CASUALTY ACTUARIAL SOCIETY FORUM

Fall 2001
Including the Reserves Call Papers

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NOTICE

The Casualty Actuarial Society *Forum*
Fall 2001 Edition
Including the Reserves Call Papers

To CAS Members:

This is the Fall 2001 Edition of the Casualty Actuarial Society *Forum*. It contains ten Reserves Call Papers, one committee report, and five additional papers.

The Casualty Actuarial Society *Forum* is a nonrefereed journal printed by the Casualty Actuarial Society. The viewpoints published herein do not necessarily reflect those of the Casualty Actuarial Society.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership. Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*:

1. Authors should submit a camera-ready original paper and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera-ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. Text and exhibits should be in solid black and white.
5. Authors should submit an electronic file of their paper using a popular word processing software (e.g., Microsoft Word, WordPerfect) for inclusion on the CAS Web Site.

The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted. The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Dennis L. Lange

Dennis L. Lange, CAS *Forum* Chairperson

The 2001 CAS Reserves Call Papers
Presented at the
2001 Casualty Loss Reserve Seminar
September 10-11, 2001
The Fairmont New Orleans
New Orleans, Louisiana

The Fall 2001 Edition of the CAS *Forum* is a cooperative effort of the CAS *Forum* Committee and the CAS Committee on Reserves. This edition of the *Forum* presents the 2001 Reserves Call Paper Program conducted by the Committee on Reserves.

The CAS Committee on Reserves presents for discussion ten papers prepared in response to its Call for 2001 Reserves Papers.

This *Forum* includes papers that will be discussed by the authors at the 2001 Casualty Loss Reserve Seminar, September 10-11, in New Orleans, Louisiana.

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The Value of Interacting with the Claim Department

Robert F. Conger, FCAS, MAAA, FCIA, and
Robert L. Grove, CPCU

THE VALUE OF INTERACTING WITH THE CLAIM DEPARTMENT

Submitted by:

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ABSTRACT

Casualty actuaries have long recognized that changes in claims patterns can create distortions in loss projections and loss reserve estimates. Various actuarial methods are used to detect, mitigate and adjust for (or avoid) these distortions. The actuarial literature provides considerable guidance and numerous techniques in this regard, and this paper does not re-cover this ground.

This paper describes and illustrates important benefits of regular and ongoing interaction between casualty actuaries and Claim Department personnel, and emphasizes that this is a two-way street.

- Qualitative and quantitative input from the Claim Department can be critical in helping the actuary understand, appropriately interpret, and even anticipate changes that affect the actuarial data and actuarial projections.

- The actuary's work, in turn, can serve as an effective diagnostic to identify potential macroscopic changes in the claims arena — including mix changes, reporting patterns, claim management issues, case reserving changes, and closure/settlement patterns. With these diagnostics in hand, the actuary plays a key role in the early identification, communication, analysis and resolution of unwanted, unintended, or unrecognized claim changes that may have important business consequences extending well beyond the Actuarial Department.

The message here is that the actuary must be an active — and interactive — part of the management team. Input ***from*** the Claims Department is arguably a necessary ingredient to the actuary's work. But when the actuary provides insight ***to*** the Claims Department, the actuary can add value to the entire organization.

INTRODUCTION

Recently, an actuary friend emerged from his annual medical checkup with a puzzled look. He had been in the doctor's office for three hours, so I became concerned that perhaps the news had been bad. "No," he said pensively in response to my inquiry, "but after three hours of poking, prodding, and running diagnostic tests, all my doctor told me was that my health was adequate for someone of my age and lifestyle. The doctor hardly asked me any questions. And, I certainly expected a bit more feedback -- some indication of changes in my body's performance, and some commentary on the positive and negative aspects of my diet, exercise, and other lifestyle choices." My friend shook his head in disappointment at the minimal value he had received from his checkup. But, being a busy consultant, he set his disappointment aside and hurried back to the office to issue a one page opinion that his client's loss reserves are adequate.

In many ways, an actuarial loss reserve review is analogous to an annual physical exam. The casualty actuary collects lots of quantitative information, runs various diagnostic tests, and reaches some conclusions that often are boiled down to a message that the insurance company's loss reserves are "adequate." Too little communication occurs at several stages.

This paper focuses on the potential value of ongoing communications between "the Actuary" and the Claims Department (referred to as "Claims" for convenience):

1. ***To support the Actuary's reserve analysis.*** Interviews with Claims can indicate operational or mix changes that affect the data used by the Actuary, and therefore may affect the Actuary's choice of analytical methods or parameters. Casualty

actuaries have long recognized the need to address these types of changes, and the literature provides various methodologies that are useful when such changes have occurred or are occurring.

2. ***Providing diagnostic feedback following the actuarial analysis.*** The actuary's results may help confirm, rebut or quantify some changes tentatively identified by Claims, or may reveal additional critical issues that are not yet understood by Claims and/or by senior management. Yet, too few actuaries highlight or communicate these insights adequately. As a result, like my friend's doctor, too few actuaries are extracting and delivering the full (even if indirect) value of their work.

3. ***Creating tools for Claims.*** In addition to providing "big-picture" feedback from specific analyses, the Actuary's access to data and tools may allow the Actuary to create or calibrate some modeling tools that help Claims perform its work on individual claims more efficiently or more effectively day-to-day. We have seen relatively few actuaries contributing in this arena.

The remainder of this paper is divided into three sections corresponding to these three different forums for actuarial communication with Claims. This paper uses brief case studies to illustrate the value of actuarial involvement and communication in each of these three forums. While these case studies are derived in various ways from real-life experience, we have modified, simplified, and combined real experiences in describing these cases. Primarily, we exercised these liberties in order to make our intended points clearer. For example, all of the numbers in the examples are well-behaved; real life, of course, is not always so well-behaved. In addition, the resulting case studies do not relate to any specific insurance company or actuary; any resemblance to a real company or actuary is purely coincidental.

ACTUARIAL COMMUNICATIONS WITH CLAIMS – TO SUPPORT THE ACTUARY'S RESERVE ANALYSIS

The most basic of actuarial reserving methods generally assume, explicitly or implicitly, a consistency over time and across market segments of claim reporting and recording; claim count definitions; claim handling; case reserving philosophies and methods; mixes of claims; coverages, limits, and deductibles; and/or payment and closure speed. These (and other) factors may fundamentally affect the behavior of claims data and therefore the actuary's understanding, analysis, and interpretation of the data. The actuarial literature provides ample discussion of basic methods that perform predictably and appropriately in a stable environment (see *Bibliography* at end of paper). Of course, the world rarely behaves in an entirely consistent manner, and these consistency assumptions often are violated in the real world and in real insurance companies.

The Casualty Actuarial Society *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves* states that "understanding the trends and changes affecting the data base is a prerequisite to the application of actuarially sound reserving methods. A knowledge of changes in ... claims handling ... affecting the experience is essential to the accurate interpretation and evaluation of observed data and the choice of reserving methods....[R]eorganization of claims responsibility or changes in claims handling practices ... are examples of operational changes that can affect the continuity of the loss experience. The computation of the reserves should reflect the impact of such changes."

In the United States, the Actuarial Standards Board's (ASB) Actuarial Standard of Practice No. 36 (section 3.5.2) provides similar direction:

> Changing Conditions – The actuary should consider the likely effect of changing conditions on the subject loss and loss adjustment expense reserves. The actuary should consider whether there have been significant changes in conditions particularly with regard to claims, losses, or exposures that are new or unusual and that are likely to be insufficiently reflected in the experience data or in the assumptions used to estimate loss and loss adjustment expense reserves. Changing conditions can arise from circumstances particular to the entity or from external factors affecting others within an industry.
>
> The actuary should also consider the relevant characteristics of the entity's exposures to the extent that they are likely to have a material effect on the results of the actuary's reserve analysis. ... The actuary should obtain information from the entity regarding the significant changes in the practices or philosophy used by the entity's claims personnel and ascertain whether such changes are likely to have a material effect on the results of the actuary's reserve analysis or on the risks and uncertainties associated with the reserves.

Comparable standards are in place in many other jurisdictions. For example, The Institute of Actuaries of Australia *Professional Standard 300*, "Actuarial Reports and Advice on Outstanding Claims in General Insurance" provides the following direction (excerpts from paragraphs 20-22,29,37):

> The actuary should be familiar with the relevant aspects of the procedures for the administration and accounting of the insurer's claims and policies.
>
> The actuary should be conversant with the general characteristics of the insurance portfolio which may have a material bearing on the estimation of the liabilities. This may include familiarity with the

contractual terms and legislated benefits payable under policies written as well as other attributes, such as deductibles, policy limits and reinsurance arrangements.

The actuary also has a responsibility to be familiar with the general economic, legal and social trends in the community which may have a bearing on the liabilities.

The analysis should take into account any special features of or changes to the experience such as changes in deductible, aggregate limits, claims handling procedures, the mix of business within the portfolio, and the impact of large claims paid and outstanding. The analysis should investigate any trends in the development of the experience, particularly those from causes other than inflation.

Appropriate allowance for future costs of administering and settling claims (in addition to those included in payments on individual claims) should be made having regard for the insurer's level of expenses, organizational structure and future administrative developments. The complexity of the approach used to determine the allowance should be commensurate with the materiality of the amount of the allowance.

Newly drafted (March 2001; not yet finalized) regulatory standards in Australia provide further enumeration on these standards (Australian Prudential Regulatory Authority *Draft Guidance Note GGN 220.1*).

Thus, the profession has long-recognized the importance of understanding and reflecting Claims-related changes. Not surprising, then, that the actuarial literature is populated with techniques for adjusting raw data to a consistent basis, and techniques of analysis, to use when one or more of the basic "consistency" assumptions is violated. The published techniques address circumstances such as:

- Changing speed of claim closure during the historical experience period;
- Changing levels of case reserving historically;

- Varying rates of inflation historically and anticipated in the future;
- Changing mixes of claim types; and
- Changing coverage definitions (e.g., deductibles or limits).
- Changing laws or legal interpretations of coverage

The *Bibliography* at the end of this paper offers a partial list of resources in the literature.

The question, then, is not whether it is appropriate to identify and address claims-related changes in the choice of methods and parameters used in an actuarial reserve review. Rather, the question is "What are the best ways for a casualty actuary to become aware of, and understand the underlying change(s)?" Granted, the consequences of many types of Claims Department changes can be observed in aggregate actuarial data without any dialog with Claims. But, we have seen that an ongoing dialog with Claims can accelerate the Actuary's recognition of changes, improve the Actuary's understanding of those changes, and help the Actuary pinpoint the data that may help measure the change and the data that is likely to be affected by the change. The dialog also can reveal if the change is complete, or still in transition.

Before the Actuary has seen the first piece of numerical data, a conversation with Claims may reveal changes in: the mix of claims being presented to the company, operational methods, the use of outside adjusters, the handling of small claims, case reserving, the definition of a claim count, change in settlement philosophy, and so on. All of these types of changes, of course, may affect the behavior and interpretation of the data upon which the Actuary relies.

The Actuary should not necessarily expect this conversation to identify all the pertinent changes, however, since the Claims practitioners may be so close to the "trees" (individual claims) that they may not see the "forest" that is revealed in aggregate data. Thus, after the Actuary's initial review of data diagnostics, follow-up conversations with Claims may provide critical insights that help explain the observed behavior of the data, and guide the way to projecting its future behavior. Such conversations are particularly important to the Actuary's understanding when the data behavior is inconsistent with the assertions by Claims, when multiple changes produced mixed signals in the data (which we frequently have found to be the case), or when the data behavior suggests some underlying changes that were not even mentioned by Claims.

The Actuary might use a multi-step process in the interaction with Claims: (1) Perform various standard diagnostic tests using the actuarial data; (2) Interview with Claims to identify any factors that Claims might be aware of that would relate to the analysis, and to discuss the interpretation of diagnostic test results; (3) Identify further investigation or analysis to be performed; (4) resolve (if possible) any outstanding issues that were subject to further investigation; (5) Completes the reserve analysis with the benefit of the information and insights; and (6) for unresolved issues, highlights the resulting increased uncertainty and identifies potential further work to resolve those issues.

A few relatively simple cases should serve to illustrate the benefits to the Actuary of a dialog with Claims. Note that these simple cases may create the impression that a fruitful dialog is easily launched. For many organizations in which Actuarial and Claims have not historically communicated, a concerted effort may be required, and initial conversations may be uncomfortable (or even seem antagonistic). For example, the Actuary may hinder communication by failing to use the vocabulary of Claims. Or, Claims, fearing that the Actuary is looking for problems to tell the CEO, may answer the

Actuary's questions narrowly, and not volunteer related useful information, rather than engaging in a full dialog. Over time, however, we almost always see the possibility for a collegial relationship to develop, and with it, increasingly effective dialog.

Case reserving

The behavior of a few basic diagnostics, such as movements in the average case reserve at different evaluation dates, or changes in the relationships between paid losses and reported losses, may serve to alert the Actuary to changes in case reserve levels. These diagnostics may even suggest the aggregate magnitude of the change in case reserve levels and imply an amount by which to adjust historical data to state it at an equivalent case reserving level. But, understanding the nature of the case reserve change allows the Actuary to tailor the response to the situation, as the following three examples illustrate.

Case 1.1: Claims implemented a new computerized case reserving tool on May 15, captured a snapshot of the database immediately before and after the change, and calculated the instantaneous effect on each age of accident year, as well the effect on different types and severities of claims. This information, which was forthcoming in an interview with Claims, facilitates the Actuary's restatement of old data to the level of the current case reserving process. This same information allows the Actuary to test an alternative set of approaches, namely to remove the effects of the recent case reserve changes from the latest evaluation, and perform the actuarial projections as though the pre-existing case reserve levels had remained stable. (Note: the likelihood that this type of information will have been captured by Claims is substantially improved if the Actuary-Claims dialog is ongoing, and the if Actuary was aware of the impending case reserve change beforehand. This knowledge can trigger the Actuary to request that key statistics be captured before and after the change. Rarely do we see Claims Departments that initially anticipate the importance of capturing and communicating this type of information.)

Case 1.2: Claims historically had put no case reserves on a particular category of small claims; now Claims is applying a formula reserve of $x. With this information in hand based on an interview with Claims, the Actuary can directly test the effect of applying a formula reserve to these same types of cases historically, and thus create an adjusted data set that reflects a consistent case reserving practice.

Case 1.3: An adverse outcome on a precedent-setting court case is going to increase the cost of many open claims and all future claims that have similar characteristics. Claims re-evaluated the case reserves (upward) for the affected claims, producing the appearance of case reserve strengthening. Based on the interviews with Claims, and further parsing of the data, the actuary is able to conclude that the current case reserves are likely to be no more adequate relative to ultimate costs than was the situation historically (even though in absolute terms the case reserves are higher), and adjust the actuarial methods and parameters accordingly. With the benefit of the interviews, the Actuary also is able to distinguish between this type of permanent change in case reserve levels, and a one-time blip in results.

Claim Closure Rates

Some of the familiar methods that adjust for changes in claims patterns use the speed of claim closing as a proxy for payment speed, and adjust the historical triangle of paid losses in proportion to a recent change in closure speed.

Case 1.4: The Actuary observes a slowdown in claims closure rates. Historically, 80% of the claims were closed at the 36 month evaluation; most recently this dropped to 60%. At the same time, average case reserves appear to be dropping, leading the actuary to fear that standard paid and incurred loss projections both will understate ultimate losses. Interviews with Claims revealed that the actual payment processes and payment speeds have not changed at all. Rather, Claims formerly coded a claim as closed when all disputed issues were resolved and all that remained was to pay out an agreed schedule of payments. Now, Claims does not count a claim as closed until the last payment has been made. The Actuary correctly concludes that there is no need to adjust the paid loss data. Further, the Actuary is able to determine that the apparent reduction in average case reserves is not due to case reserve levels, but rather to an increase in the number of claims that are being counted as open (i.e., the denominator of the average case reserve calculation).

Case 1.5: The Actuary observes an apparent acceleration in claims closure rates. Further inquiry with Claims indicates no change in processes, but reveals a change in the mix of claims. The data being examined includes several types of claims, and there has been an increase in the volume of small, fast-closing claims, producing an apparent acceleration in closure rates. However, using the closure rates as a proxy for the impact on payment patterns would overstate the adjustment, and therefore understate ultimate projections, since the dollars on these fast-closing claims are relatively small. The Actuary instead subdivides the data and analyzes the different types of claims separately.

Recent change in operations

In recent years, many insurers have sought improvements in the efficiency and effectiveness of their claims operations. Changes have taken a variety of forms, including centralization of certain functions (e.g., call-in centers); greater use of technology to support work flow, work processes, and the availability of information; greater outsourcing of some functions; re-arranging conventional claim department hierarchical personnel structures (e.g., to team-based structures); and others. While many of these changes may have been made in the interests of expenses, many others have been designed to improve and control claim costs. Many of the changes have the potential to alter future patterns in the actuarial data.

Case 1.6: Recent implementation of medical bill control techniques. The company (a slow adopter!) implemented more rigorous medical bill control techniques during the most recent calendar quarter. These techniques will apply to all future medical bills, on both old and new claims. While the company will incur additional administrative expenses, management is able to document best-estimate savings of 10% on medical bills. No appreciable change is expected in the speed of processing and paying medical bills. Due to the recent implementation of this change, no symptoms are yet visible in the actuarial data. Learning of the change through interviews with Claims, the Actuary is able to estimate – and adjust for – the mis-statement in ultimate losses that would be produced by traditional methods. Further, the Actuary is able to anticipate, monitor, and adjust for distortions in payment patterns for exposure periods for which some medical bills are paid under the old system and some are paid under the new system. The Actuary also is able to estimate the effect of the change on loss adjustment expenses.

* * * *

As illustrated by these simplified cases, in our experience, the insights gained from dialog with the Claims Department aid the Actuary in

- Identifying and understanding the types of changes that are occurring in the claims data;
- Determining the types of methodologies and adjustments that will avoid or counteract any distortions or data movements resulting from the Claims Department change;
- Identifying any special types or subdivisions of data and/or diagnostics that may be helpful in detecting and measuring the effects of these changes;
- Developing expectations as to the degree and magnitude of the effects of the changes on the different components of the data;
- Assisting in proper interpretation of observed patterns in diagnostic tests performed on the Claims/Actuarial data.
- Eliminating false explanations of movements in the data;
- Identifying data necessary in order to make adjustments for the changes that are occurring;
- Forecasting the future performance of the data; and

- Specifically identifying areas of uncertainty (e.g., unexplained behavior of diagnostic data tests; changes in claims operations not evident in the diagnostic results).

Why would any loss reserving actuary **not** want these insights?

ACTUARIAL DIAGNOSTIC FEEDBACK FOLLOWING THE ACTUARIAL ANALYSIS: FOR THE BENEFIT OF CLAIMS DEPARTMENT MANAGEMENT

Many (but by no means all) actuaries have learned to obtain the input they need from Claims in order to improve their analyses as described in the prior section. But relatively few seem to view this as a two-way street. When the reserve indication is calculated and the results are presented to management, the Actuary's work too often is considered complete. We have found that some of the by-products of the Actuary's loss reserving work ultimately are even more valuable to the insurance company than is the loss reserve indication itself.

These "by-products" may describe performance characteristics of the business; indicate how it is changing over time (short-term or long-term); identify, isolate, and quantify problematic aspects of the business; and compare company performance to peer group indicators. Time and again we have seen these types of indicators serve to focus Claims Department management attention on an element of claims practice that is not performing as desired. Further targeted diagnostic work (such as more data analysis, process reviews, and claim file reviews) typically is necessary to confirm (or modify) the original hypothesis; guide a determination of needed changes; and quantify the impact on the bottom line. But, without the Actuary's insights, the serious investigation might have been delayed, or perhaps never launched. These types of insights, while not always welcome news to Claims, may help Claims and senior management to identify and address a claims issue before it becomes a serious problem; or to recognize the need to explore alternative philosophies, methods, and procedures. And, as time goes by, further actuarial diagnostics can help monitor the intended and unintended effects of revised claims procedures, thereby providing valuable feedback to Claims as well as key insights leading into subsequent actuarial reserve analyses.

The Actuary's insights don't just identify Claims issues, of course. By-products of actuarial loss reserving engagements frequently provide the foundation for estimating current and potential future profitability of a segment; detecting issues with the way pricing tools are being used; identifying and quantifying shifts in the mix of business; and evaluating the potential performance of reinsurance products. But the following brief case studies are intended to illustrate the types of insights related to Claims that we have seen emerge from reserving engagements.

We also have observed powerful teamwork results on due diligence engagements (potential acquisitions) where actuaries and claims practitioners have partnered to analyze the effectiveness of the target company's claims operations. The data observations developed by the Actuary in the course of analyzing the target company's balance sheet loss reserves can serve to identify potential areas of examination in the claims arena. With tangible numerical indicators of trends and changes, the interviews and explorations can proceed more efficiently and can focus (in part) on identified issues.

Case Reserving

Sometimes we actuaries develop the attitude that case reserves exist only to help (or hinder!) our actuarial reserving processes. In fact, of course, case reserves serve many other purposes, including: playing an integral role in strategizing, planning, and budgeting the future course of a claim; calculating experience rates, retro rates, and dividends; and enhancing the accuracy of allocating ultimate claim costs to different policyholders or business segments for profitability analyses. Thus, if the Actuary is able to identify a changing pattern of case reserving, that insight is important to the management of the Claims Department (and Underwriting) as well as to the Actuary. A cautionary note, however: a broadcast to all claims examiners that "case reserves are inadequate" may wreak havoc as individual adjusters attempt to compensate by modifying their case reserving habits in a variety of ways. The result can be a level of case reserves that varies haphazardly over time and across adjusters. Any message to claims examiners must be filtered carefully by management of the Claims Department in order to manage consistency of adjuster performance.

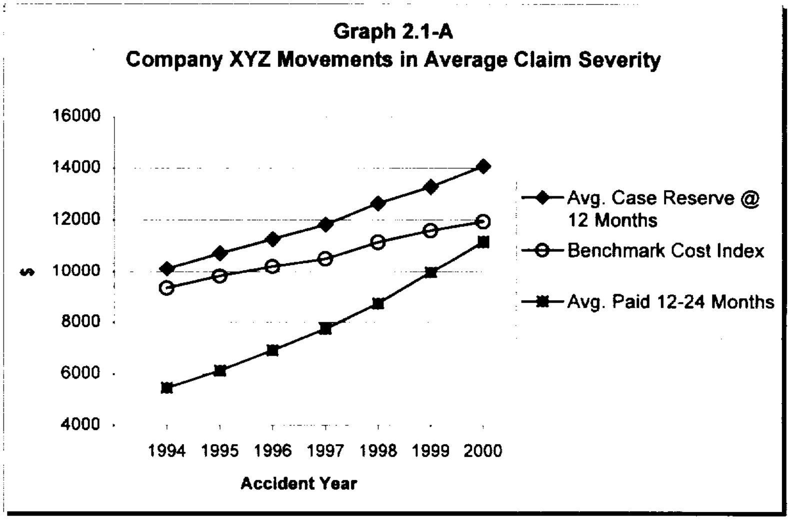
Case 2.1: Redundant case reserves. In Company XYZ, claims examiners set case reserves on liability cases based on a "worst case" scenario. This practice dated back a number of years, originally having been established in the interests of "conservatism." Over time, adjusters began being judged based on their ability to settle cases for less than the case reserve. Two adverse consequences resulted: cases were settled for more than necessary (just less than worst case), and as the cost of settling claims escalated, adjusters also gradually edged case reserves higher, creating an unfortunate cycle of claim cost escalation. During the course of a reserve review, the Actuary observed case reserve levels far in excess of competitors, although less so currently than historically. The Actuary also observed that paid claim severity trends have been running higher than trends in average case reserves, which in turn have been higher than the company's benchmark severity index for this business. See Exhibit for Case 2.1. The Actuary brought these observations to the management team within the Claims Department. While considerable additional research and analysis (involving both Actuarial and Claims) were required in order to determine what was transpiring, and to correct both the process and the metrics being used in Claims, the Actuary's communication launched the process.

We note that we have heard this "worst case" scenario many times. Sometimes, it is an accurate description. Other times, it proves to be a convenient but inaccurate explanation for deteriorating claims results that actually are attributable to poor underwriting or other non-claims factors. Drawing this distinction requires careful analysis by the Actuary and Claims working together.

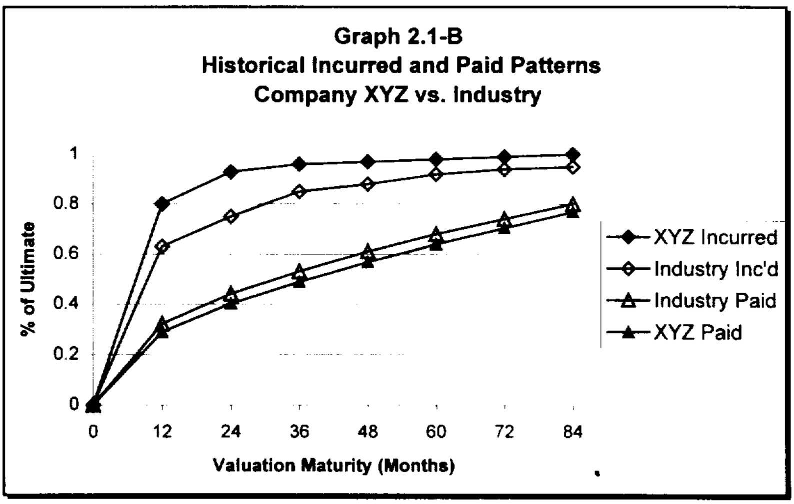
EXHIBIT FOR CASE 2.1

Actuary's observations in preparation for meeting with Vice President-Claims

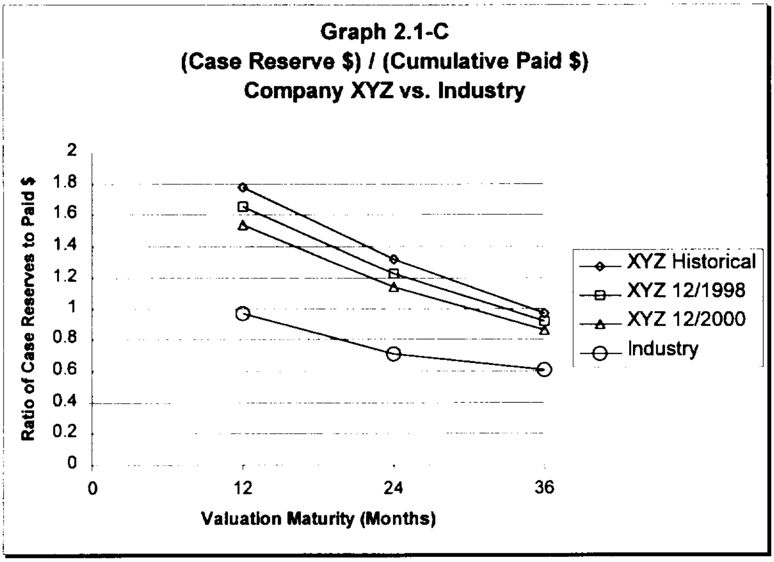
- Historically, our case incurred losses approached their ultimate level much faster than for the industry. Our payment patterns were similar to the industry's (Graph 2.1-B).
- This same information may be depicted as the relationship of our total case reserves at a particular valuation maturity, to our cumulative paid losses at the same valuation maturity. In this view (Graph 2.1-C), it appears that recently our case reserves are not as strong as historically, though still above industry levels.
- Average case reserves per claim are growing faster than our benchmark severity index for this coverage, and the average payment per claim is growing significantly faster than the index (Graph 2.1-A).
- Claim count patterns (reporting speed and closure speed) are stable (graph not shown).
- Graphs are displayed on the following pages.



Graph 2.1-A
Company XYZ Movements in Average Claim Severity
16000
14000
12000
10000
8000
6000
4000
$
1994
1995
1996
1997
1998
1999
2000
Accident Year
Avg. Case Reserve @ 12 Months
Benchmark Cost Index
Avg. Paid 12-24 Months



Graph 2.1-B
Historical Incurred and Paid Patterns
Company XYZ vs. Industry
1
0.8
0.6
0.4
0.2
0
% of Ultimate
0
12
24
36
48
60
72
84
Valuation Maturity (Months)
XYZ Incurred
Industry Inc'd
Industry Paid
XYZ Paid



Graph 2.1-C
(Case Reserve $) / (Cumulative Paid $)
Company XYZ vs. Industry
Ratio of Case Reserves to Paid $
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
12
24
36
Valuation Maturity (Months)
XYZ Historical
XYZ 12/1998
XYZ 12/2000
Industry

Case 2.2: In Company ABC, due to minimal case reserve training of new claims examiners, workers compensation case reserves were established on a stair-step basis, typically at a level just sufficient to cover the following year of expected claim payment activity. This proved to be a reasonable match for Company ABC's approach to claim management, which was simply to pay each bill as it came in, with little review or strategizing about the course of treatment and care of the injured worker. The Company's development patterns were relatively consistent over time, and the Actuary was able to perform the actuarial loss reserving analysis each year using standard methodologies. However, the Actuary observed that Company ABC's loss development patterns differed significantly from peers (see Exhibit for Case 2.2), and together with colleagues in Claims, investigated the causes and implications. One outcome was that serious claims became subject to a formal case reserving discipline, and thus were more easily highlighted for monitoring and strategy development. The most important outcome related not directly to case reserves at all, but to a realization that the claim management process required revamping.

EXHIBIT FOR CASE 2.2

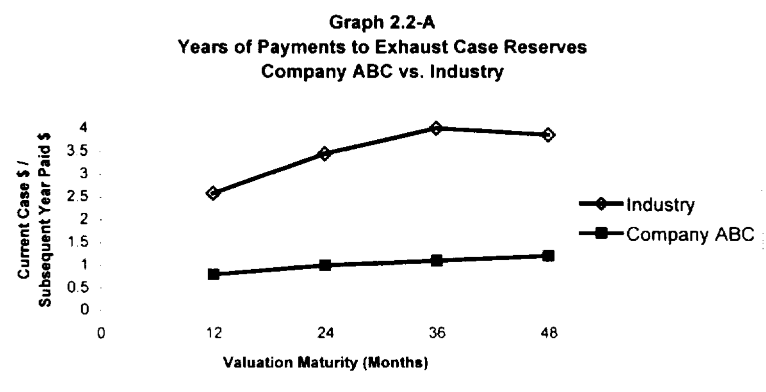
Actuary's observations in preparation for meeting with Vice President-Claims

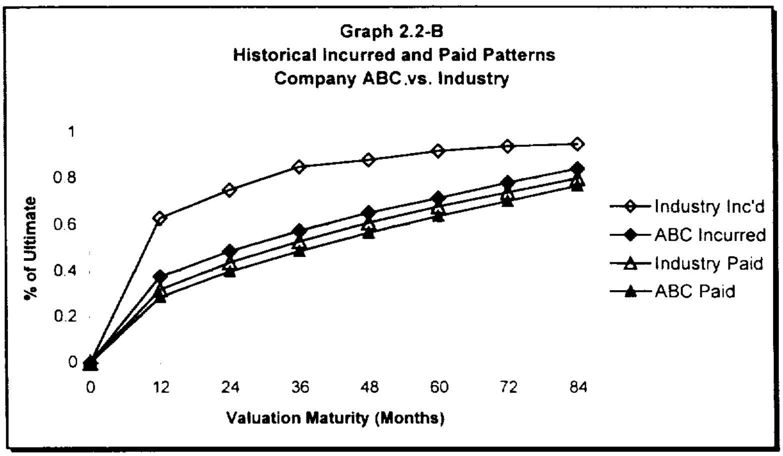
- For this long-tail line of insurance, our Company's case reserves at any point in time are just barely greater than payment activity on those claims over the following 12 months, while the industry carries case reserves equivalent to 3 or 4 years of payment activity (Graph 2.2-A).

- Our aggregate loss development patterns are relatively stable, indicating that our processes have been relatively consistent over time.

- Our company's losses develop to an ultimate level over a longer period of time than is the case for the industry (Graph 2.2-B).

- Graphs are shown on the following page.


Graph 2.2-A
Years of Payments to Exhaust Case Reserves
Company ABC vs. Industry
Current Case $ / Subsequent Year Paid $
4
3.5
3
2.5
2
1.5
1
0.5
0
0
12
24
36
48
Valuation Maturity (Months)
Industry
Company ABC


Graph 2.2-B
Historical Incurred and Paid Patterns
Company ABC vs. Industry
% of Ultimate
1
0.8
0.6
0.4
0.2
0
0
12
24
36
48
60
72
84
Valuation Maturity (Months)
Industry Inc'd
ABC Incurred
Industry Paid
ABC Paid

Case 2.3: In Company C, the Actuary unilaterally undertook an educational effort to help claims examiners understand actuarial projections. "The ultimate loss that I project ranges from 5% to 25% above the case reserves that you set," the Actuary explained, "which means that in the fullness of time we will learn that current case reserves are 5% to 25% too low." The next quarter, case reserves had risen for each adjuster who attended the training session, but by widely varying amounts. Neither Actuarial nor Claims knew how to interpret the results.

As noted earlier, this type of actuarial communication aids neither Claims nor Actuarial, but creates chaos – the equivalent of a sharp tug on the steering wheel while on a slippery road. Actuarial communications about reserve levels need to be communicated to the right level of management. The CEO, Claims, Underwriting, and Actuarial must jointly decide on the best course of action and communication.

Claim Closure Speed

Conventional wisdom has it that a closed claim can't develop adversely, and that a closed claim is therefore better than an open claim. This apparent truism does not always hold.

Case 2.4: The Actuary observed a sudden acceleration in the speed with which the Company was closing claims. This observation aligned with Claims' previous comment that it was undertaking to reduce the volume of stale claims. However, the Actuary noted that the average severity of closed claims was rising sharply, and that the closure rates primarily were accelerating on the most recent, immature accident years rather than reducing the inventory of open claims on older years. See Exhibit for Case 2.4. Further dialog revealed that management, in the interests of focusing claims examiners, had established one-dimensional goals for an 18 month period that focused entirely on reducing the total inventory of open claims. The ensuing analysis showed that adjusters had responded to the one-dimensional goal, as requested. But, the analysis also revealed that, rather than focusing on closing difficult old stale claims, the adjusters had focused on the high volume of new claims. And, the adjusters had discovered that it was relatively easy to get claims closed by offering settlement amounts more generous than had been the Company's previous practice. As a result of these findings, the Company quickly returned to a balanced scorecard concept whereby multiple aspects of claim adjuster performance were monitored and measured.

EXHIBIT FOR CASE 2.4

Actuary's observations in preparation for meeting with Vice President-Claims

- At year-end 1999, Management and Claims agreed that the company had accumulated an undesirably large open inventory of old, stale claims. Claims launched an initiative to address this by incenting adjusters to reduce the total volume of open liability claims.

- As Graph 2.4-A shows, we did reduce the volume of open claims significantly even though the volume of new claims remained stable.

- However, as Chart 2.4-B (next page) shows, the closure activity by the adjusters focused on 1999-2000 accident year claims, not on the old, stale claims.

- The average case reserve has increased 16%, versus an ongoing 5% severity trend. This is probably a result of having closed the newer claims, but more analysis is needed.

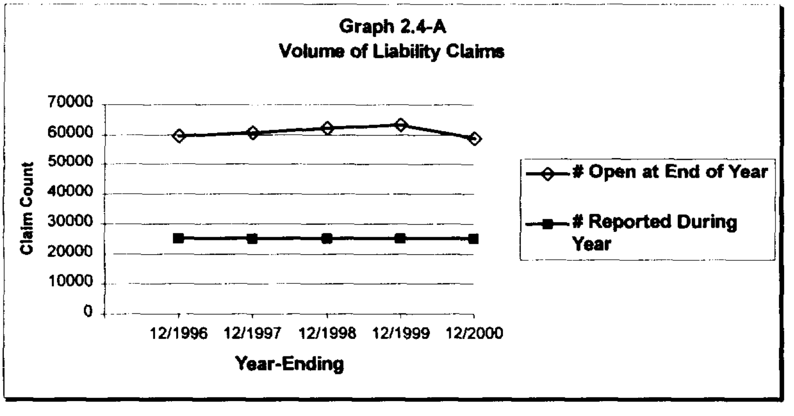


Chart 2.4-B

Accident Year	Open Claim Counts as % of Reported Claim Counts: 12/1999	12/2000: Historical Projected*	12/2000: Actual	Counts Closed Calendar Year 2000: Historical Projected*	Counts Closed Calendar Year 2000: Actual
Prior	0%	0%	0%		
1991	3	0	1	750	563
1992	10	3	5	1,750	1,313
1993	14	10	11	1,000	750
1994	20	14	16	1,500	1,125
1995	25	20	21	1,250	937
1996	32	25	27	1,750	1,313
1997	40	32	34	2,000	1,500
1998	49	40	38	2,250	2,750
1999	60	49	40	2,750	5,000
2000		60	43	10,000	14,250
Total				25,000	29,500

*Historical projected was calculated by assuming that 12/1999 open %'s would recur at 12/2000

Fraud

Patterns of fraud that may be invisible at the individual claim level may come into sharp focus for the Actuary who has access to the aggregate book of business.

