steadily declined each of the past two years, possibly the result of a lag in reflecting Industry conditions Post-1997. Finally, we should consider that as ABC's reported losses begin to pierce the applicable reinsurance retentions, we would expect more ALAE to be ceded under the reinsurance treaties of ABC (predominately pro-rata).

The claims specific model simplifies the process somewhat. Historical payments are tracked for the last 48 months for each individual claim. This monthly average is then multiplied by the number of remaining months to determine the ultimate ALAE reserve. Appropriate cession modifications are then made if future loss cessions would increase over time. Under some treaties, ALAE cessions under pro rata agreements are rounded down to the nearest whole percent in accordance with reinsurance agreements. For example, if 32.4% of losses were ceded under an "ALAE pro rata" reinsurance agreement, we would then cede 32.0% of ultimate ALAE. Adjustments have been made for the applicable treaties.

Until the end of 2001, the average ALAE payment will include some payments from the 1997 calendar year, which looked to be the tail end of the "big ALAE cost years" following the Tort Reforms of 1994. We could consider taking the latest three-year monthly average (1998-2000) or each of the latest three years averages multiplied by appropriate inflation factors. It is still unclear as to whether ALAE costs will routinely <u>increase</u> for a given claim especially if it is already classified as a PT claim, and there is no likely future settlement. Such a situation should have very little additional outside attorney involvement. In the specific claims model, we have judgmentally decided to cap each individual ALAE ultimate amount at two times the incurred to date figure. Our data shows cumulative ALAE development factors to ultimate are significantly below 2.000 for claims that are at least four years old.

IV. OTHER ISSUES

A. Unallocated Loss Adjustment Expense (ULAE)

1. Background

This section is intended to explain some of the outstanding issues, which arise in establishing ULAE reserves for runoff entities (though not necessarily comment on which one would be superior to others). Several theories exist, and are as varied as the claims reserving practice of the companies themselves or of the interpretations of the various State Insurance Departments. Some departments have even taken the approach that a runoff entity is only set up to handle claims reserves, and as such, ALL costs associated with the runoff operations should be considered as ULAE costs. Other departments have taken the approach that runoff companies are trying to reduce their handling costs, and would recommend applying a lower percentage of ULAE reserve than they had considered while an ongoing operation. There is also an Industry expectation that outsiders (attorneys, cedents, etc.) will pursue claim actions less aggressively with a runoff entity than they would with an ongoing operation. This should reduce ULAE costs, all things being equal.

In many cases, companies are looking to reduce overhead through reduction of employees or other tangible costs, and thus historical information may not always be a good indicator of the future. On the other side, companies may have voluntary attrition of its most capable claim-handling employees who may seek other longer-term opportunities. Such a situation could add additional costs and time as training of new employees becomes necessary. If a company looks to avoid employee attrition by providing retention bonuses to some of their workers, this will result in further ULAE costs.

2. Paid ULAE to Paid Loss Method

Traditional ULAE methodologies utilize the "paid ULAE to paid loss" methodology, whereby a specified percentage is applied to IBNR while half the percentage is applied to case reserves. The underlying thought process makes the assumption that half of ULAE cost is expended when a claim is first reported, while the other half is expended when a claim is closed. A company's case reserves have already been opened and would only need to be closed (the second half of the percentage). IBNR reserves need both the first and second half of the percentage (to both open and close a claim).

Under the traditional thinking, since ABC is in runoff and has only Workers Compensation reserves (which would not have IBNR claims and pipeline IBNR becomes rare over time), the formula would simplify to the selected ULAE percentage multiplied by the case reserves (factoring in something for development of known cases perhaps). This method could be distorted if a company uses substantially different "settlement value" case reserves than whatever source was used to derive the selected percentage (either Industry or the Company's historical figures). If credible enough data exists, companies can estimate ULAE percentages required to settle claims above or below a certain open case reserve thresholds ($50,000 for example), and look to apply this methodology separately for each monetary classification.

3. Reserve Based on Pre-Paid ULAE Costs

It is not uncommon for a runoff entity to hire a Third Party Administrator (TPA) to handle claim costs going forward. These costs will be reduced over time as claims become fewer and economies of scale become present. One would expect that costs to the TPA should start to decrease over time. A ULAE reserve methodology could thus be undertaken, which takes the current year's budgeted figure and decreases this amount for a set (or infinite) number of years going forward. The formula would use a geometric decay: Reserve = Year X Budget*(1/X%-1) where X is the selected yearly decrease in TPA budgetary cost. If a company determines budgetary costs based on number of open claims being handled, appropriate modifications can turn this method into a more familiar "frequency of claims*severity of payment" method in determining projected budget figures.

4. Industry ULAE/Reserve Percentages

One can determine an Industry ratio of ULAE reserves to either IBNR or total reserve, and then apply this percentage to the appropriate company denominator. If an Industry Schedule P is used from the latest Best's Aggregates and Averages (A&A) all company page, appropriate modifications should be used to assimilate the runoff entity. For example, since ABC has been in runoff for 3 years, the three most recent Schedule P years should be excluded from the A&A page in determining the selected percentage. If one were to believe that their own establishment of settlement reserves is done in a manner inconsistent with the Industry, additional steps would need to be taken.

As there is most likely great inconsistency among definitions of ULAE by company (even after the NAIC codification of 1/1/1998), one should be particularly cautious in using this methodology, and should certainly use this method in conjunction with other methods.

B. Duration for Economic Value of Company

It is sometimes necessary to derive the company's duration for economic value for many useful functions including Industry reporting requirements to A.M.Best or many financial management reports. While ongoing operations may be able to determine this figure by simply applying a selected payout or Industry payout pattern to arrive at future payment streams, runoff entities, and especially Workers Compensation runoff entities, will need to be more careful.

As seen in both the Industry payouts and ABC calendar year payments, Workers Compensation shows a pattern of heavy payment in the first couple of years, followed by a steadier stream of payments going forward. One can look at this as really consisting of two patterns: an initial payout pattern whereby the surgeries and expensive hospitalizations occur, and a secondary payout pattern whereby claimants receive consistent payouts for mostly rehabilitation and therapy costs.

As a result, a counter-intuitive observation of duration actually INCREASING for early years can be seen in the data. An example will show how this could occur. Suppose that a payout pattern followed a "bimodal hump" pattern whereby 96% is paid in the first year, followed by four successive years of 1% payment in each. Since duration is essentially a modified weighted average of payments, you would expect that the duration just before the first year would be close to 1.0, whereas after the first year, you would expect the value to lie between 2.0 and 3.0 (or between the second and third payment). If the calculation is done blindly for a runoff entity (for instance by using the Industry pattern starting at year one), we could arrive at a duration, which would not be realistic for a runoff entity.

One could consider: (1) using an average accident date for your book of business and applying the duration for this accident year only, and (2) calculating the duration for all accident years and then determining a weighted average using the total reserve amounts for each accident year. ABC, however, has now established a claims specific payment model. For likely to settle claims (a shrinking segment of the remaining claims), an assumption can be made as to future payouts. This can be as simple as assuming a uniform payout assumption over a given number of years. For the unlikely to settle claims, it would be a less judgmental exercise to simply divide all future payments into calendar year projections (under any or all escalation/recovery scenarios) and use these payment figures to determine not only an appropriate duration figure, but also any discount factors, which may be requested. Using the claims specific model would not only serve to provide more accurate and stable conclusions but also lead to more justifiable and defendable assumptions.

V. CONCLUSIONS

Traditional reserving methodologies, like all other actuarial methodologies, use historical data where available, supplemented with judgmental decisions. For a runoff situation, additional difficulty lies when remaining claimants and development patterns may be drastically different than those seen in historical data. Such items may lead one to consider setting IBNR using non-traditional reserving methods.

This paper has introduced some practical approaches for estimating reserves, for an operation, which has recently been subject to both internal and external changes. The reserving model introduced shows how a runoff entity has handled distortions, which could be significant when applying traditional methods to a dwindling book of outstanding claims. While the methods shown are not panaceas for all problems and situations associated with runoff reserving certainties and uncertainties, these methods, in conjunction with the traditional methods and actuarial judgment, can be used for the purposes of many business requirements and scopes of assignment.

Although a Florida Workers Compensation runoff situation has been used as the base model, many of the methods and assumptions used, especially in regard to items such as inuring reinsurance arrangements, can be applied to any operation. A runoff example is not unique when one thinks of an ongoing situation as consisting of two segments –

(1) the more recent years of less mature data, and (2) older groups of data, which can be considered similar to a runoff entity. Obvious adjustments will need to be made if one assumes that claimants perceive ongoing enterprises as having "deep pockets" and thus pursue claims less aggressively with runoff companies.

Methods shown at the very least can be viewed as reasonability "tests" for a company's traditional reserving methodologies. If an actuary would need to explain results to management or outside agencies, new methods can be used to supplement traditional methods. From an internal perspective, management may look to see how varying scenarios would impact their own bottom line financial figures.

The intent of the paper was to not only take the readers through this particular situational example, but rather to encourage the thought process as to what items may need to be considered for an entity's own case. In particular, Workers Compensation carriers may be subject to substantially different State laws and regulations; additionally, case-reserving philosophies among different companies' claims departments can be significantly different. It is the author's hope that the items discussed within can be readily adaptable to whatever situations may happen to arise in day-to-day operations.

Most importantly, no reserving method should ever be used in complete isolation from all others. Integration among other methodologies, systems departments, and claims operations is critical for the development of appropriate reserves for management. Considerable judgment will need to be employed in using non-traditional methods, but scenario testing of assumptions is a reasonable way to gain a comfort level for variation of ultimate reserve level. It is the ongoing thought process of applying different methods to each individual scenario, which will be instrumental in providing reasonable and justifiable conclusions given less than typical situations.

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APPENDIX

This Appendix documents the assumptions and methods underlying the abstract.

Background on 1994 Florida Tort Reforms and Special Disability Trust Fund– APPENDIX A

Calculation of Final IBNR Amount– APPENDIX B

Backup to Workers Compensation Claims Development Model – APPENDIX C

Calculation of Subrogation Recovery – APPENDIX D

Calculation of New/Reopened Claims Provision – APPENDIX E

Calculation of IBNR Amounts Exceeding Reinsurance Limits – APPENDIX F

Sample ULAE Calculations – APPENDIX G

Duration of ABC Insurance Company – APPENDIX H

Background on 1994 Florida Tort Reforms and Special Disability Trust Fund –

Appendix A

1. Key Changes to Florida Tort Reforms – 1/1/1994

Background - While the State of Florida enacted legislation effective January 1, 1994 with the intention of containing future Workers Compensation costs; the overall implementation has proven to have quite the opposite effect from what was originally intended. This Appendix will give some background on key reforms, and try to show why costs have risen not only for losses, but also for loss adjustment expense, and internal claims operations.

Workers Compensation benefits are determined by statute, and therefore, in theory should be a no-fault type of coverage. As will be seen, the Tort Reforms of 1994 gave ample opportunity to aggressively pursue (driving up ALAE costs) both better benefit reclassifications as well as more lucrative settlement opportunities (which drove up costs).

Changes in Benefits by Category – Beginning with the 1994 Reforms, *Temporary Total* (TT) benefits were reduced from a maximum duration of 260 months or the classification of Maximum Medical Improvement (MMI) to 104 weeks, regardless of whether or not MMI was declared by a physician. After the 104 weeks expired on 1/1/1996 to extend benefits, many filings took place to push for more reclassifications into Permanent Total categories. Reduced benefits after 104 weeks were subsequently awarded.

Beginning in 1994, *Permanent Partial* (PP) benefits had the lost time payments reduced from 66 2/3% of the Average Weekly Wage (AWW) to 33 1/3% of the AWW.

An attempt was made to restrict those claimants who were eligible for *Permanent Total* (PT) benefits, providing a list of eligible injury classifications as well as admittance if a claimant was already accepted for Social Security Disability benefits (SSDI).

Since the reforms, judges have begun to allow more liberal interpretations of PT eligibility with many precedents being set statewide. In many cases, judges allowed for a "temporary" allowance of Permanent Total benefits pending future observation. This becomes a difficult item to overturn as time goes on. Additionally, Social Security **potential** for acceptance, instead of **actual** acceptance has become the norm for reclassification rather than the exception.

As a result, the statistics in the state of Florida show a greater frequency of PT claims, and higher severity for its TT and PP claims. It has been theorized that originally classified TT and PP claimants may more aggressively pursue expensive treatments early on (driving up costs) in the hopes that they can be better positioned for PT classification in the future (driving up frequency). Florida PT claims also show a lower severity than the Industry, in part because many of these claimants would not be as seriously injured as PT claimants in other states (injury impairment would be considered TT or PP in other states, which would have lower severities). The costs of reclassification have contributed to Florida having an ALAE percentage of loss, which is significantly higher than the Countrywide average.

Alternative Dispute Resolution – An Office of Employee Assistance (OEA) was established to attempt to cut down on the time and costs of attorney involvement. In actuality, time and attorney costs actually ended up increasing. With many petitions being filed with the OEA, the system was not able to function without large operational costs, which sometimes resulted in retribution to attorneys for the costs of submitting the petitions.

Allowance of Medical Settlements – Prior to 1994, entities were allowed to settle indemnity (lost wages) portions of a claim, but not medical loss or loss adjustment expense. Beginning 1/1/1994, companies were allowed to settle the entire Workers Compensation claim, or any portion thereof (medical or lost wage). The allowance of additional settlements may have contributed to additional ALAE costs in the state of Florida.

2. Special Disability Trust Fund (SDTF)

The Special Disability Trust Fund (SDTF) reimburses the employer when an employee suffers an injury, directly or indirectly, as a result of a previous disability. If an employee with a pre-existing injured back was forced to take a job as a salesperson instead of a manual laborer, a subsequent injury in a car accident may make the employee eligible for SDTF recoveries. SDTF recoveries inure to the benefit of all additional reinsurance coverage.

The Florida SDTF operates on a pay-as-you-go basis whereby each Florida Workers Compensation writer contributes a percentage of current premium writings (4.5% currently).

Beginning in 1994, the State of Florida made a handful of revisions to the rules of SDTF benefit. Whereas accidents occurring prior to 1/1/1994 would receive full benefit of SDTF recovery, those accidents taking place following 1/1/1994 would only receive 50% of the eligible benefit recovery. Accidents taking place beginning 1/1/1998 are unable to receive any benefit from the SDTF. Also beginning in 1994, injuries must meet a verbal threshold of eligible injuries, in addition to a monetary deductible threshold of $10,000 per claimant.

In order to receive benefits, an affidavit must first be filed stating that the employer knew, in advance, of the ailment or previous medical condition. After securing the affidavit, a medical opinion is needed stating that the subsequent injury was made worse as a result of the pre-existing condition. Finally, a proof of claim needs to be submitted to the SDTF whereby they will have 90 days to approve or disapprove a claim for recovery. Once a claim is approved, it is very rare that a subsequent disapproval will take place.

The SDTF will pay for paid claims on a first come, first served basis. As additional payments are made on case reserves or development on known cases, additional papers (called SDF-2 forms) will need to be filed with the State.

Currently, the time from notice of claim until recovery from the Fund is received is a little more than three years as recouping assessments lag behind payment of benefits.

Calculation of Final IBNR Amount -

Appendix B

Final ABC IBNR calculation including Adustments not covered by Claims Specific Workers Compensation Model Appendix B

Scenario: (Likely to Settle at 100% of Current Reserves with 10% medical escalation years 17-21, 6% escalation thereafter) Exhibit 1

Segment	IBNR Addition/(Subtraction)
Claims Specific Model IBNR	$1.590
Subrogation	($0.030)
True IBNR Claims/Pipeline IBNR/Reopened Claims Adjustment	$6.991
Net migration of claims from Likely to Settle to Unlikely to Settle (and vice versa), etc.	$8.625
Adjustment for exceeding of reinsurance limits or impaired reinsurance	$0.665
TOTAL – All Additional Items	**$17.841**

Backup to Workers Compensation Claims Development Model -

Appendix C

ABC INSURANCE COMPANY

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Maximum Exposure with no medical escalation)

		Maximum Exposure (A)				
	# Claims	Direct	SDF-Reqstd	SDF-on Res	Ceded	Net
Closed Claims	26	0	597,525	0	(98,635)	(498,890)
Resolved Claims	1,429	8,081,983	43,385,627	0	(3,354,927)	(31,948,717)
Coverage B/coverage issues	16	702,191	0	0	449,367	252,824
Special Disability accepted claims						
Likely to Settle	99	62,080,400	4,490,180	38,964,140	11,692,622	6,933,458
Unlikely to Settle	97	39,353,217	6,650,415	27,722,435	4,026,957	953,410
Total special disability accepted claims	196	101,433,617	11,140,595	66,686,575	15,719,579	7,886,868
Maximum reinsurer reserve claims						
Likely to Settle	39	47,668,159	0	0	29,512,506	18,155,653
Unlikely to Settle	63	54,713,811	0	0	37,866,350	16,847,461
Total maximum reinsurer reserve claims	102	102,381,970	0	0	67,378,856	35,003,114
Likely to Settle						
PT	76	68,821,993	0	0	39,084,031	29,737,962
Not at MMI	100	23,600,739	0	0	10,672,883	12,927,856
Medical Maintenance	96	8,630,703	0	0	266,624	8,364,079
PT Pending	33	14,468,232	0	0	6,624,531	7,843,701
Other	385	92,228,143	0	0	40,157,993	52,070,150
Total Likely to Settle	690	207,749,810	0	0	96,806,062	110,943,748
Unlikely to Settle						
PT	45	22,137,855	0	0	7,906,520	14,231,335
Not at MMI	33	3,675,759	0	0	563,048	3,112,711
Medical Maintenance	159	10,299,084	0	0	144,499	10,154,585
PT Pending	5	1,512,003	0	0	499,372	1,012,631
Other	149	11,078,570	0	0	2,085,450	8,993,120
Total Unlikely to Settle	391	48,703,271	0	0	11,198,889	37,504,382
Grand Total	2,850	469,052,842	55,123,747	66,686,575	188,099,191	159,143,329
Likely to settle	828	317,498,369	4,490,180	38,964,140	138,011,190	136,032,859
Unlikely to settle	551	142,770,299	6,650,415	27,722,435	53,092,196	55,305,253
Grand total (excl closed/cov B/resolved)	1,379	460,268,668	11,140,595	66,686,575	191,103,386	191,338,112

ABC INSURANCE COMPANY

Appendix C

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Exhibit 1, Page 2

Summary (Maximum Exposure with no medical escalation)

		As Reported (B)				Difference = (A)-(B)			
	# Claims	Direct	SDF	Ceded	Net	Direct	SDF	Ceded	Net
Closed Claims	26	13,876	0	10,091	3,785	(13,876)	597,525	(108,726)	(502,675)
Resolved Claims	1,429	18,111,596	0	5,297,462	12,814,134	(10,029,613)	43,385,627	(8,652,389)	(44,762,851)
Coverage B/coverage issues	16	702,191	0	449,367	252,824	0	0	0	0
Special Disability accepted claims									
Likely to Settle	99	13,484,539	0	5,674,656	7,809,883	48,595,861	43,454,320	6,017,966	(876,425)
Unlikely to Settle	97	15,060,904	0	4,903,268	10,157,636	24,292,313	34,372,850	(876,311)	(9,204,226)
Total special disability accepted claims	196	28,545,443	0	10,577,924	17,967,519	72,888,174	77,827,170	5,141,655	(10,080,651)
Maximum reinsurer reserve claims									
Likely to Settle	39	11,053,502	0	5,217,631	5,835,871	36,614,657	0	24,294,875	12,319,782
Unlikely to Settle	63	20,833,189	0	13,729,879	7,103,310	33,880,622	0	24,136,471	9,744,151
Total maximum reinsurer reserve claims	102	31,886,691	0	18,947,510	12,939,181	70,495,279	0	48,431,346	22,063,933
Likely to Settle									
PT	76	12,706,345	0	3,653,508	9,052,837	56,115,648	0	35,430,523	20,685,125
Not at MMI	100	4,024,591	0	1,209,992	2,814,599	19,576,148	0	9,462,891	10,113,257
Medical Maintenance	96	3,559,253	0	0	3,559,253	5,071,450	0	266,624	4,804,826
PT Pending	33	3,124,530	0	739,757	2,384,773	11,343,702	0	5,884,774	5,458,928
Other	385	16,781,741	0	5,295,060	11,486,681	75,446,402	0	34,862,933	40,583,469
Total Likely to Settle	690	40,196,460	0	10,898,317	29,298,143	167,553,350	0	85,907,745	81,645,605
Unlikely to Settle									
PT	45	8,621,601	0	1,590,017	7,031,584	13,516,254	0	6,316,503	7,199,751
Not at MMI	33	996,314	0	169,936	826,378	2,679,445	0	393,112	2,286,333
Medical Maintenance	159	5,755,231	0	2,792	5,752,439	4,543,853	0	141,707	4,402,146
PT Pending	5	324,745	0	10,835	313,910	1,187,258	0	488,537	698,721
Other	149	5,584,773	0	985,170	4,599,603	5,493,797	0	1,100,280	4,393,517
Total Unlikely to Settle	391	21,282,664	0	2,758,750	18,523,914	27,420,607	0	8,440,139	18,980,468
Grand Total	2,850	140,738,921	0	48,939,421	91,799,500	328,313,921	121,810,322	139,159,770	67,343,829
Likely to settle	828	64,734,501	0	21,790,604	42,943,897	252,763,868	43,454,320	116,220,586	93,088,962
Unlikely to settle	551	57,176,757	0	21,391,897	35,784,860	85,593,542	34,372,850	31,700,299	19,520,393
Grand total (excl closed/cov B/resolved)	1,379	121,911,258	0	43,182,501	78,728,757	338,357,410	77,827,170	147,920,885	112,609,355

ABC INSURANCE COMPANY
Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 80% of Current Reserves with no medical escalation)

		Likely Exposure @80% of Current Reserves (A)				
	# Claims	Direct	SDF-Reqstd	SDF-on Res	Ceded	Net
Closed Claims	26	0	597,525	0	(98,635)	(498,890)
Resolved Claims	1,429	8,081,983	43,385,627	0	(3,354,927)	(31,948,717)
Coverage B/coverage issues	16	702,191	0	0	449,367	252,824
Special Disability accepted claims						
Likely to Settle	99	10,446,757	4,490,180	6,590,868	579,958	(1,214,249)
Unlikely to Settle	97	39,353,217	6,650,415	27,722,435	4,026,957	953,410
Total special disability accepted claims	196	49,799,974	11,140,595	34,313,303	4,606,915	(260,839)
Maximum reinsurer reserve claims						
Likely to Settle	39	8,842,802	0	0	3,748,859	5,093,943
Unlikely to Settle	63	54,713,811	0	0	37,866,350	16,847,461
Total maximum reinsurer reserve claims	102	63,556,613	0	0	41,615,209	21,941,404
Likely to Settle						
PT	76	10,165,076	0	0	2,922,807	7,242,269
Not at MMI	100	3,219,673	0	0	967,993	2,251,680
Medical Maintenance	96	2,847,402	0	0	0	2,847,402
PT Pending	33	2,499,624	0	0	591,806	1,907,818
Other	385	13,425,393	0	0	4,236,048	9,189,345
Total Likely to Settle	690	32,157,168	0	0	8,718,654	23,438,514
Unlikely to Settle						
PT	45	22,137,855	0	0	7,906,520	14,231,335
Not at MMI	33	3,675,759	0	0	563,048	3,112,711
Medical Maintenance	159	10,299,084	0	0	144,499	10,154,585
PT Pending	5	1,512,003	0	0	499,372	1,012,631
Other	149	11,078,570	0	0	2,085,450	8,993,120
Total Unlikely to Settle	391	48,703,271	0	0	11,198,889	37,504,382
Grand Total	2,850	203,001,200	55,123,747	34,313,303	63,135,472	50,428,678
Likely to settle	828	51,446,727	4,490,180	6,590,868	13,047,471	27,318,208
Unlikely to settle	551	142,770,299	6,650,415	27,722,435	53,092,196	55,305,253
Grand total (excl closed/cov B/resolved)	1,379	194,217,026	11,140,595	34,313,303	66,139,667	82,623,461

ABC INSURANCE COMPANY
Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 80% of Current Reserves with no medical escalation)

		As Reported (B)				Difference = (A)-(B)			
	# Claims	Direct	SDF	Ceded	Net	Direct	SDF	Ceded	Net
Closed Claims	26	13,876	0	10,091	3,785	(13,876)	597,525	(108,726)	(502,675)
Resolved Claims	1,429	18,111,596	0	5,297,462	12,814,134	(10,029,613)	43,385,627	(8,652,389)	(44,762,851)
Coverage B/coverage issues	16	702,191	0	449,367	252,824	0	0	0	0
Special Disability accepted claims									
Likely to Settle	99	13,484,539	0	5,674,656	7,809,883	(3,037,782)	11,081,048	(5,094,698)	(9,024,132)
Unlikely to Settle	97	15,060,904	0	4,903,268	10,157,636	24,292,313	34,372,850	(876,311)	(9,204,226)
Total special disability accepted claims	196	28,545,443	0	10,577,924	17,967,519	21,254,531	45,453,898	(5,971,009)	(18,228,358)
Maximum reinsurer reserve claims									
Likely to Settle	39	11,053,502	0	5,217,631	5,835,871	(2,210,700)	0	(1,468,772)	(741,928)
Unlikely to Settle	63	20,833,189	0	13,729,879	7,103,310	33,880,622	0	24,136,471	9,744,151
Total maximum reinsurer reserve claims	102	31,886,691	0	18,947,510	12,939,181	31,669,922	0	22,667,699	9,002,223
Likely to Settle									
PT	76	12,706,345	0	3,653,508	9,052,837	(2,541,269)	0	(730,701)	(1,810,568)
Not at MMI	100	4,024,591	0	1,209,992	2,814,599	(804,918)	0	(241,999)	(562,919)
Medical Maintenance	96	3,559,253	0	0	3,559,253	(711,851)	0	0	(711,851)
PT Pending	33	3,124,530	0	739,757	2,384,773	(624,906)	0	(147,951)	(476,955)
Other	385	16,781,741	0	5,295,060	11,486,681	(3,356,348)	0	(1,059,012)	(2,297,336)
Total Likely to Settle	690	40,196,460	0	10,898,317	29,298,143	(8,039,292)	0	(2,179,663)	(5,859,629)
Unlikely to Settle									
PT	45	8,621,601	0	1,590,017	7,031,584	13,516,254	0	6,316,503	7,199,751
Not at MMI	33	996,314	0	169,936	826,378	2,679,445	0	393,112	2,286,333
Medical Maintenance	159	5,755,231	0	2,792	5,752,439	4,543,853	0	141,707	4,402,146
PT Pending	5	324,745	0	10,835	313,910	1,187,258	0	488,537	698,721
Other	149	5,584,773	0	985,170	4,599,603	5,493,797	0	1,100,280	4,393,517
Total Unlikely to Settle	391	21,282,664	0	2,758,750	18,523,914	27,420,607	0	8,440,139	18,980,468
Grand Total	2,850	140,738,921	0	48,939,421	91,799,500	62,262,279	89,437,050	14,196,051	(41,370,822)
Likely to settle	828	64,734,501	0	21,790,604	42,943,897	(13,287,774)	11,081,048	(8,743,133)	(15,625,689)
Unlikely to settle	551	57,176,757	0	21,391,897	35,784,860	85,593,542	34,372,850	31,700,299	19,520,393
Grand total (excl closed/cov B/resolved)	1,379	121,911,258	0	43,182,501	78,728,757	72,305,768	45,453,898	22,957,166	3,894,704

ABC INSURANCE COMPANY

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 100% of Current Reserves with no medical escalation)

		Likely Exposure @100% of Current Reserves (A)				
	# Claims	Direct	SDF-Reqstd	SDF-on Res	Ceded	Net
Closed Claims	26	0	597,525	0	(98,635)	(498,890)
Resolved Claims	1,429	8,081,983	43,385,627	0	(3,354,927)	(31,948,717)
Coverage B/coverage issues	16	702,191	0	0	449,367	252,824
Special Disability accepted claims						
Likely to Settle	99	13,058,446	4,490,180	8,238,585	1,104,506	(774,825)
Unlikely to Settle	97	39,353,217	6,650,415	27,722,435	4,026,957	953,410
Total special disability accepted claims	196	52,411,663	11,140,595	35,961,020	5,131,463	178,585
Maximum reinsurer reserve claims						
Likely to Settle	39	10,988,688	0	0	5,213,543	5,775,145
Unlikely to Settle	63	54,713,811	0	0	37,866,350	16,847,461
Total maximum reinsurer reserve claims	102	65,702,499	0	0	43,079,893	22,622,606
Likely to Settle						
PT	76	12,706,345	0	0	3,653,508	9,052,837
Not at MMI	100	4,024,591	0	0	1,209,992	2,814,599
Medical Maintenance	96	3,048,694	0	0	0	3,048,694
PT Pending	33	3,124,530	0	0	739,757	2,384,773
Other	385	16,781,741	0	0	5,295,060	11,486,681
Total Likely to Settle	690	39,685,901	0	0	10,898,317	28,787,584
Unlikely to Settle						
PT	45	22,137,855	0	0	7,906,520	14,231,335
Not at MMI	33	3,675,759	0	0	563,048	3,112,711
Medical Maintenance	159	10,299,084	0	0	144,499	10,154,585
PT Pending	5	1,512,003	0	0	499,372	1,012,631
Other	149	11,078,570	0	0	2,085,450	8,993,120
Total Unlikely to Settle	391	48,703,271	0	0	11,198,889	37,504,382
Grand Total	2,850	215,287,508	55,123,747	35,961,020	67,304,367	56,898,374
Likely to settle	828	63,733,035	4,490,180	8,238,585	17,216,366	33,787,904
Unlikely to settle	551	142,770,299	6,650,415	27,722,435	53,092,196	55,305,253
Grand total (excl closed/cov B/resolved)	1,379	206,503,334	11,140,595	35,961,020	70,308,562	89,093,157

ABC INSURANCE COMPANY

Appendix C

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Exhibit 3, Page 2

Summary (Likely to Settle at 100% of Current Reserves with no medical escalation)

		As Reported (B)				Difference = (A)-(B)			
	# Claims	Direct	SDF	Ceded	Net	Direct	SDF	Ceded	Net
Closed Claims	26	13,876	0	10,091	3,785	(13,876)	597,525	(108,726)	(502,675)
Resolved Claims	1,429	18,111,596	0	5,297,462	12,814,134	(10,029,613)	43,385,627	(8,652,389)	(44,762,851)
Coverage B/coverage issues	16	702,191	0	449,367	252,824	0	0	0	0
Special Disability accepted claims									
Likely to Settle	99	13,484,539	0	5,674,656	7,809,883	(426,093)	12,728,765	(4,570,150)	(8,584,708)
Unlikely to Settle	97	15,060,904	0	4,903,268	10,157,636	24,292,313	34,372,850	(876,311)	(9,204,226)
Total special disability accepted claims	196	28,545,443	0	10,577,924	17,967,519	23,866,220	47,101,615	(5,446,461)	(17,788,934)
Maximum reinsurer reserve claims									
Likely to Settle	39	11,053,502	0	5,217,631	5,835,871	(64,814)	0	(4,088)	(60,726)
Unlikely to Settle	63	20,833,189	0	13,729,879	7,103,310	33,880,622	0	24,136,471	9,744,151
Total maximum reinsurer reserve claims	102	31,886,691	0	18,947,510	12,939,181	33,815,808	0	24,132,383	9,683,425
Likely to Settle									
PT	76	12,706,345	0	3,653,508	9,052,837	0	0	0	0
Not at MMI	100	4,024,591	0	1,209,992	2,814,599	0	0	0	0
Medical Maintenance	96	3,559,253	0	0	3,559,253	(510,559)	0	0	(510,559)
PT Pending	33	3,124,530	0	739,757	2,384,773	0	0	0	0
Other	385	16,781,741	0	5,295,060	11,486,681	0	0	0	0
Total Likely to Settle	690	40,196,460	0	10,898,317	29,298,143	(510,559)	0	0	(510,559)
Unlikely to Settle									
PT	45	8,621,601	0	1,590,017	7,031,584	13,516,254	0	6,316,503	7,199,751
Not at MMI	33	996,314	0	169,936	826,378	2,679,445	0	393,112	2,286,333
Medical Maintenance	159	5,755,231	0	2,792	5,752,439	4,543,853	0	141,707	4,402,146
PT Pending	5	324,745	0	10,835	313,910	1,187,258	0	488,537	698,721
Other	149	5,584,773	0	985,170	4,599,603	5,493,797	0	1,100,280	4,393,517
Total Unlikely to Settle	391	21,282,664	0	2,758,750	18,523,914	27,420,607	0	8,440,139	18,980,468
Grand Total	2,850	140,738,921	0	48,939,421	91,799,500	74,548,587	91,084,767	18,364,946	(34,901,126)
Likely to settle	828	64,734,501	0	21,790,604	42,943,897	(1,001,466)	12,728,765	(4,574,238)	(9,155,993)
Unlikely to settle	551	57,176,757	0	21,391,897	35,784,860	85,593,542	34,372,850	31,700,299	19,520,393
Grand total (excl closed/cov B/resolved)	1,379	121,911,258	0	43,182,501	78,728,757	84,592,076	47,101,615	27,126,061	10,364,400

ABC INSURANCE COMPANY

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 100% of Current Reserves with 6% medical escalation beginning year 17)

		Likely Exposure @100% of Current Reserves (A)				
	# Claims	Direct	SDF-Reqstd	SDF-on Res	Ceded	Net
Closed Claims	26	0	597,525	0	(98,635)	(498,890)
Resolved Claims	1,429	8,081,983	43,385,627	0	(3,354,927)	(31,948,717)
Coverage B/coverage issues	16	702,191	0	0	449,367	252,824
Special Disability accepted claims						
Likely to Settle	99	13,058,446	4,490,180	8,238,585	1,104,506	(774,825)
Unlikely to Settle	97	46,842,746	6,650,415	34,428,568	4,277,950	1,485,813
Total special disability accepted claims	196	59,901,192	11,140,595	42,667,153	5,382,456	710,988
Maximum reinsurer reserve claims						
Likely to Settle	39	10,988,688	0	0	5,213,543	5,775,145
Unlikely to Settle	63	67,850,440	0	0	48,264,753	19,585,687
Total maximum reinsurer reserve claims	102	78,839,128	0	0	53,478,296	25,360,832
Likely to Settle						
PT	76	12,706,345	0	0	3,653,508	9,052,837
Not at MMI	100	4,024,591	0	0	1,209,992	2,814,599
Medical Maintenance	96	3,048,694	0	0	0	3,048,694
PT Pending	33	3,124,530	0	0	739,757	2,384,773
Other	385	16,781,741	0	0	5,295,060	11,486,681
Total Likely to Settle	690	39,685,901	0	0	10,898,317	28,787,584
Unlikely to Settle						
PT	45	26,008,242	0	0	10,561,657	15,446,585
Not at MMI	33	6,282,042	0	0	1,268,292	5,013,750
Medical Maintenance	159	23,572,808	0	0	1,203,715	22,369,093
PT Pending	5	2,298,182	0	0	765,008	1,533,174
Other	149	22,457,876	0	0	4,131,716	18,326,160
Total Unlikely to Settle	391	80,619,150	0	0	17,930,388	62,688,762
Grand Total	2,850	267,829,545	55,123,747	42,667,153	84,685,262	85,353,383
Likely to settle	828	63,733,035	4,490,180	8,238,585	17,216,366	33,787,904
Unlikely to settle	551	195,312,336	6,650,415	34,428,568	70,473,091	83,760,262
Grand total (excl closed/cov B/resolved)	1,379	259,045,371	11,140,595	42,667,153	87,689,457	117,548,166

ABC INSURANCE COMPANY

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 100% of Current Reserves with 6% medical escalation beginning year 17)

		As Reported (B)				Difference = (A)-(B)			
	# Claims	Direct	SDF	Ceded	Net	Direct	SDF	Ceded	Net
Closed Claims	26	13,876	0	10,091	3,785	(13,876)	597,525	(108,726)	(502,675)
Resolved Claims	1,429	18,111,596	0	5,297,462	12,814,134	(10,029,613)	43,385,627	(8,652,389)	(44,762,851)
Coverage B/coverage issues	16	702,191	0	449,367	252,824	0	0	0	0
Special Disability accepted claims									
Likely to Settle	99	13,484,539	0	5,674,656	7,809,883	(426,093)	12,728,765	(4,570,150)	(8,584,708)
Unlikely to Settle	97	15,060,904	0	4,903,268	10,157,636	31,781,842	41,078,983	(625,318)	(8,671,823)
Total special disability accepted claims	196	28,545,443	0	10,577,924	17,967,519	31,355,749	53,807,748	(5,195,468)	(17,256,531)
Maximum reinsurer reserve claims									
Likely to Settle	39	11,053,502	0	5,217,631	5,835,871	(64,814)	0	(4,088)	(60,726)
Unlikely to Settle	63	20,833,189	0	13,729,879	7,103,310	47,017,251	0	34,534,874	12,482,377
Total maximum reinsurer reserve claims	102	31,886,691	0	18,947,510	12,939,181	46,952,437	0	34,530,786	12,421,651
Likely to Settle									
PT	76	12,706,345	0	3,653,508	9,052,837	0	0	0	0
Not at MMI	100	4,024,591	0	1,209,992	2,814,599	0	0	0	0
Medical Maintenance	96	3,559,253	0	0	3,559,253	(510,559)	0	0	(510,559)
PT Pending	33	3,124,530	0	739,757	2,384,773	0	0	0	0
Other	385	16,781,741	0	5,295,060	11,486,681	0	0	0	0
Total Likely to Settle	690	40,196,460	0	10,898,317	29,298,143	(510,559)	0	0	(510,559)
Unlikely to Settle									
PT	45	8,621,601	0	1,590,017	7,031,584	17,386,641	0	8,971,640	8,415,001
Not at MMI	33	996,314	0	169,936	826,378	5,285,728	0	1,098,356	4,187,372
Medical Maintenance	159	5,755,231	0	2,792	5,752,439	17,817,577	0	1,200,923	16,616,654
PT Pending	5	324,745	0	10,835	313,910	1,973,437	0	754,173	1,219,264
Other	149	5,584,773	0	985,170	4,599,603	16,873,103	0	3,146,546	13,726,557
Total Unlikely to Settle	391	21,282,664	0	2,758,750	18,523,914	59,336,486	0	15,171,638	44,164,848
Grand Total	2,850	140,738,921	0	48,939,421	91,799,500	127,090,624	97,790,900	35,745,841	(6,446,117)
Likely to settle	828	64,734,501	0	21,790,604	42,943,897	(1,001,466)	12,728,765	(4,574,238)	(9,155,993)
Unlikely to settle	551	57,176,757	0	21,391,897	35,784,860	138,135,579	41,078,983	49,081,194	47,975,402
Grand total (excl closed/cov B/resolved)	1,379	121,911,258	0	43,182,501	78,728,757	137,134,113	53,807,748	44,506,956	38,819,409

ABC INSURANCE COMPANY

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 100% of Current Reserves with 10% medical escalation years 17-21, 6% thereafter)

		Likely Exposure @100% of Current Reserves (A)				
	# Claims	Direct	SDF-Reqstd	SDF-on Res	Ceded	Net
Closed Claims	26	0	597,525	0	(98,635)	(498,890)
Resolved Claims	1,429	8,081,983	43,385,627	0	(3,354,927)	(31,948,717)
Coverage B/coverage issues	16	702,191	0	0	449,367	252,824
Special Disability accepted claims						
Likely to Settle	99	13,058,446	4,490,180	8,238,585	1,104,506	(774,825)
Unlikely to Settle	97	49,357,873	6,650,415	36,627,494	4,411,618	1,668,346
Total special disability accepted claims	196	62,416,319	11,140,595	44,866,079	5,516,124	893,521
Maximum reinsurer reserve claims						
Likely to Settle	39	10,988,688	0	0	5,213,543	5,775,145
Unlikely to Settle	63	71,720,199	0	0	51,443,867	20,276,332
Total maximum reinsurer reserve claims	102	82,708,887	0	0	56,657,410	26,051,477
Likely to Settle						
PT	76	12,706,345	0	0	3,653,508	9,052,837
Not at MMI	100	4,024,591	0	0	1,209,992	2,814,599
Medical Maintenance	96	3,048,694	0	0	0	3,048,694
PT Pending	33	3,124,530	0	0	739,757	2,384,773
Other	385	16,781,741	0	0	5,295,060	11,486,681
Total Likely to Settle	690	39,685,901	0	0	10,898,317	28,787,584
Unlikely to Settle						
PT	45	27,390,591	0	0	11,609,845	15,780,746
Not at MMI	33	7,132,126	0	0	1,549,889	5,582,237
Medical Maintenance	159	27,725,015	0	0	2,173,000	25,552,015
PT Pending	5	2,612,261	0	0	986,451	1,625,810
Other	149	26,438,302	0	0	5,127,068	21,311,234
Total Unlikely to Settle	391	91,298,295	0	0	21,446,253	69,852,042
Grand Total	2,850	284,893,576	55,123,747	44,866,079	91,513,909	93,389,841
Likely to settle	828	63,733,035	4,490,180	8,238,585	17,216,366	33,787,904
Unlikely to settle	551	212,376,367	6,650,415	36,627,494	77,301,738	91,796,720
Grand total (excl closed/cov B/resolved)	1,379	276,109,402	11,140,595	44,866,079	94,518,104	125,584,624

ABC INSURANCE COMPANY

Claim Specific Loss Reserve Model with Losses Evaluated as of December 31, 2000

Summary (Likely to Settle at 100% of Current Reserves with 10% medical escalation years 17-21, 6% thereafter)

		As Reported (B)				Difference = (A)-(B)			
	# Claims	Direct	SDF	Ceded	Net	Direct	SDF	Ceded	Net
Closed Claims	26	13,876	0	10,091	3,785	(13,876)	597,525	(108,726)	(502,675)
Resolved Claims	1,429	18,111,596	0	5,297,462	12,814,134	(10,029,613)	43,385,627	(8,652,389)	(44,762,851)
Coverage B/coverage issues	16	702,191	0	449,367	252,824	0	0	0	0
Special Disability accepted claims									
Likely to Settle	99	13,484,539	0	5,674,656	7,809,883	(426,093)	12,728,765	(4,570,150)	(8,584,708)
Unlikely to Settle	97	15,060,904	0	4,903,268	10,157,636	34,296,969	43,277,909	(491,650)	(8,489,290)
Total special disability accepted claims	196	28,545,443	0	10,577,924	17,967,519	33,870,876	56,006,674	(5,061,800)	(17,073,998)
Maximum reinsurer reserve claims									
Likely to Settle	39	11,053,502	0	5,217,631	5,835,871	(64,814)	0	(4,088)	(60,726)
Unlikely to Settle	63	20,833,189	0	13,729,879	7,103,310	50,887,010	0	37,713,988	13,173,022
Total maximum reinsurer reserve claims	102	31,886,691	0	18,947,510	12,939,181	50,822,196	0	37,709,900	13,112,296
Likely to Settle									
PT	76	12,706,345	0	3,653,508	9,052,837	0	0	0	0
Not at MMI	100	4,024,591	0	1,209,992	2,814,599	0	0	0	0
Medical Maintenance	96	3,559,253	0	0	3,559,253	(510,559)	0	0	(510,559)
PT Pending	33	3,124,530	0	739,757	2,384,773	0	0	0	0
Other	385	16,781,741	0	5,295,060	11,486,681	0	0	0	0
Total Likely to Settle	690	40,196,460	0	10,898,317	29,298,143	(510,559)	0	0	(510,559)
Unlikely to Settle									
PT	45	8,621,601	0	1,590,017	7,031,584	18,768,990	0	10,019,828	8,749,162
Not at MMI	33	996,314	0	169,936	826,378	6,135,812	0	1,379,953	4,755,859
Medical Maintenance	159	5,755,231	0	2,792	5,752,439	21,969,784	0	2,170,208	19,799,576
PT Pending	5	324,745	0	10,835	313,910	2,287,516	0	975,616	1,311,900
Other	149	5,584,773	0	985,170	4,599,603	20,853,529	0	4,141,898	16,711,631
Total Unlikely to Settle	391	21,282,664	0	2,758,750	18,523,914	70,015,631	0	18,687,503	51,328,128
Grand Total	2,850	140,738,921	0	48,939,421	91,799,500	144,154,655	99,989,826	42,574,488	1,590,341
Likely to settle	828	64,734,501	0	21,790,604	42,943,897	(1,001,466)	12,728,765	(4,574,238)	(9,155,993)
Unlikely to settle	551	57,176,757	0	21,391,897	35,784,860	155,199,610	43,277,909	55,909,841	56,011,860
Grand total (excl closed/cov B/resolved)	1,379	121,911,258	0	43,182,501	78,728,757	154,198,144	56,006,674	51,335,603	46,855,867

ABC INSURANCE COMPANY Appendix C

Loss Reserve Model at December 31, 2000 — Exhibit 6, Page 1

Examples

Likely to Settle at 100% of Current Reserves with 10% medical escalation years 17-21, 6% thereafter

		5a-PT unlikely to settle	5a-PT unlikely to settle	5b-Not MMI unlikely to settle	5b-Not MMI unlikely to settle	5b-Not MMI unlikely to settle	5b-Not MMI unlikely to settle	5c-Med Main unlikely to settle
Category		2a-PT	2a-PT	3a-Not MMI	3a-Not MMI	3a-Not MMI	3a-Not MMI	2c-MMM
Fund		6666	7777	6666	8888	8888	7777	8888
FY		83	96	83	89	93	96	92
Claim #		1	2	3	4	5	6	7
Accident Month/Year		Oct-82	Sep-96	Jun-82	Sep-89	Jul-93	Mar-97	Jul-92
As Reported:								
	Medical Paid	49,492	12,306	137,493	127,445	29,176	0	180,023
	Indemnity Paid	217,958	37,976	69,505	49,189	4,758	0	47,774
	Legal/other paid	6,356	21,498	18,733	8,251	111	9,159	19,688
	Medical Reserve	50,508	32,194	30,007	100,055	4,324	10,000	48,977
	Indemnity Reserve	86,042	84,024	0	0	2,000	10,000	0
	Legal/other reserve	6,644	12,502	5,267	3,749	3,889	10,841	6,312
	Total Incurred reported	417,000	200,500	261,005	288,689	44,258	40,000	302,774
	Retention-1	10,000,000	0	10,000,000	750,000	500,000	0	500,000
	Limit	10,000,000	1,000,000	10,000,000	4,750,000	1,000,000	1,000,000	1,000,000
	Percent QS	100.00%	75.00%	100.00%	100.00%	50.00%	75.00%	50.00%
	Retention-2	10,000,000	1,000,000	10,000,000	0	1,500,000	1,000,000	1,500,000
	Limit	10,000,000	10,000,000	10,000,000	0	9,000,000	10,000,000	9,000,000
	Percent QS	100.00%	100.00%	100.00%	0.00%	100.00%	100.00%	100.00%
	Ceded paid reported	0	53,835	0	0	0	6,869	0
	Ceded Incurred reported	0	150,375	0	0	0	30,000	0
	Total reserve reported	143,194	128,720	35,275	103,804	10,213	30,841	55,289
	Ceded reserve reported	0	96,540	0	0	0	23,131	0
	Net reserve reported	143,194	32,180	35,275	103,804	10,213	7,710	55,289
Maximum exposure:								
	Age	75	52	69	61	41	41	40
	Life expectancy	88	78	83	80	82	76	76
1	Remaining Months	153	308	165	228	489	419	429
2	Average monthly medical pmnts	222	224	606	917	310	0	1,698
3	Average monthly indemnity pmnts	977	690	306	354	51	0	451
4	Average monthly expense pmnts	29	391	83	59	1	183	186
5	Last 4 yrs avg monthly medical pmnts	53	153	1,685	685	8	0	1,069
6	Last 4 yrs avg monthly expense pmnts	15	433	306	24	0	191	149
	Selected annual medical average up to 8 yrs	2,663	2,685	7,268	11,002	3,725	0	20,380
	Selected annual medical average after 8 yrs	2,663	2,685	3,003	3,003	3,725	0	5,066
	Direct medical reserve override	0	0	0	0	0	0	300,000
	Maximum medical reserve	56,518	103,247	70,246	97,332	478,162	0	300,000
8=1*3	Maximum Indemnity reserve	149,541	212,665	40,417	64,548	19,803	0	0
9=1*6	Maximum expense reserve-capped at 2 x Incurred	2,231	68,000	48,000	5,515	0	40,000	0
10	Total maximum reserve	208,290	383,912	158,663	167,395	497,965	40,000	300,000
11	SDTF %	0%	0%	0%	0%	0%	0%	0%
12=7+8*11	SDTF recoverable on reserves	0	0	0	0	0	0	0
13	SDTF requests O/S on paid	0	0	0	0	0	0	0
14=12+13	Total SDTF recoverable	0	0	0	0	0	0	0
15=10-14	Gross reserves before reinsurance	208,290	383,912	158,663	167,395	497,965	40,000	300,000
16	Ceded reserves	0	287,934	0	0	15,953	30,000	14,489
17=15-16	Net maximum exposure	208,290	95,978	158,663	167,395	482,012	10,000	285,511
Difference maximum over (under) reported		65,096	63,798	123,388	63,591	471,799	2,290	230,222
Likely exposure:								
	Reserve as a % of reported if likely to settle	100%	100%	100%	100%	100%	100%	100%
	Medical reserve	56,518	103,247	70,246	97,332	478,162	0	300,000
	Indemnity reserve	149,541	212,665	40,417	64,548	19,803	0	0
	Legal/other reserve	2,231	68,000	48,000	5,515	0	40,000	0
	Total reserves	208,290	383,912	158,663	167,395	497,965	40,000	300,000
	SDTF recoverable on reserves	0	0	0	0	0	0	0
	SDTF requests O/S on paid	0	0	0	0	0	0	0
	Total SDTF recoverable	0	0	0	0	0	0	0
	Gross reserves before reinsurance	208,290	383,912	158,663	167,395	497,965	40,000	300,000
	Ceded reserves	0	287,934	0	0	15,953	30,000	14,489
	Net likely exposure	208,290	95,978	158,663	167,395	482,012	10,000	285,511
Difference likely over (under) reported		65,096	63,798	123,388	63,591	471,799	2,290	230,222

ABC INSURANCE COMPANY
Loss Reserve Model at December 31, 2000
Examples
Likely to Settle at 100% of Current Reserves with 10% medical escalation years 17-21, 6% thereafter

		5c-Med Main unlikely to settle	5c-Med Main unlikely to settle	5d-PT pending unlikely to settle	5d-PT pending unlikely to settle	5e-Other unlikely to settle	5e-Other unlikely to settle	5e-Other unlikely to settle
Category		2c-MMM	2c-MMM	3b-PT Pending	3b-PT Pending	3c-Other	3c-Other	3c-Other
Fund		9999	9999	8888	8888	8888	9999	7777
FY		86	86	95	94	95	95	96
Claim #		8	9	10	11	12	13	14
Accident Month/Year		Apr-87	May-87	Jun-96	Jun-94	Dec-95	Oct-95	Jul-96
As Reported:								
	Medical Paid	342,592	40,189	71,769	30,658	134,444	17,806	49,018
	Indemnity Paid	294,239	24,837	61,057	19,935	50,516	10,906	19,933
	Legal/other paid	66,044	8,208	6,430	25,841	5,613	6,567	4,956
	Medical Reserve	140,208	31,311	1,231	33,342	22,856	8,194	10,982
	Indemnity Reserve	0	0	2,943	55,065	20,784	5,094	5,067
	Legal/other reserve	14,956	6,292	905	4,159	9,387	5,983	474
	Total Incurred reported	858,039	110,837	144,335	169,000	243,600	54,550	90,431
	Retention-1	10,000,000	10,000,000	0	500,000	500,000	1,000,000	0
	Limit	10,000,000	10,000,000	1,000,000	1,000,000	1,000,000	10,000,000	1,000,000
	Percent QS	100.00%	100.00%	75.00%	50.00%	50.00%	100.00%	75.00%
	Retention-2	10,000,000	10,000,000	1,000,000	1,500,000	1,500,000	0	1,000,000
	Limit	10,000,000	10,000,000	10,000,000	9,000,000	9,000,000		10,000,000
	Percent QS	100.00%	100.00%	100.00%	100.00%	100.00%	0.00%	100.00%
	Ceded paid reported	0	0	104,442	0	0	0	55,431
	Ceded Incurred reported	0	0	108,251	0	0	0	67,823
	Total reserve reported	155,164	37,604	5,080	92,566	53,027	19,271	16,523
	Ceded reserve reported	0	0	3,810	0	0	0	12,393
	Net reserve reported	155,164	37,604	1,270	92,566	53,027	19,271	4,130
Maximum exposure:								
	Age	66	42	40	37	51	40	41
	Life expectancy	82	82	82	75	77	82	76
1	Remaining Months	188	478	500	461	318	500	419
2	Average monthly medical pmnts	2,027	239	1,237	374	2,101	266	845
3	Average monthly indemnity pmnts	1,741	148	1,053	243	789	163	344
4	Average monthly expense pmnts	391	49	111	315	88	98	85
5	Last 4 yrs avg monthly medical pmnts	770	55	766	178	596	4	257
6	Last 4 yrs avg monthly expense pmnts	15	0	134	448	117	44	103
	Selected annual medical average up to 8 yrs	24,326	2,871	14,849	4,486	25,208	3,189	10,142
	Selected annual medical average after 8 yrs	3,003	2,871	5,403	4,466	4,912	3,189	5,403
	Direct medical reserve override	0	0	0	0	250,000	0	0
	Maximum medical reserve	82,746	496,658	652,087	465,926	250,000	382,772	416,289
8=1*3	Maximum Indemnity reserve	0	0	421,080	89,659	0	0	0
9=1*6	Maximum expense reserve-capped at 2 x Incurred	0	0	14,671	60,000	30,000	22,200	10,861
10	Total maximum reserve	82,746	496,658	1,087,838	615,585	280,000	404,972	427,150
11	SDTF %	0%	0%	0%	0%	0%	0%	0%
12=7+8*11	SDTF recoverable on reserves	0	0	0	0	0	0	0
13	SDTF requests O/S on paid	0	0	0	0	0	0	0
14=12+13	Total SDTF recoverable	0	0	0	0	0	0	0
15=10-14	Gross reserves before reinsurance	82,746	496,658	1,087,838	615,585	280,000	404,972	427,150
16	Ceded reserves	0	0	855,138	60,815	0	0	320,363
17=15-16	Net maximum exposure	82,746	496,658	232,700	554,770	280,000	404,972	106,787
Difference maximum over (under) reported		-72,418	459,054	231,430	462,204	226,973	385,701	102,657
Likely exposure:								
	Reserve as a % of reported if likely to settle	100%	100%	100%	100%	100%	100%	100%
	Medical reserve	82,746	496,658	652,087	465,926	250,000	382,772	416,289
	Indemnity reserve	0	0	421,080	89,659	0	0	0
	Legal/other reserve	0	0	14,671	60,000	30,000	22,200	10,861
	Total reserves	82,746	496,658	1,087,838	615,585	280,000	404,972	427,150
	SDTF recoverable on reserves	0	0	0	0	0	0	0
	SDTF requests O/S on paid	0	0	0	0	0	0	0
	Total SDTF recoverable	0	0	0	0	0	0	0
	Gross reserves before reinsurance	82,746	496,658	1,087,838	615,585	280,000	404,972	427,150
	Ceded reserves	0	0	855,138	60,815	0	0	320,363
	Net likely exposure	82,746	496,658	232,700	554,770	280,000	404,972	106,787
Difference likely over (under) reported		-72,418	459,054	231,430	462,204	226,973	385,701	102,657

Appendix C
Exhibit 7, Page 1

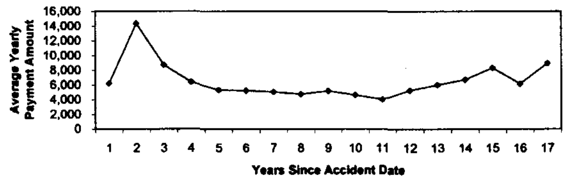
ABC Insurance Company

Medical Payments-Claims Open With Exposure Excluding Maximum Retention

Average Paid Using a 6% Annual Inflation Rate to get All Years to 1997 Values

Acc Yr	# Claims	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17
1984	20	4,761	10,263	11,450	7,029	9,191	7,326	6,550	7,916	13,764	8,737	3,732	5,120	6,354	6,632	6,856	8,676	8,944
1985	25	4,694	15,420	12,879	11,529	6,384	10,707	5,524	3,845	4,563	3,195	3,737	4,278	2,543	4,246	4,158	4,166	
1986	39	3,283	13,363	10,950	9,773	11,719	9,827	5,092	3,897	4,080	2,821	4,507	4,138	7,460	6,607	11,777		
1987	58	5,811	21,762	16,091	12,790	7,915	5,980	4,665	6,271	4,801	4,476	3,530	5,147	5,763	7,987			
1988	75	8,353	23,677	13,620	10,519	5,536	3,782	2,488	3,666	3,988	6,251	4,695	5,101	6,506				
1989	87	5,328	13,738	9,647	6,888	6,890	4,956	4,694	4,635	5,737	5,177	4,656	6,323					
1990	74	6,414	15,580	8,519	3,556	3,032	3,852	2,633	3,709	4,633	3,767	2,894						
1991	90	6,577	14,121	7,438	4,825	4,929	3,832	4,931	4,358	4,047	3,923							
1992	53	6,287	11,689	7,113	5,927	6,683	6,355	7,963	6,021	6,879								
1993	80	5,885	12,298	6,510	6,632	5,376	5,456	4,427	5,242									
1994	147	6,970	12,659	9,912	5,895	4,925	4,490	7,097										
1995	224	6,903	12,548	7,971	4,945	4,101	5,297											
1996	276	5,746	14,042	6,155	5,554	4,694												
1997	1	39	8,888	57	14,006													
Weighted Avg	1,249	6,241	14,355	8,736	6,408	5,340	5,256	5,069	4,748	5,176	4,648	4,021	5,269	6,008	6,758	8,338	6,171	8,944

ABC Average Calendar Year Payments from Accident Date for Open Claims Not Including Claims at Maximum Reinsurance Retention



Appendix C
Exhibit 7, Page 2

ABC Insurance Company

Medical Payments-Claims Open With Exposure Excluding Maximum Retention - Medical Escalation at 6% Beginning with Year 17

Average Paid

Acc Yr	# Claims	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20	Year 21	Year 22	Yr 23
1979	1					111	591	514	640	816	629	904	893	755	179	1,252	366	1,095	1,089	417	891	395	1,110	4,516
1980	2					9,789	5,001	4,111	5,111	5,576	3,705	7,977	1,958	6,412	7,104	2,234	4,266	32,065	9,017	12,954	1,416	2,864	4,260	4,516
1981	6					2,383	3,627	1,834	1,204	1,850	681	918	799	880	1,565	2,291	1,044	1,192	975	1,670	3,972	4,019	4,260	4,516
1982	7					4,753	1,776	3,407	1,634	1,948	1,237	8,634	4,155	1,065	3,430	7,373	10,758	3,419	4,581	6,502	3,792	4,019	4,260	4,516
1983	9					5,183	3,676	2,017	5,287	4,090	1,543	7,642	2,402	1,909	2,134	7,980	1,850	6,798	13,846	3,577	3,792	4,019	4,260	4,516
1984	20					4,309	3,435	3,071	3,711	6,453	4,096	1,750	2,400	2,979	3,109	3,214	4,068	4,193	3,374	3,577	3,792	4,019	4,260	4,516
1985	25					3,173	5,321	2,745	1,911	2,268	1,588	1,857	2,126	1,264	2,110	2,066	2,070	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1986	39					6,174	5,177	2,683	2,053	2,149	1,486	2,374	2,180	3,930	3,480	6,204	3,003	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1987	58					4,420	3,339	2,605	3,502	2,681	2,499	1,971	2,874	3,218	4,460	3,003	3,003	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1988	75					3,277	2,239	1,472	2,170	2,360	3,700	2,779	3,019	3,851	3,003	3,003	3,003	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1989	67					4,323	3,109	2,945	2,906	3,599	3,248	2,921	3,967	3,003	3,003	3,003	3,003	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1990	74					2,016	2,562	1,751	2,467	3,061	2,506	1,925	3,003	3,003	3,003	3,003	3,003	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1991	90					3,474	2,702	3,476	3,072	2,853	2,765	3,003	3,003	3,003	3,003	3,003	3,003	3,183	3,374	3,577	3,792	4,019	4,260	4,516
1992	53					4,994	4,748	5,951	4,499	5,140	5,066	5,066	5,066	5,066	5,066	5,066	5,066	5,370	5,693	6,034	6,396	6,780	7,187	7,618
1993	80					4,258	4,322	3,506	4,153	4,060	4,060	4,060	4,060	4,060	4,060	4,060	4,060	4,303	4,562	4,835	5,125	5,433	5,759	6,104
1994	147					4,135	3,770	5,959	4,303	4,303	4,303	4,303	4,303	4,303	4,303	4,303	4,303	4,562	4,835	5,125	5,433	5,759	6,104	6,471
1995	224					3,650	4,714	4,562	4,562	4,562	4,562	4,562	4,562	4,562	4,562	4,562	4,562	4,835	5,125	5,433	5,759	6,104	6,471	6,859
1996	276					4,428	4,835	4,835	4,835	4,835	4,835	4,835	4,835	4,835	4,835	4,835	4,835	5,125	5,433	5,759	6,104	6,471	6,859	7,270
1997	1					5,125	5,125	5,125	5,125	5,125	5,125	5,125	5,125	5,125	5,125	5,125	5,125	5,433	5,759	6,104	6,471	6,859	7,270	7,707

Accident Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
True Average Up to Year N=16	637	5,270	1,573	4,183	3,809	3,550	2,375	3,445	3,157	2,763	3,378	2,330	3,057	5,066	4,060	4,621	4,182	4,428	N/A
Selected Starting Value	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	5,066	4,060	4,303	4,562	4,835	5,125

Notes: Calendar Years 24 onward Equals (Yr N-1)*1.06
Weighted Calendar Year average of Accident Years 1979-1991 equals 3,003
Accident Years 1994 onward apply 6% trend from the 1993 Accident Year

ABC Insurance Company

Medical Payments-Claims Open With Exposure Excluding Maximum Retention - Medical Escalation at 10% Beginning with Year 17

Average Paid

Acc Yr	# Claims	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 1	Year 1	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20	Year 21	Year 22	Yr 23
1979	1					111	591	514	640	816	629	904	893	755	179	1,252	366	1,095	1,089	417	891	395	1,110	5,852
1980	2					9,789	5,001	4,111	5,111	5,576	3,705	7,977	1,958	6,412	7,104	2,234	4,266	32,065	9,017	12,954	1,416	2,664	5,320	5,852
1981	6					2,383	3,627	1,834	1,204	1,650	681	918	799	860	1,585	2,291	1,044	1,192	975	1,670	3,972	4,837	5,320	5,852
1982	7					4,753	1,776	3,407	1,634	1,948	1,237	8,634	4,155	1,085	3,430	7,373	10,758	3,419	4,581	6,502	4,397	4,837	5,320	5,852
1983	9					5,183	3,676	2,017	5,287	4,090	1,543	7,642	2,402	1,909	2,134	7,980	1,850	6,796	13,846	3,997	4,397	4,837	5,320	5,852
1984	20					4,309	3,435	3,071	3,711	6,453	4,096	1,750	2,400	2,979	3,109	3,214	4,068	4,193	3,634	3,997	4,397	4,837	5,320	5,852
1985	25					3,173	5,321	2,745	1,911	2,268	1,588	1,857	2,126	1,264	2,110	2,066	2,070	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1986	39					6,174	5,177	2,683	2,053	2,149	1,486	2,374	2,180	3,930	3,480	6,204	3,003	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1987	58					4,420	3,339	2,605	3,502	2,681	2,499	1,971	2,874	3,218	4,460	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1988	75					3,277	2,239	1,472	2,170	2,360	3,700	2,779	3,019	3,851	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1989	87					4,323	3,109	2,945	2,908	3,599	3,248	2,921	3,967	3,003	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1990	74					2,016	2,562	1,751	2,467	3,081	2,506	1,925	3,003	3,003	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1991	90					3,474	2,702	3,476	3,072	2,853	2,765	3,003	3,003	3,003	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,320	5,852
1992	53					4,994	4,748	5,951	4,499	5,140	5,066	5,066	5,066	5,066	5,066	5,066	5,066	5,573	6,130	6,743	7,418	8,160	8,976	9,873
1993	80					4,258	4,322	3,506	4,153	4,060	4,060	4,060	4,060	4,060	4,060	4,060	4,060	4,466	4,912	5,403	5,944	6,538	7,192	7,911
1994	147					4,135	3,770	5,959	4,466	4,466	4,466	4,466	4,466	4,466	4,466	4,466	4,466	4,912	5,403	5,944	6,538	7,192	7,911	8,702
1995	224					3,650	4,714	4,912	4,912	4,912	4,912	4,912	4,912	4,912	4,912	4,912	4,912	5,403	5,944	6,538	7,192	7,911	8,702	9,573
1996	276					4,428	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,944	6,538	7,192	7,911	8,702	9,573	10,530
1997	1					5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	6,538	7,192	7,911	8,702	9,573	10,530	11,583

Accident Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
True Average Up to Year N=16	637	5,270	1,573	4,183	3,809	3,550	2,375	3,445	3,157	2,763	3,378	2,330	3,057	5,066	4,060	4,621	4,182	4,428	N/A
Selected Starting Value	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	5,066	4,060	4,466	4,912	5,403	5,944

Notes: Calendar Years 24 onward Equals (Yr N-1)*1.10
Weighted Calendar Year average of Accident Years 1979-1991 equals 3,003
Accident Years 1994 onward apply 10% trend from the 1993 Accident Year

ABC Insurance Company

Medical Payments-Claims Open With Exposure Excluding Maximum Retention - Medical Escalation at 10% Years 17-21, 6% thereafter

Average Paid

Acc Yr	# Claims	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20	Year 21	Year 22	Yr 23
1979	1					111	591	514	640	816	629	904	893	755	179	1,252	366	1,095	1,089	417	891	395	1,110	5,435
1980	2					9,789	5,001	4,111	5,111	5,576	3,705	7,977	1,958	6,412	7,104	2,234	4,266	32,065	9,017	12,954	1,416	2,664	5,127	5,435
1981	6					2,383	3,627	1,834	1,204	1,650	681	918	799	860	1,585	2,291	1,044	1,192	975	1,670	3,972	4,837	5,127	5,435
1982	7					4,753	1,776	3,407	1,634	1,948	1,237	8,634	4,155	1,085	3,430	7,373	10,758	3,419	4,581	6,502	4,397	4,837	5,127	5,435
1983	9					5,183	3,676	2,017	5,287	4,090	1,543	7,642	2,402	1,909	2,134	7,980	1,850	6,796	13,846	3,997	4,397	4,837	5,127	5,435
1984	20					4,309	3,435	3,071	3,711	6,453	4,096	1,750	2,400	2,979	3,109	3,214	4,068	4,193	3,634	3,997	4,397	4,837	5,127	5,435
1985	25					3,173	5,321	2,745	1,911	2,268	1,588	1,857	2,126	1,264	2,110	2,066	2,070	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1986	39					6,174	5,177	2,683	2,053	2,149	1,486	2,374	2,180	3,930	3,480	6,204	3,003	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1987	58					4,420	3,339	2,605	3,502	2,681	2,499	1,971	2,874	3,218	4,460	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1988	75					3,277	2,239	1,472	2,170	2,360	3,700	2,779	3,019	3,851	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1989	87					4,323	3,109	2,945	2,908	3,599	3,248	2,921	3,967	3,003	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1990	74					2,016	2,562	1,751	2,467	3,081	2,506	1,925	3,003	3,003	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1991	90					3,474	2,702	3,476	3,072	2,853	2,765	3,003	3,003	3,003	3,003	3,003	3,003	3,304	3,634	3,997	4,397	4,837	5,127	5,435
1992	53					4,994	4,748	5,951	4,499	5,140	5,066	5,066	5,066	5,066	5,066	5,066	5,066	5,573	6,130	6,743	7,418	8,160	8,649	9,168
1993	80					4,258	4,322	3,506	4,153	4,060	4,060	4,060	4,060	4,060	4,060	4,060	4,060	4,466	4,912	5,403	5,944	6,538	6,931	7,346
1994	147					4,135	3,770	5,959	4,466	4,466	4,466	4,466	4,466	4,466	4,466	4,466	4,466	4,912	5,403	5,944	6,538	7,192	7,624	8,081
1995	224					3,650	4,714	4,912	4,912	4,912	4,912	4,912	4,912	4,912	4,912	4,912	4,912	5,403	5,944	6,538	7,192	7,911	8,386	8,889
1996	276					4,428	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,403	5,944	6,538	7,192	7,911	8,702	9,225	9,778
1997	1					5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	5,944	6,538	7,192	7,911	8,702	9,573	10,147	10,756

Accident Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
True Average Up to Year N=16	637	5,270	1,573	4,183	3,809	3,550	2,375	3,445	3,157	2,763	3,378	2,330	3,057	5,066	4,060	4,621	4,182	4,428	N/A
Selected Starting Value	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	3,003	5,066	4,060	4,466	4,912	5,403	5,944

Notes: Calendar Years 24 onward Equals (Yr N-1)*1.06
Weighted Calendar Year average of Accident Years 1979-1991 equals 3,003
Accident Years 1994 onward apply 10% trend from the 1993 Accident Year

ABC INSURANCE COMPANY

Sample Calculation of Permanent Total Claims with Supplemental Indemnity Reserves
Loss Amounts in ($Actual)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Claimant	Accident Date	Remaining Months	Average Monthly Indemnity Payment	Average Weekly Indemnity Payment	Maximum Exposure Indemnity Reserve	Maximum Weekly	Beginning Base Weekly	Permanent Total Reserves With Supplemental Indemnity	Addition To/ (Subtraction from) Indemnity Max Exposure
				=(4)*12/52	=(3)*(4)	(A)	(A)	(B)	=(9)-(6)
Claim 1	08/28/86	223	2,028	468	452,331	315	315	304,395	(147,936)
Claim 2	11/13/92	526	604	139	317,609	409	113	648,666	331,057
Claim 3	10/14/95	350	1,912	441	669,102	453	322	684,258	15,156
Claim 4	02/22/95	412	1,060	245	436,877	453	320	807,407	370,530
Claim 5	11/07/94	549	1,498	346	822,133	444	271	1,042,568	220,435
Claim 6	04/07/94	360	1,473	340	530,359	444	221	645,906	115,547
Claim 7	10/20/89	432	1,694	391	732,011	362	362	677,664	(54,347)
Claim 8	10/23/91	292	1,796	415	524,552	392	293	496,011	(28,541)
Claim 9	02/01/95	183	1,532	353	280,285	453	352	359,229	78,944
Claim 10	10/08/81	321	1,160	268	372,354	228	228	317,148	(55,206)
Claim 11	12/23/85	216	1,599	369	345,285	307	233	287,352	(57,933)
Claim 12	04/06/92	350	1,414	326	494,767	409	227	607,585	112,818
Claim 13	10/04/94	412	1,206	278	497,024	444	186	684,547	187,522
Claim 14	12/05/91	360	1,515	350	545,479	392	207	592,463	46,983

Notes:

(A) Determined by Florida Statute

(B) Uses Beginning Base Weekly from Column 8 and adds in an additional 5% of Column 8 yearly until the Maximum Weekly in Column 7 is reached. Procedure is applied for the remaining number of months in Column 3.

ABC INSURANCE COMPANY

Appendix C
Exhibit 9

Settled/Closed Claim Percentages of Reserved Amounts by Calendar Year of Closing

Loss Amounts in ($Actual)

	(1)	(2)	(3)	(4)	(5)
Calendar Year	Incurred at 24 months Before Closing	Incurred at 12 months Before Closing	Incurred at Closing Date	Percent of 24 months at Closing (3)/(1)	Percent of 12 months at Closing (3)/(2)
1999	129,305,963	138,569,339	115,155,115	89.1%	83.1%
2000	147,518,866	154,698,957	137,018,707	92.9%	88.6%
Total	276,824,829	293,268,296	252,173,822	91.1%	86.0%

Selected Closing Percentage of Settleable Claims	100.0%

Calculation of Subrogation Recovery -

Appendix D

ABC INSURANCE COMPANY **Appendix D**

RESERVE ANALYSIS AS OF 12/31/00 (In $000s) **Exhibit 1**

SUBROGATION

PAID SUBROGATION/PAID LOSS DEVELOPMENT METHOD

Accident Year	Pd Subro/ Paid Loss	Pd Subro/ Paid Loss Factor	Ultimate Pd Subro/ Paid Loss	Gross Paid Ind & Med Losses @12/31/00	Industry Paid CDF	Indicated Ultimate Ind & Med Losses
(1)	(2)	(3)	(4) = [(2) x (3)]	(5)	(6)	(7) = [(5) x (6)]
1979	0.100	1.000	0.100	$1,167	1.046	$1,221
1980	0.141	0.971	0.137	4,075	1.058	4,311
1981	0.056	0.961	0.054	10,802	1.073	11,590
1982	0.073	0.952	0.070	19,765	1.088	21,504
1983	0.039	0.945	0.037	22,475	1.108	24,902
1984	0.052	0.939	0.049	40,801	1.128	46,023
1985	0.017	0.931	0.016	52,305	1.151	60,203
1986	0.015	0.926	0.014	66,271	1.175	77,868
1987	0.016	0.915	0.015	87,262	1.204	105,064
1988	0.015	0.906	0.014	96,777	1.233	119,326
1989	0.025	0.903	0.022	116,924	1.268	148,259
1990	0.011	0.900	0.010	126,998	1.308	166,113
1991	0.015	0.904	0.014	133,157	1.350	179,762
1992	0.013	0.898	0.011	159,938	1.399	223,753
1993	0.018	0.918	0.016	206,041	1.452	299,171
1994	0.023	0.917	0.021	204,997	1.515	310,570
1995	0.021	0.934	0.019	191,639	1.593	305,281
1996	0.024	0.989	0.023	172,200	1.703	293,257
1997	0.009	1.033	0.010	19,733	1.889	37,276

(8) All Year Weighted Average of Paid Subro to Paid Losses using Column (7) as weights: 0.019

(9) Selected Ultimate Ratio of Paid Subro to Paid Losses: 0.019

ABC INSURANCE COMPANY **Appendix D**
RESERVE ANALYSIS AS OF 12/31/00 (In $000s) **Exhibit 2**

ABC RECEIVED SUBROGATION/ PAID LOSS

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	Received Subro to Paid Loss @12/31/00
1979						0.168	0.167	0.161	0.163	0.152	0.148	0.145	0.143	0.140	0.132	0.128	0.125	0.122	0.103	0.102	0.101	0.100	0.100
1980					0.036	0.044	0.044	0.043	0.031	0.030	0.030	0.030	0.034	0.095	0.108	0.117	0.131	0.142	0.141	0.141	0.141		0.141
1981	0.001	0.003	0.006	0.028	0.049	0.062	0.061	0.061	0.060	0.059	0.055	0.059	0.059	0.058	0.058	0.059	0.057	0.057	0.057	0.056			0.056
1982	0.004	0.005	0.039	0.057	0.063	0.065	0.071	0.075	0.077	0.076	0.074	0.075	0.075	0.074	0.077	0.076	0.075	0.075	0.073				0.073
1983	0.004	0.007	0.015	0.016	0.022	0.025	0.033	0.037	0.040	0.039	0.040	0.040	0.040	0.040	0.040	0.039	0.039	0.039					0.039
1984	0.004	0.007	0.014	0.022	0.026	0.028	0.035	0.038	0.054	0.053	0.052	0.052	0.051	0.051	0.050	0.053	0.052						0.052
1985	0.002	0.006	0.008	0.009	0.010	0.012	0.015	0.014	0.017	0.017	0.017	0.017	0.018	0.017	0.017	0.017							0.017
1986	0.001	0.006	0.008	0.010	0.015	0.016	0.017	0.017	0.017	0.016	0.016	0.016	0.016	0.016	0.015								0.015
1987	0.002	0.008	0.009	0.011	0.015	0.016	0.016	0.017	0.017	0.017	0.016	0.016	0.016	0.016									0.016
1988	0.003	0.005	0.008	0.010	0.011	0.012	0.012	0.013	0.012	0.015	0.016	0.015	0.015										0.015
1989	0.002	0.010	0.010	0.011	0.012	0.013	0.014	0.014	0.023	0.024	0.024	0.025											0.025
1990	0.003	0.005	0.007	0.008	0.011	0.011	0.011	0.011	0.011	0.011	0.011												0.011
1991	0.003	0.007	0.010	0.011	0.015	0.015	0.015	0.015	0.016	0.015													0.015
1992	0.002	0.008	0.010	0.011	0.012	0.012	0.013	0.013	0.013														0.013
1993	0.005	0.010	0.015	0.016	0.017	0.018	0.018	0.018															0.018
1994	0.005	0.016	0.017	0.019	0.020	0.021	0.023																0.023
1995	0.042	0.041	0.041	0.044	0.021	0.021																	0.021
1996	0.000	0.026	0.027	0.018	0.024																		0.024
1997	0.006	0.007	0.010	0.009																			0.009

ABC INSURANCE COMPANY | **Appendix D**
RESERVE ANALYSIS AS OF 12/31/00 (In $000s) | **Exhibit 3**

ABC RECEIVED SUBROGATION/PAID DEVELOPMENT FACTORS

Accident Year	12 24	24 36	36 48	48 60	60 72	72 84	84 96	96 108	108 120	120 132	132 144	144 156	156 168	168 180	180 192	192 204	204 216	216 228	228 240	240 252	252 264	264 ULT
1979						0.995	0.962	1.013	0.932	0.976	0.977	0.987	0.978	0.943	0.971	0.981	0.971	0.843	0.990	0.991	0.995	
1980					1.223	1.007	0.962	0.715	0.988	0.985	0.994	1.151	2.793	1.136	1.082	1.118	1.091	0.993	0.995	0.999		
1981	5.831	1.853	4.806	1.711	1.279	0.983	0.994	0.982	0.992	0.922	1.081	0.998	0.987	0.992	1.014	0.977	0.999	0.995	0.992			
1982	1.487	7.422	1.441	1.118	1.025	1.091	1.051	1.038	0.986	0.971	1.016	0.997	0.988	1.040	0.982	0.995	0.989	0.981				
1983	1.977	2.033	1.112	1.367	1.122	1.332	1.104	1.103	0.968	1.010	1.003	1.000	1.016	0.987	0.990	0.990	0.992					
1984	1.907	1.952	1.619	1.182	1.099	1.222	1.088	1.429	0.986	0.980	0.988	0.998	0.981	0.982	1.063	0.988						
1985	3.504	1.296	1.080	1.143	1.188	1.229	0.977	1.209	0.983	0.991	0.994	1.046	0.988	0.989	0.994							
1986	3.958	1.347	1.236	1.530	1.076	1.044	1.014	0.987	0.985	0.972	0.994	0.990	0.993	0.990								
1987	3.100	1.228	1.143	1.406	1.071	1.016	1.005	1.015	0.986	0.990	0.999	0.993	0.999									
1988	2.020	1.561	1.305	1.037	1.108	1.019	1.031	0.991	1.215	1.023	0.994	0.988										
1989	4.784	1.032	1.098	1.045	1.132	1.026	1.013	1.671	1.026	1.007	1.034											
1990	2.107	1.239	1.228	1.332	1.048	0.989	0.997	0.982	0.993	0.989												
1991	2.425	1.499	1.067	1.367	0.996	1.001	0.983	1.067	0.993													
1992	3.307	1.299	1.106	1.062	1.060	1.010	1.008	1.010														
1993	2.186	1.413	1.055	1.077	1.046	1.040	0.978															
1994	3.337	1.093	1.100	1.049	1.059	1.072																
1995	0.979	0.998	1.064	0.486	0.967																	
1996	298.933	1.028	0.658	1.343																		
1997	1.198	1.317	0.979																			

	12 24	24 36	36 48	48 60	60 72	72 84	84 96	96 108	108 120	120 132	132 144	144 156	156 168	168 180	180 192	192 204	204 216	216 228	228 240	240 252	252 264	TAIL
Average	20.179	1.742	1.359	1.203	1.094	1.067	1.011	1.087	1.003	0.985	1.007	1.015	1.191	1.007	1.014	1.008	1.008	0.953	0.992	0.995	0.995	
3 Yr Avg	100.370	1.114	0.900	0.959	1.024	1.041	0.989	1.019	1.004	1.006	1.009	0.990	0.993	0.987	1.016	0.991	0.993	0.990	0.992			
All Yr Ex Hi/Lo	2.875	1.413	1.176	1.218	1.089	1.054	1.008	1.069	0.990	0.987	1.002	1.001	0.993	0.997	1.009	0.988	0.993	0.987	0.992			
5 Yr Ex Hi/Lo	2.240	1.146	1.033	1.062	1.033	1.017	0.996	1.022	1.004	0.995	0.996	0.994	0.993	0.989	0.999	0.991	0.993					

Selected:	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264
Age-to-Age	1.552	1.098	1.068	1.044	1.059	1.018	0.999	1.023	0.993	1.004	0.997	0.997	0.990	0.988	0.994	0.992	0.993	0.993	0.990	0.990	0.971	1.000
Age-to-Ult	1.881	1.212	1.104	1.033	0.989	0.934	0.917	0.918	0.898	0.904	0.900	0.903	0.906	0.915	0.926	0.931	0.939	0.945	0.952	0.961	0.971	1.000

Calculation of New/Reopened Claims Provision -

Appendix E

ABC INSURANCE COMPANY

Appendix E

Exhibit 1

Calculation of IBNR for New and Reopened Claims

Loss Amounts in ($Actual)

Month	(1) Calendar Year 1997	(2) Calendar Year 1998	(3) Calendar Year 1999	(4) Calendar Year 2000	Yearly Change
January	99,792	168,117	68,147	204,587	
February	90,070	164,825	202,870	24,897	
March	131,895	149,756	339,573	82,909	
April	133,177	291,399	152,243	157,368	
May	157,488	147,232	123,982	73,755	
June	147,861	153,989	138,025	73,505	
July	181,954	189,706	86,777	39,136	
August	90,307	197,846	186,403	96,632	
September	110,346	163,048	80,604	222,456	
October	97,915	79,213	137,785		
November	140,764	153,355	140,633		
December	153,168	124,010	477,944		
Total	1,534,737	1,982,496	2,134,986	975,245	
Average	127,895	165,208	177,916	108,361	5.85%
Avg. Ex Hi Lo	126,271	161,188	158,890	103,985	6.82%
Median	132,536	158,519	139,329	82,909	15.68%
Selected	132,536	158,519	139,329	82,909	15.68%

Calculation of IBNR associated with New and Reopened Claims=

(A):	Average Monthly Payment for Current Year:	108,361
(B):	Average Yearly Payment for Current Year equals (A)*12:	1,300,327
(C):	Selected Average Yearly Decrease in Amount:	15.68%
(D):	Ultimate IBNR for New and Reopened equals (B)/(C)-(B):	6,991,030

Calculation of IBNR Amounts Exceeding Reinsurance Limits -

Appendix F

ABC Insurance Company
Calculation of IBNR Amounts Exceeding Reinsurance Limits

Appendix F
Exhibit 1

Individual Claimants with Projected Payments In Excess of Maximum Reinsurance

Claimant	Age (x)	Of 100,000 Born Alive: Number Living at Beginning of Age (x)	Of 100,000 Born Alive: Number Living at Beginning of Age (95)	Likelihood of Survival to Age (95)	Estimated Payments In Excess of Maximum Reinsurance If Alive at Age (95)	Estimated Payment by ABC
(1)	(2)	(3)	(4)	(5) = (4) / (3)	(6)	(7) = (5) x (6)
Female	38	96,891	10,914	0.1126	$5,185,665	$584,124
Male 1	56	84,009	3,799	0.0452	1,184,982	53,586
Male 2	40	92,957	3,799	0.0409	679,902	27,786
Total						$665,497

Sample ULAE Calculations -

Appendix G

ABC INSURANCE COMPANY
Calculation of Unallocated Loss Adjustment Expense using Best's Aggregates and Averages
Loss Amounts in ($000's)

Accident Year	(1) Direct and Assumed Case Loss	(2) Direct and Assumed IBNR Loss	(3) Direct and Assumed Case ALAE	(4) Direct and Assumed IBNR ALAE	(5) Direct and Assumed IBNR ULAE	(6) ULAE Reserve % of Loss +ALAE Reserve	(7) ULAE Reserve % of IBNR Reserve	(8) ABC Total Reserve	(9) ABC Booked IBNR	(10) Method 1 ULAE Reserve	(11) Method 2 ULAE Reserve
	(A)	(B)	(C)	(D)	(E)	(F)	(G)			(6)*(8)	(7)*(9)
Prior	16,136,790	7,178,145	274,816	692,260	485,310						
1991	2,238,712	1,395,334	38,431	167,273	105,387						
1992	2,117,601	1,491,083	38,189	191,619	117,342						
1993	2,021,196	1,500,264	34,860	221,699	112,556						
1994	2,032,807	1,579,728	46,547	238,468	130,070						
1995	2,259,525	1,773,433	49,026	281,701	161,427						
1996	2,507,057	1,965,472	74,280	334,622	192,902						
1997	3,374,262	2,225,059	110,670	442,325	236,765						
1998	5,009,899	2,995,728	193,847	645,373	368,768						
1999	7,106,790	4,874,449	297,310	989,744	591,015						
2000	8,458,321	10,353,546	390,735	1,498,737	1,195,736						
Total	53,262,960	37,332,241	1,548,711	5,703,821	3,697,278	3.8%	8.6%				
Ex. 98-00	32,687,950	19,108,518	666,819	2,569,967	1,541,759	2.8%	7.1%	161,388	69,588	4,521	4,949

(A): Based on 2001 Best's Aggregates and Averages Schedule P - Part 1D, Column (13)
(B): Based on 2001 Best's Aggregates and Averages Schedule P - Part 1D, Column (15)
(C): Based on 2001 Best's Aggregates and Averages Schedule P - Part 1D, Column (17)
(D): Based on 2001 Best's Aggregates and Averages Schedule P - Part 1D, Column (19)
(E): Based on 2001 Best's Aggregates and Averages Schedule P - Part 1D, Column (21)
(F): =(5)/((1)+(2)+(3)+(4))
(G): =(5)/((2)+(4))

ABC INSURANCE COMPANY

Appendix G

Exhibit 2

Calculation of Unallocated Loss Adjustment Expense using TPA Payment Schedule

Loss Amounts in ($000's)

Calendar Year	Estimated TPA ULAE Payment Schedule	Percentage Increase/ (Decrease)
1995	4,876	
1996	3,704	-24.0%
1997	2,841	-23.3%
1998	2,064	-27.3%
1999	1,575	-23.7%
2000	1,106	-29.8%

Selected Increase/(Decrease)	-25.0%
ULAE Factor = (1.0/25%)	4.00
Selected ULAE = (Factor-1.0)*(2000 Yr Paid Amount)	**3,318**

Appendix G
Exhibit 3

ABC INSURANCE COMPANY
Calculation of Unallocated Loss Adjustment Expense using Calendar Year Payments
Loss Amounts in ($000's)

	(1)	(2)	(3)	(4)	(5)	(6)
Calendar Year	Direct and Assumed Loss and ALAE	Paid ULAE	Paid ULAE/ Paid Loss Ratio	ABC Total Reserve	ABC IBNR	ABC Case Reserve
	(A)	(A)	(2)/(1)			(4)-(5)
1995	4,876	204	4.2%			
1996	3,704	165	4.5%			
1997	2,841	118	4.2%			
1998	2,064	79	3.8%			
1999	1,575	58	3.7%			
2000	1,106	43	3.9%			
Total				161,388	69,588	91,800

(7) Selected ULAE %	4.0%
(8) Selected ULAE = ((5)+((6)*0.5))*(7)	4,620

Duration of ABC Insurance Company -

Appendix H

ABC INSURANCE COMPANY

Appendix H

Calculation of Duration at Different Evaluation Points

Exhibit 1

Selected Interest Rate: 5.0%

	(1)	(2)	(3)	(4)
Year N	Selected CDF	Percentage Paid	Incremental Paid from Year N to Year N+1	Duration At Year N
	(A)	=1.0/(1)	(B)	(C)
1	7.096	14.1%	18.2%	5.311
2	3.100	32.3%	12.4%	5.925
3	2.237	44.7%	8.2%	6.516
4	1.889	52.9%	5.8%	6.926
5	1.703	58.7%	4.1%	7.145
6	1.593	62.8%	3.2%	7.127
7	1.515	66.0%	2.9%	6.976
8	1.452	68.9%	2.6%	6.762
9	1.399	71.5%	2.6%	6.502
10	1.350	74.1%	2.4%	6.270
11	1.308	76.5%	2.4%	6.007
12	1.268	78.9%	2.2%	5.775
13	1.233	81.1%	2.0%	5.515
14	1.204	83.1%	2.0%	5.214
15	1.175	85.1%	1.8%	4.942
16	1.151	86.9%	1.8%	4.629
17	1.128	88.7%	1.6%	4.349
18	1.108	90.3%	1.6%	4.027
19	1.088	91.9%	1.3%	3.748
20	1.073	93.2%	1.3%	3.370
21	1.058	94.5%	1.1%	3.022
22	1.046	95.6%	1.1%	2.597
23	1.034	96.7%	1.0%	2.190
24	1.024	97.7%	1.0%	1.753
25	1.013	98.7%	0.8%	1.373
26	1.005	99.5%	0.5%	1.000
27	1.000	100.0%		

Notes (A): Based on Florida information from NCCI Annual Statistical Bulletin with and adjusted 8th/Ult. Tail; Items below line have been estimated
(B): From Column (2)
(C): Assumes that all payments are made at end of Year N

Sample Calculation of Duration (Year 1):

$$\frac{(1)*((1.05)^{-1})*18.2+(2)*((1.05)^{-2})*12.4+\ldots\ldots+(26)*((1.05)^{-26})*0.5}{((1.05)^{-1})*18.2+((1.05)^{-2})*12.4+\ldots\ldots+((1.05)^{-26})*0.5}$$

= 5.311

Two Approaches to Calculating Correlated Reserve Indications Across Multiple Lines of Business

Gerald S. Kirschner, FCAS, MAAA,
Colin Kerley, and Belinda Isaacs

Two Approaches to Calculating Correlated Reserve Indications Across Multiple Lines of Business

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Classic Solutions

Abstract

As reserving actuaries focus more on reserve ranges and less on point estimates, the question of how to develop a reasonable reserve range in the aggregate becomes more and more relevant. When working with a single set of "best estimates", the answer is simple - assuming all the best estimates are the mean values for each block of business being analyzed, the best estimate for the total is equal to the sum of the parts. However, if the by line best estimates are other than the mean values, the sum of the parts is not the same as the best estimate for the aggregate.

This paper presents two possible approaches to developing aggregate reserve indications when looking at results other than the mean value. The approaches both rely on a simulation model. One takes in the actuary's judgment as to the correlations between the different underlying blocks of business and the second uses bootstrapping to eliminate the need for the actuary to make judgment calls about the nature of the correlations.

I. Introduction

The bar continues to be raised for actuaries performing reserve analyses. For example, the approval of Actuarial Standard of Practice # 36 for United States actuaries clarifies and codifies the requirements for actuaries producing "written statements of actuarial opinion regarding property/casualty loss and loss adjustment expense reserves."[1] A second example in the United States is the National Association of Insurance Commissioners requirement that companies begin booking management's best estimate of reserves by line and in the aggregate, effective January 2001. A third example, this one from Australia, is contained in the Australian Prudential Regulation Authority's (APRA) General Insurance Prudential Standards, applicable from July 2002 onwards. In these regulations, APRA specifically states "the Approved Actuary must provide advice on the valuation of insurance liabilities at a given level of sufficiency – **that level is 75%.**"[2]

[1] Actuarial Standard of Practice # 36, page 1.
[2] Australia Prudential Regulation Authority, Prudential Standard GPS 210, "Liability Valuation for General Insurers", July 2002 p. 2.

In this environment, it is clear that actuaries are being asked to do more than ever before with regard to reserve analyses. One set of techniques that has been of substantial interest to the paper-writing community for quite some time is the use of stochastic analysis or simulation models to analyze reserves. Stochastic methods[3] are an appealing approach to answering the questions currently being asked of reserving actuaries. One might ask, "Why? What makes stochastic methods more useful in this regard than the traditional reserving methods that I've been using for years?"

The answer is not that the stochastic methods are *better* than the traditional methods.[4] Rather the stochastic methods are more informative about more aspects of reserve indications than traditional methods. When all an actuary is looking for is a point estimate, then traditional methods are quite sufficient to the task. However, when an actuary begins developing reserve ranges for one or more lines of business, and trying to develop not only ranges on a by-line basis, but in the aggregate, the traditional methods quickly pale in comparison to the stochastic methods. The creation of reserve ranges from point estimate methods is often an ad-hoc one, such as looking at results using different selection factors or different types of data (paid, incurred, separate claim frequency and severity development, etc.) or judgmentally saying something like "my best estimate plus or minus ten percent." When trying to develop a range in the aggregate, the ad-hoc decisions become even more so, such as "I'll take the sum of my individual ranges less X% because I know the aggregate is less risky than the sum of the parts."

Stochastic methods, by contrast, provide actuaries with a structured, mathematically rigorous approach to quantifying the variability around a best estimate. This is not meant to imply that all judgment is eliminated when a stochastic method is used. There are still many areas of judgment that remain, such as the choice of stochastic method and / or the shape of the distributions underlying the method, and the number of years of data being used to fit factors. What stochastic methods do provide is (a) a consistent framework and a repeatable process in which the analysis is done and (b) a mathematically rigorous answer to questions about probabilities and percentiles. Now,

[3] In this paper we use the word stochastic to mean frameworks that are not deterministic, i.e. have a random component. This is typically done by creating a framework for the reserving technique where many previously fixed quantities are represented by random variables. Probability distributions may then be generated for claims reserves, either analytically or by Monte-Carlo simulation.

[4] When we talk about "traditional methods", we mean the time-honored tradition of analyzing a triangle of paid or incurred loss data by looking at different averages of age-to-age development factors, selecting one for each development age and projecting paid or incurred losses to "ultimate" using the selected factors. There are many variations on this basic approach that can be applied, including data adjustments (like Berquist-Sherman), factor modifications (like Bornheutter-Ferguson), trend removal, but at the end of the day, the traditional methods all produce one reserve indication with no information as to how reality might differ from that single indication .

when asked to set reserves equal to the 75th percentile, as in Australia, the actuary has a mechanism for identifying the 75th percentile. Moreover, when the actuary analyzes the same block of business a year later, the actuary will be in a position to discuss how the 75th percentile has changed, knowing that the changes are driven by the underlying data and not the application of different judgmental factors (assuming the actuary does not alter the assumptions underlying the stochastic method being used).

It cannot be stressed enough, though, that stochastic models are not crystal balls. Quite often the argument is raised that the promise of stochastic models is much greater than the benefit they provide. The arguments typically take on one or both of the following forms:

1. Stochastic models do not work very well when data is sparse or highly erratic. Or, to put another way, stochastic models work well when there is a lot of data and it is fairly regular – exactly the situation in which it is easy to apply a traditional point-estimate approach.
2. Stochastic models overlook trends and patterns in the data that an actuary using traditional methods would be able to pick up and incorporate into the analysis.

England and Verrall, in their 2002 paper, addressed this sort of argument with the response "It is sometimes rather naïvely hoped that stochastic methods will provide solutions to problems when deterministic methods fail. Indeed, sometimes stochastic models are judged on whether they can help when simple deterministic models fail. This rather misses the point. The usefulness of stochastic models is that they can, in many circumstances, provide more information which may be useful in the reserving process and in the overall management of the company."[5] This, in our opinion, is the essence of the value proposition for stochastic models. They are not intended to replace traditional techniques. There will always be a need and a place for actuarial judgment in reserve analysis that stochastic models will never supplant. Even so, as the bar is raised for actuaries performing reserve analyses, the additional information inherent in stochastic models makes the argument in favor of adding them to the standard actuarial repertoire that much more compelling.

Having laid the foundation for why we believe actuaries ought to be incorporating stochastic models into their everyday toolkit, let us turn to the actual substance of this paper – using a stochastic model to develop an aggregate reserve range for several lines of business with varying degrees of correlation between the lines.

II. Correlation – mathematically speaking and in lay terms

Before jumping into the case study, we will take a small detour into the mathematical theory underlying correlation.

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[5] England & Verrall (2002), pp. 2-3.

Correlations between observed sets of numbers are a way of measuring the "strength of relationship", between the sets of numbers. Broadly speaking, this "strength of relationship" measure is a way of looking at the tendency of two variables, X and Y, to move in the same (or opposite) direction. For example, if X and Y were positively correlated then if X gives a higher than average number, we would expect Y to give a higher than average number as well.

It should be mentioned that there are many different ways to measure correlation, both parametric (for example, Pearson's R) and non-parametric (Spearman's Rank Order, or Kendall's tau). It should also be mentioned that these statistics only give a simple view of the way two random variables behave together – to get more detailed picture, we would need to understand the joint probability density function (pdf) of the two variables.

As an example of correlation between two random variables we will look at the results of flipping two coins, and look at the relationship between correlation coefficients and conditional probabilities.

Example 1

We have two coins, each with an identical chance of getting heads (50%) or tails (50%) with a flip. We will specify their joint distribution, and so determine the relationship between the outcomes of both coins. Note that in our notation, 0 signifies a Head, 1 a Tail.

Case 1:

Case 1 Joint Distribution Table

		Coin B		
		0	**1**	**Marginal**
Coin A	**0**	0.25	0.25	0.5
	1	0.25	0.25	0.5
	Marginal	0.5	0.5	

The joint distribution table shows the probability of all the outcomes when the two coins are tossed. In the case of two coin tosses there are 4 potential outcomes, hence, there are 4 cells in the joint distribution table. For example, the probability of Coin A being a head (0) and Coin B a tail (1) can be determined by looking at the 0 row for Coin A and the 1 column for Coin B, in this example 0.25. In this case, our coins are independent. The correlation coefficient is zero, where we calculate the correlation coefficient by:

$$\text{Correlation Coefficient} = Cov(A, B) / (Stdev(A) * Stdev(B)) \quad (2.1)$$

and

$$Cov(A, B) = E[(A - mean(A)) * (B - mean(B))] = E(AB) - E(A)E(B) \quad (2.2)$$

We can also see that the outcomes of coin B are not linked in any way to the outcome of coin A. For example

P(B = 1 | A = 1) = P(A = 1, B = 1) / P(A = 1)
= 0.25 / 0.5
= 0.50
= P(B = 1)

Case 2:

Case 2 Joint Distribution Table

		Coin B		
		0	**1**	**Marginal**
Coin A	**0**	0.3125	0.1875	0.5
	1	0.1875	0.3125	0.5
	Marginal	0.5	0.5	

From this distribution we calculate the correlation coefficient to be 0.25[6].

By looking at the conditional distributions, it is clear that there is a link between the outcome of coin B and coin A :

P(B = 1 | A = 1) = P(A = 1, B = 1) / P(A = 1)
= 0.3125 / 0.5
= 0.625

P(B = 0 | A = 1) = 0.375

So we can see that with the increase in correlation, there is an increase in the chance of getting heads on coin B given coin A shows heads.

[6] Proof that the correlation coefficient for case 2 is 0.25:

E(A,B) = $\sum_{i=0}^{1}\sum_{j=0}^{1} i * j * P(A_i B_j)$ = 0 + 0 + 0 + 1*1*0.3125 = 0.3125

E(A) = 0.5 = E(B)

Cov(A,B) = E(A,B) – E(A)E(B) = 0.3125 – 0.25 = 0.0625

Var(A) = $\sum_{i=0}^{1}(i - E(A))^2 * P(A_i)$ = 0.25 = Var(B)

StDev(A) = 0.5 = StDev(B)

Correlation Coefficient = 0.0625 / (0.5 * 0.5) = 0.25

With this 2 coin example it turns out that if we want the marginal distributions of each coin to be the standard 50% heads, 50% tails then, given the correlation coefficient we want to produce, we can uniquely define the joint pdf for the coins.

We find that, for a given correlation coefficient of ρ,

$P(A = 1, B = 1) = P(A = 0, B = 0) = (1 + \rho) / 4$
$P(A = 1, B = 0) = P(A = 0, B = 1) = (1 - \rho) / 4$

We can then recover the conditional probabilities :

$P(B = 1 \mid A = 1) = (1 + \rho) / 2$
$P(B = 0 \mid A = 1) = (1 - \rho) / 2$

So for example, we can see that

$\rho = 0.50$ gives $P(B = 1 \mid A = 1) = 0.750$.
$\rho = 0.75$ gives $P(B = 1 \mid A = 1) = 0.875$.
$\rho = 1.00$ gives $P(B = 1 \mid A = 1) = 1.000$.

As expected, an increase in the correlation coefficient means the higher the chance of throwing heads on coin B, given coin A shows heads.

In lay terms, then, we would repeat our description of correlation at the start of this section, that correlation, or the "strength of relationship" is a way of looking at the tendency of two variables, X and Y, to move in the same (or opposite) direction. As the coin example shows, the more positively correlated X and Y are, the greater our expectation that Y will be higher than average if X is higher than average.

It should be noted, however, that the expected value of the sum of two correlated variables is exactly equal to the expected value of the sum of the two uncorrelated variables with the same means.

III. Significance of the existence of correlations between lines of business

Suppose we have two or more blocks of business for which we are trying to calculate reserve indications. If all we are trying to do is determine the expected value of the reserve run-off, we can calculate the expected value for each block separately and add all the expectations together. However, if we are trying to quantify a value other than the mean, such as the 75^{th} percentile, we cannot simply sum across the lines of business. If we do so, we will overstate the aggregate reserve need. The only time the sum of the 75^{th} percentiles would be appropriate for the aggregate reserve indication is when all the lines are fully correlated with each other – a highly unlikely situation! The degree to which the lines are correlated will influence the proper aggregate reserve level and the aggregate reserve range. How significant an impact will there be? That primarily depends upon two factors – how volatile the reserve ranges are for the underlying lines of business and how strongly correlated the lines are with each other. If there is not much volatility, then the strength of the correlation will not matter that much. If, however, there is considerable volatility, the strength of correlations will produce differences that could be material. This is demonstrated in the following example.

Example 2: The impact on values at the 75th percentile as correlation and volatility increase

Table 1 shows some figures relating the magnitude of the impact of correlations on the aggregate distribution to the size of the correlation. In this example, we have modeled two lines of business (A and B) assuming they were Normally distributed with identical means and variances. The means were assumed to be 100 and the standard deviations were 25. We are examining the 75th percentile value derived for the sum of A and B. The tables below show the change in the 75th percentile value between the uncorrelated situation and varying levels of correlation between lines A and B. Reading down the column shows the impact of an increasing level of correlation between lines A and B, namely that the ratio of the correlated to the uncorrelated value at the 75th percentile increases as correlation increases.

Table 1: Comparison of values at the 75th percentile as correlation increases

Correlation	Values at 75th percentile	Ratio of Values at 75th percentile
0.00	223.8	0.0%
0.25	226.7	1.3%
0.50	229.2	2.4%
0.75	231.5	3.4%
1.00	233.7	4.4%

Now let's expand the analysis to see what happens as the volatility of the underlying distributions increase. Table 2 shows a comparison of the sum of lines A and B at the 75th percentile as correlation increases and as volatility increases. The ratios in each column are relative to the value for the zero correlation value at each standard deviation value. For example, the 5.8% ratio for the rightmost column at the 25% correlation level means that the 75th percentile value for lines A + B with 25% correlation is 5.8% higher than the 75th percentile of $N(100,200)_A + N(100,200)_B$ with no correlation. As can be seen from this table, the greater the volatility, the larger the differential between the uncorrelated and correlated results at the 75th percentile.

Table 2: Comparison of values at the 75th percentile as both correlation and volatility increase

	Standard Deviation Value			
	25	50	100	200
Value for 0.00 correlation at the 75th percentile	223.8	247.7	295.4	390.8
Correlation	Ratio of values at 75th percentile			
0.25	1.3%	2.3%	3.8%	5.8%
0.50	2.4%	4.3%	7.3%	11.0%
0.75	3.4%	6.2%	10.4%	15.8%
1.00	4.4%	8.0%	13.4%	20.2%

This effect is magnified if we look at similar results but further out on the tails of the distribution, for example looking at the 95th percentiles, as is shown in Table 3.

Table 3: Comparison of values at the 95th percentile as both correlation and volatility increases

	Standard Deviation Value			
	25	50	100	200
Value for 0.00 correlation at the 75th percentile	258.1	316.3	432.6	665.2
Correlation	Ratio of values at 75th percentile			
0.25	2.7%	4.3%	6.3%	8.3%
0.50	5.1%	8.3%	12.1%	15.7%
0.75	7.3%	11.9%	17.4%	22.6%
1.00	9.3%	15.2%	22.3%	29.0%

Note that these results will also depend on the nature of the underlying distributions – we would expect different results for lines of business that were Lognormally distributed for example.

IV. Case Study

A. Background

The data used in this case study is fictional. It describes three lines of business, two long tail and one short tail. All three produce approximately the same mean reserve indication, but with varying degrees of volatility around their respective means. By having the three lines of approximately equal size, we can are able to focus on the impact of correlations between lines without worrying about whether the results from one line is overwhelming the results from the other two lines.

Appendix I contains the data triangles.

The examination of the impact of correlation on the aggregated results will be done using two methods. The first assumes the person doing the analysis can provide a *positive-definite* correlation matrix (see section B below). The relationships described in the correlation matrix are used to convert the uncorrelated aggregate reserve range into a correlated aggregate range. The process does not affect the reserve ranges of the underlying lines of business. It just influences the aggregation of the reserve indications by line so that if two lines are positively correlated and the first line produces a reserve indication that is higher than the expected reserve indication for that line, it is more likely than not that the second line will also produce a reserve indication that is higher than its expected reserve indication. This is exactly what was demonstrated in the coin examples in Section II.

The second method dispenses with what the person doing the analysis knows or thinks he or she knows. This method relies on the data alone to derive the relationships and linkages between the different lines of business. More precisely, this method assumes that all we need to know about how related the different lines of business are to each other is contained in the historical claims development that we have already observed. This method uses a technique known as bootstrapping to extract the relationships from the observed claims history. The bootstrapped data is used to generate reserve indications that inherently contain the same correlations as existed in the original data. Therefore, the aggregate reserve range is reflective of the underlying relationships between the individual lines of business, without first requiring the potentially messy step of requiring the person doing the analysis to develop a correlation matrix.

B. A note on the nature of the correlation matrix used in the analysis

The entries in the correlation matrix used must fulfill certain requirements that cause the matrix to be what is known as *positive definite*. The mathematical description of a positive definite matrix is that, given a vector *x* and a matrix A, where

$x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$ and

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$x^T A x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= a_{11}x_1^2 + a_{12}x_1x_2 + a_{21}x_2x_1 + \cdots + a_{nn}x_n^2 \qquad (4.1)$$

Matrix A is positive definite when $x^T A x > 0$
for all *x* other than $x_1 = x_2 = \cdots = x_n = 0$. (4.2)

C. Correlation matrix methodology

The methodology used in this approach is that of **rank correlation**. Rank correlation is a useful approach to dealing with two or more correlated variables when the joint distribution of the correlated variables is not normal. When using rank correlation, what matters is the ordering of the simulated outcomes from each of the individual distributions, or more properly, the *re-ordering* of the outcomes.

Rank Correlation Example

Suppose we have two random variables, A and B. A and B are both defined by uniform distributions ranging from 100 to 200. Suppose we draw five values at random from A and B. They might look as shown in Table 4:

Table 4

Index	A	B
1	155	154
2	138	125
3	164	100
4	122	198
5	107	128

Now suppose we are interested in the joint distribution of A + B. We will use rank correlation to learn about this joint distribution. We will use a bivariate normal distribution to determine which value from distribution B ought to be paired with a value from distribution A. The easiest cases are when B is perfectly correlated with A or perfectly inversely correlated with A. In the perfectly correlated case, we pair the lowest value from A with the lowest value from B, the second lowest value from A with the second lowest value from B, and so on and so forth to the highest values for A and B. In the case of perfect inverse correlation, we pair the lowest value from A with the highest value from B, etc, etc, etc. The results from these two cases are shown in Table 5.

Table 5

Perfectly Correlated			Perfect Inverse Correlation		
Rank to Use			**Rank to Use**		
A	**B**		**A**	**B**	
1	1		1	5	
2	2		2	4	
3	3		3	3	
4	4		4	2	
5	5		5	1	
Resulting Joint Distribution			**Resulting Joint Distribution**		
A	**B**	**A+B**	**A**	**B**	**A+B**
107	100	207	107	198	305
122	125	247	122	154	276
138	128	266	138	128	266
155	154	309	155	125	280
164	198	362	164	100	264
Range of Joint Distribution			**Range of Joint Distribution**		
Low		207	**Low**		264
High		362	**High**		305

When there is no correlation between A and B, the ordering of the values from distribution B that are to be paired with values from distribution A are wholly random. The original order of the values drawn from distributions A and B is one example of the no-correlation condition. When positive correlations exist between A and B, the orderings reflect the level of correlation and the range of the joint distribution will be somewhere between the wholly random situation and the perfectly correlated one.

Application of Rank Correlation Methodology to Reserve Analysis

The application of the rank correlation methodology to a stochastic reserve analysis is done through a two-step process. In the first step, a stochastic reserving technique is use to generate N possible reserve runoffs from each data triangle being analyzed. Examples of several different techniques, including bootstrapping (England 2001), application of the chain-ladder to logarithmically adjusted incremental paid data (Christofides 1990) and application of the chain-ladder to logarithmically adjusted cumulative paid data (Feldblum 1999) can be found in articles listed in the bibliography to this paper. In this case study, 5,000 different reserve runoffs were produced using the bootstrapping technique described in England (2001). This is the end of step one.

In step two, the user must specify a correlation matrix. We do not propose to cover how one may estimate such a correlation matrix in this paper, as we feel this is an important topic in its own right, the details of which would merit a separate paper. One such paper for readers who are looking for guidance in this area is "Correlation and the Aggregation of Unpaid Loss Distributions" by Paul Brehm, due to be published as part of the CAS 2002 Reserving Call Papers. In this paper, we will simply assume that the user has such a matrix, either calculated analytically, or estimated using some other approach, such as a judgmental estimation of correlation.

We generate 5,000 samples for each line of business from a multivariate normal distribution, with the correlation matrix specified by the user. We then sort the samples from the reserving method into the same rank order as the normally distributed samples. This ensures that the rank order correlations between the three lines of business are the same as the rank order correlations between the three normal distributions. The aggregate reserve distribution is calculated from the sum of the individual line reserve distributions. This resulting aggregate reserve range will be composed of 5,000 different values from which statistics such as the 75th percentile can be drawn. The range of aggregated reserve indications is reflective of the correlations entered into the correlation matrix at the start of the analysis.

For example, the ranked results from the multivariate normal process might be as follows:

Line 1, Rank	Line 2, Rank	Line 3, Rank
528	533	400
495	607	404
995	710	904
233	325	831
733	912	551
825	33	801
325	107	331
630	210	571
653	986	51
983	730	301
130	900	782

The first of the 5,000 values in the aggregate reserve distribution will be composed of the 528th largest reserve indication for line 1 + the 533rd largest reserve indication for line 2 + the 400th largest reserve indication for line 3. The second of the thousand values will be composed of the 495th largest reserve indication for line 1 + the 607th largest reserve indication for line 2 + the 404th largest reserve indication for line 3. Through this process, the higher the positive correlation between lines, the more likely it is that a value below the mean for one line will be combined with a value below the mean for a second line. At the same time, the mean of the overall distribution remains unchanged and the distributions of the individual lines remains unchanged.

D. Rank correlation results

To show the impact of the correlations between the lines on the aggregate distribution, we ran the model five times, each time with a different correlation value: zero correlation, 25% correlation, 50% correlation, 75% correlation, and 100% correlation. The correlations were chosen to highlight the range of outcomes that result for different levels of correlation, not because the data necessarily implied the existence of correlations such as these. The results are shown both numerically in Table 6 and graphically in Figure 1 and Figure 2.

Table 6: Case study results – aggregated reserve indication at different levels of correlation between underlying lines of business (all values are in thousands)

	0% corr	**25% corr**	**50% corr**	**75% corr**	**100% corr**
Mean	4,330,767	4,330,767	4,330,767	4,330,767	4,330,767
Standard Deviation	1,510,033	1,596,840	1,705,469	1,829,748	1,998,140
Minimum	2,587,213	2,293,224	2,084,841	2,086,531	1,930,725
Maximum	72,366,202	72,771,841	73,474,899	75,564,417	81,277,681
1% ile	2,995,943	2,861,958	2,695,429	2,510,514	2,408,319
5% ile	3,247,847	3,087,062	2,956,837	2,867,115	2,762,663
10% ile	3,384,401	3,241,518	3,143,080	3,033,779	2,987,948
20% ile	3,588,011	3,500,438	3,424,399	3,358,196	3,277,806
30% ile	3,782,986	3,681,105	3,615,534	3,574,383	3,522,031
40% ile	3,942,032	3,897,816	3,820,380	3,790,977	3,745,674
50% ile	4,113,146	4,078,681	4,071,349	4,027,615	3,973,908
60% ile	4,278,521	4,279,869	4,292,852	4,267,561	4,232,721
70% ile	4,493,139	4,518,971	4,547,255	4,558,175	4,560,471
80% ile	4,786,940	4,876,233	4,931,662	5,031,358	5,111,862
90% ile	5,378,096	5,475,577	5,604,519	5,679,109	5,842,125
95% ile	6,008,476	6,230,885	6,371,310	6,436,050	6,836,095
99% ile	8,286,504	8,687,785	9,310,024	10,075,891	10,322,456
Estimated 75th %ile	**4,640,039**	**4,697,602**	**4,739,459**	**4,794,767**	**4,836,166**

Figure 1: Graph of case study results – aggregated reserve indication at different levels of correlation between underlying lines of business

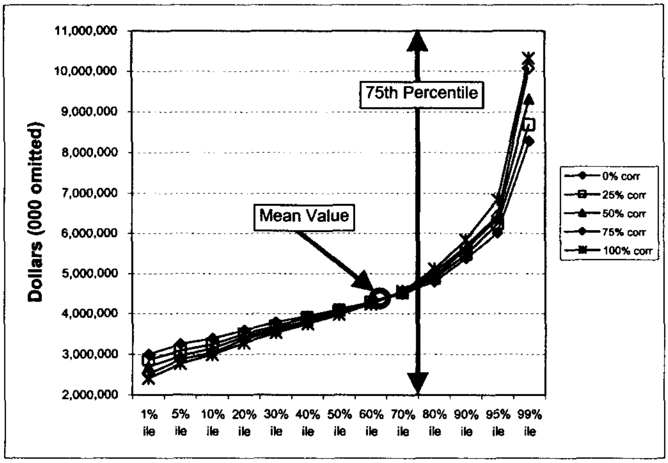
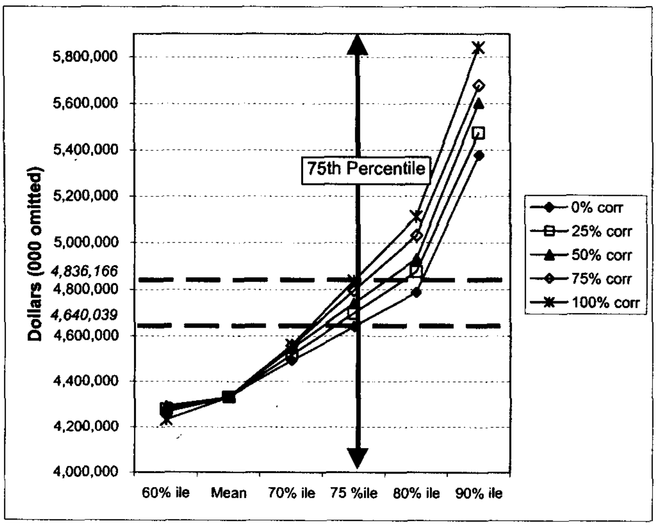


Figure 2: Graph of case study results – aggregated reserve indication at different levels of correlation between underlying lines of business – blowing up area around 75th percentile



As expected, the higher the positive correlation, the wider the aggregated reserve range. With increasingly higher positive correlations, it is less likely that a better than expected result in one line will be offset by a worse than expected result in another line. This causes the higher positive correlated situations to have lower aggregate values for percentiles below the mean and higher aggregate values for percentiles above the mean. The results of the table and graph show just this situation. For information purposes, the difference between the zero correlation situation and the perfectly correlated situation have been displayed on the graph in Figure 2.

E. Bootstrap methodology

Bootstrapping is a sampling technique that is an alternative to traditional statistical methodologies. In traditional statistical approaches, one might look at a sample of data and postulate the underlying distribution that gave rise to the observed outcomes. Then, when analyzing the range of possible outcomes, new samples are drawn from the postulated distribution. Bootstrapping, by comparison, does not concern itself with the underlying distribution. The bootstrap says that all the information needed to create new samples lies within the variability that exists in the already observed historical data. When it comes time to create the new samples, different observed variability factors are combined with the observed data to create "pseudo-data" from which the new samples are generated. A step-by-step description of the application of the bootstrap methodology to claims reserving is contained in Appendix II.

So what is bootstrapping, then, as it is applied to reserve analysis? Bootstrapping is a resampling method that is used to estimate in a structured manner, the variability of a parameter. In reserve analysis, the parameter is the difference between observed and expected paid amounts for any given accident year / development year combination. During each iteration of the bootstrapping simulation, random draws are made from all the available variability parameters. One random draw is made for each accident year / development year combination. The variability parameter is combined with the actual observation to develop a "pseudo-history" paid loss triangle A reserve indication is then produced from the pseudo-history data triangle by applying the traditional cumulative chain-ladder technique to "square the triangle". A step-by-step walkthrough of the bootstrap process is included in Appendix II.

Note that this example is using paid amounts. The bootstrap approach can equally be applied to incurred data, to generate "pseudo-history" incurred loss triangles, which may be developed to ultimate in the same manner as the paid data. Also, the methodology is not limited to working with just positive values. This is an important capability when using incurred data, as negative incrementals will be much more common when working with incurred data.

This approach is extended to multiple lines in the following manner. Instead of making random draws of the variability parameters independently for each line of business, the same draws are used across all lines of business. The variability parameters will differ from line to line, but the choice of which variability parameter to pick is the same across lines.

The example of Table 7 - Table 9 should clarify the difference between the uncorrelated and correlated cases. The example shows two lines of business, Line A and Line B. Both are 4x4 triangles. Table 7 shows the variability parameters calculated from the original data. We start by labeling each parameter with the accident year, development year and triangle from which the parameters are derived.

Table 8 shows one possible way the variability parameters might be reshuffled to create an uncorrelated bootstrap. For each Accident / Development year in each triangle A and B, we select a variability parameter from Table 7 at random. For example Triangle A, Accident Year 1, Development Year 1 has been assigned (randomly) the variability parameter from the original data in Table, Accident Year 2, Development Year 1. Note that each triangle uses the variability parameters calculated from that triangle's data, i.e. none of the variability parameters from Triangle A are used to create the pseudo-history in Triangle B. Also note that the choice of variability parameters for each Accident Year / Development Year in Triangle A is independent of the choice of variability parameter for the corresponding Accident Year / Development Year in Triangle B.

For the correlated bootstrap shown in Table 8, the choice of variability parameter for each Accident Year / Development Year in Triangle A is *not* independent of the choice of variability parameter for the corresponding Accident Year / Development Year in Triangle B. We ensure that the variability parameter selected from Triangle B comes from the same Accident Year / Development Year used to select a variability parameter from Triangle A.

The process shown in Table 9 implicitly captures and uses whatever correlations existed in the historical data when producing the pseudo-histories from which the reserve indications will be developed. The resulting aggregated reserve indications will reflect the correlations that existed in the actual data, without requiring the analyst to first postulate what those correlations might be. This method also does not require the second stage reordering process that the correlation matrix methodology required. The correlated aggregate reserve indication can be derived in one step.

Table 7

Variability Parameters Calculated from Original Data

Triangle A

AY	Development Year 1	2	3	4
1	$(1_A,1_A)$	$(1_A,2_A)$	$(1_A,3_A)$	$(1_A,4_A)$
2	$(2_A,1_A)$	$(2_A,2_A)$	$(2_A,3_A)$	
3	$(3_A,1_A)$	$(3_A,2_A)$		
4	$(4_A,1_A)$			

Triangle B

AY	Development Year 1	2	3	4
1	$(1_B,1_B)$	$(1_B,2_B)$	$(1_B,3_B)$	$(1_B,4_B)$
2	$(2_B,1_B)$	$(2_B,2_B)$	$(2_B,3_B)$	
3	$(3_B,1_B)$	$(3_B,2_B)$		
4	$(4_B,1_B)$			

Calculated Variability Parameters

Table 8

Uncorrelated Bootstrapping - Reshuffling of variability parameters in Triangle B is independent of the reshuffling in Triangle A

AY	Development Year 1	2	3	4
1	$(2_A,1_A)$	$(3_A,2_A)$	$(1_A,3_A)$	$(3_A,1_A)$
2	$(2_A,2_A)$	$(2_A,3_A)$	$(1_A,2_A)$	
3	$(3_A,1_A)$	$(1_A,1_A)$		
4	$(1_A,1_A)$			

AY	Development Year 1	2	3	4
1	$(2_B,2_B)$	$(3_B,2_B)$	$(1_B,3_B)$	$(2_B,2_B)$
2	$(3_B,1_B)$	$(2_B,3_B)$	$(2_B,2_B)$	
3	$(1_B,3_B)$	$(1_B,1_B)$		
4	$(1_B,2_B)$			

Randomly Selected Variability Parameters to be used in the creation of one possible pseudo-history

Table 9

Correlated Bootstrapping - Reshuffling of variability parameters in Triangle B is identical to the reshuffling in Triangle A

AY	Development Year 1	2	3	4
1	$(2_A,1_A)$	$(3_A,2_A)$	$(1_A,3_A)$	$(3_A,1_A)$
2	$(2_A,2_A)$	$(1_A,2_A)$	$(2_A,3_A)$	
3	$(3_A,1_A)$	$(1_A,1_A)$		
4	$(1_A,1_A)$			

AY	Development Year 1	2	3	4
1	$(2_B,1_B)$	$(3_B,2_B)$	$(1_B,3_B)$	$(3_B,1_B)$
2	$(2_B,2_B)$	$(1_B,2_B)$	$(2_B,3_B)$	
3	$(3_B,1_B)$	$(1_B,1_B)$		
4	$(1_B,1_B)$			

Randomly Selected Variability Parameters to be used in the creation of one possible pseudo-history

F. Bootstrap results

The model was run one final time using the bootstrap methodology to develop an aggregated reserve range. The bootstrap results have been added to the results shown in Table 6, Figure 1 and Figure 2. The revised results are shown in Table 10, Figure 3, and Figure 4 where we can compare the aggregate reserve distributions generated from the two different approaches.

Table 10 (all values are in thousands)

	0% corr	25% corr	50% corr	75% corr	100% corr	Bootstrap
Mean	4,330,767	4,330,767	4,330,767	4,330,767	4,330,767	4,335,587
Standard Deviation	1,510,033	1,596,840	1,705,469	1,829,748	1,998,140	1,601,469
Minimum	2,587,213	2,293,224	2,084,841	2,086,531	1,930,725	2,250,401
Maximum	72,366,202	72,771,841	73,474,899	75,564,417	81,277,681	67,405,104
1% ile	2,995,943	2,861,958	2,695,429	2,510,514	2,408,319	2,708,420
5% ile	3,247,847	3,087,062	2,956,837	2,867,115	2,762,663	3,014,557
10% ile	3,384,401	3,241,518	3,143,080	3,033,779	2,987,948	3,194,731
20% ile	3,588,011	3,500,438	3,424,399	3,358,196	3,277,806	3,443,479
30% ile	3,782,986	3,681,105	3,615,534	3,574,383	3,522,031	3,653,888
40% ile	3,942,032	3,897,816	3,820,380	3,790,977	3,745,674	3,849,489
50% ile	4,113,146	4,078,681	4,071,349	4,027,615	3,973,908	4,043,971
60% ile	4,278,521	4,279,869	4,292,852	4,267,561	4,232,721	4,271,588
70% ile	4,493,139	4,518,971	4,547,255	4,558,175	4,560,471	4,554,548
80% ile	4,786,940	4,876,233	4,931,662	5,031,358	5,111,862	4,957,356
90% ile	5,378,096	5,475,577	5,604,519	5,679,109	5,842,125	5,691,814
95% ile	6,008,476	6,230,885	6,371,310	6,436,050	6,836,095	6,471,699
99% ile	8,286,504	8,687,785	9,310,024	10,075,891	10,322,456	9,116,338
Estimated 75th %ile	**4,640,039**	**4,697,602**	**4,739,459**	**4,794,767**	**4,836,166**	**4,755,952**

Figure 3: Graph of case study results – adding bootstrapped correlation to aggregated reserve indication at different levels of correlation between underlying lines of business

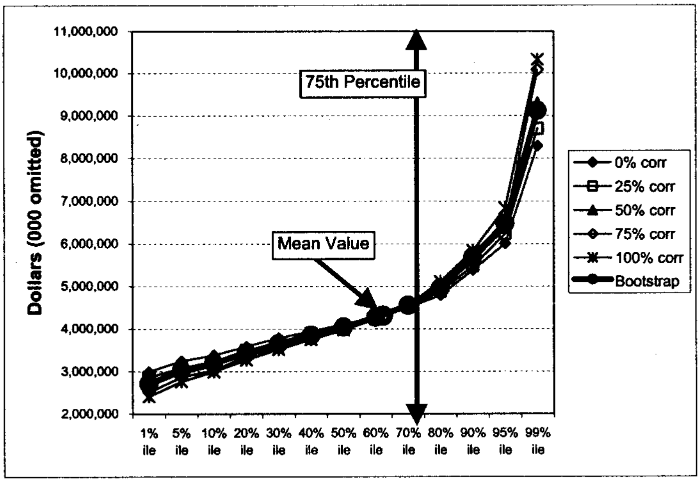
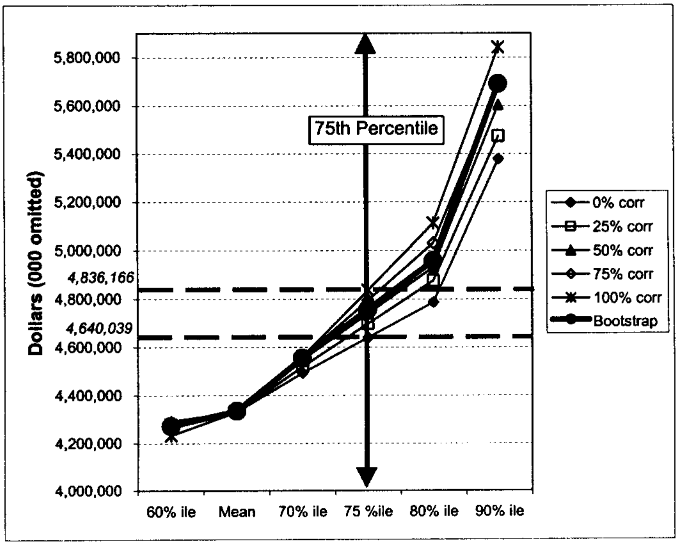


Figure 4: Graph of case study results – adding bootstrapped correlation to aggregated reserve indications at different levels of correlation between underlying lines of business – blowing up area around 75th percentile



The results shown in the preceding figures and tables provide us with the following information:

1. If we wanted to hold reserves at the 75th percentile, the smallest reserve that ought to be held is $4.640 billion and the largest ought to be $4.836 billion.
2. The maximum impact on the 75th percentile of indicated reserves due to correlation is 4.5% of the mean indication ($196 million / $4.331 billion).
3. There does appear to be correlation between at least two of the lines. The observed level of correlation is similar to what would be displayed, were there to be a 50% correlation between each of the lines. It could be that two of the lines exhibit a stronger than 50% correlation with each other and a weaker than 50% correlation with the third line so that the overall results produce values similar to what would exist at the 50% correlation level.
4. The proper reserve to book, assuming the 50% correlation is correct, is $4.75 billion.

Some level of correlation between at least two of the lines is indicated by the bootstrapped results. This is valuable information to know, even beyond the range of reserves indicated by the bootstrap methodology. With this information, company management can assess prospective underwriting strategies that recognize the interrelated nature of these lines of business, such as how much additional capital might be required to protect against adverse deviation. If the lines were uncorrelated, future adverse deviation in one line would not necessarily be reflected in the other lines. With the information at hand, it would be inappropriate to assume that adverse deviation in one line will not be mirrored by adverse deviation in one or both of the other lines. Continuing with this thought, the bootstrapped results would have been valuable even if they had shown there to be little or no correlation between the lines – because then company management could comfortably assume independence between the lines of business and make their strategic decisions accordingly.

V. Summary and conclusions

Let us move beyond the numbers of the case study to summarize what we feel to be the important general conclusions that can be drawn. To begin, calculating an aggregate reserve distribution for several lines of business requires not only a model for the distribution of reserves for each individual line of business, but also an understanding of the dependency of the reserve amounts between each of the lines of business. To get a feel for the impact of these dependencies on the aggregate distribution, we have proposed two different methods. One can use a rank correlation approach with correlation parameters estimated externally. However, this approach requires either calculating correlations using a method such as has been proposed by Brehm (2002) or by judgmentally developing a correlation matrix. Alternatively, one can use a bootstrap method that relies on the existing dependencies in the historic data triangles. This requires no external calibration, but may be less transparent in providing an understanding of the data. It also limits the calculations to reflecting only those relationships that have existed in the past in the projection of reserve indications.

Furthermore, it would appear that the correlation issue is not important for lines of business with non-volatile reserve ranges. However for volatile reserves, the impact of correlations between could be significant, particularly as one moves towards more extreme ends of the reserve range. If so, either correlation approach can provide actuaries with a way of quantifying the effect of correlations on the aggregate reserve range. Overall, the use of stochastic techniques adds value as such techniques can not only assess the volatility of reserves, but can also identify the significance of correlations between lines of business in a more rigorous manner than is possible with traditional techniques.

To conclude, we believe that stochastic quantification of reserve ranges, with or without an analysis of correlations between lines of business, is a valuable extension of current actuarial practice. Regulations such as those recently promulgated by APRA will accelerate the general usage of stochastic techniques in reserve analysis. An accompanying benefit to the use of stochastic reserving techniques is the ability to quantify the effects of correlations between lines of business on overall reserve ranges. This will help actuaries and company management to better understand how variable reserve development might be, both by line and in the aggregate, allowing companies to make better-informed decisions on the booking of reserves and the amount of capital that must be deployed to protect the company against adverse reserve development.

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Appendix I: Data sets

The data used in this case study is fictional. It describes three lines of business, two long tail and one short tail. All three produce approximately the same mean reserve indication, but with varying degrees of volatility around their respective means. The data triangles are shown in Table 11 to Table 13 below. The data is all in the format of incremental paid losses, with all dollar amounts in thousands

Table 11 – Line 1 (derived from Commercial Automobile business)

	1	2	3	4	5	6	7	8	9	10
1	20,513	78,579	65,886	57,537	59,293	11,338	10,815	7,811	1,117	11,792
2	13,847	39,035	39,375	29,884	32,754	10,298	6,276	6,924	3,835	0
3	15,785	49,135	42,672	27,920	36,399	27,828	9,596	6,781	0	0
4	20,784	62,266	47,120	59,331	41,672	20,726	16,790	0	0	0
5	108,531	115,103	187,886	90,515	149,616	86,813	0	0	0	0
6	26,097	59,195	1,786	19,780	22,835	0	0	0	0	0
7	64,819	142,577	100,694	34,304	0	0	0	0	0	0
8	44,065	53,039	8,975	0	0	0	0	0	0	0
9	20,022	39,276	0	0	0	0	0	0	0	0
10	37,163	0	0	0	0	0	0	0	0	0

When calculating ultimate indications from this data set, a tail extrapolation allowing for development up to thirty years was included in the calculations.

Table 12 – Line 2 (derived from Homeowners business)

	1	2	3	4	5	6	7	8	9	10
1	761,590	327,920	53,290	16,280	8,400	11,900	9,070	10,140	2,010	80
2	784,590	309,150	64,120	34,990	26,540	30,320	5,640	320	290	0
3	1,077,950	331,980	53,160	44,020	23,170	15,420	8,990	5,780	0	0
4	1,065,310	370,910	52,660	47,320	27,000	12,700	(800)	0	0	0
5	1,055,040	372,020	62,250	51,310	18,710	16,970	0	0	0	0
6	1,654,920	413,100	59,920	56,950	38,050	0	0	0	0	0
7	1,326,870	440,340	129,070	58,860	0	0	0	0	0	0
8	1,875,230	465,410	96,290	0	0	0	0	0	0	0
9	1,572,510	419,950	0	0	0	0	0	0	0	0
10	1,902,050	0	0	0	0	0	0	0	0	0

When calculating ultimate indications from this data set, no tail extrapolation was used. Development was assumed to end at ten years.

Table 13 – Line 3 (derived from Workers' Compensation business)

	1	2	3	4	5	6	7	8	9	10
1	36,212	115,053	140,789	115,705	111,334	26,366	20,877	19,788	6,117	16,618
2	40,885	139,674	129,071	109,117	89,906	43,988	20,551	21,526	18,368	0
3	40,322	113,355	100,782	61,491	64,420	40,803	20,580	25,214	0	0
4	38,013	69,213	56,892	75,435	49,984	29,359	25,466	0	0	0
5	37,810	60,405	85,602	33,211	53,347	35,643	0	0	0	0
6	37,159	67,486	34,465	33,121	41,478	0	0	0	0	0
7	34,415	68,634	34,427	18,653	0	0	0	0	0	0
8	37,786	40,462	24,049	0	0	0	0	0	0	0
9	35,380	73,641	0	0	0	0	0	0	0	0
10	39,866	0	0	0	0	0	0	0	0	0

When calculating ultimate indications from this data set, a tail extrapolation allowing for development up to thirty years was included in the calculations.

Appendix II: A step-by-step walkthrough of the bootstrap process used for reserve simulation

Bootstrapping is a technique broadly accepted within the statistical community. It uses the noise within the historical data to make implications about both the noise in the future and about the parameter uncertainty. Since it uses the historical noise, it is not restricted to normal error structures, but rather uses the error structure implicit within the historical data. The method used is based upon the approach outlined by England and Verrall (1999) and expanded upon by England (2001). We encourage readers who want further explanation of the theory or other examples of the methodology to read both of these papers.

The theoretical model to which this bootstrapping technique is compared is a model of incremental claims known as an "over-dispersed" Poisson distribution. This model is described by Renshaw and Verrall (1998). Using the notation from England (1999), incremental claims for origin year i in development year j are denoted C_{ij}, we have:

$$E[C_{ij}] = m_{ij} \text{ and } Var[C_{ij}] = \phi E[C_{ij}] = \phi\, m_{ij} \qquad \text{(App2.1)}$$

$$\log(m_{ij}) = \eta_{ij} \qquad \text{(App2.2)}$$

$$\eta_{ij} = c + \alpha_i + \beta_j \qquad \text{(App2.3)}$$

These equations defined a generalized linear model in which the calculated value is modeled with a logarithmic link function and the variance is proportional to the mean. The proportionality of the variance to the mean instead of the equality of the variance to the mean is the reason the model is described as an "over-dispersed" Poisson. The parameter ϕ is an unknown scale parameter that is estimated as part of the fitting procedure. England (2001) notes that "with certain positivity constraints, predicted values and reserve estimates from this model are exactly the same as those from the chain ladder model."[7]

The steps undertaken to calculate runoff using the bootstrapping method are:

1. Begin with a triangle of cumulative historical payments. We will use the data from Table 12 – Line 2 (derived from Homeowners business). This is shown in Triangle 1.

[7] England (2001), p. 3.

Triangle 1: Cumulative historical data

	1	2	3	4	5	6	7	8	9	10
1	761,590	1,089,510	1,142,800	1,159,080	1,167,480	1,179,380	1,188,450	1,198,590	1,200,600	1,200,680
2	784,590	1,093,740	1,157,860	1,192,850	1,219,390	1,249,710	1,255,350	1,255,670	1,255,960	
3	1,077,950	1,409,930	1,463,090	1,507,110	1,530,280	1,545,700	1,554,690	1,560,470		
4	1,065,310	1,436,220	1,488,880	1,536,200	1,563,200	1,575,900	1,575,100			
5	1,055,040	1,427,060	1,489,310	1,540,620	1,559,330	1,576,300				
6	1,654,920	2,068,020	2,127,940	2,184,890	2,222,940					
7	1,326,870	1,767,210	1,896,280	1,955,140						
8	1,875,230	2,340,640	2,436,930							
9	1,572,510	1,992,460								
10	1,902,050									

2. Calculate factors based upon historical payments. The factors calculated are based on the cumulative chain ladder method. The factors are weighted averages.

Development Factors

	1	2	3	4	5	6	7	8	9	10
Ave	n/a	1.3088	1.0452	1.0288	1.0156	1.0124	1.0041	1.0041	1.0009	1.0001

3. Using the cumulative factors calculated in step 2, refit the original payments.

Most recent payment period equals most recent payment period cumulated payments in the actual data.

Fitted payments (accident year r, calendar year c) all other payment periods =

$$\frac{FittedPayment(r, c+1)}{ChainLadderFactor(r, c+1)} \qquad \text{(App2.4)}$$

The results of the refitting are shown in Triangle 2.

Triangle 2: Cumulative fitted values

	1	2	3	4	5	6	7	8	9	10
1	822,235	1,076,160	1,124,783	1,157,142	1,175,141	1,189,716	1,194,624	1,199,476	1,200,600	1,200,680
2	860,149	1,125,782	1,176,647	1,210,498	1,229,327	1,244,574	1,249,708	1,254,784	1,255,960	
3	1,069,695	1,400,040	1,463,298	1,505,395	1,528,811	1,547,772	1,554,158	1,560,470		
4	1,084,109	1,418,906	1,483,016	1,525,680	1,549,412	1,568,628	1,575,100			
5	1,089,411	1,425,845	1,490,269	1,533,142	1,556,989	1,576,300				
6	1,555,371	2,035,703	2,127,682	2,188,893	2,222,940					
7	1,389,272	1,818,310	1,900,466	1,955,140						
8	1,781,437	2,331,583	2,436,930							
9	1,522,331	1,992,460								
10	1,902,050									

For example, the derivation of the row 8, column 2 value of 2,331,583 = the row 8, column 3 value of 2,436,930 from Triangle 2 divided by the column 3 average of 1.0452. The derivation of the row 8, column 1 value of 1,781,437 equals the row 8, column 2 value of 2,331,583 from Triangle 2 divided by the column 2 average of 1.3088.

4. Calculate unscaled Pearson residuals. This is the residual definition chosen by England and Verrall (1999) as being suitable for a generalized linear model of the type described by formulas App2.1 through App2.3. The formula for the Pearson residual is given by formula App2.5. The calculated unscaled residuals are shown in Triangle 3.

$$\textit{Pearson Residual}(r,c) = \frac{\textit{Actual Payment}(r,c) - \textit{Fitted Payment}(r,c)}{\sqrt{\textit{Fitted Payment}(r,c)}} \qquad \text{(App2.5)}$$

The values are unscaled in the sense that they do not include the scale parameter ϕ. The scale parameter is not needed when performing the bootstrap calculations, but it will be needed to incorporate an estimate of process error in the final results. The scale parameter will be incorporated into the calculations beginning with Step 11.

Triangle 3: Unscaled Pearson Residuals

	1	2	3	4	5	6	7	8	9	10
1	-66.88	146.84	21.16	-89.38	-71.55	-22.16	59.40	75.92	26.42	0.00
2	-81.47	84.43	58.77	6.19	56.20	122.07	7.05	-66.75	-25.83	
3	7.98	2.84	-40.15	9.37	-1.61	-25.72	32.59	-6.70		
4	-18.06	62.41	-45.22	22.54	21.22	-47.01	-90.39			
5	-32.93	61.35	-8.56	40.75	-33.27	-16.84				
6	79.82	-97.01	-105.71	-17.22	21.69					
7	-52.94	17.25	163.68	17.90						
8	70.27	-114.24	-27.90							
9	40.67	-73.18								
10	0.00									

For example, the derivation of the row 8, column 1 value is:

$$Pearson\ Residual(8,1) = 70.27 = \frac{1{,}875{,}230 - 1{,}781{,}437}{\sqrt{1{,}781{,}437}}$$

5. One adjustment must be made to the unscaled Pearson residuals before they can be used in the bootstrap algorithm. This is to adjust the residuals to account for the number of degrees of freedom in the original data triangle. This step is done so as to allow the estimation variances derived from the bootstrap model to be compared to the estimation variances that can be obtained from the over-dispersed Poisson generalized linear model. The degree of freedom adjustment is accomplished by multiplying each residual by an adjustment factor equal to:

$$\sqrt{\frac{n}{n-p}} \qquad \text{(App2.6)}$$

where n = number of data points (55 in a 10 x 10 triangle) and
p = number of parameters being estimated = (2 * number of accident years)- 1

The degrees of freedom adjustment for this data triangle = $\sqrt{\frac{55}{55-19}} = 1.236$. The adjusted residuals are shown in Triangle 4.

Triangle 4: Unscaled Pearson Residuals, adjusted for degrees of freedom in original data triangle

	1	2	3	4	5	6	7	8	9	10
1	-82.67	181.50	26.16	-110.48	-88.44	-27.38	73.42	93.83	32.66	0.00
2	-100.70	104.36	72.64	7.65	69.46	150.89	8.72	-82.51	-31.93	
3	9.87	3.52	-49.62	11.58	-1.99	-31.79	40.29	-8.28		
4	-22.32	77.14	-55.89	27.86	26.23	-58.11	-111.73			
5	-40.70	75.83	-10.58	50.36	-41.12	-20.82				
6	98.66	-119.91	-130.65	-21.29	26.81					
7	-65.44	21.33	202.31	22.13						
8	86.86	-141.21	-34.49							
9	50.27	-90.46								
10	0.00									

6. Randomly select from the adjusted Pearson residuals, excluding the cells in the top right and bottom left, as these will always be 0. An example of one possible random selection of residuals is shown in Triangle 5.