Case 2.5: In the course of a reserve analysis, the Actuary was comparing claims costs for various segments of a book of business. The Actuary began to see a pattern whereby a certain geographical region consistently displayed higher average costs than did nearby regions. Further analysis revealed a particular group of medical providers and an attorney whose involvement were significantly correlated with high claim costs. See Exhibit for Case 2.5. Unable to explain these differences, the Actuary and Claims Management turned the findings over to the Fraud Unit, which was able to establish a pattern of wrongdoing.

EXHIBIT FOR CASE 2.5

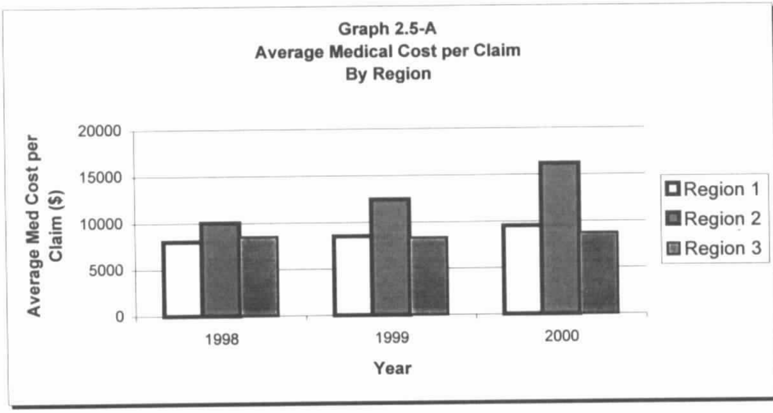
Actuary's observations in preparation for meeting with Vice President-Claims

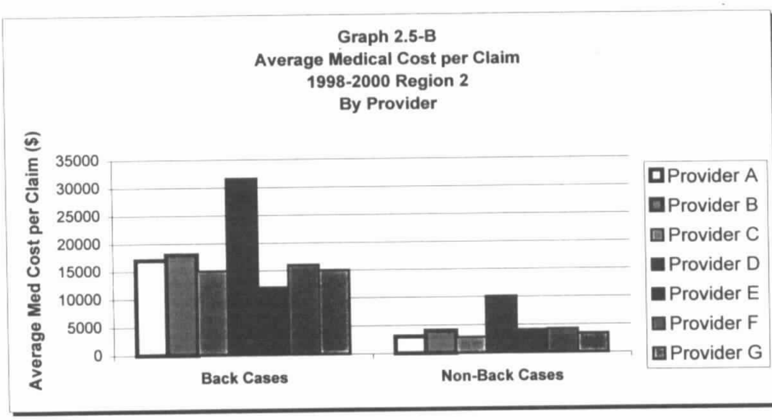
- Region 2 is producing average claim costs for liability that are well in excess of the other regions. This differential began to emerge three years ago.

- Within Region 2, Medical Provider "D" has significantly higher costs per claim than other Medical Providers. This appears to be only partially explained by the mix of claims and injuries being handled by D (higher % of back injuries), though a more thorough analysis will be required of this issue.

- The volume of claims handled by D has increased dramatically over the past few years, and an increasing percentage of the claims handled by D involve back injuries. Interestingly, similar patterns are evident for Attorney "X", and a high percentage of the claims handled by X also are handled by D, and vice versa.

- See graphs on following page.


Graph 2.5-A
Average Medical Cost per Claim
By Region
Average Med Cost per Claim ($)
20000
15000
10000
5000
0
1998
1999
2000
Year
Region 1
Region 2
Region 3


Graph 2.5-B
Average Medical Cost per Claim
1998-2000 Region 2
By Provider
Average Med Cost per Claim ($)
35000
30000
25000
20000
15000
10000
5000
0
Back Cases
Non-Back Cases
Provider A
Provider B
Provider C
Provider D
Provider E
Provider F
Provider G

Workload projections and Loss adjustment expense

Actuaries are accustomed to examining changes in the mix of claims and considering possible implications for trend, loss development patterns, pricing, loss adjustment expense reserves, and so forth. These same mix changes have implications for the claims department in planning for needed changes in staffing mix or staffing levels, use of outside vendors, etc. Similarly, comparisons of a Company's loss adjustment expense levels to peer company loss adjustment expense levels may assist the Actuary's analysis, but may be equally interesting to Claims.

Case 2.6: As part of an analysis of loss adjustment expense reserves, the Actuary models the projected future volume of reported claims, closed claims, and open claims, by calendar year. After reflecting a recent change in the mix of business written, from monoline liability to multiline, the Actuary concludes that: the volume of new claims will increase significantly; the volume of pending claims will increase only slightly; and that the mix of claims will shift from relatively complex liability claims to a combination of liability and first party property claims. The Actuary requires input from Claims to estimate the cost implications of these shifts, but also shares the results of the model to assist Claims in reviewing staffing plans. See Exhibit for Case 2.6.

EXHIBIT FOR CASE 2.6

Actuary's observations in preparation for meeting with Vice President-Claims

- As you know, we have increased the volume of Commercial Multi Peril business significantly, and expect continued growth. This is a significant change from the historical emphasis on monoline liability.

- I thought you would be interested in our projections (next page), which indicate:

  — A significant increase in the total volume of new claims, with all of the volume increase being property claims

  — Because the property claims open and close so much faster, we anticipate a relatively modest increase in the inventory of open claims.

- Let's get together to discuss these projections and to examine the cost implications, which I need to compare to our internal pricing assumptions. In addition, if I can refine these estimates or provide additional information that would assist you in your staffing projections, let me know.

Chart 2.6-A

	Actual			Projected		
	1998	**1999**	**2000**	**2001**	**2002**	**2003**
Earned Premium ($000)	$53,000	$61,000	$70,000	$87,000	$96,000	$103,000
Approx. Premium Mix						
Liability	90%	87%	84%	70%	64%	64%
Property	10	13	16	30	36	36
Volume of Arising Claims						
Liability	2,226	2,359	2,489	2,460	2,360	2,410
Property	742	1,057	1,422	3,160	3,980	4,070
Total	2,968	3,416	3,911	5,620	6,340	6,480
% Growth in Volume of Arising Claims						
Liability		6%	6%	-1%	-4%	2%
Property		42	35	122	26	2
Total		15	14	44	13	2
Inventory of Open Claims (Year-End)						
Liability	2,780	2,932	3,094	3,166	3,134	3,131
Property	147	196	266	545	755	810
Total	2,926	3,127	3,360	3,711	3,889	3,940
% Growth in Volume of Open Claims (Year-End)						
Liability		5%	6%	2%	-1%	0%
Property		33	36	105	39	7
Total		7	7	10	5	1

Unfortunately, in our experience, too few claims departments enjoy the benefit of an actuarial model of the future volume and type of claims. Such a model has the potential to allow Claims to plan and manage the staffing level and structure that will be needed in order to maintain a particular level of service, and a particular balance between claim expenditures and LAE.

Case 2.7: An Actuary's analysis of loss adjustment expense reserves includes an examination of the Company's historical levels of loss adjustment expense (relative to volumes of claim payments, numbers of claims, etc), and a comparison of those levels across several regions. At first blush, the Company's expense levels in the Midwest region appear higher than in other regions. However, further examination indicates that the Midwest region has different mix of claims than other regions: a higher proportion of liability claims. Adjusting for this mix reveals that expense levels in the Midwest region are relatively consistent with the other regions. This insight, together with a parallel analysis of the impact of business mix on claim frequency and claim cost, provides an improved baseline for evaluating and quantifying several strategic changes that are being contemplated for the Claims department.

It usually is relatively easy to develop a long list of actuarial observations, quantified as a by-product of the Actuary's loss reserving analysis, which can potentially serve as raw material for a substantial conversation with Claims. Such a list might include, for example:

- Changes in the relative adequacy or absolute level of case reserves, across the board or for particular types of claims;
- Changes in the level of claim counts, the speed of claim closure, the definitions of different types of claims, the definition of which claims will be counted or not;
- Trends or step-movements in claim severities, across accident year or evaluation, either for claims in general or for a particular type of claim;
- Shifts in the mix of business or the mix of claims, across any dimension;
- Loss adjustment expense levels, in total or for particular components of loss adjustment expense;
- Changes in the Company performance (along any indicator) versus the performance of peer companies, or consistent differences in the level of Company performance versus peer companies;
- Any quantitative observations that confirm or rebut Claim department changes that were discussed during pre-analysis interviews; and
- Any other interesting diagnostic.

Of course, for communication with Claims, the list ought to be pruned to the most interesting diagnostic results, particularly those with implications for the most critical areas of Claims.

The purpose of this paper is not to offer the reader a specific checklist of items for the Actuary to share with Claims. It certainly is not to suggest that each month the Actuary should drop a huge package of computer output on Claims – we would rather see the Actuary provide a few key items accompanied by observations and questions.

Nor is the point to suggest that the Actuary needs to be able to discern the inner workings of the Claim department based on reviewing a handful of aggregate diagnostics. Rather, the point is for the Actuary to be ever-mindful that patterns in the data observed by the Actuary may be of considerable interest to colleagues in other functional areas, and may not previously have been observed by those colleagues. It is not necessary for the Actuary to have a complete explanation of the underlying causes of the patterns prior to these conversations. In fact, it is more likely that the Actuary's resulting conversation with other Company executives will *lead to* a useful exploration, understanding, and (if necessary) treatment of the underlying causes. And, it is likely that the resulting conversation will stimulate the release of additional information and perspectives on underlying business or operational changes that have occurred or are occurring, thus aiding the Actuary in the reserve analysis. (Note, again, that the appropriate level for the conversation is Claims executive, not the front-line Claims practitioner, whose performance could become volatile if influenced directly by the observations of the Actuary.) Conversely, when the Actuary fails to share observations with Claims executives – observations that are uniquely accessible to the Actuary viewing the "big picture" – the Claims executives are left to navigate with incomplete information and insufficient feedback, and do not necessarily have an adequate foundation for identifying trends and changes that are of interest to Claims.

Actuaries, talk to your Claims executives! Tell them what you are seeing, and ask for their perspectives.

CREATING TOOLS FOR THE CLAIMS DEPARTMENT

A third general area in which actuarial-claims interaction can create value for the Company is in the development of tools for Claims. These tools might include, for example, case reserving benchmarks or algorithms to identify patterns that signal the need for expert intervention (such as patterns of potentially fraudulent behavior, claims characteristics that indicate the need for medical intervention in the process, litigation management signals).

The common thread is that the Actuary, with access to the "big picture," can extract pieces and patterns out of that big picture to support various aspects of the claim-specific focus of the claims examiner.

Case reserving tools

Claims examiners typically see one claim at a time; actuaries, of course, see the aggregation of many claims. This perspective, and the Actuary's access to the full scope of data, positions the actuary to assist with the design and development of case reserving tools.

One form of an actuarial case reserving tool is a system that estimates the ultimate cost of claim based on its current characteristics (for workers compensation, for example, these characteristics likely would include nature of injury; degree of physical and occupational disability; age; occupation; wage; type, extent, and cost of medical treatment to date; jurisdiction). The parameters used in such a model can be calibrated by the Actuary based on the known cost of past claims, adjusted to current and future cost levels; statutory benefit structures; forecasted rates of medical inflation; and so on.

While such a tool can provide very useful benchmarks for the claims examiner, critical roles remain for the claims examiner, and we do not advocate blindly abdicating case reserving responsibility to a computer algorithm. First, every case has potentially unique characteristics, and it is a useful exercise for the claims examiner to review whether the key characteristics selected by the computer algorithm, and the resulting case reserve, are pertinent to the case at hand – or whether the case presents some characteristics that suggest a different case reserve. To facilitate this type of review, the actuarial case reserving tool should publish, for each claim, the key characteristics driving the estimation of ultimate cost. In addition, further analytic work by the Actuary might permit the algorithm to identify claims and claims characteristics most likely to lead to exceptional outcomes, and thus most likely to warrant human review.

The second reason to include the human in the case reserving process is the view that case reserving is an integral part of the establishment of a game plan and strategy for management of the claim, rehabilitation of the claimant, and eventual disposition of the claim. In this view, the case reserve is essentially the quantitative summary of that planned course of events, treatments, and outcomes. Just as in a business, budgeting is an integral part of the business planning process, and really may be viewed as a financial summary of the planned business activities. Case reserves play a similar role for an individual claim.

The Actuary's involvement in developing case reserving benchmarks makes the human intervention of the claims examiner more valuable, not less. It adds another tool to the claims examiner's toolbox. The tool can improve decisionmaking by helping single out the claims most likely in need of human intervention, putting key information at the examiner's fingertips, and identifying the characteristics that are most subject to uncertainty in the evaluation process.

Identification of problematic claims

Just as an actuarial case reserving support tool can, as one of its functions, identify cases that are exceptional, or that are fitting some predetermined pattern, more generally actuaries can play a role in developing tools to highlight claims for other types of intervention. This intervention could include examination of patterns of potentially fraudulent activities, scrutiny of a litigation management plan, or review of the need for a change in medical treatment plans.

CONCLUSION

Talk to your Claims executives. Ask them questions. Listen to them. Share your insights, your tools (customized to their perspectives), and your expertise. Share your views of the "big picture"; illuminate it with the Claims view of what is happening on the front lines, and of the trends that can be observed from that perspective.

You will be able to perform your job more effectively, as well as help your Company perform more effectively.

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Loss Reserving Without Loss Development Patterns—Beyond Berquist-Sherman

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Loss Reserving without Loss Development Patterns - Beyond Berquist-Sherman

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ABSTRACT: This paper describes loss reserving techniques that may be used in situations where changes have occurred that render past years' loss development patterns inappropriate for use in estimating loss reserves for more recent years. The loss reserving issues addressed by this paper are generally the same as those covered by James R. Berquist and Richard E. Sherman in their paper Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach, PCAS LXIV, 123. The essential difference in techniques is that while Berquist-Sherman restates prior years' development patterns to be applicable to the current evaluation basis, this paper restates the current diagonal to the level implied by the older years' estimates. This restatement is done on an implied ultimate basis, thereby eliminating the need to apply loss development patterns to the less mature years.

1. INTRODUCTION

Actuaries frequently encounter situations where changes have taken place or are suspected to have taken place that make historical loss development patterns inappropriate for use in projecting ultimate losses for relatively immature years. The types of changes that can cause distortions in the analysis relate to case reserve adequacy, claim settlement rates, the legal/regulatory environment, and other internal and external factors. In situations such as these, the actuary typically makes adjustments to remove or reduce the distortions that changes in patterns would cause with traditional loss projection methods. For example, the techniques described in Berquist-Sherman produce adjusted development patterns that are estimated to be consistent with the reserve levels and settlement rates present as of the last diagonal by restating historical development data.

The techniques described in this paper do not restate historical information. Instead, they use prior years' experience (specifically, implied ultimate average case reserves by maturity and average incremental claim settlements by maturity) at historical evaluation dates to forecast the corresponding values by maturity at the current evaluation date. Regression and other techniques appropriate to the particular situation are applied to the older years' estimates - which are presumably unaffected by the pattern changes - to forecast the values for the immature years. These estimated values are combined with estimates of the corresponding claims (e.g., open and IBNR claims as of the most recent evaluation date, estimated future incremental closed claims by closure period) to forecast the indicated loss reserves.

2. APPROACH

We create loss and claim information for ten accident years at ten annual evaluation points, and demonstrate the results of various loss projection techniques applied to this data. The data for the first year is based on hypothetical ultimate claims and losses, and hypothetical claim and loss development patterns. Data for each succeeding year is derived by assuming 1% per year trend in ultimate claim counts and a 4% per year trend in ultimate severity (i.e., total ultimate loss trend of 5% per year, assuming constant exposure level).

Using this approach, we are able to know in advance the ultimate claims and losses for each year. We can then alter the case reserving and claim settlement assumptions to introduce the types of distortions to the patterns that can affect ultimate loss projections. This approach allows us to show the error that traditional, unadjusted loss projection techniques produce when there are pattern changes, and to demonstrate the relative accuracy of the alternative techniques in a given situation. The scenarios considered here include the following:

- *Scenario 1*: Stable Settlement and Reserving Patterns (also referred to as the Base Scenario). This scenario is contained in Exhibits 1 and 2;
- *Scenario 2*: Case Reserve Strengthening. Scenario 2 is documented on Exhibits 3 through 7 attached;
- *Scenario 3*: Settlement Rate Acceleration. Scenario 3 calculations are shown on Exhibits 8 through 12;
- *Scenario 4*: Case Reserve Strengthening and Settlement Rate Acceleration. This scenario is documented on Exhibits 13 through 17.

While there are several ways that these types of changes can occur, we define strengthening broadly as an increase in the percentage of ultimate losses that are reported at a given maturity. Similarly, we define settlement rate acceleration as an increase in the percentage of ultimate claims and losses that are closed or paid, respectively, at a given maturity. Our scenarios assume a six-month acceleration in the respective patterns.

3. SCENARIO 1: STABLE SETTLEMENT AND RESERVING PATTERNS

Our base scenario assumes that all patterns are stable over the entire ten-year experience period. We use this scenario to create a baseline, and to demonstrate the characteristics of traditional loss development techniques that can later be compared to the more complicated scenarios.

Exhibit 1, Sheets 1 through 5 shows the data, the implied loss development factors (LDFs), and the results of simple loss development techniques. Sheet 1 pertains to incurred losses, Sheet 2 shows the paid loss analysis, and Sheets 3, 4, and 5 show reported claims, closed claims, and claims closed with payment (referred to as paid claims), respectively.

In each case, we calculate the incremental loss development factors for each year as it ages, and several averages of the implied factors. Given the stability in the data used here, all factors for a given maturity and data type are the same. The selected incremental factors - which throughout this paper are based on the volume weighted average of the last three factors - are accumulated to factors to ultimate. These factors are applied to the latest diagonal of data to produce the projected ultimate values. As each of the Sheets of Exhibit 1 shows, the projected amounts are exactly equal to the actual ultimate losses and claims contained in the hypothetical data.

Whenever possible, it is useful to review various statistics underlying the loss data such as the average case reserve (referred to as the average outstanding losses), the average paid loss (both cumulatively and incrementally for each year), claim closure rates, and paid claim ratios. These statistics - often referred to as diagnostics - can provide insights into the underlying reporting, reserving, payment, and settlement rate patterns that can identify changes that may have occurred. Exhibit 2, Sheets 1 through 5 shows these diagnostics for the Base Scenario.

Sheet 1 of Exhibit 2 shows the average outstanding. As we would expect given the stability inherent in the Base Scenario, the annual change in the average outstanding values for each maturity is equal to the 4% per year severity trend underlying our hypothetical data. Sheet 2 shows the cumulative average paid losses per paid claim and Sheet 3 shows the incremental average paid results. As with the average outstanding experience, the average paid amounts by maturity in the Base Scenario show consistently the underlying 4% annual severity trend. Sheets 4 and 5 show claim closure rate data and paid claim ratios (paid claims divided by total closed claims). The consistency in settlement rates underlying the Base Scenario causes these statistics to show stability from year to year.

These diagnostic statistics each imply a stable reserving and settlement scenario. As will be seen in subsequent sections, these statistics will show significant variation among years in the presence of case reserve adequacy changes and claim settlement rate changes.

4. SCENARIO 2: CASE RESERVE STRENGTHENING

In the second scenario, we assume that case reserve strengthening takes place during the eighth calendar year, and that the strengthening affects the whole diagonal (i.e., all accident years). We reflect this assumption by adjusting the percentage of ultimate losses reported to be equal to the average of the Base Scenario percentage at a given maturity and the Base percentage for the subsequent maturity. We assume that the accelerated reporting pattern applies for the remainder of the experience period.

The standard loss development techniques applied to incurred and paid losses are shown on Exhibit 3, Sheets 1 and 2, respectively. As these projections show, the incurred projection is significantly above the actual ultimate losses (projected all years combined ultimate losses of $796.0 million versus actual ultimate losses of $766.5 million), and the paid projection is equal to the actual ultimate losses.

Obviously, in real life situations, one would not know in advance that there has been case reserve strengthening, so this situation would not be as simple as accepting the paid loss projection and rejecting the incurred estimates. All that would be known at this stage is that the incurred losses produce a much higher projection than the paid losses.

A look at the loss development factors, as well as the various diagnostics statistics provides additional insights. It is apparent from the incurred loss development factors calculated on Exhibit 3, Sheet 1 that something changed on the second prior diagonal (i.e., the diagonal that is as of the end of year eight). During that year, the incurred losses experienced significantly more development than during prior years. Also, development after the eighth calendar year is less than experienced during earlier time periods.

In addition, Exhibit 3, Sheet 2 shows paid loss development factors that are consistent over the entire experience period, and the average outstanding triangle on Exhibit 4, Sheet 1 shows a significant increase during calendar year eight, followed by a return to more normal trends thereafter. Lastly, the average paid claim and the claim count diagnostics (Exhibit 4, Sheets 2 to 5) show a consistent pattern.

Given the likely conclusion that case reserve adequacy has increased over the last several years, some adjustment is needed to the incurred loss projection if it is to be included in the analysis of ultimate losses. The techniques described in Berquist-Sherman would produce adjusted development patterns consistent with the stronger case reserve levels present as of the last diagonal by restating historical development data to the current case reserve level. The Berquist-Sherman techniques would provide accurate estimates of ultimate losses in this case.

Alternatives to this technique would be to use the data prior to the change in case reserve adequacy to estimate what the most recent diagonal would have been in the absence of case reserve adequacy changes. Exhibit 5, Sheets 1 and 2, and Exhibit 6 provides the details of two such alternatives. Details are as follows:

1. ***Ultimate Unclosed Claim Severity Technique (Exhibit 5, Sheets 1 and 2)*** – The general approach of this technique is to calculate the ultimate closed claim severities by maturity for prior years, and use those estimates to estimate the needed average amount per unclosed claim as of the latest evaluation point.

   Exhibit 5, Sheet 1 shows that the calculations start with an estimate of the ultimate losses based on the incurred loss development technique (note that Exhibit 5, Sheet 2 performs the same calculations, but starts with the ultimate losses implied by the paid loss development technique). The paid loss triangle (triangle (A)) is subtracted from the ultimate loss projections to create a triangle of implied ultimate unpaid losses (triangle (B)). The next step is to estimate the claim counts that remain to be closed. Specifically, the closed claim triangle (C) is subtracted from the projected ultimate reported claims to create a triangle of implied unclosed claims (i.e., open and IBNR claims; triangle (D)). The ratio of the implied ultimate unpaid losses to the estimated unclosed claims by maturity gives a triangle of estimated ultimate unclosed claim severities (triangle (E)).

Regression or other estimation techniques can be applied to each column of the ultimate unclosed claim severity triangle (triangle (E)) to estimate the values for the latest diagonal. On Exhibit 5, Sheets 1 and 2, we simplistically apply exponential regressions to all years prior to the last three diagonals to forecast the ultimate claim severities for the last diagonal. The years used in the regressions are the boxed and highlighted ranges of triangle (E) on Exhibit 5, Sheets 1 and 2.

The last step in this technique is to multiply the forecasted ultimate unclosed claim severities (Item (F)) by the estimated number of unclosed claims from the latest diagonal of triangle (D) to arrive at the indicated unpaid losses. These estimates are added to the losses paid to date to produce the estimated ultimate losses by year. As Exhibit 5, Sheet 1 shows, this technique applied to incurred losses produces estimated ultimate losses that are very close to the actual values (i.e., $768.9 million versus $766.5 million), and the paid loss projections on Exhibit 5, Sheet 2 are exactly equal to the actual values.

2. ***Incremental Closed Claim Severity Technique (Exhibit 6)*** – This approach is independent of the various projections of ultimate losses. Instead, it uses historical closed claim severities by maturity to forecast future severities by maturity. These forecasts are combined with estimates of future closed claim counts to estimate future loss payments. These estimates are combined with actual payments to date to produce estimated ultimate losses.

   Exhibit 6 shows that the calculations start with the actual paid loss triangle (triangle (A)). This data is used to calculate actual incremental paid losses by maturity (triangle (B)) by taking differences of adjacent columns of triangle (A). The next step is to use actual cumulative closed claims (triangle (C)) to calculate actual incremental closed claims by maturity (triangle (D)). The ratio of these two incremental triangles provides historical closed claim severities by maturity (triangle (E)), through the latest diagonal.

   Regression or other estimation techniques can be applied to each column of the incremental closed claim severity triangle to estimate future incremental closed claim severities for the less mature years as they age (i.e., for the boxed and highlighted area below the latest diagonal of triangle (E) of Exhibit 6). On Exhibit 6 we simplistically apply exponential regressions to all years prior to the last three diagonals to forecast the future incremental claim severities. The years used in the regressions are boxed on triangle (E) of Exhibit 6, Sheet 1.

   In addition to the forecast of future incremental closed claim severities, we need to estimate future closed claims by maturity at closing. These estimates are based on the estimated ultimate reported claims, claims closed to date, and the estimated claim closing pattern. The ultimate reported claims are derived on Exhibit 3, Sheet 3, and the claim closing pattern is derived on Exhibit 3, Sheet 4.

These values, along with claims closed to date are used to forecast the future incremental closed claims shown below the latest diagonal of triangle (D) on Exhibit 6.

The last step in this technique is to multiply the forecasted incremental closed claim severities by the estimated number of incremental closed claims to arrive at the indicated unpaid losses. These estimates are added to the losses paid to date to produce the estimated ultimate losses by year. As Exhibit 6 shows, this technique produces estimated ultimate losses that are exactly equal to the actual values.

Exhibit 7 provides a summary of the various ultimate loss projections for Scenario 2. As the summary indicates, all projection methods that are based on paid losses replicate the actual ultimate losses exactly, while the ultimate loss projections produced by the traditional incurred loss development technique are significantly overstated. The ultimate closed claim severity technique based on incurred losses, while still overstated, is significantly more accurate than the traditional incurred method.

5. SCENARIO 3 - SETTLEMENT RATE ACCELERATION

Exhibits 8 through 12 provide the same projection techniques as described above for Scenario 2. As described above, Scenario 2 assumes that instead of case reserve adequacy changes, there was acceleration in claim settlement rate and in the loss payment pattern starting in calendar year eight. As with the strengthening scenario, we reflect this assumption by adjusting the percentage of ultimate claims that are closed and losses that are paid to be equal to the average of the Base Scenario percentage at a given maturity and the Base percentage for the subsequent maturity. We assume that the accelerated patterns apply for the remainder of the experience period.

The standard loss development techniques applied to incurred and paid losses are shown on Exhibit 8, Sheets 1 and 2, respectively. As these projections show, the incurred projection is equal to the actual ultimate losses, but the paid loss projection is significantly above the actual ultimate losses (projected ultimate losses of $840.7 million versus actual ultimate losses of $766.5 million).

A look at the paid loss development factors, as well as the various diagnostics statistics provides additional insights. It is apparent from the paid loss development factors calculated on Exhibit 8, Sheet 2 that something changed on the second prior diagonal (i.e., the diagonal that is as of the end of year eight). During that year, the paid losses experienced significantly more development than during prior years. Also, paid loss development after the eighth calendar year is less than experienced during earlier time periods.

In addition, Exhibit 9, Sheet 1 shows that the average outstanding losses were actually reduced as of the end of the eighth calendar year. This development in isolation may be interpreted to mean that case reserve weakening took place, which we know is not true given the assumptions used in deriving the data. Looking further at the paid loss and claim count diagnostics completes the picture. Specifically, Exhibit 9, Sheets 2 and 3 show significant increases in average paid amounts at the same point in time that the average outstanding values were reduced. Further, Sheets 4 and 5 of Exhibit 9 point to acceleration in claim closing rates. Combined, these observations point to closure rate and loss payment acceleration.

The alternative techniques described above in the case reserving strengthening scenario are useful in the settlement acceleration case as well. Exhibit 10, Sheets 1 and 2 demonstrates the Ultimate Unclosed Claim Severity technique and Exhibit 11 shows the Incremental Closed Claim Severity method.

The results of all of the projections are summarized on Exhibit 12. As this summary shows, the adjusted methods are significantly more accurate than the paid loss projection.

6. SCENARIO 4 - CASE RESERVE STRENGTHENING AND SETTLEMENT RATE ACCELERATION

Exhibits 13 through 17 expand the hypothetical examples to combine the case reserve strengthening and closure rate acceleration assumptions underlying Scenarios 2 and 3.

The traditional LDF projections shown on Exhibit 13, Sheets 1 and 2 each significantly overstate the ultimate losses (i.e., the incurred projection is $796.0 million and the paid projection is $840.7 million versus the actual ultimate losses of $766.5 million). A look at the diagnostics on Exhibit 14, Sheets 1 through 5 shows changes along the eighth diagonal in average outstandings, average paid claims, and closure rates.

Application of the adjusted loss projection techniques is shown on Exhibits 15 and 16. The summary on Exhibit 17 shows that these methods are significantly more accurate than the traditional projection methods.

7. DISCUSSION

These techniques may not be used in all situations. Further, they often will not produce a more accurate estimate than the more traditional adjusted methods. However, the techniques described here have some advantages, including the following:

- These techniques may be more understandable to company management. The traditional adjusted methods involve restatement of significant amounts of historical data. Essentially, the whole development triangle is adjusted based on estimates of current case reserve adequacy or claim settlement rates. These adjustments are often non-intuitive to the non-actuary. The methods described here restate only the current diagonal, and the assumptions underlying the adjustments can be clearly demonstrated. This may allow for a clearer discussion with underwriting and claims management on underlying operational changes that may be contributing to the observed changes.

- The methods in this paper allow explicit reflection of cost and/or operational changes from the older years to the present. As described above, trends from historical timeframes are used to estimate the latest diagonal on a level that is consistent with the case reserving and settlement rates. Known changes in mix of business or other factors that would affect trends in average case reserves or settlement rates can be reflected explicitly in the adjustment calculations.

- The new methods allow for easy combination of case reserve adequacy adjustments and claim settlement rate adjustments. While traditional adjusted methods can be combined to adjust for both types of changes, the calculations are cumbersome. As demonstrated with Scenario 4, the methods described here easily handle the combined effects of case reserve and settlement rate changes.

- This paper's techniques can be used equally well whether the case reserve or settlement rate changes occur on a calendar year basis (i.e., affecting all accident periods as of a given evaluation date) or on an accident year basis (i.e., affecting only part of the diagonal). The traditional adjusted methods can also handle both situations, but the calculations would differ between the two situations.

- These new projection techniques are sensitive to actual claim experience through the latest evaluation point. They simply adjust for changes in case reserve adequacy and/or claim settlement rates, but are still heavily influenced by the claim frequency implied by the latest evaluation of data.

- These non-traditional methods provide a means of reasonableness testing of the results produced by more traditional reserving methods

The techniques described in this paper may often provide an alternative for handling situations where development patterns are suspected to have changed. The approach and examples used here are relatively simplistic, designed to illustrate the methodology. Refinements to be considered include the following:

- Use of unpaid claims in the adjustments instead of unclosed claims. The adjustments shown here use unclosed claims to avoid the distortions that the settlement rate acceleration causes with the projection of estimated unpaid claims. It would be possible, and desirable, to adjust the paid claim projections to remove the distortions caused by the pattern changes, and incorporate the adjusted values into the loss projections. This approach would be especially important if the paid claim ratio (i.e., paid claims to total claims) is changing.

- Use of more sophisticated forecasting techniques. The examples shown here derive estimated unclosed claim severities based on relatively simplistic regression techniques applied to historical severities. In many instances, it would be preferable to base the forecasted values on a more rigorous analysis of underlying exposure changes. For example, changes in limits of liability, classes, territory, and other aspects of the underlying business should be reflected in the forecasts of future severities.

- These techniques are best suited to situations where the change in patterns is relatively recent. If several years of credible development factors are available after the change in case reserving and/or settlement rates is suspected to have occurred, it is advisable to reflect the actual "post-change" experience in the loss estimates.

- Reasonableness testing should be performed. Specifically, it is advisable to calculate the loss development factors implied by the alternative methods' loss estimates. Also, it is important to evaluate whether the implied ultimate loss ratios, severities, and pure premiums and other factors are consistent with knowledge of the company operations, business mix, and other factors.

- Sensitivity testing. As with all projections that rely on regressions and other forecasting techniques, there can be a high degree of leverage with any of the individual assumptions used. It is therefore important to test the impact of the key assumptions on the estimates. For example, the impact on the ultimate loss projections of alternative trending of future claim severities, claim closure patterns, etc., should be evaluated.

8. CONCLUSION

This paper explores several alternative techniques that may be useful in situations where changes have taken place in case reserving and/or claim settlement rates. These methods should be viewed as additional tools for the actuary to use in evaluating the likely consequences of the suspected changes. As is always the case, these methods need to be used with care, and significant judgment and interpretation of the results is required, considering the results of many different estimation techniques, knowledge of company operations, and other factors.

Estimated Loss Development Pattern
Incurred Losses ($000)

Exhibit 1
Sheet 1

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	22,638	37,938	49,003	53,252	59,843	60,649	60,938	60,938	60,938	60,938
2	23,769	39,834	51,453	55,914	62,835	63,681	63,984	63,984	63,984	
3	24,958	41,826	54,026	58,710	65,976	66,865	67,184	67,184		
4	26,206	43,917	56,727	61,645	69,275	70,209	70,543			
5	27,516	46,113	59,564	64,728	72,739	73,719				
6	28,892	48,419	62,542	67,964	76,376					
7	30,336	50,840	65,669	71,362						
8	31,853	53,382	68,952							
9	33,446	56,051								
10	35,118									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	
2	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000		
3	1.676	1.292	1.087	1.124	1.013	1.005	1.000			
4	1.676	1.292	1.087	1.124	1.013	1.005				
5	1.676	1.292	1.087	1.124	1.013					
6	1.676	1.292	1.087	1.124						
7	1.676	1.292	1.087							
8	1.676	1.292								
9	1.676									
10										
Average Factors										
Simple Avg Last 3	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Selected Factors	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Factors to Ultimate	2.692	1.606	1.244	1.144	1.018	1.005	1.000	1.000	1.000	1.000

Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0

Estimated Loss Development Pattern
Paid Losses ($000)

Exhibit 1
Sheet 2

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	9,851	21,509	39,918	55,064	59,206	64,550				
6	10,344	22,584	41,914	57,817	62,166					
7	10,861	23,713	44,010	60,708						
8	11,404	24,899	46,210							
9	11,974	26,144								
10	12,573									

Year	Age Interval in Months									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	
2	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000		
3	2.183	1.856	1.379	1.075	1.090	1.070	1.072			
4	2.183	1.856	1.379	1.075	1.090	1.070				
5	2.183	1.856	1.379	1.075	1.090					
6	2.183	1.856	1.379	1.075						
7	2.183	1.856	1.379							
8	2.183	1.856								
9	2.183									
10										
Average Factors										
Simple Avg Last 3	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Simple Avg Last 5	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Simple Avg 3 of 5	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Selected Factors	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Factors to Ultimate	7.519	3.444	1.856	1.345	1.251	1.147	1.072	1.000	1.000	1.000

Ultimate Losses	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Projected	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0

Estimated Claim Development Pattern
Reported Claims

Exhibit 1
Sheet 3

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	
2	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000		
3	1.127	1.085	1.026	1.013	1.000	1.000	1.000			
4	1.127	1.085	1.026	1.013	1.000	1.000				
5	1.127	1.085	1.026	1.013	1.000					
6	1.127	1.085	1.026	1.013						
7	1.127	1.085	1.026							
8	1.127	1.085								
9	1.127									
10										
Average Factors										
Simple Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Selected Factors	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Factors to Ultimate	1.270	1.127	1.039	1.013	1.000	1.000	1.000	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	0	0	0	0	0	0	0	0	0	0	0

Estimated Claim Development Pattern **Exhibit 1**
Closed Claims **Sheet 4**

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	996	1,010	1,010	
3	375	610	778	912	960	979	1,006	1,020		
4	379	616	786	921	970	989	1,016			
5	382	622	793	931	980	999				
6	386	628	801	940	989					
7	390	634	809	949						
8	394	641	818							
9	398	647								
10	402									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	
2	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000		
3	1.626	1.276	1.173	1.053	1.020	1.027	1.014			
4	1.626	1.276	1.173	1.053	1.020	1.027				
5	1.626	1.276	1.173	1.053	1.020					
6	1.626	1.276	1.173	1.053						
7	1.626	1.276	1.173							
8	1.626	1.276								
9	1.626									
10										
Average Factors										
Simple Avg Last 3	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Simple Avg Last 5	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Simple Avg 3 of 5	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Selected Factors	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Factors to Ultimate	2.721	1.674	1.311	1.118	1.062	1.042	1.014	1.000	1.000	1.000

Ultimate Counts	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Projected	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	0	0	0	0	0	0	0	0	0	0	0

Estimated Claim Development Pattern
Paid Claims (I.e., Claims Closed With Payment)

Exhibit 1
Sheet 5

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	743	758	758	
3	247	393	548	678	705	732	751	765		
4	250	397	554	684	712	739	758			
5	252	401	559	691	719	746				
6	255	405	565	698	727					
7	257	409	571	705						
8	260	413	576							
9	263	417								
10	265									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	
2	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000		
3	1.588	1.396	1.236	1.041	1.037	1.026	1.019			
4	1.588	1.396	1.236	1.041	1.037	1.026				
5	1.588	1.396	1.236	1.041	1.037					
6	1.588	1.396	1.236	1.041						
7	1.588	1.396	1.236							
8	1.588	1.396								
9	1.588									
10										
Average Factors										
Simple Avg Last 3	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Simple Avg Last 5	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Simple Avg 3 of 5	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Selected Factors	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Factors to Ultimate	3.093	1.948	1.395	1.129	1.085	1.046	1.019	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	820	812	804	796	788	780	773	765	758	750	7,847
Actual	820	812	804	796	788	780	773	765	758	750	7,847
Difference	0	0	0	0	0	0	0	0	0	0	0

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Outstanding Losses ($000)										
1	14,533	20,242	16,163	7,951	11,134	7,544	4,097	0	0	0
2	15,259	21,254	16,971	8,348	11,690	7,921	4,302	0	0	
3	16,022	22,317	17,819	8,766	12,275	8,317	4,517	0		
4	16,824	23,433	18,710	9,204	12,889	8,733	4,743			
5	17,665	24,605	19,646	9,664	13,533	9,169				
6	18,548	25,835	20,628	10,147	14,210					
7	19,475	27,126	21,659	10,655						
8	20,449	28,483	22,742							
9	21,472	29,907								
10	22,545									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Number Open Claims										
1	420	290	200	93	59	40	14	0	0	0
2	424	293	202	94	59	41	14	0	0	
3	428	296	204	95	60	41	15	0		
4	433	299	206	96	60	41	15			
5	437	302	208	97	61	42				
6	441	305	210	98	62					
7	446	308	212	99						
8	450	311	214							
9	455	314								
10	459									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Average Outstanding ($000)										
1	34.602	69.801	80.813	85.261	189.834	187.416	287.526	-	-	-
2	35.972	72.565	84.013	88.638	197.352	194.839	298.913	-	-	
3	37.397	75.439	87.340	92.149	205.168	202.555	310.752	-		
4	38.878	78.427	90.799	95.798	213.293	210.577	323.059			
5	40.418	81.533	94.395	99.592	221.741	218.917				
6	42.018	84.762	98.134	103.536	230.522					
7	43.683	88.118	102.020	107.637						
8	45.413	91.608	106.061							
9	47.211	95.236								
10	49.081									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Losses ($000)										
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	9,851	21,509	39,918	55,064	59,206	64,550				
6	10,344	22,584	41,914	57,817	62,166					
7	10,861	23,713	44,010	60,708						
8	11,404	24,899	46,210							
9	11,974	26,144								
10	12,573									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claims - Cumulative										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	743	758	758	
3	247	393	548	678	705	732	751	765		
4	250	397	554	684	712	739	758			
5	252	401	559	691	719	746				
6	255	405	565	698	727					
7	257	409	571	705						
8	260	413	576							
9	263	417								
10	265									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Average Paid Claim - Cumulative ($000)										
1	33.421	45.962	61.099	68.199	70.455	74.040	77.255	81.250	81.250	81.250
2	34.745	47.782	63.519	70.900	73.245	76.972	80.314	84.468	84.468	
3	36.121	49.674	66.034	73.707	76.146	80.021	83.495	87.813		
4	37.552	51.642	68.649	76.627	79.161	83.190	86.802			
5	39.039	53.687	71.368	79.661	82.296	86.485				
6	40.585	55.813	74.195	82.816	85.556					
7	42.192	58.024	77.133	86.096						
8	43.863	60.322	80.188							
9	45.600	62.711								
10	47.406									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: Stable Settlement and Reserve Patterns

Year	Age Interval in Months									
	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
Paid Losses - Incremental ($000)										
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4,097	0	0
2	8,510	10,070	15,903	13,083	3,578	4,616	3,922	4,302	0	
3	8,935	10,574	16,698	13,737	3,757	4,847	4,118	4,517		
4	9,382	11,102	17,533	14,424	3,945	5,089	4,324			
5	9,851	11,657	18,409	15,146	4,142	5,344				
6	10,344	12,240	19,330	15,903	4,349					
7	10,861	12,852	20,296	16,698						
8	11,404	13,495	21,311							
9	11,974	14,170								
10	12,573									

Year	Age Interval in Months									
	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
Paid Claims - Incremental										
1	243	143	153	127	27	26	19	14	0	0
2	245	144	154	128	27	26	19	14	0	
3	247	145	156	129	28	26	19	15		
4	250	147	157	131	28	27	19			
5	252	148	159	132	28	27				
6	255	150	160	133	28					
7	257	151	162	135						
8	260	153	164							
9	263	154								
10	265									

Year	Age Interval in Months									
	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
Average Paid Claim - Incremental ($000)										
1	33.421	67.303	99.314	98.306	125.750	169.749	201.892	287.526	-	
2	34.745	69.968	103.247	102.200	130.730	176.472	209.888	298.913	-	
3	36.121	72.739	107.336	106.247	135.907	183.461	218.200	310.752		
4	37.552	75.620	111.587	110.455	141.290	190.727	226.842			
5	39.039	78.615	116.006	114.829	146.885	198.280				
6	40.585	81.728	120.600	119.377	152.702					
7	42.192	84.965	125.376	124.105						
8	43.863	88.330	130.342							
9	45.600	91.828								
10	47.406									

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	996	1,010	1,010	
3	375	610	778	912	960	979	1,006	1,020		
4	379	616	786	921	970	989	1,016			
5	382	622	793	931	980	999				
6	386	628	801	940	989					
7	390	634	809	949						
8	394	641	818							
9	398	647								
10	402									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Reported Claims										
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closure Rate										
1	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000	1.000	1.000
2	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000	1.000	
3	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000		
4	0.467	0.673	0.792	0.906	0.941	0.960	0.986			
5	0.467	0.673	0.792	0.906	0.941	0.960				
6	0.467	0.673	0.792	0.906	0.941					
7	0.467	0.673	0.792	0.906						
8	0.467	0.673	0.792							
9	0.467	0.673								
10	0.467									

Scenario: Stable Settlement and Reserve Patterns

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claims										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	743	758	758	
3	247	393	548	678	705	732	751	765		
4	250	397	554	684	712	739	758			
5	252	401	559	691	719	746				
6	255	405	565	698	727					
7	257	409	571	705						
8	260	413	576							
9	263	417								
10	265									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	996	1,010	1,010	
3	375	610	778	912	960	979	1,006	1,020		
4	379	616	786	921	970	989	1,016			
5	382	622	793	931	980	999				
6	386	628	801	940	989					
7	390	634	809	949						
8	394	641	818							
9	398	647								
10	402									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claim Ratio										
1	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750	0.750	0.750
2	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750	0.750	
3	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750		
4	0.660	0.644	0.705	0.743	0.734	0.747	0.746			
5	0.660	0.644	0.705	0.743	0.734	0.747				
6	0.660	0.644	0.705	0.743	0.734					
7	0.660	0.644	0.705	0.743						
8	0.660	0.644	0.705							
9	0.660	0.644								
10	0.660									

Estimated Loss Development Pattern
Incurred Losses ($000)

Exhibit 3
Sheet 1

Scenario: Case Reserve Strengthening

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
1	22,638	37,938	49,003	53,252	59,843	60,649	60,938	60,938	60,938	60,938
2	23,769	39,834	51,453	55,914	62,835	63,681	63,984	63,984	63,984	
3	24,958	41,826	54,026	58,710	65,976	67,169	67,184	67,184		
4	26,206	43,917	56,727	61,645	69,612	70,528	70,543			
5	27,516	46,113	59,564	69,037	73,092	74,054				
6	28,892	48,419	65,317	72,489	76,747					
7	30,336	58,254	68,583	76,113						
8	43,057	61,167	72,012							
9	45,210	64,225								
10	47,471									

Year	Age Interval in Months									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	
2	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000		
3	1.676	1.292	1.087	1.124	1.018	1.000	1.000			
4	1.676	1.292	1.087	1.129	1.013	1.000				
5	1.676	1.292	1.159	1.059	1.013					
6	1.676	1.349	1.110	1.059						
7	1.920	1.177	1.110							
8	1.421	1.177								
9	1.421									
10										
Average Factors										
Simple Avg Last 3	1.587	1.235	1.126	1.082	1.015	1.002	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.623	1.257	1.110	1.099	1.014	1.002	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.548	1.227	1.125	1.080	1.015	1.002	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.591	1.254	1.102	1.102	1.013	1.003	1.000	1.000	1.000	1.000
Selected Factors	1.548	1.227	1.125	1.080	1.015	1.002	1.000	1.000	1.000	1.000
Factors to Ultimate	2.346	1.515	1.235	1.098	1.016	1.002	1.000	1.000	1.000	1.000

Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	$111,370	$97,312	$88,936	$83,560	$78,005	$74,176	$70,543	$67,184	$63,984	$60,938	$796,007
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$16,836	$7,279	$3,191	$1,898	$231	$106	$0	$0	$0	$0	$29,541

Estimated Loss Development Pattern
Paid Losses ($000)

Exhibit 3
Sheet 2

Scenario: ***Case Reserve Strengthening***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	9,851	21,509	39,918	55,064	59,206	64,550				
6	10,344	22,584	41,914	57,817	62,166					
7	10,861	23,713	44,010	60,708						
8	11,404	24,899	46,210							
9	11,974	26,144								
10	12,573									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	
2	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000		
3	2.183	1.856	1.379	1.075	1.090	1.070	1.072			
4	2.183	1.856	1.379	1.075	1.090	1.070				
5	2.183	1.856	1.379	1.075	1.090					
6	2.183	1.856	1.379	1.075						
7	2.183	1.856	1.379							
8	2.183	1.856								
9	2.183									
10										
Average Factors										
Simple Avg Last 3	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Simple Avg Last 5	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Simple Avg 3 of 5	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Selected Factors	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	1.000
Factors to Ultimate	7.519	3.444	1.856	1.345	1.251	1.147	1.072	1.000	1.000	1.000

Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0

Estimated Claim Development Pattern
Reported Claims

Exhibit 3
Sheet 3

Scenario: Case Reserve Strengthening

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	
2	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000		
3	1.127	1.085	1.026	1.013	1.000	1.000	1.000			
4	1.127	1.085	1.026	1.013	1.000	1.000				
5	1.127	1.085	1.026	1.013	1.000					
6	1.127	1.085	1.026	1.013						
7	1.127	1.085	1.026							
8	1.127	1.085								
9	1.127									
10										
Average Factors										
Simple Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Selected Factors	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Factors to Ultimate	1.270	1.127	1.039	1.013	1.000	1.000	1.000	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	0	0	0	0	0	0	0	0	0	0	0

Estimated Claim Development Pattern
Closed Claims

Exhibit 3
Sheet 4

Scenario: Case Reserve Strengthening

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	996	1,010	1,010	
3	375	610	778	912	960	979	1,006	1,020		
4	379	616	786	921	970	989	1,016			
5	382	622	793	931	980	999				
6	386	628	801	940	989					
7	390	634	809	949						
8	394	641	818							
9	398	647								
10	402									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	
2	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000		
3	1.626	1.276	1.173	1.053	1.020	1.027	1.014			
4	1.626	1.276	1.173	1.053	1.020	1.027				
5	1.626	1.276	1.173	1.053	1.020					
6	1.626	1.276	1.173	1.053						
7	1.626	1.276	1.173							
8	1.626	1.276								
9	1.626									
10										
Average Factors										
Simple Avg Last 3	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Simple Avg Last 5	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Simple Avg 3 of 5	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Selected Factors	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	1.000
Factors to Ultimate	2.721	1.674	1.311	1.118	1.062	1.042	1.014	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	0	0	0	0	0	0	0	0	0	0	0

Estimated Claim Development Pattern
Paid Claims (I.e., Claims Closed With Payment)

Exhibit 3
Sheet 5

*Scenario: **Case Reserve Strengthening***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	743	758	758	
3	247	393	548	678	705	732	751	765		
4	250	397	554	684	712	739	758			
5	252	401	559	691	719	746				
6	255	405	565	698	727					
7	257	409	571	705						
8	260	413	576							
9	263	417								
10	265									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	
2	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000		
3	1.588	1.396	1.236	1.041	1.037	1.026	1.019			
4	1.588	1.396	1.236	1.041	1.037	1.026				
5	1.588	1.396	1.236	1.041	1.037					
6	1.588	1.396	1.236	1.041						
7	1.588	1.396	1.236							
8	1.588	1.396								
9	1.588									
10										
Average Factors										
Simple Avg Last 3	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Simple Avg Last 5	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Simple Avg 3 of 5	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Selected Factors	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	1.000
Factors to Ultimate	3.093	1.948	1.395	1.129	1.085	1.046	1.019	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	820	812	804	796	788	780	773	765	758	750	7,847
Actual	820	812	804	796	788	780	773	765	758	750	7,847
Difference	0	0	0	0	0	0	0	0	0	0	0

Scenario: ***Case Reserve Strengthening***

Outstanding Losses ($000)

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	14,533	20,242	16,163	7,951	11,134	7,544	4,097	0	0	0
2	15,259	21,254	16,971	8,348	11,690	7,921	4,302	0	0	
3	16,022	22,317	17,819	8,766	12,275	8,621	4,517	0		
4	16,824	23,433	18,710	9,204	13,225	9,052	4,743			
5	17,665	24,605	19,646	13,974	13,886	9,504				
6	18,548	25,835	23,403	14,672	14,581					
7	19,475	34,541	24,573	15,406						
8	31,653	36,268	25,802							
9	33,236	38,081								
10	34,898									

Number Open Claims

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	420	290	200	93	59	40	14	0	0	0
2	424	293	202	94	59	41	14	0	0	
3	428	296	204	95	60	41	15	0		
4	433	299	206	96	60	41	15			
5	437	302	208	97	61	42				
6	441	305	210	98	62					
7	446	308	212	99						
8	450	311	214							
9	455	314								
10	459									

Average Outstanding ($000)

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	34.602	69.801	80.813	85.261	189.834	187.416	287.526	-	-	-
2	35.972	72.565	84.013	88.638	197.352	194.839	298.913	-	-	
3	37.397	75.439	87.340	92.149	205.168	209.956	310.752	-		
4	38.878	78.427	90.799	95.798	218.861	218.271	323.059			
5	40.418	81.533	94.395	144.004	227.529	226.915				
6	42.018	84.762	111.335	149.707	236.540					
7	43.683	112.204	115.744	155.636						
8	70.294	116.648	120.328							
9	73.078	121.267								
10	75.972									

Annual Percent Change

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	7.8%	4.0%	-		
4	4.0%	4.0%	4.0%	4.0%	6.7%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	50.3%	4.0%	4.0%				
6	4.0%	4.0%	17.9%	4.0%	4.0%					
7	4.0%	32.4%	4.0%	4.0%						
8	60.9%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: ***Case Reserve Strengthening***

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Paid Losses ($000)										
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	9,851	21,509	39,918	55,064	59,206	64,550				
6	10,344	22,584	41,914	57,817	62,166					
7	10,861	23,713	44,010	60,708						
8	11,404	24,899	46,210							
9	11,974	26,144								
10	12,573									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Paid Claims - Cumulative										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	743	758	758	
3	247	393	548	678	705	732	751	765		
4	250	397	554	684	712	739	758			
5	252	401	559	691	719	746				
6	255	405	565	698	727					
7	257	409	571	705						
8	260	413	576							
9	263	417								
10	265									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Average Paid Claim - Cumulative ($000)										
1	33.421	45.962	61.099	68.199	70.455	74.040	77.255	81.250	81.250	81.250
2	34.745	47.782	63.519	70.900	73.245	76.972	80.314	84.468	84.468	
3	36.121	49.674	66.034	73.707	76.146	80.021	83.495	87.813		
4	37.552	51.642	68.649	76.627	79.161	83.190	86.802			
5	39.039	53.687	71.368	79.661	82.296	86.485				
6	40.585	55.813	74.195	82.816	85.556					
7	42.192	58.024	77.133	86.096						
8	43.863	60.322	80.188							
9	45.600	62.711								
10	47.406									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario:	***Case Reserve Strengthening***

Year	Age Interval in Months									
	0-12	**12-24**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-96**	**96-108**	**108-120**
Paid Losses - Incremental ($000)										
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4,097	0	0
2	8,510	10,070	15,903	13,083	3,578	4,616	3,922	4,302	0	
3	8,935	10,574	16,698	13,737	3,757	4,847	4,118	4,517		
4	9,382	11,102	17,533	14,424	3,945	5,089	4,324			
5	9,851	11,657	18,409	15,146	4,142	5,344				
6	10,344	12,240	19,330	15,903	4,349					
7	10,861	12,852	20,296	16,698						
8	11,404	13,495	21,311							
9	11,974	14,170								
10	12,573									

Year	Age Interval in Months									
	0-12	**12-24**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-96**	**96-108**	**108-120**
Paid Claims - Incremental										
1	243	143	153	127	27	26	19	14	0	0
2	245	144	154	128	27	26	19	14	0	
3	247	145	156	129	28	26	19	15		
4	250	147	157	131	28	27	19			
5	252	148	159	132	28	27				
6	255	150	160	133	28					
7	257	151	162	135						
8	260	153	164							
9	263	154								
10	265									

Year	Age Interval in Months									
	0-12	**12-24**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-96**	**96-108**	**108-120**
Average Paid Claim - Incremental ($000)										
1	33.421	67.303	99.314	98.306	125.750	169.749	201.892	287.526	-	-
2	34.745	69.968	103.247	102.200	130.730	176.472	209.888	298.913	-	
3	36.121	72.739	107.336	106.247	135.907	183.461	218.200	310.752		
4	37.552	75.620	111.587	110.455	141.290	190.727	226.842			
5	39.039	78.615	116.006	114.829	146.885	198.280				
6	40.585	81.728	120.600	119.377	152.702					
7	42.192	84.965	125.376	124.105						
8	43.863	88.330	130.342							
9	45.600	91.828								
10	47.406									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: Case Reserve Strengthening

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	996	1,010	1,010	
3	375	610	778	912	960	979	1,006	1,020		
4	379	616	786	921	970	989	1,016			
5	382	622	793	931	980	999				
6	386	628	801	940	989					
7	390	634	809	949						
8	394	641	818							
9	398	647								
10	402									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Reported Claims										
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closure Rate										
1	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000	1.000	1.000
2	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000	1.000	
3	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000		
4	0.467	0.673	0.792	0.906	0.941	0.960	0.986			
5	0.467	0.673	0.792	0.906	0.941	0.960				
6	0.467	0.673	0.792	0.906	0.941					
7	0.467	0.673	0.792	0.906						
8	0.467	0.673	0.792							
9	0.467	0.673								
10	0.467									

Scenario: ***Case Reserve Strengthening***

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Paid Claims										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	743	758	758	
3	247	393	548	678	705	732	751	765		
4	250	397	554	684	712	739	758			
5	252	401	559	691	719	746				
6	255	405	565	698	727					
7	257	409	571	705						
8	260	413	576							
9	263	417								
10	265									

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	996	1,010	1,010	
3	375	610	778	912	960	979	1,006	1,020		
4	379	616	786	921	970	989	1,016			
5	382	622	793	931	980	999				
6	386	628	801	940	989					
7	390	634	809	949						
8	394	641	818							
9	398	647								
10	402									

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Paid Claim Ratio										
1	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750	0.750	0.750
2	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750	0.750	
3	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750		
4	0.660	0.644	0.705	0.743	0.734	0.747	0.746			
5	0.660	0.644	0.705	0.743	0.734	0.747				
6	0.660	0.644	0.705	0.743	0.734					
7	0.660	0.644	0.705	0.743						
8	0.660	0.644	0.705							
9	0.660	0.644								
10	0.660									

Scenario: Case Reserve Strengthening

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	Projected Ultimate Losses ($000) (Incurred LDF)	12	24	36	48	60	72	84	96	108	120
1	$60,938	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	63,984	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	67,184	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	70,543	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	74,176	9,851	21,509	39,918	55,064	59,206	64,550				
6	78,005	10,344	22,584	41,914	57,817	62,166					
7	83,560	10,861	23,713	44,010	60,708						
8	88,936	11,404	24,899	46,210							
9	97,312	11,974	26,144								
10	111,370	12,573									
	$796,007										

(B) Implied Ultimate Unpaid Losses at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	52,833	43,242	28,097	15,637	12,229	7,832	4,097	0	0	0
2	55,474	45,404	29,502	16,418	12,840	8,224	4,302	0	0	
3	58,248	47,675	30,977	17,239	13,482	8,635	4,517	0		
4	61,161	50,058	32,526	18,101	14,156	9,067	4,743			
5	64,325	52,668	34,258	19,113	14,971	9,627				
6	67,661	55,421	36,091	20,188	15,839					
7	72,699	59,846	39,550	22,852						
8	77,532	64,037	42,726							
9	85,337	71,168								
10	98,797									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ultimate Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	996	1,010	1,010	
3	1,020	375	610	778	912	960	979	1,006	1,020		
4	1,030	379	616	786	921	970	989	1,016			
5	1,041	382	622	793	931	980	999				
6	1,051	386	628	801	940	989					
7	1,062	390	634	809	949						
8	1,072	394	641	818							
9	1,083	398	647								
10	1,094	402									
	10,462										

(D) Implied Ultimate Unclosed Claims at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	633	403	238	106	59	40	14	0	0	0
2	639	407	240	107	59	41	14	0	0	
3	645	411	242	108	60	41	15	0		
4	652	415	245	109	60	41	15			
5	658	419	247	110	61	42				
6	665	423	250	111	62					
7	671	427	252	112						
8	678	432	255							
9	685	436								
10	692									

(E) Implied Ultimate Unclosed Claim Severity at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	**83.530**	**107.434**	**118.303**	**147.863**	**208.504**	**194.590**	**287.526**	-	-	-
2	**86.838**	**111.689**	**122.988**	**153.719**	**216.761**	**202.297**	298 913	-	-	
3	**90.277**	**116.112**	**127.859**	**159.807**	**225.346**	210.308	310 752	-		
4	**93.853**	**120.711**	**132.922**	**166.136**	234 271	218.637	323.059			
5	**97.731**	**125.745**	**138.617**	173 683	245 293	229.838				
6	**101.782**	**131.008**	144.586	181 638	256 947					
7	**108.277**	140.069	156 875	203 571						
8	114 333	148.394	167 795							
9	124.597	163.285								
10	142 821									
(F) Fitted Last Diagonal:	$121.279	$147.359	$155.845	$186.667	$253.194	$227.296	$323.059	-	-	-

(G) Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Implied Ultimate Outstanding:	$83,896	$64,226	$39,683	$20,954	$15,607	$9,520	$4,743	$0	$0	$0	
Paid to Date:	12,573	26,144	46,210	60,708	62,166	64,550	65,800	67,184	63,984	60,938	
Implied Ultimate Losses:	$96,469	$90,370	$85,893	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$768,886
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	$1,935	$338	$148	$0	$0	$0	$0	$0	$0	$0	$2,421

Scenario: ***Case Reserve Strengthening***

Year	Projected Ultimate Losses ($000) (Paid LDF)										
		(A) Paid Losses at Evaluation Age in Months ($000s)									
		12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
1	$60,938	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	63,984	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	67,184	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	70,543	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	74,070	9,851	21,509	39,918	55,064	59,206	64,550				
6	77,773	10,344	22,584	41,914	57,817	62,166					
7	81,662	10,861	23,713	44,010	60,708						
8	85,745	11,404	24,899	46,210							
9	90,032	11,974	26,144								
10	94,534	12,573									
	$766,465										

(B) Implied Ultimate Unpaid Losses at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	52,833	43,242	28,097	15,637	12,229	7,832	4,097	0	0	0
2	55,474	45,404	29,502	16,418	12,840	8,224	4,302	0	0	
3	58,248	47,675	30,977	17,239	13,482	8,635	4,517	0		
4	61,161	50,058	32,526	18,101	14,156	9,067	4,743			
5	64,219	52,561	34,152	19,006	14,864	9,520				
6	67,430	55,189	35,860	19,957	15,607					
7	70,801	57,949	37,652	20,954						
8	74,341	60,846	39,535							
9	78,058	63,888								
10	81,961									

Year	Projected Ultimate Reported Claims										
		(C) Closed Claims at Evaluation Age in Months									
		12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	996	1,010	1,010	
3	1,020	375	610	778	912	960	979	1,006	1,020		
4	1,030	379	616	786	921	970	989	1,016			
5	1,041	382	622	793	931	980	999				
6	1,051	386	628	801	940	989					
7	1,062	390	634	809	949						
8	1,072	394	641	818							
9	1,083	398	647								
10	1,094	402									
	10,462										

(D) Implied Ultimate Unclosed Claims at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	633	403	238	106	59	40	14	0	0	0
2	639	407	240	107	59	41	14	0	0	
3	645	411	242	108	60	41	15	0		
4	652	415	245	109	60	41	15			
5	658	419	247	110	61	42				
6	665	423	250	111	62					
7	671	427	252	112						
8	678	432	255							
9	685	436								
10	692									

(E) Implied Ultimate Unclosed Claim Severity at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	**83.530**	**107.434**	**118.303**	**147.863**	**208.504**	**194.590**	**287.526**	-	-	-
2	**86.838**	**111.689**	**122.988**	**153.719**	**216.761**	**202.297**	298 913	-	-	
3	**90.277**	**116.112**	**127.859**	**159.807**	**225.346**	210 308	310 752	-		
4	**93.853**	**120.711**	**132.922**	**166.136**	234 271	218 637	323 059			
5	**97.570**	**125.491**	**138.187**	172 716	243 549	227 296				
6	**101.434**	**130.461**	143 659	179 556	253 194					
7	**105.451**	135 628	149 349	186 667						
8	109 627	140 999	155 264							
9	113 969	146 584								
10	118 483									
(F) Fitted Last Diagonal:	**$118.483**	**$146.584**	**$155.264**	**$186.667**	**$253.194**	**$227.296**	**$323.059**	-	-	-

(G) Ultimate Losses	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Implied Ultimate Outstanding:	$81,961	$63,888	$39,535	$20,954	$15,607	$9,520	$4,743	$0	$0	$0	
Paid to Date:	12,573	26,144	46,210	60,708	62,166	64,550	65,800	67,184	63,984	60,938	
Implied Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0

Scenario: Case Reserve Strengthening

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	59,682	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	58,549	62,666	67,184		
4	9,382	20,485	38,017	52,441	56,386	61,476	65,800			
5	9,851	21,509	39,918	55,064	59,206	64,550				
6	10,344	22,584	41,914	57,817	62,166					
7	10,861	23,713	44,010	60,708						
8	11,404	24,899	46,210							
9	11,974	26,144								
10	12,573									

(B) Incremental Paid Losses in Age Interval in Months ($000s)

Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4,097	0	0
2	8,510	10,070	15,903	13,083	3,578	4,616	3,922	4,302	0	
3	8,935	10,574	16,698	13,737	3,757	4,847	4,118	4,517		
4	9,382	11,102	17,533	14,424	3,945	5,089	4,324			
5	9,851	11,657	18,409	15,146	4,142	5,344				
6	10,344	12,240	19,330	15,903	4,349					
7	10,861	12,852	20,296	16,698						
8	11,404	13,495	21,311							
9	11,974	14,170								
10	12,573									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ult Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	996	1,010	1,010	
3	1,020	375	610	778	912	960	979	1,006	1,020		
4	1,030	379	616	786	921	970	989	1,016			
5	1,041	382	622	793	931	980	999				
6	1,051	386	628	801	940	989					
7	1,062	390	634	809	949						
8	1,072	394	641	818							
9	1,083	398	647								
10	1,094	402									
	10,462										

(D) Incremental Closed Claims in Age Interval in Months

Year	Est'd Unclosed Claims	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Implied Future Closed Claims
1	0	368	230	165	132	47	18	26	14	0	0	0
2	0	371	232	167	133	48	19	26	14	0	**0**	0
3	0	375	235	168	134	48	19	27	15	**0**	**0**	0
4	15	379	237	170	136	49	19	27	**15**	**0**	**0**	15
5	42	382	239	172	137	49	19	**27**	**15**	**0**	**0**	42
6	62	386	242	173	138	50	**19**	**27**	**15**	**0**	**0**	62
7	112	390	244	175	140	**50**	**20**	**28**	**15**	**0**	**0**	112
8	255	394	247	177	**141**	**50**	**20**	**28**	**15**	**0**	**0**	255
9	436	398	249	**179**	**143**	**51**	**20**	**28**	**15**	**0**	**0**	436
10	692	402	**252**	**180**	**144**	**52**	**20**	**28**	**16**	**0**	**0**	692
	1,613											

(E) Incremental Paid Claim Severity in Age Interval in Months ($000s)

Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Implied Future Severity
1	22 054	41 698	91.790	94.575	72.353	238.940	143.654	287.526	-	-	-
2	22 927	43 350	95 425	98.321	75.218	248.403	149.343	298 913	-	-	-
3	23.835	45 067	99.204	102.215	78.197	258.241	155 258	310.752	-	-	-
4	24 779	46.851	103.133	106 263	81 294	268.468	161 407	**323.059**	-	-	323.059
5	25.760	48.707	107.218	110.471	84 514	279.101	**167.799**	**336.853**	-	-	227.296
6	26.780	50.636	111.464	114.847	87.861	**290.154**	**174.444**	**349.154**	-	-	253.194
7	27.841	52 641	115.878	119.395	**91.340**	**301.646**	**181.353**	**362.982**	-	-	186 667
8	28 944	54 726	120.467	**124.123**	**94.958**	**313.592**	**188.535**	**377.358**	-	-	155 264
9	30.090	56.893	**125.238**	**129.039**	**98.718**	**326.011**	**196.002**	**392.302**	-	-	146 584
10	31.282	**59.147**	**130.198**	**134.150**	**102.628**	**338.923**	**203.765**	**407.839**	-	-	118.483

(F) Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Implied Future Payments:	$81,961	$63,888	$39,535	$20,954	$15,607	$9,520	$4,743	$0	$0	$0	
Paid to Date:	12,573	26,144	46,210	60,708	62,166	64,550	65,800	67,184	63,984	60,938	
Implied Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0

Summary of Estimates **Exhibit 7**

Scenario: ***Case Reserve Strengthening***

Year	Actual Ultimate Losses	Loss Development Technique: Incurred Losses	Loss Development Technique: Paid Losses	Ultimate Unclosed Claim Severity Technique on: Incurred Losses	Ultimate Unclosed Claim Severity Technique on: Paid Losses	Incremental Closed Claim Severity
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Estimated Ultimate Losses						
1	$60,938	$60,938	$60,938	$60,938	$60,938	$60,938
2	63,984	63,984	63,984	63,984	63,984	63,984
3	67,184	67,184	67,184	67,184	67,184	67,184
4	70,543	70,543	70,543	70,543	70,543	70,543
5	74,070	74,176	74,070	74,070	74,070	74,070
6	77,773	78,005	77,773	77,773	77,773	77,773
7	81,662	83,560	81,662	81,662	81,662	81,662
8	85,745	88,936	85,745	85,893	85,745	85,745
9	90,032	97,312	90,032	90,370	90,032	90,032
10	94,534	111,370	94,534	96,469	94,534	94,534
Total	$766,465	$796,007	$766,465	$768,886	$766,465	$766,465
Difference: Estimated vs Actual						
1		$0	$0	$0	$0	$0
2		0	0	0	0	0
3		0	0	0	0	0
4		0	0	0	0	0
5		106	0	0	0	0
6		231	0	0	0	0
7		1,898	0	0	0	0
8		3,191	0	148	0	0
9		7,279	0	338	0	0
10		16,836	0	1,935	0	0
Total		$29,541	$0	$2,421	$0	$0

Notes:

(2) Based on hypothetical assumptions.
(3) Ultimate losses from Exhibit 3, Sheet 1. Difference = (3) minus (2)
(4) Ultimate losses from Exhibit 3, Sheet 2. Difference = (4) minus (2)
(5) Ultimate losses from Exhibit 5, Sheet 1. Difference = (5) minus (2)
(6) Ultimate losses from Exhibit 5, Sheet 2. Difference = (6) minus (2)
(7) Ultimate losses from Exhibit 6. Difference = (7) minus (2).