Triangle 5: One possible random selection of residuals

	1	2	3	4	5	6	7	8	9	10
1	50.27	-55.89	-119.91	-31.93	-1.99	-31.93	86.86	-49.62	-22.32	69.46
2	-58.11	9.87	-119.91	-141.21	-40.70	8.72	-82.51	50.27	77.14	
3	104.36	3.52	-31.93	-40.70	-34.49	21.33	-10.58	75.83		
4	-31.93	21.33	75.83	69.46	21.33	-65.44	-130.65			
5	93.83	3.52	181.50	-27.38	-10.58	-41.12				
6	-111.73	7.65	8.72	-31.93	-31.93					
7	32.66	-41.12	-119.91	150.89						
8	22.13	-27.38	69.46							
9	-100.70	75.83								
10	73.42									

7. Calculate a "false history" based on the randomly selected residuals from step 6.

$$False\ History(r,c) = Random\ Residual(r,c) * \sqrt{Fitted\ Payment(r,c)} + Fitted\ Payment(r,c)$$

(App2.7)

Triangle 6: False history based on random residuals in Triangle 5

	1	2	3	4	5	6	7	8	9	10
1	867,818	225,759	22,183	26,615	17,732	10,720	10,994	1,395	376	701
2	806,259	270,717	23,823	7,870	13,244	16,323	(778)	8,657	3,821	
3	1,177,634	332,366	55,226	33,746	18,138	21,898	5,540	12,337		
4	1,050,861	347,137	83,310	57,012	27,017	10,145	(4,039)			
5	1,187,350	338,473	110,492	37,203	22,213	13,596				
6	1,416,033	485,637	94,622	53,311	28,155					
7	1,427,768	402,103	47,787	89,955						
8	1,810,969	529,835	127,892							
9	1,398,085	522,125								
10	2,003,312									

An example of a false history is shown in Triangle 6, using the residuals shown in Triangle 5. For example, the derivation of the row 8, column 1 value is:

$$False\ History(8,1) = 1,906,677 = 93.83 * \sqrt{1,781,437} + 1,781,437$$

8. Recalculate the weighed average cumulative chain ladder factors using cumulated false history from Triangle 6.

Development Factors from false history in Triangle 6

	1	2	3	4	5	6	7	8	9	10
Ave	n/a	1.310	1.045	1.028	1.014	1.010	1.002	1.006	1.002	1.001

9. Use the projection ratios from Step 8 to square the triangle from Step 7 using the traditional cumulative chain ladder method, as is shown in Triangle 7.

Triangle 7: Squaring the false history triangle

	1	2	3	4	5	6	7	8	9	10
1	867,818	1,093,577	1,115,760	1,142,375	1,160,107	1,170,827	1,181,821	1,183,216	1,183,592	1,184,293
2	806,259	1,076,977	1,100,800	1,108,670	1,121,914	1,138,237	1,137,459	1,146,116	1,149,938	1,150,619
3	1,177,634	1,510,000	1,565,226	1,598,972	1,617,110	1,639,008	1,644,548	1,656,885	1,659,871	1,660,854
4	1,050,861	1,397,998	1,481,309	1,538,321	1,565,338	1,575,483	1,571,444	1,580,320	1,583,168	1,584,106
5	1,187,350	1,525,824	1,636,315	1,673,518	1,695,732	1,709,328	1,712,954	1,722,629	1,725,733	1,726,756
6	1,416,033	1,901,670	1,996,292	2,049,602	2,077,757	2,098,848	2,103,301	2,115,181	2,118,992	2,120,248
7	1,427,768	1,829,872	1,877,659	1,967,614	1,994,932	2,015,182	2,019,457	2,030,864	2,034,523	2,035,728
8	1,810,969	2,340,804	2,468,695	2,538,749	2,573,996	2,600,124	2,605,639	2,620,357	2,625,079	2,626,634
9	1,398,085	1,920,209	2,005,844	2,062,763	2,091,401	2,112,631	2,117,112	2,129,071	2,132,907	2,134,171
10	2,003,312	2,624,319	2,741,354	2,819,144	2,858,284	2,887,298	2,893,423	2,909,766	2,915,009	2,916,737

To the left of the heavy black line is the false history data from Triangle 6, to the right is the squaring of the false history data using the link ratios from Step 8.

At this point, the bootstrapping methodology has quantified a measure of the estimation error, but not the process variance. In order to obtain the full prediction error, a measure of process variance must be included in the simulation process. To incorporate process variance in the calculations, England proposes the simulation of incremental payments from a series of Gamma distributions. Each projected incremental payment is assumed to have its own Gamma distribution with mean equal to the incremental projected payments that can be derived from Step 9. The variance is equal to the incremental projected payment multiplied by the scale parameter ϕ that was previously mentioned in Step 4. As a practical measure we have extended this method to allow negative incrementals by modeling the absolute incremental projected payment with the Gamma, and then applying the appropriate sign.

10. Calculate incremental projected payments from the squared triangle. The absolute values of these incremental projected payment amounts will be used as the mean values in each Gamma distribution.

Triangle 8: Calculating incremental projected payments from the squared triangle

	1	2	3	4	5	6	7	8	9	10
1										
2										681
3									2,986	983
4								8,876	2,848	938
5							3,626	9,676	3,104	1,022
6						21,091	4,452	11,881	3,811	1,255
7					27,318	20,250	4,275	11,407	3,659	1,205
8				70,053	35,247	26,128	5,515	14,718	4,722	1,555
9			85,634	56,919	28,639	21,230	4,481	11,959	3,836	1,264
10		621,007	117,035	77,790	39,140	29,014	6,125	16,344	5,243	1,727

11. Calculate the scale parameter ϕ. The scale parameter is estimated as the Pearson chi-squared statistic divided by the degrees of freedom. The Pearson chi-squared statistic is equal to the sum of the squares of the unscaled Pearson residuals that were calculated in Step 4. The degrees of freedom equal n – p, where n and p were calculated in Step 5. The scale parameter is the same for all projected incremental payment periods.

For the example shown here, the sum of the squares of the unscaled Pearson residuals from Triangle 3 equals 203,397, and the degrees of freedom equals (55 – 19), or 36. The scale parameter ϕ = 5,650.

12. For each incremental future payment, draw a random sample from a Gamma distribution whose mean is equal to the absolute value of the incremental payment calculated in Step 10 and whose variance equals the product of ϕ (as calculated in Step 11) and the absolute value of the incremental payment calculated in Step 10. Set the sign of the random sample so as to be the same as the original incremental payment calculated in Step 10.

Triangle 9: One possible example of random draws from Gamma distributions to simulate payments that include process error as well as parameter error

	1	2	3	4	5	6	7	8	9	10
1										
2										145
3									95	22
4								5,961	683	637
5							3,853	3,621	2,558	599
6						11,919	2,636	38,594	2,461	96
7					32,750	6,270	10,371	17,807	3,472	0
8				55,382	34,696	19,754	3,525	12,698	1,055	1,674
9			98,708	77,614	34,395	20,533	4,794	190	1,214	0
10		624,837	136,808	80,000	63,599	30,370	217	20,878	1,773	9,111

In this example, the value for row 9, column 3 was drawn from a Gamma distribution with a mean of 85,634 and a variance equal to 5,650 * 85,634. The value for row 10, column 3 was drawn from a Gamma distribution with a mean of 117,035 and a variance equal to 5,650 * 117,035.

13. Sum the incremental future payments calculated in Step 12 to arrive at the final reserve estimate for this particular simulation. In the example shown in Triangle 9, this equals 1,478,376.

14. Repeat steps 5 through 12 N times, producing a different simulated reserve indication each time. At the end of the N simulations, examine the resulting distribution of reserves to arrive at the overall reserve range and reserve indications at different percentiles.

Results from 5,000 simulations

Mean	1,425,665
Standard Deviation	136,233
Minimum	970,631
Maximum	2,055,375
1% ile	1,125,108
5% ile	1,206,925
10% ile	1,253,563
20% ile	1,308,459
30% ile	1,353,153
40% ile	1,389,518
50% ile	1,424,384
60% ile	1,457,631
70% ile	1,492,174
80% ile	1,535,347
90% ile	1,596,733
95% ile	1,656,242
99% ile	1,760,099
Indicated reserve, based on squaring the original data triangle	1,416,460

Management's Best Estimate of Loss Reserves

Rodney E. Kreps, FCAS, MAAA

Management's Best Estimate of Loss Reserves

Rodney Kreps
Guy Carpenter Instrat

Abstract

An economically rational way for management to set reserve estimates is to utilize the future change in the value of the company as a statistical decision function and then to choose the reserve estimate so as to minimize the average value of this function. The mean of the reserve distribution is almost surely too low as an outcome.

Introduction

Management is required[1] to provide a best estimate of loss reserves. In the opinion of this author, actuarial practices[2] strongly suggest the estimate be the mean value of the distribution of loss reserves. It will be argued that the problem of the estimate is best approached by statistical decision theory, and that all the usual statistical estimates can be produced in such a fashion. Further, there is an economic basis for choosing a decision function, which then determines the estimate. Desirable characteristics for a decision function are discussed, and a candidate function is proposed. A simplified example and a spreadsheet are provided. One general conclusion that emerges is that the mean is probably not a good estimate, as it is almost surely low.

Statement of Statutory Accounting Principles

The SSAP #55 effective January 1, 2001 says[3] in part "For each line of business and for all lines of business in the aggregate management shall record its best estimate of its liabilities ..." and "Management's analysis of the reasonableness of ... reserve estimates shall include an analysis of the amount of variability in the estimate." Not to put too much into a single word, but please note that it is "in the estimate" rather than "of the estimate." The author believes that actuaries have tended to place too much attention on the differences between different estimates and not enough on the variability of actual results.

SSAP 55 goes on to say "Management's range [of estimates] shall be realistic and, therefore, shall not include the set of all possible outcomes but only those outcomes that

[1] Statutory Statement of Accounting Principles #55 effective January 1, 2001
[2] ASOP # 36, especially section 3.6.3
[3] This and subsequent quotes in this section are taken from SSAP 55, page 55-6, sections 10 and 11.

are considered reasonable." In other words, weight scenarios by their probabilities: use a distribution. But how?

In the next section SSAP 55 says that in the case of a range with all values equally likely, choose the middle of the range; but if the equally likely values do not form a range "management should determine its best estimate of the liability." Again, there is not a lot of help here on how to actually do it. Let us see what the actuaries have to say.

Actuarial Standard of Practice

ASOP 36 – Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves - has quite a bit to say about reserves and uncertainty. We will assume that the hard work of evaluating trends, court climates, and other sources of both process and parameter uncertainty has been done and that we have a best candidate loss reserve distribution which includes all the outcomes and best estimates of their probabilities.

This is, of course, a major assumption seldom made explicit in practice but often used implicitly. For example, section 3.6.3 is entitled "Expected Value Estimate" and says "In evaluating the reasonableness of reserves, the actuary should consider one or more expected value estimates of the reserves, except when such estimates cannot be made based on available data and reasonable assumptions." Expected value, apart from the fact that you never actually expect to see it happen, is the mean of a distribution. So "one or more expected value estimates" is saying "find different ways of getting the mean of our unknown but implicitly present distribution."

The same section goes on to say "Other statistical values such as the mode (most likely value) or the median (50th percentile) may not be appropriate measures for evaluating loss and loss adjustment expense reserves, such as when the expected value estimates can be significantly greater than these other estimates." Here the author sees the innate, carefully cultivated, and experientally substantiated conservatism of the actuaries expressed as "Let's go for the higher value." Curiously enough, this paper will argue that the mean itself is too low.

The section continues "The actuary may use various methods or assumptions to arrive at expected value estimates. In arriving at such expected value estimates, it is not necessary to estimate or determine the range of all possible values, nor the probabilities associated with any particular values." So, although ASOP wants mean value estimates, it does not want a distribution. Most of the techniques for doing reserve estimates, even including those which are used for sparse or missing data, have an underlying statistical model and an implied distribution. We are invited to use the models and forget the distributions.

The next section 3.6.4 is entitled "Range of Reasonable Reserve Estimates." It begins "The actuary may determine a range of reasonable reserve estimates that reflects the uncertainties associated with analyzing the reserves. A *range of reasonable estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable." Clearly, something like this needs to be in the ASOP in order to give the actuary room not to be forced by a formula. This is saying, look at least implicitly at several reasonable distributions (alternatively, models), get the mean from each one, and make a weighted choice. The author would prefer that the judgment calls be in the creation of the best predictive distribution. For example, if there are two equally valid distributions, weight them equally. The mean will be the average of the individual means. Of course, this does require going from no distributions to three.

This same section continues "The actuary may include risk margins in a range of reasonable estimates, but is not required to do so, except as required by ASOP No. 20. A range of reasonable estimates, however, usually does not represent the range of all possible outcomes." So, while allowing that there really is a distribution with outcomes of differing probabilities, the ASOP wants to make sure that the reserve estimate is well inside the range while not giving any guidance on how to get to a risk margin or what might be appropriate. Presumably, the margin should be for the risk that there will be, in the words of section 3.3.3, an "amount of adverse deviation that the actuary judges to be material with respect to the statement of actuarial opinion.." Unless the distribution is very narrow, this seems quite likely to be the case.

Section 3.6.5 on "Adverse Deviation" makes the same point, but does not suggest a risk margin. It only says "The actuary should consider whether the future paid amounts are subject to significant risks and uncertainties that could result in a material adverse deviation." Section 4.6(g) says that it is up to the actuary to include mention of this in the report . In addition, section 4.8 says "An actuary must be prepared to justify the use of any procedures that depart materially from those set forth in this standard and must include, in any actuarial communication disclosing the results of the procedures, an appropriate statement with respect to the nature, rationale, and effect of such departures." Perhaps this justification could be "and I plunked down another 35% because this line is all over the place."

The author's sense of the ASOP writing on uncertainty is that it gives management some idea of how much wiggle room there is in the creation of management's best estimate, at least according to the actuaries. But none of this actually gives an economic basis for how the estimate should be made.

Statistical Decision Theory – the short version

An economic basis for the creation of an estimate needs a way to combine a probability distribution of outcomes with an economic function describing the pain that will be felt when the realized random outcome differs from the estimate. The simplest recipe is to go for Least Pain, as follows:

(1) Create a pain function based on economic reality. This function will be the economic decision function which will drive the results. "Pain function" actually is the technical term because this function is meant to represent how unpleasant adverse outcomes of

various sorts may be. This function will depend on the estimate and the random variable representing possible outcomes. Typically it will be zero when random outcome and estimate are equal.

(2) Average the pain function over the probability distribution of outcomes for every fixed estimate.

(3) Find the value of your estimate which makes the average pain smallest.

It is easy enough to see that such a prescription satisfies SSAP 55, and that the description of the pain function represents management's logic in creating the estimate.

The context is in hand is setting the reserves and then a year later making adjustments for development on old years. More precisely, we assume that the assessment a year later is "correct" and represents a random realization of the underlying distribution.

What we will show next is that all the usual estimates can be represented as being derived from pain functions. The comparison of pain functions gives us a way to speak about the relevance of the estimates in a business and economic context. Pain functions can be thought of as negative utility functions.

Following that we will argue for the general characteristics of a pain function for loss reserving.

Mathematical Representation of the recipe

The general case is that we have a probability density function f(x) with support from 0 to infinity. We also have a pain function p(m,x) which is a function of the estimate m and the random variable x. We denote the average of p over f as P(m):

$$P(m) = E(p) = \int_0^\infty p(m,x) f(x) dx$$

It is reasonable to ask that the integral exists, and that p >= 0 everywhere. We want m to be such that P(m) is the smallest, so we choose the value for m which makes

$$0 = \frac{d}{dm} P(m) = \int_0^\infty \frac{\partial}{\partial m} p(m,x) f(x) dx$$

Sometimes in practice p will be discontinuous at x = m. In that case define

$$p(m,x) = \begin{cases} p_-(m,x) & \text{for } x \le m \\ p_+(m,x) & \text{for } x \ge m \end{cases}$$

which makes

$$\frac{dP(m)}{dm} = \int_0^m \frac{\partial p_-(m,x)}{\partial m} f(x)dx + \int_m^\infty \frac{\partial p_+(m,x)}{\partial m} f(x)dx$$
$$+ p_-(m,m) - p_+(m,m)$$

Usually the last difference is zero and in fact the individual terms are usually zero.

Notice that the scale and absolute value of P(m) do not enter into the calculation for m. You can add a constant and multiply by any constant and m does not change. The pain functions given are the only ones known to the author which give the usual statistics for all distributions.

Example: the mean

For the mean, the pain function is a quadratic about the estimate:

$$p(m,x) = (m-x)^2$$

The average pain is

$$P(x) = \int_0^\infty (m-x)^2 f(x)dx$$

In this particular case, we can do the integrals in terms of the mean and variance of the distribution:

$$\begin{aligned} P(m) &= \int_0^\infty (m^2 - 2mx + x^2) f(x)dx \\ &= m^2 - 2m * mean + (Var + mean^2) \\ &= Var + (m - mean)^2 \end{aligned}$$

As a function of m, this clearly has a minimum when m equals the mean of the distribution.

At this point, we should pause and ask ourselves "Why do I want a quadratic decision function? What is so good about squared dollars?" The symmetry of the pain function about the estimate implies that for the reserves to come in lower than our estimate is as bad as having them come in higher. The quadratic form implies that two dollars low is four times as bad as one dollar low.

Example: the median

For the median, the pain function is linear about the estimate with equal slope on both sides, and with a discontinuity in the derivative at x = m:

$$\begin{aligned} p(m,x) &= abs(m-x) \\ &= \begin{cases} m-x & \text{for x } \le m \\ x-m & \text{for x } \ge m \end{cases} \end{aligned}$$

Although this function is still symmetric about the estimate, it says that it is dollars, rather than squared dollars, that are of interest; and that two dollars off is only twice as bad as one dollar off. This has some plausibility.

For the evaluation, the partial derivative is

$$\frac{\partial p(m,x)}{\partial m} = \begin{cases} 1 & \text{for } x < m \\ -1 & \text{for } x > m \end{cases}$$

So the equation to be solved for m is

$$0 = \int_0^m f(x)dx - \int_m^\infty f(x)dx$$
$$= F(m) - [1 - F(m)]$$

where F(x) is the cumulative distribution function. This requires

$$F(m) = \tfrac{1}{2}$$

That is to say, m is the median.

Example: the fixed percentile

For an arbitrary fixed percentile, the pain function is linear about the estimate, but with different slopes on the high and low side:

$$p(m,x) = \begin{cases} m - x & \text{for } x \le m \\ \alpha(x - m) & \text{for } x \ge m \end{cases}$$

Again it is dollars that are of interest but here it is a factor of α worse for x to be high rather than low. Take α to be some constant, say 3.

For the evaluation, the partial derivative is

$$\frac{\partial p(m,x)}{\partial m} = \begin{cases} 1 & \text{for } x < m \\ -\alpha & \text{for } x > m \end{cases}$$

So the equation to be solved for m is

$$0 = \int_0^m f(x)dx - \int_m^\infty \alpha f(x)dx$$
$$= F(m) - \alpha[1 - F(m)]$$

where again F(x) is the cumulative distribution function. This requires

$$F(m) = \frac{\alpha}{\alpha + 1}$$

That is to say, m is the α /□ α +1 percentile value. For $\alpha = 3$, this is the 75th percentile.

It will be argued that a decision function that gives more weight to the high side than the low is desirable for loss reserving. It is usually worse to come in above your estimate than below it.

Example: the mode

Here the decision function is one outside of some (preferably small) interval around the estimate, and zero within it. The economic interpretation of this decision function is that for the reserves to come in outside of this interval is equally bad no matter where it happens. This means that high or low, just outside or very far away are all the same. This does not seem reasonable. On the other hand, this will provide the single best guess for an interval of given size to contain the result.

The pain function is

$$p(m,x) = \begin{cases} 1 & \text{for } x \le m-\varepsilon \\ 0 & \text{for } m-\varepsilon < x < m+\varepsilon \\ 1 & \text{for } x \ge m+\varepsilon \end{cases}$$

and 2ε is the size of the interval. The average pain is the probability that the random variable is realized outside of the interval:

$$P(m) = F(m-\varepsilon) + \left[1 - F(m+\varepsilon)\right]$$

Setting the derivative to zero,

$$0 = f(m-\varepsilon) - f(m+\varepsilon) \approx -2\varepsilon \frac{df(m)}{dm}$$

For small interval, this says that the density function is a maximum at m. That is, m is the mode.

Fundamental considerations

It is clear that all of the usual estimates can be phrased as resulting from a particular choice of decision function and that infinitely many decision functions are possible. What is needed is to construct the decision function from the economic or other forces which impact the entity setting the reserve levels. This will be the appropriate decision function.

One possibility is a purely subjective estimate of how, say, the CFO feels about various sizes of future difference of reserves from the stated estimate. Slightly better would be to use the reserves committee as input, in a Delphi method.

Another possibility is to examine the fundamental economic consequences which result from the reserves (at least as appearing in next year's Annual Statement) coming in different from the reserve level which is currently set. A good candidate for the decision function is the **decrease in the net economic worth of the company** as a result of the reserve changes. While estimates of this may involve subjective judgments, at least something of definite and measurable economic value is being considered.

Interested parties who may affect the economic value of the company include policyholders, stockholders, agents, regulators, rating agencies, IRS, investment analysts, and lending institutions.

If the reserves come in slightly higher than the estimate, there perhaps is not much market reaction. The industry as a whole has had the reputation of being under-reserved. It may also be that some managements will like being slightly under-reserved because then they are able to have overstated earnings the previous year. However, if the increase in reserve levels is significant enough that surplus is significantly impacted then a number of effects come into play: The capacity to write business is impaired; the firm's credit rating may become impaired, increasing the cost of capital; rating agencies, investor analysts, and the IRS become more concerned; future renewals (the "goodwill" of the firm) become more problematical. And if the change is large enough then IRIS tests begin to be triggered and regulatory authorities are involved, which definitely will decrease the value of the firm. The economic consequences would seem to be rapid and non-linear in the reserve increase.

On the other hand suppose the reserves come in significantly below the estimate. This means that the company has been over-reserved and consequently[4] is less competitive than it could have been; that the IRS has ammunition for its audit; that dividends could have been larger; participatory plans could have been more generous; that there is a danger of losing future business from over-pricing. These effects would seem to be less immediately significant than the results from under-reserving, at least in the short term.

Each of these situations will generate a negative effect on the net worth of the company compared to its value if the reserves came in as stated. However, intuition suggests that the effect will be much stronger on the under-reserving side than on the over-reserving side, and will be non-linear, especially as particular analyst, rating agency, and regulatory tests reach trigger points.
The immediate consequence is that a symmetric pain function such as that for the mean would be over-estimating the negative impact of reserve decreases and consequently the estimate is intrinsically too low.

An approximation to the correct function

As a crude approximation which has some of the properties just suggested, consider a decision function which is quadratic around the estimator but linear (using the tangent to the parabola) at some value below it. Call it "semi-quadratic." This function is mathematically well-behaved, as the value and the first derivative are both continuous. This function will also satisfy that being high (in the outcome) is never better than being low and that the high side is quadratic while the low side is linear. The choice of distance below determines the slope of the line; the closer it is to the estimator the smaller the slope.

We make the decision function have the dimensions of dollars to mimic the economic value. S is the company surplus, since that is the appropriate scale for many tests.

The decision function is
$$p(m,x) = \begin{cases} 2(m-x) - \alpha S & \text{for } x \le m - \alpha S \\ \dfrac{(x-m)^2}{\alpha S} & \text{for } x \ge m - \alpha S \end{cases}$$

[4] Assuming that the pricing and reserving actuaries actually talk with each other

The parameter α is dimensionless and reflects management attitude. The pain is equal to αS when the outcome is αS above or below the estimator. A small α reflects a relatively low pain for over-reserving and conversely a higher pain for under-reserving.

An α of 3% is used below, which says that a reserve change of 3% of surplus downward is the same pain as the same change upward. However, 10% under-reserved is about twice as bad as 10% over-reserved; 20% under-reserved is about 3.6 times as bad as 20% over-reserved. Again, as α gets smaller management is less tolerant of under-reserving.

The partial derivative is
$$\frac{\partial p(m,x)}{\partial m} = \begin{cases} -2 & \text{for } x \le m-\alpha S \\ \dfrac{2(x-m)}{\alpha S} & \text{for } x \ge m-\alpha S \end{cases}$$

and the equation to be solved is
$$0 = -\int_0^{m-\alpha S} f(x)dx + \frac{1}{\alpha S}\int_{m-\alpha S}^{\infty} (x-m) f(x)dx$$

Or,
$$F(m-\alpha S) = \frac{1}{\alpha S}\left\{\left[mean - F_1(m-\alpha S)\right] - m\left[1-F(m-\alpha S)\right]\right\}$$

Where $F_1(x)$ is the integral of $xf(x)$ – the first moment distribution.

In order to work entirely with dimensionless variables, it is convenient to measure the estimate and the mean in units of surplus. Then the above equation holds with S = 1.

Just to make explicit the kind of results that might be seen, as an example take F(x) to be a lognormal distribution with known *mean* and coefficient of variation **c**. Both F(x) and $F_1(x)$ can be explicitly calculated in terms of the normal distribution function

$$N(x) \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz:$$

We have
$$F(x) = N\left(\frac{\ln(x)-\mu}{\sigma}\right)$$

and
$$F_1(x) = mean\, N\left(\frac{\ln(x)-\mu}{\sigma} - \sigma\right).$$

with the usual $\sigma = \sqrt{\ln(1+c^2)}$ and $\mu = \ln(mean) - \sigma^2/2$.

For a company which has a mean reserve to surplus ratio of 3.5 with a coefficient of variation in the reserves of 10%, an α of 3% results in setting the reserve estimator at 11.5% above the mean. This estimator is at about the 87.2% level of the reserve CDF. See the accompanying worksheet SemiQuadraticExample.xls for details and other values of α.

Why bother?

Almost everything actuarial that goes into actually setting reserves is assumed in this discussion. In particular, the explicit existence of a distribution is problematical. Not impossible, just hard.

In many companies the current reserve-setting process probably already done on a least-pain basis. However, the pain is not future pain but present pain. Reserves are set with an eye on what was set in the past and on current analyst expectations. The reserve committee is always playing catch-up. The procedure discussed here assumes that the reserves are set on old years (how the random variable comes in) with no concern for current politics – clearly a naïve assumption.

Also, the author is not aware of a line of business decision function that would make sense. Perhaps some of the capital allocation methodologies could be helpful.

Similarly, no one actually knows how the value of the company will decrease; but experienced players have some sense of it. There have been enough examples in the last few years to show that reserve changes can have significant impact. It is also clear that what may impact a given company's pain function may be quite different from another's, and that the emphasis may very well change from year to year. Allowing this would be a problem for regulators.

Still, it would be a useful exercise for the reserve committee at a company to get together and try to build, even crudely, their pain function for the year. They could perhaps begin with some standardized event (lose 10% of surplus) which has enough pain to work as a comparison with other possibilities, and then fill out both the high and low sides at a convenient and realistic set of values[5]. Then they could ask the actuary to do the numerical calculation for the estimate, and have a much better idea of what the increase in average pain would be from using an estimator not at the minimum if they choose to do so. And, in the process they would come to be able to explain how they arrived at their estimate. Another interesting exercise[6] would be to put the management incentive plan as an input to the pain function.

All of this makes reserving by formula (e.g., use the mean) impossible. But it really is, anyway.

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[5] For example, let loss 10% of surplus = 1 pain unit then make up the rest: lose 25% = 10; lose 20% = 8; lose 15% = 3; lose 5% = 0.5; lose 0 = 0; gain 5% = 0; gain 10% = 0.5; gain 15% = 1; gain 20% = 2.

[6] This is probably implicitly happening already.

Considerations Regarding Materiality and Range of Reserves in Connection With Actuarial Standard of Practice #36

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Abstract

The Actuarial Standard of Practice No. 36 has highlighted several issues which have been implicitly considered by property/casualty actuaries for years. For the first time, the types of statements of actuarial opinion have been standardized and listed for categorization by property/casualty actuaries. However, many other areas which the actuary needs to be familiar with are not documented in standard actuarial literature. This paper examines the interrelationship of materiality and range of reasonable reserves. Some common rules of thumb are formulated in regards to the range of reasonable reserve estimates. Accounting literature, such as The American Institute of Certified Public Accountants ("AICPA") Professional Standards and the Security Exchange Commission Staff Accounting Bulletins, are referenced in order to provide the actuary some reference materials while issuing opinions. In addition, some practical considerations regarding necessary work steps needed to issue a statement of actuarial opinion are outlined.

Introduction

The Actuarial Standard of Practice No. 36 has highlighted several issues which have been implicitly considered by property/casualty actuaries for years. For the first time, the types of statements of actuarial opinion ("SAO") have been standardized and listed for categorization by property/casualty actuaries. However, many other areas which the actuary needs to be familiar with are not documented in standard actuarial literature. This paper examines the interrelationship of materiality and range of reasonable reserves. Some common rules of thumb are formulated in regards to the range of reasonable reserve estimates. Accounting literature, such as American Institute of Certified Public Accountants ("AICPA") AICPA Professional Standards and the Security Exchange Commission Staff Accounting Bulletins, are referenced in order to provide the actuary some reference materials while issuing opinions. In addition, some practical considerations regarding necessary work steps needed to issue a statement of actuarial opinion are outlined.

I. Actuarial Standards of Practice

Actuarial Standards of Practice are promulgated by the American Academy of Actuaries' Actuarial Standards Board ("ASB"). In the Preface to Actuarial Standards of Practice, it is stated that the ASB is provided with the sole discretion of promulgating actuarial standards of practice. The objectives of the ASB include direction, management, exposition and promulgation of actuarial standards of practice by its operating committees, and to provide continuous reviews of existing standards of practice.

Since 1990, the National Association of Insurance Commissioners ("NAIC') requires that for most property casualty insurers (with some minor exceptions) a statement of opinion be signed by a *qualified actuary*, as outlined in the NAIC Instructions for Completing the statutory Property/Casualty Annual Statement blank. This statement contains an opinion expressed by a qualified actuary regarding the reasonableness of the carried statutory loss and loss adjustment expense reserves as shown in the statutory annual statement blank. The focus of this paper is in regards to the NAIC required opinions for statutory purposes.

There are numerous other situations which require statements of actuarial opinion, some of which are:

- Through December 31, 2001, the state of Minnesota statutory insurance laws triennially require an independent actuary to opine on the reasonableness of the carried loss and loss adjustment expense reserves of non-Minnesota domiciled carriers.
- Underwriting pools and associations may require an actuarial opinion regarding the carried loss and loss adjustment expense reserves for the benefit of the members of the pool.
- The state of Vermont requires a statement of actuarial opinion regarding the reasonableness of the loss and loss adjustment expense reserves of Vermont domiciled captive insurance companies.
- The U.S. Department of the Treasury has requested statement of actuarial opinions in connection with insurance companies which write surety bonds.

The widespread use of these statement of actuarial opinions contributed to the need for additional guidance for statements of actuarial opinion. ASOP36 is the professional standard of practice which governs the issuance of these actuarial opinions as well.

II. General Overview of Actuarial Standard of Practice #36 ("ASOP36")

ASOP36 was adopted for written statements of actuarial opinion with respect to loss and loss adjustment expense reserves valued on or after October 15, 2000 (Section 1.4 of ASOP36). Several definitions are provided in Section 2 of ASOP36 which can guide the actuary through the Standard of Practice. The standard introduced many new requirements. Some of the more important new features are

- a requirement that the actuary evaluate whether there are specific risks and uncertainties which could result in material adverse deviation in the loss and loss adjustment expense reserves (Section 3.3.3);
- a requirement that the actuary evaluate materiality in the evaluation of loss and loss adjustment expense reserves , with consideration to the intended uses for the statements of actuarial opinion (Section 3.4); and
- specific guidance as to the nature and extent of disclosures required for statements of actuarial opinion. (Section 4).

III. Discussion of Three Areas of Interest: Types of ASOP36

A. Types of Opinions (3.3.2)

ASOP36 defines five types of statements of actuarial opinion, with conditions for each noted:

a. Determination of Reasonable Provision

When the stated reserve amount is within the actuary's range of reasonable reserve estimates

b. Determination of Deficient or Inadequate Provision

When the stated reserve amount is less than the minimum amount that the actuary believes is reasonable, the actuary should disclose the additional amount necessary to equal the minimum amount that the actuary believes is reasonable (4.6.d)

c. Determination of Redundant or Excess Provision

When the stated reserve amount is greater than the maximum amount that the actuary believes is reasonable, the actuary should disclose the amount by which the stated amount exceeds the maximum amount that the actuary believes is reasonable (4.6.e)

d. Qualified Opinion

When the stated reserve amount includes a certain item or items in question because they cannot be reasonably estimated or the actuary is unable to render an opinion on those items

An opinion on the liabilities associated with the stated reserve except for the qualified item(s) should be rendered in accordance with a. through c.

If the item(s) are not believed to be material, a qualified opinion is not required

e. No Opinion

If no opinion can be reached due to deficiencies in data, analyses, assumptions, or related information

Comments Regarding Types of Actuarial Opinions and Potential Impact on Actuarial Work Processes

Before the issuance of ASOP36, no binding professional guidance existed for types of actuarial opinions. The "Property and Casualty Practice Note" as published by the Committee on Property and Liability Financial Reporting ("COPLFR") of the American Academy of Actuaries for the past several years provides guidance regarding Statements of Actuarial Opinion for statutory loss and loss adjustment expense reserves; however, the practice note "...has not been promulgated by the Actuarial Standards Board nor is it binding on any actuary".

The determination of the reasonable provision, which states that carried reserves must be within a reasonable range of reserves (discussed below), introduces precision recently not required. Before ASOP36, an actuary might opine that a Company whose carried reserve is "slightly" above the high end of a reasonable range "conservative"; under ASOP36, the actuary must opine that the Company's reserves are redundant or excessive, and quantify the amount.

Another example shows an additional potential impact of ASOP36. Before ASOP36, the actuary might have stated that loss and loss adjustment expense reserves were "reasonable but conservative " if a company's carried loss and loss adjustment expense reserves were slightly below the maximum amount that the actuary believes is reasonable. Under the guidance of ASOP36, such an opinion would now be "reasonable".

Additionally, the disclosure of the amounts of the deficiencies or redundancies (III. A. b. and III. A. c. above) necessitate possible changes in work processes for opining actuaries under NAIC statutory regulations. Currently, March 1 is the statutory filing due date for financial statements ending December 31 of the prior calendar year, accompanied by the statement of actuarial opinion. Subsequently, the due date for the delivery of the actuarial report to the company is May 1 (or prior to May 1 within two weeks of the request by the state insurance department). Prior to ASOP36, the actuary could determine by the statutory filing date the type of opinion which would be rendered, and subsequently refine precisely the range and point estimate (if determined) by the due date for the delivery of the report. Due to ASOP36's requirement that the disclosure of the precise amount of deficiency or redundancy be included for deficient/redundant opinions, the actuary must now determine the precise low end of range (for deficient provisions/opinions) or the precise high end of range (for redundant provisions/opinions) by the statutory filing date.

B. Range of Reasonable Reserve Estimates (3.6.4)

Following the definitions of the types of opinions as outlined above, ASOP36 iterates that the actuary may determine a range of reasonable reserve estimates that reflect the uncertainties associated with analyzing reserves. "A *range of reasonable estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable. The actuary may include risk margins in a range of reasonable estimates, but is not required to do so, except as may be required by ASOP No. 20. A range of reasonable estimates, however, usually does not represent the range of all possible outcomes."

Discussion of Range

Accounting literature has discussed methods to account for contingencies which are of interest to the actuary. Statement of Financial Accounting Standards No. 5 ("FASB 5") [of the Financial Accounting Standard Board's ("FASB") *Statement of Standards*] "Accounting for Contingencies" establishes accounting requirements for U.S. Generally Accepted Accounting Principles ("GAAP") which are relate to property casualty loss reserve liabilities. Paragraph 8 of FASB5, "Accrual of Loss Contingencies", states that an estimated loss from a contingency shall be accrued by a charge to income as long both of the following conditions are met:

a. It is probable that an asset has been impaired or a liability has been incurred at the date of the financial statements, and
b. The amount of loss can be reasonably estimated

An important interpretation of FASB5 has impacted the concept of range, and the way that accountants view the "range of reasonable reserve liabilities". "FASB Interpretation No. 14, Reasonable Estimation of the Amount of a Loss an interpretation of FASB Statement No. 5", states that " When some amount within the range appears at the time to be abetter estimate than any other amount within the range, that amount shall be accrued. When no amount within the range is a better estimate than any other amount, however, the minimum amount in the range shall be accrued."

The difference between an actuary's view of a best estimate and range can be differentiated from the perspective of an accountant. When an actuary determines a point estimate as well as a range of reasonable reserve estimates, that point estimate has a higher degree of certainty than other points within the range. Similarly, under FASB5, that greater degree of certainty implies that the point estimate should be established.

However, the accountant might view all points in a reasonable range of reserves as equally likely. However, an actuary may opine that the point estimate is the most likely scenario, with points within a reasonable range of reserves becoming less probable as one moves towards either end of the range. This distinction is important to be noted in actuarial and accounting interactions.

Similar guidance relating to accrual of liability for statutory purposes is outlined in the *NAIC Accounting Practices and Procedures Manual*, effective January 1, 2001. The adoption of this statutory accounting framework culminated a multi-year effort of the NAIC Accounting Policies and Procedures Task Force to "codify" statutory accounting policies. The Manual is embodied in a series of Statements of Statutory Accounting Principles ("SSAP's"), which introduced some significant changes in the statutory accounting practices for many property/casualty insurance companies. The NAIC codification principles also discuss the concept of range with respect to carried loss and loss adjustment expense reserves. SSAP 55 states that the Company should accrue "Management's Best Estimate" of its liabilities for unpaid claims, unpaid losses and loss/claim adjustment expenses for each line of business and for all lines of business in the aggregate. Management may consider a range of reserve estimates; the range shall not include the set of all possible outcomes but only those outcomes that are considered reasonable. When no estimate within the range is better than any other, the midpoint of the range (as opposed to the minimum from the FASB Interpretation No. 14 of FASB 5) is to be accrued. [1]

Current actuarial literature is rich with examples regarding methods to determine ranges of loss reserves. In the spring of 1994, an entire Casualty Actuarial Society Discussion Call Paper Program ("Variability in Reserves") was devoted to various methodologies to determine ranges of reasonable reserves. The Thomas Mack paper titled "Measuring the Variability of Chain Ladder Reserve Estimates", and the Daniel Murphy paper entitled "Unbiased Loss Development Factors" in PCAS 1994 are two such papers. A more recent paper written by Chandu Patel and Alfred Raws titled "Statistical Modeling Techniques for Reserve Ranges: A Simulation Approach" in the 1998 Fall Forum Reserving Call Papers compares various approaches for establishing reasonable

[1] If management's best estimate is different from the estimate of the Company's appointed actuary, some accountants believe that management should offer reasons as to why the difference has occurred, i.e., the factors that the actuary's estimates have not considered which are captured in Management's determination of the loss and loss adjustment expense reserve amount which is carried.

ranges of reserves, and connects those reasonable range of reserves with testing of confidence level factors.

Disclosure of the Reasonable Range of Reserve Estimates

ASOP36 does not require the range of reasonable reserve estimates to be disclosed in the opinion. Commentary in Appendix 2, "Comments on the 1999 Third Exposure Draft and Subcommittee Responses" provides reasoning as follows: "The subcommittee believes that the actuary may be able to consider a range of reasonable estimates for purposes of the opinion without having to specify the end points of the range. This is acceptable because the actuary could be basing the opinion on various methods and estimates that produce results not much different from the stated reserve amount. Consequently, disclosure of a specific range is unnecessary."

However, the Documentation section (Section 4.2) states that the actuary should be guided by the provisions of ASOP No. 9 ("ASOP9"), *Documentation and Disclosure in Property and Casualty Insurance Ratemaking, Loss Reserving, and Valuations.* The explicit ASOP36 requirement from 3.3.2.a. that "When the stated amount is within the actuary's range of reasonable reserve estimates (see Section 3.6.4), the actuary should issue a statement of actuarial opinion that the stated amount makes a reasonable provision..." appears to imply that the actuary must per se already have developed a reasonable range of reserves in order to issue a "reasonable" actuarial opinion. Consequently, ASOP9 would imply that the specific amount of the reasonable range of reserves should be at the very least in the actuarial workpapers. Prior to the issuance of ASOP36, an actuary could issue a reasonable opinion if the indicated reserves were "close", (for example, within 5%, of the carried reserves); ASOP36 appears to necessitate an explicit range calculation notwithstanding the distance of the indicated reserves from the carried reserves.

In the case of the NAIC required actuarial report supporting the actuarial opinion, disclosing the specific range of reasonable reserves would appear to be a logical conclusion resulting from the Documentation section of ASOP36. For example, if the stated reserve amount was within an actuary's reasonable range of reserves but close to either end of the actuary's reasonable range of reserves, disclosure of a specific range in the actuarial report could be especially useful for the regulator. The disclosure of risk of material adverse deviation (discussed in this paper's Section II.C. below) effectively exposes the high end of the range of reserves and its relation to the carried reserves, in those cases where risk of material adverse deviation is thought to exist.

C. Materiality (3.4), Significant Risks and Uncertainties (Explanatory Paragraph) [3.3.3], and Adverse Deviation (3.6.5)

The AICPA Professional Standards section entitled "U.S. Auditing Standards" defines methodology to evaluate materiality in a manner similar to the guidance provided by ASOP36. The AICPA section goes one step further: it defines Materiality. That definition is useful for our guidance in evaluating materiality standards. Section 312.10 of the AICPA code states the following:

> "The auditor's consideration of materiality is a matter of professional judgment and is influenced by his or her perception of the needs of a reasonable person who will rely on the financial statements. The perceived needs of a reasonable person are recognized in the discussion of materiality in Financial Accounting Standards Board Statement of Financial Accounting Concepts No. 2, Qualitative Characteristics of Accounting Information, which defines materiality as 'the magnitude of an omission or misstatement of accounting information that, in the light of surrounding circumstances, makes it probable that the

judgment of a reasonable person relying on the information would have been changed or influenced by the omission or misstatement.' That discussion recognizes that materiality judgments are made in light of surrounding circumstances and necessarily involve both quantitative and qualitative considerations."

In Section 3.4 of ASOP36, it is stated that the actuary should consider the purposes and intended uses for the SAO in evaluating materiality. The ASOP36 states that the actuary should evaluate materiality based upon:

1. Professional judgment,
2. Materiality guidelines or standards applicable to the SAO, and
3. The actuary's intended purpose for the SAO. The actuary should understand which financial values are usually important to the intended users of the statement of actuarial opinion and how those financial values are likely to be affected by changes in the reserves and future payments for losses and loss adjustment expense reserves.

ASOP36 provides three examples of materiality which actuaries could reference in Statements of Actuarial Opinion:

1. "Specified reserve amount for which an opinion is given"; i.e. as a percentage of net loss and loss adjustment expense reserves.

2. "The Company's reported surplus"; this materiality standard would be appropriate for a SAO for an insurance company to be used for financial reporting to insurance regulators.

3. "The Company's net worth and annual net income" could be the bases used in an actuarial appraisal.

Section 3.3.3, Significant Risks and Uncertainties (Explanatory Paragraph) states that the actuary should include an explanatory paragraph when the actuary reasonably believes that there are significant risks and uncertainties that could result in material adverse deviation. The explanatory paragraph should contain a) "the amount of adverse deviation that the actuary judges to be material with respect to the statement of actuarial opinion"; and b) "a description of the major factors or particular conditions underlying risks and uncertainties that the actuary believes could result in material adverse deviation".

Section 3.6.5 discusses and defines Adverse Deviation. "An adverse deviation occurs when such a variation results in paid amounts higher than provided for in the reserves. The actuary should consider whether the future paid amounts are subject to significant risks and uncertainties that could result in a **material** adverse deviation" (emphasis added).

Quantitative Percentages to Assess Materiality

From the above discussion, ASOP36 provides three bases against which to assess materiality: loss reserves, surplus and net income. However, ASOP36 does not provide numerical percentages relating to materiality measures. The following are some broad quantitative measures, not meant to be all-encompassing, which can provide the actuary with some guidance on selecting those bases.

- Loss and loss adjustment expense reserves. The author of this paper has seen 5% and 10% of loss and loss adjustment expense reserves as materiality percentage amounts. The range of reasonable reserve estimates has important implications regarding the amounts of material adverse deviation and tests of materiality. The range also provides some guidance in selecting materiality standards. The interrelationship between the reasonable range of reserves and materiality is discussed below in Section IV.

- Reported Surplus. Richard Roth demonstrated in his paper "Analysis of Surplus and Rate of Return without Using Leverage Ratios" that the reserves to surplus ratio has remained relatively constant at 2:1 from 1975 through 1990. For companies writing at that ratio, this suggests materiality percentages of 10% and 20% of surplus would be equal to the 5% and 10% reserve percentages from the above paragraph.

- Net Income. In August, 1999, the Securities Exchange Commission ("SEC") released Staff Accounting Bulletin ("SAB") No. 99. In the SAB, the views of the staff were expressed that relying on certain quantitative benchmarks to assess materiality in preparing financial statements were inappropriate; further, misstatements are not immaterial simply because they fall beneath a numerical threshold. However, the Financial Accounting Standards Board ("FASB") did note that in certain limited circumstances the SEC and other bodies had issued quantitative materiality guidance (discussed in SAB 99's Section 7213, Materiality). The SEC quoted "contradictory studies", one study which suggested widespread use of a "rule of thumb" of 5% to 10% of net income. Although the FASB rejected the formulaic approach, this example is another place to start in assessing materiality in percentage form, after evaluating the quantitative and qualitative considerations discussed below.

Professional Guidance regarding Materiality

The CAS Valuation, Finance and Investments Committee ("VFIC") has published "Materiality and ASOP No. 36: Considerations for the Practicing Actuary. (The VFIC was included as Appendix 7 in the December 31, 2001 Property and Casualty Practice Note developed by COPLFR). Materiality in accounting contexts is also discussed, with references to the NAIC Accounting Practices and Procedures Manual. The VFIC paper is a good resource, summarizing the qualitative and quantitative measures that the actuary should consider. Quantitative measures in addition to the percentages of loss reserves, surplus, net income and worth presented above are: absolute magnitude of item that represents a correction or different result, absolute magnitude of item for which data is not available, and the impact of an item on IRIS ratios and Risk-based Capital results.

SAB 99, introduced above, stated that numerical quantitative values for rules of thumb have no basis in law or accounting literature.[2] However, quantitative rules are simpler to understand, and are stated above to reflect some common rules of thumb in regards to materiality.

In summary, the above sources provide guidance in terms of either qualitative or quantitative measures important in assessing materiality:

- VFIC: "Requiring the use of professional judgment and placing importance on intended purpose both emphasize the role of qualitative considerations in evaluating materiality."

[2] Note that the SEC only has regulatory authority regarding publicly traded companies. The Supplementary Information for each SAB states that "the statements in the staff accounting bulletins are not rules or interpretations of the Commission, nor are they published as bearing the Commission's official approval.

- SAB 99: "But quantifying, in percentage terms, the magnitude of a misstatement is only the beginning of an analysis of materiality; it cannot appropriately be used as a substitute for a full analysis of all relevant considerations".

Significant Risks and Uncertainties

The explanatory paragraph in the opinion discussing significant risks and uncertainties is not required if material adverse deviation is deemed not to exist. However, the "maximum" amount of material adverse deviation is not required to be disclosed if material adverse deviation is deemed to exist; only the presence of this "minimum" material hurdle amount is disclosed. Finally, broad statements about risks and uncertainties due to economic changes, judicial decisions, regulatory actions are generally not the types of risks envisioned by the requirement to disclose risk of material adverse deviation, nor is an exhaustive list of all potential sources required to be mentioned.

IV. Specific Examples: The Connection between Material Risk of Adverse Deviation and Reasonable Range of Reserves

A. Definition of Material Adverse Deviation in Relation to Range

In Appendix 2 of ASOP36, it was noted that comment letters included a request that the ASOP provide more guidance by giving examples in the various sections of the ASOP. From the discussion above, the role of judgment in assessing materiality is listed as primary. The central purposes of this paper are to connect the concepts of reasonable range of reserves and materiality, and to demonstrate that the actuary's inherent ideas of what constitutes the width of an "average"

They represent interpretations and practices followed by the Division of Corporation Finance and the Office of the Chief Accountant in administering the disclosure requirements of the Federal securities laws."

reasonable range of reserves can influence the materiality level that is chosen by the actuary. For example, if two actuaries believed that materiality standards are 10% and 20% of losses, respectively, then the amount of times that the "10% actuary" expresses material adverse deviation exists would be above that of the "20% actuary". This point is demonstrated below using industry development.

First, the concept of the connection between reasonable range and materiality is presented below. Simple general examples are then presented to demonstrate the concepts. Then, specific examples are constructed below using a confidence level approach outlined in the paper Unbiased Loss Development Factors by Daniel Murphy in PCAS 1994. It has already been discussed, but is worth reiterating here, that qualitative measures regarding the risk of adverse material deviation should be considered in addition to the quantitative section below.

Section II.C. showed the considerable latitude that ASOP36 provides the actuary in determining materiality. There is however, a connection between the risk of material adverse deviation, the amount of materiality, and the maximum end of the range of reasonable reserves. The following illustrates that the risk of material adverse deviation exists if:

a. The difference between the High end of the Range and Carried Reserves is greater than:

b. The Materiality Amount

The following uses terminology as presented by Robert Butsic in his paper "Solvency Measurement for Property-Liability Risk Based Capital Applications to clarify the discussion.

Given:

Assets	A	cash (realizable value is certain)
Loss Reserve	L	unpaid loss (realizable value is a random variable)
Capital	C	assets – loss reserve (realizable value is a random variable)

For a discrete loss size probability distribution, when assets are certain, the Expected Policyholder Deficit ("EPD") is

$$D_L = \sum_{x>A} p(x)\,(x-A), \qquad (1)$$

where $p(\bullet)$ is the probability density for losses ($0 \le x < \infty$). The EPD ratio is $d_L = D_L / L$

In words, Butsic defines the term D_L / L as:

- the ratio of capital to the expected valued of the risk element ("L"), and
- the coefficient of variation, the ratio of the standard deviation of the risk element to its mean

Let us define

Maximum Loss (G) in our range of reasonable reserves	L_G
Materiality measure as a percentage of losses	m
Total materiality amount in dollars	M=m*L

For a discrete loss size probability distribution, when assets are certain, the Expected Material Deviation ("EMD") is:

$$D_{MD} = \sum_{x>L+M} p(x)\,(x-(L+M)), \qquad (2)$$

where $p(\bullet)$ is the probability density for losses ($0 \le x < \infty$). Risk of material adverse deviation exists when $L_G > L+M$; the amount of material adverse deviation is equal to $L_G - (L+M)$. The EMD ratio is therefore defined as $d_{md} = D_{MD} / L$. In words, the term $d_{md} = D_{MD} / L$ can be defined as the ratio of the expected material deviation to the expected valued of the risk element ("L").

Similar distributions exist for continuous distributions:

$$D_L = \int_A^{\infty} (x-A)\, p(x)dx, \qquad (3)$$

And,

For a reasonable range of reserves,

$$D_{MD} = \int_{L+M}^{\infty} (x-(L+M))\, p(x)dx, \qquad (4)$$

Where p(•) is the probability density for losses ($0 \leq x < \infty$), risk of material adverse deviation exists when $L_G > L+M$.

The above examples show that material adverse deviation can be considered a special subset of the concept of capital when applied to insurance situations. As capital is reserved for deviations in excess of loss reserves, material adverse deviation can be considered when the deviation amount places losses higher than the top end of the range. The amount of capital is available to absorb that amount.

Note that ASOP36 requires the actuary to disclose the materiality threshold ("M") if the actuary believes that the risk of material adverse deviation exists. However, the amount of adverse deviation, defined as $[L_G - (L+M)]$, need not be disclosed; effectively, the actuary is expressing the opinion that $[L_G - (L+M)] > 0$.[3]

[3] If $[L_G - (L+M)] > 0$, then $[L_G - L > M]$; the amount M must be disclosed, but not the amount $L_G - L$

B. General examples of Range of Reasonable Reserve Estimates in Relation to Risk of Material Adverse Deviation

To continue the above example, the following is a range of reasonable reserves for an insurance company, with the materiality threshold expressed in terms of 10% of carried reserves[4]:

Low	Point	High	Carried	Materiality	High End - Carried
90	100	110	105	10.5	5

Since the high end of the reasonable range – carried reserves (5) is less than the materiality standard (10.5), risk of material adverse deviation does not exist. An alternative presentation of the same test is as follows:

Low	Point	High	Carried	Carried + Materiality
90	100	110	105	115.5

Since the high end of our reasonable range of reserves is below the carried reserves plus the materiality standard, then risk of material adverse deviation does not exist. Restated, in this case, the high end of the actuary's *range of reasonable estimates* is 110, an amount that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable. The risk of material adverse deviation does not exist because the amount

[4] If the standard was expressed as a percentage of statutory surplus, the standard could be converted to reserves using a reserves to capital percentage, and applied as described above.

of the carried reserves plus the material standard (115.5) is greater than the highest amount the actuary deems reasonable (110).

To continue the above example, the following is the same example, with carried reserves of 95:

Low	Point	High	Carried	Materiality	High End - Carried
90	100	110	95	9.5	15

Since the high end of the reasonable range – carried reserves (15) is greater than the materiality standard (9.5), risk of material adverse deviation exists. An alternative presentation of the same test is as follows:

Low	Point	High	Carried	Carried + Materiality
90	100	110	95	104.5

Since the high end of our reasonable range of reserves is above the carried reserves plus the materiality standard, then risk of material adverse deviation exists. Restated, in this case, the high end of the actuary's *range of reasonable estimates* is 110, an amount that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable. Since the amount of the carried reserves plus the material standard (104.5) is greater than the highest amount the actuary deems reasonable (110), the risk of material adverse deviation does exist, and can be quantified to be 5.5 (110-104.5).

As defined above, the EMD ratio is equal to 5.8% (5.5/95). This ratio, the ratio of the expected material deviation to the expected valued of the risk element ("L"), should be examined within the context of the NAIC IRIS Ratio Tests, Risk Based Capital percentages, or similar impairment of capital measures.

Note that ASOP36 does not mandate that the materiality amount, 9.5 in this example, be disclosed, as well as the risk of material adverse deviation. However, the **amount** of material adverse deviation implied by the high end of the range, equal to 5.5 (110-104.5), need not be disclosed.

C. Specific examples of Range of Reasonable Reserve Estimates

In his 1994 paper, Daniel Murphy presents a paper in which confidence intervals are constructed using regression techniques and ranges of reasonable reserves. Mr. Murphy's paper provides one convenient triangle-based approach to determine confidence intervals. However, it is silent regarding the concept of "reasonable range of reserves". The above paper cited by Mr. Patel and Mr. Raws provided an example where "..the range can be defined as the values encompassed in the 5^{th} and the 95^{th} percentile…". The following are the results of Mr. Murphy's methodology applied to 2000 A.M. Best's data for several lines of business, with the assumption that the 5%

and 95% confidence levels define a reasonable range of reserves.

Line of Business	5%	50%	95%	Carried
Medical Malpractice	7,566	8,808	10,051	8,111
	-14.1%		+14.1%	
Workers Compensation	48,589	52,702	56,814	51,097
	-7.8%		+7.8%	
Private Passenger Auto Liability	52,800	56,587	60,374	63,534
	-6.7%		+6.7%	
General Liability	34,107	37,578	41,049	34,037
	-9.2%		+9.2%	
Commercial Auto Liability	18,813	20,437	22,061	18,894
	-7.9%		+7.9%	
All Lines	286,162	304,677	323,192	297,039
	-6.1%		+6.1%	

If a materiality standard of 10% of carried reserves was applied to the above lines of business, risk of material adverse deviation could be tested as follows:

LOB	10% of Carried Reserves (A)	High End Minus Carried Reserves (B)	Risk (B>A?)
Medical Malpractice	811	1,940	Yes
Workers Comp	5,110	5,717	Yes
PP AL	6,353	(3,160)	No
GL	3,404	7,012	Yes
CAL	1,889	3,167	Yes
Total	29,704	26,153	No

The workers compensation and private passenger automobile situations illustrate that the degree of margin or deficiency can influence the determination of risk of material adverse deviation. Although the workers compensation high end of range of 7.8% was only 1.1% above the private passenger automobile range, the deficiency of the line (relative to the private passenger auto liability redundancy) caused the risk of material adverse deviation to be present.

The amount of material adverse deviation, and the EMD ratios define above, are as follows:

LOB	Material Adverse Deviation (B)-(A)	EMD Ratios
Medical Malpractice	1,129	14% (1,129/8,111)
Workers Comp	607	1%(607/51,097)
GL	3,698	11% (3,698/34,037)
CAL	1,278	7% (1,278/18,894)

The results of the above test confirm several preconceived notions regarding whether risk of material adverse deviation exists by line of business. Given the low frequency, high severity nature of medical malpractice, and the relatively long-tail payout of the line of business, the width of the range of the reasonable reserves produces an opinion regarding the presence of risk of material adverse deviation. The redundancy of private passenger automobile does not allow risk of material adverse deviation to be achieved.

By raising the materiality standard to 20%, the test results change as follows:

LOB	20% of Carried Reserves (A)	High End Minus Carried Reserves (B)	Risk (B>A?)
Medical Malpractice	1,622	1,940	Yes
Workers Comp	10,220	5,717	No
PP AL	12,706	(3,160)	No
GL	6,808	7,012	Yes
CAL	3,778	3,167	No
All	59,408	26,153	No

The above shows that the width of the "average" reasonable range of reserves can influence the materiality standard selected. For example, one actuary might believe that a reasonable materiality standard should be 10%, based upon the idea that a preconceived "average" width of the high end of a reasonable range of reserves is 10%. Consequently, that actuary would express that the risk of material adverse deviation exists more often (4 out of 6 times for the above lines of business) than the actuary with a 20% materiality standard (2 out of 6 times for the above lines of business).

V. Conclusions

This paper has demonstrated the following points:

1. The range of reasonable reserves and the amount of material adverse deviation are related. Reasonable ranges of reserves can be generated to support a reasonable opinion, as well as to test for the risk of material adverse deviation.

2. The amount of material adverse deviation can be quantified, as the high end of the range less the carried reserves plus the materiality standard.
3. The width of what the actuary deems to be an "average" reasonable range of reserves may be an additional factor to be considered when selecting the materiality amount, and the "average" frequency that the risk of material adverse deviation will be cited by the actuary.
4. Although the range of reasonable reserves need not be disclosed in the actuarial opinion, other Actuarial Standards of Practice (such as ASOP9) under certain circumstances could imply the necessity to disclose the range in the actuarial report.
5. The risk of material adverse deviation can be supported by qualitative as well as quantitative tests.

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The Runoff Environment—Considerations for the Reserving Actuary

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CAS Paper

The Runoff Environment – Considerations for the Reserving Actuary

Abstract

As companies experience poor results or enter into weakened financial condition, particular lines of business or entire books of business may be cancelled or non-renewed and the loss reserves for this business placed into runoff. In such circumstances, it is possible that traditional reserving methodologies used to estimate IBNR reserves may produce distorted results, as the assumptions of a stable environment upon which most methodologies rely fail to hold. In this paper, we review some of the causes of such distortions and discuss some adjustments the reserving actuary may want to consider when evaluating a runoff book of business.

Specifically, we focus on the medical professional liability, or medical malpractice, line of business, using loss and claim data from two previously large writers of this line, both of which are now in runoff. We compare certain results of these two writers post-run off with their own results prior to entering runoff and with a compilation of on-going peer companies. The key statistics examined are (1) average closed claim severity, (2) claim closing patterns, and (3) loss payment patterns. We attempt to quantify the impact on these statistics of entering runoff, explore reasons for these changes, and suggest adjustments to be made to standard reserving methods. We also discuss the role of insurance guaranty associations (IGAs) and the impact of runoff on excess of loss reinsurance.

I. Introduction/Background

In this paper, we have focused on the impact of runoff on reserving for one particular line of business – medical malpractice. We chose to focus on one line because the effort necessary to evaluate the impact for every line of business in depth would be too monumental for a paper such as this. We are hopeful that many of the comments made here would apply to other lines of business as well, and perhaps others can test this with further research.

The medical malpractice insurance market has undergone dramatic changes since the mid 1970's. The market has been characterized by severe swings, from market corrections with significant rate increases and capacity shortages to periods of intense competition. Medical malpractice insurance is a difficult line of business to write because of the potential for bursts of large severe claims as well as high claim frequency periods. Problems can be more dramatic than those for other lines because of the long-term nature of claims reporting and settlement. A recent history of troubled property-casualty insurance companies includes many that either wrote medical malpractice liability as their primary line or one of their primary lines.

Financial difficulties for an insurer can impact many parties, including policyholders, claimants, creditors, regulators and reinsurers. In a liquidation procedure or even in voluntary runoff, the reserving process can take on additional significance, as estimates produced may have an impact on the way the assets of the company or estate are used. To equitably runoff or close a book of business, it is important for all sides to have accurate information and projections. Assumptions that the troubled insurance companies' operations and financial trends continue as before the troubles arose could lead to misleading conclusions. It is important to consider that revelations of difficulties within a company create a new environment along with new perceptions. Also, it is reasonable to believe that the operation of the company prior to the onset of financial difficulties may have undergone some changes that led to the problems that occurred.

In this paper, we discuss both the empirical data we have reviewed from two runoff entities that had previously written a large amount of medical malpractice business and the qualitative input we have gathered from our experience and the experience of others in the industry we have interviewed. Based on our analysis, we have drawn the following conclusions regarding considerations for actuaries when evaluating a runoff entity:

1. The speed at which claims are reported and settled is likely to change. In some cases, there may be a "stay" in place that freezes claim activity, slowing down the reporting and closing of claims for a period of time. Absent such a stay, or once one in place is lifted, there is the possibility of a "run-on-the-bank" situation, where claimants rush to report and settle claims to avoid the possibility of receiving reduced recoveries or possibly no recovery at all.

2. The average amount paid per claim is likely to decrease. During a "run-on-the-bank" situation, not only do claimants want to settle their claims quickly, but they also are generally willing to accept a lower settlement amount in order to do so. Furthermore, in some cases, the limited assets of the company or the limits of guarantee funds may result in lower amounts paid per claim. These lower amounts paid per claim may result in higher net-to-gross ratios for claims paid after runoff than those paid before runoff, as these lower values may decrease the frequency and severity of claims piercing excess of loss reinsurance retentions.

3. In some cases, the ultimate loss ratios for the latest years are likely to be significantly higher than prior years. At the start of financial difficulties, there is sometimes pressure put on underwriters to generate inward cash flow. This can result in business being placed on the books that is less profitable than usual.

Without considering the three issues listed above, traditional actuarial techniques can produce inaccurate results. Any development method would be impacted by changes in the speed of the reporting and closing of claims; any counts and averages method would be impacted by changes in the average amount paid per claim; and any expected loss ratio or Bornhuetter-Ferguson method using an expected loss ratio would be impacted by changes in the expected loss ratio. Furthermore, actuaries sometimes apply techniques to estimate unpaid losses on either a gross basis or a net basis with respect to reinsurance and use a net-to-gross ratio to estimate the other; use of a historical ratio may be inappropriate.

II. Analysis

Data

For our analysis, we relied on the medical malpractice data for two companies that recently placed their business into runoff. We will keep the names of the companies confidential, referring to them as "Company X" and "Company Y". Both companies wrote large amounts of physician claims-made policies with only limited hospital or long-term care facility exposure. Company X is in liquidation while Company Y voluntarily placed its medical malpractice line into runoff after experiencing significant financial difficulties. As shown in the data tables included later, we have reviewed three years of post-runoff data for Company X and 1 year of post-runoff data for Company Y, with these results compared to the years prior to runoff for each company.

For purposes of comparison, we created an industry medical malpractice benchmark database using composite information from member companies of the Physician Insurers Association of America (PIAA). This is a group of mutual medical malpractice writers that wrote similar business to Companies X and Y. The data was compiled from the 2000 Annual Statements for the majority of companies currently in the PIAA.

Statistics

We examined changes in the following statistics:

- Incremental claims closed with payment (CWIPs)
- Incremental loss payments
- Average claim payments

Note that we have only used paid loss and closed claim data. Due to the financial difficulties of both companies, we believe distortions may have existed in the case reserves such that historic loss development would not be indicative of future development.

III. Understanding the Results

Before providing quantitative results, we believe it is necessary to understand the process, dynamics and motivations of parties involved with financially troubled insurers, such as within a liquidation, to properly understand and interpret the results that follow.

Signs of Financial Problems

While there is usually no single factor that causes an insurer to enter financial distress, the following characteristics (either alone or in some combination), have been exhibited in the majority of insolvencies: (1) deficient loss reserves, (2) rapid growth, (3) overstated assets, (4) alleged fraud, (5) significant change in business, (6) reinsurance failure, and (7) catastrophic loss. These characteristics may be voluntarily disclosed by the company or discovered by external entities, such as state regulators, rating agencies or financial analysts.

Liquidation Process and Effect on Claims

In the event that a company's financial standing deteriorates beyond repair, it may be liquidated. The process of liquidation varies from state to state because of local statutes and enhancements to the NAIC model laws. After an order of liquidation is obtained, the appointed liquidator in the state of domicile notifies all insurance departments in other states where the company wrote business and insurance guaranty associations (IGAs). IGAs represent solvent insurers in each state and absorb the losses of claimants against insolvent insurers. Not all lines of insurance are eligible for coverage by IGAs with restrictions that vary by state.

The liquidator is also responsible for notifying all agents, policyholders, and any others who might have claims against the company in its insolvency. The liquidator also usually issues a "bar date" on future claims. The bar date is established to cut off future claims and is usually set for 1 year after the liquidation begins. The liquidator may also place a stay on all pending litigation against the insurer. This stay will allow the liquidator and IGAs time to review claim files and, if necessary, prepare an adequate defense.

After liquidation, the IGAs in each state where there are policyholders for the insolvent company become responsible for handling claims against those policyholders. The IGAs have the authority to settle claims for a limited amount, usually between $100,000 and $300,000 per claim. Any claims that settle for amounts greater than these limits must be approved by the liquidator. If approved, the amounts greater than the IGA limits are submitted to the estate of the insurer as a "Class 2" claim against the estate. (The priority of claims is defined by state statute but usually runs from Class 1 to Class 9. Class 1 claims include administrative expenses, both for the IGAs and the estate to liquidate assets and are the first to be paid with any assets held by the estate. Class 9 claims are the claims of shareholders or other owners and are the final claims paid by the estate, if possible.)

The estate, through the liquidator, works to generate as much cash as possible quickly through sale of assets and recoveries from third parties such as reinsurers. Often the amounts compiled for the estate are not enough to cover all the claims submitted by all classes. In this case, the liquidator may declare a "dividend", allocating money to each of the different classes. The dividend amounts are often less than the full amount requested.

IGAs will recover payments made under the limit of their authority through an assessment of solvent companies in the state that operate in the line of business of the claim paid. Usually, the assessments are generated after the insolvency and are charged to insurers (and passed onto policyholders) as a percent of net written premium. IGAs are usually exempt from any litigation in their work in regard to bad faith negotiating.

Claim Process (From the Other Side)

To gain a complete picture of the impact of financial troubles for insurers on claim settlements, it is important to view the claims process from an alternative to our traditional industry position – the view of the claimant. As in any type of liability insurance, the start of the claim process is an event or incident where there is an injury and the perception of responsibility for the injury by a third party. For medical malpractice, the third party is usually a physician or other health-care provider. The physician involved often will file a report of any incidents with their insurer and a file may be created with, perhaps, a small default reserve attached. Some states have a mandatory requirement for incident reporting. As the majority of incidents or events do not turn into claims, most of these files are closed within 180 days without any paid loss.

A claim is defined as any written or oral demand for compensation in the form of money or services, with no legal papers having been filed in court. Many claims that are unresolved become suits. A suit is formal litigation that alleges an error or omission on the part of one or more defendants. Only approximately 1 in 3 malpractice claims results in an indemnity payment to the plaintiff. Only a small number of cases are resolved as the result of a jury verdict.

If the perception of malpractice exists, the claimant/patient will more often than not approach a plaintiff's attorney. Prior to preparing a formal claim, the attorney must evaluate each incident to determine (1) if there was negligence and (2) what damages, if any, were incurred. If the attorney believes the case has merit, he will file a notice of claim with the physician or facility involved. The notice will include a request for discovery of documents. Discovery is a standard part of most litigation and allows parties access to information held by the other side. One of the documents usually requested is proof of insurance. The proof of insurance document will include all material insurance information for the target of the claim such as the name of the insurers involved and limits provided in the policy.

This information is critical to the pursuit of the claim. Except in unusually strong cases, most attorneys will pursue damages only up to the maximum insurance limit purchased. If the attorney seeks recoveries from the physician involved for amounts greater than the physician's carried insurance limit, and if there is no excess coverage or other facility in place, the attorney will need to seek to attach personal assets of the physician involved. The processing of attaching personal assets can be a long and difficult process and the attorney has to weigh the cost of pursuing these assets against the benefit of accepting a cash settlement from an insurer.

The listing of insurers involved is an important piece of information. Astute plaintiff attorneys will often track the rating of insurers and this may influence their decision to settle. Also, attorneys involved in a claim against an insolvent insurer will be notified regarding any stays of litigation and the financial status of the insurance company involved. If a company enters dire financial straits, it is often in the interest of the claimant to seek a quick settlement. This may avoid a long expensive legal battle in which the claimant, if victorious, may only receive pennies on the dollar of any settlement, due to previous settlements diluting any remaining assets.

IV. Discussion of Results

In the initial phase of our analysis, we examined three statistics for the companies pre- and post-runoff.

1. Claim Closure Rates

The first statistic we examined was the ratio of claims closed with payment (CWIP) in a given year to those closed with payment in the prior year for a given report year. We hoped to learn from this statistic whether claims were settling faster. For both Company X and Company Y, the ratios increased after the business was placed into runoff. For example, before runoff, the number of claims closed with payment in the third year of experience (24-36 months after the beginning of the report year reviewed) was slightly more than the number closed in the second year of experience – 25% more (ratio of 1.25) for Company X, 3% more (1.03 ratio) for Company Y. After runoff, this ratio increased dramatically for Company X – from 1.25 to 9.33, 5.74 and 3.71 for the following three years. The ratio for Company Y also increased, but by a smaller amount (from 1.03 to 1.32).

Similar trends can be observed in Table 1 for other age periods. After 60 months, the number of claims closed for these companies decreased to a number too small to use in such comparisons.

Table 1
Ratio of Claims Closed with Payment in a Given Calendar Year to those Closed with Payment in the Prior Calendar Year for a Given Report Year

	Company X				Company Y	
Period	Pre-runoff	Post-Runoff			Pre-runoff	Post-Runoff
(Months)	3-Year Avg	Year 1	Year 2	Year 3	3-Year Avg	Year 1
12-24/0-12	2.17	14.50	14.00	-	2.90	7.00
24-36/12-24	1.25	9.33	5.74	3.71	1.03	1.32
36-48/24-36	0.61	5.00	2.87	0.73	0.59	0.85
48-60/36-48	0.61	3.50	3.17	0.50	0.73	0.74

What causes these changes? There are several items that we have identified:

1. When an insurance company has financial problems, claimants and their attorneys are more likely to look for a faster settlement. As described above, claimants may be aware of the problems and the potential for reduced recoveries.

2. In runoff situations, there is pressure from within the insurance company itself to settle claims faster as well. Uncertainty is reduced as the number of open claims is reduced.

3. In the case of Company X, there was a 6-month stay on litigation in place at the time the company was placed into liquidation. (This stay took place the year before the "Post-runoff Year 1" shown in Table 1, but after the "Pre-Runoff 3-Year Avg" period.) As this would depress the number of claims closed in the year before Post-Runoff Year 1, this would cause the ratios in the Post-Runoff Year 1 column above to be unusually high. Some of the large number of claims closing are those that would have settled in prior years had there been no stay. (No such stay exists for Company Y.)

Another observation for Company X is that the ratios in Table 1 appear to decline in Post-Runoff Year 3, close to the Pre-Runoff levels. At some point, after the initial pressure of settling claims faster subsides, perhaps there could be a return to "normal" development patterns. Or, it is possible that claims are continued at an elevated pace, but just not as high as the previous year.

When evaluating a company in runoff, the actuary may want to investigate these issues. Conversations with the claims and legal departments may be able to shed some light on such issues. Any methodology that involves the development of claim counts, such as for use in a counts and averages method, may need to be adjusted. The adjustments would include recognizing some estimate of a speed-up in claim closing.

2. Average Payment per Claim

The second statistic we examined was the size of the average loss paid per claim in a calendar year. We hoped to use this statistic to determine whether the financial difficulties of the companies were impacting the amount paid for claims.

Changes in the speed of claim settlement as observed in Table 1 would likely also have some impact on the amount of losses paid. Table 2 compares the average amounts of loss paid per claim closed with payment. The period represents the time elapsed since initial report or the age of a claim.

Table 2
Average Losses Paid per Claim Closed with Payment in a Given Calendar Year by Age of Reported Claim
($ thousands)

	Company X				Company Y	
Period	Pre-runoff	Post-Runoff			Pre-runoff	Post-Runoff
(Months)	3-Year Avg	Year 1	Year 2	Year 3	3-Year Avg	Year 1
0-12	34.6	12.0			185.3	120.3

12-24	159.0	82.1	90.4	192.1	257.1	212.7
24-36	195.0	105.4	118.3	137.2	292.5	248.3
36-48	175.3	133.1	150.4	132.9	237.9	221.9
48-60	173.8	93.1	102.2	105.9	206.7	103.9

As shown in Table 2, the average amount paid per claim decreased for both Company X and Company Y after their business was placed into runoff. Note that the pre-runoff data shown in Table 2 has not been trended to account for inflation; if it had been, the decrease would appear even more dramatic.

Some of the factors that could have contributed to the above decreases include:

1. Claims are being settled quicker, as shown in Table 1. As a result, when a claim that would normally result in future payments is settled, a discount for the time value of money would likely be applied, so that the amount paid represents the present value of what would otherwise have been paid.

2. Concerns about the financial condition of the insurance company could lead to claimants accepting less than they normally would.

3. IGA limits may have an impact. As noted earlier, IGAs have limits of $100,000 to $300,000 on their authority to settle claims and the majority of claims handled do settle within these limits.

4. Although not exhibited here, at some point, a decrease would exist in the final closeout of an estate when the liquidator must allocate any remaining assets to Class 2 claims. If the remaining assets are less than the outstanding claim reserves, then full payment of claims will not be made, reducing average payment size.

The actuary may want to review average claim statistics such as these when reviewing a company in runoff. Certainly any counts and averages method may need to be adjusted.

3. Incremental Paid Loss Development

The final statistic examined was incremental paid loss development. We hoped to use this statistic to examine changes in the payout pattern.

A speedup in the rate at which claims are closed (as shown in Table 1) could result in payments being made faster. However, a decrease in the average paid per claim (as shown in Table 2) can somewhat mitigate this effect. Table 3 compares the total amount paid in a given period to the prior period.

Table 3
Ratio of Paid Loss in a Given Calendar Year to Paid Loss in the Prior Calendar Year by Age of Claim Report Year

	Company X				Company Y	
Period	Pre-runoff	Post-Runoff			Pre-runoff	Post-Runoff
(Months)	3-Year Avg	Year 1	Year 2	Year 3	3-Year Avg	Year 1
12-24/0-12	15.82	15.87	105.47		3.80	7.69
24-36/12-24	1.71	5.25	8.26	5.64	1.05	1.93
36-48/24-36	0.63	3.71	4.09	0.82	0.52	0.72
48-60/36-48	0.54	4.09	2.43	0.35	0.69	0.35

Similar observations can be made here as were made from the CWIP data in Table 1. In the years after business is placed into runoff, the change in the annual paid losses for these companies was significantly higher than historical levels. The reasons for these changes are the same as those reasons discussed in the CWIP section.

Consideration of possible changes in the paid loss development pattern is important, as paid loss development is a common technique used by actuaries when evaluating this business, and reliance on past patterns can be problematic when changes such as these shown in Table 4 occur. This is important not only when estimating reserves, but when estimating future payout streams as well.

V. Testing of Results

Next, we tested our results to attempt to see if the change in examined statistics occurred as a result of the business being placed into runoff or if the changes were because of the external claims environment, which may have impacted similar on-going business as well. Our tests compared the Company X and Y results (prior to and post runoff) to those of the ongoing industry, as represented by the PIAA.

Table 4
Ratio of Claims Closed with Payment in a Given Calendar Year to those Closed with Payment in the Prior Calendar Year for a Given Report Year.

	Company X				PIAA	
Period	Pre-runoff	Post-Runoff			3-Year Avg Pre-Runoff of X	3-Year Avg Post-Runoff of X
(Months)	3-Year Avg	Year 1	Year 2	Year 3		
12-24/0-12	2.17	14.50	14.00	-	2.75	2.93
24-36/12-24	1.25	9.33	5.74	3.71	0.99	1.12
36-48/24-36	0.61	5.00	2.87	0.73	0.76	0.68
48-60/36-48	0.61	3.50	3.17	0.50	0.58	0.57

	Company Y		PIAA	
Period	Pre-runoff	Post-Runoff	3-Year Avg Pre-Runoff of Y	First Year Post-Runoff of Y

(Months)	3-Year Avg	Year 1		
12-24/0-12	2.90	7.00	2.45	3.71
24-36/12-24	1.03	1.32	1.05	1.12
36-48/24-36	0.59	0.85	0.68	0.65
48-60/36-48	0.73	0.74	0.64	0.68

Table 5
Average Losses Paid per Claim Closed with Payment in a Given Calendar Year
By Age of Reported Claim ($thousands)

	Company X				PIAA	
Period	Pre-runoff	Post-Runoff			3-Year Avg Pre-Runoff of X	3-Year Avg Post-Runoff of X
(Months)	3-Year Avg	Year 1	Year 2	Year 3		
0-12	34.6	12.0	-	-	92.6	136.8
12-24	159.0	82.1	90.4	192.1	161.6	209.2
24-36	195.0	105.4	118.3	137.2	180.5	209.5
36-48	175.3	133.1	150.4	132.9	166.2	206.1
48-60	173.8	93.1	102.2	105.9	173.0	234.5

	Company Y		PIAA	
Period	Pre-runoff	Post-Runoff	3-Year Avg	First Year
(Months)	3-Year Avg	Year 1	Pre-Runoff of Y	Post-Runoff of Y
0-12	185.3	120.3	132.9	126.7
12-24	257.1	212.7	200.4	205.3
24-36	292.5	248.3	198.1	221.0
36-48	237.9	221.9	199.8	222.7
48-60	206.7	103.9	209.2	257.6

Table 6
Ratio of Paid Loss in a Given Calendar Year to Paid Loss in the Prior Calendar Year
By Age of Claim Report Year

	Company X				PIAA	
Period	Pre-runoff	Post-Runoff			3-Year Avg Pre-Runoff of X	3-Year Avg Post-Runoff of X
(Months)	3-Year Avg	Year 1	Year 2	Year 3		
12-24/0-12	15.82	15.87	105.47	-	4.92	4.51
24-36/12-24	1.71	5.25	8.26	5.64	1.17	1.17
36-48/24-36	0.63	3.71	4.09	0.82	0.73	0.70
48-60/36-48	0.54	4.09	2.43	0.35	0.60	0.67

	Company Y		PIAA	
Period	Pre-runoff	Post-Runoff	3-Year Avg	First Year
(Months)	3-Year Avg	Year 1	Pre-Runoff of Y	Post-Runoff of Y
12-24/0-12	3.80	7.69	4.20	4.98
24-36/12-24	1.05	1.93	1.19	1.08
36-48/24-36	0.52	0.72	0.70	0.69
48-60/36-48	0.69	0.35	0.59	0.85

As each of the tables above demonstrate, the data for the PIAA is reasonably close to that of Company X and Y prior to runoff. The PIAA statistics remain consistent (or rise expectedly as for the average paid claim amounts) for the years after each of the companies placed its medical professional liability business into runoff. Based on this information, we concluded that our comparisons of Company X and Y data prior to and post runoff were not biased by any external events that would have affected the industry as a whole.

VI. Impact of Results

The knowledge of these results and conclusions stated above may affect the different entities involved in different ways:

Claimants: Claimants may receive lower settlements than if their claims were filed with an on-going insurer, particularly if the company is in liquidation. If the company is not in liquidation, the lowered settlement amount maybe offset somewhat by receiving the settlement faster.

Estate Managers/Company Management: These entities may benefit from lower settlements, offset somewhat by faster payment of claims. Lower settlements may result in more money available for larger claims or for other creditors. Benefits may also arise to the extent there are any profit-sharing provisions in reinsurance arrangements that have not been previously exhausted.

Other Solvent Insurers: On-going insurers must often foot the bill for insolvencies through IGAs. Lower settlement values benefit these insurers, somewhat offset by earlier payments.

Reinsurers: When an insurer enters into financial difficulty, and in particular becomes insolvent, the nature of the insurer's relationship with its reinsurers may change drastically. The insurer may no longer be viewed by reinsurers as a source for future business, but rather more a sink of administrative expenses and demands for accelerated payments. A liquidator or receiver of an insurer may also be more likely to turn to litigation or arbitration when problems arise, adding to the friction in the new relationship and perhaps damaging the reputation of the reinsurer.

While no reinsurer intentionally seeks business from troubled insurers, it appears from our work that there may be some surprising benefits. As we have described, once a company enters liquidation, there may be a stay on all litigation for a period of 3 to 6 months. This may be followed by a longer period in which the IGAs receive claim files and start processing claims. As a result, reinsurers may not be forwarded claims for recoveries until a year or more after anticipated at contract origination. This period may elongate, as reinsurers also may not be accommodating in forwarding timely recoveries. Reinsurers may scrutinize claim settlements and coverage decisions more closely, as again, there is no future relationship to potentially jeopardize. The total delay in payment results in greater investment income accruing to the reinsurers.

Also, the number of claims may be affected in a liquidation, as a claim bar is often in place 18 months or less after liquidation. This limits the total number of claims eligible for coverage and reinsurance. Also, as we have shown, excess of loss reinsurers may benefit, as claim payments appear to settle for lower amounts post liquidation.

VII. Putting Results to Work

After reviewing the results presented above, it is important to understand how to put them to use when estimating loss reserves. A summary of suggestions to adjust basic loss reserving methods based on our experience is presented below.

- Consider relying on paid methods in addition to incurred methods, if possible. The case reserve levels for runoff lines, particularly for companies in financial difficulty, are often distorted due to inattention or even intentional underreserving. While there may be some distortion in the historic paid losses, it is probably more reliable than incurred loss data as it is more difficult to intentionally manipulate paid loss data.

- One may want to use historic claim frequency levels, perhaps adjusted higher to reflect a possible deterioration in underwriting if there is evidence of such, to estimate ultimate claims, rather than simply developing the claims paid or reported to date. As shown above, a runoff situation can cause acceleration in the timing of claim closings. When observed, this increase should not necessarily be interpreted as an increase in the ultimate frequency of claims. The historic frequency level for the book of business may be a reasonable a priori value to use when estimating ultimate claims, especially for medical malpractice, where claim frequency trends have been relatively low lately. The frequency may need to be adjusted upward to reflect any deterioration in the experience that caused the company's financial difficulties. The downside to using historic frequency to estimate ultimate claims is that it may be difficult to establish appropriate exposures for more recent years.

- For a "counts and averages" reserving method, one may want to adjust the projected paid claim severities to reflect the affects of potential discounting and the involvement of IGAs in settlements. As shown above, there is an impact on the average paid claim size because of the runoff environment. To account for this, for a company in liquidation, one may want to lower projected paid loss severities using increased limit factors based on the average IGA limits in effect in the states where the company wrote most of its business and where its claims will ultimately be settled and other factors.

- For the paid loss development method, one may want to restate the historic paid loss triangle to address the speed-up in claims closing and decrease in average claim payments before selecting a payment pattern. This can be accomplished in a manner similar to that presented in other CAS papers, such as that by Berquist and Sherman, that adjust historic loss experience for speed-up in claims closing and reserve strengthening. Specifically, the amount (in terms of time) of speed-up in payments can be measured by calculating the percentage of ultimate claims closed, using ultimate claims calculated as described above. For example, while historically 25% of all claims may be closed in the first 24 months, now 50% of estimated ultimate claims may be closed due to the speed-up in settlement. If historically, it took 48 months for 50% of claims to close, there is an approximate 2-year speed-up in payments. Historic paid loss severities for the 48-month evaluation can be discounted 2 years and used for the loss severity at 24 months. Adjustment can also be made to severities for the impact of IGAs on claim settlements and other factors as described above for the counts and averages methods. These severities can be multiplied by the estimated number claims closed for each evaluation to determine total paid losses for each evaluation.

VIII. General Conclusions

We have analyzed the impact on selected claim and loss statistics for medical malpractice insurance when this line of business is placed into runoff. The impact observed in the two companies reviewed is a measurable speed-up in the settlement of claims with a corresponding decline in the average amount of paid loss severity. This information should perhaps be recognized in any methods used when performing loss projections for this type of business.

IX. Limitations

The lack of available data prevented us from performing further tests. It is important to note while performing loss projections for runoff companies, that the data may be inaccurate or intentionally distorted. Upon liquidation (and sometimes a factor that can lead to liquidation), many files are misplaced or lost along with institutional knowledge as staff departs. We have made our best effort to limit the impact of any distortions in the data used in this analysis.

X. Further Research

Our analysis focused on the impact of the runoff environment on medical malpractice claims. It would be interesting to test the same statistics presented here for other lines of business. Also, another possible variation would be to test these same statistics for on-going insurers not in financial difficulties that place lines such as medical malpractice into runoff. With a healthy balance sheet and active claims department, the results may prove to be very different.

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Accounting Rule Guidance Statement of Financial Accounting Standards No. 113—Considerations in Risk Transfer Testing

CAS Valuation, Finance, and Investments Committee

Accounting Rule Guidance
Statement of Financial Accounting Standards No. 113
Considerations in Risk Transfer Testing

Valuation, Finance, and Investments Committee (VFIC).

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1. Synopsis.

Genesis.
In an effort to provide some considerations to the CAS membership on risk transfer testing, the CAS Valuation, Finance, and Investment Committee (VFIC) conducted a research project. This paper is the culmination of VFIC's work.

The demonstration of risk transfer for a reinsurance contract is required by FAS 113 in order for the contract in question to receive reinsurance accounting treatment for GAAP purposes. However, there is little supporting literature from which to draw guidance on risk transfer testing methodology, risk metrics, or threshold values; hence this paper.

Approach
After a brief introduction, this paper begins with an overview of FAS 113 (§3) and other related risk transfer statements (§4). VFIC conducted a brief survey of risk transfer practices, which is presented in §5. Next, a series of examples are presented (§6) to illustrate the data requirements, methodology, and considerations involved in approaches commonly used today to demonstrate risk transfer in reinsurance contracts. The remaining sections of the paper (§7-8) are devoted to the discussion of other risk metrics that actuaries could use to characterize the level of risk present in a reinsurance contract.

Conclusions.
Methodology. FAS 113 states that risk transfer testing of reinsurance contracts must include 1) a thorough understanding of contract provisions, 2) a model of the incidence of cash flows between parties, 3) a single, appropriate discount rate, and 4) insurance risk only. By their absence, these requirements preclude consideration of income taxes, reinsurer expenses, brokerage, or credit risk in the determination of risk transfer. To meet the FAS 113 requirements we recommend that risk transfer analysis include a view of the distribution of expected contract losses, identification of an appropriate risk metric and threshold values, and duration-matched or immunized yields as the appropriate discount rates.

Risk Metric. Current practice tends to split risk transfer analysis into separate tests of probability (of an adverse result) and significance (magnitude of the result). A measure of loss at a given probability is called value at risk, or VaR.

While FAS 113 couches risk transfer in words like "*reasonable possibility*" and "*significant loss,*" the broader issue is whether a particular contract transfers risk. In this vein, a variety of other risk metrics were explored. VFIC analyzed expected deficit measures (such as expected policy holder deficit, or EPD), tail value at risk (TVaR), and distributional transforms such as the exponential and Wang transforms. Some of the positive and negative aspects of each of these are discussed in this paper.

Threshold or Critical Values. Over time, common practice seems to have concluded that a 10% chance represents a 'reasonable probability,' and a 10% loss represents a 'significant loss.' That is, the critical value for VaR is –10% at a probability of 10%. Thus we have what many term the 10-10 rule. In practice, other critical values are commonly used. It must be stressed that such rules-of-thumb are used in practice, but FAS 113 itself does not dictate critical values.

Our analysis of TVaR suggested that critical values in the range of –25% would represent minimal risk transfer. The discussion of distribution transforms proposes a critical value for the Wang transform of –10% that is wholly consistent with the 10-10 rule.

Regardless of the model employed or the risk metric used, judgment is still required as to where to establish the threshold or critical values for what constitutes risk transfer and what does not.

Intuitively, it seems natural to judge risk transfer for a reinsurance contract by analyzing whether the cedant has transferred (reduced) risk, not, as FAS 113 requires, by whether the reinsurer has assumed risk. While the answers to these two questions may be the same when focusing on a single transaction (as done in FAS113), on an enterprise-wide basis, they can be different. It should be noted that the recommendation on Index Securitization proposed the opposite to FAS 113: analysis is done from the cedant's perspective on an enterprise-wide basis. This could lead to different accounting treatments for reinsurance products and index securitizations, unless both tests are required for securitization and industry loss triggers.

2. Introduction.

The Valuation, Finance, and Investment Committee (VFIC), a CAS research committee, was asked by CAS membership to investigate and recommend considerations regarding risk transfer testing for reinsurance contracts due to the requirements set forth by FAS 113. This paper is the result of VFIC's research and discussions on the subject. The intent of this paper is to illustrate how risk transfer could be tested given the requirements set forth.

FAS 113 dictates the conditions, namely risk transfer, required for a reinsurance contract to be accounted for as reinsurance for GAAP purposes. Failing these conditions, the contract receives deposit accounting treatment. The statement itself does not provide specific guidelines for the quantification of risk transfer; FASB never intended to provide such specific guidance.

Numerical guidelines for measuring risk transfer—such as the well-known 10-10 rule – have become widely used. While often used in an audit context, auditors are not the only audience for risk transfer, however. Regulators, rating agencies and securities analysts all may want to evaluate whether or not a deal has enough risk transfer to meet FAS 113 requirements, and typical audit criteria may not suit their purposes.

The next section is a review of FAS 113 and related requirements. This is followed by a brief review of current practice. Examples of risk transfer testing are given, shedding light on key considerations. We then look more broadly at how risk transfer might be viewed by actuaries.

3. Overview of FAS 113

Statement. The stated purpose of FAS 113 is as follows.

> "This statement establishes the conditions required for a contract with a reinsurer to be accounted for as reinsurance and prescribes accounting and reporting standards for those contracts."

It is clear from the stated intent that FASB did not intend to make 113 a prescription of methodology.

The summary of FAS 113 goes on to portray the essence of risk transfer:

> "Contracts that do not result in the *reasonable possibility* that the reinsurer may realize a *significant loss* from the *insurance risk* assumed generally do not meet

> the conditions for reinsurance accounting and are to be accounted for as deposits." [emphasis added]

The phrases *reasonable possibility* and *significant loss* are clearly the key considerations in the analysis of risk transfer, but they are largely undefined. The terms *reasonable* and *significant* indicate that FASB is inviting the application of informed judgment. In the measurement methods discussed below, a line has to be drawn to define a cutoff between enough risk for 113 and not enough. It is not the primary intent of this paper to draw those lines, instead different methods of measuring risk that could provide a consistent framework for applying such judgment are emphasized.

Risk Transfer Tests. Property-casualty reinsurance contracts are covered by paragraphs 9 – 11 of FAS 113 – "Reinsurance of Short-Duration Contracts." Paragraph 9 of FAS 113 defines risk transfer conditions as follows.

> "Indemnification of the ceding enterprise against loss or liability relating to insurance risk in reinsurance of short duration contracts requires both of the following, unless the condition in paragraph 11 is met:
>
> > "a. The reinsurer assumes significant insurance risk under the reinsured portions of the underlying reinsurance contracts.
> > "b. It is reasonably possible that the reinsurer may realize a significant loss from the transaction."

Paragraph 9 is clear that risk due to "loss" refers only to insurance risk, i.e. (a) ultimate amount of net cash flows between the parties, **and** (b) the timing of the receipt of cash. Risk factors do not include recognition of reinsurer costs, investment risk, taxes, or credit risk to name a few.

The 'condition in paragraph 11' referred to above states, "(failing tests a and b) the ceding enterprise shall be considered indemnified against a loss or liability relating to insurance risk only if substantially all the insurance risk relating to the reinsured portions of the underlying insurance contracts has been assumed by the reinsurer." (For the sake of discussion, we will refer to this as test c.) The condition described in test c covers fronting arrangements, where a deal may appear highly lucrative, but the assuming party does, in fact, assume virtually the entire risk.

So, in essence, to answer the question of risk transfer affirmatively, the reinsurance contract must meet either test **c** or tests **a &.**

Except in the extreme case of **c**, where the cedant ends up with virtually no risk on the ceded portions, the criteria for risk transfer does not look at whether or not the ceding insurer reduces its risk. Rather the test **a & b** is on whether on not the reinsurer assumes risk[1].

The closest FAS 113 comes to a definition of *significant insurance risk* is in footnote 4 to paragraph 11, which references FAS 97. Here, "*insignificant*" is defined as "having little or no importance; trivial." Presumably a failure to be insignificant would connote significance.

Neither does FAS 113 elaborate on what constitutes a *reasonable possibility*. The term *reasonably possible* is used in FASB Statement No. 5, "Accounting for Contingencies," to mean the scenario's "probability is more than remote." 'Remote' is not defined further in the statement. Based on FAS 5, it can be concluded that the test is applied to the scenario as a whole, not to the individual assumptions in a scenario. Thus, the entire set of assumptions must be reasonably possible.

Tests **a & b**: are discussed in paragraphs 9, 10 and 11 of FAS 113. In paragraph 9, test **a** is characterized by

> "A reinsurer shall not be considered to have assumed significant insurance risk under the reinsured contracts if the probability of a significant variation in either the *amount or timing* of payments by the reinsurer is *remote*. Contractual provisions that delay timely reimbursement to the ceding enterprise would prevent this condition from being met."[2]

This is the more clear-cut of the two tests, in that the reinsurer does not have to be able to lose money to meet it but just have uncertainty about both the timing and amount of payments. Again, "remote" is not defined further.

Paragraph 10 discusses test **b** in more detail. It appears that an examination of *reasonably possible outcomes* is anticipated in order to show that this test is met.

> "The ceding enterprise's evaluation of whether it is reasonably possible for a reinsurer to realize a significant loss from the transaction shall be based on the present value of all cash flows between the ceding and assuming enterprises under reasonably possible outcomes, without regard to how the individual cash flows are characterized. The same interest rate shall be used to compute the present value of the cash flows for each reasonably possible outcome tested."

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1 This is in contrast to the issue of securitization and reinsurance based on parametric triggers – for example when the insurer gets a pre-defined recovery if a force 4 hurricane hits Florida. The tests the NAIC is considering for statutory accounting in such cases are based on whether or not the cedant gets a reduction in underwriting risk from entering into such a contract. A number of tests of risk reduction have been proposed to test this. However these are not directly relevant to risk transfer under FAS 113, as the test here is on the reinsurer increasing risk, not on the insurer reducing risk.

2 This clause was added to avoid contacts that cede losses but allow actual reimbursements according to a schedule in such a way that the reinsurer locks in a profit based on the float of funds.

A simulation of randomly generated outcomes would be one way to carry out test **b**. "Reasonably possible" would then be defined using the probability of observing a result equal to or worse than some critical value based on simulation output. This would be the likely basis of the "10% chance" measure widely used today.

For the set of outcomes examined, the evaluation of whether or not there is a significant loss is one where the present value of the payments to the cedant exceeds the present value of the payments to the reinsurer by a threshold amount. This is never stated so directly, however. This section creates the companion measure of "10% loss," i.e., the net present value of losses ceded is 10% greater than the net present value of the consideration paid. However, when payments are based on netting out of offsetting items, it can be difficult to distinguish the consideration paid from losses and expense credits. For instance, reinstatement premium is very similar to a loss participation.

Paragraph 10 does provide some explicit guidance on risk transfer testing. Namely, it is based on 1) the net present values of cash flows, 2) on cash flows between the parties (e.g., no taxes, no consideration of reinsurer expenses), 3) using a constant interest rate.

Paragraph 11 specifies that the test of significance of loss is relative to the amounts ceded to the reinsurer. Thus presumably the significance of a given loss amount, say $10,000, might be different given different ceded premiums, say $100,000 vs. $1 billion. Thus we put the two parts of the test together and have a "10% chance of a 10% loss," as opposed to a test in dollar terms.

It would be easier to interpret paragraphs 10 and 11 if they could be used to separate the test of a reasonable possibility of a significant loss into two independent steps: generate a lot of scenarios and first test each to see if it generates a significant loss. Then see how many did so, and test to see if enough did. You would need a test of significance to do the first step and a test of reasonable possibility to do the second step, and these could be independent.

However, the wording of these two sections keeps *reasonably possible* and *significant loss* intertwined. It seems completely consistent with these paragraphs to require a stricter standard for *reasonably possible* when *significant loss* is interpreted more broadly, and vice versa. Thus a 5% chance of a loss of 100% of premium might provide as much or more *reasonable possibility of significant loss* as a 10% chance of a loss of 25% of premium, for example.

In fact this kind of linkage might actually be implied by the lack of separation of the two phrases. Under this viewpoint one would still count loss scenarios as part of the test, but the test of reasonable possibility would not be independent of the test of significant loss.

Thus to sum up tests **a** & **b**:

- test **a** is met if the reinsurer has risk of variation in both timing and amount of payments, and payments must be timely to meet this criterion;

- test **b** requires an examination of possible outcomes. To meet this test, at least some of the outcomes have to produce a loss for the reinsurer, where a loss is determined using present values of all cash flows. The significance of losses is to be evaluated relative to the present value of payments to the reinsurer. The test is of reasonable possibility of significant loss, and it would be appropriate, though not required, to evaluate reasonability and significance conjointly.

Looking at test **c**, the reference to *reinsured portions* of the underlying insurance contracts is potentially ambiguous. It could mean *reinsured percentage*, as in a quota share contract, or *reinsured sections*, as in the liability portion of a homeowner's policy. These are actually both rather narrow interpretations of *portions* and probably are consistent with the intent of FAS 113. For example, if a company writes a very profitable book of auto collision insurance, so profitable that it virtually cannot have an underwriting loss, but reinsures some of this on a quota share basis in order to meet financial ratio tests, the reinsurer probably will not be able to meet test **b**. But test **c** would be satisfied so this deal would qualify for reinsurance accounting. Here the reinsurer and ceding insurer share the risk on an equal basis.

A broader interpretation of *portions* would allow a portion of a homeowner's book to constitute all losses on all policies in all events where the insurer's event loss is less than $100 million. If this qualifies as a portion, then there might be cases where a reinsurer could write a capped quota share in which it would be virtually guaranteed a profit even though the cedant could suffer a major loss on the retained book, and this would qualify for reinsurance accounting under test **c**. This broad a definition of *portion* could probably be stretched to fit in any reinsurance deal, and so would negate the need for tests **a & b**.

Thus a more narrow definition of *portions* is implied. Interpreting *reinsured portions* as *reinsured percentage* seems to be well within the intent of FAS 113. The same might apply to *reinsured sections*, particularly if there is a separately identifiable premium for the sections under consideration. Conditions that do not refer to individual policy provisions but rather the insurer's experience on a book of policies would seem to stretch the intend of *portions* beyond what FAS 113 seems to consider.

To sum up test **c**: a portion of policies has to be fully ceded, where *portion* probably is restricted to percentage or section, or something similar, and the only risk the cedant can retain on this portion must be trivial, having no importance. This situation describes fronting sorts of relationships and straight unrestricted quota share reinsurance.

4. Related statements.

Statutory Accounting. In statutory accounting, reinsurance is primarily addressed in Chapter 22 of the *NAIC Accounting Practices and Procedures Manuals for Property and Casualty Insurance Companies*. Amendments were made after the GAAP adoption of FAS 113. As a result, the statutory accounting principles established regarding risk transfer and reinsurance accounting are generally consistent with GAAP. Chapter 22 states:

> "Reinsurance Contracts Must Include Transfer of Risk
> The essential ingredient of a reinsurance contract is the shifting of risk. The essential element of every true reinsurance contract is the undertaking by the reinsurer to indemnify the ceding insurer (i.e., reinsured company), not only in form but in fact, against loss or liability by reason of the original insurance. Unless the so-called reinsurance contract contains this essential element of risk transfer, no credit whatsoever shall be allowed on account thereof in any accounting financial statement of the ceding insurer."

SSAP 62, as part of codification, provides the following guidance, drawing heavily on FAS 113:

> [§11] Determining whether an agreement with a reinsurer provides indemnification against loss or liability (transfer of risk) relating to insurance risk requires a complete understanding of that contract and other contracts or agreements between the ceding entity and related reinsurers. A complete understanding includes an evaluation of all contractual features that (a) limit the amount of insurance risk to which the reinsurer is subject (e.g., experience refunds, cancellation provisions, adjustable features, or additions of profitable lines of business to the reinsurance contract) or (b) delay the timely reimbursement of claims by the reinsurer...
>
> [§12] Indemnification of the entity company against loss or liability relating to insurance risk in reinsurance requires both of the following:
> a. The reinsurer assumes significant risk under the reinsured portions of the underlying insurance agreements; and
> b. It is reasonably possible that the reinsurer may realize a significant loss from the transaction.

IASB. The International Accounting Standards Board's (IASB) Insurance Steering Committee has drafted a statement of principles on accounting for insurance contracts. As the statement is not final, it may well be modified before being officially released to the public. With these caveats in mind, it is instructive to compare the IASB's views on risk transfer to FAS 113.

As currently construed, the IASB's Principle 1.2 defines an insurance contract. Reinsurance is simply treated as a sub-set of insurance contracts. Principle 1.3 defines the uncertainty required for a contract to qualify as an (re)insurance contract. This principle, then, is closely related to the risk transfer requirement in FAS 113. Principle 1.3 does introduce the word "*material*" in describing uncertainty or risk transfer, much like FAS 113 refers to "*significant.*" Principle 1.3, however, does not distinguish between underwriting risk and timing risk as does FAS 113.

5. Current Practices.

As risk transfer tests are only defined in broad conceptual terms, practitioners of risk transfer testing are left to model insurance processes as they think best and define key terms such as "*remote*" and "*significant*" operationally. In practice, if the cedant's analysis passes muster with their auditor, reinsurance accounting is granted. Thus auditors, and sometimes the cedant's consultant, need to be able to recognize risk transfer when they see it.

VFIC conducted a brief, informal poll of actuaries at two major consulting firms and three major audit firms regarding their risk transfer testing. In particular, the practitioners were asked 1) does your firm have an official policy regarding risk transfer testing, 2) what threshold value do you use for determining *reasonably possible*, 3) how big of a loss is *significant*, and 4) what methods are used. A brief summary of the interviews follows.

	Respondent 1	Respondent 2	Respondent 3	Respondent 4	Respondent 5
Official Policy?	No	No	Yes	Don't know	Don't know
Probability	5% or 10%	10% or 20%	"Reasonable worst case chance"	20%	10%
Significance	5% or 10%	10% or 20%	10%	20%	10%
Method	Establish a probability distribution of expected losses, reflecting the timing thereof. Compare to the present value of premium.	Compare expected value of present value of losses to expected value of present value premiums by scenario	Scenario testing	NA	Net present value of all cash flows.

While there are certainly differences in practices indicated above, there are also some common themes. First, while probability threshold ("possibility") is rarely codified, 5%, 10%, and 20% are typical; 10% is in fact the most typical. The critical value defining significance is almost always the same as the probability threshold, i.e., 5%-5%, 10%-10%, 20%-20%. Again, 10% is the most typical, and thus we have what has become known as the "10-10 rule," whereby if the reinsurer has a 10% chance of suffering a 10% loss, then the contract is deemed to have transferred risk.

It must be emphasized that this 10-10 rule has become a *de facto* practice. FAS 113 makes no reference to it, nor does the statement define "*remote*" and "*significant*" thresholds with any numbers, let alone 10% and 10%. Furthermore, the 10-10 rule has not been officially propagated by anyone.

The 10-10 rule is a test utilizing value-at-risk (VaR) as the risk measure. That is to say, the ceding company must demonstrate a VaR of 10% at the 90th percentile of the distribution of the net present value of underwriting losses on the contract in question. And, in practice, a VaR test makes sense given the construct of FAS 113, i.e., the explicit reference to probability and significance gives rise to viewing risk in two parts – frequency and severity.

There are some other common practices, as well. First, the view is always prospective in nature. Second, "loss" as respects the reinsurer is always measured as the net present value of future cash flows. Finally practitioners interviewed are consistent in their view that reinsurer expenses, taxes, investment risk, and credit risk are not subject of the risk analysis.

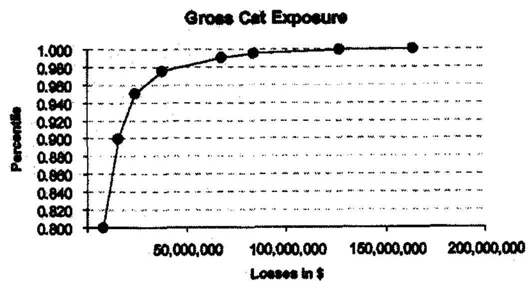
One problem with the 10-10 rule is that many standard reinsurance contracts, ones that everyone would acknowledge are highly risky, would not pass the test. Typical high layer property catastrophe treaties are but one example. Although these can be handled on an exception basis, it would be useful to have methods of measuring risk that agree with the assessments of experienced practitioners. The next section uses a series of examples to highlight this issue as well as to illuminate considerations required in traditional risk transfer testing.

6. Examples and considerations.

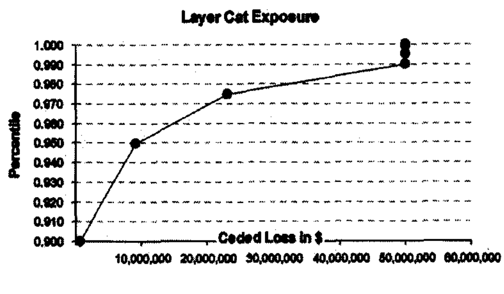
Given currently accepted practice, how could the practitioner prove that there is a less-than-remote-chance that their reinsurers could suffer a significant loss? Following are a series of numerical examples, designed to illustrate the basic data requirements and analysis of present day risk transfer testing. While such analysis presumably suffices for purposes of FAS 113, the examples will serve to show the inadequacies of a simple 10-10 rule (or VaR tests in general).

Example 1. Property Catastrophe Excess of Loss
An insurance company has exposure to southeastern U.S. hurricanes. Standard industry catastrophe models were applied, and the following catastrophe loss event cumulative distribution function was produced:

Probability	Loss
0.001	63
0.005	85
0.010	528
0.025	2,877
0.050	26,160
0.100	95,939
0.200	303,325
0.300	607,426
0.400	1,146,366
0.500	2,001,899
0.600	3,185,892
0.700	4,925,404
0.800	8,150,810
0.900	15,632,088
0.950	24,206,066
0.975	38,072,833
0.990	67,451,525
0.995	83,683,074
0.999	126,792,315
0.9999	163,627,870



Assume the company is content with a $15 million retention, roughly absorbing up to the one-in-ten-year event. Assume, too, that the company accepts a $50 million layer, thereby going through the top on a one-in-one-hundred-year event. Catastrophe losses were simulated according to the above distribution, and layer losses were calculated.



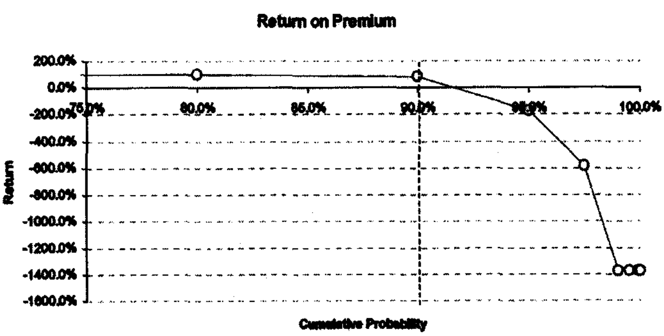
The above distributions produce an expected gross catastrophe loss of $6 million and an expected ceded loss of $1.625 million.

Assume for simplicity that the reinsurance market is pricing catastrophe covers to a 50% loss ratio (premium equals $3.25 million). For this purpose we will ignore reinstatements. Further assume that premiums are paid in full at the beginning of the year and losses are paid in full at the end of the year. As we are dealing with short duration losses, a discount rate of 4% was used.

Given the data and assumptions, the net present value of cash flows between the cedant and the reinsurer can be calculated (shown below as ROP – Return on Premium).

Probability	Gross Loss	Ceded Loss	Loss Ratio	Reinsurer NPV	ROP
0.001	63	-	0.0%	0.0%	100.0%
0.005	85	-	0.0%	0.0%	100.0%
0.010	528	-	0.0%	0.0%	100.0%
0.025	2,877	-	0.0%	0.0%	100.0%
0.050	26,160	-	0.0%	0.0%	100.0%
0.100	95,939	-	0.0%	0.0%	100.0%
0.200	302,299	-	0.0%	0.0%	100.0%
0.300	607,426	-	0.0%	0.0%	100.0%
0.400	1,146,366	-	0.0%	0.0%	100.0%
0.500	2,001,899	-	0.0%	0.0%	100.0%
0.600	3,185,892	-	0.0%	0.0%	100.0%
0.700	4,925,404	-	0.0%	0.0%	100.0%
0.800	8,150,810	-	0.0%	0.0%	100.0%
0.900	15,632,088	632,088	19.4%	18.7%	81.3%
0.950	24,206,066	9,206,066	282.9%	272.1%	-172.1%
0.975	38,072,833	23,072,833	709.1%	681.8%	-581.8%
0.990	67,451,525	50,000,000	1536.7%	1477.6%	-1377.6%
0.995	83,683,074	50,000,000	1536.7%	1477.6%	-1377.6%
0.999	126,792,315	50,000,000	1536.7%	1477.6%	-1377.6%
0.9999	163,627,870	50,000,000	1536.7%	1477.6%	-1377.6%

The reinsurer's "profit curve," the trace of the ROP versus the cumulative probability looks as follows.



A catastrophe example was deliberately chosen as the first example. No one would dispute the clear risk transfer that exists between cedant and reinsurer in a property catastrophe excess of loss program. Yet the above graph clearly demonstrates that the sample transaction fails the 10-10 rule. At the 90th percentile the reinsurer makes an 82% return on premium, thus it is not true that there is at least a 10% chance of at least a 10% loss. Perhaps this can be rectified by simply choosing a different probability to reflect the "*reasonable possibility*," for at the 95th percentile, the reinsurer suffers a 172% loss.

The first example illustrates a number of key points.

1. Key considerations in this analysis included:
   - A thorough understanding of the reinsurance contract,
   - A probability distribution of expected losses, as determined by the cedant,
   - Incidence or timing of cash flows between the parties,
   - A duration-appropriate discount rate.

2. Elements that were not and should not be considered include:
   - Reinsurer expenses,
   - Brokerage, and
   - Taxes

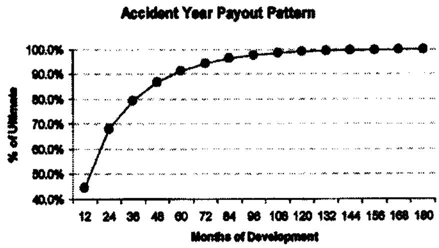
3. A VaR test may work, but risk transfer cannot be judged on a single, simple rule such as 10%-chance-of-a-10%-loss. The whole of the reinsurer's profit and loss curve is important to consider. In this case, while the reinsurer is still in a profit position at the 90th percentile, there is clearly a precipitous and deep drop shortly thereafter. In this situation, the reinsurer or reinsurers stand to lose a considerable amount of money relative to the premium revenue.

Example 2: Quota Share Reinsurance Example
In this example, an insurance company seeks a 50% quota share protection on its accident year results. Even though test **c** may apply, it may be interesting to see how tests **a** and **b** would view this type of contract under different risk measures.

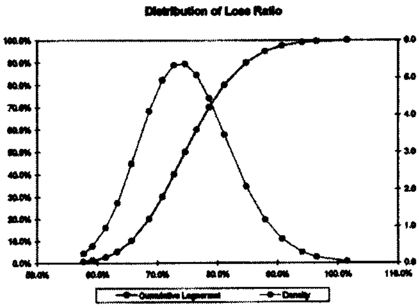
For the upcoming year, this company forecasts:

Written Premium	$1,000
Earned Premium	1,000
Accident Year Loss Ratio	75%
Expense Ratio	32%
Combined Ratio	107%

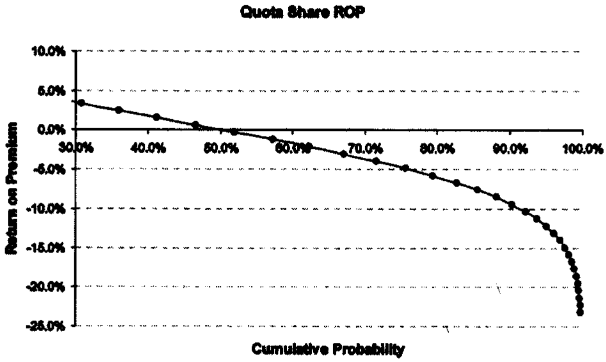
To complete this example, we assume that the insurance company in question is an industry-typical, all lines writer and has an accident year loss payout pattern that mirrors the industry total[3]:



The company has estimated the distribution of the upcoming accident year loss ratio as part of its normal forecasting process. We assume the loss ratio is distributed lognormally with a mean of 75% and a coefficient of variation of 10%.



The quota share treaty has a 30% ceding commission. Premiums and commissions are paid evenly through out the year. Under these assumptions, the reinsurer's profit/loss curve looks as follows.



At the 90.4th percentile, the reinsurer suffers a 9.5% of premium loss. It does not literally pass the 10-10 rule test. However, given the precipitous drop in profitability in the tail, and given the inherent uncertainties of the analysis itself, it should be evident that there are "*reasonable possibilities*" of "*significant losses.*"

[3] Source: 1999 Industry total Schedule P, all lines paid triangle from A.M. Best's.

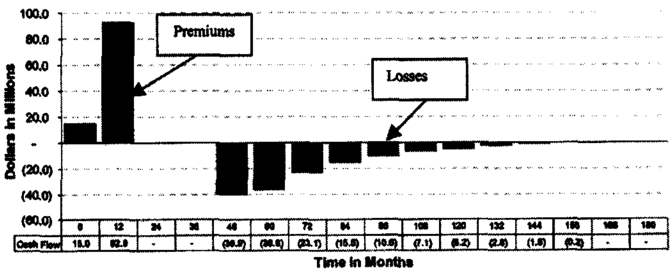
Example 3: Finite Reinsurance Example.
Finite reinsurances are often the principal source of risk transfer questions. In this example, all underlying numbers are the same as in the quota share example. This time, however, the cedant is seeking protection in excess of the planned loss ratio up to a 5%-point limit (i.e., the corridor from 75% to 80%).

Assume the reinsurer charges an up front premium (often called the deposit premium, minimum and deposit premium, the reinsurance premium, or the margin) of $15. As is typical in finite transactions, for every dollar of loss ceded, an additional premium (AP) is charged, in this case 65% of the ceded loss. Because additional premium is ceded, the net expense ratio will deteriorate with increasing cessions. To compensate for the expense ratio effect, losses are typically "over ceded" such that the net combined ratio (or underwriting result) is immunized. So, here ceded losses are grossed up by dividing by 1-AP. The ceding rule is:

<u>If the actual loss ratio is</u>:	<u>Cede</u>:
<75%	0
>75%	(LR-75%)/(1-.65)

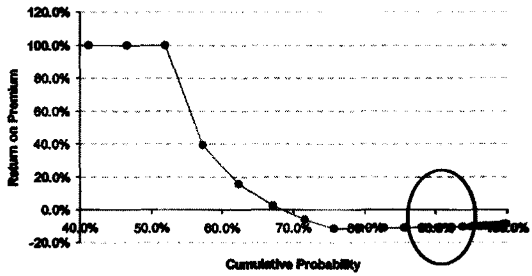
subject to a maximum of the grossed up 5% limit – 5/(1-.65).

To compute the incidence of the cash flows, we assume that the deposit premium is paid at the beginning of the year, and that the AP is paid in full at the end of the year. A recoverable is established on the company's statutory and GAAP balance sheets immediately when the expected ultimate exceeds the retention. Loss recoveries are not made until the *paid loss ratio* exceeds the retention. For a loss ratio of 80%, the cash flows between the cedant and the reinsurer would look as follows.



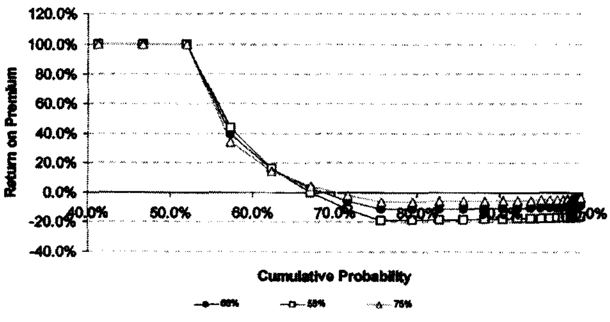
The cash flow graph above highlights the zeal behind using aggregate stop loss contracts, especially in a soft market. A ceded recoverable is established for the full, nominal dollar loss reserves above a certain loss ratio, but due to the time lag in receiving recoveries, the reinsurance price reflects a sizable discount. The difference between the discount and the nominal value of the reserves in question becomes income for statutory or GAAP purposes. Economically speaking, no value is really created nor destroyed beyond the reinsurer's margin.

Cash flows as shown above were produced for loss ratios ranging from 70% to 100%. For each loss ratio, the net present value of cash flows was calculated using a 5% discount rate. Net present values were graphed as a function of cumulative probability (of the loss ratio) to produce the reinsurer's profit/loss curve.



This finite example was produced to demonstrate the 10-10 rule almost exactly. Here there is a chance of a 10% loss or more at the 90.4th percentile, almost exactly satisfying the 10-10 rule.

This same graph was re-drawn for the above base case as well as cases with a 55% AP and a 75% AP:



In the above graph, the 75% AP program would presumably not pass risk transfer under a 10-10 rule test. The 55% program would pass. Even in the 65% example, however, consideration must be given to the entire profit/loss curve, not just the 90th percentile. How much profit is made on the upside? How bad is the downside?

Aggregate stop loss deals specifically and finite reinsurance in general can be considerably more complicated than this example. It is critically important here to have a thorough understanding of the contract terms. Some common variations include:

- Funds held arrangements[4],
- Commutation provisions,
- Capacity charges,
- Margin charges,
- Inclusion of expenses, and
- Caps on economic loss.

Summary of Considerations in Applying VaR tests.
Risk transfer testing requirements are prospective in nature. Thus the mean result (loss ratio, statutory underwriting result, GAAP underwriting result...) is a forecast of a future period. The actuary must account for pricing changes, loss trends, credibility, etc., i.e., all of the typical on-leveling adjustments ordinarily made to historic data.

Practitioners must go beyond the mean. The distribution associated with the mean result should be calculated in accordance with the model employed for the forecasting. Distributions can be estimated by methods applied to loss triangles, collective risk theory models, or variances estimated from time series of relevant results

A model of the incidence of cash flows is required. The model must distinguish between funds held and funds transferred between parties. Dependencies between cash flows and the magnitude of the loss must be accounted for, e.g., the effect of catastrophes on an assumed loss payout pattern. Cash flows should be discounted at the same, appropriate rate. A risk free rate is specified, preferably a pre tax, immunized yield

In the end, a discounted cash flow model, perhaps a dynamic model should suffice. Clearly a thorough understanding of the contract terms is required for a thorough analysis.

"Remote" results can be judged on the basis of closed form distributions of results, simulations, or through scenario testing. Significance is defined by the magnitude of the net present value of cash flows between parties as a percent of revenues.

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[4] Funds held arrangements, wherein the cedant holds the loss fund and earns the associated investment income. Here the actuary must consider what constitutes the basis for measuring the 10% loss. Is premium the appropriate base? On one hand, it would seem not, as it is not cash between the parties. On the other hand, FAS 113 states, "Payments and receipts under a reinsurance contract may be settled net. The ceding enterprise may withhold funds...Determining the amounts paid or deemed to have been paid (hereafter referred to as "amounts paid") for reinsurance requires and understanding of all contract provisions."

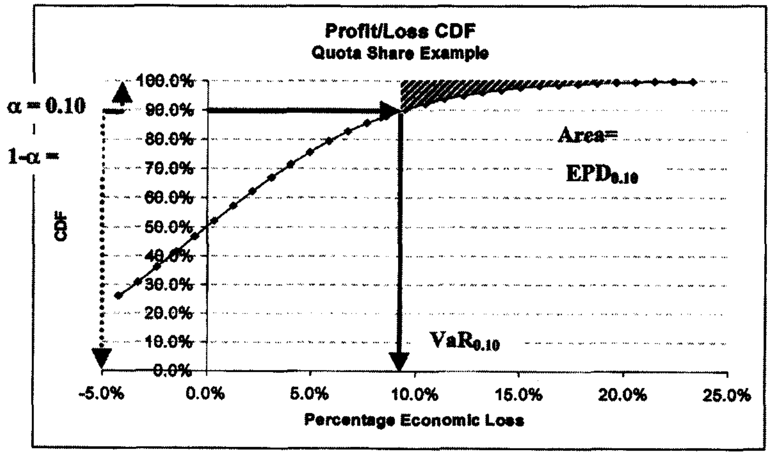
7. Beyond VaR Tests.

FAS 113 does not prescribe a specific method to test for risk transfer. Furthermore, given a model, FAS 113 does not precisely define whether the model output would imply that the contract in question passed or failed. While we must meet the considerations of FAS 113, actuaries needn't demonstrate risk transfer using the 10-10 rule or VaR test more generally.

Expected Deficit Methods.
The examples presented above suggest that a single point of *remote probability* and a single critical value for *significance* maybe inadequate, e.g., 10-10. Instead risk/reward is perhaps better viewed across the entire spectrum of profit and loss (consider the property catastrophe example). That is, there is a trade-off between probability and significance.

The 10-10 rule is used as a rule of thumb, for simplicity or as a starting point. Assume for the moment that a 10% chance of a 10% loss is, in fact, evidence of risk. It is simply not an exclusive evidence of risk. What if risk was defined by the trace of a line – almost akin to an efficient frontier – of those points that, by their combination of probability and magnitude, define risk transfer: 10-10, 5-20, 1-100, 0.1-1000? From such a set of points, one coordinate measuring probability, one measuring the magnitude of the loss, we can construct a single risk measure: the expected policyholder deficit (or in this case, the expected reinsurer's deficit).

The graph below compares the 10-10 rule ($VaR_{\alpha=10}$) with EPD. This graph was drawn using the data from the quota share example provided above.



In the continuous case, expected reinsurer's deficit (ERD) is defined as

$$\int_{NPV(loss)>NPV(premium)}^{\infty}[NPV(premium)-NPV(loss)]f(x)dx$$

In the discrete case, the expected reinsurer's deficit is

$$\sum_{NPV(loss)>NPV(premium)}^{\infty}[NPV(premium)-NPV(loss)]\Pr(x)$$

That is, the expected reinsurer's deficit is the average, or expected, deficit over all values where a deficit exists. If the NPV's above are divided by premiums (or cash to the reinsurer) the expected deficit is per unit of revenue. Using the pairs of numbers above, assuming these were our only loss scenarios, the ERD = (.10*-.10) + (.05*-.20) + (.01*-1.0) + (.001*-10) = -.04 or -4%. For comparison, the ERD's calculated for the three examples previously are as follows.

- Property Catastrophe = -40%
- Quota Share = -3%
- Finite = -3%

This metric has some appeal in that it is well grounded in actuarial theory concerning the measurement of risk. It also overcomes the 10-10 rule weakness (or VaR rules in general) of relying on a singular point to define risk transfer. We still have the problem of critical values, however: in this instance, what ERD defines risk transfer? In the above examples, property catastrophe has a –40% ERD, a number significant enough to likely be granted worthy of risk transfer (even though it didn't pass the 10-10 rule test). The quota share and finite examples have –3% ERDs. Here it is less clear that there is meaningful risk transfer.

Tail Value at Risk.
More recently, VaR and EPD measures have come under criticism in actuarial and finance circles because they are not coherent measures of risk. Given random losses X and Y, a risk measure, ρ, is considered coherent if it conforms to the following properties[5].

1. Sub-additivity: For variables X and Y, $\rho(X+Y) \leq \rho(X)+\rho(Y)$
2. Monotonicity: If $X \leq Y$, $\rho(X) \leq \rho(Y)$
3. Positive Homogeneity: for $\lambda \geq 0$, $\rho(\lambda X)=\lambda\rho(X)$
4. Translation Invariance: $\rho(X+a) = \rho(X)+a$

The sub-additivity property simply requires that the combination of two risk factors does not create additional risk; in fact, risk is the same or less. Value at Risk, despite its popularity, violates this axiom.

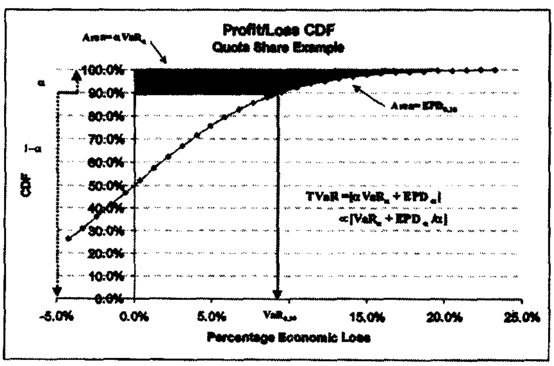
In the alternative, Tail Value at Risk, or TVaR, is a coherent risk measure. TVaR is equal to the expected value of a loss variable, say X, given that X exceeds the critical value VaR_α, i.e.,

$$TVaR_\alpha = E[X \mid x > VaR_\alpha]$$

If α is the probability of default, then VaR_α is the total assets, and TVaR may be expressed as:

$$TVaR_\alpha = \alpha\text{*assets} + EPD\text{, or } TVaR \propto \text{assets} + EPD_\alpha/\alpha$$

As in the EPD case, above, TVaR can be represented graphically as follows.



[5] See the discussion in Meyers [2]

TVaR's were calculated for each of the three examples above at the 90th percentile.

- Property Catastrophe = -319%
- Quota Share = -42%
- Finite = -23%

Recall from the previous section that the "ERD" did not discriminate between the quota share contract and the finite contract. TVaR does, and indicates that the quota share contract has more risk.

We do not have enough research, or perhaps even the prerogative, to suggest a threshold TVaR that implies a contract passes risk transfer. However, in the examples presented here, a finite contract, that by all accounts only marginally passes more traditional, 10-10 test and has no meaningful downside beyond the 10% loss, has a TVaR of -23%. Perhaps this suggests a threshold value in the 20-25% range or less would reflect minimal risk transfer.

Other Coherent Risk Measures

Coherent risk measures are characterized statistically as expected values of outcomes under adjusted probability distributions. For instance, TVaR, is expressed as:

$$E[X \mid x > VaR_{\alpha}]$$

This could equally well be expressed as the adjusted expected value of X under transformed probabilities, where the transformed probability is zero for $X < VaR_{\alpha}$ and is the actual probability adjusted to sum to unity otherwise.

This particular measure has been criticized on at least two grounds (e.g., see Wang (2001) *A Risk Measure that Goes Beyond Coherence*, Institute of Insurance and Pension Research, Research Report No. 18, University of Waterloo). First, it ignores all results below VaR_{α}. Second, it just measures losses above VaR_{α} on an expected basis, which is an under-weighting compared to moment-based measures, which use higher powers to represent the extreme risks of extreme events.

An alternative probability adjustment, which produces an alternative coherent risk measure that addresses these concerns, is provided by the Wang transform. This transform adjusts each scenario probability u by first calculating the normal-distribution percentile of u, then applying a functional transform to that percentile, and finally taking the normal probability of the transformed percentile. In mathematical notation:

Let $\Phi(x)$ be the standard normal cumulative distribution function, and $\Phi^{-1}(u)$ be its inverse, the percentile function, which applied to a probability u gives the corresponding percentile. Let h(x) be the percentile distortion function. Then the probability transform applied to a cumulative loss probability u is $v = g(u) = \Phi[h(\Phi^{-1}(u))]$. A simple example is to take h(x) linear, such as bx+a, or even an additive constant, such as x+a.

One use of risk measures is to calculate the market price of risk transfer. Wang has shown that prices of risk in a number of markets, including catastrophe bonds, corporate bonds, and stock options can be approximated fairly closely by choosing the appropriate h function for each market. (Risk pricing may vary across markets in part due to the degree of hedging and liquidity available, as well as to the degree to which financial results are subject to sudden large drops.) The key issue to getting the right h function is applying enough probability distortion in the tails of the distributions to capture the market reaction to tail events. However, even a linear h function provides a non-linear price effect in the tails, and thus can be used for benchmarking.

Quantifying the market price of the risk inherent in a given transaction could be an alternative method for determining if there is enough risk transfer to satisfy the requirements of FAS 113. Even if a contract is priced above the market value of the risk it has, it still might meet the FAS requirements for risk transfer. However, as *significant loss* is to be interpreted relative to ceded premium, a deal could fail risk transfer, but pass if the premium is reduced. Thus there is a pricing continuum from weak pricing to strong pricing to excessive pricing to not enough risk transfer for 113 to no risk at all.

As an example of the application of the Wang transform to risk transfer, let h(x) = 0.7x – 1.3. This gives prices quite a bit above market standards, but might be in the area between excessive pricing and no risk transfer. To apply this to risk transfer testing, a number of scenarios can be simulated showing the present-value profitability to the reinsurer for each scenario, and resorted into a cumulative probability distribution. The expected value of the profit should be positive under this distribution, or the reinsurer would not be interested. But if you distort the probabilities with the Wang transform to give more weight to the adverse scenarios, the transformed expected value could be negative. If it is negative with the target h function selected, then risk transfer would be deemed to be established.

With the linear h assumed, the 50 excess 15 catastrophe cover in Example 1 would pass risk transfer, with a transformed mean of –440%, and would still barely pass (with a mean of –2%) with the premium increased to as much as $25M., which gives a 1% probability of a 92% loss. This premium is well above typical market standards, but may be in the gray area between no risk transfer and excessive pricing. Setting the h function would be the judgment part of this approach. With these values, the quota share from Example 2 easily passes risk transfer with a transformed mean return of –19%.

Premium for the catastrophe cover much above $25M would fail risk transfer by this standard. It might seem unusual to find a catastrophe cover not meeting risk transfer, but grossly overpriced catastrophe covers could be used as payback or to add the appearance of risk to basically cosmetic deals. An actuarial risk-measurement procedure should be able to identify them.

Exponential Transform

Oakley Van Slyke and Rodney Kreps, in an unpublished manuscript [2], suggest another possible approach to testing risk transfer through measuring the capital cost inherent in a reinsurance transaction. This is based on the work of Karl Borch, 1962 on quantifying risk costs. Borch shows that under certain assumptions the only risk-reflecting pricing transform that properly measures risk cost is an exponential transform. His assumptions - as discussed in Giuseppe Russo and Oakley E. Van Slyke [4] are essentially:

- There are no arbitrage opportunities. That is, the cedant would never pay more to cede a loss than the amount of the loss. In turn, no one would be able to sell insurance for a premium greater than the amount of the exposure.

- The evaluation of an alternative is robust with respect to the input data. That is, a small change in an input parameter should not lead to a large change in the evaluation of an alternative.

- The evaluation of an alternative is robust with respect to the analytical process one is using. For example, making small refinements to a particular scenario should not drastically change the evaluation of a particular alternative.
- The evaluation of an alternative is robust to changes in the time scale. For example, changing the time intervals of the analysis from quarterly to monthly should not have a significant change in the evaluation of an alternative.

- If there is no risk, one can determine the present value of a stream of future cash flows by discount factors derived from the term structure of interest rates.

These assumptions lead to establishing an equivalent constant risk-adjusted value (RAV) of a risky deal, subject to the risk capacity c that is carried. First let X represent the random loss from the deal, prior to any premium payments Then the Risk Adjusted Value of liabilities for risk-carrying capacity c > 0 is:

$$RAV(c) = c\ln\left\{E\left[e^{X/c}\right]\right\}$$

this emphasizes large losses, more so as c is small and less so as c is large.

The risk load to take on these liabilities = RAV(c) – E[X], is then expressed as:

$$\pi = c\ln\left\{E\left[e^{X/c}\right]\right\} - E[X]$$

Van Slyke and Kreps then impose the condition that the capacity available is a multiple of the risk load:

$$c = \pi/s \Rightarrow \pi = \pi/s\ \ln\left\{E\left[e^{sX/\pi}\right]\right\} - E[X]$$

If you subtract a constant premium p from X and then evaluate the risk in the deal, E[X] and the RAV also decrease by p. Thus the risk load to package and resell the whole deal is the same as that for the losses alone. Then taking the financial scale as multiples of p would make X the negative of the return on premium. Taking Y = –X as the return on premium gives:

$$\pi = E[Y] + (\pi/s)\ln E[e^{-sY/\pi}]$$

as the equation for the risk load as a percent of premium for reselling the entire deal. If the market s is known, this equation can be solved numerically for π, which then can be used to compute the risk adjusted value of the deal. If the RAV is positive, the price is below market levels. If RAV is slightly negative, the deal is priced above the market, but still could be fairly risky. As with the Wang transform, however, when the RAV is too negative, the pricing eventually crosses the line between excessive pricing and no risk transfer.

Van Slyke did some other research that suggests that s = 0.4 would fairly represent pricing in a number of financial markets. This value will be assumed in the discussion which follows.

Taking the RAV cutoff point for return on premium as RAV = –70% would be similar to the Wang transform values illustrated above. For Example 1, the RAV would be about positive 75%, which would suggest that the postulated pricing is light in terms of market risk pricing. With the premium increased to $25M, the RAV drops to – 67.2%, so barely passes risk transfer by this standard. For the quota share Example 2, the RAV is about 25%, which suggests there is considerable risk remaining in this deal.

The Borch approach is based on somewhat different market assumptions than the transformed distribution approach. Although these are consistent for independent risks, there could be inconsistencies for correlated risks. For example, see G.G. Venter, *Premium Calculation Implications of Reinsurance without Arbitrage*, ASTIN Bulletin 21, #2, November 1991, where it is shown that arbitrage-free pricing for both correlated and independent risks can be done only with expected values from transformed distributions. This was one of the precursors of Wang's work. However by just focusing on the ending distribution and ignoring intermediate changes in value, distribution transforms fail to account for the sudden drops in value that are modeled in stochastic financial pricing methods. The potential for discontinuous price drops seems to require more risk premium, possibly because dynamic hedging strategies are less effective. Thus although probability transforms on ending distributions can produce good benchmarking rules, they are not as fundamental as the financial stochastic process models, and have to be calibrated separately to each market studied.

Transformed 10 – 10 Rule

If the 10 – 10 rule is accepted for normal distributions, then a transformation can provide an equivalent standard for skewed distributions.

To see this, let X represent the ROP (return on premium) of the contract to the reinsurer, when this is negative and zero otherwise. *For this variable X with distribution F, define a new risk-measure as follows:*

1. *For a pre-selected security level* α=10%, *let* $\lambda = \Phi^{-1}(\alpha) = -1.282$, which is the α-th percentile of the standard normal distribution.
2. *Apply the Wang Transform*: $F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$.
3. *Calculate the expected value under* F^*: $WT(\alpha) = E^*[X]$.
4. If $WT(\alpha) < -10\%$, it passes the test, otherwise it fails the test.

When X has a Normal(μ,σ^2) distribution, $WT(\alpha)$ is identical to the 100α-th percentile. This serves as a base or benchmark for 10-10-rule. For distributions that are non-normal, $WT(\alpha)$ may correspond to a percentile higher or lower than α, depending on the shape of the distribution.

For Example 1, the catastrophe layer, these values of the transform are a little less strict than the tests evaluated above, with premium as high as $34M for the layer meeting the test. For Example 2, the quota share, $WT(0.10) = -14.39\% < -10\%$, so it passes the transformed 10-10-rule.

In conclusion, at its core, FAS 113 requires only that risk transfer be present to gain reinsurance accounting treatment. FAS 113 does not require a 10-10 rule in gauging the risk transfer. The preceding sections offered some alternative measures such as TVaR, the Wang Transform, and the exponential transform for judging the degree of risk.

8. Beyond FAS 113.

Insights from the Securitization Task Force.
As configured, FAS 113 requires that the cedant establish that the reinsurer has assumed some amount of risk. If one were to consider the evaluation of risk transfer beyond that which is described in FAS 113, it would seem preferable that the cedant demonstrate a complementary concept: that they have, in fact, ceded risk. Thus, risk transfer would not be defined based on cash flows between parties, but rather the changed risk of the cedant – before and after application of the contract in question. This is essentially the logic the Index Securitization Task Force has used in proposing methods and metrics for companies to justify whether or not a hedge should qualify for reinsurance accounting.

The Index Securitization Task Force, in its paper [1], *Evaluating the Effectiveness of Index-Based Derivative in Hedging Property/Casualty Insurance Transactions*, describes potential quantitative measures of hedge effectiveness. These include change in Expected Policyholder Deficit, change in Value at Risk, change in Standard Deviation, coverage ratio and correlation. Of these, the first three examine the reduction of risk attributable to the hedge. At the request of the task force, VFIC narrowed this list to two measures that best demonstrated a reduction in exposure to loss, thus enabling a hedge to receive underwriting accounting treatment versus investment accounting treatment. These measures are: reduction in Tail Value at Risk and reduction in Standard Deviation.

As discussed above, Tail Value at Risk is defined as the average of all loss scenarios over the 100^p th percentile, where p is a selected probability level, such as .90. One can consider this measure a melding of the expected policyholder deficit and value at risk measures. The tail value at risk measure captures both the probability and magnitude of large under-recoveries. Based on empirical studies, the committee found that tail value at risk produced more consistent results than value at risk when the probability levels were varied.

The other measure the committee recommended, reduction in standard deviation, distinguishes between true hedges and speculative investments since it is sensitive to both upside deviation and downside risk.

With respect to the degree of risk reduction, one may consider that risk has been transferred if both or either of these measures demonstrates that their value is less following the application of the hedge or reinsurance contract. A more conservative view would set specific thresholds by some predefined amount.

Given this application of risk measurement for gauging the effectiveness of a hedge for reinsurance accounting treatment, it is not inconceivable that the same sort of standard be utilized to gauge risk transfer in reinsurance contracts. In fact, in the absence of consistent treatment, there is the potential for different standards and approaches to be applied when evaluating a reinsurance contract for risk transfer versus evaluating hedge effectiveness for index-based securitization.

9. Conclusions.

In order to garner reinsurance accounting treatment for GAAP accounting purposes, a reinsurance contract must meet the requirements set forth in FAS 113. FAS 113 requires that a reinsurance contract transfer risk. There is little supporting literature to find guidance in what constitutes an acceptable demonstration of the existence of risk in a reinsurance contract. In an effort to provide some guidance to the CAS membership on risk transfer testing, VFIC conducted a research project on risk transfer. Based on this research and analysis, VFIC concludes:

1. **Statement**. FAS 113 requires the reinsurer to be exposed to a "*reasonable possibility*" of a "*significant loss*" from the "*insurance risk,*" but it stops short of prescribing methodology for testing, metrics for measuring, or specific thresholds to judge risk transfer against. This is appropriate given the diversity and complexity of reinsurance transactions.

2. **Methodology**. Regarding methodology, FAS 113 articulates that risk transfer testing include:
    - A thorough understanding of contract provisions,
    - A model of the incidence of cash flows between parties,
    - Cash flows should be discounted at the same, appropriate rate, and
    - Incorporating insurance risk only

    These requirements preclude consideration of income taxes, reinsurer expenses, brokerage, or credit risk in the determination of risk transfer.

    To meet the FAS 113 requirements, we recommend that risk transfer analysis include:
    - "*Reasonable possibility*" requires a view of the distribution of expected contract losses,
    - Identification of threshold values for "*reasonable possibility*" of a "*significant loss*" based on the loss distribution, and
    - Duration-matched or immunized yields as the appropriate discount rates,

3. **Metrics**. Current practice, born out of the phrases "*reasonable possibility*" of a "*significant loss,*" splits risk transfer analysis into separate tests of probability and significance. Using a singular loss metric for a given probability is a metric known as Value at Risk, or VaR. This paper offered examples of three types of reinsurance contracts and calculated a VaR for each using 10% as the "*reasonable possibility.*"

    One weakness of VaR is that it does consider only a single point on the loss distribution. While FAS 113 literally speaks to the existence of a "*reasonable possibility*" of a "*significant loss,*" the broader issue involved with FAS 113 is whether a particular contract transfers risk. In this vein, VFIC explored risk metrics other than VaR. First among these was expected policyholder deficit (EPD). Expected deficit methods were able to illustrate risk transfer for a property catastrophe example where the standard VaR measure (with α=10%) was not.

Both VaR and EPD measures have been criticized as risk measures because they are not coherent. Tail Value at Risk (TVaR) is a coherent risk measure. TVaR was analyzed, as well, and was found in simple examples to discriminate risk levels between contract types where EPD and VaR did not. Even TVaR has been criticized as a risk measure in that it ignores losses below VaR_α and loss above VaR_α are treated on an expected basis only.

Distributional transforms were researched as alternatives to traditional risk measures. Transforms are coherent and address the shortcomings of TVaR noted above. The exponential and Wang transforms provide risk transfer metrics founded in the risk load required for a market-based transaction to transfer the risk.

4. **Thresholds or Critical Values**. Over time, common practice seems to have concluded that a 10% chance represents a reasonable probability, and a 10% loss represented a significant loss. Thus we have what many term the 10-10 rule. This rule-of-thumb is really just a statement of the critical values associated with a VaR risk measure. There are clearly exceptions to this "rule," as other critical values are frequently used in practice.

   A sample finite reinsurance contract, designed to have minimal risk transfer, generated a TVaR of -23%. While this represents limited research, it may suggest a minimal threshold value for demonstrating risk transfer with this measure.

   Section 7 proposes a transformed 10-10 rule for the Wang transform, suggesting a critical value of –10% from the mean of the transformed distribution as an adequate demonstration of risk transfer.

   Regardless of the model employed or the risk metric used, judgment is still required as to where to establish the threshold values for probability (frequency) and significance (severity) for VaR tests or for pass/fail more generally for other risk measures. .