Estimated Loss Development Pattern
Incurred Losses ($000)

Exhibit 8
Sheet 1

Scenario: Settlement Rate Acceleration

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	22,638	37,938	49,003	53,252	59,843	60,649	60,938	60,938	60,938	60,938
2	23,769	39,834	51,453	55,914	62,835	63,681	63,984	63,984	63,984	
3	24,958	41,826	54,026	58,710	65,976	66,865	67,184	67,184		
4	26,206	43,917	56,727	61,645	69,275	70,209	70,543			
5	27,516	46,113	59,564	64,728	72,739	73,719				
6	28,892	48,419	62,542	67,964	76,376					
7	30,336	50,840	65,669	71,362						
8	31,853	53,382	68,952							
9	33,446	56,051								
10	35,118									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	
2	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000		
3	1.676	1.292	1.087	1.124	1.013	1.005	1.000			
4	1.676	1.292	1.087	1.124	1.013	1.005				
5	1.676	1.292	1.087	1.124	1.013					
6	1.676	1.292	1.087	1.124						
7	1.676	1.292	1.087							
8	1.676	1.292								
9	1.676									
10										
Average Factors										
Simple Avg Last 3	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Selected Factors	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	1.000
Factors to Ultimate	2.692	1.606	1.244	1.144	1.018	1.005	1.000	1.000	1.000	1.000

Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0

Estimated Loss Development Pattern
Paid Losses ($000)

Exhibit 8
Sheet 2

Scenario: ***Settlement Rate Acceleration***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	9,851	21,509	39,918	57,135	61,878	66,820				
6	10,344	22,584	49,865	59,991	64,972					
7	10,861	33,861	52,359	62,991						
8	18,152	35,555	54,977							
9	19,059	37,332								
10	20,012									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	2.183	1.856	1.379	1.075	1.090	1.070	1.072	1.000	1.000	
2	2.183	1.856	1.379	1.075	1.090	1.109	1.035	1.000		
3	2.183	1.856	1.379	1.075	1.129	1.071	1.035			
4	2.183	1.856	1.379	1.124	1.080	1.071				
5	2.183	1.856	1.431	1.083	1.080					
6	2.183	2.208	1.203	1.083						
7	3.118	1.546	1.203							
8	1.959	1.546								
9	1.959									
10										
Average Factors										
Simple Avg Last 3	2.345	1.767	1.279	1.097	1.096	1.084	1.047	1.000	1.000	1.000
Simple Avg Last 5	2.280	1.802	1.319	1.088	1.094	1.080	1.047	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	2.221	1.709	1.267	1.096	1.095	1.083	1.046	1.000	1.000	1.000
Simple Avg 3 of 5	2.108	1.753	1.321	1.080	1.087	1.084	1.047	1.000	1.000	1.000
Selected Factors	2.221	1.709	1.267	1.096	1.095	1.083	1.046	1.000	1.000	1.000
Factors to Ultimate	6.535	2.943	1.722	1.359	1.241	1.133	1.046	1.000	1.000	1.000

Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	$130,780	$109,865	$94,687	$85,617	$80,602	$75,712	$71,330	$67,184	$63,984	$60,938	$840,698
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$36,246	$19,833	$8,942	$3,954	$2,829	$1,642	$787	$0	$0	$0	$74,233

Estimated Claim Development Pattern
Reported Claims

Exhibit 8
Sheet 3

Scenario: Settlement Rate Acceleration

Year	12	24	36	48	60	72	84	96	108	120
	Evaluation Age in Months									
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
	Age Interval in Months									
1	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	
2	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000		
3	1.127	1.085	1.026	1.013	1.000	1.000	1.000			
4	1.127	1.085	1.026	1.013	1.000	1.000				
5	1.127	1.085	1.026	1.013	1.000					
6	1.127	1.085	1.026	1.013						
7	1.127	1.085	1.026							
8	1.127	1.085								
9	1.127									
10										
Average Factors										
Simple Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Selected Factors	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Factors to Ultimate	1.270	1.127	1.039	1.013	1.000	1.000	1.000	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	0	0	0	0	0	0	0	0	0	0	0

Estimated Claim Development Pattern
Closed Claims

Exhibit 8
Sheet 4

Scenario: ***Settlement Rate Acceleration***

Year	Evaluation Age in Months: 12	24	36	48	60	72	84	96	108	120
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	1,003	1,010	1,010	
3	375	610	778	912	960	988	1,013	1,020		
4	379	616	786	921	983	998	1,023			
5	382	622	793	945	993	1,008				
6	386	628	868	954	1,003					
7	390	715	877	964						
8	470	722	885							
9	475	730								
10	480									

Year	Age Interval in Months: 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	
2	1.626	1.276	1.173	1.053	1.020	1.035	1.007	1.000		
3	1.626	1.276	1.173	1.053	1.029	1.025	1.007			
4	1.626	1.276	1.173	1.067	1.015	1.025				
5	1.626	1.276	1.191	1.051	1.015					
6	1.626	1.382	1.099	1.051						
7	1.833	1.226	1.099							
8	1.536	1.226								
9	1.536									
10										
Average Factors										
Simple Avg Last 3	1.635	1.278	1.130	1.057	1.020	1.028	1.010	1.000	1.000	1.000
Simple Avg Last 5	1.631	1.277	1.147	1.055	1.020	1.028	1.010	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.623	1.273	1.128	1.056	1.020	1.028	1.010	1.000	1.000	1.000
Simple Avg 3 of 5	1.596	1.259	1.148	1.052	1.018	1.029	1.010	1.000	1.000	1.000
Selected Factors	1.623	1.273	1.128	1.056	1.020	1.028	1.010	1.000	1.000	1.000
Factors to Ultimate	2.605	1.606	1.261	1.118	1.058	1.038	1.010	1.000	1.000	1.000

Ultimate Counts	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Projected	1,250	1,171	1,117	1,078	1,062	1,046	1,033	1,020	1,010	1,000	10,787
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	157	89	44	16	11	6	2	0	0	0	324

Estimated Claim Development Pattern
Paid Claims (I.e., Claims Closed With Payment)

Exhibit 8
Sheet 5

Scenario: ***Settlement Rate Acceleration***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	750	758	758	
3	247	393	548	678	705	741	758	765		
4	250	397	554	684	726	749	765			
5	252	401	559	705	733	756				
6	255	405	632	712	740					
7	257	490	638	719						
8	336	495	644							
9	340	499								
10	343									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	
2	1.588	1.396	1.236	1.041	1.037	1.036	1.010	1.000		
3	1.588	1.396	1.236	1.041	1.051	1.023	1.010			
4	1.588	1.396	1.236	1.060	1.032	1.023				
5	1.588	1.396	1.261	1.039	1.032					
6	1.588	1.561	1.128	1.039						
7	1.902	1.303	1.128							
8	1.470	1.303								
9	1.470									
10										
Average Factors										
Simple Avg Last 3	1.614	1.389	1.172	1.046	1.038	1.027	1.013	1.000	1.000	1.000
Simple Avg Last 5	1.604	1.392	1.198	1.044	1.038	1.027	1.013	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.589	1.378	1.169	1.046	1.038	1.027	1.013	1.000	1.000	1.000
Simple Avg 3 of 5	1.548	1.365	1.200	1.040	1.035	1.028	1.013	1.000	1.000	1.000
Selected Factors	1.589	1.378	1.169	1.046	1.038	1.027	1.013	1.000	1.000	1.000
Factors to Ultimate	2.889	1.818	1.319	1.129	1.079	1.040	1.013	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	991	908	850	812	799	786	775	765	758	750	8,195
Actual	820	812	804	796	788	780	773	765	758	750	7,847
Difference	171	96	46	16	11	6	2	0	0	0	348

Diagnostics **Exhibit 9**
Average Outstanding **Sheet 1**

Scenario: Settlement Rate Acceleration

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Outstanding Losses ($000)										
1	14,533	20,242	16,163	7,951	11,134	7,544	4,097	0	0	0
2	15,259	21,254	16,971	8,348	11,690	7,921	2,151	0	0	
3	16,022	22,317	17,819	8,766	12,275	6,258	2,259	0		
4	16,824	23,433	18,710	9,204	10,344	6,571	2,372			
5	17,665	24,605	19,646	7,593	10,861	6,899				
6	18,548	25,835	12,676	7,973	11,404					
7	19,475	16,978	13,310	8,371						
8	13,702	17,827	13,976							
9	14,387	18,719								
10	15,106									

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Number Open Claims										
1	420	290	200	93	59	40	14	0	0	0
2	424	293	202	94	59	41	7	0	0	
3	428	296	204	95	60	32	7	0		
4	433	299	206	96	47	32	7			
5	437	302	208	83	48	32				
6	441	305	144	84	48					
7	446	227	145	85						
8	374	229	146							
9	378	231								
10	381									

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Average Outstanding ($000)										
1	34.602	69.801	80.813	85.261	189.834	187.416	287.526	-	-	-
2	35.972	72.565	84.013	88.638	197.352	194.839	298.913	-	-	
3	37.397	75.439	87.340	92.149	205.168	197.887	310.752	-		
4	38.878	78.427	90.799	95.798	219.688	205.724	323.059			
5	40.418	81.533	94.395	91.551	228.389	213.871				
6	42.018	84.762	88.280	95.177	237.434					
7	43.683	74.828	91.776	98.947						
8	36.644	77.791	95.411							
9	38.096	80.872								
10	39.605									

	Evaluation Age in Months									
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	1.6%	4.0%	-		
4	4.0%	4.0%	4.0%	4.0%	7.1%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	-4.4%	4.0%	4.0%				
6	4.0%	4.0%	-6.5%	4.0%	4.0%					
7	4.0%	-11.7%	4.0%	4.0%						
8	-16.1%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: *Settlement Rate Acceleration*

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Losses ($000)										
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	9,851	21,509	39,918	57,135	61,878	66,820				
6	10,344	22,584	49,865	59,991	64,972					
7	10,861	33,861	52,359	62,991						
8	18,152	35,555	54,977							
9	19,059	37,332								
10	20,012									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claims - Cumulative										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	750	758	758	
3	247	393	548	678	705	741	758	765		
4	250	397	554	684	726	749	765			
5	252	401	559	705	733	756				
6	255	405	632	712	740					
7	257	490	638	719						
8	336	495	644							
9	340	499								
10	343									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Average Paid Claim - Cumulative ($000)										
1	33.421	45.962	61.099	68.199	70.455	74.040	77.255	81.250	81.250	81.250
2	34.745	47.782	63.519	70.900	73.245	76.972	82.411	84.468	84.468	
3	36.121	49.674	66.034	73.707	76.146	81.780	85.675	87.813		
4	37.552	51.642	68.649	76.627	81.213	85.019	89.068			
5	39.039	53.687	71.368	81.005	84.429	88.386				
6	40.585	55.813	78.960	84.213	87.773					
7	42.192	69.158	82.087	87.548						
8	53.961	71.897	85.338							
9	56.098	74.744								
10	58.320									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	6.7%	4.0%	4.0%	
3	4.0%	4.0%	4.0%	4.0%	4.0%	6.2%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	6.7%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	5.7%	4.0%	4.0%				
6	4.0%	4.0%	10.6%	4.0%	4.0%					
7	4.0%	23.9%	4.0%	4.0%						
8	27.9%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: ***Settlement Rate Acceleration***

Year	Age Interval in Months									
	0-12	**12-24**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-96**	**96-108**	**108-120**
Paid Losses - Incremental ($000)										
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4,097	0	0
2	8,510	10,070	15,903	13,083	3,578	4,616	6,073	2,151	0	
3	8,935	10,574	16,698	13,737	3,757	6,906	4,318	2,259		
4	9,382	11,102	17,533	14,424	6,490	4,707	4,533			
5	9,851	11,657	18,409	17,217	4,743	4,942				
6	10,344	12,240	27,281	10,126	4,980					
7	10,861	23,000	18,497	10,632						
8	18,152	17,403	19,422							
9	19,059	18,273								
10	20,012									

Year	Age Interval in Months									
	0-12	**12-24**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-96**	**96-108**	**108-120**
Paid Claims - Incremental										
1	243	143	153	127	27	26	19	14	0	0
2	245	144	154	128	27	26	26	7	0	
3	247	145	156	129	28	36	17	7		
4	250	147	157	131	41	23	17			
5	252	148	159	146	28	23				
6	255	150	227	81	28					
7	257	232	148	82						
8	336	158	150							
9	340	160								
10	343									

Year	Age Interval in Months									
	0-12	**12-24**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-96**	**96-108**	**108-120**
Average Paid Claim - Incremental ($000)										
1	33.421	67.303	99.314	98.306	125.750	169.749	201.892	287.526	-	-
2	34.745	69.968	103.247	102.200	130.730	176.472	234.641	298.913	-	
3	36.121	72.739	107.336	106.247	135.907	192.603	258.471	310.752		
4	37.552	75.620	111.587	110.455	157.275	205.774	268.707			
5	39.039	78.615	116.006	117.925	172.001	213.924				
6	40.585	81.728	120.241	125.247	178.813					
7	42.192	99.051	124.799	130.207						
8	53.961	110.048	129.742							
9	56.098	114.406								
10	58.320									

Year	Evaluation Age in Months									
	12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	16.2%	4.0%	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	9.1%	10.2%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	15.7%	6.8%	4.0%			
5	4.0%	4.0%	4.0%	6.8%	9.4%	4.0%				
6	4.0%	4.0%	3.7%	6.2%	4.0%					
7	4.0%	21.2%	3.8%	4.0%						
8	27.9%	11.1%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: ***Settlement Rate Acceleration***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	1,003	1,010	1,010	
3	375	610	778	912	960	988	1,013	1,020		
4	379	616	786	921	983	998	1,023			
5	382	622	793	945	993	1,008				
6	386	628	868	954	1,003					
7	390	715	877	964						
8	470	722	885							
9	475	730								
10	480									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Reported Claims										
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closure Rate										
1	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000	1.000	1.000
2	0.467	0.673	0.792	0.906	0.941	0.960	0.993	1.000	1.000	
3	0.467	0.673	0.792	0.906	0.941	0.969	0.993	1.000		
4	0.467	0.673	0.792	0.906	0.954	0.969	0.993			
5	0.467	0.673	0.792	0.919	0.954	0.969				
6	0.467	0.673	0.858	0.919	0.954					
7	0.467	0.759	0.858	0.919						
8	0.557	0.759	0.858							
9	0.557	0.759								
10	0.557									

Scenario: Settlement Rate Acceleration

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claims										
1	243	385	536	664	691	717	736	750	750	750
2	245	389	543	671	698	724	750	758	758	
3	247	393	548	678	705	741	758	765		
4	250	397	554	684	726	749	765			
5	252	401	559	705	733	756				
6	255	405	632	712	740					
7	257	490	638	719						
8	336	495	644							
9	340	499								
10	343									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	1,003	1,010	1,010	
3	375	610	778	912	960	988	1,013	1,020		
4	379	616	786	921	983	998	1,023			
5	382	622	793	945	993	1,008				
6	386	628	868	954	1,003					
7	390	715	877	964						
8	470	722	885							
9	475	730								
10	480									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claim Ratio										
1	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750	0.750	0.750
2	0.660	0.644	0.705	0.743	0.734	0.747	0.748	0.750	0.750	
3	0.660	0.644	0.705	0.743	0.734	0.750	0.748	0.750		
4	0.660	0.644	0.705	0.743	0.738	0.750	0.748			
5	0.660	0.644	0.705	0.747	0.738	0.750				
6	0.660	0.644	0.728	0.747	0.738					
7	0.660	0.685	0.728	0.747						
8	0.715	0.685	0.728							
9	0.715	0.685								
10	0.715									

Scenario: Settlement Rate Acceleration

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	Projected Ultimate Losses ($000) (Incurred LDF)	12	24	36	48	60	72	84	96	108	120
1	$60,938	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	63,984	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	67,184	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	70,543	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	74,070	9,851	21,509	39,918	57,135	61,876	66,820				
6	77,773	10,344	22,584	49,865	59,991	64,972					
7	81,662	10,861	33,861	52,359	62,991						
8	85,745	18,152	35,555	54,977							
9	90,032	19,059	37,332								
10	94,534	20,012									
	$766,465										

(B) Implied Ultimate Unpaid Losses at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	52,833	43,242	28,097	15,637	12,229	7,832	4,097	0	0	0
2	55,474	45,404	29,502	16,418	12,840	8,224	2,151	0	0	
3	58,248	47,675	30,977	17,239	13,482	6,576	2,259	0		
4	61,161	50,058	32,526	18,101	11,612	6,905	2,372			
5	64,219	52,561	34,152	16,935	12,192	7,250				
6	67,430	55,189	27,908	17,782	12,802					
7	70,801	47,801	29,303	18,671						
8	67,594	50,191	30,769							
9	70,973	52,700								
10	74,522									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ultimate Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	1,003	1,010	1,010	
3	1,020	375	610	778	912	960	988	1,013	1,020		
4	1,030	379	616	786	921	983	998	1,023			
5	1,041	382	622	793	945	993	1,008				
6	1,051	386	628	868	954	1,003					
7	1,062	390	715	877	964						
8	1,072	470	722	885							
9	1,083	475	730								
10	1,094	480									
	10,462										

(D) Implied Ultimate Unclosed Claims at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	633	403	238	106	59	40	14	0	0	0
2	639	407	240	107	59	41	7	0	0	
3	645	411	242	108	60	32	7	0		
4	652	415	245	109	47	32	7			
5	658	419	247	96	48	32				
6	665	423	183	97	48					
7	671	346	185	98						
8	602	350	187							
9	608	353								
10	614									

(E) Implied Ultimate Unclosed Claim Severity at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	**83.530**	**107.434**	**118.303**	**147.863**	**208.504**	**194.590**	**287.526**	-	-	-
2	**86.838**	**111.689**	**122.988**	**153.719**	**216.761**	**202.297**	298.913	-	-	
3	**90.277**	**116.112**	**127.859**	**159.807**	**225.346**	207.954	310.752	-		
4	**93.853**	**120.711**	**132.922**	**166.136**	246.610	216.189	323.059			
5	**97.570**	**125.491**	**138.187**	176.512	256.376	224.751				
6	**101.434**	**130.461**	152.497	183.503	266.530					
7	**105.451**	138.024	158.537	190.770						
8	112.331	143.490	164.815							
9	116.780	149.173								
10	121.405									
(F) Fitted Last Diagonal:	**$118.483**	**$146.584**	**$155.264**	**$186.667**	**$253.194**	**$227.296**	**$323.059**	-	-	-

(G) Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Implied Ultimate Outstanding:	$72,728	$51,785	$28,985	$18,270	$12,161	$7,332	$2,372	$0	$0	$0	
Paid to Date:	20,012	37,332	54,977	62,991	64,972	66,820	68,171	67,184	63,984	60,938	
Implied Ultimate Losses:	$92,740	$89,118	$83,962	$81,261	$77,133	$74,152	$70,543	$67,184	$63,984	$60,938	$761,014
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	($1,794)	($915)	($1,783)	($402)	($641)	$82	$0	$0	$0	$0	($5,453)

Scenario: Settlement Rate Acceleration

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	Projected Ultimate Losses ($000) (Paid LDF)	12	24	36	48	60	72	84	96	108	120
1	$60 938	8 105	17 695	32,841	45,301	48 709	53 105	56 840	60,938	60,938	60,938
2	63 984	8 510	18 580	34,483	47,566	51,144	55 761	61 833	63,984	63,984	
3	67 184	8 935	19 509	36 207	49,944	53 701	60 607	64 925	67 184		
4	[illegible]	9 362	20 485	38 017	52,441	56,931	63 638	68,171			
5	[illegible]	9 651	21 509	39 918	57,135	61 878	66 820				
6	[illegible]	10 344	22,584	49 865	59,991	64 972					
7	[illegible]	10,861	33 861	52 359	62,991						
8	[illegible]	18 152	35,555	54 977							
9	109 565	19,059	37,332								
10	130 780	20 012									
	$840 698										

(B) Implied Ultimate Unpaid Losses at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	52 833	43 242	28,097	15 637	12,229	7,832	4,097	0	0	0
2	55 474	45 404	29,502	16 415	12,840	8,224	2,151	0	0	
3	58 248	47,675	30,977	17,239	13,482	6,576	2,259	0		
4	61 948	50 845	33,313	18,888	12 399	7,692	3 159			
5	65 661	54 204	35 794	18 575	13 834	8 893				
6	70 258	58 018	30,737	20,611	15 630					
7	74,755	51,755	33 258	22,525						
8	76,535	59,132	39 710							
9	90,806	72,533								
10	110 768									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ultimate Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1 000	1,000	1 000
2	1 010	371	603	770	903	951	969	1 003	1 010	1 010	
3	1 020	375	610	778	912	960	988	1 013	1,020		
4	1 030	379	616	786	921	983	998	1 023			
5	1 041	382	622	793	945	993	1 008				
6	1 051	386	628	868	954	1,003					
7	1 062	390	715	877	964						
8	1 072	470	722	885							
9	1 083	475	730								
10	1,094	480									
	10,462										

(D) Implied Ultimate Unclosed Claims at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	633	403	238	106	59	40	14	0	0	0
2	639	407	240	107	59	41	[illegible]	0	0	
3	645	411	242	108	60	32	[illegible]	0		
4	652	415	245	109	47	32	[illegible]			
5	658	419	247	96	48	32				
6	665	423	183	97	48					
7	671	346	185	95						
8	602	350	187							
9	608	353								
10	614									

(E) Implied Ultimate Unclosed Claim Severity at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	**83.530**	**107.434**	**118.303**	**147.863**	**208.504**	**194.590**	**287.526**			
2	**86.838**	**111.689**	**122.988**	**153.719**	**216.761**	**202.297**	298 913			
3	**90.277**	**116.112**	**127.859**	**159.807**	**225.346**	207 954	310 752			
4	**95.061**	**122.609**	**136.139**	**173.361**	263 327	240 834	430 284			
5	**100.065**	**129.412**	**144.832**	193 630	290 912	275 663				
6	**105.689**	**137.148**	167 953	212 692	325 419					
7	**111.341**	149 442	179 931	231 174						
8	127 191	169 053	212 712							
9	149 413	205 312								
10	180 453									

(F) Fitted Last Diagonal:

$127.970	**$157.572**	**$167.039**	**$199.823**	**$253.194**	**$227.296**	**$323.059**

(G) Ultimate Losses	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Implied Ultimate Outstanding:	$78,552	$55,667	$31,184	$19,557	$12,161	$7,332	$2,372	$0	$0	$0	
Paid to Date:	20,012	37,332	54,977	62,991	64,972	66,820	68,171	67,184	63,984	60,938	
Implied Ultimate Losses:	$98,564	$93,000	$86,160	$82,548	$77,133	$74,152	$70,543	$67 184	$63,984	$60,938	$774 205
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67 184	$63,984	$60,938	$766 465
Difference:	$4,030	$2,967	$415	$886	($641)	$82	$0	$0	$0	$0	$7 739

Scenario: *Settlement Rate Acceleration*

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	8,105	17,695	32,841	45,301	48 709	53,105	56,840	60,938	60,938	60 938
2	8,510	18,580	34,483	47,566	51 144	55,761	61,833	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	9,382	20,485	38,017	52,441	58,931	63 638	68,171			
5	9,851	21,509	39,918	57,135	61,878	66 820				
6	10,344	22,584	49,865	59,991	64,972					
7	10,861	33,861	52 359	62,991						
8	18,152	35,555	54 977							
9	19,059	37,332								
10	20,012									

(B) Incremental Paid Losses in Age Interval in Months ($000s)

Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4 097	0	0
2	8,510	10,070	15 903	13 083	3,578	4,616	6,073	2,151	0	
3	8,935	10,574	16 698	13 737	3,757	6,906	4,318	2,259		
4	9 382	11,102	17,533	14,424	6 490	4,707	4,533			
5	9 851	11,657	18,409	17,217	4 743	4,942				
6	10 344	12,240	27,281	10,126	4 980					
7	10,861	23,000	18,497	10,632						
8	18,152	17,403	19,422							
9	19,059	18,273								
10	20,012									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ult Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	1 003	1,010	1,010	
3	1,020	375	610	778	912	960	988	1,013	1,020		
4	1 030	379	616	786	921	983	998	1,023			
5	1,041	382	622	793	945	993	1,008				
6	1,051	386	628	868	954	1,003					
7	1,062	390	715	877	964						
8	1,072	470	722	885							
9	1,083	475	730								
10	1,094	480									
	10,462										

(D) Incremental Closed Claims in Age Interval in Months

Year	Est'd Unclosed Claims	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Implied Future Closed Claims
1	0	368	230	165	132	47	18	26	14	0	0	0
2	0	371	232	167	133	48	19	33	7	0	**0**	0
3	0	375	235	168	134	48	28	24	7	**0**	**0**	0
4	7	379	237	170	136	62	15	25	**7**	**0**	**0**	7
5	32	382	239	172	151	48	15	**24**	**8**	**0**	**0**	32
6	48	386	242	240	86	49	**16**	**23**	**8**	**0**	**0**	48
7	98	390	325	161	87	**47**	**17**	**25**	**9**	**0**	**0**	98
8	187	470	252	163	**91**	**46**	**17**	**24**	**9**	**0**	**0**	187
9	353	475	254	**159**	**95**	**47**	**18**	**25**	**9**	**0**	**0**	353
10	614	480	**238**	**170**	**101**	**50**	**19**	**27**	**9**	**0**	**0**	614
	1,339											

(E) Incremental Paid Claim Severity in Age Interval in Months ($000s)

Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Implied Future Severity
1	22 054	41 698	91 790	94.575	72 353	238 940	143 654	287.526	-	-	-
2	22 927	43.350	95 425	98 321	75.218	248 403	181.515	298 913	-	-	-
3	23.835	45.067	99 204	102 215	78 197	244 846	177.276	310 752	-	-	-
4	24 779	46.851	103.133	106.263	104 894	310.761	184 296	**323.059**	-	-	323 059
5	25 760	48 707	107.218	113.867	98 022	323.069	**167.799**	**335.863**	-	-	211 622
6	26 780	50 636	113.660	117.603	101 904	**290.154**	**174.444**	**349.154**	-	-	243 979
7	27 841	70 751	114.545	122 261	**91.340**	**301.645**	**181.353**	**362.982**	-	-	176 135
8	38 588	69 073	119 081	**124.123**	**94.958**	**313.592**	**188.535**	**377.358**	-	-	154 238
9	40 116	71 808	**125.238**	**129.039**	**98.718**	**326.011**	**196.002**	**392.302**	-	-	144.503
10	41 705	**59.147**	**130.198**	**134.150**	**102.628**	**338.923**	**203.765**	**407.839**	-	-	114.905

(F) Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Implied Future Payments:	$70,532	$51,050	$28,794	$17,239	$11,719	$6,827	$2,372	$0	$0	$0	
Paid to Date:	20,012	37,332	54,977	62,991	64,972	66,820	68,171	67,184	63,984	60,938	
Implied Ultimate Losses:	$90,544	$88,383	$83,771	$80,230	$76,690	$73,646	$70,543	$67,184	$63,984	$60,938	$755,912
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	($3 990)	($1,650)	($1,975)	($1,432)	($1,083)	($424)	$0	$0	$0	$0	($10,554)

Scenario: Settlement Rate Acceleration

Year	Actual Ultimate Losses	Loss Development Technique: Incurred Losses	Loss Development Technique: Paid Losses	Ultimate Unclosed Claim Severity Technique on: Incurred Losses	Ultimate Unclosed Claim Severity Technique on: Paid Losses	Incremental Closed Claim Severity
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Estimated Ultimate Losses						
1	$60,938	$60,938	$60,938	$60,938	$60,938	$60,938
2	63,984	63,984	63,984	63,984	63,984	63,984
3	67,184	67,184	67,184	67,184	67,184	67,184
4	70,543	70,543	71,330	70,543	70,543	70,543
5	74,070	74,070	75,712	74,152	74,152	73,646
6	77,773	77,773	80,602	77,133	77,133	76,690
7	81,662	81,662	85,617	81,261	82,548	80,230
8	85,745	85,745	94,687	83,962	86,160	83,771
9	90,032	90,032	109,865	89,118	93,000	88,383
10	94,534	94,534	130,780	92,740	98,564	90,544
Total	$766,465	$766,465	$840,698	$761,014	$774,205	$755,912
Difference: Estimated vs Actual						
1		$0	$0	$0	$0	$0
2		0	0	0	0	0
3		0	0	0	0	0
4		0	787	0	0	0
5		0	1,642	82	82	(424)
6		0	2,829	(641)	(641)	(1,083)
7		0	3,954	(402)	886	(1,432)
8		0	8,942	(1,783)	415	(1,975)
9		0	19,833	(915)	2,967	(1,650)
10		0	36,246	(1,794)	4,030	(3,990)
Total		$0	$74,233	($5,453)	$7,739	($10,554)

Notes

(2) Based on hypothetical assumptions.
(3) Ultimate losses from Exhibit 8, Sheet 1. Difference = (3) minus (2).
(4) Ultimate losses from Exhibit 8, Sheet 2. Difference = (4) minus (2).
(5) Ultimate losses from Exhibit 10, Sheet 1. Difference = (5) minus (2).
(6) Ultimate losses from Exhibit 10, Sheet 2. Difference = (6) minus (2).
(7) Ultimate losses from Exhibit 11. Difference = (7) minus (2).

Estimated Loss Development Pattern
Incurred Losses ($000)

Exhibit 13
Sheet 1

Scenario: Strengthening and Acceleration

Year	Evaluation Age in Months: 12	24	36	48	60	72	84	96	108	120
1	22,638	37,938	49,003	53,252	59,843	60,649	60,938	60,938	60,938	60,938
2	23,769	39,834	51,453	55,914	62,835	63,681	63,984	63,984	63,984	
3	24,958	41,826	54,026	58,710	65,976	67,169	67,184	67,184		
4	26,206	43,917	56,727	61,645	69,612	70,528	70,543			
5	27,516	46,113	59,564	69,037	73,092	74,054				
6	28,892	48,419	65,317	72,489	76,747					
7	30,336	58,254	68,583	76,113						
8	43,057	61,167	72,012							
9	45,210	64,225								
10	47,471									

Year	Age Interval in Months: 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000	1.000	
2	1.676	1.292	1.087	1.124	1.013	1.005	1.000	1.000		
3	1.676	1.292	1.087	1.124	1.018	1.000	1.000			
4	1.676	1.292	1.087	1.129	1.013	1.000				
5	1.676	1.292	1.159	1.059	1.013					
6	1.676	1.349	1.110	1.059						
7	1.920	1.177	1.110							
8	1.421	1.177								
9	1.421									
10										
Average Factors										
Simple Avg Last 3	1.587	1.235	1.126	1.082	1.015	1.002	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.623	1.257	1.110	1.099	1.014	1.002	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.548	1.227	1.125	1.080	1.015	1.002	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.591	1.254	1.102	1.102	1.013	1.003	1.000	1.000	1.000	1.000
Selected Factors	1.548	1.227	1.125	1.080	1.015	1.002	1.000	1.000	1.000	1.000
Factors to Ultimate	2.346	1.515	1.235	1.098	1.016	1.002	1.000	1.000	1.000	1.000

Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	$111,370	$97,312	$88,936	$83,560	$78,005	$74,176	$70,543	$67,184	$63,984	$60,938	$796,007
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$16,836	$7,279	$3,191	$1,898	$231	$106	$0	$0	$0	$0	$29,541

stimated Loss Development Pattern
aid Losses ($000)

Exhibit 13
Sheet 2

Scenario: Strengthening and Acceleration

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
1	8.105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	9,851	21,509	39,918	57,135	61,878	66,820				
6	10,344	22,584	49,865	59,991	64 972					
7	10,861	33,861	52,359	62,991						
8	18,152	35,555	54,977							
9	19,059	37,332								
10	20,012									

Year	Age Interval in Months									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	2 183	1.856	1 379	1 075	1 090	1 070	1.072	1.000	1 000	
2	2.183	1.856	1.379	1 075	1.090	1.109	1.035	1.000		
3	2.183	1 856	1.379	1 075	1 129	1 071	1 035			
4	2 183	1.856	1.379	1.124	1 080	1 071				
5	2 183	1 856	1.431	1.083	1 080					
6	2.183	2 208	1.203	1.083						
7	3.118	1 546	1 203							
8	1 959	1 546								
9	1 959									
10										
Average Factors										
Simple Avg Last 3	2 345	1.767	1.279	1 097	1.096	1.084	1.047	1.000	1.000	1.000
Simple Avg Last 5	2 280	1.802	1.319	1 088	1 094	1.080	1.047	1.000	1 000	1.000
Vol Wt'd Avg Last 3	2 221	1.709	1.267	1 096	1 095	1.083	1.046	1.000	1.000	1 000
Simple Avg 3 of 5	2 108	1 753	1 321	1.080	1 087	1.084	1 047	1.000	1 000	1.000
Selected Factors	2 221	1 709	1.267	1.096	1 095	1.083	1 046	1 000	1.000	1.000
Factors to Ultimate	6 535	2 943	1.722	1.359	1 241	1.133	1 046	1 000	1.000	1.000

Ultimate Losses	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Projected	$130,780	$109,865	$94,687	$85,617	$80,602	$75,712	$71,330	$67,184	$63,984	$60,938	$840,698
Actual	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference	$36,246	$19,833	$8,942	$3,954	$2,829	$1,642	$787	$0	$0	$0	$74,233

Estimated Claim Development Pattern
Reported Claims

Exhibit 13
Sheet 3

Scenario: Strengthening and Acceleration

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	
2	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000		
3	1.127	1.085	1.026	1.013	1.000	1.000	1.000			
4	1.127	1.085	1.026	1.013	1.000	1.000				
5	1.127	1.085	1.026	1.013	1.000					
6	1.127	1.085	1.026	1.013						
7	1.127	1.085	1.026							
8	1.127	1.085								
9	1.127									
10										
Average Factors										
Simple Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg Last 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Simple Avg 3 of 5	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Selected Factors	1.127	1.085	1.026	1.013	1.000	1.000	1.000	1.000	1.000	1.000
Factors to Ultimate	1.270	1.127	1.039	1.013	1.000	1.000	1.000	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	0	0	0	0	0	0	0	0	0	0	0

Estimated Claim Development Pattern
Closed Claims

Exhibit 13
Sheet 4

Scenario: Strengthening and Acceleration

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	1,003	1,010	1,010	
3	375	610	778	912	960	988	1,013	1,020		
4	379	616	786	921	983	998	1,023			
5	382	622	793	945	993	1,008				
6	386	628	868	954	1,003					
7	390	715	877	964						
8	470	722	885							
9	475	730								
10	480									

Year	Age Interval in Months									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.626	1.276	1.173	1.053	1.020	1.027	1.014	1.000	1.000	
2	1.626	1.276	1.173	1.053	1.020	1.035	1.007	1.000		
3	1.626	1.276	1.173	1.053	1.029	1.025	1.007			
4	1.626	1.276	1.173	1.067	1.015	1.025				
5	1.626	1.276	1.191	1.051	1.015					
6	1.626	1.382	1.099	1.051						
7	1.833	1.226	1.099							
8	1.536	1.226								
9	1.536									
10										
Average Factors										
Simple Avg Last 3	1.635	1.278	1.130	1.057	1.020	1.028	1.010	1.000	1.000	1.000
Simple Avg Last 5	1.631	1.277	1.147	1.055	1.020	1.028	1.010	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.623	1.273	1.128	1.056	1.020	1.028	1.010	1.000	1.000	1.000
Simple Avg 3 of 5	1.596	1.259	1.148	1.052	1.018	1.029	1.010	1.000	1.000	1.000
Selected Factors	1.623	1.273	1.128	1.056	1.020	1.028	1.010	1.000	1.000	1.000
Factors to Ultimate	2.605	1.606	1.261	1.118	1.058	1.038	1.010	1.000	1.000	1.000

Ultimate Counts	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Projected	1,250	1,171	1,117	1,078	1,062	1,046	1,033	1,020	1,010	1,000	10,787
Actual	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000	10,462
Difference	157	89	44	16	11	6	2	0	0	0	324

Estimated Claim Development Pattern
Paid Claims (I.e., Claims Closed With Payment)

Exhibit 13
Sheet 5

Scenario: ***Strengthening and Acceleration***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	750	758	758	
3	247	393	548	678	705	741	758	765		
4	250	397	554	684	726	749	765			
5	252	401	559	705	733	756				
6	255	405	632	712	740					
7	257	490	638	719						
8	336	495	644							
9	340	499								
10	343									

Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1	1.588	1.396	1.236	1.041	1.037	1.026	1.019	1.000	1.000	
2	1.588	1.396	1.236	1.041	1.037	1.036	1.010	1.000		
3	1.588	1.396	1.236	1.041	1.051	1.023	1.010			
4	1.588	1.396	1.236	1.060	1.032	1.023				
5	1.588	1.396	1.261	1.039	1.032					
6	1.588	1.561	1.128	1.039						
7	1.902	1.303	1.128							
8	1.470	1.303								
9	1.470									
10										
Average Factors										
Simple Avg Last 3	1.614	1.389	1.172	1.046	1.038	1.027	1.013	1.000	1.000	1.000
Simple Avg Last 5	1.604	1.392	1.198	1.044	1.038	1.027	1.013	1.000	1.000	1.000
Vol. Wt'd Avg Last 3	1.589	1.378	1.169	1.046	1.038	1.027	1.013	1.000	1.000	1.000
Simple Avg 3 of 5	1.548	1.365	1.200	1.040	1.035	1.028	1.013	1.000	1.000	1.000
Selected Factors	1.589	1.378	1.169	1.046	1.038	1.027	1.013	1.000	1.000	1.000
Factors to Ultimate	2.889	1.818	1.319	1.129	1.079	1.040	1.013	1.000	1.000	1.000

Ultimate Counts	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Projected	991	908	850	812	799	786	775	765	758	750	8,195
Actual	820	812	804	796	788	780	773	765	758	750	7,847
Difference	171	96	46	16	11	6	2	0	0	0	348

Scenario: ***Strengthening and Acceleration***

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
Outstanding Losses ($000)										
1	14,533	20,242	16,163	7,951	11,134	7,544	4,097	0	0	0
2	15,259	21,254	16,971	8,348	11,690	7,921	2,151	0	0	
3	16,022	22,317	17,819	8,766	12,275	6,562	2,259	0		
4	16,824	23,433	18,710	9,204	10,680	6,890	2,372			
5	17,665	24,605	19,646	11,902	11,214	7,234				
6	18,548	25,835	15,451	12,498	11,775					
7	19,475	24,393	16,224	13,122						
8	24,906	25,613	17,035							
9	26,151	26,893								
10	27,459									

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
Number Open Claims										
1	420	290	200	93	59	40	14	0	0	0
2	424	293	202	94	59	41	7	0	0	
3	428	296	204	95	60	32	7	0		
4	433	299	206	96	47	32	7			
5	437	302	208	83	48	32				
6	441	305	144	84	48					
7	446	227	145	85						
8	374	229	146							
9	378	231								
10	381									

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
Average Outstanding ($000)										
1	34.602	69.801	80.813	85.261	189.834	187.416	287.526	-	-	-
2	35.972	72.565	84.013	88.638	197.352	194.839	298.913	-	-	
3	37.397	75.439	87.340	92.149	205.168	207.496	310.752	-		
4	38.878	78.427	90.799	95.798	226.833	215.714	323.059			
5	40.418	81.533	94.395	143.514	235.817	224.257				
6	42.018	84.762	107.604	149.197	245.156					
7	43.683	107.505	111.866	155.106						
8	66.609	111.763	116.296							
9	69.247	116.189								
10	71.990									

Year	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	-	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	6.5%	4.0%	-		
4	4.0%	4.0%	4.0%	4.0%	10.6%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	49.8%	4.0%	4.0%				
6	4.0%	4.0%	14.0%	4.0%	4.0%					
7	4.0%	26.8%	4.0%	4.0%						
8	52.5%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: Strengthening and Acceleration

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Losses ($000)										
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	9,851	21,509	39,918	57,135	61,878	66,820				
6	10,344	22,584	49,865	59,991	64,972					
7	10,861	33,861	52,359	62,991						
8	18,152	35,555	54,977							
9	19,059	37,332								
10	20,012									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claims - Cumulative										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	750	758	758	
3	247	393	548	678	705	741	758	765		
4	250	397	554	684	726	749	765			
5	252	401	559	705	733	756				
6	255	405	632	712	740					
7	257	490	638	719						
8	336	495	644							
9	340	499								
10	343									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Average Paid Claim - Cumulative ($000)										
1	33.421	45.962	61.099	68.199	70.455	74.040	77.255	81.250	81.250	81.250
2	34.745	47.782	63.519	70.900	73.245	76.972	82.411	84.468	84.468	
3	36.121	49.674	66.034	73.707	76.146	81.780	85.675	87.813		
4	37.552	51.642	68.649	76.627	81.213	85.019	89.068			
5	39.039	53.687	71.368	81.005	84.429	88.386				
6	40.585	55.813	78.960	84.213	87.773					
7	42.192	69.158	82.087	87.548						
8	53.961	71.897	85.338							
9	56.098	74.744								
10	58.320									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	6.7%	4.0%	4.0%	
3	4.0%	4.0%	4.0%	4.0%	4.0%	6.2%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	6.7%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	5.7%	4.0%	4.0%				
6	4.0%	4.0%	10.6%	4.0%	4.0%					
7	4.0%	23.9%	4.0%	4.0%						
8	27.9%	4.0%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: ***Strengthening and Acceleration***

Year	Age Interval in Months 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
Paid Losses - Incremental ($000)										
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4,097	0	0
2	8,510	10,070	15,903	13,083	3,578	4,616	6,073	2,151	0	
3	8,935	10,574	16,698	13,737	3,757	6,906	4,318	2,259		
4	9,382	11,102	17,533	14,424	6,490	4,707	4,533			
5	9,851	11,657	18,409	17,217	4,743	4,942				
6	10,344	12,240	27,281	10,126	4,980					
7	10,861	23,000	18,497	10,632						
8	18,152	17,403	19,422							
9	19,059	18,273								
10	20,012									

Year	Age Interval in Months 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
Paid Claims - Incremental										
1	243	143	153	127	27	26	19	14	0	0
2	245	144	154	128	27	26	26	7	0	
3	247	145	156	129	28	36	17	7		
4	250	147	157	131	41	23	17			
5	252	148	159	146	28	23				
6	255	150	227	81	28					
7	257	232	148	82						
8	336	158	150							
9	340	160								
10	343									

Year	Age Interval in Months 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
Average Paid Claim - Incremental ($000)										
1	33.421	67.303	99.314	98.306	125.750	169.749	201.892	287.526	-	-
2	34.745	69.968	103.247	102.200	130.730	176.472	234.641	298.913	-	
3	36.121	72.739	107.336	106.247	135.907	192.603	258.471	310.752		
4	37.552	75.620	111.587	110.455	157.275	205.774	268.707			
5	39.039	78.615	116.006	117.925	172.001	213.924				
6	40.585	81.728	120.241	125.247	178.813					
7	42.192	99.051	124.799	130.207						
8	53.961	110.048	129.742							
9	56.098	114.406								
10	58.320									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Annual Percent Change										
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	16.2%	4.0%	-	
3	4.0%	4.0%	4.0%	4.0%	4.0%	9.1%	10.2%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	15.7%	6.8%	4.0%			
5	4.0%	4.0%	4.0%	6.8%	9.4%	4.0%				
6	4.0%	4.0%	3.7%	6.2%	4.0%					
7	4.0%	21.2%	3.8%	4.0%						
8	27.9%	11.1%	4.0%							
9	4.0%	4.0%								
10	4.0%									

Scenario: Strengthening and Acceleration

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	1,003	1,010	1,010	
3	375	610	778	912	960	988	1,013	1,020		
4	379	616	786	921	983	998	1,023			
5	382	622	793	945	993	1,008				
6	386	628	868	954	1,003					
7	390	715	877	964						
8	470	722	885							
9	475	730								
10	480									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Reported Claims										
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000
2	795	896	972	997	1,010	1,010	1,010	1,010	1,010	
3	803	905	982	1,007	1,020	1,020	1,020	1,020		
4	811	914	992	1,017	1,030	1,030	1,030			
5	819	924	1,002	1,028	1,041	1,041				
6	828	933	1,012	1,038	1,051					
7	836	942	1,022	1,048						
8	844	952	1,032							
9	853	961								
10	861									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closure Rate										
1	0.467	0.673	0.792	0.906	0.941	0.960	0.986	1.000	1.000	1.000
2	0.467	0.673	0.792	0.906	0.941	0.960	0.993	1.000	1.000	
3	0.467	0.673	0.792	0.906	0.941	0.969	0.993	1.000		
4	0.467	0.673	0.792	0.906	0.954	0.969	0.993			
5	0.467	0.673	0.792	0.919	0.954	0.969				
6	0.467	0.673	0.858	0.919	0.954					
7	0.467	0.759	0.858	0.919						
8	0.557	0.759	0.858							
9	0.557	0.759								
10	0.557									

Scenario: ***Strengthening and Acceleration***

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claims										
1	243	385	538	664	691	717	736	750	750	750
2	245	389	543	671	698	724	750	758	758	
3	247	393	548	678	705	741	758	765		
4	250	397	554	684	726	749	765			
5	252	401	559	705	733	756				
6	255	405	632	712	740					
7	257	490	638	719						
8	336	495	644							
9	340	499								
10	343									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Closed Claims										
1	368	598	763	894	941	960	986	1,000	1,000	1,000
2	371	603	770	903	951	969	1,003	1,010	1,010	
3	375	610	778	912	960	988	1,013	1,020		
4	379	616	786	921	983	998	1,023			
5	382	622	793	945	993	1,008				
6	386	628	868	954	1,003					
7	390	715	877	964						
8	470	722	885							
9	475	730								
10	480									

Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120
Paid Claim Ratio										
1	0.660	0.644	0.705	0.743	0.734	0.747	0.746	0.750	0.750	0.750
2	0.660	0.644	0.705	0.743	0.734	0.747	0.748	0.750	0.750	
3	0.660	0.644	0.705	0.743	0.734	0.750	0.748	0.750		
4	0.660	0.644	0.705	0.743	0.738	0.750	0.748			
5	0.660	0.644	0.705	0.747	0.738	0.750				
6	0.660	0.644	0.728	0.747	0.738					
7	0.660	0.685	0.728	0.747						
8	0.715	0.685	0.728							
9	0.715	0.685								
10	0.715									

Scenario: Strengthening and Acceleration

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	Projected Ultimate Losses ($000) (Incurred LDF)	12	24	36	48	60	72	84	96	108	120
1	$60,938	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	63,984	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	67,184	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	70,543	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	74,176	9,851	21,509	39,918	57,135	61,878	66,820				
6	78,005	10,344	22,584	49,865	59,991	64,972					
7	83,560	10,861	33,861	52,359	62,991						
8	88,936	18,152	35,555	54,977							
9	97,312	19,059	37,332								
10	111,370	20,012									
	$796,007										

(B) Implied Ultimate Unpaid Losses at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	52,833	43,242	28,097	15,637	12,229	7,832	4,097	0	0	0
2	55,474	45,404	29,502	16,418	12,840	8,224	2,151	0	0	
3	58,248	47,675	30,977	17,239	13,482	6,576	2,259	0		
4	61,161	50,058	32,526	18,101	11,612	6,905	2,372			
5	64,325	52,668	34,258	17,042	12,299	7,357				
6	67,661	55,421	28,139	18,013	13,033					
7	72,699	49,698	31,201	20,569						
8	70,785	53,382	33,960							
9	78,253	59,979								
10	91,358									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ultimate Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	1,003	1,010	1,010	
3	1,020	375	610	778	912	960	988	1,013	1,020		
4	1,030	379	616	786	921	983	998	1,023			
5	1,041	382	622	793	945	993	1,008				
6	1,051	386	628	868	954	1,003					
7	1,062	390	715	877	964						
8	1,072	470	722	885							
9	1,083	475	730								
10	1,094	480									
	10,462										

(D) Implied Ultimate Unclosed Claims at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	633	403	238	106	59	40	14	0	0	0
2	639	407	240	107	59	41	7	0	0	
3	645	411	242	108	60	32	7	0		
4	652	415	245	109	47	32	7			
5	658	419	247	96	48	32				
6	665	423	183	97	48					
7	671	346	185	98						
8	602	350	187							
9	608	353								
10	614									

(E) Implied Ultimate Unclosed Claim Severity at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	**83.530**	**107.434**	**118.303**	**147.863**	**208.504**	**194.590**	**287.526**	-	-	-
2	**86.838**	**111.689**	**122.988**	**153.719**	**216.761**	**202.297**	298.913	-	-	
3	**90.277**	**116.112**	**127.859**	**159.807**	**225.346**	207.954	310.752	-		
4	**93.853**	**120.711**	**132.922**	**166.136**	246.610	216.189	323.059			
5	**97.731**	**125.745**	**138.617**	177.622	258.615	228.051				
6	**101.782**	**131.008**	153.761	185.690	271.347					
7	**108.277**	143.503	168.803	210.158						
8	117.634	152.613	181.908							
9	128.757	168.778								
10	148.832									
(F) Fitted Last Diagonal:	**$121.279**	**$147.359**	**$155.845**	**$166.667**	**$253.194**	**$227.296**	**$323.059**	-	-	-

(G) Ultimate Losses	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Implied Ultimate Outstanding:	$74,445	$52,059	$29,094	$18,270	$12,161	$7,332	$2,372	$0	$0	$0	
Paid to Date:	20,012	37,332	54,977	62,991	64,972	66,820	68,171	67,184	63,984	60,938	
Implied Ultimate Losses:	$94,457	$89,392	$84,070	$81,261	$77,133	$74,152	$70,543	$67,184	$63,984	$60,938	$763,113
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	($77)	($641)	($1,675)	($402)	($641)	$82	$0	$0	$0	$0	($3,354)

Scenario: Strengthening and Acceleration

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	Projected Ultimate Losses ($000) (Paid LDF)	12	24	36	48	60	72	84	96	108	120
1	$60,938	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	63,984	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	67,184	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	71,330	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	75,712	9,851	21,509	39,918	57,135	61,878	66,820				
6	80,602	10,344	22,584	49,865	59,991	64,972					
7	85,617	10,861	33,861	52,359	62,991						
8	94,687	18,152	35,555	54,977							
9	109,865	19,059	37,332								
10	130,780	20,012									
	$840,698										

(B) Implied Ultimate Unpaid Losses at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	52,833	43,242	28,097	15,637	12,229	7,832	4,097	0	0	0
2	55,474	45,404	29,502	16,418	12,840	8,224	2,151	0	0	
3	58,248	47,675	30,977	17,239	13,482	6,576	2,259	0		
4	61,948	50,845	33,313	18,888	12,399	7,692	3,159			
5	65,861	54,204	35,794	18,578	13,834	8,893				
6	70,258	58,018	30,737	20,611	15,630					
7	74,755	51,755	33,258	22,626						
8	76,535	59,132	39,710							
9	90,806	72,533								
10	110,768									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ultimate Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	1,003	1,010	1,010	
3	1,020	375	610	778	912	960	988	1,013	1,020		
4	1,030	379	616	786	921	983	998	1,023			
5	1,041	382	622	793	945	993	1,008				
6	1,051	386	628	868	954	1,003					
7	1,062	390	715	877	964						
8	1,072	470	722	885							
9	1,083	475	730								
10	1,094	480									
	10,462										

(D) Implied Ultimate Unclosed Claims at Evaluation Age in Months

Year	12	24	36	48	60	72	84	96	108	120
1	633	403	238	106	59	40	14	0	0	0
2	639	407	240	107	59	41	7	0	0	
3	645	411	242	108	60	32	7	0		
4	652	415	245	109	47	32	7			
5	658	419	247	96	48	32				
6	665	423	183	97	48					
7	671	346	185	98						
8	602	350	187							
9	608	353								
10	614									

(E) Implied Ultimate Unclosed Claim Severity at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	**83.530**	**107.434**	**118.303**	**147.863**	**208.504**	**194.590**	**287.526**	-	-	-
2	**86.838**	**111.889**	**122.988**	**153.719**	**216.761**	**202.297**	298.913		-	
3	**90.277**	**116.112**	**127.859**	**159.807**	**225.346**	207.954	310.752			
4	**95.061**	**122.609**	**136.139**	**173.361**	263.327	240.834	430.284			
5	**100.065**	**129.412**	**144.832**	193.630	290.912	275.663				
6	**105.[illegible]**	**137.148**	167.953	212.692	325.419					
7	**111.341**	149.442	179.931	231.174						
8	127.191	169.053	212.712							
9	149.413	205.312								
10	180.453									

(F) Fitted Last Diagonal:	$127.970	$157.572	$167.039	$199.823	$253.194	$227.296	$323.059

(G) Ultimate Losses	Year 10	Year 9	Year 8	Year 7	Year 6	Year 5	Year 4	Year 3	Year 2	Year 1	All Years
Implied Ultimate Outstanding:	$78,552	$55,667	$31,184	$19,557	$12,161	$7,332	$2,372	$0	$0	$0	
Paid to Date:	20,012	37,332	54,977	62,991	64,972	66,820	68,171	67,184	63,984	60,938	
Implied Ultimate Losses:	$98,564	$93,000	$86,160	$82,548	$77,133	$74,152	$70,543	$67,184	$63,984	$60,938	$774,205
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	$4,030	$2,967	$415	$886	($641)	$82	$0	$0	$0	$0	$7,739

Scenario: Strengthening and Acceleration

(A) Paid Losses at Evaluation Age in Months ($000s)

Year	12	24	36	48	60	72	84	96	108	120
1	8,105	17,695	32,841	45,301	48,709	53,105	56,840	60,938	60,938	60,938
2	8,510	18,580	34,483	47,566	51,144	55,761	61,833	63,984	63,984	
3	8,935	19,509	36,207	49,944	53,701	60,607	64,925	67,184		
4	9,382	20,485	38,017	52,441	58,931	63,638	68,171			
5	9,851	21,509	39,918	57,135	61,878	66,820				
6	10,344	22,584	49,865	59,991	64,972					
7	10,861	33,861	52,359	62,991						
8	18,152	35,555	54,977							
9	19,059	37,332								
10	20,012									

(B) Incremental Paid Losses in Age Interval in Months ($000s)

Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1	8,105	9,591	15,145	12,460	3,408	4,397	3,735	4,097	0	0
2	8,510	10,070	15,903	13,083	3,578	4,616	6,073	2,151	0	
3	8,935	10,574	16,698	13,737	3,757	6,906	4,318	2,259		
4	9,382	11,102	17,533	14,424	6,490	4,707	4,533			
5	9,851	11,657	18,409	17,217	4,743	4,942				
6	10,344	12,240	27,281	10,126	4,980					
7	10,861	23,000	18,497	10,632						
8	18,152	17,403	19,422							
9	19,059	18,273								
10	20,012									

(C) Closed Claims at Evaluation Age in Months

Year	Projected Ult Reported Claims	12	24	36	48	60	72	84	96	108	120
1	1,000	368	598	763	894	941	960	986	1,000	1,000	1,000
2	1,010	371	603	770	903	951	969	1,003	1,010	1,010	
3	1,020	375	610	778	912	960	988	1,013	1,020		
4	1,030	379	616	786	921	983	998	1,023			
5	1,041	382	622	793	945	993	1,008				
6	1,051	386	628	868	954	1,003					
7	1,062	390	715	877	964						
8	1,072	470	722	885							
9	1,083	475	730								
10	1,094	480									
	10,462										

(D) Incremental Closed Claims in Age Interval in Months

Year	Est'd Unclosed Claims	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Implied Future Closed Claims
1	0	368	230	165	132	47	18	26	14	0	0	0
2	0	371	232	167	133	48	19	33	7	0	**0**	0
3	0	375	235	168	134	48	28	24	7	**0**	**0**	0
4	7	379	237	170	136	62	15	25	**7**	**0**	**0**	7
5	32	382	239	172	151	48	15	**24**	**8**	**0**	**0**	32
6	48	386	242	240	86	49	**16**	**23**	**8**	**0**	**0**	48
7	98	390	325	161	87	**47**	**17**	**25**	**9**	**0**	**0**	98
8	187	470	252	163	**91**	**46**	**17**	**24**	**9**	**0**	**0**	187
9	353	475	254	**159**	**95**	**47**	**18**	**25**	**9**	**0**	**0**	353
10	614	480	**238**	**170**	**101**	**50**	**19**	**27**	**9**	**0**	**0**	614
	1,339											

(E) Incremental Paid Claim Severity in Age Interval in Months ($000s)

Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Implied Future Severity
1	22.054	41.698	91.790	94.575	72.353	238.940	143.654	287.526	-	-	-
2	22.927	43.350	95.425	98.321	75.218	248.403	181.515	298.913	-	-	-
3	23.835	45.067	99.204	102.215	78.197	244.846	177.276	310.752	-	-	-
4	24.779	46.851	103.133	106.263	104.894	310.761	184.296	**323.059**	-	-	323.059
5	25.760	48.707	107.218	113.867	98.022	323.069	**167.799**	**335.863**	-	-	211.622
6	26.780	50.636	113.660	117.603	101.904	**290.164**	**174.444**	**349.164**	-	-	243.979
7	27.841	70.751	114.545	122.261	**91.340**	**301.645**	**181.353**	**362.982**	-	-	176.135
8	38.588	69.073	119.081	**124.123**	**94.958**	**313.592**	**188.535**	**377.388**	-	-	154.238
9	40.116	71.808	**126.238**	**129.039**	**98.718**	**326.011**	**196.002**	**392.302**	-	-	144.503
10	41.705	**59.147**	**130.198**	**134.150**	**102.628**	**338.923**	**203.765**	**407.839**	-	-	114.905

(F) Ultimate Losses	**Year 10**	**Year 9**	**Year 8**	**Year 7**	**Year 6**	**Year 5**	**Year 4**	**Year 3**	**Year 2**	**Year 1**	**All Years**
Implied Future Payments:	$70,532	$51,050	$28,794	$17,239	$11,719	$6,827	$2,372	$0	$0	$0	
Paid to Date:	20,012	37,332	54,977	62,991	64,972	66,820	68,171	67,184	63,984	60,938	
Implied Ultimate Losses:	$90,544	$88,383	$83,771	$80,230	$76,690	$73,646	$70,543	$67,184	$63,984	$60,938	$755,912
Actual Ultimate Losses:	$94,534	$90,032	$85,745	$81,662	$77,773	$74,070	$70,543	$67,184	$63,984	$60,938	$766,465
Difference:	($3,990)	($1,650)	($1,975)	($1,432)	($1,083)	($424)	$0	$0	$0	$0	($10,554)

Summary of Estimates **Exhibit 17**

Scenario: Strengthening and Acceleration

	Actual	Loss Development Technique		Ultimate Unclosed Claim Severity Technique on		Incremental
	Ultimate	Incurred	Paid	Incurred	Paid	Closed Claim
Year	Losses	Losses	Losses	Losses	Losses	Severity
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Estimated Ultimate Losses						
1	$60,938	$60,938	$60,938	$60,938	$60,938	$60,938
2	63,984	63,984	63,984	63,984	63,984	63,984
3	67,184	67,184	67,184	67,184	67,184	67,184
4	70,543	70,543	71,330	70,543	70,543	70,543
5	74,070	74,176	75,712	74,152	74,152	73,646
6	77,773	78,005	80,602	77,133	77,133	76,690
7	81,662	83,560	85,617	81,261	82,548	80,230
8	85,745	88,936	94,687	84,070	86,160	83,771
9	90,032	97,312	109,865	89,392	93,000	88,383
10	94,534	111,370	130,780	94,457	98,564	90,544
Total	$766,465	$796,007	$840,698	$763,113	$774,205	$755,912
Difference: Estimated vs Actual						
1		$0	$0	$0	$0	$0
2		0	0	0	0	0
3		0	0	0	0	0
4		0	787	0	0	0
5		106	1,642	82	82	(424)
6		231	2,829	(641)	(641)	(1,083)
7		1,898	3,954	(402)	886	(1,432)
8		3,191	8,942	(1,675)	415	(1,975)
9		7,279	19,833	(641)	2,967	(1,650)
10		16,836	36,246	(77)	4,030	(3,990)
Total		$29,541	$74,233	($3,354)	$7,739	($10,554)

Notes:

(2) Based on hypothetical assumptions.
(3) Ultimate losses from Exhibit 13, Sheet 1. Difference = (3) minus (2).
(4) Ultimate losses from Exhibit 13, Sheet 2. Difference = (4) minus (2).
(5) Ultimate losses from Exhibit 15, Sheet 1. Difference = (5) minus (2).
(6) Ultimate losses from Exhibit 15, Sheet 2. Difference = (6) minus (2).
(7) Ultimate losses from Exhibit 16. Difference = (7) minus (2).

Reserving for Construction Defects

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ABSTRACT

Construction Defects: Property and Casualty insurers and actuaries cringe at the very mention of those two words. Insurers are troubled by the high frequency of construction defect claims while actuaries have encountered countless struggles with finding an appropriate and reasonable method for projecting the emergence of construction defect losses. As actuaries, it is our job to help our clients understand the issues at hand and to provide them with estimates with which they can feel comfortable given the great deal of uncertainty embedded in the market.

In this paper, we give the reader an overview of the issues surrounding an actuarial analysis of construction defects. We provide background information, including relevant legal decisions and defining characteristics of construction defects. We discuss items that should be considered when performing an actuarial analysis of construction defect data and present a few of the tailored methodologies that we have employed in recent years. Finally, we offer our thoughts on current trends as well as what we might expect to see in the future.

BACKGROUND

The issue of construction defects stems primarily from a building boom in California that began in the late 1970's. At the time, California real estate was the most sought after in all of the country. During the 1980's, the Golden State experienced a population growth rate more than double that of the nation as a whole. See Exhibit 1 for a comparison of growth rates between states. As a result, what ensued would eventually come to haunt insurance companies who wrote mono-line and package policies for both general contractors and subcontractors doing business in that state. [1]

The high demand for housing wreaked havoc on the construction industry. Contractors found themselves with too many projects and a limited amount of skilled labor. To keep up with the extraordinary demands for real estate, many contractors began cutting corners in the construction process by doing the following:

Hiring individuals who lacked the qualifications and experience necessary for producing quality workmanship

Foregoing proper supervision on location at many construction sites

Building cheaply and quickly with the focus of moving onto the next project

In addition to the changes in construction quality, there was also a significant shift in the types of residential structures being erected. The population growth, coupled with the price of real estate, caused the construction market to turn largely to town homes and condominiums (multi-unit dwellings).

These actions laid the groundwork for the construction defect lawsuits that emerged in California. Lawyers were very aggressive in getting homeowners associations to sue the contractors responsible for defects arising in multi-unit dwellings. Homeowners associations offered an excellent target for the law firms because they had more financial backing and the ability to take more risk in terms of filing a lawsuit than did most individual homeowners. Furthermore, if the association board was initially reluctant to sue the contractors, the board could have potentially been sued by one of the homeowners, thus forcing the board to move forward with the suit against the contractor. As an additional incentive, a successful verdict was likely to be a large, highly publicized event, thus encouraging other homeowner associations to file lawsuits in hopes of reaching a similar conclusion.

Due to the sudden onslaught of construction defect claims, insurance companies were forced to take action against future claims. To protect themselves, they did one of the following:

Raised their premiums for contractors

Became more selective about the contractors to which they would issue policies

Attempted to exclude coverage for losses already known to the insured at policy inception through specific Montrose exclusions

Many contractors who had been able to purchase insurance before found themselves either unable to obtain coverage at all or facing unaffordable premiums. Those who could purchase a policy were forced to pass along the severe premium increases to homebuyers, thus contributing to the rapid escalation of real estate prices. As a result, the number of new multi-unit dwellings decreased significantly during the late 1980's and early 1990's. See Exhibit 2. [2]

Many construction defect lawsuits presented questions regarding apportionment of financial responsibility among insurers and defendant insureds. Which policy should be triggered? For most insurance coverages, the date of the accident is used to determine which insurance policy to assign the claim. However, the nature of construction defects makes it difficult to determine when an "accident" has occurred. Prior to 1995, insurance companies tended to follow the manifestation trigger theory. The manifestation date is the date at which the defect makes itself known. It was typically identified as the filing date of the construction defect complaint. However, this date was not interpreted consistently between insurers. Therefore, when a coverage lawsuit was filed, an insurance company would often vigorously contest the insured contractor's claim for defense and indemnity by denying that the manifestation date was during their policy period.

THE MONTROSE DECISION

The ambiguity of responsibility was about to be changed in July 1995 by a precedent setting decision brought down in a chemical pollution case that would soon filter into construction defect litigation. Montrose Chemical Corporation of California v. Admiral Insurance Company (The Montrose case) was a pollution liability coverage case that determined that a continuous (coverage) trigger applied during the time that the pollution occurred, effectively triggering all policies in force during that time period. The California Supreme Court also rejected insurer defenses of "known loss" and "loss in progress" doctrines. Plaintiff attorneys have successfully applied the Montrose decision to construction defect cases.

More than a dozen occurrence trigger theories have also been advanced. At the time of the Montrose decision, the court considered the three other major trigger theories: exposure (injury occurs when claimant is exposed to injury causing event), injury-in-fact (injury occurs when claimant first suffers injury), and manifestation (injury occurs on the date the injury becomes manifest or discoverable).

Among the earliest applications of the continuous trigger concept to construction defect cases was the decision in the case of Stonewall Insurance Company v. City of Palos Verdes Estates (June 1996). In this case, homeowners in Palos Verdes Estates sued the city for the damage to their homes due to the sinking of the land. The court ruled in favor of the Montrose allocation of the damages to all years during the damage period. [3]

The Montrose Decision, while providing some clarity on the issue of coverage allocation, caused frequencies to increase dramatically because multiple insurers were named on virtually every lawsuit filed. At the same time, severities generally decreased because each insurer was deemed only partially involved.

In the Post-Montrose environment, the insured liability exposure is usually allocated among all insurance companies who have written coverage for the insured during the continuous trigger period. This "trigger spread" approach to allocation refers to the time period of an insured's exposure, and recognizes the extant tendency of courts to allocate losses "horizontally", meaning that carriers are required to respond to latent claims on a pro rata or shared basis.

The continuous coverage trigger may, or may not, be beneficial to the insured. By spreading the losses to all policies in force from the commencement of construction to manifestation, the insured's available coverage is maximized. However, insureds with large deductible policies are penalized. Each policy is triggered, and the attachment point on any one of the policies is unattainable until the insured paid each deductible. In this way, an insurer who writes large deductible policies is insulated. A similar case can be made for the insulation of reinsurers to construction defect claims. The continuous coverage trigger causes high frequency and low severity type claims, which are less likely to reach an excess of loss reinsurance attachment point.

In many jurisdictions, the coverage allocation process ascribes apportioned responsibility only to insurance companies. Accordingly, during those times that the insured did not have coverage, the gaps in coverage do not dilute allocations to the insurers. In most cases, the indemnity portion of the claim is prorated based on the time on the risk. Loss adjustment expense is prorated based on the number of carriers unless a carrier prefers to retain their own counsel, in which case they will not participate in the shared attorney cost.

In the mid 1990's, some insurance companies were forced out of the market because the abrupt infiltration of claims proved too overwhelming to continue writing policies with potential construction defect exposures. Many of those who continued to write policies implemented Montrose exclusions into the policy language to avoid being cited in a situation where damages were known to the insured prior to the beginning date of the policy.

WHAT IS A CONSTRUCTION DEFECT?

Posed to different sources, this question may produce different answers. It is difficult to find a clear, concise definition. Broadly speaking, when presented the question, courts have concluded that virtually any condition that reduces the value of a building, home, condominium or common area may be legally recognized as a defect in design or workmanship. Major defects may be related to landslides or subsidence, but the spectrum includes poor drainage, leaky roofs, defective plumbing, wiring and a host of other real and potential problems such as "sick buildings".

Insurance companies may have their own way of defining a construction defect for the purpose of coverage interpretation. Among the many coverage issues that may be relevant to an insurer's defense or indemnity obligations are:

> Does the claim involve "property damage" as defined in the commercial general liability (CGL) policy? Some components of construction defect claims are clearly "physical injury to tangible property". Others, such as diminution in value and costs of preventing future damage, present difficult coverage interpretation problems.
>
> Is the claim excluded under the work exclusion? CGL policies generally exclude coverage for "work performed" by the insured with the rationale that liability policies are not intended to guarantee adequate construction. The Broad Form Property Damage endorsement broadens coverage and narrows the effect of the exclusion by saying that the work exclusion does not apply if a contractor or subcontractor performed the damaged work or the work out of which the damage arises on behalf of the named insured.

Does the claim fall under any other non-standard policy provisions? The Subsidence Exclusion is one such provision which purports to eliminate coverage for property damage caused by the subsidence of land and arising out of, or attributable to, any operation of the insured.

Contractors and homeowners also have differing and self-serving opinions on what constitutes a defect. Ultimately, it is often up to the courts to decide the issue on individual lawsuits.

There are two types of defects: patent and latent. Patent defects are those that are detectable through reasonable inspection. In most jurisdictions, the Statute of Limitations for filing suit for patent defects is two to four years. On the other hand, latent defects are those that are not detectable through reasonable inspection and are manifested over a period of time. Most construction defect claims fall into this latent category. The time limit for presenting latent claims is often governed by a state's Statute of Repose, which begins running on the date that construction is completed. In California, aside from certain cross-complaint situations, which may enlarge the time for perfecting a claim, suits are barred ten years after the construction is completed.

Construction defects come from a variety of sources. Some defects are attributed to faulty workmanship. Most often, these defects are related to the following:

Plumbing / Drainage / Irrigation

Improper Materials

Structural Failure or Collapse

Electrical Wiring

Insulation

Other defects are a result of landslides and earth settlement conditions. Examples of these conditions include:

Expansive Soils

Underground Water

Vertical Settlement

Earthquakes

As an actuary, it is important to understand how your company or your client is defining construction defects. Knowing what types of claims are being included in your data will enhance the assumptions you make about development patterns and tail selection.

GATHERING DATA

It is important to understand what is included in the data you have gathered before beginning any construction defects analysis. Interviews with people from various departments in the company may be necessary to ensure that, to the extent possible, the correct data is retrieved and appropriately understood by those working with it. An attempt should be made to get answers to the following questions regarding any construction defects data set.

What is the definition of a construction defect claim?

How is the accident date determined?

What reinsurance agreements are in place?

Which states have construction defects exposures?

What is the mix of exposure for general contractors, designer/builders, and subcontractors?

Is the exposure residential or commercial construction?

Is exposure information available? (Earned premium, number of contractors insured, etc.)

Are there any policy provisions or enhancements, such as presence or absence of the broad form property damage endorsement?

It may also be appropriate to experiment with different segmentations of the data when performing an analysis. This may provide a deeper understanding of frequencies and severities for different types of business, as well as be able to offer added insight to your client. The following segmentations should be considered if the data is available.

California and Non-California (or other specific states)

The legal environment in California has proved to be unique. Separating California from the rest of the states may enhance the analysis.

General Contractors vs. Subcontractors

We recommend that the data be segmented between general and subcontractors, whenever possible. General contractors appear to have significantly higher severities than subcontractors. In some cases, the severities are as much as five times higher. We attribute this phenomenon to the fact that the general contractors are in control of the entire project, while the subcontractors are only performing a portion of the work on each project and therefore may not be subject to the total claim value. While producing higher severities, the claim count emergence is lower for general contractors than for subcontractors. Again, we believe that the larger number of projects that a subcontractor works on gives rise to the higher number of claims.

Report Year Data

In the next section, we discuss in more detail the difficulties of establishing an accident year for each claim. Because of these difficulties, we have found that it enhances our analysis to use report year data and methods. Report year data is beneficial for two reasons. The first is that the report date will be consistently applied to all claims. The second is that report year data allows the number of claims in each year to be set. Development on these claims is more readily determinable.

DIFFICULTIES WITH TRADITIONAL RESERVING METHODS

Due to the changing environment surrounding construction defects, problems arise with the application of traditional reserving methods to general liability or commercial multiple peril lines of business that contain construction defect claims. The most commonly used method to determine ultimate losses is the accident year loss development method. The following assumptions are inherent in the loss development method:

The accident date is clearly identifiable and consistently applied

Future emergence of an accident year can be determined from the emergence of historical accident years

Ultimate loss is a function of current loss to date

With the application of this method to construction defect claims, these key assumptions may be violated.

The first point of difficulty with any accident year development method is the determination of the appropriate accident date for a construction defect claim. As previously mentioned, the Montrose decision changed the theory underlying the date of loss from a manifestation trigger theory to a continuous trigger theory. The continuous trigger period can begin as early as the date the work contract is signed and continue until the repairs are made. The continuous trigger theory allowed multiple insurers to experience loss on a single occurrence. Under the Post-Montrose continuous trigger theory, the determination of the accident date varies by company and frequently varies within a single company. This is particularly noticeable when companies do not have a dedicated construction defect claims unit established.

There are two main philosophies when determining the accident date of a construction defect claim under the continuous trigger theory. The first method is to assign a claim to each accident year where there is believed to be potential exposure. The second method is to determine one appropriate accident year to which the claim would be coded. For example, a company may dictate that each construction defect claim should be coded to the accident year two years after the completion of the project in question. It is also possible that a company would decide to use some combination of these two methods when coding claims to an accident year. While neither method is preferable over the other, it is important that one method be applied consistently. It is also important for the actuary to have an understanding of the accident date determination used in a particular company. It may require interviews with claims handlers and other construction defect claims specialists within the company.

A second difficulty with applying the loss development method is the determination of the future development pattern. The loss emergence patterns appear to be lengthening due to the change in trigger theory and the Statute of Limitations. Under the Pre-Montrose environment, the plaintiff attorneys in California tended to file lawsuits within three years of the manifestation date, most likely because of the Statute of Limitations for patent defects. On the other hand, latent defects are subject to the Statute of Repose. In California, a plaintiff is allowed up to ten years from the building's date of completion to resolve a potential claim, or a lawsuit must be filed to prevent the Statute of Limitations from barring recovery. In the current environment, where the continuous trigger applies, insurers that may not have otherwise been affected by the manifestation trigger theory are experiencing late reported claims.

Another reason that it is difficult to determine the loss development pattern is that the effects of the litigation surrounding construction defects affect an accident year triangle on the diagonal. Due to the Montrose Decision, an influx of claims is normally observed in recent calendar years. The distortion of the calendar year diagonal in an accident year triangle leads to higher development factors along the diagonal from which to select. These factors may not be appropriate to be applied to losses at the current evaluation date. There is also simply a lack of historical data. As the Montrose Decision was in 1995, there have not been many years to observe how the change will impact the emergence of loss.

Determining the tail development factor is also difficult when applying the loss development method. Again, the future construction defect environment is so uncertain that it is extremely difficult to develop a deep enough understanding of the loss emergence to determine at what point any tail factor would become unreasonable. In California, it seems reasonable to assume that there will be no more claims reported after 13 years of development for any accident year.

This is because there is a ten-year Statute of Limitations for reporting the discovery of a defect with the potential for an additional 3 years to file the lawsuits for indemnity. However, there is not yet substantial data to support this theory.

NON-TRADITIONAL RESERVING METHODOLOGIES

This section describes three approaches that we have used to estimate the construction defect claim ultimate losses.

Montrose Adjustment Method

Transactional Count / Incremental Paid Loss Method

Report Year Analysis (pure IBNR estimated using a selected exposure distribution)

The Montrose Adjustment Method is a derivation of the traditional loss development approaches while the other two methods segment the losses into two components: frequency and severity, which are estimated separately.

Montrose Adjustment Method

With the application of the Montrose Decision on the construction defect claims, there has been a significant calendar year impact on the traditional accident period loss development methods. Prior to the decision in 1995, the historical loss and claim count triangles had considerably less volume. Subsequent to the decision, the volume has increased dramatically along each calendar year thus causing the link ratios in a traditional development method to rise initially. In almost every instance, these link ratios have remained above expected levels. An example of this can be seen in the link ratio method displayed in Exhibit 3. This calendar year occurrence affects the accident year triangle on the diagonal. The magnitude of this phenomenon will be different by company as there are three variables that can influence the pattern:

Volume of business written in each year

Type of business written

Claims handling procedures

This phenomenon makes the selection of a reasonable tail factor extremely difficult, if not impossible. This is because we traditionally depend on observed development just prior to the end of the triangle to aid in the selection of the tail factor. However, as you can see in Exhibit 3, the development usually seen with construction defects does not decrease even after many months of development. The development remains at a high level because the claim emergence prior to calendar year 1995 is significantly below that seen after 1995 and, thus, the new claims emerging are over leveraging the development pattern.

The Montrose Adjustment Method attempts to mitigate the effect that the calendar year emergence has on the development factors by recasting the volume of the pre-Montrose years to mimic the type of development those years would have experienced if Montrose had happened many years ago. This approach can be used for losses, allocated loss adjustment expense (ALAE), or claim counts. We have used this method with reported counts in our examples.