5. Intuitively, it seems natural to judge risk transfer for a reinsurance contract by analyzing whether the cedant has transferred (reduced) risk, not, as FAS 113 requires, by whether the reinsurer has assumed risk. On an enterprise-wide basis, the two can be different. On a single transaction, as FAS 113 addresses, the two perspectives may be the same. However, it should be noted that the recommendation on Index Securitization proposed the opposite: analysis is done from the cedant's perspective on an enterprise-wide basis. This could lead to

different accounting treatments for reinsurance products and index securitizations, unless both tests are required for securitization and industry loss triggers.

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Fitting Moments with Weights

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Abstract: *This note investigates ways to fit individual claim loss data to a prior known "underlying severity level" by adjusting the relative importance, or weight, assigned to each claim. Here, "underlying severity level" is measured by the weighted mean cost per case. The paper also generalizes the approach to accommodate fitting higher moments of the loss distribution, especially the variance. It establishes the existence of an optimal reweighting, but whose calculation may be too difficult for practical application. To address this, the paper describes two easier calculations, one designed to fit only the mean and another to fit both mean and variance.*

Section I: Setup and Notation

Let X be any finite set, by a *weight on* X we simply mean a non-negative real-valued function $\omega : X \to [0,\infty)$. In this case will also refer to ω as a *weight* and refer to the pair (X,ω) is a *weighted set.* But we will often abuse this formality and just refer to X as weighted by ω. For any finite set X, we let $|X|$=number of elements in X. When X is weighted by ω, we use the notation:

$$|A|_\omega = \sum_{x \in A} \omega(x), \text{ for any subset } A \subseteq X.$$

We note two simple properties that a weight ω on X may or may not have:

ω is *positive* if and only if $\omega(x) > 0$ for every $x \in X$

ω is a *probability weight* if and only if $|X|_\omega = 1$.

It is clear that the concept of a discrete probability density on X exactly coincides with what we are here calling a probability weight.

Now let $X \subset \mathrm{R}$ be any finite set of real numbers and ω a weight on X. By combining the weights of elements of X that are equal, we can without any loss of generality write $X = \{x_1 < x_2 < ... < x_n\}$ as a series of n distinct numbers in ascending order. Think of the x_i as representing the distinct loss amounts from the claim sample X, arranged in increasing order to facilitate a size of loss analysis. Now take any $z \in \mathrm{R}$ with $x_1 < z < x_n$. It is intuitively clear that there exits a weight υ on X for which z is the weighted mean:

$$z = \frac{\sum_{x \in X} \upsilon(x)x}{\sum_{x \in X} \upsilon(x)} = \frac{\sum_{x \in X} \upsilon(x)x}{|X|_\upsilon} = \mu_\upsilon(X).$$

If we define yet a third weight ρ on X by setting $\rho = \frac{\upsilon|X|_\omega}{\omega|X|_\upsilon}$. Then we can think of ρ as a multiplicative adjustment factor to the weight ω that reweights the weighted set X to give it the given mean z while holding the total weight constant.

Section II: Moments of Finite Claim Samples

This paper pursues the question of how to come up with an appropriate υ. For this purpose, we introduce the *formal moments* of X, relative to any *function* $\upsilon : X \to \mathrm{R}$

$$\mu_k = \mu_k(X,\upsilon) = \frac{\sum_{x \in X} \upsilon(x)x^k}{|X|_\upsilon}, \quad 0 \le k \le n-1.$$

Observe that when υ is a probability weight, this is just the usual first $n-1$ moments of the claim sample X. It turns out that for any vector of potential formal moments of X, say $m = (1, m_1, ..m_{n-1})$, there is a uniquely defined function $\upsilon(m) : X \to \mathrm{R}$ such that:

(*) $$m_k = \mu_k(X,\upsilon), \quad 0 \le k \le n-1.$$

To verify this, recall the $n \times n$ Van der Monde matrix:

$$V = V(X) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}.$$

whose determinant:

$$Det(V) = \prod_{1 \le j < i \le n} (x_i - x_j) > 0.$$

provides a standard exercise in introductory linear algebra textbooks. The verification is by induction on n. Case $n = 1$ holds vacuously and case $n = 2$ is clear. Regard the x_i as constants and construct the $n-1$ degree polynomial:

$$p(y) = Det\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ x_1 & x_2 & \cdots & x_{n-1} & y \\ \vdots & \vdots & & \vdots & \vdots \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_{n-1}^{n-1} & y^{n-1} \end{bmatrix}.$$

Note that substituting y by any of $x_1, \ldots, x_{n-1}$ results in a matrix with two identical columns. But then clearly $p(y)$ has the distinct roots $x_1, \ldots, x_{n-1}$, and we may write $p(y) = a\prod_{1 \le i < n} (y - x_i)$, where the constant a is the coefficient of y^{n-1}. But expanding the determinant along column n and invoking the induction hypothesis:

$$a = Det\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_{n-1} \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{n-2} & x_2^{n-2} & \cdots & x_{n-1}^{n-2} \end{bmatrix} = \prod_{1 \le j < i \le n-1} (x_i - x_j).$$

Whence:

$$Det(V) = p(x_n) = a\prod_{1\le i<n}(x_n - x_i) = \prod_{1\le j<i\le n-1}(x_i - x_j)\prod_{1\le i<n}(x_n - x_i) = \prod_{1\le j<i\le n}(x_i - x_j),$$

that completes the induction.

Now we can naturally identify any function $\upsilon : X \to \mathrm{R}$ with the row vector $(\upsilon(x_1), \upsilon(x_2),...\upsilon(x_n))$. With this notation, observe that (*) is just the matrix equation: $V\upsilon^T = m^T$. Since the matrix V is nonsingular, the function $\upsilon : X \to \mathrm{R}$ can be calculated from $\upsilon^T = V^{-1}m^T$, establishing both existence and uniqueness of υ. In theory, this provides a way of determining whether a weight υ exists on X that reweights the claims to fit the given set of n moments, and even provides a way to calculate it. In practice, however, the claim sample may be very large and this may not be very practical.

More likely, we are only concerned with fitting the first few moments of the claim sample X to a set of moment values derived from empirical data, say $\hat{m} = (\hat{m}_0 = 1, \hat{m}_1,...\hat{m}_k), k \le n$. The moments must be reasonable in relation to X, for example we clearly must have:

$$x_1 m_j \le m_{j+1} \le x_n m_j \ 1 \le j \le k.$$

Which would be assured, say, if all the empirical claim costs fell within the range of X.

When $k = 1$ it is clear that the set of "possible" moments over all probability weights on X is just:

$$M_1(X) = \{(1, m_1) \mid x_1 \le m_1 \le x_n\}.$$

The case $k = 2$, which corresponds to fitting both the mean and standard deviation, is more complicated and so we consider sub-cases.

Sub-case $k = n = 2$, here the reader can easily verify that:

$$M_2(X) = \{(1, m_1, m_2) \mid x_1 \le m_1 \le x_2,\ m_2 = x_2(m_1 - x_1) + m_1 x_1\}.$$

Sub-case $k = 2, n = 3$, here we claim that

$$M_2(X) = \left\{ (1, m_1, m_3) \middle| \begin{array}{c} x_1 \le m_1 \le x_3 \\ Max(x_2(m_1 - x_1) + m_1 x_1, x_3(m_1 - x_2) + m_1 x_2) \le m_2 \le x_3(m_1 - x_1) + m_1 x_1 \end{array} \right\}$$

To verify this, consider the set of 2 simultaneous equations:

$$\begin{aligned} m_1 &= \omega_1 x_1 + \omega_2 x_2 + (1 - \omega_1 - \omega_2) x_3 \\ m_2 &= \omega_1 x_1^2 + \omega_2 x_2^2 + (1 - \omega_1 - \omega_2) x_3^2 \end{aligned}$$

which may be rewritten as:

$$\begin{aligned} x_3 - m_1 &= \omega_1 (x_3 - x_1) + \omega_2 (x_3 - x_2) \\ x_3^2 - m_2 &= \omega_1 (x_3^2 - x_1^2) + \omega_2 (x_3^2 - x_2^2) \end{aligned}$$

Considering ω_1, ω_2 as unknowns, we know from the above that there is a unique solution to these equations. In fact, we let the reader verify that the solution is:

$$\omega_1 = \frac{m_2 - x_3^2 + (x_3 - m_1)(x_3 + x_2)}{(x_3 - x_1)(x_2 - x_1)}, \quad \omega_2 = \frac{x_3^2 - m_2 - (x_3 - m_1)(x_3 + x_1)}{(x_3 - x_2)(x_2 - x_1)}$$

Note too that

$$\omega_1 + \omega_2 = \frac{x_3^2 - m_2 - (x_3 - m_1)(x_2 + x_1)}{(x_3 - x_2)(x_3 - x_1)}$$

Considering ω_1, ω_2 as weights, we see that they define a probability density with moment vector $(1, m_1, m_2) \in M_2(X)$ exactly when $\omega_1 \geq 0, \omega_2 \geq 0$ and $\omega_1 + \omega_1 \leq 1$. Now the reader can easily check that:

$$\begin{aligned} x_2(m_1 - x_1) + xx_1 \leq m_2 \quad &\Leftrightarrow \quad \omega_1 + \omega_1 \leq 1 \\ x_3(m_1 - x_2) + xx_2 \leq m_2 \quad &\Leftrightarrow \quad \omega_1 \geq 0 \\ m_2 \leq x_3(m_1 - x_1) + m_1 x_1 \quad &\Leftrightarrow \quad \omega_2 \geq 0 \end{aligned}$$

from which our claim follows. There remains:

Sub-case $k = 2, n \geq 4$,

$$M_2(X) = \{(1, m_1, m_2) \mid x_1 \leq m_1 \leq x_n, x_n(m_1 - x_{n-1}) + m_1 x_{n-1} \leq m_2 \leq x_n(m_1 - x_1) + m_1 x_1\}.$$

To prove this, let $(1, m_1, m_2) \in M_2(X)$ and let $\omega(x_i) = \omega_i$ be the corresponding probability weight. As before, we consider the set of two simultaneous equations:

$$\begin{aligned} x_n - m_1 &= \omega_1(x_n - x_1) + \omega_2(x_n - x_2) + \ldots + \omega_{n-1}(x_n - x_{n-1}) \\ x_n^2 - m_2 &= \omega_1(x_n^2 - x_1^2) + \omega_2(x_n^2 - x_2^2) + \ldots + \omega_{n-1}(x_n^2 - x_{n-1}^2). \end{aligned}$$

Eliminating the "unknown" ω_{n-1} gives:

$$\begin{aligned} x_n^2 - m_2 - (x_n + x_{n-1})(x_n - m_1) = \omega_1 &(x_n^2 - x_1^2 - (x_n + x_{n-1})(x_n - x_1)) \\ + \omega_2 &(x_n^2 - x_2^2 - (x_n + x_{n-1})(x_n - x_2)) \\ &\vdots \\ + \omega_{n-2} &(x_n^2 - x_{n-2}^2 - (x_n + x_{n-1})(x_n - x_{n-2})) \end{aligned}$$

This can be rewritten as

$$m_2 = x_n(m_1 - x_{n-1}) + m_1 x_{n-1} \quad + \omega_1(x_{n-1} - x_1)(x_n - x_1)$$
$$+ \omega_2(x_{n-1} - x_2)(x_n - x_2)$$
$$\vdots$$
$$+ \omega_{n-2}(x_{n-1} - x_{n-2})(x_n - x_{n-2})$$

Since the probability weights $\omega_i \geq 0$, this clearly implies that:

$$m_2 \geq x_n(m_1 - x_{n-1}) + m_1 x_{n-1}$$

Observe too that

$$(x_{n-1} - x_1)(x_n - x_1) > (x_{n-1} - x_2)(x_n - x_2) > \cdots > (x_{n-1} - x_{n-2})(x_n - x_{n-2}) > 0.$$

It follows that m_2 is maximized by assigning as much weight as possible to x_1, i.e. by making ω_1 as big as possible. Now, for fixed weighted mean m_1, the minimum x_1 gets maximum weight when it is required to offset all by itself the maximum x_n. Note that in that event:

$$m_1 = \hat{\omega}_1 x_1 + (1 - \hat{\omega}_1) x_n \quad \Rightarrow \hat{\omega}_1 = \frac{x_n - m_1}{x_n - x_1}$$

From this, we see that:

$$\begin{aligned} m_2 &\leq x_n(m_1 - x_{n-1}) + m_1 x_{n-1} + \hat{\omega}_1(x_{n-1} - x_1)(x_n - x_1) \\ &= x_n(m_1 - x_{n-1}) + m_1 x_{n-1} + (x_{n-1} - x_1)(x_n - m_1) \\ &= x_n(m_1 - x_1) + m_1 x_1 \end{aligned}$$

And we have shown that

$$M_2(X) \subseteq \{(1, m_1, m_2) \mid x_1 \leq m_1 \leq x_n,\ x_n(m_1 - x_{n-1}) + m_1 x_{n-1} \leq m_2 \leq x_n(m_1 - x_1) + m_1 x_1\}.$$

Conversely, let $(1, m_1, m_2)$ belong to the right hand side and let

$$z_1 = x_1 < z_2 = x_{n-1} < z_3 = x_n.$$

Then we find that

$$\begin{aligned} z_2(m_1 - z_1) + m_1 z_1 &= x_{n-1}(m_1 - x_1) + m_1 x_1 \\ &= x_1(m_1 - x_{n-1}) + m_1 x_{n-1} \le x_n(m_1 - x_{n-1}) + m_1 x_{n-1} = z_3(m_1 - z_2) + m_1 z_2 \\ &\le m_2 \\ &\le x_n(m_1 - x_1) + m_1 x_1 = z_3(m_1 - z_1) + m_1 z_1. \end{aligned}$$

It then follows from the case $n = 3$ that $(1, m_1, m_2) \in M_2(X)$ whence:

$$\{(1, m_1, m_2) \mid x_1 \le m_1 \le x_n,\ x_n(m_1 - x_{n-1}) + m_1 x_{n-1} \le m_2 \le x_n(m_1 - x_1) + m_1 x_1\} \subseteq M_2(X)$$

and the proof is complete for the sub-case $k = 2, n \ge 4$.

That argument readily extends to:

Case $3 \le k \le n$:

$$M_k(X) \subseteq \left\{ (1, m_1, \ldots, m_k) \left| \begin{array}{c} x_1 \le m_1 \le x_n \\ x_n^j - (x_n - m_1)\left(\dfrac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}}\right) \le m_j \le x_n^j - (x_n - m_1)\left(\dfrac{x_n^j - x_1^j}{x_n - x_1}\right),\ 2 \le j \le k \end{array} \right. \right\}$$

To prove this, again let $(1, m_1, m_2, \ldots, m_k) \in M_k(X)$ and $\omega(x_i) = \omega_i$ be the corresponding probability weight. We have a set of k simultaneous equations:

$$\begin{aligned} x_n - m_1 &= \omega_1(x_n - x_1) + \omega_2(x_n - x_2) + \ldots + \omega_{n-1}(x_n - x_{n-1}) \\ x_n^2 - m_2 &= \omega_1(x_n^2 - x_1^2) + \omega_2(x_n^2 - x_2^2) + \ldots + \omega_{n-1}(x_n^2 - x_{n-1}^2) \\ &\vdots \\ x_n^k - m_k &= \omega_1(x_n^k - x^k) + \omega_2(x_n^k - x_2^k) + \ldots + \omega_{n-1}(x_n^k - x_{n-1}^k) \end{aligned}$$

Now fix $j, 1 \le j \le n$ and define the function

$$f(x) = x_n^j - x^j - (x_n - x)\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}}.$$

Letting $\rho = \frac{x_n}{x_{n-1}} > 1$, we see that for $x_1 < x < x_{n-1}$:

$$\frac{df}{dx} = -jx^{j-1} + \frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}} = -jx^{j-1} + \frac{x_{n-1}^j}{x_{n-1}}\frac{\rho^j - 1}{\rho - 1} = \left(1 + \rho + \ldots + \rho^{j-1}\right)x_{n-1}^{j-1} - jx^{j-1} > 0.$$

And so $f(x)$ is an increasing function on (x_1, x_{n-1}). Since $f(x_{n-1}) = 0$, we see that

$$0 < -f(x_{n-2}) < -f(x_{n-3}) < \ldots < -f(x_2) < -f(x_1).$$

Eliminating the "unknown" ω_{n-1} between the two equations involving m_1 and m_j gives:

$$x_n^j - m_j - (x_n - m_1)\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}} = \sum_{i=1}^{n-2} \omega_i f(x_i)$$

$$m_j = x_n^j - (x_n - m_1)\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}} + \sum_{i=1}^{n-2}\left(-f(x_i)\right)\omega_i$$

This clearly implies that

$$x_n^j - (x_n - m_1)\left(\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}}\right) \le m_j.$$

It follows, as before, that for any fixed m_1, m_j is maximized by assigning as much weight as possible to x_1, i.e. by making ω_1 as big as possible. And again, for fixed weighted mean m_1, the minimum x_1 gets maximum weight when it is required to offset all by itself the maximum x_n. Recall that:

$$m_1 = \hat{\omega}_1 x_1 + (1 - \hat{\omega}_1)x_n \quad \Rightarrow \hat{\omega}_1 = \frac{x_n - m_1}{x_n - x_1}$$

and from this, we see that:

$$m_j \le x_n^j - (x_n - m_1)\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}} - \hat{\omega}_1 f(x_1)$$

$$= x_n^j - (x_n - m_1)\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}} - \left(\frac{x_n - m_1}{x_n - x_1}\right)\left(x_n^j - x_1^j - (x_n - x_1)\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}}\right)$$

$$= x_n^j - \left(\frac{x_n - m_1}{x_n - x_1}\right)\left(x_n^j - x_1^j\right) = x_n^j - (x_n - m_1)\left(\frac{x_n^j - x_1^j}{x_n - x_1}\right)$$

We have shown that

$$M_k(X) \subseteq \left\{ (1, m_1, \ldots, m_k) \left| \begin{array}{c} x_1 \le m_1 \le x_n \\ x_n^j - (x_n - m_1)\left(\frac{x_n^j - x_{n-1}^j}{x_n - x_{n-1}}\right) \le m_j \le x_n^j - (x_n - m_1)\left(\frac{x_n^j - x_1^j}{x_n - x_1}\right) 2 \le j \le k \end{array} \right. \right\}$$

for $3 \le k \le n$, as required.

The point of this discussion, as regards using weights on a set X to fit pre-assigned moments, is that the number of elements of the set X limits the number of moments and the minimum and maximum values of the set X determines the allowable range of the moments. In particular, it may be advisable to arrange for X to encompass outliers, even at the expense of X being representative of claims experience, especially since it will be reweighted anyway and by design such outliers do not "adversely" impact the mean.

Section III: Finding the Weight

On the other hand, now suppose that (X, ω) was built to be representative of the kind of claims we are investigating and so we want to stay as "close as possible" to weight ω, in some sense.

Define the subset

$$\mathsf{P} = \left\{ (\omega_1, \omega_2, \cdots, \omega_n) \mid 0 \le \omega_i, \sum_{i=1}^{n} \omega_i = 1 \right\} \subset \mathsf{R}^n$$

that corresponds to probability weight functions. Note that this subset is closed, convex, and compact. Consider the $(k+1)\times n$ matrix (of maximum rank):

$$V_k = V_k(X) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{\,k} & x_2^{\,k} & \cdots & x_n^{\,k} \end{bmatrix}$$

Suppose we are given a vector $\hat{m} = (1, \hat{m}_1, ...\hat{m}_k)$ presumably derived from empirical data, and we are assured (or we refer to a characterization of covered moment vectors, as above, and augment the range of X if necessary) that the solution set

$$\mathsf{S} = \mathsf{P} \cap \left\{ \upsilon \in \mathsf{R}^n \mid V_k \upsilon^T = \hat{m}^T \right\}$$

is not empty—in fact it is convex and compact. Since the norm function is continuous, it then follows that there is some $\upsilon_0 \in \mathsf{S}$ such that

$$\|\omega - \upsilon_0\| = Min \left\{ \|\omega - \upsilon\| \mid \upsilon \in \mathsf{S} \right\}$$

making $\upsilon_0 \in \mathsf{S}$ in some sense an optimal reweighting of X, inasmuch as it fits the required moments while staying as close as possible to the original weight ω. We have verified that there exists a well-defined "best" solution, not necessarily unique, to the task of reweighting (X, ω) to fit a given set of moments.

Even though the set S is convex and compact and fairly well described, in general it is no picnic finding $\upsilon_0 \in \mathsf{S}$ that minimizes the distance to a point. We conclude this paper with two simpler approaches that, while lacking in theoretical appeal, are simple to implement.

Approach 1: Suppose, as above, it remains a priority to use a weight as near as practical with the original weight ω but we are only concerned with fitting the weighted mean to a given value $\hat{m}$, which we assume satisfies $x_1 < \hat{m} < x_n$. Consider the piecewise linear function:

$$f(\lambda,t) = Max(0, 2\lambda t + 1 - \lambda) \quad \lambda \in \mathbb{R}, t \in [0,1]$$

Notice the following limits:

$$\lim_{\lambda \to +\infty} f(\lambda,t) = \begin{cases} 0 & t \in [0,1) \\ +\infty & t = 1 \end{cases}$$

$$\lim_{\lambda \to -\infty} f(\lambda,t) = \begin{cases} 0 & t \in (0,1] \\ +\infty & t = 0 \end{cases}.$$

Consider the 1-parameter family of weights:

$$\omega_\lambda(x_i) = f\left(\lambda, \frac{x_i - x_1}{x_n - x_1}\right)\omega(x_i).$$

Note that $\omega_0 = \omega$. Define $g(\lambda) = \mu_1(X, \omega_\lambda)$. The reader can verify that g is a continuous, increasing function of λ with:

$$\lim_{\lambda \to -\infty} g(\lambda) = x_1 \quad \lim_{\lambda \to +\infty} g(\lambda) = x_n$$

It follows that there is a unique number λ_0 with $g(\lambda_0) = \hat{m}$. We remark that λ_0 can be readily found in practice with the use of a binary search algorithm. The weight $\upsilon = \frac{\omega_{\lambda_0} |X|_\omega}{|X|_{\omega_{\lambda_0}}}$ on X has the same total weight as the original weight ω with $\mu_1(X,\upsilon) = \mu_1(X,\omega_{\lambda_0}) = g(\lambda_0) = \hat{m}$.

Approach 2: Suppose, we are concerned with fitting both the weighted mean and variance, but it is not a priority to use a weight near the original weight ω. Suppose we are given a target mean $\hat{m}$ and variance $\hat{s}^2$. This approach exploits the Beta density, and we use the notation of [1]. We let $x_0 = 0 \le x_1$ and $L = x_n$. As in [1], the probability density function $g(z)$ of the two-parameter Beta density of mean $\hat{m}$ and variance $\hat{s}^2$ on the interval $(0, L)$ can be determined as:

$$c = \frac{\hat{s}}{\hat{m}} \qquad \alpha = \frac{L - \hat{m} - c^2\hat{m}}{Lc^2} > 0 \qquad \beta = \left(\frac{L - \hat{m}}{\hat{m}}\right)\alpha > 0$$

$$g(z) = g(\alpha, \beta; z) = \frac{z^{\alpha-1}(L-z)^{\beta-1}}{\mathrm{B}(\alpha,\beta)L^{\alpha+\beta+1}} \quad z \in (0, L).$$

Then define:

$$\upsilon(x_j) = \int_{x_{j-1}}^{x_j} g(z)dz \quad 1 \le j \le n.$$

We have:

$$|X|_\upsilon = \sum_{j=1}^{n} \upsilon(x_j) = \sum_{j=1}^{n} \int_{x_{j-1}}^{x_j} g(z)dz = \int_0^L g(z)dz = 1$$

and so υ is a probability weight on X. We also have:

$$\begin{aligned}\mu_k = \mu_k(X,\upsilon) = \frac{\sum_{x \in X} \upsilon(x)x^k}{|X|_\upsilon} &= \sum_{j=1}^{n} x_j{}^k \int_{x_{j-1}}^{x_j} g(z)dz = \sum_{j=1}^{n} \int_{x_{j-1}}^{x_j} x_j{}^k g(z)dz \\ &> \sum_{j=1}^{n} \int_{x_{j-1}}^{x_j} z^k g(z)dz = \int_0^L z^k g(z)dz \\ &= \begin{cases} \hat{m} & k = 1 \\ \hat{m}^2 + \hat{s}^2 & k = 2 \end{cases}\end{aligned}$$

Which indicates that while the weighted moments are greater, in most cases they should reasonably well approximate their target.

References:

1. Corro, Dan, *Fitting Beta Densities to Loss Data,* CAS Forum, Summer 2002.

Completing and Using Schedule P

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Completing and Using Schedule P

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Abstract

Schedule P is a complex section of the Annual Statement, demanding much expertise to complete and to understand. The cross checks performed by the NAIC compare the Schedule P figures within its various parts, with other pages of the Annual Statement, and with Schedule P data from the preceding year. The NAIC uses Schedule P Summary data for three of the Insurance Regulatory Information System ("IRIS") tests, and it uses the detailed line of business data to determine the reserving risk and the written premium risk charges in the risk-based capital formula. Investment analysts and rating agencies use the schedule to measure the adequacy of a company's held reserves and thereby estimate its financial strength and expected market value. The IRS uses the schedule to determine loss reserve discounts, anticipated salvage and subrogation, and the discounts for anticipated salvage and subrogation. Actuaries and accountants need a thorough understanding of this Schedule, both to complete it for their own company or client and to evaluate the performance of peer companies.

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Preface

by Richard J. Roth, Jr.[1]

Few people probably remember what Schedules O and P were like in the 1980s when they contained little more than loss and loss adjustment expense development. The insurance department regulators needed more detailed information by line in order to monitor the solvency of the insurance companies. The information in the Annual Statement is the only information that the regulators have between the on-site financial examinations. Furthermore, the investment community, the rating agencies, agents, and the insurance industry observers wanted more financial disclosure. In the 1980s, the personal computer was coming into common use as a powerful analytical tool.

In the middle 1980's, I decided to make a proposal to combine Schedules O and P into a completely redesigned Schedule P. The intent was to include all of the basic actuarial statistics necessary to make a wide variety of actuarial analyses using the personal computer. There would be no analyses or projections in Schedule P, only the data to make the analyses and projections in a form that could be readily used in a personal computer.

Today, it is difficult to imagine how much opposition I faced, which came mainly from the larger insurance companies. The larger insurance companies did not want any more disclosure in the Annual Statement. They argued that their businesses were so complicated that the additional Schedule P information would be "meaningless." We regulators had to keep reminding the larger companies that the insurance world is really made up of hundreds of small and medium size companies, which require constant monitoring.

Only after the third major effort before the NAIC Blanks Committee was I able to get Schedule P substantially changed, even though I had widespread support among actuaries. Additional features in Schedule P were added in subsequent years, such as Parts 5, 6 and 7.

Even today, any changes to Schedule P in terms of additional reporting usually meet with fierce opposition from the larger insurers and reinsurers.

Schedule P is the actuarial portion of the Annual Statement and is critical to monitoring the solvency of insurers. The Casualty Actuarial (technical) Task Force of the NAIC is charged with maintaining and preserving Schedule P. Only small changes are likely to be made in the future, to reflect changes in the industry or risk based capital. The main concern in the future will be to prevent the NAIC Blanks Committee from weakening Schedule P by eliminating

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[1] Richard J. Roth, Jr., former Chief Casualty Actuary of the California Insurance Department and Chair of the NAIC Casualty Actuarial Task Force, was the architect of the new Schedule P in 1989.

information. Certain large insurers and reinsurers would argue that the NAIC financial reporting should be "modernized" or "simplified" or "deregulated" and that "unnecessary and wasteful reporting should be eliminated." These are code words for attempts to eliminate information on reinsurance transactions and claim counts.

As this text by Sholom Feldblum so clearly shows, there is a wealth of information in Schedule P, but most of the information could be easily lost if the NAIC and the actuarial profession are not constantly vigilant to attacks to reduce what has been fought so hard to obtain. This is the constant challenge.

Each year, the information in Schedule P is in wide demand. The NAIC, rating agencies and private companies distribute Schedule P data by CD's shortly after the Annual Statements are filed. This information is used by a wide range of users, including rating agencies, stock analysts in New York, competing companies, and, of course, regulators. Consulting actuaries have developed software programs for sale that will produce analyses of the Schedule P data.

I wish to thank my friend and fellow actuary, Sholom Feldblum, for the extraordinary job that he has done over the years in writing this text and in teaching how to use Schedule P. His efforts are now greatly helping the property/casualty insurance industry in the United States stand apart from the rest of the industries in terms of financial reporting.

– Richard J. Roth, Jr., June 2002

Introduction

MAJOR PURPOSES OF THE SCHEDULE

Property-casualty insurance is a highly regulated industry. Insurers exchange promises for premiums; they promise to indemnify losses that may not be settled until years after the policy premiums are collected.

If a manufacturing firm becomes insolvent, its owners and creditors lose. If an insurance company becomes insolvent, its customers – the policyholders – bear the brunt of the loss.

The public relies upon insurers to fulfill their promises, and state regulators are entrusted with safeguarding insurance solvency. Other industries, such as public utilities, may be regulated because they are not sufficiently competitive. Insurers are regulated (in part) because they are extremely competitive – and the rough and tumble of the marketplace may leave their promises unfulfilled.

For some industries, solvency regulation is an accounting task. Regulators audit the company's books to ensure that assets and liabilities are properly accounted for. For property-casualty insurance, solvency regulation is a highly specialized actuarial skill. Solvency risks may be unanticipated by the company, and they may be discerned only by trained analysis of the company's financial statements and historical loss experience. Schedule P is perhaps the most useful tool that regulators have to monitor company solvency and safeguard the public trust in the property-casualty insurance industry.

Schedule P is designed to measure loss and loss adjustment expense reserve adequacy, both retrospectively and prospectively. Schedule P displays historical triangles of losses, claims, and premiums, showing the observed development over the past ten years and facilitating the estimates of future development.

Part 1 of Schedule P provides a comprehensive view of the company's current loss reserve structure, including gross and ceded reserves by line of business and type of reserve (loss vs expense and case versus bulk) on an accident year basis.[2] Part 2 provides a retrospective test, by accident year and line of business, of reserves held in prior years. The totals from the

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[2] The term "loss" in this paper often signifies both loss and allocated loss adjustment expenses (identified as defense and cost containment expenses in Schedule P).

one year and two year adverse development exhibits, shown in the Part 2 Summary exhibit, are used for IRIS tests 9, 10, and 11, the NAIC retrospective and prospective early warning tests of reserve adequacy.

The historical exhibits in Parts 2 through 6 provide data for several prospective tests of loss reserve adequacy. Part 3 displays paid loss development triangles, and the difference between Parts 2 and 4 provides case incurred (or reported) loss development triangles. Average severities, both incurred and paid, may be derived from the claim count figures in Part 5 combined with the loss figures in Parts 2, 3, and 4.

Other Purposes

Schedule P has numerous other functions:

- It shows payments and reserves for losses and loss adjustment expenses by line and by accident year, thereby isolating blocks of business with good or poor experience. In addition, the accident year figures provided in Schedule P show the effects of changes in loss reserve margins on the calendar year results reported in the Underwriting and Investment Exhibit.
- It provides the loss payment patterns for the federal income tax loss reserve discounting procedure. In addition, it provides the disclosures needed for "grossing up" losses (i) for interest discounts and (ii) for anticipated salvage and subrogation.
- It provides the data for computing the reserving risk and the written premium risk charges in the risk-based capital (RBC) formula, thereby setting the insurer's capital requirements. It also provides the loss payment patterns for the investment income offsets in the formula.
- It shows the percentage of premiums and losses associated with loss-sensitive contracts for the loss-sensitive contract offset in the risk-based capital formula, and it shows the sensitivity of premiums and of reinsurance commissions to losses on these contracts.
- It separates occurrence from claims-made experience for three lines of business, as needed for the risk-based capital claims-made offset.
- It supports the opinion of the Appointed Actuary on loss and loss adjustment expense reserve adequacy.
- It shows the development of exposure year premiums resulting from audits and retrospective adjustments, allowing a more accurate comparison of loss ratios by accident year. This development is also needed to determine the tax basis earned premium for lines with audits or retrospective adjustments.
- It shows direct plus assumed versus ceded experience, so that the effects of reinsurance transactions on accident year loss ratios can be examined.
- It shows claim count development patterns and changes in average claim severity by year, allowing better analysis of claims department performance.

Schedule P is not limited to solvency regulation and tax filings. It is used extensively by actuaries and financial analysts to estimate a company's net worth. For an experienced reserving actuary, Schedule P provides more information than the SEC's Form 10K.

Schedule P was revised extensively for the 1989 Annual Statement, with further modifications in subsequent years. This paper explains what data are required for the schedule, and how the exhibits should be completed. It describes how to use Schedule P data for prospective analyses of loss reserve adequacy, using both paid and incurred loss development procedures. It discusses the use of Schedule P information for other reporting requirements, such as the risk-based capital formula, the Statement of Actuarial Opinion, and the IRS loss reserve discounting procedure.

Historical Experience

Schedule P shows experience for all lines of business, though the grouping of lines differs from the grouping in the Underwriting and Investment Exhibit. The long-tailed lines show the 10 most recent accident years of data plus a prior years row. These lines are primarily casualty lines or lines that have a significant casualty component.

A. Homeowners/Farmowners
B. Private Passenger Auto Liability/Medical
C. Commercial Auto/Truck Liability/Medical
D. Workers' Compensation
E. Commercial Multiple Peril
f. Medical Malpractice (occurrence policies in section 1 and claims-made policies in section 2)
G. Special Liability (Ocean Marine, Aircraft [All Perils], Boiler and Machinery)
H. Other Liability (occurrence in section 1 and claims-made in section 2)[3]
M. International
R. Products Liability (occurrence in section 1 and claims-made in section 2)

The short-tailed lines show the two most recent accident years of data plus a prior years row. These are primarily first party property lines of business.

I. Special Property (Fire, Allied Lines, Inland Marine, Earthquake, Glass, Burglary & Theft)
J. Auto Physical Damage
K. Fidelity / Surety

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[3] According to the Annual Statement *Instructions*, "Business reported on the Aggregate write-ins for other lines of business of the Underwriting and Investment Exhibit and the State Page should be included in the Other Liability sections of Schedule P." This seems strange; perhaps the intention is to include the aggregate write-in lines under the "other" exhibits, not the "Other Liability" exhibits.

L. Other (Including Credit, Accident and Health)
S. Financial Guaranty / Mortgage Guaranty

The data reported in the prior years row differ among the sections of Schedule P, as explained below.

REINSURANCE EXPERIENCE

Proportional reinsurance, or pro-rata reinsurance (quota share and surplus share), is shown as assumed or ceded premiums, losses, and loss adjustment expenses in the exhibits for the primary lines of business. A 50% quota share treaty for personal automobile liability business is reflected in the assumed and ceded columns of Parts 1B, 2B, etc.

Assumed non-proportional reinsurance, or excess-of-loss reinsurance, is split into three categories and shown separately from the primary lines of business: non-proportional property, non-proportional casualty, and financial lines (exhibit categories N, O, and P).[4] These reinsurance lines use the 10-year casualty format.

Ceded non-proportional reinsurance is reported in the same exhibit as the underlying business. A primary company which cedes part of its workers' compensation business on an excess-of-loss treaty records the experience in the ceded columns of Part 1D, and the reinsurer who assumes the business includes it in Part 1O (Part "one-oh," not Part "ten"). A reinsurance company which retrocedes part of its workers' compensation business that it assumed on a non-proportional treaty shows the retrocession in Part 1O as well.

If a reinsurance treaty contains both proportional and non-proportional sections, the premiums and losses for the sections must be divided and reported on the appropriate lines: the proportional parts for both the ceding company and the assuming company in the exhibits for the underlying lines of business, the non-proportional parts for the ceding company in the exhibits for the underlying lines of business, and the non-proportional parts for the assuming company in the exhibits for the underlying lines of business. This is analogous to the treatment of reinsurance treaties that are prospective with regard to some claims and retroactive with regard to other claims: the premiums and losses for the two sets of claims must each be treated according to their appropriate statutory rules.[5]

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4 Lines of business with both property and casualty components, such as homeowners, commercial multiple peril, and aircraft, are included in Reinsurance B (liability reinsurance). Financial reinsurance includes reinsurance on fidelity and surely contracts. The Annual Statement *Instructions* list the elements of each reinsurance line in more detail.

5 See SFAS 113, paragraph 25: "When practicable, prospective and retroactive provisions included within a single contract shall be accounted for separately"; SSAP 62, paragraph 24: "Prospective and retroactive provisions included within a single agreement shall be accounted for separately." For summaries of GAAP and statutory accounting, see Yoheved and Sarason [2002].

Intercompany pooling agreements are reported differently; see the full discussion later in this paper.

Prospective vs Retroactive Reinsurance

Only prospective reinsurance affects the Schedule P figures. Retroactive reinsurance is not reflected in the Schedule P exhibits.

Retroactive reinsurance is defined in SSAP 62, paragraph 21, as "reinsurance in which a reinsurer agrees to reimburse a ceding entity for liabilities incurred as a result of past insurable events covered under contracts subject to the reinsurance." The NAIC *Instructions* to the Statement of Actuarial Opinion (section 11) provide a three-fold definition:

> *For the purpose of this instruction, "retroactive reinsurance" refers to any agreement which increases the transferring insurer's Surplus to Policyholders as a result of the transferee undertaking any loss obligation already incurred and for which the consideration paid by the transferring insurer is derived from present value or discounting concepts.*

Retroactive reinsurance affects the special surplus entry on the liability side of the statutory balance sheet (page 3 of the Annual Statement), but it is not reflected in the Annual Statement exhibits and schedules, such as Schedule P.[6] It affects statutory income in the same fashion as prospective reinsurance does, except that it is coded under "other income" on the statutory statement of earnings (SSAP 62, paragraph 28i). It has a full effect on policyholders' surplus, though not on the unassigned portion of surplus. It affects GAAP income, GAAP equity, and taxable income.

Reinsurance and Risk-Based Capital

Risk-based capital adjusted surplus includes special surplus funds. The adjusted surplus used to compute the risk-based capital ratio does not depend on whether the reinsurance is classified as prospective or retroactive.

The risk-based capital *ratio* is slightly reduced if the reinsurance is coded as retroactive instead of prospective. The RBC ratio equals adjusted surplus divided by the risk-based capital requirements. The RBC reserving risk charge is greater than the charge for

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[6] SSAP No. 62, "Reinsurance," paragraph 28, says with regard to retroactive reinsurance agreements:

1. The ceding entity shall record, without recognition of the retroactive reinsurance, loss and loss expense reserves on a gross basis on the balance sheet and in all schedules and exhibits.
2. The assuming entity shall exclude the retroactive reinsurance from loss and loss expense reserves and from all schedules and exhibits.

reinsurance recoverables, particularly after the covariance adjustment. Prospective reinsurance reduces risk-based capital requirements and decreases the denominator of the risk-based capital ratio. Retroactive reinsurance does not have this effect.

Illustration: Companies A and B have the same initial surplus and capital requirements. Company A prospectively reinsures a book of general liability business. Company B retroactively reinsures an identical book of business. Companies A and B have the same ending surplus, though some of company B's surplus is coded as special surplus funds. Company B has more loss reserves shown on the balance sheet, in the Underwriting and Investment Exhibit, and in Schedule P than Company A has; company A has a write-in contra-liability for reinsurance recoverables which Company B does not have. Company B has greater RBC requirements than Company A has, since the reserving risk charge is greater than the credit risk charge (over 30% versus 10%) and the margin effect of the reserving risk charge is much greater than the marginal effect of the credit risk charge; see Feldblum [1996: RBC] for the RBC risk charges and Feldblum [2002: Comm] for estimating the effects of retroactive reinsurance on RBC requirements.

Reinsurance and Surplus Relief

The statutory treatment of retroactive reinsurance is more conservative than the GAAP treatment in that it does not allow a reduction of statement reserves. It is more liberal than the GAAP treatment in that it allows full "up-front" surplus relief, whereas GAAP recognizes the profit from retroactive reinsurance ratably over the lifetime of the claims.

Illustration: On December 31, 20XX, the ABC Insurance Company has $100 million of loss reserves which it retrospectively reinsures for $80 million. Both its policyholders' surplus and its GAAP equity are $200 million on that date. By December 31, 20XX+1, $25 million of the original loss reserves have been settled. It has no other underwriting or investment operations.

ABC's statutory financial statements show $20 million of other income on December 31, 20XX, $20 million of special surplus funds, and no change in unassigned surplus. During 20XX+1, ABC shows a $25 million reduction in loss reserves and a $25 million reduction in the write-in contra-liability for recoverable from retroactive reinsurance.

ABC's GAAP financial statements show no income on December 31, 20XX, and no change in GAAP equity. During 20XX+1, ABC shows income of $25/$100 × $20 million = $5 million as well as a $5 million increase in GAAP equity.

Summary Exhibits

The Summary exhibits show 10 accident years of data plus a prior years row for all lines of business combined. Ten accident years of data, as well as a prior years line, must be kept for all lines of business, since all 10 years are used for the Summary exhibits.[7]

Illustration: For the 20X9 Annual Statement, Schedule P, Part 1J, "Auto Physical Damage," shows two individual accident years, 20X8 and 20X9, along with a prior years row. For incorporation of the auto physical damage experience into the Schedule P, Part 1 Summary exhibit, the company must keep auto physical damage data for accident years 20X0 through 20X9, along with a prior years row suitable for the 10 year exhibits. The entries in the prior years row in the Part 1J exhibit do *not* equal the data for the prior years row used for the summary exhibit plus the data for accident years 20X0 through 20X7. Separate data must be kept.

IRIS tests 9 and 10, the one-year and two-year retrospective tests of reserve adequacy; are based on the Part 2 Summary exhibit. IRIS test 11, the prospective test of reserve adequacy, uses the one and two-year adverse developments from the Part 2 Summary exhibit as inputs (along with other data).

The Schedule P Exhibits

Part 1 shows cumulative experience by accident year at the Statement date. Most of the figures in Part 1 are audited by an independent CPA, and the Statement of Actuarial Opinion should reconcile to the data in Part 1.

Parts 2 through 6 show the supporting historical triangles. Parts 2 through 5 are cumulative accident year data. Part 6 is cumulative exposure year data, which is the premium equivalent of accident year losses and expenses. The Part 7 policy year exhibits are not intended to support the Part 1 information; see the discussion below.

For the individual years shown on the exhibits, Part 1 shows calendar year premiums that are not changed for subsequent earned but unbilled premiums or accrued retrospective premiums. The losses and expenses are cumulative accident year figures.

Illustration: In the 20X9 Schedule P, the 20X5 paid loss and expense figures in columns 4 through 10 represent payments from January 1, 20X5, through December 31, 20X9, for

[7] The Schedule P *Instructions* say: "Since the Summary of each part contains ten years of historical data, the information from the "prior" line for the Property Lines, Sections I through L, and Financial Guaranty/Mortgage Guaranty, Section S, must be supplemented for the eight accident years preceding the two most recent years."

accident year 20X5. The 20X5 unpaid loss and expense reserves in columns 13 through 23 are the reserves held on December 31, 20X9.

The treatment of losses and expenses is similar for Parts 2 through 5.

Prior Years Rows

The "prior years" row differs among the various Schedule P parts.

- No calendar year earned premiums are shown for the prior years row in Part 1; the cells are "XXX"ed out. The exposure year earned premiums in the Part 6 prior years row reflects the current calendar year contributions to the old exposure years.

- For the Part 1 prior years row, the loss and expense payments and the salvage and subrogation reimbursements are those made or received in the most recent calendar year only. This is not a cumulative amount. This is the same procedure as that used for the exposure year earned premiums in Part 6.

- For the Part 3 prior years row, the loss and expense payments are those made since January 1 of the *second* calendar year shown along the column headings. For the 20X9 Annual Statement, these are payments made since January 1, 20X1 (not January 1, 20X0.) The top-left corner cell is "XXX"ed out in these exhibits.

- The unpaid loss and expense reserves in the prior years rows are the reserves for old accident years evaluated at the current statement date for Part 1 and at each December 31 for Parts 2 and 4 and for outstanding claims in Part 5.

- The reported claim triangles and closed claim triangles in Part 5 use the Part 3 format, not the Part 1 or Part 6 format.

Data Types

Part 1 shows data separately for "direct and assumed" and for "ceded," so that the analyst may determine the effects of reinsurance recoverables on the company's experience. If the direct and assumed loss ratio is significantly higher than the net loss ratio, the business ceded may be unprofitable. The reinsurers may cancel treaties, raise reinsurance rates, or underwrite facultative business more carefully in future years.

- The *net* loss ratio is influenced by the reinsurance market at the current time: in soft reinsurance markets, the net loss ratio appears better than in hard markets.

- The direct and assumed loss ratio reflects the quality of the primary insurer's book of business, and it may be a good predictor of both the direct and net loss ratios in future years.[8]

Parts 2, 3, and 4 show historical loss triangles for *net* losses and "defense and cost containment" (DCC) expenses; there are no corresponding triangles for direct business.

Part 5 shows historical claim count triangles for direct and assumed business.

Part 6 shows historical development of direct and assumed exposure year earned premium (in section 2) and of ceded exposure year earned premium (in section 2).

Part 7 shows policy year earned premiums, losses, and reinsurance commissions for business written on loss-sensitive contracts. These policy year figures pertain to a portion of the company's business only; the data cannot be reconciled with other Annual Statement exhibits. Part 7 was designed for the loss-sensitive contract offset in the risk-based capital formula, and it need be completed only by companies seeking this offset.

The DCC expenses in Parts 1 through 4 are the current NAIC version of the old "allocated loss adjustment expenses" (ALAE). "Allocated loss adjustment expenses" is the standard insurance term for loss expenses associated with particular claims, such as legal defense costs and expert medical testimony. Before 1998, loss adjustment expenses in Schedule P were divided between allocated and unallocated. For the 1998 and subsequent Annual Statements, loss adjustment expenses were divided more rigorously between (i) defense and cost containment (DCC) and (ii) adjusting and other (AAO); see below. In general, DCC corresponds to ALAE and AAO corresponds to ULAE.

In theory, historical loss triangles for direct and assumed business can be formed by joining the Part 1 exhibits from successive years. The effort involved usually outweighs the benefits, and this analysis is not commonly performed. Changes in intercompany pooling agreements and discrepancies between the Schedule P exhibits of different years distort these analyses and further diminish their value.

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[8] Richard Roth, who designed the current Schedule P, writes [1986], page 86: "Surprisingly, very few companies – particularly small companies – have any idea how profitable or whether they are making money or whether the business being ceded is profitable or not profitable. Once they pay that reinsurance premium they don't care, it's just gone. . . . Well, what happens is if the business that is being ceded is consistently unprofitable, we know that two or three years down the line they're not going to have any reinsurance. Also, it says that the business that they're writing is probably underpriced and that they will soon have problems."

Part 1 – Current Valuation

PREMIUMS

Part 1 premiums are recorded by calendar year.[9] Once entered, they are "frozen," and they are not adjusted for subsequent earned but unbilled premiums stemming from exposure audits or accrued retrospective premiums stemming from retrospective premium adjustments.

> *Illustration:* An insurance company issues retrospectively rated workers' compensation policies. Worse than expected adverse development on a block of business raises the loss figures and the associated premium figures at subsequent valuations.
>
> - The additional losses are assigned to the appropriate accident years in Parts 1 – 4.
> - In Part 1, the additional premiums received are assigned to the current calendar year, not to the years when the policies were issued or the premium was earned.

Part 1 of Schedule P shows overstated loss ratios for the year when the losses occurred and understated loss ratios for the year in which the additional premiums are billed.[10]

The overstatement and understatement discussed above relates to over- and under-estimation of the retrospective premium adjustments. The initial estimate of the future retrospective premium adjustment *is* included in the calendar year earned premiums.

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9 The latest calendar year net earned premium shown in Schedule P, Part 1, column 3, row 11 (total for all accident years), for each line of business should equal the net earned premium shown on page 7, "Underwriting and Investment Exhibit," Part 2, "Premiums Earned," column 4. Premium figures from earlier years should agree with the figures in the preceding years' Annual Statements. If there is an intercompany pooling agreement that has changed over time, the comparison with earlier Annual Statements can be done only on a consolidated basis. See the discussion in the text on intercompany pooling.

10 Salzmann [1967], pages 120-121, notes that "calendar/accident year loss ratios are theoretically less accurate than policy year loss ratios," but she adds that "the primary purpose of Schedule P is to assist in the determination of adequate reserve levels — not the precise measurement of loss ratios." This is correct for the chain ladder loss reserving techniques, which do not rely on premium figures. It is less true for other reserving techniques, such as the Stanard-Bühlmann expected loss technique or the Brosius least squares technique; see the discussions of Parts 2 and 3 below.

Upon reviewing an earlier (pre-1996) draft of this paper, Richard Roth commented: "An acknowledged weakness of Schedule P is the mismatch between losses and premiums by year, especially for reinsurance and workers' compensation. Early drafts of Schedule P addressed this problem; however, the problem is not that easy to solve. It is not enough just to add a column for policy year premiums. Whole triangles of premiums must be reported." These triangles are now shown in Part 6 of Schedule P.

More accurate "exposure/accident year" loss ratios and loss ratio development can be obtained by combining the information in Parts 2 and 6. The illustrations in the discussion below of Part 6 show the difference between the initial estimates of future retrospective adjustments and subsequent revisions of these estimates.

In Part 1 of Schedule P, the prior years row shows payments made or received in the current year, or reserves held on open cases as of the statement date. No figures are shown for premiums on the prior years row, since current calendar year adjustments do not affect previous calendar year premiums.

LOSS AND LOSS EXPENSE PAYMENTS

Columns 4 through 11 show loss and loss expense payments by accident year. For the individual accident years, these are cumulative payments. For accident year 20XX, column 4 shows loss payments on direct and assumed business from January 1, 20XX through the statement date.

For the prior years row, the payments are those made in the current calendar year only. For the 20XX Annual Statement, these are the payments made from January 1, 20XX through December 31, 20XX.

Salvage and Subrogation Received

Column 4 (direct and assumed loss payments) and column 5 (ceded loss payments) are *net* of salvage and subrogation received.

- *Salvage:* The insurer settles an automobile physical damage claim by paying the $10,000 blue book value of the car. It sells the damaged car to a repair shop for $2,000. The company shows $8,000 as the loss paid in column 4 and $2,000 as the salvage received in column 10.
- *Subrogation:* The insurer settles an automobile physical damage claim by paying the $10,000 blue book value of the car. The driver of the other vehicle is negligent and liable for the damages. The company collects the full $10,000 from the driver of the other vehicle or the driver's insurer. The company shows $0 as the loss paid in column 4 and $10,000 as the subrogation received in column 10.

Column 10, salvage and subrogation received, is for information only (termed a "memorandum" column in the Annual Statement *Instructions*); it is not used to calculate subsequent columns. Column 11, the total net paid column, equals columns 4 – 5 + 6 – 7 + 8 – 9; it does not involve column 10.

Salvage and subrogation is most material for automobile physical damage (Part 1J). Some companies show significant amounts of subrogation for automobile liability (Part 1B) and workers' compensation (Part 1D) as well.

Illustration A: The insurer makes a $40,000 personal injury protection [PIP] loss payment to its own insured injured in an auto accident in a no-fault state. The driver of the other vehicle was negligent and the damages exceed the tort threshold. The company collects $25,000 by subrogation from the negligent driver or the negligent driver's auto insurer. The net loss payment in Part 1B, personal auto liability/no-fault, is $15,000; the subrogation is $25,000.

Illustration B: The insurer makes a $40,000 workers' compensation loss payment to its own insured injured in an auto accident stemming from a work-related accident. The driver of the other vehicle was negligent and is liable for the damages. The company collects $25,000 by subrogation from the negligent driver or the negligent driver's auto insurer. The net loss payment in Part 1B, personal auto liability/no-fault, is $15,000; the subrogation is $25,000.

Calendar Year Reconciliation

Schedule P, Part 1, shows cumulative paid losses by accident year. The Underwriting and Investment Exhibit, Part 3, shows paid losses in the most recent calendar year. The Annual Statement cross-checks determine the calendar year paid losses from figures in the current Schedule P and that of the previous year, and they compare these figures with those in the Underwriting and Investment Exhibit.[11]

Illustration: The reconciliation for the 20X9 Annual Statement is as follows.

a. In the 20X9 Schedule P, Part 1, column (4) minus column (5), total row (row 12), shows cumulative net loss payments at December 31, 20X9, for accident years 20X0 through 20X9 plus the calendar year 20X9 loss payments for accident years prior to 20X0.
b. In the 20X8 Schedule P, Part 1, column (4) minus column (5), the sum of rows 3 through 11, shows cumulative net loss payments at December 31, 20X8, for accident years 20X0 through 20X8. We do not include the prior years row or the first individual accident year row (the year prior to 20X0).
c. The difference between (a) and (b) is the calendar year 20X9 loss payments.

The calendar year payments for loss plus defense and cost containment (DCC) expenses can be derived from Part 3 of the current year's Schedule P. Part 3 of the Underwriting and Investment Exhibit shows pure loss payments, without the DCC payments.

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[11] A cross-check reconciles entries in different exhibits of the Annual Statement or in Annual Statements of different years. These are computer cross-checks performed on the electronic submission; they are not done by pencil and paper.

For the accident year to which losses are assigned, see the discussion below of occurrence versus claims-made business.

LOSS ADJUSTMENT EXPENSES

Before 1998, loss adjustment expenses were divided between allocated loss adjustment expenses (ALAE) and unallocated loss adjustment expenses (ULAE).

- ALAE were adjustment expenses related to particular claims, such as legal defense fees paid to outside counsel.
- ULAE were adjustment expenses that were not related to individual claims, such as claims department rent, utilities, and similar overhead costs.

For pricing insurance policies, most companies include ALAE with losses, using data subdivided by accident year (or policy year), subline, state, and various other dimensions. ULAE is generally included as a loading on losses plus ALAE.

The expenses included in ALAE or ULAE differed somewhat by company. For instance, a company using outside legal counsel may include the defense costs with ALAE, whereas a company using in-house legal counsel may include the defense costs with ULAE. This presented no problems for individual company ratemaking, though it created difficulties for bureau ratemaking and for accounting supervision.

The problem was particularly severe for rating bureaus. If some companies code defense costs as ALAE because they use outside legal counsel and other companies code defense costs as ULAE because they use in-house legal counsel, the aggregate industry data compiled by the rating bureau contains a mixture of definitions and might not be appropriate for any of the companies.

In the 1990's, the rating bureaus, particularly the National Council on Compensation Insurance (NCCI), began standardizing the coding of ALAE vs ULAE. Expenses would be classified by type of expense to promote similar coding among companies. For instance, legal defense costs would be coded as ALAE, whether inside or outside counsel was used. Companies that used in-house legal counsel would allocate the salaries and overhead costs of their attorneys to individual claims.

DCC and AAO: Principles

Some companies were concerned that new NCCI classification rules might not be consistent with statutory accounting requirements, which still defined ALAE as loss adjustment expenses that were related to particular claims. In 1997, the NAIC Casualty Actuarial (Technical) Task Force (CATF) proposed new definitions of ALAE and ULAE that classified by type of expense. The new definitions were adopted by the NAIC for the 1998 and subsequent Annual

Statements. To avoid any confusion between the old and new definitions, revised terms were adopted as well (in 1999): defense and cost containment (DCC) for ALAE and adjusting and other (AAO) for ULAE.

Three principles govern the 1998 definitions of loss adjustment expenses:

2. The classification is by type of expense, regardless of whether the expense relates to specific claims.[12]
3. The classification is uniform for all companies. No discretion is permitted for the classification of loss adjustment expenses.[13]
4. The new definitions divide expenses into two groups: (i) expenses that vary with the amount of loss are coded as defense and cost containment and (ii) expenses that vary with the number of claims, or which do not vary with either the amount of loss or the number of claims, are coded as adjusting and other.[14]

The first two principles are determinative if they conflict with the third principle. If an expense is classified by the NAIC as defense and cost containment, the company does not have the option of coding the expense as adjusting and other, even if the company believes that the expense varies with the number of claims and not with the amount of loss.

Schedule P Interrogatory number 2 requires the company to acknowledge that it is using the new definitions:

> *2. The definition of allocated loss adjustment expenses (ALAE) and, therefore, unallocated loss adjustment expenses (ULAE), was changed effective 1/1/98. This change in definition applies to both paid and unpaid expenses. Are these expenses (now reported as "Defense and Cost Containment" and "Adjusting and Other") reported in compliance with these definitions in the statement?*

The expenses classified as defense and cost containment include legal defense fees, the costs of expert witnesses, and fees to professionals working in defense of a claim. The

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[12] The Annual Statement *Instructions* say that "it is the character of the expenses that is most important, not whether the expenses were internal or external to the insurer."

[13] The statutory accounting principles *Statement of Concepts*, paragraph 31, says: "The regulators' need for meaningful, comparable financial information to determine an insurer's financial condition requires consistency in the development and application of statutory accounting principles." The consistency principle was a dominant stimulus for the new definitions of DCC and AAO.

[14] The Annual Statement Instructions say: *The loss adjustment expenses are separated with the intent of identifying the "Defense and Cost Containment" expenses as those which are correlated with the loss amounts, and the "Adjusting and Other" as those expenses which are correlated with claim count or are general loss adjustment expenses.*

expenses classified as adjusting and other include adjustors' fees as well as fees to other professionals working as adjustors. General claim department overhead which can not be grouped into a DCC category, such as rent, is classified as adjusting and other.[15]

Illustration: A company uses in-house attorneys to handle the legal defense of routine claims. For statutory financial statements besides Schedule P, the salaries and other employee costs of these attorneys is coded as defense and cost containment, and classified by line of business and by calendar year. For Schedule P, these costs must be subdivided by accident year (in addition to line of business and calendar year). The legal department must allocate the salaries and other employee costs, including the related portions of legal department overhead, to the relevant claims.[16]

Declaratory Judgment Actions

The environmental impairment (pollution) liabilities facing the insurance industry are potentially great. The remediation of abandoned toxic waste sites is a major component of pollution liability costs, but the responsibility for these costs is disputed by insurance companies and

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[15] The formal definitions are as follows (SSAP No. 55, "Unpaid Claims, Losses, and Loss Adjustment Expenses," paragraph 5(c): "Defense and cost containment includes

1. Surveillance expenses;
2. Fixed amounts for cost containment expenses;
3. Litigation management expenses;
4. Lost adjustment expenses for participation in voluntary and involuntary market pools if reported by accident year;
5. Fees or salaries for appraisers, private investigators, reinspectors and fraud investigators, if working in defense of a claim, and fees or salaries for rehabilitation nurses, if such cost is not included in losses;
6. Attorney fees incurred owing to a duty to defend, even when other coverage does not exist; and
7. The cost of engaging experts.

Adjusting and other includes

1. Fees of adjusters and settling agents (but not if engaged in a contentious defense);
2. Loss adjustment expenses for participation in voluntary and involuntary market pools if reported by calendar year;
3. Attorney fees incurred in the determination of coverage, including litigation between the insurer and the policyholder; and
4. Fees or salaries for appraisers, private investigators, hearing representatives, reinspectors and fraud investigators, if working in the capacity of an adjuster."

[16] Allocation of legal department overhead costs to individual claims or accident years is explicitly required by the Annual Statement *Instructions*: "The fees charged for insurer employees should include overhead, just as an outside firm's charges would include." The company may *not* classify the salaries as DCC and the related employee expense costs as AAO.

their policyholders. The primary issue is whether the pre-1986 Commercial General Liability (CGL) policy provided coverage for these liabilities.

After the passage of the CERCLA legislation in 1980 by the Congress, insurers and their policyholders turned to the courts for declaratory judgment regarding the incidence of liability. The courts were asked to judge (to "declare") which party must pay the remediation costs. Most legal defense costs for pollution cases in the 1980's and early 1990's related to attorney fees for these declaratory judgment (DJ) actions.

Before codification, there were three views regarding the allocation of these attorney fees:

1. Insurance companies were paying these legal defense costs to absolve themselves of liability. Legal defense costs for both third party and first party claims are coded as ALAE. The same coding should be used for defense costs stemming from DJ actions.
2. The DJ costs are related not to the defense of claims but to the determination of coverage for specific types of claims. They are similar to other adjusting costs and should be coded as ULAE.
3. The DJ costs are not related to claims handling but to policy interpretation. They should be coded as general expenses, not as loss adjustment expenses.

Of these three types of expenses – ALAE, ULAE, and general expenses – only ALAE affects the one-year and two-year adverse loss reserves development tests (IRIS tests 9 and 10), since only ALAE is included in the Schedule P, Part 2, Summary exhibit. The declaratory judgment actions were unanticipated costs, and (for some companies) the adverse development was large.

Under the revised NAIC definitions of loss adjustment expenses, DJ legal fees are coded as adjusting and other, not as defense and cost containment. They do not affect the adverse loss development for IRIS tests 9 and 10.[17]

DISTRIBUTION OF ADJUSTING AND OTHER EXPENSES

Most defense and cost containment (DCC) expenses are related to specific claims and can be assigned to accident years. Adjusting and other expenses in columns 8 and 9 are claims department overhead and salaries; they are assigned to accident year by formula.

Schedule P contains columns both for direct and assumed and for ceded adjusting and other expenses. In practice, adjusting and similar claims department expenses are rarely ceded

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[17] The Annual Statement *Instructions* say that DCC expenses "exclude expenses incurred in the determination of coverage" (i.e., declaratory judgment action expenses). These expenses are explicitly included in AAO, which include "attorney fees incurred in the determination of coverage, including litigation between the insurer and the policyholder."

in reinsurance contracts, since they can not be easily measured and associated with individual losses, policies, policy years, or underwriting years.

The distinction between (i) direct + assumed and (ii) ceded applies only to accident years 1997 and subsequent. For accident years prior to 1997, the net adjusting and other expenses are shown in the direct + assumed column. There was only a single "net" column for unallocated loss adjustment expenses before 1997, and it would have been difficult for companies to restate the old experience between direct + assumed and ceded portions.

Until 1997, Schedule P had a mandated statutory formula for distributing ULAE to accident years. (ULAE was the precursor of the current adjusting and other expenses.) In 1997, the distribution rules were changed; there is no longer a set statutory procedure, but there is general guidance on the permitted procedures. Some companies are still using the old procedure, which remains permissible; other companies have switched to new methods. Both approaches are explained below.

Previous Statutory Procedure

The old statutory procedure, which governed the distribution of paid ULAE from calendar years before 1997, is still used by many companies. This approach was defined in the pre-1997 Schedule P Interrogatory #4 as follows:

> *The unallocated loss expense payments paid during the most recent calendar year should be distributed to the various years in which losses were incurred as follows: (1) 45 percent to the most recent year, (2) 5 percent to the next most recent year, and (3) the balance to all years, including the most recent, in proportion to the amount of loss payments paid for each year during the most recent calendar year. If the distribution in (1) or (2) produces an accumulated distribution to each year in excess of 10 percent of the premiums earned for such year, disregarding all distributions made under (3) such accumulated distribution should be limited to 10 percent of premiums earned and the balance distributed in accordance with (3).*

The assumptions underlying this procedure are

- Half of unallocated loss adjustment expenses are incurred when the claim is reported (costs of setting up files and initial investigations), and half are incurred when the claim is settled (costs of issuing checks and final negotiations).

- 90% of claims are reported during the year when the accident occurred, and 10% are reported the following year.

Unallocated expenses related to claim reporting are assigned to the two most recent accident years in a 9 to 1 proportion, and unallocated expenses related to claim settlement are allocated in proportion to loss payments.[18]

Illustration: Distribution of AAO Expenses

Suppose the company has the following 2005 experience for a line of business, all of whose claims are settled within five years:

Exhibit 1.1: Prior Method of Distributing Unallocated Loss Expenses by Accident Year ($000)

Cal/Acc Year	Earned Premium	Losses Paid in 2005	
2001	8,000	200	Calendar year 2005 unallocated
2002	8,500	500	loss adjustment expenses paid: 600
2003	9,000	800	
2004	9,000	2,000	
2005	9,500	2,500	

The calendar year 2005 unallocated loss adjustment expenses (now AAO expenses) are $600,000. 45% of $600,000, or $270,000, is allocated to 2005, and 5% of $600,000, or $30,000, is allocated to 2004. The remaining $300,000 is allocated in the same proportion as paid losses. Exhibit 1.2 shows the full distribution of unallocated loss adjustment expenses to accident year.

Exhibit 1.2: Prior Method of Distributing Unallocated Loss Expenses by Accident Year ($000)

Cal/Acc Year	Losses Paid in 1995	Paid Loss percentage	Unallocated Expense Distribution: Steps 1 & 2	Step 3	Total
2001	200	3%	0	10	10
2002	500	8	0	25	25
2003	800	13	0	40	40
2004	2,000	33	30	100	130
2005	2,500	42	270	125	395
Total:	6,000	100%	300	300	600

[18] This distribution also assumes that the dollar amount of closed claims equals the dollar amount of reported claims. See Kittel [1991] and Bill [1991] for the effects of exposure growth and inflation on the distribution of ULAE by accident year.

Many medical malpractice, products liability, professional liability, non-proportional reinsurance claims are not reported until years after the accident date, and insurers providing this coverage spend much time negotiating settlements and handling the claims. The old statutory distribution procedure assigned at least 45% of the calendar year unallocated loss adjustment expenses to the most recent accident year. This percentage is too high for lines of business with long reporting lags.

In addition, the old statutory procedure assumed that half of the ULAE was proportional to the amount of the loss settlement. Many components of ULAE, such as setting up claim files, are more closely related to the number of claims than to the size of the loss.

> *Illustration:* In the late 1990's and early 2000's, hundreds of thousands of asbestos claims have been filed. The associated AAO expenses are large. All of these claims relate to the prior years row in the products liability or other liability exhibits.

Revised Method

The old statutory procedure had long been recognized as arbitrary before the 1997 changes.[19] By the late 1980's, many companies were using more sophisticated ULAE reserving procedures, which associated claims department expenses more accurately with policy years or accident years. In 1989, W. Johnson published a reserving method that associated ULAE entirely with claim reporting and settlement patterns, not with loss payment patterns.

The third Schedule P Interrogatory now says:

> *The adjusting and other expense payments and reserves should be allocated to the years in which the losses were incurred based on the number of claims reported, closed and outstanding in those years. When allocating adjusting and other expense between companies in a group or a pool, the adjusting and other expenses should be allocated in the same percentage used for the loss amounts and the claim counts. For reinsurers, adjusting and other expense assumed should be reported according to the reinsurance contract. For adjusting and other expense incurred by reinsurers, or in those situations where suitable claim count information is not available, adjusting and other expense should be allocated by a reasonable method determined by the company and described in Interrogatory 7, below. Are they so reported in this Statement?*

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[19] Troxel and Breslin [1983], page 130, comment: ". . . the unpaid ULAE for a workers' compensation claim will probably be less than 50 percent since a large reserve is often established for related monthly payments which incur little ULAE." See also Salzmann [1967], page 125: "The present percentages used to distribute unallocated claims expense . . . in Schedule P are arbitrary. Industry studies might be undertaken to determine unallocated claims expense distributions by size of claim and by age of claim." For further explanation of the prior procedure, see Salzmann [1988], page 83.

The Interrogatory seems to mandate an allocation method. The Annual Statement Instructions clarify that the method alluded to is preferred but not mandatory:

> *The "Adjusting and Other" expenses can be assigned in any justifiable way among the accident years. The preferred way is to apportion these expenses in proportion to the number of claims reported, closed, or outstanding each year.*

The Schedule P Interrogatory cited above gives a general procedure without specifying the specifics, which may differ by line of business and by company. Part 5 of Schedule P provides histories of claim count information by accident year, facilitating the use of claim counts to distribute adjusting and other expense payments by year.

Illustration: Revised Method of AAO Distribution

Suppose the company determines that for other liability claims, the average 20XX adjusting and other expense costs per claim, based on a random sample of claims, were as follows:

- claims reported during the year (initial investigation and setting up files): $500
- claims settled during the year (final investigation and payment expenses): $300
- claims closed during the year with no payment (final investigation): $200
- claims open at year-end but not reported in the year (general expense): $100

In this sample, all the claims reported during the year remained outstanding at year-end.

This expense study is simplistic, and it may easily be refined. For example:

a. Some adjusting and other expense depends on the amount of loss. Large claims receive more attention from claims department personnel than small claims receive.
b. The adjusting and other expenses vary by characteristics of the claims. More complex claims require more investigative work and incur more AAO expenses.

The dollar amounts per claim depend on the time period of the sample, and they increase with inflation in subsequent years. We convert the dollar amounts to relativities, which are not affected by inflation.

We distribute the adjusting and other expenses to accident years in three steps.

1. We determine expense relativities by type of claim, based on the sample data.
2. We compile the number of claims reported, outstanding, and closed with and without payment by accident year from Schedule P, Part 5.
3. We distribute the calendar year adjusting and other expense payments to accident years by the claim count figures and the relativities.

RELATIVITIES

To avoid the distorting effects of inflation, we express adjusting costs in relativities. We denote the cost of maintaining an outstanding claim through the end of the year as one unit of adjusting expense. The cost of closing a claim without payment is two units of adjusting expense, and the cost of settling a claim with payment is three units of adjusting expense.

A reported claim either remains open at the end of the year or is closed (with or without payment) during the year. The costs of reported claims in the sample overlaps with the cost of claims open at year end and claims closed during the year. The average AAO cost of a reported claim should be differentiated according to its status at the end of the year. For simplicity, let us assume that all reported claims in the sample were outstanding at the end of the year. The cost of reporting itself is four units of adjusting expense, so the total cost of a claim reported during the year is five units of adjusting expenses.[20]

CLAIM HISTORY

Schedule P, Part 5, shows three types of cumulative accident year direct plus assumed claim count triangles: (i) closed with payment, (ii) outstanding, and (iii) reported. A triangle of claims closed without payment may be formed by subtraction:

> *cumulative claims reported – cumulative claims outstanding at year end – cumulative claims closed with payment = cumulative claims closed without payment.*

The historical Schedule P triangles show cumulative claim counts; incremental (calendar year) claim counts are needed for distributing adjusting and other payments. The incremental claim counts are determined as the difference between the cumulative claim counts at the current valuation date and the cumulative claim counts in the preceding column. For the prior years row, the entry in the final column is the incremental amount, not the cumulative amount, so no further calculation is needed.

Suppose we must distribute $10 million of calendar year 20X9 adjusting and other expense payments by accident year. We calculate the following incremental 20X9 claim count figures:

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[20] A more rigorous analysis would determine the distribution of reported claims by their status at the end of the year. This distribution, along with the average AAO costs, might be as follows:

• reported during the year and still outstanding at the end of the year:	50%	$450
• reported during the year and closed without payment:	25%	$500
• reported during the year and closed with payment:	25%	$600

This distribution would be used to further refine the analysis in the text.

Exhibit 1.3: Reported, Outstanding, and Closed Claims by Accident Year

Acc Year	Reported	Closed with Payment	Closed w/o Payment	Out-standing	Weighted Claims	Distri-bution
Prior	0	5	0	5	20	0.20%
20X0	0	10	0	10	40	0.40%
20X1	0	15	0	20	65	0.65%
20X2	0	25	0	30	105	1.05%
20X3	10	40	5	55	225	2.25%
20X4	15	60	10	80	340	3.40%
20X5	25	80	10	120	480	4.80%
20X6	50	100	10	180	700	7.00%
20X7	125	150	15	300	1,280	12.80%
20X8	275	215	50	400	2,245	22.45%
20X9	800	200	100	500	4,500	45.00%
Total	1,300	900	200	1,700	10,000	100.00%

The "weighted claims" entry for each accident year equals the sum of the entries in the four preceding columns times the relativities for each type of claim. For example, the weighted claims for accident year 20X9 is

$$800 \times 4 + 200 \times 3 + 100 \times 2 + 500 \times 1 = 4{,}500.$$

DISTRIBUTION

The distribution of adjusting and other payments by accident year is proportional to the distribution of weighted claims by accident year. The total calendar year 20X9 adjusting and other expense payments is $10 million, and the total incremental weighted claims for all accident years at December 31, 20XX, is 10,000. The distribution of AAO payments to accident year 20X9 is

$$\$10{,}000{,}000 \times 4{,}500 / 10{,}000 = \$4{,}500{,}000.$$

The $4,500,000 is the incremental AAO for accident year 20X9 in calendar year 20X9. Similarly, the incremental AAO for accident year 20X8 in calendar year 20X9 is $2,245,000.

Schedule P requires cumulative figures. The cumulative AAO for accident year 20X8 at statement date December 31, 20X9 equals $2,245,000 plus the cumulative AAO for accident year 20X8 at statement date December 31, 20X8. Similar computations are done for the individual accident years. The prior years row in Schedule P, Part 1 shows the current calendar year activity, so the entry is 0.20% × $10,000,000 = $20,000.

From the Schedule P interrogatory, it might seem that the old statutory distribution method is no longer permitted, since it is not "based on the number of claims reported, closed and outstanding." This is not the intention. Dick Roth, who drafted the new interrogatory, explains that the old statutory method is indeed based on the number of claims reported, closed and outstanding. However, it also makes assumptions about the way that AAO is paid: 50% when the claim is reported and 50% when the loss is paid. The current procedure no longer requires companies to make this assumption.[21]

POOLING AND REINSURANCE AAO

When allocating AAO among companies in a pool, one should use the same method used to allocate losses and claims to the participating companies, not the "number of claims reported, closed and outstanding." Suppose that Companies A and B participate in a pool. If Company A gets 40% of the losses and Company B gets 60% of the losses, then Company A gets 40% of the AAO and Company B gets 60% of the AAO. Companies A and B then allocate their respective percentages of the AAO to accident years according to a claim count method.

The amount of AAO assumed by a reinsurance company depends upon the reinsurance contract. If the contract is a 50% pro-rata treaty, the contract may specify that the reinsurer also assumes 50% of the AAO. Unallocated loss adjustment expenses are generally not included in reinsurance treaties, so this issue is rarely material.

In reinsurance arrangements, the reinsurance company may not have the claim counts of the underlying business. If so, the reinsurer may use another method to distribute AAO to accident years.

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[21] The Annual Statement *Instructions* say: "The "Adjusting and Other" expenses can be assigned in any justifiable way among the accident years. The preferred way is to apportion these expenses in proportion to the number of claims reported, closed, or outstanding each year." Any distribution method may be used, as long as it can be justified.

Claim Counts

Column 12 shows the number of claims reported on direct and assumed business. The lines of business may be grouped into three categories with respect to claim count coding:

- Reported claim counts are shown for nine lines of business: homeowners/farmowners, personal auto liability, commercial auto liability, workers' compensation, commercial multiple peril, other liability, medical malpractice, auto physical damage, and products liability. For these nine lines, claims outstanding are also shown in column 25 and claims closed with and without payment are shown in Schedule P, Part 3. Reported claim counts are not shown for lines combining different types of coverage, such as special liability, special property, international, and non-proportional reinsurance.
- The remaining *primary* lines of business show the number of claims outstanding in column 25, but they need not show the reported claims or the number of claims closed with and without payment.
- The non-proportional reinsurance lines (A, B, and C) need not show even the number of outstanding claims.

Illustration: Claim counts are difficult to assign to non-proportional reinsurance, as the following examples show.

- An explosion in a large factory may reverberate through several excess reinsurance layers and their retrocession agreements. Rules for the percentage of a claim shown by each reinsurer would be arbitrary.
- An aggregate retention in an excess-of-loss treaty would cause a reinsurance recoverable stemming from the complete book of business. There is no claim count.

Average Claim Severities

Claim count information can be used in several ways.

Cumulative losses paid to date divided by cumulative claims closed with payment provides the average paid claim cost. A comparison of a carrier's trend in average claim cost by accident year for a given line of business with either industry averages or inflation indices may help identify deteriorating or improving books of business.

Similar ratios may be formed from other claim count figures. As examples,

- Cumulative losses reported to date divided by the sum of claims closed with payment and outstanding claims shows the average incurred claim cost for non-frivolous claims.

- Outstanding case reserves divided by outstanding claims shows the average size of case reserves. A comparison of trends in this ratio with trends in average paid claim costs may identify strengthening or weakening case reserve adequacy.[22]

Claims may be counted either "per claim" (i.e., "per accident") or "per claimant." Automobile liability insurance illustrates the difference. If an insured driver causes an accident and injures three other persons, each of whom seeks bodily injury compensation, are there three claims or one claim? Carriers may use either definition, but they must be consistent for all lines. The choice is reported in Schedule P Interrogatory 6:

6. Claim count information is reported (check one): (a) per claim _____
(b) per claimant _____

Direct and Assumed vs Ceded

Claim count information in Schedule P uses direct and assumed business, not ceded or net business. The assumed business on the primary lines of business is assumed proportional business, whereas the ceded business on the primary lines of business includes ceded non-proportional business.

Assumed claim counts on proportional reinsurance arrangements uses the same proportion as losses. With regard to intercompany pooling agreements, for instance, the Annual Statement *Instructions* say

> *Claim counts should be reported in accordance with the pooling arrangement and should reflect the company's proportionate share of the total number of claims. If the company's losses are 40% of the pool, then 40% of the claim counts should be reported.*

The same procedure is used for proportional reinsurance arrangements between unaffiliated entities.[23] For non-proportional reinsurance, there is no simple way to determine the number of claims ceded or assumed, since the percentage of a claim that is ceded depends on the

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[22] Actuarial expertise is essential in such analyses. Average claim cost ratios (paid, reported, and outstanding) depend on the maturity of the data. All three ratios increase with the development period, though they increase at different rates. See Salzmann [1984] on the importance of using data at the same maturity when comparing accident years.

[23] In past years, the NAIC *Instructions* were unclear regarding assumptions from non-affiliated ceding companies. The previous version of this paper, written in 1996, cited the Annual Statement Instructions then applicable and noted Richard Roth's recommendations for completing the exhibit. The issues have since been resolved as stated in the text, and the Annual Statement *Instructions* have been changed to accord with Mr Roth's recommendations. Companies that are still using the old claim count method for non-affiliated proportional reinsurance should switch to the procedure outlined in this paper.

size of the claim. For this reason, ceded and net claim counts are not shown for any line, and assumed claim counts are not shown for non-proportional reinsurance.

LOSS AND LOSS EXPENSE RESERVES

Columns 13 through 25 show data by accident year on unpaid amounts: losses, loss expenses, anticipated salvage and subrogation, and claims.

Before 1989, Schedule P, Part 1F showed IBNR reserves separately from case reserves. It was unclear whether the development on reported cases should be classified as IBNR or as case reserves, and insurers chose different definitions of IBNR. To avoid inconsistency among companies, Schedule P divides reserves between case reserves and bulk + IBNR reserves. All actuarial reserves, whether for development on reported cases or emergence of unreported cases, comprise the "bulk + IBNR" reserves.[24]

Actuarial bulk reserves for reported claims are not necessarily a sign of under-reserving, as long as the company sets proper total reserves.

> *Illustration:* A workers' compensation carrier reports 1,000 claims for lower back sprains and strains. Most workers with such injuries return to work within a few weeks, though some become permanently disabled. The insurance company can not identify the claims that will develop into permanent cases. Some companies augment the individual case reserves to fund the claims that develop adversely; other companies use bulk reserves. Neither method is intrinsically better.

Many claims examiners set a single case reserve for both losses and defense and cost containment expenses. For these companies, columns 17 and 18, the case basis "direct plus assumed" and ceded reserves for defense and cost containment expenses unpaid would be zero. Zero entries in columns 17 or 18 are acceptable, as long as the appropriate bulk reserves are recorded in columns 19 and 20.

> *Illustration:* A claims adjuster sets a $1 million reserve for a general liability claim. The reserving actuary estimates that 20% of the amount will be used for defense and cost containment expenses. The appropriate entries in Schedule P would be +$1 million as the case basis losses unpaid, –$200,000 as the bulk losses unpaid, and +$200,000 as the bulk defense and cost containment expenses unpaid.[25]

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[24] See SSAP No. 55, "Unpaid Claims, Losses, and Loss Adjustment Expenses," paragraph 5(b): "Bulk provisions are reserves included with other IBNR reserves to reflect deficiencies in known case reserves."

[25] In practice, the entries for this claim would be mixed with the entries of other claims, and the negative loss reserve would not be noticeable.

Retroactive Reinsurance

Prospective reinsurance is the transfer of the risk of loss from exposures that have not yet been earned. Retroactive reinsurance is the transfer of losses that have already occurred, though they have not yet been settled and some have not even been reported yet.

Retroactive reinsurance is sometimes used to circumvent statutory requirements to hold full value (undiscounted) reserves.

> *Illustration:* A block of unpaid losses has an ultimate (full) value of $100 million and a present value of $75 million. The primary company transfers the losses to a reinsurer with a payment of $80 million. The reinsurer gains $5 million of economic income, and the primary company gains $20 million of statutory income.

GAAP recognizes both prospective and retroactive reinsurance. For GAAP financial statements, the equity gain from retroactive reinsurance is recognized ratably over the settlement lifetime of the claims.

For statutory accounting, retroactive reinsurance increases total surplus, but it does not immediately affect unassigned surplus. Retroactive reinsurance has no effect on the loss reserves shown on Annual Statement "exhibits or schedules" (i.e., the Underwriting and Investment Exhibit and Schedule P). The reinsurance is not coded as ceded business in Schedule P and it does not reduce loss reserves on line 1 of page 3. Instead, the reinsurance recoverable is coded as a write-in contra-liability on line 22 of page 3 and an offsetting entry on line 24, "aggregate write-ins for special surplus funds."

RETROACTIVE REINSURANCE ACCOUNTING ILLUSTRATION

On December 31, 20XX, $100 million of loss reserves are reinsured retrospectively for $80 million. The accounting entries are as follows:

December 31, 20XX:

		Debit	Credit
Balance sheet:	Case loss reserve:		(No change)
Income statement:	Other income:		$20,000,000
Balance sheet:	Cash paid:		$80,000,000
Balance sheet:	Contra-liability for reinsurance recoverable:		-$100,000,000
Balance sheet:	Special surplus funds:		$20,000,000[26]

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[26] The $20,000,000 in the special surplus line is a segregation of surplus, not an accounting entry.

Details of retroactive reinsurance transactions are shown in note 22F to the Annual Statement. Note 22F discloses the following five items, by calendar year, for all retroactive reinsurance agreements "that have already occurred and that will generate special surplus transactions":

a. Reserves transferred;
b. Consideration paid or received;
c. Paid losses reimbursed or recovered;
d. Special surplus from retroactive reinsurance; and
e. The cedants and reinsurers included in items (a) through (d)

For explanation and illustration of this note, see Yoheved and Feldblum [2002: notes].

Anticipated Salvage and Subrogation

Before 1991, statutory accounting required insurers to hold loss reserves gross of anticipated salvage and subrogation, whereas GAAP statements showed reserves net of anticipated salvage and subrogation.[27]

> *Illustration: The company's policyholder incurs an automobile collision claim. The car is severely damaged, and the company expect to pay the "blue book" value of $5,000. The company expects to receive salvage of $2,000 on the damaged vehicle.*
>
> *For GAAP statements, the company sets up a loss reserve of $3,000, whereas for statutory statements, the company sets up a loss reserve of $5,000. The salvage was not recognized until it was received.*

The Internal Revenue Service bases taxable income on Annual Statement figures. In 1991, the Treasury amended its deduction for incurred losses to permit only reserves *net* of salvage and subrogation anticipated as an offset to taxable income (see Rev. Proc. 91–48 1991–34 I.R.B. 1), just as it allows only discounted reserves as an offset to taxable income. It presumed that Schedule P reserves were gross of anticipated salvage and subrogation, and it reduced these figures to a net basis. The Treasury determines anticipated salvage and subrogation on a formula basis, just as it determines the loss reserve discount on a formula basis.

For many insurers, Schedule P reserves were net of anticipated salvage and subrogation even before 1991, despite the statutory regulation to the contrary.[28] To avoid a double reduction for anticipated salvage and subrogation, with the corresponding overstatement of

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[27] See page 22 for definitions of these terms and for the statutory accounting treatment of salvage and subrogation received.

[28] A survey of 14 major property-casualty insurance companies in 1990 found that 13 were offsetting their reserves, either partially or fully, for anticipated salvage and subrogation.

taxable income and of the federal income tax liability, the NAIC allowed insurers to report reserves net of anticipated salvage and subrogation for the 1991 and subsequent Annual Statements and to "gross up" the reserves for federal income tax purposes.[29]

The Treasury allows insurers to "gross up" their loss reserves for anticipated salvage and subrogation only if the amount of the reduction is disclosed in the Annual Statement.[30] Column 23, "salvage and subrogation anticipated," shows this disclosure. It is not used in the Schedule P calculation of the net incurred losses, since loss reserves in column 24 are already net of the anticipated salvage and subrogation amounts in column 23, just as the Schedule P paid losses are net of salvage and subrogation received.[31] A similar disclosure of anticipated salvage and subrogation is made in the Statement of Actuarial Opinion Regarding Loss and Loss Adjustment Expense Reserves, paragraph 9(a).[32]

Companies may use either of two practices to report anticipated salvage and subrogation: the case reserves in columns 13 and 14 may be shown net of anticipated salvage and subrogation, or the case reserves may be shown gross of anticipated salvage and subrogation, and the anticipated amounts (for both reported and IBNR claims) may be an offset to the bulk reserves in columns 15 and 16.

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[29] For statutory financial statements, the reporting entity may choose not to reduce loss reserves for anticipated salvage and subrogation. For GAAP financial statements and for tax purposes, the reduction for anticipated salvage and subrogation is required.

As Ruth Salzmann has pointed out to me, the major purpose of Part 2 of Schedule P is to show favorable or adverse loss development. If reserves are gross of anticipated salvage and subrogation, but payments are net of salvage and subrogation received, the Part 2 triangles show apparent favorable development, because salvage and subrogation is not recognized until it is received. Reporting reserves net of anticipated salvage and subrogation improves the accuracy of the Schedule P retrospective tests of reserve adequacy.

[30] Compare Treasury regulations 2001FED 26,153, §1.832-4, paragraph 14.D(2): "A company. . . is allowed to increase the unpaid losses shown on its annual statement only if the company . . . discloses on its annual statement, by line of business and accident year, the extent to which estimated salvage recoverable is taken into account in computing the unpaid losses shown on the annual statement filed by the company for the calendar year ending with or within the taxable year of the company." Alternatively, a separate disclosure statement may be filed with regulatory authorities.

[31] Similarly, the paid losses in column 4 are already net of the salvage and subrogation received in column 10.

[32] The disclosure wording of the Statement of Actuarial Opinion, paragraph 9A, is as follows:

Anticipated salvage and subrogation included as a reduction to loss reserves as reported in Schedule P – Analysis of Losses and Loss Expenses, Underwriting and Investment Exhibit – Part 3A and on Page 3 – Liabilities, Surplus, and Other Funds, Line 1, $_____.

For tax purposes, the anticipated salvage and subrogation is discounted just as the gross loss reserves are discounted. The Treasury procedures for estimating and discounted anticipated salvage and subrogation are discussed below in conjunction with the discounting procedures for loss reserves.

Distributing Unallocated Expense Reserves

Property-casualty insurance companies often place less emphasis on estimating reserves for adjusting and other expenses (unallocated loss adjustment expenses) for several reasons:

1. The amount of the reserve for adjusting and other expenses is relatively small, and it is not subject to large uncertainty.
2. The reserves for adjusting and other expenses are not included in the NAIC retrospective reserve adequacy tests (IRIS tests 9 and 10; see below), and there is no cross-check in the Annual Statement for the amount of these reserves.
3. Some companies do not appreciate the rationale for holding reserves for adjusting and other expenses. They reason as follows:

   Losses are an expense of the period during which the loss occurred, so loss reserves are set up when the loss occurs, even if the loss has not yet been paid or even reported. Defense and cost containment expenses (ALAE) are associated with particular claims, so they have the same accounting treatment as those claims. But adjusting and other expenses are claims department overhead. Just as underwriting department overhead flows through income when it is incurred, so claims department overhead should flow through income when it is incurred.

This reasoning is not correct. The underwriting department overhead is incurred for policies written during that time period, so the expense flows through the income statement for that time period. The claims department overhead is incurred (in part) for claims that occurred during previous accounting periods. A reserve must be established when the claims occur, not when these expenses are incurred.[33] [34]

Because adjusting and other expenses are not associated with particular claims or particular accident years, and because this reserve may not be of major concern, some companies determine a general reserve that is not associated with specific accident years. Before 1997, Schedule P had no instructions for distributing unallocated loss adjustment expenses unpaid to accident year.

A common procedure for this distribution was to use the rationale for the distribution of unallocated expense payments to accident years, and to assume that the "bulk + IBNR" reserves consist of pure IBNR, not development on known cases.

The unallocated expense reserves were distributed in the same proportion as case reserves plus twice the IBNR reserve. Because of its simplicity, this procedure is still used by many companies.[35]

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[33] Both GAAP and statutory accounting allocate all loss adjustment expenses to the period when the claims occurred. SFAS 60, paragraph 20, says:

A liability for all costs expected to be incurred in connection with the settlement of unpaid claims (claim adjustment expenses) shall be accrued when the related liability for unpaid claims is accrued. Claim adjustment expenses include costs associated directly with specific claims paid or in the process of settlement, such as legal and adjusters' fees. Claim adjustment expenses also include other costs that cannot be associated with specific claims but are related to claims paid or in the process of settlement, such as internal costs of the claims function.

Statutory accounting has the same rule; see SSAP No. 55.

[34] Total loss adjustment expense reserves should reconcile with the Underwriting and Investment Exhibit. Schedule P, Part 1, line 12 (total for all accident years), columns 17 – 18 + 19 – 20 + 21 – 22 should equal the corresponding line of business entries in Part 3A of the Underwriting and Investment Exhibit, column 9, "unpaid loss adjustment expenses." The Underwriting and Investment Exhibit does not subdivide the unpaid loss adjustment expenses between defense and cost containment expenses and adjusting and other expenses.

[35] Salzmann [1988], pages 83–84, describes this procedure in more detail:

"By combining the intent and arithmetic of the footnote to the schedules, the total unallocated LAE liability is the sum of two products: (1) the liability for reported losses times the paid/paid ratio @ 50%, and (2) the IBNR liability times the paid/paid ratio @ 100%.

These two calculations can be reduced to one:

Unallocated LAE liability = .5 paid/paid ratio *x* (Total loss liability + IBNR liability)."

[Before 1989, the procedure for distributing unallocated loss adjustment expense payments to accident years was described in a footnote to Schedule P, Part 1 and not in the Annual Statement instructions. Salzmann's

These assumptions are not entirely accurate. In particular, much IBNR is development on reported cases, so the second assumption over-weights the proportion of the reserves for adjusting and other expenses associated with IBNR reserves.

Schedule P Interrogatory #3 now requires reserves for adjusting and other expense payments to be allocated to accident years based on "the number of claims reported, closed, and outstanding."[36] Reserving methods patterned on the procedure recommended by Johnson [1989] are used by some companies. The parameters of the reserving method, such as the percentage of adjusting and other expense costs to be ascribed to claim reporting or to claim payment, vary by line of business and by company. There is no standard method of estimating AAO reserves or of spreading them to accident year.

Claims Outstanding

Column 25 shows the number of claims outstanding on direct and assumed business. Column 25 must be completed for all primary lines of business, though not for the three reinsurance lines.[37] The ratio of case reserves in column 13 (or case reserves plus DCC reserves in columns 13 plus 17) to column 25 shows the average value of an outstanding claim. This ratio must be used with caution, for two reasons:

1. Lines such as workers' compensation, automobile no-fault, and accident & health provide periodic payments during the duration of a disability. The case reserves show only the remaining unpaid losses, not the entire benefits, so the ratio of case reserves to claims outstanding understates the value of an outstanding claim. This distortion increases as the claims mature.

2. Smaller, simpler cases are settled more rapidly than larger, more complex cases, particularly in the tort liability lines of business.

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paid/paid ratio is the ratio of "unallocated loss adjustment expense paid to losses paid for the most recent calendar year(s)."]

As Ruth Salzmann has explained to me, "The method is not put forward on its own merits; rather, it is appropriate only because it is consistent with the *assumption* underlying the formula allocation of paid unallocated loss expenses by accident year. Thus, the method does no more than anticipate future *formula* allocations." Claim reporting and settlement patterns allow a better distribution of both paid and unpaid unallocated expenses by accident year; see the following footnote.

[36] As noted above, this should be interpreted as a recommendation, not as a requirement. The Annual Statement Instructions explain that "the Adjusting and Other expenses can be assigned in any justifiable way among the accident years. The preferred way is to apportion these expenses in proportion to the number of claims reported, closed, or outstanding each year."

[37] These exhibits show assumed non-proportional business. Since the reinsurer is assuming a layer of loss, not the entire loss, the number of outstanding claims is not a meaningful figure.

3. Loss development on reported cases is included in the bulk reserves shown in column 15, not in the case reserves of column 13. One can not include column 15 in calculating the average value, since this column includes IBNR reserves, and IBNR claims are not included in column 25. If there is significant loss development on reported cases, then the ratio noted above understates the value of an outstanding claim.

ILLUSTRATION: OUTSTANDING CLAIM SEVERITY

Lest readers underestimate the difficulties of using Schedule P average outstanding claim severities, we show an example of workers' compensation premiums, unpaid losses and loss adjustment expenses, outstanding claim counts, and average outstanding claim severities.

Exhibit 1.4: Outstanding Claim Severity

Year	Net Premium	Net Unpaid Loss + LAE	Direct + Assumed Outstanding Claims	Outstanding Claim Severity
Prior		$800,000	13,650	$58,608
20X0	$1,800,000	$230,000	2,600	$88,462
20X1	$2,650,000	$320,000	3,400	$94,118
20X2	$2,800,000	$330,000	4,400	$75,000
20X3	$2,800,000	$360,000	5,400	$66,667
20X4	$2,650,000	$325,000	6,600	$49,242
20X5	$2,500,000	$530,000	8,800	$60,227
20X6	$2,250,000	$650,000	10,000	$65,000
20X7	$2,000,000	$715,000	14,250	$50,175
20X8	$1,650,000	$750,000	23,000	$32,609
20X9	$1,300,000	$880,000	42,000	$20,952

The progression of average outstanding claim severities reflects the company's operations and the nature of workers' compensation claims.

- ☞ For accident years 20X2 and prior, almost all the outstanding claims are lifetime pension cases. The increasing severities reflect inflation and the partial (weekly) payments on these claims. The pension claims in the prior years row stem from old years; the severities reflect only the amount still remaining to be paid.
- ☞ For accident years 20X2 through 20X4, the pension cases are increasingly mixed with temporary cases, and the average outstanding claim severities decrease.
- ☞ The company began switching business to large dollar deductible policies in 20X4, as the decline in net earned premium indicates. The rise in average outstanding claim severities in 20X5 and 20X6 reflects the higher average costs of excess claims.

☞ The claims in accident years 20X7 through 20X9 are increasingly dominated by temporary cases, and much of the reserves are bulk reserves, not case reserves. The average outstanding claim severities decline rapidly, despite the increasing use of large dollar deductible policies.

Loss Ratios

Columns 26 through 31 are calculated figures.

- Column 26, "Total losses and loss expenses incurred, direct and assumed," equals the sum of columns 4, 6, 8, 13, 15, 17, 19, and 21.
- Column 27 (ceded) equals the sum of columns 5, 7, 9, 14, 16, 18, and 20.
- Column 28 (net) equals column 26 minus column 27, or the sum of columns 11 and 24.
- Columns 29 through 31, "Loss and loss expense percentage (Incurred / Premiums Earned)" for direct and assumed, ceded, and net business are the ratios of columns 26 through 28 to columns 1 through 3, respectively.

Industry-wide averages by line of business of column 31 for 1983-1992 were used to determine the written premium charge in the NAIC risk-based capital formula. The individual company ratios in column 31 are used for the company adjustment to the written premium risk charge in the risk-based capital submission; see Feldblum [RBC: 1996].

These ratios are gross of non-tabular discount and net of tabular discount. They are used by financial analysts to assess the underwriting performance of insurance enterprises (i) in absolute terms, (ii) in comparison with other insurers, and (iii) in comparison with past performance.

Loss Reserve Discounting

Columns 32 and 33 show the non-tabular discount for losses and loss adjustment expenses, respectively. These columns provide a reconciliation of the Schedule P figures with the entries in the Underwriting and Investment Exhibit, which are reproduced in columns 35 and 36 of Schedule P.

In general, property-casualty loss reserves are shown at undiscounted values on statutory accounting statements, with the exception of tabular discounts.[38] The statutory undiscounted values must include the effects of expected inflation from the statement date to the settlement date, but they may not include the effects of discount rates.

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[38] Undiscounted values are also termed nominal values or ultimate values. Discounted values are also termed market values or fair values.

Loss reserve valuation in other accounting systems – GAAP, tax, risk-based capital, and international accounting – are noted below.

- For property-casualty insurance, GAAP discounting rules follow the statutory accounting procedures, with minor exceptions.
- For federal income tax purposes, only discounted reserves are offsets to taxable income. The discounted reserves are determined from Schedule P entries; see page 171 below.
- The risk-based capital formula determines the reserving risk charge and the written premium risk charge based on discounted reserves. The RBC formula uses the IRS discounting procedures and loss payment patterns, though with a flat 5% discount rates instead of the 60 month moving average of the federal mid-term rates. The RBC loss reserve discount factors were established in 1993 and have not been changed since then.
- Currently evolving international insurance accounting standards use fair value (i.e., discounted value) for loss reserve valuation.

TABULAR AND NON-TABULAR DISCOUNTS

There are two types of loss reserve discounts: tabular discounts and non-tabular discounts. Tabular discounts are discounts based upon a mortality or morbidity table. Under statutory accounting, they may be applied only to the indemnity (i.e., wage replacement) portion of workers' compensation pension cases or to long-term disability claims. They may not be applied to the medical benefits or loss adjustment expenses associated with these claims.[39] This is similar to the reserve valuation for an immediate annuity, except that the beneficiary of a workers' compensation pension case is a disabled life.

Non-tabular discounts are determined from the aggregate payment patterns of the book of business or other information, generally using historical paid loss data. See the section below on the IRS loss reserve discount factors for an illustration.

> *Illustration:* A construction worker is permanently paralyzed after a fall from a scaffold. The weekly workers' compensation indemnity benefits are $1,000 for life. Based on the injured worker's age, sex, and health status, the expected future lifetime is 40 years. The undiscounted reserve is 40 years x 52 weeks x $1,000 per week = $2.08 million. The discounted reserve, which would be substantially less, is shown in Schedule P.
>
> The workers' compensation insurer also pays for daily home health care visits, rehabilitation treatment, and periodic nursing and physician care. The current cost is

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[39] SSAP No. 65, "Property and Casualty Contracts," paragraph 11, says: "Tabular reserves are indemnity reserves that are calculated using discounts determined with reference to actuarial tables which incorporate interest and contingencies such as mortality, remarriage, inflation, or recovery from disability applied to a reasonably determinable payment stream. Tabular reserves shall not include medical loss reserves or loss adjustment expense reserves."

about $600 a week. These costs are expected to increase with inflation and with deterioration of the worker's condition as he or she ages. Based on actuarial analyses of future inflation rates and development patterns, the undiscounted reserve is $3.5 million and the discounted reserve is $1.1 million. This is classified as a non-tabular reserve discount, even though it is based on an individual claim.

DISCOUNTING AND RISK-BASED CAPITAL

The distinction between tabular and non-tabular reserve discounts affects the risk-based capital ratio. The RBC ratio is the company's "adjusted surplus" divided by its risk-based capital requirements. Adjusted surplus is policyholders' surplus minus non-tabular reserve discounts, along with other adjustments applicable primarily to life insurance companies. Tabular reserve discounts do not have this effect. Since the risk-based capital ratio is seen as an indicator of financial strength, companies have an incentive to reclassify non-tabular reserve discounts as tabular reserve discounts.[40]

Illustration: For the ABC Insurance Company, policyholders' surplus is $500 million, loss reserves are $800 million, the tabular discount is $100 million, the non-tabular discount is $50 million, and the risk-based capital requirements are $300 million. The RBC ratio is

($500 million – $50 million) / $300 million = 150%.

DISCOUNTING AND STATUTORY REPORTING

The treatment of discounting in the Annual Statement exhibits and schedules is as follows:

- Loss and expense reserves on the balance sheet (page 3, lines 1, 2, and 3), in the Underwriting and Investment Exhibit, Parts 3 and 3A (pages 10 and 11), and in other parts of the Annual Statement (such as the Page 15 state pages) are net of both tabular and non-tabular discounts.

- Schedule P, Part 1, is net of tabular discount and gross of non-tabular discount. In order to reconcile Part 1 of Schedule to the rest of the Annual Statement, non-tabular discounts are disclosed in columns 32 and 33.

- Schedule P, Parts 2 and 4 are gross of both tabular and non-tabular discounts. The reconciliation between Part 1 of Schedule P and Parts 2 and 4 of Schedule P is in the Note to the Financial Statements titled "Discounting of Liabilities for Unpaid Losses and Unpaid Loss Adjustment Expenses" [Note 28, section (1), in 2001]. Disclosure of loss

[40] The exclusion of discounts on medical benefits and loss adjustment expenses from classification as tabular discounts was established by Mr. Vincent Laurenzano in May 1994 in conjunction with the final draft of the property-casualty risk-based capital formula, and it was subsequently adopted into statutory accounting.

reserve discounts in or with the Annual Statement is necessary for the company to "gross up" its discounted reserves for federal income tax purposes, thereby reducing its tax liability for the year; see the section on IRS loss reserve discounting further below.

The "amortization of discount," or the "unwinding of the interest discount," in loss triangles that are net of discount shows up as apparent adverse loss development. Part 2 of Schedule P is intended to show true adverse loss development, so it is reported gross of all discounts.

For lines of business which do *not* have tabular discounts (that is, for all lines except workers' compensation and accident & health), the reconciliation between Part 1 and Part 2 of Schedule P is as follows: for each accident year,

Part 1, columns 28 – 21 + 22 – 8 + 9 equals Part 2, column 10.

Net incurred losses and loss adjustment expenses by accident year (Part 1, column 28) minus net adjusting and other expense reserves (column 21 minus column 22) minus net cumulative adjusting and other expenses paid (column 8 minus column 9) equals net incurred losses and defense and cost containment expenses at the current statement date (Part 2, column 10). This reconciliation does not work for lines of business that have tabular discounts. The Part 2, column 10 figures are higher by the amount of the tabular discount.

Dynamic Discount Rates

Life insurance and annuity policy reserves are held at discounted values on statutory financial statements. The maximum allowable discount rate that is prescribed by statutory regulation is dynamic in that it varies with the yield on investment grade corporate bonds minus a specified margin that varies with the characteristics of the insurance product; see the 1990 Standard Valuation Law for life insurance products.

Similarly, the 2001 statutory accounting codification rules limit the maximum interest rate for *non-tabular* reserve discounts when discounting is permitted. The maximum permitted interest rate is the lower of (i) the yield on five year Treasury notes and (ii) the company's investment yield minus 1.5 percentage points. The company's investment yield is

a. The company's average yield on *invested* assets if invested assets exceed the loss reserves plus the unearned premium reserves, or
b. The company's average yield on *total* assets if invested assets are less than the loss reserves plus the unearned premium reserves.[41]

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[41] See SSAP Number 65, "Property and Casualty Contracts," paragraph 12:

When establishing discounted loss reserve liabilities prescribed or permitted by the state of domicile using a non-tabular method . . . the rate used [shall not] exceed the lesser of the following two standards:

The maximum permitted discount rate is dynamic in that it varies with the current yields on Treasury securities and with the company's own investment results. Prior to the codification of statutory accounting in the late 1990's, most states that permitted discounting in specific instances used static maximum discount rates, which were absolute rates coded in the law.[42]

No maximum discount rate is specified by statutory accounting for tabular discounts. However, the discount rate used, both for tabular and non-tabular discounts, must be disclosed in the notes to the financial statements.

ILLUSTRATION: MAXIMUM DISCOUNT RATE

A property-casualty insurance company discounts certain reserves using a non-tabular method. The maximum permitted discount rate is based on the following data.

December 31, 20XX loss reserves:	$120 million
December 31, 20XX unearned premium reserves:	$50 million
December 31, 20XX statutory invested assets:	$160 million
Average investment yield on invested assets during 20XX:	9.5% per annum
December 31, 20XX total statutory assets:	$210 million
20XX investment income earned (line 8 of U&IE):	$14 million
5 year Treasury note rate on December 31, 20XX	7.5% per annum

The yield on five year Treasury notes is 7.5% per annum. On the statement date, the company holds $170 million of loss plus unearned premium reserves, and it has $160 million of invested assets. Since the invested assets are less than the reserves, we examine the yield on total assets, which is $14 million / $210 million = 7.0%. Subtracting the statutory margin of 1.5% gives 5.5%. The maximum permitted statutory discount rate is the lower of 5.5% and 7.5%, or 5.5% per annum.

This company has a large percentage of non-invested assets, such as premiums receivable, accrued retrospective premiums, deferred tax assets, and non-investment real estate. If the company's invested assets were greater than the loss plus unearned premium reserves of $170, we would use the yield on invested assets minus 1.5% percentage points, to give 8.0%

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a. If the reporting entity's statutory invested assets are at least equal to the total of all policyholder reserves, the reporting entity's net rate of return on statutory invested assets, less 1.5%, otherwise, the reporting entity's average net portfolio yield rate less 1.5% as indicated by dividing the net investment income earned by the average of the reporting entity's current and prior year total assets; or

b. The current yield to maturity on a United States Treasury debt instrument with maturities consistent with the expected payout of the liabilities.

[42] Statutory accounting retains the static perspective in the IRIS test on the company's investment yield, which uses fixed numbers as the bounds (currently 4.5% to 10.0%).

per annum. The maximum permitted loss reserve discount rate would be the lower of 8.0% and 7.5%, or 7.5% per annum.

DISCOUNTING DISCLOSURES AND RECONCILIATION

Columns 35 and 36 show the effect of the non-tabular discount on the loss and loss adjustment expense reserves. If no discount is used, column 35 equals columns 13 – 14 + 15 – 16 (net case reserves plus net bulk reserves), and column 36 equals columns 17 – 18 + 19 – 20 + 21 – 22 (net case DCC reserves plus net bulk DCC reserves plus net AAO reserves). If a non-tabular discount is used, the discount figures in columns 32 and 33 must be subtracted from these sums to obtain columns 35 and 36.

Schedule P, Part 1 loss reserves are net of tabular discounts. No adjustment for tabular discounts is needed to reconcile the figures with the Underwriting and Investment Exhibit.

Both tabular and non-tabular discounts are disclosed in Note 28 to the financial statements. For the tabular discounts, the note shows

a. The table used;
b. The discount rates;
c. The amount of the discounted reserves; and
d. The amount of the discount.

The amount of the discount is subdivided by line of business and by type of reserve: case reserves vs bulk and IBNR reserves.

Tabular discounts on known claims (case reserves) are easily determined. Given the required input data, such as the age, sex, and impairment status of the annuitant (the claimant), the weekly benefit, the discount rate, and the mortality table, the discounted reserve is determined by actuarial formula.

Tabular discounts on IBNR reserves are more complex. The reserving actuary determines the expected number of permanent disability or fatal cases to emerge on existing business, the expected subdivision by sex, and the average age, impairment status, and weekly benefit amounts. These projections, together with the discount rate, the mortality table, and the actuarial formulas, give the discounted reserves.[43]

The tabular discount shown in Note 28 should reconcile with the difference in loss reserves at the statement date between (i) Schedule P, Part 1, loss plus LAE reserves but not including AAO reserves, and (ii) Schedule P, Part 2 minus Part 3.

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[43] Workers' compensation IBNR for pension cases is not the emergence of unreported claims but the re-evaluation of temporary total claims or permanent partial claims into permanent total claims.

For non-tabular discounts, Note 28 to the financial statements shows

a. The discount rates and their basis (i.e., their rationale);
b. The amount of the discounted reserves; and
c. The amount of the discount.

The amount of the discount is subdivided by line of business and by type of reserve: case reserves vs bulk and IBNR reserves vs defense and cost containment expenses vs adjusting and other expenses. Non-tabular discounts may be applied to loss adjustment expenses; tabular discounts may not be applied to loss adjustment expenses.

The non-tabular discounts in Note 28 should reconcile with the entries shown in Schedule P, Part 1, columns 35 and 36 for losses and loss adjustment expenses, respectively.

INTERCOMPANY POOLING

Column 34 shows the intercompany pooling participation percentage, if applicable. Member companies of an insurance group often redistribute premiums, losses, and loss adjustment expenses according to participation formulas. Column 34 shows the individual company's share of the group figures.

Intercompany pooling agreements are used primarily for rating purposes.

> *Illustration:* A private passenger automobile insurer wishes to differentiate between high-risk, moderate risk, and low-risk drivers. It does not have a risk classification plan filed and approved in all jurisdictions that matches the judgment of its underwriters. The insurer sets up three affiliated legal entities, Companies X, Y, and Z, to write substandard, standard, and preferred risks at rates appropriate for each type of driver.
>
> A single management team runs all three legal entities, and they desire a single set of underwriting results for the corporate group as a whole. Each legal entity cedes all its business to the lead company, which then retrocedes a percentage of the pooled business back to each legal entity.

For Schedule F and for other parts of the Annual Statement, each legal entity's percentage of the pooled business is assumed business, not direct business. The cessions to the lead company appear as ceded reinsurance in Schedule F, Part 3, and the assumptions of a percentage of the pooled business appear as assumed business in Schedule F, Part 1.[44]

[44] Schedule F explicitly differentiates between reinsurance transactions with affiliated companies and those with unaffiliated companies; see Feldblum [2002: SchF].

For Schedule P, each legal entity's percentage of the pooled business is direct business, not assumed business. The intercompany pooling agreement does not create cessions or assumptions for Schedule P. To complete Schedule P, one constructs first a pooled schedule, and each legal entity takes its appropriate percentage of every entry. The "appropriate" percentage is the percentage in the *current* intercompany pooling agreement, not the percentage for the year in which the losses occurred; see the discussion below.

The intercompany pooling agreement relates to underwriting revenues and expenditures: premiums, losses, loss adjustment expenses, and underwriting expenses. It does not affect assets, investment income, or surplus. Asset transactions may be handled by a single investment department, but the assets and investment income of each legal entity are kept distinct.

The coding of cessions to unaffiliated reinsurers and assumptions from unaffiliated companies depends on whether the cessions or assumptions are classified as pooled business.

Illustration: Intercompany Pooling

The coding of intercompany pooling transactions varies with the circumstances. The illustration below shows the more common transactions. The prose documentation is followed by a table listing the entries.

Illustration (Step 1): Companies X, Y, and Z are affiliated members of an insurance fleet that writes private passenger automobile insurance. Companies X, Y, and X write substandard, standard, and preferred risks, respectively. For marketing purposes, most risks are classified as preferred. In 20XX, Companies X, Y, and Z write $10 million, $20 million, and $70 million of premium.

By an intercompany pooling agreement, companies X and Z cede all their premium to company Y. Company Y is termed the "lead company." Company Y retrocedes 30% of the business to Company X and 20% of the business to Company Z. For the Schedule P entries, company X shows 30% of the pooled earned premiums and incurred losses as direct business, Company Y shows 50% of the pooled earned premiums and incurred losses as direct business, and Company Z shows 20% of the pooled earned premiums and incurred losses as direct business. To complete the individual company Schedule P's, we construct the pooled Schedule P and take percentages.

Illustration (Step 2): We add three sets of transactions.

- Before pooling, Company X reinsures its business under a 50% pro-rata treaty.
- Before pooling, Company Z assumes $30 million of private passenger automobile premium from an unaffiliated insurer.

- After pooling but before retroceding business to Companies X and Z, Company Y has an excess of loss reinsurance treaty above a $100,000 retention with a 10% reinsurance rate on subject premium.

i. The $5 million of ceded premium by Company X is ceded pooled premium, which is shared in the 30%, 50%, 20% percentages by the three companies.
ii. The $30 million of assumed premium by Company Z is assumed pooled premium, which is shared in the 30%, 50%, 20% percentages by the three companies. The total written premium by Company Z which is ceded to the pool is $70 million + $30 million = $100 million, of which Companies X, Y, and Z get 30%, 50%, and 20%.
iii. Ceded premiums under the excess of loss treaty by Company Y are ceded pooled premiums. The total pooled written premium is $5 million from Company X, $20 million from Company Y, and $100 million from Company Z, for a total of $125 million. Before pooling by the 30%, 50%, and 20% percentages, the excess of loss premiums and losses are removed.

We add one additional step to this illustration, which slightly changes the figures.

Illustration (Step 3): The intercompany pooling agreement does not include New Jersey business.

- Company X writes $2 million of New Jersey written premium, of which it ceded $1 million by its quota share treaty.
- Company Z writes directly $3 million of New York written premium, and it assumes $1 million of New Jersey written premium as part of the totals shown earlier.

For Schedule P,

i. Company X cedes only $8 million to the pool, half of which is then ceded pooled business. The remaining $2 million of New Jersey premium is direct business for company X, of which $1 million is ceded.
ii. The $3 million + $1 million = $4 million of New Jersey written premium written directly or assumed by company Z is not pooled.

Illustration (Step 4): Company Y's assets have been depleted by poor investments. After pooling, company Y cedes 20% of its resulting business for surplus relief. This transaction does not affect the Schedule P entries of companies X and Z.

The components of this illustration are shown in the chart below. Figures are in millions of dollars.

Exhibit 1.5: Intercompany Pooling Agreement (Figures in Millions of Dollars)

	Affiliated Companies			
	X	Y	Z	Pooled
Direct WP, rest of country (pooled)	$8	$20	$67	
Direct WP, New Jersey (not pooled)	$2	$0	$3	
Assumed WP, rest of country (pooled)	$0	$0	$29	
Assumed WP, New Jersey (not pooled)	$0	$0	$1	
Direct + assumed pooled WP	$8	$20	$96	$124
Ceded pooled WP	$4	$0	$0	$4
Net pooled WP, before excess of loss	$4	$20	$96	$120
Pool excess of loss cession to non-affiliates				$12
Net pooled WP, after excess of loss				$108
Pool retrocessions to affiliates	$21.6	$54.0	$32.4	
Post-pooling cessions to non-affiliates	$0	$10.8	$0	
WP affecting Schedule P	$21.6	$43.2	$32.4	

POOLING RESTATEMENTS

The Annual Statement *Instructions* say, "The pooling percentage is to reflect the company's participation in the pool as of year-end." If an insurance group modifies the pooling arrangement, there may be an apparent change in the incurred or paid loss development because of the intercompany agreement, not because of changes in claims handling or reserving procedures. Therefore, "any retroactive change in pooling participation will require appropriate restatement of Schedule P."

Illustration: A member company of an insurance group receives 40% of the pooled business in 20XX. In 20XX+1, its pooling participation percentage changes to 70%. Leaving the original 40% participation for 20XX may distort the loss development patterns: its loss payments and reserves were 40% of the group total in 20XX, but its payments and reserves were 70% of the total in 20XX+1. Its loss triangles would show jumps in both payments and reserves between 20XX and 20XX+1. To facilitate the use of the loss development patterns, the company restates all past figures to a 70% participation percentage.

If the pooling percentage changes, the individual company historical figures in the current Schedule P will not agree with the entries of previous years. Rather, for any accident year, the Schedule P entries divided by the pooling percentage in column 34 should reconcile with the Schedule P entries in previous years divided by the pooling percentage.[45]

The need for loss triangles to forecast accurately future development argues for even more comprehensive restatements of past experience.

Illustration: An insurer incorporates a new subsidiary in 20XX and gives it 40% of its total business. Premiums and losses for this subsidiary were zero before 20XX, as the company did not yet exist. But if the parent company gets 100% of the business before 20XX, but only 60% in 20XX and subsequent years, its loss development triangles will be distorted. According to Richard Roth, the subsidiary should be given 40% of the business for all years, even when it did not exist, and the parent company should be given 60% of the business for all years.

Treaty commutations affect both the reported and the paid loss development patterns. The same logic would dictate that both the ceding and assuming carriers restate their experience after a treaty commutation. Carriers commute individual claims in addition to whole treaties, such as lifetime pension claims in workers' compensation, long term disability claims in accident and health insurance, and structured settlements in other liability. The analyst completing *Schedule P* is not always aware of these commutations, and restating past history is an onerous task.

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[45] The text follows Richard Roth's explanations. Mr. Roth designed the current Schedule P, and he was chairman of the NAIC Casualty Actuarial (Technical) Task Force, so his interpretation was determinative, at least until his retirement in 2001. The Annual Statement *Instructions* themselves are ambiguous.

- The *Instructions* say that any *retroactive* change in pooling participation will require appropriate restatement of Schedule P (emphasis added). According to Mr. Roth, *any* change in pooling participation requires restatement of Schedule P.
- Schedule P provides separate column 34 entries (intercompany pooling participation percentage) for each accident year. According to Mr. Roth, the percentage for each accident year should be the current participation percentage.
- If the pooling percentages change on a calendar year basis or a policy year basis (and include the development from past accident years), the accident year loss development patterns would be distorted. If the pooling percentages change on an accident year basis, the accident year loss development patterns would not necessarily be distorted.

In theory, when a commutation affects the loss development patterns, the effects should at least be disclosed in Question 7 of the *Schedule P* Interrogatories. In practice, a company may note that commutations have occurred, but it would rarely try to quantify the effects.

Similar problems exist for primary companies when their reinsurers become insolvent. Loss reserves are shown net of reinsurance recoverables in the Schedule P historical triangles. If a reinsurer becomes insolvent, the ceded reserve drops to $0 and the net reserve increases. Even if the primary company had been aware of the potential insolvency, the loss reserves are net of the recoverable, and the provision for reinsurance separately adjusts the company's surplus for the expected uncollectible amounts (see Feldblum [2002: SchF]. When reinsurance recoverables are written off, disclosure in Schedule P Interrogatory Number 7 is appropriate.

OCCURRENCE AND CLAIMS-MADE BUSINESS

In 1993, the old claims-made business exhibit was removed from Schedule P, three lines of business were segmented into occurrence and claims-made portions, and the disclosure of extended loss and expense reserves was put into a Schedule P interrogatory.
Occurrence policies provide coverage for accidents that occur during the policy period, regardless of when the claims are reported. Claims-made policies provide coverage for accidents that are reported during the policy period. Most claims-made policies limit coverage to accidents that occur subsequent to the "retroactive date," or the date that claims-made coverage was first issued to the policyholder. Claims-made coverage is used primarily for medical malpractice insurance, certain other professional liability insurance, and some products liability insurance.[46]

Tail Coverage

The coverage restrictions on claims-made policy forms can inhibit movement from one insurance company to another.

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[46] Statutory accounting principles for claims-made policies is covered in SSAP Number 65, "Property and Casualty Contracts," paragraphs 4 through 9.

Illustration: A physician is covered during 20XX under a claims-made policy with one insurer. On January 1, 20XX+1, the physician switches to a claims-made policy with a second insurer. The new claims-made policy has a retroactive date of 1/1/XX+1.

The first insurer will not indemnify claims that are reported after the switch to the second insurer on 1/1/XX+1. The second insurer will not indemnify claims that occurred while the physician was covered by the first insurer since they occurred before the retroactive date.

To cover claims that occur during the claims-made period with the first insurer but are reported subsequent to its termination, the physician purchases tail coverage from the first insurer. The tail coverage covers claims that occur during the claims-made period but are reported after its termination.

Extended Tail Coverage

Tail coverage is also used if the physician leaves his or her practice and longer needs full insurance coverage.

> *Illustration:* A physician leaves private practice to join an HMO. The HMO has medical malpractice coverage for its staff, and the physician no longer needs an individual policy. The physician may still need a tail policy to cover accidents that occurred before the physician joined the HMO.

If a physician stops practicing because of retirement, disability, or death, he or she (or the estate) still needs tail coverage for late reported claims. Medical malpractice coverage is expensive, and its importance may not be appreciated when the physician stops working.

To avoid burdening the retired or disabled physician (or the estate) with the heavy costs of tail coverage, some insurers spread this cost over the term of the claims-made coverage and provide free tail coverage in the event of retirement, disability, or death.

> *Illustration:* The cost of annual claims-made coverage for a certain physician is $10,000. The insurer may charge $12,000, and use the extra $2,000 a year to build up a reserve for free tail coverage in the event of retirement, disability, or death. This is not a loss reserve, since the insurance company does not yet have any liability for claims. It is not shown in the Schedule P exhibits. It is akin to life insurance policy reserves, or to an active life reserve in disability insurance.[47] It is shown on the insurer's balance sheet as a write-in

[47] See SSAP #65, "Property and Casualty Contracts," paragraph 8: "Some claims made policies provide extended reporting coverage at no additional charge in the event of death, disability, or retirement of a natural person insured. In such instance, a policy reserve is required to assure that premiums are not earned prematurely. The amount of the reserve should be adequate to pay for all future claims arising from these

line on page 3, but there is no exhibit in the property-casualty Annual Statement that discloses it. Instead, the extended loss and expense reserves by accident year and by line of business (for medical malpractice, other liability, and products liability) are shown in the first Schedule P interrogatory.

RBC Underwriting Risk Charges

The separate occurrence and claims-made exhibits for medical malpractice, other liability, and products liability stem from the risk-based capital underwriting risk charges. The paragraphs below provide a brief summary; see Feldblum [1996: RBC] for further explanation.

The reserving risk and written premium risk charges in the risk-based capital formula are determined from Schedule P information. Reserving risk is the risk that unanticipated events may increase the company's obligations for past claims above the amounts expected at the statement date.

> *Illustration:* A company has $100 million of medical malpractice loss reserves. In a "worst case" adverse scenario, as defined by the NAIC's risk-based capital formula, the reserves may develop adversely by 56.5% to $156.5 million. The present value of medical malpractice loss reserves is 80.8% of the undiscounted value in the RBC formula. The company needs $156.5 million × 80.8% = $126.5 million of assets to guard against unanticipated adverse development.

The adverse loss development may result from two causes: (a) the emergence of late reported claims, or pure IBNR loss emergence, and (b) increases in the loss estimates for reported claims, or development on known claims. Claims-made business has no pure IBNR loss emergence. Some companies argued that claims-made business should show less adverse loss development, and it needs a smaller reserving risk charge.

To quantify the difference in adverse loss development between occurrence and claims-made business, the NAIC segmented the Schedule P exhibits for three lines of business into occurrence and claims-made portions in 1993. These three lines – medical malpractice, other liability, and products liability – include almost all the claims-made business written in the property-casualty insurance industry.

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coverage features, after recognition of future premiums to be paid by current insureds for these benefits. The reserve, entitled 'extended reporting endorsement policy reserve' shall be classified as a component part of the unearned premium reserve considered to run more than one year from the date of the policy." Before this rule became effective (in 2001), the extended loss and expense reserves could be placed in either the loss reserves or the unearned premium reserves, at the option of the company.

Post Codification Tail Coverage Accounting

Tail coverage converts claims-made coverage into occurrence coverage. Like occurrence policies, it covers losses which occur during a certain period, regardless of when they are reported. Tail coverage is appended to claims-made policies, but it is included with the Schedule P occurrence exhibits, not the claims-made exhibits.

Post-codification statutory accounting for tail coverage on claims-made policies depends on the duration of the tail period.

If the tail has an indefinite term, the full premium is earned on the date the policy is issued and a bulk loss reserve is established for the estimated future losses. There is no unearned premium reserve, and all reserves are shown in Schedule P.

If the tail has a definite (limited) term, the premium is earned over the term of the tail coverage. An unearned premium reserve is established on the effective date of the policy, and it is amortized over the term of the coverage. Case loss reserves are established as the losses are reported. Bulk loss reserves are needed for adverse development on known case reserves, not for the emergence of IBNR claims. The only reserves shown in Schedule P are those for known cases.

Illustration: A physician with medical malpractice coverage under a claims-made policy switches from Insurer A to Insurer B on January 1, 20XX. To cover potential liability for claims occurring before January 1, 20XX, but reported on or after January 1, 20XX, the physician purchases tail coverage from Insurer A on December 31, 20XX–1 for a premium of $15,000.

If the tail policy has an unlimited duration (an "indefinite term"), the earned premium on December 31, 20XX–1 is $15,000. A bulk reserve is established on December 31, 20XX–1, for the expected future claims, which may be more or less than $15,000.[48] The bulk reserve is shown in Schedule P for accident year 20XX–1.

If the tail policy has a three year term (a "definite term"), the written premium on December 31, 20XX, is $15,000, the unearned premium reserve is $15,000, and the earned premium is $0. The unearned premium reserve is amortized over three years, either ratably over the policy term ($5,000 each year) or in proportion to the expected protection. If the insurer

[48] Since the average medical malpractice loss may be paid several years in the future, the present value of the losses may be 50% or less of the nominal value. If the discount factor is 50%, the $15,000 premium may cover $25,000 of undiscounted losses plus $2,500 of underwriting expenses and profit. A premium of $15,000 coupled with a loss reserve of $25,000 may indicate a long tail, not under-pricing.

expects the claims over the three years to be *reported* in a 7:5:3 proportion, the amortization schedule may be $7,000 in 20XX, $5,000 in 20XX+1, and $3,000 in 20XX+2.

As claims are reported, case loss reserves are established. There is no bulk reserve for IBNR claims that are expected to be reported during the three year period, since these claims are covered by the unearned premium reserve. If the insurer believes that the unearned premium reserve for the block of business is inadequate, a premium deficiency reserve is established; no bulk reserve is used.[49] Bulk reserves are needed only for adverse development on known claims.

Under post codification statutory accounting rules, tail coverage with an indefinite term is like occurrence coverage, and tail coverage with a definite term is like claims-made coverage. In theory, tail coverage with an indefinite term should be reported on the occurrence exhibits, and tail coverage with a definite term should be reported on the claims-made exhibits. Tail coverage with a three year term is like a three year claims-made policy.

The Schedule P rules stipulate that all tail coverage is reported on the occurrence exhibits. The Schedule P rules pre-date the post codification accounting principles for claims-made coverage: the Schedule P rules were made in 1993, whereas the post codification statutory accounting rules for claims-made policies were not effective until 2001.

Loss Date

The caption of Part 1, column 1 says "years in which premiums were earned and losses were incurred," and the captions in Parts 2 through 6 are similar. Part 7 uses policy year experience, so its caption is "Years in which policies were issued." There is no reference to "accident year" in the column captions, though we speak of Schedule P as an accident year schedule. The date when losses are incurred means the date the insurer incurs the obligation for the loss under the coverage provided by the contract. This date differs by type of policy:

- For occurrence policies, this is the date that the loss occurs.
- For claims-made policies, this is the date that the loss is reported to the insurer.[50]
- For tail coverage, this is the date that the policy is issued.

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[49] See SSAP #65, "Property and Casualty Contracts," paragraph 9: "When the anticipated losses, loss adjustment expenses, and maintenance costs anticipated to be reported during the extended reporting period exceed the recorded unearned premium reserve for a claims made policy, a premium deficiency reserve shall be recognized."

[50] See SSAP Number 55, "Unpaid Claims, Losses, and Loss Adjustment Expenses," paragraph 4: "For claims made type policies, the covered or insured event is the reporting to the entity of the incident that gives rise to a claim."

- For fidelity and surety, this is the date that the loss is discovered.[51]

Illustration: An accident covered by a medical malpractice policy occurs in 1993 and is reported in 1997.

- If the physician had an occurrence policy in 1993, this loss is recorded in Schedule P as an accident year 1993 loss.
- If the physician had claims-made coverage from 1993 through 1997, this loss is recorded in Schedule P as an accident year 1997 loss.
- If the physician had claims-made coverage from 1993 through 1995, and then purchased tail coverage on December 31, 1995, this loss is recorded in Schedule P as an accident year 1995 loss.

EXCESS STATUTORY RESERVES

Until the codification of statutory accounting in 2001, excess of statutory over statement reserves were determined in Schedule P for certain long-tailed lines of business whose reported experience in the most recent accident years seemed overly optimistic. The statutory reserves did not affect statutory income, taxable income, GAAP income, or GAAP equity.

The excess of statutory over statement reserves, known as the "Schedule P penalty," was eliminated in 2001. The formula used to calculate the excess of statutory over statement reserves was not considered to be an accurate predictor of loss reserve adequacy. Continued use of this formula contravened the recognition principle of post-codification statutory accounting, which stipulates that liabilities be recognized when they are incurred.

Instead, the adequacy of Schedule P reserves is monitored as follows.

i. The Statement of Actuarial Opinion requires a qualified actuary to opine on the reasonableness of the company's reserves. The report of the Appointed Actuary must reconcile the opinion with the entries in Schedule P, Part 1.

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[51] A fidelity policy covers a firm for losses resulting from embezzlement by its employees. Common fidelity loss scenarios involve (i) company officers with check writing privileges, such as claims adjustors and procurement officers, who might embezzle funds by writing checks to friends or relatives or (ii) members of accounting or investment departments who might divert funds to their own accounts. The embezzlement may continue for many years before the employer becomes aware of it; much embezzlement is never discovered. If the occurrence of the theft were the date of accident, it would be time-consuming and perhaps impossible to ascertain whether the accidents were covered by a given policy. If the date of report were the date of accident, firms may delay reporting the embezzlement until they had purchased or upgraded their fidelity insurance coverage. Instead, the date of accident is the date of discovery, or the date that the embezzlement is assumed to have been discovered.

ii. The reserve adequacy tests performed with the historical loss triangles in Schedule P, Parts 2, 3, 4, and 5 provide actuarial tests of reserve adequacy.
iii. Periodic financial examinations by the state insurance departments using more extensive data provide additional tests of reserve adequacy.

The actuarial tests of reserve adequacy obviated the need for rote statutory formulas.

STRUCTURED SETTLEMENTS

Retroactive reinsurance does not affect the Schedule P entries, since it may be misused to implicitly discount reserves and circumvent statutory accounting reserving philosophy. Structured settlements are similar to retroactive reinsurance. However, structured settlements are used primarily for the benefit of claimants, not to implicitly discount the statutory reserves.

Regulatory authorities and courts often encourage the use of structured settlements. Casualty insurance contracts indemnify policyholders for their liability under tort compensation systems. The policyholder may be liable for negligent operation of a motor vehicle or for negligent manufacture of a harmful product.

Most casualty insurance damages are paid in lump sums. Damages received as compensation for accidents are exempt from federal income taxation, by specific exemption in the Internal Revenue Code. The subsequent investment income on the compensation received is subject to taxation just like any other investment income.

If the lump sum award is used by the claimant to purchase a life annuity, a percentage of each life annuity payment is subject to federal income taxation. The percentage depends on the annuitant's life expectancy and the type of annuity.

> *Illustration:* A life annuity with benefits of $80,000 a year is purchased for a premium of $1,000,000. The annuitant has a life expectancy of 20 years. The expected nominal benefits are $1,600,000 in total, and the premium is ⅝ of this amount. For each benefit payment, ⅜ is subject to federal income taxation and ⅝ is exempt from taxation.

Governments and courts are concerned that lump sum awards may not always be in the best interests of accident victims, particularly if the victim is not competent to manage the funds wisely. In a structured settlement, the insurance company pays the damages as an annuity, either a life annuity or as a combination of a life annuity, an annuity certain, or lump sum payments.

If a structured settlement is properly constructed, *all* the benefits are exempt from federal income taxation. A properly constructed structured settlement has significant tax advantages over a lump sum payment.

Illustration: A young child is permanently disabled by a negligently constructed toy. To avoid potential mismanagement of a lump sum award by the victim or by the victim's guardians, the court awards damages of $5,000 a month ($60,000 a year) for the child's lifetime. The estimated total benefits are $3,000,000, given the child's age, sex, physical condition, and expected life. To fund the award, the casualty insurance company purchases a $5,000 per month life annuity from a life insurance company for $1,000,000. The casualty insurance company owns the life annuity, with the child as the measuring life.

Statutory Accounting for Structured Settlements

The statutory accounting for the structured settlement depends on the terms of the life annuity.

Scenario A: The casualty insurance company designates itself as the payee of the life annuity, and it assigns the payments to the child. The casualty insurance company retains its liability to the child if the life insurance company that issues the annuity fails to pay benefits.

A loss reserve of $3,000,000 is reported in Schedule P, and the $1,000,000 life annuity is a fixed-income financial asset shown on the balance sheet. As the benefits are paid, the Schedule P reserves are reduced by the nominal payments, and the reported value of the life annuity is amortized in accordance with its remaining value.[52]

Illustration: A structured settlement is effected on December 31, 20XX. The accounting entries for 20XX and 20XX+1 are as follows. This illustration assumes that the value of the life annuity decreases by $20,000 during the first year, since the annuitant has a shorter remaining life expectancy.

December 31, 20XX:

		Debit	Credit
Balance sheet:	Case loss reserve:		$3,000,000
Income statement:	Incurred loss:	$3,000,000	
Balance sheet:	Cash paid:		$1,000,000
Balance sheet:	Life annuity:	$1,000,000	

The case reserve on the balance sheet (a credit) balances the incurred loss on the income statement (a debit). The reduction in the cash asset on the balance sheet (a credit) balances the life annuity on the balance sheet (a debit).

52 The value of the life annuity at each subsequent date is based on the illustrative policy values.

December 31, 20XX+1:

		Debit	Credit
Balance sheet:	Loss reserve decrease:	$60,000	
(Cash flow statement:	Paid loss:	$60,000)	
Balance sheet:	Cash paid:		$60,000
Balance sheet:	Cash received from annuity:	$60,000	
Balance sheet:	Life annuity reduction:		$20,000
Income statement:	Miscellaneous income:		$40,000

The reduction in the loss reserve on the balance sheet, a debit, balances the reduction in value of the life annuity and the miscellaneous income, which are credits. The cash received from the annuity balances the cash paid to the claimant. The paid loss is neither a debit nor a credit. It is a cash flow statement entry, not a balance sheet or income statement entry.

Scenario B: The claimant is the payee of the annuity, as well as the measuring life. The cost of the annuity is coded as a paid loss, and the original loss reserve is eliminated. The casualty insurance company has completed its obligations to the claimant by purchasing the annuity. The life insurance company that issued the annuity has the obligation to ensure timely and continued payments. The following are the accounting entries for the same structured settlement if the claimant is the payee.

December 31, 20XX:

		Debit	Credit
Incurral of loss:			
Balance sheet:	Case loss reserve:		$3,000,000
Income statement:	Incurred loss:	$3,000,000	
Structured settlement and purchase of annuity:			
Balance sheet:	Cash paid:		$1,000,000
(Cash flow statement:	Paid loss:	$1,000,000)	
Income statement:	Incurred loss:	–$2,000,000	
Balance sheet:	Case loss reserve:		–$3,000,000
Net of the two transactions:			
Balance sheet:	Case loss reserve:		$0
Income statement:	Incurred loss:	$1,000,000	
Balance sheet:	Cash paid:		$1,000,000

Subsequent payments from the life insurance company to the claimant do not affect the balance sheet or the income statement of the property-casualty insurance company.

A structured settlement with the claimant as the payee causes a sharp decline in the ultimate incurred loss and an increase in the paid loss on the date of settlement or purchase. This affects the observed development patterns in Schedule P, Parts 2, 3, and 4. Structured settlements should be noted in Schedule P, Interrogatory 7.

COMMUTATIONS

Commutations are the reverse of retroactive reinsurance. They have the opposite effect on Schedule P observed loss development as structured settlements have.

In a commutation, the primary insurance company "buys back" a reserve that had been ceded to a reinsurance company. The reserve is generally for long term disability benefits or for workers' compensation indemnity losses. The primary company and the reinsurer may agree that the primary company can more efficiently handle the periodic loss payments to the claimant.

> *Illustration:* One of the claims ceded under a workers' compensation excess of loss reinsurance treaty is a lifetime pension claim with $1,000 weekly benefit payments. Ten years after the inception of the underwriting year, this is the only claim still outstanding.

The remaining life expectancy of the injured worker is 20 years, and the undiscounted loss reserve is $1,040,000. The primary company commutes the claim by *accepting* a payment of $400,000 from the reinsurer and relieving it of its liability. (The primary company buys the reserve by accepting cash; the reinsurer sells the reserve by paying cash. The reserve is a liability, the opposite of an asset.)

The primary company shows the following accounting entries on the date of the commutation:

		Debit	Credit
Commutation of reserve:			
Balance sheet:	Case loss reserve:		$1,040,000
Income statement:	Incurred loss:	$640,000	
Balance sheet:	Cash received:	$400,000	

There is an increase in the reported losses on the date of commutation, which distorts the Schedule P loss development pattern. Structured settlements should be disclosed in Schedule P, Interrogatory 7. In practice, a workers' compensation insurer which effects numerous commutations each year may not consider them sufficiently material for disclosure.

Auxiliary Exhibits

SCHEDULE P TRIANGLES

Schedule P provides several historical triangles for each line of business: three loss triangles, three claim count triangles, and two premium triangles.

- Part 2 shows net incurred losses and defense and cost containment (DCC) expenses.
- Part 3 shows net paid losses and DCC expenses.
- Part 4 shows net bulk and IBNR reserves for losses and DCC expenses.
- Part 5 shows direct and assumed claims closed with loss payment (section 1), claims outstanding (section 2), and claims reported (section 3).
- Part 6 shows earned premiums by exposure year in two formats: direct and assumed (section 1) and ceded (section 2).

Schedule P, Part 7 shows triangles of policy year premiums and losses and of reinsurance commissions. These triangles show transactions on loss sensitive business only. They are designed for the risk-based capital submission, not for monitoring reserve adequacy.

Derived Triangles

Other loss exhibits can be formed from these data. The incurred losses in Part 2 are the sum of paid losses, case reserves, and bulk reserves. A triangle of reported losses (also termed case incurred losses, or paid losses plus case reserves) can be formed as the Part 2 triangle minus the Part 4 triangle. A triangle of outstanding case reserves can be formed as the Part 2 triangle minus the Part 4 triangle minus the Part 3 triangle.

The other commonly used triangles for loss reserve adequacy monitoring are the following:

- Net exposure year earned premium formed as the Part 6 direct plus assumed exposure year earned premium minus the Part 6 ceded exposure year earned premium.
- Total direct plus assumed claims closed (both with payment and without payment) formed as Part 5 direct plus assumed reported claims minus Part 5 direct plus assumed outstanding claims.
- Net loss ratios formed in one of two fashions: (a) Part 2 net incurred losses divided by Part 1 net earned premium, or (b) Part 2 net incurred losses divided by net exposure year earned premium from Part 6. The net exposure year earned premium is the difference between the direct plus assumed premium and the ceded premium.

Average severity triangles are also frequently used in loss reserve adequacy testing:

Exhibit 3.3: Completing the 20X9 Part 3X "Prior" Line

	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8
Prior	0	180	280	355	405	440	490	515	535

The 20X9 payment is the amount in Part 1X, columns 4 – 5 + 6 – 7 (= columns 11 – 8 + 9). For the prior row, this is $23,000 – $2,000 + $0, or $21,000. This figure is added to the cumulative payments through 20X8 in Part 3X to give the cumulative payments through 20X9, or $535,000 + $21,000 = $556,000.

Loss Reserve Adequacy Testing – Prospective Valuation

The primary purpose of Schedule P is to provide the data for prospective tests of loss reserve adequacy. The historical exhibits in Parts 2, 3, 4, 5, and 6 are designed to facilitate these tests. This paper describes the types of tests that may be applied, and it provides illustrations of the major ones.

Schedule P is used by state regulators, tax examiners, rating agencies, financial analysts, and other analysts. This paper provides a complete explanation of the actuarial reserving methods, their strengths, and their potential pitfalls.

FUNDAMENTAL PRINCIPLE OF RESERVE ESTIMATION

The fundamental principle of actuarial loss reserve estimation techniques is that there are stable patterns of loss reporting or loss settlement, and that the observed historical experience is a valid predictor of future expected experience. These patterns may relate to dollar amounts of losses, number of claims, or to ratios of losses to claims, losses to other losses, or losses to premiums. For instance, a chain ladder paid loss development method assumes that the ratio of the paid losses at one evaluation date to the paid losses at the previous evaluation date is relatively stable from accident year to accident year. Observed ratios from past accident years are a valid predictor of expected ratios in future accident years.

The insurance environment is always changing, and company claims practices change as well. The actuarial reserving techniques described here are designed not only to apply the observed historical patterns to future periods but also to modify these patterns in accordance with known or anticipated changes in the insurance environment and company claims practices.

Homogeneity and Stability

Actuarial reserving methods rely on certain assumptions which are not perfectly fulfilled by the Schedule P data:

The stability of the patterns assumed in loss reserve projections varies with the homogeneity of the data. Claims of the same type (homogeneous data), such as indemnity benefits on workers' compensation lower back sprains, show more stable patterns. A mix of claims of varied types, such as all workers' compensation claims, may show less stable patterns.

Heterogeneous data are most problematic when the mix of claim types changes. The private passenger automobile liability exhibits in Schedule P include jurisdictions with both tort liability compensation systems and no-fault compensation systems. When a state changes its compensation system, or when the company changes its mix of business by state, the stability of the reserving patterns is impaired.

Similarly, the workers' compensation exhibits in Schedule P include numerous types of policies, such as first dollar coverage, retrospectively rated policies, and large dollar deductible policies, as well as numerous types of benefits and claims, ranging from physicians' fees for non-disabling injuries to lifetime indemnity payments for permanent total disabilities. When the types of policies issued or the types of benefits change, the stability of the reserving patterns is impaired.

The Schedule P exhibits are a compromise between a simple, unrefined view of the company's total reserves and a refined analysis by homogeneous loss groupings. The analyst working with Schedule P should understand the uses to which the data can be put and the limitations on the reserve indications that are produced.

Reserve estimation requires a good understanding of the external financial environment and of the company's claims handling practices. Changes in claim settlement rates, case reserve adequacy, or inflation must be incorporated into the reserve indications.

Several methods of testing for changes in the company's claims handling practices can be used to make the reserve indications more accurate. Schedule P provides the data needed for some of these tests. Actuaries and regulators should understand the types of tests commonly used and the adjustments needed to improve the reserve indications.

Schedule P, Part 3 is particularly useful for external evaluations of loss reserve adequacy, since it is not dependent upon company reserving policies. It is not affected by changes in the company's case reserve adequacy, about which regulators and outside analysts may have little information.[54] It is most effective for lines of business where there are substantial loss payments in the first year or two and claim settlement rates are stable; examples are personal automobile liability and workers' compensation. It is less useful for lines with long lags in claims reporting and settlement, where the proportion of loss payments is small in the first year or two, and where claim settlement rates fluctuate widely; examples are products liability and non-proportional reinsurance.