The objective of this method is to adjust the pre-Montrose incremental claim activity so that the link ratios in later months of development will appear more reasonable and a tail factor will be easier to estimate. This adjustment consists of building additional counts into the earlier months of development of the incremental triangle and re-cumulating the triangle. Ideally, we want to add enough claim counts to the early development of the accident years prior to 1995 so that the resulting development pattern will be comparable across all years.

We begin with the triangle of incremental reported counts as displayed on Exhibit 4-A. We have included a diagonal line in the incremental count triangle above which are the counts that will be restated. We have also displayed, on the same exhibit, a triangle of link ratios that show the ratio of incremental reported counts from one period to another. We have included a line after accident 1994 on this triangle because accident years 1995 and subsequent are Post-Montrose. Therefore, we are assuming that the development in these accident years is indicative of future development. Link ratios should be selected from the Post-Montrose ratios.

The Pre-Montrose incremental counts are restated as though they were Post-Montrose by dividing the Post-Montrose incremental counts at the earliest age of development by the appropriate link ratio. For example, in accident year 1990, 53 claims were reported between 60 and 72 months of development. This is shown as the earliest Post-Montrose development on Exhibit 4-A. Prior to that period, 20 claims were reported between 48 and 60 months. By dividing 53 by 1.2, we now have 45 claims in the development period between 48 and 60 months for accident year 1990. This process continues for all of the Pre-Montrose development periods. The restated incremental triangle is displayed in Exhibit 4-B.

The restated reported counts can be re-cumulated and used with the traditional link ratio method. See Exhibit 4-C. Notice that the Pre-Montrose development periods have identical development factors. These should not be considered when selecting your link ratios. They should be, however, comparable with the more recent ratios in the triangle. It is now more apparent that the ratios decrease in later development periods, allowing an easier selection of more mature link ratios and tail factor. When selecting the tail factor, one may also scenario test the selection to account for the statute of limitations.

Because our triangle now has more claim counts than it did previously, it is not possible to simply apply the cumulative development factor to the latest diagonal to produce ultimate counts. It is necessary to subtract one from the cumulative development factor before applying it to the adjusted counts and add this development to the original case reported counts. See Exhibit 4-D.

The Montrose Adjustment method assumes that the current level of claim activity is now a normal occurrence in this type of data and is not a spike up of activity associated with the Montrose Decision. The method can often produce volatile results, particularly in the initial stages of claim emergence, because the claims department will be making initial determinations as to the internal processes to be used in the coding of claims, as well as the philosophy of handling those claims. It may be beneficial to begin the recasting of information using a year more recent than 1995 to account for this initial volatility. For instance, if your company began to see construction defect claims in 1995 but waited until 1997 to set up a special claims unit to handle these claims, you may choose to use 1997 as your base year for this approach since it may be more representative future emergence.

Given the assumptions underlying this method, the results will likely lead to a conservative estimate of the liabilities, particularly without accounting for the statute of limitations in the selection of the tail factor. While conservative, this can be particularly useful in helping to bracket a range of reasonable liabilities and demonstrating to management what the high end of the liabilities might be.

Transactional Count / Incremental Paid Method

This method is similar to the incremental paid loss method developed by Adler/Kline. [4] The difference between our incremental method and the one that Adler/Kline developed is the way in which ultimate counts are determined and distributed to each development period. We have called this method of determining ultimate counts a "transactional" count method.

The goal of the transactional method is to create an incremental closed with payment claim triangle that has been "squared" to ultimate. This triangle can then be multiplied by the corresponding severities selected at each development period. To create this triangle, we begin with reported counts and attempt to estimate the portion of these claim counts that will close with payment and the portion that will close without payment at each development period. Therefore, we make two selections of disposal rates: closed with payment disposal rates and closed without payment disposal rates. These disposal rates are not based on ultimate counts, as they are in the Adler/Kline paper. They are based on the number of claims that were open at the end of the prior period plus those that were reported during the current period.

Exhibit 5-A displays a reported count triangle that has been "squared", which is the starting point for this method. Estimate the number of claim counts that will ultimately be reported is an important step in this method and may tend to drive the results. Ultimate reported counts could be determined by the approach described in the Montrose Adjustment Method. We used the results of the Montrose Adjustment Method in Exhibit 4 to create the reported count triangle displayed in Exhibit 5-A.

To determine ultimate reported counts, we have also employed a method that decays calendar year reported counts over time. When using a calendar year approach, the resulting counts must be distributed back to accident year for use in our transactional count method.

The lower half of Exhibit 5-B displays a triangle of claim counts labeled "Active Counts during Period". This triangle is created by adding the counts that were open at the end of the prior period (displayed on the upper portion of Exhibit 5-B) and the incremental counts that were reported during the period, shown on the lower half of Exhibit 5-A.

The triangle of active counts will be used to create disposal rates for the claims that will close with payment and the claims that will close without payment. A triangle of the historical closed with payment disposal rates can be created by dividing the incremental closed with payment by the active counts during the period, and a triangle of the historical closed without payment disposal rates can be created by dividing the incremental closed without payment claims by the active counts during the period. The cumulative triangle of closed with payment counts and closed without payment counts are displayed on Exhibit 5-C. The incremental triangles are displayed on Exhibit 5-D. The historical disposal rates and selections are displayed on Exhibit 5-E. We have made the selections of disposal rates based on observed historical patterns.

Once the disposal rates have been selected, it is possible to "square" the triangles of counts open at the end of the prior period, active counts during the period, closed with payment counts, and closed without payment counts. Each of these triangles builds off of the others. The number of claims that will close during the period can be determined by applying the disposal rates to the active counts during the period. After subtracting the number of claims that close during the period, you can determine the number of claims that will be open at the end of the period, and so on. The "squared" triangles are displayed on Exhibits 5-F and 5-G.

The final step in this approach is to multiply the incremental closed with payment claim count "triangle" by the incremental severities. We typically make selections from the historical incremental severities and trend them into future periods. Generally, we have found that the severities have been relatively stable, so it is the estimate of ultimate counts that ultimately tends to drive the variability of the results. Exhibit 5-H displays the incremental closed with payment counts and severities. Exhibit 5-I shows the multiplication of the two triangles in Exhibit 5-H. Outstanding loss is calculated by adding the incremental paid loss in future development periods, or below the diagonal line.

Report Year Analysis

This last method is the report period year approach. There are two major components necessary for this type of analysis: the first is the development of reported loss on known claims, and the second is the estimation of the pure IBNR loss.

The first component of this analysis is relatively straightforward. The traditional loss development methods can be applied to both paid and incurred losses on a report year basis to develop an estimate of ultimate losses. We also estimate ultimate claim counts on a report year basis. We have found that applying the development method to incurred counts, where incurred counts are defined as closed with payment plus open counts, produces a reasonable estimate of ultimate counts. See Exhibit 6.

To estimate the IBNR claim counts, we begin by attempting to estimate the company's remaining exposure to construction defect claim experience. We have used the general liability contractors written premium as an exposure base for construction defects. To determine the number of claims that will be reported in future calendar years, we must determine the portion of exposure that continues to exist from the year the policies were written. We have chosen to decay the exposure from each underwriting year to future years with a selected distribution. This distribution is based on observed patterns of reported counts. See Exhibit 7-A. The exposure to construction defect claims of future report years can be determined by adding together the appropriate amounts from each underwriting year. See Exhibit 7-B.

Once the report year exposure has been estimated, future reported counts are determined by selecting a frequency for future report years. These can be selected from observed historical frequencies. The historical frequencies are the comparison of our selected ultimate claim counts from our report year methods to the report year estimated exposure to construction defect claims for those years. Based on these observed frequencies, a future frequency can be selected and applied to the future report year exposure to obtain a pure IBNR claim count estimate. See Exhibit 7-C.

Finally, total estimated IBNR losses are estimated by multiplying these claim counts by a selected severity. The severity can be estimated by observing the severities implied by the results of the report year development methods for loss and claim counts. Total ultimate losses are then found by adding the results of the report year loss development methods and the pure IBNR loss estimate. See Exhibit 7-D.

As with any methodology, this one has its advantages and disadvantages. One advantage of this approach is that because claims are aggregated on a report year basis, the number of claims attaching to a particular year is known. The resulting development patterns for the emergence and settlement patterns are considerably shorter than on an accident year basis and, therefore, are easier to select. Conversely, the IBNR can be somewhat more difficult because the future claim emergence and associated costs must be estimated. In fact, determining IBNR is the essence of the difficulty with projecting ultimate losses for construction defects. Furthermore, report year results can be difficult to compare with accident year results unless the future liabilities can be converted back to an accident year basis. Nonetheless, we believe that this method or some adaptation of it has produced the most reasonable and consistent results for our clients.

ADDITIONAL CONSIDERATIONS AND CURRENT TRENDS

Current Trends in Frequencies

Between 1994 and 1999, there was a continual rise in claim activity in California related to construction defects. During the last several years, there has been an increasing belief that the claim frequency will begin to subside as the statute of limitations runs out on reportable claims. During 2000, many companies began to see a flattening of claim activity, which could be caused by the statute of limitations or just random fluctuation. As 2001 unfolds, the industry is anxiously awaiting whether companies will continue to see a stabilization of claim emergence or even begin to see a decrease in claim activity or whether it will begin to rise again.

Current Trends in Severities

Unlike the large increase in claim activity and the highly publicized large verdicts as the construction defects came to the forefront of the insurance industry, the average severity has remained relatively stable through 1999. During 2000, a few companies have seen a slight decrease in severity as they continue to refine their stance on the claim handling approach. Additionally, when analyzing historical paid severities by age of claim, the severities appear to be stable as well. This has substantiated the notion, that this is primarily a frequency issue. Up to this point, this notion appears to have been correct. However, companies should continue to closely monitor the severity trend, particularly given the continued uncertainty of the claim count emergence and each company's stance on handling claims. In addition, it is still unknown whether the claims in the tail will be larger than the claims paid to date.

ALAE to Loss Ratios

Unlike the stability of loss severities, the ratio of ALAE to loss has continued to increase over time. We recommend that ALAE be analyzed separately for the following reasons:

Claim departments continue to modify their stance on the handling of claims

Companies have attempted to control the costs by entering into either a specified charge per claim or a fixed fee arrangements with outside law firms

When multiple companies are involved in the litigation of the claim, they frequently share in the cost of one law firm

In addition, we recommend that ALAE for general contractors and sub-contractors be analyzed separately as well, because they have shown considerable differences in the ultimate ratio.

WHAT LIES AHEAD?

California Landscape

Currently, the situation in California is troubling. There remains a shortage of skilled construction workers and real estate prices are astronomically high with a shortage of affordable housing (condominiums and town homes) being built. In addition, with the size and impact of the construction defect problem on the insurance industry, the state faces an insurance availability crisis. Eventually, the increased pressure arising from the current situation will begin to force changes. Potential changes on the horizon could come from many different sources, legislative, judicial, or economic.

The California legislature has attempted to ease the situation by passing legislative items such as the Calderon Act that became effective January 1, 1996. This act applies only to multi-unit dwellings. It attempts to implement mandatory mediation sessions with the homeowners association and the builder to attempt to resolve lawsuits before they are filed. While it was highly touted as a significant step at the time of passage, to date, it appears to have had little impact on the number of lawsuits filed or the settlement process. [5]

In December 2000, the California Supreme Court ruled on a construction defected related case, Alan O. Aas v. Superior Court. The impact of this ruling is that the Supreme Court has supported a lower court decision that plaintiffs could not seek damages for construction defects that had not yet caused property damage. It is too soon to quantify the impact of this decision, however, it is speculated that this decision will significantly reduce the exposure developers, contractors and sub-contractors face in the construction industry. [6]

The past several decades have seen a substantial rise in the population growth in California. This has been driven by a number of items, not the least of which is the dot-com boom. As the current boom appears to be subsiding, the continued pressure for affordable housing may ease slightly.

Other States

There continues to be speculation that what has transpired in California will transfer to other areas of the country, specifically where the population has been increasing rapidly. Baby-boomers are retiring to the south and west regions of the country to states such as Nevada, Florida, Texas, Arizona, and Colorado. While there has been an increase in the number of construction defect suits in these and other areas, the legal landscape is different that California. In most states, the statute of limitations is much shorter that California, and other states have not adopted the same continuous trigger theory that California has on these claims.

The issues discussed above have helped keep the situation in other areas from rapidly running out of control. However, there continues to be increased pressure from lawyers and homeowners, and claim frequency is rising in these states. Other states should be monitored closely both from a claim environment and a legal environment to ensure that both the construction and the insurance industries are prepared, in the event the situation changes.

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Exhibit 1

Population Ten Year Growth Rates

	California	U.S
1970-1980	19%	11%
1980-1990	26%	10%
1990-2000	14%	13%

From Census 2000

Exhibit 2

Building Permits Issued in California

Calendar Year	Total New Housing Units	1 Unit Single Family	2 Units	3 & 4 Units	5+ Units	5+ Structures	Total Structures	Total Excluding Single Family	Proportion Multi-unit of Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9) = (8) / (1)
1984	224,689	112,920	6,496	13,434	91,839	8,214	128,220	15,300	6.8%
1985	271,396	113,647	6,390	13,765	137,594	11,255	132,030	18,383	6.8%
1986	314,641	145,692	6,366	14,498	148,085	11,811	164,828	19,136	6.1%
1987	251,824	134,691	4,924	11,822	100,387	8,152	148,683	13,992	5.6%
1988	253,369	160,735	4,366	8,955	79,313	6,154	171,631	10,896	4.3%
1989	237,694	162,981	4,148	7,838	62,727	5,462	172,756	9,775	4.1%
1990	163,175	104,843	3,926	5,746	48,660	3,991	112,439	7,596	4.7%
1991	105,956	73,885	2,342	4,554	25,175	2,036	78,393	4,508	4.3%
1992	97,781	76,332	1,886	3,934	15,629	1,382	79,781	3,449	3.5%
1993	84,341	69,568	1,406	2,390	10,977	953	71,907	2,339	2.8%
1994	96,982	77,795	1,382	3,100	14,705	1,178	80,550	2,755	2.8%
1995	83,864	68,148	1,170	2,880	11,666	1,002	70,558	2,410	2.9%
1996	92,060	73,532	1,138	2,457	14,933	1,042	75,845	2,313	2.5%
1997	109,589	84,149	1,180	2,298	21,962	1,401	86,797	2,648	2.4%
1998	123,653	92,933	1,366	2,689	26,665	1,677	96,061	3,128	2.5%
1999	138,039	102,750	1,134	2,460	31,695	1,820	105,840	3,090	2.2%
2000	143,216	103,991	1,196	2,780	35,249	1,871	107,254	3,263	2.3%
Total	2,792,269	1,758,592	50,816	105,600	877,261	69,401	1,883,572	124,980	

Link Ratio Method Exhibit 3

Reported Counts

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Case Reported Counts	CDF	Ultimate Reported Counts
	Months of Development													
1990	52	61	72	83	103	156	306	567	927	1,345	1,671	1,671	1.36	2,272
1991	73	84	97	132	350	647	998	1,460	2,029	2,584		2,584	1.69	4,365
1992	68	76	99	339	610	965	1,386	1,861	2,337			2,337	2.20	5,134
1993	94	144	373	714	1,076	1,483	1,889	2,398				2,398	3.13	7,515
1994	103	412	864	1,211	1,552	1,925	2,465					2,465	4.64	11,447
1995	93	484	921	1,255	1,648	2,142						2,142	6.42	13,756
1996	135	668	1,033	1,382	1,894							1,894	8.83	16,732
1997	90	349	605	888								888	12.08	10,734
1998	31	83	140									140	16.83	2,359
1999	18	34										34	28.90	989
2000	20											20	96.93	1,939
												16,573		77,243

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-Ult
	Age-to-Age										
1990	1.17	1.18	1.15	1.24	1.52	1.96	1.85	1.64	1.45	1.24	
1991	1.15	1.15	1.36	2.65	1.85	1.54	1.46	1.39	1.27		
1992	1.12	1.30	3.42	1.80	1.58	1.44	1.34	1.26			
1993	1.53	2.59	1.91	1.51	1.38	1.27	1.27				
1994	4.00	2.09	1.40	1.28	1.24	1.28					
1995	5.21	1.90	1.36	1.31	1.30						
1996	4.94	1.55	1.34	1.37							
1997	3.89	1.73	1.47								
1998	2.68	1.69									
1999	1.90										
Average	3.35	1.72	1.39	1.37	1.38	1.38	1.48	1.43	1.30	1.24	
Factor to Ultimate	96.93	28.90	16.83	12.08	8.83	6.42	4.64	3.13	2.20	1.69	1.36

Exhibit 4-A

Montrose Adjustment Method

Incremental Reported Counts

Accident Year	Months of Development										
	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
1990	52	9	11	11	20	53	150	261	360	417	326
1991	73	11	13	35	218	297	350	463	569	554	
1992	68	8	23	240	271	355	421	476	476		
1993	94	50	229	341	362	408	406	509			
1994	103	309	452	347	341	373	540				
1995	93	391	437	334	393	494					
1996	135	533	365	349	512						
1997	90	259	256	284							
1998	31	52	57								
1999	18	16									
2000	20										

Accident Year	Age-to-Age										
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-Ult
1990	0.17	1.22	1.00	1.82	2.67	2.80	1.74	1.38	1.16	0.78	
1991	0.15	1.18	2.69	6.24	1.36	1.18	1.32	1.23	0.97		
1992	0.12	2.88	10.44	1.13	1.31	1.18	1.13	1.00			
1993	0.53	4.59	1.49	1.06	1.13	1.00	1.25				
1994	3.00	1.46	0.77	0.98	1.10	1.45					
1995	4.21	1.12	0.76	1.18	1.26						
1996	3.94	0.69	0.96	1.47							
1997	2.89	0.99	1.11								
1998	1.68	1.09									
1999	0.90										
Avg Below Line	2.73	0.97	0.94	1.32	1.26						
Selected	2.73	0.97	0.94	1.20	1.20						

Montrose Adjustment Method Exhibit 4-B

Adjusted Incremental Counts

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-Ult
1990	15	41	39	37	45	53	150	261	360	417	326
1991	73	199	193	182	218	297	350	463	569	554	
1992	96	262	255	240	271	355	421	476	476		
1993	87	236	229	341	362	408	406	509			
1994	114	309	452	347	341	373	540				
1995	93	391	437	334	393	494					
1996	135	533	365	349	512						
1997	90	259	256	284							
1998	31	52	57								
1999	18	16									
2000	20										

Montrose Adjustment Method Exhibit 4-C

Restated Cummulative Triangle

Accident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132
1990	15	55	95	132	177	230	380	641	1,001	1,418	1,744
1991	73	272	465	647	865	1,162	1,513	1,975	2,544	3,099	
1992	96	359	613	853	1,124	1,479	1,900	2,376	2,852		
1993	87	323	552	893	1,255	1,662	2,068	2,577			
1994	114	423	874	1,222	1,562	1,936	2,475				
1995	93	484	921	1,255	1,648	2,142					
1996	135	668	1,033	1,382	1,894						
1997	90	349	605	888							
1998	31	83	140								
1999	18	34									
2000	20										

Accident Year	Age-to-Age 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-Ult
1990	3.73	1.71	1.39	1.34	1.30	1.65	1.69	1.56	1.42	1.23	
1991	3.73	1.71	1.39	1.34	1.34	1.30	1.31	1.29	1.22		
1992	3.73	1.71	1.39	1.32	1.32	1.28	1.25	1.20			
1993	3.73	1.71	1.62	1.41	1.32	1.24	1.25				
1994	3.73	2.07	1.40	1.28	1.24	1.28					
1995	5.21	1.90	1.36	1.31	1.30						
1996	4.94	1.55	1.34	1.37							
1997	3.89	1.73	1.47								
1998	2.68	1.69									
1999	1.90										
Average	3.35	1.72	1.39	1.34	1.29	1.28	1.37	1.35	1.25	1.23	
Factor to Ultimate	65.86	19.64	11.44	8.22	6.12	4.73	3.70	2.70	2.00	1.60	1.30

Montrose Adjustment Method Exhibit 4-D

Calculation of Ultimate Reported Counts

Accident Year	Case Reported Counts	Restated Reported Counts	Cumulative Development Factor	Additional Counts	Ultimate Counts
	(1)	(2)	(3)	(4)	(5)
1990	1,671	1,744	1.30	523	2,194
1991	2,584	3,099	1.60	1,855	4,439
1992	2,337	2,852	2.00	2,847	5,185
1993	2,398	2,577	2.70	4,376	6,774
1994	2,465	2,475	3.70	6,689	9,154
1995	2,142	2,142	4.73	7,987	10,129
1996	1,894	1,894	6.12	9,705	11,599
1997	888	888	8.22	6,411	7,300
1998	140	140	11.44	1,463	1,603
1999	34	34	19.64	638	672
2000	20	20	65.86	1,297	1,317
Total	16,573	17,865		43,792	60,364

Transactional Count Method

Exhibit 5-A

Reported Counts

Accident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	Ultimate
1990	52	61	72	83	103	156	306	567	927	1,345	1,671	2,194
1991	73	84	97	132	350	647	998	1,460	2,029	2,584	3,296	4,439
1992	68	76	99	339	610	965	1,386	1,861	2,337	3,050	3,870	5,185
1993	94	144	373	714	1,076	1,483	1,889	2,398	3,300	4,170	5,169	6,774
1994	103	412	864	1,211	1,552	1,925	2,465	3,386	4,575	5,721	7,039	9,154
1995	93	484	921	1,255	1,648	2,142	2,736	3,754	5,068	6,335	7,791	10,129
1996	135	668	1,033	1,382	1,894	2,453	3,133	4,298	5,804	7,255	8,922	11,599
1997	90	349	605	888	1,192	1,544	1,971	2,705	3,653	4,566	5,615	7,300
1998	31	83	140	195	262	339	433	594	802	1,003	1,233	1,603
1999	18	34	59	82	110	142	182	249	336	420	517	672
2000	20	67	115	160	215	279	356	488	659	824	1,013	1,317

Incremental Reported Counts

Accident Year	Months of Development 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-92	92-108	108-120	120-132	132-Ult
1990	52	9	11	11	20	53	150	261	360	417	326	523
1991	73	11	13	35	218	297	350	463	569	554	712	1,143
1992	68	8	23	240	271	355	421	476	476	713	819	1,315
1993	94	50	229	341	362	408	406	509	902	870	1,000	1,604
1994	103	309	452	347	341	373	540	921	1,189	1,146	1,317	2,115
1995	93	391	437	334	393	494	594	1,018	1,314	1,267	1,456	2,337
1996	135	533	365	349	512	559	680	1,166	1,505	1,451	1,667	2,677
1997	90	259	256	284	304	352	428	734	947	913	1,049	1,685
1998	31	52	57	55	67	77	94	161	208	201	230	370
1999	18	16	25	23	28	32	39	68	87	84	97	155
2000	20	47	48	45	55	63	77	132	171	165	189	304

Open Counts at End of Period

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
1990					95	121	189	282	378	411	377	
1991				110	280	306	365	461	551	608		
1992			71	227	308	399	461	513	564			
1993		127	262	342	391	437	445	592				
1994	85	261	438	421	408	411	590					
1995	73	316	399	321	393	572						
1996	106	409	362	383	549							
1997	68	183	254	326								
1998	21	44	54									
1999	18	18										
2000	18											

Active Counts During Period

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
1990					103	148	271	450	642	795	737	
1991				132	328	577	657	828	1,030	1,105		
1992			99	311	497	663	820	937	989			
1993		144	357	603	704	799	843	954				
1994	103	394	712	786	761	781	950					
1995	93	465	753	734	714	887						
1996	135	639	774	711	895							
1997	90	328	438	538								
1998	31	73	101									
1999	18	34										
2000	20											

Transactional Count Method

Exhibit 5-C

Cumulative Closed with Payment Counts

Accident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	Ultimate
1990					8	11	59	161	313	499	691	
1991				19	23	158	347	572	808	1,082		
1992			23	31	135	282	484	717	965			
1993		15	28	181	368	584	812	1,040				
1994	6	42	166	399	582	817	1,019					
1995	7	57	238	429	595	768						
1996	11	121	298	486	688							
1997	3	51	129	245								
1998	5	15	37									
1999	-	7										
2000	-											

Cumulative Closed without Payment

Accident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	Ultimate
1990					-	24	59	124	236	435	603	
1991				3	47	183	285	427	670	893		
1992			5	82	166	284	440	631	808			
1993		2	83	191	316	463	632	766				
1994	12	109	259	391	562	698	856					
1995	13	111	284	505	660	802						
1996	18	139	373	513	657							
1997	18	116	222	318								
1998	5	24	49									
1999	-	10										
2000	2											

Transactional Count Method Exhibit 5-D

Incremental Closed with Payment Counts

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
1990					8	3	47	103	152	186	192	
1991				19	4	135	189	225	236	274		
1992			23	8	104	147	202	233	248			
1993		15	13	153	187	215	228	228				
1994	6	36	124	233	183	235	202					
1995	7	51	181	191	166	173						
1996	11	109	178	187	202							
1997	3	47	78	116								
1998	5	10	23									
1999	-	7										
2000	-											

Incremental Closed without Payment

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
1990					-	24	34	65	112	199	168	
1991				3	44	135	103	142	243	223		
1992			5	77	85	117	156	191	178			
1993		2	82	108	126	147	170	134				
1994	12	97	150	132	171	135	158					
1995	13	98	173	222	155	142						
1996	18	121	235	140	143							
1997	18	98	106	96								
1998	5	20	24									
1999	-	10										
2000	2											

Transactional Count Method

Closed with Payment Disposal Rate

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Ultimate
					Months of Development							
1990					8%	2%	17%	23%	24%	23%	26%	
1991				14%	1%	23%	29%	27%	23%	25%		
1992			23%	3%	21%	22%	25%	25%	25%			
1993		10%	4%	25%	27%	27%	27%	24%				
1994	6%	9%	17%	30%	24%	30%	21%					
1995	7%	11%	24%	26%	23%	19%						
1996	8%	17%	23%	26%	23%							
1997	4%	14%	18%	22%								
1998	16%	13%	23%									
1999	0%	19%										
2000	0%											
Selected Disposal Rate		15%	21%	26%	23%	24%	24%	25%	24%	25%	25%	25%

Closed without Payment Disposal Rate

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Ultimate
					Months of Development							
1990					0%	16%	13%	14%	18%	25%	23%	
1991				2%	13%	23%	16%	17%	24%	20%		
1992			5%	25%	17%	18%	19%	20%	18%			
1993		1%	23%	18%	18%	18%	20%	14%				
1994	12%	25%	21%	17%	22%	17%	17%					
1995	14%	21%	23%	30%	22%	16%						
1996	13%	19%	30%	20%	16%							
1997	20%	30%	24%	18%								
1998	16%	27%	24%									
1999	0%	29%										
2000	8%											
Selected Disposal Rate		25%	24%	18%	19%	18%	17%	16%	20%	20%	20%	20%

Transactional Count Method

Exhibit 5-F

Open Counts at End of Period

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
					Months of Development							
1990					95	121	189	282	378	411	377	495
1991				110	280	306	365	461	551	608	726	1,028
1992			71	227	308	399	461	513	564	702	837	1,184
1993		127	262	342	391	437	445	592	843	942	1,068	1,470
1994	85	261	438	421	408	411	590	889	1,173	1,276	1,426	1,948
1995	73	316	399	321	393	572	683	1,000	1,307	1,416	1,579	2,154
1996	106	409	362	383	549	643	775	1,141	1,494	1,620	1,808	2,466
1997	68	183	254	326	364	415	494	722	942	1,021	1,138	1,553
1998	21	44	54	60	74	87	106	157	206	224	250	341
1999	18	18	23	26	31	37	45	66	86	94	105	143
2000	18	39	48	52	62	73	88	129	170	184	205	280

Active Counts During Period

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
					Months of Development							
1990					103	148	271	450	642	795	737	900
1991				132	328	577	657	828	1,030	1,105	1,320	1,869
1992			99	311	497	663	820	937	989	1,277	1,522	2,152
1993		144	357	603	704	799	843	954	1,494	1,713	1,942	2,672
1994	103	394	712	786	761	781	950	1,511	2,078	2,320	2,593	3,541
1995	93	465	753	734	714	887	1,166	1,701	2,315	2,574	2,872	3,917
1996	135	639	774	711	895	1,108	1,323	1,940	2,646	2,945	3,287	4,485
1997	90	328	438	538	630	716	843	1,228	1,669	1,855	2,070	2,823
1998	31	73	101	109	127	151	181	267	365	407	454	620
1999	18	34	42	46	54	63	76	112	153	171	190	260
2000	20	65	87	93	107	125	150	220	300	334	373	509

Transactional Count Method

Exhibit 5-G

Cummulative Closed with Payment Counts

Accident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	Ultimate
1990					8	11	59	161	313	499	691	916
1991				19	23	158	347	572	808	1,082	1,412	1,880
1992			23	31	135	282	484	717	965	1,284	1,665	2,203
1993		15	28	181	368	584	812	1,040	1,396	1,825	2,310	2,978
1994	6	42	166	399	582	817	1,019	1,392	1,888	2,468	3,116	4,001
1995	7	57	238	429	595	768	1,051	1,471	2,024	2,667	3,385	4,364
1996	11	121	298	486	688	956	1,277	1,757	2,388	3,124	3,946	5,067
1997	3	51	129	245	391	564	769	1,073	1,471	1,935	2,452	3,158
1998	5	15	37	66	95	132	176	242	329	431	544	699
1999	-	7	15	28	40	55	74	102	138	181	228	293
2000	-	10	28	52	77	107	144	198	270	353	447	574
Selected Disposal Rate		15%	21%	26%	23%	24%	24%	25%	24%	25%	25%	25%

Cummulative Closed without Payment

Accident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	Ultimate
1990						24	59	124	236	435	603	783
1991				3	47	183	285	427	670	893	1,157	1,531
1992			5	82	166	284	440	631	808	1,064	1,368	1,799
1993		2	83	191	316	463	632	766	1,060	1,403	1,791	2,326
1994	12	109	259	391	562	698	856	1,105	1,514	1,978	2,497	3,205
1995	13	111	284	505	660	802	1,002	1,282	1,738	2,252	2,827	3,610
1996	18	139	373	513	657	854	1,081	1,401	1,921	2,510	3,168	4,065
1997	18	116	222	318	437	564	708	911	1,239	1,610	2,024	2,589
1998	5	24	49	69	93	120	151	195	267	348	439	563
1999	-	10	20	28	39	50	63	81	112	146	184	236
2000	2	18	39	56	76	99	124	161	220	287	361	463
Selected Disposal Rate		25%	24%	18%	19%	18%	17%	16%	20%	20%	20%	20%

Incremental Method

Exhibit 5-H

Incremental Closed With Payment Counts

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
					Months of Development							
1990					8	3	47	103	152	186	192	225
1991				19	4	135	189	225	236	274	330	467
1992			23	8	104	147	202	233	248	319	380	538
1993		15	13	153	187	215	228	228	356	428	485	668
1994	6	36	124	233	183	235	202	373	496	580	648	885
1995	7	51	181	191	166	173	284	420	552	643	718	979
1996	11	109	178	187	202	268	322	479	631	736	822	1,121
1997	3	47	78	116	147	173	205	303	398	464	517	706
1998	5	10	23	28	30	36	44	66	87	102	114	155
1999	-	7	9	12	13	15	19	28	37	43	48	65
2000	-	10	18	24	25	30	36	54	72	84	93	127

Incremental Paid Severity

Trend Factor 1.05

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail
					Months of Development							
1990					-	11,880	12,847	20,332	21,046	17,285	19,295	25,000
1991				-	125,288	10,290	19,374	17,622	21,800	16,809	25,000	26,250
1992			-	22,224	19,508	13,264	19,024	24,444	27,226	25,000	26,250	27,563
1993		-	16,261	9,395	14,434	21,794	23,238	22,291	25,000	26,250	27,563	28,941
1994	-	8,245	8,367	16,084	15,802	23,139	13,833	25,000	26,250	27,563	28,941	30,388
1995	10,144	9,621	18,062	12,996	14,270	11,997	20,000	26,250	27,563	28,941	30,388	31,907
1996	7,489	4,750	13,446	16,546	11,917	20,000	21,000	27,563	28,941	30,388	31,907	33,502
1997	2,127	9,188	6,801	30,437	15,000	21,000	22,050	28,941	30,388	31,907	33,502	35,178
1998	4,365	3,781	16,179	15,000	15,750	22,050	23,153	30,388	31,907	33,502	35,178	36,936
1999		15,135	15,000	15,750	16,538	23,153	24,310	31,907	33,502	35,178	36,936	38,783
2000		10,000	15,750	16,538	17,364	24,310	25,526	33,502	35,178	36,936	38,783	40,722
Selected Severity		10,000	15,000	15,000	15,000	20,000	20,000	25,000	25,000	25,000	25,000	25,000

Incremental Method

Exhibit 5-1

Incremental Paid Loss (000's)

Accident Year	12	24	36	48	60	72	84	96	108	120	132	Tail	Outstanding Loss
1990					-	41	607	2,088	3,190	3,212	3,711	5,624	5,624
1991				-	479	1,392	3,663	3,964	5,152	4,603	8,251	12,267	20,519
1992			-	177	2,035	1,946	3,845	5,698	6,745	7,980	9,985	14,828	32,794
1993		-	207	1,440	2,706	4,689	5,303	5,087	8,912	11,242	13,380	19,335	52,869
1994	-	300	1,037	3,749	2,885	5,431	2,796	9,332	13,017	15,983	18,763	26,902	83,995
1995	66	486	3,268	2,478	2,372	2,073	5,674	11,027	15,223	18,621	21,816	31,244	103,605
1996	85	519	2,389	3,102	2,409	5,355	6,760	13,210	18,275	22,372	26,221	37,561	129,753
1997	7	434	532	3,523	2,199	3,633	4,525	8,775	12,104	14,801	17,337	24,827	88,201
1998	21	37	369	423	466	803	1,022	2,007	2,781	3,407	3,994	5,722	20,625
1999		99	135	190	207	355	451	884	1,225	1,500	1,759	2,519	9,225
2000		98	290	399	431	735	931	1,821	2,522	3,087	3,619	5,185	19,118
													391,805

Link Ratio Method Exhibit 6

Incurred Counts

Report Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	Case Incurred Counts	CDF	Ultimate Incurred Counts
1990	-	-	-	-	293	282	266	243	228	207	186	186	1.00	186
1991	-	-	-	325	307	284	254	232	204	176		176	0.85	149
1992	-	-	534	502	464	424	392	356	323			323	0.75	243
1993	-	790	839	772	706	658	609	561				561	0.68	384
1994	790	1,090	989	890	826	759	700					700	0.62	437
1995	1,271	1,135	1,003	923	836	761						761	0.58	438
1996	1,451	1,315	1,236	1,152	1,081							1,081	0.53	577
1997	1,323	1,244	1,169	1,107								1,107	0.49	548
1998	1,238	1,163	1,109									1,109	0.46	508
1999	1,516	1,461										1,461	0.43	629
2000	1,352											1,352	0.41	552
												8,817		4,651

Report Year	Age-to-Age 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-Ult
1990					0.96	0.94	0.91	0.94	0.91	0.90	
1991				0.94	0.93	0.90	0.91	0.88	0.86		
1992			0.94	0.93	0.91	0.93	0.91	0.91			
1993		1.06	0.92	0.92	0.93	0.93	0.92				
1994	1.38	0.91	0.90	0.93	0.92	0.92					
1995	0.89	0.88	0.92	0.91	0.91						
1996	0.91	0.94	0.93	0.94							
1997	0.94	0.94	0.95								
1998	0.94	0.95									
1999	0.96										
Average	0.95	0.94	0.93	0.93	0.93	0.92	0.91	0.91	0.89	0.85	
Factor to Ulti	0.41	0.43	0.46	0.49	0.53	0.58	0.62	0.68	0.75	0.85	1.00

Exposure Count Method

Exhibit 7-A

Distribution of Exposures

Months of Development	Selected Distribution of Exposures	1992	1993	1994	1995	1996	1997	1998	1999	2000
		Underwriting Year								
12	15%	750	750	750	750	600	600	450	-	-
12-24	25%	1,250	1,250	1,250	1,250	1,000	1,000	750	-	-
24-36	20%	1,000	1,000	1,000	1,000	800	800	600	-	-
36-48	10%	500	500	500	500	400	400	300	-	-
48-60	8%	400	400	400	400	320	320	240	-	-
60-72	6%	300	300	300	300	240	240	180	-	-
72-84	5%	250	250	250	250	200	200	150	-	-
84-96	4%	200	200	200	200	160	160	120	-	-
96-108	3%	150	150	150	150	120	120	90	-	-
108-120	2%	100	100	100	100	80	80	60	-	-
120-132	1%	50	50	50	50	40	40	30	-	-
132-144	1%	50	50	50	50	40	40	30	-	-
144-156	0%	3	3	3	3	2	2	2	-	-
Total Written Premium	100%	5,003	5,003	5,003	5,003	4,002	4,002	3,002	-	-

Exposure Count Method

Exhibit 7-B

Allocation of Exposure to Report Year

Report Year	Underwriting Year 1992	1993	1994	1995	1996	1997	1998	1999	2000	Total RY Exposure
1992	750									750
1993	1,250	750								2,000
1994	1,000	1,250	750							3,000
1995	500	1,000	1,250	750						3,500
1996	400	500	1,000	1,250	600					3,750
1997	300	400	500	1,000	1,000	600				3,800
1998	250	300	400	500	800	1,000	450			3,700
1999	200	250	300	400	400	800	750	-		3,100
2000	150	200	250	300	320	400	600	-	-	2,220
2001	100	150	200	250	240	320	300	-	-	1,560
2002	50	100	150	200	200	240	240	-	-	1,180
2003	50	50	100	150	160	200	180	-	-	890
2004	3	50	50	100	120	160	150	-	-	633
2005		3	50	50	80	120	120	-	-	423
2006			3	50	40	80	90	-	-	263
2007				3	40	40	60	-	-	143
2008					2	40	30	-	-	72
2009						2	30	-	-	32
2010							2	-	-	2
2011								-	-	-
2012									-	-

Exposure Count Method

Exhibit 7-C

Selection of Ultimate Counts

Report Year	RY Exposure	Ultimate Incurred Claims	Indicated Frequency	Selected Frequency	Ultimate Claims
1992	750	243	3.24	3.24	243
1993	2,000	384	1.92	1.92	384
1994	3,000	437	1.46	1.46	437
1995	3,500	438	1.25	1.25	438
1996	3,750	577	1.54	1.54	577
1997	3,800	548	1.44	1.44	548
1998	3,700	508	1.37	1.37	508
1999	3,100	629	2.03	2.03	629
2000	2,220	552	2.49	2.49	552
2001	1,560			2.75	429
2002	1,180			2.75	325
2003	890			2.75	245
2004	633			2.75	174
2005	423			2.75	116
2006	263			2.75	72
2007	143			2.75	39
2008	72			2.75	20
2009	32			2.75	9
2010	2			2.75	0
2011	-			2.75	-
2012	-			2.75	-
Total					5,745

Exposure Method Exhibit 7-D

Determination of IBNR Loss

Report Year	Pure IBNR Claims	Selected Severity	Pure IBNR Loss
2001	429	30,000	12,870
2002	325	31,500	10,222
2003	245	33,075	8,095
2004	174	34,729	6,041
2005	116	36,465	4,237
2006	72	38,288	2,764
2007	39	40,203	1,575
2008	20	42,213	836
2009	9	44,324	390
2010	0	46,540	19
2011	-	48,867	-
2012	-	51,310	-
Total	1,429		47,049

A Dynamic Method for the Valuation of Fair Value Insurance Liabilities

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A Dynamic Method for the Valuation of Fair Value Insurance Liabilities

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Abstract

This paper presents a dynamic method to estimate fair value insurance liabilities for the whole book (with separate but correlated lines) of business. The model studies the aggregate liability without assuming independence of individual losses. A non-traditional approach is proposed which estimates the fair value liability based on a stochastic model of individual losses. Using the contingent claim analysis, the fair value liability are approximated by solving a partial differential equation. Parameters estimation, correlations measurement and applications of the model are also discussed in the study. Comparisons of the proposed method to the existing methods are given for application purpose.