LOSS RESERVING TECHNIQUES

Loss reserving techniques can be classified along several dimensions:

1. Estimates of dollars of loss vs separate estimates of claim frequency and claim severity;
2. Paid amounts vs reported amounts; the amount may be either dollars of loss or the number of claims;
3. Chain ladder techniques vs expected loss techniques; the techniques may be applied to both paid amounts and reported amounts and to both dollars of loss and the number of claims; and
4. Estimation along rows (development techniques) vs estimation down columns (trend techniques).

Some reserving methods use combinations of these techniques.

1. The Brosius least squares approach uses a credibility weighted combination of a chain ladder estimate and an expected loss estimate.
2. Some reserving methods estimate claim counts by development techniques and average claim severity by trend techniques.

We explain the reserving techniques that can be done with Schedule P data, so that regulators and other analysts can make optimal use of the information provided. The exposition in this paper does not assume a prior knowledge of the actuarial methods described here, though practical reserving experience is helpful for making efficient use of these techniques.

We begin with a paid loss development using dollars of loss; this is probably the most common method of evaluating the reserve adequacy of other companies when the available information is limited to Schedule P data. The intuition for this reserving technique is that the pattern of payments is stable from accident year to accident year. For instance, if the ultimate

[54] Changes in the company's case reserve adequacy can be estimated from a combined analysis of Parts 2, 3, 4, and 5 of Schedule P, as discussed below in the text. These are estimates gleaned from reported data; they are not as valuable as discussions with claims department personnel that internal actuaries use. For a checklist of the types of information relevant to the reserving actuary, see the appendix to Berquist and Sherman, [1977].

paid losses for an accident year are 250% of the losses paid through the first 12 months in the past, we assume the ratio of 250% holds true for future accident years as well.

ACTUARIAL RESERVING PRINCIPLES

The following principles underlie actuarial reserving methods:[55]

1. *Use of all available data:* We could base the loss development factors on mature years for which we know the ultimate loss payments. This was a common technique in the first half of the twentieth century, and it is still used for short-tailed health insurance reserves (see the calculation of claim completion ratios in Bluhm [2000], chapter 30). For long-tailed lines of business, mature years are old years, and the ratio of ultimate losses to losses paid within the first 12 months may have changed in the intervening time. By using only mature years, we ignore the most recent data, which generally provides the most relevant information.

   For the chain ladder development procedures, we use link ratios, or age-to-age factors. The link ratios compare figures at adjacent development ages, such as 12 months and 24 months, or 24 months and 36 months. The 12 to 24 month paid loss link ratio is the cumulative accident year paid losses evaluated at 24 months of development divided by the cumulative paid losses at 12 months of development for the same accident year. The loss development factor from a given valuation date to ultimate is the cumulative product of the link ratios from that date to ultimate. The development factor from 12 months to ultimate is the cumulative product of the link ratios from 12 months to 24 months, from 24 months to 36 months, from 36 months to 48 months, and so forth.

2. *Stability:* A chain ladder loss development procedure can be implemented with incremental loss payments (or loss reportings) or with cumulative loss payments (or loss reportings). In later development periods, the incremental figures are small, and the ratios of incremental figures are increasingly unstable. To provide greater stability, we use cumulative figures in all the chain ladder development procedures.

3. *Extrapolation and Smoothing:* Loss reserve indications are most important for the long-tailed commercial casualty lines of business, such as workers' compensation, general liability, products liability, medical malpractice, and excess of loss reinsurance, and particularly for lines of business with high volatility in claim reporting and settlement practices. For these lines, claim settlement patterns extend well beyond ten years, which is the limit of the Schedule P loss triangles.

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55 See Feldblum [2002: SB] for a more complete presentation of the principles underlying actuarial reserving methods.

- Average paid loss severity formed as the Part 3 net paid losses divided by the Part 5 direct plus assumed closed claims, either in total or closed with payment only.
- Average reported claim severity formed as the net reported losses (Part 2 minus Part 4 triangles) divided by the Part 5 direct plus assumed reported claims.
- Average outstanding case reserves formed as the net outstanding case reserves (Part 2 minus Part 4 minus Part 3 triangles) divided by the Part 5 direct plus assumed outstanding claims.

LOSS ADJUSTMENT EXPENSES

Each loss triangle includes defense and cost containment expenses, but not adjusting and other expense.[53] The Underwriting and Investment Exhibit does not differentiate between these two types of loss adjustment expenses. Rather, the combined unpaid loss adjustment expenses are shown by line of business in the Underwriting and Investment Exhibit, Part 3A, page 11, column 9. The division between unpaid DCC and unpaid AAO loss adjustment expenses by line of business can also be found in the Insurance Expense Exhibit, columns 15 and 17, in both Part 2 (net business) and Part 3 (gross business).

Before 1998 the NAIC differentiated between allocated loss adjustment expenses (ALAE) and unallocated loss adjustment expenses (ULAE). In general, ALAE became DCC and ULAE became AAO. For some companies, the differences can be material.

The adoption of the new expense classification in 1998 could be by calendar year or by accident year, at the company's option.

- If calendar year adoption is used, the historical triangles contain allocated loss adjustment expenses for the pre-1998 calendar year columns and defense and cost containment expenses for the 1998 and subsequent calendar year columns.
- If accident year adoption is used, the historical triangles contain allocated loss adjustment expenses for the pre-1998 accident year rows and defense and cost containment expenses for the 1998 and subsequent accident year rows.

There is no simple way to obtain completely homogeneous loss triangles.

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[53] Before 1989, the *Schedule P* historical triangles included all loss adjustment expenses. This format was criticized on the grounds that the statutory distribution of unallocated loss adjustment expenses (now adjusting and other expenses) to accident year is arbitrary and lessens the usefulness of the historical loss triangles; see Otteson [1967].

Net vs Direct Experience

The historical loss triangles show net experience. Historical triangles of direct plus assumed business can be formed by combining Annual Statements of successive years, using figures from Schedule P, Part 1.

> *Illustration:* In March 20XX+1 one can compile historical development exhibits of direct plus assumed business from the 20XX and preceding years' Schedule P's, using direct plus assumed columns from Part 1.

The claim count triangles in Part 5, as well as the claim count columns in Part 3, show direct plus assumed experience. Net claim counts are not shown in Schedule P. The Part 5 claim count triangles are shown only for eight lines of business. These are the nine lines for which claim counts are shown in Schedule P, Part 1 minus auto physical damage, which has only a two year exhibit.

The exposure year earned premium triangles in Part 6 show direct plus assumed experience and ceded experience separately. Net experience is the difference between these triangles. For the rationale of showing separate direct plus assumed triangles and ceded triangles instead of net triangles, see the discussion of the Part 6 triangles below.

Several other items are shown in the Schedule P auxiliary exhibits. Part 2 shows one and two year loss developments in columns 11 and 12. Part 3 shows the number of claims closed, with and without loss payments, for nine lines of business, in columns 11 and 12.

The paid loss triangles in Part 3 are easier to compile than the loss triangles in Part 2. They are also less affected by changes in company claims department practices (such as changes in case reserve adequacy), and they are more likely to contain accurate figures. They are commonly used by actuaries to analyze reserve adequacy of peer companies and by regulators to analyze reserve adequacy of domestic companies. We begin the discussion with Part 3.

Part 3 – Paid Losses

Part 3 shows cumulative paid losses and DCC expenses by accident year and development date. The same accident years are shown as in Part 1: ten years for the long-tailed (liability and assumed non-proportional reinsurance) lines of business, and two years for the short-tailed property lines. Ten years of data must be gathered for all lines of business, since they are all included in the ten year Part 3 Summary exhibit; see the discussion of Part 1 above.

The paid loss figures for the current year's Part 3 exhibits can be derived from the Part 3 exhibits of the prior year's Schedule P and Part 1 of the current year's Schedule P.

- Historical data for individual accident years — that is, all figures except those in the first row (prior years) and the right-most column (the current valuation) — are unchanged from those in the previous year's Part 3 exhibit.
- The figures in the right-most column of the Part 3 exhibits are the current valuation. These entries should equal columns 4 – 5 + 6 – 7 (net paid losses plus net paid DCC expenses) in Part 1. This computation is equal to columns 11 – (8 – 9), or total paid loss and loss adjustment expenses minus paid AAO expenses.

The prior years row must be handled separately, as explained below.

THE "PRIOR" YEARS ROW

The Part 3 "prior years" entries can be obtained from the previous year's Annual Statement, after a suitable modification of the figures. The cell in the upper left hand corner of Schedule P, Part 3, which is the first calendar year column for the prior years row, always contains a zero entry. Some printed versions of the Annual Statement place "XXX" in this cell.

> *Illustration:* In the 2010 Annual Statement, the 2001 accident year row begins with loss payments in calendar year 2001. The prior years row, which includes accident years 2000 and prior, begins with loss payments in calendar year *2002*. The rationale for this format is that the prior years row shows the development on the year-end (December 31) 2001 reserve. This development begins with payments in calendar year 2002.

When computing the entries for the prior years row for the 20XX Annual Statement based on the entries in the 20XX–1 Annual Statement, one must take into account the different accident years included in the prior years row and the different starting date for the cumulative loss payments.

The 20XX Schedule P, Part 3, prior years line shows the cumulative loss and DCC payments in calendar years 20XX–8 and subsequent for accident years 20XX–10 and prior. The 20XX–1 Schedule P, Part 3, prior years line shows the cumulative loss and DCC payments in calendar years 20XX–9 and subsequent for accident years 20XX–11 and prior. In the 20XX–1 Schedule P, the 20XX–10 accident year row shows the cumulative payments for that accident year starting in 20XX–10. We explain the calculations by means of an illustration.

Illustration: Completing the Prior Years Row

To complete the prior years row in the 20X9 Schedule P, we follow the steps outlined below.

- We take the prior years row and the 20X8–10 row from the 20X8 Schedule P, subtract from each figure in these two rows the cumulative paid losses and DCC through 20X0, and add the two rows.
- We discard the cumulative paid losses and DCC through 20X0–1 (which is now negative), keep the next entry (a zero) as the first figure in the new prior line, and enter the remaining figures in the rest of the row.
- For the last figure in the row, we add the calendar year 20X9 paid losses and DCC for accident years prior to 20X0 to the last cumulative total. The calendar year 20X9 paid losses and DCC for accident years prior to 20X0 are shown in the 20X9 Schedule P, Part 1, column 11 minus column 9 plus column 8, prior row.

Illustration: The 20X8 Schedule P, Part 3X contains the entries shown in Exhibit 3.1. Figures are in thousands of dollars.

Exhibit 3.1: 20X8 Schedule P, Part 3X, First Two Rows

	X0–1	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8
Prior	0	220	350	400	425	450	460	470	475	480
X0–1	375	600	650	700	750	775	800	840	860	875

In the 20X9 Part 1X exhibit for this line of business, the prior years row shows $23 thousand in column 11 ("Total net paid"), $2 thousand in column 8 ("Adjusting and other payments, direct and assumed"), and $0 in column 9 ("Adjusting and other payments, ceded").

To complete the 20X9 Part 3X exhibit, the cumulative payments through 20X0 are subtracted from the first two rows in the 20X8 Part 3X exhibit. In the example, $220 thousand is subtracted from the 20X8 prior row and $600 thousand is subtracted from the second row (accident year 20X0–1) giving the following entries (Exhibit 3.2):

Exhibit 3.2: Adjustments to the 20X8 Part 3X "Prior" Line

	X0–1	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8
Prior	–220	0	130	120	205	230	240	250	255	260
X0–1	–225	0	50	100	150	175	200	240	260	275

The two rows are summed, and the (calendar year) 20X0–1 column is dropped, as shown in Exhibit 7:

Actuarial Averages

We determine averages of the most recent three and the most recent five link ratios, and we select prospective factors from the historical figures and expectations about future conditions. In this illustration, the selected link ratios lie between the three and five year averages.

The method of taking averages differs among reserving actuaries. The following methods are the most common:

1. Straight averages (equal weighted averages).
2. Straight averages after eliminating the high and low values (an "ex-high-low" average).
3. Weighted averages, where the weights are the paid losses in the earlier of the two valuations. This is equivalent to using the sum of the dollar amounts at the later valuation divided by the sum of the dollar amounts at the earlier valuation.
4. Weighted averages, where the weights increase from the older accident years to the more recent accident years.

There are two distinct rationales for using weighted averages.

Rationale 1: The rationale for using weighted averages where the weights are the paid losses in the earlier of the two valuations is that years with more exposure should be given greater credibility.

- When changes in volume stem from monetary inflation, the simple averages are proper.
- When the changes in volume stem from changes in exposure, the weighted averages are proper.

The former reason for changes in volume (that is, monetary inflation) is more common, so generally simple average should be used.

Rationale 2: When more recent experience is a better predictor of future expected link ratios, weighted averages should be used, where the weights increase from the older accident years to the more recent accident years. The optimal weights can be determined using statistical techniques; see Mahler [1990; 1997].

The elimination of high and low values has both advantages and drawbacks.

- An ex-high-low average may be useful when the data are sparse and random loss fluctuations lead to unreasonable expected link ratios. In addition, Schedule P data are not always "clean." Unusual link ratios may stem from incorrect coding of loss amounts, not from actual payment fluctuations.

- The elimination of high and low values leaves out important information about potential fluctuations in reserve development. The use of ex-high-low averages makes it seem like future development is more stable than it truly is.

A statistical bias may be introduced by using an ex-high-low average, and an existing bias may be corrected by an ex-high-low average. These biases are particularly important when the data are sparse or when the loss distribution is skewed.

The distribution of paid loss link ratios is skewed, since a large court award may result in an unusually high link ratio but the link ratios generally do not fall below unity.[59] An ex-high-low average eliminates the very high link ratios, but it has little effect on the low link ratios. This may create a bias in the projected link ratios, since the high observations are removed.

The preceding paragraph seems to imply that the removal of high and low link ratios may create a bias. The converse may also be true, since the chain ladder method is inherently biased, and the removal of high and low link ratios may partially offset that bias. See Stanard [1985] and Wu [1999] for discussion of the bias in the chain ladder reserving method.

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[59] Recoveries from reinsurance, salvage, and subrogation sometimes cause link ratios below unity. If all figures are properly coded net of recoveries, whether from reinsurance, salvage, or subrogation, and if actual recoveries equal expected recoveries, the net paid loss link ratios should be equal to or greater than unity. If actual recoveries are greater than expected, or if the figures are not coded net of anticipated recoveries, the net net paid loss link ratios may be less than unity.

Exhibit 3.6: Paid Loss Development Test of Reserve Adequacy (dollars in thousands)

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9–10
Averages									
3 year	2.415	1.352	1.171	1.102	1.060	1.044	1.033		
5 year	2.322	1.335	1.162	1.094	1.059				
Select	2.350	1.340	1.170	1.100	1.060	1.040	1.030	1.030	1.020
Cumulative	4.835	2.057	1.535	1.312	1.193	1.125	1.082	1.051	1.020
Pd to Date	$156	$406	$485	$546	$542	$534	$434	$403	$454
Developed	$754	$835	$746	$716	$647	$601	$470	$423	$463
Ultimate	$830	$919	$819	$788	$711	$661	$517	$466	$509
Reserve	$674	$513	$334	$242	$169	$127	$83	$63	$55

PAID LOSS DEVELOPMENT FACTORS

The *cumulative* link ratios, or paid loss development factors, are the cumulative products of the appropriate link ratios (age-to-age factors) in adjacent columns. For instance, the cumulative link ratio from seven to ten years, or 1.082, is the product of 1.030, 1.030, and 1.020, which are the link ratios from seven to eight, eight to nine, and nine to ten years.

The cumulative losses *paid to date* are taken from the last column of Exhibit 3.4: $156,000 is the cumulative accident year 20X9 paid losses at December 31, 20X9, $406,000 is the cumulative accident year 20X8 paid losses at December 31, 20X9, and so forth. The 20X9 paid losses are at one year of maturity; they are placed below the development factor for one to ten years. Similar placement is used for paid losses of other accident years. The next row in Exhibit 3.6, "developed," shows losses developed to ten years of maturity.

PAID LOSS DEVELOPMENT TAIL FACTORS

In the long-tailed (commercial casualty and reinsurance) lines of business, payments continue after ten years. The percent of losses still unpaid after ten years may be estimated in several ways. We can use (i) data reported in Schedule P itself , (ii) external factors, or (iii) curve-fitting techniques.

Schedule P Data

Schedule P data show paid losses at 10 years of maturity for the oldest accident year and the incurred losses at the same maturity in Part 2. If the bulk + IBNR reserves at ten years of development are fully adequate, the Part 2 cumulative incurred losses at ten years of development for the most mature accident year are a reasonable estimate of ultimate losses. The ratio of the paid losses to the incurred losses at that date may be used as the paid loss development tail factor from ten years to ultimate.

Not all companies set fully adequate bulk + IBNR reserves at late development dates, since the statutory margin in undiscounted reserves for late-paying claims may offset the apparent reserve inadequacy.[60] In addition, unanticipated loss development may occur even at late maturities. Examination of the one-year and two-year development in the prior years row should help the analyst determine the statutory reserve adequacy at ten years of development.

This estimate is sensitive to random loss fluctuations, since it uses one ratio to determine a development factor that affects all accident years. As an alternative, the analyst may decompose the paid loss tail factor into two parts: (i) the ratio of paid losses to reported losses at ten years of development and (ii) the ratio of reported losses to incurred losses at ten years of development. The first ratio can be determined from prospective chain ladder developments of paid losses and of reported losses. The second ratio may be estimated from the oldest accident year or the oldest two accident years shown in Schedule P.

The one- and two-year adverse loss developments for the prior years row from the Part 2 exhibits are helpful for selecting a reported loss tail factor. The one-year adverse loss development divided by the reported losses at ten years of maturity for the oldest accident year shown in Schedule P is sometimes used as an estimate of the reported loss tail factor from ten years to ultimate. A similar estimate is provided by one half of the two-year adverse loss development divided by the reported losses at ten years of maturity for the oldest accident year shown in Schedule P. This type of estimate was used by the NCCI for workers' compensation until the mid-1990's (see Feldblum [1992: WCR]).

CAVEATS

This estimate must be used with caution, since various circumstances may distort the expected patterns.

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[60] The "statutory margin" is the difference between statutory reserves and fair value reserves. For most lines of business, the this is the difference between undiscounted and discounted reserves. For workers' compensation and long term disability insurance, this is the difference between reserves valued using tabular discounts only and reserves valued using both tabular and non-tabular discounts.

For the long-tailed lines of business, the analyst develops an actuarial model that reflects the reporting or settlement pattern of the losses. One may construct the model with Schedule P data themselves, or one may adopt the results of other models. The model provides estimates of the expected loss development beyond ten years.

Analysts' views vary regarding the importance of tail effects on loss development. On the one hand, the choice of a tail factor affects all accident years, and it has a leveraged effect on the overall reserve indication. On the other hand, payments made many years in the future have a lower present value than payments made in the near future.

Most reserving methods use standard techniques for (i) tail development factors, (ii) changes in loss cost inflation, and (iii) selection rules for link ratios. We show the most common procedures in the discussion below.

Outline: Paid Loss Development

The format of a paid loss development analysis is as follows.[56] Link ratios, or the ratios of cumulative paid losses at one valuation to cumulative paid losses at the preceding valuation, are calculated for each accident year and valuation date. A prospective link ratio for each development interval is selected from the historical observations, using averages, weighted averages, trends, or other projection techniques.

No single procedure for determining prospective link ratios is appropriate for all lines and companies. One common approach is to use the average of the most recent three to five link ratios, adjusted for random outliers and known or suspected trends. Unusual results should be checked for data errors, and the final selected factors should be smoothed to form a consistent progression. The prospective link ratios show the expected development between adjoining valuation points.

Development factors from each valuation point to 10 years of maturity are the cumulative products of the adjoining link ratios. For example, the development factor from six years to ten years is the product of the link ratios from (a) six to seven years, (b) seven to eight years, (c) eight to nine years, and (d) nine to ten years.

At the current statement date, each accident year shows cumulative paid losses at a different development age. The product of these cumulative paid losses and the paid loss development factors from that development age to ultimate are the estimated ultimate losses by accident year.

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[56] Introductory treatments of paid loss development reserving procedures may be found in Salzmann [1984], Peterson [1981], pages 181–196, and Wiser [2001: FCAS].

The tail factor, or the loss development factor from the last observed development age to ultimate, is determined by statistical modeling techniques or by the adoption of external information. It is often shown separately in the worksheets, so that readers can see the method used to estimate it; see the exhibits below.

Paid loss development procedures may be distorted by changes in inflation rates. The simple method illustrated below is standard actuarial practice. It is appropriate only when inflation rates have remained steady for the entire experience period, and they are expected to remain at the same level in the immediate future.

A better procedure – now commonly used in the actuarial community – is to remove the effects of inflation from the historical loss triangles, perform the paid loss development analysis on "real dollar" amounts, and add back in expected future inflation. The expected future inflation may be either a deterministic rate or a set of stochastic interest rate paths.[57]

ILLUSTRATION: PAID LOSS DEVELOPMENT

We illustrate this procedure with simulated data for a long-tailed line of business (workers' compensation). Exhibit 3.4 shows the Part 3D entries as they would appear in the 20X9 Schedule P for accident years 20X0 through 20X9.[58]

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[57] For the treatment of inflation using a deterministic procedure, see Hodes, Feldblum, and Neghaiwi [1999]; for the treatment using a stochastic procedure, see Hodes, Feldblum, and Blumsohn [1999].

[58] These data are based on actual Schedule P entries for a large commercial lines insurer that was acquired by a peer company in the mid-1990's. The figures have been disguised, and the accident years have been changed.

Exhibit 3.4: 20X9 Schedule P, Part 3D ($000)

Part 3	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	103	226	294	334	363	384	398	412	422	433
20X1		111	238	309	356	387	409	428	442	454
20X2			108	221	286	328	354	375	391	403
20X3				111	238	311	357	392	416	434
20X4					135	299	394	458	504	534
20X5						146	314	418	490	542
20X6							159	343	463	546
20X7								146	353	485
20X8									152	406
20X9										156

Paid Loss Link Ratios

Paid loss link ratios are the ratios of

i cumulative paid losses for a specific accident year at a given valuation date to
ii cumulative paid losses for the same accident year at a valuation date one year earlier.

For instance, the paid loss link ratio from two years to three years of development for accident year 20X6 is $463,000 divided by $343,000, or 1.350. The complete set of link ratios is shown in the table below.

Exhibit 3.5: 20X9 Schedule P, Paid Loss Link Ratios

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 – 10
20X0	2.194	1.301	1.136	1.087	1.058	1.036	1.035	1.024	1.026
20X1	2.144	1.298	1.152	1.087	1.057	1.046	1.033	1.027	
20X2	2.046	1.294	1.147	1.079	1.059	1.043	1.031		
20X3	2.153	1.301	1.148	1.098	1.061	1.043			
20X4	2.215	1.318	1.162	1.100	1.060				
20X5	2.151	1.331	1.172	1.105					
20X6	2.157	1.350	1.179						
20X7	2.418	1.374							
20X8	2.671								

The row labels are accident years; the column captions are development intervals. The caption "2 to 3" means from two years of development to three years of development. We have rotated the triangle, turning the diagonals in Exhibit 3.4 into the columns in Exhibit 3.5.

i The second column in Exhibit 3.4 shows cumulative paid amounts on December 31, 20X1.
ii The second column in Exhibit 3.5 shows paid loss development from 1 year after the inception of the accident year to 2 years after the inception of the accident year.

Each column of Exhibit 3.5 is the ratio of two *diagonals* in Exhibit 3.4. The diagonals in Exhibit 3.4 represent development ages. The diagonals in Exhibit 3.5 represent calendar years.

> *Illustration:* The second column in Exhibit 3.5 shows paid loss development from 1 year to 2 years, or from 12 months to 24 months. (Reserving actuaries generally speak in months of development, not years of development.) The link ratio of 2.671 for accident year 20X8 is the ratio of 406 to 152. The link ratio of 2.418 for accident year 20X7 is the ratio of 353 to 146.

No link ratio is calculated for the 20X9 accident year, since there is only one valuation. No link ratios are shown for the prior row, since the claims in this row stem from different accident years. For the prior years row, the time since inception of the accident year varies by claim.

not experienced any development past five years in the past, and it does not expect any development past five years in the future.

The illustration gives enough data to determine three historical link ratios for each development period. We use the straight average of the three observed link ratios.

The loss development factors are the backwards cumulative products of the link ratios. All link ratios past five years of development are unity, so the loss development factor from five years to ultimate is unity.

Reserve Adequacy

The indicated ultimate losses for each accident year equal the cumulative paid losses at the valuation date times the appropriate paid loss development factor to ultimate. For instance, accident year 20X9 has cumulative paid losses of $187,200 as of 12 months of development. We multiply by the loss development factor of 3.863 from 1 year to ultimate: $187,200 × 3.863 = $723,200.

For the four most recent accident years, the incurred losses shown in Part 2 are less than the indicated reserves from the paid loss development analysis. The table compares indicated ultimate losses to held ultimate losses. This is the same as comparing indicated reserves to held reserves, since the paid loss component of the two is the same. The total reserve deficiency is $270,000.

Expected Loss Reserve Estimation

Chain ladder paid loss development procedures require a credible base of paid losses in each accident year from which to estimate the future loss payments for that accident year. The chain ladder estimation procedure is less useful when most claims are not settled until several years after the occurrence date or when claim sizes are highly volatile. In these situations, the claims that have already been paid in the most recent accident years do not form a sufficiently credible base for estimation of future loss payments.

An alternative set of reserve estimation procedures relies on expected losses instead of payments made to date. The conceptual difference between chain ladder and expected loss reserving procedures is explained in the illustration below.[62]

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[62] The expected loss reserving method was first introduced by Bornhuetter and Ferguson [1972]. Textbook summaries of this method may be found in Salzmann [1974], Wiser [2001], and Peterson [1981]. The conceptual differences between chain ladder method and expected loss methods are examined in Brosius [1993] and Feldblum [2002: SB].

Illustration: The 20XX private passenger automobile earned premium is $100 million. The expected loss ratio for accident year 20XX is 80%, and 25% of the claim payments are expected to be made during the accident year.

The paid loss development factor from 12 months to ultimate is 4.000, since ¼ of the claim payments are expected during the first 12 months and ¾ are expected subsequently.

Scenario A: In January 20XX+1, the company shows $20 million of claim payments in accident year 20XX. It is estimating reserves for its 20XX Schedule P, Part 1B.

- The chain ladder paid loss development procedure indicates that ultimate 20XX losses are $20 million × 4.000 = $80 million. The indicated reserve = $80 million – $20 million = $60 million.
- The expected loss reserving method says that 75% of estimated ultimate losses will be paid after 12 months since inception of the accident year. Since the expected loss ratio is 80%, the indicated reserves are 75% × 80% × $100 million = $60 million.

Since the claim payments to date equal the expected claim payments to date, the chain ladder and expected loss reserving methods provide the same reserve indication.

Scenario B: In January 20XX+1, the company shows $25 million of claim payments in accident year 20XX. It is estimating reserves for its 20XX Schedule P, Part 1B.

- The chain ladder paid loss development procedure indicates that ultimate 20XX losses are $25 million × 4.000 = $100 million. The indicated reserve = $100 million – $25 million = $75 million.
- The expected loss reserving method says that 75% of estimated ultimate losses will be paid after 12 months since inception of the accident year. Since the expected loss ratio is 80%, the indicated reserves are 75% × 80% × $100 million = $60 million.

For the expected loss reserving method, we use the expected loss ratio when the book of business is priced, not the expected loss ratio at the reserve date.

> *Illustration:* The expected losses after the accident year has expired are $25 million + $60 million = $85 million, and the expected loss ratio is 85%. For the expected loss reserving method, we use the original 80% expected loss ratio.

The differing reserve indications reflect different perspectives on the higher than expected claim payments in the first 12 months since inception of the accident year.

- The chain ladder method assumes that the higher claim payments in the first 12 months reflect higher expected losses in total. Just as the claim payments to date are 25% higher than expected [25% = ($25 million – $20 million)/$20 million], the ultimate incurred losses are 25% higher than originally expected [($100 million – $80 million)/$80 million = 25%].

- The expected loss method assumes that the higher claim payments in the first 12 months reflect random loss fluctuations; they do not reflect higher total incurred losses. We continue with the original expected loss ratio to estimate future loss payments.

The proper interpretation of the higher than expected claim payments in the first 12 months depends on the type of claim.

- Workers' compensation indemnity benefits are paid weekly. Most of the payments in the first 12 months are partial payments, paid in accordance with statutory benefit schedules. Higher than expected partial payments are indicative of higher than expected incurred losses in total.
- In contrast, medical malpractice claims are relatively independent of each other. Higher than expected claim payments in the first 12 months probably reflect a few unusual settlements, not necessarily higher than expected total incurred losses.[63]

The expected loss reserving method requires an initial estimate of incurred losses for each accident year. The reserving actuary would normally use an expected loss ratio provided by the pricing actuary.

ILLUSTRATION: EXPECTED LOSS RESERVING METHOD

We develop reserve indications using the expected loss method for the workers' compensation illustration used earlier. The net workers' compensation earned premium in Schedule P, Part 1D are shown below (figures are in millions of dollars). For 20X1 through 20X5, the expected loss ratio for workers' compensation was 75%. In 20X6, marketplace competition worsened, and the expected loss ratio from 20X6 through 20X9 was 80%.

Year	Premium	Year	Premium	Year	Premium
20X1	600	20X4	850	20X7	1000
20X2	650	20X5	900	20X8	1100
20X3	700	20X6	950	20X9	1100

Data

[63] The chain ladder and expected loss reserving methods may be viewed as ideal cases: the chain ladder method gives full credibility to the observed experience as an estimator of the remaining loss payments, and the expected loss method gives no credibility to the observed experience as an estimator of the remaining loss payments. The Stanard-Bühlmann method and the least squares method give partial credibility to the observed experience; see below.

For lines of business with significant audits or retrospective premiums, the reserving actuary may use the net exposure year earned premium instead of the net calendar year earned premium. The net earned premium by exposure year equals Schedule P, Part 6D, section 1, column 11, minus Schedule P, Part 6D, section 2, column 11; see the discussion of Part 6 below in this paper.

The expected loss ratio by calendar year or by exposure year is not shown in the Annual Statement. The reserving actuary would use an estimate provided by the pricing actuary.

Expected Loss Factors

The expected loss factor at K months of development is the percentage of ultimate losses that will be paid between K months of development and ultimate. In the private passenger automobile illustration above, the expected loss factor at 12 months of development is 75%.

The expected loss factors may be derived from the loss development factors.

> *Illustration:* The paid loss development factor from 24 months to ultimate is 2.500. This implies that for each dollar of loss paid up through 24 months of development, an additional $1.50 will be paid after 24 months. The expected loss factor is $1.50/($1.00+$1.50) = 60%.

If LDF is the loss development factor from K months to ultimate, the expected loss factor at K months is (LDF−1)/LDF = 1 − 1/LDF.

Exhibit 3.10 shows the loss development factors, the expected loss factors, and the indicated reserves for the workers' compensation illustration earlier in this paper.

Illustration: Through most of the 1990's, insurers believed that asbestos claim activity had subsided. A surge in asbestos claims in 1999 and 2000 caused enormous adverse loss development for the prior years row for some insurers.

If the company has changed its mix of business or the type of policy forms over the ten year historical period, the estimated tail factor may not be appropriate for current conditions.

Illustration: The company may have switched its workers' compensation business from first dollar policies (or retrospectively rated policies) to large dollar deductible policies. These policy types have different expected tail factors: they are much higher for large dollar deductible policies than for first dollar policies or retrospectively rated policies.

Illustration: In the 1980's, companies switched from occurrence policies to claims-made policies in medical malpractice, and they added an absolute pollution exclusion in comprehensive general liability policies. Both these changes reduced the expected paid loss tail factor.

External Factors

The analyst may assume that the paid loss tail factor does not differ significantly by company at ten years of maturity. This may be a reasonable assumption for workers' compensation, where benefits are mandated by the state compensation system and are not tied to the policy form. The analyst may use a paid loss tail factor determined from rating bureau industry data or from another company's data. The caveats mentioned above should be considered.

Curve-Fitting

The standard actuarial technique for selecting paid loss tail factors is to fit a curve to the observed paid loss link ratios and to extend the curve past the most mature development interval. The inverse power curve is often used for this purpose; see Sherman [1984] and Hodes, Feldblum, and Blumsohn [1999].

This is the method of choice for actuarial analyses. Two caveats for this method should be considered:

- Two or more types of curves may provide a good fit to the observed data, but they may give different projections for the paid loss tail factor.
- If claims which settle quickly differ significantly from claims which remain open at ten years of maturity, the curve fitting should begin after several years of development.

Illustration: In workers' compensation, temporary total claims settle quickly, and they have little expected development at ten years of maturity. Most claims outstanding at ten years of maturity are lifetime pension cases. The curve-fitting should begin at three or four years of maturity so as to eliminate the majority of temporary total claims.

Ultimate Losses

For the illustration in this section, we use a paid loss tail factor of 1.10 from ten years to ultimate. We can think of this as development continuing for nine more years in the following pattern:

- another two years of 1.020 link ratios
- three years of 1.010 link ratios
- four years of 0.005 link ratios

The "ultimate" losses in Exhibit 3.6 are the developed losses increased by 10 percent. These may be compared with the final incurred losses shown in Part 2, column 10. (The ultimate reported losses are shown as the sum of the "paid to date" and the "reserves" rows in Exhibit 3.6.) The ultimate paid losses total $6,221,000, and the incurred losses shown on Part 2 total $6,244,000. The Part 3 prospective test shows adequate reserves.[61]

ILLUSTRATION: PROSPECTIVE VALUATION

We show a private passenger automobile illustration that uses similar techniques. The 20X9 Schedule P, Parts 2B and 3B, show the following data (figures in thousands of dollars).

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[61] In practice, reserve adequacy tests often show large discrepancies between indicated reserves and held reserves, particularly for the long-tailed lines of business. Compared to other liability, products liability, medical malpractice, and non-proportional reinsurance, though, workers' compensation reserves are more stable, since the benefits are fixed by statute, both in magnitude and in timing. The major uncertainty in indemnity benefits is the duration of disability on non-permanent cases and the mortality rates on permanent cases. For sufficiently large blocks of business, both of these have relatively compact distributions. The major uncertainty for medical benefits is the rate of inflation and the extent of utilization of medical services. Over a large enough block of business, these risks also have relatively compact distributions. Butsic [1988, p. 179] summarizes this view by saying that "Workers' Compensation reserves should have a lower risk than Other Liability reserves, even though the average payment durations are about the same, because Workers' Compensation loss reserves consist partly of fixed, more predictable, life pension benefits."

Exhibit 3.7: Extract from Part 2B - Private Passenger Automobile Liability: Incurred Losses & Defense and Cost Containment Expenses Reported at Year-End

Loss Year	'X4	'X5	20X6	20X7	20X8	20X9
20X0	–	–	477.6	477.6	477.6	477.6
20X1	–	–	490.8	500.8	490.8	490.8
20X2	–	–	460.2	460.2	460.2	460.2
20X3	–	–	476.0	470.4	470.4	470.4
20X4	–	–	591.0	609.4	603.3	603.3
20X5	–	–	579.7	627.0	691.8	650.4
20X6	–	–	738.2	775.1	784.8	783.1
20X7	–	–	–	584.0	601.1	599.4
20X8	–	–	–	–	608.0	631.4
20X9	–	–	–	–	–	624.0

Exhibit 3.8: Extract from Part 3B - Private Passenger Automobile Liability: Cumulative Paid Losses & Defense and Cost Containment Expenses at Year-End

Loss Year	'X4	'X5	20X6	20X7	20X8	20X9
20X0	–	–	477.6	477.6	477.6	477.6
20X1	–	–	490.8	490.8	490.8	490.8
20X2	–	–	460.2	460.2	460.2	460.2
20X3	–	–	428.4	470.4	470.4	470.4
20X4	–	–	472.8	548.4	603.3	603.3
20X5	–	–	376.8	501.6	588.4	650.4
20X6	–	–	206.7	465.1	627.9	728.4
20X7	–	–	–	175.2	390.7	527.4
20X8	–	–	–	–	182.4	410.4
20X9	–	–	–	–	–	187.2

Using the Schedule P data, we test the adequacy of the company's private passenger automobile liability loss and DCC reserves with a paid loss chain ladder analysis.

Link Ratios

We form paid loss link ratios for each development period. The diagonals of Schedule P have been converted into columns in the table below.

Exhibit 3.9: Private Passenger Automobile Liability:
Paid Loss Link Ratios and Reserve Indications

Loss Year	1 – 2 years	2 – 3 years	3 – 4 years	4 – 5 years	5 – 6 years	6 – 7 years	7 – 8 years	8 – 9 years
20X0	–	–	–	–	–	–	1.000	1.000
20X1	–	–	–	–	–	1.000	1.000	1.000
20X2	–	–	–	–	1.000	1.000	1.000	–
20X3	–	–	–	1.098	1.000	1.000	–	–
20X4	–	–	1.160	1.100	1.000	–	–	–
20X5	–	1.331	1.173	1.105	–	–	–	–
20X6	2.250	1.350	1.160	–	–	–	–	–
20X7	2.230	1.350	–	–	–	–	–	–
20X8	2.250	–	–	–	–	–	–	–
20X9	–	–	–	–	–	–	–	–
Average	2.243	1.344	1.164	1.101	1.000	1.000	1.000	1.000
Cum LDF	3.863	1.722	1.282	1.101	1.000	1.000	1.000	1.000
Cum Paid	187.2	410.4	527.4	728.4	650.4	603.3	470.4	460.2
Indicated	723.2	706.7	676.1	801.9	650.4	603.3	470.4	460.2
Held	624.0	631.4	599.4	783.1	650.4	603.3	470.4	460.2
Adequacy	–99.2	–75.3	–76.7	–18.8	0	0	0	0

In practice, liability payment patterns extend for many years. This illustration is simplified, with no development past five years. The paid loss link ratio past five years are unity, and by the fifth year, all the cumulative paid loss figures equal the incurred loss figures. The company has

a. For accident year 20X9, $120 million of premium (30% × $400 million) has been processed so far, and $75 million of losses have been paid.
b. For accident year 20X8, $187.5 million of premium (50% × $375 million) has been processed so far, and $185 million of losses have been paid.

We do this for all ten accident years. The total processed premium is $2117.5 million. The total paid losses are $1700 million. The total premium that remains to be processed is $817.5 million. We form the equation

$2117.5 million : $1700 million :: $817.5 million : X

We solve for X, the total loss reserve, as X = $1700 × $817.5 ÷ $2117.5 = $656.3 million.

WORKERS' COMPENSATION ILLUSTRATION: STANARD-BÜHLMANN

We apply the Stanard-Bühlmann reserving method to the workers' compensation illustration. The earned premiums are not necessarily at the same adequacy level for all accident years. We used different expected loss ratios for the Bornhuetter-Ferguson expected loss method in the previous section of this paper. For the Stanard-Bühlmann application, we assume that we have no information about the adequacy level of the earned premiums, so we use the unadjusted premium figures from Schedule P, Part 1D.

Exhibit 3.13: Stanard-Bühlmann Method using Paid Losses (dollars in thousands)

Months	12	24	36	48	60	72	84	96	108
LDF to10 yrs	4.835	2.057	1.535	1.312	1.193	1.125	1.082	1.051	1.020
LDF to ultimate	5.319	2.263	1.689	1.443	1.312	1.238	1.190	1.156	1.122
Processing factor	0.188	0.442	0.592	0.693	0.762	0.808	0.840	0.865	0.891
Premium	$1100	$1100	$1000	$950	$900	$850	$700	$650	$600
Processed premium	$207	$486	$592	$658	$686	$687	$588	$562	$535
Unprocessed prem	$893	$614	$408	$292	$214	$163	$112	$88	$65
Losses paid to date	$156	$406	$485	$546	$542	$534	$434	$403	$454

We form totals from the nine accident years.

- The total processed premium is $5,001.
- The total unprocessed premium is $3,960.
- The total losses paid to date are $2,849.

We solve for the losses to be paid in the future, or the reserve:

$5,001 : $2,849 :: $3,960 : Reserve
Reserve = $2,849 × $3,960 / $5,001 = $2,256.

For completing Schedule P, we must allocate the reserves to accident year. We assume that the ratio of unprocessed premium to unpaid loss is constant from year to year. If the unprocessed premium for accident year "i" is UP_i, the reserve for accident year "i" is

$$\text{Reserve}_i = \text{UP}_i \times \text{Reserve}_{total} / \text{UP}_{total}$$

where UP_{total} is the total unprocessed premium and Reserve_{total} is the total reserve.

Illustration: In the example above, the total unprocessed premium is $3,960,000, and the total reserve $2,256,000. The unprocessed premium for accident year 20X9 is $893,000. The indicated reserve for accident year 20X9 is $893,000 × $2,256,000 / $3,960,000 = $508,739.

The least squares reserving method was developed by Dr J. Eric Brosius [1993] as a combination of the chain ladder reserving method and the expected loss reserving method. The least squares method uses linear regression to determine the optimal weighting of the chain ladder method and the expected loss method in each development period.

- The chain ladder reserving method assumes that the ultimate losses in each accident year equal the cumulative paid losses in that accident year times the loss development factor. The reserves in that accident year equal the ultimate losses minus the cumulative paid losses, or reserves = cumulative paid losses × (LDF – 1).
- The expected loss reserving method assumes that the reserves in any accident year equal the expected losses in that accident year times the expected loss factor.

Each method is reasonable in certain circumstances. For long term disability insurance, the benefits remaining to be paid to disabled policyholders is best estimated as a percentage of the benefits already paid. For casualty excess of loss reinsurance treaties, the losses to be paid in the future have little relation to the losses already paid in that accident year.

For most lines of business, some of the unpaid losses are better estimated as a function of the losses already paid and some of the unpaid losses are better estimated as a function of initial expected losses.

Let LDF_t = the loss development factor at development date "t"
XLF_t = the expected loss factor at development date "t"
$Paid_{i,t}$ = the paid losses for accident year "i" at development date "t"
XL_i = the expected losses for accident year "i"
$Unpd_{i,t}$ = the unpaid losses for accident year "i" at development date "t"

- The chain ladder reserving method says that $Unpd_{i,t} = (LDF_t - 1) \times Paid_{i,t}$.
- The expected loss reserving method says that $Unpd_{i,t} = XLF_t \times XL_i$.

The least squares reserving method uses a weighted average of these two estimates. If the weight for the chain ladder reserving method is "w," the indicated reserve is

$$Unpd_{i,t} = (1 - w) \times XLF_t \times XL_i + w \times (LDF_t - 1) \times Paid_{i,t}$$

Exhibit 3.10: Expected Loss Method using Paid Losses (dollars in millions)

Months	12	24	36	48	60	72	84	96	108
Premium	$1100	$1100	$1000	$950	$900	$850	$700	$650	$600
Expected loss ratio	80%	80%	80%	80%	80%	75%	75%	75%	75%
Expected losses	$880	$880	$800	$760	$720	$638	$525	$488	$450
LDF to10 yrs	4.835	2.057	1.535	1.312	1.193	1.125	1.082	1.051	1.020
LDF to ultimate	5.319	2.263	1.689	1.443	1.312	1.238	1.190	1.156	1.122
Expected loss factor	0.812	0.558	0.408	0.307	0.238	0.192	0.160	0.135	0.109
Loss reserve	$715	$491	$326	$233	$171	$122	$84	$66	$49

- The top row shows the months of development. The dollar figures and percentages are for the accident year at that age of development on December 31, 20X9. In the column marked "12," the LDF to ultimate is the loss development factor from 12 months to ultimate. The premium of $1,100 million is the premium for accident year 20X9, which is now at 12 months of development.

- The row labeled premium is the net earned premium in Schedule P, Part 1, column 3. If the premiums are subject to audits or retrospective adjustments, the analyst may use the exposure year net earned premiums from Schedule P, Part 6. For any exposure year, these are the cumulative direct earned premiums in Part 6, section 1 minus the cumulative ceded earned premiums in Part 6, section 2.

- The expected loss ratios are internal company estimates. The expected losses are the product of the earned premium and the expected loss ratio.

- The loss development factors to 10 years of development are taken from the chain ladder illustration above. The loss development factors to ultimate assume a tail factor of +10%: $4.836 \times 1.100 = 5.319$.

- The expected loss factor equals unity minus the reciprocal of the loss development factor: $1-1/5.319 = 0.812$.

- The loss reserve equals the expected losses times the excess loss factor: $880,000 × 0.812 ≈ $715,000.

The total reserve for these ten accident years is $2,258,000. The chain ladder paid loss development method used earlier gave a reserve indication of $2,260,000.

Expected loss reserving methods are frequently used either instead of chain ladder methods or as complements to chain ladder methods. These methods are equally applicable to paid losses or reported losses; see the discussion of reported loss reserve indications further below in this paper. The Stanard-Bühlmann reserving method and the least squares reserving method are expected loss reserving methods that are particularly useful for outside analysts.

THE STANARD-BÜHLMANN RESERVING PROCEDURE

Outside analysts do not know the expected loss ratio. Even the in-house expected loss ratio may not be sufficient to estimate the expected losses. The earned premium times the expected loss ratio is a suitable estimate only when the indicated premium is also the premium charged. The estimate must be adjusted when the premium in the rate manual is not the pricing actuary's indicated premium. It must be further adjusted when underwriters provide schedule credits and debits, as is commonly done in the commercial lines of business.

The Stanard-Bühlmann reserving method derives the expected losses from the historical experience. We explain first the intuition for this method by means of an illustration, and we then apply the method to the Schedule P workers' compensation data used above.

Percentages Paid

The Stanard-Bühlmann reserving method uses expected patterns of percentages paid at each development date. A paid loss development factor of LDF from "k" months to ultimate means that 1/LDF of total losses have been paid by the development date and (LDF–1)/LDF of total losses are expected to be paid subsequent to the development date.

> *Illustration:* The paid loss development factor from 12 months to ultimate is 5.000. The cumulative paid losses at 12 months of development is 1/5 = 20%. The percentage of losses expected to be paid after 12 months of development is (5–1)/5 = 80%.

For a Stanard-Bühlmann reserving method using reported losses instead of paid losses, we substitute reported losses for paid losses in all the computations.

- A paid loss reserving method gives the total (case + bulk + IBNR) reserve indication.
- A reported loss reserving method gives the bulk + IBNR reserve indication.

We show first the intuition for the Stanard-Bühlmann method, and we then apply the method to the workers' compensation illustration used in this paper.[64]

ILLUSTRATION: STANARD-BÜHLMANN RESERVING METHOD

We have determined the following percentages of losses that are paid by each development date from the inception of the accident year.

Exhibit 3.11: Stanard-Bühlmann Loss Lags

Loss Lag	Percent Paid	Loss Lag	Percent Paid
12 mos	30%	72 mos	85%
24 mos	50%	84 mos	90%
36 mos	65%	96 mos	94%
48 mos	75%	108 mos	97%
60 mos	80%	120 mos	99%

At December 31, 20X9, we have the following data on premiums and cumulative paid losses for the ten most recent accident years from Schedule P, Part 1.

Exhibit 3.12: Adjusted Premiums and Paid Losses by Accident Year

Year	Adjusted Premiums	Paid Losses	Year	Adjusted Premiums	Paid Losses
20X0	200 million	150 million	20X5	300 million	185 million
20X1	220 million	155 million	20X6	320 million	205 million
20X2	240 million	200 million	20X7	340 million	155 million
20X3	260 million	175 million	20X8	375 million	185 million
20X4	280 million	215 million	20X9	400 million	75 million

[64] Complete documentation of the Stanard-Bühlmann reserving method may be found in Stanard and Feldblum [2002: SB]. The Stanard-Bühlmann Practitioner's Guide explains the rationale for the expected loss reserving methods, the needed adjustments to premium, and illustrations of the method in various scenarios.

Premiums and Losses

In theory, the premiums should be adjusted for rate level changes and for loss cost trends so that the premiums are at the same level of adequacy for all accident years. If we have the needed information, we can make the appropriate adjustments. State regulators and outside analysts would not have the needed information, and even an in-house actuary might find the adjustments too difficult. We assume here that we lack the information needed for adjusting the earned premium, so we use the raw data in Schedule P, Part 1.[65]

To see the intuition for the Stanard-Bühlmann reserving method, consider year 20X9. The premium is $400 million. By 12 months from the inception of the accident year, 30% of the premium, or $120 million, has been processed into paid losses. The other 70% of the premium, or $280 million, has not yet been processed into paid losses.

The word "processed" warrants explanation. The premium does not *become* paid losses. Rather, there is some relationship between the $400 million of premium and the ultimate paid losses. We don't know this relationship, since we don't know the expected loss ratio and we don't know the level of premium adequacy. We know only that at 12 months of development, 30% of the losses should have been paid. The $120 million of "processed" premium has the same relationship to the losses that have already been paid as the other $280 million of premium has to the losses that are yet to be paid.

The chain ladder reserving method uses the accident year information to determine the relationship. The $120 million of premium that has already been processed corresponds to $75 million of paid losses. This implies that

$120 million : $75 million :: $280 million : X

X = $75 million × $280 million / $120 million = $175 million.

This method gives high credibility to the $75 million of paid losses in accident year 20X9. If losses are volatile, we don't want to give too much credence to the $75 million of losses that have been paid as of 12 months for accident year 20X9.

Instead, we would like to combine the various accident years. For most purposes, we can not add dollars from two different years, since a dollar from year X is worth more than a dollar from year X+1 when the inflation rate is positive. But here we are comparing premiums to losses. To add the figures from different years, we assume that the change in premiums from year to year is about the same as the change in expected losses from year to year. We combine the processed premium from each year, and we combine the paid losses from each year.

[65] For a complete review of the Stanard-Bühlmann reserving method, see Stanard and Feldblum [2002].

We assume that the premium is at the same level of adequacy for each accident year – that is, the expected loss ratio is the same for all accident years.[66] We divide by the premium:

$$\text{Unpaid loss ratio}_{i,t} = (1 - w) \times XLF_t \times ELR + w \times (LDF_t - 1) \times \text{Paid Loss Ratio}_{i,t}$$
$$\text{Unpaid loss ratio}_{i,t} = \alpha + \beta \times \text{Paid Loss Ratio}_{i,t}$$

We use least squares regression analysis to estimate optimal value of α and ß.

Illustration: Least Squares Reserving Method

We show a simple illustration of the least squares reserving method, and we then apply the technique to the workers' compensation Schedule P exhibits used earlier.

The earned premiums and cumulative paid losses by accident year and development date are shown below.

Exhibit 3.14: Least Squares Reserving Method

Accident	Earned Prem	Paid Losses ($000's)			
Year	($000's)	12 mos.	24 mos.	36 mos.	48 mos.
20X2	1,700	595	935	1,156	1,275
20X3	1,900	760	950	1,140	1,330
20X4	2,000	600	1,100	1,400	1,600
20X5	2,200	1,100	1,320	1,430	
20X6	2,500	1,000	1,500		
1997	2,600	1,300			

We estimate the paid losses for accident year 20X5 at 48 months of development.

We have three mature accident years, 20X2-20X4, having losses at both 36 months and 48 months of development. Let "x" be the 36 month losses and "y" be the 48 month losses. We estimate y as a linear function of x:

$$y = a + b \times x$$

[66] This is the same assumption used for the Stanard-Bühlmann reserving method. If the premiums are not at the same adequacy level for all accident years, further adjustments would be necessary. See Stanard and Feldblum [2002: SB] for explanation of the premium adjustments.

We use least squared regression to estimate the parameters "a" and "b."

The regression analysis is proper only if the units in which "x" and "y" are expressed are not changing over time. Monetary inflation causes the value of a dollar to change over time, so we can't use the dollar amount of losses in our regression equation. The 20X2 losses are in dollars that are worth less in real terms than the dollars of 20X3 losses.

If the premiums are at the same level of adequacy in each accident year, the loss ratios are expected to remain the same over time. We perform the regression on the loss ratios, not on the absolute dollars of loss.[67]

We redefine the variables: "x" is the reported loss ratio at 36 months and "y" is the reported loss ratio at 48 months. The reported loss ratios at 36 months and at 48 months for accident years 20X2 – 20X4 are shown below:

Exhibit 3.15: Least Squares Estimation

Acc Yr	x	y	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$
20X2	68%	75%	0.000400	0.000000
20X3	60%	70%	0.003600	0.003000
20X4	70%	80%	0.001600	0.002000
Average	66%	75%	0.001867	0.001667

For accident year 20X2, *"x" = 1,156 / 1,700 = 68% and "y" = 1,275 / 1,700 = 75%.*

The minimum least squares parameters are

$$y = a + bx, \quad b = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2}, \quad a = \bar{y} - b\bar{x}$$

Using the values in the table above, we get

b = 0.001667 / 0.001867 = 89.29%
a = 75% – 89.29% × 66% = 16.07%

[67] One often sees regression analyses spanning several years performed on nominal dollars. Such analyses are generally flawed; they would be improved by deflating the nominal dollars to a common base.

In accident year 20X5, the 36 month paid loss ratio is 65%. Using the values of "a" and "b" above, the 48 month paid loss ratio is estimated as

$$16.07\% + 89.29\% \times 65\% = 74.11\%$$

Since the earned premium for accident year 20X5 is $2.2 million, the estimated paid losses at 48 months equal $2.2 million × 74.11% = $1.63 million.

ILLUSTRATION: WORKERS' COMPENSATION DATA

We apply the least squares reserving method to the workers' compensation Schedule P triangle. The linear regression requires a credible set of data points. Performing the linear regression on two or three mature accident years often generates constant terms that are less than zero or slope terms that are less than unity. To avoid these problems, we generate the paid losses at 120 months of development by the chain ladder reserving method for accident years 20X0 through 20X4. We use the least squares reserving method to generate the paid losses at 120 months of development for the subsequent accident years.

We estimate the paid losses at 120 months of development from the paid losses at an earlier maturity by the linear regression equation below:

$$\text{paid loss at 120 months} = \alpha + \beta \times \text{paid loss at earlier maturity}$$

Illustration: For accident years 20X0 through 20X4, we estimate the paid losses at 120 months of maturity with the chain ladder reserving method. We then form the linear regression

$$\text{paid loss at 120 months} = \alpha + \beta \times \text{paid loss at 60 months}$$

We derive the values of α and β by a least squares linear regression analysis. Using the values of α and β together with the accident year 20X5 paid losses at 60 months of development, we derive the paid losses at 120 months for accident year 20X5.

MONETARY INFLATION

Monetary inflation changes the units in each accident year. Because the regression analysis is done over a period of years, the units in which α is expressed differ by year. For accident year 20X0, the units are 20X0 dollars; for accident year 20X1, the units are 20X1 dollars.

To convert the dollar values in the different years into comparable figures, we divide the paid loss amounts by that year's earned premium. If the premium figures are at the same adequacy level for each year, the loss ratios are in comparable units.[68]

We use the earned premium figures from Schedule P, Part 1 to determine the loss ratios. The earned premiums, in thousands of dollars, are the same as the premiums used for the expected loss reserving method and the Stanard-Bühlmann reserving method.

Exhibit 3.16: Earned Premium by Accident Year for Least Squares Reserving Method

Accident Year	Earned Premium	Accident Year	Earned Premium
20X0	$560	20X5	$900
20X1	$600	20X6	$950
20X2	$650	20X7	$1,000
20X3	$700	20X8	$1,100
20X4	$850	20X9	$1,100

We divide the paid losses by the earned premiums to convert the cumulative paid loss triangle into a paid loss ratio triangle.

[68] This is the same assumption used for the Stanard-Bühlmann reserving method. A rigorous analysis would adjust the earned premiums for their expected adequacy levels; see Standard and Feldblum [2002].

Exhibit 3.17: 20X9 Schedule P, Part 3D Loss Ratios

Pd LR	12 mo	24 mo	36 mo	48 mo	60 mo	72 mo	84 mo	96 mo	108 m	120 m
20X0	18.73	41.09	53.45	60.73	66.00	69.82	72.36	74.91	76.73	78.73
20X1	18.50	39.67	51.50	59.33	64.50	68.17	71.33	73.67	75.67	
20X2	16.62	34.00	44.00	50.46	54.46	57.69	60.15	62.00		
20X3	15.86	34.14	44.43	51.00	56.00	59.43	62.00			
20X4	15.88	35.18	46.35	53.88	56.29	62.82				
20X5	16.22	34.89	46.44	54.44	60.22					
20X6	16.74	36.11	48.74	57.47						
20X7	14.60	35.30	48.50							
20X8	13.82	36.91								
20X9	14.18									

For accident years 20X1 through 20X4, we derive the value in the "120 months" column using the paid loss development factors.

- For accident year 20X1, the paid loss development factor from 108 months to 120 months is 1.020, so the paid loss ratio at 120 months is 75.67% × 1.020 = 77.18%.
- For accident year 20X2, the paid loss development factor from 96 months to 120 months is 1.051, so the paid loss ratio at 120 months is 62% × 1.051 = 65.14%.
- For accident year 20X3, the paid loss development factor from 84 months to 120 months is 1.082, so the paid loss ratio at 120 months is 62% × 1.082 = 67.09%.
- For accident year 20X4, the paid loss development factor from 72 months to 120 months is 1.125, so the paid loss ratio at 120 months is 62.82% × 1.125 = 70.70%.

Using accident years 20X0 through 20X4 as a starting point, we estimate the indicated reserves for accident years 20X5 through 20X9 with the least squares method.

For accident years 20X0 through 20X4, we estimate the least squares regression with the 60 month cumulative paid loss as the independent variable and the 120 month cumulative paid loss as the dependent variable:

paid loss at 120 months ("y") = $\alpha + \beta \times$ paid loss at 60 months ("x")

The observed values for accident years 20X0 through 20X4 are shown below.

Exhibit 3.18: Least Squares Reserving Method for Schedule P Illustration

Year	Loss Ratio 60 mos: "x"	Loss Ratio 120 mos: "y"	x^2	$x * y$
20X0	0.66000	0.78730	0.00354025	0.00414239
20X1	0.64500	0.77180	0.00198025	0.00240834
20X2	0.54460	0.65140	0.00312481	0.00370505
20X3	0.56000	0.67090	0.00164025	0.00189459
20X4	0.59290	0.70700	0.00005776	0.00008117
Average	0.60050	0.71768	0.00206866	0.00244631

The minimum least squares parameters are

$$y = \alpha + \beta x, \quad \beta = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}, \quad \alpha = \bar{y} - \beta \bar{x}$$

Using the values in the table above, we get

$\beta = 0.00244631 / 0.00206866 = 1.18256$
$\alpha = 0.71768 - 0.60050 \times 1.18256 = 0.00755 = 0.755\%$

We derive the values of α and β by a least squares linear regression analysis. Using the values of α and β together with the accident year 20X5 paid loss ratio at 60 months of development, we derive the paid loss ratio at 120 months for accident year 20X5.

- The paid loss ratio at 60 months of development is 60.22%.
- The projected paid loss ratio at 120 months of development is 0.755% + 1.18256 × 60.22% = 71.969%.

We continue in this fashion for all accident years. We explain the sequence of calculations below, though we do not show the arithmetic for the remaining accident years.

For accident year 20X6, we seek to project the paid loss ratio at 120 months of development from the paid loss ratio at 48 months of development. We use accident years 20X0 through 20X5 to determine the linear relationship between these two paid loss ratios that minimizes the sum of squared errors, exactly as we did above for paid loss ratios at 60 months of developments and 120 months of development using accident years 20X0 through 20X4.

The new values of α and β are α = 2.4512% and β = 1.261487. For accident year 20X6, the paid loss ratio at 48 months of development is 57.4737%. The projected paid loss ratio at 120 months of development is 2.4512% + 57.4737% × 1.261487 = 74.95%.

The table below shows the values of α and β for each accident year from 20X5 through 20X9, as well as the projected paid loss ratios for 120 months of development.

Exhibit 3.19: Least Squares Reserving Method for Schedule P Illustration

Accident Year	Alpha (A)	Beta (ß)	Paid Loss Ratio at 120 Mos	Earned Premium	Cum Pd Losses at 120 Mos
20X5	0.7504%	1.182611	71.97%	$900,000	$647,730
20X6	2.4512%	1.261487	74.95%	$950,000	$712,025
20X7	6.7305%	1.369435	73.15%	$1,000,000	$731,500
20X8	15.6391%	1.562814	73.32%	$1,100,000	$806,520
20X9	53.1247%	1.184733	69.93%	$1,100,000	$769,230

The calculated values of α are low for old accident years and high for recent accident years. This means that the expected loss method is appropriate for recent accident years and the chain ladder method is appropriate for older accident years. This conforms well to our intuition about reserving methods. For recent accident years, the cumulative paid losses to date are still sparse, and they provide only limited information about the projected ultimate losses. For older accident years, the cumulative paid losses to date are more credible, and we can use them to project ultimate losses.

We multiply the paid loss ratios at 120 months of development by the earned premium in each year to get the cumulative paid losses at 120 months of development. The remaining steps in the least squares reserving method are the same as for other reserving methods discussed earlier. We multiply the cumulative paid losses at 120 months of development by a 120 months to ultimate paid loss tail factor to get ultimate losses by accident year. These projected ultimate losses are compared with the reported losses by accident year to determine whether the reported loss reserves are adequate.

Considerations for Reserve Adequacy Testing

The Schedule P historical loss triangles are not always sufficiently homogeneous for accurate projections of reserve adequacy. Several illustrations are noted below, reflecting changes in the expected inflation rate, the insurance environment, state compensation systems, policy deductibles, policy exclusions, and company growth.

INFLATION ADJUSTMENTS

The paid loss development patterns in the illustration combine true development with the effects of inflation. Inflation is implicit in each paid loss link ratio. We assume that future inflation will be the same as the inflation implicit in the projected link ratio.

The effects of inflation on the loss payment pattern depends on the line of business. For workers' compensation, as for most lines, inflation affects *medical* benefits through the payment date. If inflation rises between the accident date and the time that the medical services are provided, we expect the workers' compensation medical benefits to be higher.

The effects of inflation on workers' compensation indemnity benefits depend on the state. In about half of the U.S. jurisdictions, indemnity payments that extend beyond two years have cost of living adjustments (COLA's), so inflation affects the indemnity reserves as well.

If inflation is expected to be higher in future years than it was during the historical Schedule P experience period, the paid loss chain ladder reserve indication is understated. Conversely, If inflation is expected to be lower in future years than it was during the experience period, the reserve indication is overstated.

These distortions are large even for small changes in the inflation rate. If the average loss is paid 5 years after the accident date, a one percentage point change in the inflation rate causes a 5 percentage point change in the combined ratio.

After the first two or three years, the workers' compensation reserves are dominated by lifetime pension cases, which take many years to settle. The average time to settlement of workers' compensation reserves is eight years. A one percentage point change in the future inflation rate causes a 8% change in the reserve indication.

CORRECTING FOR INFLATION BIAS

The following steps correct for the inflation bias in the reserve indication:

1. Select an appropriate inflation index for the line of business. For workers' compensation, we might use the medical CPI for medical benefits and wage inflation for indemnity benefits.[69] We assume that medical benefits are 100% sensitive to inflation, and indemnity benefits are 50% sensitive to inflation. Workers' compensation benefits are split relatively evenly between indemnity and medical benefits. Since medical benefits are

[69] Masterson [1968; 1992] illustrates econometric correlations of insurance loss cost trends with various inflation indices. The National Council on Compensation Insurance continues his work at the present time.

paid (on average) sooner than indemnity benefits, the reserves are split about 70% indemnity and 30% medical.[70]

2. Relate the workers' compensation benefit trends to the chosen index. The medical CPI shows changes in a fixed basket of goods. It does not incorporate the effects of increasing utilization of medical services, more complex medical procedures, and more expensive medical equipment. In general, workers' compensation medical benefit trends exceed the medical CPI by about four percentage points a year (see Feldblum [2002: wcr]). The overall workers' compensation benefit trend might be modeled as

$$30\% \times (\text{medical CPI} + 4\%) + 70\% \times \text{wage inflation}.$$ [71]

3. From the cumulative Schedule P, Part 3 payment triangles, determine the corresponding incremental payment triangles by taking first differences between adjacent development periods. Detrend the incremental payments with the benefit trend index developed in the previous step to form a triangle of "real dollar" incremental payments.

4. Determine the corresponding cumulative "real dollar" paid loss triangle from the incremental triangle. Perform the paid loss chain ladder analysis to select projected link ratios in "real dollar" terms.

5. From expected future inflation estimates made by outside economists or by the analyst working with the Schedule P data, estimate future benefit trends. Combine the estimated future benefit trends with the real dollar link ratios to obtain the expected future nominal dollar link ratios. Use these adjusted link ratios to project reserve indications.

There are numerous variations on this procedure. Financial engineers tend to use stochastic interest rate paths and inflation rate paths. For an illustration of a deterministic inflation adjustment to reserve indications, see Hodes, Feldblum, and Neghaiwi (1999). For an illustration of the stochastic inflation adjustment, see Hodes, Feldblum, and Blumsohn (1999).

During periods of inflation rate stability, the chain ladder procedure shown earlier in this paper is sufficient. During periods of inflation rate volatility, the inflation adjustment described here is necessary to avoid large biases in the reserve indications.

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[70] Much of the medical costs are expended on temporary total cases and on "medical-only" cases, which are paid rapidly and may not even appear in the year-end reserves. In contrast, indemnity benefits may account for 90% or more of a long-term lifetime pension case, which remains in the reserves year after year.

[71] We are examining trend on a development period basis, so we make no adjustments for expected changes in the mix of business by classification. The trend examines changes in the cost per exposure, not changes in the type of exposures. In contrast, an accident year trend must also consider the effects of shifting mixes of business, such as the changes caused by shifts from a manufacturing to a service economy.

INSURANCE ENVIRONMENT

During the past thirty years, workers' compensation medical costs have increased more rapidly than indemnity costs, rising from 30% of benefits in the 1960's to about 50% in 2000. High medical care inflation, increasing use of physicians' services, more sophisticated and expensive medical equipment, and the absence of any limit on medical benefits in the workers' compensation system contribute to this. In addition, increasing deductibles and coinsurance levels in group health insurance plans, along with more sophisticated cost containment efforts by health insurers, have led to cost-shifting to workers' compensation.

Medical benefits are paid quickly, whereas indemnity benefits are paid as the income loss accrues; the paid loss link ratios are therefore higher for indemnity benefits. A rote Schedule P, Part 3 loss reserve adequacy projection uses link ratios developed from experience dominated by indemnity losses and applies them to experience with a higher percentage of medical losses, thereby distorting the results (see Woll [1981]).

Offsetting this effect are the increasing trends in paid loss link ratios for both medical and indemnity benefits, probably stemming from lengthening durations of disability and perhaps increasing attorney involvement in workers' compensation claims (NCCI [1992]). The lengthening durations of disability are abetted by more liberal decisions on the compensability of stress claims and of occupational illnesses (Millus [1987]; [1988]). They may be seen both in average disability by type of injury and by the shift of claims from temporary disability to permanent partial disability (Gardner [1989]). These trends have stopped and perhaps even reversed in the 1990's, probably because of the system reforms enacted in many jurisdictions.

The actuarial solution to these problems is to create more homogeneous data sets by dividing the workers' compensation experience (i) between medical and indemnity benefits and (ii) between long-term pension cases and other cases. Schedule P data do not show these splits.

STATE COMPENSATION SYSTEMS

Several jurisdictions, such as Massachusetts, New Jersey, and Pennsylvania, have revised their personal automobile no-fault compensation systems by increasing personal injury protection (PIP) benefits, modifying the tort threshold, or providing policyholder options (Marter and Weisberg [1991]; Musick and Szczepanski [1992]). PIP and residual bodily injury (RBI), although combined in Schedule P, have different paid loss development patterns. A change in the mix of benefits may distort the estimates of reserve adequacy.

Illustration: State X changes from a low monetary tort threshold to a strong verbal tort threshold. For a given amount of PIP benefits paid in the first year or two years since policy inception, we expect a greater volume of residual bodily injury (RBI) losses to appear at later maturities when the compensation system has a weak monetary threshold than when it has a strong verbal threshold. The paid loss link ratios should decline. A rote Schedule P, Part

3 loss reserve adequacy analysis may show a reserve deficiency even if none exists.

DEDUCTIBLES

In the 1990's, many insurers switched from first dollar workers' compensation policies and retrospectively rated policies to large dollar deductible (LDD) policies. With an LDD policy, the insurer handles all claims, but it assesses the employer (the insured) for the cost of benefits below the retention for each claim (see Feldblum [2002: wcr], Teng [1994], Brown and Schmitz [2000]).

LDD policies decrease state premium taxes and residual market assessments, both of which are levied on direct written premium in most states.[72] They avoid "dollar trading" on small claims, which pose no financial risk to the insured, and they provide greater incentives to the insured to provide a safe workplace and adequate safety procedures. They allow the insured to keep the premium dollars until they are needed to pay claims, enhancing the cash flow of the insured.

Similarly, many insurers use large dollar deductibles on general liability and commercial automobile policies, particularly when these lines are written in combination with workers' compensation for a large account.

Excess Development

Loss development factors (if they are more than unity) increase for higher layers of loss (Pinto and Gogol [1987]), for several reasons:

- If the loss amount before development exceeds the retention, all the development appears in the upper layer and none appears in the lower layer.
- If the loss amount before development is below the retention but after development it exceeds the retention, the development below the retention is capped and the development above the retention is infinite.
- In some lines of business, emergence of IBNR claims often involves large and complex claims. This occurs most often with latent disease claims, stress claims, and psychological injury claims (cf Feldblum [2002: wcr]).
- The size-of-loss distribution changes as the claims mature, becoming increasingly skewed towards high cost claims (see Pinto and Gogol [1987]).

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[72] Several large states are exceptions, notably New York, Massachusetts, and California.

Illustration: We examine a set of workers' compensation claims that remain open for two years or longer. We keep the illustration simple to highlight the effect of a retention.

The payment per claim at 12 months is uniformly distributed between $10,000 and $60,000. The cumulative payments for each claim are twice as large at 24 months. The LDD deductible is $50,000 per claim.

Total Development: The average size of a claim is $35,000 at 12 months and $70,000 at 24 months. The loss development link ratio from 12 months to 24 months is 2.000.

Excess Development: One fifth of the claims exceed the retention at 12 months of development. For these claims, the average amount above the retention is $5,000. The average cost per claim to the LDD insurer is 1/5 × $5,000 = $1,000.

At 24 months of development, the claims are uniformly distributed between $20,000 and $120,000. Claims between $50,000 and $120,000 has portions above the retention; these are 7/10 of the claims. Of the claims with portions above the retention, the average benefit paid by the insurer is $35,000, or ½ × ($120,000 – $50,000). The average cost per claim at 24 months of development is 7/10 × $35,000 = $24,500. The loss development link ratio from 12 months to 24 months is 24.500.

The use of historical experience based on first dollar policies to project loss development for LDD policies may severely understate the required loss development factors. For recommendations on developing LDD blocks of business, see Siewert [1996].

POLICY EXCLUSIONS

Insurance policy forms show two trends.

- For common perils, policies have become broader, covering various additional hazards.
- For unusual perils that affect only a small number of insureds but that may cause expensive losses, insurers tend to exclude the coverage from the basic policy and to use special endorsements or policies.

  Illustration: Before 1986, the standard CGL (Commercial General Liability) policy form used a pollution exclusion known as exclusion "f," which excluded liabilities resulting from pollution except when it was "sudden and accidental." In the 1980's, after passage of the CERCLA Act of 1980 which put the costs of remediation of abandoned toxic waste sites on the companies which has deposited wastes there, exclusion "f" was termed ambiguous by several state courts and interpreted to mean all unintentional pollution. In 1986, insurers replaced exclusion "f" with an absolute pollution exclusion and covered the potential liabilities under separate environmental impairment policies.

Illustration: Asbestos hazards were originally covered under standard products liability forms. Asbestos exposures have proved far more expensive than imagined; even in the year 2000, there have been hundreds of thousands of asbestos claims related to policies written 30 or more years earlier. By the 1970's, many insurers had eliminated coverage of asbestos hazards from their unendorsed products liability forms and cover the exposures under special asbestos policies. Similar trends may occur for exposures related to firearms, cigarettes, and pharmaceutical drugs (such as Prozac).

Illustration: Terrorism losses were covered under most insurance policies before the World Trade Center losses of September 11, 2001. Several reinsurers may exclude terrorism losses from their reinsurance treaties. Primary insurers may do the same, and they may cover the hazard under separate forms. Terrorism coverage is needed primarily by large accounts in major urban centers, just as hurricane coverage is needed by insureds in costal areas, flood insurance is needed in flood plains, and earthquake insurance is needed along earthquake fault lines.

In each illustration above, the subsequent exclusion of coverage causes the historical experience to be extraordinary and unlikely to be repeated in the future. Users of Schedule P should examine the causes of high development in past years and check whether policy form changes lower the probability of these events occurring in the future.

COMPANY GROWTH AND DECLINE

If an insurer expands its writings over the course of a year, more claims are incurred in the latter part of the year than in the earlier. Paid loss development factors are higher for accident years with later average loss occurrence dates. A change in the rate of business growth may distort the projection of reserve adequacy.

Illustration: To avoid complex mathematics, we assume that claims are reported six months after occurrence of the accident and that case reserves are adequate. If the volume of business is steady over the course of the accident year, the reported loss development factor from 12 months since inception of the accident year to 24 months since inception of the accident year is 2.000. The rationale for this development factor is that claims which occur in the first six months of the accident year (half the claims) are reported by 12 months. Claims which occur in the latter six months of the accident year (the other half of the claims) are reported after 12 months but before 24 months.

If the volume of business during the accident year increases linearly over time, starting at $0 on January 1, one quarter of the claims occur in the first six months and three quarters of the claims occur in the latter six months.[73] The reported loss development factor from 12 months to 24 months is 4.000, not 2.000. The user of Schedule P should examine the growth or decline of business over the historical period and adjust the loss development factors appropriately.

In summary, Part 3 of Schedule P is the major publicly available document for estimating reserve adequacy. However, one must be aware of the potential distortions caused by the lack of data homogeneity and shifts in mix of benefits to properly evaluate the statistical indications.

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[73] Suppose the rate of loss occurrence is $0 per year at inception of the accident year, $100,000 per year at 6 months, and $200,000 per year at 12 months. The average rate of loss occurrence during the first six months is ½ × ($0 + $100,000) = $50,000, and the average rate of loss occurrence during the last six months is ½ × ($100,000 + $200,000) = $150,000. One quarter of the losses occur during the first six months and are reported by 12 months. Three quarters of the losses occur during the latter six months and are reported after 12 months.

Part 2 - Incurred Losses

Part 2 shows a triangle of net incurred losses and defense and cost containment expenses (DCC) by accident year and evaluation date. The Part 2 entries are the sum of paid amounts, case reserves, and bulk + IBNR reserves for both losses and DCC. Each entry in Part 2 equals the corresponding entry in Part 3 plus the loss and DCC reserves at that date.

Part 2 is designed as a retrospective test of loss reserve adequacy. If the insurer sets adequate reserves, the incurred losses for each accident year should show neither upward nor downward development.[74] The NAIC uses the Part 2 Summary exhibit for the loss reserve development tests in the Insurance Regulatory Information System (IRIS).

IRIS Loss Development Tests

For any accident year, column 10 of Part 2 shows incurred losses valued at the statement date, and column 9 shows the corresponding valuation one year earlier. If the insurer has reserved adequately, payments during the year are offset by a reduction of reserves, and there should be no change in incurred losses between valuation dates. Column 11 shows the latest year's change in incurred losses for each accident year except the most recent one (there is no "previous" valuation for the most recent accident year). Column 12 shows the change over the last two years in incurred losses for each accident year except the most recent two years.

Illustration: Exhibit 2.1 below shows a Schedule P, Part 2 triangle for workers' compensation.

- For accident year 20X1, the one-year adverse loss development is \$520,000 – \$521,000 = –\$1,000; the two-year adverse loss development is \$520,000 – \$500,000 = +\$2,000.
- For accident year 20X6, the one-year adverse loss development is \$787,000 – \$786,000 = +\$1,000; the two-year adverse loss development is \$787,000 – \$761,000 = +\$26,000.

The Part 2 Summary exhibit shows data for all lines of business combined. The one- and two-year adverse loss developments in the Summary exhibit are summed over all accident years (including the prior years row) and shown on row 12.

IRIS Retrospective Tests 9 and 10

[74] This generalization assumes that the reserves are not discounted. It is also true if any discounts on the statement reserves are disclosed, so that Schedule P, Part 2 shows the undiscounted amounts. The generalization is not correct if the reserves contain an implicit interest discount, since the unwinding of the discount, or the amortization of the discount, shows up as apparent adverse development.

IRIS Tests 9 and 10 compare the one and two year adverse development to policyholders' surplus at the inception date of the development.

- IRIS Test 9 divides the one year reserve development from row 12 of the Summary exhibit by the policyholders' surplus at the end of the prior year.
- IRIS Test 10 divides the two year reserve development by the policyholders' surplus at the end of the second prior year.

A ratio of 20% or greater on either test is an exceptional score. Four or more exceptional scores on IRIS tests serves as a warning of potential financial weakness and may trigger a financial examination. An exceptional score on any of the three loss reserve adequacy tests (IRIS tests 9, 10, and 11) must be commented upon in the Statement of Actuarial Opinion.

The "Five Year Historical Data" exhibit of the Annual Statement, lines 68 through 72, shows the one and two year developments and the ratios for tests 9 and 10 for the five most recent Annual Statements.

Exhibit 2.1: One and Two Year Loss Development

One Year Loss Development	20XX	20XX–1	20XX–2	20XX–3	20XX–4
Development in estimated losses and loss expense incurred prior to current year					
Percent of development of loss and loss expense incurred to policyholders' surplus of prior year end					
Two Year Loss Development					
Development in estimated losses and loss expense incurred two years before the current year and prior year					
Percent of development of loss and loss expense incurred to reported policyholders' surplus of second prior year end					

IRIS Prospective Test 11

IRIS Test 11 is a prospective test of reserve adequacy. It is a regulatory test, not an actuarial test; it does not use the actuarial principles mentioned earlier. It is a simple formula that requires no independent judgment in selecting or smoothing factors.

IRIS Test 11 compares the outstanding loss ratios of three years. The *outstanding loss ratio* is the ratio of outstanding losses and loss adjustment expenses at a given statement date to the earned premium in that statement year. IRIS Test 11 updates the outstanding loss ratios from the past two years by means of the one- and two-year reserve developments, and compares these ratios with the current year's outstanding loss ratio.

The losses and premiums in this ratio are not matched.

- The numerator is unpaid loss and loss adjustment expenses for all accident years.
- The denominator is earned premium for the current calendar year.

This mismatch constrains the usefulness of IRIS Test 11, since business volume growth or decline, changes in the mix of business between property and liability lines, and changes in the types of policies issued distort the "outstanding" loss ratio (Salzmann [1981], page 175).

Unpaid losses and loss adjustment expenses are reported on page 3, "Liabilities, Surplus and Other Funds," lines 1, 2, and 3.

- Line 1 shows total net loss reserves. It includes reinsurance payable on unpaid losses for business assumed by the reporting company. It is reduced for reinsurance recoverables on unpaid losses for business ceded by the reporting company.
- Line 2 shows reinsurance payable on paid losses for business assumed by the reporting company.
- Line 3 shows reserves for unpaid loss adjustment expenses (both DCC and AAO).
- Earned premium is shown on page 4, "Underwriting and Investment Exhibit: Statement of Income," line 1, column 1.

Illustration: The outstanding loss ratio for December 31, 20XX equals the outstanding loss and loss adjustment expenses at December 31, 20XX shown on page 3, lines 1 + 2 + 3, divided by the earned premium for 20XX shown on page 4, column 1, line 1.

TEST 11 OVERVIEW

To test reserve adequacy in 20XX, IRIS Test 11 examines the outstanding loss ratios in 20XX–1 and 20XX–2. An outstanding loss ratio in 20XX that is lower than the average of the outstanding loss ratios in the two preceding years may be a symptom of under-reserving.

Illustration: The outstanding loss ratios in 20XX, 20XX+1, and 20XX+2 are 125%, 120%, and 105%. The 20XX+2 outstanding loss ratio of 105% is well below the average 122.5% outstanding loss ratio of the preceding two years. The company may be experiencing financial problems, and it may weakened its loss reserves.

This simple computation may highlight instances of reserve weakening, but it does not uncover instances of persistently weak reserves. To correct this problem, the prior two years' outstanding loss ratios are adjusted for the one and two-year adverse loss development in the current year's Schedule P to determine *restated* outstanding loss ratios.

- The one year reserve development is added to the unpaid losses and loss adjustment expenses for the prior year. This sum is divided by the prior year's earned premium. The necessary figures are taken either from the "previous year" column in the current Annual Statement, pages 3 and 4, or from the "current year" column in the previous year's Annual Statement.
- The two year reserve development is added to the unpaid losses and loss adjustment expenses for the second prior year. This sum is divided by the second prior year's earned premium. The necessary figures are taken either from the "previous year" column in the previous year's Annual Statement, pages 3 and 4, or from the "current year" column in the second prior year's Annual Statement.

The average of these two restated outstanding loss ratios is multiplied by the current year's earned premium (from page 4, column 1, line 1, of the current year's Annual Statement) to determine the indicated outstanding losses and loss adjustment expenses. This figure, minus the reported unpaid losses and loss adjustment expenses (from page 3, column 1, lines 1+2+3), is the indicated reserve deficiency. A deficiency greater than 25 percent of policyholders' surplus (page 3, line 32, or page 4, line 20) indicates an exceptional score.

Illustration: IRIS Test 11

The 20X5 Schedule P, Part 2, Summary shows a one year adverse loss development of $3 million and a two year adverse loss development of $4 million. The following data are taken from the current and the two previous Annual Statements to compute the results of IRIS Test 11 (figures are in thousands of dollars).

Exhibit 2.2: IRIS Test 11: Input Data

	20X3	20X4	20X5
Earned premium	$12,000	$12,500	$19,000
Loss reserves	9,000	10,000	16,000
Reinsurance payable on paid losses	500	1,000	2,500
Loss adjustment expense reserves	2,500	4,000	4,500
Policyholders' surplus	$7,850	$8,900	$12,150

RESTATED OUTSTANDING LOSS RATIOS

The restated outstanding loss ratios for 20X3 and 20X4 are the restated loss reserves divided by the earned premium. The restated loss reserves are defined as the sum of loss reserves, LAE reserves, reinsurance payable on paid losses, and Schedule P, Part 2, Summary adverse loss development.

For 20X3, the restated loss reserves are

\$9 million + \$0.5 million + \$2.5 million + \$4 million = \$16 million,

and the restated outstanding loss ratio is \$16 million ÷ \$12 million = 133.3%.

For 20X4, the restated loss reserves are

\$10 million + \$1 million + \$4 million + \$3 million = \$18 million,

and the restated outstanding loss ratio is \$18 million ÷ \$12.5 million = 144.0%.

STATUTORY INDICATED RESERVES

The average restated outstanding loss ratio is (1.333 + 1.440) ÷ 2 = 1.386. The 20X5 earned premiums are \$19 million, so the indicated unpaid losses are 1.386 x \$19 million = \$26.347 million. The held reserves at December 31, 20X5 are \$23 million [= \$16 million + \$2.5 million + \$4.5 million]. The indicated reserve deficiency is \$26.347 million – \$23 million = \$3.347 million. Policyholders' surplus in 20X5 is \$12,150,000. The ratio of \$3.347 million to \$12.15 million is 27.55%, which constitutes an exceptional score for IRIS Test 11. The figures are summarized in the table below.

Table 2.3: IRIS Test 11: Estimated Reserve Deficiency
(Figures in Thousands of Dollars)

Statement date	20X3	20X4	20X5
Loss reserves	$9,000	$10,000	$16,000
Reinsurance payable on paid losses	$500	$1,000	$2,500
Loss adjustment expense reserves	$2,500	$4,000	$4,500
Adverse loss development	$4,000	$3,000	—
Restated loss reserves	$16,000	$18,000	
Earned premium	$12,000	$12,500	
Restated outstanding loss ratio	1.333	1.440	
Average restated O/S loss ratio			1.387
Earned premium, current year			$19,000
Indicated loss reserves			$26,347
Held reserves at December 31, 20X5			$23,000
Indicated reserve deficiency			$3,347

Second Illustration

To ensure comprehension of the IRIS reserve adequacy test, we show a second illustration in abbreviated form. Readers are encourage to develop the IRIS Test 11 result and compare it to the procedure below.

The Annual Statements for years 20XX–2, 20XX–1, and 20XX show the following figures in millions of dollars.

Exhibit 2.4: IRIS Test 11, Illustration 2, Input Data

	20XX–2	20XX–1	20XX
Earned Premium (page 4, line 1)	$1,100	$1,500	$1,750
Loss Reserves (page 3, line 1)	1,500	2,000	3,000
Reinsurance payable on paid losses	200	300	300
LAE Reserves (page 3, line 2)	1,000	1,500	1,700
Policyholders' Surplus	$8,000	$9,000	$10,000

The 20XX Schedule P, Part 2 Summary shows one-year adverse loss development of 600 and two-year adverse loss development of 1,500. We calculate the IRIS Test 11 results for the 20XX statement date.

We determine the restated outstanding loss ratios for the two prior years: 20XX–2 and 20XX–1. We multiply the average of the two restated outstanding loss ratios by the 20XX earned premium to derive the Test 11 indicated reserves. We subtract the 20XX held reserves from the indicated reserves, and we divide the result by the 20XX policyholders' surplus. A quotient greater than 25% is an exception value for Test 11.

The restated outstanding losses in each prior year equals the sum of:

i. the loss reserves;
ii. the reinsurance payable on paid losses;
iii. the LAE reserves; and
iv. the adverse loss development from that prior year to the current statement date as indicated in the Schedule P, Part 2, Summary.

For 20XX–2, the restated outstanding losses are $1500 + $200 + $1000 + $1500 = $4,200 million. The restated outstanding loss ratio is the restated outstanding losses divided by the earned premium, or $4,200 / $1,100 = 3.818.

For 20XX–1, the restated outstanding losses are $2,000 + $300 + $1,500 + $600 = $4,400 million. The restated outstanding loss ratio is $4,400 / $1,500 = 2.933.

The average restated outstanding loss ratio in the two prior years is (3.818 + 2.933) / 2 = 3.376. The Test 11 indicated reserves for 20XX are 3.376 × $1,750 = $5,908 million.

The held reserves in 1998 are $3,000 + $300 + $1,700 = $5,000 million. The excess of the indicated reserves over the held reserves equals $5,908 – $5,000 = $908 million.

The ratio of the Test 11 indicated reserve deficiency to policyholders' surplus is $908 / $10,000 = 9.08%. This is less than 25%, so the Test 11 results are not exceptional.

Distortions

IRIS Test 11 uses all lines combined data, which is an extremely heterogeneous mixture. The test results are distorted by (i) company growth, (ii) changes in the mix of business by line, and (iii) changes in policy types. Each of these effects is explained below.

Growth

Rapid growth after a period of stability may indicate a reserve deficiency even if reserves are adequate, particularly if the company writes long-tailed lines of business. For workers' compensation or medical malpractice, the outstanding loss ratio may be as high as 300%, since losses are paid several years after the premium is collected.

The example above illustrates this problem. The company grew rapidly in 20X5, increasing its premium volume from $12.5 million to $19.0 million, an increase of $6.5 million. In comparison, the 20X4 increase in premium was only $12.5 million – $12.0 million = $0.5 million.

Loss reserves increased in 20X5 by $23 million – $18 million (after restatement for adverse loss development) = $5 million. This is the increase that we would expect for an additional $6.5 million of earned premium. Nevertheless, the company shows an exceptional score on IRIS Test 11.

The NAIC realizes that changes in premium volume may distort the results. Business growth overstates the reserve deficiency, though the NAIC believes the effect is not great: "Within the normal range of variations in premium from year to year, the distortion from changes in premium is not significant" (NAIC *IRIS Manual*, Test 11).

The outstanding loss ratio does not properly match losses in the numerator with premiums in the denominator. IRIS Test 11 is a holdover from days when many state insurance departments did not have actuarial or financial staff who could perform reserve adequacy tests and when reserving software to perform actuarial reserve adequacy tests was not available. The Annual Statements are now filed electronically (in addition to the hardbound copies), and software is available for complete actuarial analyses of reserve adequacy. It is hard to justify the continued use of IRIS Test 11.

Mix by Line

A change in the mix of business from long-tailed liability lines to short-tailed property lines may lead to an exceptional score on IRIS Test 11, even if reserves are adequate. Conversely, a change in the mix of business from short-tailed property lines to long-tailed liability lines may prevent an exceptional score on IRIS Test 11, even if reserves are deficient.

> *Illustration:* A company writes workers' compensation and commercial fire insurance.
>
> - The expected outstanding loss ratio for workers' compensation is 250%.
> - The expected outstanding loss ratio for commercial fire is 20%.
>
> With a 50%–50% mix of business, the overall expected outstanding loss ratio is
>
> $$50\% \times 250\% + 50\% \times 20\% = 135\%.$$
>
> If the company shifts to a mix of 60% workers' compensation and 40% commercial fire, the overall expected outstanding loss ratio is 60% x 250% + 40% x 20% = 158%.[75] If the company shifts to a mix of 40% workers' compensation and 60% commercial fire, the overall expected outstanding loss ratio is 40% x 250% + 60% x 20% = 112%. A 10% shift in the mix of business leads to a 23% difference in the expected outstanding loss ratio.

If the company shifts from short-tailed property lines to long-tailed liability lines, a steady outstanding loss ratio may mask a reserve deficiency problem. Conversely, if the company shifts from long-tailed liability lines to short-tailed property lines, a decreasing outstanding loss ratio is expected, and it does not necessarily indicate a reserve deficiency problem.

The NAIC realizes that changes in the mix by line may distort the results. The NAIC recommends that "For companies which have had major shifts in product mix, the estimated reserve deficiency or redundancy should be calculated separately for the major product groups. . . ." (*ibid.*).

Policy Type

A shift in the mix of policy type may have an effect similar to a shift in the mix of business by line. For instance, a shift from first dollar workers' compensation policies to large dollar deductible workers' compensation policies may prevent an exceptional score on IRIS Test 11, even if reserves are deficient. A book of large dollar deductible workers' compensation policies has an exceedingly high outstanding loss ratio, since the losses are paid many years after the premium is collected. See the discussion above regarding mix of business for further discussion.

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[75] This is the steady-state expected outstanding loss ratio. The change in the observed outstanding loss ratio after a change in the mix of business is gradual, extending over several years.

Case Incurred (Reported) Loss Reserve Adequacy Tests

Part 2 includes bulk + IBNR reserves in addition to case reserves and paid losses. Actuaries project indicated reserves from historical experience, such as loss payments and reserves set by claims examiners, not from previous actuarial forecasts.

Part 4 of Schedule P shows the bulk + IBNR reserves carried by the company in past years in the same format as in Part 2. The difference between Parts 2 and 4 reflects the historical claims experience of the company. The case incurred (or reported) loss development patterns derived from this experience can be used to prospectively estimate reserve adequacy.

ILLUSTRATION: REPORTED LOSS DEVELOPMENT

We continue the illustration from the discussion of Part 3, using data from the same company.

Exhibit 2.5: 20X9 Schedule P, Part 2 ($000)

Part 2	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	563	524	514	501	494	482	485	486	486	486
20X1		578	554	528	526	519	518	518	521	520
20X2			487	495	486	478	478	476	475	475
20X3				523	519	520	517	520	522	522
20X4					603	637	649	661	666	667
20X5						708	708	700	708	707
20X6							740	761	786	787
20X7								800	800	802
20X8									860	866
20X9										898

For a well reserved company, Part 2 should show little upward or downward development along the rows. This illustration shows no significant development for accident years 20X2, 20X3, 20X5, 20X7, and 20X8.; downward development for accident years 20X0 and 20X1; and slight upward development for accident years 20X4 and 20X6. For all accident years combined, there is a 0.5 percent decline in incurred losses from the first report to the statement date.

Part 4 shows bulk and IBNR reserves. Since bulk and IBNR reserves are replaced by case reserves and payments as claims are reported and settled, we expect a steady decline along the rows.

Exhibit 2.6: 20X9 Schedule P, Part 4 ($000)

Part 4	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	348	177	114	82	61	41	36	26	20	12
20X1		326	190	119	85	62	47	35	28	20
20X2			265	166	113	76	60	46	40	31
20X3				296	167	114	81	60	50	38
20X4					328	194	131	95	74	58
20X5						410	231	142	100	62
20X6							438	246	170	118
20X7								462	246	146
20X8									515	238
20X9										560

The difference between Parts 2 and 4 shows case incurred (or reported) losses plus DCC. This new triangle may be used for a prospective test of loss reserve adequacy.

Exhibit 2.7: 20X9 Schedule P, Part 2 – Part 4 ($000)

Pt 2-4	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	215	347	399	419	433	442	449	460	466	474
20X1		252	363	409	441	457	471	483	493	500
20X2			222	329	373	402	418	430	435	444
20X3				227	352	406	436	460	471	484
20X4					275	443	518	566	592	609
20X5						298	477	558	608	645
20X6							302	515	616	670
20X7								338	554	656
20X8									345	628
20X9										338

LINK RATIOS AND DEVELOPMENT FACTORS

The reported loss link ratios shown below are formed in the same manner as the paid loss link ratios discussed earlier.

Exhibit 2.8: 20X9 Schedule P, Reported Loss Link Ratios

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 – 10
20X0	1.614	1.150	1.050	1.033	1.021	1.016	1.024	1.013	1.017
20X1	1.440	1.127	1.078	1.036	1.031	1.025	1.021	1.014	
20X2	1.482	1.134	1.078	1.040	1.029	1.012	1.021		
20X3	1.551	1.153	1.074	1.055	1.024	1.028			
20X4	1.611	1.169	1.093	1.046	1.029				
20X5	1.601	1.170	1.090	1.061					
20X6	1.705	1.196	1.088						
20X7	1.639	1.184							
20X8	1.820								

Loss reserve projections that rely on reported (case incurred) loss development patterns are aided by knowledge of the insurer's case reserving practices, as well as of changes in these practices during the experience period. The three year average reported loss link ratios are higher than the corresponding five year averages for the first three maturities, so we have selected the three year averages as estimates for the future.

Exhibit 2.9: Reported Loss Development Test of Reserve Adequacy (dollars in $000)

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9–10
Averages									
3 year	1.722	1.183	1.090	1.054	1.027	1.022	1.022		
5 year	1.675	1.175	1.084	1.048	1.027				
Select	1.720	1.180	1.090	1.050	1.030	1.020	1.020	1.010	1.010
Cumulative	2.539	1.476	1.251	1.148	1.093	1.061	1.041	1.020	1.010
Reported	$338	$628	$656	$670	$645	$609	$484	$444	$500
Developed	$858	$927	$821	$769	$705	$646	$504	$453	$505
Bulk Res	$520	$299	$165	$99	$60	$37	$20	$9	$5
Total Res	$702	$521	$336	$223	$163	$112	$70	$50	$51

For all accident years combined, the estimated ultimate incurred loss plus DCC is $6,188 thousand, and the reported incurred amounts on Part 2 are $6,244 thousand. The difference of less than 1 percent indicates accurate reserving.

UPDATING THE PART 2 EXHIBITS

The figures for individual accident years in Part 2, except for those in the right-most column and the prior years row, may be copied from the corresponding entries in the previous Annual Statement. For the prior years row, a modification is required.

- The entries in the previous year's Schedule P for the prior row and for the first accident year should be divided between reserves and paid losses: paid losses are in Part 3 and reserves equal Part 2 minus Part 3.
- The reserves from the first two rows in the previous year's Schedule P are added together and posted directly to the current Schedule P.
- The current Schedule P payments are taken from Part 3.
- The sum of the reserves and the payments is the current year's prior years row on Part 2.

Illustration: We are completing the prior years row for the 2010 Schedule P. We illustrate the derivation of the 2007 statement date figure: the accident years 2000 and prior as of December 31, 2007. Assume the following figures are given in the 2009 Schedule P, Part 2, and in the 2010 Schedule P, Part 3:

- The 2009 Schedule P, Part 2, prior years row, shows $35 million at December 31, 2007, and the 2000 accident year shows $15 million at December 31, 2007.
- The 2009 Schedule P, Part 3, prior years row, shows $10 million at December 31, 2007, and the 2000 accident year shows $8 million at December 31, 2007.
- The 2010 Schedule P, Part 3, prior years row, shows $12 million at December 31, 2007.

We derive the figures for the 2010 Schedule P, Part 2 prior years row as follows:

- The reserves for the 2009 prior years row at December 31, 2007 are $35 million – $10 million = $25 million
- The cumulative paid losses for the 2009 prior years row at December 31, 2007 are $15 million – $8 million = $7 million.
- The required entry for the 2010 Schedule P, Part 2, prior years row, evaluated at December 31, 2007 is $25 million + $7 million + $12 million = $44 million.

The entries for the right-most column can be copied from Part 1. For each accident year, Part 2, column 10 equals columns 11 – 8 + 9 + 24 – 21 + 22 from Part 1. Columns 11 and 24 in Part 1 show total paid and unpaid losses plus loss adjustment expenses. Since Part 2 does not include AAO adjustment expenses, one must subtract the net AAO expenses. Columns 8 – 9 equal the net AAO paid (direct plus assumed minus ceded) and columns 21 – 2 equal the net AAO unpaid.

If the Part 1, column 24 entries are net of tabular discounts, one must add the tabular discounts by accident year to obtain the Part 2, column 10 entries. The tabular discounts are shown in Note 28 to the Annual Statement.

Reported loss development reserve procedures are particularly important for long tailed lines of business whose loss payments are small at early maturities, such as products liability and excess of loss reinsurance.

ILLUSTRATION: UPDATING THE TWO-YEAR LINES

The following illustration puts together the procedures for Part 3, Part 2, and intercompany pooling agreements, and it clarifies the difference between the two year exhibits and the ten year exhibits.

The 20X3 Schedule P for XYZ Insurance shows the following data for auto physical damage.

Exhibit 2.11: Auto Physical Damage Incurred Losses

Schedule P – Part 2J – Auto Physical Damage			
	Incurred Losses and DCC		
Year in which	Reported at Year End (in $000)		
Losses were Incurred	20X1	20X2	20X3
Prior	400	380	370
20X2	XXX	1000	1020
20X3	XXX	XXX	800

Exhibit 2.12: Auto Physical Damage Cumulative Paid Losses

Schedule P – Part 3J – Auto Physical Damage			
	Cumulative Paid Losses and DCC		
Year in which	at Year End (in $000)		
Losses were Incurred	20X1	20X2	20X3
Prior	000	300	330
20X2	XXX	780	960
20X3	XXX	XXX	580

During 20X4, XYZ establishes an insurance subsidiary, ABC Insurance, and enters into a intercompany pooling arrangement in which each company gets 50% of the combined business. Neither company has any other reinsurance ceded or assumed.

For 20X4, the companies record the following:

Exhibit 2.13: Auto Physical Damage Calendar Year Transactions

Year in Which Losses Were Incurred	Losses and DCC Paid During 20X4		Reserves for Losses and DCC at the end of 20X4	
	XYZ	ABC	XYZ	ABC
Prior	16	0	10	0
20X2	40	0	20	0
20X3	200	0	50	0
20X4	400	300	150	100

We construct Schedule P, Part 2J and Part 3J for XYZ's 20X4 Annual Statement.

INTERCOMPANY POOLING

We begin with the pooling rules. We must form the 20X4 Schedule P exhibits for XYZ Insurance. In 20X4, XYZ formed a subsidiary, ABC Insurance, and established a 50%

intercompany pooling arrangement. Premiums and losses for XYZ and for ABC are pooled, and each company takes half.

In this illustration, ABC Insurance was not in existence during 20X2 and 20X3. This is not relevant for Schedule P. The Schedule P exhibits are restated to reflect the current intercompany pooling arrangements as if they were in effect in all prior years, even during years when one or more of the companies was not in existence.

Loss Payments

We begin with the prior years row in Part 3J, the paid loss triangle. In the paid loss triangles, the prior years row begins with "000." The 10-year exhibits for the long-tailed liability lines of business differ from the 2-year exhibits for the short-tailed property lines of business in the following respect.

- In the two year exhibits for the short-tailed property lines, the prior years row has two cells with figures preceded by one cell with "000." The entries begin in the same calendar year (20X3 in the illustration) as the entries for the oldest individual accident year (accident year 20X3 in the illustration).
- In the ten year exhibits, the "prior years" row has one cell with "000" followed by *nine* cells with figures. The oldest *individual* accident year has *ten* cells with figures. The entries for the prior years row begin one year after the entries for the oldest individual accident year.

We may restate this by saying that in the two year exhibits, the prior years row begins one year earlier in comparison to the ten year exhibits.

The same is true for the loss reserves. For the two year exhibits, Parts 2 and 4 begin with loss reserves for the prior years row at the date two years prior to the current statement date. Had we formed the two year exhibits by analogy to the ten year exhibits, we would have started the loss reserves for the prior years row at a date one year prior to the current statement date.

The rationale is that we need two years of real entries to show the two year development for IRIS Test 10. For the ten year lines of business, the entries needed for the two year adverse development test are available without the need to extend development back one year.

Prior Years Row

We form the prior years row in the 20X4 Part 3J:

- The prior years row in the 20X3 Part 3J says that
  A. \$300 was paid in calendar year 20X2 for accident years 20X1 and prior, and
  B. \$330 – \$300 = \$30 was paid in calendar year 20X3 for accident years 20X1 and prior.

- For the prior years row in the 20X4 Part 3J we need to determine
  A. How much was paid in calendar year 20X3 for accident years 20X2 and prior, and
  B. How much was paid in calendar year 20X4 for accident years 20X2 and prior.

- From the 20X3 Part 3J we determine the amount paid in calendar year 20X3 for accident year 20X2 as $960 – $780 = $180. We add to this the $30 paid in calendar year 20X3 for accident years 20X1 and prior to get a total of $210 for Item 2.A above.

- The amount paid in calendar year 20X4 for accident years 20X2 and prior is $16 + $40 = $56. We add $56 and $210 to get $266, which is the final entry in the prior years row in the 20X4 all companies combined Part 3J (before the allocation to member companies).

INDIVIDUAL ACCIDENT YEARS

The remaining 20X4 Part 3J entries are straight-forward. Since the two companies participate in an intercompany pooling agreement, we first format the combined (all companies) Schedule P exhibits.

☞ The calendar year 20X3 payments for accident year 20X3 are the same as in the 20X3 Part 3J, or $580.

☞ The calendar year 20X4 payments are taken from the 20X4 experience:
  ✓ For accident year 20X3, the payments are $200. The cumulative accident year 20X3 payments as of December 31, 20X4 are $580 + $200 = $780.
  ✓ For accident year 20X4, the combined payments for the two companies are $400 + $300 = $700.

The 20X4 Part 3J for the two companies combined looks as follows:

Exhibit 2.14: Pooled Companies Cumulative Paid Losses

Years in which Losses were Incurred	Cumulative Paid Losses and DCC at Year End ($000) 20X2	20X3	20X4
Prior	000	210	266
20X3	XXX	580	780
20X4	XXX	XXX	700

Since the current pooling arrangement is 50% for each company, the 20X4 Part 3J for each company (XYZ and ABC) is exactly half of the combined Part 3J, as shown below.

Exhibit 2.15: Individual Companies Cumulative Paid Losses

Years in which Losses were Incurred	Cumulative Paid Losses and DCC at Year End ($000) 20X2	20X3	20X4
Prior	000	105	133
20X3	XXX	290	390
20X4	XXX	XXX	350

Loss Reserves

To form the 20X4 Part 2J, we add the loss reserves to the loss payments. From the 20X3 Parts 2J and 3J we form a triangle of loss reserves only, which is the difference between these two triangles:

Exhibit 2.16: Pooled Companies Loss Reserves

Year in which Losses were Incurred	Loss Reserves at Year End ($000) 20X1	20X2	20X3
Prior	400	80	40
20X2	XXX	220	60
20X3	XXX	XXX	220

For the 20X4 Part 2J, the entry in the upper-left corner is the loss reserves at December 31, 20X2 for accident years 20X2 and prior. From the triangle directly above, this is $80 + $220 = $300. In a similar fashion, we construct a triangle of loss reserves only for the 20X4 Schedule P, as shown below.

Exhibit 2.17: Pooled Companies Loss Reserves, Updated

Year in which Losses were Incurred	Loss Reserves at Year End ($000) 20X1	20X2	20X3
Prior	300	100	30
20X3	XXX	220	50
20X4	XXX	XXX	250

The December 31, 20X4 reserves are taken from the 20X4 experience exhibit. For accident year 20X4, we combined the reserves being held for XYZ and for ABC.

We add these reserves to the 20X4 Part 3J paid losses to get the 20X4 Part 2J (before the allocation to company).

Exhibit 2.18: Pooled Companies Incurred Losses

Years in which Losses were Incurred	Incurred Losses and DCC at Year End ($000) 20X2	20X3	20X4
Prior	300	310	296
20X3	XXX	800	830
20X4	XXX	XXX	950

We now allocate this triangle 50% to ABC and 50% to XYZ. The final Part 2J for each company looks as follows:

Exhibit 2.19: Individual Companies Incurred Losses

Years in which Losses were Incurred	Incurred Losses and DCC at Year End ($000) 20X2	20X3	20X4
Prior	150	155	148
20X3	XXX	400	415
20X4	XXX	XXX	475

PART 4 - BULK + IBNR RESERVES

Part 4 shows bulk + IBNR, or "actuarial," reserves, by accident year and evaluation date. These are reserves "for incurred but not reported claims, for reopened claims, for development on case reserves of reported claims, and for aggregate reserves on newly reported claims without specific case reserves" (Annual Statement *Instructions*). The use of Part 4 to derive case incurred (or reported) loss figures is described above.

Part 5 – Claim Counts

Parts 2, 3, and 4 of Schedule P allow the analyst to perform prospective reserve analyses using absolute dollar techniques, using either paid loss methods (Part 3) or reported loss methods (Parts 2 and 4).

Part 5 allows the analyst to perform claim count and average claim severity reserving analyses. Three claim count triangles are shown for the nine lines of business mentioned earlier in this paper (see page 35):

- Section 1: Cumulative direct plus assumed claims closed with loss payment
- Section 2: Direct plus assumed claims outstanding
- Section 3: Cumulative direct plus assumed claims reported

A triangle of direct plus assumed claims closed without loss payment can be derived from the three triangles shown, since reported claims equal the sum of

- claims outstanding,
- claims closed with loss payment, and
- claims closed without loss payment.

The total number of claims closed, both with and without loss payment, is the number of reported claims minus the number of claims outstanding.

AVERAGE SEVERITY RESERVE ANALYSES

Absolute dollar reserve analyses may be distorted by changes in (a) the volume of business, (b) inflation, or (c) case reserve adequacy. Claim count development is more stable. Average severity changes from accident year to accident year can be compared with inflation indices.

> *Illustration:* Required reserves for the most recent accident year are difficult to quantify in long-tailed lines of business. Claim count / average severity reserving methods are often used for lines of business where losses take long to settle and are subject to substantial random fluctuation, as might be caused by unpredictable jury awards. (Medical malpractice provides a good example.) If the ultimate claim count can be estimated, and if average severities in the most recent accident year are assumed to be 8% higher than those in the previous accident year, the estimated ultimate losses are 108% × the ratio of claim counts in the most recent accident year to claim counts in the previous accident year. The ultimate losses minus the paid losses and the case reserves equals the indicated bulk + IBNR reserve.

Part 5 serves as an effective regulatory monitoring tool as well. Distressed insurers seeking to avoid regulatory scrutiny may artificially strengthen their surplus by (a) reducing case reserves, (b) not setting up compensating bulk reserves, and (c) paying claims more slowly. Whereas analyses of reserve adequacy based on absolute dollar figures may not uncover the problems, the claim count triangles would show two results:

- Reported claim counts and outstanding claim counts are not affected by reserve strengthening or weakening. The lower case and bulk + IBNR reserves depress the average severities, revealing the potential reserve problems.
- The slower claim settlement patterns are reflected by lower paid-to-reported claim ratios and higher open-to-reported claim ratios.

Illustration A: A company shows a 10% annual trend in average closed claim severities and a 2% annual trend in average outstanding claim severities. There has been no changes in claim settlement speed. A regulator might suspect that the company has weakened its loss reserves.

Illustration B: A paid loss development analysis does not show a reserve deficiency problem. However, the ratio of outstanding claims to reported claims at 12 months of development has increased from 30% to 45%. A regulator might suspect that the company is masking a reserve deficiency problem by delaying the settlement of claims.

ADJUSTING THE HISTORICAL TRIANGLES

Actuaries have developed techniques to correct for distortions caused by case reserve strengthening or weakening and for changes in claim settlement patterns (see Berquist and Sherman [1977]):

Changes in Claim Settlement

The paid loss development analysis using the Part 3 historical triangles uses "chronological" development ages: 12 months, 24 months, and so forth. If the company's loss settlement patterns are changing, the analyst may use "settlement" ages. Instead of using cumulative paid losses through 12 months, 24 months, and so forth, the analyst may use cumulative paid losses through the period of time when 25% of claims have been settled, 35% of claims have been settled, and so forth. This type of analysis requires the paid claim count histories of Part 5 (claims closed with payment).

CHANGES IN CLAIM SETTLEMENT: ILLUSTRATION

In past years, 50% of the automobile insurance liability accident year claims are settled during the accident year, another 30% are settled in the next 12 months, and the final 20% are settled in the next 12 months. We summarize the cumulative claim settlement pattern as 50%-80%-100% for the first three years.

In an effort to control costs, the company has striven to settle claims more quickly during the most recent calendar year. The new pattern is 70%-90%-100% for the first three years. That is, 70% of the claims are settled during the accident year, another 20% are settled in the next 12 months, and the final 10% are settled in the next 12 months.

Small and simpler claims are settled more rapidly than large, complex claims. The pattern of loss payments is slower than the pattern of percentage of claims closed. In past years (before the recent revision of claim settlement practices), the pattern of loss payments was 33.3%, 66.7%, 100% in the first three years. That is, one third of losses were paid during the accident year, another third were paid in the next 12 months, and the final third were paid in the next 12 months.

In past years, the paid loss development factor from 12 months to ultimate in past years was 100% / 33.3% = 3.000, and the paid loss development factor from 24 months to ultimate was 100% / 66.7% = 1.500.

These historical paid loss development factors are too high for the current paid losses. Since a larger percentage of claims have been closed at each chronological age, a larger percentage of the losses have been paid by each development date, and there should be less paid loss development in subsequent periods.

BERQUIST-SHERMAN ADJUSTMENT

Instead of using cumulative paid losses, the reserving actuary may use the past experience to estimate the paid loss development factors from "50% of claims closed to ultimate" = 3.000 and from "80% of claims closed to ultimate" = 1.500. The analyst interpolates between these two figures to estimate the paid loss development factor from "70% of claims closed to ultimate." Simple linear interpolation gives a loss development factor of 2.000. In truth, the relationship between claims closed and losses paid is not linear, so the simple interpolation is not appropriate. Berquist and Sherman [1977] illustrate the procedure with an exponential relationship between claim settlement and loss payment.

The adjustment from chronological age to settlement age is important for actuarial loss reserving, but it must be used with caution. Many factors can cause changes in the claim settlement patterns without necessarily changing the loss payment pattern. For instance, an increase in nuisance claims, as had occurred in private passenger automobile and workers' compensation in the 1970's and 1980's, caused a large increase in the claim counts and the claim settlement speed, but no significant increase in the percentage of losses paid; see Conners and Feldblum [1998].

Changes in Case Reserve Adequacy

The case incurred (i.e., reported) loss development using Schedule P, Parts 2 and 4, may be distorted by changes in case reserve adequacy, even when these changes are compensated for by offsetting changes in bulk + IBNR reserve levels. To circumvent the problems caused by varying case reserve adequacy levels, one may restate the past case reserves based on the assumed inflation rate between the accident years.

> *Illustration:* The average severity of personal automobile liability open claims at 12 months of development is $20,000 for accident year 20X9. The loss cost trend for private passenger automobile liability from accident year 20X8 to accident year 20X9 is 8%. We would expect the average outstanding clam severity in 20X8 at 12 months to have been about $20,000 / 1.08 = $18,519, or about $18,500. If the average value of open claims in 20X8 at 12 months differs significantly, the average case reserve adequacy level may have changed between 20X8 and 20X9.[76]
>
> If the average value of open claims in accident year 20X8 at 12 months of development was $16,500, we might suspect that the claims department has been strengthening case reserves during the most recent calendar year (20X9). Replacing the observed value with the $18,500 expected value corrects for the distortion caused by changing reserve adequacy levels.
>
> Suppose that the reported loss link ratio from 12 months to 24 months was 1.500 for accident year 20X8, when average case reserves were $16,500 at 12 months of development. This link ratio is too high for the 20X9 reported losses at 12 months of development, since the case reserves are stronger.

BERQUIST-SHERMAN ADJUSTMENT

To illustrate the necessary procedure to correct the distortion, suppose that the case reserves at 12 months of development in the previous year were composed of $100 million of paid losses and $100 million of case reserves. We separate the reported loss link ratio of 1.500 into two parts. The losses already paid in the first 12 months don't change in the next 12 months. The remaining reported losses – the case reserves – may be paid in the next 12

[76] To keep the illustrations in this paper simple, we do not consider credibility issues. In practice, we would not infer a change in the average case reserve adequacy level from a single observation of average outstanding claim severities. We would examine the relationship between average outstanding claim severities in calendar years 20X8 and 20X9 at various ages of development. If the difference between them is consistently different from the expected difference based on claim cost inflation, we might infer a change in case reserve adequacy levels.

months or may be re-estimated for a different amount. In addition, additional claims may be reported as IBNR losses.

Since the total reported losses increased by 50% from $200 million at 12 months to $300 million at 24 months and the losses already paid ($100 million) did not change, the case reserves increased by 100% from $100 million at 12 months to $200 million at 24 months.

The current case 20X9 reserves are more adequate by a factor of $18,500 / $16,500 = 1.121. We expect the current 20X9 case reserves to increase by 200% x $16,500 / $18,500 – 100% = 78.4% from 12 months to 24 months. The intuition for this is that the increase in the 20X8 case reserves from 12 months at December 31, 20X8 to 24 months at December 31, 20X9 was composed of two pieces: a +12.1% reserve strengthening and a +78.4% development increase from 12 months to 24 months.

The revised reported loss link ratio from 12 months to 24 months is (1.000 + 1.784) / 2 = 1.392. The figures are summarized in the table below, which assumes no change in exposure levels between accident years 20X8 and 20X9, but changes in reserve adequacy and monetary inflation.

Exhibit 5.1: Berquist-Sherman Adjustment for Changing Reserve Adequacy

Accident year	Paid loss at 12 months	Case reserves at 12 months	Reported losses at 12 months	Reported losses at 24 months	Reported loss development factor
20X8	$100 million	$100 million	$200 million	$300 million	1.500
20X9	$108 million	$121 million	$229 million	$324 million	1.415

A complete description of this procedure is presented by Berquist and Sherman [1977].

NET VS DIRECT PLUS ASSUMED

The claim count triangles in Part 5 show direct plus assumed business. The loss triangles in Parts 2, 3, and 4 show net business. If the company has a significant amount of ceded business, and if the ceding percentages of proportional treaties or the retentions in non-proportional treaties have changed over time, then the average severity analyses will be distorted.

The original rationale for showing the claim count triangles in Part 5 as direct and assumed business instead of net business was threefold.

First, it is difficult to measure net claim counts for business with ceded non-proportional reinsurance.

Illustration: A property is insured for $10 million, with an excess of loss reinsurance cover of $8 million above a $2 million retention. A claim is incurred for $5 million. The primary company pays $2 million and the reinsurer pays $3 million. There is one direct claim for the primary company. The reinsurer should presumably code this as one assumed claim. But is there 1 net claim, 0 net claims, or some intermediate number for the primary company? Coding net claims in the same percentage as net losses – 40% of a claim for the primary company – would be enormous work for little or no benefit.

Second, before 1993 the only claim counts shown in Schedule P were for the current valuation. These are the Part 1 and Part 3 claim count columns. The intention of these columns is to match the direct plus assumed claim counts with the direct plus assumed loss statistics. The difficulties in matching net vs. direct plus assumed business arose with the addition of historical claim count triangles to Schedule P in 1993.

Finally, when Part 5 was first formed, there was no reduction in claim counts for non-affiliated proportional ceded reinsurance. With the exception of intercompany pooling agreements, the Schedule P definition of net claim counts was the same as the Schedule P definition of direct plus assumed claim counts. This is no longer the case, since all proportional reinsurance reduces the direct plus assumed claim counts; see the discussion earlier in this paper.

AVERAGE VALUES OF OUTSTANDING CLAIMS

Both Part 1, column 25, "Number of Claims Outstanding," and Part 5, Section 2 allow one to determine the average value of an outstanding claim. A triangle of net case reserves by accident year may be formed as Part 2 – Part 3 – Part 4. This triangle includes both case loss reserves and case reserves for defense and cost containment (if the company holds such case reserves).

Direct plus assumed case reserves by accident year are in Part 1, column 13, which shows only the figures for the current statement date. The case reserves divided by the number of claims outstanding is the average value of an open case. A comparison of these values by accident year shows trends in average loss costs.

The trend in average outstanding claim severity is important for monitoring case reserve adequacy. An open claim loss severity trend lower than the closed claim loss severity trend may indicate case reserve weakening.

Illustration: The average loss cost trend for claims closed with payment is +8% per annum. This estimate is derived from the ratio of the Schedule P, Part 3 triangle to the Schedule P, Part 5, Section 1 triangle (closed with loss payment).

The average loss cost trend for open claims is +3% per annum. This estimate is derived from the ratio of the Schedule P, Part 2 – 3 – 4 triangle to the Schedule P, Part 5, Section 2 triangle (outstanding claims).

The 5 percentage point difference in the loss cost trends may indicate case reserve weakening, particularly if the company shows other signs of financial weakness.

COMPLETING THE PART 5 EXHIBITS

Part 5 has three sections: Section 1 shows cumulative claims closed with loss payment; section 2 shows claims outstanding; and section 3 shows cumulative reported claims.

Section 1 of Part 5 is similar to Part 3 for the individual accident years: Part 5 shows cumulative claims and Part 3 shows cumulative loss payments. The entries for the"prior years" row are different. In Part 3, the individual accident years show cumulative figures, and the "prior years" row shows cumulative loss payment beginning with the second calendar year shown along the top of the exhibit.

> *Illustration:* In the 2010 Annual Statement, the "prior years" row in Schedule P, Part 3 shows cumulative loss payments beginning from January 1, *2002* for accident years 2000 and prior.

Column 11 of Part 3, "number of claims closed with loss payment," shows the cumulative number of claims through the statement date for the individual accident years. Column 12 shows the corresponding number of claims closed without loss payment.

> *Illustration:* In the "prior years" row in the 2010 Annual Statement, Schedule P, Part 3, column 11 shows the cumulative number of claims closed with loss payment from January 1, 2002 through December 31, 2010 for accident years 2000 and prior.

Section 1 of Part 5 shows the cumulative number of claims closed with loss payment for the individual accident years at each December 31. For the individual accident years, column 10 of Part 5, Section 1 equals column 11 of Part 3. For the "prior years" row, Section 1 of Part 5 shows *incremental closings* in each calendar year, not the cumulative total. For the "prior years" row, column 10 of Section 1 of Part 5 does *not* equal column 11 of Part 3.[77]

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[77] Neither the NAIC *Instructions* nor the Schedule P exhibits mention this difference, though one item of the formatting of the exhibits alludes to it. The upper left hand cell of the Part 3 exhibits contains "000," indicating that the cumulative payments begin with the second column. These are the payments from the reserves held at the year-end date corresponding to the first column. The "prior years" closed claims shown in columns 11 and 12 of Part 3 correspond to the cumulative paid losses at the current statement date in column 11. In Section 1 of Part 5, the first cell in the "prior years" row does *not* contain "000," indicating that this row shows incremental closed claims, not cumulative closed claims. [I am indebted to Richard Roth for

Illustration: For accident year 2001, there are 5,000 claims closed with payment in each calendar year from 2001 through 2010. For accident years 2000 and prior, there are 10,000 claims closed with payment in each calendar year from 2001 through 2010.

For accident year 2001 in Part 5, Section 1, the counts are cumulative, so the company reports 5,000 in the 2001 column (column 1), 10,000 in the 2002 column (column 2), 15,000 in the 2003 column (column 3), and so forth, ending with 50,000 in the 2010 column (column 10). For the "prior years" row, the counts are incremental, so the company reports 10,000 claims in each column.

In column 11 of the Part 3 exhibit for accident year 2001, the company shows the cumulative count at the current statement date, or 50,000, as in Part 5, Section 1, column 10. For the "prior years" row, column 11 shows the cumulative claims closed since January 1, 2002, or 90,000, which differs from the entry in Part 5, Section 1, column 10 (which is 10,000).

Part 5, Section 2 shows claims outstanding at each year end. This figure is affected by the company's small claims handling procedures. Not all companies set up claim files for small claims that are expected to be settled quickly, as often occurs in personal automobile physical damage.

Part 5, Section 3, is similar to Part 5, Section 1. The individual accident years show the cumulative claims reported. The "prior years" row shows the incremental claims reported in each calendar year interval. The relationship that

cumulative reported claims = cumulative paid claims plus outstanding claims

holds for the individual accident years, but not for the "prior years" row.

For claims-made coverage, the Schedule P "incurral date" is the report date. Year 20XX in the left-most column means claims reported in 20XX, not claims with accident dates in 20XX. For the individual years shown in the left-most column, the figures in the initial diagonal are carried unchanged along each row. The entries in the "prior years" row should all be zero.

"Claims in transit" are a minor exception to these rules. A claim that is reported to the company on December 28, 20XX, may not be entered into the company's electronic files until January 10, 20XX+1. If the company prepares Schedule P immediately after the end of the year, the claim belongs in the year 20XX row but it may not show up until the year 20XX+1 column.

Electronic data processing files are not prevalent, and claims in transit for so long that they are not entered in time to the company's files are rare.

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explaining this to me.]

Part 6 – Premium Development

Part 6 shows the development of earned premium by exposure year, similar to the development of incurred losses by accident year in Part 2. Exposure year earned premium is not required elsewhere in the Annual Statement, and not all companies compile the requisite data.[78]

Accrued retrospective premiums and earned but unbilled premiums are most commonly analyzed by policy period, not by exposure year. Policyholders are concerned with the experience on their own contracts; the segmentation by exposure year is of little concern to them. Insurers are concerned with the effects of retrospective rating provisions and competitive adjustments on premiums. They use policy period data, not exposure year data.

The distribution of the current calendar year's earned premium to exposure years is shown in the right-most column (column 11), along with a reconciliation of the earned premium figures to those in Part 1 of Schedule P. Reconciliation to calendar year earned premium of earlier years uses the entries on the bottom row of Part 6; see below.

Exposure year premium figures are important for lines of business where premiums are affected by exposure audits, retrospective rating adjustments, or accounting lags in booking premiums. These lines are workers' compensation, other liability, products liability, commercial automobile, and reinsurance.

The text of this section deals with the Schedule P, Part 6 exhibits and the statutory accounting procedures directly tied to these exhibits. The post codification statutory accounting rules for audits and retrospective adjustments are complex. They are important background information for understanding Part 6 of Schedule P and the related Annual Statement pages, but a complete explanation of the issues would be too long for this chapter. We have placed this material in Appendix B. Readers may find this appendix helpful for mastering Part 6 of Schedule P.

Part 7 of Schedule P shows policy year triangles of premiums and losses on loss sensitive contracts. The concepts discussed here for the Part 6 exhibits are applicable to the Part 7 exhibits as well. To avoid repetition, we discuss both exposure year premiums and policy year premiums in the text below.

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[78] Only exposure years 1993 and subsequent need be reported in Schedule P, though companies may report entries for earlier years if they have the data. This provision has little effect now on the Part 6 exhibits.

The tax regulations in January 2000 regarding expected audits and retrospective premium adjustments affect the statutory accounting practices of many insurers. We explain the tax accounting rules in Appendix B as well.

PRINCIPLES

1. For most personal insurance policies, the premium is fixed at policy inception based on a known exposure base, such as car-years or house-years. For most commercial insurance policies, the premium depends on the activity of the insured during the policy period. Workers' compensation premium depends on the payroll during the policy term; products liability premium depends on the sales during the policy term. The written premium at policy inception is only a deposit premium.

2. For retrospectively rated policies, the premium depends on the loss experience of the insured, which is not known with certainty until the losses have been settled.

3. The insurer's estimated ultimate premium may differ from the written premium initially charged. For policies subject to audits, the initial premium may be below the estimated ultimate premium for competitive reasons. For retrospectively rated policies, the insurer expects to return premium to the employer at the first retrospective adjustment and to collect additional premiums at second and subsequent adjustments. Actual cash flow patterns and premium billing patterns differ by company and by policy. The Schedule P, Part 6 premium triangles show the premium billing patterns by line of business.

4. If the estimated ultimate premium differs from the premium actually billed, the insurer accrues the difference as a return premium (a retro debit) or as expected additional premium (a retro credit).[79] The premium triangles in Part 6 of Schedule P reflect the combined effects of exposure audits, retrospective rating, insurer accruals of return or additional premiums, and changes over time in these accruals.

Illustration: Retrospective Rating

A retrospectively rated workers' compensation policy is issued on January 1, 20XX with a premium rate of $1 per $100 of payroll. The premium rate is used for the deposit premium and the standard premium. On January 1, the insured employer estimates $200,000,000 of payroll for the coming year, so the initial written premium is $2,000,000. The retrospective rating formula for this illustration is

net premium = 20% × standard premium + 1.10 × reported losses.

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[79] The accrual rules were revised in 2002 for statutory accounting and in 2000 for tax accounting. The company's tax department may use the actuarial worksheets for Schedule P, Part 6 for the tax filing; see the appendix on accounting for audits and retrospective premium adjustments.

On February 15, 20XX+1, after the policy term has expired, the insurer audits the employer's payroll records. The true payroll for 20XX was $250,000,000, and the insurer bills the employer for an additional $500,000 of premium. The standard premium is $2.5 million.

Insurers often use low estimates for the coming year's payroll as competitive tools to produce low initial premium estimates. The final premium is revised upward in accordance with the payroll audit done after policy expiration. The insurer loses the investment income on the premium that is not collected until the end-of-year audit, but it retains the policy (Feldblum [1992: WCR]).

On July 1, 20XX+1, the first retro adjustment is processed. The retrospective rating formula uses reported losses, consisting of paid losses and case reserves; IBNR losses are not included. At the first retro adjustment, losses are still immature. The indicated retrospective premium is generally less than the estimated ultimate premium, resulting in a return premium to the employer (Berry [1980], Teng and Perkins [1996], Feldblum [1997: TP]).

At 18 months after policy inception (July 1, 20XX+1), the reported losses are $1,200,000, giving a retrospective premium of 20% × $2.5 million + 1.1 × $1.2 million = $1.82 million. The insurer returns $2,500,000 – $1,820,000 = $680,000 to the employer.

At second and subsequent adjustments, the reported losses increase as they develop to maturity, and the insurer collects additional premium from the employer. At 30 months after policy inception (July 1, 20XX+2), the reported losses may be $1,500,000, giving a retrospective premium of 20% × $2.5 million + 1.1 × $1.5 million = $2.15 million. The insurer bills the employer for an additional $2,150,000 – $1,820,000 = $330,000 of premium.

CALENDAR YEAR, EXPOSURE/ACCIDENT YEAR, AND POLICY YEAR

Earned premium may be recorded by calendar year, exposure year, or policy year, and incurred losses may be recorded by calendar year, accident year, report year, or policy year. The Annual Statement reporting procedures are as follows:

Earned Premium

- Most accounting exhibits use calendar year premiums. These include the income statement; the Underwriting and Investment Exhibit, Part 2; the page 15 state exhibits; Schedule F; Schedule P, Part 1; Schedule T; and the Insurance Expense Exhibit.
- Schedule P, Part 6, shows exposure year earned premium. The reconciliation of Schedule P, Part 6 to Schedule P, Part 1 is shown in the last line of the right-most column of Part 6 (see below).
- Schedule P, Part 7 shows policy year earned premium for loss-sensitive contracts only.

Incurred losses

- Most accounting exhibits use calendar year losses. These include the income statement; the Underwriting and Investment Exhibit, Part 3; the page 15 state exhibits; Schedule F; Schedule T; and the Insurance Expense Exhibit.
- Schedule P, Parts 1 through 4 show accident year incurred losses for occurrence policies and report year losses for claims-made policies. The reconciliation of accident year incurred losses to calendar year incurred losses is not shown explicitly, but it can be derived in the same manner as for earned premiums.[80]
- Schedule P, Part 7 shows policy year incurred losses for loss-sensitive contracts only.

AUDIENCES

The four data types – calendar year, policy year, exposure/accident year, and report year – serve different audiences.

1. Calendar year data, which is final at the end of the year, is used for accounting statements in the United States. No actuarial estimates are needed. Calendar year data eliminates potential biases caused by consistent over- or under-reporting of initial estimates. However, calendar year data are the subject to smoothing of reported results.
2. Policy year data are used for policy pricing, particularly when policy conditions that affect the premiums and losses change over time, as is true for retrospectively rated policies.
3. Accident year and exposure year data are used for reserving, which requires data that are homogeneous in the age since the accident or since policy inception.
4. Report year data are used by claims personnel. Claims department efficiency is often measured by the lag between report and settlement of the claim.

ILLUSTRATION: DATE TYPES

The Part 6 triangles incorporate the effects of accrued retrospective premium estimates and the changes over time in these estimates. Some users of Schedule P are not aware that these estimates are included, and they mistakenly presume that the premium development patterns should reflect the progression of billed premiums.

Part 6 shows earned premium triangles; Part 7 shows both earned premium triangles and premium reserve triangles. Exposure year earned premiums are like accident year incurred losses, so Schedule P, Part 6 is the premium equivalent of the losses in Part 2. Accrued retrospective premium reserves and earned but unbilled premium reserves are bulk reserves,

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[80] The reconciliation is complicated by the differing treatments of loss adjustment expenses. In the historical triangles of Schedule P (Parts 2, 3, and 4), defense and cost containment adjustment expenses are combined with losses, and adjusting and other adjustment expenses are not shown. In the Underwriting and Investment Exhibit, loss adjustment expenses are shown only in total (i.e., DCC + AAO), separate from losses. In addition, Schedule P is gross of reserve discounts, whereas the other statutory exhibits are net of discounts.

like the loss reserves in Schedule P, Part 4. The earned premiums minus the premium reserves equal the billed premiums, which are similar to the reported losses shown as the difference between Part 2 and Part 4.

To highlight the effects of premium reserve estimates, we show the illustration in two parts. Part A assumes no estimates of IBNR losses, of future audits, or of accrued retrospective premiums. Part B includes these estimates.

Part 6 shows exposure year triangles; Part 7 shows policy year triangles. The illustration in this section shows the derivation of both exposure year and policy year premiums.

Illustration: A retrospectively rated annual workers' compensation policy is issued on October 1, 2003. The standard premium is $10,000, and the maximum premium is equal to 150% of the standard premium. The following transactions occur in this illustration:

- One loss occurs on March 1, 2004, with an initial reserve estimate of $8,000.
- On December 15, 2004, the payroll audit indicates that an additional $1,000 of premium should be billed. The standard premium is now $11,000, and the maximum premium is changed to $16,500.
- On November 1, 2005, the case reserve is revised to $25,000.
- On July 1, 2005, the first retrospective adjustment shows no additional or return premiums.
- On July 1, 2006, the second retrospective adjustment calls for an additional premium of $5,500. The $17,000 increase in the incurred losses results in only a $5,500 increase in the retrospective premium because of the premium maximum in the policy.

The contract is loss-sensitive, so its premium and loss amounts appear in both Part 6 and Part 7. The appropriate earned premium and incurred loss figures are as follows.

Calendar Year Accounting

Calendar year incurred losses equal calendar year paid losses plus the change in the reserve from the beginning of the year to the end of the year. In this illustration, there are no paid losses, but there are reserve changes.

- The 2003 incurred losses are zero.
- The 2004 incurred losses are $8,000 – $0 = $8,000.
- The 2005 incurred losses are $25,000 – $8,000 = $17,000.

Calendar year earned premiums (Schedule P, Part 1) equals written premium minus the change in the unearned premium reserves.

- The initial premium is split $2,500 for calendar year 2003 and $7,500 for calendar year 2004, reflecting the pro-rata earning of premium over the coverage period. The calendar year 2003 written premium is $10,000 and the unearned premium reserve at December 31, 2003, is $2,500, so the calendar year 2003 earned premium is $10,000 – ($7,500 – $0) = $2,500.

- The audit premium of $1,000 is recorded as 2004 earned premium when it is billed. This is the meaning of the statutory accounting dictum that "audit premiums are earned when written." This dictum is correct when there are no estimates of future audits or when these estimates are not being considered.
- In practice, earned but unbilled premium reserves and accrued retrospective premium reserves are bulk reserves, set by the actuary. The audit premiums are earned in the accounting period when they are written. The decrease in the premium reserve is a negative earned premium in the period when the reserves are taken down.
- The retrospective premium of $5,500 is recorded as 2006 earned premium when it is billed.

This accounting treatment presumes that the premiums resulting from the exposure audit and the retrospective adjustment are unanticipated, and that the increase in losses is not anticipated in the IBNR reserve. If reserves are held for earned but unbilled (EBUB) premiums or accrued retrospective premiums, the accounting is different; see the discussion below.

Policy Year Accounting

Policy year incurred losses (Schedule P, Part 7, Section 2) are allocated to the effective date of the policy, regardless of the dates of loss occurrence or reporting.

- At December 31, 2003, policy year 2003 incurred losses are zero.
- At December 31, 2004, policy year 2003 incurred losses are $8,000.
- At December 31, 2005, policy year 2003 incurred losses are $25,000.

Policy year earned premium (Schedule P, Part 7, Section 4) is allocated to the effective date of the policy.

- At December 31, 2003, the (estimated ultimate) 2003 earned premiums are $10,000. Only ¼ of the premium has been earned by December 31, so the policy year 2003 earned premium as of December 31, 2003 is $2,500.
- At December 31, 2004, the revised 2003 earned premiums are $11,000 (written premium plus audit). The 2004 earned premiums (from this policy) are zero. All premium is coded to policy year 2003, regardless of when the premium is billed. At December 31, 2005, the policy year 2003 earned premium is still $11,000.
- At December 31, 2006, the revised 2003 earned premiums are $16,500 (written premium plus audit plus retrospective premium); the 2004, 2005, and 2006 earned premiums (from this policy) are zero.

Accident/Exposure Year Accounting

Accident year incurred losses (Schedule P, Parts 1, 2, 3, and 4) are coded to the date the loss occurrs (for occurrence policies) or to the date the loss is reported (for claims-made policies).

- At December 31, 2003, accident year 2003 incurred losses are zero.
- At December 31, 2004, accident year 2003 incurred losses are zero, and accident year 2004 incurred losses are $8,000.
- At December 31, 2005, accident year 2003 incurred losses are zero; accident year 2004 incurred losses are $25,000; and accident year 2005 incurred losses are zero.

Exposure year earned premiums (Schedule P, Part 6) are similar to accident year incurred losses. The earned premium is allocated by year based on the exposures in each year.

- At December 31, 2003, exposure year 2003 earned premiums are $2,500. If there are audits or retrospective adjustments in 2003 relating to policies that were issued and earned in previous years, they are coded as exposure year premium (for Part 6) or as policy year premium (for Part 7) relating to earlier years.
- At September 31, 2004, exposure year 2003 earned premiums are $2,500 and exposure year 2004 earned premiums are $7,500. The December 15, 2004 audit is distributed over the policy term, so on December 31, 2004, the exposure year 2003 earned premiums are $2,750, and the exposure year 2004 earned premiums are $8,250
- The $5,500 retrospective premium stems from a March 2004 loss, and one might presume that it should be coded to exposure year 2004. In practice, it is too complex to allocate retrospective premiums to exposure years based on the accidents which led to the premiums.[81] Instead, the retrospective premiums are allocated to exposure years as the audit premiums are allocated (in proportion to the coverage period): $1,375 to exposure year 2003 and $4,125 to exposure year 2004.

We will incorporate premium and loss reserves in this illustration after explaining the statutory accounting rules for exposure year premiums.

Accounting for Exposure Year Premiums

Part 6 shows premium development triangles separately for direct plus assumed business (Section 1) and for ceded business (Section 2). Net premium development is the difference between these two triangles. Direct plus assumed business is shown separately from ceded business since audit premiums and accrued retrospective premiums are more easily and accurately recorded for direct premiums than for ceded premiums.

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[81] In addition, the maximum premium caps the full policy year retrospective premium. It would be difficult to spread this cap by exposure year.

The historical loss triangles in Parts 2, 3, and 4 show *net* losses. For companies with significant reinsurance transactions, one must take care to compare net losses with net premiums. This is particularly true if there have been material changes in the ceded reinsurance arrangements during the historical period.

The accounting rules for Schedule P, Part 6 are as follows:

1. The individual exposure years show cumulative earned premiums. The earned premiums include (i) collected premiums, (ii) billed but uncollected premiums, (iii) earned but unbilled premiums, and (iv) accrued retrospective premiums. Only the earned portion of these components is included in the Part 6 exhibits.

   Illustration: A policy is written on July 1, 20XX for a written premium of $10,000. On December 31, 20XX, the actuary expects a final audit premium to be billed around September 20XX+1 for $2,000. The 20XX earned premium is

   50% x $10,000 + 50% x $2,000 = $6,000.

2. The "prior years" row shows *incremental calendar year changes* to the earned premium for the prior exposure years. The Part 6 exhibits are like the Part 5 exhibits in this respect, not like the Part 3 exhibits.

   Illustration: The cumulative earned premiums at December 31, 2009, are $20 million apiece for exposure years 2000 and 2001. In 2010, there is an unanticipated retrospective adjustment of +$20,000 for an annual policy with a July 1, 2000, effective date. No other calendar year 2010 retrospective adjustments affect any exposure years 2001 and prior.

   For the 2010 Schedule P, column 10 of Part 6 shows (i) the cumulative total for the individual exposure years 2001 through 2010, and (ii) the calendar year transactions for exposure years 2000 and prior. The +$20,000 retrospective adjustment is divided evenly between exposure year 2000 and exposure year 2001, since the policy was in force from July 1, 2000 through June 30, 2001. Exposure year 2001 shows $20,010,000 in row 2, column 10, of Part 6. The prior year figure in row 1, column 10 is $10,000.

3. For all but the current calendar year, earned premiums need be distributed only to exposure years 1993 and subsequent. The distribution for earlier exposure years may be shown if the company desires and has the data. This rule becomes moot for the 2003 and subsequent Annual Statements; for most companies, it is no longer material for the 2001 and 2002 Annual Statements as well.

4. The distribution of the current calendar year's earned premiums to all exposure years (including the "prior years" row) is shown in column 11 of Part 6, to facilitate the

reconciliation with calendar year earned premiums. The entries in this column are incremental figures, not cumulative figures. The reconciliation procedure is explained below.

5. The final row of the Part 6 exhibits shows the Schedule P, Part 1 calendar year earned premiums. This facilitates the reconciliation of exposure year earned premiums with calendar year earned premiums.

To clarify the contents of the historical premium triangles in Part 6 and Part 7 of Schedule P, we show first a simple example of earned but unbilled premium and accrued retrospective premium, followed by the complete illustration of calendar year, exposure year, and policy year premiums that we began earlier.

Part 6 Illustration

A company issues a retrospectively rated workers' compensation policy with a deposit premium of $100,000 on January 1, 20XX. This illustration is deliberately simplified, so that the exposure year is the same as the policy year. Several large losses occur in 20XX. On December 31, 20XX, the company expects to collect an additional $40,000 in future retrospective adjustments, and it puts up an accrued retrospective premium asset (or contra-liability) of $40,000.

The company can collect additional premium only for reported losses, not for IBNR losses or for expected development on known claims. At the first retrospective adjustment on July 1, 20XX+1, the company collects $30,000 from the insured employer and reduces the accrued retrospective premium reserve to $10,000.

During the third quarter of 20XX+1, there is unexpectedly high development on the reported claims. By December 31, 20XX+1, the company raises the accrued retrospective premium reserve to $20,000.

The reporting in Parts 6 and 7 of Schedule P is as follows:

A. The 20XX exposure year earned premium in Part 6, as well as the 20XX policy year earned premium in Part 7, Section 4 is the written premium minus the change in the unearned premium reserve. The accrued retrospective premium reserve is a contra-liability, which went from $0 on January 1 to $40,000 on December 31. The 20XX earned premium is

$$\$100{,}000 - (-\$40{,}000 - \$0) = \$140{,}000.$$

B. The "net reserve for premium adjustments and accrued retrospective premiums at year end" in Section 5 of Schedule P, Part 7 shows the contra-liabilities as positive figures. The figure for policy year 20XX is $40,000 at December 31, 20XX.

C. In calendar year 20XX+1, Parts 6 and 7 of Schedule P show cumulative figures. The cumulative 20XX earned premium is the $130,000 paid plus the $20,000 remaining reserve, or $150,000.

   An alternative view is helpful for the reconciliation with calendar year earned premium. The 20XX+1 calendar year earned premium is the written premium minus the change in reserves, or $30,000 – [–$20,000 – (–$40,000)] = $10,000.

   This $10,000 is added to the $140,000 exposure year 20XX earned premium at December 31, 20XX to give a cumulative amount of $150,000 at December 31, 20XX+1.

D. The "net reserve for premium adjustments and accrued retrospective premiums at year end" in Section 5 of Schedule P, Part 7 for policy year 20XX at December 31, 20XX+1 is $20,000.

ACTUARIAL ESTIMATES

As a final illustration, we rework the example presented earlier in this section, using the company's estimates of earned but unbilled premiums and accrued retrospective premiums.

A retrospectively rated annual workers' compensation policy is issued on October 1, 2003. The standard premium is $10,000, and the maximum premium is equal to 150% of the standard premium. The following transactions occur in this illustration:

- On December 31, 2003, the reserving actuary estimates that the payroll audit at policy expiration will add $2,000 of premium.
- One loss occurs on March 1, 2004, with an initial reserve estimate of $8,000.
- On December 15, 2004, the payroll audit indicates that an additional $1,000 of premium should be billed. The standard premium is now $11,000, and the maximum premium is changed to $16,500.
- On December 31, 2004, the reserving actuary estimates bulk reserves for this policy of $6,000; this is primarily adverse development on known claims. The actuary also estimates an accrued retrospective premium reserve of $4,000.
- On July 1, 2005, the first retrospective adjustment shows no additional or return premiums.
- On November 1, 2005, the case reserve is revised to $25,000. On December 1, 2005, the claim is settled for $25,000.
- On December 31, 2005, the reserving actuary, using an aggregate bulk reserving method, changes the accrued retrospective premium reserve to $12,000.