1. Introduction

This study addresses the evaluation of insurance liabilities on a fair value basis. The fair value of liabilities is, as stated in the white paper by the Casualty Actuarial Society's Task Force on Fair Value Liabilities: "the fair value of the market value, if a sufficiently active market exists, OR an estimated market value, otherwise"(CAS 2000).

Fair value estimates of insurance liability reflect expected cash flows, the time value of money and an adjustment for risk. Over last fifteen years, many methods for estimating the fair value of property/casualty insurance liabilities has been introduced. All of these methods have their own advantages and disadvantages as summarized in the Casualty Actuarial Society's Task Force white paper (CAS 2000). Among various methods, there are two major approaches used to compute risk loads for the fair value liability that are represented in the literatures: the finance approach and the actuarial approach. The classical finance approach, is used in such methods as CAPM (D'Arcy and Doherty (1988), Fairley (1987), Feldblum (1990), Mahler (1998), and Myers and Cohn (1987)), the internal rate of return (Cummins (1990)), the single-period risk-adjusted discount method (Butsic (1988), and D'Arcy (1988)), the method based on underwriting data (Myers and Cohn (1987)), and the direct estimation of market values method (Allen, Cummins and Philips (1998), Ronn and Verma (1986)). The finance approach evaluates systematic risk by measuring the correlation between insurance companies returns from underwriting and market returns on its shareholder's equity.

The traditional actuarial approach is to use the aggregate probability distribution-based risk loads for the market risk adjustment of the liabilities. The actuarial based methods often explicitly incorporate process (diversifiable) and parameter (nondiversifiable) risk components into the risk load formulas. For a multiple line insurance company, liability (includes aggregate claim and expenses, taxes, et.c.) analysis estimates the total random losses for a book of insurance product line by studying possible aggregate claim distributions. Such distributions are probability distributions of the total dollar amount of loss under one or more insurance policies. They combine the separate effects of the underlying frequency and severity distributions. Assuming families of distributions (e.g. lognormals or shifted gammas) such that if each separate distribution is a member of these families, a closed form and elegant solution is possible. These methods can also be used to value unearned premium reserve and incurred but not reported reserves. (See Beard, Pesonen and Pentikâinen (1984), Bhlmann (1970), Embrechts (1995), Hayne (1989), Heckman and Meyers (1983), Heckman (1999), Kreps (1990 and 1998), Meyers and Nathaniel (1983), Meyers (1991, 1994 and 1998), Panjer (1992), Philbrick (1994), Wang (1997)).

Among all the existing methods, this approach is most widely used in actuarial practice and it continues to develop. The method can be used with company-specific data and can be used by line to reflect unique line of business risks. As indicated in the Casualty Actuarial Society's Task Force white paper, there are some unsolved problems associated with this approach such as measuring correlations of lines or segments of the business with other segments, estimating/ calibrating model parameters, and establishing a guideline for the applications of available methods. This paper presents a dynamic method to estimate the fair value of insurance liabilities for the whole book (with separate but correlated multiple lines) of business. The model studies the aggregate liability without assuming independent individual losses based on a non-traditional version of the collective risk theory. A new approach is proposed which estimates the fair value of insurer's liability based on a stochastic model of individual losses. To reflect the changing of the aggregate liability over time, a continuous model is presented using contingency claim analysis. By using the contingent claim analysis, the fair value liablity are approximated by solving a partial differential equation. Parameters estimation, correlations measurement and applications of the model are also discussed in the study.

The paper is organized as follows: The mathematical model for fair value of liability is presented in the next section. Several applications of the model and case studies are presented in Section 3. In the following section, the comparison of the new method to the existing methods will be addressed. Section 5 summarizes and concludes the paper.

2. Theory

This section presents the mathematical model for the valuation of fair value liability. To reflect the changing of the aggregate liability over time, a continuous model is presented using contingency claim analysis. We begin with the simplest case, where it is assumed that correlation among the classes of business are all a result of one underlying force (risk source) that affects different classes.

2.1 Mono-line of Business

For a specific line of business and a specifiic accident year t, we define $\{X(t), t \geq 0\}$, as the instantaneous ulltimate loss (includes claim, expenses and taxes) process, and $\{L(t), t \geq 0\}$, as the aggregate of fair value liability process over the period of $[0, t]$.

Assume the instantaneous loss amount $X(t)dt$ between time t and time $t+dt$ is described by a general stochastic process of the form:

$$dX = \mu(t, X)dt + \sigma(t, X)dW \qquad (2.1)$$

where μ is the drift of X , W is a standard Brownian motion (Wiener process),
and the local volatility σ is a deterministic function that may depend on both the loss X and the time t.

Over the time period [0,T], the aggregate of fair value liability $L(T)$ is defined by the equation

$$L(T) = \int_0^T X(\xi)e^{-r\xi}d\xi + F(X(T))e^{-rT},$$

where r is the discount rate (see Section 3.1 for the detail discussion), and F is assumed to be a continuous terminal function.

Remark: In many cases, there may be some delay in claims: information might not be available until the end of the evaluation period (time T). Therefore, in our definition, F is introduced, as a function of $X(T)$, to reflect situations like this. Notice that if F is the zero function, the definition above is the same as the conventional definition for the present value of the aggregate loss. Notice also, that it is possible for $X(t)$ to be negative, reflecting the release of reserves upon deaths of annuitants. Similarly, the aggregate of fair value liability over $[0,t]$, $L(t)$, is defined as

$$L(t) = \int_0^t X(\xi)e^{-r\xi}d\xi + F(X(t))\,e^{-rt}.$$

Remark: The claim reserve process is $R(t) = L(t) - C$ where C is either the claims paid to date or the case incurred claims to date. Since C is a known value, so we focus our analysis on L in this paper.

Next, we define the function $u(t,x)$ as the expected present value of the fair value liability over [0,t],

$$u(t,x) = E[L(t)\,|\,X(0) = x] \qquad (2.2)$$

where $x = X(0)$.

Remark: The function $u(t,x)$ is the conditional expectational of the aggragate of fair value liability, conditioned by $X(0) = x$. When $t = T$, $u(T,x)$ is the expected present value of the fair value liability over $[0,T]$.

THEOREM 1 *Suppose that σ and μ satisfy the linear growth condition*

$$|\mu(t,x)|^2 + |\sigma(t,x)|^2 \le K^2(1 + |x|^2) \qquad (2.3)$$

for every $0 \le t < \infty$, $x \in R$,

$$|F(x)| \le K^2(1 + |x|^2)$$

for every $x \in R$, where K is a positive constant; and

suppose that $u(t,x)$ is continuous and is of class $C^{1,2}([0,T] \times R)$. Then the expected present value of the fair value liability $u(t,x)$ can be calculated by solving the following Cauchy problem

$$u_t = \frac{1}{2}\sigma^2 u_{xx} + \mu u_x - ru + x; \quad in\,[0,T) \times R \qquad (2.4)$$

and

$$u(0,x) = F(x); \quad x \in R \qquad (2.5)$$

as well as the polynomial growth condition:

$$max_{0 \leq t} \quad |u(t,x)| \leq M(1+|x|^{2\eta}); \quad x \in R \tag{2.6}$$

for some $M > 0, \eta \geq 1$.

Proof This is a special case of Theorem 2, when $d = 1$. See the proof of Theorem 2.

In the following examples, we consider several simple applications of Theorem 1.

EXAMPLE 1

We first consider a mono-line liability reserve with the amount of cash flows being certain: the instantaneous loss amount $X(t)dt$ satisfy $dX = \mu_0 X dt$, where μ_0 is a constant.

Therefore $\mu(t,X) = \mu_0 X$, and $\sigma = 0$ in equation (2.1). We also ignore the investment income, i.e. $r = 0$. Furthemore, we assume $F(x) = 0$.

According to equation (2.2), given that $x = X(0)$, the expected present value of the fair value liability is

$$u(t,x) = E[\int_0^t X(\xi)d\xi \,|X(0) = x] = \int_0^t (xe^{\mu_0 \xi})\, d\xi = \frac{x}{\mu_0}(e^{\mu_0 t} - 1).$$

The following figure (Figure 1) provides a graphic view of $X(t)$ and $u(t,x)$ in this example.

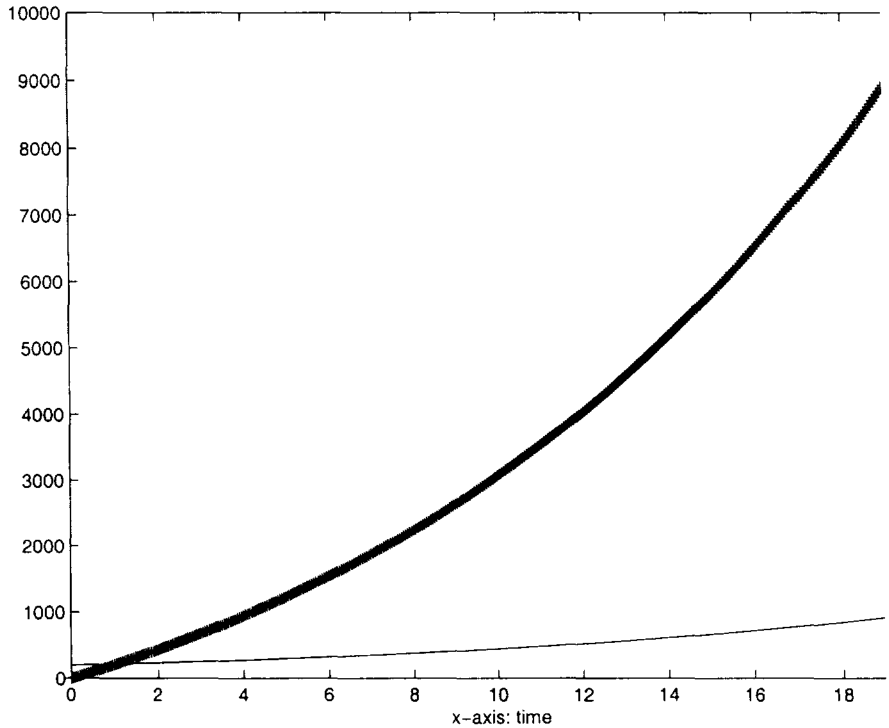

Figure 1. The expected fair value liability u(t,x) ('+++') v.s. the individual claims X(t) ('—,).

$u(t, x)$ satisfies

$$u_t = xe^{\mu_0 t},$$

$$u_x = \frac{1}{\mu_0} e^{\mu_0 t},$$

$$u_{xx} = 0.$$

It follows that

$$\frac{1}{2}\sigma^2 u_{xx} + \mu u_x - ru + x = 0 + \mu_0 x u_x - 0 + x = \mu_0 x u_x + x = u_t,$$

and

$$u(0, x) = 0 = F(x).$$

Therefore, Equation (2.4) and (2.5) hold.

Accoording to Theorem 1, the fair value liability can be estimated by solving the partial differential equations:

$$u_t = \mu_0 x u_x + x,$$

and

$$u(0, x) = 0.$$

EXAMPLE 2

Consider a mono line liability reserve with uncertain cash flows:

$\mu = 0, \sigma = 1$ in Equation (2.1).

In this case, we have $dX = dW$.

Furthemore, ignore investment income ($r = 0$) and assume $F(x) = F_0$, a constant function.

According to Equation (2.2),

$$u(t, x) = E[\int_0^t W(\xi) d\xi + F_0 \,|X(0) = x] = xt + F_0.$$

It is easy to see that
$u_t = x$, $u_{xx} = 0$, and $u(0, x) = F_0$.

Therefore, $u(t,x)$ satisfies $u_t = \frac{1}{2}u_{xx} + x$ and $u(0,x) = F_0$ which are Equations (2.4) and (2.5) when $r = \mu = 0, \sigma = 1$.

Accoording to Theorem 1, the fair value liability can be estimated by solving the partial differential equations:

$$u_t = \frac{1}{2}u_{xx} + x,$$

and

$$u(0,x) = F_0.$$

EXAMPLE 3

Consider a monoline liability reserve with uncertain cash flows, when $\mu = 0, \sigma = 1$ and $r = 0$.

Let $F(x)$ be a bounded and continuous function, and consider a special case of Equation (2.2):

$$u(t,x) = E[\int_0^t W(\xi)\,d\xi + F(W(t))\,|X(0) = x] = xt + E[F(x + W(t))]$$

First,

$$u(t,x) = xt + \int_{-\infty}^{\infty} F(y)\,p(t;x,y)dy,$$

where

$$p(t;x,y) = \frac{1}{\sqrt{2\pi t}}\,e^{\frac{-(y-x)^2}{2t}}$$

is the transition density of the one-dimensional Brownian family.

Then $u(t,x)$ satisfies Equations (2.5):

$$u(0,x) = \lim_{t\to 0, y\to x} u(t,y) = F(x).$$

Next, one can varifies that

$$u_t = x + \int_{-\infty}^{\infty} F(y)\, p_t(t;x,y)dy \;=\; x + \int_{-\infty}^{\infty} F(y)\, p_{xx}(t;x,y)\, dy.$$

Therefore

$$u_t \;=\; x \;+\; \frac{1}{2}\, u_{xx},$$

which is Equation (2.4) when $u(t,x) \;=\; xt + \int_{-\infty}^{\infty} F(y)\, p(t;x,y)dy$ (see the proof of Theorem 2 as to why (2.4) reduces to $u_t = \frac{1}{2}u_{xx} + x$ in this case).

Accoording to Theorem 1, the fair value liability can be estimated by solving the partial differential equations:

$$u_t \;=\; \frac{1}{2}u_{xx} \;+\; x,$$

and

$$u(0,x) \;=\; \lim_{t\to 0, y\to x} u(t,y) \;=\; F(x).$$

2.2 Multi-line of Business

In general, the correlation among the lines of business might be a result of several underlying forces that affect different classes in different ways. For example, risk sources might include economic inflation, judicial climate, tort reform, property catastrophes, health of the economy, and rate levels.

We now discuss multiple line business with correlated risk by generalizing the results in Sectin 2.1.

For a class of business consisting of n lines, we define

$$X(t) = (x^{(1)}(t), x^{(2)}(t), \cdots, x^{(n)}(t))^T \;=\; \begin{pmatrix} x^{(1)}(t) \\ x^{(2)}(t) \\ \cdots \\ x^{(n)}(t) \end{pmatrix},$$

as the instantaneous loss process at time $t, t \geq 0$.

Assume the loss amount $X(t)$ at time t is described by a n-dimensional stochastic process of the form:

$$dx^{(i)} \;=\; \mu_i(t,X)dt \;+\; \sum_{j=1}^{d} \sigma_{i,j}(t,X)dW_j \tag{2.7}$$

for $i = 1, 2, \cdots, n$, where

$\mu = (\mu_1(t, X), \mu_2(t, X), \cdots, \mu_n(t, X))$
is the drift of X,

W is a d-dimensional Wiener process,

and the local volatility $\sigma = (\sigma_{i,j}(t, X))$ is a n-by-d matrix that may depend on both the claim X and the time t.

Next, let $L(t), t \geq 0$, be the present value of aggregate fair value liability over the period of $[0, t]$, defined as

$$L(t) = \int_0^t (\sum_{i=1}^n x^{(i)}(\xi))e^{-r\xi}d\xi + F(\sum_i^n x^{(i)}(t))e^{-rt},$$

and let $u(t, X) = E^X[L]$ be the expected value of the fair value liability given that $X = X(0)$.

As a general case of one risk source (equation (2.2)), $u(t, X)$ is defined as

$$u(t, X) = E[\int_0^t (\sum_i^n x^{(i)}(\xi))e^{-r\xi}\, d\xi + F(\sum_i^n x^{(i)}(t))e^{-rt} \,|\, X = X(0)] \quad (2.8)$$

where $X = (x^{(1)}(0), x^{(2)}(0), \cdots, x^{(n)}(0))$ is the vector of losses at time 0 from the n risk sources.
Let $a(t, X) = (a_{i,j}(t, X))$ be a $n \times n$ matrix defined as $a(t, X) = \sigma\sigma^T$:

$$a_{i,j}(t, X) = \sum_{k=1}^d \sigma_{i,k}(t, X)\, \sigma_{k,j}(t, X),$$

$$g(X) \equiv \sum_{i=1}^n x^{(i)}(0),$$

and

$$\mathcal{A}u \equiv \frac{1}{2} \sum_{i,k=1}^n a_{i,k}(t, X)\, u_{x_i x_k} + \sum_{i=1}^n \mu_i(t, X)\, u_{x_i} \quad (2.9)$$

THEOREM 2 *Suppose that σ and μ satisfy the linear growth condition*

$$||\mu(t,X)||^2 + ||\sigma(t,X)||^2 \leq K^2(1+||X||^2) \qquad (2.10)$$

for every $0 \leq t < \infty$, $x \in R^n$,

$$|F(X)| \leq K^2(1+||X||^2)$$

for every $x \in R$,
where K is a positive constant; and assume that $u(t,X)$ is continuous, and is of class $C^{1,2}([0,T] \times R^n)$.
Then
$u(t,X)$ satisfies the Cauchy problem

$$u_t = \mathcal{A}u - ru + g(X); \quad in\,[0,T) \times R^n \qquad (2.11)$$

and

$$u(0,X) = F(g(X)); \quad X \in R^n \qquad (2.12)$$

as well as the polynomial growth condition:

$$max_{0 \leq t} |u(t,X)| \leq M(1+||X||^{2\eta}); X \in R^n \qquad (2.13)$$

for some $M > 0, \eta \geq 1$.

The proof of the Theorem 2 is given in Appendix 1.

Theorem 2 indicates that an estimate for the fair value insurance liability could be obtained by solving a partial differential equation (2.11)-(2.12).

The model presented here is a dynamic model: the fair value liability can be evaluated in a multi-period setting. Consider a sequence of time periods: $[0,T_1], [T_1,T_2], \ldots, [T_{k-1},T_k]$ and apply our model in every one of the k periods, a system of partial differential equations like (2.11) – (2.12) can be solved sequentially for the valuation of the fair value liability over the k periods.

Finally to conclude the section, we present a mathematical formula for the solution of partial differential equation (2.11)-(2.12).

2.3 Theoretical solution

To derive a closed-form solution, several conditions are introduced.

First, let us define

(i) *Uniform ellipticity*: There exists a positive constant δ such that

$$\sum_{i,k=1}^{n} a_{i,k}(t,x)\,\eta_i\eta_k \geq \delta||\eta||^2 \tag{2.14}$$

holds for every $\eta \in R^d$ and $(t,x) \in [0,\infty) \times R^d$.

(ii) *Boundedness*:
The functions $a_{i,k}(t,x)$ and $\mu_i(t,x)$ are bounded in $[0,T] \times R^d$.

(iii) *Hölder continuity*:
The functions $a_{i,k}(t,x)$ and $\mu_i(t,x)$ are Hölder-continuous in $[0,T] \times R^d$.

THEOREM 3 *Under the conditions (i)-(iii) and (2.10), $u_t = \mathcal{A}u - ru$ has a unique fundamental solution $G(t,x;\tau,\xi)$;*
the solution of equations (2.11)-(2.12) is

$$u(t,X) = \int_{R^d} G(t,X;0,\xi)F(\,g(X)\,)\,d\xi + \int_0^t \int_{R^d} G(t,X;\tau,\xi)\,g(X)\,d\xi d\tau \tag{2.15}$$

The proof of the Theorem 3 is given in Appendix 2. Theorem 3 provides a theoretical basis for the solution of equations (2.11)-(2.12). In practice, however, numerical solution of equations (2.11)-(2.12) should be seeked for any fair value liability valuation.

3. Applications

In this section, we consider the implementation issues of the model presented in previous section and its applications.

3.1 Discount Rate

We start with discussion on the discount rate, r, used in defining fair value liability process

$$L(t) = \int_0^t (\sum_{i=1}^n x^{(i)}(\xi))e^{-r\xi}d\xi + F(\sum_i^n x^{(i)}(t))e^{-rt}. \quad (3.1)$$

The discount rate is the interest rate at which the investment funds earn interest. The simplest way to implement the model is to use the risk-free interest rate as the discount rate r. Although the risk-adjusted rate is not used directly, the estimated fair value liability $u(t, X)$ is risk adjusted. The equation (2.11) is risk adjusted since its coefficients includes the covariance matrix $a(t, X)$ (see the definition of $\mathcal{A}$ in equation (2.9)).

The discount rate r can also be risk-adjusted as

$$r = r_f + \pi$$

by assuming that the short rate $R(t)$ follows process

$$dR(t) = rR(t)dt + \sigma_R(t, R)dW$$

where π is the market risk premium and σ_R is the local volatility of $R(t)$. There are many literatures in finance and economics on valuation and hedge of interest rate risk. Examples inlcude Duffie (1992), Hull (2000), Heath, Jarrow and Morton (1992).

3.2 Parameter Estimation

In order to solve equations (2.11) – (2.12), the parameters $\{\mu_i, i = 1, \ldots, n;\}$ and $\{a_{i,k}, i, k = 1, \ldots, n\}$ in Equation (2.11) need to be selected first. Simulation techenique are the methods most widely used today by actuaries to solve this problem. Recent advance in computing technology has significantly increased the accuracy and reduced the cost of the simulation. Patel and Raws (1999) presented a simulation approach in reserve valuation. As far as the data used for the simulation, we recommend a weighted average of simulation base on public data and company-specific data.

3.3 Case Studies

We now show some numerical examples of estimating fair value liability by solving equation (2.11) – (2.12) in case studies.

Case Study of Mono-line Business

We first consider a mono-line liability reserve with uncertain cash flows: assuming the instantaneous loss amount $X(t)dt$ satisfy

$$dX = 0.08dt + 2dW.$$

Assume that the investment return is 4% ($r = 4\%$) and $F(X) = X^{1.5}$. Using Theorem 1, we calculated the fair value liability by solving Equation (2.4) and (2.5). We used finite differences method to solve $(2.4) and (2.5)$ numerically. The estimated fair value liability with different initial individual loss levels are given in Figure 2.

Next, we consider a mono-line liability paid out over a longer period of time has higher uncertainty:

instead of constant volatility, we consider varying volatility:

assuming the instantaneous loss amount $X(t)dt$ satisfy

$$dX = 0.08dt + \sigma(t) = 2\sqrt{1+t}dW,$$

with all the other parameters remaining unchanged.

Figure 3 presents the computed values of fair value liability in this case.

Our estimates show that the fiar value liability with nonconstant volatility is more sensitive to the initial claim levels. Figure 4 makes a comparison of the two situations.

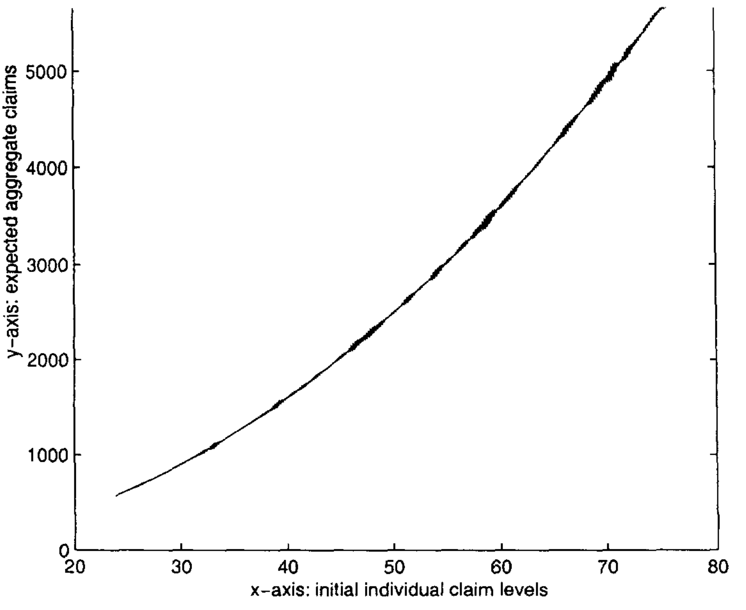

Figure 2. The expected fair value liability with variance $\sigma = 2.0$.

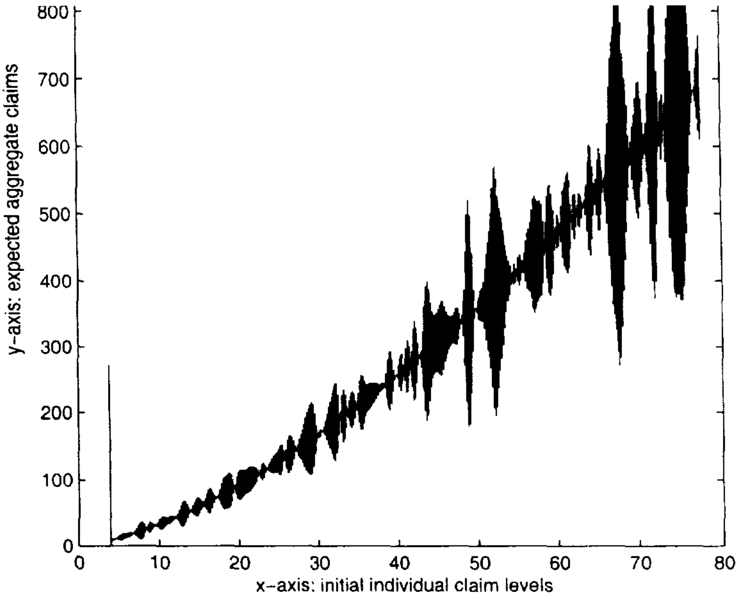

Figure 3. The expected fair value liability with variance $\sigma(t) = 2\sqrt{1+t}$.

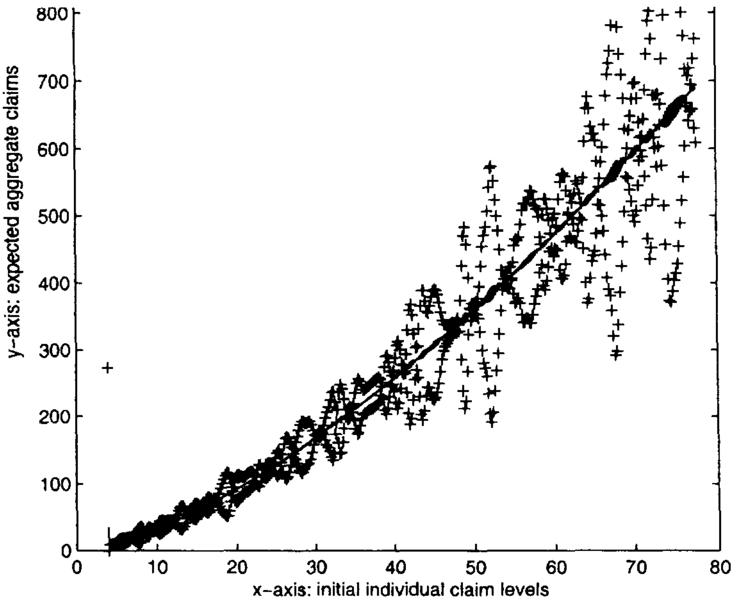

Figure 4. The differences in the expected fair value liability with covariance 2.0

Case Study of Multi-line Business

Assume an insurer writes two lines of business with uncertain cashflows. Let the loss process be:

$$X = (X^{(1)}(t), X^{(2)}(t)),$$

Assume $X^{(1)}(t)$ represent a property reserve with drift $\mu = 0.08$ and local volatility of $\sigma = 2$. Assume $X^{(2)}(t)$ represent a liability reserve with drift $\mu = 0.1$ and local volatility of $\sigma = 5$. Assume the correlation between the property reserve and the liability reserve be 1.5.

Therefore the drift μ and the covarance matrix $\sigma(t, X)$ are

$$\mu(t, X) = \begin{pmatrix} .08 \\ .1 \end{pmatrix}.$$

$$\sigma(t, X) = \begin{pmatrix} 2 & 1.5 \\ 1.5 & 5 \end{pmatrix}.$$

Let the discount rate remain at 4% and the function F be defined as

$$F(X) = ((x^{(1)})^3 + (x^{(2)})^{1.5})^2.$$

Using Theorem 2 in Section 2.2, we calculated the fair value liability by solving Equations (2.11), (2.12).

Again, we used a finite difference method to calculate the estimated fair value liability. Figure 5 shows the computed values of the fair value liability.

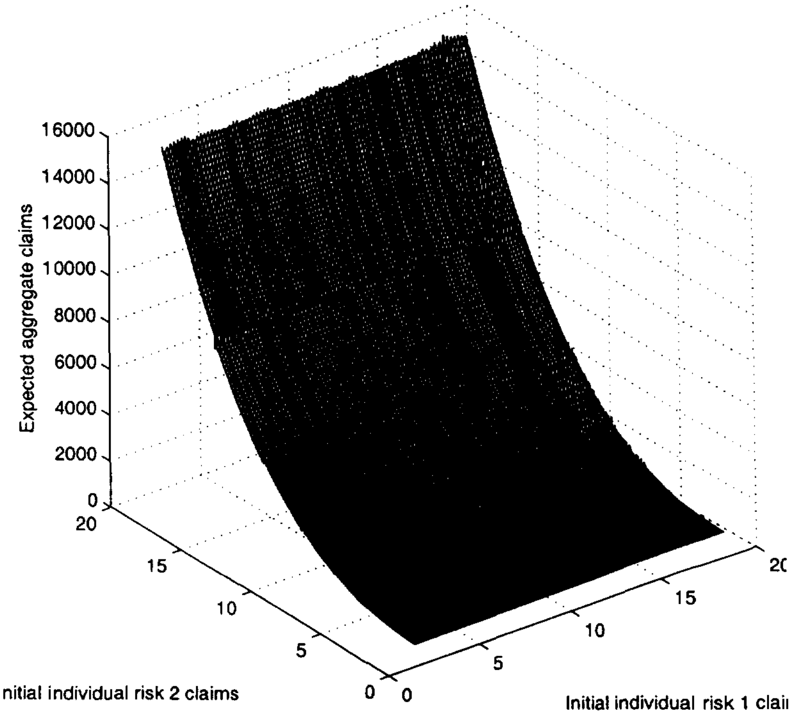

Figure 5. The fair value liability with constant variance.

Next, we checked how different levels of the correlation affect the estimated liabilities. As indicated in Table 1 , our estimates show that, in majority of cases, the fair value liability are lower when the loss claims between the lines of business are less correlated.

Table 1. Expected Fair Value Liability

(x_1, x_2)	$\sigma_{12} = 0$	$\sigma_{12} = 0.5$	$\sigma_{12} = 1.5$
(5, 5)	290.8	291.4	303.0
(5, 10)	330.4	332.7	372.9
(5, 15)	404.5	409.1	463.5
(5, 18)	434.6	434.4	427.6
(10, 5)	1956.9	1927.4	1657.5
(10, 10)	1908.7	1993.1	1565.11
(10, 15)	2171.7	2205.5	2438.1
(10, 18)	2283.8	2346.1	2902.2
(15, 5)	6687.2	6675.2	6583.5
(15, 10)	7134.8	7179.2	7540.8
(15, 15)	6903.7	6904.1	6947.6
(15, 18)	6845.7	6835.2	6759.6

Table 1 also shows that, for a fixed level of covariance, the calculated fair value liability increase as the initial loss amounts increase.

Finally, we considered the case when volatility varied with time. Assume all the other parameters remain the same and let

$$\sigma(t, X) = \begin{pmatrix} .08\sqrt{1+t} & .5 \\ .5 & 2(.5+t)^{\frac{1}{3}} \end{pmatrix}.$$

The estimated liabilities are shown in Figure 6.

The comparision of the estimated fair value liability (when the initial risk 1 claim level is x=9) between the constant volatility and non-constant volatility is shown in Figure 7.

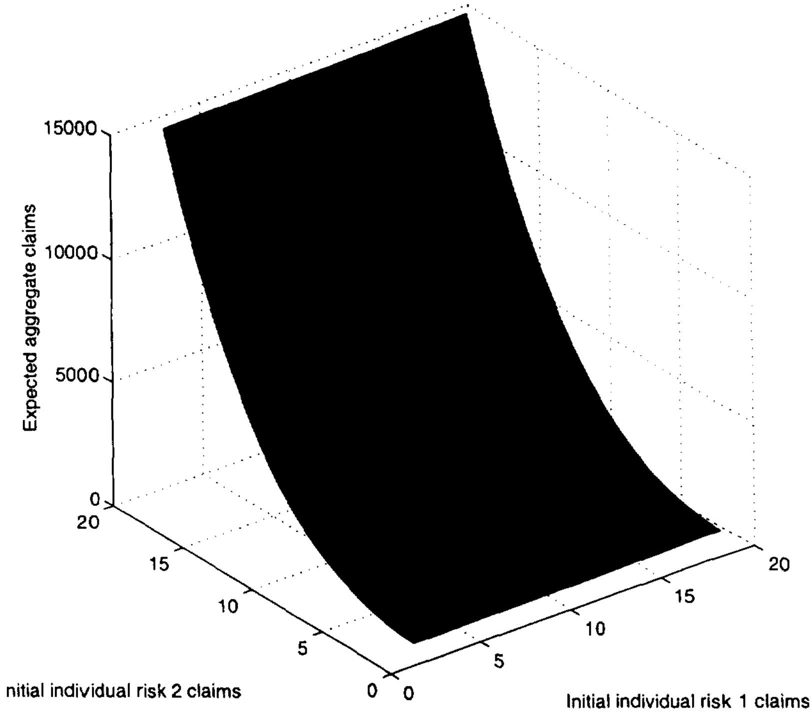

Figure 6. The fair value liability with NON constant variance.