- On July 1, 2006, the second retrospective adjustment calls for an additional premium of $5,500. The $17,000 increase in the incurred losses results in only a $5,500 increase in the retrospective premium because of the premium maximum in the policy.

In this illustration, we speak of the reserving actuary developing reserve indications for a single policy or for a single claim. In practice, this is rarely done. The reserve indications are based on aggregate data. They are estimated for accident years or policy years, not for individual claims or policies.

The illustration is heuristic. We show the component pieces of paid amounts, case reserves, and bulk reserves to clarify the statutory accounting principles. The accrued retrospective premium reserve of December 31, 2005 is an example of this. The reserving actuary used an aggregate reserving method, whereby the premium reserve is about ⅔ of the bulk loss reserve. Had the actuary used a per policy reserving method, the premium reserve would have been capped at $5,500.

Estimated Payroll Audit

The actuary's estimate of the earned but unbilled premium is included in the earned premium for the year. At December 31, 2003, the estimated earned premium for the policy is $10,000 deposit premium + $2,000 audit premium = $12,000. One quarter of the policy has been earned by December 31, so the 2003 earned premium is $3,000. The expected earned premium for 2004, as of December 31, 2003, is $9,000.

On September 30, 2004, the policy expires. The additional $9,000 of earned premium is charged to calendar year 2004 earned premium, exposure year 2004 earned premium, and policy year 2003 earned premium.

On December 15, 2004, the payroll audit yields only $1,000, not $2,000. The net earned premium from the payroll audit is the billed premium plus the change in reserve,[82] or

$$\$1{,}000 + (\$0 - \$2{,}000) = -\$1{,}000.$$

- For calendar year earned premiums, the net earned premium from the payroll audit of –$1,000 is allocated to 2004.
- For policy year earned premiums, the net earned premium from the payroll audit of –$1,000 is allocated to 2003.
- For exposure year earned premiums, the net earned premium from the payroll audit of –$1,000 is allocated ¼ to 2003 and ¾ to 2004.

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[82] We refer to the premium asset as the reserve, as is common practice in the industry. Were we to speak of the premium liability as the reserve, the earned premium would be the billed premium *minus* the change in the reserve.

The same procedure is used for all other bulk reserves.

On December 31, 2004, the bulk reserves for this policy are $6,000 for losses and $4,000 for premiums. The actuary is using an aggregate premium reserving method with a ⅔ sensitivity factor.

- For calendar year accounting, both the bulk reserve for losses and the bulk reserve for premiums are assigned to 2004.
- For policy year accounting, both the bulk reserve for losses and the bulk reserve for premiums are assigned to 2003.
- For accident year accounting, the bulk reserves for losses are assigned to 2004.
- For exposure year accounting, the ¼ of the bulk reserve for premiums is assigned to 2003 and ¾ is assigned to 2004.

On December 31, 2005, the bulk reserve for premiums is revised to $12,000. The change in the bulk reserve is $12,000 – $4,000 = $8,000. This is assigned to calendar year 2005 and to policy year 2003. For exposure year accounting, ¼ is assigned to 2003 and ¾ is assigned to 2004.

On July 1, 2006, the bulk reserve for premiums is changed to a billed premium of $5,500. The net earned premium resulting from the retrospective adjustment is

$$\$5,500 + (\$0 - \$12,000) = -\$6,500.$$

This net earned premium is assigned to calendar year 2006 and to policy year 2003. For exposure year accounting, ¼ is assigned to 2003 and ¾ is assigned to 2004.

COMPLETING THE PART 6 EXHIBITS

An illustration should help clarify the reporting of premiums in Part 6 and the reconciliation with Part 1. Since the earned premium entries include the earned but unbilled premium and accrued retrospective premium reserves, a company which sets reserves accurately should show little development along the rows. Upward development indicates conservatism; downward development indicates over-optimistic reserves.

Exhibit 6.1: 20X9 Schedule P, Part 6 ($000's)

Part 6	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9	(A)*
prior	25	15	10	8	6	5	4	4	3	3	3
20X0	500	480	485	488	490	495	495	497	498	499	1
20X1		520	525	523	520	530	540	538	540	542	2
20X2			550	555	555	560	555	550	552	555	3
20X3				580	585	590	592	595	595	597	2
20X4					620	630	700	690	700	700	0
20X5						700	710	720	720	730	10
20X6							750	750	740	760	20
20X7								800	820	810	-10
20X8									850	860	10
20X9										900	900
(B)**	525	515	570	594	630	740	841	802	878	941	941

* "A" = "Current Year Premiums Earned"

** "B" = "Earned premiums, Schedule P, Part 1"

The final row in the exhibit shows the calendar year earned premiums from Schedule P, Part 1, column 2 (direct plus assumed earned premiums). Consider calendar year 20X4. Of the $630,000 in earned premium, $620,000 is allocated to exposure year 20X4. $5,000 is allocated to exposure year 20X3, which is the difference between the cumulative figures of $585,000 and $580,000. A negative $3,000 is allocated to exposure year 20X1; in other words, the take down in the accrued retrospective premium reserve for exposure year 20X1 between 12/31/20X3 and 12/31/20X4 exceeded the additional premiums collected in this period for exposure year 20X1.

For the prior years row, the Part 6 entries are the incremental values themselves. The reconciliation is as follows:

The calendar year "X" earned premium =
the sum of the calendar year "X" column entries for individual exposure years
– the sum of the calendar year "(X–1)" column entries for individual exposure years
\+ the calendar year "X" entry for the "prior years" row.

This reconciliation is possible only if the company shows entries for all exposure years. If entries are shown only for exposure years 1993 and subsequent, then any changes in earned premium associated with previous exposure years simply disappear. The right-most column in the exhibit shows incremental premium changes for all exposure years during the current calendar year, to enable a reconciliation with the current calendar year earned premium.

Approximations

Part 6 is similar to Part 2; both show development of incurred amounts. In Part 2, however, payments and case reserves are related to particular losses, which are associated with specific accident years. Similarly, bulk reserves are generally determined by the development of accident year paid losses or reported losses, so bulk reserves also relate to specific accident years.

Return premium and additional premiums are associated with policies. The earned but unbilled premium reserves and the accrued retrospective premium reserves are generally determined from policy year triangles, not exposure year triangles. Most companies will convert the return premiums, additional premium collections, and reserve changes from a policy year basis to an exposure year basis by approximations. Nevertheless, since the primary purpose of Part 6 is to allow the computation of accurate exposure/accident year loss ratios, Part 6 uses exposure years, not policy years.

Part 7 – Loss Sensitive Contracts

Parts 1 through 6 of Schedule P were designed to monitor loss reserve adequacy. Part 7 was designed by the American Academy of Actuaries Task Force on Risk-Based Capital (RBC), and it has two purposes: (i) to determine the company's percentage of written premium and of reserves related to loss-sensitive contracts, and (ii) to determine the sensitivity of premiums and of reinsurance commissions to losses on these contracts.

- Parts 1 through 6 show experience on the company's entire book of business. Part 7 shows experience on loss sensitive contracts only.

- Part 7 is optional. It must be completed only if the company claims a reduction for loss sensitive contracts in its risk-based capital reserving risk charge or written premium risk charge. All other exhibits in Schedule P must be completed by all companies.

- Parts 1 through 6 show data by line of business. Section 1 of Part 7A and Part 7B use the same subdivision by line of business. Sections 2 through 5 of Part 7A and sections 2 through 7 of Part 7B are on an all lines combined basis. Loss sensitive contracts sold to large accounts often combine several lines of business, and it might be difficult to separate the premium sensitivity by line.

- The losses and claim counts in Parts 1 through 5 are on an accident year basis, and the earned premiums in Part 6 are on an exposure year basis. Accident year and exposure year are equivalent data types, though the former refers to losses or claims and the latter refers to premiums. The losses and premiums in Part 7 are on a policy year basis. No other exhibit in the Annual Statement is on a policy year basis.[83]

- Part 7A shows net experience on primary loss-sensitive contracts, and Part 7B shows net experience on reinsurance loss-sensitive contracts. The direct business is shown separately from the reinsured business because the RBC loss sensitive contract offset is 30% for primary policies and 15% for reinsurance treaties. The rationale for this difference is that workers' compensation retrospectively rated policies often have wider swings than sliding scale commissions have on reinsurance treaties.

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[83] Premium sensitivity is dependent on the retrospective rating plan parameters, which are analyzed on a policy year basis; see the "formula approach" in Teng and Perkins [1996].

RBC UNDERWRITING RISK CHARGES

For most companies, the reserving risk charge (R_4) and the written premium risk charge (R_5) contribute the largest portions of total capital requirements. The risk-based capital formula provides a reduction in these charges for business written on loss sensitive contracts.

The reserving risk charge is the amount of capital needed to guard against unanticipated adverse development on existing reserves in a "worst case" scenario. The risk-based capital formula determines the worst case scenario based on industry-wide Schedule P experience from 1983 through 1992. The capital needed is reduced for the expected investment income on the assets backing the loss reserves.

Illustration: Based on historical industry-wide Schedule P data from 1983 through 1992, the risk-based capital formula estimates that workers' compensation loss reserves may develop adversely by 27.3% in a worst case scenario. The average discount factor for workers' compensation loss reserves in the risk-based capital formula is 87.2%.

In a worst case scenario, $100 of workers' compensation loss reserves may develop into $127.30 of paid losses. The assets needed now to fund $127.30 when the losses are paid equal $127.30 x 87.2% = $111.01. The reserving risk charge for workers' compensation is 11.0% of the held reserves.

Premium Sensitivity

If the workers' compensation policy is retrospectively rated – that is, the policy is a loss sensitive contract – the adverse development on loss reserves is at least partially offset by additional premium. Less capital is needed to guard against a worst case scenario.

> *Illustration:* Suppose that for each dollar of additional loss, the insurer expects 40¢ of additional premium (retrospective premium credits). If $100 of loss reserves develops into $127.30 of paid losses, the insurer expects to collect additional premium of $27.30 x 40% = $10.92. The insurer needs $11.01 – $10.92 = $0.09 of capital for adverse development on a dollar of held reserves.

The sensitivity of retrospective premiums to losses varies widely among retrospectively rated policies. The retrospective rating formula itself generally has a sensitivity of at least unity. A dollar of loss may lead to a $1.10 or $1.15 of retrospective premium, where the extra ten or fifteen cents covers loss adjustment expenses and other loss related charges, such as state premium taxes and involuntary market burdens.

In practice, losses are capped in most retrospective rating plans, and retrospective premiums are limited by a maximum. The actual premium sensitivity depends on the parameters of the retrospective rating plan, the shape of the insured's size-of-loss distribution, and the amount of the standard premium.

The premium sensitivity also depends on the maturity of the losses. The first losses to be reported are rarely capped, and the insured generally has not reached the maximum premium. The premium sensitivity is about unity. In contrast, adverse development on mature loss reserves generally occurs on large losses, which may have already been capped by the parameters of the retrospective rating plan. In a worst case scenario, the insured may also have reached the maximum premium. The premium sensitivity may be quite low, such as 20¢ or 30¢ for each additional dollar of loss reported.[84]

In 1993, the NAIC Working Group on Risk-Based Capital decided on conservative levels of premium sensitivity: 30% for primary contracts and 15% for reinsurance contracts. Companies which write retrospectively rated workers' compensation policies for large accounts have argued that the premium sensitivity on their books of business is significantly greater.[85] Sections 2 through 5 of Schedule P, Part 7A, and section 2 through 7 of Schedule P, Part 7B, are designed to provide the data for more accurate estimates of premium sensitivity.

LOSS RESERVE ADEQUACY AND RBC OFFSETS

Schedule P, Part 7 is not related to the earlier parts of Schedule P. The ostensible reasons for the inclusion of the loss sensitive contracts exhibits in Schedule P are that

- The risk-based capital underwriting risk charges use the Schedule P line division, and the RBC loss sensitive contract offset uses the same line division.
- Premium sensitivity relates to reserve development, which is the subject matter of Schedule P.

The historical motivation for including these exhibits in Schedule P was more direct. The NAIC was concerned that companies might not properly classify their contracts if they report the figures in the risk-based capital submission. Companies would be less likely to classify

[84] For discussions of premium sensitivity and its determinants, see Bender [1994], Mahler [1996], Teng and Perkins [1996], and Feldblum [1997: PDLD].

[85] The premium sensitivity depends on the types of plans sold by the insurance company. For a workers' compensation carrier selling wide-swing plans to large accounts, the sensitivity may be between 80% and 85% for the written premium risk loss sensitive contract offset and between 60% and 65% for the reserving risk loss sensitive contract offset. For a company selling narrow swing plans to small risks, the offsets are much smaller. For an analysis of premium sensitivity on plans sold to small accounts, see Bender [1994] and the discussion by Mahler.

a contract as loss sensitive if in fact it were not loss sensitive if the reporting were in Schedule P, since most companies treat the Schedule P submission with more care and diligence.[86]

PREMIUMS, COMMISSIONS, DIVIDENDS

Loss sensitive contracts are of three types:

- For retrospectively rated primary contracts, the final premium depends on the losses incurred by the insured, subject to loss limits and premium maximums and minimums.
- For sliding scale reinsurance treaties, the reinsurance commission depends on the loss ratio experienced on the assumed book of business, subject to a maximum and minimum (see Clark [1996]).
- For many policyholder-dividend plans, the dividend payable to each insured depends on that insured's loss ratio or on the loss ratio of a classification group.

The risk-based capital principles are as follows:

- If the premium varies with losses and is sufficiently responsive, the policy is considered "loss sensitive."
- If the primary policy's commission varies with losses (e.g., contingent commissions), the policy is not considered a loss sensitive contract. Contingent commissions on direct business generally have narrow swings, so the sensitivity to losses is limited.
- If the reinsurance treaty's commission varies with losses (e.g., sliding scale commissions), the policy may be considered a loss sensitive contract. However, since the average responsiveness of reinsurance commissions and premiums to losses differs from the average responsiveness of primary premiums to losses, separate offsets are used for direct and for assumed business, and separate Part 7 exhibits are shown for primary business and for reinsurance contracts.
- Varying dividend rates do not make a policy loss sensitive. Policyholder dividends are generally optional, not contractual.

DEFINITION OF LOSS-SENSITIVE CONTRACTS

The risk-based capital underwriting risk factors are applied to loss reserves and to written premium, so Section 1 of Parts 7A and 7B determines the percentage of loss reserves and of written premium by line of business that relates to loss-sensitive business. Since the risk-based capital requirements are lower for loss-sensitive business, distressed companies have an incentive to classify their business as loss-sensitive, even if the loss-sensitivity is

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86 The inability to reconcile the Part 7 data with other statutory exhibits make regulators especially uneasy. Vincent Laurenzano, in particular, advocated the inclusion of these exhibits in Schedule P to ensure the accuracy of the figures.

minimal. To prevent such abuse, a contract must fulfill the following six criteria to be classified as loss-sensitive:

1. An increase in losses can lead to an increase in net payment for that policy. If the loss sensitive item is not a monetary transaction, the contract is not loss sensitive.

2. The loss sensitive payment must be at least 75% of the loss on primary business and at least 50% of the loss on reinsurance treaties before the application of any limits. In other words, if losses on a retrospectively rated workers' compensation policy increase by $10,000, the retrospective premium must increase by at least $7,500 before the application of loss limits or maximum premium caps.

3. Maximum and minimum premiums, loss limits, and upper and lower bounds on the reinsurance commission may constrain an otherwise "loss sensitive" contract. For a contract to be classified as loss sensitive, the "swing" of the plan must be at least 20% for primary business and 10% for reinsurance treaties. The net amount payable when the loss experience is the worst possible must be at least 20% greater than the net amount payable when the loss experience is the best possible.

   Illustration: A retrospectively rated workers' compensation policy with a minimum premium of $9,000 and a maximum premium of $10,000 would not qualify as loss sensitive.

4. The maximum net payment must be at least 15% greater than the expected net payment for primary business and at least 7.5% greater than the expected net payment for reinsurance treaties.

   Illustration: A retrospectively rated workers' compensation policy with a minimum premium of $5,000, an expected premium of $10,000, and a maximum premium of $11,000 does not qualify as loss sensitive.

5. The loss sensitive payments must be either premiums or commissions. A policy with loss sensitive policyholder dividends does not quality as "loss sensitive."

6. The losses and the corresponding loss sensitive payments must flow through the income statement and the balance sheet.

   Illustration: A workers' compensation policy has a large dollar deductible of $100,000. For losses below $100,000, the insurance company settles the claims and pays the benefits, but the insured reimburses the insurer for these payments. One might characterize this policy as loss sensitive, since the greater the losses paid by the insurer, the greater the payments made by the insured. However, these amounts do not flow through the income statement as incurred losses and as premiums, so the contract does not qualify as loss sensitive.

PART 7 HISTORICAL EXHIBITS

The Part 7 historical exhibits provide the historical data to quantify the sensitivity of premiums and reinsurance commissions to losses on an all-lines combined basis. These are Sections 2 through 5 for primary contracts and Sections 2 through 7 for reinsurance contracts.

- Sections 2 and 3 show incurred losses and bulk + IBNR loss reserves. They are similar to the Part 2 and Part 4 exhibits, except that the experience is subdivided by policy year, not by accident year.
- Section 4 shows earned premiums. It is similar to Part 6, except that policy year experience is shown, not exposure year experience. For the prior years row, see below.
- Section 5 shows bulk premium reserves. In general, companies do not hold "case basis" premium reserves. They hold "policy basis" unearned premium reserves reflecting the actual premiums they have recorded as written on each policy. Bulk premium reserves are the equivalent of the Section 3 bulk loss reserves, reflecting additional premiums (positive or negative) anticipated due to audits and other retrospective adjustments.
- Sections 6 and 7 of Part 7B show reinsurance commission exhibits. These sections are similar to the premium exhibits in Sections 4 and 5.

The premium and loss triangles show cumulative values.[87]

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[87] Some analysts have construed the Annual Statement *Instructions* to imply that the commission triangles show incremental values.

For earned premiums, the *Instructions* say:

> *Each reported estimate should be the estimate of the net earned premium as of each year-end, not the incremental amounts earned during each calendar year.*

For the commission triangles (Part 7B, Sections 6 and 7), the *Instructions* say:

> *An entry denoting the expectation of future additional commissions to be paid should be displayed as a negative value. An entry denoting the expectation of future earned commissions should be displayed as a positive value.*

It is likely that the NAIC intended no difference between the premium and commission triangles. The *Instructions* mean that an *expectation of future additional commissions to be paid should be displayed as a negative value* in the bulk commission reserve, just as expectation of future premiums to be returned are displayed as negative values in the bulk premium reserve. The full text of the Annual Statement *Instructions*

PREMIUM SENSITIVITY

Sections 2 through 5 of Part 7A and sections 2 through 7 of Part 7B were designed to quantify premium sensitivity.[88] We explain the intended use of these exhibits.

The risk-based capital reserving risk charge is based on the loss reserves – both case basis reserves and bulk + IBNR reserves – shown by the company's Schedule P, Part 2 minus Part 3. The reserving risk charge quantifies how much these reserves might develop adversely in a worst-case scenario. The loss sensitive contract offset factor quantifies how much additional premium would be expected if reserves develop adversely in this fashion.

Illustration – Premium Sensitivity

Illustration: The exhibits below show extracts from Schedule P, Part 7A, sections 2 through 5 (figures are in thousands of dollars). The actual exhibits contain ten policy years by ten development periods, but these extracts suffice to illustrate the quantification techniques. We quantify the premium responsiveness from 24 to 36 months and from 36 to 48 months.

Exhibit 7.1: Incurred Loss and DCC on Loss-Sensitive Contracts

Section 2	20X4	20X5	20X6	20X7
20X4	$1,000	$2,200	$2,400	$2,500
20X5		$1,100	$2,500	$2,650
20X6			$1,200	$3,000
20X7				$1,500

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makes this clear:

> *In Part 7B of Schedule P, for all reinsurance contracts where the commission paid to the cedant varies with losses, display the development of that commission in Section 6 and display any assets or liabilities accrued with respect to that commission in Section 7. An entry denoting the expectation of future additional commissions to be paid should be displayed as a negative value. An entry denoting the expectation of future earned commissions should be displayed as a positive value. An entry denoting the expectation of future return commissions should be displayed as a positive value.*

Although some readers of the Annual Statement *Instructions* perceive a difference between the premium and commission triangles, we do not see this difference. We advise companies to treat premiums and commissions in the same fashion.

[88] The term "premium sensitivity," as used in this paper, stems from the term "loss-sensitive contracts." Other actuaries use the term "premium responsiveness" to refer to the same phenomenon.

Exhibit 7.2: IBNR plus Bulk Loss and DCC on Loss-Sensitive Contracts

Section 3	20X4	20X5	20X6	20X7
20X4	$350	$550	$300	$200
20X5		$400	$600	$450
20X6			$450	$650
20X7				$500

Exhibit 7.3: Earned Premium on Loss-Sensitive Contracts

Section 4	20X4	20X5	20X6	20X7
20X4	$1,500	$3,150	$3,300	$3,350
20X5		$1,650	$3,600	$3,700
20X6			$1,800	$4,200
20X7				$2,000

Exhibit 7.4: Accrued Retrospective Premiums on Loss-Sensitive Contracts

Section 5	20X4	20X5	20X6	20X7
20X4	$0	$200	$150	$110
20X5		$0	$210	$155
20X6			$0	$220
20X7				$0

PART 7 DATA

These exhibits show policy year data, not accident year losses (as in Parts 2, 3, and 4 of Schedule P) or exposure year premiums (as in Part 6 of Schedule P). For each policy year in Section 2 of Part 7, the incurred losses as of 24 months are about twice the incurred losses as of 12 months.

The policy year 20X4 incurred losses as of 12 months are those losses on policies written in 20X4 that have occurred by December 31, 20X4. These are about half the policy year 20X4 losses, if policies are written evenly over the course of the year. By December 31, 20X5, all of the policy year 20X4 losses have occurred (though they have not necessarily all been reported by this time), so the 24 month figure is about twice as great as the 12 month figure.[89]

The same comments about losses are true for Section 4, which shows the policy year earned premiums. By the end of the policy year, all the premiums have been written (though not necessarily collected yet), but only about half of these premiums have been earned.

INITIAL DEPOSITS

This example assumes that the initial written premiums are the estimated ultimate net premiums. We have done this for heuristic purposes, to simplify the expected cash flows. Although this is sometimes true, it is not always standard practice, for several reasons:

- *Payroll and sales estimates:* Some insureds provide understated payroll or sales projections to lower the deposit premiums.
- *Competition:* Insurers tend to accept understated exposure estimates to keep their deposit premiums competitive. For Schedule P, Part 7 the reporting company should use the estimated ultimate premium (not the premium used by the underwriter).
- *Taxes:* Companies may book a low written premium estimate to defer state premium taxes or federal income taxes.[90] State premium taxes are based on direct written premiums. The tax liability is not incurred until the company records the written premium.

If the initial written premium is the estimated ultimate net premium, there is no retrospective premium reserve at policy inception. At the first retrospective adjustment, some premiums are returned to policyholders, since not all losses have yet been recorded, even though the insurer expects some development on the reported losses. The accrued retrospective premium asset becomes positive after the first adjustment. For companies that charge initial

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[89] The actual distribution of insurance policy effective dates for large commercial accounts is skewed. Many corporations align the policy years on their insurance contracts with their internal accounting fiscal years, so that they can close their books at the end of one fiscal year and begin new books at the start of a new fiscal year. Insurance policies for large commercial accounts tend to have effective dates of January 1 (corresponding to a January – December fiscal year) or another quarter beginning date. At times, an insured requests an effective date of December 31 for tax purposes, if there is reason to allocate the insurance premium to a previous tax year.

[90] Even though the company reports its best estimate of ultimate earned premium, the low written premium estimate affects taxable income through the revenue offset provision (see the Appendix on revenue offset). As of the January 5, 2000 Treasury regulations, this manner of reducing taxable income is no longer permissible. Many underwriters and actuaries and not yet aware of this change in the tax regulations, and they continue to provide low written premium estimates. This is acceptable for statutory accounting, as long as the earned premium estimates are correct; see SSAP No. 53, "Property-casualty Contracts – Premiums." The company's tax officer, mindful of tax avoidance penalties, will "gross-up" the written premium, using the actuarial estimates of earned but unbilled premiums and accrued retrospective premiums. See Sarason, *et al.* [2002] for the tax regulations and Yoheved and Sarason [2002] for the statutory accounting rules.

premiums below the estimated ultimate net premium (for competitive reasons), the accrued retrospective premium asset will be positive from policy inception.

QUANTIFYING THE SENSITIVITY

The illustration is constructed to demonstrate the intended use of these exhibits. Consider first the premium sensitivity from 24 to 36 months. Only policy years 20X4 and 20X5 in the illustration are mature enough to show this sensitivity. For policy year 20X4, losses develop from $2.20 million at 24 months to $2.40 million at 36 months, for a change of $0.20 million. Premiums develop from $3.15 million at 24 months to $3.30 million at 36 months, for a change of $0.15 million. The premium sensitivity is $0.15 million / $0.20 million, or 75%.

For policy year 20X5, losses develop from $2.50 million at 24 months to $2.65 million at 36 months, for a change of $0.15 million. Premiums develop from $3.60 million at 24 months to $3.70 million at 36 months, for a change of $0.10 million. The premium sensitivity is $0.10 million / $0.15 million, or 67%.

As the estimated premium sensitivity from 24 months to 36 months, we might take the average of these two numbers. Alternatively, we might give more weight to the 20X5 policy year, particularly if the rating plan parameters had changed in 20X5.[91]

For the premium sensitivity from 36 months to 48 months, only policy year 20X4 is sufficiently mature to provide the needed figures. Losses develop from $2.40 million at 36 months to $2.50 million at 48 months, for a change of $0.10 million. Premiums develop from $3.30 million at 36 months to $3.35 million at 48 months, for a change of $0.05 million. The premium sensitivity is $0.05 million / $0.10 million, or 50%.

This is consistent with the operation of loss sensitive contracts. As reserves mature, premium sensitivity declines, since more losses are censored by the loss limit and more premiums are capped by the premium maximum. In addition, some retrospective rating plans are closed at late maturities.

This example was designed to illustrate the intended use of the Schedule P exhibits; it would rarely be encountered in practice. The incurred losses here develop smoothly upward, and the premiums follow them equally smoothly. A company which is well reserved should show flat incurred losses along development periods, and similarly flat earned premiums. The incurred losses in these triangles include IBNR and bulk reserves, and the earned premiums

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[91] The manner of selecting projected factors differs between loss reserving and premium sensitivity. Link ratios for loss emergence and settlement are largely beyond the insurer's control. The analyst may use a straight average or a weighted average of the observed link ratios. The factors for premium sensitivity depend on the plan parameters. If the lower sensitivity for the 20X5 policy year stems from a change in the plan parameters, the analyst may give dominant weight to the latest ratio.

include the accrued retrospective premium asset. The changes in incurred losses from period to period would be sometimes small and sometimes large, sometimes positive and sometimes negative, resulting primarily from random loss fluctuations. The changes in earned premiums from period to period would be equally variable, resulting again from random loss fluctuations as well as from censoring by loss limits and capping by the premium maximums.[92]

A well-reserved company would have two series of variable figures with means of zero, since favorable and adverse development are equally likely. The ratios of these series will be even more variable – sometimes very high, sometimes very low, sometimes positive, sometimes negative. These ratios may not reveal much about premium sensitivity. In fact, aggregate industry statistics from these sections of Schedule P, Part 7 have not yielded meaningful figures for tests of premium sensitivity.

Reported Losses and Billed Premium

Premium sensitivity does not deal with the relationship of changes in total earned premium to changes in total incurred losses. Rather, it deals with the relationship of changes in *billed premium* to changes in *reported losses*. Schedule P, Part 7 allows this analysis as well.

- Section 2 of Part 7 shows incurred losses, and Section 3 shows IBNR and bulk reserves. The difference between Sections 2 and 3 represents reported losses.
- Section 4 shows total earned premiums, and Section 5 shows the net reserve for premium adjustments and accrued retrospective premiums. The difference between Sections 4 and 5 represents billed premium.

We repeat the calculations for premium sensitivity using the simulated Schedule P, Part 7 exhibits provided above. For the premium sensitivity from 24 months to 36 months, we have data from both policy year 20X4 and policy year 20X5.

For policy year 20X4, reported losses develop from ($2.2 million – $0.55 million) = $1.65 million at 24 months to ($2.4 million – $0.3 million) = $2.1 million at 36 months, for a change of $2.1 million – $1.65 million = $0.45 million. Billed premium develops from ($3.15 million – $0.2 million) = $2.95 million at 24 months to ($3.3 million – 0.15 million) = $3.15 million at 36 months, for a change of $3.15 million – $2.95 million = $0.20 million. Premium sensitivity from 24 months to 36 months is $0.20 million / $0.45 million = 44.4%.

[92] The date of recognition of additional losses or additional accrued retrospective premium reserves would add to the variability in the two series of changes, one of incurred losses and one of earned premiums. The reserving actuary may recognize the potential increase in ultimate losses in one year, but may not book the corresponding increase in the accrued retrospective premium reserves until some time later. See the discussion below regarding the time lag between premiums and losses.

For policy year 20X5, reported losses develop from ($2.50 million – $0.60 million) = $1.90 million at 24 months to ($2.65 million – 0.45 million) = $2.20 million at 36 months, for a change of $2.20 million – $1.90 million = $0.30 million. Billed premium develops from ($3.6 million – $0.21 million) = $3.39 million at 24 months to ($3.70 million – $0.155 million) = $3.545 million at 36 months, for a change of $3.545 million – $3.39 million = $0.155 million. Premium sensitivity from 24 months to 36 months is $0.155 million / $0.30 million = 51.7%.

Anticipated Emergence versus Unanticipated Development

The risk-based capital reserving risk charge seeks to quantify the amount of capital needed to guard against unanticipated adverse development of loss reserves. If the company's reserves would develop adversely by $15 million in a "worst-case" (but still reasonable) scenario, the company should hold $15 million of capital to ensure its solvency.

The figures calculated in the preceding section show the responsiveness of the retrospective premiums to the emergence of reported losses. They do not show the responsiveness of the retrospective premiums to the unanticipated adverse development of the incurred losses.

> *Illustration:* We are examining the premium sensitivity from 24 months to 36 months on a workers' compensation retrospectively rated book of business. The reported losses are $100 million at 24 months, and the anticipated reported losses at 36 months are $120 million. The expected ultimate losses are $150 million.
>
> Suppose we have estimated a historical premium sensitivity for this period of 50%. When reported losses increase by $20 million, the billed premium increases by $10 million (on average). We must infer the effects for large and unanticipated adverse loss development, as envisioned in the risk-based capital "worst case year" scenario. For example, if the ultimate losses are re-estimated at $180 million at 36 months instead of $150 million, will the accrued retrospective premium asset increase by an additional $15 million, or 50% of the additional losses of $30 million?

DECOMPOSITION OF ADVERSE DEVELOPMENT

We decompose the development of reported losses from $100 million at 24 months to $120 million at 36 months into two parts.

- Some temporary cases last a few months longer, and some medical benefits cost more than expected. This development is "rateable," and premium sensitivity is high.
- Some temporary total cases, such as lower back sprains, are reclassified into lifetime pension cases, when it becomes clear that the injured employee will not return to work. Only some of this development is "rateable." The rest of this development is truncated by the loss limits or the maximum premiums.

Large and unanticipated adverse loss development has a heavy proportion of this "non-rateable" element. The re-estimation of the ultimate losses from $150 million to $180 million may result from the reclassification of several back sprains as severe and permanent disabilities, or a judicial or legislative decision that certain disease claims are compensable. These claims are large and they are paid over a long period. A large part of these claims may not be rateable.

The premium sensitivity depends on the maturity of the losses as well as on the average loss ratio in the block of business. The emergence of anticipated losses differs from the unanticipated adverse development of the expected losses in that

- the anticipated losses are generally paid sooner than the unanticipated losses, and
- the anticipated losses occur at a lower loss ratio than do the unanticipated losses.

Since the anticipated losses are generally paid sooner and are generally in a lower loss ratio environment, they are associated with a stronger premium sensitivity. The figures derived from the historical triangles in Part 7 may not fully address the risk-based capital concerns.

Premium Billing Lags

When quantifying premium sensitivity, it is important to use corresponding premiums and losses. Premium billing occurs about 3 months after the retrospective adjustment. This implies that the premium billing lags the loss occurrence by 3 to 15 months.

> *Illustration:* A policy is effective from July 1, 20XX, through June 30, 20XX+1. Retrospective adjustments are done six months after the policy's expiration and every 12 months subsequently. For this policy, the retrospective adjustments will be done on each January 1, starting with January 1, 20XX+2. The resulting retrospective premium adjustments are billed to the policyholder (or returned to the policyholder) on each April 1.

Each retrospective premium is driven by losses that are reported between 15 months and 3 months prior to the premium billing date. For this policy, losses that are reported between January 1 and December 31 affect the premium adjustment that will be billed on April 1.

Illustration: The average lag between loss occurrence and premium billing is ½ × (3+15) = 9 months. If one does not use any lag, the results are distorted. To see this most clearly, suppose that

- the retrospective premium billing is done on July 1,
- all losses occur on July 1,
- there is 100% premium responsiveness, and
- the incurred losses alternate between $1,000 and $0 in succeeding years.

The Schedule P, Part 7, test of premium sensitivity would show the following:

Exhibit 7.5: Premium Sensitivity and Reporting/Billing Lags

Year	1	2	3	4	5	6
Change in incurred losses	$1,000	$0	$1,000	$0	$1,000	$0
Change in billed premium	$0	$1,000	$0	$1,000	$0	$1,000

The premium billing shows up a year after the loss occurs. In truth, there is 100% premium sensitivity, but Schedule P, Part 7 shows a −100% premium sensitivity.[93]

Simplistic tests of premium sensitivity may yield negative regression coefficients or seemingly random regression coefficients. The reserving actuary may think that the data are incorrect, when the problem is an improper matching of the premiums and losses. Actual examination of aggregate industry Schedule P, Part 7 data has not yielded meaningful information.

Prior Years Row

The Annual Statement *Instructions* comment on the data for the prior years rows in Sections 2, 3, 4, and 5 of Part 7A and Part 7B as follows:

> *[for losses:] The "prior" row should display the reported estimate of ultimate losses and defense and cost containment expense on a policy year basis for all policy years ten or more years older than the current policy year.*
>
> *[for premiums:] The "prior" row should display the reported estimate of net earned premium on a policy year basis for all policy years ten or more years older than the current policy year.*

These instructions do not make sense. Companies do not keep records of earned premiums and incurred losses on loss sensitive contracts written years ago. None of the Schedule P prior years rows asks for such data. The prior years rows use one of three types of data:

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[93] The date of recognition of additional losses or additional accrued retrospective premium reserves would add to the variability in the two series of changes. The reserving actuary may recognize the potential increase in ultimate losses in one year, but may not book the corresponding increase in the accrued retrospective premium reserves until some time later.

The actual calculations of the premium sensitivity use successive calendar years at the same adjustment date for successive blocks of business, not successive adjustments for a single block of business. The underlying concepts are the same, though the representation is more complex.

- Current reserves for all old years (the reserves in Part 2, Part 4, and Part 1).
- Current calendar year payments or receipts related to old years (payments in Part 1; claims in Part 5; and premiums in Part 6).
- Cumulative payments since the second calendar year in the triangle (Part 3).

The only procedure which makes sense for the incurred and earned triangles in Part 7 is the procedure used for Part 2 of Schedule P. This is a combination of the first method for the reserves and the third method for the payments.

The Part 2 procedure is useful for reserve adequacy testing. It is not helpful for quantifying premium sensitivity, which is the purpose of the Part 7 exhibits. The quantification methods described here do not make use of the prior years rows, since the premiums and losses stem from different policy years.

The format of the Part 7 exhibits is taken directly from the other parts of Schedule P. The designers of Schedule P, Part 7 had no intentions for the prior years row. This row is not used for quantifying premium sensitivity. Companies should not spend time trying to figure out the data needed for this row. The data are not used or checked by the NAIC.

Federal Income Taxes

The 1986 Tax Reform Act introduced several federal income tax provisions that are specific to property-casualty insurance. This section focuses on tax provisions and related statutory accounting requirements that rely on Schedule P. For a general treatment of federal income taxes relating to property-casualty insurance companies, with emphasis on items of particular concern to casualty actuaries, see Sarason, *et al.* [2002].

DATA SOURCES

The computation of federal income taxes relies on the following Annual Statement exhibits:

1. The tax computation begins with statutory pre-tax income from the Underwriting and Investment Exhibit: Part 1 for investment income and Parts 2, 2A, and 3 for underwriting income.[94]
2. The additional tax liability resulting from the revenue offset provision is calculated from Part 2 of the Underwriting and Investment Exhibit. The January 2000 tax regulations and the statutory accounting codification changes effective on January 1, 2001 affect the recognition of taxable revenue from earned but unbilled premiums and accrued retrospective premiums. The Schedule P, Part 6 exhibits may be used to adjust statutory income to taxable income; see Sarason, *et al.* [2002].
3. Schedule P, Part 1 is used to calculate the additional tax liability resulting (i) from the IRS loss reserve discounting provision and (ii) from anticipated salvage and subrogation

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[94] For most industries, the federal income tax liability is based on the generally accepted accounting (GAAP) statements of the company. For the property-casualty insurance industry, the federal income tax liability is based on statutory income. See the Treasury regulations, 2001FED 26,153, §1.832-4(a)(1): "Gross income means the gross amount of income earned during the taxable year from interest, dividends, rents, and premium income, computed on the basis of the underwriting and investment exhibit of the annual statement."

The Internal Revenue Code lists numerous adjustments, of which the following are the most important:

1. The earlier incurral of the tax liability resulting from revenue offset and loss reserve discounting.
2. The effects of anticipated salvage and subrogation and the discounting provisions relating thereto.
3. The reduction of the tax liability resulting from municipal bond income and the dividends received deduction, along with the limitation thereon.
4. The difference in the incurral dates of the tax liability resulting from the amortization and accrual rules for fixed-income securities.

In addition, the alternative minimum income tax provisions may cause an earlier incurral of the tax liability. All changes in the incurral dates of the tax liabilities may lead to deferred tax assets and liabilities on the statutory balance sheet.

recoveries. Schedule P, Part 3 may be used to determine the non-admitted portion of the deferred tax asset stemming from the loss reserve discounting.

4. Schedule D is used to determine the reduction in the tax liability resulting from municipal bond income and the dividends received deduction, as well as any additional or reduced tax liability resulting from the difference between statutory amortization of fixed-income securities and tax amortization of these securities. The company's optimal investment strategy depends on the anticipated taxable underwriting income, which depends on the Schedule P calculations.

This section covers IRS loss reserve discounting and the non-admitted portion of the resulting statutory deferred tax asset.

Loss Reserve Discounting

For statutory financial statements, calendar year incurred losses equal the losses paid during the year plus the change in the full value loss reserves from the beginning of the year to the end of the year. For federal income tax purposes, the incurred losses during the tax year equal the losses paid during the year plus the change in the *discounted* loss reserves from the beginning of the year to the end of the year.

The determination of discounted loss reserves relies on Schedule P. The valuation actuary may be asked to compute (i) the discounted loss reserves, (ii) the amount of the discount, (iii) the effects of bulk reserve changes on taxable income and the tax liability, (iv) whether the company should elect its own loss payment pattern, and (v) the optimal investment strategy for a given amount of bulk reserves or level of reserve adequacy.

The cost of capital is a major factor for the pricing of insurance contracts. The double taxation of the investment income on capital funds is a significant component of this cost. The IRS loss reserve discounting provisions and the statutory deferred tax asset affect the cost of holding capital for insurers.

Investment Income and Amortization

For long-tailed lines of business, the statutory accounting rules cause an underwriting loss during the policy term when losses occur. After policy expiration, the investment income on the assets backing the loss reserves provide steady and positive net income. For tax accounting, the expected investment income on the assets backing the loss reserves offsets the expected amortization of the interest discount in the reserves. The underwriting gain or loss is realized during the policy term, with no expected net gain or loss in subsequent years.

Complete (exact) offsetting depends on the following conditions:

a. There are no implicit (undisclosed) discounts in the statutory loss reserves.

b. The IRS discount rate equals the investment yield of the company.
c. The IRS loss payment pattern equals the actual liquidation pattern for the block of business.
d. The company holds fully discounted reserves, with disclosure of the amount of discount.

These conditions are not consistent with current statutory requirements, so complete offsetting is not expected. Nonetheless, they clarify the heuristic illustration below.

Illustration: Offsetting

A one day policy is written on December 31, 20XX for a net premium of $10,000. One loss occurs on December 31, 20XX, which is paid for $12,100 on December 31, 20XX+2. The term structure of interest rates is flat at 10% per annum. To simplify the illustration, we assume that the IRS loss payment pattern is the same as the actual loss payment pattern here.

In 20XX, statutory accounting shows an underwriting loss of $10,000 – $12,100 = $2,100. The $10,000 net premium is invested at 10% per annum. The investment income is $10,000 × 10% = $1,000 in 20XX+1 and $11,000 × 10% = $1,100 in 20XX+2. There is no underwriting gain or loss in 20XX+1 or 20XX+2, so these are the statutory income amounts.

If we assume a two year IRS loss payment pattern and a discount rate of 10% per annum, the discounted loss reserves are $12,100 / 1.100^2 = $10,000 at December 31, 20XX. Tax accounting shows no underwriting gain or loss in 20XX and a tax liability of $0 for 20XX.

In 20XX+1, investment income is $1,000. The discounted loss reserve on December 31, 20XX+1 is $12,100 / 1.100 = $11,000. The underwriting loss (or the offset to underwriting income) for tax year 20XX+1 equals the amortization of the interest discount on the loss reserves, or $11,000 – $10,000 = $1,000. The underwriting loss just offsets the investment income. The net taxable income is $0, and the tax liability is $0.

In 20XX+2, investment income is $1,100. The incurred loss offset to taxable underwriting income in 20XX+2 is the paid loss plus the change in the discounted loss reserve, or

$12,100 (paid on December 31, 20XX+2) + $0 – $11,000 = –$1,100.

This is the amortization of the interest discount on the 12/31/20XX+1 reserve of $11,000. It offsets the investment income in 20XX+2. The taxable income is $0, and the tax liability is $0.[95]

[95] Some insurance personnel speak of the post-1986 federal income tax incurral pattern as a "prepayment of taxes by the insurance industry." This is correct from a statutory or GAAP perspective. The IRS would take the opposite view; before 1986 the Treasury helped fund the conservative insurance accounting practices.

DISCOUNTING PRINCIPLES

The discounted loss reserves are determined from three components:

- The undiscounted loss reserves, as shown in Schedule P, Part 1;
- The loss reserve discount rate, which is promulgated each year by the Treasury; and
- The loss payment pattern by line of business, which is determined from Schedule P data.

Illustration: The December 31, 20XX undiscounted loss reserves are $100 million. The loss reserve discount rate is 8% per annum. The $100 million of reserves will be paid in three parts: 50% on December 31, 20XX+1, 30% on December 31, 20XX+2, and 20% on December 31, 20XX+3.[96] The discounted loss reserves equal

$\$100 \text{ million} \times (50\%/1.08 + 30\%/1.08^2 + 20\%/1.08^3) = \$100 \text{ million} \times 0.879 = \$87.9 \text{ million}.$

Undiscounted Loss Reserves

The Treasury assumes that the loss reserves in Schedule P, Part 1 are undiscounted values. If discounted values are shown, the losses may be "grossed up" to undiscounted amounts before application of the IRS loss reserve discounting procedure. The "gross-up" is permitted only if the amount of the discount is disclosed in (or with) the Annual Statement.[97]

Illustration: Schedule P, Part 1 is gross of non-tabular discount and net of tabular discount.

- A company incurs $10,000,000 of accident year 20XX workers' compensation losses, including lifetime pension claim reserves with a tabular discount of $1,000,000.
- The IRS loss reserve discount factor for workers' compensation accident year 20XX reserves is 85%.

If the company does not disclose the tabular discount in the Annual Statement, the offset to taxable income is $10 million x 85% = $8.5 million. If the company does disclose the tabular discount in the Annual Statement, the offset to taxable income is ($10 million + $1 million)

96 This illustration is simplified. The actual procedure assumes mid-year payments and a longer loss payment pattern.

97 See section 846(b)(2) of the Internal Revenue Code: "Adjustment If Losses Discounted on Annual Statement: If the amount of unpaid losses shown in the annual statement is determined on a discounted basis, and *the extent to which the losses were discounted can be determined on the basis of information disclosed on or with the annual statement*, the amount of the unpaid losses shall be determined without regard to any reduction attributable to such discounting."

× 85% = \$9.35 million. The difference in taxable income is \$9.35 million – \$8.5 million = \$0.85 million, and the difference in the tax liability is \$0.85 million × 35% = \$297,500.

DISCLOSURE AND TIMING COSTS

The difference between statutory income and taxable income in the illustration above is a timing difference; it will reverse in subsequent years. The cost to the company is the present value of the expected after-tax investment yield on this money.

Illustration: Suppose the pension reserves are paid (on average) twelve years after policy expiration, and the after-tax investment yield is 6% per annum. The cost to the company is

$$\$297{,}500 \times [(1.06^{12} - 1) / 1.06^{12}] = \$297{,}500 \times 0.503 = \$149{,}651.61.$$ [98]

The required disclosure of *non-tabular* discounts by accident year and by line of business is provided in columns 34 (losses) and 35 (loss adjustment expenses) of Schedule P, Part 1. The required disclosure of *tabular* discounts is shown in note 28 (in the 2001 Annual Statement) to the financial statements, "Discounting of Liabilities for Unpaid Losses or Unpaid Loss Adjustment Expenses." For tabular discounts, the reporting company shows four items by line of business: (i) the mortality table used, (ii) the discount rate used, (iii) the amount of discounted reserves, and (iv) the amount of the tabular discount.[99]

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[98] Because of the statutory deferred tax asset and the capital requirements imposed on insurance companies, the actual cost to equityholders is somewhat different; see Kelly, *et al.*, [2002] for a full discussion.

[99] The footnote does not require disclosure of the discount by accident year. Companies provide this information anyway, since it is needed to gross up the undiscounted reserves for tax purposes.

Limitation

The IRS is concerned that a company might claim such a large discount for its statutory loss reserves that the discounted tax-basis loss reserves would be greater than the Annual Statement loss reserves, thereby reducing the tax liability by means of discounting instead of increasing the tax liability. To prevent this, the discounted IRS loss reserves may not be greater than the loss reserves shown in the Annual Statement.[100]

Statutory accounting allows only limited discounting: tabular discounts and exceptional cases of non-tabular discounts. For tabular discounts, most companies use conservative interest rates, such as 3.5% or 4% per annum. For non-tabular discounts, the permissible discount rate for statutory accounting is rarely greater than the discount rate used for IRS loss reserve discounting; see SSAP No. 65 on "Property and Casualty Contracts."

In summary, the statutory loss reserves are rarely lower than the IRS discounted loss reserves. The workers' compensation "prior years" row (Part 1D) is an exception. These reserves are primarily indemnity reserves for lifetime pension cases, and many companies use tabular discounts. For this row, the "composite discount factor" used in the IRS discounting calculations assumes (on average) three more years of payment, whereas the pension cases in these reserves may have (on average) a future expected lifetime of 10 to 20 years.

ILLUSTRATION: THE LIMITATION

The workers' compensation prior years row shows unpaid losses and loss adjustment expenses of $30 million. In the Notes to the Financial Statements, the company reports a $10 million tabular discount for these claims. The IRS composite discount factor applicable to these reserves is 90%.

Without the limitation discussed above, the gross loss reserves are $30 million + $10 million = $40 million. The IRS discounted loss reserves are 90% x $40 million = $36 million. Since this exceeds the $30 million of statutory loss reserves, the IRS discounted loss reserves are capped at $30 million.

[100] See the Internal Revenue Code §846(a)(3): "In no event shall the amount of the discounted unpaid losses with respect to any line of business attributable to any accident year exceed the aggregate amount of unpaid losses with respect to such line of business for such accident year included on the annual statement."

Discount Rate

The discount rate varies by accident year. For each accident year, the discount rate is the 60 month moving average of the federal mid-term rates ending on the December 1 preceding the accident year. This rate is frozen and applies to that accident year's losses in all future calendar years. In tax parlance, the discount rate is "vintaged." The federal mid-term rate is the average rate on Treasury securities with 3 to 9 years remaining maturity.[101]

The federal mid-term rate is promulgated by the Treasury each month.[102] The 60 month moving average applicable to an accident year is promulgated by the Treasury during the accident year, and it can be determined as soon as the last federal mid-term rate has been announced.

Illustration: The loss reserve discounting rate for accident year 20X9 is the 60 month average of the federal mid-term rates from January 1, 20X4, through December 1, 20X8. It can be computed in December 20X8, before the inception of accident year 20X9, so that companies can effectively determine their tax strategies during 20X9.

Yield Projections

The market values of future cash flows are based on the current term structure of interest rates. The date that the liability was incurred is not relevant. In contrast, the IRS bases the discount rate on the incurral year of the liability. The rationale is that the insurance company uses the premium cash flows from the policy to purchase fixed-income securities to fund the future loss payments. The yield on the fixed-income securities is determined at the date of purchase. If the duration of the assets backing the reserves matches the duration of the loss liabilities,

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[101] See section 846(c)(2) of the Internal Revenue Code: "Determination of Annual Rate: The annual rate determined by the Secretary under this paragraph for any calendar year shall be a rate equal to the average of the applicable Federal mid-term rates (as defined in section 1274(d) but based on annual compounding) effective as of the beginning of each of the calendar months in the test period. The test period is the most recent 60-calendar-month period ending before the beginning of the calendar year for which the determination is made."

The federal mid-term rates are expressed as bond equivalent yields, since bond coupons are paid semi-annually in the United States. (A bond equivalent yield is a yield with semi-annual compounding.) The IRS loss reserve discounting procedure uses annual compounding, since it assumes that losses are paid in mid-year (i.e., once a year). The bond equivalent yields are converted to effective annual yields before averaging, using the formula $r_a = (1 + r_s/2)^2 - 1$, where r_a is the effective annual yield and r_s is the bond equivalent yield with semi-annual compounding. If the bond equivalent yield is 8% per annum, the equivalent effective annual rate is $(1 + 0.08/2)^2 - 1 = 8.16\%$.

[102] The yield among mid-term securities varies with the remaining maturity, in accordance with the term structure of interest rates. More recently issued securities tend to have slightly lower yields, since they are more marketable. The Secretary of the Treasury selects an appropriate average rate.

the losses will be paid from the coupon income and the principal repayment from these securities. The yield during the accident year is the relevant investment yield throughout the life of the policies.[103]

Loss Payment Pattern

The IRS determines the expected loss payment pattern by line of business from Schedule P, Part 1. Most discussions of the IRS loss reserve discounting procedure show the mechanics of the computation, with no explanation of the rationale. The approach here is the opposite. We consider first the rationale for the IRS procedure before explaining the mechanics of its computation. We use the figures in the prospective paid loss chain ladder development illustration earlier in this paper.

To determine the discounted reserves, we must we estimate the percentage of these reserves that will be paid in each subsequent calendar year. We use a sequence of three illustrations to clarify the procedure.

Illustration: We are computing the loss payment pattern for the 20X9 accident year reserves shown below. We use the historical data to estimate the percentages to be paid in each future calendar year.

[103] Whether a moving average rate or the current rate is a better predictor for future rates is an open question. Accountants often prefer average rates, on the assumption that the most recent monthly figure may be abnormally high or low. Some financial analysts presume that interest rates revert towards a long-term mean, and a 60 month moving average may be a better reflection of this mean. Other analysts presume that interest rates form a random walk, and the present term structure of interest rates is the best reflection of expected future rates. The dominant view is that the current rate is a better estimator of the rate during the next 12 months than the 60 month moving average is; see Dr Jonathan Benjamini and S. Feldblum, *Dynamic Financial Analysis: a Primer for the Practicing Actuary* [2002].

Exhibit Tx.1: 20X9 Schedule P, Part 3D ($000)

Part 3	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	103	226	294	334	363	384	398	412	422	433
20X1		111	238	309	356	387	409	428	442	454
20X2			108	221	286	328	354	375	391	403
20X3				111	238	311	357	392	416	434
20X4					135	299	394	458	504	534
20X5						146	314	418	490	542
20X6							159	343	463	546
20X7								146	353	485
20X8									152	406
20X9										156

Mature Accident Year

Consider a single accident year. The 20X0 accident year, with estimated total losses of $486 thousand, shows the following percentages paid in calendar years 20X0 through 20X9:

Exhibit Tx.2: Loss Payment Pattern from the Single Accident Year 20X0 ($000)
(Data from Schedule P, Part 3, of the 20X9 Annual Statement)

Part 3	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
1. 20X0	$103	$226	$294	$334	$363	$384	$398	$412	$422	$433
2. perc'nt	0.212	0.465	0.605	0.687	0.747	0.790	0.819	0.848	0.868	0.891
3. incr'tl	0.212	0.253	0.140	0.082	0.060	0.043	0.029	0.029	0.021	0.023

- Row 1: The row labeled "20X0" shows the cumulative dollars (in thousands) of accident year 20X0 losses paid by December 31 of each calendar year from 20X0 through 20X9.
- Row 2: The row labeled "perc'nt" shows the cumulative percentages of accident year 20X0 losses paid by December 31 of the calendar year in each column.
- Row 3: The row labeled "incr'tl" shows the incremental percentages of accident year 20X0 losses paid in each calendar year.

The final row in the table above tells us that 21.2% of an accident year's incurred losses are paid during the accident year, another 25.3% are paid in the 12 months following the accident year, 14.0% are paid in the subsequent 12 months, and so forth. The final 1 – 89.1%, or 10.9%, are paid more than 10 years after the inception of the accident year.

This procedure relies on a single accident year that is already 10 years old. It has the following drawbacks:

- Settlement of large losses may distort the payment pattern in any one accident year.
- The loss payment pattern does not reflect any changes in the intervening nine years.
- This method ignores the information embedded in the observed liquidation of accident years 20X1 through 20X8.

The method used by the IRS differs in several respects, as explained below.

RECENT DATA

To use the most recent data, we examine the dollars paid in calendar year 20X9 divided by the total incurred losses for each accident year. The paid loss development illustration used earlier in this paper shows the following figures from Schedule P, Parts 2 and 3.[104]

Exhibit Tx.3: Loss Payment Pattern from Successive Accident Years ($000's)
(Data from Schedule P, Parts 2 and 3, from the 20X9 Annual Statement)

AccYr	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
Row 1	$422	$442	$391	$416	$504	$490	$493	$353	$152	$0
Row 2	$433	$454	$403	$434	$534	$542	$546	$485	$406	$156
Row 3	$11	$12	$12	$18	$30	$52	$53	$132	$254	$156
Row 4	$486	$520	$475	$522	$667	$707	$787	$802	$866	$898
Row 5	2.3%	2.3%	2.5%	3.4%	4.5%	7.4%	6.7%	16.5%	29.3%	17.4%

The rows show the following figures:

Row (1): Cumulative dollars of loss paid through December 31, 20X8 (from Part 3).
Row (2): Cumulative dollars of loss paid through December 31, 20X9 (from Part 3).
Row (3): Incremental dollars of loss paid in 20X9 (= row (2) minus row (1)).
Row (4): Incurred losses (from Part 2).

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[104] The accident years are shown along the horizontal axis of the table. In the exhibits used for the paid loss chain ladder development method, the accident years are shown along the vertical axis.

Row (5): Incremental dollars of loss paid as a percentage of incurred losses (row 3 / row 4).

Consider the column for accident year 20X4:

Row 1: $504,000 has been paid by 12/31/20X8, or 60 months since inception of the accident year.
Row 2: $534,000 has been paid by 12/31/20X9, or 72 months since inception of the accident year.
Row 3: $30,000 has been paid between 60 months and 72 months.
Row 4: The total accident year 20X4 incurred losses are $667,000.
Row 5: 4.5% (or $30,000 / $667,000) of the incurred losses are paid between 60 months and 72 months since inception of the accident year.

This procedure uses figures from Schedule P, Part 3, which shows cumulative paid losses at the current valuation date and the previous valuation date. The IRS used figures from Part 1, perhaps because Part 1 is an audited exhibit whereas Part 3 is not an audited exhibit. In addition, the Part 1 figures are for the current valuation date, so they are more easily verified than are historical figures from the previous valuation date.

Incremental Percentages and Cumulative Differences

For the lines of business with ten year exhibits, the IRS makes one additional change. The procedure outlined above uses the incremental paid loss percentages in each accident year to estimate the percentage of losses paid in each time interval. The IRS uses the difference in the cumulative paid loss percentages between successive accident years.

Exhibit Tx.4: Loss Payment Pattern Between Accident Years ($000's)
(Data from Schedule P, Parts 2 and 3, from the 20X9 Annual Statement)

AccYr	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
Row 1	$433	$454	$403	$434	$534	$542	$546	$485	$406	$156
Row 2	$486	$520	$475	$522	$667	$707	$787	$802	$866	$898
Row 3	89.1%	87.3%	84.8%	83.1%	80.1%	76.7%	69.4%	60.5%	46.9%	17.4%
Row 4	1.8%	2.5%	1.7%	3.1%	3.4%	7.3%	8.9%	13.6%	29.5%	17.4%

- Row (1) shows the cumulative paid losses at December 31, 20X9 for each accident year.
- Row (2) shows the incurred losses at December 31, 20X9 for each accident year.
- Row (3) shows the ratio of cumulative paid losses to incurred losses.
- Row (4) shows the differences in successive ratios. For accident year 20X9, nothing is paid before calendar year 20X9, so 17.4% of incurred losses are paid in the first 12 months. For losses paid between 12 months and 24 months, we reason as follows.

- ✓ From the 20X8 accident year, we infer that 46.9% of incurred losses are paid by 24 months since inception of the accident year.
- ✓ From the 20X9 accident year, we infer that 17.4% of incurred losses are paid by 12 months since inception of the accident year.
- ✓ This implies that 46.9% – 17.4% = 29.5% of incurred losses are paid between 12 months and 24 months since inception of the accident year.[105]

The figures in row (4) sum to 89.1%. This is the ratio of cumulative paid losses to incurred losses for accident year 20X0 . All the figures are available from Schedule P, Part 1.

This is the procedure used by the IRS, with one difference in the data used.

- The Part 1 figures used by the IRS include all loss adjustment expenses.[106]
- The Part 3 figures shown here include only defense and cost containment expenses.

IRS RATIONALE

We summarize the computations as follows:

i. For each accident year in Schedule P, Part 1, we calculate the cumulative paid losses at the current valuation date as a percentage of the incurred losses for that accident year.
ii. We take the difference between successive accident years to determine the expected percentage of incurred losses paid in each 12 month interval.
iii. We use this procedure for the ten accident years shown in Part 1. If the cumulative paid losses for the oldest year equal 100% of the incurred losses, we stop here. If the cumulative paid losses for the oldest year are less than 100% of the incurred losses, we extend the loss payment pattern for additional years, as described below.

The cumulative paid losses as of the current valuation date are shown in Part 1, column 11, "total net paid." The incurred losses at the current valuation date are shown in column 28, "total losses and loss expense incurred."

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105 Rounding to a single decimal places causes the apparent discrepancy in some of these figures.

106 See section 846(f)(2) of the Internal Revenue Code: *The term "unpaid losses" includes any unpaid loss adjustment expenses shown on the annual statement.*

ILLUSTRATION: PERSONAL AUTO LIABILITY

Although the concepts are straight-forward, the implementation is complex. We explain the details with two illustrations.

The ABC Insurance Company elected to use its own loss payment pattern in the 2007 determination year. This election applies to accident years 2009 through 2013. It is now January 2011, and ABC is computing the loss payment pattern for computing discounted reserves for accident year 2011. The following figures are taken from ABC's 2009 Annual Statement, Schedule P, Part 1B (private passenger automobile liability).

Exhibit Tx.5: Private Passenger Automobile Liability Paid and Incurred Losses

Accident Year	Losses + LAE Paid (col 11)	Losses + LAE Incurred (col 28)
Prior	$250,000	$250,000
2000	270,000	275,500
2001	300,000	316,000
2002	320,000	348,000
2003	340,000	386,500
2004	350,000	421,500
2005	370,000	480,500
2006	380,000	550,500
2007	360,000	610,000
2008	330,000	687,500
2009	200,000	571,500

The 60 month rolling average of the federal mid-term rate, from January 2006 through December 2010, is 7% per annum.

Determination Year and Company Election

Year 2011 is not a determination year, so industry aggregate Schedule P data for a valuation date of December 31, 2009 would not be used to determine loss payment patterns. Determination years end in a "2" or a "7," and they use aggregate industry Schedule P data for statement dates ending in a "0" or a "5." As examples,

- For determination year 20X2, Schedule P data as of December 31, 20X0 are used.
- For determination year 20X7, Schedule P data as of December 31, 20X5 are used.

Once every five years (determination years), the company makes an election to use either

- the loss reserve discount factors developed by the Treasury, which are based on industry aggregate Schedule P data, or
- its own loss reserve discount factors, which are based on its own Schedule P data

The election is made with the company's tax filing for the determination year. It applies to that year and to the succeeding four years.

If the company elects to use its own payment patterns, it uses Schedule P data that are available before the beginning of each tax year. These are the Schedule P data from two years earlier.

In this illustration, the company makes an election with its 2007 tax filing to use its own Schedule P data. The election applies to the 2007 through 2011 accident years.

The 2009 Schedule P data for computation of the loss payment patterns for the 2011 accident year are available by March 1, 2010. The loss reserve discount rate is not available until mid-December 2010. The computation in this illustration is done between mid-December 2010 and early 2012 (before the 2011 tax filing).

The election to use one's own data applies to all applicable lines of business. These are all the Schedule P lines except (i) international and (ii) lines for which the company does not have ten years of experience. The old rule that the company's reserves for the line of business must be at least as great as those of 10% of the companies in the industry was explicitly revoked by the Treasury in 1991 on the grounds that it discriminated against small companies.[107]

An election lasts for five years – until the next determination year. The company can revoke its election to use its own Schedule P data before the completion of the five year period only with the acquiescence of the Treasury.

A company that has elected to use its own Schedule P data recomputes the loss payment patterns each year, based on the most recent Schedule P data filed with regulatory authorities before the beginning of the tax year. The most recent Schedule P filed before the beginning of tax year 2011 was filed by March 1, 2010, containing data as of valuation date December 31, 2009. The industry payment patterns are computed only once every five years.

The 60 month rolling average of federal mid-term rates ends with the month preceding the accident year, not with the month preceding the Schedule P valuation date. In this illustration, the 60 month rolling average ends with December 2010, not December 2008 or 2009.

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[107] Because of an oversight in the Insurance Accounting and Systems Association insurance accounting textbook, this rule still remains on the CAS examination syllabus in 2002.

VINTAGING

The loss reserve discount factors computed here are used for accident year 2011 only. The discount factors for previous accident years at every future valuation date have already been determined and frozen. In tax parlance, they are vintaged. They are not revised in subsequent calendar years.

We determine between 11 and 15 discount factors for accident year 2011. The first ten discount factors are used at valuation dates December 31, 2011 through December 31, 2020. The final one to five development factors are used at subsequent valuation dates. The development factors are combined into a composite development factor for the prior years row for valuation dates 2021, 2022, and subsequent; see below. The discount factors all use the 7% discount rate and the loss payment pattern computed here.

The application of the loss reserve discount factors is not clear from a cursory reading of the Internal Revenue Code. The chart below shows the valuation dates and accident years for the various loss reserve discount factors.

Exhibit Tx.6: Valuation Dates for Loss Reserve Discount Factors

Discount Factor	Accident Year	Individual / Composite	Tax Year (Valuation Date)	Schedule P Accident Year
12 mos	2011	individual	2011	2009
24 mos	2011	individual	2012	2008
...	...	...	...	...
120 mos	2011	individual	2020	2000
132 mos	2011	composite	2021	AY+10 **
144 mos	2011	composite	2022	AY+11 **
...	...	...	...	...

** There years are referred to in the Internal Revenue Code as AY+10 through AY+15. They appear as part of the Schedule P prior years row.

The first ten discount factors computed here apply to accident year 2011 only. They are used at valuation dates between 12 months and 120 months from inception of the accident year. These correspond to tax years 2011 through 2020. For subsequent valuation dates, the discount factor computed here is combined with discount factors for other accident years and applied to the Schedule P prior years row.

The final column matches the accident years in Schedule P with the future valuation dates at which time the loss reserve discount factors are applied. For example, the cumulative paid percentage for accident year 2008 at valuation date December 31, 2009 equals the expected cumulative paid percentage for accident year 2011 at valuation date December 31, 2012. Similarly, the cumulative paid percentage for accident year 2000 at valuation date December 31, 2009 equals the expected cumulative paid percentage for accident year 2011 at valuation date December 31, 2020.

Discounting Sequence

The loss reserve discount factor computation can be divided into a series of steps.

Step 1: We calculate the nominal (undiscounted) amounts for

Cumulative percentages paid,
Incremental percentages paid, and
Percentages unpaid.

Step 2: We calculate the adjustments for long-tailed lines of business – i.e., those with ten year Schedule P exhibits[108] – showing

Adjusted incremental percentages paid,
Long-tail extension of payments, and
Adjusted percentages unpaid.

Step 3: We apply the appropriate discount rate to obtain the

Discounted percentages unpaid, and
Loss reserve discount factors.

The logic of the procedure was outlined above. This section proceeds through the specific computations mandated by the Internal Revenue Code.

UNDISCOUNTED PERCENTAGES

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[108] The term "long-tailed" has three meanings, depending on context:

- The Schedule P meaning: lines with ten year exhibits versus two year exhibits.
- The actuarial meaning, denoting average length of time between premium collection and loss payment.
- The IRS meaning: a line is "long-tailed" if the losses unpaid after ten years exceed the losses assumed paid in the tenth year.

All the Schedule P long-tailed lines of business are likely to be classified by the IRS as long-tailed. The length of the tail for IRS purposes depends on the Schedule P entries for the ninth and tenth oldest year; see below.

The loss reserve discount factors for this illustration are calculated in Exhibit Disc.1 on page 196, 242.

- Column 2 shows the cumulative net paid losses and loss adjustment expenses by accident year at the current statement date, from Schedule P, Part 1, column 11.[109]

- Column 3 shows the incurred net losses and loss adjustment expenses by accident year at the current statement date, from Schedule P, Part 1, column 28. These entries include paid losses and loss adjustment expenses, case reserves, and bulk + IBNR reserves.

- Column 4 shows the cumulative percentage paid from inception of the accident year to the current statement date, or column 2 divided by column 3. For accident year 2009, the percentage is $200,000 / $571,500 = 35.00%. For accident year 2008, the percentage is $330,000 / $687,500 = 48.00%.

Assumed Incremental Percentage Paid

- Column 5 shows the expected incremental percentage paid in each 12 month period. These entries are the first differences of the series in the previous column:

  ✓ For accident year 2009, the cumulative percentage paid at 12 months since inception of the accident year is 35.00%. For the most recent accident year, the incremental percentage paid equals the cumulative percentage paid.

  ✓ For accident year 2008, the cumulative percentage paid at 12 months since inception of the accident year is 48.00%. This implies that 48.00% – 35.00% = 13.00% of incurred losses are paid between 12 months and 24 months since inception of the accident year.

Schedule P shows 10 accident years of data, from which we estimate 10 twelve-month intervals of expected loss payments. If any losses remain unpaid at the end of 10 years – that is, if the cumulative paid losses for the oldest accident year does not equal the incurred losses for that accident year – we assume that all these losses are paid in the eleventh year, with the following limitation.

The amount assumed to be paid in the eleventh year is capped by the amount assumed to be paid in the tenth year. The excess amount is assumed to be paid in the twelfth year, but it is also capped at the same limit. The remaining excess is assumed to be paid in the thirteenth year, and so forth. We continue in this fashion through the fifteenth year. The remaining excess is assumed to be paid in the sixteenth year, with no limit. The next illustration (other

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[109] The net paid losses and net incurred losses are net of reinsurance recoverables and return premiums.

liability) shows the computation of an extended loss payment pattern. We defer further explanation of the procedure until we get to that illustration.

The Schedule P entries for the "prior years" row are not used in the computation of the loss reserve discount factors. The reserves and payments in this row relate to various accident years. A "composite" discount factor is used to determine the discounted loss reserves for the prior rows in Schedule P; see the discussion below.

In this illustration, the cumulative percentage paid for the ninth year (2001) is 94.94%, and the cumulative percentage paid for the tenth year (2000) is 98.00%. (The "n^{th}" year here means the "n^{th}" year working backwards from the current valuation date.)[110] The amount assumed to be paid from the end of the ninth year to the end of the tenth year is 98.00% – 94.94% = 3.06%. The amount still unpaid after 10 years is 100.00% – 98.00% = 2.00%. Since 2.00% is less than 3.06%, the full 2.00% is assumed to be paid in the eleventh year. No losses are assumed to be paid after 11 years.

Several of the commercial casualty lines of business have loss payment patterns extending beyond ten years; this is especially true for workers' compensation, other liability, products liability, and medical malpractice. For these lines of business, we don't expect the cumulative paid losses at the end of the tenth year to equal the incurred losses for that year.[111] The next illustration shows the adjustments used for these long-tailed lines of business.

DISCOUNTING COMPUTATIONS

- Column 6 shows the percentage of losses unpaid at the end of the accident year, which equals the complement of the cumulative percentage of losses paid. For accident year 2009 in the personal automobile illustration, the cumulative percentage of losses paid is 35.00%, and the percentage of losses unpaid at the end of the accident year is 100% – 35.00% = 65.00%.

- Column 7 shows the discounted percentage of losses unpaid at the end of the accident year. To compute these figures, we assume that all losses are paid at mid-year. We may

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[110] We estimate the amounts to be paid in future calendar years by looking at old accident years. The difference in the cumulative percentages paid between the n^{th} past accident year and the $(n+1)^{rst}$ past accident year is the percentage assumed to be paid between the end of the n^{th} calendar year from inception of the accident year to the end of the $(n+1)^{rst}$ calendar year from inception of the accident year. The n^{th} accident year working backwards from the most recent accident year corresponds to the n^{th} calendar year working forewards from the current statement date.

[111] The IRS computation of the loss reserve discount factors for all years is heavily influenced by the Schedule P entries for the ninth oldest accident year and the tenth oldest accident year. By random loss fluctuations, any long-tailed line of business may have an 11 year loss payment pattern one year and a 16 year loss payment pattern the next year.

use either (i) an iterative method, working backwards from the oldest accident year or (ii) a formula method.

We use the figures in the personal automobile illustration to explain the methods.[112]

Iterative Method

Two percent of the incurred losses are assumed to be paid in the eleventh year, labeled "AY + 10" in the exhibit. We assume that they are paid in mid-year. With a 7.0% discount rate, the discounted value of these losses at the preceding December 31 is $2\%/(1.070)^{0.5} = 1.93\%$.

Going backwards in accident years corresponds to going forwards in calendar years. The "current accident year" in this Schedule P exhibit is 2009, though the computed loss payment pattern is used for accident year 2011, not accident year 2009. The current valuation date for accident year 2011 for which this discount factor applies is December 31, 2011. Accident year AY+1 corresponds to calendar year 2011+1 = 2012. Accident year AY+10 corresponds to calendar year 2011 + 10 = 2021.[113]

To determine the discounted percentage of losses unpaid at the end of the ninth year, we combine two pieces:

- The percentage of losses assumed to be paid in the tenth year – which are assumed to be paid at mid-year – discounted for half a year to the end of the ninth year.
- The discounted percentage of losses unpaid at the end of the tenth year, discounted for an additional year to the end of the ninth year.

In the illustration, the two pieces are as follows.

- 3.07% of accident year 2011 losses are assumed to be paid in the middle of the tenth year, or July 1, 2020. They are discounted for half a year to December 31, 2019: $3.07\%/1.070^{0.5} = 2.97\%$.
- The discounted percentage of accident year 2011 losses unpaid at the end of the tenth year (December 31, 2020) is discounted for a full year: 1.93%/1.070 = 1.80%.

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[112] The assumption that all losses are paid at mid-year is a proxy for an even distribution of paid losses during the year. In truth, losses are paid (on average) earlier than the middle of the year, particularly for losses paid in the 2 or 3 years following the inception of the accident year. The IRS procedure provides a slightly longer discount period than is warranted. This reduces the offset to taxable income and increases the income tax liability. This bias is offset by the shorter payment patterns implicit in the IRS extension past ten years.

[113] For an excellent explanation of this technique, see Salzmann [1984], who uses a similar version to develop a reserving method for allocated loss adjustment expenses.

The sum of 2.97% and 1.80% is 4.77%. We continue in this fashion for all accident years. This is the iterative method.

Formula Method

Alternatively, formulas may be used for each year. the formula for the 2009 accident year in the Schedule P exhibit, which corresponds to accident year 2011 valued at December 31, 2011, is

$$(13.00\% \div 1.07^{0.5}) + (11.02\% \div 1.07^{1.5}) + \ldots + (3.07\% \div 1.07^{8.5}) + (2.00\% \div 1.07^{9.5}) = 52.26\%.$$

Loss Reserve Discount Factors

Column 8 shows the loss reserve discount factors used in the tax calculation. These factors are the discounted percentage of unpaid losses at the end of each year divided by the undiscounted percentage of unpaid losses at the end of that year. For accident year 2009 in the illustration, the loss reserve discount factor is 52.26% / 65.00% = 80.3944%. This corresponds to the loss reserve discount factor for accident year 2011 valued at December 31, 2011. Since these factors are used to determine the tax liability, the IRS demands 6 decimal place accuracy.[114]

These loss reserve discount factors apply to the 2011 accident year only. Suppose that at December 31, 2011, the accident year 2011 undiscounted reserves are $450,000. The corresponding discounted reserves are $450,000 x 80.3944% = $361,775.

The loss reserve discount factor in the preceding row, 81.6659%, is applied to the accident year 2011 reserves on December 31, 2012, not to the reserves of any other accident year. If the 2012 Schedule P reserves for accident year 2011 are $350,000, the 2012 discounted reserves for accident year 2011 are $350,000 x 81.6659% = $281,380.

[114] Six decimal place accuracy is not necessarily meaningful if the Schedule P entries have fewer than six significant digits. Since the paid and incurred entries are in thousands of dollars, they have fewer than six significant digits for small companies.

ILLUSTRATION: OTHER LIABILITY

The following figures are taken from the 2009 Annual Statement, Schedule P, Part 1H (other liability), of a company that has elected to use its own loss payment pattern for computing discounted reserves for accident year 2011.

Exhibit Tx.7: Other Liability Paid and Incurred Losses

Accident Year	Losses + LAE Paid (col 11)	Losses + LAE Incurred (col 28)
Prior	$235,000	$250,000
2000	50,000	55,500
2001	55,000	62,000
2002	60,000	70,000
2003	65,000	80,000
2004	70,000	96,000
2005	65,000	103,000
2006	60,000	115,000
2007	50,000	125,000
2008	35,000	140,000
2009	15,000	180,000

The 60 month rolling average of the federal mid-term rate from January 2006 through December 2010 is 7.0% per annum.

Extension of Payments

We retain the same accident years and discount rate from the previous illustration. We focus on the extension of payments for long-tailed lines of business.

The loss reserve discount factors are used for accident year 2011 only. In this illustration, we determine 15 separate loss reserve discount factors. The first ten discount factors are used for valuation dates December 31, 2011 through December 31, 2020. The 11th through the 15th discount factors are used at valuation dates December 31, 2021 through December 31, 2025 as part of the composite discount factor for accident years more than 10 years old (the "prior years" row in Schedule P). The calculation of the composite discount factor is explained below.

CAPPING

The amount assumed to be paid in the eleventh year is capped by the amount assumed to be paid in the tenth year. In this illustration, 90.09% – 88.71% = 1.38% of incurred losses are assumed to be paid in the tenth year. The amount remaining unpaid after 10 years is 100.00% – 90.09% = 9.91% of the incurred losses. Only 1.38% is assumed to be paid in the eleventh year. The remaining 9.91% – 1.38% = 8.53% is assumed to be unpaid at the end of the eleventh year.

The 1.38% cap affects the subsequent years as well. The amount assumed to be paid in each of the five years immediately following the tenth year is the lesser of (i) the amount unpaid at the end of the previous year and (ii) the 1.38% cap. We show first an illustration with a loss payment pattern that does not extend through the 16th year before returning to the other liability illustration here.

> *Illustration:* Suppose that the IRS loss reserve discounting procedure for commercial automobile indicates that 90.90% is paid within 10 years and 88.10% is paid within nine years. This implies that 90.90% – 88.10% = 2.80% is paid in the tenth year. The amounts assumed to be paid in the 11th, 12th, and 13th years are also 2.80%. Only 9.10% – 3 × 2.8% = 0.70% remains unpaid after thirteen years. This is the amount assumed to be paid in the 14th year.

Whatever remains after 15 years is assumed to be paid in the 16th year, even if it exceeds the 1.38% cap.

> *Illustration:* In the other liability example above, 9.91% – 5 × 1.38% = 3.01% remains unpaid after 15 years, so 3.01% is assumed to be paid in the sixteenth year.[115]

EXTENDED DEVELOPMENT

As for the personal auto illustration, we show the iterative procedure and the formula method

Iterative Procedure: We begin the computation of the discounted percentages unpaid at the December 31 preceding the final loss payment. For this (other liability) illustration, the loss payment pattern extends through 16 years, so we begin the computation of the discounted percentage unpaid with the end of the fifteenth year.

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[115] See the Internal Revenue Code §§ 846(d)(3)(C) and (D), "Special rule for certain long-tail lines": In the case of any long-tail line of business, the period taken into account shall be extended (but not by more than 5 years), and the amount of losses which would have been treated as paid in the 10th year after the accident year shall be treated as paid in such 10th year and each subsequent year in an amount equal to the amount of the losses treated as paid in the 9th year after the accident year (or, if lesser, the portion of the unpaid losses not theretofore taken into account). To the extent such unpaid losses have not been treated as paid before the last year of the extension, they shall be treated as paid in such last year. The term "long-tail line of business" means any line of business if the amount of losses which would be treated as paid in the 10th year after the accident year exceeds the losses treated as paid in the 9th year after the accident year.

3.01% of the accident year 2011 incurred losses are assumed to be paid in the middle of the 16th year, or July 1, 2026. The discounted loss reserve at the end of the 15th year (or December 31, 2025) is $3.01\% / 1.070^{0.5} = 2.91\%$.

The discounted percentage unpaid at the end of the 14th year equals the sum of (i) the 2.91% discounted percentage unpaid at the end of the 15th year discounted for an additional full year and (ii) the 1.38% of the incurred losses assumed to be paid on July 1 of the 15th year discounted for half a year. This is $2.91\% / 1.070 + 1.38\% / 1.070^{0.5} = 4.05\%$. (The 0.01 percentage point difference from the figure in the exhibit is a rounding discrepancy.)

Formula Method: We calculate each discounted percentage unpaid by formula. For the 2011 valuation date for the 2011 accident year, the discounted percentage unpaid equals

$$(16.67\% \div 1.07^{0.5}) + (15.00\% \div 1.07^{1.5}) + \ldots + (1.38\% \div 1.07^{13.5}) + (3.01\% \div 1.07^{14.5}) = 71.32\%.$$

ILLUSTRATION: ACCIDENT YEARS AND VALUATION DATES

Associating particular accident years and valuation dates with the appropriate Schedule P exhibits and average discount rates is the most confusing part of the calculations. The following illustration highlights the relationships.

Best's *Aggregates and Averages* shows the following data for private passenger auto liability from the 2005 industry aggregate Schedule P, Part 1B (in thousands of dollars).

Exhibit Tx.8: Loss Reserve Discounting Valuation Dates: Input Data

Accident Year	Losses Paid	Losses Incurred
Prior	$15,871,690	$15,968,279
1996	11,959,296	12,024,227
1997	13,496,724	13,613,803
1998	15,261,632	15,431,377
1999	17,079,431	17,381,876
2000	17,960,909	18,514,492
2001	19,922,828	21,136,036
2002	20,799,050	23,244,356
2003	21,050,478	26,110,739
2004	19,316,816	29,486,820
2005	10,735,738	31,281,287

The 12 month averages of the federal mid-term rate for calendar years 2000 through 2009 are shown below:

Exhibit Tx.9: Federal Mid-Term Rates

Year	Average	Year	Average	Year	Average
2001	6.7%	2004	7.0%	2007	7.0%
2002	6.6%	2005	7.3%	2008	7.3%
2003	6.8%	2006	7.4%	2009	6.9%

We determine the loss reserve discount factor for accident year 2009 private passenger automobile liability reserves at the December 31, 2017 valuation date.

DISCOUNT RATE

The discount rate is the 60 month moving average from January 1, 2004 through December 1, 2008. This equals 1/5 × (7.0% + 7.3% + 7.4% + 7.0% + 7.3%) = 7.2%.

The discount rate is computed once and used for all accident year 2009 discount factors.

LOSS PAYMENT PATTERN

Year 2009 is not a determination year. The preceding determination year is 2007. The data used is from the 2005 Schedule P from Best's *Aggregates and Averages.* This loss payment pattern is used for accident years 2007 through 2011.

Valuation date December 31, 2017 is nine years after the inception of accident year 2009. The loss reserve discount factor depends on the percentages assumed to be paid in 2010, 2011, and subsequent years. These are the same percentages as the percentages of accident year 2007 reserves at December 31, 2015, assumed to be paid in 2016, 2017, and the subsequent years.

In the terms used in this paper, these are the percentages assumed to be paid in years 10, 11, and so forth, up through year 16. The exact number of years, as well as the percentages, depends on the loss payment pattern.

The 1996 accident year in the 2005 Schedule P exhibit shows that $11,929,296,000 / $12,024,227,000 = 99.46% of incurred losses are paid by the end of ten years. The remaining 0.54% of incurred losses is distributed to years 11 and subsequent.[116]

If 0.54% or more of losses were assumed to be paid in the tenth year, we would allocate the entire 0.54% of remaining losses to the eleventh year. To determine the amount assumed to be paid in the tenth year, we compare the 1996 and 1997 accident years. From the 1997 accident year, we infer that $13,496,724,000 / $13,613,803,000 = 99.14% of incurred losses are paid in the first nine years. The percentage assumed to be paid during the tenth year is 99.46% – 99.14% = 0.32%.

The cap on payments for years 11 through 15 is 0.32%, with payments exceeding the cap rolled over to the next year. 0.54% is unpaid after ten years. The maximum that can be allocated to year 11 is 0.32%. The remainder, or 0.22%, is allocated to year 12.

LOSS RESERVE DISCOUNT FACTORS

At December 31, 2017, 99.14% of the 2009 accident year losses are assumed to have been paid. The remaining 0.86% of losses are assumed to be paid on July 1 in years 2018, 2019, and 2020 in the ratio 0.32%:0.32%:0.22%. In other words,

- (0.32%) ÷ (0.86%) of the reserves will be paid on July 1, 2018;
- (0.32%) ÷ (0.86%) of the reserves will be paid on July 1, 2019; and
- (0.22%) ÷ (0.86%) of the reserves will be paid on July 1, 2020.

We discount these assumed payments back to December 31, 2017, at a 7.2% discount rate. The discount factor for year 10 is $1/(1.072)^{0.5}$, since these loss payments are being discounted from 7/1/2018 back to 12/31/2017. The total discounted reserves, as a percentage of the undiscounted reserves, are

Year 10 (7/1/2018)	$[(0.32\%) \div (0.86\%)] \div (1.072)^{0.5}$	= 35.94%
Year 11 (7/1/2019)	$[(0.32\%) \div (0.86\%)] \div (1.072)^{1.5}$	= 33.52%
Year 12 (7/1/2020)	$[(0.22\%) \div (0.86\%)] \div (1.072)^{2.5}$	= 21.50%

The loss reserve discount factor is 35.94% + 33.52% + 21.50% = 90.96%.

Patterns

In the other liability illustration, the loss reserve discount factors are similar for the ten accident years that are separately reported in Schedule P, ranging from 77% to 80%. Some actuaries

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[116] The calculations here use percentages with two decimal places. In practice, six significant digits are used for the IRS discount factors.

presume that loss reserve discount factors should be lowest (i.e., furthest below unity) at inception and should increase towards unity as the reserves become more mature. This presumption is that the amount of the discount as a percentage of the remaining reserves is greatest at early maturities and declines to zero at later maturities.

This presumption is correct for the true discount factor for an individual loss. Suppose a loss occurs on July 1, 20X1, and it will be paid on July 1, 20X9. The amount of the discount is greatest on December 31, 20X1, and it declines steadily thereafter.

This presumption is not correct for an accident year. If loss payments follow an exponential decay, as modeled by McClenahan [1975] and Butsic [1981], the loss reserve discount factor remains relatively constant as long as some claims remain unpaid. The expected discount factor depends on the rate of decay and the discount rate, not on the development period. As Butsic [1981] shows, if the loss payments follow an exponential decay, the average remaining time to settlement is constant over the lifetime of the reserves.[117]

The loss reserve discount factors in the other liability illustration increase steadily in the final six years, from 80% to about 97%. This is caused by the IRS assumption of a constant percentage of incurred losses paid in each development period during the extended part of the loss payment pattern, instead of the declining percentage of incurred losses assumed by an exponential decay pattern.[118] For instance, the other liability illustration uses a 1.38% figure for each development period. The assumption of a final lump sum payment in the last year, whether or not the payment pattern is extended, augments the upward trend in the loss reserve discount factors for mature periods.

Negative Percentages Paid

Because different accident years are used for the cumulative paid percentages, the incremental paid percentages paid may be negative. The negative assumed payments can appear in any year except the most recent one, though they are more likely to occur in the more mature accident years. For the IRS loss reserve discounting procedure, they are most problematic (i) when they occur in the most mature accident year, causing a negative cap for

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[117] For workers' compensation, the decay is slower than exponential. Temporary total claims dominate the early payments; most of these claims are settled within a year or two. Permanent partial disability and permanent total disability claims dominate the reserves for mature years. These claims may remain open for 30 or 40 years. The loss payment pattern is rapid initially but it is very slow by ten years of maturity. See also the discussion above of the Sherman inverse power curve estimate of the paid loss development tail factor.

This is a general statistical result. If we combine distributions with exponential decays, each with a different rate of decay, the combination has a decreasing rate of decay.

[118] The exponential decay assumes that a constant percentage of the remaining reserves (not of the total incurred losses) is paid in each development period.

the assumed payments in the extended loss payment pattern, or (ii) when they cause a negative loss reserve discount factor.[119]

NEGATIVE CAP

If the negative assumed loss payment occurs in the oldest accident year individually reported in Schedule P, the cap on the extended payments in all subsequent years would also be negative. To avoid this situation, a negative assumed loss payment in the oldest accident year is replaced by the average of the assumed loss payments in the three oldest years.[120]

Illustration: The 2009 Schedule P for a given line has the cumulative paid losses and incurred losses shown in Exhibit Tx.10.

Exhibit Tx10: Negative Assumed Loss Payments

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio: (4)	Incremental Paid/Incurred Ratio: (5)	Undiscounted Percentage Unpaid: (6)
2000	$280,000	$300,000	93.33%	-3.64%	6.67%
2001	320,000	330,000	96.97%	9.47%	3.03%
2002	315,000	360,000	87.50%	5.92%	12.50%
2003	310,000	380,000	81.58%	6.58%	18.42%
2004	300,000	400,000	75.00%	7.25%	25.00%

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119 The true problem is not simply that negative assumed payments are unreasonable. The problem is that the prevalence of negative assumed payments highlights the inaccuracy of the entire calculation method. No estimation procedure is perfect. But the standard actuarial technique for determining loss payment patterns is reasonably accurate. An actuarial estimate of the future loss payments in a given year of 10% might mean a 90% confidence interval that the true expected loss payment is between 8% and 12%. In contrast, the IRS procedure is much less accurate. An estimate of the future loss payments in a given year of 10% might mean a 90% confidence interval that the true expected loss payment is between 5% and 20%. The accuracy diminishes for estimated loss payments close to 0%. An IRS estimate of the future loss payments in a given year of 1% might mean a 90% confidence interval that the true expected loss payment is between 0% and 15%.

120 See the Internal Revenue Code §846(d)(3)(G): "If the amount of the losses treated as paid in the 9th year after the accident year is zero or a negative amount, subparagraphs (C)(ii) and (D) shall be applied by substituting the average of the losses treated as paid in the 7th, 8th, and 9th years after the accident year for the losses treated as paid in the 9th year after the accident year." A literal reading of this paragraph implies that the cap for the years subsequent to the tenth year is changed, but the negative assumed payment for the tenth year remains.

The negative assumed payment in the oldest accident year (2000) stems from a statistical fluctuation – slightly lower claims remaining open in the ninth year (2001) than expected.

The average assumed loss payment in the three oldest accident years is ⅓ × (–3.64% + 9.47% + 5.92%) = 3.92%. This figure replaces the –3.64% in accident year 2000 before the extended payment pattern is computed.

If the average of the three oldest accident years is still negative, the average of the four oldest accident years is used. If this average is still negative, the average of the five oldest accident years is used. One continues in this fashion until one comes to the average of all ten accident years. This average cannot be negative.

NEGATIVE DISCOUNT FACTORS

It is possible for one or more of the computed accident year discount factors to be zero or negative. A negative discount factor can result only if the assumed amount paid is also negative in some year. However, most negative assumed amounts paid do not cause negative discount factors.

A negative discount factor has no financial meaning. A discount factor of 80% means that the present value of a $100,000 future cash flow is $80,000. A rational investor with no risk aversion would be indifferent between $80,000 paid now and the $100,000 cash flow when it is actually paid. By the same interpretation, a discount factor of –40% would mean that an investor is indifferent between paying $40,000 now and receiving the $100,000 when the cash flow is actually received. This does not make sense.

Because a negative discount factor is not reasonable, the negative factor is replaced by an interpolated factor between the nearest positive discount factors on both sides. Simple linear interpolation is used.

Illustration: The computed loss reserve discount factors for accident years AY+7, AY+8, and AY+9 are +80%, –35%, and +85%. The negative discount factor of –35% is replaced by the interpolated factor of +80% + ½ (85% – 80%) = 82.5%.

Illustration: The computed loss reserve discount factors for accident years AY+6, AY+7, AY+8, and AY+9 are +70%, –35%, –45%, and +85%. The negative discount factors of –35% and –45% are replaced by the interpolated factors of +70% + ⅔ (85% – 70%) = 80% and +70% + ⅓ (85% – 70%) = 75%.

NEGATIVE DISCOUNT FACTORS AND NEGATIVE PAYMENTS

Negative discount factors stem from negative assumed loss payments. The negative assumed loss payments stem from the quirks of the IRS loss reserve discounting procedure, not from negative loss payments or from data errors in the company's historical records.[121]

If the negative assumed loss payment occurs in the oldest accident year shown in Schedule P, it is replaced by the average of the assumed loss payments in the three oldest accident years. If the negative assumed loss payment occurs in any other accident year, it is not changed. Only negative loss reserve discount factors are replaced by positive ones.

TAX LIABILITIES AND REFUNDS

If the computed loss reserve discount factors for accident years AY+7, AY+8, and AY+9 are +80%, +10%, and +85%, no change is made, though the +10% discount factor for year AY+8 is unreasonable. This sequence of discount factors causes a large tax liability in one year followed by a tax refund in the subsequent year for a given line of business.

Illustration: The expected loss reserves for accident year 20X1 are $50 million, $45 million, and $40 million at year-end 20X7, 20X8, and 20X9, with expected payments of $5 million in each year. The statutory incurred loss from the runoff of the reserves is $0 in each year. If the loss reserve discount factors are 80%, 10%, and 85%, the tax basis incurred losses are as follows (figures in millions of dollars).

Exhibit Tx.11: Unreasonable Loss Reserve Discount Factors (dollars in millions)

Calendar Year	Paid Loss	Change in Loss Reserve	Incurred Loss
20x8	$5	$45 x 10% – $50 x 80% = –$35.5	–$30.5
20x9	$5	$40 x 85% – $45 x 10% = $29.5	$34.5

This effect is submerged within the other tax liabilities and tax refunds of the company, and it is generally not noticeable. The majority of negative assumed loss payments produce positive but unreasonable loss reserve discount factors.

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[121] Negative loss payments are possible, though they are rare. They can result from unanticipated salvage and subrogation or from unanticipated reinsurance recoverables. They can also result from a failure to accrue anticipated salvage and subrogation or a failure to accrue anticipated reinsurance recoverable.

The company's interpretation of "net amounts" also affects the figures. Net paid losses in Schedule P means direct plus assumed paid losses minus ceded paid losses. One might presume that this means the direct plus assumed losses paid minus the reinsurance recoverables actually received. This is not the case. Net paid losses means the direct plus assumed losses paid minus the reinsurance recoverables received or anticipated on these loss payments. See SSAP No. 53, "Property-Casualty Contracts – Premiums," SSAP No. 62, "Property and Casualty Reinsurance," Yoheved and Sarason [2002], and Feldblum [2002: Schedule F].

Exhibits A.3 and A.4 on pages 244 and 245 illustrate this. Exhibit A.4 uses the same other liability illustration worked out above, with a change in the paid losses for accident year 2002 from $60,000 to $69,000. The incremental paid to incurred ratio for accident year 2001 becomes –9.86%, the discounted loss reserve for accident year 2002 becomes –1.36%, and the loss reserve discount factor for accident year 2002 is –95%. The negative loss reserve discount factor is replaced a positive factor of ½ × (82.5189% + 77.4439%) = 79.9814%.

Exhibit A.3 shows the same scenario, but the accident year 2002 paid losses are changed to $68,000, not $69,000. The incremental paid to incurred ratio for accident year 2001 becomes –8.43%, the discounted loss reserve for accident year 2002 becomes 0.02%, and the loss reserve discount factor for accident year 2002 is less than 1% (0.6645%). This scenario is also unreasonable, but it is retained by the IRS rules.[122]

COMPOSITE DISCOUNT FACTORS

The loss reserve discount factors calculated above are applied to the unpaid losses for the appropriate accident year. Schedule P shows loss reserves by accident year only for the ten most recent years, to which ten separate loss reserve discount factors are applied. The 11th through 15th loss reserve discount factors are applied to the reserves in the Schedule P prior years row, which is not divided into the component accident years.

The IRS loss reserve discounting procedure assumes that all losses are paid no later than the 16th year. The prior years row in Schedule P contain losses that will be paid in the 12th through the 16th year, which use the loss reserve discount factors for years AY+11 through AY+15. A composite discount factor is formed from the five individual discount factors for application to the prior years row.

Each discount factor is the ratio of discounted reserves to undiscounted reserves for a given accident year at a given valuation date. For instance, the "tenth" accident year 2010 discount factor for AY+10 represents the discounted reserves for accident year 2010 at December 31, 2020, divided by the undiscounted reserves for accident year 2110 at December 31, 2020. This discount factor is computed in tax year 2010, not in tax year 2020.

We explain the calculation of the composite discount factor by illustration.

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[122] The disregard for financial reason evident in these IRS rules diminishes the public's respect for the IRS loss reserve discounting procedure. The IRS would do well to amend the procedures in accordance with sound actuarial techniques. The Casualty Actuarial Society and the American Academy of Actuaries would do well to formally recommend the necessary changes to the U.S. Treasury. An interim correction would be to smooth the pattern of assumed payments. This would at least eliminate unreasonable factors, though ideally a sound actuarial procedure should be used instead.

ILLUSTRATION: COMPOSITE DISCOUNT FACTORS

For tax year 2019, Schedule P shows ten individual accident years: 2010 through 2019. Previous accident years – 2009 and prior – are grouped in the prior years row. Since the IRS loss reserve discounting procedure assumes that all losses are paid by the 16th year, we assume that the loss reserves in the prior years row represent losses from accident year 2005 through 2009.

We form a composite discount factor based on the following discount factors:

- Accident year 2005 discount factor for a valuation date 15 years after inception of year.
- Accident year 2006 discount factor for a valuation date 14 years after inception of year.
- Accident year 2007 discount factor for a valuation date 13 years after inception of year.
- Accident year 2008 discount factor for a valuation date 12 years after inception of year.
- Accident year 2009 discount factor for a valuation date 11 years after inception of year.

Some of these loss reserve discount factors use the same loss payment pattern. However, they all use different discount rates, and they are computed in separate years.

Suppose these five loss reserve discount factors are as shown below:

Exhibit Tx.12: Composite Discount Factor

Accident Year (1)	Valuation Date (2) **	Undiscounted Reserve (3)	Discounted Reserve (4)	Discount Factor (5)
2005	AY + 15	5.0%	4.8%	96.9%
2006	AY + 14	7.2%	6.8%	93.9%
2007	AY + 13	9.1%	8.3%	91.0%
2008	AY + 12	11.7%	10.3%	88.2%
2009	AY + 11	13.3%	11.4%	85.4%
Total	prior years row	46.3%	41.6%	89.8%

** For all the accident years, the valuation date is December 31, 2020. The Internal Revenue Code refers to this as AY+15 for accident year 2005, AY+14 for accident year 2006, etc.

The calculation of the individual discount factors has been explained earlier. Each discount factor in column 5 is the ratio of the discounted reserves in column 4 to the undiscounted reserves in column 3. The reserve figures in columns 3 and 4 are expressed as percentages of the corresponding year's incurred losses. We compute the total of the five percentages for the discounted reserves and the undiscounted reserves. We divided these totals to obtain the composite discount factor for the prior years row.

Using a simple average to obtain the "total" row assumes that each year has the same volume of incurred losses. It might seem better to weight the discount factors by the actual percentage of incurred losses by accident year in the prior years row. However, the IRS bases the loss reserve discounting procedure on information contained in the Annual Statement. The distribution of the prior years row reserves by accident year is not found in the Annual Statement.

Taxpayer's Election

The Secretary of the Treasury revises the line of business loss payment patterns every five years, using aggregate (industry-wide) Schedule P data. The first loss payment patterns were determined in early 1987 for the 1987 through 1991 tax years.[123] The industry-wide Schedule P data were those contained in the most recent Best's *Aggregates and Averages* that was available in early 1987. This was the 1986 edition of Best's *Aggregates and Averages*, containing data from the 1985 Annual Statements.

The Treasury redetermines the loss payment patterns every five years; these are the years 1992, 1997, 2002, 2007, and so forth. The loss payment patterns apply to that year and the four subsequent years (1992-1996; 1997-2001; and so forth).

The loss payment patterns are determined once every five years, but the discount rate is recomputed each year. The loss reserve discount factors change each year, since the discount rate changes, even though the loss payment patterns may remain the same.

The Treasury recognizes that the aggregate industry loss payment patterns may not be appropriate for some insurers.

> *Illustration:* The aggregate industry-wide other liability loss payment pattern assumes a long average lag between the occurrence of accidents and the settlement of claims. Insurer ABC writes relatively quick settling premises and operations coverage for offices, showrooms, and retail stores. Since its claims settle more quickly than the industry averages, it should be able to use discount factors closer to unity, thereby giving higher discounted loss reserves, a greater offset to taxable income, and lower tax liabilities.

At determination years, each insurer may elect to use its own data to compute the loss payment patterns for the next five years. The election is made with the tax return for the determination year, which is filed a few months after the end of the year.

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[123] Preliminary loss payment patterns were determined in 1996 for review by the IRS staff and Congressional committees.

Illustration: On its 2007 tax return, filed in early 2008, Insurer ABC may elect to use its own data for the loss payment patterns used to compute the loss reserve discount factors for accident years 2007 through 2011.

If the insurer elects to use its own data, it recomputes the loss payment pattern each year, though each accident year's loss reserve discount factors are still "vintaged," or "frozen." For each accident year's loss reserve discount factors, the insurer uses the most recent Schedule P data that has been filed before the beginning of the accident year.

> *Illustration:* On its 2007 tax return, Insurer ABC elects to use its own data for the loss payment patterns used to compute the loss reserve discount factors for accident years 2007 through 2011. For the 2007 accident year, it uses 2005 Schedule P data; for the 2008 accident year, it uses 2006 Schedule P data; and so forth. This is the same "two year lag" as occurs with industry-wide loss reserve discount factors.

An election to use one's own data applies to all lines of business. An insurer may not elect to use its own data for some lines of business and the industry data for other lines.[124]

Election Restrictions

For two types of business an insurer must use the industry-wide loss reserve discount factors and may not use its own data:

- An insurer may not use its own data for the international line of business or for the reinsurance lines of business.[125]
- An insurer's election to use its own data does not apply to any line of business for which it "does not have sufficient historical experience to determine a loss payment pattern" [IRC §846(e)(4)(A)].

The 1986 conference reports, as well as the 1988 Treasury regulation 88-100, interpreted the latter provision to mean that an insurer whose reserves in a given line of business were smaller than those of 90% of other insurers may not use its own data to determine the loss payment patterns.[126] Small companies complained that this provision discriminated against

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[124] See Treasury regulations 2001FED 26,330C, §1.846-2, Election by taxpayer to use its own historical loss payment pattern: "A taxpayer making the election must use its own historical loss payment pattern in discounting unpaid losses for each line of business that is an eligible line of business in that determination year."

[125] See Internal Revenue Code §846(e)(3) "No election under this subsection shall apply to any international or reinsurance line of business"; see also § 846(d)(3)(E).

[126] See Regulation 88-100, §III: "Until further guidance is issued, such statistically significant amount is business in at least the 10th percentile of industry-wide reserves for a line of business for the determination year

them. In 1991, the Secretary of the Treasury specifically revoked this provision. Instead, the insurer must have data for all ten accident years shown in Schedule P to use its own data for that line of business.[127]

The adequacy of an insurer's loss reserves has a large effect on its election to use its own data. An insurer with less adequate loss reserves than those of the industry is more likely to gain from using its own data.

> *Illustration:* In 20X9, the industry-wide Schedule P for a given line of business shows accident year 20X9 cumulative paid losses of $100 million and incurred losses of $400 million, indicating that 25% of losses are paid in the first 12 months. Insurer ABC, which holds less adequate loss reserves, shows $3 million of accident year 20X9 cumulative paid losses and $10 million of incurred losses, indicating that 30% of losses are paid in the first 12 months. Insurer ABC seems to pay its losses more rapidly, so its discount factor should be closer to unity, its offset to taxable income should be larger, and its tax liability should be smaller. In truth, insurer ABC may have the same loss payment pattern as the industry has, but it may be holding less adequate loss reserves.

ANTICIPATED SALVAGE AND SUBROGATION

The loss reserves that are an offset to taxable income must be net of anticipated salvage and subrogation.[128] If the insurer does not disclose that the unpaid losses in Schedule P are net of anticipated salvage and subrogation, the IRS assumes they are gross of anticipated salvage and subrogation and requires a reduction for the anticipated amounts. Column 23 of Schedule P, Part 1 provides this disclosure by accident year and by line of business.[129]

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with respect to which the election is made."

[127] See Treasury regulation 2001FED 26,330C, §1.846-2, Election by taxpayer to use its own historical loss payment pattern: "A line of business is an eligible line of business in a determination year if . . . the taxpayer reports losses and loss expenses incurred . . . for at least the number of accident years for which losses and loss expenses incurred for that line of business are required to be separately reported on that annual statement."

[128] See IRC §846(e): *An insurance company is required to take estimated salvage recoverable (including that which cannot be treated as an asset for state statutory accounting purposes) into account in computing the deduction for losses incurred.*

[129] See the Internal Revenue Code, section 846(2) "A company is allowed to increase the unpaid losses shown on its annual statement only if the company . . . discloses on its annual statement, by line of business and accident year, the extent to which estimated salvage recoverable is taken into account in computing the unpaid losses shown on the annual statement . . ."

The anticipated salvage and subrogation that must be subtracted from the unpaid losses is the discounted anticipated salvage and subrogation. The discount factors are determined by the Treasury. Companies may elect to use their own discount factors for loss reserves, but they must use the Treasury discount factors for anticipated salvage and subrogation.[130]

COMPUTATIONAL SEQUENCE

The sequence for determining the offset to taxable income from loss reserves is as follows:

Step 1. Total net losses and expenses unpaid are taken from Schedule P, Part 1, column 24.
Step 2. The salvage and subrogation anticipated from Schedule P, Part 1, column 23, is added.
Step 3. The tabular discounts for loss reserves from Note 27 are added. This amount is the unpaid losses gross of all discounts and of anticipated salvage and subrogation.
Step 4. The Schedule P, Part 1, loss reserves are gross of the non-tabular discounts shown in Schedule P, Part 1, columns 32 and 33. However, these non-tabular discounts must be disclosed as well.
Step 5. The gross loss reserves are discounted using either (i) the industry loss reserve discount factors published by the Treasury or (ii) the company's own loss reserve discount factors, depending on the election made by the company in the most recent determination year.
Step 6. The gross anticipated salvage and subrogation is discounted using the Treasury discount factors.
Step 7. The discounted anticipated salvage and subrogation is subtracted from the discounted loss reserves to give the discounted reserves net of anticipated salvage and subrogation. The change in these discounted reserves is the loss offset to taxable income.[131]

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[130] Until recently, companies did have the option of their own discount factors for anticipated salvage and subrogation. Treasury regulation 2001FED 26,153, §1.832-4, says that "except as otherwise provided in guidance published by the Commissioner in the Internal Revenue Bulletin, estimated salvage recoverable must be discounted either (1) by using the applicable discount factors published by the Commissioner for estimated salvage recoverable; or (2) by using the loss payment pattern for a line of business as the salvage recovery pattern for that line of business and by using the applicable interest rate for calculating unpaid losses under section 846(c)." Guidance explicitly revoking this choice was issued in 2001.

[131] The IRS has issued extensive rules relating to the 1991 "fresh start" for anticipated salvage and subrogation, with insurers classified as "grossers" and "netters," and to various permitted discount factors in earlier years. These rules are not relevant to current and future tax years.

Deferred Tax Assets

The computation of the admitted portion of the deferred tax asset stemming from IRS loss reserve discounting is based on two items:

- the loss reserve discount factors by accident year and by line of business for the current valuation date and for the valuation date 12 months hence, and
- the company's loss payment pattern by line of business.

The IRS loss payment pattern is used to compute the loss reserve discount factors. The actuary's estimated loss payment pattern is used to compute the admitted portion of the deferred tax asset.

Of all the changes in the NAIC's codification project, the deferred tax asset stemming from IRS loss reserve discounting has the greatest effect on policy pricing and company valuation; see Kelly, *et al.* [2002: prms]. We present first the requisite background explanations of deferred tax assets and liabilities, and we illustrate the loss reserve discounting procedures that rely on Schedule P data.

CURRENT TAXES VS DEFERRED TAXES

There are two ways of accounting for federal income taxes:

- The incurred tax liability is the tax liability actually incurred by the taxpayer, based on the provisions of the Internal Revenue Code, or
- The accrued tax liability is the tax liability implied by the company's balance sheet, whether GAAP or statutory.

Current taxes are the incurred tax liability. The current year's change to the deferred tax asset or liability is the difference between the incurred tax liability and the accrued tax liability.[132] The change to the deferred tax asset or liability is a direct charge or credit to surplus shown on line 24 of the NAIC Annual Statement.

Before 2001, insurers could not admit any deferred tax asset or liabilities on the statutory balance sheet. In contrast, GAAP recognizes deferred tax assets and liabilities if they are

[132] This definition uses a retrospective computation. SFAS 109 requires a prospective computation, which may be different if the tax rate changes or if there are other changes in tax regulations. For simplicity, we use the retrospective viewpoint at first. We explain the prospective viewpoint further below.

expected to be realized; see SFAS 109. With the implementation of codification in 2001, statutory accounting recognizes deferred tax liabilities and a portion of deferred tax assets.

Permanent Differences and Timing Differences

Tax accounting differentiates between permanent differences and timing differences, as defined below.

- *Permanent differences* are differences that do not reverse in later accounting periods. The tax exemption for municipal bond interest is a permanent difference.
- *Timing differences* are differences that reverse in later accounting periods. The revenue offset provision creates a timing difference between statutory income and taxable income.

An alternative perspective is to view permanent differences as differences in the tax rates applicable to different sources of income; see Kelly, *et al.* [2002: prms]. For property-casualty insurers, both corporate bond income and municipal bond income are taxable income, but the former has a 35% tax rate and the latter has a 5.25% tax rate.

Income Statement vs Balance Sheet

It is tempting to define timing differences as differences in the timing of income between the book income statement (i.e., GAAP or statutory) and the tax income statement. This is not correct.

> *Timing differences are differences between the tax income statement and the income statement implied by the GAAP or statutory balance sheet.*[133]

UNREALIZED CAPITAL GAINS AND LOSSES

For each accounting year, we compute the difference between the book value and the cost of the financial asset. The change in this difference from the previous year to the current year is the unrealized capital gain or loss. For common stocks, the book value is the market value.

Unrealized capital gains and losses are admitted on the statutory (as well as GAAP) balance sheet, though they do not flow through the income statement. They are direct charges and credits to surplus, not a portion of net income.

For tax purposes, capital gains and losses are not part of income until they are realized.

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[133] This definition is particularly relevant to the deferred tax liabilities and assets stemming from unrealized capital gains and losses. For the deferred tax assets stemming from revenue offset and loss reserve discounting, we could use the difference between statutory income and taxable income.

- Unrealized capital gains increase the book value of common stocks on the statutory balance sheet. There is no tax liability in the current tax year. Instead, the reporting company shows a deferred tax liability.
- Similarly, unrealized capital losses decrease the book value of common stocks on the statutory balance sheet. There is no tax refund in the current tax year. Instead, the reporting company shows a deferred tax asset.

Illustration

ABC Insurance Co buys common stock for $50 million on December 31, 20XX.

- On December 31, 20XX+1, the common stock are worth $40 million;
- On December 31, 20XX+2, the common stock are worth $60 million; and
- On December 31, 20XX+3, the common stock are worth $80 million.

The federal income tax rate is 35%. On December 31, 20XX+3, the ABC Insurance Company sells the common stock. We calculate the following accounting entries:

- The unrealized capital gains and losses in years 20XX+1, 20XX+2, and 20XX+3.
- The realized capital gains and losses in years 20XX+1, 20XX+2, and 20XX+3.
- The deferred tax assets and liabilities in years 20XX+1, 20XX+2, and 20XX+3.

Tax year 20XX+1

The market value of the stock has decreased by $10 million. The stock has not been sold yet, so the capital loss is unrealized. There are no realized capital gains and losses.

- On December 31, 20XX, book value – cost = $50 million – $50 million = $0.
- On Dec 31, 20XX+1, book value – cost = $40 million – $50 million = –$10 million.
- The unrealized capital gain or loss = –$10 million – $0 million = –$10 million.

The current balance sheet shows a decline of $10 million. When the stocks are sold, ABC Insurance Company will have an income loss of only $6.5 million, since the capital loss can offset other capital gains, and the company's tax liability will be reduced by $3.5 million. There is a $3.5 million deferred tax asset on the 20XX+1 balance sheet.

Tax year 20XX+2

The stock prices have increased. The unrealized capital gain is the *change* in the difference between book value and cost of the stocks. The unrealized capital gain for 20XX+2 is $20 million. The realized capital gain is again zero, since the stocks have not been sold.

- On December 31, 20XX+1, book value – cost = $40 million – $50 million = –$10 million.

- On December 31, 20XX+2, book value – cost = $60 million – $50 million = +$10 million.
- The unrealized capital gain or loss = +$10 million – (–$10 million) = +$20 million.

The company's balance sheet is $20 million stronger than it was a year ago. However, if the stocks were sold now, the company would realize a gain of only $13 million, since $7 million would go to taxes. The *change* in the deferred tax assets and liabilities is a credit of $7 million. Since we began with a deferred tax asset (a debit) of $3.5 million, we now have a deferred tax liability (a credit) of $3.5 million.

Tax year 20XX+3

The company sells the stock. The difference between market value and cost of the stocks is now $0 (since there are no more stocks on the balance sheet), so the unrealized capital gain is –$10 million.

- On December 31, 20XX+2, book value – cost = $60 million – $50 million = +$10 million.
- On December 31, 20XX+3, book value – cost = $0 million – $0 million = $0 million.
- The unrealized capital gain or loss = $0 million – ($10 million) = –$10 million.

The realized capital gain, which is defined as the sale price minus the purchase price, is +$30 million. The deferred tax assets and liabilities are now zero.[134]

Statutory Recognition of Deferred Tax Assets

All deferred tax liabilities are recognized on the statutory balance sheet. For most deferred tax assets, the admitted statutory portion equals the entire asset, and statutory accounting is the same as GAAP.[135] In certain instances, only a portion of the deferred tax assets are

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[134] Unrealized capital gains and losses give rise to deferred tax liabilities and assets, respectively. Realized capital gains and losses affect current taxes; they do not give rise to deferred tax assets and liabilities. An exception stems from the rule that capital losses can offset capital gains but not operating gains.

If capital losses exceed capital gains, the company may carry forward the unused capital losses. The tax rate times the unused capital loss is a deferred tax asset, not a deduction in current tax liabilities.

Capital losses can be carried forward a limited number of years. If during these years the company has not realized sufficient capital gains to offset all the capital losses, the remaining capital losses expire unused, and the deferred tax asset is removed.

[135] There are two potential differences between GAAP and statutory accounting even when the full deferred tax asset passes the 12 month test:

- Some companies use a valuation allowance on the GAAP balance sheet for deferred tax assets and liabilities that may not reverse.
- Some companies use fair values, or discounted values, for deferred tax assets and liabilities that may

recognized on the statutory balance sheet. This applies particularly to the deferred tax asset stemming from IRS loss reserve discounting for medium- and long-tailed lines of business.

SSAP No. 10, "Income Taxes," paragraph 10, says:

Gross DTAs shall be admitted in an amount equal to the sum of:

a Federal income taxes paid in prior years that can be recovered through loss carrybacks for existing temporary differences that reverse by the end of the subsequent calendar year;

b The lesser of:

i. The amount of gross DTAs, after the application of paragraph 10 a., expected to be realized within one year of the balance sheet date; or

ii. Ten percent of statutory capital and surplus as required to be shown on the statutory balance sheet of the reporting entity for its most recently filed statement with the domiciliary state commissioner adjusted to exclude any net DTAs, EDP equipment and operating system software and any net positive goodwill; and

iii. The amount of gross DTAs, after application of paragraphs 10 a. and 10 b., that can be offset against existing gross DTLs.

A gross deferred tax asset is admissible if it will reverse within one year, as required by paragraph (a) and by paragraph (b.i).

The limitation of 10% of surplus in paragraph (b.ii) is rarely applicable. Few companies have deferred tax assets that will reverse in the coming year and that exceed 10% of policyholders' surplus. The deferred tax asset stemming from IRS loss reserve discounting is large, but most of this deferred tax asset does not reverse within one year.

The offsetting against existing gross deferred tax liabilities mentioned in paragraph (b.iii) is relevant for companies with large unrealized capital gains from common stock holdings. The actuary should take this provision into account when quantifying the admitted portion of the deferred tax asset.

Common stock that has suffered an unrealized capital loss may be sold within the next 12 months to realize the tax benefits. A literal reading of the SSAP would permit the recognition of the deferred tax asset only if the company expects to realize the capital loss during the coming calendar year. In practice, most auditors do not require an explicit company expectation to realize the loss in order to admit the deferred tax asset.

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not reverse for many years.

Revenue Offset

The deferred tax asset stemming from revenue offset is similar to the deferred tax asset stemming from loss reserve discounting. For annual policies, the entire deferred tax asset will reverse during the coming year, and it is fully admitted on the statutory balance sheet.

BACKGROUND

All acquisition expenses flow through the statutory income statement when they are incurred. No deferred policy acquisition cost (DPAC) asset is entered on the statutory balance sheet.

On GAAP financial statements, acquisition expenses are capitalized on the balance sheet and amortized through the income statement over the term of the policy. The DPAC asset depends on the actual expenses incurred by the company.

For tax purposes, 20% of the written premium is treated as acquisition expenses that are capitalized and amortized over the term of the policy.[136] More precisely, the revenue offset provision defines the taxable earned premium.

- Statutory earned premium equals written premium minus the change in the unearned premium reserves.
- Taxable earned premium equals written premium minus 80% of the change in the unearned premium reserves.

ILLUSTRATION: DPAC OF 20%

An annual policy with a premium of $1,000 and acquisition expenses of $200 is written on December 31, 20XX.

- The statutory balance sheet shows a loss of $200. The written premium of $1,000 is offset by the unearned premium reserve of $1,000, and the incurred acquisition cost of $200 flows through the income statement.
- For tax purposes, the $1,000 written premium is offset by only $800 of unearned premium reserves, leaving a $200 gain. This $200 gain combined with the $200 acquisition cost yields a $0 net gain or loss.

The income implied by the statutory balance sheet – taxable income = –$200 – $0 = –$200.

In 20XX+1, statutory earned premium is $1000, since the entire unearned premium reserve is taken down over the course of the year. The taxable income is $800, since only 80% of the

[136] Life and health insurers and annuity writers have a similar "DAC-tax."

change in the unearned premium reserve is considered. For 20XX+1, the income implied by the statutory balance sheet – taxable income equals $1000 – $800 = $200.

At the end of 20XX+1, the statutory balance sheet equals the implied tax balance sheet. Both show net cash received of $1000 – $200, or the written premium minus the acquisition expense. The temporary balance sheet difference at December 31, 20XX fully reverses by December 31, 20XX+1.

At December 31, 20XX, taxable income is $200 greater than the income implied by the statutory balance sheet. The tax liability for 20XX is 35% × $200 = $70 greater than the tax liability that would be determined from the statutory balance sheet. Since the $70 difference will reverse over the coming 12 months, it is recognized as a deferred tax asset on the statutory balance sheet.

The deferred tax asset on the statutory balance sheet does not depend on the amount of actual acquisition expenses. In contrast, the deferred tax asset on the GAAP balance sheet depends on the size of the GAAP deferred policy acquisition cost asset relative to the 20% assumption in the revenue offset provision.

ILLUSTRATION: DPAC OTHER THAN 20%

A company writes and collects a $1000 annual premium on December 31, 20XX. Acquisition expenses of $250 are incurred (and paid) on December 31, 20XX. The marginal tax rate on underwriting income is 35%. All acquisition costs are deferrable under GAAP.

Taxable underwriting income for 20XX is $200 (taxable premium income from revenue offset) – $250 (acquisition expenses) = –$50. The tax outflow is a negative $17.50 (or a tax refund of $17.50).[137]

The taxable premium income may be evaluated in either of two ways.

- Taxable earned premium = written premium minus 80% of the change in the unearned premium reserves = $1000 – 80% × $1000 = $200.
- Taxable earned premium = statutory earned premium plus 20% of the change in the unearned premium reserves = $0 + 20% × $1000 = $200.

The tax liability is 35% times the taxable income: 35% × ($200 – $250) = –$17.50.

Taxable underwriting income for 20XX+1 equals $800 of taxable premium income. The tax outflow is $800 × 35% = $280.00. Written premium during the year is $0 and the unearned

[137] The tax refund stemming from negative taxable income offsets tax liabilities stemming from positive taxable income on other insurance contracts. There is no need to presume tax carrybacks or carryforwards.

premium reserve declines from $1000 to $0. We use the same two computation methods: (i) $0 – 80% × (–$1000) = $800, or (ii) $1000 + 20% × (–$1000) = $800.

A deferred tax asset of $70 stemming from the revenue offset provision is entered on the balance sheet on December 31, 20XX, and it is amortized over the course of the policy term. The full deferred tax asset from revenue offset is recognized on the statutory balance sheet, since it reverses within 12 months of the balance sheet date (for annual policies).

On GAAP financial statements, the book income for 20XX is $1000 – $0 = $1000, since all acquisition expenses are capitalized. The taxable income is –$50 (as above), and the tax liability is –$17.50 (i.e., a refund). GAAP shows a deferred tax liability (not an asset) of $17.50, exactly offsetting the tax refund.

LOSS RESERVE DISCOUNTING

The statutory incurred losses are the paid losses plus the change in the undiscounted loss reserves. The taxable incurred losses are the paid losses plus the change in the *discounted* loss reserves. The difference between statutory and taxable incurred losses is a timing difference. The change in the deferred tax asset is 35% of this difference.

Illustration: A policy is issued on January 1, 20XX, for a premium of $1000 and expenses of $200. Losses of $800 are incurred in 20XX, of which half are paid in 20XX and half are paid in 20XX+1. The IRS loss reserve discount factor at the 12 month valuation is 90%. For simplicity, we assume that the companies earns no investment income.

- The statutory incurred losses in 20XX are $400 of paid losses plus $400 of loss reserve change = $800. Statutory income is $1000 – $200 – $800 = $0. The accrued taxes on income of $0 is $0.
- The taxable incurred losses in 20XX are $400 of paid losses plus $360 of change in discounted loss reserves = $760. Taxable income is $1000 – $200 – $760 = $40. The tax liability on $40 is $14.

The difference between the income implied by the statutory balance sheet and taxable income is $0 – $14 = –$14. The gross deferred tax asset is $14.

Only the portion of the deferred tax asset that reverse within 12 months is admitted on the statutory balance sheet. We examine the statutory income and taxable income for 20XX+1.

- The statutory incurred losses in 20XX+1 are $400 of paid losses plus –$400 of loss reserve change = $0. There is no premium or expense in 20XX+1, so statutory income is $0. The accrued taxes on income of $0 is $0.

- The taxable incurred losses in 20XX+1 are \$400 of paid losses plus –\$360 of change in discounted loss reserves = \$40. There is no premium or expense in 20XX+1, so taxable income is \$0 – \$40 = –\$40. The tax liability is 35% × (–\$40) = –\$14.

The full difference between statutory and taxable income reverses in 20XX+1, so the full deferred tax asset of \$14 is admitted on the statutory balance sheet.

Twelve Month Reversal

We present the formula for computing the admitted portion of the deferred tax asset stemming from loss reserve discounting. The computations are done separately by line of business and by accident year.

Illustration: For accident year 20XX in a given line of business, the loss reserve discount factors are Z_1 at December 31, 20YY, and Z_2 at December 31, 20YY+1. Let "R" be the held loss reserves at December 31, 20YY. Let "P" be the percentage of accident year 20XX reserves that will be paid during calendar year 20XX.

- At December 31, 20YY, the difference between statutory and taxable income for accident year 20XX is $R \times (1 - Z_1)$. The gross deferred tax asset is $35\% \times R \times (1 - Z_1)$.
- At December 31, 20YY+1, the difference between statutory and taxable income for accident year 20XX is $R \times (1 - P) \times (1 - Z_2)$. The gross deferred tax asset is $35\% \times R \times (1 - P) \times (1 - Z_2)$.
- The admitted portion of the deferred tax asset on the statutory balance sheet at December 31, 20YY is $35\% \times R \times [(1 - Z_1) - (1 - P) \times (1 - Z_2)]$.

The value of "P" depends on the actuary's best estimate of the loss payment pattern. It is not the same as the IRS loss payment pattern. To estimate the pattern, we must derive actuarially justified discount factors.

Actuarial Discount Factors

The percentage of losses expected to be paid by each valuation date is the reciprocal of the paid loss development factor.[138] We reproduce below the paid loss development factors from the illustration earlier in this paper.

[138] See Feldblum [2002: SB] for a full discussion of this topic.

Exhibit DTA.1: Paid Loss Development Test of Reserve Adequacy

	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	6 yrs	7 yrs	8 yrs	9 yrs
Pd LDF's	4.835	2.057	1.535	1.312	1.193	1.125	1.082	1.051	1.020
LDF w/ tail	5.319	2.263	1.689	1.443	1.312	1.238	1.190	1.156	1.122
Reciprocal	18.8%	44.2%	59.2%	69.3%	76.2%	80.8%	84.0%	86.5%	89.1%
Incr'tl Pd %	18.8%	25.4%	15.0%	10.1%	6.9%	4.6%	3.2%	2.5%	2.6%

The rows in the table are described below.

- The "Pd LDF's" are the paid loss development factors from each development date to 10 years of maturity, derived from Schedule P, Part 3, data. The paid loss development factor from 1 year to 10 years of maturity is 4.835.
- The "LDF w/ tail" is the paid loss development factors from each development date to ultimate, using a tail factor of +10%. The paid loss development factor from 1 year to ultimate is 5.319.
- The "Reciprocal" of the paid loss development factor to ultimate shows the percentage of losses paid by the development date. The cumulative losses paid by 1 year after the inception of the accident year is 1/5.319 = 18.8% of ultimate paid losses.
- The "Incr'tl Pd %" is the incremental paid losses during each development period as a percentage of ultimate paid losses. The losses paid between 1 year and 2 years after inception of the accident year are 44.2% – 18.8% = 25.4% of ultimate paid losses.

Loss Reserve Discounting

For GAAP financial statements, the deferred tax asset from loss reserve discounting is treated in the same fashion as the deferred tax asset from revenue offset. Both are fully recognized on the balance sheet.

ILLUSTRATION

In the other liability loss reserve discounting illustration in this paper, the accident year 2009 loss reserves for statutory and GAAP balance sheets on December 31, 2009 are $180,000 – $15,000 = $165,000. The corresponding discounted tax basis loss reserves are

$165,000 × 77.8022% = $128,373.63.

The difference between the GAAP loss reserves and the tax basis loss reserves is

$165,000.00 – $128,373.63 = $36,626.37.

The addition to taxable income stemming from loss reserve discounting for accident year 2009 at December 31, 2009 is \$36,626.27 × 35% = \$12,819.23. This is the deferred tax asset on the GAAP balance sheet.

The admitted portion of the deferred tax asset on the statutory balance sheet depends on the portion of the loss reserve that will still be unpaid in one year's time. This is an actuarial estimate; it is not the IRS provision used in the loss reserve discounting calculation. We may estimate this amount from Schedule P, Part 3, as discussed earlier.

Suppose the projected paid loss link ratios for other liability are 8.000 at 12 months of development and 5.000 at 24 months of development.

- At 12 months of development, 1/8.000 = 12.5% of incurred losses have been paid and 1 – 1/8.000 = 87.5% of incurred losses are still unpaid.
- At 24 months of development, 1/5.000 = 20.0% of incurred losses have been paid and 1 – 1/5.000 = 80.0% of incurred losses are still unpaid.

We expect 80.0% / 87.5% = 91.428571% of the December 31, 2009, accident year 2009 loss reserves to remain unpaid at December 31, 2010. This amount is \$165,000 × 91.4285714% = \$150,857.14. The expected IRS discounted reserves at December 31, 2010 equal this amount times the IRS loss reserve discount factor for accident year 2009 at 24 months of development, or 78.7611% in the other liability illustration:

\$150,857.14 × 78.7611% = \$118,816.75.

Schedule P Interrogatories

The Schedule P Interrogatories contain seven questions. Five of these have been discussed above along with the relevant Schedule P exhibits:

- *Interrogatory 1* shows a ten year exhibit of extended loss and expense reserves on claims-made policies for medical malpractice, other liability, and products liability. The caption for the first column says: "years in which premiums were earned and losses were incurred." The extended loss and expense reserves are policy reserves. They do not relate to earned premiums or incurred losses. It is unclear how the extended loss and expense reserves should be distributed by year in this exhibit.

- *Interrogatory 2* discloses whether the company has followed the revised NAIC definitions of defense and cost containment and adjusting and other adjustment expenses.

- *Interrogatory 3* relates to the distribution of adjusting and other adjustment expenses payments and reserves by accident year.

- *Interrogatory 4* discloses loss reserve discounting and the resulting difference between the Underwriting and Investment Exhibit and Schedule P.

- *Interrogatory 5* discloses the net premiums in force for fidelity and surety business. Some companies use premiums in force to estimate the reserves for these lines of business.

- *Interrogatory 6* discloses whether claim counts are per claim or per claimant.

The seventh interrogatory relates to estimates of reserve adequacy:

> *Interrogatory 7: The information provided in Schedule P will be used by many persons to estimate the adequacy of the current loss and expense reserves, among other things. Are there any especially significant events, coverage, retention or accounting changes which have occurred which must be considered when making such analyses?*

The NAIC left this question quite general, so that companies would describe freely any changes in their experience. The Annual Statement *Instructions* list several items that should be described in this interrogatory:

- A change in the method of counting claims.
- The intercompany pooling of only a portion of the business.
- Changes in the intercompany pooling arrangement.

Other material changes should also be noted. For example, if a company revises its tabular discount for permanent disability indemnity benefits on workers' compensation claims, either by changing the interest rate or by discounting different blocks of claims, this should be noted.

Reserve Margins

Calendar year underwriting results for the long-tailed lines of business are influenced by the adequacy of reserve margins for previously reported claims and by the strength of case and bulk reserves for newly reported claims.

- If the held reserves at the beginning of the calendar year were inadequate, the adverse loss development will dampen the current year's reported results.
- If inadequate reserves are set up for newly reported claims, the calendar year reported results will look better than they actually are.

Ultimate claim costs in the liability lines of business are difficult to predict, since they are influenced by numerous external factors, such as unexpected judicial decisions, new causes of action, and social developments affecting claims consciousness and jury awards. Moreover, insurance underwriting cycles may lead company managements to smooth reported earnings by alternately strengthening and weakening loss reserves.

Schedule P allows one to compare reported calendar year results with actual accident year results. Calendar year results are shown in the Underwriting and Investment Exhibits for net earned premiums on Part 2, column 4, and net losses incurred on Part 3, column 7. The calendar year loss ratio is shown in Part 3, column 8.

The corresponding calendar/accident year figures are shown in Schedule P, Part 1, columns 3 and 28. A triangle of accident year / exposure year loss ratios may be formed from the Part 2 and Part 6 triangles. As noted earlier, one should adjust the Part 6 triangles to a net of reinsurance basis.

RESERVE MARGIN CONTROVERSY

Reserve margins have long been a controversial topic among casualty actuaries. Some actuaries maintain that discretionary reserve margins are improper, since they hamper accurate analysis of reserve adequacy. Other actuaries believe that reserve margins are sometimes appropriate or unavoidable.

Illustration: A company may change its bulk reserves gradually from quarter to quarter, reducing the volatility in the actuarial reserve estimates. The smooth progression of liabilities over time may better reflect management's reserving philosophy than the actuary's statistical estimates. As long as the held reserves exceed the minimum reserves considered to be reasonable, management's actions are not necessarily improper.

Illustration: The statutory full-value reserves depend on the inflation rate between the accident date and the payment date of the losses. As the inflation rate changes, the full-value reserves should change as well, though the present value of the reserves does not necessarily change if the discount rate moves in tandem with the inflation rate. The company may use a long-term average inflation rate in its reserve estimates, and it may not revise the estimates with quarterly or yearly changes in the inflation rate.[139]

Changes in reserve margins may also stem from smoothing of calendar year results over underwriting cycles. The Schedule P, Part 2 triangles allow an analysis of reserve margin changes over calendar years. Casualty actuaries have used these triangles to examine three commonly held propositions about reserve margins.

- Companies tend to move together. Some years, many companies are strengthening reserves; in other years, many companies exhibit reserve weakening.
- Lines of business tend to move together. A company may seek to smooth overall operating results, not line of business results.
- Reserve margin changes tend to offset earnings volatility over the course of the underwriting cycle.

None of these propositions is universally accepted. Schedule P allows regulators and financial analysts to examine the industry's response to underwriting cycles, interest rate changes, and inflation rate changes.

[139] The risk-based capital formula uses a flat 5% discount rate for its investment income offset. Since it uses a fixed discount rate, it should use a fixed long-term average inflation rate for the losses well.

UNDER-RESERVING

Loss reserves may be inadequate for various reasons:

- Unforeseeable future developments cause mis-estimation of reserve indications. The surge in asbestos claims in 1999 and 2000 is an example of severe adverse development that was not expected by insurance industry actuaries or lawyers.
- Companies in financial distress may hide their weakness by reducing the bulk reserves for long-tailed lines of business.
- Companies writing long-tailed lines of business may value their reserves at a non-zero valuation rate.

This paper takes no position on the general adequacy of industry reserves. Several studies, however, have seen under-reserving as a contributing factor to many insurance insolvencies (Best's [1991]; AAA [1991], page 166; Hartman [1992]).

ACTUARIAL OPINION

Schedule P reflects the company's estimates of indicated reserves, which is not necessarily the same as the reserving actuary's estimate. To shift the onus of ensuring accurate reserve estimates, the NAIC requires that a "Statement of Actuarial Opinion Regarding Loss and Loss Adjustment Expense Reserves" accompany the Annual Statement.

The Statement of Actuarial Opinion is signed by a qualified actuary appointed by the company's Board of Directors. Each year, the actuary presents a report to the Board of Directors explaining the procedures used to arrive at the opinion and the conclusions embodied in it (NAIC Blanks Task Force, Attachment N of October 1991 meeting; Lamb [1991; 1992]; Witcraft [1992]). The American Academy of Actuaries Committee on Property-Liability Financial Reporting publishes a *Practice Note* each year providing guidance to actuaries in completing the Statement of Actuarial Opinion.

The Statement of Actuarial Opinion must comment on the reasonableness of the reserves for six items, three of which are taken from Schedule P (paragraph 8 of the NAIC *Instructions*):

A. Reserve for unpaid losses (page 3, line 1)
B. Reserve for unpaid loss adjustment expenses (page 3, line 3)
C. Reserve for unpaid losses – direct and assumed (Schedule P, Part 1, Summary exhibit, totals from columns 13 and 15)

D. Reserve for unpaid loss adjustment expenses – direct and assumed (Schedule P, Part 1, Summary exhibit, totals from columns 17, 19, and 21)
E. The extended loss and expense reserves (Schedule P Interrogatory 1)
F. The Page 3 write-in item reserve, "Retroactive reinsurance reserve ceded or assumed."

If the company writes certain types of property-casualty policies with durations of 13 months or longer, the actuary must also opine on the unearned premium reserves for these policies. This requirement relates to product warranty and mechanical breakdown policies; see SSAP No. 65, "Property and Casualty Contracts," paragraphs 21-31. The unearned premium reserves for these contracts depends on actuarial estimates of future losses and expenses.

Schedule P contains management's best estimate of the indicated reserves. The Statement of Actuarial Opinion does not contain the Appointed Actuary's estimate. Rather, it contains the Appointed Actuary's opinion whether management's estimate is reasonable.

> *Illustration:* Management's best estimate of the indicated reserves is $8 billion; this is the amount shown on page 3, line 1. The Appointed Actuary believes that the best estimate of the indicated reserves is $8.5 billion. However, the actuary considers the range of reasonableness to be $7.5 billion to $9.5 billion. The actuary would issue an unqualified opinion.[140]

SCOPE OF THE STATEMENT

Annual Statement *Instruction* 12, paragraph (11), describes the scope of the statement:

> *The actuary should comment in the scope section on each of the following topics, describing the effect of each on loss or loss expense reserves: retroactive reinsurance, financial reinsurance, and reinsurance collectibility, asbestos exposures and environmental exposures. The actuary should also comment on and describe the effects of any additional topics, such as discounting, salvage/subrogation, and underwriting pools and associations which in the actuary's judgment materially affect loss or loss expense reserves. If the company reserves will create exceptional values using the NAIC IRIS tests 9 (One Year Reserve Development to Surplus), 10 (Two Year Reserve Development to Surplus) and 11 (Estimated Current Reserve Deficiency to Surplus), the actuary should include an explanation.*

The following topics relate to the Schedule P entries:

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[140] The American Academy of Actuaries Committee on Property-Liability Financial Reporting defines the range of reasonable estimates as the "range of estimates that would be produced by alternative sets of assumptions that the actuary judges to be reasonable, considering all information reviewed by the actuary. . . The range of reasonable estimates is narrower, perhaps considerably, than the range of possible outcomes of the ultimate settlement value of the reserve."

Anticipated salvage and subrogation: Management's estimate of the anticipated salvage and subrogation is shown in column 23 of Part 1. The Appointed Actuary should comment on the reasonableness of this estimate if it affects reserve adequacy.

Discounts: Non-tabular loss reserve discounts are shown in Part 1, columns 32 and 33. Tabular discounts are disclosed in Note 28 to the Annual Statement. In the Statement of Actuarial Opinion, the actuary should comment on both tabular discounts, which may affect workers' compensation and accident & health loss reserves and non-tabular discounts, which might be used for all lines of business.

Pools and Associations: The ceded and assumed entries in Schedule P include amounts for voluntary and involuntary pools. Some of these ceded and assumed entries may be large, such as those for workers' compensation residual market pools in the late 1980's and early 1990's. The Appointed Actuary must comment whether the company uses the pool's estimates of required or booked reserves, or whether the company independently estimates the needed reserves.

Retroactive reinsurance denotes the transfer of financial obligation with the following three attributes:

- the losses have already occurred
- the primary company's surplus is increased and
- the consideration paid to the reinsurer is determined by present value techniques.[141]

Retroactive reinsurance is not reflected in the Schedule P exhibits, though it affects policyholders' surplus and statutory income (see Feldblum [2002: SchF]).

Financial reinsurance refers to arrangements in which the reinsurance company does not incur timing and underwriting risk; see SSAP No. 62, "Property and Casualty Reinsurance." A lack of timing risk or underwriting risk precludes a transaction from being considered reinsurance in statutory reports.

Reinsurance Collectibility: Part 1 of Schedule P shows both gross and net loss reserves, but it does not indicate the expected collectibility of reinsurance recoverables. Schedule F imposes statutory penalties for unauthorized and slow-paying reinsurers and for overdue reinsurance (Simon and Visner [1992]; Feldblum [2002]). The Statement of Actuarial Opinion

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[141] The NAIC *Instructions* to the Statement of Actuarial Opinion (section 11) provide this three-fold definition: "For the purpose of this instruction, "retroactive reinsurance" refers to any agreement which increases the transferring insurer's Surplus to Policyholders as a result of the transferee undertaking any loss obligation already incurred and for which the consideration paid by the transferring insurer is derived from present value or discounting concepts." See also SSAP No. 62, "Property and Casualty Reinsurance," paragraph 22.

should comment on any anticipated collection problems on reinsurance recoverables.[142] The actuary completing Schedule P should be familiar with the provision for reinsurance from Schedule F and with other information about reinsurance collectibility.

Paragraph 10 of the NAIC *Instructions* to the Statement of Actuarial Opinion require the opinion actuary to write that

> *In forming my opinion on the loss and loss adjustment expense reserves, I relied upon data prepared by the responsible officers or employees of the company or group to which it belongs. I evaluated that data for reasonableness and consistency. I also reconciled that data to Schedule P Part 1 of the company's current annual statement.*

The Practice Note issued by the Committee on Property-Liability Financial Reporting explains the reconciliation to Schedule P as follows:

A. *each of the following types of data, if relied on significantly in forming the actuarial opinion (on a net or direct plus assumed basis), were reconciled to Schedule P: paid losses, incurred (case basis) losses, paid defense and cost containment expenses, incurred (case basis) defense and cost containment expenses, paid adjusting and other expenses, and earned premiums;*

B. *the reconciliation consisted of comparing the changes from the prior year-end values (e.g., current calendar year paid losses and changes in case basis loss reserves), in detail by line of business and year in which losses were incurred to the extent that such detail was relied upon significantly and is provided in Schedule P; . . .*

The Appointed Actuary keeps work papers showing the reconciliation to Schedule P for seven years from the date of the opinion.

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[142] The NAIC *Instructions* to the Statement of Actuarial Opinion say: "Before commenting on reinsurance collectibility, the actuary should solicit information from management on any collectibility problems, review ratings given to reinsurers by a recognized rating service, and examine Schedule F for the current year for indications of regulatory action or reinsurance recoverable on paid losses over 90 days past due. The comment should also reflect any other information the actuary has received from management or which is publicly available about the capability or willingness of reinsurers to pay claims. The actuary's comments do not imply an opinion on the financial condition of any reinsurer."

Appendix A: Accounting for Audits and Retrospective Adjustments

The Schedule P, Part 6 exhibits may be used by the IRS to ensure that companies are complying with the January 2000 tax regulations regarding the recording of expected audit premiums and retrospective adjustments. Companies must book the estimated ultimate premiums on the policy effective date for tax purposes. An understanding of both statutory and tax accounting for audits and retrospective adjustments is essential for tax compliance. This appendix is background information on the accounting rules.

General Principles

A. Statutory accounting has two methods of recording written premium and computing the earned premium for policies with audits or retrospective adjustments, which we label "Method 1" and "Method 2" below.[143]
B. For statutory accounting purposes, companies may use either method. For tax purposes, companies must use Method 1 (the adjustment to written premium).
C. There are two financial statement reporting procedures for earned but unbilled premiums and for accrued retrospective premiums. The Annual Statement uses one procedure for the income statement and the other procedure for the balance sheet. Both sets of figures are supported by the Underwriting and Investment Exhibit, Parts 2, 2A, and 2B.
D. There are two methods for determining the non-admitted portion of the accrued retrospective premiums (SSAP No. 66, "Retrospectively Rated Contracts," paragraph 9).

A. ACCOUNTING METHODS

SSAP Number 53, "Property-casualty Contracts – Premiums," paragraph 9 says:

> *Adjustments to the premium charged for changes in the level of exposure to insurance risk (e.g., audit premiums on workers' compensation policies) are generally determined based upon audits conducted after the policy has expired. Reporting entities shall estimate audit premiums, the amount generally referred to as earned but unbilled (EBUB) premium, and shall record the amounts as an adjustment to premium, either through written premium or as an adjustment to earned premium. The estimate for EBUB may be determined using actuarially or statistically supported aggregate calculations using historical company unearned premium data, or per policy calculations.*

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[143] The methods are shown in SSAP Number 53, "Property-Casualty Contracts – Premiums," paragraphs 9-12, and SSAP Number 66, "Retrospectively Rated Contracts," paragraphs 6-8.

- Method 1 records the earned but unbilled premium through written premium.
- Method 2 records the earned but unbilled premium as an adjustment to earned premium.

Illustration: A workers' compensation policy with a written premium of $10,000 is issued on January 1, 20XX. On December 31, 20XX, the company's actuary anticipates that an additional $2,000 of premium will be billed at the final audit.

The estimated earned premium in 20XX is $12,000. The calendar year earned premium is calculated as the written premium minus the change in the unearned premium reserve. The additional $2,000 in earned premium must stem from either an additional $2,000 of written premium or a decrease of $2,000 in the unearned premium reserve.

Method 1: The $2,000 expected audit premium is coded as 20XX written premium, giving a total written premium of $12,000. All premium has been earned by December 31, and the unearned premium reserve at the end of the year is $0. The earned premium equals the written premium minus the change in the reserve, or $12,000 – ($0 – $0) = $12,000.

Method 2: The 20XX written premium remains $10,000. The $2,000 audit premium will be coded as a 20XX+1 written premium when it is billed, not as a 20XX written premium when it is estimated. The $2,000 earned but unbilled premium is treated as a contra-liability, or a negative unearned premium reserve. The traditional end-of-year unearned premium reserve resulting from the $10,000 deposit premium is $0. The net unearned premium reserve is $0 – $2,000 = –$2,000. The earned premium is the written premium minus the change in reserve, or $10,000 – (–$2,000 – $0) = $12,000.

Companies may use Method 1 for some policies and Method 2 for other policies. The two methods produce different written premiums and unearned premium reserves. The differences offset each other, and the earned premiums are the same for each method. The final Schedule P, Part 6 entries should not depend on the accounting method, though the means of computing the figures depends on the accounting method.

B. Statutory vs Tax Accounting

Method 2 defers some of the written premium until the audit is billed or the retrospective adjustment is processed. Taxes and assessments based on written premiums, such as state premium taxes and state assessments, are similarly deferred.

Method 1 shows a higher written premium than Method 2 and a correspondingly higher unearned premium reserve. Since only 80% of the unearned premium reserve is an offset to taxable income ("revenue offset"), Method 1 speeds up the income tax liability. Until January 2000, this was an additional incentive to use Method 2. The tax regulations of January 5, 2000 require companies to use Method 1 to compute the unearned premium reserve for federal income tax purposes; Method 2 is not acceptable tax accounting.[144] Nonetheless, Method 2 remains the more common method for statutory accounting.

The effects of the two methods of the federal income tax liability is illustrated below. Method 1 gives the higher tax liability, and it is now mandated by the IRS.

PREMIUM ACCOUNTING ILLUSTRATION

A workers' compensation policy with a written premium of $10,000 is issued on July 1, 20XX. On December 31, 20XX, the company's actuary anticipates that an additional $2,000 of premium will be billed at the final audit.

The estimated earned premium in 20XX is $6,000. The statutory earned premium is calculated as the written premium minus the change in the unearned premium reserve. The tax-basis earned premium is calculated as the written premium minus 80% of the change in the unearned premium reserve.

Method 1: The $2,000 expected audit premium is coded as 20XX written premium, giving a total written premium of $12,000. Half of the premium has been earned by December 31, and the unearned premium reserve at the end of the year is $6,000. The statutory earned premium equals the written premium minus the change in the reserve, or $12,000 – ($6,000 – $0) = $6,000. The tax-basis earned premium equals the written premium minus 80% of the change in the reserve, or $12,000 – 80% × ($6,000 – $0) = $7,200.

Method 2: The 20XX written premium remains $10,000. The earned but unbilled premium equals $1,000, since only 50% of the audit premium is earned. This $1,000 is treated as a negative unearned premium reserve. The traditional unearned premium reserve at the end of the year resulting from the $10,000 deposit premium is $5,000. The net unearned premium reserve is $5,000 – $1,000 = $4,000. The statutory earned premium is the written premium minus the change in reserve, or $10,000 – ($4,000 – $0) = $6,000. The tax-basis earned premium equals the written premium minus 80% of the change in the reserve, or $10,000 – 80% × ($4,000 – $0) = $6,800.

Taxable income is $400 greater in Method 1 than in Method 2. Method 2 is no longer permitted for tax accounting by the January 2000 tax regulations.

C. FINANCIAL STATEMENT REPORTING PROCEDURES

The statutory income statement shows earned premiums, for which there is no difference between Method 1 and Method 2. For Method 1, the accrued retrospective premiums are an

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[144] The January 2000 tax regulations were proposed in January 1997, but they were not put into final form until January 2000.

addition to written premiums. For Method 2, the accrued retrospective premiums are an offset to the unearned premium reserves.

Before the Tax Reform Act of 1986, both methods had the same effect on taxable income. The revenue offset provision in the 1986 Act unduly increased the tax on companies using Method 1, since no acquisition expenses had yet been paid on the anticipated audits.

Many companies use Method 2 to compute the premiums earned from audits. Similarly, the statutory income statement computes the earned premium from an unearned premium reserve that is *net* of earned but unbilled premiums and accrued retrospective premiums.

The statutory balance sheet shows the unearned premium reserve *gross* of earned but unbilled premiums and accrued retrospective premiums, and it shows separate assets for earned but unbilled premiums and accrued retrospective premiums. This is true regardless of the method used to calculate the earned premiums. The rationale is to provide additional disclosure and to facilitate the computation of the non-admitted portion of the earned but unbilled premium and accrued retrospective premium assets.

ILLUSTRATION – INCOME STATEMENT AND BALANCE SHEET

We use the same scenario as in a previous illustration. A workers' compensation policy with a written premium of $10,000 is issued on January 1, 20XX. On December 31, 20XX, the company anticipates that an additional $2,000 of premium will be billed at the final audit.

The statutory income statement uses Method 2. The unearned premium reserve is –$2,000, the written premium is $10,000, and the earned premium is $12,000.

The statutory balance sheet shows the earned but unbilled premiums and the accrued retrospective premiums as separate assets, so that the non-admitted portion may be deducted. The gross asset is added back to the net unearned premium reserve for the balance sheet liability. The entries shown for this illustration are as follows:

- Earned but unbilled premiums, gross (page 2, line 10.2, column 1): $2,000
- Earned but unbilled premiums, non-admitted (page 2, line 10.2, column 2): $200
- Earned but unbilled premiums, net (page 2, line 10.2, column 3): $1,800
- Unearned premium reserves (page 3, line 10): $0

D. NON-ADMITTED ASSET

There are two methods of computing the non-admitted portion of the accrued retrospective premium asset:[145]

1. Ten percent of the unsecured accrued retrospective premium asset is not admitted.
2. The non-admitted portion varies by policy, depending on the credit rating of the insured.

Companies must use the same method for all policies. A company may not use the second method for insureds with high credit ratings and the first method for insureds with low credit ratings.

Schedule P, Part 6 uses the gross accrued retrospective premiums, not the net admitted amounts.

Accrued Retrospective Premium Reserves

For tax purposes, companies must establish reserves for audit premiums and accrued retrospective premiums; they generally show the reserves on their statutory statements as well. The Annual Statement has three terms for such premium reserves.

- *Earned but unbilled* (EBUB) premiums are primarily audit premiums for past exposures that have not yet been billed by the insurer.[146] They are shown (in total) on the balance sheet, page 2, line 10.2, and by line of business in the Underwriting and Investment Exhibit, Part 2A, "Recapitulation of All Premiums," page 8, column 3.

- *Accrued retrospective premiums based on experience* (ARP's) are the net additional premiums expected from future retrospective adjustments on retrospectively rated contracts (see SSAP, Number 66, "Retrospectively Rated Contracts"). Net additional premiums means expected future additional premiums minus expected future return premiums. They are shown in total on the balance sheet, page 2, line 10.3, and in the Underwriting and Investment Exhibit, Part 2A, "Recapitulation of All Premiums," page 8, line 35, column 5.

- *Reserve for rate credits and retrospective premiums based on experience* is the accrued retrospective premiums plus rate credits given on group accident and health insurance.

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[145] The gross accrued retrospective premium asset is shown on page 2, column 1, line 10.3. The non-admitted portion is shown in column 2 of line 10.3, and the net admitted portion is shown in column 3. The statutory surplus shown on the liability side of the balance sheet is based on the net asset. The change from the previous year to the current year in the non-admitted portion of the accrued retrospective premium asset is a direct charge to surplus on page 4, line 25.

[146] SSAP Number 53, "Property-Casualty Contracts – Premiums," paragraph 9, says that *reporting entities shall estimate audit premiums, the amount generally referred to as earned but unbilled (EBUB) premium.*

It is shown by line of business in the Underwriting and Investment Exhibit, Part 2A, "Recapitulation of All Premiums," page 8, column 4.[147]

Some actuaries use the term *earned but not reported* (EBNR) premiums, based on the acronym for incurred but not reported (IBNR) losses.

If the company expects to return premium to the insured at the retrospective adjustment, the premium reserve is a liability. If the company expects to collect additional premium from the insured at the retrospective adjustment, the premium reserve is an asset. Generally, expected future premium collections exceed expected premium returns. The premium reserve is used here to refer to the net asset; this is the statutory usage in the Annual Statement.

Statutory Accounting Principles

The statutory accounting principles are as follows:

If accounting method 1 is used for earned but unbilled premiums (see above), the earned but unbilled premium affects the written premium and the unearned premium reserves. The earned but unbilled premiums are included in columns 1 and 2 of the Underwriting and Investment Exhibit, Part 2A, "Recapitulation of All Premiums," page 8. They are not included in column 3, "earned by unbilled premium."

If accounting method 2 is used for earned but unbilled premiums (see above), the earned but unbilled premium do affect the written premium or the unearned premium reserves. The earned but unbilled premiums are included in column 3 of the Underwriting and Investment Exhibit, Part 2A, "Recapitulation of All Premiums," page 8. These entries are negative amounts; they offset the unearned premium reserves shown in columns 1 and 2.

The reserve for rate credits or retrospective adjustments based on experience are negative amounts showing the net accrued retrospective premiums and the accident and health insurance rate credits in column 4 of the Underwriting and Investment Exhibit, Part 2A, "Recapitulation of All Premiums," page 8. These entries are also negative amounts; they offset the unearned premium reserves shown in columns 1 and 2.

The net unearned premium reserves shown in column 5 of the Underwriting and Investment Exhibit, Part 2A, are the sum of columns 1 through 4. These adjusted unearned premium reserves are used to calculate the earned premiums in Part 2 of the Underwriting and Investment Exhibit ("Premiums Earned" on page 7). The total earned premiums for all lines of business combined is carried to line 1 of the statutory income statement (page 4).

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[147] Not all companies agree on the definitions of these terms, and this paper makes no attempt to clarify the differences of opinion.

The "accrued retrospective premium based on experience" for all lines of business combined is removed from the unearned premium reserve on line 35 of Part 2A of the Underwriting and Investment Exhibit, and the net amount (the "balance") is shown on line 37. Since the "accrued retrospective premium" is a contra-liability, though it shown as a positive figure in the Annual Statement, line 37 should equal line 34 plus line 35. [In contrast, the "reserves for rate credits or retrospective adjustments based on experience" shown in column 4 of Part 2A are shown as negative figures when they are contra-liabilities.]

The line 37 unearned premium reserve is carried to the liability side of the balance sheet, page 3, line 9: "unearned premium reserves." The accrued retrospective premiums on line 35 of Part 2A are carried to the asset side of the balance sheet, page 2, column 1, line 10.3. The non-admitted portion is deducted in column 2, and the net admitted portion is shown in column 3.[148]

[148] For most other items, the incurred amount on the income statement equals the paid or received amount on the cash flow statement plus or minus the change in reserves on the balance sheet. For premiums, this relationship does not hold, since there are different treatments of accrued retrospective premiums in the income statement and on the balance sheet.

Appendix B: Revenue Offset

For other industries, sales constitute revenues for income tax purposes. Similarly, premium due is the taxable revenue (as well as the statutory and GAAP revenue) for life insurance companies. For property-casualty insurance companies, earned premium is the revenue for both statutory and taxable income, not written premium or collected premium.

For the statutory income statement, earned premium equals written premium minus the change in the unearned premium reserves. For taxable income, earned premium equals written premium minus 80% of the change in the unearned premium reserves.[149] [150]

- A change in written premium with no change in earned premium does not affect statutory income, whereas
- A change in written premium with no change in earned premium affects the unearned premium reserve and changes the tax liability by means of the revenue offset provision.

Statutory and taxable income also differ in their treatments of accrued retrospective premiums (see Appendix A).

Illustration: Single Policy

An insurer writes a policy with a $10,000 written premium on December 31, 20XX, and it pays $2,000 in agents' commissions on that day. Losses of $8,000 are incurred and paid evenly through the policy term. There are no other expenses or losses on this policy. We assume that losses are paid when they are incurred so that we need not deal with IRS loss reserve discounting.

The unearned premium reserve for this policy is $0 on January 1, 20XX, and $10,000 on December 31, 20XX. The change in the unearned premium reserve during the year is $10,000. The earned premium in 20XX is $10,000 of written premium minus the $10,000

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[149] See the Treasury regulations, 2001 FED 26,153, §1.832-4(a)(3): "The determination of premiums earned on insurance contracts during the taxable year begins with the insurance company's gross premiums written on insurance contracts during the taxable year, reduced by return premiums and premiums paid for reinsurance. This amount is increased by 80 percent of the unearned premiums on insurance contracts at the end of the preceding taxable year, and is decreased by 80 percent of the unearned premiums on insurance contracts at the end of the current taxable year."

[150] Life insurance companies and annuity writers are subject to a DAC-tax that is identical in concept though more complex than the property-casualty tax provision explained here; see Atkinson and Dallas [2000], chapter 9.

change in the unearned premium reserve, or \$0. Expenses during 20XX are \$2,000, and statutory income during 20XX is –\$2,000. Without revenue offset, the federal income tax liability would be 35% × –\$2,000 = –\$700, or a \$700 tax refund.

The unearned premium reserve on December 31, 20XX+1, is \$0. The change in the unearned premium reserve during 20XX+1 is –\$10,000. The earned premium in 20XX+1 is \$0 of written premium minus the –\$10,000 change in the unearned premium reserve, or \$0 – (–\$10,000) = +\$10,000. Losses of \$8,000 are incurred and paid in 20XX+1. The statutory income is \$10,000 – \$8,000 = \$2,000. The tax liability (ignoring revenue offset) would be 35% × \$2,000 = \$700.

Statutory accounting recognizes a loss at policy inception and a gradual profit during the remainder of the policy lifetime, thereby preventing companies from recognizing income until it has been fully earned.[151]

Were there no revenue offset provision in the tax code, the U.S. Treasury would fund part of the initial underwriting loss at policy inception. The illustration above shows a tax refund of \$700 in 20XX and a tax liability of \$700 in 20XX+1. Before 1987, statutory accounting helped the insurance industry defer its tax liabilities. Steady growth (in nominal dollar terms) led to persistent deferral of tax liabilities.

Direct and Indirect Methods

The Tax Reform Act of 1986 introduced the revenue offset provision of the Internal Revenue Code. The provision may be stated in two equivalent ways. These two perspectives are used in the two fashions of computing taxable income and the federal income tax liability, which are termed here the "direct method" and the "indirect method." The direct method is easier to understand; the indirect method is the method actually used in the Internal Revenue Code for computing taxable income.

1. *Direct method:* The taxable earned premium equals the taxable written premium minus 80% of the change in the unearned premium reserve. This may be stated as "only 80% of the change in the unearned premium reserve is an offset to taxable income."

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[151] Some analysts see a conservative bend in statutory accounting's write-off of pre-paid acquisition costs when they are incurred, particularly in comparison with GAAP's capitalization and amortization of the deferred policy acquisition cost asset. This is not quite correct. Statutory accounting is correct accounting from a tangible asset perspective, since the pre-paid acquisition costs are often incurred whether or not the company retains the policy. International accounting standards follow statutory accounting on this issue. GAAP capitalizes an "imaginary" asset called DPAC to match revenues and expenses and show a better portrayal of the company's profitability. However, statutory accounting is unduly conservative in its double treatment of underwriting expenses: once when they are incurred and a second time in the gross unearned premium reserves. See Yoheved and Sarason [2002] for further discussion of GAAP and statutory accounting of property-casualty insurance companies.

2. *Indirect method:* Twenty percent of the change in the unearned premium reserve is an *addition* to statutory income for computing taxable income.

We can use either method for the illustration.

Direct method: The taxable earned premium in 20XX equals the taxable written premium minus 80% of the change in the unearned premium reserve, or $10,000 – 80% × ($10,000 – $0) = $2,000 in 20XX. Agents' commissions are $2,000 on December 31, 20XX. Taxable income is $2,000 – $2,000 = $0, and the tax liability is $0.

In 20XX+1, the taxable earned premium equals $0 – 80% × ($0 – $10,000) = $8,000. The losses incurred and paid in 20XX+1 are $8,000. The taxable income is $8,000 – $8,000 = $0, and the tax liability is $0.

Indirect method: Twenty percent of the change in the unearned premium reserve in 20XX is 20% × ($10,000 – $0) = $2,000. The statutory income in 20XX is –$2,000. Taxable income is –$2,000 + $2,000 = $0, and the tax liability is $0.

In 20XX+1, twenty percent of the change in the unearned premium reserve is 20% × ($0 – $10,000) = –$2,000. The statutory income in 20XX+1 is +$2,000. The taxable income is +2,000 – $2,000 = $0, and the tax liability is $0.

Illustration B: Two Years

An insurer writes a policy with a $10,000 written premium on July 1, 20XX, and it pays $2,000 in agents' commissions on that day. Losses of $8,000 are incurred evenly over the policy term, and they are paid when they are incurred. On July 1, 20XX+1, the insurer renews the policy for a written premium of $15,000, and it pays $3,000 in agents' commissions on that day. Losses of $12,000 are incurred evenly over the policy term, and they are paid when they are incurred. There are no other expenses on these policies.

Illustration B shows the importance of computing the *change* in the unearned premium reserve during the year. The statutory unearned premium reserve equals $0 on December 31, 20XX–1, $5,000 on December 31, 20XX, $7,500 on December 31, 20XX+1, and $0 on December 31, 20XX+2.

Calendar Year 20XX

Statutory earned premium is $10,000 written premium minus the ($5,000 – $0) = $5,000 change in the unearned premium reserve; the earned premium is $5,000. Expenses are $2,000, and incurred losses are $4,000. The statutory income in 20XX is $5,000 – $2,000 – $4,000 = –$1,000. There are two methods to calculate the taxable income.

a. *Direct method:* The taxable earned premium is taxable written premium minus 80% of the change in the unearned premium reserve, or \$10,000 – 80% × (\$5,000 – \$0) = \$6,000. The taxable income is \$6,000 – \$2,000 – \$4,000 = \$0, and the tax liability is \$0.
b. *Indirect method:* Twenty percent of the change in the unearned premium reserve is 20% × (\$5,000 – \$0) = \$1,000. The statutory income in 20XX is –\$1,000. The taxable income is –\$1,000 + \$1,000 = \$0, and the tax liability is \$0.

CALENDAR YEAR 20XX+1

Statutory earned premium is \$15,000 written premium minus the (\$7,500 – \$5,000) = \$2,500 change in the unearned premium reserve; the earned premium is \$12,500. Expenses incurred and paid on January 1, 20XX+1 are \$3,000, and incurred losses during the year are \$4,000 (first six months) + \$6,000 (latter six months) = \$10,000. The statutory income is \$12,500 – \$3,000 – \$10,000 = –\$500. There are two methods to calculate taxable income.

a. *Direct method:* The taxable earned premium is the taxable written premium minus 80% of the change in the unearned premium reserve, or \$15,000 – 80% × (\$7,500 – \$5,000) = \$13,000. Expenses and losses are the same as for statutory income. The taxable income is \$13,000 – \$3,000 – \$10,000 = \$0, and the tax liability is \$0.
b. *Indirect method:* Twenty percent of the change in the unearned premium reserve is 20% × (\$7,500 – \$5,000) = \$500. The statutory income in 20XX+1 is –\$500. The taxable income is –\$500 + \$500 = \$0, and the tax liability is \$0.

CALENDAR YEAR 20XX+2

Statutory earned premium is \$0 written premium minus the (\$0 – \$7,500) = –\$7,500 change in the unearned premium reserve, or \$7,500. Expenses incurred in 20XX+2 are \$0, and incurred losses during the year are \$6,000. Statutory income is \$7,500 – \$6,000 = \$1,500.

There are two methods to calculate the taxable income.

a. *Direct method:* The taxable earned premium is \$0 – 80% × (\$0 – \$7,500) = \$6,000. The taxable income is \$6,000 – \$6,000 = \$0, and the tax liability is \$0.
b. *Indirect method:* Twenty percent of the change in the unearned premium reserve is 20% × (\$0 – \$7,500) = –\$1,500. The statutory income in 20XX+2 is \$1,500. The taxable income is \$1,500 + –\$500 = \$0, and the tax liability is \$0.

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Exhibit A.1: Private Passenger Automobile Loss Reserve Discount Factors

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio (4)	Incremental Paid/Incurred Ratio (5)	Undiscounted Percentage Unpaid (6)	Discounted Percentage Unpaid (7)	Loss Reserve Discount Factor (8)
AY + 15							
AY + 14							
AY + 13							
AY + 12							
AY + 11							
AY + 10				2.00%	100.00%	0.00%	
2000	$270,000	$275,500	98.00%	3.07%	2.00%	1.93%	96.6735%
2001	$300,000	$316,000	94.94%	2.98%	5.06%	4.77%	94.1800%
2002	$320,000	$348,000	91.95%	3.99%	8.05%	7.34%	91.2271%
2003	$340,000	$386,500	87.97%	4.93%	12.03%	10.71%	89.0399%
2004	$350,000	$421,500	83.04%	6.03%	16.96%	14.78%	87.1281%
2005	$370,000	$480,500	77.00%	7.98%	23.00%	19.65%	85.4281%
2006	$380,000	$550,500	69.03%	10.01%	30.97%	26.07%	84.1740%
2007	$360,000	$610,000	59.02%	11.02%	40.98%	34.04%	83.0660%
2008	$330,000	$687,500	48.00%	13.00%	52.00%	42.47%	81.6659%
2009	$200,000	$571,500	35.00%	35.00%	65.00%	52.26%	80.3944%

Exhibit A.2: Other Liability Loss Reserve Discount Factors

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio (4)	Incremental Paid/Incurred Ratio (5)	Undiscounted Percentage Unpaid (6)	Discounted Percentage Unpaid (7)	Loss Reserve Discount Factor (8)
AY + 15			100.00%	3.01%	0.00%	0.00%	
AY + 14			96.99%	1.38%	3.01%	2.91%	96.6736%
AY + 13			95.61%	1.38%	4.39%	4.05%	92.3385%
AY + 12			94.23%	1.38%	5.77%	5.12%	88.7803%
AY + 11			92.85%	1.38%	7.15%	6.12%	85.6177%
AY + 10	$235,000	$250,000	91.47%	1.38%	8.53%	7.06%	82.7122%
2000	$50,000	$55,500	90.09%	1.38%	9.91%	7.93%	79.9988%
2001	$55,000	$62,000	88.71%	3.00%	11.29%	8.74%	77.4439%
2002	$60,000	$70,000	85.71%	4.46%	14.29%	11.07%	77.4718%
2003	$65,000	$80,000	81.25%	8.33%	18.75%	14.66%	78.1822%
2004	$70,000	$96,000	72.92%	9.81%	27.08%	21.76%	80.3309%
2005	$65,000	$103,000	63.11%	10.93%	36.89%	29.82%	80.8185%
2006	$60,000	$115,000	52.17%	12.17%	47.83%	38.44%	80.3644%
2007	$50,000	$125,000	40.00%	15.00%	60.00%	47.69%	79.4828%
2008	$35,000	$140,000	25.00%	16.67%	75.00%	59.07%	78.7611%
2009	$15,000	$180,000	8.33%	8.33%	91.67%	71.32%	77.8022%

Exhibit A.3: Other Liability Loss Reserve Discount Factors

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio (4)	Incremental Paid/Incurred Ratio (5)	Undiscounted Percentage Unpaid (6)	Discounted Percentage Unpaid (7)	Loss Reserve Discount Factor (8)
AY + 15			100.00%	3.01%	0.00%	0.00%	
AY + 14			96.99%	1.38%	3.01%	2.91%	96.6736%
AY + 13			95.61%	1.38%	4.39%	4.05%	92.3385%
AY + 12			94.23%	1.38%	5.77%	5.12%	88.7803%
AY + 11			92.85%	1.38%	7.15%	6.12%	85.6177%
AY + 10	$235,000	$250,000	91.47%	1.38%	8.53%	7.06%	82.7122%
2000	$50,000	$55,500	90.09%	1.38%	9.91%	7.93%	79.9988%
2001	$55,000	$62,000	88.71%	-8.43%	11.29%	8.74%	77.4439%
2002	$68,000	$70,000	97.14%	15.89%	2.86%	0.02%	0.6645%
2003	$65,000	$80,000	81.25%	8.33%	18.75%	15.38%	82.0371%
2004	$70,000	$96,000	72.92%	9.81%	27.08%	22.43%	82.8251%
2005	$65,000	$103,000	63.11%	10.93%	36.89%	30.45%	82.5297%
2006	$60,000	$115,000	52.17%	12.17%	47.83%	39.03%	81.5980%
2007	$50,000	$125,000	40.00%	15.00%	60.00%	48.24%	80.4018%
2008	$35,000	$140,000	25.00%	16.67%	75.00%	59.59%	79.4482%
2009	$15,000	$180,000	8.33%	8.33%	91.67%	71.80%	78.3276%

Exhibit A.4: Other Liability Loss Reserve Discount Factors

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio (4)	Incremental Paid/Incurred Ratio (5)	Undiscounted Percentage Unpaid (6)	Discounted Percentage Unpaid (7)	Loss Reserve Discount Factor (8)
AY + 15			100.00%	3.01%	0.00%	0.00%	
AY + 14			96.99%	1.38%	3.01%	2.91%	96.6736%
AY + 13			95.61%	1.38%	4.39%	4.05%	92.3385%
AY + 12			94.23%	1.38%	5.77%	5.12%	88.7803%
AY + 11			92.85%	1.38%	7.15%	6.12%	85.6177%
AY + 10	$235,000	$250,000	91.47%	1.38%	8.53%	7.06%	82.7122%
2000	$50,000	$55,500	90.09%	1.38%	9.91%	7.93%	79.9988%
2001	$55,000	$62,000	88.71%	-9.86%	11.29%	8.74%	77.4439%
2002	$69,000	$70,000	98.57%	17.32%	1.43%	-1.36%	-95.3447%
2003	$65,000	$80,000	81.25%	8.33%	18.75%	15.47%	82.5189%
2004	$70,000	$96,000	72.92%	9.81%	27.08%	22.52%	83.1368%
2005	$65,000	$103,000	63.11%	10.93%	36.89%	30.53%	82.7436%
2006	$60,000	$115,000	52.17%	12.17%	47.83%	39.10%	81.7523%
2007	$50,000	$125,000	40.00%	15.00%	60.00%	48.31%	80.5167%
2008	$35,000	$140,000	25.00%	16.67%	75.00%	59.65%	79.5341%
2009	$15,000	$180,000	8.33%	8.33%	91.67%	71.86%	78.3932%

The Minimum Bias Procedure— A Practitioner's Guide

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The Minimum Bias Procedure

A Practitioner's Guide

prepared by

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The Minimum Bias Procedure

This *Practitioner's Guide* is geared to the practicing actuary who would like to optimize classification relativities. It provides the intuition underlying the minimum bias procedure and the alternative methods that have been proposed subsequently. It uses a simple illustration to show the computations required for each method, and to evaluate their advantages and drawbacks.

All the procedures discussed here can be easily coded in modern spreadsheets using the built-in functions provided by vendors. Practicing actuaries should be able to quickly implement minimum bias procedures, and the intuition here should enable students to readily master the methods.

BACKGROUND

The minimum bias procedure was first introduced in a 1960 *Proceedings* paper by Robert Bailey and LeRoy Simon, "Two Studies in Automobile Insurance." Bailey and Simon examined models with two types of arithmetic functions (multiplicative and additive), two types of bias functions (balance principle and χ-squared), and two data types (loss costs and loss ratios).[1]

Bailey and Simon used their procedure (i) to judge the merits of an additive versus a multiplicative classification model for Canadian private passenger automobile business and (ii) to choose optimal rate relativities.[2] The 1960 Bailey and Simon paper discusses the rationale for the minimum bias procedure, the characteristics of a suitable rating model, and the rating scenarios that fit the various types of models.

The 1960 Bailey and Simon paper concluded that: (i) the additive model fits the Canada private passenger automobile data better than the multiplicative model, and (ii) the χ-squared function is the optimal bias function. The first conclusion was based on a goodness-of-fit test; the second conclusion was based on the credibility assigned by the χ-squared function.

In a 1963 *Proceedings* paper, "Insurance Rates with Minimum Bias," Robert Bailey summarized the minimum bias theory, outlining the considerations that support the use of the balance principle as the bias function and explaining when loss ratios serve better than loss

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[1] References to the *Proceedings* are to the *Proceedings of the Casualty Actuarial Society.*

[2] The minimum bias procedure deals with loss cost relativities, which we refer to here as classification relativities. In practices, actuaries determine rate relativities. The two types of relativities may differ if expenses are not a fixed percentage of premiums. These issues are discussed in a subsequent section of this *Guide.*

costs. Bailey's 1963 *Proceedings* paper was on the CAS examination syllabus for many years, serving as a teaching text for a generation of actuaries.

In a 1988 *Proceedings* paper, "Minimum Bias with Generalized Linear Models", Robert Brown expanded the minimum bias method to use two additional types of bias functions. Brown retained the balance principle and χ-squared functions from the Bailey and Simon papers. He added a least squares function, which is similar to the χ-squared function, and a maximum likelihood function, which assumes certain distributions of claim frequency or claim severity in the insured population.

Brown also examined generalized linear models (GLM), which have potential statistical advantages and may accomplish the same objectives as the minimum bias procedures. He did not find that the generalized linear models produced more accurate results. For the Canadian private passenger automobile business, Brown found the multiplicative model superior to the additive model.

In 1990, Gary Venter published a thorough discussion of Brown's paper, in which he introduced several extensions of the existing procedures, such as a combined additive and multiplicative model, along with an analysis of credibility consideration and other modeling issues. Brown's *Proceedings* paper, along with Venter's discussion, replaced the 1963 Bailey paper on the actuarial syllabus in the mid-1990's.

Bailey's 1963 paper, Brown's paper, and Venter's discussion have proved difficult for practicing actuaries to understand and for actuarial candidates to master. These authors wrote for experienced actuaries who were familiar with the ratemaking issues and proficient with the mathematical methods.

Dr J. Eric Brosius and Sholom Feldblum have taught the minimum bias procedure to several hundred actuarial candidates since 1995. They have developed heuristic illustrations and intuitive explanations that clarify the theory.

This paper is a practitioner's guide to the minimum bias procedure. It combines the theory of the original actuarial papers with the teaching material prepared by Brosius and Feldblum. It explains the rationale for the procedure and it shows its applications. It teaches the method to new actuaries and it gives them the background to read the original *Proceedings* papers.

The title of this paper is the "Minimum Bias Procedure," since that name is now common in the U.S. actuarial profession. The subject of this paper should more properly be described as the development of multi-dimensional classification systems. This subject is broad. The paper covers part of this subject, of which one component is the minimum bias procedure and the alternative methods discussed here.

This *Practitioner's Guide* does not cover generalized linear models. Generalized linear models are commonly used in the United Kingdom and in continental Europe for multi-dimensional classification ratemaking. We treat generalized linear models in a companion *Practitioner's Guide.*

THE PRACTITIONER'S GUIDE

Practicing actuaries are unique professionals. Their goal is to manage business endeavors, not simply to provide statistical advice. Yet their expertise rests on a large body of theoretical knowledge, not just on experience. The role of this *Practitioner's Guide* is to transform theoretical knowledge into practical business situations.

The pure actuary concentrates on fundamental theory, confident that sound theory will find its way into multiple applications. Many actuarial papers are written from this perspective.

The practicing actuary, in contrast, begins with the business problem and works backward to find theoretical solutions. Similarly, this Practitioner's Guide begins with the business need for multi-dimensional classification relativity systems. It unveils the intuition underlying the statistical methods and examines their suitability for the business scenarios.

THE WORLD BEFORE BAILEY AND SIMON

Before Bailey and Simon introduced the minimum bias procedure, classification relativities were determined one dimension at a time. The appendices to Charles McClenahan's "Ratemaking" chapter and Robert Finger's "Risk Classification" chapter in the CAS *Foundations of Casualty Actuarial Science* textbook illustrate the procedure. This remains the dominant classification ratemaking system for many lines of business in the United States. In other countries, such as Great Britain, actuaries have made more use of generalized linear models (GLM) to develop classification relativities; see the companion paper on GLM.

If a line of business has a one-dimensional classification system, the minimum bias procedure adds nothing to the traditional calculation. Workers' compensation, for example, uses industry as the only dimension in the classification system. Insurers are now examining other classification dimensions for workers' compensation; the minimum bias procedure and generalized linear models may prove valuable in this analysis.

The minimum bias procedure is useful when the classification system has two or more dimensions. Throughout this paper, we use examples of two dimensions. The extension to three or more dimensions is straight-forward, but the arithmetic becomes cumbersome and it is more difficult to format the arrays on a two-dimensional page. These problems are eliminated by spreadsheet implementations of the procedure.

There are several reasons for using a procedure which looks at all dimensions of the classification system simultaneously; we provide the intuition further below. The primary statistical issue relates to the optimal bias function. The 1960 Bailey and Simon paper emphasizes the credibility argument, from which the authors concluded that the χ-squared function is the optimal bias function. The 1963 Bailey paper emphasizes the bias argument, from which Bailey inferred that the balance principle is the optimal bias function. Some statisticians argue that the maximum likelihood function is inherently superior to the other bias functions; this method is discussed in Brown's paper.

We define the terms, explain the statistical procedures, and review the intuition underlying each method further below. It is hard for some readers to grasp the intuition until they have a working knowledge of the methods. We provide the explanations alongside a series of heuristic illustrations.

We are setting pure premiums. We do not deal with expenses by classification or with gross premiums.[3] We base the pure premiums upon the empirical observations in each cell of an

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[3] For expense loadings by classification and development of gross premiums in competitive markets, see S. Feldblum [1996: PAP].

array. For a two dimensional classification system, this means each cell in a matrix. The observations can be average loss costs (i.e., pure premiums), loss frequencies, or loss ratios. In practice, the data would consist of losses and exposures (for loss costs), claim counts and exposures (for loss frequencies), or losses and premiums (for loss ratios).

ILLUSTRATIONS

A series of illustrations forms the backbone of this *Practitioner's Guide*. The basic illustration has two dimensions with two values in each dimension. This prevents the intuition from getting submerged under tedious mathematics. The illustrations are constructed so that they are not conceptually different from real scenarios. For practical work, the minimum bias procedure is most important for multi-dimensional classification systems that have multiple entries in each dimension.

Most of the illustrations show only one iteration. (The meaning of an "iteration" is provided below.) In practice, multiple iterations are needed for convergence, since the procedures do not have closed form solutions. The work would be tedious were it done by hand. With current spreadsheet applications, the work is elementary. Some spreadsheets have built-in iterative functions, such as "goal-seek" and "solver" in Excel. Some software packages, such as SAS, have built-in routines for generalized linear models. Once the intuition is clear, the required programming is not difficult.

Illustration: A classification system for private passenger automobile insurance has two dimensions: (i) urban vs rural and (ii) male vs female. A company insures exactly four drivers, one in each cell, with the following observed loss costs:[4]

	Urban	Rural
Male	$800	$400
Female	$400	$200

We seek to determine pure premium relativities.[5] We compare all males with all females, or $1,200 for two exposures compared to $600 for two exposures. This gives a pure premium relativity of

Male to Female = 2 to 1

We do the same for urban versus rural, and we get the same relativity. We choose "rural female" as the base class, and we get the following set of relativities:

Male:	2.00	=	s_1
Female:	1.00	=	s_2
Urban:	2.00	=	t_1
Rural:	1.00	=	t_2

An urban male driver has an indicated pure premium of the base rate times the urban relativity times the male relativity, or $200 × 2.00 × 2.00 = $800. More generally, the pure premium in cell (i,j) is $\$200 \times s_i \times t_j$.

4 We defer for the moment our discussion of credibility. If the observed loss costs were fully credible – that is, if they were fully accurate and unbiased estimators of the expected pure premiums within each cell of the matrix – we would use the observed loss costs as the pure premiums, and we could dispense with the classification ratemaking problem.

5 We sometimes refer to the pure premium relativities as rate relativities, since this is the more common actuarial term. For the conversion of pure premium relativities into rate relativities, see Feldblum [1996: PAP, pages 231-237].

In this illustration, the indicated pure premiums match the observed loss costs. The minimum bias method is not needed for this case.[6]

Assumptions

Two assumptions underlie this analysis.

1. Instead of using the observed loss costs as the indicated pure premiums, we convert them into a system of classification relativities. There are both practical and theoretical reasons for using classification relativities.

   a. *Practical:* In many lines of business, there are several classification dimensions with numerous classes in each. Using separate rates for each cell of the array is unwieldy.
   b. *Theoretical:* Using relativities improves the accuracy of the rate indications, since we use all the information regarding each cell's expected pure premium. This reason comes under the general rubric of credibility considerations.

   The practical reason (reason "a") was once compelling, though the development of on-line premium quoting system has reduced its importance. If a cell has low volume, credibility considerations justify basing future rates more heavily on classification relativities than on observed data in that cell. We discuss this further below.

2. We assumed that the relativities model is multiplicative. A multiplicative model means that the relativity for a given cell is the product of the relativities in its row and column. Robert Finger [1976] justifies this assumption (in another context) by stating that several independent factors interact multiplicatively to determine the liability claim size. For automobile insurance, these factors include

   - the speed of the vehicles before impact
   - the health of the injured party
   - the protection (e.g., with seat belts, interior padding) of the victim
   - the income of the victim
   - the skill of the plaintiff's attorney, and
   - the skill of the defendant's claims adjusters.

   This reasoning would be more persuasive if these factors were also the classification dimensions. They are not; instead, the classification dimensions are driver characteristics, garaging territory, and use of the vehicle.

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[6] The McClenahan and Finger chapters of the *Foundations of Casualty Actuarial Science* textbook uses this procedure to determine rate relativities.

The multiplicative model implicitly assumes that the classification dimensions are independent. It is less appropriate when classification dimensions are correlated. The pricing actuary must determine – both empirically and logically – what model is best. The minimum bias procedure aids this determination.

THE ADDITIVE MODEL

The indicated pure premiums may differ from the observed loss costs for two reasons:

- The model structure may be incorrect.
- Random loss fluctuations influence the observed loss costs.

We treat the first reason, the model structure, in this section. We are determining pure premium relativities, but the observed loss costs are as shown below.

	Urban	Rural
Male	\$700	\$500
Female	\$400	\$200

We begin in the same fashion as we did before. We compare all males to all females, giving a pure premium relativity of \$1,200 to \$600, or 2 to 1. We compare all urban to all rural, giving a pure premium relativity of \$1,100 to \$700, or 1.571 to 1. We choose rural females as the base class.

The indicated pure premium relativities no longer match the observed loss costs. The indicated pure premium for rural males is \$200 × 2.000 = \$400, but the observed loss costs are \$500. The indicated pure premium for urban females is \$200 × 1.571 = \$314, but the observed loss costs are \$400. The differences are significant.

No multiplicative factors work perfectly. In urban territories, the relationship of male to female is \$700 to \$400, or 1.75 to 1. In rural territories, the relationship of male to female is \$500 to \$200, or 2.50 to 1. A male to female relativity appropriate for the urban territories is not optimal for the rural territories.

Similarly, for male drivers, the urban to rural relativity is \$700 to \$500, or 1.4 to 1. For female drivers, the urban to rural relativity is \$400 to \$200, or 2 to 1. An urban to rural relativity appropriate for male drivers is not optimal for female drivers.

The discussion in the paragraphs above assumes that the rating model is multiplicative. In this illustration, an additive is more appropriate. We add or subtract a dollar amount for each class instead of multiplying by a factor.

Let the base class be rural females, with a base rate of $200, and let the relativities be as shown below:

- Male: +$300
- Female: +$0
- Urban: +$200
- Rural: +$0

The rate for any cell is the base rate plus the male/female relativity plus the territory relativity.

The indicated pure premiums now match the observed loss costs. Rural male = $200 + $300 + $0 = $500. Urban male = $200 + $300 + $200 = $700.

The additive method provides an exact match because

1. The dollar difference between males and females is the same for the rural column as for the urban column. In both columns, the dollar difference is $300.
2. The dollar difference between urban and rural is the same for the male row as for the female row. In both rows, the dollar difference is $200.

ADDITIVE INTUITION

Some casualty actuaries implicitly assume that pure premium relativities should be multiplicative, not additive. Current multi-dimensional classification systems for the casualty lines of business generally use multiplicative factors.

Regulators sometimes castigate insurers for using multiplicative factors that "unduly" increase the rates for high-risk insureds. Some actuaries assume that this criticism is purely political, not actuarial. This is often true, but it is not always correct. When two or more dimensions of the classification system are correlated, multiplicative systems may be biased. For some types of insurance, multiplicative systems may be biased even when classification dimensions are not correlated.

Life insurance rating systems provide an example. If smokers have twice the mortality of non-smokers, and persons with high-blood pressure have twice the mortality of persons with average blood pressure, should high-blood pressure smokers have four times the mortality of average blood pressure non-smokers? Life insurance underwriters employ judgment to assess the rating for applicants with multiple causes of high mortality. A pure multiplicative rating system would not be appropriate.

We discuss multiplicative and additive models throughout this *Practitioner's Guide*. It is useful to understand the circumstances that justify the use of each type of system.

We use the illustration presented in the 1960 Bailey and Simon paper. We have two rating dimensions: (i) class of driver and (ii) merit rating class.

1. Class of driver refers to the driver characteristics, such as age, sex, and marital status, as well as use of the vehicle, such as pleasure use or business use.

2. Merit rating class refers to the number of immediately preceding accident free years, ranging from 0 to 3.

These two rating dimensions are partially correlated. Young, unmarried male drivers have a high average class relativity. Because these drivers either are new drivers or (if not new) are more likely to have had an accident in the past year, they have relatively few accident free years.

Mature female drivers have a low class relativity. Because they are more experienced drivers with fewer past accidents, they also have (on average) merit rating class credits. The two rating dimensions are not independent.

To choose between a multiplicative model and an additive model, we first find an optimal model of each type. We use a minimum bias procedure to select the optimal multiplicative factors for the multiplicative model and the optimal additive factors for the additive model. We then compare the goodness-of-fit of the indicated pure premiums from each model to the observed loss costs. Both Rob Brown and Bailey and Simon did this analysis for the Canadian merit rating plan factors. As Venter notes in his discussion of Brown's paper, some private passenger automobile insurance carriers used a combined additive and multiplicative model; see below.

This teaching guide does not advocate any particular rating method. Once readers are comfortable with the procedures described here, they should be well equipped to optimize the classification relativities for any scenario.

BIAS FUNCTIONS

In practice, the indicated pure premiums do not perfectly match the observed loss costs for either an additive model or a multiplicative model. We illustrate with the same 2 by 2 classification system. The observed loss costs are shown in the table below:

	Urban	Rural
Male	$800	$500
Female	$400	$200

Neither an additive model nor a multiplicative model provides a perfect match. If we use a model that does not perfectly match the observed data, we must determine how to minimize the mismatch between the observed loss costs and the indicated pure premiums. A "bias function" is a means of comparing two or more models to see which fits the data with the smallest degree of mismatch.[7] To choose the optimal model, we proceed along three steps:

1. We choose a rating method, such as an additive model, a multiplicative model, or a combined model.
2. We select a bias function to optimize the rating method. This *Practitioner's Guide* discusses the balance principle, least squares, χ^2, and maximum likelihood bias functions.
3. For each optimized rating method, we examine the goodness-of-fit of the indicated pure premiums with the observed loss costs.

For models using a maximum likelihood bias function, we must also choose a probability distribution function for losses within each cell.

We begin with the balance principle, since it is the bias function most commonly used in practice. The 1963 Bailey paper provides a compelling justification for the balance principle.[8]

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[7] The "bias function" is not a standard statistical term, and the balance principle is not a standard statistical principle. As used in this *Practitioner's Guide*, the bias function is the means of determining how "close" the indicated pure premiums are to the observed loss costs or how great the "mismatch" is between these two sets of data. The sum of the squared deviations and the χ-squared deviation are common statistical bias functions. The balance principle, which was introduced by Bailey and Simon in 1960 and then strongly endorsed by Bailey in 1963, minimizes the bias along the dimensions of the classification system, thereby leading to the term "minimum bias."

[8] In contrast, Brown [1988] and Mildenhall [1999] argue that the standard statistical functions, such as least squares, χ-squared, and maximum likelihood, along with generalized linear models, should be considered in place of the balance principle.

THE BALANCE PRINCIPLE

The balance principle means that

the sum of the *indicated pure premiums* = the sum of the *observed loss costs*

along *every row* and *every column*.

We examine the balance principle for both the additive and the multiplicative models in our simplified illustration. On the left are the observed loss costs; on the right are the indicated pure premiums. We begin with the multiplicative model.[9]

	Urban	*Rural*		*terr₁*	*terr₂*
Male	$800	$500	*sex₁*	$200 \times s_1 \times t_1$	$200 \times s_1 \times t_2$
Female	$400	$200	*sex₂*	$200 \times s_2 \times t_1$	$200 \times s_2 \times t_2$

To balance along the first row (the "male" row), we must have

$$800 + 500 = 200 \times s_1 \times t_1 + 200 \times s_1 \times t_2$$

To balance along the second row (the "female" row), we must have

$$400 + 200 = 200 \times s_2 \times t_1 + 200 \times s_2 \times t_2$$

To balance along the first column (the "urban" column), we must have

$$800 + 400 = 200 \times s_1 \times t_1 + 200 \times s_2 \times t_1$$

To balance along the second column (the "rural" column), we must have

$$500 + 200 = 200 \times s_1 \times t_2 + 200 \times s_2 \times t_2$$

We have two rows and two columns, for a total of four equations. We have four variables, so we can solve the equations.

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[9] To keep the notation simple, we use rating dimensions of male vs female and urban vs rural throughout this *Practitioner's Guide*. For the formulas in the illustrations, we use $sex_1 = s_1$ = male, $sex_2 = s_2$ = female, $terr_1 = t_1$ = urban, and $terr_2 = t_2$ = rural. The recursive equations use variable names of *x*, *y*, and *z*, and rating dimensions of *i* and *j*.

Although we have four equations in four unknowns, we do not have a unique solution. There are two special considerations we must be aware of. These two considerations offset each other in such a way as to yield a unique set of indicated pure premiums.

1. Dependence among the equations: These equations are related by a totality constraint. Using any three of these equations we can derive the fourth, since the sum of the rows equals the sum of the columns. For instance, the fourth equation equals the first equation plus the second equation minus the third equation.

More generally, the equation for any column equals the sum of the equations for the rows minus the sum of the equations for the other columns. The equation for any row equals the sum of the equations for the columns minus the sum of the equations for the other rows.

2. Invariance under reciprocal scalar multiplication: We can set one of the variables arbitrarily, and we can still solve the system of equations. To see this most clearly, suppose that we have solved these equations for values of the four variables s_1, s_2, t_1, and t_2. Another solution is $2s_1$, $2s_2$, $\frac{1}{2}t_1$, and $\frac{1}{2}t_2$. We could use any constant in place of "2." No matter which set of relativities we pick, the values in the cells remain the same. The values in the cells are the product of an "s" relativity and a "t" relativity, so the additional constant cancels out.

We have an additional variable. The pure premium in each cell depends on the "base rate" (actually, the "base pure premium"). If the relativities s_1, s_2, t_1, and t_2 optimize the rating model for a base pure premium of $200, the relativities $2s_1$, $2s_2$, t_1, and t_2 optimize the rating model for a base pure premium of $100.[10]

The minimum bias procedure optimizes the relationship of the rating variables along each dimension of the classification system. If $s_1 = 2 \times s_2$ for a given base rate and a given set of territorial relativities, then $s_1 = 2 \times s_2$ for any other base rate and territorial relativities.

By convention, we choose a base class in each classification dimension. This is often the largest class, though any class may be used. The base class in each classification dimension is given a relativity of unity. This determines the values of all other rating variables as well as the value of the base rate.

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[10] With so much leeway in choosing the classification relativities, one might ask what we are "optimizing." We are optimizing the indicated pure premiums. Each set of classification relativities give the same indicated pure premiums. The optimization is relative to the bias function. The optimal pure premiums have the least bias, the least sum of squared deviations, the least χ-squared value, or the greatest likelihood.

Solving the Equations

The equations are *not* linear, so there is no closed form solution. We begin with an arbitrary (but reasonable) choice of relativities for one dimension, and we solve the equations iteratively.[11]

Illustration: We choose a set of relativities for urban and rural. Suppose we choose 2.00 for urban and 1.00 for rural.

	Urban	*Rural*		$terr_1 = 2$	$terr_2 = 1$
Male	$800	$500	sex_1	$200 \times s_1 \times 2$	$200 \times s_1 \times 1$
Female	$400	$200	sex_2	$200 \times s_2 \times 2$	$200 \times s_2 \times 1$

The balance equation for the first row (the "male" row) says that

$$800 + 500 = 200 \times s_1 \times 2 + 200 \times s_1 \times 1$$

This give us: $1{,}300 = 600 \times s_1$, or $s_1 = 13/6$.

Balancing along the second row (the "female" row) gives

$$400 + 200 = 200 \times s_2 \times 2 + 200 \times s_2 \times 1,$$
$$\text{or } s_2 = 600/600 = 1.$$

We now have intermediate values for the male and female relativities of 13/6 and 1. We discard the initial values for the urban and rural relativities of 2.00 and 1.00, and we solve for new intermediate values by balancing along the columns.

The balance equation for the first column (the "urban" column) says that

$$800 + 400 = 200 \times (13/6) \times t_1 + 200 \times 1 \times t_1$$

This implies that $1{,}200 = 633.33\ t_1$, or $t_1 = 1.895$.

Balancing along the second column (the "rural" column) gives

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[11] Iterative methods were originally adopted because there are no closed form solutions. They are ideal for spreadsheet applications, which have eliminated the hand calculations.

$$500 + 200 = 200 \times (13/6) \times t_2 + 200 \times 1 \times t_2,$$
$$\text{or } t_2 = 1.105.$$

We continue in this fashion. We discard the previous male and female relativities, and we solve for new ones:

Balancing along the first row (the "male" row) gives

$$800 + 500 = 200 \times s_1 \times 1.895 + 200 \times s_1 \times 1.105$$

and balancing along the second row (the "female" row) gives

$$400 + 200 = 200 \times s_2 \times 1.895 + 200 \times s_2 \times 1.105$$

We solve these two equations for new values of the male and female relativities, we discard the previous values of the urban and rural relativities, and we balance along the columns for new values of the urban and rural relativities.

We continue in this fashion until the relativities converge. Convergence means that the change in the relativities from an additional iteration is not material. Pencil and paper calculation of the minimum bias relativities is tedious, particularly if there are many classes within each dimension. The built-in iterative functions in standard spreadsheet packages eliminate this problem.

In practice, we begin with starting values determined by the simple rate relativities procedure. In this illustration, the urban to rural relativity is 12 to 7. If we choose a pure premium relativity of 1.000 as the starting value for the rural class, we would choose a starting value of $12 \div 7 = 1.714$ for the urban class. The starting values have no effect on the final rates in each cell.

Once the series converges, the common practice is to normalize the base class relativities to unity. We normalize by changing the base rate. For instance, suppose that the series above converged after a single iteration. (It does not actually converge after a single iteration; we simply want to show the normalization technique.)

A. The territorial relativities are 1.895 for urban and 1.105 for rural. If the rural territory is the base class, we change the rural relativity to 1.000, we change the urban relativity to $1.895 \div 1.105 = 1.715$, and we change the base rate to $\$200 \times 1.105 = \$221$.

B. The male/female relativities are 13/6 for males and 1.000 for females. If females are the base class, these relativities are already normalized. If males are the base class, we change the male relativity to 1.000, the female relativity to 6/13, and we multiply the base rate by 13/6.

THE ADDITIVE MODEL

There are several equivalent formulas for the additive model. The rate in cell x_iy_j, or row "i" and column "j," is

A. Base rate + x_i + y_j,
B. Base rate × (1 + u_i + v_j), or
C. Base rate × (p_i + q_j)

To see the equivalence of these formulas, suppose the base rate in formula "A" is $10.

- In formula "B," the base rate is also $10, each "u" value is one tenth the corresponding "x" value in formula "A," and each "v" value in formula "B" is one tenth the corresponding "y" value in formula "A": $u_i = 0.1 \times x_i$ and $v_j = 0.1 \times y_j$.
- Formula "C" is equivalent to formula "B," except that either the "p" values are all increased by 1, the "q" values are all increased by 1, or the "p" values are increased by a constant (*c*) and the "q" values are increased by the complement of that constant (*1–c*): $p_i = 1 + u_i$ <u>or</u> $q_j = 1 + v_j$ (both not both) or p_i = "c" + u_i <u>and</u> q_j = "1–c" + v_j.

We use the first form – formula "A" – for our illustrative example, since it shows the additive method most clearly. In practice, formula "C" might be preferred, since only the base rate need be increased for inflation. In formula "A," the base rate and all the relativities must be increased for inflation. The inflation adjustments would necessitate new rate pages each year.

We choose initial values for the urban and rural relativities: say, $250 and $0.[12] These initial values are based on the simple rate relativities procedure, since the average differential between the urban and rural observed loss costs is $250.[13]

	Urban	*Rural*		$terr_1 = 250$	$terr_2 = 0$
Male	$800	$500	sex_1	$200 + s_1 + 250$	$200 + s_1 + 0$
Female	$400	$200	sex_2	$200 + s_2 + 250$	$200 + s_2 + 0$

Balancing along the first row (the "male" row) gives

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12 The term "relativities" is more appropriate for a multiplicative model, where the relativities are multiplicative factors. An additive model uses dollar relativities that are added to the base rate.

13 The average differential is ½ × [(800 – 500) + (400 – 200)] = 250.

$$800 + 500 = 200 + s_1 + 250 + 200 + s_1 + 0,$$
$$\text{or } s_1 = 650/2 = 325.$$

Balancing along the second row (the "female" row) gives

$$400 + 200 = 200 + s_2 + 250 + 200 + s_2 + 0,$$
$$\text{or } s_2 = -50/2 = -25.$$

We discard the initial values for the urban and rural relativities, and we balance along the columns, using the intermediate values of the male and female relativities, to get new values for the urban and rural relativities. We continue the iterative process until the series converges.

The negative relativity of –$25 for females seems odd at first. In truth, the relativity for female drivers is not inherently negative; this is an artifact of the base rate and the starting values.

We could make the relativity for females positive by adding a constant to the male and female relativities and subtracting the same constant from the rural and urban relativities. For instance, we could add $75 to the male and female relativities to get relativities of $400 and $50, and we would subtract $75 from the rural and urban relativities.

We can make all the relativities positive or negative by adjusting the base rate. For instance, by choosing a base rate of $1,000, we obtain negative relativities for male drivers, female drivers, rural drivers, and urban drivers. Companies may do this for marketing reasons. All drivers get discounts from the base rate, so all drivers feel they are gaining from the classification system.

Alternatively, we may set a relativity of $0 for the base class in each classification dimension. This determines the base rate and all other relativities.

Exposures

The illustrations assume one driver in each cell or the same number of drivers in each cell. In practice, there may be different numbers of risks in each cell.

Two types of adjustments are needed: an adjustment to the bias function and an adjustment for credibility.

- We adjust the bias function for the relative volume of business in each cell (see below), not for the absolute volume of business.
- We may make a credibility adjustment based on the absolute volume of business in a cell.

Illustration: Insurer A has 100 exposures in each cell; insurer B has 10,000 exposures in each cell. Insurer A may rely more heavily on the minimum bias procedure. Insurer B may give greater weight to the empirical observations.

We deal with the adjustment to the bias function in this section. We defer the adjustment for credibility until later.

We deal with multiple exposures in a cell as though there were multiple cells laid on top of each other. We set the sum of the observed loss costs in each row or column equal to the sum of the indicated pure premiums in the corresponding row or column. If there are two drivers in a given cell, we double both the observed loss costs and the indicated pure premiums in that cell. If there are "n" drivers in a given cell, we multiply both the observed loss costs and the indicated pure premiums in that cell by "n."

In the illustrations above, we used two matrices: a 2 by 2 matrix of observed loss costs and a 2 by 2 matrix of indicated pure premiums. This is sufficient when there is exactly one driver in each cell, or when the number of drivers in each cell is the same. When the number of drivers varies by cell, we need a matrix of the number of drivers in each cell.

For the multiplicative model, suppose that the number of drivers were as follows:

- male urban: 1200
- male rural: 600
- female urban: 1000
- female rural: 800

We include the number of drivers in the matrices:

	Urban	*Rural*		*$terr_1$*	*$terr_2$*
Male	$1200 \times \$800$	$600 \times \$500$	*sex_1*	$1200 \times 200 \times s_1 \times t_1$	$600 \times 200 \times s_1 x t_2$
Female	$1000 \times \$400$	$800 \times \$200$	*sex_2*	$1000 \times 200 \times s_2 \times t_1$	$800 \times 200 \times s_2 x t_2$

We modify the balance equations to include exposures.

To balance along the first row (the "male" row), we must have

$$1200 \times 800 + 600 \times 500 = 1200 \times 200 \times s_1 \times t_1 + 600 \times 200 \times s_1 \times t_2$$

To balance along the second row (the "female" row), we must have

$$1000 \times 400 + 800 \times 200 = 1000 \times 200 \times s_2 \times t_1 + 800 \times 200 \times s_2 \times t_2$$

To balance along the first column (the "urban" column), we must have

$$1200 \times 800 + 1000 \times 400 = 1200 \times 200 \times s_1 \times t_1 + 1000 \times 200 \times s_2 \times t_1$$

To balance along the second column (the "rural" column), we must have

$$600 \times 500 + 800 \times 200 = 600 \times 200 \times s_1 \times t_2 + 800 \times 200 \times s_2 \times t_2$$

This paper proceeds simultaneously along three paths: illustrations, rigor, and intuition. Readers new to this topic should focus on the illustrations. Readers seeking to implement these methods should focus on the rigor. The rigor presents the equations which must be incorporated into a spreadsheet or program as well as the goodness-of-fit tests that indicate which models are superior.

More experienced actuaries should focus on the intuition. The intuition explains why we are using these procedures and it provides the rationale for the methods. The skilled actuary must understand not only the mechanics of a procedure but also what the procedure attempts to accomplish and why a particular procedure is appropriate for a given scenario.

We begin with observed loss costs. One might wonder: *Why can't we use these figures, appropriately developed and trended, as the indicated pure premiums for the coming policy period? Instead of fitting either multiplicative or additive models to the observed data, let us use $800 as the indicated pure premium for urban male drivers, $400 as the indicated pure premium for urban female drivers, $500 as the indicated pure premium for rural male drivers, and $200 as the indicated pure premium for rural female drivers.*

The common answer is that the individual cells are "not fully credible." This answer is correct, but the terminology is not ideal. The term "credible" has a vague connotation. Let us be more precise, so that we grasp the intuition.

Credibility is a relative concept. No cell is inherently credible or not credible. The cell's credibility depends on the value of its own experience versus the information provided by the values in other cells.

Consider the basic illustration with the following observed loss costs:

	Urban	*Rural*
Male	\$800	\$500
Female	\$400	\$200

The urban male observed pure premium of $800 is a mixture of expected losses and random loss fluctuations. How might we judge whether it is biased upwards or downwards?

Let us suppose first that the rating values combine additively to generate the expected losses. The urban male observed loss cost of $800 is $300 more than the rural male observed loss cost of $500. This suggests that the urban attribute of the vehicle's location adds about $300 to the expected loss costs.

However, the urban female observed loss cost of $400 is only $200 more than the rural female observed loss cost of $200. This suggests that the extra cost associated with the urban attribute is only $200, not $300. This implies that the urban male loss cost of $800 might be biased upwards.

We perform a similar analysis for male versus female. The urban male observed loss cost of $800 is $400 more than the urban female observed loss cost of $400. This suggests that the male attribute adds about $400 to the expected loss costs.

However, the rural male observed loss cost of $500 is only $300 more than the rural female observed loss cost of $200. This suggests that the extra cost associated with the male attribute is only $300, not $400. In other words, the urban male loss cost of $800 might be biased upwards.

The $800 observed loss cost in the urban male cell does not tell us how much of this observed loss cost is expected and how much is distorted by random loss fluctuations. If we know the mathematical function linking the cells – that is, if the characteristics of the driver and the vehicle have some additive or multiplicative relationship – we can use additional cells to provide information about the true expected costs for urban male drivers. In this illustration, the other cells imply that the $800 observed loss cost might be biased upwards.

If we assume that the cells are linked by a multiplicative relationship, our inferences change. The urban male observed value of $800 is 160% of the rural male observed value of $500. This suggests that the urban attribute of the vehicle's location adds about 60% to the expected loss costs.

The urban female observed loss cost of $400 is twice the rural female observed loss cost of $200. This suggests that the extra cost associated with the urban attribute is +100%, not +60%. The urban male loss cost of $800 might be biased downwards.

We use a similar analysis for male versus female. The urban male observed loss cost of $800 is twice the urban female observed loss cost of $400. This suggests that the male attribute adds about 100% to the expected loss costs.

The rural male observed loss cost of $500 is 250% of the rural female observed loss cost of $200. This suggests that the extra cost associated with the male attribute is +150%, not +100%. The urban male loss cost of $800 might be biased downwards.

If the cells are linked additively, we infer that the urban male observed loss costs of $800 might be biased upwards. If the cells are linked multiplicatively, we infer that the urban male observed loss costs of $800 might be biased downwards.[14]

With a 2 by 2 matrix, there are 4 cells in total. If the exposures are evenly distributed among the cells, each cell contains 25% of the total exposures, whether there is 1 car in cell or 10,000 cars in each cell. We give much credence to the observed value in that cell compared to our inferences from other cells. With a larger array, such as a 10 by 10 by 10 array, there are many more cells. The average cell contains only 0.1% of the total exposures. We give less credence to the observed loss costs in that cell compared to our inferences from other cells.

This is the intuition for classification ratemaking in general and for the minimum bias procedure in particular. The rating model – such as additive, multiplicative, or combined – tells us the type of relationship joining the cells. The bias function – such as balance principle, χ-squared, least squared error, or maximum likelihood – provides a method of drawing inferences from one cell to another.

CREDIBILITY

The original papers on the minimum bias procedure discuss credibility considerations, but they come to differing conclusions. Insurers and rating bureaus using versions of the minimum bias procedure sometimes add credibility enhancements.

We do not discuss credibility in depth until we cover additional bias functions, goodness-of-fit tests, and classification models. We introduce here the general credibility issues.

The 1960 Bailey and Simon paper uses credibility considerations to pick a bias function. The authors' view of the appropriate credibility for each cell led them to choose the χ-squared bias function over the balance principle.

The 1963 Bailey paper, which advocates the balance principle, has no explicit discussion of credibility. The balance principle has an implicit credibility component. The credibility of a cell is proportional to the exposures in that cell. If we double the exposures in a cell, we double the credibility of that cell.

We conceive of a cell with multiple exposures as a stack of cells each containing a single exposure. These cells are all at the same location in the array, but there is nothing special joining these cells into a group. Instead of one cell with two exposures and an average loss

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[14] In practice, the direction of the bias rarely depends on the type of rating model. The more common scenario might show an observed loss cost of $600, an additive model indicated pure premium of $550, and a multiplicative model indicated pure premium of $530. We might infer that the observed loss costs are biased upwards by random loss fluctuations.

cost of $500, we conceive of two cells with one exposure in each. The observed loss costs in the two cells have an average value of $500.

Venter looks at credibility from a different angle. We said above that the $800 observed loss cost for urban male drivers might be overstated for an additive model or understated for a multiplicative model. The over- or understatement stems from random loss fluctuations. If there is a single exposure in each cell, an overstatement or understatement is likely. If there are 10,000 exposures in each cell, the degree of overstatement or understatement is likely to be smaller.

If there is single exposure in each cell, we might attribute the$800 observed loss cost to random loss fluctuations. If there are 10,000 exposures in the urban male cell, we might attribute the observed loss cost to the special hazards of being male and living in a city. The number of exposures in the cell may suggest how much credence to give to the observed value in that cell versus the inferences from other cells.

ITERATIVE FORMULAS

We have presented simple illustrations and intuitive explanations. To program this procedure on a spreadsheet or in source code, we need general formulas.

We derive the iterative formulas for the multiplicative balance principle model. For the balance principle, we balance along the rows and the columns until we achieve convergence. Convergence means that an additional iteration does not materially change the relativities. With modern spreadsheets, the speed of convergence is not a concern. If the insurance premium is expressed in whole dollars, we might define convergence such that the indicated pure premiums stay the same to the nearest dollar.

We designate the number of exposures in row *i* and column *j* by n_{ij}, and the observed pure premiums in row *i* and column *j* by r_{ij}. The balancing equations for the multiplicative model are

$$\sum_j (n_{ij} r_{ij}) = \sum_j (n_{ij} x_i y_j)$$

In this equation, *x* is a row relativity and *y* is a column relativity.[15] The equation assumes a base rate of unity. We solve this equation for x_i to get

[15] In the illustrations, we use *s* for the row relativity and *t* for the column relativity as acronyms for the classification dimensions (sex and territory). The variables *x* and *y* are commonly used in the literature.

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j}$$

We sum over the j subscript when we balance along the rows (the i subscripts). We do this separately for each i. When we balance along the columns, we sum over the i subscripts and we do this separately for each j. When the series converges, we set the relativity for the base class in each classification dimension to unity, and we adjust the base rate to offset this.

For readers who wish to proceed to the original actuarial papers, this formula is the fifth expression on page 10 in the 1963 Bailey paper. In Brown's 1988 paper, this is formula (1.1) on page 189, formula (2.3) on page 192, and formula (3.3) on page 195. It is also formula (5.20) on page 201, which is Brown's multiplicative Poisson model with maximum likelihood estimation. This is also the final equation on page 44 of the 1999 Holler, Sommer, and Trahair paper (*CAS Forum*, Winter 1999), which follows Brown's derivation.

We used two dimensions in this formula. One might assume that the two dimensions correspond to the two variables x and y. That is not correct. The two dimensions correspond to the two subscripts i and j. The x and y variables correspond to two sets of relativities. A dimension can have two or even more sets of relativities.

Illustration: The classification system has two dimensions: male vs female and territory A vs territory B. Territory A has more attorneys than territory B has, resulting in a higher propensity to sue and higher loss costs. Territory B has several blind intersections, leading to more accidents. We might presume that the higher attorney involvement in territory A increases the cost of all claims, so a multiplicative factor is appropriate, whereas the blind intersections in territory B adds additional hazards, so an additive factor is appropriate. The rating model might take the form

$$\textit{indicated pure premium} = x_i \times y_j + z_j,$$

where i represents the male/female classification dimension and j represents the territory dimension. To optimize this rating model, the balance principle is not sufficient; we would have to employ one of the other bias functions.[16]

[16] The balance principle provides $i+j$ equations, but we have $i+2j$ variables. The other bias functions discussed in this *Practitioner's Guide* provide $i+2j$ equations.

The arithmetic is similar for any number of dimensions. The multiplicative model has one set

$$x_i = \frac{\sum_{j,k} n_{ijk} r_{ijk}}{\sum_{j,k} n_{ijk} y_j z_k}$$

of relativities for each dimension. With three dimensions, the iterative formula is

Similarly, we develop the general formula for the balance principle additive model by assuming a base rate of \$0. The balance principle equation is

$$\sum_j (n_{ij} r_{ij}) = \sum_j n_{ij}(x_i + y_j)$$

and the iterative formula is

$$x_i = \frac{\sum_j n_{ij}(r_{ij} - y_j)}{\sum_j n_{ij}}$$

MULTIPLICATIVE MODEL

Illustration: A multiplicative rating model with two dimensions and two classes in each dimension. The observed loss costs and exposures in each class are shown below. We use the balance principle to optimize the pure premium relativities.

Using a base rate of 100 and starting values of 1.00 for y_1 and 1.500 for y_2, we compute the first iterative values of y_1 and y_2.

Loss Costs		y_1	y_2
	x_1	300	300
	x_2	200	400

Exposures		y_1	y_2
	x_1	100	150
	x_2	100	100

Iterations

We are given starting values for y_1 and y_2. To compute the first iterative values of y_1 and y_2, we must first compute the intermediate values for x_1 and x_2.

- From the starting values for y_1 and y_2, we compute values for x_1 and x_2.
- From the computed values of x_1 and x_2, we compute new values for y_1 and y_2.

Since the base pure premium is \$100, the indicated pure premiums are \$100 $\times x_i \times y_j$. To simplify the mathematics, we compute all values in units of \$100. The indicated pure premiums are $x_i \times y_j$, and the observed loss costs are \$3, \$3, \$2, and \$4.

We form a matrix of observed loss costs and indicated pure premiums:

	y_1	y_2		y_1	y_2
x_1	3	3	x_1	$x_1 \times y_1$	$x_1 \times y_2$
x_2	2	4	x_2	$x_2 \times y_1$	$x_2 \times y_2$

We multiply each of the figures by the number of exposures in the cell:

	y_1	y_2		y_1	y_2
x_1	100×3	150×3	x_1	$100 \times x_1 \times y_1$	$150 \times x_1 \times y_2$
x_2	100×2	100×4	x_2	$100 \times x_2 \times y_1$	$100 \times x_2 \times y_2$

The starting values for y_1 and y_2 are 1.00 and 1.50. We use the balance principle to obtain values for x_1 and x_2:

$$100 \times 3 + 150 \times 3 = 100 \times x_1 \times 1.00 + 150 \times x_1 \times 1.5,$$
$$\text{or } 300 + 450 = 100 \times x_1 + 225 \times x_1,$$
$$\text{or } x_1 = 2.308.$$

and

$$100 \times 2 + 100 \times 4 = 100 \times x_2 \times 1.00 + 100 \times x_2 \times 1.5,$$
$$\text{or } 200 + 400 = 100 \times x_2 + 150 \times x_2,$$
$$\text{or } x_2 = 2.400.$$

We now have intermediate values for x_1 and x_2. We discard the initial values for y_1 and y_2, and we balance along the columns.

$$100 \times 3 + 100 \times 2 = 100 \times 2.308 \times y_1 + 100 \times 2.400 \times y_1,$$
$$\text{or } 300 + 200 = 230.8 \times y_1 + 240 \times y_1,$$
$$\text{or } y_1 = 1.062.$$

and

$$150 \times 3 + 100 \times 4 = 150 \times 2.308 \times y_2 + 100 \times 2.400 \times y_2,$$
$$\text{or } 450 + 400 = 346.2 \times y_2 + 240 \times y_2,$$
$$\text{or } y_2 = 1.450.$$

This completes the mathematics. We comment on several items in this exercise.

Data and Assumptions

The number of exposures in each cell may be viewed as a credibility item. We give 50% more credence to the observed loss costs in the x_1/y_2 cell. Intuitively, we believe the relationships involving cell x_1/y_2 more than we believe other relationships.

- The observed loss costs in the x_1 row indicate that there is no difference between y_1 and y_2. The observed loss costs in the x_2 row indicate that the y_2 class should have twice the pure premium as the y_1 class. We give more credence to the first of these two relationships.

- The observed loss costs in the y_1 column indicates that the x_2 class should have a pure premium 33% *lower* than the x_1 class. The observed loss costs in the y_2 column indicates that the x_2 class should have a pure premium 33% *higher* than the x_1 class. We give more credence to the second of these two relationships.

Since there are only four cells in the array, the indicated pure premiums should be close to the observed loss costs. If the relationships implied by the various rows and columns were consistent, the number of exposures in each cell would have little effect on the indicated pure premiums.[17] In this exercise, the relationships implied by the different rows and columns are not consistent. The higher exposures in the x_1/y_2 cell tilts the indications toward the relationships involving that cell.

We see this in the computed values for x_1 and x_2. If the exposures were the same in each cell, we would have no reason to rate x_1 differently from x_2: the y_1 column suggests that the x_2 class should have a 33% lower pure premium and the y_2 column indicates that the x_2 class should have a 33% higher pure premium. Since we give more credence to the second of these two relationships, the x_2 relativity is slightly higher than the x_1 relativity.

Rating Model

In practice, the pricing actuary is optimizing the pure premium relativities within a given rating model, such as the multiplicative model in this exercise. In theory, the pricing actuary might use the minimum bias procedure to determine the optimal rating model.

The general rule is that a multiplicative model is indicated when the observed loss costs are more dispersed, and an additive model is indicated when the observed loss costs are less dispersed.[18] More precisely, a multiplicative model is indicated when the high rated classifications stem from multiple high relativities. An additive model is indicated when the combination of high relativities does not result in very high observed loss costs.

Sometimes the type of dispersion is evident, such as high pure premiums for young unmarried urban male drivers and low pure premiums for mature suburban female drivers. In this exercise, the degree of dispersion is not evident.

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[17] By "consistent," we mean that the relationship among the column relativities are the same in each row.

[18] One can phrase this in statistical terms as a multiplicative model works better when the coefficient of variation is high and an additive model works better when the coefficient of variation is low. The generalizations are intended to guide practicing actuaries in choosing a model. In truth, the actuary must test the goodness-of-fit of each model to see which works best.

ADDITIVE MODEL

An additive model with two dimensions has the observed loss costs shown below. Each cell has 1000 exposures. The base lost cost is 100. The formula for loss costs by cell is Loss $Cost_{ij}$ = (Base Loss Cost) × ($x_i + y_i$). We use the starting values shown below to compute intermediate values for y_1 and y_2.

- Loss Costs:

	y_1	y_2
x_1	500	750
x_2	250	475
x_3	150	400

- Starting Values:

x_1	4.500
x_2	3.000
x_3	2.000

Computations

The number of exposures is the same in each cell. To simplify the computations, we may assume that there is a single exposure in each cell, since the factor of "1,000" cancels out of every equation.

The base rate is $100. To simplify the mathematics, we use units of $100 and a base pure premium of unity. The indicated pure premiums are $x_i + y_i$, and the observed loss costs are divided by 100.

The matrix of observed loss costs and indicated pure premiums is shown below:

	Observed Values			Indicated Values	
	y_1	y_2		y_1	y_2
x_1	5	7.5	x_1	$x_1 + y_1$	$x_1 + y_2$
x_2	2.5	4.75	x_2	$x_2 + y_1$	$x_2 + y_2$
x_3	1.5	4	x_3	$x_3 + y_1$	$x_3 + y_2$

We balance along the columns. For the first column, we have

$$5.00 + 2.50 + 1.50 = (x_1 + y_1) + (x_2 + y_1) + (x_3 + y_1)$$

We substitute the starting values of the "x"s to get

$$5.00 + 2.50 + 1.50 = (4.50 + y_1) + (3.00 + y_1) + (2.00 + y_1),$$
$$\text{or } 3\, y_1 = 9.00 - 9.50, \text{ or } y_1 = -0.167.$$

For the second column, we have

$$7.50 + 4.75 + 4.00 = (x_1 + y_2) + (x_2 + y_2) + (x_3 + y_2)$$

We substitute the starting values of the "x"s to get

$$7.50 + 4.75 + 4.00 = (4.50 + y_2) + (3.00 + y_2) + (2.00 + y_2),$$
$$\text{or } 3\, y_2 = 16.25 - 9.50, \text{ or } y_2 = 2.25.$$

We have finished balancing along the columns. The next step is to balance along the rows. We take the new *y* values, $y_1 = -0.167$ and $y_2 = +2.25$, and we compute new values for x_1 and x_2 by balancing along each row. We continue this process – alternately balancing along rows and columns – until the new values at the end of an iteration do not differ significantly from the old values at the beginning of that iteration along both dimensions.

Balance

It is helpful to see the convergence of the iterative equations.

- During the iterative process – before convergence – the plan is alternately balanced along the rows or along the columns, but not along both.
- Once the series converges, the plan is balanced along both the rows and the columns.

We have just balanced along the columns. To see that we are not yet balanced along the rows, we examine the first row:

$$5.00 + 7.50 = (x_1 + y_1) + (x_1 + y_2).$$

Substituting the starting values of the "x"s and the first iterative values of the "y"s, we get

$$12.50 = 4.50 + (-0.167) + 4.50 + 2.25 = 11.083.$$

The equality does not hold, since the plan is not yet balanced. Since we are still far from convergence, balancing along the columns distorts any balance along the rows.

Other Classification Dimensions

The basic illustrations use the minimum bias procedure to set pure premium relativities simultaneously for the male/female dimension and the urban/rural dimension. There may be other dimensions to the classification plan as well, such as age of driver, marital status, type of vehicle, use of the car, driver education, prior accident history, and so forth.

How do these other dimensions affect our analysis?

Ideally, we would use a multi-dimensional minimum bias procedure to set all classification relativities simultaneously. In practice, this may not be possible. Some rate relativities may be set on a statewide basis, whereas other rate relativities may be set on a countrywide basis. Some rate relativities may be analyzed each year, whereas other rate relativities may be analyzed every several years.

Territorial analyses must be done on a state by state basis. Certain driver characteristics and vehicle characteristics may be analyzed on a countrywide basis, for two reasons:

1. The relativities are not expected to vary by state, as long as the states use the same insurance compensation system.[19] The effects of type of vehicle, such as sedan, SUV, or sports car, on insurance loss costs should not vary by state. The same is true for the age, sex, and marital status of the driver.

2. Some classification cells would have few exposures in a state analysis, and the results may be distorted by random loss fluctuations. The countrywide analysis uses more data, providing more credible results. For example, we may wish to analyze driver age in yearly increments: age 17, age 18, age 19, and so forth. Single state data may be too sparse to give credible results.

Some classification dimensions, such as driver education, have a minor effect on overall loss costs. We may analyze these classification dimensions every five years or so, not every year.

Suppose that we analyze the male/female dimension and the urban/rural dimension on a statewide basis, while relativities for other classification dimensions are set on a countrywide basis. We use a minimum bias method for the statewide analysis.

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[19] The countrywide analysis may actually be done on all tort liability states or all no-fault states. The type of vehicle may have different effects, depending on the compensation system. The bodily injury rate relativities may be higher for SUV's (sports utility vehicles) than for sedans in tort liability states. The reverse may be true in no-fault states.

If all the classification dimensions are independent, the analysis should work well. If one or more of the other classification dimensions is correlated with the male/female or urban/rural dimensions, the rating analysis may be distorted.

Illustration: Suppose that young people migrate to urban areas, for university education, work opportunities, and the glamor of urban social activities. Older people move to the suburbs and rural areas, to buy homes and raise families away from the vices of urban areas. The age of the driver is correlated with the urban/rural garaging location.

The statewide analysis may indicate an urban to rural relativity of 2 to 1. The countrywide analysis may indicate a relativity for young unmarried male drivers of 3 to 1 when compared to adult drivers.

The relativity for young unmarried urban male drivers is not 6 to 1, even if a multiplicative model is appropriate for automobile insurance. Many of the young unmarried male drivers in the countrywide analysis live in urban areas, and many of the urban drivers are young and unmarried.

The optimal solution is to use a complete multi-dimensional statewide analysis, including age and marital status of the driver. This is not always practicable, since there are too many possible inter-relationships. For instance, the effect of vehicle type on bodily injury loss costs would probably be analyzed only on a countrywide basis, since there are too many vehicle types to give credible results on a statewide basis. But suburban married women with children are more likely to drive SUV's, and young unmarried urban male drivers are more likely to drive sports cars.

Loss Ratios

The common solution is to use loss ratios instead of loss costs in the minimum bias procedure. More precisely, we use loss ratios adjusted to the base rates for the classification dimensions included in the minimum bias analysis.

We use the basic illustration to develop the intuition. The first illustrations clarify the concepts. We then use a more involved setting to show the application of the procedure for private passenger automobile rating.

Suppose the empirical experience consists of loss ratios by classification, not loss costs. We observe the following loss ratios for four drivers for a multiplicative model:

	Urban	*Rural*
Male	75%	85%
Female	90%	80%

We could take either of two approaches:

First Approach: We treat the unadjusted loss ratios as though they were loss costs. Instead of using pure premium relativities, we use loss ratio relativities. These relativities are in addition to whatever pure premium relativities are embedded in these loss ratios.

In this scenario, the minimum bias procedure will indicate about equal loss ratio relativities for urban vs rural and slightly higher loss ratio relativities for females than for males. This does not mean that urban risks are similar to rural risks, or that female drivers have more accidents than male drivers have. If the current rate relativities are reasonable, we would expect the loss ratios in all cells to be about equal. In this scenario, the current male to female rate relativity might be 2.4 to 1. Since the average female loss ratio of 85% is higher than the average male loss ratio of 80%, the loss ratio relativities would indicate that we should slightly reduce the male to female rate relativity.

Second Approach: We convert the raw loss ratios to base class loss ratios. Suppose the current rate relativities are 2.4 to 1 for male to female and 1.8 to 1 for urban to rural. We must divide the male premiums by 2.4 and the urban premiums by 1.8. This is equivalent to multiplying the male loss ratios by 2.4 and the urban loss ratios by 1.8. In sum, we multiply the raw loss ratios by the current classification relativities, as shown in the table below.

	Urban	*Rural*
Male	75% × 2.4 × 1.8 = 324%	85% × 2.4 × 1.0 = 204%
Female	90% × 1.0 × 1.8 = 162%	80% × 1.0 × 1.0 = 80.0%

We apply the minimum bias procedure to the adjusted loss ratios. The resulting loss ratio relativities would be the same as the indicated rate relativities.

To see this, suppose that the base rate is $100. We determine the observed loss costs in each cell:

- For the male/urban cell, the premium is $100 × 2.4 × 1.8 = $432. The observed loss ratio is 75%, so the loss cost is 75% × $432 = $324.
- For the male/rural cell, the premium is $100 × 2.4 × 1.0 = $240. The observed loss ratio is 85%, so the loss cost is 85% × $240 = $204.

- For the female/urban cell, the premium is $100 × 1.0 × 1.8 = $180. The observed loss ratio is 90%, so the loss cost is 90% × $180 = $162.
- For the female/rural cell, the premium is $100 × 1.0 × 1.0 = $100. The observed loss ratio is 80%, so the loss cost is 80% × $100 = $80.

As we mentioned above, the common practice is to set the rate relativity to unity for the base class in each classification dimension. To facilitate this procedure, we divide each adjusted loss ratio in the matrix by the adjusted loss ratio for the base class. The resulting loss ratios in this illustration are shown below.

	Urban	Rural
Male	324% / 80% = 405.0%	204% / 80% = 255.0%
Female	162% / 80% = 202.5%	80.0% / 80% = 100.0%

LOSS RATIO INTUITION

We have shown how to convert loss ratios to reflect the loss costs in each cell. This might be useful if the observed data were loss ratios and we wanted to use loss costs for the minimum bias procedure. But the observed data are loss costs, not loss ratios. We must first convert the observed loss costs to loss ratios before converting back to loss costs.

The purpose of this conversion from loss costs to loss ratios and then back to loss costs is to eliminate the potentially distorting effects of classification dimensions that are not being analyzed in the minimum bias procedure.

We explain by illustration. We have average observed bodily injury loss costs for four groups of drivers, with 1000 drivers in each cell.

	Urban	Rural
Male	$800	$500
Female	$400	$200

There are other dimensions in the classification system.

Type of Vehicle: For bodily injury, cars are subdivided into various groups. SUV's (sports utility vehicles) and similar vehicles, such as light trucks, are larger and sturdier. They provide better protection for their occupants, but they cause greater damage to others. Sedans and small cars cause less damage to others. Sedans and small cars are more common in urban areas; SUV's and light trucks are more common in rural areas.

The distribution of vehicles types between urban and rural areas, along with the appropriate surcharge or discount for each type of vehicle, affects the observed loss costs. The pricing actuary may not have this distribution for the state under review. This is not necessary; the use of loss ratios instead of loss costs corrects for the effects on vehicle type.

It is hard to follow abstract intuition. To keep the mathematics clear, we assume that there are only two types of vehicles – SUV's and sedans – and that SUV's receive a +20% surcharge for bodily injury coverage. In this state, SUV's comprise 40% of the rural vehicles and 10% of the urban vehicles.

Age of Driver: The male/female rate relativity applies to all male and female drivers. Unmarried male drivers under the age of 21 receive additional surcharges, ranging from +25% for 20 year old drivers to +125% for 16 year old drivers. There is no corresponding surcharge for unmarried female drivers under the age of 21.[20] These surcharges are determined from a countrywide analysis.

The pricing actuary performing the minimum bias analysis does not have a distribution of male drivers by age and marital status. This is not necessary; the loss ratios are sufficient. To clarify the mathematics, however, we assume that 10% of male drivers are unmarried and under the age of 21. The average surcharge for these drivers is +50%.

DOUBLE COUNTING AND OFFSETTING

If we do not take vehicle type and driver age into account, we will overcharge male drivers and rural drivers.

Male Drivers: The male/female relativity is determined from the statewide analysis. The surcharges for young unmarried male drivers is determined from a separate countrywide analysis. The poor driving experience of young unmarried male drivers is counted twice: once at the countrywide level for the surcharges and once at the statewide level for the male/female

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[20] For many years, it was unclear why young males are such hazardous drivers. Maturity or temperament were often given as vague explanations. For life insurance, the higher mortality rates of males is assumed to be physiological, but there seemed to be no similar relationship between male drivers and auto accidents.

Advances in biological science have removed much of the puzzle. Young males have high levels of testosterone, which often leads to aggressive, risk-taking behavior. Much of the adventurous and often dangerous activities of young males, which previous generations ascribed to social acculturation, may have the same biological roots as facial hair and deep voices. Auto insurance rating by testosterone level may be far too intrusive; rating by age, sex, and marital status is a more acceptable proxy.

Lest readers misunderstand our comments, we note that young female drivers have higher loss costs than adult female drivers, but the differences are much smaller than they are for male drivers. Driving experience, maturity, and temperament have an effect, though they account for a relatively small portion of young unmarried male driver loss costs.

relativity. To accurately determine the male/female relativities, we must remove the hazardous effects of being young and unmarried from the male driver classification.

Rural Drivers: Rural drivers are less hazardous than urban drivers, but they drive more dangerous vehicles. The vehicle surcharge is determined in the countrywide analysis. To properly determine the urban/rural relativities, we must remove the effects of vehicle type from the statewide experience.

Removing these effects is not easy. It might seem reasonable to include additional dimensions in the statewide minimum bias analysis. We could include vehicle type as a third rating dimension and avoid its potential distorting effects on the minimum bias procedure.

For two reasons, this is not a practical or a reasonable solution.

A. Credibility: The experience for certain types of vehicles may be sparse in the statewide data. Rate relativities determined from sparse data may reflect random loss fluctuations.
B. Consistency: The relativity for a given vehicle type should be the same in all states. If separate statewide analyses are done, the relativities by type of vehicle will vary by state.

The countrywide analysis gives more credible relativities that are consistent from state to state.

To remove the effects of vehicle type and age of driver from the statewide analysis, we assume that the countrywide relativities are accurate. We examine each risk in the minimum bias procedure. We divide the actual loss costs by the type of vehicle relativity and by the driver age relativity. This gives the loss costs that we would have expected to see were the vehicle evenly distributed over all other rating dimensions.

Illustration: Suppose a four door sedan is the base vehicle type and age 21+ is the base age. A two door compact has a bodily injury discount of 10% and an unmarried 20 year old male driver has a surcharge of 25%.

Suppose the observed loss costs for a 20 year old unmarried male driver of a two door compact car are $450. The loss costs adjusted for age and vehicle type equal

$$\$450 / (0.90 \times 1.25) = \$400.$$

It is not practicable to make these adjustments car by car. There may be several classification dimensions that might distort the statewide analysis.

Using loss ratios adjusts for all classification dimensions simultaneously. Using observed loss ratios instead of observed loss costs adjusts for driver age, driver sex, territory, vehicle types, and all other rating dimensions. We add back in the current rating relativities for classification dimensions that we are analyzing, or male/female and urban/rural in this illustration.

We show the effects for this illustration. The calculations below are heuristic. We need not perform them for the minimum bias analysis. They reveal the intuition underlying the use of loss ratios instead of loss costs.

Illustration: The average observed loss costs for the 1000 drivers in each of four classes are displayed above. The current relativities are 2.4 to 1 for male to female and 1.8 to 1 for urban to rural. The average SUV to sedan relativity is 1.2 to 1. SUV's comprise 40% of rural cars and 10% of urban cars. Unmarried males under the age of 21 comprise 10% of male drivers, and their average surcharge is +50%. We convert the observed loss costs to adjusted loss costs for the minimum bias analysis.

Let us suppose that the pure premium for a female driving a sedan in a rural territory is $200. The choice of the base rate does not affect the results, since the same multiplicative factor affects all four cells. We work out the premium in each cell.

- Rural/female: We adjust for vehicle type with a multiplicative factor of 1 + 20% × 40% = 1.08. The average pure premium is $200 × 1.08 = $216.00.

- Urban/female: We adjust for vehicle type with a multiplicative factor of 1 + 20% × 10% = 1.02. The average pure premium is $400 × 1.02 = $408.00.

- Rural/male: We adjust for vehicle type with a multiplicative factor of 1 + 20% × 40% = 1.08 and for driver age with a multiplicative factor of 1 + 10% × 50% = 1.05. The average pure premium is $500 × 1.08 × 1.05 = $567.00.

- Urban/male: We adjust for vehicle type with a multiplicative factor of 1 + 20% × 10% = 1.02 and for driver age with a multiplicative factor of 1 + 10% × 50% = 1.05. The average pure premium is $800 × 1.02 × 1.05 = $856.80.

We divide the average loss costs in each cell by the average pure premium in that cell to get the observed net loss ratios in the cell.[21]

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[21] These loss ratios are net of expenses; they are losses divided by pure premiums. (These net loss ratios are sometimes called "experience ratios.") In practice, we have gross premiums, not pure premiums, so we use traditional loss ratios, not net loss ratios. The traditional loss ratios are slightly distorted by expense flattening procedures. Loss costs show pure premium relativities, whereas traditional loss ratios show rate relativities. In many cases, the distortion is not material. When the potential distortion is material, offsetting adjustments must be made. These adjustments depend on the expense flattening procedure; a full explanation would take us too far afield. On rate relativities versus pure premium relativities, see Feldblum [1996: PAP].

	Urban	Rural
Male	\$800 / \$856.80 = 93.37%	\$500 / \$567.00 = 88.18%
Female	\$400 / \$408.00 = 98.04%	\$200 / \$216.00 = 92.59%

We have removed the effects of all classification dimensions from the observed loss costs. We multiply by the current pure premium relativities for male/female and urban/rural to restore these effects to the observed data.

	Urban	Rural
Male	93.37% x 2.4 x 1.8 = 403.36%	88.18% x 2.4 x 1.0 = 211.64%
Female	98.04% x 1.0 x 1.8 = 176.47%	92.59% x 1.0 x 1.0 = 92.59%

The necessary adjustments are now done. If we want the base class in each dimension to have a relativity of unity, we divide by the base class adjusted loss ratio to get relative loss ratios, as shown below.

	Urban	Rural
Male	403.36% / 92.59% = 435.63%	211.64% / 92.59% = 228.57%
Female	176.47% / 92.59% = 190.59%	92.59% / 92.59% = 100.00%

If we wish, we can convert the relative loss ratios to adjusted loss costs by multiplying the cells by the base rate.

	Urban	Rural
Male	435.63% x \$200 = \$871.26	228.57% x \$200 = \$457.14
Female	190.59% x \$200 = \$381.18	100.00% x \$200 = \$200

Loss Ratio Approach

A classification system with two classes in each of two dimensions shows the observed data in the four matrices below.

- The relativities matrix shows the current pure premium relativities for male to female and urban to rural.
- The loss costs matrix shows the average loss costs per driver.
- The exposures matrix shows the number of cars in each cell. The company writes predominantly rural business.
- The premium matrix shows the premium collected in each cell.

The base pure premium is$66.67 for rural females.

Relativities	Urban	Rural
Male	4	2
Female	2	1

Loss Costs	Urban	Rural
Male	180	120
Female	100	40

Exposures	Urban	Rural
Male	100	1000
Female	100	1000

Earned Premium	Urban	Rural
Male	25000	125000
Female	13333	66667

We use the loss ratio approach to adjust the observed loss costs for the effects of other rating factors, and we calculate the first iteration for the minimum bias analysis, using the balance principle and a multiplicative model. We start with initial relativities of 1.5 for urban and 0.75 for rural.

Intuition

The use of loss ratios instead of pure premiums adjusts for potential distortions resulting from an uneven mix of business by classification. Let us review the rationale for the adjustments. We explain why we don't just use the given loss costs for the minimum bias procedure.

The loss cost approach (or the pure premium approach) implicitly assumes that the distribution of all other classification dimensions is homogeneous across class and driving record. When the distribution is not even, the loss ratio approach corrects the problem, as long as relativities for the unanalyzed classification dimensions are accurate.

We reason through the exercise. The matrix of relativities shows that the relativity for male drivers is twice the relativity for female drivers The loss costs per car in the first column of the loss costs matrix for male drivers are 1.8 times those for female drivers.

If we had only the matrix of observed loss costs, we would presume that the relativities should be adjusted. The male to female relativity should be 1.8 to 1, not 2 to 1.

The exposures and premium matrices show the error in this reasoning. Two cells in the first column (urban) have the same number of exposures: 100 for each cell. If there were an even distribution along other classification dimensions, then the premium for urban males would be twice the premium for urban females. The actual premiums are 25,000 and 13,333, for a ratio of 1.875 to 1.

To aid the intuition, let us assume that the only other classification dimension is vehicle type. We expect premium in the ratio of 2 to 1 for urban males compared to urban females. We actually have premium in the ratio of 1.875 to 1. That means that more females than males are driving high rated vehicles. If we evened out the distribution among territories, then the ratio of average loss costs between these males and females would increase from 1.8 to something greater.

Let us consider how much greater. Our first impulse is to say that the observed loss costs should be 2 to 1, not 1.8 to 1, since the pure premium relativities are 2 to 1, not 1.8 to 1. This is not correct, for two reasons.

- The pure premium relativities are the current relativities, not the indicated relativities. We do not know if these relativities are correct. Perhaps they were correct several years ago, but they are no longer correct now. Perhaps they were never exact, but they were chosen as round numbers.
- Even if the relativities are correct, random loss fluctuations may distort the observed loss costs. From these data alone, we can not determine if the relativities are correct and the

observed loss costs are distorted by random loss fluctuations or the observed loss costs are correct and the current relativities are not accurate.

We must adjust the observed loss costs from the experience data, not from our current relativities. The adjustment factor is based on the relativities, the exposures, and the premium. If the 100 urban males drove the same vehicles as the 100 urban females, their expected loss costs would increase by 2.000/1.875. The observed loss cost relativity would be 1.800 × (2.000 ÷ 1.875) = 1.920.

To clarify the intuition, let us revise the figures in the problem. Suppose that we change the premium for urban females to $25,000. Urban males and urban females both have premiums of $25,000, both have exposures of 100, yet males have a relativity twice that of females.

Let us further suppose that all urban males drive sedans and all urban females drive SUV's. There are no other classification dimensions besides sex of driver, territory, and vehicle type.

If sedans and SUV's had the same rates, the premium for urban males would be twice the premium for urban females. Since the urban males and the urban females have the same premium, we infer that the premium rate for SUV's is twice the premium rate for sedans.

If we assume that the rates for vehicle type are indicative of expected loss costs, part of the observed loss cost relativity between urban males and urban females stems from the different types of cars that these two groups drive. If all the urban females exchanged their SUV's for sedans, their observed loss costs should drop in half. The observed loss cost relativity between urban males and urban females would be 3.6, not 1.8.

Similarly, if all the urban males exchanged their sedans for SUV's, their observed loss costs should double. The observed loss cost relativity between urban males and urban females would be 3.6, not 1.8. This is what we mean when we say: "What would be the observed loss cost relativity if the effects of other classification dimensions were eliminated?"

We restate this as follows. The premium for urban females ($13,333) is higher than expected ($12,500) compared to the premium for urban males based on the number of exposures and the male/female relativity. This implies that the urban females are driving more hazardous vehicles than the urban males are driving. More generally, this implies that the group of urban female drivers have other characteristics that are raising their premiums and loss costs relative to those of the urban male drivers.

The current male to female relativity is 2.00. In the formula

$$1.80 \times (2.00 \div 1.875) = 1.92$$

the rate relativity for the classification dimension which we are reviewing appears in the numerator. The denominator is the rate relativity of the two groups (urban males and urban females), based on the premium ratio divided by the exposures ratio.

Let us revise the figures again to clarify the intuition for the denominator. Suppose that the male to female rate relativity were 1 to 1, the exposures in the two cells were equal, but the premiums were \$25,000 and \$13,333. Suppose also that the only other classification dimension is vehicle type, and that all the urban males drive sedans and all the urban females drive SUV's.

If sedans and SUV's had the same premium rate, then the premiums for urban males and for urban females would be the same. Since the premium for urban males is 1.875 × the premium for urban females, we infer that the premium rate for sedans is 1.875 × the premium rate for SUV's.

Part of the observed loss cost difference between urban males and urban females stems from the different vehicle types. If all the urban females drove sedans, their premiums and their expected losses would increase by a factor of 1.875. The ratio of observed loss costs for urban males compared with urban females would be reduced by a factor of 1.875.

The general procedure is straightforward. We remove the effect of *all* classification dimensions from the array of observed loss costs by converting it into an array of loss ratios. In our intuitive reasoning, this is accomplished by dividing by 1.875, since

loss ratios = average loss costs × exposures / premiums, or
loss ratios = average loss costs / (premiums / exposures)

The average effect of all rate relativities in a cell is proportional to the premium in that cell divided by the exposures in that cell. This is equivalent to saying that the premium for any car is the base rate times the product of the rate relativities for each classification dimension. To remove the effects of all dimensions from the array of loss costs, we divide by the ratio of premiums to exposures; that is, we multiply by the ratio of exposures to premium.

We restore the effects of the classification dimensions which we are reviewing. In our intuitive reasoning, this is accomplished by multiplying by the male to female relativity of 2.000.

Each cell contains loss costs × exposures ÷ premiums. These are the loss ratios in the cells. We now multiply by the array of rate relativities for the classification dimensions that we are analyzing. This restores the effect of these classification dimensions.

This leaves us with the loss costs relativities that would be observed were there complete homogeneity in the distribution of exposures of the other classification dimensions.

An additional simplification is to form relative loss ratios to the base class and then multiply by the base rate. This additional step is not essential. In a multiplicative system, multiplying all figures by a constant does not affect the final rate relativities.

Let us keep an eye on this relationship. We want to revise the matrix so that the loss cost relationship is not 1.80 to 1 but 1.80 × 2.00 ÷ 1.875 = 1.92 to 1. The same is true for all other relationships in the matrix of loss costs. Since we have many relationships, we use a systematic method for adjusting them. We use the following sequence of computations.

Form loss ratios: We form loss ratios, using the (i) loss costs per exposure, (ii) the exposures in each cell, and (iii) the earned premium in each cell, as

loss ratio = loss costs per exposure × exposure ÷ premium.

This gives us a matrix of loss ratios, as shown below:

Absolute Loss Ratios:	*Urban*	Rural
Male	0.72	0.96
Female	0.75	0.6

We want to form a matrix of adjusted loss costs. These are the expected loss costs were there no other classification dimensions affecting the experience. We multiply by the base rate of the base class, which is the class of rural females.

It is easiest to do this with relative loss ratios (not absolute loss ratios), with the relativity for the base class being 1.000. We divide the matrix of loss ratios by the base class loss ratio of 60% to get the matrix of relative loss ratios.

Relative Loss Ratios:	*Urban*	Rural
Male	1.2	1.6
Female	1.25	1

We want the expected loss costs in each cell. We multiply the relative loss ratios by the classification differentials for each cell to get the relative loss costs by cell. These are relative loss costs by cell, not absolute loss costs by cell.

We are given the current differentials in the relativities matrix.

Current Relativities:	*Urban*	Rural
Male	4	2
Female	2	1

We multiply the relative loss ratios by the current class relativities to get the relativities implied by the observed data. These relativities are like the observed loss costs in the earlier illustrations in this *Practitioner's Guide.*

Observed Relativities	*Urban*	Rural
Male	4.8	3.2
Female	2.5	1

Let us check our work. $4.80 \div 2.50 = 1.92$, which equals $1.80 \times 2.00 \div 1.875$. We have now adjusted the figures so that we can perform the minimum bias calculation.

We can proceed in either of two fashions.

(i) We can multiply by the pure premium for the base class and proceed with the pure premium approach to the minimum bias procedure.

(ii) We can do the minimum bias analysis, and then multiply by the base class pure premium.

The two methods are mathematically equivalent. We use the latter method here.

We set up the matrices of observations and of indicated relativities, as shown below.

	Observed Relativities			*Indicated Relativities*	
	Urban (t_1)	Rural (t_2)		Urban (t_1)	Rural (t_2)
s_1	4.8	3.2	s_1	$s_1 \times t_1$	$s_1 \times t_2$
s_2	2.5	1	s_2	$s_2 \times t_1$	$s_2 \times t_2$

The initial values of urban (t_1) and of rural (t_2) are 1.5 and 0.75. We balance along the rows, and we multiply each cell by the corresponding number of exposures:

$$100 \times 4.80 + 1{,}000 \times 3.20 = 100 \times s_1 \times 1.5 + 1{,}000 \times s_1 \times 0.75,$$
$$\text{or } s_1 = 3680 / 900 = 4.089.$$

$$100 \times 2.50 + 1{,}000 \times 1.00 = 100 \times s_2 \times 1.5 + 1{,}000 \times s_2 \times 0.75,$$
$$\text{or } s_2 = 1250 / 900 = 1.389.$$

We use these relativities for male and female (s_1 and s_2), and we discard the starting values for urban/rural (t_1 and t_2). We balance along the columns.

$$100 \times 4.80 + 100 \times 2.50 = 100 \times t_1 \times 4.089 + 100 \times t_1 \times 1.389,$$
$$\text{or } t_1 = 730 / 547.8 = 1.333.$$

$$1000 \times 3.20 + 1000 \times 1.00 = 1000 \times t_2 \times 4.089 + 1000 \times t_2 \times 1.389,$$
$$\text{or } t_2 = 4200 / 5478 = 0.767.$$

These are the pure premium relativities after one iteration. If expenses are proportional to premiums, they are also the rate relativities.[22] To convert to actual rates, we multiply by the base rate of $66.67.

New Rates	Urban	Rural
Male	$363.39	$209.09
Female	$123.44	$71.03

[22] If all expenses are proportional to the gross premium, as is true for state premium taxes, the expenses are also proportional to the pure premium, and the pure premium relativities equal the rate relativities.

LOSS RATIO INTUITION

The loss ratio method is commonly used by practicing actuaries. The mathematics is not complex, but the intuition sometimes seems elusive We show another illustration, where we focus on the intuition, not the arithmetic.

The previous illustration showed the loss costs and exposures in each cell. We needed only the incurred losses and earned premiums by cell to determine the loss ratios. We used the loss costs by cell only to show the intuition of the method. In practice, the practicing actuary may have only incurred losses and earned premiums by cell, but not exposure counts by cell. For the exposition in this section, we speak of loss ratios and rate relativities, not loss costs.

We are using loss ratios for their theoretical benefits, though the practical benefits may be equally great. Accurate exposure counts by cell are not always available to the pricing actuary. When we lack exposure counts by cell, we can not use a straight-forward loss costs method, even if the distribution of insureds by other rating dimensions is even.

We focus here on transforming the data into loss cost relativities by cell. Once we have the loss cost relativities by cell, we can use any of the models in this *Practitioner's Guide* to determine rate relativities. We explain the intuition in steps.

Illustration: Loss Ratio Intuition

Incurred Losses		
	Urban	*Rural*
Male	$2,700	$2,000
Female	$1,500	$1,200

Earned Premium		
	Urban	*Rural*
Male	$3,000	$4,000
Female	$2,400	$1,600

The current relativities by sex of driver and by garaging location are as follows:

Male:	1.50	Urban:	1.20
Female:	1.00	Rural:	1.00

Three Causes

To correct for potential distortions caused by an uneven distribution of insureds by other classification dimensions, we use loss ratios instead of loss costs. The underlying principle is that if the current premiums are actuarially correct – that is, if the current rate relativities are actuarial proper – the loss ratios in each cell should all be equal, except for random loss fluctuations.

The reader might wonder: *If all the current rate relativities were actuarially correct, we would not need to perform a rate relativities analysis. We perform the analysis because we want to examine whether the rate relativities are correct. What exactly are we assuming here?*

We restate this question as follows:

- We need data with no uneven distribution of insureds by classification dimension to determine accurate rate relativities.
- We must assume that the rate relativities are accurate to correct for uneven distributions of insureds by classification dimension.

The logic in these two statements seem circular at first. The exposition below explains what we are assuming and provides the justification for the assumption.

We assume that the rate relativities are correct for all classification dimensions besides those being examined now. We make no assumptions about the current rate relativities for the classification dimensions being examined. We are forming new rate relativities for the classification dimensions being examined now, and we ignore the current rate relativities.

We examine this in the numerical illustration. The observed loss ratios, or the ratios of the incurred losses to the earned premiums in each cell, are shown in the matrix below.

Loss Ratios		
	Urban	*Rural*
Male	90.00%	50.00%
Female	62.50%	75.00%

If all current rate relativities were actuarially proper and there were no random loss fluctuations, all cells would have the same loss ratio. They do not have the same loss ratios in this illustration. There are three possible causes.

Cause 1: The differences may be caused by random loss fluctuations. The importance of random loss fluctuations is a credibility issue. The credibility of the data increases with the volume of data and decreases with the dispersion of the loss distribution of these data. Credibility issues are important, but they are distinct from the minimum bias issues.

This *Practitioner's Guide* includes a short section on credibility. Most of the exposition assumes either that the data are fully credible or that the pricing actuary has already made (or will make) whatever adjustments are warranted by credibility considerations.

Cause 2: The differences are caused by improper rate relativities in other classification dimensions and there is an uneven distribution of insureds by these other classification dimensions. For example, perhaps the rates for a certain type of vehicle are too low, and the proportion of urban males driving that type of vehicle is greater than the proportions of the insureds in the other cells driving that type of vehicle.

If this is the cause of the differences, we are stymied. However, as long as the uneven distribution of insureds by the other classification dimension is not too serious, an inaccuracy in the rates will not distort our analysis too greatly. We may restate our assumption as follows:

> *For other classification dimensions, either the current rate relativities are accurate or the distribution of the insureds that we are examining is relatively even across these other dimensions.*

We may rephrase this assumption to fit the illustration as follows. Either the along other classification dimensions, such as vehicle type, are actuarially correct, or if a certain vehicle types has too high or too low a classification relativity, the proportion of males and females driving that type of vehicle is the same.

In many instances, this assumption is not perfect. Nevertheless, even if the use of loss ratios does not perfectly correct distortions caused an uneven distribution of insureds along other classification dimensions, it corrects the distortions at least partially. This assumption may not be perfect, but it makes our analysis better.

Cause 3: The final cause of the differences in the loss ratios by cell is inaccuracies in the rate relativities for the two classification dimensions that we are examining: sex and territory. This can be corrected by the minimum bias procedure, since the loss ratios by cell times the current relativities by cell equal the relative loss costs by cell.

Illustration: Suppose the loss ratio for male drivers is 90% and the loss ratio for female drivers is 75%. If the current male to female rate relativity is 2 to 1, the male to female loss cost relativity is 2 × 90% to 1 × 75% = 2.4 to 1.

For the illustration in this section, we form a matrix of relativities by sex and territory:

Current Rate Relativities		
	Urban	*Rural*
Male	1.80	1.50
Female	1.20	1.00

The relative loss costs by sex and territory are the product of the matrix of relativities and the matrix of loss ratios:

Loss Cost Relativities		
	Urban	*Rural*
Male	1.62	0.75
Female	0.75	0.75

We now proceed to determine optimal rate relativities by any of the minimum bias models discussed in this *Practitioner's Guide.*

Combined Models

Throughout this *Practitioner's Guide*, we have used simple multiplicative and additive models. In part, this reflects insurance practice, since most lines of business use simple multiplicative and additive models.

In truth, the business practice reflects ratemaking capabilities. Actuaries did not have simple procedures to optimize combined models, so these models did not gain wide acceptance.

The rationale for combined models is strong. Since the least squares and χ-squared bias functions provide simple recursive equations for many combined models, they may become more popular in the future.[23]

Illustration: Rating territory may have a variety of effects on insurance loss costs.

1. High crime areas may have a greater incidence of car theft and claim fraud. Thefts would raise comprehensive pure premiums, and fraud would raise liability pure premiums.
2. Areas with more sophisticated medical facilities may have higher loss costs for bodily injury claims.
3. Territories with a higher incidence of attorneys per capita often experience a higher incidence of bodily injury claims per physical accident.[24]

The first of these three effects argues for an additive model; the last of these three effects argues for a multiplicative model; and the second of these three effects may have both additive and multiplicative components.

Intuition alone is rarely sufficient to optimize a rating model. The minimum bias method allows the pricing actuary to determine the optimal rating structure from the observed loss costs.

COMBINED MODEL

Suppose a classification system has two dimensions: sex of driver and rating territory. Each classification dimension has two values: male vs female and urban vs rural. The male/female rating dimension has a multiplicative effect on loss costs. The rating territory dimension has both a multiplicative and an additive effect on loss costs. We show the structure of this rating model, and we explain how it can be optimized.

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[23] Generalized linear models allows the optimization of even more complex rating models. We hope to provide a companion *Practitioner's Guide* on the use of generalized linear models for classification ratemaking.

[24] See Conners and Feldblum [1998] for the effects on private passenger automobile insurance.

For the male/female classification dimension, we use rate relativities of s_1 and s_2. For the urban/rural dimension, each class has two relativities: a multiplicative relativity denoted by t_1 and t_2, and an additive relativity denoted by z_1 and z_2. We denote the base rate as B.

The indicated pure premium for any class is $B \times (s_i \times t_j + z_j)$. The subscripts "i" and "j" denote the classification dimension. The indicated pure premiums are shown in the table below.

Observed Loss Costs				*Indicated Pure Premiums*	
	Urban	*Rural*		t_1, z_1	t_2, z_2
Male	r_{11} = \$800	r_{12} = \$500	*Male*	$B \times (s_1 \times t_1 + z_1)$	$B \times (s_1 \times t_2 + z_2)$
Female	r_{21} = \$400	r_{22} = \$200	*Female*	$B \times (s_2 \times t_1 + z_1)$	$B \times (s_2 \times t_2 + z_2)$

If we use the balance principle as the bias function, we balance along the two rows and the two columns. This gives four equations, of which only three are independent, since there is a totality constraint. We must solve for six classification relativities.

We can not solve for the optimal solution by straight-forward iterative methods. We show the procedure used earlier to highlight the problems.

Using the methods explained earlier, we choose starting values for the "t" relativities and the "z" relativities. We balance along the first row to determine the intermediate value for the s_1 (male) relativity, and we balance along the second row to determine the intermediate value for the s_2 (female) relativity.

We discard the values for t_1 and t_2, but we retain the starting values of z_1 and z_2. We balance along the first column to obtain the intermediate value for t_1 (urban), and we balance along the second column to obtain the intermediate value for t_2 (rural).

We discard the values for z_1 and z_2, but we retain the intermediate values of t_1 and t_2. We balance again along the first column to obtain the intermediate value for z_1 (urban), and we balance along the second column to obtain the intermediate value for z_2 (rural).

We have no problem doing the calculations. We show the general equations below, where $r_{i,j}$ is the observed loss cost for class (i,j) and $n_{i,j}$ is the number of exposures for class (i,j).

We multiply both the observed loss costs and the pure premiums by the number of exposures. Balancing along the first row gives us

$$\sum_j n_{1j} \, r_{1j} = \sum_j n_{1j} \, (s_1 \times t_j + z_j)$$

We are solving for s_1, going across the row, and using all the *j* values. We transpose this equation to give

$$s_1 = \sum_j n_{1j} (r_{1j} - z_j) \div \sum_j n_{1j} t_j$$

For the general equation, we substitute the *i* subscript for the "1" subscript in the *s, n,* and *r* variables, to give

$$s_i = \sum_j n_{ij} (r_{ij} - z_j) \div \sum_j n_{ij} t_j$$

We balance along the columns, using the first column as our example:

$$\sum_i n_{i1} r_{i1} = \sum_i n_{i1} (s_i \times t_1 + z_1)$$

We transpose this equation to solve for either *t* or for *z*. Solving the equation for t_j yields

$$t_j = \sum_i n_{ij} (r_{ij} - z_j) \div \sum_i n_{ij} s_i.$$

This looks like the equation for the *s* variables, with one difference. When we solve for *s*, we have z_j in the formula. When we solve for s_i, the formula uses all the *z* values. When we solve for *t*, we have a particular *z* value in the formula (e.g., when we solve for t_2, we have z_2 in the formula). Similarly, we can solve for the *z* variables to get

$$z_j = \sum_i n_{ij} (r_{ij} - s_i t_j) \div \sum_i n_{ij}$$

The series will not necessarily converge. If the series does converge, it does not necessarily have a unique limit. In a multi-dimensional combined multiplicative and additive model, there are many more relativities than there are equations when the balance principle is used as the bias function.

If we use a least squares or a χ-squared bias function, the combined model is not conceptually different from a simple multiplicative or additive model. We form the least squares or χ-squared expression in the same manner as did above. We set the partial derivative with respect to each rating variable equal to zero. We have the same number of equations as we have rating variables.

Caveats

The use of the minimum bias procedure with combined models is a powerful rating tool. But as the rating models grow more complex, the potential rating errors become more serious.

If there are a large number of exposures in each class, the optimization procedure is less likely to be distorted by random loss fluctuations. As the number of exposures in each cell decreases, the effects of random loss fluctuations become more serious.

OUTLIERS

The least squares and χ-squared bias functions are sensitive to outliers. Outliers are observed values that differ substantially from the expected values because of random loss fluctuations. Distortions stemming from random loss fluctuations can be controlled in several ways.

- Losses can be capped at basic limits or similar retentions.
- Low volume classes can be assigned limited credibility.
- The data in each cell can be examined for unusual values.

The use of low retentions or low credibility conflicts with the objective of basing rates on observed experience as much as possible. The examination of the observed data for unusual values is too time consuming for the exigencies of practical work.

Rather, the bias function should be chosen so that the results are not too sensitive to outliers.

Illustration: The classification system has two dimensions: male/female along one dimension and ten territories along the other dimension. The current driver relativities are 1.000 for female and 2.000 for male. The current territorial relativities are 1.000, 2.000, . . ., 10.000 for the ten territories, labeled (01, 02, . . . , 10). The base rate is $100, and a multiplicative model is used.

Scenario A: The observed loss costs are shown below, in units of 100 dollars.

Territory:	01	02	03	04	05	06	07	08	09	10
Male	$2	$4	$6	$8	$10	$12	$14	$16	$18	$20
Female	$1	$2	$3	$4	$5	$6	$7	$8	$9	$10

The observed loss costs exactly match the indicated pure premiums in the current rating system. All three bias functions discussed so far – balance principle, least squares, and χ-squared – would indicate retention of the current relativities.

Scenario B: Because of a random large loss, the observed loss costs for the males in territory 10 are $10,000 instead of $2,000. The "territory 10 / male" cell shows $100 instead of $20. This type of random loss fluctuation is common in classification analysis for small populations.

We have starting values of (1.000, 2,000, . . ., 10.000) for the ten territories. We determine the intermediate value for the male relativity.

The balance principle selects the male relativity "s_1" such that

$$(s_1 \times t_1) + (s_1 \times t_2) + \ldots + (s_1 \times t_{10}) = r_{1,1} + r_{1,2} + \ldots + r_{1,10}$$
$$s_1 \times \$55 = \$190$$
$$s_1 = 3.455$$

The least squares bias function selects the male relativity to minimize the squared error

$$\sum (r_{1,i} - s_1 \times terr_i)^2$$

We set the partial derivative with respect to s_1 equal to zero:

$$\sum (r_{1,i} - s_1 \times terr_i) \times (-terr_i) = 0$$

$$s_1 = \sum (r_{1,i} \times terr_i) / \sum terr_i^2 =$$

$$[(1\times2) + (2\times4) + (3\times6) + \ldots + (9\times18) + (10\times100)] / [1^2 + 2^2 + 3^2 + \ldots 9^2 + 10^2] = 4.078$$

Compared with the balance principle, the least squares bias function exacerbates the distortion caused by random loss fluctuations. In this instance, the χ-squared bias function magnifies the distortion less than the least squares bias function does. In other instances, the χ-squared bias function magnifies the distortion more than the least squares bias function does. Since combined models are more sensitive to random loss fluctuations than simple models are, and since the least squares or χ-squared bias function must be used, the pricing actuary must be particularly careful to exclude outliers from the data.

Other Bias Functions

Summary: We examine other bias functions, beginning with the χ-squared function and the squared error function. We continue with our simple 2 by 2 illustration for both additive and multiplicative models using these bias functions.

We review arguments for and against specific bias functions. We examine two goodness-of-fit tests – average absolute error and χ-squared – and we consider the relationship between the bias function chosen and the goodness-of-fit test.

We review the maximum likelihood bias function and the distributions commonly used with it. We discuss some of the potential advantages and drawbacks of the more sophisticated models compared to the balance principle.[25]

SQUARED ERROR AND X-SQUARED

Let us return to the simple illustration with which we began, as reproduced below.

	Urban	Rural		$terr_1$	$terr_2$
Male	\$800	\$500	sex_1	$200 \times s_1 \times t_1$	$200 \times s_1 \times t_2$
Female	\$400	\$200	sex_2	$200 \times s_2 \times t_1$	$200 \times s_2 \times t_2$

The left-hand side of the matrix shows the observed loss costs; the right-hand side shows the indicated pure premiums. Our objective is to pick classification relativities such that the indicated pure premiums are "as close as possible" to the observed loss costs.

We used the balance principle earlier to fit the classification relativities. We did not attempt to justify the balance principle; we provide its justification further below.

Statisticians would normally fit the classification relativities using other methods, such as:

1. Minimize the average absolute error between the indicated and observed figures.

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[25] This *Practitioner's Guide* summarizes three seminal papers on the minimum bias procedure:

- Bailey and Simon [1960] recommends the χ-squared bias function, based on a credibility argument.
- Bailey [1963] recommends the balance principle, based on a bias argument.
- Brown [1988] investigates other bias functions, based on statistical arguments.

This *Guide* highlights the implications of the various bias functions, though it leaves conclusions to the reader.

2. Minimize the sum of the squared errors between the indicated and observed figures (i.e., the least squares bias function)
3. Minimize the sum of the relative squared errors between the indicated and observed figures (i.e., minimize the χ-squared error).
4. Maximize the likelihood of obtaining the observations given the classification relativities.

Minimizing the average absolute error makes sense to practicing actuaries. Minimizing the average absolute error is rarely used in statistics, perhaps because it was once thought to be mathematically intractable.[26] The three other methods result in relatively simple iterative equations for the minimum bias procedure.

We use the average absolute error as one of the goodness-of-fit tests. Given a set of classification relativities, it is easy to calculate the average absolute error. It is not easy to determine the set of classification relativities that minimizes the average absolute error.

Methods 2 and 3 – least squares and χ-squared are similar. We first show the procedures, and then we discuss the intuition for each.

Squared Error

The squared error for each cell is the square of the difference between the observed pure premium and the indicated pure premium. For urban male drivers in our basic illustration, this number is $(\$800 - \$200 \times s_1 \times t_1)^2$.

We sum the squared errors for the four cells to get (SE = sum of squared errors):

SE	$= (\$800 - \$200 \times s_1 \times t_1)^2$	urban male
	$+ (\$500 - \$200 \times s_1 \times t_2)^2$	rural male
	$+ (\$400 - \$200 \times s_2 \times t_1)^2$	urban female
	$+ (\$200 - \$200 \times s_2 \times t_2)^2$	rural female

To minimize the sum of the squared errors, we take partial derivatives with respect to each variable and set them equal to zero. For the "male" classification relativity ("s_1"), we have

$$0 = \partial SE/\partial s_1 = 2 \times (\$800 - \$200 \times s_1 \times t_1) \times (-\$200 \times t_1) + 2 \times (\$500 - \$200 \times s_1 \times t_2) \times (-\$200 \times t_2)$$

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[26] Compare Cook [1967], page 200: "Why then do we use the method of least squares? Simply because absolute values are alleged to be mathematically inconvenient." Cook provides an algorithm for minimizing the average absolute error, which is simple to compute by pencil and paper and even easier to program in a spreadsheet. It would be useful to compare a minimum bias procedure based on Cook's algorithm with the methods in this *Practitioner's Guide*. [Charles Cook, "The Minimum Absolute Deviation Trend Line," Proceedings of the CAS, vol 80 [1967], pages 200-204.

We need to consider the cells only in the male (s_1) row. The other cells do not have an s_1 term in the squared error, so the partial derivative with respect to s_1 is zero.

Taking partial derivatives with respect to each of the classification relativities gives us four equations in four unknowns. The equations are not linear, so we use iteration to solve them.

Let us choose the same starting values for the squared error bias function as we chose for the balance principle (namely $t_1 = 2$ and $t_2 = 1$):

	Urban	Rural		$terr_1 = 2$	$terr_2 = 1$
Male	\$800	\$500	sex_1	$200 \times s_1 \times 2$	$200 \times s_1 \times 1$
Female	\$400	\$200	sex_2	$200 \times s_2 \times 2$	$200 \times s_2 \times 1$

Using the squared error bias function, we solve for the male relativity (s_1):

$$0 = \partial SE/\partial s_1 = 2 \times (\$800 - \$200 \times s_1 \times 2) \times (-\$200 \times 2) + 2 \times (\$500 - \$200 \times s_1 \times 1) \times (-\$200 \times 1)$$

To avoid dealing with multiples of 10, we choose a base rate of \$2 and we evaluate the observed pure premiums in multiples of \$100.

$$0 = \partial SE/\partial s_1 = 2 \times (\$8 - \$2 \times s_1 \times 2) \times (-\$2 \times 2) + 2 \times (\$5 - \$2 \times s_1 \times 1) \times (-\$2 \times 1)$$

$$-64 + 32s_1 - 20 + 8s_1 = 0$$
$$40s_1 = 84$$
$$s_1 = 2.1$$

Similarly, we solve for the female relativity (s_2):

$$0 = \partial SE/\partial s_2 = 2 \times (\$400 - \$200 \times s_2 \times 2) \times (-\$200 \times 2) + 2 \times (\$200 - \$200 \times s_2 \times 1) \times (-\$200 \times 1)$$

Simplifying as before, we get

$$0 = \partial SE/\partial s_2 = 2 \times (\$4 - \$2 \times s_2 \times 2) \times (-\$2 \times 2) + 2 \times (\$2 - \$2 \times s_2 \times 1) \times (-\$2 \times 1)$$

$$-32 + 32s_2 - 8 + 8s_2 = 0$$
$$-40 + 40s_2 = 0$$
$$s_2 = 1$$

We now discard the starting values of $t_1 = 2$ and $t_2 = 1$. Using the intermediate values of $s_1 = 2.1$ and $s_2 = 1$, we set the partial derivatives of the sum of the squared errors with respect to t_1 and t_2 equal to zero and we solve for new values of t_1 and t_2. We continue in this fashion until the series converges.

Squared Error Intuition

The properties of squared error minimization in the minimum bias procedure are unlike the properties of squared error minimization in other statistical problems, as explained below. We note first that the bias function makes a difference, even in this simple illustration.

A. The balance principle bias function gives $s_1 = 13/6 = 2.167$ and $s_2 = 1$.
B. The squared error bias function gives $s_1 = 2.100$ and $s_2 = 1$.

The balance principle looks at the errors; the squared error bias function looks at the square of the errors. The χ-squared bias function looks at the square of the errors relative to the expected value. The squared error and χ-squared bias functions place more weight on outlying cells, where the squares of the errors are large. The balance principle and the squared error bias function place more weight on the cells with large dollar values. The χ-squared bias function weights all cells more evenly.

Illustration: A classification system with two dimensions has male vs female in one dimension and territories 1, 2, and 3 in the other dimension. The starting relativities are 1.00, 2.00, and 3.00 for territories 1, 2, and 3. The observed loss costs for the three territories in the male row are $2, $4, and $12, with equal exposures in each cell.

	territory 1 (1.00)	territory 2 (2.00)	territory 3 (3.00)
male	$2.00	$4.00	$12.00
female	—	—	—

We want to determine the indicated relativity for males. Our concern here is not to solve this problem but to understand the effects of the different bias functions. We examine the effects of different choices for the male relativity.

- If the male relativity is 2.00, the indicated pure premiums are $2, $4, and $6. The first two cells have a perfect fit, and the third cell is too low by $6.

- If the male relativity is 4.00, the indicated pure premiums are $4, $8, and $12. The first two cells are too high by a total of $6, and the third cell has a perfect fit.

The balance principle considers the first power of the errors. To achieve balance, we choose a male relativity of 3.00. The indicated pure premiums are \$3, \$6, and \$9. The first two cells are too high by a total of \$3, and the third cell is too low by \$3.

The squared error bias function is more concerned with large errors than with small errors. We are more concerned with the error for territory 3, which is relatively large, than with the errors for territories 1 or 2, which are relatively small. To minimize the sum of squared errors, we increase the male relativity slightly, thereby reducing the error in territory 3 and increasing the errors in territories 1 and 2.

To solve this problem using the squared error bias function, we minimize the following expression:

$$(2-x)^2 + (4-2x)^2 + (12-3x)^2.$$

Taking the partial derivative with respect to "x" and setting it equal to zero gives

$$2(2-x)(-1) + 2(4-2x)(-2) + 2(12-3x)(-3) = 0$$

$$4 + 16 + 72 = 2x + 8x + 18x$$
$$92 = 28x$$
$$x = 92 / 28 = 3.286$$

SQUARED ERROR MINIMIZATION

Upon reflection, the illustration above seems odd to some statisticians. We are choosing a value to minimize the squared error among a series of observations. An elementary statistical theorem, which we review below, is that the average minimizes the sum of the squared errors. This seems inconsistent with the comments above.

Were we dealing with a single classification dimension, squared error minimization indeed produces the arithmetic average. The following illustration explains this statement.

Illustration: We are measuring a patient's fever with an old thermometer that is in poor working order. The thermometer is unbiased, but it is very inaccurate, and the observed readings are highly distorted by sampling error. We perform nine trials, and we observe readings of (100.1, 100.2, . . . , 100.9). The readings were not in this order, so there is no observed trend; we have simply arranged them in ascending numerical order. Using the least squared error function, we wish to determine the best estimate of the patient's temperature.

We rephrase the illustration mathematically. We have observed values of $z_1, z_2, \ldots, z_n$, and we must choose a single value for the z's – call it z^* – to minimize the squared error.

The sum of the squared errors is $\sum (z_i - z^*)^2$. The partial derivative of this sum with respect to z^* is $\sum 2(z_i - z^*)(-1)$. Setting this equal to zero gives $z^* = \sum z_i \div n$. The indicated z^* is the average of the z_i's.

In the temperature measurement illustration, the average of the nine observations is 100.5. This is the solution using the squared error bias function.

If we had chosen instead some other value, such as 100.3, we could correct this estimate by the average of the errors. The error in each observation is the observation minus 100.3. This is the series (–0.02, –0.01, 0, +0.01, . . . , +0.06). The average is +0.02. The corrected estimate is 100.3 + 0.02 = 100.5.

Multi-Dimensional Systems

This is not true for multi-dimensional systems. In a multiplicative model with two dimensions, the z_i's are the observed values. The z^* is the indicated relativity for one of the two dimensions. The other dimension has relativities of $y_1, y_2, \ldots, y_n$.

The sum of the squared errors is $\sum (z_i - y_i \times z^*)^2$. The partial derivative of this sum with respect to z^* is $\sum 2(z_i - y_i \times z^*)(-y_i)$.

Setting this equal to zero gives $z^* = \sum z_i \div \sum y_i^2$.

The indicated z^* is no longer the average of the z_i's. Rather, this result is the solution to the minimum bias procedure using the squared error bias function, as we show next.

BALANCE PRINCIPLE OPTIMIZATION

Let us contrast squared error minimization with the balance principle. When we deal with a single classification dimension, squared error minimization produces the arithmetic average. The balance principle selects the multi-dimensional equivalent to the mean.

The balance principle provides the economically optimal solution to the minimum bias problem. This is the economic corollary to Bailey's 1963 statement that the balance principle provides the only unbiased solution; see below.[27]

GENERAL SOLUTION

Throughout this study note, we solve the elementary 2 by 2 illustration before deriving the general formulas. The general formulas require readers to keep too many subscripts in mind. Although this is not difficult once one is accustomed to the minimum bias procedure, it hampers the initial grasp of the intuition.

Let us consider now a more general two dimensional classification system.

- We still assume one exposure per cell or the same number of exposures per cell. We deal with varying exposures per cell when we deal with credibility.
- The extension to more than two dimensions is straight-forward, though the additional subscripts obscure the intuition.

Suppose we have two dimensions, age of driver and territory, with "n" age classes and "m" territories. The observed loss cost in the i^{th} age class and the j^{th} territory is r_{ij}. The indicated pure premium in the i^{th} age class and the j^{th} territory is $x_i \times y_j$. This is the standard notation for the minimum bias computations.

The squared error in any cell is $(r_{ij} - x_i \times y_j)^2$. The sum of the squared errors is

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[27] By economically optimal, we mean the bias function that maximizes the expected income of the firm in most scenarios. Clearly, there are exceptional scenarios when a different bias function may be better. In a jurisdiction that places restrictions on risk classification, the bias function may have to be changed to accommodate these restrictions. If the ratemaking data contain data errors, these errors should be corrected before the bias function is applied. If the insurer seeks to expand in certain classifications for competitive or marketing reasons, the minimum bias procedure may not accommodate the insurer's strategy. In most scenarios, however, the balance principle serves the economic interests of the firm.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (r_{ij} - x_i y_j)^2$$

We take partial derivatives with respect to each variable and set them equal to zero. We have a total of (n+m) variables, and we have a total of (n+m) equations. The constraints for least squares minimization are the same as the constraints for the balance principle. There is one totality constraint, since taking the sum of the squared errors along the rows is the same as taking the sum of the squared errors along the columns. This means that we have only (n+m−1) equations, since the (n+m) equations are not independent. In addition, we can multiply all the relativities along any dimension by a constant and divide the base rate by the same constant.

The (n+m) equations are not linear, so we must search for a solution by numerical methods. We choose starting values for one dimension – say, the y_j's. To solve for the value of x_1, we take the partial derivative with respect to x_1 and set it equal to zero:

$$\sum_{J=1}^{m} 2(r_{1j} - x_1 y_j)(-y_j) = 0$$

This gives us

$$x_1 = \sum (r_{1j} \times y_j) \div \sum y_j^2.$$

The x_1 is a variable. The summation signs in the last two equations above are over the *j* subscript. The *y* values are fixed; they are not variables once we have assigned starting values to the *y* values.

We repeat this procedure to solve for $x_2, x_3, \ldots, x_n$. Having solved for all the *x* values, we discard the starting *y* values and solve for new values of the *y* variables using the same procedure as for the *x* variables.

ADDITIVE MODEL

We can use an additive model with the least squares bias function. We show first the results for the elementary 2 by 2 illustration.

We repeat the observed loss costs and the indicated pure premiums for the additive model in the 2 by 2 illustration.

	Urban	Rural		$terr_1$	$terr_2$
Male	\$800	\$500	sex_1	$200 + s_1 + t_1$	$200 + s_1 + t_2$
Female	\$400	\$200	sex_2	$200 + s_2 + t_1$	$200 + s_2 + t_2$

As mentioned earlier, there are three mathematically equivalent ways of defining the additive model; the solution method is the same for each of them. The rate in cell x_iy_j is

A. Base rate + x_i + y_j,
B. Base rate × (1 + u_i + v_j), or
C. Base rate × (p_i + q_j)

We use the first of these three equations for the intuition here, though we would use one of the other two methods in practice, thereby avoiding the need to adjust the relativities for inflation each year. A multiplicative relationship between the base rate and the relativities does not make the model multiplicative. Since the relationship among the factors is additive, the model is additive. A combined multiplicative and additive model has relationships among the relativities that are both multiplicative and additive; see below.

For the male urban cell, the squared error is $(\$800 - \$200 - s_1 - t_1)^2$. The sum of the squared errors for all four cells is

$$
\begin{aligned}
& (\$800 - \$200 - s_1 - t_1)^2 \\
+ \; & (\$500 - \$200 - s_1 - t_2)^2 \\
+ \; & (\$400 - \$200 - s_2 - t_1)^2 \\
+ \; & (\$200 - \$200 - s_2 - t_2)^2
\end{aligned}
$$

We take partial derivatives with respect to each variable and set them equal to zero. The partial derivative with respect to s_1 is

$$2(\$800 - \$200 - s_1 - t_1)(-1) + 2(\$500 - \$200 - s_1 - t_2)(-1) = 0.$$

or

$$s_1 = (\$900 - t_1 - t_2) \div 2.$$

For the additive model with the least squares bias function, the simultaneous equations are linear, and we can solve them directly. Nevertheless, it is easier to program the solution using numerical methods.

If we choose starting values of t_1 = \$250 and t_2 = \$0, we get s_1 = \$325.

GENERAL FORMULA

For the general formula, we let B = the base rate. The sum of the squared errors is

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(r_{ij} - B - x_i - y_j)^2$$

We take the partial derivative with respect to x_1 and set it equal to zero:

$$\sum_{j=1}^{m} 2(r_{1j} - B - x_1 - y_j)(-1) = 0$$

or

$$x_1 = \sum(r_{1j} - y_{1j})/m - B$$

where the summation is over the *j* subscript.

THE BIAS FUNCTION

The optimal relativities depend on the choice of the bias function. The choice of bias function can be viewed from three perspectives.

1. Mathematical tractability
2. Social equity
3. Economic optimization

Mathematical tractability is of most concern when some bias functions give simple relationships and some bias functions give equations that defy simple solutions. For the minimum bias procedure, we get relatively simple equations for the bias functions discussed in this paper. We do not get simple equations if we use the average absolute error as the bias function, so we do not consider that method.[28]

Social equity is subjective, though it is vital to the success of a highly regulated industry like insurance. The balance principle sometimes results in large errors for outlying cells. The errors are particularly large in absolute value for high rated cells. If a multiplicative model is

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[28] With modern spreadsheets, the average absolute error no longer poses tractability issues. Just like the solution for the balance principle is the mean, the solution for the average absolute error is the median. It is not uncommon for actuaries to use the median instead of the mean in practical problems.

used when an additive model is more appropriate, the errors for outlying cells are frequently overcharges. The squared error bias function reduces these large errors.[29]

Of the bias functions which we consider in this paper, the squared error bias function is the best at reducing large overcharges for individual cells. Analysts who are concerned with large overcharges might prefer the squared error bias function. Ferreira's critique of insurance industry classification systems in Massachusetts illustrates this social position.[30]

Economic optimization drives the behavior of firms in free markets. There is disagreement regarding these economic forces in complex markets, but the major attributes of these forces can be described.

We take the perspective of the firm (the insurer), not the perspective of the consumer. Firms seek to maximize profits and to minimize losses (among other firm objectives). Suppose an insurer issues 3 policies. It must choose between two rating systems.

A. Under rating system A, it loses $1 each on the first two policies and it breaks even on the third policy.
B. Under rating system B, it breaks even on the first two policies and it loses $1.50 on the third policy.

Rating system A is off by $2 using the balance principle while rating system B is off by $1.50. Using the squared error bias function, rating system A is off by $2 while rating system B is off by $2.25. The balance principle says we should choose rating system B, and the squared error bias function says we should choose rating system A.

The economic principle of profit maximization (or loss minimization) says we should choose rating system B, as the balance principle says. Using our simple assumptions, the profit maximization principle generally agrees with the balance principle.

Economic forces are not trivial. There are many economic reasons for avoiding large errors, including consumer dissatisfaction, consumer switching, and public relations. In democratic

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29 This is the same as saying that the least squares bias function is sensitive to outliers, since a single large outlier can significantly change the results when the least squares bias function is used.

30 See Ferreira [1978], as well as Cummins *et al.*, *Risk Classification in Life Insurance*, chapter 4, pages —. We are not endorsing Ferreira's views, which are inconsistent with competitive insurance markets. See the continuing discussion in the text of this *Practitioner's Guide*. [Ferreira, Joseph Jr., "Identifying Equitable Insurance Premiums for Risk Classes: An Alternative to the Classical Approach," in Andrew F. Giffin, Vincent Travis, and William Owen (eds.), *Automobile Insurance Risk Classification: Equity and Accuracy* (Boston: Massachusetts Division of Insurance, 1978), pages 74-120.][Cummins, J. David, Barry D. Smith, R. Neil Vance, and Jack L. VanDerhei, *Risk Classification in Life Insurance* (Boston: Kluwer Academic Publishers, 1983).]

systems where social opinion and political pressures are strong, firms may sacrifice short-term profit maximization to achieve other ends, such as workforce diversity and environmental protection. Furthermore, manager incentives may encourage the pursuit of other goals, such as corporate growth instead of profit maximization. Nevertheless, profit maximization remains the dominant corporate goal. The pricing actuary should keep these social and economic desiderata in mind when choosing a bias function for the minimum bias procedure.

X-SQUARED

The χ-squared bias function is similar to the squared error bias function, except that each "bias" is divided by the expected value.

Let us return to the simple illustration with which we began, as reproduced below.

	Urban	Rural		$terr_1$	$terr_2$
Male	$800	$500	sex_1	$200 \times s_1 \times t_1$	$200 \times s_1 \times t_2$
Female	$400	$200	sex_2	$200 \times s_2 \times t_1$	$200 \times s_2 \times t_2$

The χ-squared value for each cell is (the square of the difference between the observed loss cost and the indicated pure premium) divided by the indicated pure premium. For urban male drivers in our basic illustration, this number is

$$(\$800 - \$200 \times s_1 \times t_1)^2 \div (\$200 \times s_1 \times t_1).$$

We sum the squared errors for the four cells to get the sum of χ-squared values:

$$\begin{aligned}
&= (\$800 - \$200 \times s_1 \times t_1)^2 \div (\$200 \times s_1 \times t_1) \text{ urban male}\\
&+ (\$500 - \$200 \times s_1 \times t_2)^2 \div (\$200 \times s_1 \times t_2) \text{ rural male}\\
&+ (\$400 - \$200 \times s_2 \times t_1)^2 \div (\$200 \times s_2 \times t_1) \text{ urban female}\\
&+ (\$200 - \$200 \times s_2 \times t_2)^2 \div (\$200 \times s_2 \times t_2) \text{ rural female}
\end{aligned}$$

To minimize the sum of the squared errors, we take partial derivatives with respect to each variable and set them to zero. We use the quotient rule:

$$\text{If } y(x) = f(x)/g(x), \text{ then } \partial y/\partial x = [g(x) \times \partial f/\partial x - f(x) \times \partial g/\partial x] / g^2(x).$$

For the "male" classification relativity ("s_1"), we have

$$\begin{aligned}
0 = \partial SE/\partial s_1 &= [\, (\$200 \times s_1 \times t_1) \times 2 \times (\$800 - \$200 \times s_1 \times t_1) \times (-\$200 \times t_1)\\
&\quad - (\$800 - \$200 \times s_1 \times t_1)^2 \times (\$200 \times t_1)\,] / (\$200 \times s_1 \times t_1)^2\\
&\quad + [\, (\$200 \times s_1 \times t_2) \times 2 \times (\$500 - \$200 \times s_1 \times t_2) \times (-\$200 \times t_2)\\
&\quad - (\$500 - \$200 \times s_1 \times t_2)^2 \times (\$200 \times t_2)\,] / (\$200 \times s_1 \times t_2)^2
\end{aligned}$$

Although the arithmetic looks cumbersome, the equation can be reduced to a simple form. To avoid needless arithmetic, we show the general solution, and we resume the illustration after deriving the appropriate recursive equation.

χ-SQUARED RECURSIVE EQUATIONS

We show the general recursive equations for the χ-squared bias function. The horizontal axis is the "j" dimension, and the vertical axis is the "i" dimension. We show two dimensions with two classes in each dimension to aid visualization of the example. The equations themselves have no constraints on the number of classes in each dimension. The extension of the equations to three or more dimensions is straight-forward.

	Urban	Rural		$terr_1$	$terr_2$
Male	\$800	\$500	sex_1	$200 \times s_1 \times t_1$	$200 \times s_1 \times t_2$
Female	\$400	\$200	sex_2	$200 \times s_2 \times t_1$	$200 \times s_2 \times t_2$

We form the χ-squared bias function as a double summation covering all the cells in the array.

$$\sum \sum (n_{ij} r_{ij} - n_{ij} x_i y_j)^2 / n_{ij} x_i y_j$$

We factor out the number of exposures from the equation to give

$$\sum \sum n_{ij} (r_{ij} - x_i y_j)^2 / x_i y_j$$

We seek to minimize the χ-squared value.

Given starting values for either dimension we determine the intermediate values for the other dimension. Assume we have chosen starting values for the "y" relativities and we are solving for the intermediate value of x_i. Only the cells in the "i"th row have terms withe x_i in them. We take the partial derivative of this row with respect to x_i, and we set it equal to 0.

We use the quotient rule for taking derivatives: if $f(x) = g(x)/h(x)$, then $\partial f/\partial x = [h(x) \times \partial g/\partial x + g(x) \times \partial h/\partial x] / h^2(x)$.[31]

In the equation below, we take the summation over the "j" dimension. The value of "i" is fixed.

$$\sum n_{ij} [x_i y_j 2(r_{ij} - x_i y_j) \times (-y_j) - (r_{ij} - x_i y_j)^2 y_j] / (x_i y_j)^2 = 0$$

The value $x_i = 0$ will not minimize this equation, so we can multiply both sides of the equation by $(x_i)^2$. We separate the left side of the equation into two fractions, and we factor out $(y_j)^2$ from the first fraction and y_j from the second fraction:

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[31] This is the same as the product rule for taking derivatives, with $f(x) = g(x) \times (1/h(x))$.

$$\sum -2 \times n_{ij}\, x_i\, (r_{ij} - x_i\, y_j) - (n_{ij} / y_j)\, (r_{ij} - x_i\, y_j)^2 = 0$$

We expand the square and we combine like terms:

$$\sum -2 \times n_{ij}\, x_i\, r_{ij} + 2 \times n_{ij}\, x_i^2 y_j - (n_{ij} / y_j)\, (r_{ij}^2 - 2\, r_{ij}\, x_i\, y_j + x_i^2\, y_j^2) = 0$$

$$\sum -2 \times n_{ij}\, x_i\, r_{ij} + 2 \times n_{ij}\, x_i^2 y_j - (n_{ij} / y_j)\, (r_{ij}^2) + 2 \times n_{ij}\, x_i\, r_{ij} - n_{ij}\, x_i^2\, y_j = 0$$

$$\sum\ n_{ij}\, x_i^2 y_j - (n_{ij} / y_j)\, (r_{ij}^2) = 0$$

This gives a relatively simple expression for each x_i in terms of the y_j values:

$$x_i = [\ \sum\ (n_{ij} \times r_{ij}^2 / y_j) / \sum\ n_{ij}\ y_j\ ]^{0.5}$$

For the illustration, there is one exposure in each cell. The starting values are $y_1 = 2$ and $y_2 = 1$. We use a base rate of \$200, and we divide all cells by \$200.

	Urban	Rural		$terr_1 = 2$	$terr_2 = 1$
Male	\$4	\$2.5	sex_1	$s_1 \times 2$	$s_1 \times 1$
Female	\$2	\$1	sex_2	$s_2 \times 2$	$s_2 \times 1$

Using the χ-squared bias function along the first row, we get

$$s_1 \text{ (male relativity)} = [(½ \times 4^2 + 1 \times 2.5^2) / (2 + 1)]^{0.5} = 2.179.$$

Using the χ-squared bias function along the second row, we get

$$s_2 \text{ (female relativity)} = [(½ \times 2^2 + 1 \times 1^2) / (2 + 1)]^{0.5} = 1.000.$$

The male to female relativity is 2.179 to 1.

The least squares bias function gave a relativity of 2.1 to 1. The dollar values in the urban-male cell are larger than the dollar values in the rural-male cell, so the least squares bias function gives more weight to the urban-male cell as compared to the rural-male cell than the χ-squared bias function does.

ADDITIVE MODEL WITH Χ-SQUARED

The χ-squared bias function can be used with any type of model, whether multiplicative, additive, or combined. If an additive model is used, we minimize the following expression:

$$\sum \sum n_{ij} \, (r_{ij} - x_i - y_j)^2 \, / \, (x_i + y_j)$$

We set the partial derivative with respect to each relativity equal to zero. It is easiest to solve the resulting set of simultaneous equations by iteration. Bailey and Simon [1960], followed by Brown [1988] give the recursive equations as

$$\Delta x_i = \frac{\sum_j n_{i,j} \left(\frac{r_{i,j}}{x_i + y_j} \right)^2 - \sum_j n_{i,j}}{2 \sum_j n_{i,j} \left(\frac{r_{i,j}}{x_i + y_j} \right)^2 \left(\frac{1}{x_i + y_j} \right)}$$

The form of the recursive equation is not as simple as that for other rating models, but the work needed to implement this model in a spreadsheet is not significantly greater.[32]

[32] We have not used this model in our own applications, and we have not attempted to verify the recursive equation or to examine the appropriateness of this model for specific scenarios.

GOODNESS-OF-FIT

There are various rating models, and there are various bias functions. For a given rating model and bias function, the minimum bias procedure optimizes the relativities. We now wish to optimize the rating system by choosing the best rating model and bias function.

The optimal procedure depends on two items.

- The choice of rating model, such as multiplicative, additive, or combined, depends on the characteristics of the observed loss costs. For some types of coverage, a multiplicative model is preferred; for other types of coverage, an additive model is preferred.
- The choice of the bias function depends on the objective.
  - The statistician seeking the best fit might use a maximum likelihood function if a tractable distribution function is appropriate for this coverage or a χ-squared function if the probability distribution function is not known or not tractable.
  - The regulator seeking to avoid large dollar mismatches between observed loss costs and indicated pure premiums might use a least squares function.
  - The insurer seeking to avoid monetary losses might use the balance principle.

Each bias function may be associated with a particular use. The preferences listed above are possibilities; other preferences are also possible. In particular, a regulator might prefer the balance principle to provide the most efficient rating system.

- The objective of avoiding large dollar overcharges and undercharges is a dubious goal. It is not always compatible with free markets, and introduces inefficiencies into the insurance system.
- A pricing actuary working with a new line of business might prefer the χ-squared function to examine whether the rating system chosen is compatible with the observed loss costs.

Empirical Tests

We can test the choice of rating model empirically.

Illustration: We are using a χ-squared bias function to optimize classification relativities. We do not know whether a multiplicative model or an additive model is more appropriate.

We perform the minimum bias procedure twice: once with the multiplicative model and a χ-squared bias function and once with an additive model and a χ-squared bias function. After optimizing the relativities for each model, we compare the final χ-squared difference between the observed loss costs and the indicated pure premiums for each model. The model with the lower χ-squared value is preferred.

Illustration: We are using the balance principle to optimize classification relativities. We do not know whether a multiplicative model or an additive model is more appropriate.

We perform the minimum bias procedure twice: once with the multiplicative model and the balance principle and once with an additive model and the balance principle. After optimizing the relativities for each model, we compare the average absolute difference between the observed loss costs and the indicated pure premiums for each model. The model with the lower average absolute difference is preferred.

We can not empirically test the suitability of the bias function. The illustration below explains why.

Illustration: We are using a multiplicative model, and we are deciding between the balance principle and the χ-squared function.

We perform the minimum bias procedure twice: once with the multiplicative model and the balance principle and once with the multiplicative model and a χ-squared bias function.

If we use a χ-squared function to measure the difference between the observed loss costs and the indicated pure premiums to test the performance of the two models, the χ-squared bias function does better. This result is tautological, since the χ-squared bias function minimized the χ-squared difference between the observed loss costs and the indicated pure premiums.

If we use the average absolute difference between the observed loss costs and the indicated pure premiums to test the performance of the two models, the balance principle does better.[33] The χ-squared bias function minimizes large percentage errors. The balance principle and the average absolute difference minimize dollar differences.

The choice of bias function is a qualitative choice, depending on the objectives of the rating system. It is not subject to a quantitative test of suitability. We examine these qualitative issues in the following section of this *Practitioner's Guide.*

Empirical tests of actual insurance rating systems may help dispel some of the rancor in public policy decisions. Some persons have criticized the insurance industry for using multiplicative models that overcharge high rated classifications. It has been suggested that an additive model might be more equitable.

Many insurers tend to view this criticism as politically motivated, intended to curry support among urban voting blocs. An empirical test of a multiplicative model against an additive model should help resolve some of the actuarial questions.

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[33] This result is (perhaps) not always true, but exceptions are rare.

The relative merits of a multiplicative versus an additive model are unclear. In their 1960 *Proceedings* paper, Bailey and Simon concluded that an additive model was preferred to a multiplicative model for the Canadian private passenger automobile data. In his 1988 *Proceedings* paper, Rob Brown concluded that a multiplicative model was preferred to an additive model for the Canadian private passenger automobile data. In his discussion of Brown's paper, Gary Venter has suggested that a combined multiplicative and additive model might be superior to either of the models tested.

SQUARED ERROR VS X-SQUARED

The squared error bias function is similar to the χ-squared bias function. We examine the relative advantages of each.

The χ-squared test looks at percentage differences; the squared error test looks at absolute differences. For fitting distributions, statisticians often prefer the χ-squared test to a least squares test.

Illustration: We are fitting a distribution to two empirical data points.

- Point A has an observed value of $101 and a fitted value of $100.
- Point B has an observed value of $1.50 and a fitted value of $1.00.

We examine the errors for each point.

- The squared error is $(101 - 100)^2 = 1.00$ for point A and $(1.50 - 1.00)^2 = 0.25$ for point B. Point B fits better.
- The χ-squared value is $(101 - 100)^2 / 100 = 0.01$ for point A and $(1.50 - 1.00)^2 / 1.00 = 0.25$ for point B. Point A fits better.

The statistician might prefer the χ-squared test to the squared error test. The practical businessperson might argue that the insurance enterprise is not concerned with optimizing a statistical fit. It is concerned with optimizing net income. At point A, the insurer has a gain or loss of $1.00. At point B, the gain or loss is $0.50. The squared error test is preferred.[34]

This argument does not fully reflect the purpose of the minimum bias procedure. The argument would be correct if we fully believed the observed values – that is, if the observed values were fully credible. But if the observed values were fully credible, we would have no need to use the minimum bias procedure; we would just use the rates indicated by the observed loss costs in each cell.

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[34] This is similar to the argument that we discussed earlier when comparing the balance principle bias function and the squared error bias function.

We are using the minimum bias procedure because the individual observed values are not fully credible, and we believe that the relationships among all the cells in the observed matrix provides useful information for choosing the true expected values.

When we say that a particular fit "X" has less of an error than another fit "Y," we do not mean that fit "X" will produce a smaller error in the future period. Rather, we mean that fit "X" is probably closer to the true values of the cells, and so it is a better pricing procedure than fit "Y" is. Our assumption is that we don't know the true expected loss costs. The χ-squared bias function does a better job showing us the true expected loss costs than the squared error bias function does.

The 1960 Bailey and Simon paper says (page 10) in defense of the χ-squared bias function:

> *The indication of each group should be given a weight inversely proportional to the standard deviation of the indication.*

This is a traditional justification for classical credibility, as Bailey and Simon continue:

> *The standard deviation of the indication is inversely proportional to the square root of the expected number of losses for the group.*[35]

BALANCE PRINCIPLE VS X-SQUARED

In the preceding sections, we compared the χ-squared bias function to the squared error bias function, and the balance principle to the squared error bias function. We now compare the χ-squared bias function with the balance principle.

We can not give an unequivocal answer. The 1960 Bailey and Simon paper prefers the χ-squared bias function, whereas the 1963 Bailey paper argues for the balance principle.[36]

- The χ-squared bias function uses proportional departures; the balance principle does not use proportional departures.

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[35] After the writings of Hans Bühlmann, Gary Venter, Howard Mahler, and others, this statement is no longer accepted uncritically. We do not attempt to judge it more rigorously in this paper.

[36] Another bias function is the absolute proportional departure of the indicated pure premiums from the observed loss costs. The absolute proportional departure is not as mathematically tractable as the other two.

- The balance principle uses the first order departure, which is economically optimal.[37] The χ-squared bias function uses the squared departure, which is not economically optimal.
- The balance principle is unbiased; the χ-squared bias function is not unbiased.

The last statement above warrants explanation. The 1963 Bailey paper argues that the balance principle constrains the relativities so that the total indicated pure premiums along any dimension equal the total observed loss costs along that dimension.

Illustration: If the balance principle is used as the bias function, the total indicated pure premiums for all urban drivers equals the total observed loss costs for all urban drivers. Similarly, the total indicated pure premiums for all male drivers equals the total observed loss costs for all male drivers.

Common Practice

Common practice among casualty actuaries is to use the balance principle, not the χ-squared bias function. One might argue that since more effective procedures drive out less effective procedures in a competitive market, this is an argument in favor of the balance principle.

In truth, many ratemaking procedures were selected for ease of implementation, not necessarily for their mathematical accuracy. The balance principle was easier before the widespread use of desktop computers, and it gained widespread acceptance. Few actuaries have tried the χ-squared bias function or the least squares bias function. No conclusions should be drawn from the common practice among actuaries.

Credibility

Many practitioners combine the minimum bias procedure with credibility weighting of the indicated pure premiums either with the observed loss costs or with the underlying pure premiums. We show illustrations of each method.

Indicated and Observed

The minimum bias procedure gives the indicated pure premiums for each class in an array. The pure premiums used for the final rates is a weighted average of the indicated pure premiums and the observed loss costs for that class. The credibility for the observed loss

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[37] As noted earlier, no bias function is economically optimal in all scenarios. We mean here simply that the balance principle would be the best bias function in most scenarios for a firm seeking to maximize profits.

costs is a function of the volume of business in the class. Classes with greater volume place more weight on the observed loss costs.[38]

Various credibility parameters are used; the classical credibility formulas are most common. Classes with a certain volume of claims or of exposures are given full credibility. The square root rule is used for classes with lower volume of claims or exposures.

Illustration: Suppose that classes with exposure of 10,000 or more car-years are accorded full credibility. A class with 3,600 car-years of exposure, an \$800 observed loss cost, and a \$700 indicated pure premium, is accorded $(3{,}600/10{,}000)^{0.5} = 60\%$ credibility. The credibility weighted pure premium is $60\% \times \$800 + (1 - 60\%) \times \$700 = \$760$.

INDICATED AND UNDERLYING

For premises and operations ratemaking, ISO uses a balance principle minimum bias procedure with observed loss ratios to determine the indicated changes to class group and type of policy relativities.[39]

- An indicated relativity change of 1.08 for type of policy 12 means that the existing relativity for type of policy 12 should be increased by 8%.
- The full credibility standard is based on the number of claims in the class during the experience period. These standards are 2,500 claims for OL&T BI, 3,000 claims for M&C BI, and 7,500 claims for M&C PD.
- Partial credibility is based on the square root rule. For example, 1,080 claims in M&C BI gives $(1{,}080/3{,}000)^{0.5} = 60\%$ credibility.
- The indicated relativity change for the class is raised to the power of the credibility. If the indicated relativity change is 1.08 and the credibility is 60%, the credibility weighted relativity change is $1.080^{0.6} = 1.047$.

These two illustration show different uses of credibility. ISO credibility weights the indicated classification *relativities* with the current classification relativities to dampen the changes from year to year. The first illustration credibility weights the observed loss costs with the indicated pure premiums to increase the accuracy of the final rates.[40]

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[38] See Venter's [1990] discussion of Brown's 1988 paper cited earlier.

[39] See Nancy C. Graves and Richard Castillo, "Commercial General Liability Ratemaking for Premises and Operations," *Pricing* (Casualty Actuarial Society 1990 Discussion Paper Program), Volume II, pages 631-696, for a more complete discussion of the ISO procedure.

[40] Compare Gary Venter's distinction between classical credibility, which is used to minimize rate fluctuations from year to year, and Bayesian-Bühlmann credibility, which is used to increase the accuracy of the estimate.

Embedded Credibility

The minimum bias procedure has credibility embedded in the calculations, since each cell is weighted by the number of exposures in that cell. The traditional credibility weighting in classification ratemaking is embedded in the procedure; it need not be added a second time.

A comparison with the single-dimensional classification ratemaking procedure should clarify this. Suppose there are three territories in a state with the experience shown below. The exposures are car-years, and the dollar figures are in thousands.

	Exposures	Claims	Premium	Losses	Loss Ratio	Indication
Terr 01	$5,000	$500	$5,000	$3,500	70.0%	0.972
Terr 02	$10,000	$1,000	$15,000	$10,800	72.0%	1.000
Terr 03	$2,000	$200	$4,000	$2,980	74.5%	1.035
Total	$17,000	$1,700	$24,000	$17,280	72.0%	

The observed data suggest that

- Territory 01 should have a reduction of 2.8% in its base rate.
- Territory 02 should have no change in its base rate.
- Territory 03 should have an increase of 3.5% in its base rate.

The indications in the table above take no account of the number of exposures or the number of claims in each territory. Since territory 03 has only 200 claims in the experience period, the +3.5% indication may be distorted by random loss fluctuations. To adjust for the volume of business in each territory, the raw indications may be credibility weighted with the overall average of unity, where the credibility depends on the number of exposures or the number of claims.

In the minimum bias procedure, the number or exposures in each cell affects the computation. The weight accorded to the observed loss costs in the cell is proportional to the number of exposures in the cell. From this perspective, credibility weighting the observed loss costs by the number of exposures would be applying credibility twice.

Nevertheless, some justification remains for a credibility adjustment. To determine the indicated pure premium for a cell, the minimum bias procedure uses all the cells in the array and the type of rating model. The credibility embedded in the minimum bias procedure deals with random loss fluctuations. A second credibility adjustment deals with model specification risk. We explain these concepts with an illustration.

Illustration: We are setting classification relativities with a minimum bias procedure. The observed loss costs for young unmarried urban male drivers is $2,500 per car. The indicated pure premiums for these drivers is $3,000 per car. There are two explanations for the difference.

1. The rating model is correct, and random loss fluctuations account for the difference. Random loss fluctuations may have reduced the observed loss costs for this cell, or random loss fluctuations in neighboring cells may have increased the indicated pure premium for this cell. If random loss fluctuations are the cause of the difference between the observed loss costs and the indicated pure premium, the credibility embedded in the minimum bias procedure is sufficient. No additional credibility adjustment should be used.

2. There were no random loss fluctuations causing the difference, but the rating model is not correct. The minimum bias procedure may be using a multiplicative model, which produces high indicated pure premiums for high risk drivers, when an additive model is proper, which would lead to lower pure premiums for these drivers. This is model specification risk, and a second credibility adjustment is warranted.

Classical credibility procedures are not an ideal compensation for model specification risk. The ideal approach is to use several models, such as multiplicative, additive, and combined models, and to test the goodness-of-fit for each model. Time constraints preclude this ideal approach in most cases, and a credibility adjustment may be a reasonable alternative.

Rate Fluctuations

The use of credibility to temper rate fluctuations from year to year is a dubious practice. In practice, most actuaries conceive of credibility as a means to price more accurately. Although Venter correctly notes that the stated rationale for classical credibility deals with tempering rate fluctuations, even classical credibility does serve the objective of increasing the accuracy of the rate indications.[41]

When rating bureaus made advisory rates, they had more incentive to temper rate fluctuations from year to year than private insurers have. Since the objective is rarely well defined, the credibility procedures are often arbitrary. ISO's credibility procedure may not have had firm statistical justification, but it fulfilled the objective of tempering the requested rate changes.

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[41] See Venter's chapter of "Credibility" in any of the first three editions of the CAS textbook, *Foundations of Casualty Actuarial Science.*

MAXIMUM LIKELIHOOD

Some statisticians prefer a maximum likelihood test to either a χ-squared test or a least squares test when fitting a distribution to observed data. In his 1988 *Proceedings* paper, Rob Brown illustrated the use of a maximum likelihood test to optimize classification relativities.

The use of a maximum likelihood test requires an assumption about the distribution of values in each class. The appropriate distribution for loss costs is not evident. It probably is not a simple mathematical distribution that would be amenable to the procedure discussed here, such as an exponential distribution or a Poisson distribution. If the appropriate distribution is not known, the statistical merits of a maximum likelihood test are less clear.

The maximum likelihood test is rarely used in practical work, and not all actuaries are familiar with it. We explain the use of the maximum likelihood test by a series of illustrations.

> *Illustration:* We are fitting an exponential curve to a set of insurance losses. The exponential distribution function says that the likelihood of a loss of size "x" is proportional to $e^{-\lambda x}$. We first determine the constant of proportionality.
>
> Given that a loss has occurred, the likelihood that the loss is between zero and infinity is 1. If "k" is the constant of proportionality, the integral of $ke^{-\lambda x}$ between 0 and infinity equals k/λ. For this to be unity, the constant of proportionality must be λ. The exponential distribution function is $\lambda e^{-\lambda x}$.

LIKELIHOOD AND PROBABILITY

We use the term likelihood, not the term probability. If the exponential distribution function has a λ of 0.0001, the likelihood of a loss of size \$20,000 is $0.0001 \times e^{-2}$.

If losses are spread throughout the positive numbers, the probability of a loss exactly equal to \$20,000 is zero.[42] We may conceive of the likelihood that a loss is equal to \$X as the probability that the loss is between $\$X - \epsilon$ and $\$X + \epsilon$, divided by $2 \times \epsilon$. The limit of this ratio as ϵ tends to zero is the likelihood.

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[42] In practice, losses cluster at round dollar figures, so the probability of loss exactly equal to \$20,000 is greater than zero. The statement in the text assumes an ideal model, where losses can be any amount, down to fractions of a penny, with no rounding to dollar amounts.

Before showing the use of the maximum likelihood test, we examine the mean of the exponential distribution function. The mean equals

$$\int_0^\infty x\lambda e^{-\lambda x}\,dx = 1/\lambda$$

We resume the illustration. We would like to fit an exponential curve to a set of insurance losses. We seek to determine the value of λ.

We have four methods of doing this. We show the full procedure only for the maximum likelihood method.

METHOD OF MOMENTS

The mean of the exponential distribution is 1/λ. We take the average of the observations, and we set λ equal to the reciprocal of this average.

LEAST SQUARES

We divide the loss sizes into ranges, such as $0 to $5,000, $5,001 to $25,000, $25,001 to $100,000, and so forth. We calculate the percentage of observed losses which fall into each range. For any given λ, we determine the percentage of theoretical losses that would fall into each range.

For each range, we calculate the squared difference between the observed percentage and the theoretical percentage. We sum the squared differences over all the ranges. The result is a function of λ. To minimize this squared difference, we set the partial derivative with respect to λ equal to zero.

χ-*SQUARED*

The χ-squared procedure is similar to the least squares procedure, but instead of taking the squared difference we take the χ-squared difference. For each range, we divide the squared difference by the expected value.

MAXIMUM LIKELIHOOD

We dispense with the ranges. Suppose we have observed five losses, with sizes of $3,000, $5,000, $15,000, $20,000, and $80,000. For a given value of λ, the likelihood of a loss equal to $3,000 is $\lambda e^{-\lambda \times 3{,}000}$. The likelihood of five losses for the values listed above is the product of the likelihoods of each individual loss, or

$$\lambda e^{-\lambda \times 3{,}000} \times \lambda e^{-\lambda \times 5{,}000} \times \lambda e^{-\lambda \times 15{,}000} \times \lambda e^{-\lambda \times 20{,}000} \times \lambda e^{-\lambda \times 80{,}000}$$

The sum of the five losses is \$123,000. We simplify the likelihood to $\lambda^5 e^{-\lambda \times 123{,}000}$. To find the optimal λ, we must choose the value that gives the greatest likelihood. To do this, we set the partial derivative with respect to λ equal to zero.

Before taking the partial derivative, we make one simplification. Maximizing the likelihood is the same as maximizing the logarithm of the likelihood. The logarithm of the likelihood is

$$5 \ln \lambda - 123{,}000 \times \lambda.$$

Setting the partial derivative with respect to λ to zero gives $5/\lambda - 123{,}000 = 0$, or $\lambda = 5/123{,}000$.

The method of moments provides the same answer. The mean of the five losses is 123,000/5, and λ is the reciprocal of the mean. In many cases, the method of moments is not practicable, or it gives a different answer than the maximum likelihood procedure.

Maximum Likelihood and Minimum Bias Procedure

The rating model uses the classification relativities to determine the expected loss in each cell, or the mean loss in each cell. The maximum likelihood test is most practicable as a bias function when

- a single parameter distribution is used
- the mean of the distribution equals this parameter or some simple function of this parameter, such as its reciprocal
- the distribution extends over the positive real numbers
- the distribution is a reasonable reflection of some insurance process.

The exponential distribution and the Poisson distribution meet these conditions.

We illustrate a multiplicative model with the exponential distribution function. We use the same illustration as for the other models. The observed loss costs are shown below.

	Urban	Rural		$terr_1$	$terr_2$
Male	\$800	\$500	sex_1	$200 \times s_1 \times t_1$	$200 \times s_1 \times t_2$
Female	\$400	\$200	sex_2	$200 \times s_2 \times t_1$	$200 \times s_2 \times t_2$

Each class has an exponential distribution of loss costs. If the indicated pure premium is \$200, we don't expect every driver in that class to have losses of \$200 each year. Rather, we expect the observed losses to follow an exponential distribution with a mean of \$200.

The λ differs by cell. The reciprocal of λ is the indicated pure premium in that cell.

Illustration: For the urban/male cell, the loss costs have an exponential distribution with the parameter λ equal to $1/(\$200 \times s_1 \times t_1)$.

We choose starting values for $t_1 = 2.00$ and $t_2 = 1.00$. We determine the likelihood of the observed loss costs. The value of λ for the urban/male cell is $1/(200 \times s_1 \times t_1) = 1/(400 \times s_1)$. The likelihood of the \$800 loss cost in the urban/male cell is

$$1/400s_1 \times \exp(-800/400s_1) = 1/(400s_1) \times \exp(-2/s_1).$$

The value of λ for the rural/male cell is $1/(200 \times s_1 \times t_2) = 1/(200s_1)$. The likelihood of the \$500 loss cost in the rural/male cell is

$$1/200s_1 \times \exp(-500/200s_1) = 1/(200s_1) \times \exp(-2.5/s_1).$$

The likelihoods of the observed values in the female cells are determined in the same manner.

To maximize the likelihood, we maximize the logarithm of the likelihood, also knows as the loglikelihood.

- The likelihood of the set of four observed values is the *product* of the four individual likelihoods.
- The loglikelihood of the set of four observed values is the *sum* of the four individual loglikelihoods.

The partial derivative of the loglikelihood with respect to s_1 depends on the loglikelihoods in the male row only. This is the same simplification that we used for the least squares method and the χ-squared method.

The loglikelihood of the values in the male row is $-\ln(400s_1) - 2/s_1 - \ln(200s_1) - 2.5/s_1$. The partial derivative with respect to s_1 is $-1/s_1 + 2s_1^{-2} - 1/s_1 + 2.5s_1^{-2}$. We set this equal to zero.

$$-1/s_1 + 2s_1^{-2} - 1/s_1 + 2.5s_1^{-2} = 0$$
$$-s_1 + 2 - s_1 + 2.5 = 0$$
$$s_1 = 2.25.$$

The likelihood of the \$400 loss cost in the urban/female cell is

$$1/400s_2 \times \exp(-400/400s_2) = 1/(400s_2) \times \exp(-1/s_2).$$

The likelihood of the $200 loss cost in the rural/female cell is

$$1/200s_2 \times \exp(-200/200s_2) = 1/(200s_2) \times \exp(-1/s_2).$$

The loglikelihood of the values in the female row is $-\ln(400s_2) - 1/s_2 - \ln(200s_2) - 1/s_2$. The partial derivative with respect to s_2 is $-1/s_2 + 1s_2^{-2} - 1/s_2 + 1s_2^{-2}$. We set this equal to zero.

$$-1/s_2 + 1s_2^{-2} - 1/s_2 + 1s_2^{-2} = 0$$
$$-s_2 + 1 - s_2 + 1 = 0$$
$$s_2 = 1.00.$$

JUSTIFICATION

The maximum likelihood method has strong statistical justification. If the distribution of loss costs is a simple mathematical function, such as a Poisson distribution, a normal distribution, a lognormal distribution, or an exponential distribution, we can solve for simple recursive equations; see Brown [1988].

In practice, we don't know the proper distributions. The distributions that have been suggested for use in the minimum bias procedure, such as the exponential distribution, the Poisson distribution, and the normal distribution, are not assumed to be the correct distribution. They are tractable distributions that allow simple recursive functions.

If the tractable distribution is not a reasonable reflection of the true distribution, the use of the maximum likelihood method adds bias to the indicated relativities. The Poisson distribution and the normal distribution are not reasonable reflections of the true loss costs distribution. The true distribution has much fatter tails than these two mathematical functions indicate. Using these distributions with the maximum likelihood bias function may distort the solution.

RECURSIVE FUNCTIONS: PRACTICE SUMMARY

This *Practitioner's Guide* emphasizes intuition. Modern spreadsheet software has built-in iterative functions that perform the needed calculations. Mildenhall [1999] shows that the commonly used minimum bias recursive functions are equivalent to certain multiple regression equations or generalized linear models. Commercially available GLM models can be adapted to perform the minimum bias calculations.[43]

For each model discussed in this *Guide*, there are simple iterative functions. The task of the pricing actuary is to determine the type of rating function – multiplicative, additive, or combined – and the type of bias function (balance principle, least squares, χ-squared, or maximum likelihood). If the maximum likelihood bias function is used, the actuary must also select a probability distribution function for the loss costs (or other values) in each cell.

The type of data in each cell will generally be either loss costs or loss ratios. If the pricing actuary is using all the dimensions of the classification system in the minimum bias analysis, it is easiest to use loss costs. If there are significant classification dimensions that are not included, and if there may be an uneven distribution of exposures along these other classification dimensions, the pricing actuary may prefer to use loss ratios.

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[43] As Mildenhall points out, the minimum bias procedures discussed here are a subset of the potential generalized linear models that can be used for classification ratemaking. Actuaries without a strong statistical background may find it difficult to understand the intuition for generalized linear models. We hope to complete the companion *Practitioner's Guide* on generalized linear models in the near future.

The table below shows the models which have been proposed for insurance use, followed by the recursive equations.

Rating Model	*Bias Function*	*Distribution Function*
multiplicative	balance principle	N/A
additive	balance principle	N/A
multiplicative	least squares	N/A
additive	least squares	N/A
multiplicative	χ-squared	N/A
additive	χ-squared	N/A
multiplicative	maximum likelihood	normal
additive	maximum likelihood	normal
multiplicative	maximum likelihood	exponential
multiplicative	maximum likelihood	Poisson

Multiplicative model, balance principle:

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j}$$

Multiplicative model, balance principle, three dimensions:

$$x_i = \frac{\sum_{j,k} n_{ijk} r_{ijk}}{\sum_{j,k} n_{ijk} y_j z_k}$$

Additive model, balance principle:

$$x_i = \frac{\sum_j n_{ij}(r_{ij} - y_j)}{\sum_j n_{ij}}$$

Multiplicative model, least squares:

$$x_i = \sum (n_{ij} \times r_{ij} \times y_j) \div \sum (n_{ij} \times y_j^2).$$

Additive model, least squares:

$$x_1 = \sum(r_{1j} - y_{1j})/m - B$$

Multiplicative model, χ-squared:

$$x_i = [\sum (n_{ij} \times r_{ij}^2 / y_j) / \sum n_{ij} y_j]^{0.5}$$

Additive model, χ-squared:

$$\Delta x_i = \frac{\sum_j n_{i,j}\left(\frac{r_{i,j}}{x_i + y_j}\right)^2 - \sum_j n_{i,j}}{2\sum_j n_{i,j}\left(\frac{r_{i,j}}{x_i + y_j}\right)^2\left(\frac{1}{x_i + y_j}\right)}$$

Multiplicative model, maximum likelihood, normal density function:

$$x_i = \frac{\sum_j n_{ij}^2 r_{ij} y_j}{\sum_j n_{ij}^2 y_j^2}$$

Additive model, maximum likelihood, normal density function:

$$x_i = \frac{\sum_j n_{ij}^2 (r_{ij} - y_j)}{\sum_j n_{ij}^2}$$

Multiplicative model, maximum likelihood, exponential density function:

$$x_i = \frac{\sum_j \frac{r_{ij}}{y_j}}{k}$$

where "k" is the number of classes in the "j" dimension.

The recursive functions for a multiplicative model, maximum likelihood, Poisson distribution function are the same as those for the multiplicative model, balance principle.

Derivations of the formulas for the maximum likelihood models may be found in Brown [1988].

Conclusion

Accurate classification systems are the bedrock of insurance pricing. Accurate and unbiased rating systems enable insurers to attain competitive advantages over their peer companies. Inaccurate rating systems lead to unsatisfactory profits and losses of market share.

As competition in the insurance industry increases, and as companies are forced to rely on their own pricing prowess instead of bureau rates, the need for more accurate ratemaking procedures increases. The minimum bias procedure can be used to optimize a variety of rating models.

In the past, the iterative computational methods and the lack of clear documentation hindered many practicing actuaries from using minimum bias methods. The availability of built-in functions to perform iterative calculations in popular spreadsheets and programming languages has removed the major obstacle to effective use of minimum bias methods. This *Practitioner's Guide* provides clear documentation for actuaries desiring to implement minimum bias methods.

Reinsurance Accounting: Schedule F

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REINSURANCE ACCOUNTING: SCHEDULE F

prepared by
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REINSURANCE ACCOUNTING: SCHEDULE F

Introduction

Schedule F discloses an insurer's reinsurance transactions for both ceded business and assumed business. It is one of the most complex schedules in the Annual Statement, having grown from its original focus on unauthorized reinsurance to cover overdue loss recoverables, amounts in dispute, and a restatement of the statutory balance sheet. The complete rewrite of Schedule F for the 1993 Annual Statement heightened the need for clear documentation of these statutory exhibits, which this paper provides.

Reinsurance transactions are an important consideration in monitoring a company's financial strength, as demonstrated by the emphasis on reinsurance arrangements and collectibility in the NAIC *IRIS Tests*, the risk-based capital requirements, the *Statement of Actuarial Opinion* and the Canadian *Report of the Actuary*.

This paper explains the structure and purposes of Schedule F, as well as the relationship of this schedule to other statutory statements. This paper also contains illustrations of

- Calculating the statutory penalty for
  - i. Recoverables from unauthorized reinsurers (Part 5),
  - ii. Overdue recoverables (Part 6), and
  - iii. Recoverables from "slow-paying" authorized reinsurers (Part 7).
- Completing the restated balance sheet (Part 8), and

Both the insurance industry and its consumers benefit from efficient regulation that promotes insurance company solvency. The paper concludes with an analysis of the objectives of reinsurance regulation, the success of Schedule F in meeting these objectives, and suggestions for improving the schedule and the associated regulation.

STRUCTURE OF SCHEDULE F

Schedule F serves several purposes:

- Parts 1-3 provide the *supporting data* for the company's assumed and ceded reinsurance accounting entries. Part 1 shows assumed premiums and losses by type of reinsured, and Part 3 shows ceded premiums and losses by type of reinsurer. Part 2 shows an exhibit of premiums (but not losses) on portfolio reinsurance transactions that were effected during the most recent calendar year.

- Parts 4-7 develop the *provision for reinsurance*. Part 4 shows an aging schedule for recoverables on paid losses and loss adjustment expenses. Part 5 calculates the statutory provision for reinsurance recoverables from unauthorized companies: unsecured total recoverables, overdue recoverables, and amounts in dispute. Parts 6 and 7 calculate the statutory provision in the same three categories for reinsurance recoverables from authorized companies: for non-slow-paying authorized reinsurers in Part 6 and from slow-paying authorized reinsurers in Part 7.

- Statutory accounting is on a "net of reinsurance" basis, with reinsurance recoverables serving as offsets to direct liabilities. Part 8 of Schedule F *restates the statutory balance* sheet from a net to a gross basis.

Most insurance exhibits and schedules in the NAIC financial statements show data by line of business. This is the format in the Underwriting and Investment Exhibit, the Page 15 state exhibits, Schedule P, and the Insurance Expense Exhibit. Reinsurance transactions in Schedule F are on a line of business basis as well: by primary line for ceded business and for assumed proportional business and by reinsurance line (property, casualty, and financial) for assumed non-proportional business.

Schedule F shows figures for all lines of business combined, split by reinsurance company for ceded business and by reinsured company for assumed business.

Part 1: Assumed Reinsurance

Part 1 of Schedule F shows a listing of assumed reinsurance relationships by reinsured company. The listing is subdivided, where appropriate, by affiliated versus unaffiliated company, U.S. versus alien company, and type of company (mandatory pools versus voluntary pools versus other companies).[1]

The assumed reinsurance in Part 1 of Schedule F and the ceded reinsurance in Part 3 of Schedule F are prospective reinsurance only. Retroactive reinsurance affects the special surplus entry on the liability side of the statutory balance sheet (page 3 of the Annual Statement), but it is not reflected in the exhibits and schedules, such as Schedule F.[2]

Assumed reinsurance entries are of four types:

- Losses payable to the reinsured company on paid losses and on case reserves;

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[1] A *domestic* company is one domiciled in the state under consideration. A U.S. company domiciled in another state is a *foreign* company. A company domiciled outside the U.S. is an *alien* company.

[2] SSAP No. 62, "Reinsurance," paragraph 28, says with regard to retroactive reinsurance agreements:

a. The ceding entity shall record, without recognition of the retroactive reinsurance, loss and loss expense reserves on a gross basis on the balance sheet and in all schedules and exhibits.
b. The assuming entity shall exclude the retroactive reinsurance from loss and loss expense reserves and from all schedules and exhibits.

Retroactive reinsurance does not reduce the loss reserves reported in the Annual Statement for the ceding company. However, it affects statutory income in the same fashion as prospective reinsurance does, except that it is booked under "other income" on the statutory statement of earnings. It has a full effect on policyholders' surplus, though not on the unassigned portion of surplus. It fully affects GAAP income, GAAP equity, and taxable income.

The risk-based capital ratio is slightly reduced if the reinsurance is coded as retroactive instead of prospective. The risk-based capital ratio, which determines the RBC action level, is the ratio of risk-based capital adjusted surplus to the risk-based capital requirements for the company.

- The risk-based capital adjusted surplus includes special surplus funds just as it includes unassigned surplus funds. The adjusted surplus used to compute the risk-based capital ratio does not depend on whether the reinsurance is classified as prospective or retroactive.
- The RBC reserving risk charge is greater than the charge for reinsurance recoverables, particularly after the covariance adjustment. Prospective reinsurance reduces risk-based capital requirements and decreases the denominator of the risk-based capital ratio.

Prospective reinsurance reduces the denominator of the risk-based capital ratio and increases the ratio itself. Retroactive reinsurance does not have this effect.

- Premiums assumed from the reinsured company, the unearned portion of the assumed premiums, and assumed premiums that are still uncollected;
- Contingent commissions receivable from or payable to the reinsured company; and
- Security, or funds deposited with the reinsured company and letters of credited provided for the benefit of the reinsured company.

LOSSES PAYABLE

Losses payable to the reinsured company are divided between reserves on loss already paid by the ceding company (column 6) and reserves on reported but unpaid losses of the ceding company (column 7). Column 6 agrees with line 2 of page 3 (the statutory balance sheet), which has an explicit reference to Schedule F, Part 1, column 6.

Column 7 of Schedule F, Part 1 is similar to the entry on the Underwriting and Investment Exhibit, Part 3A, column 2. However, the Schedule F entry includes loss adjustment expense whereas the Underwriting and Investment Exhibit entry does not, so there is no exact reconciliation.

The reporting company must also hold reserves for IBNR losses of the ceding companies.[3] These are shown in the Underwriting and Investment Exhibit, Part 3A, column 6, which reports the total for all ceding companies by line of business. The reporting company's reserves on IBNR losses are not subdivided by ceding company, so they are *not* shown in Schedule F, Part 1.[4]

Reinsured losses paid during the year are not shown in Schedule F. They are shown by line of business for all ceding companies combined in the Underwriting and Investment Exhibit, Part 3, column 2.

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[3] The reporting company is the company preparing Schedule F.

[4] Schedule F, Part 3, shows reinsurance loss recoverables subdivided between loss and LAE and between recoverables on case reserves and those on IBNR reserves. The ceding company generally estimates the IBNR recoverables by reinsurer, so that it may offset its direct loss reserves. Similarly, it estimates the recoverable separately for losses and for loss adjustment expenses, so that it may offset its unpaid losses and LAE on lines 1 and 3 of page 3. The assuming company has no need for these separate estimates.

PREMIUMS AND COMMISSIONS

Column 9 shows contingent commissions payable; column 10 shows assumed premiums receivable; and column 11 shows unearned premium. The column 10 entry is net of regular commissions, which do not appear in column 9. The column 9 entry is for contingent commissions (sometimes called profit commissions) only, and it may be either a positive or negative figure. A positive figure means that the reporting company expects to pay additional contingent commissions to the ceding company. A negative figure means that the reporting company expects to receive back some contingent commissions previously paid to the ceding company.

Illustration: Suppose the reporting company has two reinsurance treaties, both with a gross premium of $1,000,000. One treaty has a fixed commission rate of 30% of gross premiums. If no premium has yet been received, the column 10 entry would be $700,000, since "the amounts reported should be net of commissions payable" (*Instructions*).[5] The column 9 entry would be $0, since the treaty has no contingent commissions.

The other treaty has a sliding scale contingent commission arrangement, where the commission depends on the loss ratio of the assumed business: 30% minus one half of the difference between the actual loss ratio and 70%, or

$$30\% - 0.5 \times (\text{actual loss ratio} - 70\%),$$

bounded between 10% (for an actual loss ratio of 110%) and 50% (for an actual loss ratio of 30%). At the last meeting between the reinsurer and the ceding company, the loss ratio was estimated at 60%, so a 35% commission was paid. Since that time, additional reported losses indicate that the true loss ratio is 80%, so the final contingent commission should be 25%. The contingent commission payable is a negative 10% of $1,000,000, or –$100,000.

FUNDS WITHHELD AND LETTERS OF CREDIT

A reinsurer may provide funds or letters of credit to secure the balances payable to the ceding company.

- If the reinsurer is not authorized to transact reinsurance business in the state of domicile of the ceding company, the ceding company must post a statutory liability called the

[5] SSAP No. 62, "Reinsurance," paragraph 50, explains: *Commissions payable on reinsurance assumed business shall be included as an offset to Agents' Balances or Uncollected Premiums. Commissions receivable on reinsurance ceded business shall be included as an offset to Ceded Reinsurance Balances Payable.*

provision for reinsurance to offset the reinsurance recoverables. In common parlance, the reinsurance recoverables are not admitted to reduce the net loss liability unless the recoverables are secured (see the subsequent discussion of Schedule F, Part 5).[6]

- If the reinsurer is authorized but triggers the "slow-paying" test in Schedule F, Part 4, it is classified as a slow paying reinsurer, and a provision for reinsurance equal to (at least) 20% of the reinsurance recoverables must be posted, unless the recoverables are secured (see the subsequent discussion of Schedule F, Part 7).

- Even if the reinsurer is both authorized and not slow-paying, the ceding company may request letters of credit to ensure that its losses will be reimbursed.

Part 1 of Schedule F shows these securing amounts as follows:

- Column 12 shows "funds held by or deposited with reinsured companies." These assets, owned by the reinsurer but held by the ceding company, are shown on line 11 of page 2 of the reinsurer's balance sheet (assets) and on line 12 of page 3 of the ceding company's balance sheet (liabilities).

- Column 13 shows "letters of credit posted." The letter of credit may be issued by a bank or other financial institution to secure recoverables from the reinsurer. The letter of credit does *not* affect the reinsurer's balance sheet, but it reduces the provision for reinsurance on the ceding company's balance sheet (if the reinsurer is unauthorized or slow-paying).

- Column 14 shows "amount of assets pledged or compensating balances to secure letters of credit." The commercial bank issuing the letter of credit may demand that the reinsurer hold a compensating balance in an account with the bank to secure the letter of credit. Suppose the ceding company wants a letter of credit to secure the recoverables from an unauthorized reinsurer. A commercial bank might charge a high fee to provide the letter of credit. To reduce the fee, the reinsurer transfers cash from another financial institution to the bank issuing the letter of credit. The reinsurer is restricted from using these funds as long as the bank's obligation on the letter of credit remains outstanding.

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[6] In more rigorous statutory accounting terms, the reinsurance recoverables are always admitted, since they reduce the statement reserves of the ceding company, whether on the balance sheet (page 3, line 1), the Underwriting and Investment Exhibit, or Schedule P, regardless of whether the reinsurer is authorized or slow-paying. Unsecured recoverables from unauthorized reinsurers, 20% of unsecured recoverables from slow-paying authorized reinsurers, 20% of loss recoverables more than 90 days overdue from all reinsurers, and 20% of amounts in dispute from unauthorized reinsurers and from non-slow paying authorized reinsurers must be reported as a statutory provision for reinsurance on the liability side of the balance sheet (page 3, line 15).

Part 2: Portfolio Reinsurance

Part 2 of Schedule F shows "Premium portfolio reinsurance effected or cancelled during the current year." Reinsurance ceded by portfolio is shown on the top half of the page, and reinsurance assumed by portfolio is shown on the bottom half of the page.

The information in Part 2 of Schedule F relates to premiums only, shown separately by reinsuring or ceding company. The entries are

- Columns 1-3: Company identification (federal ID number, NAIC company code, name)
- Column 4: Date of Contract
- Column 5: Amount of Original Premium
- Column 6: Amount of Reinsurance Premiums

Portfolio reinsurance is defined in the Annual Statement Instructions as "the transfer of the entire liability of an insurer for in force policies as respects a described segment of the insurer's business." No guidance is provided for the entries in columns 5 and 6.

Originally (in 1989), Part 2 of Schedule F dealt with premiums on loss portfolio transfers. In the early 1990's, the term "loss portfolio transfers" was changed to retroactive reinsurance, as was the title of Schedule F, Part 2. Retroactive reinsurance is defined in SSAP, paragraph 21, as "reinsurance in which a reinsurer agrees to reimburse a ceding entity for liabilities incurred as a result of past insurable events covered under contracts subject to the reinsurance." The NAIC *Instructions* to the Statement of Actuarial Opinion (section 11) provide a three-fold definition:

> *For the purpose of this instruction, "retroactive reinsurance" refers to any agreement which increases the transferring insurer's Surplus to Policyholders as a result of the transferee undertaking any loss obligation already incurred and for which the consideration paid by the transferring insurer is derived from present value or discounting concepts.*

Portfolio reinsurance appears to include both prospective and retroactive reinsurance, though the primary policies must be "in force." SSAP No. 62, "Reinsurance," paragraph 29, defines portfolio reinsurance as "the transfer of an insurer's entire liability for in force policies *or outstanding losses*, or both, of a segment of the insurer's business" (emphasis added). The "in force" qualification in the Annual Statement *Instructions* is not determinative.

The intention of columns 5 and 6 is unclear. Since column 6 refers to the reinsurance premium, column 5 seems to refer to the primary premium. One insurer, which ceded the prospective part of policies in force for a block of business, entered the primary premium for the entire block as the original premium in column 5 and the reinsurance premium in column 6.

The treatment of loss portfolio transfers in the Annual Statement changed in the early 1990's, and there was a need for regulators to track these agreements to ensure that proper statutory accounting was being followed. The purpose of an exhibit showing the premium on portfolio reinsurance is unclear.

Part 3: Ceded Reinsurance

Part 3 of Schedule F shows a listing of ceded reinsurance relationships by reinsurance company. The listings are subdivided by affiliated versus unaffiliated company, authorized versus unauthorized company, U.S. versus alien company, and type of company (mandatory pools versus voluntary pools versus other companies).

The authorized versus unauthorized status of the reinsurer is essential for Schedule F, whose primary purpose is to determine the provision for reinsurance. Unaffiliated reinsurers may clearly be either authorized or unauthorized. An affiliated reinsurer may also be unauthorized. A domestic company may have an unauthorized off-shore affiliate in a tax haven. Reinsurance ceded to the unauthorized affiliate may be used to reduce tax liabilities or to circumvent U.S. restrictions on loss reserve discounting.

The columns in Part 3 emphasize the amounts recoverable from assuming reinsurers and the offsetting funds that secure the recoverables. This information is used to derive the provision for reinsurance by type of reinsurer. We explain each column below.

Fronting Companies

Column 5 identifies "insurance contracts ceding 75% or more of direct premiums written." There is a cost to buying reinsurance. If the reinsurance contract cedes 75% or more of the primary premium, one might wonder why the primary company wrote the business in the first place.

Regulators are often suspicious of such reinsurance arrangements. It is true that some risks are too large or too risky for the primary company. A primary insurance company may bid on a $50 million commercial office building and then cede most of the exposure to larger reinsurers. Even in these scenarios, it is unusual for the ceded premium to be 75% or more of the primary premium. Excess of loss reinsurance premiums are rarely that large, and quota share reinsurance cessions of 75% or more of the exposure are not common.

Fronting arrangements are used by insurers seeking to write direct business in jurisdictions where they are not licensed, particularly if the jurisdiction has extraterritorial regulation. Suppose the ABC Insurance Company wises to write business in New York, but it does not wish to subject its operations to New York insurance requirements, and it is not licensed in New York. The XYZ Insurance Company is licensed in New York and writes business there. The ABC Insurance Company may have the XYZ Insurance Company write the business and cede the premium to ABC. The XYZ Insurance Company gets a fronting company fee for its services from the ABC Insurance Company, and ABC gets to write the business without supervision of the New York Insurance Department.

Regulators do not always like such fronting arrangements. Ideally, the regulator would like to monitor the accounts of the ABC Insurance Company, but ABC is not licensed in the jurisdiction and its books are not shown to the regulator. Instead, the regulator monitors the accounts of the fronting company, which is licensed in the jurisdiction.

This is the general regulatory perspective in much of Schedule F. Ideally, the regulator would like to monitor the accounts of unauthorized reinsurers and of authorized reinsurers in financial distress with overdue accounts payable. But it does not have access to the accounts of unauthorized reinsurers, and companies in financial distress may not present a "full and true statement" of their accounts. Instead, regulators seek the relevant information from companies domiciled or licensed in their states.

The Annual Statement *Instructions* say that

> *Each individual contract, except those listed below, which provides for the cession of 75% or more of direct premiums written under such cession during the year, should be identified by inserting a 2 in this column. The reinsurance transactions so identified shall include both treaty and facultative cessions of direct business written by the company.*

Possible fronting arrangements can be ascertained from the entry in this column.

EXCEPTIONS

Four types of reinsurance contracts are exempt from identification in this column.

(1) *Affiliated transactions:* Intercompany reinsurance transactions with affiliates are exempt from identification in this column. Sister companies A, B, and C may participate in an intercompany pooling agreement, whereby companies A and C cede all their business to company B. Company B retrocedes one third of the pooled business back to company A and one third back to company C. These transactions appear as affiliated reinsurance cessions in Schedule F. These are not fronting arrangements. Insurers use fleets of companies for rating purposes: one company may have rates for preferred insureds and another company may have rates for substandard insureds.

(2) *Pools:* Insurers participate in various involuntary market pools and joint underwriting associations, particularly for workers' compensation and commercial automobile business. One or more companies act as servicing carriers for the pool. They write the involuntary business and cede everything to the pool, keeping only an expense allowance for their acquisition costs and underwriting costs. These are not fronting arrangements, and they are exempt from identification in this column. The Annual Statement *Instructions* say

Exclude: Reinsurance transactions involving any group, association, pool, or organization of insurers which engage in joint underwriting activities and which are subject to examination by any state regulatory authority or which operate pursuant to any state or federal statutory or administrative authorization.

(3) *Small Amounts:* A reinsurance transaction in which the annual gross premium ceded is less than 5% of policyholders' surplus is exempt from identification in this column. Regulators are concerned about companies that serve as fronting insurers for other companies. A small reinsurance transaction may result from the ceding company leaving a line of business or a geographic area when it has little remaining business.

(4) *Captives:* Reinsurance transactions involving captive insurance companies are exempt from identification in this column. An insurance company can deduct loss reserves from its taxable income. A non-insurance company can deduct only paid losses from its taxable income, not loss reserves. To gain the tax benefits of insurance while avoiding the expense costs of commercial insurance, a large policyholder may form an insurance subsidiary to write coverage on the parent company's exposures.

It is expensive for insurance companies to hold capital, partly because of double taxation costs.[7] To avoid holding capital, the captive may cede the business to the parent company, to other affiliates of the parent company, or to unaffiliated reinsurers.

LOSS AND LOSS ADJUSTMENT EXPENSES

Columns 7 and 8 of Schedule F, Part 3 show "reinsurance recoverable on paid losses and on paid LAE" (respectively). Reinsurance recoverables on paid losses and loss adjustment expenses are balance sheet assets on both statutory and GAAP financial statements; see SFAS 113 and SSAP No. 62, paragraph 19.[8] The total of columns 7 plus 8 should equal the entry on page 2, column 3, line 14, "Reinsurance recoverables on loss and loss adjustment expense payments."[9]

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[7] See Feldblum [DCCS] on the costs to an insurance company of holding capital.

[8] Both authorized and unauthorized reinsurance recoverables are admitted on the asset side of the balance sheet. SSAP No. 62, paragraph 19, says: "Reinsurance recoverable on loss payments is an admitted asset. . . . Unauthorized reinsurance is included in this asset and reflected separately as a liability to the extent required." The asset for reinsurance recoverables does not depend on the authorized status of the reinsurer. The provision for reinsurance on the liability side of the balance sheet *does* depend on the authorized status of the reinsurer.

[9] On the asset side of the statutory balance sheet, column 1 shows the gross asset, column 2 shows the non-admitted portion, and column 3 shows the net admitted asset, all for the current year. Column 4 shows the net admitted asset for the previous year. Schedule F shows only the net admitted amounts. All of these figures are gross of the provision for reinsurance. The balance sheet is in dollars whereas Schedule F is in

Reinsurance recoverables on unpaid losses and LAE are divided into four groups:

- Column 9 – Recoverables on known case loss reserves
- Column 10 – Recoverables on known case LAE reserves
- Column 11 – Recoverables on IBNR loss reserves
- Column 11 – Recoverables on IBNR LAE reserves

Reinsurance recoverables on unpaid losses and loss adjustment expenses are contra-liabilities on the statutory balance sheet; they offset the direct loss and LAE reserves on page 3, lines 1, 2, and 3.[10] The Underwriting and Investment Exhibit, Part 3A, "Unpaid losses and loss adjustment expenses" shows ceded loss reserves divided between reported losses in column 3 (or "case reserves") and incurred but not reported losses in column 7. The totals for all lines of business combined in the Underwriting and Investment Exhibit, Part 3A, for columns 3 and 7 should equal the totals for all reinsurers combined in Part 3 of Schedule F, columns 9 and 11, respectively.

For loss adjustment expenses, the Underwriting and Investment Exhibit shows the net amount in column 9, but not the direct, assumed, and ceded pieces of the net amount. There is no formal cross-check for columns 10 and 12 of Schedule F, Part 3.[11]

UNEARNED PREMIUMS

Column 13 shows unearned premiums. The unearned premium reserves held by the assuming reinsurers are similar to the loss recoverables due from these reinsurers, since if the reinsurer cancels the contract or if it becomes insolvent, the unearned premium reserves must be returned to the ceding company. For calculating the provision for reinsurance from unauthorized and slow-paying reinsurers, the unearned premium reserves and contingent commissions are included with loss recoverables.

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thousands of dollars. The Schedule F entry must be multiplied by 1000 before comparison with the balance sheet.

[10] See SSAP No. 62, paragraph 26: "Reinsurance recoverables on unpaid case-basis and incurred but not reported losses and loss adjustment expenses shall be netted against the liability for gross losses and loss adjustment expenses."

[11] Schedule P shows both loss and loss adjustment expense liabilities for ceded business. However, Schedule P uses a different allocation of reinsurance to direct, assumed, and ceded categories than Schedule F does, so the figures may differ between the schedules. See footnote 40 below, as well as the letter from Martin F. Carus of the New York Insurance Department to Robert Solitro of October 28, 1991 regarding Part 1A of Schedule F (*Proceedings of the NAIC*, 1992, Volume IA, page 351).

When Schedule F was revised in 1993, a simplified estimation procedure was permitted for the unearned premium reserves column in Part 3.[12] The current Annual Statement *Instructions* do not mention this approximation; we do not know if companies may still use it. We explain the approximation below, without judging whether it is still allowed.

For unauthorized reinsurers, the actual unearned premium must be calculated, since this contra-liability must be offset by a provision for reinsurance unless the funds are secured. When there are many authorized reinsurers involved, the unearned premium reserves associated with each company may be estimated as follows:

A. Calculate the total unearned premium reserve (UEPR) for all reinsurers combined.

B. Calculate the unearned premium reserve for each unauthorized reinsurer. The sum of these reserves is the aggregate UEPR for unauthorized reinsurers.

C. The difference between "A" and the aggregate in "B" is the unearned premium reserve associated with authorized reinsurers.

D. Spread the aggregate unearned premium reserve for authorized reinsurers to companies in proportion to the premium in force for each reinsurer.

If $UEPR_{com}$ is the unearned premium reserve for a given authorized reinsurer,
$UEPR_{tot}$ is the aggregate unearned premium reserve for all authorized reinsurers,
PIF_{com} is the premium in force for this authorized reinsurer, and
PIF_{tot} is the aggregate premium in force for all authorized reinsurers,

then $UEPR_{com}$ may be estimated as

$$UEPR_{com} = UEPR_{tot} \times PIF_{com} \div PIF_{tot}.$$

12 See the NAIC *Proceedings*, 1991, Volume 1A, page 368.

COMMISSIONS

Column 14 shows reinsurance recoverables on contingent commissions. They may be either positive or negative amounts: positive if the reinsurance experience is favorable and the reporting company expects additional contingent commissions, and negative if the reinsurance experience is unfavorable and the reporting company must return some of the contingent commissions already received.

Regular commissions are netted with the ceded balances payable in column 16. In other words, the ceded premium balances are net of regular commissions. Suppose the ceding company has a quota share reinsurance treaty with a 30% commission rate. If the gross premium balance is $1,000,000, the ceding company would show $700,000 in column 16. Amounts stemming from profit commissions or contingent commissions, whether positive or negative, are shown in column 14, not in column 16.

The total reinsurance recoverables for contingent commissions, whether positive or negative, should agree with the figure in Note 22 to the Financial Statements, section C.2:

> *Report the additional or return commissions, predicated on loss experience or on any other form of profit sharing arrangements in this annual statement as a result of existing contractual arrangements.*

The Commission Footnote

A company may use reinsurance as surplus relief for statutory statements. This is acceptable practice, since the reinsurance reduces the underwriting risk of the company.

A company may structure the reinsurance agreement to provide more surplus relief than is warranted. State regulators frown upon such practices, since they may be indicative of financial trouble. The commission footnote to Part 3 of Schedule F (reproduced below) seeks to identify instances of this practice. The footnote requests disclosure of the five largest provisional commission rates in reinsurance treaties. The provisional commission rate is the commission rate before application of loss sensitive contract features, such as sliding scale commissions and retrospective rating; see the second illustration below.

Reinsurance commissions from involuntary pools and joint underwriting associations are not included in the footnote disclosure. The involuntary pools may provide a high commission allowance to servicing carriers because of the difficulty of servicing the small, high risk insureds who comprise much of the pool population. The commission allowance is set by state regulation or by an industry rating bureau. The servicing carrier is not using the pool for surplus relief.[13]

Martin F. Carus (a member of the NAIC Reinsurance Study Group which developed the current format of Schedule F) explains the rationale of the footnote as follows:[14] *The purpose of the footnote is to detail the five largest commission rates (or where contingent commission clauses exist, the provisional commission rates) for the cedent's treaties so that it can be discerned if any treaties have inordinately high rates. Examination and internal financial analyses have found that some insurers were masking their leverage ratios and true underwriting performance by increasing the ceded premium and commission levels in their ceded reinsurance agreements.*[15] We explain Mr. Carus's comment with two illustrations after showing the text of the note.

NOTE: Report the five largest provisional commission rates included in the cedent's reinsurance treaties. The commission rate to be reported by contract with ceded premium in excess of $50,000.

	(1) *Name of Company*	*(2)* *Commission Rate*	*(3)* *Ceded Premium*
(1)			
(2)			
(3)			
(4)			
(5)			

Two illustrations clarify the purpose of this footnote. (I am indebted to Mr. Carus for both illustrations.[16])

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[13] See the Annual Statement *Instructions* for Schedule F, Part 3: "Disclosure of the five largest provisional commission rates should exclude mandatory pools and joint underwriting associations."

[14] Personal communication in a letter of January 25, 1994.

[15] The disclosure in Note 22 to the Financial Statements, section C.1 quantifies the total surplus relief provided by reinsurance commissions. SSAP No. 62, "Reinsurance," paragraph 70(a), "Reinsurance Assumed and Ceded" explains that *the financial statements shall disclose the maximum amount of return commission which would have been due reinsurers if all reinsurance were canceled with the return of the unearned premium reserve.* In general, this surplus relief is proper accounting, though an inordinate amount of surplus relief may arouse regulatory suspicion of financial weakness. The purpose of the commission footnote in Part 3 of Schedule F is to identify possibly *improper* reinsurance commission arrangements.

[16] The text of Mr. Carus's letter follows: "For instance, company A enters into an excess of loss treaty with a premium based on 6% of gross net (i.e., gross of direct commissions but net of other ceded premium)

ILLUSTRATION 1: Excess of loss reinsurance is generally priced without a ceding commission. Suppose the ABC Insurance Company has $100 million of subject written premium on January 1, 20XX, and its policyholders' surplus has fallen to $30 million. The 3.33 to 1 ratio of written premium to policyholders' surplus is above the trigger of the NAIC IRIS test, and the company may be subject to additional regulatory attention.

If the company purchases an excess of loss reinsurance treaty for a premium of $6 million, or 6% of the subject premium, the net written premium is $94 million. Surplus remains at $30 million, since the reduction in cash of $6 million is offset by a reduction in unearned premium reserves of $6 million. The written premium to policyholders' surplus ratio is 3.13 to 1, which is still too high.

Instead, the company purchases the same excess of loss reinsurance treaty, but the treaty calls for a reinsurance premium of $12 million (or a 12% reinsurance premium rate) along with a 50% ceding commission. The cash flows in the reinsurance treaty have not changed – the net reinsurance premium is still $6 million – but the statutory accounting presentation is different. The ceding company shows $88 million of net written premium and $36 million of policyholders' surplus. The $12 million of reinsurance premium is offset by a $12 million reduction of the unearned premium reserves, and the $6 million of ceding commission is a

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written or earned premium. Company B writing the same block of business obtains the same excess of loss treaty but pays 12% of gross net premiums and earns a commission of 50%. Both insurers have protected themselves equally in terms of exposure but company B's leverage position is markedly improved over that of company A. The commissions are earned immediately by company B while company A must earn its retained premium ratably over the underlying policy terms. Moreover, company B's net premiums written or earned are artificially decreased which makes its premium leverage ratios look better than company A's. This is inappropriate considering the companies' equal exposure. Regulators' concerns are generated when companies change their ceded reinsurance programs from year 1 to year 2 or going from a company A position to a company B position.

Similar examples can be constructed relative to quote share arrangements where provisional commission rates, adjustable based on developing loss experience, are used to accomplish the same thing. Compare a 20% quota share arrangement with a 10% provisional commission rate to a 40% quote share arrangement with a 55% provisional commission rate on the same block of business. If ultimately the exposures' results work out evenly, the net compensation to the reinsurer will be the same; however, the temporary masking effects and income generation features described above hinder accurate financial condition assessment by regulators.

This is the reason for the disclosures at the bottom of Schedule F – Part 3. Using the word "provisional" in the footnote connotes the intention of looking at the rates as the treaties are initiated and not after their development because it is at initiation that the manipulation of leverage and income is generated. It is also intended that a regulator will be looking to the cedent's acquisition cost ratios in Part 4 of the Underwriting and Investment Exhibit to see if there are wide divergences with footnoted commission rates. This has particular reference to the example with excess treaties. If these are found to exist, undoubtedly the cedent will be queried by the regulator.

The purpose of this disclosure is to enable regulators to monitor whether a ceding company is masking an unduly high leverage ratio by means of reinsurance treaties with high commission rates."

revenue. The ratio of net written premium to policyholders' surplus is \$88 million / \$36 million = 2.44, which is well below the IRIS trigger of 3.

ILLUSTRATION 2: Quota share reinsurance is priced with a ceding commission. Varying the ceding commission changes the effective reinsurance premium rate, so the accounting sleight-of-hand is more subtle. The reinsurance treaty can set the ceding commission as a contingent commission, with a high provisional commission rate to provide surplus relief.

Suppose the ABC Insurance Company has \$100 million of subject written premium on January 1, 20XX, and its policyholders' surplus has fallen to \$20 million. The 5 to 1 ratio of written premium to policyholders' surplus is so high that the company might attract regulatory examination.

The company's business is so poor and its financial condition is so weak that reinsurers might be reluctant to provide aid. Instead, the company might purchase a 20% quota share reinsurance treaty with a 10% provisional ceding commission that has a 1 for 1 sliding scale. The 10% provisional ceding commission assumes a 90% loss ratio. If the actual loss ratio is higher, such as 95%, the ceding commission is reduced to 5%; if the actual loss ratio is lower, such as 80%, the ceding commission is increased to 20%. This is finite reinsurance. The reinsurer has little underwriting risk; the purpose of the reinsurance treaty is surplus relief.[17]

The net cash flow at inception of the treaty is \$20 million × (1 – 10%) = \$18 million. The net written premium is \$80 million and the adjusted surplus is \$22 million. At inception, the revised ratio of written premium to policyholders' surplus is \$80 million / \$22 million, or 3.64. This is still too high.

To solve its surplus problem, ABC Insurance Company purchases a 40% quota share reinsurance treaty with a 55% provisional ceding commission. The cash flow at inception of the treaty is exactly the same as in the previous scenario. The net cash flow at inception is \$40 million × (1 – 55%) = \$18 million. But the net written premium is \$60 million and the adjusted surplus is \$42 million. At inception, the revised written premium to policyholders' surplus ratio is \$60 million / \$42 million, or 1.43. This appears excellent.

This "solution," of course, is accounting gimmickery. The high 55% ceding commission is just an accounting fiction, since it will be revised 1 for 1 with the actual loss ratio. Yet the apparent

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[17] A small amount of insurance risk would have to be retained to pass the transfer of risk tests in SFAS 113 and SSAP No. 62.

written premium to surplus ratio of 1.43 at inception provides the relief which the ABC Insurance Company needs.[18]

These scenarios illustrate the potential use of high reinsurance commission rates or high provisional commission rates to circumvent statutory accounting intentions and portray higher premium to surplus ratios than is warranted by the economics of the business. The Part 3 footnote identifies instances of high reinsurance commission rates, so that the state regulator can re-examine the reinsurance treaties involved. Each case may be different, and no set rules are prescribed.

Part 4: Aging of Ceded Reinsurance

SUMMARY

Before 1989, there was no statutory penalty for authorized reinsurance, regardless of its presumed collectibility. In 1989, a statutory penalty for loss recoverables more than 90 days past due and for all recoverables from slow-paying ("triggering") authorized reinsurers was implemented, and a payment schedule was added to Part 1A of Schedule F. In 1993, the aging schedule was revised, the aging rules were changed, and the aging exhibit was made into the current Part 4. The aging schedule determines the percentage of the reinsurer's loss recoverables that are more than 90 days past due and whether the reinsurer should be classified as slow-paying, thereby triggering the provision for reinsurance in Part 7 of Schedule F.

THE DUE DATE

Non-insurance commercial contracts generally specify the date by which payment must be made. Traditionally, many reinsurance treaties were "gentlemen's agreements." They relied on the contracting parties to remit funds as the liabilities emerged, without specifying payment dates. The complexities of reinsurance agreements and the reliance on the "utmost good faith" of the contracting parties argued against specific payment schedules in the contracts.

In addition, ceding companies may not always bill their reinsurers immediately for small losses. They may wait until the recoverables accumulate above a certain level, such as $50,000, and then bill the reinsurer for the total amount.

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[18] In theory, the ABC Insurance Company must set up a statutory liability of $22 million for potential return commission. It would be difficult for regulators to recognize the need for this statutory liability, and a financially distressed company may be unlikely to post it voluntarily.

To accommodate these attributes of reinsurance agreements, the Annual Statement *Instructions* say:

> *For purposes of completing Columns 5 through 9, a paid loss and paid loss adjustment expense recoverable is due pursuant to original contract terms (as the contract stood on the date of execution).*
>
> *Where the reinsurance agreement specifies or provides for determination of a date at which claims are to be paid by the reinsurer, the aging period shall commence from that date.*
>
> *Where the reinsurance agreement does not specify a date for payment by the reinsurer, but does specify or provide for determination of a date at which claims are to be presented to the reinsurer for payment, the aging period shall commence from that date.*
>
> *Where the reinsurance agreement does not specify or provide for the determination of either of such dates, the aging period shall commence on the date on which the ceding company enters in its accounts a paid loss recoverable which, with respect to the particular reinsurer, exceeds $50,000. If the amount is less than $50,000 it should be reported as currently due.*

Examples of Due Dates

The following scenarios illustrate the Annual Statement *Instructions*:

1. The reinsurance contract may specify a date by which time recoverables are due, such as "thirty days from the time of notice to the reinsurer." Suppose that

   - A loss occurs on March 15;
   - The loss is paid by the ceding company on August 15;
   - The ceding company bills the reinsurer on September 15 (the date of notice); and
   - The reinsurance contract specifies that recoverables are due within thirty days of the time of notice.

   The recoverable is due on October 15. If it is not paid by December 31, the recoverable is 75 days (two and a half months) overdue.[19]

2. Suppose the dates of loss occurrence, payment, and billing as the same as above, but the reinsurance contract does not specify a date by which time recoverables are due. Instead,

[19] For simplicity, we use an assumption of 30 day months in this illustration. The actual statutory rules have no such assumption, and an exact day count is (presumably) intended.

the reinsurance contract says that claims are to be presented to the reinsurer for payment within 30 days of the date the loss is paid by the ceding company. The recoverable is due on September 15. If it is not paid by December 31, the recoverable is 105 days (three and a half months) overdue.[20]

3. Suppose the reinsurance contract specifies neither the due date nor the presentation date. Moreover; suppose the loss was for $100,000, and when the ceding company paid the claim, it entered on its books a paid loss recoverable of $100,000. The aging period starts from August 15. If the recoverable is not paid by December 31, the recoverable is 135 days (four and a half months) overdue.

4. Suppose the facts are as described in the paragraph above, but the loss was for $15,000, and it was the only loss recoverable from this reinsurer. To avoid excessive transaction expenses for small claims, the ceding company waits until several such claims have accumulated before seeking recovery from the reinsurer, and it does not bill the reinsurer in that year. The claim would remain current through December 31.

Small Claims

A small claim remains current as long as the aggregate amount of such claims for a reinsurer remains below $50,000. However, no claim may remain current for more than one year. The Annual Statement *Instructions* say

> *Any such amounts so reported [i.e., as currently due] in a prior year's annual statement and is still outstanding as of the date of this annual statement must be reported under Column 9 and included in Column 10.*

Any item listed as a loss recoverable in the 20XX Annual Statement – whether as currently due or as overdue – and still unpaid by the reinsurance company at December 31, 20XX+1 must be reported as overdue more than 120 days (i.e., Column 9 of Part 4) in the 20XX+1 Annual Statement.

Reinsurance Intermediaries

Direct writing reinsurers have their own (captive) agency force; independent agency reinsurers use brokers and reinsurance intermediaries. When a broker or a reinsurance intermediary is involved, the ceding company's dealings may be with the broker or the intermediary, not with the reinsurance company. In such cases, notification of the claim or presentation of the

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[20] The specified presentation date become the due date. An earlier draft of this statutory rule set the due date as 30 days after the specified presentation date. This accounts for the "more than 120 days past due" column in Schedule F, Part 4; see footnote 21

claim to the broker or intermediary is equivalent to notification or presentation to the reinsurance company.

THE AGING SCHEDULE

Part 4 of Schedule F shows the following numerical columns:

- Column 5. Currently due recoverables (i.e., not yet overdue)
- Column 6 – 10. Overdue recoverables
  - Column 6. 1 to 29 days
  - Column 7. 30 to 90 days
  - Column 8. 91 to 120 days
  - Column 9. Over 120 days
  - Column 10. Total overdue (cols. 6 + 7 + 8 + 9)
- Column 11. Total due (cols. 5 + 10)
- Column 12. Percentage overdue (col 10 ÷ col. 11)
- Column 13. Percentage more than 120 days overdue (col. 9 ÷ col. 11)

Columns 12 and 13 show the percentages of loss recoverables that are overdue (i.e., not current) and that are overdue more than 120 days. For the statutory provision for reinsurance, the relevant ratio is the percentage more than 90 days overdue; see Part 5, column 13 and Part 6, column 4). These amounts are used to determine the statutory penalty for overdue recoverables and to determine whether the reinsurer should be classified as slow-paying (see below). Column 13 in Part 4, which shows the percentage more than 120 days overdue, is not used in the statutory calculations.[21]

We explain the use of the aging schedule in the discussion below of Schedule F, Part 6.

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[21] The column 13 ratio is included because the aging schedule for certain recoverables was speeded up by 30 days in 1993 compared to 1992. The industry advisory committee to the NAIC reinsurance study group recommended that the cutoff date for the statutory provision be increased from 90 days to 120 days. The NAIC study group kept the cutoff date at 90 days, but it provided columns to monitor the difference between a 90 day and a 120 day cutoff date.

The Statutory Provisions for Reinsurance

Statutory accounting imposes "provisions" (or penalties) for certain types of reinsurance recoverables:

- unsecured recoverables from unauthorized reinsurers,
- unsecured recoverables from slow-paying (authorized) reinsurers,
- overdue recoverables from both authorized and unauthorized reinsurers, and
- recoverables in dispute from unauthorized reinsurers and from non-slow-paying authorized reinsurers.

These statutory provisions for reinsurance appear on line 15 of page 3 of the Annual Statement: "15. Provision for reinsurance (Schedule F, Part 7)."

On the statutory balance sheet, reinsurance recoverable on paid losses and loss adjustment expenses is shown as an asset (line 14 of page 2). Reinsurance recoverable on unpaid losses and loss adjustment expenses is shown as a contra-liability to gross unpaid losses and loss adjustment expenses (lines 1 and 3 of page 3). Ceded unearned premium reserves are shown as a contra-liability to gross unearned premium reserves (line 9 of page 3). The provision for reinsurance is a liability that relates to all of these items.

The provision for reinsurance does not affect the loss reserves on line 1 of page 3, which are net of all reinsurance. It does not affect the loss reserves in the Underwriting and Investment Exhibit or in Schedule P, where no distinctions are made between authorized and unauthorized reinsurers and between slow-paying and non-slow-paying authorized reinsurers.

The provision for reinsurance serves as a minimum bound for uncollectible reinsurance.[22] If the reporting company believes that the uncollectible reinsurance recoverables are greater than the Schedule F provision for reinsurance, it must hold the full estimated uncollectible amount as its provision for reinsurance.[23]

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[22] SSAP No. 62, "Reinsurance," paragraph 52, makes this explicit: "The . . . Provision for Overdue Reinsurance provides for a minimum reserve for uncollectible reinsurance with an additional reserve required if an entity's experience indicates that a higher amount should be provided." But see page 84 for a more critical analysis of this issue.

[23] The Annual Statement *Instructions* say that "if the company's experience indicates that a higher amount should be provided, such higher amount should be entered."

The year to year change in the provision for reinsurance is a direct charge or credit to surplus; it does not flow through the statutory income statement.[24]

By reducing statutory surplus, the provision for reinsurance also reduces risk-based capital adjusted surplus and lowers the risk-based capital ratio.[25]

GAAP financial statements have no provision for reinsurance. GAAP statements show all reinsurance recoverables as assets, not as contra-liabilities, and they reduce the assets for expected uncollectible amounts, just as for other receivables. Similarly, the A. M. Best rating agency removes the provision for reinsurance from net liabilities when calculating its adjusted leverage ratios.[26]

Note 22D to the statutory financial statements, "Uncollectible Reinsurance," discloses "uncollectible reinsurance written off during the year" by reinsurer, in four categories: (i) losses incurred, (ii) loss adjustment expenses incurred, (iii) premiums earned, and (iv) other. (See the Annual Statement *Instructions* and SSAP No. 62, "Reinsurance," paragraph 67). The

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[24] SSAP No. 62, "Reinsurance," paragraph 52, says: "The minimum reserve Provision for Reinsurance is recorded as a liability and the change between years is recorded as a gain or loss directly to unassigned funds (surplus)."

Statutory accounting is more complex if the company holds an additional reserve. The SSAP says that "any reserve over the minimum amount shall be recorded on the statement of income by reversing the accounts previously utilized to establish the reinsurance recoverable." The provision for reinsurance remains a direct charge or credit to surplus. The excess of the estimated uncollectible amount over the statutory provision for reinsurance flows through the income statement. For example, if the statutory provision for reinsurance is $10 million but the reporting company holds a $15 million liability instead, the excess $5 million portion flows through the income statement. It is a part of underwriting income, since it "reverses the accounts previously utilized to establish the reinsurance recoverable"; it is not a component of other income.

Glenda Channel, Finance Reporting Manager of the NAIC, has pointed out to me that the statutory accounting rules are not consistent. The "excess portion" flows through the income statement. But the entire estimated uncollectible amount replaces the provision for reinsurance on line 15 of page 3. The change in the amount recorded on line 15 of page 3 from the previous year to the current year is a direct charge or credit to surplus. The excess portion is thereby counted twice: once as an income statement flow and once as a direct charge.

Ms. Channel notes that "the Annual Statement Instructions (or cross references) might need to be modified" (email, 26 November 2001). In the meantime, companies should avoid this double charge to surplus by choosing whether to run the excess amount through the income statement or accounting for it as a direct charge to surplus.

[25] The provision for reinsurance slightly reduces the risk-based capital requirements, since only reinsurance recoverables that are not offset by the provision for reinsurance are "subject to RBC." This effect is minor; it does not change the statement in the text. See page 29 below for a more complete discussion of the risk-based capital effects of the provision for reinsurance.

[26] See the introduction to Best's *Key Rating Guide*.

amount of such write-offs is not directly related to the provision for reinsurance. However, the write-offs are a check on the adequacy of the company's provision for reinsurance. A company with write-offs consistently exceeding its provision for reinsurance may be underestimating its future liabilities.

The company's Appointed Actuary must discuss reinsurance collectibility and its potential effect on loss reserve adequacy in the Statement of Actuarial Opinion. The Appointed Actuary should use the Schedule F exhibits as one source of information on potential collectibility problems. The NAIC *Instructions* to the Statement of Actuarial Opinion, section 11, say

> *Before commenting on reinsurance collectibility, the actuary should solicit information from management on any actual collectibility problems, review ratings given to reinsurers by a recognized rating service, and examine Schedule F for the current year for indications of regulatory action or reinsurance recoverable on paid losses over 90 days past due.*

An estimate of uncollectible reinsurance is distinct from the statutory provision for reinsurance. There may be a large provision for reinsurance despite no anticipated reinsurance collectibility problem.

Relationships

The relationships among the statutory liability, the contra-asset, and the disclosures are summarized below.

1. *Prospective vs retrospective:*

   i. Note 22 to the financial statements is a retrospective disclosure, identifying the statutory write-off during the past year for uncollectible reinsurance recoverables.
   ii. The provision for reinsurance, the GAAP offset for expected uncollectible recoverables, and the actuary's disclosure in the Statement of Actuarial Opinion are prospective estimates.

2. Basis of Estimate:

   i. Note 22 is an objective accounting fact.
   ii. The Schedule F provision for reinsurance is a formula driven figure.
   iii. The GAAP financial statements provide management's best estimate of future reinsurance uncollectibility.
   iv. The Statement of Actuarial Opinion is the Appointed Actuary's estimate of future reinsurance uncollectibility. Although the Appointed Actuary may be an officer of the company, and the Appointed Actuary should take into account the views of company management regarding potential uncollectible reinsurance recoverables, the actuary's

opinion is an independent professional view which may not agree with management's opinion.

SAP-GAAP Accounting Philosophies

The GAAP vs. statutory accounting approach to measuring the potential uncollectibility of reinsurance recoverables reflects the different underlying philosophies of these accounting systems.

GAAP

The primary goal of GAAP financial statements is to provide potential investors in the corporate enterprise, whether equityholders or creditors, with unbiased information about future expected income. The company's management is the source of most GAAP estimates; this is also true for estimates of uncollectible reinsurance recoverables.

The estimate is audited by an independent accountant. Misrepresentation by management is constrained by the potential lawsuits that such action might cause. Potential investors are assumed to be sufficiently sophisticated that they can interpret and evaluate management estimates. GAAP emphasizes going-concern enterprises, since these are the enterprises of most interest to investors.

STATUTORY ACCOUNTING

The primary goal of statutory financial statements is to assure policyholders that the insurance obligations will be fulfilled. This is particularly important for policyholders of companies in financial distress. Since these companies have incentives to avoid disclosure of uncollectible accounts or similar financial problems, statutory accounting relies heavily on formulas, not on management estimates alone. The formulas are generally conservative; they are intentionally biased, and they are not best estimates.

Most policyholders are unsophisticated. They are unable to independently evaluate management actions or disclosures, and they pose little threat of lawsuits for intentional misrepresentation. Regulators serve as the policyholders' agents to monitor the financial statements of potentially distressed companies. Statutory accounting emphasizes run-off accounting, since the danger to policyholders comes from expiring companies, not from continuing companies.

The chart below summarizes the differing objectives of GAAP and statutory accounting.

	GAAP	*Statutory Accounting*
audience served	investors	policyholders
focus (topic)	future profitability	current obligations
focus (financial statement)	income statement	balance sheet
nature of estimate	unbiased	conservatively biased
basis of estimate	company management	statutory formula
users	sophisticated	not as sophisticated
companies targeted	going concern companies	cos. in financial distress

Federal Income Taxes

The provision for reinsurance is a statutory liability, not a statement liability. It appears on the statutory balance sheet, but the change in the provision for reinsurance does not flow through the statutory income statement.

The change in the provision for reinsurance from the previous year to the current year appears as a direct charge or credit to policyholders' surplus on page 4, line 26. An increase in the provision for reinsurance from last year to the current year causes a decrease in policyholder surplus, and a decrease in the provision for reinsurance from last year to the current year causes an increase in policyholder surplus.

A change in the provision for reinsurance has no effect on taxable income, just as it has no effect on statutory income or GAAP income. Thus, a change in the provision for reinsurance causes a timing (temporary) difference between the statutory balance sheet and the implied tax balance sheet.

In other cases, an increase in a non-admitted asset causes an addition to the deferred tax asset on the statutory balance sheet, like other increases in timing differences between statutory income and taxable income. We illustrate with examples. In all scenarios, the insurance company writes a policy on July 1, 20XX for a premium of $1,000, with a $200 commission paid on July 1.

1. *Revenue Offset:* Statutory income for 20XX is earned premium of $500 – commission expense of $200 = $300. Taxable income for 20XX adds revenue offset of 20% × change in unearned premium reserve or 20% × $500 = $100, so taxable income = $400. The

federal income tax on the $100 difference between taxable income and the income that is implied by the statutory balance sheet is 35% x ($400 – $300) = $35. The deferred tax asset on the statutory balance sheet is $35.

2. *Agents' Balances:* Suppose that the entire net premium or $1,000 – $200 = $800 was due on July 1, 20XX, but the agent remitted only $650. The remaining $150 is more than 90 days past due and it is not admitted on the statutory balance sheet. The taxable income for 20XX remains $400. The income implied by the statutory balance sheet is derived as follows:

   - Cash received = $650
   - Unearned premium reserve = $500
   - Income implied by statutory balance sheet = $650 – $500 = $150

   The income shown on the statutory income statement is $300, not $150. The calculation of the deferred tax asset relies on the income *implied* by the statutory balance sheet, not the income shown on the statutory income statement.[27] The difference between taxable income and implied statutory income is $400 – $150 = $250. The deferred tax asset is 35% x $250 = $87.50.

3. *Provision for Reinsurance:* Suppose that the company included this policy under its 60% proportional reinsurance treaty. A loss of $100 occurs on July 15, which the primary company pays on August 1. It enters the reinsurance recoverable of $60 on its ledger on that date as well, but the recoverable is not paid by the reinsurer until the next year. The recoverable is more than 90 days past due by December 31, and a provision for reinsurance of $12 is set up.[28] The taxable income from this loss is –$100 + $60 = –$40. The income implied by the statutory balance sheet is derived as follows:

   - Cash paid (loss paid) = $100
   - reinsurance recovered (asset) = $60
   - Provision for reinsurance (statutory liability) = $12
   - Income implied by the statutory balance sheet = –$100 + $60 – $12 = –$52.

Following the reasoning in the previous examples, we should say that the difference between taxable income and the income implied by the statutory balance sheet is –$40 – (–$52) = $12.

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27 The deferred tax assets and liabilities depend on the timing difference between actual taxable income and the statutory income implied from the statutory balance sheet. This is identical to the timing differences between the actual statutory balance sheet and the balance sheet entries implied by taxable income. The latter definition – the "balance sheet perspective – is the definition used by the FASB.

28 For simplicity, we are working with small numbers, and we ignore the $50,000 minimum for overdue claims. Assume that there are other loss recoverables from this reinsurer whose total exceeds $50,000.

The federal income tax on this amount is 35% × \$12 = \$4.20, which ought to be shown as a deferred tax asset.

This is *not* the procedure actually used by statutory accounting. SSAP No. 10, "Income Taxes," section 6B, specifically excludes the provision for reinsurance (the Schedule F penalty) from affecting deferred tax assets or liabilities:

> *Temporary differences include unrealized gains and losses and nonadmitted assets but do not include asset valuation reserve (AVR), interest maintenance reserve (IMR), Schedule F penalties . . .*

The rationale for this treatment is that the provision for reinsurance – like the asset valuation reserve and the interest maintenance reserve – is a policyholder safeguard, not a timing difference. It may be necessary for companies in financial distress and inclined to dissemble in their estimates of reinsurance collectibility, but it is unduly conservative for most companies. Statutory accounting does not anticipate a different timing of the reinsurance payment pattern than tax accounting anticipates. The tax on the reinsurance recoverable is not expected to reverse in future years. Rather, the provision for reinsurance for a particular reinsurance contract is expected to diminish as the recoverables are collected and the need for conservative valuation dissipates.

The same is true for the asset valuation reserve and the interest maintenance reserve. They do not reflect a different statutory perspective on the actual value of financial assets. They serve to safeguard the company's ability to pay claims even in adverse financial scenarios. For all of these items, a deferred tax asset would simply reduce the conservatism of the statutory balance sheet.

RISK-BASED CAPITAL REQUIREMENTS

The NAIC's risk-based capital formula sets capital requirements for property-casualty insurance companies based on the amounts and types of risk that they face. To guard against the potential uncollectibility of reinsurance recoverables, the risk-based capital formula includes a risk charge equal to 10% of reinsurance recoverables "subject to RBC."

An admitted reinsurance recoverable increases policyholders' surplus, and the provision for reinsurance reduces policyholders' surplus. If surplus has been reduced by the provision for reinsurance, there is no need to set a capital requirement for the collectibility of the reinsurance recoverables involved. Contrast the two scenarios below.

- If the primary company has a $1 million loss recoverable from a quick-paying authorized reinsurer, and if the recoverable is *not* 90 days or more past due, the full $1 million offsets the gross loss reserve and increases policyholders' surplus. The RBC formula imposes a risk charge of $100,000 to guard against the possibility that the recoverable may not be collected. This risk charge is not the expected uncollectible amount, and it is not a minimum bound for this amount. The risk charge is the potential uncollectible amount in an (unanticipated) adverse scenario.

- If the primary company has a $1 million unsecured loss recoverable from an unauthorized reinsurer, the full $1 million is included in the provision for reinsurance. The loss recoverable does not increase policyholders' surplus, and there is no need for a risk charge to guard against potential collectibility problems in adverse scenarios.

Reinsurance recoverables subject to RBC equal the total recoverables minus the provision for reinsurance (see Feldblum: RBC [1996]). Security held for reinsurance recoverables reduces the provision for reinsurance but it does not reduce the RBC risk charge on the secured recoverables. If the primary company has a $1 million fully secured loss recoverable from an unauthorized reinsurer, the full $1 million reduces the net loss reserve and increases policyholders' surplus. Even if the primary company is holding $1 million as funds withheld from the unauthorized reinsurer, it must hold an additional $100,000 of capital to satisfy the RBC risk charge.[29]

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[29] The ceding company is holding double security for 10% of the recoverables: the funds withheld from the reinsurer and the risk-based capital charge. This is excessive, since the security is greater than the total recoverable. The NAIC justifies this double charge by the disincentive that might otherwise occur to using authorized reinsurers. If secured receivables from unauthorized reinsurers had no risk-based capital charge, ceding companies might be tempted to reinsure their business with unauthorized reinsurers who provided full security; see Feldblum [RBC: 1996].

The covariance adjustment in the property-casualty risk-based capital formula reduces the capital charge for reinsurance recoverables. The risk charges are grouped into six categories, R_0 through R_5, and the covariance adjustment is a function of these risk categories. The 10% charge for reinsurance recoverables subject to RBC is included in the R_3 (credit risk) category. Half the R_3 charge is moved to the R_4 (reserving risk) category before application of the covariance adjustment.

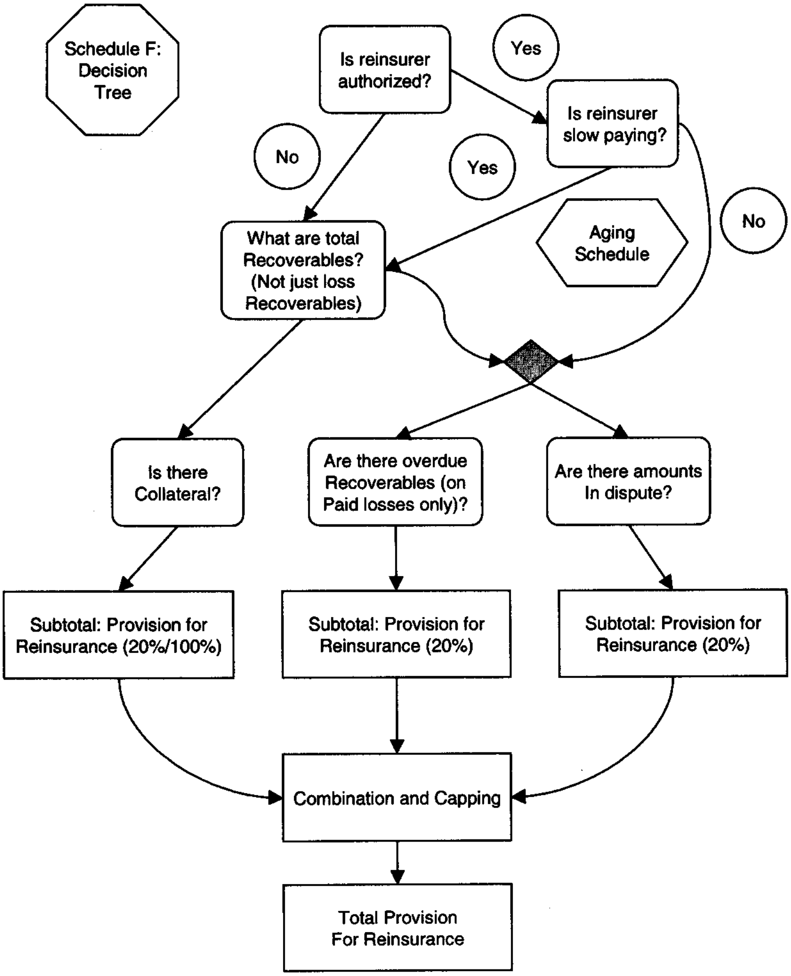
The covariance adjustment reduces the individual category charges in inverse proportion to the size of the category charge. Alternatively stated, the post-covariance marginal effect of the risk charges is in direct proportion to the size of the charges in the risk category.

Illustration: If the R_4 charge is $100 million for a given company and the R_3 charge is $20 million, each dollar of R_4 charge has approximately five times the effect on overall capital requirements as each dollar of R_3 charge. If $1 is added to the R_4 risk charge, the effect on overall capital requirements is about 5 times the effect of adding $1 to the R_3 risk charge.

For most companies, the reserving risk charge (R_4) is large, so the reduction for covariance is small, while the credit risk charge (R_3) is small, so the reduction for covariance is large. The average reduction is about 90 to 95% for the credit risk charge and about 40 to 50% for the reserving risk charge, giving an overall reduction to the charge for reinsurance recoverables of about 45%. On average, the marginal risk-based capital charge for reinsurance recoverables is about 4.5% of the recoverables subject to RBC, not 10%. See Feldblum (RBC: 1996) for a more complete analysis of the effects of the covariance adjustment.

DECISION TREE

Calculating the provision for reinsurance can be complex. The decision tree below shows the elements that affect the provision for reinsurance:



One objective of this paper is to assist in completion of the Annual Statement blank, so the text of this paper follows the format of the Schedule F exhibits. The exhibits are hard to follow, and the computation of the provision for reinsurance seems complex. In fact, there are only a half dozen decision rules, as the graphic above indicates. The following list summarizes these decision rules.

1. If the reinsurer is not authorized, (i) there is no need to test for speed of payments, (ii) 100% of unsecured recoverables are included in the provision for reinsurance, and (iii) we follow the left hand side of the decision tree graphic. (Only if the reinsurer is authorized do we test for the speed of payment.)

2. If the reinsurer is not authorized, the provision for reinsurance is the sum of three parts:

   - 100% of the unsecured (total) recoverables
   - 20% of the loss recoverables more than 90 days past due
   - 20% of the amounts in dispute

   Security has no effect on the provision for reinsurance for loss recoverables more than 90 days overdue and for amounts in dispute.

3. The provision for reinsurance is capped by the amount of total recoverables. Part 5 of Schedule F has a three pronged capping procedure, of which the first two prongs are redundant.

4. If the reinsurer is authorized, we test for speed of payment.

5. If the authorized reinsurer is slow-paying, we treat the slow-paying authorized reinsurer like an unauthorized reinsurer, with three differences.

   - We use 20% of the unsecured total recoverables instead of 100% of the unsecured total recoverables.
   - We use the *greater* of (i) 20% of the unsecured total recoverables (including amounts in dispute) and (ii) 20% of the loss recoverables more than 90 days past due, not the sum of these two parts. There is no need for a capping procedure.
   - We do not examine amounts in dispute separately.

6. If the authorized reinsurer is not slow-paying, the provision for reinsurance is the sum of:

   - 20% of the loss recoverables more than 90 days past due, and
   - 20% of the amounts in dispute.

Part 5: Unauthorized Reinsurers

Part 5 of Schedule F calculates the provision for reinsurance with unauthorized companies. The provision consists of three parts:

- 100% of unsecured (total) recoverables,
- 20% of overdue loss recoverables, and
- 20% of amounts in dispute.

Before 1989, the statutory provision for reinsurance applied only to unsecured *unauthorized* reinsurance recoverables. In 1991, a provision for overdue recoverables from *authorized* reinsurers was added. Security, such as funds withheld and letters of credit, reduced the provision for reinsurance for total recoverables from unauthorized reinsurers, but it did not reduce the provision for reinsurance for loss recoverables over 90 days past due from authorized reinsurers.

Between 1989 and 1991, the only statutory penalty for unauthorized reinsurance was for *unsecured* total recoverables. The provision for recoverables more than 90 days past due from authorized reinsurers applied even if the recoverables were secured. Some authorized reinsurers claimed that they were being penalized more harshly than unauthorized reinsurers if all recoverables were secured. To avoid a possible disincentive to using authorized reinsurance, the provision for recoverables more than 90 days past due was added for unauthorized reinsurers as well.

Recoverables in dispute are not considered overdue, since the cause for non-payment is uncertainty about the reinsurer's liability, not tardiness. Regulators noted that a ceding company could avoid the penalty for overdue recoverables by classifying the recoverables as "in dispute."[30] A provision of 20% of recoverables in dispute was therefore added in 1993.

Penalty for Unsecured Recoverables

Part 5 shows the following figures for unauthorized reinsurers.

- Column 5 shows total recoverables, consisting of net unearned premiums, all loss recoverables, and all commissions. This figure should agree with the corresponding entry in column 15 of Part 3 of Schedule F for unauthorized reinsurers.

Columns 6 through 10 show the funds securing the recoverables, consisting of

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[30] Written notification by the reinsurer that it disputed the claim is sufficient to classify the recoverable as an amount in dispute; actual litigation or arbitration proceedings are not necessary.

- funds held by the company under reinsurance treaties (column 6),
- letters of credit (column 7),
- ceded balances payable (column 8),
- miscellaneous balances (column 9), and
- other allowed offset items (column 10).

Column 11 is the sum of columns 6 through 10. The amount of securitizing funds is capped at the amount of recoverables; that is, column 11 may not exceed column 5. Column 5 minus column 11, shown in column 12, is the amount of unsecured recoverables from unauthorized reinsurers.

Securing agreements are not fail-safe. The subdivision by type of credit allows the reader to better analyze the types of securing funds held by the primary company on behalf of unauthorized reinsurers.[31] Funds withheld are better security than letters of credit, for several reasons:

- The bank issuing the letter of credit may not renew its obligation if the reinsurer's financial condition deteriorates.

  Illustration: A reinsurer obtains a one-year letter of credit on February 1, 20XX, when it is financially healthy. A hurricane in September 20XX produces severe losses for the reinsurer, and impairs its financial condition. Its old recoverables are secured by the letter of credit, and no provision for reinsurance is imposed on the 20XX Annual Statements of its reinsured companies. On February 1, 20XX+1, the bank that issued the letter of credit declines to renew it, leaving the ceding companies exposed to potential collectibility problems.

  Statutory accounting requires that the letter of credit be "evergreen" in order for it to offset the provision for reinsurance. That is, the letter of credit must contain a provision that the issuing bank may not decline to renew it as long as the recoverables remain outstanding.

- If a reinsurer with a letter of credit becomes insolvent, the bank that issued the letter of credit may claim that the letter of credit is invalidated by misrepresentations made by the reinsurer on the application. The ceding company must examine the letter of credit carefully to verify that it is not contingent upon the veracity of representations made by the reinsurance company.

Overdue Recoverables

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[31] For an example of potential problems with letters of credit, see Greene [1988]. Howard W. Greene, "Retrospectively-Rated Workers Compensation Policies and Bankrupt Insureds," *Journal of Risk and Insurance*, Volume 7, No. 1 (September 1988), pages 52-58.

The amount of overdue recoverables *not in dispute* are shown in column 13 of Part 5: "Recoverable paid losses and LAE expenses over 90 days past due not in dispute." 20% of recoverables that are more than 90 days past due are subject to a provision for reinsurance, whether or not they are secured. The number of days the recoverables are overdue is based on the aging schedule in Part 4 of Schedule F.

The total provision for reinsurance may not exceed the total reinsurance recoverables. Schedule F implements this upper bound by specifying that the provision for recoverables more than 90 days past due may not exceed the amount of funds securitizing the total recoverables.

Illustration: Suppose that there are $100 million of recoverables from an unauthorized reinsurer, $50 million of which are more than 90 days past due, and there are letters of credit totaling $5 million. The amount of unsecured recoverables is $100 million – $5 million = $95 million, and twenty percent of the overdue amount is $10 million. Without the cap, the total provision for reinsurance would be $105 million, which is unreasonable since the total recoverables are only $100 million. The penalty for overdue recoverables is therefore limited to the amount of securitizing funds, so the total penalty in this case is $100 million (= $95 million + $5 million).[32]

The provision is shown in columns 14 and 15. Column 14 shows 20% of the recoverables more than 90 days past due in column 13. Column 15 shows the "smaller of col. 11 (= total security) or col. 14."

Amounts in Dispute

Amounts in dispute are not included in column 13 (the recoverables more than 90 days past due), but they are included in column 5 (the total recoverables). "Dispute" is defined as litigation, arbitration, or notification, where notification means "a formal written communication from a reinsurer denying the validity of coverage" (NAIC Annual Statement *Instructions* and SSAP No. 62). The treatment of amounts in dispute is the same as the treatment of loss recoverables more than 90 days past due: 20% of the amounts in dispute are included in the provision for reinsurance.

As is true for loss recoverables more than 90 days past due, the provision for reinsurance for amounts in dispute is limited by the amount of securing funds. The penalty is shown in column 16: "Smaller of col. 11 or 20% of amount in dispute included in col. 5." (Column 11 is the amount of securing funds.)

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[32] By limiting the provision for reinsurance for overdue amounts to the amount of security, the total provision for reinsurance is limited to the total recoverables.

Security does not offset the provisions for amounts in dispute or for recoverables more than 90 days past due. Security guarantees that insolvency of the reinsurer will not prevent payment of the claim.

- If the reinsurer does not admit liability for the claim, the security is not applicable to that claim.
- If the recoverable is more than 90 days past due, we presume that the reinsurer may deny liability for the claim, rendering the security worthless.

Column 17 shows the sum of the three provisions: unsecured total recoverables, 20% of recoverables more than 90 days past due, and 20% of amounts in dispute. This sum is limited by the total recoverables. Column 17 reads:

"Total provision for unauthorized reinsurance: smaller of column 5 (= total recoverables) or columns 12 + 15 + 16 (= the sum of the three provisions for reinsurance)."[33]

- Column 5 is the total recoverables.
- Column 12 is the unsecured recoverables.
- Column 15 is 20% of the recoverables more than 90 days past due.
- Column 16 is 20% of the amounts in dispute.

This penalty is carried to footnote (6) of Part 7: "Provision for unauthorized reinsurance: Schedule F – Part 5, column 17 x 1000." Part 5 of Schedule F is in thousands of dollars whereas the provision for reinsurance is in dollars, so Part 5, column 17 is multiplied by a factor of 1000.

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[33] The limitation in column 17 makes the limitations in columns 15 and 16 redundant. Schedule F grew incrementally, with different provisions being added one by one. Sometimes the final calculation makes an intermediate step unnecessary.

ILLUSTRATIONS

The provision for reinsurance is a fixed formula that is easily applied. The chart below shows several examples. Figures are in thousands of dollars.

Reinsurer:	A	B	C
1. Total Recoverables	$1,000	$1,000	$1,000
2. Securing Funds	$0	$1,200	$600
3. Provision for reinsurance (#1)	$1,000	$0	$400
4. Recoverables > 90 days due	$200	$200	$200
5. Provision for reinsurance (#2)	$40	$40	$40
6. Amounts in dispute	$100	$100	$100
7. Provision for reinsurance (#3)	$20	$20	$20
8. Total provision for reinsurance (uncapped)	$1,060	$60	$460
9. Total provision for reinsurance (capped)	$1,000	$60	$460

- Line 1 includes unearned premium reserves, contingent commissions, loss recoverables on paid losses, and loss recoverables on unpaid losses.

- Line 2 includes all securing funds. The offset is limited to the total recoverables from that reinsurer. Securing funds from one reinsurer can not offset the provision for reinsurance for another reinsurer.

- Line 3: The first provision for reinsurance is the unsecured total recoverables, bounded below by $0.

- Line 4 shows loss recoverables more than 90 days past due.

- Line 5: The second provision for reinsurance is 20% of the loss recoverables more than 90 days past due. Part 5 of Schedule F applies the capping procedure three times, beginning with this line. The chart applies the capping procedure a single time at the end.

- Line 6 shows the amounts in dispute. The amounts in dispute are also included in line 1, the total recoverables. The recoverables on line 1 are offset by securing funds. The amounts in dispute on line 6, like the loss recoverables more than 90 days past due, are not offset by securing funds.

- Line 7: The third provision for reinsurance is 20% of the amounts in dispute.
- Line 8: The total provision for reinsurance is the sum of the three pieces on lines 3, 5, & 7.
- Line 9: The total provision for reinsurance is capped by the amount of total recoverables.

Part 6: Overdue Authorized Reinsurance

Part 6 of Schedule F calculates the statutory provision for recoverables more than 90 days past due from authorized reinsurers that are *not* classified as slow-paying.[34] [35]

Recoverables that are more than 90 days past due are treated equally among all reinsurers, whether authorized or unauthorized and whether slow-paying or not slow-paying. The provision for reinsurance is 20% of these amounts, and security has no effect on the statutory liability. With regard to other recoverables, authorized reinsurers are similar to unauthorized reinsurers only if they are slow-paying authorized reinsurers, though their provision for reinsurance is 20% of the total unsecured recoverables, not 100% of the total unsecured recoverables.

The percentage of loss recoverables more than 90 days past due is calculated for each authorized reinsurer. This percentage is the ratio of the following two amounts:

- loss recoverables more than 90 days overdue to
- all recoverables on paid losses and loss adjustment expenses plus amounts received in the last 90 days of the statement year.

This "overdue ratio" is shown in column 7. If the ratio is 20% or greater, the reinsurer is classified as slow-paying; otherwise, the reinsurer is not slow-paying.[36]

The data used to calculate this ratio are reported in columns 4, 5, and 6. Recoverables on paid losses and LAE that are more than 90 days past due are shown in column 4 and total recoverables on paid losses and LAE are shown in column 5. Amounts in dispute are excluded from both the overdue recoverables and from the total recoverables. The recoverables more than 90 days past due in column 4 of Part 6 equal the sum of the entries in Part 4, column 8 ["91 to 120 days overdue"] and Part 4, column 9 ["Over 120 days overdue"]

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[34] The Annual Statement provides no term to differentiate slow-paying authorized reinsurers. Robert Graham has noted to me that the industry advisory committee to the NAIC reinsurance study group used the term "triggering company" to indicate a company that exceeds the 20% test and *triggers* an additional statutory provision for reinsurance.

[35] The subtitles for Parts 6 and 7of Schedule F are not helpful for understanding their content. The Part 6 subtitle reads "provision for overdue authorized reinsurance," and the Part 7 subtitle reads "provision for overdue reinsurance." From the Annual Statement *Instructions* and the column captions, the reader can discern that authorized reinsurer that are slow-paying are included in Part 7 and authorized reinsurers that are not slow-paying are included in Part 6.

[36] If the overdue ratio is exactly 20%, the reinsurer is classified as slow-paying.

minus the amounts in dispute [see footnote (a) in Part 6]. The total recoverables in column 5 of Part 6 equal the sum of the entries in Part 3, column 7 ["Recoverables on paid losses"] and Part 3, column 8 ["Recoverables on paid LAE"] minus the amounts in dispute [see footnote (b) in Part 6].

The recoverables received in the last 90 days of the statement year, as reported in column 6, are not shown elsewhere in the Annual Statement.

ILLUSTRATION: Suppose that

- The primary company collected $15 million in loss and loss adjustment expense payments from a reinsurer between October 1, 20XX, and December 31, 20XX.
- The remaining recoverables on paid losses and loss adjustment expenses on December 31, 20XX from this reinsurer are $75 million.
- Of these recoverables, $25 million are more than 90 days past due.

The ratio in column 7 of Part 6 is [$25 million ÷ ($75 million + $15 million)] = 27.78%. This reinsurer would be classified as slow-paying.

INCENTIVES

The purpose of including the "amounts received in the prior 90 days" in the denominator of the test ratio described above is to avoid discouraging the settlement of reinsurance claims. Suppose that on December 15, a primary company has $10 million of recoverables on paid losses from a reinsurer. Half of the recoverables ($5 million) are for routine claims; none of these is more than 90 days overdue. Half of the recoverables are for more complex claims. Of these, $1.5 million are more than 90 days overdue.

In this scenario, only 15% of the recoverables are more than 90 days overdue. On December 15, the reinsurer would not be classified as slow-paying. Now suppose that the reinsurer, seeking to settle its accounts by the end of the year, pays $5 million to the primary company to settle the routine claims in the last two weeks of December. It leaves $5 million of recoverables of which $1.5 million are more than 90 days past due, for a 30% overdue ratio.

This is a common scenario, since many companies settle routine accounts by year end. By speeding up the payments on the routine claims, the reinsurer moved from a 15% overdue ratio to a 30% overdue ratio. The ceding company would prefer to delay the settlement of these claims to avoid the provision for reinsurance and the reduction in policyholders' surplus.

To encourage companies to settle reinsurance accounts, the NAIC incorporated the "amounts received in the prior 90 days" in the denominator of the test ratio. Payment of claims during the final quarter of the statement year may have a beneficial effect on the test ratio if some of these claims would have been more than 90 days past due by year end. Payment of claims in the fourth quarter of the statement year can not have an adverse effect on the test ratio.

ILLUSTRATION: The primary company has $10 million of recoverables on paid losses and LAE from a reinsurer on December 15, of which $2.5 million are more than 90 days past due. This reinsurer would be classified as slow-paying on December 15. In the last two weeks of December, the reinsurer pays $5 million to settle claims, including $1 million of claims that are more than 90 days past due. The overdue ratio at December 31 is $1.5 million / $10 million or 15%, and the reinsurer is no longer classified as slow-paying.

For reinsurers that are *not* slow-paying, the provision for reinsurance is 20% of the recoverables that are more than 90 days past due plus 20% of the amounts in dispute that are more than 90 days past due. The column entries in Part 6 of Schedule F are as follows. For reinsurer that are *not* slow-paying, the amounts in column 4 are carried to column 8. The amounts in dispute that are not included in the column 4 total recoverables are shown in column 9. Twenty percent of the column 9 amount is reported in column 10. To this figure is added 20% of the amount in column 8, and the sum is reported in column 11. This penalty is carried to footnote (3) on Part 7.

There is no provision for reinsurance for amounts in dispute that are not yet 90 days past due. A more accurate statement of the provision for reinsurance for authorized reinsurers that are not slow-paying would be "20% of the loss recoverables more than 90 days past due whether or not they are in dispute."[37]

Since Part 6 includes only authorized reinsurers that are not slow-paying, there is no provision for unsecured total recoverables. The amount of security is not relevant for authorized reinsurers that are not slow-paying, since security has no effect on the provision for overdue recoverables or for amounts in dispute. There is no "capping" procedure on the total provision for reinsurance, since there is no provision for total recoverables.

[37] It is not clear if the regulators who designed Schedule F intended any provision for reinsurance for amounts in dispute that are not yet 90 days past due. The comments in the text follow the wording of the Schedule F exhibits.

Part 7: Slow-Paying Authorized Reinsurers

Reinsurers that are slow-paying are treated like unauthorized reinsurers, except that the statutory penalty is the *greater* of 20% of the unsecured recoverables and 20% of the recoverables that are more than 90 days past due, not the sum of these two amounts.

For slow-paying authorized reinsurers, the unsecured recoverables include amounts in dispute. For unauthorized reinsurers and for authorized reinsurers that are not slow-paying, security does not offset the provision for reinsurance for amounts in dispute, since the security does not apply unless the reinsurer admits that it is required to pay the claim. For authorized reinsurers that are classified as slow-paying, security has the same effect on amounts in dispute as on other recoverables. We offer no rationale for this; it may be an oversight in the present format of Schedule F.

The calculations are shown in Part 7 of Schedule F. Columns 4 through 11 have the same format as columns 5 through 12 of Part 5, which computes the provision for reinsurance for unauthorized reinsurers. Column 11 of Part 7 shows the unsecured total recoverables for slow-paying authorized reinsurers, just as column 12 of Part 5 shows the unsecured total recoverables for unauthorized reinsurers. For slow-paying authorized reinsurers, only 20% of this amount is included in the provision for reinsurance. The 20% factor is applied in footnote 2, not in the column entries.

Column 11 of Part 7 is the total unsecured recoverables and column 12 is the "greater of column 11 or Schedule F, Part 4, columns 8 and 9." Part 4, columns 8 plus 9, is the loss recoverables that are more than 90 days past due. The column 12 total is carried to footnote (1), 20% of which is carried to footnote (2). Footnote (2) is the provision for slow-paying authorized reinsurers.

The Provision for Reinsurance

The footnotes in Part 7 show the provisions for reinsurance.

- Footnote 2 shows the provision for slow-paying authorized reinsurers.
- Footnote 3 shows the provision for authorized reinsurers that are not slow-paying.
- Footnote 4 shows the total provision for authorized reinsurers [= footnotes 2 + 3].
- Footnote 5 shows the provision for unauthorized reinsurers.
- Footnote 6 shows the total provision for reinsurance [= footnotes 4 + 5], which is carried to page 3, line 15.

OTHER ESTIMATES

The statutory penalty is a minimum. If the primary company believes that the uncollectible recoverables are more than the statutory provision for reinsurance, it should hold the larger amount instead of the provision for reinsurance.

The change in the provision for reinsurance is a direct charge or credit to surplus on line 26 of page 4; it does not flow through the statutory income statement. If the company books a liability in excess of the provision for reinsurance because it believes that the uncollectible amount is greater than the provision for reinsurance, the excess amount flows through the statutory income statement. See page 22 for a complete discussion of this topic.

RESIDUAL MARKETS

The NAIC *Instructions* regarding Part 4 note that "all recoverables from mandatory pools should be reported . . . as being current." Servicing carriers for residual market pools, as are used for workers' compensation, commercial automobile, and Massachusetts personal automobile, cede the involuntarily written business to the pool. Pools are often slow payers, since they may make only quarterly transactions with servicing carriers and with pool members. The servicing carriers may find that much of the recoverables are more than 90 days past due and would lead to a provision for reinsurance on the statutory financial statements. This would be a disincentive for insurers to act as servicing carriers, thereby exacerbating availability problems in these lines of business. To avoid such problems, the NAIC imposes no statutory reinsurance penalties for business ceded to residual market pools.

Part 8: Restatement of Balance Sheet

Part 8 of Schedule F was added with the 1992 Annual Statement. This exhibit is the statutory counterpart to the accounting changes made by SFAS 113, "Accounting and Reporting for Reinsurance of Short-Duration and Long-Duration Contracts," issued in December 1992.

Page 3 of the NAIC statement, the statutory balance sheet, is on a "net of reinsurance" basis. Line 1 of page 3, "losses," shows the loss reserves net of reinsurance recoverable on unpaid losses. Line 9 of page 3, "unearned premiums," is net of unearned premiums for ceded reinsurance.[38]

An insurer with a $1,000,000 unpaid loss which is fully reinsured shows a net unpaid loss liability of $0 on line 1 of page 3. But reinsurance arrangements rarely reduce an insurer's legal liability to claimants. The insurer's obligation to the claimant is independent of the reinsurance transaction.

SFAS 60, which controlled reinsurance accounting on GAAP financial statements until 1993, used the same offsetting of reinsurance recoverables with direct business as statutory accounting uses. SFAS 113, which controls reinsurance accounting after 1992, requires that the insurer show the full $1,000,000 loss reserve liability, along with a corresponding $1,000,000 asset for the anticipated reinsurance recoverables. This enables readers of the financial statements to differentiate between

- A $0 net loss liability consisting of a $0 direct loss liability and a $0 recoverable, and
- A $0 net loss liability consisting of a $1,000,000 direct loss liability and a $1,000,000 reinsurance recoverable.

The statutory balance sheet on page 3 of the Annual Statement remains on a net basis. Part 8 of Schedule F shows a restated balance sheet on a gross of reinsurance basis, with the net amount due from reinsurers combined into a single asset.

Part 8 of Schedule F changes the format of the balance sheet to the GAAP format. The reinsurance recoverables are assets, not contra-liabilities. The content of the entries remains the statutory content; the provision for reinsurance remains on the balance sheet.

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[38] In 1992, Line 10 of page 2, "Agents' balances," showed the balances due from agents net of ceded premium balances due to reinsurers. The 2001 codification changes separated the direct agents' balances asset from the reinsurance balances liability, which is now shown separately on line 11 of page 3.

An Illustration

Statutory accounting for reinsurance can be complex. Let us follow a simplified reinsurance transaction to illustrate the effects on page 2, page 3, and Schedule F, Part 8.

Suppose an insurer writes a commercial automobile policy with a $10,000 premium on December 31, 2001, and includes the contract under its 40% quota share reinsurance treaty with a non-affiliated authorized reinsurer. It incurs one loss for $5,000 on October 1, 2002, which it pays on July 1, 2003. It collects the recoverable from its reinsurer on March 1, 2004. For simplicity, assume that all premium is paid on the policy effective date, the primary company incurs no expenses, and there is no reinsurance commission on this treaty.

FIRST YEAR – UNEARNED PREMIUMS

On December 31, 2001, the primary company collects $10,000 from the insured and pays $4,000 to the reinsurer. In its 2001 Annual Statement, the company shows $10,000 of direct premiums in the "Underwriting and Investment Exhibit," Part 2B, "Premiums Written," column 1. It shows $4,000 of ceded premium in column 5 of Part 2B. The "net premium written" in column 6 of this exhibit is $10,000 – $4,000, or $6,000.

Since the earned premium on December 31, 2001 is $0, the unearned premium reserve is $10,000 gross of reinsurance and $6,000 net of reinsurance. The net unearned premium reserve is carried to page 8, Part 2A, "Recapitulation of all Premiums," and to page 7, Part 2, "Premiums Earned."

SECOND YEAR – LOSS RESERVES

On December 31, 2002, the entire policy premium has been earned, so both the gross and the net unearned premium reserves are $0. Since a $5,000 loss was incurred on October 1, 2002, and remains unpaid as of December 31, 2002, there is a gross loss reserve of $5,000. The primary company has a 40% quota share treaty, so the net of reinsurance loss reserve is $3,000.

In Schedule P, Part 1C the company shows:

- $10,000 of "direct and assumed" earned premium in column 2,
- $4,000 of ceded earned premium in column 3, and
- $6,000 of net earned premium in column 4.

It shows $5,000 of "direct and assumed" case basis losses unpaid in column 13, and $2,000 of ceded unpaid losses in column 14. The net unpaid loss is $3,000.

Page 11, Part 3A, "Unpaid losses and loss adjustment expenses," shows the direct loss reserve of $5,000 in column 1, the ceded loss reserve of $2,000 in column 3, and the net loss reserve of $3,000 [= $5,000 – $2,000] in column 4. The net loss reserve flows through to the "Underwriting and Investment Exhibit," Page 10, Part 3, "Losses paid and incurred," column 5, "net losses unpaid current year."

THIRD YEAR – PAID LOSSES

On December 31, 2003, the primary company has paid $5,000 to the claimant, but it has not yet recovered any money from the reinsurer. Both the direct and ceded loss reserves on Part 3A of the "Underwriting and Investment Exhibit" (columns 1 and 3, respectively) are set to zero. Part 3 of the "Underwriting and Investment Exhibit," column 1, "losses paid less salvage on direct business," shows $5,000, while column 3, "reinsurance recovered," shows $2,000.[39] The reinsurance recoverable appears as an asset on page 2, line 14, "Reinsurance recoverable on loss and loss adjustment expense payments," not as a contra-liability.

The gross of reinsurance unearned premium reserve is not shown on these exhibits. The gross of reinsurance loss reserve may be determined from Schedule P, except that the Schedule P definition of reinsurance differs from the Schedule F definition of reinsurance.[40]

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[39] The "reinsurance recovered" entry is the full recoverable, even though there has been no cash transaction. James Anastasio, Vice President and Treasurer at the American Re-Insurance Company, explains that:

> *Insurance accounting dictates that an entry be made to reflect the reinsurance recovered regardless of the fact that the cash has not been received. In lieu of cash, a receivable asset is created called "reinsurance receivable on losses and loss adjustment expenses.*

Once the entry is posted to reflect this "reinsurance recovered," the contra-liability "reinsurance recoverable on unpaid losses" in the amount of $2,000 is taken down.

Insurance uses accrual accounting. The occurrence of a loss is an income statement debit, not the payment of the loss. When the loss occurs, the net (of reinsurance) loss reserve is the income statement debit. When the direct loss is paid to the claimant, the loss reserve becomes a paid loss and the offsetting contra-liability called reinsurance recoverable on unpaid losses becomes a reinsurance recovered; there is no effect on the income statement. When the recoverable is collected, the asset called reinsurance recoverable on paid losses becomes an asset called cash; there is no effect on the income statement.

[40] When an insurance group has an intercompany pooling agreement among affiliated carriers, Schedule P treats the premiums and losses as direct business, not as ceded and assumed business, regardless of which company's paper the business is written on. Schedule F, however, treats the business as ceded and assumed, depending on which company issued the policy. Other exhibits in the Annual Statement follow the Schedule F definition, not the Schedule P definition.

The net figures in Schedule P equal the net figures elsewhere in the Annual Statement, but the "direct and assumed" and the "ceded" figures do not necessarily equal the corresponding figures in other exhibits. For instance, the "net earned premium" in the Schedule P, Part 1 Summary, column 4, line 11 [= current year]

FOURTH YEAR – REINSURANCE RECOVERIES

By December 31, 2004, the primary company has received payment from the reinsurer. The page 2 asset, "Reinsurance recoverable on loss and loss adjustment expense payments," is eliminated, having been replaced by cash (or other assets).

This illustration is used below to explain the entries in the Schedule F, Part 8 exhibit.

THE PART 8 EXHIBIT

Schedule F, Part 8, "Restatement of Balance Sheet to Identify Net Credit for Reinsurance," allows the Annual Statement user to see the effects of ceded reinsurance transactions on the company's balance sheet. All items from pages 2 and 3 are carried to this exhibit, though only the lines most relevant to reinsurance transactions are shown separately. Other lines are combined as "other assets" (line 5 in Schedule F, Part 8) and "other liabilities" (line 15 in Schedule F, Part 8).

Cessions to an involuntary pool or a joint underwriting association are not shown in Part 8. These are programs mandated by state governments to provide coverage for risks that might not otherwise be insured by private insurers. The uncollectibility risk is assumed to be insignificant, since the liabilities of the pools are backed by state assessments on all insurance companies writing business in the state for the line of business handled by the pool.

The involuntary pools for certain lines of business have been large in some years. In the latter half of the 1980's, shortly before Part 8 of Schedule F was formed, the workers' compensation reinsurance pools covered over 25% of the total business in some states. Including the involuntary cessions in Part 8 of Schedule F would have masked the effects of voluntary ceded

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should equal the net premiums earned in the current year on page 7, "Underwriting and Investment Exhibit," Part 2, "Premiums Earned," column 4, line 34 [= totals]. The net losses unpaid excluding loss adjustment expenses in the "Underwriting and Investment Exhibit," Part 3A, column 8, line 34 ("totals") should equal the net losses unpaid from the Schedule P, Part 1 Summary, line 12 ("totals"), columns 13 – 14 + 15 – 16. But the component pieces, the "direct and assumed" and the "ceded," may not correspond between Schedule P and the "Underwriting and Investment Exhibit" if there is an intercompany pooling agreement among affiliated carriers. In other words, Schedule P, columns 14 + 16 may not equal the "Underwriting and Investment Exhibit," Part 3B, columns 1 + 2 + 5 + 6. Similarly, Schedule P, columns 14 + 16 may not equal the "Underwriting and Investment Exhibit," Part 3A, columns 3 + 7. According to the Annual Statement *Instructions* to the "Underwriting and Investment Exhibit," the Part 3A, column 3 totals ("reinsurance recoverable from authorized and unauthorized reinsurers") should equal the Schedule F, Part 3, column 9 totals ("reinsurance recoverables on known case loss reserves"), and the Part 3A, column 7 totals ("ceded IBNR losses") should equal the Schedule F, Part 3, column 11 totals ("reinsurance recoverable on IBNR loss reserves"). There are no such references to the ceded amounts in Schedule P. For further discussion, see Sholom Feldblum, "Completing and Using Schedule P," Third Edition, in *Regulation and the Casualty Actuary*, edited by Sholom Feldblum and Gregory Krohm (NAIC, 1997); revised Fourth Edition [2002] available in electronic form on the CAS web site.

reinsurance. The voluntary reinsurance, which has potential collectibility problems, is the important component of the Part 8 disclosure.

Schedule F, Part 8, has the entries shown below. The "Item" numbers refer to the line numbers on pages 2 and 3 of the Annual Statement (the statutory balance sheet). Column 3 of page 2 shows the net admitted assets, or the total assets minus the non-admitted assets.

Restatement of Balance Sheet to Identify Net Credit for Reinsurance

ASSETS (Page 2, column 3)

1. Cash and invested assets (Item 9)
2. Agents' balances or uncollected premiums (Item 10)
3. Funds held by or deposited with reinsured companies (Item 11)
4. Reinsurance recoverables on loss and LAE payments (Item 14)
5. Other assets (Items 11 and 12 and 15 through 24)
6. Net amount recoverable from reinsurers

7. Totals (Item 25)

LIABILITIES (Page 3)

8. Losses and loss adjustment expenses (Items 1 through 3)
9. Taxes, expenses, and other obligations (Items 4 through 8)
10. Unearned premiums (Item 9)
11. Dividends declared and unpaid (Items 10.1 and 10.2)
12. Funds held by company under reinsurance treaties (Item 12)
13. Amounts withheld or retained by company for account of others (Item 13)
14. Provision for reinsurance (Item 15)
15. Other liabilities (Items 14 and 16 through 22)

16. Total Liabilities (Item 23)

17. Surplus as regards policyholders (Item 32)

18. Totals (Item 33)

For each entry, there are three columns:

1. As Reported (net of ceded)
2. Reinsurance Adjustments
3. Restated (gross of ceded)

PURPOSE OF PART 8

Part 8 of Schedule F shows the net effect of ceded reinsurance transactions on the statutory balance sheet. The balance sheet itself shows various entries relating to ceded reinsurance, some of which are placed on separate lines and some of which are offsets to gross figures. Part 8 consolidates all the entries into a single asset, termed "net amount recoverable from reinsurers."

There are four types of adjustments made in Part 8 of Schedule F:

- Some asset entries remain an asset entry, though the name is changed. For instance, the asset called "reinsurance recoverables on loss and LAE payments" is shifted into "net amount recoverable from reinsurers."

- Contra-liabilities resulting from ceded reinsurance are changed into assets. For instance, the reinsurance recoverable on unpaid losses, which is an offset to unpaid losses on page 3 of the Annual Statement, is added to the "net amount recoverable from reinsurers."

- Liabilities stemming from ceded reinsurance, such as the "funds held by company under reinsurance treaties," are *offsets* to the "net amount recoverable from reinsurers."

- On the statutory balance sheet, the provision for reinsurance counter-balances the assets or contra-liabilities stemming from ceded reinsurance. On Part 8 of Schedule F, the provision for reinsurance reduces the "net amount recoverable from reinsurers."

The adjustments are described individually below, and an illustration is provided towards the end of this paper.

GAAP and Statutory Accounting

Statutory accounting shows unpaid losses and unearned premium reserves net of reinsurance recoverables on the balance sheet. SFAS 60 used the same procedure for GAAP financial statements until 1993.

SFAS 113, paragraph 3, citing APB Opinion No. 10, Omnibus Opinion –1966, paragraph 7 states, "It is a general principle of accounting that the offsetting of assets and liabilities in the balance sheet is improper except where a right of setoff exists." The criteria for offsetting are specified in FASB Interpretation No. 39, "Offsetting of Amounts Related to Certain Contracts." SFAS 113 notes that "those criteria include the requirement that the reporting party have the legal right to set off the amount owed to one party with an amount receivable from that same party."

SFAS 113, paragraph 14, explains that:

reinsurance contracts in which a ceding enterprise is not relieved of the legal liability to its policyholder do not result in removal of the related assets and liabilities from the ceding enterprise's financial statements. Ceding enterprises shall report estimated reinsurance receivables arising from those contracts separately as assets. Amounts paid to the reinsurer relating to the unexpired portion of reinsured contracts (prepaid reinsurance premiums) also shall be reported separately as assets.

GAAP financial statements now show two balance sheet items:

- unpaid losses and unearned premium reserves gross of reinsurance recoverables on the liability side of the balance sheet and
- the total recoverables from reinsurers on paid losses, unpaid losses, and unearned premium reserves on the asset side of the balance sheet.

NAIC Statement of Statutory Accounting Principles 62, Property & Casualty Reinsurance, section on "Accounting for Prospective Reinsurance Agreements" keeps the offsetting procedure. Paragraph 26 says:

Reinsurance recoverables on paid losses shall be reported as an asset, reinsurance recoverables on loss and loss adjustment expense payments, in the balance sheet. Reinsurance recoverables on unpaid case-basis and incurred but not reported losses and loss adjustment expenses shall be netted against the liability for gross losses and loss adjustment expenses.

The justification for retaining the net accounting procedures was to avoid a major change in statutory balance sheets. Many insurance accountants consider the GAAP procedure a more informative presentation of the company's financial position. Schedule F, Part 8 shows the statutory balance sheet as it would look if offsetting were not permitted.

The balancing item in Part 8 of Schedule F, "Net amount recoverable from reinsurers," is lower than the corresponding entry on the GAAP financial statements by the amount of the provision for reinsurance minus the GAAP provision for uncollectible reinsurance recoverables. In this sense, the provision for reinsurance is a non-admitted asset.

Restatement of Liabilities

We begin with the "Liabilities" section of this exhibit. Line 8, "losses and loss adjustment expenses (Items 1 through 3 of page 3)," may be illustrated with the commercial auto example above. Column 1 of Part 8 shows the net of reinsurance amounts that are reported on page 3. Column 2 of Part 8 shows the required adjustment to exclude the effects of ceded reinsurance. Column 3 of Part 8 shows the gross of reinsurance amounts.

In the example given earlier, for the 2002 statement, the net 2002 losses unpaid of $3,000 would be shown in the first column, the ceded amount of $2,000 would be shown in the second column, and the gross amount of $5,000 would be shown in the third column. The figures in column 2 for these lines are generally positive amounts, since only ceded reinsurance (not assumed reinsurance) is considered.[41]

Line 10 ("item 9"), "Unearned premiums," is similar. In the commercial auto example, for the 2001 statement, the net 2001 unearned premium reserves of $6,000 would be shown in the first column, the ceded amount of $4,000 would be shown in the second column, and the gross amount of $10,000 would be shown in the third column.

Item 12, "Funds held by company under reinsurance treaties," and item 15, "Provision for reinsurance," are positive liabilities on page 3. If the company had no ceded reinsurance, it would have zeros on these lines. Column 2 of this exhibit therefore has negative amounts which fully offset any positive amounts in column 1, leaving zeros in column 3.[42]

Items 12 and 15 have similar treatment, but they are different types of entries. Item 12 is a real liability. The funds are owned by the reinsurance companies, though they are held by the ceding company. The ceding company shows a liability for the amounts which it holds but are owned by other parties, similar to the liability shown on line 13 of page 3.

Item 15 represents a statutory liability. The provision for reinsurance is not owed to a third party. It represents a statutory adjustment to cancel other assets or contra-liabilities that are not admitted assets on the statutory balance sheet.

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[41] Insurance accounting differs among companies, and there are numerous exceptional situations that do not conform with the general rules presented here. There are companies which show negative amounts in some of these cells. Similarly, few of the general rules mentioned later in the text are true for all companies.

[42] The NAIC Annual Statement *Instructions* say that *these liabilities become offsets to the overall asset "net amount recoverable from reinsurers."* In other words, the full amount in column 1 is reversed in column 2. Since line 6, "net amount recoverable from reinsurers," is a balancing item, they are "offsets" to line 6.

Item 13, "amounts withheld or retained by company for account of others," does not relate to reinsurance. These are funds which the reporting company owes to other parties. Two common examples are FICA taxes at the end of the year and uncashed checks to claimants.[43]

- An employer pays FICA taxes to the U.S. Treasury on the earnings of its employees. The FICA taxes are deposited within 15 days of the end of the month into a commercial bank or other depository institution to cover the payroll of that month. At the end of December, the reporting company still holds the cash in its own accounts, but it owes the money to the Treasury (for the benefit of its employees). A liability for the amount of the December FICA taxes is shown on line 13 of page 3.

- If a claimant does not cash a claim check drawn by the insurance company, the company must eventually remit the funds to the state. At the end of December, the reporting company may have various uncashed claim checks, but it has not yet remitted the funds to the state. A liability for these funds is shown on line 13 of page 3.

These funds are unrelated to ceded reinsurance, and it is unclear why line 13 of page 3 is broken out separately on Part 8 of Schedule F. The prominent display of this line is confusing to insurance accountants. Readers of the Annual Statement would be better served if this line were subsumed under the "other liabilities" entry in Part 8 of Schedule F.

The other lines in the liabilities section of this exhibit are less commonly used, though the analyst must consider any additional effects of reinsurance treaties. For example, the policyholder dividends declared and unpaid may be changed if a proportional reinsurance treaty contributes a percentage of the dividend.

For line 17, "surplus as regards policyholders," column 2 is "X-ed out." On page 3, surplus is the balancing item; that is, it is the difference between reported assets and reported liabilities. In Schedule F, Part 8, line 6, "net amount recoverable from reinsurers," is the balancing item. Policyholders' surplus does not change. The Part 8 exhibit changes the accounting *presentation* of the company's balance sheet. It does not change the overall *result* of the balance sheet.

Restatement of Assets

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[43] D. Keith Bell, "Other Liabilities, Capital and Surplus," in Insurance Accounting and Systems Associations, Inc., *Property-Liability Insurance Accounting* (Durham, NC, 1994), chapter 6, page 6-9, describes the two major components of this liability:

- Deductions from employees or agents for payroll taxes, group insurance premiums, pensions, and similar items.
- Policyholder or claimant funds held by the company (e.g., uncashed checks).

Line 4, "reinsurance recoverable on loss and loss adjustment expense payments (item 14 of page 2)," relates to ceded reinsurance. The column 1 entry is offset by a negative entry in column 2, leaving a zero in column 3. Part 8 is transferring the asset from a recoverable on paid losses to part of the total recoverable from reinsurers.

Line 3, "funds held by or deposited with reinsured companies (item 11 of page 2)," relates to assumed reinsurance, not ceded reinsurance. These are the funds owned by the reporting company (whose Annual Statement we are considering) but held by its reinsured companies. This entry has nothing to do with the ceded reinsurance transactions of the reporting company. Most companies show a zero in column 2 for this line. No other entry makes sense; there is no Annual Statement *Instruction* for this line. The separate display of this line is confusing to some insurance accountants.

Line 2, "agents' balances or uncollected premiums" (item 10 of page 2), is a carry-over from the pre-2001 Annual Statement. The line and the Annual Statement *Instructions* pertaining to it will presumably be changed by the NAIC Blanks Committee as soon as the error is noted. The Annual Statement *Instructions* say that

> *This asset should be increased by the ceded reinsurance balances payable (reversing the parenthetical decrease on page 2, line 10) which is offset against the "net amount recoverable from reinsurers."*

This was correct for the 2000 and prior Annual Statements. The 2001 NAIC codification changed the reinsurance premium balances payable

- *from* a contra-asset to agents' balances receivable
- *to* a separate liability on line 11 of page 3: "ceded reinsurance balances payable (net of ceding commissions)."

For the year 2001 Annual Statement, companies should ignore the Annual Statement *Instructions*. They should leave this item unchanged. They should reverse the balance sheet entry of Page 3, Item 11. Presumably, they should do this on the "other liabilities" line of Schedule F, Part 8, though the official designation of this line is "items 14 and 16 through 22." By oversight, line 11 of page 3 is nowhere shown on Schedule F, Part 8.

Line 1, "cash and invested assets," and line 5, "other assets," are used by some companies, while other companies show zeros in column 2 for these lines. For instance, one company shows the line 1 adjustment as a balancing item to the line 12 adjustment. Line 12 shows "funds held under reinsurance treaties." If there were no ceded reinsurance, the primary company would not have these funds, so "cash and invested assets" are reduced by the same amount.[44]

Line 6, "net amount recoverable from reinsurers," is the balancing item. Mathematically, it is the amount needed so that line 7, "total assets," column 2, equals line 16, "total liabilities," column 2. Conceptually, it is the net asset representing the "assets plus the contra-liabilities minus the liabilities" on the statutory balance sheet relating to ceded reinsurance.

[44] This reasoning is not correct. As noted earlier in the text, Part 8 does not change the reinsurance arrangements of the company; it changes only the accounting presentation of these arrangements. The liability called "funds held by company under reinsurance treaties" is transformed into a contra-asset. There is no change to the assets held by the reporting company. When completing the statutory exhibits, readers are advised to use the standard practices recommended in this paper.

Illustrations

The exhibits in Schedule F are sparsely documented in the NAIC *Instructions* to the Annual Statement. An unfortunate result is that many company statements in recent years have contained errors in the Schedule F entries.

Much of the exposition in the preceding sections is abstract. The following sections present examples that demonstrate the mechanics of completing these schedules.

I. Restatement of Balance Sheet

You are the reinsurance officer for a medium size commercial lines insurer that has substantial reinsurance transactions, and you have been asked to complete Schedule F, Part 8 of the Annual Statement. You have filled in the entries in the first column, using the figures from pages 2 and 3 of the statutory blank, as shown on the exhibit on the following page. The reinsurance accounting department in your company provides you with the following additional information:

1. The total reinsurance recoverables on paid and unpaid losses are $160,000,000.

2. The unearned premium reserves are $50,000,000 on direct business and $10,000,000 on assumed business.

3. The "ceded reinsurance balances payable" on line 11 of page 3 are $5,000,000.

Your company's management asks you what figures will appear in the boxes labeled A, B, C, and D in the third column ("restated") on the exhibit:

A. Recoverable from reinsurers.
B. Total assets.
C. Total liabilities.
D. Surplus as regards policyholders.

Schedule F, Part 8: Initial Exhibit

Schedule F, Part 8: Restatement of Balance Sheet to Identify Net Credit for Ceded Reinsurance ($000,000's)			
ASSETS (page 2, column 3)	As Reported	Adjustment	Restated
1. Cash and invested assets	$200		
2. Agents' balances or uncollected premiums	$10		
3. Funds held by or deposited with reinsured companies	$30		
4. Reinsurance recoverable on loss and LAE payments	$40		
5. Other assets	$20		
6. Net amount recoverable from reinsurers			A
7. Total Assets	$300		B
LIABILITIES (page 3)			
8. Losses and loss adjustment expenses	$100		
9. Taxes and other expenses	$3		
10. Unearned premiums	$40		
11. Dividends declared and unpaid	$2		
12. Funds held by company under reinsurance treaties	$20		
13. Amounts withheld or retained for account of others	$1		
14. Provision for reinsurance	$15		
15. Other liabilities	$9		
16. Total liabilities	$190		C
17. Surplus as regards policyholders	$110		D
18. Total liabilities plus surplus	$300		

Completing Part 8

The completed exhibit is shown below, along with explanation of each entry. We proceed line by line, stating the assumptions and showing the derivation of the values.

Certain adjustments depend upon the particularities of each case, for which there is insufficient information in this example (e.g., lines 1, 5, 9, and 11 below). We assume that no adjustments are needed for these lines unless information requiring an adjustment is provided.

For certain other items, there are differences of opinion among insurance accountants about the proper adjustments. The illustration here should not be taken to imply that other methods of completing this exhibit are necessarily wrong.

Assets

1. Line 1 in the exhibit, "Cash and invested assets," needs no adjustment. The entry in column 3 is $200,000,000.

2. The Part 8 exhibit along with its *Instructions* assume pre-codification statutory accounting for agents' balances. The exhibit and its instructions will presumably be corrected to conform with the current statutory balance sheet. (By the time this paper is printed, the corrections noted here should have been placed on the NAIC web site.)

We explain first the intention of the Annual Statement *Instructions*, which assume the pre-2001 balance sheet format. We then show the appropriate accounting entries for 2001 and subsequent years.

Before 2001, the following balance sheet items were net of reinsurance ceded: (i) loss reserves, (ii) loss adjustment expense reserves, (iii) unearned premium reserves, and (iv) agents' balances. We deal with loss reserves, loss adjustment expense reserves, and unearned premium reserves in the appropriate sections below. The agents' balances entry was direct agents' balances and premiums receivable net of reinsurance balances payable. For instance, if direct agents' balances were $15 million and premium balances owed to assuming reinsurers were $5 million, the agents' balances entry on line 10.1 of page 2 was $10 million. The pre-2001 line label for agents' balances on the statutory balance sheet was "Premiums and agents' balances in course of collection (after deducting ceded reinsurance balances payable of ______)."

The NAIC codification project prohibited the netting of premiums receivable with premiums payable, though it retained the net accounting for loss reserve and unearned premium reserves. Statutory issue paper No. 75, "Property and Casualty Reinsurance," paragraph 5 says that "ceded reinsurance premiums payable (net of ceding commission) shall be classified as a liability." The current line 10.1 on page 2 reads "Premiums and agents' balances in course of collection," and line 11 on page 3 reads "ceded reinsurance balances payable (net of ceding commissions)."

The Schedule F, Part 8 exhibit has not been updated to reflect this change. The 2001 *Instructions* for agents' balances on the Schedule F, Part 8 exhibit still read:

> *Line 2 – AGENTS' BALANCES OR UNCOLLECTED PREMIUMS: This asset should be increased by the ceded reinsurance balances payable (reversing the parenthetical decrease on Page 2, Line 10) which is offset against "net amount recoverable from reinsurers."*

This Annual Statement *Instruction* is no longer valid by January 1, 2001, when codification of statutory accounting was effective. There should be no adjustment for line 2. Instead, the statutory liability on line 11 of page 3 for ceded reinsurance balances payable should be reversed in column 2 of Schedule F, Part 8. There is no separate line for this in Part 8, so the reversal should be made on line 15, "other liabilities." The label for line 15 says "items 14 and 16 through 22," which does not include item 11 on page 3. The item numbers should be disregarded in this instance.

3. Line 3 in the exhibit, "Funds held by or deposited with reinsured companies," relates to assumed reinsurance, not ceded reinsurance. The company provides reinsurance to other carriers, as shown by the $10,000,000 of unearned premium reserves on assumed business. Since Part 1 of Schedule F relates to ceded reinsurance only, there is no adjustment on this line, and the restated entry remains $30,000,000.

4. Line 4 in the exhibit, "Reinsurance recoverable on loss payments," is the recoveries from reinsurers on losses that have already been paid. If there were no ceded reinsurance, there would be no recoverables from reinsurers. This entry is reversed by an adjustment of –$40,000,000, yielding a zero in the restated column. The whole amount is offset by an opposite entry in line 6, "Net amount recoverable from reinsurers."

5. Line 5 in the exhibit, "Other assets," are not affected by ceded reinsurance transactions except in exceptional circumstances. We assume that no such exceptions are involved here, so we enter a zero for the adjustment and $20,000,000 in the "restated" column.

6. Line 6 in the exhibit, "Net amount recoverable from reinsurers," is the balancing item. We can not determine it until we have completed the "liabilities" portion of this exhibit.

7. Line 7 is the total assets. This is the sum of lines 1 through 6. Since line 6 is not yet known, we skip line 7 as well.

Liabilities

8. We are told that "the total reinsurance recoverables on paid and unpaid losses are $160,000,000." The recoverables on paid losses are $40,000,000 (line 4), so the recoverables on unpaid losses (line 8) are $120,000,000. We enter $120,000,000 as the adjustment on line 8, and $220,000,000 as the restated amount.

9. Most of the items included in line 9 (lines 4 through 8 of page 3) are not directly affected by reinsurance transactions. For instance, line 6 on page 3, "taxes, licenses, and fees" is based on direct premium written, not on net premium written. Similarly, income taxes, borrowed money, and interest (lines 7, 8, and 9 on page 3) are not related to the manner in which reinsurance transactions are presented.[45] Contingent commissions (included in line 3 on page 3) may sometimes be affected by reinsurance transactions. Since we are given no information about this, we assume that no adjustment is needed here. We enter a zero for the "adjustment," yielding $3,000,000 in the restated column.

10. Line 10 in this exhibit shows $40,000,000 of net unearned premium reserves. Since the direct unearned premium reserve is $50,000,000 and the assumed unearned premium reserve is $10,000,000 (as stated by the reinsurance accounting department), the ceded unearned premium reserve is $20,000,000, which is the adjustment for this line. The entry in the restated column is $60,000,000.

11. Line 11 in this exhibit, "dividends declared and unpaid," relates generally to direct business, not to net business. The adjustment for this line is zero, and the restated amount is $2,000,000.[46]

12. Line 12 in this exhibit, "Funds held under reinsurance treaties," are monies owned by reinsurers but held by the primary company. If there were no ceded reinsurance, the primary company would not be holding any funds belonging to reinsurers. The entry is reversed by an adjustment of –$20,000,000, leaving a zero in the restated column.

13. Line 13 in this exhibit, "Amounts withheld for account of others," is generally not related to reinsurance transactions. The adjustment is zero, leaving 1 million in the restated column.

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[45] One might suppose that federal income taxes depend on reinsurance transactions, since if the reinsurer indemnifies an incurred loss, the tax liability should increase. This is not relevant to the Part 8 exhibit. When we restate the accounting presentation of the statutory balance sheet, the tax liability does not change.

[46] In some instances, particularly on quota share treaties, the reinsurer may pay a part of the policyholders' dividend. In other treaties, there is no policyholders' dividend liability incurred by the reinsurer. Rather, the expected policyholder dividend may be included in the ceding commission, it may be paid to the primary company (not to the policyholders), or it may be included in a sliding scale commission arrangement. For simplicity, this illustration assumes that there is no ceded portion of the policyholders' dividend liability.

14. Line 14 in this exhibit, "Provision for reinsurance," is the statutory penalty for recoverables from unauthorized reinsurers, recoverables from slow-paying reinsurers, loss recoverables more than 90 days past due, and amounts in dispute. If there were no ceded reinsurance, there would be no provision for reinsurance. The entire amount is eliminated on a "gross of reinsurance" basis. The adjustment is –$15,000,00, and the restated amount is zero.

15. Line 15 in this exhibit, "Other liabilities," generally do not relate to reinsurance transactions. This entry is comprised of the following items from page 3:

- Remittances and items not allocated;
- Net adjustments to assets and liabilities due to foreign exchange rates;
- Drafts outstanding;
- Payable to parent, subsidiaries, or affiliates;
- Payable for securities;
- Liability for amounts held under uninsured accident and health plans;
- Capital notes and interest thereon; and
- Aggregate write-ins for liabilities.[47]

As noted above, item11 from the balance sheet, "ceded reinsurance balances payable, net of ceding commission," should be included in this line. This balance sheet entry is reversed, so we enter –$5,000,000 for the "adjustment," leaving $4,000,000 in the restated column.

16. Line 16 in this exhibit, "Total liabilities," is the sum of lines 8 through 15. For the adjustments, we have (in millions of dollars)

$$120 + 0 + 20 + 0 - 20 + 0 - 15 - 5 = \$100 \text{ million},$$

and for the restated column we have

$$220 + 3 + 60 + 2 + 0 + 1 + 0 + 4 = \$290 \text{ million}.$$

17. Line 17 in this exhibit, "surplus as regards policyholders," is not affected by this calculation for Schedule F, Part 8. Reclassifying the balance sheet accounts changes the accounting presentation; it does not change surplus. Column 2, the adjustment," is "X-ed out" in the blank. The restated amount is the same as the reported amount: $110,000,000.

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[47] The aggregate write-ins for liabilities may include a contra-liability for recoverables on retroactive reinsurance; see SSAP No. 62, "Reinsurance," paragraph 28.

BALANCING ITEMS

We return to the two lines that we did not complete in the asset section of this exhibit. Since the total liability adjustment is $100,000,000, the total asset adjustment must also be $100,000,000 (column 2 of line 7). The total asset adjustment is the sum of the individual asset adjustments. The one asset adjustment in this illustration is –$40,000,000 on line 4 (reinsurance recoverable on loss payments). A balancing adjustment of $140,000,000 [= $100,000,000 – (–$40,000,000)] is entered for line 6 (net amount recoverable from reinsurers).

The entries in the restated column are the sum of the entries in the as reported and adjustment columns. For the cells labeled A, B, C, and D, we have

A. For line 6 (recoverable from reinsurers), the restated amount is *$0 + $140,000,000 = $140,000,000.*

B. For line 7 (total assets), the restated amount is *$300,000,000 + $100,000,000 = $400,000,000.*

C. For line 16 (total liabilities), the restated amount is *$190,000,000 + $100,000,000 = $290,000,000.*

D. For line 17 (surplus as regards policyholders), the adjustment is always $0 and thhe restated amount equals the "as reported" amount.

The completed Part 8 exhibit is shown on the following page.

Schedule F, Part 8: Completed Exhibit

Schedule F, Part 8: Restatement of Balance Sheet to Identify Net Credit for Ceded Reinsurance ($000,000's)			
ASSETS (page 2, column 3)	As Reported	Adjustment	Restated
1. Cash and invested assets	$200	—	$200
2. Agents' balances or uncollected premiums	$10	$0	$10
3. Funds held by or deposited with reinsured companies	$30	—	$30
4. Reinsurance recoverable on loss and LAE payments	$40	($40)	$0
5. Other assets	$20	—	$20
6. Net amount recoverable from reinsurers		$140	$140
7. Total Assets	$300	$100	$400
LIABILITIES (page 3)			
8. Losses and loss adjustment expenses	$100	$120	$220
9. Taxes and other expenses	$3	—	$3
10. Unearned premiums	$40	$20	$60
11. Dividends declared and unpaid	$2	—	$2
12. Funds held by company under reinsurance treaties	$20	($20)	$0
13. Amounts withheld or retained for account of others	$1	—	$1
14. Provision for reinsurance	$15	($15)	$0
15. Other liabilities	$9	($5)	$4
16. Total liabilities	$190	$100	$290
17. Surplus as regards policyholders	$110	xxx	$110
18. Total liabilities plus surplus	$300	$100	$400

II. Provision for Unauthorized Reinsurance

We show several illustrations of the provision for reinsurance, beginning with a single, unauthorized reinsurer with no securitization of the recoverables and proceeding to more complex illustrations involving payment schedules and overdue receivables.

FLEDGLING INSURANCE

You are the reinsurance officer for the Fledgling Insurance Company, a small, newly capitalized personal automobile insurer. All your business is 100% reinsured with the XYZ Reinsurance Company, which is not licensed or authorized in your domiciliary state.

Written premium during the year was $50 million, and earned premium was $40 million. The unearned premium reserve at the end of the year is $20 million. These amounts are also 100% reinsured by XYZ Reinsurance.

Reported but unpaid losses are $25 million, along with $6 million of unpaid loss adjustment expenses associated with these claims. Incurred but not reported losses are $10 million, along with $4 million of unpaid loss adjustment expenses. These amounts are 100% reinsured by XYZ Reinsurance.

$35 million was paid to claimants this past year, along with $10 million in loss adjustment expenses. For these claims, Fledgling still awaits recovery of $15 million in losses and $5 million in loss adjustment expenses from XYZ Reinsurance.

XYZ Reinsurance has denied liability for $5 million of these losses. Fledgling Insurance expects a full recovery, and the matter is in litigation.

XYZ Reinsurance has not provided Fledgling Insurance with any security, whether letters of credit, trust agreements, or funds withheld.

Fledgling assumes no reinsurance business from other primary writers, and it cedes no business to other reinsurers.

You have been asked to complete the entries for the following items on Fledgling's balance sheet (pages 2 and 3 of the Annual Statement):

Page 2:

Line 11	Funds held by or deposited with reinsured companies
Line 14	Reinsurance recoverables . . .

Page 3:

Line 1	Losses
Line 2	Reinsurance payable . . .
Line 3	Loss adjustment expenses
Line 9	Unearned premiums
Line 12	Funds held by company under reinsurance treaties
Line 15	Provision for reinsurance

What are the appropriate entries for each of these lines?

Balance Sheet Entries

Since XYZ Reinsurance is not authorized and provides no offsetting funds or letters of credit, all recoverables from XYZ are included in the provision for reinsurance. There is no need for a payment schedule to determine amounts more than 90 days past due.

All balance sheet items are net of reinsurance, with no differentiation between authorized and unauthorized reinsurers, slow-paying and quick-paying reinsurers, and loss recoverables more than 90 days past due versus other loss recoverables. Line 15 on page 3 shows the aggregate provision for reinsurance, relating to recoverables on paid losses, unpaid losses, unearned premium reserves, and commissions.

Because Fledgling is 100% reinsured, it has no net liabilities. Because XYZ Reinsurance is unauthorized and it provides no security, all recoverables are included in the provision for reinsurance. Proceeding line by line . . .

- Page 2, line 11, "Funds held by or deposited with reinsured companies," refers to funds owned by Fledgling that are held by primary companies that have ceded business to Fledgling. Since Fledgling does not assume any reinsurance, it has not deposited funds with any ceding companies, and this amount is $0.

- Page 2, line 14, "Reinsurance recoverables, on loss and loss adjustment expense payments," relates to recoverables from XYZ Reinsurance on losses and loss adjustment expenses already paid by Fledgling. This amount is $20 million, or $15 of loss plus $5 of defense and cost containment expenses.

  XYZ's unauthorized status does not affect this asset. Insurance personnel sometimes speak of unauthorized reinsurance recoverables as non-admitted assets, but there is no "non-admitted" adjustment to this asset. Even XYZ's denial of liability does not affect this asset, as long as Fledgling expects to receive the money. Rather, the asset is offset by a corresponding liability on line 15 of page 3. In GAAP statements, which do not include a provision for reinsurance, Fledgling would disclose in a footnote the disputed amount.

- Page 3, line 1 shows loss reserves net of reinsurance, whether the reinsurance is authorized or not. This entry is $0, since all of Fledgling's business is reinsured.

- Page 3, line 2, "Reinsurance payable on paid losses," shows Fledgling's liabilities for assumed reinsurance losses. Since Fledgling assumes no business from other primary carriers, this entry is $0.

- Page 3, line 2, shows loss adjustment expense reserves net of reinsurance, whether the reinsurance is authorized or not. This entry is $0, since all of Fledgling's business is reinsured.

- Page 3, line 9, shows unearned premium reserves net of reinsurance, whether the reinsurance is authorized or not. This entry is $0, since all of Fledgling's business is reinsured.

- Page 3, line 12, "Funds held by company under reinsurance treaties," shows funds owned by XYZ Reinsurance that are held by Fledgling as security for its recoverables. Since XYZ Reinsurance has provided no security to Fledgling, this entry is $0.

- Page 3, line 15, "Provision for reinsurance," includes all the recoverables from XYZ Reinsurance. The recoverables relate to

• the unearned premium reserve	$20 million
• paid losses	15 million
• paid allocated loss adjustment expenses	5 million
• unpaid reported losses	25 million
• unpaid IBNR losses	10 million
• unpaid defense and cost containment expenses	10 million
Total	$85 million

The entry for line 15 is $85 million.

The provision for reinsurance from unauthorized reinsurers includes a provision for paid loss recoverables more than 90 days past due and for amounts in dispute in addition to the provision for unsecured total recoverables. The total provision for reinsurance is limited by the total recoverables. In this problem, the limit is reached by the provision for total unsecured recoverables, since no security has been provided. No additional provision need be made for paid loss recoverables more than 90 days past due or for amounts in dispute.

III. Overdue Reinsurance

The Stable Insurance Company, a commercial fire carrier specializing in property coverage for large risks over a high self-insured retention, has a 100% quota share reinsurance treaty with the Secure Reinsurance Company, which is licensed to conduct business in Stable's domiciliary state. During the past year, Secure has denied liability for two large claims and has been slow in paying on several other claims. Stable Insurance Company has asked Secure Reinsurance Company for a letter of credit of $40 million, which Secure provided on November 15. The letter of credit applies to recoverables on paid losses, recoverables on unpaid losses, and unearned premiums, but not to the two claims for which Secure has denied liability.

The reinsurance payment schedule from Secure Reinsurance is shown on the next page. Claim amounts are in thousands of dollars. For instance, the second line shows a claim with an accident date of January 12. Stable paid the claimant $1.6 million on March 3, and it received reimbursement from Secure on July 17.

Stable has filed suit to recover the $12 million relating to the January 4 claim, and the case is currently in litigation. Stable is discussing the March 10 claim with Secure, but no suit has yet been filed. Stable also has $8 million of unearned premium reserves ceded to Secure.

The "due date" for recoverables depends on contract provisions. If the reinsurance treaty does not define the due date or the date on which claims are to be presented to the reinsurer for payment, then recoverables are assumed to be due when the paid loss recoverable is entered on the ceding company's books. In this illustration, assume that no due date or presentation date is stated in the reinsurance treaty, and that the paid loss recoverable is entered on the ceding company's books when the direct loss payment is made.

What provision for reinsurance must Stable Insurance Company hold on its balance sheet (line 15 of page 3) at December 31?

Reinsurance Payment Schedule
(figures in thousands of dollars)

Amount of Claim	Accident Date	Payment Date (Stable to Claimant)	Payment Date (Secure to Stable)
12,000	Jan 4	Feb 5	(unpaid; Secure denies liability)
1,600	Jan 12	Mar 3	July 17
1,500	Feb 26	July 20	(unpaid)
4,400	Mar 9	June 2	Aug 1
6,500	Mar 10	Apr 14	(unpaid; Secure denies liability)
3,000	Apr 16	May 17	Oct 29
3,500	May 8	June 13	Sept 29
2,500	June 3	July 19	(unpaid)
1,000	June 8	June 28	Dec 12
4,000	Aug 22	Nov 4	(unpaid)
6,000	Aug 9	(unpaid)	(unpaid)
10,000	Sept 2	Oct 21	(unpaid)
11,200	Nov 18	(unpaid)	(unpaid)
3,800	Dec 5	(unpaid)	(unpaid)

Aging Schedule

- If an authorized reinsurer is *not* slow-paying, the provision for reinsurance is 20% of the recoverables more than 90 days past due plus 20% of the amounts in dispute, with no offset for funds withheld or letters of credit.

- If the reinsurer is classified as slow-paying, the provision for reinsurance is 20% of the larger of (i) the total recoverables, with an offset for funds withheld or letters of credit, and (ii) the recoverable more than 90 days past due.

To determine whether Secure is slow-paying, we divide the claims into six categories:

A. Claims for which reinsurance recoveries were received more than 90 days prior to the statement date;
B. Claims for which reinsurance recoveries were received within the 90 days preceding the statement date;
C. Claims paid by Stable for which the reinsurance recoverables are less than or equal to 90 days overdue;
D. Claims paid by Stable for which the reinsurance recoverables are more than 90 days overdue (and not in dispute);
E. Claims in dispute; and
F. Claims still unpaid by Stable.

Classification as a slow-paying reinsurer depends on the ratio D ÷ (B + C + D). This is the ratio of

- the amounts more than 90 days overdue to
- the amount receivable on paid claims plus the amounts received in the past 90 days.

The reinsurer is classified as slow-paying if this ratio exceeds 20%.

Using the payment schedule shown above, we have

A. $1.6 million + $4.4 million + $3.5 million = $9.5 million (January 12, March 9, and May 8 claims).
B. $3 million + $1 million = $4 million (April 16 and June 8 claims).
C. $4 million + $10 million = $14 million (August 22 and September 2 claims).
D. $1.5 million + $2.5 million = $4 million (February 26 and June 3 claims).
E. $12 million + $6.5 million = $18.5 million (January 4 and March 10 claims).
F. $6 million + $11.2 million + $3.8 million = $21 million (August 9, November 18, and December 5 claims).

The ratio of D to (B + C + D) equals $4 million ÷ ($4 million + $14 million + $4 million) = 18.2%. Since this ratio is less than 20%, Secure is not a slow-paying reinsurer.

20% of the overdue recoverables, 20% of $4 million, or $800,000 is included in the provision for reinsurance. In addition, there are $18.5 million of recoverables in dispute, 20% of which is $3.7 million. The total provision for reinsurance is $0.8 million + $3.7 million = $4.5 million.

The letter of credit provided by Secure does not affect the statutory provision for amounts more than 90 days past due or for amounts in dispute. The provision for reinsurance which appears in the Schedule F, Part 7, footnote and on line 15 of page 3 is $4,500,000.

In this example, the statutory provision for reinsurance is $4,500,000, whereas the amounts in dispute are $18.5 million. It is possible that the expected uncollectible amount exceeds the provision for reinsurance determined by the statutory formula. If so, the statutory provision for reinsurance should be increased to cover the expected uncollectible amounts. The excess of the expected uncollectible amount over the statutory provision for reinsurance flows through the income statement and affects taxable income as well.

In any case, Stable should disclose the potential effects of an adverse outcome of these disputes in the Notes to the Financial Statements. These potential adverse outcomes are classified as loss contingencies. As long as their likelihood is not remote, their effects should be disclosed in the notes. See SSAP No. 5, "Liabilities, Contingencies and Impairments of Assets," paragraph 14 (copied from SFAS 5):

> *If a loss contingency or impairment of an asset is not recorded . . . or if exposure to a loss exists in excess of the amount accrued pursuant to the provisions described above, disclosure of the loss contingency or impairment of the asset shall be made in the financial statements when there is at least a reasonable possibility that a loss or an additional loss may have been incurred. The disclosure shall indicate the nature of the contingency and shall give an estimate of the possible loss or range of loss or state that such an estimate cannot be made.*

IV. Slow-Paying Reinsurers

The Standard Reinsurance Company is licensed to conduct reinsurance business in the domiciliary state of the primary insurance company. The Schedule F, Part 4 entries for Standard are shown below.

Column 4	Current	$40 million
Column 5	1-29 days overdue	$25 million
Column 6	30-90 days overdue	$50 million
Column 7	91-120 days overdue	$20 million
Column 8	over 120 days overdue	$55 million

On Part 3 of Schedule F, the entries for Standard Reinsurance are as follows:

Column 1, "Reinsurance premium ceded,"	$210 million
Column 2, "Recoverables on paid losses,"	$175 million
Column 3, "Recoverables on paid LAE,"	$15 million
Column 4, "Recoverables on known case loss reserves,"	$160 million
Column 5, "Recoverables on known case LAE reserves,"	$20 million
Column 6, "Recoverables on IBNR loss reserves,"	$100 million
Column 7, "Recoverables on IBNR LAE reserves,"	$10 million
Column 8, "Unearned premiums,"	$75 million
Column 9, "Commissions,"	$5 million

In the past 90 days, Standard has made payments of $75 million for losses and loss adjustment expenses. Standard has provided a letter of credit for $200 million to secure its recoverables.

We compute the provision for reinsurance for the Standard Reinsurance Company.

Overdue Ratio

Since Standard is authorized, we determine whether it is a slow-paying reinsurer. We consider the ratio of (i) the amounts more than 90 days past due to (ii) the total amount receivable on paid claims plus the amounts received in the past 90 days, after eliminating all items in dispute from the total due and the amount more than 90 days past due. Standard is classified as slow-paying if this ratio exceeds 20%.

The information provided above shows

- $75 million more than 90 days overdue (columns 7 + 8 of Part 4),

- $190 million of total recoverables on paid losses and loss adjustment expenses (the sum of columns 4 through 8 of Part 3), and
- $75 million of recoverables received in the past 90 days.

The ratio is $75 million ÷ ($190 million + $75 million) = 28.3%. The Standard Reinsurance Company is classified as a slow-paying reinsurer.

The total recoverables from Standard are

- $190 million of recoverables on paid losses and loss adjustment expenses;
- $180 million of recoverables on unpaid "case basis" losses and LAE;
- $110 million of recoverables on unpaid IBNR losses and LAE;
- $75 million of ceded unearned premium reserves; and
- $5 million of commissions.

for a total of $560 million.

Standard Reinsurance has provided a letter of credit to secure $200 million of these recoverables, so the unsecured recoverables are $360 million. The provision for reinsurance considers two elements:

- Twenty percent of the unsecured amount, or $72 million (= $360 million × 20%), and
- Twenty percent of the amount more than 90 days past due, or $15 million (= $75 million × 20%).

The provision for reinsurance is the larger of these two amounts, or $72 million.

V. Provision for Reinsurance by Type of Reinsurer

We calculate the provision for reinsurance for recoverables from the reinsurers listed below (dollar amounts are in millions).

	Reinsurer A	Reinsurer B	Reinsurer C
Authorized status	Unauthorized	Authorized	Authorized
Reinsurance recoverable (all items)	$100	$100	$100
Funds held by reporting company under reinsurance treaties	10	10	10
Letters of credit	75	0	0
Recoverables on paid loss & LAE over 90 days due, not in dispute	20	5	5
Recoverables on paid loss & LAE over 120 days due, not in dispute	10	2	2
Recoverables on paid loss & LAE, total	50	32	32
Amount in dispute included above	25	10	10
Amounts company received from reinsurer in last 90 days of statement year	5	5	0

Unauthorized Reinsurance

We begin with Reinsurer A. Since Reinsurer A is not authorized, we determine the total unsecured recoverables.

- Total recoverables = $100
- Collateral is the sum of letters of credit ($75) and funds withheld ($10) = $85
- Unsecured total recoverables = 100 – $85 = $15

We then consider the overdue recoverables and the amounts in dispute.

- Loss recoverables more than 90 days past due = $20
- Amounts in dispute = $25

Recall that amounts in dispute are part of total recoverables but not of overdue recoverables.

The provision for reinsurance includes all recoverables from unauthorized reinsurers unless they are collateralized by letters of credit or funds withheld. The collateral does not help for overdue recoverables or for amounts in dispute, so 20% of these latter two items is added to the provision for reinsurance to the extent that it does not exceed the amount of collateral.

The formula for the total provision for reinsurance, including the capping rule is

total recoverables – collateral
+ lesser of (a) 20% of overdue recoverables + 20% of amounts in dispute and
(b) the amount of collateral

In this illustration, the figures are

$100 – $85
+ lesser of (a) 20% × $20 + 20% × $25 and (b) $85

= $15 + $4 + $5 = $24.

We have stated the capping rule as it appears in the Schedule F exhibits. We may rephrase the capping rule to say that the provision for reinsurance is limited to the total reinsurance recoverables.

Authorized Reinsurers

Reinsurer B is authorized, so we determine whether it is slow-paying. A slow-paying reinsurer has an overdue ratio exceeding 20%.

The overdue ratio equals the ratio of recoverables more than 90 days past due to the sum of the total recoverables on paid losses and LAE that are *not* in dispute and the recoverables received in the past 90 days.

The figures in this illustration are

recoverables more than 90 days past due	= $5
total recoverables on paid loss and LAE	= $32
amount in dispute	= $10
recoverables received in the past 90 days	= $5

The overdue ratio is

$5 ÷ ($32 – $10 + $5) = $5 ÷ $27 = 18.5%.

Since the ratio does not exceed 20%, the insurer is not slow-paying.

Non-Slow-Paying Reinsurers

Since reinsurer B is not slow-paying, the provision for reinsurance is

20% of overdue recoverables + 20% of amounts in dispute.

The figures are

20% × $5 (overdue recoverables) + 20% × $10 (amounts in dispute) = $1 + $2 = $3.

Slow-Paying Reinsurers

Reinsurer C has the same recoverables as Reinsurer B, but it paid no claims in the last 90 days of the statement year. This affects the overdue ratio test; it does not change the recoverables.

The overdue ratio is

$$\$5 \div (\$32 - \$10 + \$0) = \$5 \div \$22 = 22.73\%.$$

Since the ratio exceeds 20%, the insurer is slow-paying.

The provision for reinsurance for slow-paying authorized reinsurers is similar to the provision for unauthorized reinsurance, except that the provision is only 20% of the unsecured recoverables, not 100%. (The other differences, such as the "greater than" provision, are noted below.)

The unsecured total recoverables are $100 – $10 = $90, and 20% of the unsecured recoverables are 20% × ($90) = $18.

The loss recoverables that are more than 90 days past due are $5, and 20% of $5 = $1.

The greater of $18 and $1 is $18, which is the provision for reinsurance for Reinsurer C.

The final provision for reinsurance is the sum of the provisions for the three reinsurers, or $24 + $3 + $18 = $45.

COLLATERAL

For slow-paying reinsurers, amounts in dispute are included in total recoverables, and they are not considered separately. As noted in the text of the paper, this may be an oversight by the regulators who designed Schedule F, since collateral should not offset the provision for amounts in dispute.[48]

[48] The rules for slow-paying authorized reinsurers are particularly strange. If collateral does not offset the provision for overdue recoverables, it surely should not offset the provision for amounts in dispute.

In this exercise, the authorized reinsurer has not provided any collateral. This makes sense, since for an authorized reinsurer, collateral helps only if the reinsurer is slow-paying. But what reinsurer assumes at the outset that it is going to be slow-paying?

For slow-paying authorized reinsurers, collateral plays a role in the "greater than" expression used to compute the total provision for reinsurance. Suppose that this insurer had

- $100 of total recoverables;
- $100 of collateral; and
- $ 50 of overdue recoverables.

- The amount of uncollateralized recoverables is $0, so 20% of that is also $0.
- The collateral does not help for overdue recoverables.
- 20% of $50 is $10, which is greater than the $0 derived above.
- The final provision for reinsurance is $10.

VI. Slow-Paying Authorized Reinsurers

Given the following entries from Schedule F, we determine the provision for reinsurance for the reinsurer shown. Dollar amounts are in millions.

XYZ Reinsurance Company is authorized in the domiciliary state of the ceding company. No amounts are in dispute. XYZ has made payments of $45 in the past 90 days and has a letter of credit securing recoverables of $250.

Data from Schedule F, Part 4, "Aging of Ceded Reinsurance"	
Name of Reinsurer	XYZ
Current Recoverables	80
Recoverables 1 – 29 days overdue	15
Recoverables 30 – 90 days overdue	5
Recoverables 90 – 120 days overdue	20
Recoverables over 120 days overdue	40

Data from Schedule F, Part 3, "Ceded Reinsurance"	
Name of Reinsurer	XYZ
Reinsurance Premium Ceded	100
Recoverables on paid losses	125
Recoverables on paid LAE	35
Recoverables on known case loss reserves	30
Recoverables on known case LAE reserves	50
Recoverables on IBNR loss reserves	70
Recoverables on IBNR LAE reserves	25
Recoverables on Unearned Premiums	75
Recoverables on Commissions	3

Aging Schedule

The XYZ Reinsurance Co. is authorized. We use the aging schedule to test if it is slow paying.

The illustration in the text of the paper (Secure Insurance and Stable Reinsurance) provides a list of claims and their payment dates to determine the slow-paying status of the reinsurer. This exercise provides the Schedule F entries in Parts 3 and 4.

Current recoverables are recoverables that are still before the due date. This is most common when a due date is specified in the reinsurance contract. If there is a due date specified in the reinsurance contract, and the due date is the date on which the reinsurance recoverable is entered in the financial statements of the reporting company, then if the reinsurance recoverable is entered when the primary loss is paid, few recoverables on paid losses are current.

The overdue ratio is defined as

recoverables on paid losses and LAE more than 90 days past due

divided by the sum of

(i) all recoverables on paid losses and LAE and
(ii) recoverables paid in the last 90 days of the statement year.

Both the numerator and the denominator of this ratio exclude amounts in dispute. In this exercise, there are no amounts in dispute.

The recoverables more than 90 days past due are $20 + $40 = $60 (Schedule F, Part 4).

The total recoverables on paid losses and LAE are shown both in Part 3 and in Part 4.

- From Part 4 we have: $80 + $15 + $5 + $20 + $40 = $160.
- From Part 3 we have: $125 + $35 = $160.

The recoverables on paid losses and LAE that were paid in the last 90 days of the statement year are $45.

The overdue ratio is $60 / ($160 + $45) = $60/$205 = 29.3%. The XYZ Reinsurance Company is slow paying.

Provision for Reinsurance

The provision for reinsurance is the greater of

i. 20% of the unsecured total recoverables (not just on paid losses, and including amounts in dispute) and
ii. 20% of the recoverables on paid losses that are more than 90 days past due.

Security reduces the total unsecured recoverables, but it does not reduce the recoverables more than 90 days past due.

Total recoverables are

the ceded unearned premium reserves
+ the recoverables on paid losses and LAE
+ the recoverables on unpaid losses and LAE
+ expected commissions.

The commissions are contingent commissions or profit commissions. Regular reinsurance commissions are deducted from the premium balances and not paid by the ceding company to the reinsurer, so they would not be recoverable.

The figures are shown in Part 3 of Schedule F

- the ceded unearned premium reserves = $75.
- the recoverables on paid losses and LAE = $160.
- the recoverables on unpaid losses and LAE = $30 + $50 + $70 + $25 = $175
- the recoverable commissions = $3.
- The total recoverables = $75 + $160 + $175 + $3 = $413.
- The letter of credit is for $250. The total unsecured recoverables = $413 – $250 = $163.
- 20% of $163 = $32.60.

20% of the recoverables that are more than 90 days past due = 20% × $60 = $12.

The provision for reinsurance is the greater of $32.60 and $12, or $32.60.

VII. Decision Tree Rules

One final illustration shows the inter-relationship between overdue amounts and security.

The ABC Insurance Company has $10 million recoverable from an unauthorized reinsurer, $5 million of which is overdue. There are letters of credit totaling $6 million.

We determine the provision for reinsurance. The reinsurer is not authorized. The provision for reinsurance is the unsecured total recoverables plus 20% of the overdue amount plus 20% of the amount in dispute. In this exercise, there are no amounts in dispute. Security is not relevant for overdue amounts. The provision for reinsurance is

($10 million – $6 million) + 20% x $5 million = $4 million + $1 million = $5 million.

One must check the limitation. In this example, the provision for overdue recoverables, or $1 million, is less than the amount of security ($6 million), so there is no limitation.

Schedule F: Objectives and Evaluation

Schedule F is a series of complex exhibits, requiring considerable effort to complete. The provision for reinsurance may have a significant effect on policyholders' surplus, and it influences reinsurance practices for both domestic and international transactions.

The previous sections of this paper deal with the accounting entries required to complete the exhibits of Schedule F. The following sections evaluate the benefits and costs of Schedule F in light of the objectives of state insurance regulation.

- What are the objectives of Schedule F, and how well does Schedule F meet them?
- Are there alternative means of meeting these objectives?
- Are these objectives aligned with regulatory responsibilities to the insuring public?
- How might the regulatory responsibilities best be met?

Insurance is a highly regulated industry. Much regulation is beneficial to insurance consumers and effectively performed by state insurance departments. Some regulation may be unduly burdensome or inefficient. The task for regulators and industry professionals is to strengthen the efficient regulation and to revise or eliminate the wasteful regulation.

The primary objective of state insurance regulation is defined in the *Statutory Accounting Principles Statement of Concepts*, "objectives of statutory financial reporting," paragraph 27:

> *The primary responsibility of each state insurance department is to regulate insurance companies in accordance with state laws with an emphasis on solvency for the protection of policyholders. . . . The cornerstone of solvency measurement is financial reporting. Therefore, the regulator's ability to effectively determine relative financial condition using financial statements is of paramount importance to the protection of policyholders. . . .*

We examine the financial reporting in Schedule F in light of the regulatory responsibility in the Statement of Concepts cited above.

Parts 4 through 7 of Schedule F serve to determine the provision for reinsurance, whose purpose is described in the NAIC Practices and Procedures Manual (SSAP No. 62, "Reinsurance," paragraph 52) as a "minimum reserve for uncollectible reinsurance."[49]

General accounting statements also estimate uncollectible reinsurance recoverables. GAAP requires the management of the insurance company to disclose its best estimate of all receivables that may not be collected, not just reinsurance recoverables. These uncollectible amounts serve as offsets to the receivable accounts. The balance sheet accounts with bad debt or uncollectible offsets include premiums receivable, agents' balances, collateral loans, and reinsurance recoverables.

For each of these balance sheet accounts, statutory accounting uses fixed formulas instead of relying on management disclosure. The assets not admitted by the statutory formula are still shown on the balance sheet, and they flow through the income statement. These amounts are shown as non-admitted assets in column 3 of the statutory balance sheet, and the year-to-year change in these non-admitted assets is a direct charge or credit to policyholders' surplus on line 25 of page 4 of the Annual Statement (carried from line 6, column 3, of Exhibit 1).

For instance, an estimate of agents' balances that may not be collected – but that have not yet been written off – is shown as a "bad debt" offset to premiums receivable in GAAP financial statements. On statutory statements, agents' balances more than 90 days past due are non-admitted assets.[50]

A similar format applies to other receivable accounts. On GAAP financial statements, the accrued retrospective premium asset is offset by management's estimate of the amount that may not be collected. On statutory statements, 10% of the unsecured accrued retrospective premiums are not admitted.[51]

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[49] The full paragraph 52 reads as follows: "The NAIC Annual Statement Instructions for Property and Casualty Companies for Schedule F—Provision for Overdue Reinsurance: provide for a minimum reserve for uncollectible reinsurance with an additional reserve required if an entity's experience indicates that a higher amount should be provided. The minimum reserve Provision for Reinsurance is recorded as a liability and the change between years is recorded as a gain or loss directly to unassigned funds (surplus). Any reserve over the minimum amount shall be recorded on the statement of income by reversing the accounts previously utilized to establish the reinsurance recoverable."

[50] If the company uses direct billing to the insured, only the premium balances more than 90 days past due in excess of the unearned premium reserve are not admitted.

[51] Statutory accounting provides for an alternative quantification of the non-admitted portion of the accrued retrospective premium asset, based on the credit ratings of the insureds. See SSAP No. 66, "Retrospectively Rate Contracts," paragraph 9, subsection "d."

These examples reflect a fundamental difference in the GAAP versus SAP perspectives on the purpose of financial statements.

- GAAP financial statements are geared to current and potential investors in going-concern enterprises who seek information about the future profitability of the firm. Investors want unbiased estimates; they do not want conservative estimates or optimistic estimates. The firm's management has the understanding and information to provide good estimates. The fixed formulas used in statutory statements do not always provide unbiased estimates, and they might be misleading in a GAAP context.

- Statutory financial statements are geared to regulatory authorities. Regulators are not concerned about the profitability of going-concern firms; they are concerned about the potential insolvency of firms in financial distress. Distressed firms might have an incentive to overstate their assets or understate their liabilities, since unbiased estimates might provoke regulatory intervention in their operations. For these firms, regulators would not be fulfilling their responsibilities if they relied on the opinions of company management. Instead, they rely upon fixed formulas.

The U.S. capital markets and its legal system constrain a firm from entering misleading information into its general purpose (GAAP) financial statements. These constraints are strong, even if they are not perfect.

1. Firms depend on financial analysts to report on their stock prices, and financial analysts carefully review their financial statements. The retrospective accuracy of uncollectible offsets may be seen from a comparison of Note 22 to the Financial Statements with the provision for reinsurance. Consistently misleading entries in past financial statements may cause analysts to distrust management entries in current financial statements. In the long run, misleading accounting estimates may depress a firm's stock price.

2. A firm that knowingly misstates its general purpose financial statements is exposed to SEC penalties and to shareholder lawsuits. The personal assets of the firm's officers are not exposed to company losses, but they may be exposed to shareholder suits.

3. General purpose financial statements are audited by independent public accountants, who may be employees or officers of multi-national accounting firms. Both the assets and the reputations of the accounting firms are exposed to shareholder lawsuits resulting from misleading financial statements.

These constraints generally suffice for the financial statements of profitable and financially healthy firms. Distressed firms are less likely to feel constrained by the capital markets, and they are more willing to risk potential lawsuits.

Statutory Objectives

The rationale for the GAAP accounting philosophy is clear. The rationale for the statutory accounting philosophy is more problematic, for several reasons.

SUPPLEMENT VS REPLACEMENT

The statutory provision for reinsurance does not just supplement management's estimate of uncollectible reinsurance recoverables; the provision for reinsurance *replaces* management's estimate. Because the provision for reinsurance is conservative and its calculation is sometimes arbitrary, many users of statutory financial statements add back the provision for reinsurance to reported policyholders' surplus to determine a more realistic value for the firm.

Consider two insurers, Company A and Company B. Both companies have a $100 million provision for reinsurance. Company A estimates the true uncollectible to be $10 million. Company B estimates the true uncollectible to be $90 million.

GAAP financial statements reflect this difference in the estimated uncollectibles. On statutory financial statements, both companies show the same $100 million provision for reinsurance as an offset to policyholders' surplus. Neither company shows any offset to statutory income. Neither company discloses its true estimate of uncollectible reinsurance recoverables.

Readers of the statutory financial statements – including state insurance regulators – are interested in the true estimates of uncollectibility. Oftentimes, a result of the complex Schedule F formula may be to obscure more relevant estimates of uncollectibility.

Changing the provision for reinsurance into a supplement to management's estimate of potential uncollectibility instead of a replacement for management's estimate is not favored by some regulators. A supplemental format might encourage the perception that the GAAP procedure is correct and the statutory procedure is an arbitrary addition. A large difference between management's estimate and the provision for reinsurance may encourage readers of the Annual Statement to ignore the provision when evaluating company financial stability.

UNINTENDED CONSEQUENCES

The provision for reinsurance provides three intended incentives for insurance companies.

- The provision encourages ceding companies to prefer authorized reinsurers over unauthorized reinsurers, particularly if the latter do not fully collateralize their recoverables.
- The provision encourages ceding companies to seek collateral from unauthorized reinsurers and from slow-paying authorized reinsurers.
- The provision encourages ceding companies to demand timely payment of reinsurance recoverables.

These incentives are favorably viewed by many regulators. Some of the effects, such as more timely payment of reinsurance recoverables, are also desired by primary insurers.

The provision for reinsurance has some unintended consequences. Appropriate reinsurance arrangements are an effective means for an insurance company to manage its risk exposures. Unauthorized reinsurers sometimes provide better reinsurance arrangements or less expensive reinsurance arrangements than authorized reinsurers do. If the provision for reinsurance induces a ceding company to forego optimal reinsurance arrangements, the provision harms insurance consumers.[52]

Securing reinsurance recoverables with letters of credit is not a costless panacea. A letter of credit may be expensive, particularly if the reinsurer's financial condition in an adverse scenario can not be foreseen. The increased cost associated with letters of credit may raise the price for the primary policy or may force the primary company to forego the purchase of reinsurance. Neither of these results serves the interest of insurance consumers.

The market for reinsurance is complex; ceding companies carefully weigh costs, risks, and accounting effects when choosing among reinsurance proposals. The incentives and disincentives listed above are not absolute; they must be considered among the other objectives of ceding companies.

ACCURACY

The Schedule F provision for reinsurance is a generic formula, and it may not always serve as a reasonable proxy for uncollectible reinsurance recoverables. The sharp demarcations (i) between authorized and unauthorized reinsurers and (ii) between slow-paying and non slow-paying reinsurers does not seem justified by complex and fluid reinsurance markets.

Illustration: Reinsurers A and B have similar capital structures and mixes of business; both reinsurers settle their claims in a timely fashion; and neither one provides any security backing its reinsurance liabilities. Reinsurer A is authorized in the primary company's domiciliary state and Reinsurer B is not authorized. The provision for reinsurance for the recoverables from Reinsurer A is negligible, whereas the provision for reinsurance for the recoverables from Reinsurer B is large. The Schedule F formula may not be an accurate reflection of potential uncollectibility problems.

The trigger for classification as a slow-paying reinsurer is viewed by some analysts as an arbitrary dividing line amidst a spectrum of reinsurers. A reinsurer with an overdue ratio of

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[52] Unauthorized reinsurers domiciled in tax havens or in countries with less stringent insurance regulation are particularly likely to offer less expensive reinsurance coverage, though U.S. regulators generally frown on their activities.

21% is classified as slow-paying, whereas a reinsurer with a ratio of 19% is not slow-paying. The difference in the provision for reinsurance is greater than the empirical data justify.

The parameters for the aging schedule and the overdue ratio were chosen subjectively; they were not based on statistical or actuarial analysis. The use of 90 days past due instead of 120 days past due, the trigger of a 20% overdue ratio, and the 20% factor for the provision for reinsurance are subjective choices. This contributes to the perception that the provision for reinsurance does not properly measure the potential reinsurance uncollectibility exposure.

INDICATORS OF UNCOLLECTIBILITY

Were the provision for reinsurance merely an unsuitable proxy for uncollectibility problems, the provision may have little benefit but it would also cause little harm. But the complex Schedule F calculations may foster a misleading aura of precision while obscuring more relevant indicators of potential uncollectibility. In the long run, Schedule F may hinder regulators from properly monitoring reinsurance uncollectibility problems. This is a serious drawback.

Two of the primary indicators of potential uncollectibility problems are (i) the capital structure of the reinsurer and (ii) the extent of the reinsurer's potential liabilities in an adverse scenario:

- Reinsurers with high ratios of capital to the amount of insurance in force are less likely to default on their reinsurance obligations.
- Reinsurers with high potential exposures to the same event through multiple channels are more likely to default on their reinsurance obligations.

The "multi-channel" effect illustrates the importance of accurate assessments of potential reinsurance obligations. A reinsurer may have prudently limited its exposures to windstorm claims from its own reinsureds. But if the reinsurer has accepted retrocessions from other reinsurers, or if it has participated in layers of coverage written by other reinsurers, its total exposure in an adverse scenario may not be manageable.[53]

The experience of mortgage lenders and of bond rating organizations illustrates the use of financial ratios to estimate probabilities of default. Mortgage lenders consider (i) the ratio of equity in the home to the debt on the home and (ii) the ratio of the homeowner's monthly income to the monthly mortgage payment. Writers of mortgage insurance use these ratios, along with similar factors, to price mortgage guarantee insurance contracts.

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[53] On multi-channel effects, see Daykin, Pentikäinen, and Pesonen [1994]. (Daykin, Chris D., Teivo Pentikäinen, and M. Pesonen, *Practical Risk Theory for Actuaries*, First Edition (Chapman and Hall, 1994).)

The economist's inference is that competitive markets provide incentives to accurately quantify risk. A statutory formula that is not well correlated with actual default rates may interfere with these incentives and lead to less efficient markets.

Bond rating organizations use a host of quantitative and qualitative factors to assign credit ratings to bond issues. The interest of creditors in commercial bond ratings parallels the interest of insurance regulators in estimates of reinsurance recoverables. Creditors (bondholders) adjust the interest rate in the bond indenture in anticipation of potential future default probabilities, just as primary insurance companies hold capital to guard against potential reinsurance uncollectibility problems.

Although bond ratings are not perfect, they correlate reasonably well with empirical default costs. Arbitrage opportunities in efficient capital markets force this outcome. To the extent that bond ratings deviate from the expected probabilities of default, market credit spreads widen or narrow.[54]

The aging schedule of the reinsurer is simple to compute, but it may be less relevant to future uncollectibility problems than the capital structure of the reinsurer and its potential exposures in an adverse scenario. The NAIC should spend its resources exploring better predictors of uncollectibility problems instead of revising and enhancing the Schedule F exhibits.

THE REACH OF REGULATION

Experienced regulators are aware of these issues. The problem is not the accuracy of the formula but the reach of regulation.

To estimate reinsurance collectibility, regulators would prefer to examine the reinsurers, not the reinsureds. But reinsurance is a global market, and most large reinsurers are domiciled abroad. U.S. regulators lack authority to affect the operations of reinsurers that are not licensed in their states, even if the reinsurers are authorized to do business. They lack the information to examine the capital structures of these reinsurers or to estimate their potential liabilities after a major catastrophe.

U.S. regulators can examine the insurance operations of reinsurers domiciled or licensed in their states, and they do this when a domestic reinsurer seems financially troubled. But aggressive regulation of domestic reinsurers may hamper their ability to compete with their

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[54] Deviations from empirical default costs are often externally imposed. For instance, many pension funds and other institutional fiduciaries do not purchase bonds that are below investment grade. These institutional investors may bid up the price of BBB bonds (the lowest investment grade rating) and bid down the price of BB bonds (the highest non-investment grade rating). This is particularly true when a BBB bond is downgraded to BB or when a BB bond is upgraded to BBB. As a result, BB bonds have slightly higher net returns (i.e., default adjusted returns) than do BBB bonds. In most financial markets, these effects are small.

European peers. Aggressive regulation may force domestic reinsurers to flee abroad to the Bahamas, the Cayman Islands, and similar sanctuaries.

Instead, state regulators regulate the reinsureds, not the reinsurers. By imposing a provision for reinsurance on unsecured recoverables from unauthorized reinsurers, regulators provide incentives to alien companies to seek authorization to sell reinsurance in the states or to provide collateral if they wish to remain unauthorized. This is a round-about means of reinsurance regulation, but it may be the best that state regulators can accomplish.

Securities regulation suggests that this not the best that insurance regulators can accomplish. Firms commonly open their books to rating agencies, such as Moody's or Standard & Poor's, and even pay for the financial examination, because they benefit from a good rating and because the financial examination is no more intrusive than it has to be. Many unauthorized reinsurers may do the same, if the state insurance examination is efficient and non-intrusive (unless warranted). Regulars would do well to seek the optimal methods to ensure financially sound reinsurance arrangements.

PROSPECTIVE VS RETROSPECTIVE RISKS

The major criticism of the provision for reinsurance is its misplaced focus. The most serious and controllable solvency risk for insurance companies is the lack of adequate reinsurance arrangements. This risk is a prospective one; it is the risk that the primary company has not adequately hedged its exposures to natural catastrophes or unforeseen claims.

Adequate reinsurance arrangements are the bedrock of insurance risk management. Many insurance company failures can be traced to poor handling of reinsurance, such as excessive retentions, inadequate limits, and failure to cover significant exposures. These are all pre-loss issues; once the loss has occurred, a regulator can do little to salvage a distressed company.

Neither the NAIC Annual Statement blank nor the NAIC risk-based capital formula attempts to measure the risks stemming from poor reinsurance arrangements.[55] Techniques for evaluating insurance company risk exposures are well established in private insurance and brokerage markets, even if they are sometimes hard to implement. Supervision of solvency risks should emphasize over-concentration of property exposures (a) in catastrophe prone areas, such as the Gulf Coast states of Florida, Taxes, and Louisiana, (b) along known earthquake fault lines, or (c) within major urban areas. Excessive retentions and insufficient limits in excess-of-loss reinsurance treaties may reflect the ceding company's acceptance of undue risk in the hope of lowering its reinsurance costs and maximizing its net income.

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[55] This is not a criticism of insurance regulators. The actuaries on the American Academy of Actuaries risk-based capital task force spent a year and a half discussing the risks of natural catastrophes and inappropriate reinsurance arrangements without producing any suggestions.

Overuse or underuse of facultative reinsurance placements may reflect underwriting inexperience, timidity, or overconfidence.

Once the loss has occurred and the Annual Statement has been filed, the damage has been done. Most losses from a September hurricane will have been settled by the end of the year. If the reinsurance protection was not adequate, the primary company may already be impaired; further monitoring of reinsurance recoverables has little benefit.

The zealous quantification of aging schedules and overdue amounts may distract regulators from monitoring the risks stemming from improper reinsurance arrangements. Instead of the current Part 3 of Schedule F, regulators would be better served by an exhibit showing the terms of the proportional and the non-proportional reinsurance treaties and the facultative placements of the reporting company. Such an exhibit would require considerable underwriting skill to interpret, but it would contain the information that regulators need to ensure the sound reinsurance arrangements that promote long-term insurance solvency.

Some companies may argue that a listing of reinsurance treaties and facultative placements without corresponding information about the amounts of insurance and the concentrations of risk by line of business and by geographic region is not sufficient to judge the adequacy of the reinsurance program. This argument has some truth, but it misses the role of regulation. Accounting entries by themselves are rarely sufficient to monitor insurance risks. The primary value of the accounting information is to highlight possible areas for further investigation.

- A primary company with low policy limits in its reinsurance treaties or with restrictive policy provisions may have exposures that reinsurers are reluctant to accept.
- A primary company with reinsurance ceded predominantly to off-shore reinsurers or to weakly capitalized reinsurers may have been unable to find domestic companies or financially stronger companies willing to accept the exposures.
- A primary company with reinsurance cessions significantly lower than the industry average for the size of its direct business may be retaining too much of its exposure.
- A primary company with its reinsurance concentrated in facultative placements instead of general treaties may have inadvertent gaps in its coverage.

Financially distressed companies often have their reinsurance treaties canceled. Established reinsurers may refuse to provide coverage at affordable prices. Reinsurance programs of financially distressed companies may exhibit several of the characteristics listed above. This information is highly valuable to solvency regulators.

Some industry personnel dislike public filings that reveal corporate underwriting strategies, treaty pricing, and reinsurance contract provisions. This is a valid concern. The most useful information for solvency monitoring is also the most proprietary information. Distressed companies have little incentive to publicize their distress. Public filings should include only aggregate data that are of limited use to competitors. More accurate and revealing information is most suitable for non-public filings.

This alternative reinsurance schedule is appropriate for statutory financial statements, not for GAAP financial statements, for several reasons:

1. Reinsurance is peculiar to the insurance industry. GAAP statements are geared to general accounting, not to industry specific schedules.
2. A listing of reinsurance treaties and facultative placements requires considerable expertise to understand. It is valuable to insurance regulators and their staffs; it is of limited value to most security analysts. A security analyst with expertise in reinsurance contract terms can turn to the statutory blank.
3. The purchase of reinsurance is a trade-off between risk and return. Reinsurance reduces solvency risk, but it also reduces expected return. Equity investors are not necessarily displeased by companies pursuing aggressive and potentially risky strategies that generate high expected returns at the expense of higher default risk. Efficient diversification is done by the equity investor, not by company management. Investors often seek firms that pursue their strengths, not firms that diversify away from their core competencies. In contrast, policyholders are concerned with insolvency risk, not with the long-term expected return to the insurance company. Insurance regulators serve the interests of policyholders, not the interests of investors.

Reinsurance is the primary company tool for managing insurance risk, and reinsurance regulation is at the core of solvency regulation. The importance that state regulators place on reinsurance is reflected in the comprehensive exhibits of Schedule F.

Yet Schedule F is not perfect, and its exhibits are not necessarily the most effective means of reinsurance regulation. Better tools for solvency regulation are already available in actuarial models and in the underwriting practices of many companies. Both the industry and the public would gain from joint efforts by the actuarial and regulatory communities to enhance the reinsurance schedule in the Annual Statement.

The Stanard-Bühlmann Reserving Procedure— A Practitioner's Guide

Sholom Feldblum, FCAS, FSA, MAAA

Stanard-Bühlmann Reserving Procedure

The Stanard-Bühlmann procedure is a major advance in casualty actuarial loss reserving methods. It has proved especially useful for reinsurers lacking the pricing data to perform Bornhuetter-Ferguson analyses. Primary companies may benefit equally from this technique, particularly if the pricing actuary's expected loss ratio is not consistent with actual experience.

The Stanard-Bühlmann technique is similar to two other modified expected loss procedures: the Cape Cod method and the "adjustment to total known losses" (see Stanard [1985]). These are all methods of using the expected loss procedure when expected loss ratios are not available. We explain the similarities in this practitioner's guide, so that reserving actuaries may more knowledgeably choose among the techniques.

The Stanard-Bühlmann procedure is an intuitive procedure. Its European genealogy and its reinsurance provenance give it an undeserved aura of mathematical complexity, deterring some actuaries from its chárms. If fact, it is a simple and sensible technique, forming an excellent adjunct to the common chain ladder procedures.

STRUCTURE OF THIS GUIDE

We explain the intuition underlying chain ladder reserving techniques and expected loss reserving techniques. We show the algebraic extension of the Bornhuetter-Ferguson expected loss reserving technique to the Stanard-Bühlmann technique.

This practitioners' guide emphasizes the concepts; the algebraic implementation is straight-forward. We examine the intuition underlying the Stanard-Bühlmann method and the required adjustments to premium. We present several illustrations to show the simplicity of this method and the various reserving applications for which it is applicable.

RESERVING PRINCIPLES

The fundamental principle underlying most actuarial reserving techniques is that certain loss reporting patterns or loss settlement patterns remain relatively stable over time. The past observations, adjusted (if necessary) for changes in the insurance environment and company claims practices, are a reasonable predictor of future experience.

Examples of this principle are statements such as

- Case incurred losses as of 24 months since inception of the accident year are expected to be 50% higher than case incurred losses as of 12 months for that accident year.
- Cumulative paid losses as of 48 months since inception of the accident year are expected to be 20% higher than cumulative paid losses as of 36 months for that accident year.

The format of the two statements above is that the cumulative losses (of whatever type) as of development period $i+1$ are X% greater or lower than the same cumulative losses as of development period i, is the application of the principle to a specific reserving technique, the chain ladder procedure. The format differs for other reserving techniques, such as the Stanard-Bühlmann method.

The fundamental principle is that there is stability in the loss reporting pattern or in the loss settlement pattern. The loss reserving methods differ in the base against which we measure the stability.

ILLUSTRATION: PATTERNS OF STABILITY

Past observations indicate that losses of $550,000 would be paid over a five year period in the following fashion:

Development Months	0 – 12	12 – 24	24 – 36	36 – 48	48 – 60
Paid Losses	$100,000	$200,000	$150,000	$75,000	$25,000

We formulate the observed pattern in several ways.

A. *Incremental Development:* Losses paid between 12 months and 24 months are equal to twice the losses paid between 0 months and 12 months. Losses paid between 24 months and 36 months are equal to ¾ of the losses paid between 12 months and 24 months.

B. *Cumulative Development:* The cumulative losses paid between 0 months and 24 months are equal to 3 times the cumulative losses paid between 0 months and 12 months. The cumulative losses paid between 0 months and 36 months are equal to 1.500 times the cumulative losses paid between 0 months and 24 months.

C. *Percentages of Ultimate:* Of the $550,000 total paid losses, 18.2% are paid in the first 12 months, and 36.4% are paid in the next 12 months.

These patterns differ in the measurement base. The patterns are shown in the table below:

Development Months	0 – 12	12 – 24	24 – 36	36 – 48	48 – 60
Paid Losses	$100,000	$200,000	$150,000	$75,000	$25,000
Incremental ratio		2.000	0.750	0.500	0.333
Cumulative ratio		3.000	1.500	1.167	1.047
Percent of ultimate	0.182	0.364	0.273	0.136	0.045

The manner of expressing the pattern depends on the measurement base.[1]

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[1] Brosius [1993: "Loss Development Using Credibility"] presents a statistical procedure for selecting the base. The Brosius procedure allows for multiple bases – such as 60% of one base plus 40% of another base, and it determines the optimal percent of each.

EXPECTED LOSSES VS. ACTUAL LOSSES

For the expected losses, the bases in the example above can be converted into one another. If we are told the "incremental ratio" pattern, we can derive the "cumulative ratio" pattern and the "percent of ultimate" pattern.

The chain ladder method uses the cumulative ratio basis. The paid loss link ratio from 36 to 48 months is the 1.167 in the 36 to 48 months column of the cumulative ratio row. The cumulative product of the link ratios from a given development date forward is the loss development factor. The loss development factor from 36 months to ultimate is $1.167 \times 1.047 = 1.222$.

The percent of ultimate row is used for both the Bornhuetter-Ferguson method and the Stanard-Bühlmann method. The Bornhuetter-Ferguson factor is the sum of the percent of ultimate figures from a given development date forward. For instance, the Bornhuetter-Ferguson factor from 36 months to ultimate is $0.136 + 0.045 = 0.181$.

The Bornhuetter-Ferguson factor equals 1 – (1 divided by the link ratio). In this example, $0.181 = 1 - (1 \div 1.222)$.

The loss lag used in the Stanard-Bühlmann method is the complement of the Bornhuetter-Ferguson factor. The loss payment lag at 36 months is $1 - 0.181 = 0.819$.

DETERMINING THE PATTERNS

There are various ways of determining these patterns. The prospective future pattern is generally based on the historical observed patterns, with several adjustments.

- *Averages:* We might use either unweighted averages or weighted averages of historical observations, such as the cumulative link ratios in the experience period. When the weights are the same as the measurement base – e.g., the weights are the losses at the start of the period for the chain ladder link ratios – the weighted average may be computed by taking the totals for several years.[2]

- *Outliers:* We might eliminate outliers. For instance, we might use averages which discard

[2] Weighted averages are preferable when the differences in volume stem from differences in exposures. Unweighted averages are preferable when the differences in volume stem from monetary inflation. The Mahler paradigm of shifting risk parameters implies that more weight be given to the more recent years. Mahler's advances in credibility theory are particularly applicable to loss reserving, since the covariance matrix can be estimated from the experience.

the high value and the low value.

- *Inflation:* Changing inflation rates may bias the projected pattern. To correct for changes in the inflation rate, we might deflate the historical triangle for past inflation, perform the actuarial analysis on "real dollar" figures, and project forward with future expected inflation or stochastic inflation rate paths (cf. Hodes, Feldblum, and Blumsohn [1999]).

- *Trend and other Adjustments:* When the insurance environment is changing, we might trend the historical figures. Examples with significant effects are changing attorney involvement in private passenger automobile claims and changing claims management practices in workers' compensation insurance. For small insurers, we might weight company averages with industry averages, or state averages with countrywide averages.

Once we determined any one pattern, we have determined the other patterns as well. One sometimes hears that all of these methods start with the expected link ratios. We could equally well say that all these methods start with the expected Bornhuetter-Ferguson factors or with the expected loss lags.

FUTURE VALUES

The determination of the factors is the same for all three reserving methods. The use of the factors differs among the reserving methods.

- For chain ladder methods, we apply the expected factors to the cumulative paid or reported losses for each experience year. We do not need the estimated ultimate losses for the block of business.
- For the Bornhuetter-Ferguson method, we apply the factors to the estimated ultimate losses for the block of business. We do not need the cumulative paid or reported losses for each experience year.

In the example above, the paid losses in the first 12 months equal 18.2% of the estimated ultimate paid losses. Suppose we are using this historical pattern to estimate the needed reserves for a more recent accident year. What if the paid losses in the first 12 months of this accident year equal 25% of the estimated ultimate losses, not 18.2% of the ultimate losses?

- The chain ladder method says: "Use the cumulative paid losses in the first 12 months; ignore the estimated ultimate losses."

- The Bornhuetter-Ferguson method says: "Use the estimated ultimate losses; ignore the cumulative paid losses in the first 12 months."

CHOICE OF METHOD

Brosius, following Hugh White's discussion of the Bornhuetter-Ferguson paper, explains the differing philosophy of these two alternatives.

- The chain ladder method assumes that unusually high or low cumulative paid losses to date is indicative of correspondingly high or low paid losses in future development periods.
- The Bornhuetter-Ferguson method assumes that unusually high or loss cumulative paid losses to date reflects random loss fluctuations. This is not indicative of unusually high or low paid losses in the remaining development periods.

As Brosius points out, the truth is generally in between these two alternatives.

Yet the extreme cases interest us, because certain attributes of the insurance scenario argue for one or the other of these cases.[3]

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3 See Bornhuetter-Ferguson [1972] and Brosius [1993].

- When losses are very immature, or when loss severity is large but loss frequency is low, or when the variability of losses is unusually great, the Bornhuetter-Ferguson expected loss method may be favored.
- When losses are mature, or when loss severity is low but loss frequency is high, or when the variability of losses is small, the chain ladder method may be favored.

Excess of loss reinsurance has the former attributes, so many reinsurance actuaries are inclined to use expected loss reserving procedures. But there is a problem with expected loss procedures as applied to reinsurance.

ESTIMATED ULTIMATE LOSSES

The Bornhuetter-Ferguson method needs an estimate of the ultimate losses. For primary companies, this may not be a problem. Pricing actuaries estimate ultimate losses to set premium rates. The reserving actuary can use the estimate provided by the pricing actuary.

The estimated ultimate losses equal the premium times the expected loss ratio. This estimate is suitable when the indicated premium is also the premium charged. The estimate must be adjusted when the premium in the rate manual is not the pricing actuary's indicated premium. It must be further adjusted when underwriters provide schedule credits and debits to individual insureds, as is commonly done in the commercial lines of business. These adjustments demand business acumen, but a knowledgeable reserving actuary can sometimes make a reasonable estimate of the ultimate losses.

The reinsurer's reserving actuary does not have the data needed for this. The reinsurer's reserving book of business may consist of disparate pieces with different expected loss ratios. The reinsurer does not have the information to adjust for the adequacy level of the primary premiums or for schedule credits and debits provided by the primary underwriters.

This is also true for primary insurance enterprises if the reserving actuary does not have access to the pricing actuary's estimates, to manual deviations from indicated rates, or to the underwriters' discretionary price modifications. This is often the case for large commercial lines insurers.

Stanard-Bühlmann

Two eminent actuaries, James Stanard and Hans Bühlmann, provided a solution. If we have sufficient past experience, they argued, we don't need to know the expected loss ratio. We simply need to adjust all premiums in the historical period to the same level of adequacy.

The needed adjustments to the premiums are straightforward. However, these adjustments will divert us from the intuition underlying their reserving technique. For the moment we skip these adjustments; we explain them further below. For our first set of illustrations, assume that premiums are at the same level of adequacy for each year.

Let us clarify the assumption. We do not know the expected loss ratio for any year. But whatever the expected loss ratio is, it is the same for all years.

We use a numerical example to illustrate the Stanard-Bühlmann method. In practice, this method is most useful for long-tailed lines of business with relatively little reported loss or paid loss in the first 2 or 3 years of development. For heuristic purposes, we use a simpler example.

Determining the Pattern

The first task is to determine the pattern that is assumed to remain stable. For the Stanard-Bühlmann method, as for the other expected loss methods, the "percent of ultimate" pattern is assumed to remain relatively stable from year to year.

Stable percentages of ultimate is the assumption that we use to determine the outstanding losses. It is *not* necessarily the assumption we use to *determine the pattern.*

We said above that if we determine the incremental ratios or the cumulative ratios, we know the percentages of ultimate. Conversely, if we determine the percentages of ultimate, we know the incremental ratios and the cumulative ratios. We ask: "Which is the easiest pattern to determine?" not "Which pattern do we want to use?"

This question is surprisingly easy to answer. If we try to determine the percentages of ultimate, we can't use all the data at our disposal. In particular, we can't use any of the most current data. If we try to determine the incremental ratios or the cumulative ratios, we use all the historical data, including the most recent data.

Let us explain. If we try to determine the percentages of ultimate, we can use only mature accident years that have developed to ultimate. The patterns may have changed in the intervening years, as the social, economic, and insurance environments changed.

If we use incremental ratios or cumulative ratios, we can use all accident years, including even the most recent calendar year information in each accident year. This was the advance in casualty loss reserving theory that gave rise to the chain ladder method.[4]

We still must choose between the incremental ratios and the cumulative ratios. At early development periods, both methods work reasonably well. At later development periods, the incremental reported losses and even the incremental paid losses are relatively small. Small figures in the numerator of the ratios do not distort the estimation procedure. But small figures in the denominator of the ratios cause ratios that may be unrealistically large, reducing the accuracy of the results and adding significant bias.

Illustration: The table below shows reported loss development from ten years to 12 years. The table has five accident years and five columns, showing

- cumulative reported losses at ten years of development,
- incremental reported losses in year 11,
- cumulative reported losses at eleven years of development,
- incremental reported losses in year 12, and
- cumulative reported losses at 12 years of development.

All figures are in thousands of dollars.

Accident Year (1)	Reported Losses at Ten Years (2)	Incremental Losses in Year Eleven (3)	Reported Losses at Eleven Yrs (4)	Incremental Losses in Year Twelve (5)	Reported Losses at Twelve Yrs (6)
20X0	100,000	100	100,100	1,100	101,200
20X1	110,000	1,100	111,100	0	111,100
20X2	120,000	0	120,000	1	120,001
20X3	130,000	–100	129,900	1,100	131,000
20X4	140,000	1	140,001	100	140,101

The age-to-age link ratio from year 11 to year 12 is stable when using cumulative reported losses but is not stable when using incremental reported losses.

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[4] Health actuaries often use "claim completion percentages," which are chain ladder paid loss development factors that rely on mature years only. Since medical claims are settled quickly, the reliance on mature experience periods is not onerous; see Bluhm, *Group Insurance*, chapter 30. For a typology of reserving procedures, see Saltzman [1984].

Accident Year (1)	Age-to-Age Factor Using Cumulative Reported Losses (7) = (6) / (4)	Age-to-Age Factor Using Incremental Reported Losses (8) = (5) / (3)
20X0	1.011	11.000
20X1	1.000	0.000
20X2	1.000	∞
20X3	1.008	–11.000
20X4	1.001	100.000

This is the rationale for the method of determining the pattern. All three reserving procedures – chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann – begin by estimating link ratios (or cumulative age-to-age factors).

Loss development factors are determined as the cumulative products of the link ratios. The loss lags used in the Stanard-Bühlmann procedure, as well as the Bornhuetter-Ferguson factors, are percent of ultimate ratios.

- The reported loss lag is the percent of expected ultimate losses that have been reported by the development date.
- The paid loss lag is the percent of expected ultimate losses that have been paid by the development date.
- The loss lag equals the reciprocal of the loss development factor.
- The Bornhuetter-Ferguson factor is the complement of the loss lag, or "1 – loss lag."

Illustration: Reported loss link ratios for a block of business are shown below. We compute the corresponding loss development factors, loss lags, and Bornhuetter-Ferguson factors.

Development Months	12 – 24	24 – 36	36 – 48	48 – 60	60 – ult
Link ratio	1.500	1.250	1.100	1.050	1.020

The loss development factors are the cumulative products of the link ratios. The loss development factor from 12 months to ultimate equals

$$1.500 \times 1.250 \times 1.100 \times 1.050 \times 1.020 = 2.209.$$

The loss lag at 12 months equals 1 / 2.209 = 0.453. The Bornhuetter-Ferguson factor at 12 months equals 1 – 0.453 = 0.547.

Development Months	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.
Link ratio	1.500	1.250	1.100	1.050	1.020
Loss development factor	2.209	1.473	1.178	1.071	1.020
Loss lag	0.453	0.679	0.849	0.934	0.980
B-F factor	0.547	0.321	0.151	0.066	0.020

ALGEBRA

We show first the algebraic derivation of the Stanard-Bühlmann method from the Bornhuetter-Ferguson method, which is better known to many readers. The algebra is straightforward. The elegance of the technique is the intuition, which we discuss next.

Illustration: We have determined the following percentages of losses that are reported by each development date from the inception of the accident year. The slow loss reporting pattern is characteristic of casualty excess-of-loss reinsurance, products liability, and professional liability.

Loss Lag	Percent Reported	Loss Lag	Percent Reported
12 mos	30%	72 mos	85%
24 mos	50%	84 mos	90%
36 mos	65%	96 mos	94%
48 mos	75%	108 mos	97%
60 mos	80%	120 mos	99%

At December 31, 20X9, we have the following data on premiums and reported losses for the ten most recent accident years.

Year	Adjusted Premiums	Reported Losses	Year	Adjusted Premiums	Reported Losses
20X0	200 million	150 million	20X5	300 million	185 million
20X1	220 million	155 million	20X6	320 million	205 million
20X2	240 million	200 million	20X7	340 million	155 million
20X3	260 million	175 million	20X8	375 million	185 million
20X4	280 million	215 million	20X9	400 million	75 million

Premiums and Losses

Adjusted premiums are premiums on the same level of adequacy for all accident years. We have not yet explained what adjustments are needed to bring premiums to the "same level of adequacy"; we deal with that issue further below. We comment here on two items.

1. There is no need for an absolute level of adequacy. The premiums may be 20% inadequate in each year, or they may be 10% redundant in each year. It won't make any difference for the reserve indication.

2. For the reserving technique to be useful, the reserving actuary must be able to make the needed adjustments. If the actuary had to examine past rate reviews to determine the adequacy of the rates, the reserving technique would have only limited applicability.

   We do not mean that knowledge of the underlying data is irrelevant. No matter what reserving procedure is used, an understanding of the underlying data improves the reserve indications. We are saying only that this knowledge not any more essential for the Stanard-Bühlmann technique than it is for other reserving techniques.

The premium adjustments are relatively easy, if the intuition for the adjustments is clear. We return to this subject below.

The Stanard-Bühlmann technique may be used with either reported losses or paid losses and with either dollars of loss or with number of claims.[5] The type of premium adjustment differs for dollars of loss versus number of claims; see below.

[5] The separate quantification of loss frequency and loss severity allows for estimation of loss frequency along development rows and estimation of average severity by inflation indices.

SIMULTANEOUS EQUATIONS

To keep the intuition clear, we use a pair of simultaneous linear equations. The mathematics can be reduced to a single expression.

If the premiums are at the same adequacy level, then the multiplicative factor needed to arrive at the expected losses is the same for all years. For instance, if the premiums are all 20% inadequate, then the expected losses in each year equal

premium × 1.200 × expected loss ratio.[6]

Let Z = the expected loss ratio times the factor needed to bring premiums to adequate levels.
Let Y_i = the bulk reserves for year *"i."*
Let Y = the total bulk reserve; that is, $Y = \sum Y_i$.

The index *"i"* ranges from 0 to 9, corresponding to accident years 20X0 through 20X9.

We write the Bornhuetter-Ferguson expected loss equations for years 0 through 9. For any year, the bulk loss reserves equal

premium at an adequate level × the expected loss ratio × expected percentage unreported.

For year 20X0, the expected percentage already reported is 99%, so the Bornhuetter-Ferguson estimate of the bulk reserves is

$$\$200 \text{ million} \times Z \times (1 - 99\%) = Y_0$$

We do the same for each accident year. For the 20X9 accident year, the estimate is

$$\$400 \text{ million} \times Z \times (1 - 30\%) = Y_9$$

We sum all 10 equations to get

$$Z \times [\$200 \text{ million} \times (1 - 99\%) + \ldots + \$400 \text{ million} \times (1 - 30\%)] = \sum Y_i = Y.$$

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6 The terms "premium adequacy" and "expected loss ratio" have numerous interpretations. When used in a pricing context, premium adequacy generally has an economic meaning: premiums are adequate if they provide a reasonable return to the insurance enterprise. Statutory reserving uses undiscounted losses. By "premium adequacy" and "expected loss ratio" in this paper we mean figures such that ultimate (undiscounted) losses equal adequate premiums times the expected loss ratio.

This is a linear equation in two variables, Z and Y.

From the definition of the expected loss ratio, we know that over a long period of time, the total reported losses plus the total bulk reserves should be close to the total expected losses. We write the equation for this statement as

$$[\$150 \text{ million} + \ldots + \$75 \text{ million}] + Y = Z \times [\$200 \text{ million} + \ldots + \$400 \text{ million}]$$

This is also a linear equation in two unknowns, Y and Z.

To solve this pair of linear equations, we compute the three sums in these equations.

- The sum of the adjusted premiums is $2,935 million
- The sum of the reported losses is $1,700 million
- The sum of the adjusted premiums × the Bornhuetter-Ferguson factors is $817.5 million

The two equations are

$$Z \times \$817.5 \text{ million} = Y$$
$$\$1700 \text{ million} + Y = Z \times \$2935 \text{ million}$$

We can solve these two equations for the values of "Y" and "Z." We need to find "Y," the total bulk reserve. We eliminate Z by writing Z = Y ÷ $817.5 million. We write

$$\$1700 \text{ million} + Y = Y \times \$2935 \text{ million} \div \$817.5 \text{ million}$$
$$\$1700 \text{ million} \times \$817.5 \text{ million} = Y \times \$2117.5 \text{ million}$$
$$Y = \$1700 \text{ million} \times \$817.5 \text{ million} \div \$2117.5 \text{ million}$$

Let us stop here. The algebra is straightforward. Our goal is to derive an equation that we can write down from intuition alone. We turn now to the intuition.

INTUITION

Consider year 20X9. The adjusted premium is $400 million. By 12 months from the inception of the accident year, 30% of the adjusted premium, or $120 million, has been processed into reported losses. The other 70% of the adjusted premium, or $280 million, has not yet been processed into reported losses.

The word "processed" warrants explanation. The adjusted premium does not *become* reported losses. Rather, think of the verb "process" as connoting emergence or development or settlement. We need a general term that denotes the relationship between the premium collected and the loss activity.

There is some relationship between the $400 million of premium and the ultimate reported losses. We don't know this relationship, since we don't know the expected loss ratio and we don't know the level of premium adequacy. We know only that at 12 months of development, 30% of the losses should have been reported. $120 million of premium has the same relationship to the losses that have already been reported as the other $280 million of premium has to the losses that are yet to be reported.

The reader might think: "We have solved the reserving problem." The relationship is the same for the $120 million of premium that has already been processed as for the $280 million of premium that has yet to be processed. The $120 million of premium that has already been processed corresponds to $75 million of reported losses. We form the equation

$120 million : $75 million :: $280 million : X

We solve for X, the bulk reserve, as X = $75 million x $280 million / $120 million, or X = $175 million.

That is not right. The logic makes sense; it is the logic of the chain ladder loss development technique. We can see this in two ways.

1. Using this logic, the bulk reserve is directly dependent on the losses that have been reported so far. If the reported losses at 12 months were twice as high ($150 million instead of $75 million), the bulk reserve would be twice as large. We verify this by writing

   $120 million : $150 million :: $280 million : X
   X = $350 million.

2. If this is the chain ladder loss development procedure, there must be a loss development factor hidden here somewhere. We solved for X in the previous equation as X = $175

million. This says that X = bulk loss reserves =

reported losses × expected losses unreported / expected losses already reported
= reported losses × (1 – loss lag) / (loss lag).

The loss lag is the reciprocal of the loss development factor. We rewrite the expression above:

$$(1 - \text{loss lag}) / (\text{loss lag}) = (1 - 1/LDF) / (1/LDF) = LDF - 1.$$

For the chain ladder reserving method, the reported losses times (LDF – 1) equals the bulk loss reserve.

This is the result that we are trying to avoid. Losses are volatile, and we don't want to give too much credence to the $75 million of losses that have been reported as of 12 months for accident year 20X9.

ACTUARIAL PRESENT VALUES

We would like to use all the available data by combining the various accident years. We can not add dollars from two different years, since a dollar from year X is worth more than a dollar from year X+1 when the inflation rate is positive.

We can add present values of dollars if the dollars have been discounted or accumulated to the same date. If we know the present value of 20X1 premiums as of a given date and the present value of 20X2 premiums as of the same date, we can add them to get the present value of the combined premiums as of that date.

It seems as though we need the present values of the premiums and losses to add figures from different years. We don't have these present values. In fact, we can't possibly have these present values, since the premiums for a given accident year may be paid at different times. Similarly, the losses for a given accident year may be paid at different times.

But we don't need the present values. We are comparing premiums to losses. We require only that the change in premiums from year to year should equal the change in expected losses from year to year. Two conditions suffice for this:

i. The expense ratio stays constant from year to year, and
ii. The premiums are at the same level of adequacy from year to year.

The premium adjustment ensures the same adequacy level from year to year. The constancy of the expense ratio is less critical. Expense ratios don't change much from year to year, and we assume that they stay constant. A significant change in expense ratios would necessitate additional premium adjustments. Such changes are not common.

We said above that "we don't need present values." Perhaps that is an overstatement. We might rephrase this to say that

> *since we are comparing premiums to losses, we can get away with adding nominal amounts from different years. We are not adding apples and oranges; we are adding golden delicious apples with McIntosh apples. It's not perfect, but it's the best we can do. The cost of getting present values is greater than the improved accuracy we might obtain.*

Combining Years

We combine the processed premium from each year, and we combine the reported losses from each year.

A. For accident year 20X9, $120 million of premium (30% × $400 million) has been processed so far, and $75 million of losses have been reported.
B. For accident year 20X8, $187.5 million of premium (50% × $375 million) has been processed so far, and $185 million of losses have been reported.

We do this for all ten accident years. The total processed premium is $2117.5 million. The total reported losses are $1700 million. The total premium that remains to be processed is $817.5 million. We form the equation

$2117.5 million : $1700 million :: $817.5 million : X

We solve for X, the total bulk reserve, as X = $1700 × $817.5 ÷ $2117.5 = $656.3 million. This is the equation that we derived earlier using the Bornhuetter-Ferguson method.

ILLUSTRATION: BASIC FORMULA

Additional examples are helpful to the practicing actuary. We provide a few further illustrations before proceeding. We estimate the IBNR from the figures below.

Calendar/ Accident Year	Adjusted Earned Premium	Aggregate Reported Loss	Aggregate Reported Loss Lag
1993	200	150	75%
1994	250	200	67%
1995	300	100	40%
1996	350	50	10%
Total	1,100	500	

Intuition

From the loss lags and the reported losses, we compute the premium that has already been processed for each accident year and the premium that has not yet been processed.

(1)	(2)	(3)	(4)	(5)	(6)
Calendar/ Accident Year	Adjusted Earned Premium	Aggregate Reported Loss	Aggregate Reported Loss Lag	Processed Premium (2) × (4)	Remaining Premium (2) × [1−(4)]
1993	200	150	75%	150.0	50.0
1994	250	200	67%	167.5	82.5
1995	300	100	40%	120.0	180.0
1996	350	50	10%	35.0	315.0
Total	1,100	*500*		*472.5*	*627.5*

We use the entries from the "total" line shown in italics. If Y = the bulk reserve, we have

$$500 : Y :: 472.5 : 627.5$$
$$\text{or } Y = 500 \times 627.5 \div 472.5 = 664.$$

Algebra

The Stanard-Bühlmann method is a Bornhuetter-Ferguson method where the expected loss ratio is derived from the observed data. We show the relationship of the two methods.

The Bornhuetter-Ferguson factors equal "1 – the loss lag" or [1 – (4)] in the table above. The bulk reserve for each year is the expected total losses times the Bornhuetter-Ferguson factor.

Since the adjusted premiums are at the same adequacy level, the adjusted premium times a constant equals the expected losses in each year. We denote this constant as "ELR."

The bulk reserve for year 1993 using the Bornhuetter-Ferguson method equals

$$\$200 \times [1 - 75\%] \times ELR.$$

We form a similar equation for all the years in the historical period. The total bulk reserve for all years combined is \$627.5 × ELR.

The total reported losses are \$500. The total incurred losses equal \$500 + \$627.5 × ELR. The ELR is the total incurred losses divided by the total adjusted premium, or

$$[\$500 + \$627.5 \times ELR] \div \$1100 = ELR.$$
$$\$500 = \$472.5 \times ELR$$
$$ELR = \$500 \div \$472.5$$

Since the bulk reserve equals \$627.5 × ELR, we have

$$\text{bulk reserve} = \$500 \times \$627.5 \div \$472.5 = \$664.$$

VOLATILE LOSSES

We estimate the IBNR loss reserve using the figures below.

Cal./Acc. Year	Adjusted Earned Premium	Aggregate Reported Loss	Aggregate Reported Loss Lag
1993	10,000	1,000	95%
1994	10,000	6,000	85%
1995	10,000	5,000	70%
1996	10,000	5,000	50%
1997	10,000	4,000	30%

Reported Losses vs. Expected Losses

The Stanard-Bühlmann technique is most useful when losses are highly volatile and we don't have a good feel for the expected loss ratio.

- For 1997, adjusted premiums are $10,000, and $4,000 of losses have been reported.
- For 1993, adjusted premiums are $10,000, and $1,000 of losses have been reported.

At the current valuation date, the loss lags suggest a 95% to 30% reporting ratio for 1993 compared with 1997. The observed reporting ratio is 1 to 4.

(1)	(2)	(3)	(4)	(5)	(6)
Cal./Acc. Year	Adjusted Earned Premium	Aggregate Reported Loss	Aggregate Reported Loss Lag	Processed Premium (2) × (4)	Remaining Premium (2) × [1−(4)]
1993	10,000	1,000	95%	9,500	500
1994	10,000	6,000	85%	8,500	1,500
1995	10,000	5,000	70%	7,000	3,000
1996	10,000	5,000	50%	5,000	5,000
1997	10,000	4,000	30%	3,000	7,000
Total	50,000	21,000		33,000	17,000

Letting Y = the bulk loss reserve, we have

$$21{,}000 : Y :: 33{,}000 : 17{,}000$$
$$Y = 10{,}818$$

IBNR RESERVES

We determine the IBNR reserves from the following data:

Accident Year	Adjusted Earned Premium	Incurred Losses December 31, 2000
1998	$25,781	$16,500
1999	$28,125	$9,000
2000	$30,469	$3,900

Age (months)	Percent Reported
12	16.0%
24	40.0%
36	80.0%
48	100.0%

Processing Ratio

(1)	(2)	(3)	(4)	(5)	(6)
Cal./Acc. Year	Adjusted Earned Premium	Aggregate Reported Loss	Aggregate Reported Loss Lag	Processed Premium (2) × (4)	Remaining Premium (2) × [1−(4)]
1998	$25,781	$16,500	80%	$20,624.80	$5,156.20
1999	$28,125	$9,000	40%	$11,250.00	$16,875.00
2000	$30,469	$3,900	16%	$4,875.04	$25,593.96
Total	$84,375	$29,400		$36,742.84	$47,617.16

Letting Y = the bulk loss reserve, we have

$$\$47{,}617.16 : Y :: \$36{,}742.84 : \$29{,}400$$
$$Y = \$38{,}101.15$$

WEIGHTED VS. UNWEIGHTED AVERAGES

We said above that using nominal values instead of present values is like mixing golden delicious apples with McIntosh apples. We explain what we mean by this.

Suppose we are determining age-to-age factors (link ratios) from three years of experience. Should we use the simple average of the three years or should we use a weighted average? The weights for the weighted average are normally the loss amounts at the earlier of the two valuations, though we might also use the premium volume for the three experience years. The weighted average gives more weight to the years with a greater volume of experience. The unweighted average gives equal weight to all years.

This question has baffled generations of reserve actuaries, though the answer (in most cases) is not difficult.

1. A more recent year is a better predictor of future experience than a less recent year. Mahler [1990] refers to this as "shifting risk parameters." He shows the implications for ratemaking and for experience rating; the same logic applies to reserving. The more recent accident years should receive more weight than the older accident years. This is particularly important when potential trends appear in the columns of age-to-age factors.

   The greater weight that should be assigned to more recent years does not depend on the volume of business. Our question is different. Besides the greater weight that should be applied to more recent years, should we apply greater weight to years with greater volume?

2. The answer is that we should assign weights in proportion to the real volume of business. The loss amounts in each year differ for two reasons: (i) the "real dollar" amount of losses may differ, and (ii) inflation causes the nominal amount of losses to differ even though the "real dollar" amount of losses may be the same among the years.

   Ideally, we should weight the accident years by the deflated dollar amount of losses.[7] Using deflated losses as weights is complex; the following rule is a reasonable proxy. When the dollar amount of losses is consistent with monetary inflation, we should use unweighted averages. When the dollar amount of losses is considerably different from monetary inflation, we should use weighted averages.

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[7] Ideally, we should perform the entire reserve analysis using deflated losses, to avoid distortions caused by varying inflation rates. For a complete discussion, see Hodes, Feldblum, and Blumsohn [1999].

Adjusted Premiums

We have not yet discussed the premium adjustments. The premium adjustments depend on the type of loss data. If we use reported losses or paid losses, we use one type of adjustment. If we use reported claims or paid claims, the adjustment is different.

Before stating the general rule, we provide a set of illustrations. Each illustration is so simple that the adjustment is trivial. The series of illustrations covers all the relevant scenarios.

ILLUSTRATION #1: RATE CHANGE

> We have two accident years, 20X1 and 20X2. There is no expected loss trend; that is, the loss trend is 0% per annum.
>
> Earned premium is $100 million in 20X1 and $120 million in 20X2. All policies are effective on January 1. On January 1, 20X2, there was a +10% rate change. The exposure base is not inflation sensitive.
>
> We adjust the 20X1 and 20X2 earned premiums to the same adequacy level for a Stanard-Bühlmann procedure dealing with reported losses or paid losses.

This scenario has no loss trend: neither a loss severity trend nor a loss frequency trend. We took a +10% rate change on January 1, 20X2.

We can conceive of this in various ways:

1. The 20X1 premiums are exactly adequate. If so, the 20X2 premiums are 10% redundant. To bring the premiums to the same adequacy level for the two years, we divide the 20X2 premiums by 1.100.

2. The 20X2 premiums are exactly adequate. If so, the 20X1 premiums are deficient by a factor of 1 ÷ 1.100. To bring the premiums to the same adequacy level for the two years, we multiply the 20X1 premiums by 1.100.

   These two scenarios give the same result in the Stanard-Bühlmann technique. Multiplying the numerator of a ratio by a constant has the same effect as dividing the denominator of the ratio by the same constant.

3. There are a variety of other possibilities. The 20X1 premiums might be deficient by 5%,

or by 15%, or they might be redundant by 5%, or by 15%. They all lead to the same Stanard-Bühlmann result.

Given the various possibilities, which should we choose? The actuarial convention is to leave the most recent year unadjusted and to adjust prior years to the level of the most recent year.

This is a general actuarial convention, with the following rationale. The readers of the reserving actuary's report may not understand the Stanard-Bühlmann technique. In most situations, other company personnel believe that the current year is "correct." It is easier to explain an adjustment of prior years to the adequacy level of the current year than to explain an adjustment of the current year to the adequacy level of past years.

Thus, we multiply the 20X1 premium by 1 plus the January 1, 20X2, rate change amount.

We said above that "these two scenarios give the same result in the Stanard-Bühlmann technique." We show this explicitly.

We said earlier that if all premiums are at the same adequacy level, we can multiply all premiums by a constant "Z" to convert premiums into expected losses.

Illustration: Suppose the expected loss ratio is 70%, the 20X1 premiums are exactly adequate, and the 20X2 premiums are 10% redundant.

1. If we multiply the 20X1 premium by 1.100, the premiums in both years are 10% redundant. The value of "Z" is 70% / 1.100. In combination, we have multiplied the 20X1 premium by 1.100 × 70% / 1.100 = 70%. We have multiplied the 20X2 premium by 70% / 1.100.

2. If we divide the 20X2 premium by 1.100, the premiums in both years are exactly adequate. The value of "Z" is 70%. In combination, we have multiplied 20X1 premium by 70%. We have multiplied the 20X2 premium by 70% / 1.100.

We get the same result in both cases. This is true for all scenarios.

ILLUSTRATION #2: LOSS TRENDS

We assume a loss severity trend of +10% per annum, and we eliminate the rate change.

> We have two accident years, 20X1 and 20X2. The loss severity trend is +10% per annum. The claim frequency per exposure unit is the same in both years, though the number of exposure units may be different.
>
> Earned premium is$100 million in 20X1 and $120 million in 20X2. All policies are effective on January 1. There have been no rate changes, and the exposure base is not inflation sensitive.
>
> We adjust the 20X1 and 20X2 earned premiums to the same adequacy level for a Stanard-Bühlmann procedure dealing with reported losses or paid losses.

Let us review the possible scenarios.

A. The 20X1 premium is exactly adequate. Since losses increased by 10% per exposure unit in 20X2 and there was no rate change, the 20X2 premiums are deficient by 10%. We must multiply the 20X2 premiums by 1.100 to bring them to an adequate level.

B. The 20X2 premium is exactly adequate. Since the 20X1 losses were 9.09% [= 1 – (1 ÷ 1.100.)] less per exposure unit, and there was no rate change between the two years, the 20X1 premiums were redundant. We must divide the 20X1 premiums by 1.100 to bring them to an adequate level.

C. We can use any premium adequacy level we desire; there is no difference in the Stanard-Bühlmann result. By convention, we keep the premiums the same in the most recent year. We adjust other premiums to the adequacy level of the most recent year.

This example assumes that we are dealing with reported losses or paid losses, which are affected by both frequency and severity trends. When we deal with reported claims or paid claims, we must differentiate between the loss severity trend and the loss frequency trend.

The general rule: we determine the loss cost trend factors to bring prior years' losses to the level of the most recent year. We *divide* the prior years' premiums by the trend factors.

ILLUSTRATION #3: RATE CHANGES AND LOSS TRENDS

We assume both a loss severity trend of +10% per annum and a rate change of +10% on January 1, 20X2.

We have two accident years, 20X1 and 20X2. The loss severity trend is +10% per annum. The claim frequency per exposure unit is the same in both years, though the number of exposure units may be different. Earned premium is$100 million in 20X1 and $120 million in 20X2. All policies are effective on January 1. We took a rate change of +10% on January 1, 20X2. The exposure base is not inflation sensitive. We adjust the 20X1 and 20X2 earned premiums to the same adequacy level for a Stanard-Bühlmann procedure dealing with reported losses or paid losses.

We skip the scenarios, since the illustration is straightforward. Losses went up by 10% between the two years and the premium per exposure unit went up 10% between the two years. The premiums are at the same adequacy level. They might both be exactly adequate; they might both be deficient; they might both be redundant.

We can use the general rules that we stated above. We multiply the 20X1 premium by 1.100 for the rate change, and we divide the 20X1 premium by 1.100 for the loss trend. The net adjustment is no change.

ILLUSTRATION #4: EXPOSURE TRENDS

We add an exposure trend.

We have two accident years, 20X1 and 20X2. The loss severity trend is +10% per annum. Earned premium is$100 million in 20X1 and $120 million in 20X2. All policies are effective on January 1. We took a rate change of +10% on January 1, 20X2. The exposure base is inflation sensitive, and the exposure trend is 10% per annum. We adjust the 20X1 and 20X2 earned premiums to the same adequacy level for a Stanard-Bühlmann procedure dealing with reported losses or paid losses.

The exposure trend of +10% per annum exactly offsets the loss cost trend of +10% per annum. We conceive of an exposure trend as the reciprocal of a loss cost trend. The net trend is 0% per annum. This illustration is the same as the first illustration.

THE GENERAL RULES

Premiums: We bring all premiums to the current rate level. The illustrations above have policies effective on January 1 and rate changes effective on January 1. That is not necessary. Rather, we determine calendar year on-level factors to bring the earned premium in each calendar year to the current rate level.

Suppose the years in the experience period run from January 1 to December 31, and we took a rate change on July 1 of the most recent experience year.

> We have two accident years, 20X1 and 20X2. The loss severity trend is 0% per annum.
>
> Earned premium is$100 million in 20X1 and $120 million in 20X2. Policies are written evenly through the year. We took a rate change of +10% on July 1, 20X1. The exposure base is not inflation sensitive.
>
> We adjust the 20X1 and 20X2 earned premiums to the same adequacy level for a Stanard-Bühlmann procedure dealing with reported losses or paid losses.

The calendar year on-level factors are 1.075 for 20X1 and 1.025 for 20X2. We multiply the 20X1 premium by 1.075 and the 20X2 premium by 1.025.

The Stanard-Bühlmann technique is commonly used by reinsurance actuaries. Most excess-of-loss reinsurance treaties are effective on January 1, and reinsurance rate changes are effective on January 1 as well. This eases the required calculations.[8]

Losses: We want to trend all losses to a common date with the net trend factors. The net trend equals the loss frequency trend × the loss severity trend ÷ the exposure trend. However, we adjust the premiums, not the losses. Therefore, after determining the net trend factors to apply to the losses, we *divide* the premiums by these net trend factors.

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[8] The underlying policies written by the ceding company may be written evenly during the year, and the ceding company's rate changes may have occurred during the year. The on-level factors are taken into account to determine the reinsurance rate changes; they need not be recomputed for the reserve estimate.

CLAIM COUNTS

The Stanard-Bühlmann technique can be used with reported claims in place of reported losses. It can also be used with paid claims, though the use of paid claims for reinsurance reserving is less common than the use of reported claims.

Let us first understand why we would use claim counts instead of loss dollars. Suppose a line of business has claims that are reported quickly but claim severities that are highly variable and that may remain uncertain for many years. The reserving actuary may project ultimate claims by a development procedure and the average claim severity by a trend procedure.

Illustration: A severe workers' compensation permanent disability claim is reported quickly, though it may take years before the severity of the injury is clear. The claims are paid over the remaining lifetime of the injured worker. Both the indemnity (loss of income) benefits and the medical benefits extend over decades, and they are difficult to estimate.

The reserving actuary may project ultimate claim counts by a development year procedure and ultimate claim severities by an accident year trend. Suppose we are estimating accident year 20X9 workers' compensation reserves for permanent disability claims. Within a year or two after the expiration of the 20X9 accident year, we have a preliminary estimate of the ultimate claim count.[9] Since we have only a year or two of payments on these claims – each of which may extend for 20 or 30 years – we can not estimate claim severities from the 20X9 data.

Instead, we estimate ultimate claim severities for the more mature accident years, such as 20X0 through 20X7. We use the workers' compensation loss cost trend factors derived from shorter-term injuries to extend the claim severity trend through 20X9.

This procedure is particularly well suited for workers' compensation excess-of-loss reinsurance reserving, since most of the claims are permanent injuries.

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9 A common reserving procedure for these claims is to project the future permanent disability claims as a percentage of the reported total indemnity claims, or as a percentage of the reported back injury claims.

Reported and Unreported Claims

When we deal with reported losses, the fundamental equation is

processed premium : unprocessed premium :: reported losses : unreported losses.

The unreported losses are the bulk reserve. When we deal with reported claims, the corresponding equation is

processed premium : unprocessed premium :: reported claims : unreported claims.

The mathematics is the same, with one difference in the premium adjustments. We explain by means of an illustration.

ILLUSTRATION #5: CLAIM FREQUENCIES

We have two accident years, 20X1 and 20X2. The loss severity trend is +10% per annum. The claim frequency per exposure unit is the same in both years, though the number of exposure units may change. Earned premium is$100 million in 20X1 and $120 million in 20X2. All policies are effective on January 1. There have been no rate changes, and the exposure base is not inflation sensitive. We are using the Stanard-Bühlmann technique to estimate ultimate claim frequencies. We adjust the 20X1 and 20X2 earned premiums to the same adequacy level.

The term "same adequacy level" requires explanation. We normally speak of premium adequacy with respect to dollars of loss, not with respect to claim counts.

Conceive of the level of premium adequacy with respect to claim counts as the claim frequency with respect to premiums. If the expected claim frequency is 100 claims for each $1 million of premium in 20X1, then 20X2 has the same level of premium adequacy if the expected claim frequency is still 100 claims for each $1 million of premium.

In the illustration above, there were no rate changes in 20X1 or 20X2, and there were no changes in claim frequency. The premiums in 20X1 and 20X2 are at the same level of premium adequacy with respect to claim frequency.

Since the average loss severity rose by 10% from 20X1 to 20X2, the premiums in the two years are not at the same level of adequacy with respect to losses. For the Stanard-Bühlmann method, we use a premium adjustment if we are dealing with reported losses. We make no premium adjustment in this case if we are dealing with reported claims.

ILLUSTRATION #6: FREQUENCY AND SEVERITY TRENDS

We have two accident years, 20X1 and 20X2. The loss cost trend is +10% per annum, consisting of 7.8% claim severity trend and a 2.0% claim frequency trend. Earned premium is$100 million in 20X1 and $120 million in 20X2. All policies are effective on January 1. There have been no rate changes, and the exposure base is not inflation sensitive. We are using the Stanard-Bühlmann technique to estimate both ultimate losses and ultimate claim frequencies. We adjust the 20X1 and 20X2 earned premiums to the same adequacy level.

To estimate ultimate losses, we use the total loss cost trend of +10% per annum. To estimate ultimate claim counts, we use the claim frequency trend of 2.0% per annum.

Pricing actuaries have learned to be wary of claim frequency trends. In most lines of business, claim frequency does not follow simple exponential growth patterns. Econometric modeling of claim frequency has generally been disappointing. One might wonder how useful the claim frequency trends would be for the Stanard-Bühlmann reserving technique.

The pricing actuary and the reserving actuary use the trend factors for different purposes. The pricing actuary is projecting future claim frequency; most trend estimates have been poor predictors. The reserving actuary is quantifying the change between two past years. The claim frequency is a historical figure; it is not better or worse than the historical loss cost trend.

THE GENERAL RULE

The general rule for claim counts is similar to the rule for dollars of loss, with one difference. When we deal with claim counts, we adjust only for claim frequency trends, not for claim severity trends.

1. If we are given both claim frequency trends and claim severity trends, we use the product of these trends when we deal with dollars of loss. When we deal with claim counts, we use only the claim frequency trends.

2. If we have a single loss cost trend, we must use the claim frequency portion of the trend. We do not always know the claim frequency portion. If we can estimate the claim severity portion from other indices, we can "back out" the claim severity portion to derive the claim frequency portion.

3. The loss frequency trends in the historical data may reflect shifts in the mix of business, not real changes in claim frequency. Such trends may not be used in pricing, though they may be appropriate for aggregate reserving analyses.

4. For some lines of business, the exposure trends offset the loss severity trends, and the net trend is not material. When we are dealing with claim counts, we ignore loss severity trends but we still include exposure trends to calculate the premium adjustments.

   Illustration: Payroll in 20X1 is $100 million. The workers' compensation premium rate is 2% of payroll, giving a premium of $2 million. The real activity at the insured's workplace stays the same for 20X2, but wage inflation is 10% per annum, so payroll is $110 million and the workers' compensation premium is $2.2 million. Nothing has changed in the physical plant, and we expect the same number of claims. We increase the 20X1 premiums by a factor of +10% to bring them to the adequacy level of the 20X2 premiums.

ILLUSTRATION: REPORTED CLAIMS

We illustrate the premium adjustments by calculating the IBNR claim count from the figures below. All policies have effective dates of January 1; all rate changes occur on January 1.

Cal/Acc Year	Earned Risk Pure Premium (000)	Estimated Claim Report Lag	Reported Claims @ 12/31/91
1987	$40,000	38.0%	9
1988	$44,000	28.0	8
1989	$40,000	18.0	8
1990	$45,000	9.0	5
1991	$50,000	2.0	1

Annual loss trends and rate changes are shown below. There is no exposure trend.

Loss Cost Trends		Rate Changes	
1986 to 1987	15.0%	1/1/87	30.0%
1987 to 1988	12.5	1/1/88	10.0%
1988 to 1989	10.0	1/1/89	–10.0%
1989 to 1990	10.0	1/1/90	0.0%
1990 to 1991	10.0	1/1/91	5.0%

Premium Adjustments

There are two premium adjustments.

- We bring all premiums to the same rate level.
- We divide by the factors needed to bring all claim counts to the same claim level.

The rate change effective on January 1, 1987 is not relevant, since it affects all year equally. Conceive of the 1987 rate level as the base rate level, or 1.000. We use the other rate changes to bring premiums to the current rate level.

We can ignore the January 1, 1987, rate change only because all policies are effective on January 1. If we had any other distribution of policy effective dates during the year, we would have to consider the January 1, 1987, rate change as well.

Date	Rate Change	Rate Level Index	On-Level Factor
1/1/87	30.0%	1.0000	1.0395
1/1/88	10.0%	1.1000	0.9450
1/1/89	−10.0%	0.9900	1.0500
1/1/90	0.0%	0.9900	1.0500
1/1/91	5.0%	1.0395	1.0000

The rate level index is the cumulative downward product of the rate changes. If the policy effective dates are distributed through the year and the rate changes occur on different dates, the rate level index is the average rate level during the year. We set the rate level index for 1987 to unity. The on-level factor is the current rate level index divided by the rate level index for the accident year under consideration.

We multiply the earned risk pure premiums by the on-level factors to put all premiums on the same adequacy level.

We are given loss cost trends with no division into frequency and severity components. We assume that the trends reflect loss severity, and that the claim frequency trend is not material. No adjustment is made to premiums for trend.

Cal/Acc Year	Earned Risk Pure Premium (000)	Premium On-Level Factor	Adjusted Earned Premium
1987	$40,000	1.0395	$41,580
1988	$44,000	0.9450	$41,580
1989	$40,000	1.0500	$42,000
1990	$45,000	1.0500	$47,250
1991	$50,000	1.0000	$50,000

31 claims are reported by December 31, 1991. We determine the total processed premium and the total unprocessed premium.

Cal/Acc Year	Estimated Report Lag	Processed Premium @ 12/31/91	Unprocessed Premium @ 12/31/91
1987	38.0%	$15,800.40	$25,779.60
1988	28.0	$11,642.40	$29,937.60
1989	18.0	$7,560.00	$34,440.00
1990	9.0	$4,252.50	$42,997.50
1991	2.0	$1,000.00	$49,000.00
Total		$40,255.30	$182,154.70

The claims expected to emerge in the future, Y, is computed as

$$\$40{,}255.30 : \$182{,}154.70 :: 31 : Y,$$
$$\text{or } Y = 140$$

The reserve indication is for five accident years only. For the oldest year in the experience period, only 38% of claims have been reported so far. We still expect much claim emergence for prior years. We are using a frequency-severity reserving procedure for the more recent accident years, where the reported claim severities are not credible. For previous years, we use other reserving techniques.

For accident years 1987 through 1991, the reserve indication has great uncertainty. From 31 claims that have been reported so far, we are estimating future emergence of 140 claims.

The volatility of the reported claim counts can be seen by a comparison of accident years 1987 and 1989. As of December 31, 1991, the processed adjusted premium for 1987 is $15,800 and 9 claims have been reported, while the processed adjusted premium for 1989 is $7,560 and 8 claims have been reported.

Loss Cost Trends

Let us revise the scenario to incorporate the loss cost trends. If we use the Stanard-Bühlmann technique to estimate dollars of losses, what are the adjusted premiums?

We form an index of relative loss costs, using 1987 as the base year. We ignore the loss trend from 1986 to 1987, since it affects all years equally. The index value for 1987 is unity, the index value for 1988 is 1.125, and so forth. The trend factor is the index value for the most recent year divided by the index value for the year under consideration.

Were we adjusting losses to the current level, we would multiply by these trend factors. Since we are adjusting premiums, we divide by these trend factors.

Period	Loss Trend	Index Value	Trend Factor
1986 to 1987	15.0%	1.0000	1.497
1987 to 1988	12.5	1.1250	1.331
1988 to 1989	10.0	1.2375	1.210
1989 to 1990	10.0	1.3613	1.100
1990 to 1991	10.0	1.4974	1.000

Calendar Year Emergence

The reserving actuary is often asked to show the expected emergence and payment of losses by development period (i.e., by calendar year) subsequent to the valuation date. The emergence and payment patterns have several uses.

1. *Reserving:* The expected loss emergence and loss payment in the next calendar period provides a check on the accuracy of the reserve indication. The reserve indication itself is difficult to judge, since the losses may not emerge or settle for many years. By comparing the actual emergence or settlement in the next calendar quarter or year with the estimates implied by the reserving procedure, the company gets a better feel for the accuracy and the bias inherent in the reserve estimate.

2. *Investments:* The expected emergence and settlement of claims is necessary for asset liability management. The insurer's investment department seeks expected liability cash flows in the coming months to optimize its investment strategy. Many insurers structure their investment portfolio in accordance with their insurance liabilities, selecting security types, fixed-income durations, and investment quality to best manage their overall risk. The reserving actuary provides the settlement patterns for the loss reserve portfolio.

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PRINCIPLES OF EMERGENCE

We have examined so far the future emergence of losses, or the bulk reserve, and the future payment of losses, or the total (case + bulk) reserve. We now consider the emergence or payment of losses by development period.

- The bulk reserve as of December 31, 20XX, equals the losses expected to emerge in calendar years 20XX+1 and subsequent for accident years 20XX and prior.
- The expected emergence in 20XX+1 equals the losses expected to emerge in calendar years 20XX+1 *only* for accident years 20XX and prior.

We illustration the method using the example directly above. We calculate the number of claims expected to emerge for accident years 1988 through 1991 during calendar year 1992.

We estimate the amount of adjusted premium that will be processed in 1992. For any accident year "X," the adjusted premium that will be processed in 1992 is the total adjusted premium for that accident year times the difference in the claim report lag between that accident year and the previous accident year.

Cal/Acc Year	Adjusted Earned Premium	Estimated Report Lag	Premium Processed in 1992 (000)
1987	$41,580	38.0%	—
1988	$41,580	28.0	$4,158
1989	$42,000	18.0	$4,200
1990	$47,250	9.0	$4,253
1991	$50,000	2.0	$3,500
1988-1991			$16,111

For instance, the 1988 adjusted premium that will be processed in 1992 equals

$$\$41{,}580 \times (38.0\% - 28.0\%) = \$4{,}158.$$

The total adjusted premium for accident years 1988 through 1991 that will be processed in 1992 equals $16,111. We form the standard equation as

$$\$40{,}255.30 : \$16{,}111 :: 31 : Y,$$
$$\text{or } Y = 12.4 \text{ claims.}$$

SUMMARY

The Stanard-Bühlmann reserving technique is a simple, intuitive procedure that combines the chain ladder loss development method with the expected loss method. It works well even in situations that don't lend themselves to easy estimates, such as reserving for high layers of loss. The Stanard-Bühlmann technique has been adopted by many reinsurance actuaries. This practitioners' guide should encourage its use by primary company actuaries as well.

Insurance Applications of Bivariate Distributions

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INSURANCE APPLICATIONS OF BIVARIATE DISTRIBUTIONS

DAVID L. HOMER AND DAVID R. CLARK

Abstract

A technique is demonstrated for aggregating bivariate claim size distributions using a two-dimensional Fast Fourier Transform. Three insurance applications are described in detail relating to: 1) individual risk rating, 2) loss and allocated expenses, and 3) Dynamic Financial Analysis.

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1. INTRODUCTION

1.1. The Basic Problem

When pricing insurance contracts it is useful to estimate not only the average insured loss but also the insured loss distribution. Although an initial approach may include only an estimate of the mean, risk measures generally require an estimate of the distribution. This problem is often solved by modeling losses as a sum of individual claims. A frequency distribution describes the number of claims N, a severity distribution describes the size of each claim X_k. The individual claim sizes are usually assumed to be independent and identically distributed (iid) as well as independent from the claim counts. This model is known as the *Collective Risk Model* [3]. The aggregate loss dollars Z are the sum of the individual claim sizes.

$$Z = X_1 + ... + X_N \tag{1.1}$$

The expectation and variance of Z are easily expressed in terms of the frequency and severity components.

$$E(Z) = E(X)E(N) \tag{1.2}$$

$$Var(Z) = Var(X)E(N) + E(X)^2 Var(N) \tag{1.3}$$

Estimating the aggregate loss distribution requires more work, but there are numerous techniques available: simulation, Fast Fourier Transform, continuous Fourier Transform [1], recursion [4, 8], and moment matching [5, 9]. In this paper, the Fast Fourier Transform (FFT) will be used. The FFT has been described in detail by Robertson [7] and Wang [10].

1.2. A Problem that Includes Dependencies Between Loss Components

The collective risk model as outlined above is sufficient to describe most insurance policies. One example in which this model is not sufficient arises in individual risk rating. A policy may provide Specific Excess coverage above a per-claim retention, and may also provide coverage in excess of an aggregate amount for the retained losses. The excess of aggregate cover is commonly called a *Stop Loss* cover.

The distributions for either the Specific Excess or Stop Loss covers can be estimated using the collective risk model. However, it is more difficult to estimate the distribution for the sum of the two covers because there is a dependence between the pieces. One trivial element of the dependence is easily seen—if there are no retained losses then there are no losses in excess of the retention.

Section 2 provides a more detailed description of this problem.

1.3. Aggregating with the FFT—A brief Review

Before introducing the complication of the dependence between two coverages, we will briefly review the Fast Fourier Transform (FFT) technique for evaluating a standard collective risk model.

In order to compute the aggregate loss distribution using the FFT, the severity distribution is expressed as a probability vector[1] $x = (x_0, x_1, ..., x_n)$. Each element x_k is the probability of a claim having size ck, where c is a scaling constant.

The distribution of the claim counts N is incorporated with the use of its Probability Generating Function (PGF).

$$PGF(t) = E(t^N) \tag{1.4}$$

The frequency and severity components are put together using a standard FFT technique. Denoting the FFT and its inverse as $FFT(x)$ and $IFFT(x)$ respectively, the probability vector for the aggregate losses is computed as

$$z = (z_0, z_1, ..., z_n) = IFFT(PGF(FFT(x))). \tag{1.5}$$

The PGF is applied *elementwise*, i.e., with some abuse of notation,

$$PGF((t_0, t_1, ..., t_n)) = (PGF(t_0), PGF(t_1), ..., PGF(t_n)). \tag{1.6}$$

[1] x is indexed starting at zero. x_0 is the probability of a claim of size zero.

The vector size n must be large enough that the probability of aggregate losses greater than cn is negligible. Any probability mass for losses greater than cn will wrap around, i.e., mass for losses greater than cn will be treated as though it is mass for the available claim sizes $(0, c, 2c, ..., nc)$. The wrap-around problem is typically avoided by padding the vector with zeros as discussed in Robertson [7] and Wang [10].

1.4. Building a Bivariate Loss Distribution

The goal is to obtain a bivariate distribution of aggregate retained losses and aggregate excess losses. This will be represented as a probability matrix[2] M_z where $M_z(j,k)$ is the probability that aggregate retained losses are $c_1 j$ and aggregate excess losses are $c_2 k$. As before, c_1 and c_2 are constant scale factors.

For a single claim this matrix is easily constructed. Suppose $x = (.4, .3, .3)$ and $c = 1,000$. Then for a 1,000 deductible, with $c_1 = c_2 = c = 1,000$,

$$M_x = \begin{bmatrix} .4 & 0 & 0 \\ .3 & .3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.7}$$

The matrix M_x fully specifies the probabilities and dependencies of losses in the retained and excess layers. The sum across rows (.4,.6,0) produces the distribution of the retained losses; the sum down the columns (.7,.3,0) produces the distribution of the excess losses.

The advantage at this point is that the same FFT technique can be used to calculate aggregate losses for M_x that we used to calculate aggregate losses for x. With $FFT()$ and $IFFT()$ now representing the two dimensional FFT and its inverse, and with $PGF()$ as before, we compute the aggregate loss matrix M_z.

$$M_z = IFFT(PGF(FFT(M_x))) \tag{1.8}$$

[2] M_z indices start from zero.

As in the one-dimensional treatment, the PGF is applied elementwise and the matrix M_x must have sufficient padding so that M_z can hold the significant mass. Appendix A provides an example of the two-dimensional FFT using publicly available software.

The FFT technique is not the only way to aggregate M_x. Sundt [8] shows that M_x can be aggregated using a recursive technique.

The aggregation of bivariate severity matrices can be applied to other problems as well. In what follows three specific examples will be explored. In the first, the combined distribution of losses on specific excess and aggregate excess is considered. In the second, bivariate loss and ALAE distributions are computed, and in the third example, a problem with a simulation technique often used in DFA analysis is reviewed and corrected.

2. PER-OCCURRENCE AND EXCESS-OF-AGGREGATE COVERS IN INDIVIDUAL RISK RATING

The first problem that we will review is common in individual risk rating.

A fictional large insured, Dietrichson Drilling, is interested in retaining the majority of their "predictable" Workers Compensation losses, and mainly seeks to purchase insurance to cover individual large claims. For example, they may choose to retain the first $600,000 of each loss occurrence. At the same time, they may have a concern that the number of occurrences could also be higher than expected, and therefore seek protection on the total dollars of retained loss.

Our company, Pacific All Risk Insurance Company, has been asked to provide coverage on a Per-Occurrence basis of $400,000 excess of $600,000, and then also a Stop Loss cover to pay in the event that their total retained loss exceeds $3,000,000. The underwriter at Pacific All Risk has proposed the structure shown in Table 2.1.

As the Pacific All Risk actuary, you have selected frequency and severity distributions, and have estimated the expected losses for each of these cov-

TABLE 2.1

POLICY STRUCTURE FOR DIETRICHSON DRILLING.

Named Insured:	Dietrichson Drilling
Insurance Company:	Pacific All Risk Insurance Co.
Per-Occurrence Layer:	400,000 xs 600,000
Stop Loss Layer:	5,000,000 xs 3,000,000
Allocated expenses included in the definition of "loss"	

erages. In order to calculate the needed risk load on the program, however, you need to estimate the distribution of the sum of the two coverages.

The company's Fast Fourier Transform (FFT) model allows you to estimate a distribution for either the Per-Occurrence or the Stop Loss layer with no problem, but you recognize that there is likely to be a strong dependence between the results of the two covers and you want to reflect this in your pricing.

We will consider a simplified version of this problem. First, we will assume that the loss distribution can be reasonably approximated using only a five-point discretized severity distribution. In practice, a curve of more than a hundred points would be needed in order to accurately capture the true shape. For our example, the simpler distribution shown in Table 2.2 will be used.

Consistent with this loss distribution, our average severity is estimated to be $480,000 and the average in the 400,000 xs 600,000 layer is $78,200.

We have also estimated that the expected number of claims is 5, with a variance of 6; and the frequency will be modeled using a Negative Binomial distribution. The overall loss pick is therefore $2,400,000(5 × $480,000). Our aggregate model calculates expected losses of $123,529 in the proposed Stop Loss layer above $3,000,000.

TABLE 2.2

SEVERITY DISTRIBUTION FOR DIETRICHSON DRILLING

Probability	Loss Amount	Excess Loss
0.00%	0	0
37.80%	200,000	0
23.50%	400,000	0
14.60%	600,000	0
9.10%	800,000	200,000
15.00%	1,000,000	400,000
Average	480,000	78,200

The first step in calculating the overall loss distribution is to create a bivariate severity distribution of primary and excess losses. This is shown in Table 2.3.

TABLE 2.3

SINGLE CLAIM PRIMARY & EXCESS LOSS BIVARIATE DISTRIBUTION

Primary 600,000	Loss Excess of 600,000					
	0	200,000	400,000	600,000	800,000	1,000,000
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
200,000	**37.80%**	0.00%	0.00%	0.00%	0.00%	0.00%
400,000	**23.50%**	0.00%	0.00%	0.00%	0.00%	0.00%
600,000	**14.60%**	**9.10%**	**15.00%**	0.00%	0.00%	0.00%
800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

From Table 2.3, we can observe a strong dependence structure between the primary and excess losses: we can only have an excess loss if the primary 600,000 retention is hit.

This bivariate severity matrix becomes the input for the FFT model, and may be denoted M_x. The matrix of the of the aggregate distribution may be denoted M_z and is produced using the two-dimensional Fast Fourier Transform calculation:

$$M_z = IFFT(PGF(FFT(M_x)))$$

$$PGF(t) = (1.2 - .2t)^{-25}$$

For the bivariate matrix M_x shown in Table 2.3, the resulting M_z is given in Table 2.4.

An additional step is needed in order to calculate the estimated results in the Stop Loss layer above 3,000,000. For that calculation, the rows of Table 2.4 for all amounts 3,000,000 or less are summed to compute the probabilities of no excess-of-aggregate losses. The remaining rows are intact but the row labels are reduced by 3,000,000. The result is Table 2.5.

From Table 2.5 several statistics of interest can be calculated. The expected loss to the Stop Loss layer is $123,529 and the probability that the Stop Loss is hit is 15.08%. The average loss amount conditional upon the Stop Loss being hit is $819,210.

More dramatic from a risk management perspective is the dependence between the Per-Occurrence and Stop Loss covers. The expected loss to the Per-Occurrence layer is $391,000 (5 × $78,200), but this increases to $830,334 when we include only the scenarios in which the Stop Loss is also hit. This dependence needs to be considered in the decision to write the contract: *on average, when the Stop Loss is hit we will also be paying about twice the expected amount in the Per-Occurrence layer.*

The two-dimensional matrix above can be used to verify the expected loss pricing for either coverage individually. The probabilities associated with the Stop Loss program are found by summing across rows; the probabilities associated with the Per-Occurrence excess layer are found by summing

TABLE 2.4
Aggregate Primary & Aggregate Excess Loss Bivariate Distribution

Primary 600,000	Loss Excess of 600,000						
	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	1.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
200,000	1.65%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
400,000	2.38%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
600,000	3.09%	0.40%	0.66%	0.00%	0.00%	0.00%	0.00%
800,000	3.34%	0.65%	1.07%	0.00%	0.00%	0.00%	0.00%
1,000,000	3.39%	0.96%	1.58%	0.00%	0.00%	0.00%	0.00%
1,200,000	3.22%	1.27%	2.16%	0.26%	0.21%	0.00%	0.00%
1,400,000	2.86%	1.40%	2.44%	0.44%	0.36%	0.00%	0.00%
1,600,000	2.43%	1.45%	2.59%	0.66%	0.54%	0.00%	0.00%
1,800,000	1.97%	1.40%	2.57%	0.90%	0.78%	0.09%	0.05%
2,000,000	1.54%	1.26%	2.38%	1.02%	0.92%	0.15%	0.08%
2,200,000	1.17%	1.09%	2.12%	1.08%	1.01%	0.24%	0.13%
2,400,000	0.86%	0.90%	1.80%	1.08%	1.05%	0.33%	0.20%
2,600,000	0.62%	0.72%	1.47%	1.00%	1.01%	0.38%	0.24%
2,800,000	0.43%	0.55%	1.16%	0.88%	0.93%	0.42%	0.27%
3,000,000	0.29%	0.41%	0.89%	0.75%	0.82%	0.43%	0.29%
3,200,000	0.20%	0.30%	0.66%	0.61%	0.70%	0.41%	0.29%
3,400,000	0.13%	0.21%	0.48%	0.48%	0.57%	0.37%	0.28%
3,600,000	0.08%	0.15%	0.34%	0.37%	0.45%	0.32%	0.25%
3,800,000	0.05%	0.10%	0.24%	0.27%	0.35%	0.27%	0.22%
4,000,000	0.03%	0.07%	0.16%	0.20%	0.26%	0.22%	0.19%
4,200,000	0.02%	0.04%	0.11%	0.14%	0.19%	0.17%	0.15%
4,400,000	0.01%	0.03%	0.07%	0.10%	0.14%	0.13%	0.12%
4,600,000	0.01%	0.02%	0.05%	0.07%	0.10%	0.10%	0.09%
4,800,000	0.00%	0.01%	0.03%	0.04%	0.07%	0.07%	0.07%
5,000,000	0.00%	0.01%	0.02%	0.03%	0.04%	0.05%	0.05%
5,200,000	0.00%	0.00%	0.01%	0.02%	0.03%	0.03%	0.04%
5,400,000	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%	0.03%
5,600,000	0.00%	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%
5,800,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
6,000,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
6,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
6,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
8,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

TABLE 2.5

AGGREGATE PRIMARY EXCESS & AGGREGATE EXCESS LOSS BIVARIATE DISTRIBUTION

Stop Loss	Loss Excess of 600,000						
	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	30.28%	12.45%	22.89%	8.07%	7.65%	2.04%	1.26%
200,000	0.20%	0.30%	0.66%	0.61%	0.70%	0.41%	0.29%
400,000	0.13%	0.21%	0.48%	0.48%	0.57%	0.37%	0.28%
600,000	0.08%	0.15%	0.34%	0.37%	0.45%	0.32%	0.25%
800,000	0.05%	0.10%	0.24%	0.27%	0.35%	0.27%	0.22%
1,000,000	0.03%	0.07%	0.16%	0.20%	0.26%	0.22%	0.19%
1,200,000	0.02%	0.04%	0.11%	0.14%	0.19%	0.17%	0.15%
1,400,000	0.01%	0.03%	0.07%	0.10%	0.14%	0.13%	0.12%
1,600,000	0.01%	0.02%	0.05%	0.07%	0.10%	0.10%	0.09%
1,800,000	0.00%	0.01%	0.03%	0.04%	0.07%	0.07%	0.07%
2,000,000	0.00%	0.01%	0.02%	0.03%	0.04%	0.05%	0.05%
2,200,000	0.00%	0.00%	0.01%	0.02%	0.03%	0.03%	0.04%
2,400,000	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%	0.03%
2,600,000	0.00%	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%
2,800,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
3,000,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
3,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
3,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

down columns. By summing across rows or down columns, we calculate the marginal distributions.

In order to calculate the distribution of the sum of the two coverages combined, we sum the probabilities along each diagonal. Table 2.6 shows this calculation.

TABLE 2.6

PROBABILITIES FOR AGGREGATE PRIMARY EXCESS PLUS AGGREGATE EXCESS LOSS

Loss & ALAE	Probability	Calculation
0	30.28%	= 30.28%
200,000	12.64%	= 0.20% + 12.45%
400,000	23.31%	= 0.13% + 0.30% + 22.89%
600,000	9.02%	= 0.08% + 0.21% + 0.66% + 8.07%
800,000	8.94%	= 0.05% + 0.15% + 0.48% + 0.61% + 7.65%
⋮	⋮	⋮

3. DISTRIBUTION FOR LOSS ONLY SUBJECT TO AGGREGATE LIMIT PLUS UNLIMITED ALLOCATED LOSS ADJUSTMENT EXPENSE (ALAE)

Our insured, Dietrichson Drilling, requests a General Liability policy on a traditional guaranteed cost basis. Our company, Pacific All Risk Insurance Company, is willing to offer a standard policy form with a $1,000,000 Per-Occurrence limit and a $2,000,000 General Policy Aggregate.

Both the Per-Occurrence limit and the General Aggregate limit apply to the indemnity loss only. All defense costs and associated expenses (allocated loss adjustment expense - ALAE) are covered in addition to these limits. The Pacific All Risk policy is summarized in Table 3.1. The loss distribution is approximated in Table 3.2.

TABLE 3.1

POLICY STRUCTURE FOR DIETRICHSON DRILLING.

Named Insured:	Dietrichson Drilling
Insurance Company:	Pacific All Risk Insurance Co.
Per-Occurrence Limit:	1,000,000
General Aggregate Limit:	2,000,000
Allocated expenses paid in addition to loss	

TABLE 3.2

SEVERITY DISTRIBUTION FOR DIETRICHSON DRILLING

Probability	Loss Amount
10.00%	0
45.00%	200,000
9.00%	400,000
9.00%	600,000
9.00%	800,000
18.00%	1,000,000
Average	432,000
Average ALAE %	37.29%

As the Pacific All Risk actuary, you have been asked to estimate the aggregate distribution of the sum of the loss and ALAE combined. The first step in calculating the overall loss distribution is to assemble the bivariate severity distribution of loss and ALAE. This is shown in Table 3.3.

TABLE 3.3

SINGLE CLAIM LOSS & ALAE BIVARIATE DISTRIBUTION

Loss Amount	ALAE						
	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	8.39%	1.47%	0.13%	0.01%	0.00%	0.00%	0.00%
200,000	27.98%	13.29%	3.16%	0.50%	0.06%	0.01%	0.00%
400,000	4.15%	3.21%	1.25%	0.32%	0.06%	0.01%	0.00%
600,000	3.07%	3.30%	1.77%	0.64%	0.17%	0.04%	0.01%
800,000	2.28%	3.13%	2.15%	0.99%	0.34%	0.09%	0.02%
1,000,000	3.37%	5.65%	4.73%	2.64%	1.11%	0.37%	0.10%
1,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

It may not be obvious at a quick glance, but there is a strong dependence between loss and ALAE in this table; approximately a .500 linear correlation coefficient for Dietrichson Drilling. Larger losses are assumed to have larger dollars of associated expense, even though the ALAE percent decreases. It is also the case that the loss severity curve does not extend beyond the 1,000,000 Per-Occurrence limit, whereas the ALAE curve does not have an explicit cap. By convention, we are including closed-without-pay claims in this analysis, at least to the extent that they are likely to have contributed ALAE.

This bivariate severity matrix becomes the input for the FFT model, and will again be denoted as M_x. The matrix of aggregate distributions, M_z, is again given by the formula:

$$M_z = IFFT(PGF(FFT(M_x)))$$

$$PGF(t) = (2-t)^{-4}$$

The frequency distribution is assumed to be Negative Binomial, with a mean of 4 and a variance of 8.

The final matrix of aggregate distributions is shown in Table 3.4. In order

TABLE 3.4

AGGREGATE LOSS & AGGREGATE ALLOCATED LOSS ADJUSTMENT EXPENSE JOINT DISTRIBUTION

Loss Amount	ALAE						
	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	7.42%	0.23%	0.02%	0.00%	0.00%	0.00%	0.00%
200,000	4.33%	2.23%	0.59%	0.11%	0.02%	0.00%	0.00%
400,000	2.22%	2.10%	1.01%	0.33%	0.08%	0.02%	0.00%
600,000	1.41%	1.82%	1.20%	0.54%	0.18%	0.05%	0.01%
800,000	1.06%	1.71%	1.40%	0.77%	0.33%	0.11%	0.03%
1,000,000	1.11%	2.09%	1.98%	1.27%	0.62%	0.25%	0.08%
1,200,000	0.69%	1.59%	1.84%	1.42%	0.83%	0.39%	0.16%
1,400,000	0.40%	1.07%	1.45%	1.32%	0.90%	0.50%	0.23%
1,600,000	0.25%	0.75%	1.15%	1.19%	0.92%	0.57%	0.30%
1,800,000	0.16%	0.56%	0.95%	1.08%	0.93%	0.64%	0.37%
2,000,000	0.12%	0.44%	0.82%	1.02%	0.96%	0.72%	0.46%
2,200,000	0.07%	0.30%	0.61%	0.84%	0.87%	0.72%	0.50%
2,400,000	0.04%	0.19%	0.43%	0.65%	0.73%	0.66%	0.50%
2,600,000	0.03%	0.13%	0.31%	0.50%	0.60%	0.59%	0.48%
2,800,000	0.02%	0.09%	0.22%	0.38%	0.50%	0.52%	0.45%
3,000,000	0.01%	0.06%	0.16%	0.30%	0.41%	0.46%	0.42%
3,200,000	0.01%	0.04%	0.11%	0.22%	0.32%	0.38%	0.37%
3,400,000	0.00%	0.02%	0.07%	0.15%	0.24%	0.30%	0.32%
3,600,000	0.00%	0.01%	0.05%	0.11%	0.18%	0.24%	0.27%
3,800,000	0.00%	0.01%	0.03%	0.08%	0.13%	0.19%	0.22%
4,000,000	0.00%	0.01%	0.02%	0.05%	0.10%	0.14%	0.18%

to cap the loss only exposure at the 2,000,000 General Aggregate, we sum the probabilities for losses above 2,000,000 into a single row. The result is Table 3.5. Finally, we can create a single distribution from this matrix by

TABLE 3.5

AGGREGATE LOSS CAPPED AT 2,000,000 & AGGREGATE ALLOCATED LOSS ADJUSTMENT EXPENSE JOINT DISTRIBUTION

Loss Amount	ALAE						
	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	7.42%	0.23%	0.02%	0.00%	0.00%	0.00%	0.00%
200,000	4.33%	2.23%	0.59%	0.11%	0.02%	0.00%	0.00%
400,000	2.22%	2.10%	1.01%	0.33%	0.08%	0.02%	0.00%
600,000	1.41%	1.82%	1.20%	0.54%	0.18%	0.05%	0.01%
800,000	1.06%	1.71%	1.40%	0.77%	0.33%	0.11%	0.03%
1,000,000	1.11%	2.09%	1.98%	1.27%	0.62%	0.25%	0.08%
1,200,000	0.69%	1.59%	1.84%	1.42%	0.83%	0.39%	0.16%
1,400,000	0.40%	1.07%	1.45%	1.32%	0.90%	0.50%	0.23%
1,600,000	0.25%	0.75%	1.15%	1.19%	0.92%	0.57%	0.30%
1,800,000	0.16%	0.56%	0.95%	1.08%	0.93%	0.64%	0.37%
2,000,000	0.31%	1.31%	2.88%	4.41%	5.28%	5.31%	4.73%
2,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

TABLE 3.6

PROBABILITIES FOR LIMITED LOSS PLUS ALAE

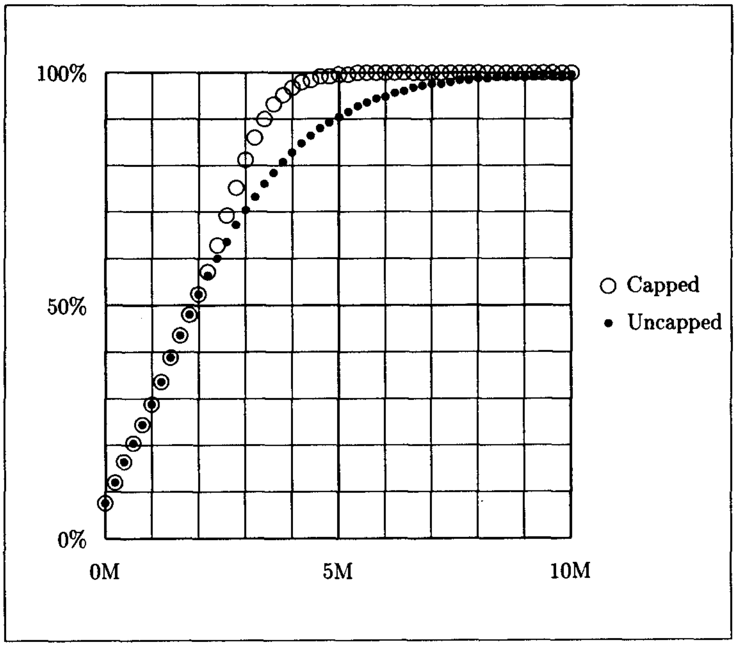
Combined Loss+ALAE	Probability	Calculation
0	7.42%	= 7.42%
200,000	4.56%	= 4.33% + 0.23%
400,000	4.47%	= 2.22% + 2.23% + 0.02%
600,000	4.09%	= 1.41% + 2.10% + 0.59% + 0.00%
800,000	3.99%	= 1.06% + 1.82% + 1.01% + 0.11% + 0.00%
⋮	⋮	⋮

summing along each diagonal to obtain Table 3.6

It is also instructive to show a graph of the distribution of the combined loss and ALAE both before and after the General Aggregate cap. In Graph 3.1 we can see that the "tail" of the cumulative distribution is greatly reduced by imposing a 2,000,000 General Aggregate. However, we note that there is still a non-remote probability of loss even above 3,000,000 due to the inclusion of ALAE on an unlimited basis.

GRAPH 3.1

CUMULATIVE DISTRIBUTION FUNCTIONS FOR CAPPED AND UNCAPPED LOSS & ALAE



4. DYNAMIC FINANCIAL ANALYSIS

As the actuary for Pacific All Risk, you have now completed your pricing work for individual insurance contracts. As a reward for your hard work, you have been rotated to the actuarial team that runs the company's Dynamic Financial Analysis (DFA) model, called Pacific Enterprise Risk Model (PERM).

The goal of the PERM team is to model the distribution of results for Pacific All Risk Insurance Company as a whole. Included in this analysis is sensitivity testing for interest rates and various complex reinsurance structures. The PERM is a giant simulation model, which needs to be parameterized for the business actually written.

A simplification made in the PERM is that the model separately simulates an aggregate value for all "small" losses and then simulates individual "large" losses. A truncation point of 1,000,000 has been selected for segregating large from small losses.

An early version of the PERM made the assumption that the small and large losses are independent. That is, the small and large losses were simulated separately and then the results were summed. However, this independence assumption was found to be false, resulting in understated variability and unrealistically low probabilities in the tail of the combined distribution.

In fact, the aggregate distributions of the small and large losses are generally not independent. If a single frequency distribution is used to generate the overall number of losses N, then the covariance[3] can be written explicity.

$$Cov(S,L) = p\mu_S(1-p)\mu_L(\sigma_N^2 - \mu_N) \tag{4.1}$$

Where,

S = aggregate small losses

L = aggregate large losses

[3] Sundt shows a more general formula in [8].

μ_S = mean small claim size,

μ_L = mean large claim size,

p = probability that a given claim is small,

σ_N^2 = variance of the claim counts, and

μ_N = mean of the claim counts.

The sign of the covariance term is driven by the claim count distribution. For the commonly used Negative Binomial this is positive; for the Poisson it is zero[4]. Equation 4.1 is derived in Appendix B.

In order to model the losses for Pacific All Risk, we begin by approximating the total loss distribution with a few discrete points (Table 4.1). As in the previous examples, a five-point distribution is used here, but would need to be expanded to a greater number of points in a more realistic application.

TABLE 4.1

SEVERITY DISTRIBUTION

Probability	Loss Amount
0.00%	0
43.80%	200,000
24.60%	400,000
13.80%	600,000
7.80%	800,000
10.00%	1,000,000
Average	431,200

This single severity curve is then reconfigured into Table 4.2 a bivariate matrix M_x. The first column defines the severity of the "small" loss distribution. The first row is a single point containing the probability of a "large"

[4] In the case of the Poisson it can be shown that the large and small claims are actually independent.

loss.

This format is a bit different than the previous examples, since the vertical and horizontal axes are in different units: the vertical in dollars and the horizontal in counts. This illustrates the flexibility in the FFT technique to allow for different scale factors for the two dimensions.

TABLE 4.2

SINGLE CLAIM SMALL LOSS & LARGE COUNTS JOINT DISTRIBUTION

Small Loss	Large Loss Counts					
	0	1	2	3	4	5
0	0.00%	**10.00%**	0.00%	0.00%	0.00%	0.00%
200,000	**43.80%**	0.00%	0.00%	0.00%	0.00%	0.00%
400,000	**24.60%**	0.00%	0.00%	0.00%	0.00%	0.00%
600,000	**13.80%**	0.00%	0.00%	0.00%	0.00%	0.00%
800,000	**7.80%**	0.00%	0.00%	0.00%	0.00%	0.00%
1,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

For a frequency distribution, we use a Negative Binomial with mean 10 and variance 20. For an actual insurance company, the overall frequency is likely to be much higher but we continue with this simplified assumption for clarity. The aggregate distribution matrix M_z is again given by the expression:

$$M_z = IFFT(PGF(FFT(M_x)))$$

$$PGF(t) = (2 - t)^{-10}$$

The resulting aggregate distribution matrix M_z is in Table 4.3. Like the

TABLE 4.3

AGGREGATE CLAIM SMALL LOSS & LARGE COUNTS BIVARIATE DISTRIBUTION

Small Loss	Large Loss Counts						
	0	1	2	3	4	5	6
0	0.10%	0.05%	0.01%	0.00%	0.00%	0.00%	0.00%
200,000	0.21%	0.12%	0.04%	0.01%	0.00%	0.00%	0.00%
400,000	0.38%	0.22%	0.07%	0.02%	0.00%	0.00%	0.00%
600,000	0.58%	0.36%	0.12%	0.03%	0.01%	0.00%	0.00%
800,000	0.82%	0.53%	0.18%	0.05%	0.01%	0.00%	0.00%
1,000,000	1.07%	0.71%	0.26%	0.07%	0.01%	0.00%	0.00%
1,200,000	1.31%	0.91%	0.34%	0.09%	0.02%	0.00%	0.00%
1,400,000	1.54%	1.11%	0.43%	0.12%	0.03%	0.00%	0.00%
1,600,000	1.74%	1.30%	0.52%	0.15%	0.03%	0.01%	0.00%
1,800,000	1.90%	1.46%	0.60%	0.18%	0.04%	0.01%	0.00%
2,000,000	2.02%	1.60%	0.68%	0.20%	0.05%	0.01%	0.00%
2,200,000	2.09%	1.71%	0.75%	0.23%	0.06%	0.01%	0.00%
2,400,000	2.12%	1.78%	0.80%	0.25%	0.06%	0.01%	0.00%
2,600,000	2.11%	1.82%	0.84%	0.27%	0.07%	0.02%	0.00%
2,800,000	2.06%	1.83%	0.86%	0.29%	0.08%	0.02%	0.00%
3,000,000	1.98%	1.81%	0.87%	0.30%	0.08%	0.02%	0.00%
3,200,000	1.88%	1.75%	0.87%	0.30%	0.08%	0.02%	0.00%
3,400,000	1.76%	1.68%	0.85%	0.30%	0.09%	0.02%	0.00%
3,600,000	1.62%	1.59%	0.82%	0.30%	0.09%	0.02%	0.00%
3,800,000	1.48%	1.49%	0.79%	0.29%	0.09%	0.02%	0.00%
4,000,000	1.34%	1.38%	0.74%	0.28%	0.08%	0.02%	0.00%
4,200,000	1.20%	1.26%	0.70%	0.27%	0.08%	0.02%	0.00%
4,400,000	1.06%	1.14%	0.64%	0.25%	0.08%	0.02%	0.00%
4,600,000	0.94%	1.03%	0.59%	0.24%	0.08%	0.02%	0.00%
4,800,000	0.82%	0.91%	0.54%	0.22%	0.07%	0.02%	0.00%
5,000,000	0.71%	0.81%	0.48%	0.20%	0.07%	0.02%	0.00%

original bivariate severity, this matrix has units in dollars for the "small" losses, and counts for the "large" losses. The marginal distribution for the aggregate small losses is found by summing the probabilities in each row.

The simulation procedure first simulates an aggregate amount for the "small" losses, and then finds a conditional frequency distribution for the "large" loss counts. The conditional large loss frequency distributions are created by rescaling each row of M_z to total 100%. This is shown in Table 4.4.

The conditional matrix shown in Table 4.4 is also instructive in itself, because it clearly shows the dependence between large and small losses. Simply put, an increase in frequency means more losses in both the large and small categories.

The final simulation procedure for the PERM is then:

- Simulate the aggregate dollars of small losses out of its marginal distribution.
- Simulate the number of large losses from the corresponding conditional frequency distribution.
- Simulate a severity amount for each of the large losses.

This procedure allows us to efficiently simulate losses without the need to individually simulate every small loss, and at the same time preserves the dependence structure between the large and small losses.

TABLE 4.4

CONDITIONAL DISTRIBUTIONS OF LARGE COUNTS GIVEN AGGREGATE SMALL LOSSES

Small Loss	Large Loss Counts						
	0	1	2	3	4	5	6
0	59.87%	29.94%	8.23%	1.65%	0.27%	0.04%	0.00%
200,000	56.88%	31.28%	9.39%	2.03%	0.36%	0.05%	0.01%
400,000	54.91%	32.07%	10.18%	2.33%	0.43%	0.07%	0.01%
600,000	53.26%	32.68%	10.87%	2.60%	0.50%	0.08%	0.01%
800,000	51.80%	33.17%	11.49%	2.85%	0.57%	0.10%	0.01%
1,000,000	50.37%	33.62%	12.11%	3.12%	0.64%	0.11%	0.02%
1,200,000	49.03%	34.01%	12.70%	3.39%	0.72%	0.13%	0.02%
1,400,000	47.77%	34.34%	13.26%	3.65%	0.80%	0.15%	0.02%
1,600,000	46.55%	34.63%	13.81%	3.92%	0.89%	0.17%	0.03%
1,800,000	45.38%	34.87%	14.34%	4.19%	0.97%	0.19%	0.03%
2,000,000	44.26%	35.09%	14.86%	4.47%	1.07%	0.22%	0.04%
2,200,000	43.17%	35.26%	15.37%	4.74%	1.16%	0.24%	0.04%
2,400,000	42.12%	35.41%	15.86%	5.02%	1.26%	0.27%	0.05%
2,600,000	41.10%	35.53%	16.34%	5.31%	1.37%	0.30%	0.06%
2,800,000	40.11%	35.62%	16.80%	5.59%	1.47%	0.33%	0.06%
3,000,000	39.16%	35.69%	17.25%	5.88%	1.58%	0.36%	0.07%
3,200,000	38.22%	35.73%	17.69%	6.17%	1.70%	0.39%	0.08%
3,400,000	37.32%	35.75%	18.12%	6.46%	1.82%	0.43%	0.09%
3,600,000	36.44%	35.75%	18.54%	6.75%	1.94%	0.47%	0.10%
3,800,000	35.58%	35.73%	18.94%	7.05%	2.07%	0.51%	0.11%
4,000,000	34.74%	35.69%	19.33%	7.34%	2.19%	0.55%	0.12%
4,200,000	33.93%	35.63%	19.71%	7.64%	2.33%	0.59%	0.13%
4,400,000	33.14%	35.56%	20.08%	7.94%	2.46%	0.64%	0.14%
4,600,000	32.37%	35.47%	20.44%	8.24%	2.61%	0.69%	0.16%
4,800,000	31.61%	35.36%	20.78%	8.54%	2.75%	0.74%	0.17%
5,000,000	30.88%	35.25%	21.12%	8.83%	2.90%	0.79%	0.19%

5. CONCLUSION

Aggregating a bivariate severity distribution is a useful technique. Two severity components are separately aggregated while preserving their dependence structure. This technique can be applied when pricing a policy with a per claim retention and a Stop Loss on the aggregate retention. It can also be applied more generally. The two random variables can be different items such as dollars and counts.

In this paper we aggregate the bivariate distribution using the FFT, but it is possible to do this with the continuous Fourier Transform or simulation. Sundt [8] shows that this can be done with recursive techniques. It may sometimes be preferable to utilize a mix of techniques.

This technique can be extended to n dimensions by developing a multivariate distribution M_x. With the claim count PGF and an n-dimensional FFT, the aggregate multivariate array M_z is obtained as,

$$M_z = IFFT(PGF(FFT(M_x))).$$

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APPENDIX A SAMPLE TWO DIMENSIONAL FAST FOURIER TRANSFORM USING R

It is convenient to compute FFT's using preprogrammed software. An excellent piece of software that includes FFT functions is based on the S language and is publicly available for free. It is called "R" [2]. Versions of R for various operating systems can be found by following 'http://cran.r-project.org/'. R is copyrighted software made publicly available under the GNU General Public License which is available at 'http://www.gnu.org/copyleft/gpl.html'. The FFT function is also available in commercial software packages, e.g., MATLAB and S-Plus.

A listing from a session with R shows how easy it is to compute two dimensional FFTs. Lines typed by the user begin with ">". The inverse of a matrix `M` is obtained with "`fft(M,T)/n`," where n is the number of elements in the matrix.

```
> ms<-matrix(c(.4,0,0,.3,.3,0,0,0,0),3,3,byrow=T)
> ms
     [,1] [,2] [,3]
[1,]  0.4  0.0    0
[2,]  0.3  0.3    0
[3,]  0.0  0.0    0
> f<-fft(ms)
> f
                [,1]            [,2]            [,3]
[1,] 1.0+0.0000000i 0.55-0.2598076i 0.55+0.2598076i
[2,] 0.1-0.5196152i 0.10+0.0000000i 0.55-0.2598076i
[3,] 0.1+0.5196152i 0.55+0.2598076i 0.10+0.0000000i
> f*f
                 [,1]             [,2]             [,3]
[1,]  1.00+0.0000000i 0.235-0.2857884i 0.235+0.2857884i
[2,] -0.26-0.1039230i 0.010+0.0000000i 0.235-0.2857884i
```

```
[3,] -0.26+0.1039230i 0.235+0.2857884i 0.010+0.0000000i
> ma<-fft(f*f,T)/9
> ma
        [,1]             [,2]             [,3]
[1,] 0.16+0i 1.652685e-18+0i 2.301894e-17+0i
[2,] 0.24+0i 2.400000e-01+0i 2.467162e-17+0i
[3,] 0.09+0i 1.800000e-01+0i 9.000000e-02+0i
```

For those wishing to program their own algorithms see [6]. Note that when the object to be transformed consists only of real numbers, there are symmetries that can be used to decrease the amount of computing required.

APPENDIX B CORRELATION OF LARGE AND SMALL LOSSES

Consider the Collective Risk Model with aggregate losses represented by the sum of individual claims

$$Z = X_1 + ... + X_N.$$

The X_i are independent and identically distributed (iid) random variables denoting claim sizes. Claim counts are denoted by the random variable N which is independent from each X_i. It is further assumed that the first moment of X_i is finite and that the second moment of N is finite.

Let T denote the threshold for distinguishing between small claims and large claims, i.e., X_i is small if $X_i \leq T$. Define a small loss indicator, $I_i = 1$ for $X_i \leq T$, and 0 otherwise. Then we have small aggregate losses

$$Z_S = X_1 I_1 + ... + X_N I_N$$

and large aggregate losses

$$Z_L = X_1(1 - I_1) + ... + X_N(1 - I_N).$$

Let p be the probability that $X_i \leq T$. Denote the conditional means for small and large claim sizes with

$$\mu_S = E[X_i | X_i \leq T]$$

$$\mu_L = E[X_i | X_i > T].$$

Denote the claim count mean and variance with

$$\sigma_N^2 = Var[N]$$

$$\mu_N = E[N].$$

Proposition

$$Cov[Z_S, Z_L] = p\mu_S(1 - p)\mu_L(\sigma_N^2 - \mu_N). \tag{B.1}$$

Proof

$$\begin{aligned}
E[Z_S Z_L] &= E[(\sum_{i=1}^{N} X_i I_i)(\sum_{i=1}^{N} X_i(1-I_i))] \\
&= E_N E_X[(\sum_{j=i}^{N} X_i I_i)(\sum_{j=i}^{N} X_i(1-I_i))] \\
&= E_N E_X[(\sum_{i=j} X_i I_i X_j(1-I_j)) + (\sum_{j\neq i} X_i I_i X_j(1-I_j))] \\
&= E_N[N(N-1)E_X[XI]E_X[X(1-I)]], \text{ since } I_i(1-I_j)=0 \text{ for } i=j \\
&= E_N[N(N-1)\mu_S p \mu_L(1-p)] \\
&= (E(N^2)-\mu_N)\mu_S p \mu_L(1-p).
\end{aligned}$$

$$E[Z_L]E[Z_S] = (\mu_N \mu_S p)(\mu_N \mu_L(1-p)) = \mu_N^2 \mu_S p \mu_L(1-p).$$

These yield the result since,

$$Cov[Z_S, Z_L] = E[Z_S Z_L] - E[Z_S]E[Z_L].$$