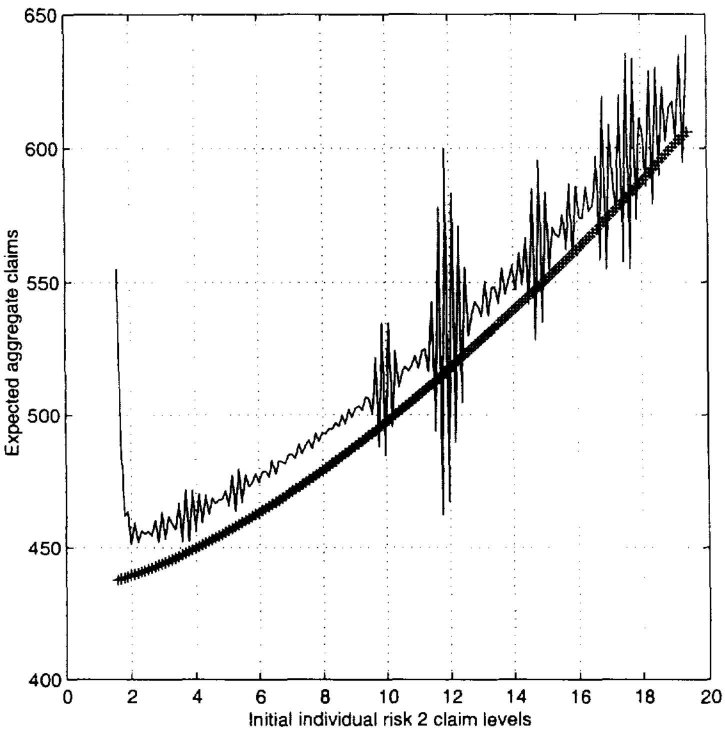

Figure 7. The fair value liability with constant variance (x=9): —
The aggregate claims with NON constant variance (x=9): +++

3.4 Applications in Reinsurance

In Section 2, a new method is provided for the estimation of the expected fair value liability without assuming independence of the individual losses. There are a number of applications of the method other than estimating fair value insurance liability. In the following, we discuss the applications of our method in reinsurance.

First we consider the problem of calculating stop-loss premiums.

Let p be the stop-loss premium, K be the cap. and L the fair value liability as defined in section 2.1:

$$L(t) = \int_0^t X(\xi)\, e^{-r\xi} d\xi + F(X(t))e^{-rt}$$

Assume L follows

$$dL(t) = \rho(t, L)dt + \nu(t, L)dW \qquad (3.2)$$

At time T, the benefit is $max\{0, L(T) - K\} \equiv (L - K)^+$.

Define $v(t, L) = E[e^{-r(T-t)}\,(L - K)^+\,|L(0) = L]$,
where r is the risk-free interest rate.

Then the fair value of the stop-loss premium should be $p = v(0, L)$. Using the analogue of Theorem 2 in Section 2, $v(t, L)$ is solved from the following:

$$v_t = \frac{1}{2}\nu^2 v_{LL} + \rho v_L - rv, \qquad (3.3)$$

$$v(0, L) = (L - K)^+. \qquad (3.4)$$

Remark: Note that the above partial differential equation is different from the Black-Scholes' partial differential equation or its type. Since L is not tradable, there is no risk neutral measure. Therefore ρ can't be replaced by a riskfree rate in equation (3.3).

Remark: In theory, p can be calculated from equations (3.3) – (3.4). However, there is no explicit formula to estimate ρ and ν without assuming the independence or some specific form of the dependence of the individual claims. One can, however, use the solution of (2.11)-(2.12) as an estimate of ρ.

In the following, we show a numerical example of calculating the stop-loss premiums, $p = U(0, S)$, and assume there is one risk source.

Recall that in the Case Study of Mono-line Business, where we consider a mono-line liability reserve with uncertain cash flows: assuming the instantaneous loss amount $X(t)dt$ satisfy

$$dX = 0.08dt + 2dW.$$

Assume that the investment return is 4% ($r = 4\%$) and $F(X) = X^{1.5}$.

Assume the initial individual claim is $x_0 = 30.8$. Using the estimates calculated in Section 3.3 as an approximation for ρ: $\rho = 178.4952$. We solved Equations (3.3) and (3.4) numerically. For the stop-loss cap $K = 160$, the stop-loss premiums calculated based on different aggregate claim levels are given in Figure 8.

We again looked at the case that the liability cash flows are more uncertian. Figure 9 compares the stop-loss premiums with constant volatility and varying volatility.

Finally, we tested how much change in stop-loss premium is due to the change of the value of ρ which is presented in Figure 10.

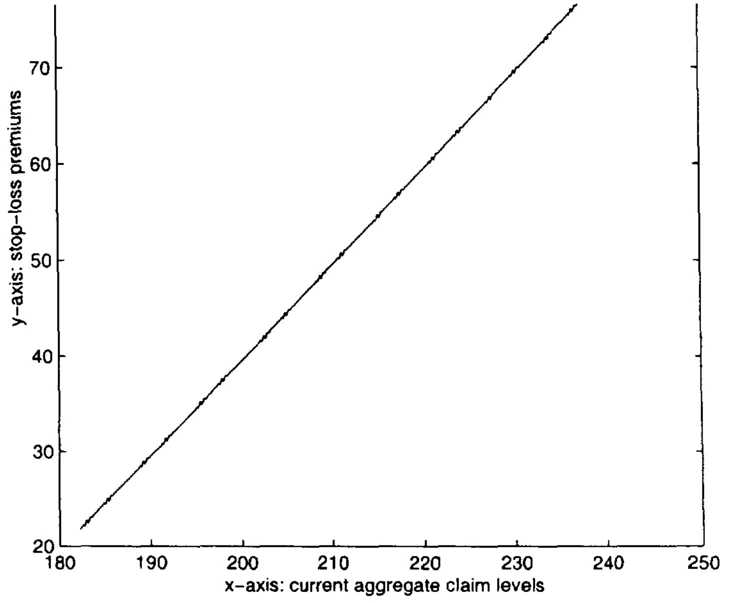

Figure 8. The stop-loss premiums with constant variance

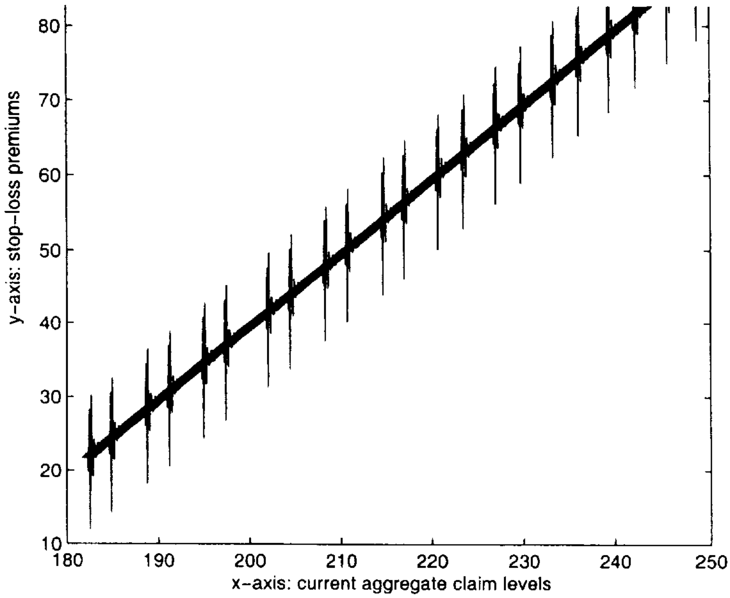

Figure 9. The stop-loss premiums with NON constant variance.

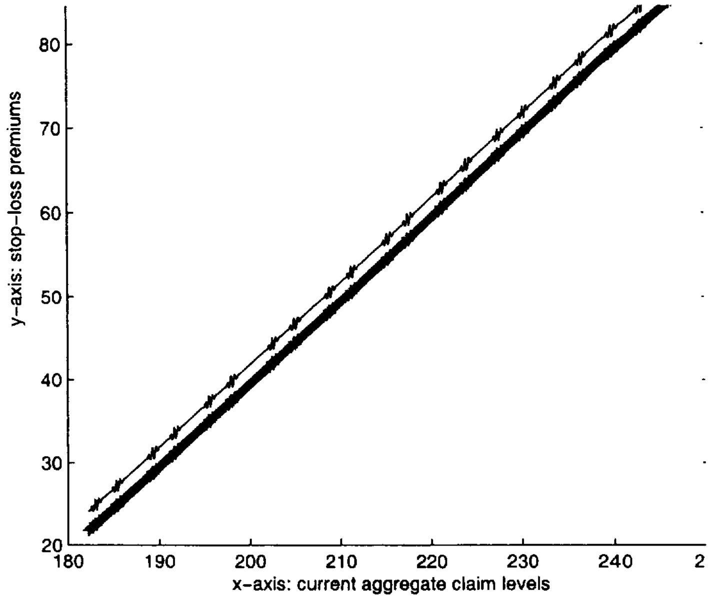

Figure 10. The differences in stop-loss premiums with changes in drifts.

Another application in reinsurance is the valuation of CATS index options. The price of a Catastrophe Insurance Futures and options (CATS) could be estimated using this approach. For a detailed discussion, see Guo (2000).

4. Discussion of the Method

In this section, we provide our view on the comparision between our method and the existing methods.

Our method provides a direct estimation of fair value liability. It used a combination of the financial approach and the actuarial approach. Unlike the method of Allen, Cummins and Phillips (1998), our method considers the impact of a particular company at issue or even specific lines of business of the company. It doesn't rely on the CAPM model, which may not accurately predict returns for insurance firms and no need to estimate the underwriting betas. There is a component of risk-adjusted discount method in our approach when the discount rate r in Equation (2.11) is risk-adjusted. The derivation of our method start with study individual loss risk process like actuarial distribution-based risk loads methods. Instead of calculating the risk loadhowever, our method estimate the risk-loaded fair value liability directly using the contigent-claim analysis in modern financial theory. Finally, the application of our method in valuation of stop-loss premium and CATS premium might provide some connection to the method of using the reinsurance market to estimate the fair value of liabilities.

5. Summary

This study provided a new dynamic method to estimate $E[L(T)]$, the expected fair value liability for a multiple line business.

The paper adopted the contingent claim analysis in modern finance theory to model the aggregate fair value liability for multiple lines of business. An important feature of the method is to concentrate on calculating the risk-loaded expectation of the aggregate liability instead of attempting to find the actual liability distribution in a complicated economic environment. The fair value liability was derived by solving a partial differential equation. Finite difference method was used to obtain the numerical solution as shown in the examples. The dynamic feature of the method make it possible to evaluate the fair value liability over the multiple periods by solving a system of partial differential equations sequentially. The effects of non-constant variance matrix on the liability estimate were discussed in the numerical examples. The paper also addressed some applications of the method including the evaluation of stop-loss premiums among others. The paper presents only the preliminary result of our study. A case study for the implementation of the new method and the comparison of other existing methods is under the way. Future research areas include creating a highly efficient and flexible simulation algorithm for the parameter estimation; deriving more accurate and stable numerical method for the partial differential equation; estimating the fair value liability with a stochastic interest rate process $\{r(t), 0 \leq t \leq T\}$; and extending the loss process to a more general risk process including a jump process, etc.

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6. Appendix 1

This appendix presents the proof for the Theorem 2 in Sectin 2.

Theorem 2

Suppose that σ and μ satisfy the linear growth condition

$$||\mu(t,X)||^2 + ||\sigma(t,X)||^2 \leq K^2(1+||X||^2) \tag{6.1}$$

for every $0 \leq t < \infty$, $x \in R^n$,

$$|F(X)| \leq K^2(1+||X||^2)$$

for every $x \in R$,
where K is a positive constant; and assume that $u(t,X)$ is continuous, and is of class $C^{1,2}([0,T] \times R^n)$.
Then
$u(t,X)$ satisfies the Cauchy problem

$$u_t = Au - ru + g(X\cdot); \quad in\,[0,T) \times R^n \tag{6.2}$$

and

$$u(0,X) = F(g(X)); \quad X \in R^n \tag{6.3}$$

as well as the polynomial growth condition:

$$max_{0\leq t}\,|u(t,X)| \leq M(1+||X||^{2\eta}); X \in R^n \tag{6.4}$$

for some $M > 0, \eta \geq 1$.

PROOF

Suppose v is a solution of (6.2) – (6.3). We apply the Itô lemma and integration by parts to the process
$v(t-\xi, X_\xi)e^{-r\xi}; \xi \in [0,t]$, in conjunction with (2.11):

$$d[v(t-\xi,X_\xi)e^{-r\xi}] = e^{-r\xi}[-g(X_\xi)d\xi + \sum_{i=1}^{d} v_{x_i}(t-\xi,X_\xi)\sigma_i dW_{(i)}].$$

Let $\tau_n \equiv inf\{\xi \geq 0; ||X_\xi|| \geq n\}$;
we obtain

$$v(t,X) = E[F(g(X))e^{-rt}1_{\{\tau_n > t\}}|X(0) = X] + E[\int_0^{t\wedge\tau_n} g(X(\xi))e^{-r\xi}d\xi|X(0) = X]$$
$$+E[v(\tau_n, X_{\tau_n})e^{-r\tau_n}1_{\{\tau_n \leq t\}}|X(0) = X] \qquad (6.5)$$

7. Appendix 2

This appendix presents the proof for the Theorem 3 in Sectin 2.

Theorem 3 Under the conditions (i)-(iii) and (2.10), $u_t = \mathcal{A}u - ru$ has a unique fundamental solution $G(t, x; \tau, \xi)$;
the solution of equation (2.11)-(2.12) is

$$u(t, X) = \int_{R^d} G(t, X; 0, \xi) F(\,g(X)\,)\, d\xi + \int_0^t \int_{R^d} G(t, X; \tau, \xi)\, g(X)\, d\xi d\tau \tag{7.1}$$

PROOF

Under the conditions (i)-(iii), there is a fundamental solution $G(t, x : \tau, \xi)$ of

$$u_t = \mathcal{A}u - ru; \quad in\,[0, T) \times R^n \tag{7.2}$$

and

$$u(0, X) = F(X); \quad X \in R^n \tag{7.3}$$

(see Friedman (1975, pp141, 148 and Friedman (1964) Chapter I). For fixed $(\tau, \xi) \in (0, T] \times R^d$, the function $G(t, x : \tau, \xi)$ is of class $C^{1,2}((0, T] \times R^d)$ and

$$u(t, X) = \int_{R^d} G(t, X; 0, \xi) F(X)\, d\xi$$

satisfies (7.2) – (7.3). We recall from Theorem 2 that the solution of (7.2) – (7.3), with $r = 0$, is given by

$$u(t, X) = E[F(X(t))|X(0) = x]$$

This leads to the conclusion that any fundamental solution $G(t, x : \tau, \xi)$ is also the transition probability density for the process X; i.e.,

$$P[X(\tau)|X(t) = x \in A] = \int_A G(t, x : \tau, \xi) d\xi;\ 0 \le t < \tau \le T.$$

In particular, under the condition (2.10), this fundamental solution is unique, and

$$u(t, X) = E[\int_0^t (\sum_i^n x^{(i)}(\xi))e^{-r\xi}\, d\xi + F(\sum_i^n x^{(i)}(t))e^{-rt} \,|\, X = X(0)],$$

the solution to equation (2.11) and (2.12) now takes the form

$$u(t, X) = \int_{R^d} G(t, X; 0, \xi) F(\, g(X)\,)\, d\xi + \int_0^t \int_{R^d} G(t, X; \tau, \xi)\, g(X)\, d\xi d\tau.$$

Evaluating Reserves in a Changing Claims Environment

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Evaluating Reserves in a Changing Claims Environment

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Abstract
Recent insurance industry emphasis on claims "best-practices" requires the reserving actuary to identify and measure the emerging effects of Claims Department initiatives. Several of these initiatives will be reviewed from both an actuarial and claims personnel perspective. Adjustments to generally accepted actuarial methodologies as well as potential metrics to measure the impact of these initiatives will be presented.

Evaluating Reserves in a Changing Claims Environment

Section 1 – Introduction

Insurers are regularly reviewing their claims handling procedures to identify areas for improving this vital function. This activity intensified during the 1990's with a number of insurers introducing focused initiatives to reengineer their claims processes. While it is difficult to pinpoint the drivers behind these actions for individual insurers, our experience has shown that this trend can generally be attributed to a number of factors:

- Improvements in cellular and mobile technology have enabled carriers to accelerate the recognition and adjustment of claims. Advanced intelligence or "smart" systems allow claim adjusters to evaluate the settlement value of claims more quickly as well;

- Competitive cost pressures have forced insurers to identify the "fair value" of claims and to take all necessary actions to settle claims expeditiously and control their claim costs. Loss adjustment costs have also received considerable attention, and innovative alternative contractual arrangements and other strategies have been developed to reduce LAE expenses without jeopardizing control on losses;

- Companies have invested heavily to develop fraud detection systems. Claims suspected to be fraudulent or claim demands that seem inconsistent with available information are tagged and specific strategies are developed to address them.

These initiatives have commonly changed the ways in which claims are reported, recognized, and settled and have therefore introduced significant distortions into the historical actuarial data used for reserving. Several CAS papers have been written to address situations in which changes in claims handling procedures have to be recognized in the reserving process. Methodologies commonly "adjust" the historical data to simulate what the experience would have looked like in the new claims handling environment.

However, these papers have generally focused on changes in case reserve adequacy and the rate at which claims are closed. More complex changes of the variety noted above have received less attention. For example, what if the strategies introduced to handle suit claims are considerably different than the strategy to handle claims suspected to be fraudulent? What if entirely new contracts are drawn up to compensate outside attorneys on a fixed fee basis? Such changes require more elaborate refinement of standard actuarial approaches to evaluate reserves appropriately.

This paper will focus on several specific claims initiatives and the actuarial methodologies we have utilized in situations where these initiatives have distorted the historical database. Section 2 provides a detailed description of illustrative operational changes. Section 3 examines why these changes can have a distortive effect on the actuarial reserving data. Finally, Section 4 provides examples of the actuarial methods that can be adapted for these changes. Section 4 also highlights some of the additional uncertainty that is introduced into the reserving process as a result of these changes.

Section 2 – The Changing Claims Environment

Reengineering was a commonly touted initiative of many business practices during the 1990's. The process of reengineering starts with a disciplined dissection of business procedures to reveal and isolate base underlying elements of the targeted process. This is followed by an equally disciplined examination toward optimizing the treatment and handling of these base elements. Varying degrees of such reengineering efforts were employed throughout the business world. The casualty claims environment was no exception. The implementation and success of these reengineering efforts varied throughout the business world. Again, the casualty claims environment was no exception. The ability to track and monitor the results of reengineering efforts can prove very difficult. This was, and is, especially true of the casualty claims environment. The reserving professional is severely challenged in identifying, understanding and quantifying the impacts of these changes on both loss and claim expense development patterns.

Internal changes in the Claims environment are tied closely to this approach of dissection and optimization. Dissection, in this case, is the heightened awareness and recognition of the differences in casualty claims. In a macro sense, the reserving professional has historically recognized the importance of segmenting, for example, the loss statistics of bodily injury, uninsured motorist, underinsured motorist, and personal injury claims. There has also been common recognition of different loss and expense development patterns between tort and no-fault states. Internal reengineering efforts have identified additional layers of segmentation: subjective injury versus objective injury; attorney representation versus non-representation; claims "in-suit" versus non-suit; low-impact subjective injuries; "express" (low severity) claims; and suspected fraudulent claims. The identification and comprehension of these subsets of casualty claims has naturally led to multiple sets of "best practice" protocols that govern their disposition. In addition, many insurance companies have taken the natural progression toward enhanced claim-type segmentation by introducing specialization to the structure of their Claims organizations. Individuals are trained to handle each of these specialized sets of claims. The combination of specialization and "best practice" protocols are essentially the second piece of the reengineering process, optimization.

The extent to which Claims operations have been able to refine this concept of dissection and optimization is directly correlated to advances in technology. Technology should be viewed as the prime enabler of the intensified differentiation in casualty claims handling. Cellular and mobile technology has enabled faster adjustment of claims. Database technology has enabled desktop access to extreme quantities of claim information that can be parsed down to specific components. In turn, this component information is used to compile different, more detailed, operational analytics for monitoring claims performance. Advanced intelligence claims systems are an ever-broadening tool in the area of liability determination and damages evaluation. Databases and intelligence systems are also the cornerstones of fraud detection strategies.

As mentioned above, many carriers have enacted significant changes in the structure of their Claims operations. In most cases, specialization has become the norm. Claim teams have been formed to align with the different segments. The goals, or benchmarks, of each team are aligned with the "best practices" protocols that govern the optimal disposition of that segment's claims. Accountability and performance measurement becomes more localized, per se. For instance, an "express" unit would handle claims that fall below a pre-determined dollar threshold, have little or no cause for liability and damage investigation, and have low probability of fraud. The accountability of this unit is most likely to center on low pending levels and high customer satisfaction. Cost control measures would be secondary given the low-severity trigger that already defines claims within this segment. Traditional actuarial claim statistics are potentially impacted by the accelerated disposition of low severity claims, a different composition of the remaining pending claims, and a different age-to-age paid loss development pattern. There is also the possibility of a decrease in the ratio of claims closed without payment due to the non-investigation protocol of this particular unit.

As a contrast, consider the accountabilities and performance measures for claim adjustment personnel dedicated to claims alleging subjective, soft-tissue injuries from incidental automobile contact. Even though these claims are also of the lower severity variety, there would likely be a greater appetite for rigorous arguments against the merits of these claims. In fact, there would be operational modifications in the end-to-end handling of claims within this segment: clear-cut selection criteria for identification of appropriate claims; stronger investigation and verification of damages; more consistent and objective evaluations of liability and damages; elevated preparedness for potential negotiations; increased willingness to try all cases where settlement cannot be reached; and flexibility in settlement methods. The expected actuarial impacts would be delayed pending disposition, increased allocated defense costs, increases in the ultimate percentage of denials, and lower average paid losses on those claims settled with payment. However, a long-term result could possibly be the elimination of these claims altogether.

Another good example of internal claim initiatives and the corresponding impact on actuarial analysis is the issue of contact time and litigation avoidance. The over-arching operational goal of a Claim department is the fair and timely resolution of all claims. Competitive cost pressures have forced insurers to take all necessary actions to settle claims expeditiously and control their claim costs. Paramount to the attainment of these objectives is the claim adjusters' ability to establish good rapport with the claimant. Operational activities expected to help drive the desired results would include:

- rapid initial contact to educate the claimant on the insurer's approach to fair claim settlement;
- anticipation and resolution of a broad range of claimant needs in a genuine and empathetic manner,
- rapid liability investigation and amicable resolution of property damage issues,
- reduction of unnecessary claimant and file transfers between claims personnel,
- regular follow-up claimant contact, and appropriate settlement offers.

All in all, it is a continuous process of relationship building interactions. The critical measures of success in this area would be contact time (average time elapsed between date of report and date the claimant is first contacted) and attorney representation rates (percentage of third party claimants represented by an attorney). The potential statistical impacts would entail: acceleration of claim notice counts, faster settlements, change in pending disposition trends, reduction in expected ultimate loss costs, and reduction in allocated legal expenses.

"Smart" claims systems, are a prime area where computing technology has enabled casualty insurers to enhance their objectivity, consistency, and negotiation strategies in the course of evaluating and settling claims. There are three key elements to this process: 1) strengthened file investigation and development; 2) objective/consistent value calculation methodology; and 3) verdict database. The strengthened file investigation is merely the execution of structured "best practices". This would include items such as liability assessments, documentation of relevant findings, structured diagnostic analysis, and structured investigation guidelines. The objective/consistent valuation process would begin with a comprehensive breakdown of claim value components. A historical database of such components would serve as a baseline for subsequent damage evaluations. In addition, a checklist of subjective factors would help ensure proper consideration is given other variables in the evaluation process. Lastly, a verdict database provides a factual understanding of attorney economics within various geographic markets. Combining these three elements produces a "smart" system that attempts to introduce consistency and objectivity within the claim evaluation process.

How, then, would such "smart" systems impact the actuarial analysis of the reserving professional? Objectivity and consistency in the liability determination and damage evaluation processes should lead to a higher level of confidence that proper claims adjusting has taken place. This would then lead to earlier settlement offers, regardless of the dollar amount involved. It is quite possible, then, to see acceleration in the settlement of higher severity objective-injury type claims.

In contrast, "smart" systems may permit insurance carriers to take a tougher negotiating stance on lower-severity, subjective claims. Insurers are more and more willing to let these claims pend longer than before, armed with the belief that their settlement offer is fair and reasonable. The fact that a slight increase in the offer at the negotiation table could bring about closure becomes less material.

"Smart" systems have also provided the tool for effective and efficient data mining of claim detail to identify areas and individuals with suspected fraudulent activity. This is one more area where insurers are putting additional focus of loss cost containment processes. Statistically, the insurer utilizing "smart" systems would likely exhibit an increase in pending claim counts for lower-severity subjective claims, an increase in the ratio of claims closed without payment, an increase in loss adjustment expenses, and a decrease in average losses paid.

One final area of change in the internal Claims environment that is worth discussion is the relatively recent attention being paid to the control of claim expenses, and in particular, legal costs.

For many carriers, legal fees contribute 50% to 70% of their overall allocated claim expenditures. During the 1990's, the trend in average legal claim expenses far outpaced liability severity trends for most coverages. It is only natural then for companies to seek approaches that enable them to control these costs.

A recent survey of corporate attorneys outside of the insurance industry echoed similar sentiment:

> "The costs of litigation are rising ... a new business model will be mandated for corporate legal departments, which must operate more efficiently to counter rising litigation costs and bottom-line pressures(.)"[1]

In controlling costs, the corporate insurance attorneys pointed to:

- closely monitoring bills, billing audits and budgets;
- early settlement, discussions/faster case settlement;
- reducing outside (attorney) costs;
- handling cases in-house.

These approaches are not unique. In fact they are very much like the legal expense cost containment initiatives commonly found within the insurance industry.

[1] KPMG LLP, "Litigation Survey" September 2000

Companies seeking to gain control over their legal expenses may begin with a review of their authorized outside or "panel" attorney firms. Consolidating the number of authorized firms permits the company to negotiate from a stronger position in that they are offering the remaining firms a larger number of potential cases. Web-based auction sites for legal services have also increased the purchasing power of insurance companies, requiring the attorneys to, in essence, compete for business.

Alternative fee arrangements between panel firms and insurance companies have also become quite common, and offer still another complication for the actuary. Flat fee agreements typically compensate the attorney a fixed amount based upon the type and complexity of the case. The timing and amount of the payments will generally follow a set schedule regardless of the actual time commitment of the attorney. Often, the panel firm and the company will agree to a set listing or "matrix" of payments covering a range of possible claim types.

Retainer agreements are another form of alternative fee arrangements in which a fixed amount is paid to the firm to handle a group of claims until their conclusion. In essence, the ultimate legal expense cost on these claims is limited to the retainer fee. In situations where the retainer is exhausted, the attorney remains responsible for servicing the claim.

Other alternative legal fee arrangements include:

Reverse Contingency Fee: additional sums paid to the attorney by the insurance company depending upon settlement outcome.

Shared Savings: defense attorney paid a percentage of savings below reserve/settlement value.

Bonus for Prompt Disposition: additional sum paid for speedy resolution.

Internally, companies are also changing the way they pursue litigation and litigation costs. Many claim organizations now employ formal litigation guidelines which detail their preferred approach to handling claims in suit. These guidelines have been established in order to promote consistency in legal philosophy. Companies have also begun to utilize alternative dispute resolution or ADR as a means for settling claims while reducing legal fees.

Companies have also increased the utilization of staff attorneys as an alternative to more expensive panel firms. Staff counsel attorneys often have the right of first refusal on handling suits, although there may be situations were they are precluded from servicing a case due to a conflict of interest.

From a statistical perspective, each of the legal expense cost containment initiatives can have a significant impact on the both the actuary's data and their expense reserve methodologies. For example, the introduction of flat fee or retainer agreements may produce an apparent acceleration in legal cost expenditures as up-front expenses are paid. However, over time, these alternative fee arrangements should produce less legal expense development than existed for previous accident years.

Section 3 – Actuarial Implications of The Changing Claims Environment

As evident in Section 2, the recent and rapid introduction of significant changes in the casualty claims environment has required the reserving actuary to become far more conversant in the "language" of claims then ever before. No longer can the actuary rely upon anecdotal descriptions of general changes in claims handling philosophy. Instead, the actuary must seek to fully understand the anticipated effects and interactions of the claims initiatives in order to accurately reflect them in the reserve analysis. In this section, we will further investigate many of the significant changes impacting the Claims environment, as well as discuss many of the potential actuarial implications resulting from these changes. Particular emphasis will be paid to translating "claims-speak" to actuarial jargon.

Historically, conversations between the actuary and the claims department occurred when the actuary sought explanations for unusual claim development. Armed with the response from the Claims Vice President that "*we're settling claims faster, and case reserves are better* ", the actuary went back to their office to adjust their triangles for settlement speed-up, and perhaps for reserve strengthening.

The complex interaction between individual claim initiatives, as well as between the initiatives and actuarial statistics no longer permits this type of limited actuarial involvement. Further, from a financial management perspective, the costs incurred in implementing these changes necessitate a more careful evaluation of their success (or failure). To illustrate this point, we will examine the following changes to the Claims environment, and discuss alternative actuarial approaches to evaluate reserves in each of these environments:

- Changes to Settlement Rates that Vary by Type of Claim

- Changes to the Mix of Claims Settled
- Interaction of Internal Initiatives and External Influences
- Changes to Claim Expense Philosophy

A. Changes to Settlement Rates that Vary by Type of Claim

A typical discussion with the claims professionals of a multi-line company might reveal several of the following initiatives. Each of these initiatives will potentially have various degrees of impact on the overall settlement rate (and perhaps, reporting pattern) of claims.

1. Formation of a Minor Injury Unit

Commonly referred to as MIST (Minor Injury Soft Tissue) or LIST (Low Impact Soft Tissue) claims, the emphasis of this initiative is on reducing improper bodily injury payments on accidents where there is a minimal amount of physical damage to the vehicle.

Several carriers have taken a much harder-line with claimants and their attorneys when the physical facts of the accident do not support the possibility of a bodily injury. As a result, the actuary may expect an initial slow-down in the settlement of these claims, coupled with a reduction in overall severity. However, these observations may change as the program matures. Depending upon the success of the program, claimants and their attorneys may become hesitant to file such claims, which could have a further impact on the overall disposal rate of claims.

2. Introduction of a Contact Time Requirement

It has become a common best practice of claims departments to seek contact with all first-party and potential third-party claimants within a day or two. This rapid contact serves several purposes. First, for the simpler claim, it encourages a quicker settlement. Second, by quickly establishing lines of communication between the carrier and the claimant, the potential of a lawsuit being filed appears to be reduced. Finally, beginning the fact-finding portion of the claim adjustment process earlier can lead to faster identification of all loss exposures, and more accurate case reserving.

3. Increased Claim Staffing

While changes in claim staffing levels should directly impact the settlement rate of claims, it is important to understand how the staffing of the department is configured as well as the responsibilities of the adjusters. For example, increasing the number of property claim adjusters should, at its surface, have a minimal impact on the settlement of more costly and complex liability claims. However, segmentation of responsibilities by claim type may allow senior claim adjusters to spend a greater percentage of their time handling complex claims. As a result, a reduction in bodily injury pending rates may be experienced.

4. Implementation of an Expert Claims Evaluation System

Among the more controversial of initiatives, several carriers are utilizing expert claim systems to assist in evaluating a range of reasonable settlement values for a claim. Typically, these systems require the capturing of specific data elements concerning the injury, possibly lengthening the settlement process. However, as previously discussed, the use of these systems can lead to more rapid settlement of higher severity objective-type injuries.

5. Use of alternative dispute resolution ("ADR")

In an effort to close claims more rapidly as well as reduce legal expenditures, companies have increasingly utilized alternative dispute mechanisms. These may include on-line settlement sites as well as traditional ADR with an impartial third-party. Each of these mechanisms will exert a change on a particular group of claims, emphasizing the need for the actuary to not only understand the approach, but to also identify the impacted claims in their reserving database.

Clearly, a review of settlements rates for all claims combined will fail to uncover the subtle shifts that have occurred for subsets of the population. Only a detailed discussion with claims operational professionals will identify possible ways to segment the data and test for shifts in the settlement patterns for each segment.

B. Changes to the Mix of Claims Settled

The migration towards specialization within the claims department has fostered an environment in which the concept of a universal claims handling philosophy is no longer applicable. In its place we now find a series of approaches, each tailored to a specific subset of claims.

For example, it would not be uncommon for there to be an emphasis on more rapid settlement of severe claims on which both the liability and damages are reasonably determinable. At the same time, the Company may choose to hold fast on minor claims on which the liability is questionable. Further, the Company may employ different settlement philosophies based on whether the claimant has legal representation.

Specialization has also led to the development of subject matter experts within the claims department. Where historically, you might find personal lines adjusters handling a wide variety of claim types, specialization has permitted experienced adjusters to focus more of their time on complex claim issues.

From an actuarial perspective, changing settlement philosophies by claim type require the actuary to question many of the traditional diagnostics they historically have relied upon. For example, one of the underlying premises of the Berquist – Sherman[2] adjustment for changing settlement

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[2] Berquist, J.R. and Sherman, R.E., "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach", PCAS, Vol. CXIV, 1997, Pg. 123-184

rates is that of an increasing incremental paid severity. Stated more simply, larger claims will generally settle later than smaller claims. However, as mentioned above, it is not unusual to observe an acceleration in the payment of a segment of larger claims, coupled with a delay in the closing of smaller claims. If this change in settlement philosophy results in an overall settlement speed-up across all claims, the traditional Berquist-Sherman methodology may lead to an overstated ultimate loss indication when applied to the un-segmented data.

Conversely, the actuary must also be aware of situations in which an overall settlement acceleration is driven mainly by "cherry-picking" or an increased emphasis on the settlement of small, relatively insignificant claims.

C. Interaction of Internal Initiatives and External Influences

Throughout most of the mid to late 1990's the personal automobile insurance industry was the beneficiary of favorable trends in bodily injury claim costs. Not surprisingly, these favorable trends overlapped with the introduction of many of the claim initiatives previously discussed. During this same time period, the insurance industry also benefited from the positive influence of several external or "environmental" cost drivers. A few of these external trends included:

- Reductions in annual medical inflation rates
- Increased use of seat belts
- Increased use of airbags, and other safety features
- Decreases in the use of alcohol / DWI convictions
- Increases in average car size
- Proportional reduction in youthful drivers

It is reasonable to believe that the improvement in results many companies experienced was a function of both internal claim initiatives and these external influences as well. This combination of factors poses additional challenges to the actuary in both the interpretation and projection of historical claim information.

When faced with numerous options for changes within the claims organization, the actuary may also be called upon to evaluate the potential benefit of one initiative versus another. As many of these initiatives require significant upheaval to personnel and systems, the ability to segment the impact of various initiatives becomes critical. Companies benefiting from favorable environmental conditions may also question whether or the not the incremental value received from internal changes offsets the actual cost of those changes.

D. Changes to Claim Expense Philosophy

The myriad of claims department initiatives has not been limited to only the indemnity portion of the claim. Numerous programs have been developed targeting expenses, primarily legal costs.

At their core, most of the recently implemented legal expense cost containment initiatives seek earlier recognition and payment of legal costs, ultimately leading to reduced overall expenditures. If successful, these initiatives should generally result in truncated expense cost development (relative to historical averages). For example, a successful fixed fee or retainer program should reduce the future legal expenditures on the covered claims in exchange for a guaranteed up-front cost. However, traditional development approaches may tend to overstate ultimate legal costs due to this front-loading of expenses.

Agreements between claim departments and outside panel firms may also impact the timing of expense payments. A movement from end of case billing to quarterly or monthly invoicing could easily be misinterpreted as a deterioration in ultimate expense costs. Likewise, a shift to end of case billing may result in understated expense ultimates utilizing traditional paid expense development techniques.

The actuary must also be aware of the potential distorting effects of a shift from outside legal (or "panel" firms) to internal staff counsel positions. These distortions may include changes to the average expense cost per claim relative to panel firms, as well as issues concerning the allocation of staff counsel costs (primarily salary and benefits) to individual claims. As such, the actuary needs to recognize that a shift between panel and staff counsel utilization can have substantial impact on their reserving statistics.

Legal bill auditing (or bill review) offers another complication to the actuary's expense reserve analysis. While the utilization of legal bill review has been challenged in some areas as a violation of attorney-client privilege, many claims professionals contend that bill review is a critical step in controlling escalating outside legal fees. The actuary needs to be aware that in addition to potential savings, the application of bill review may result in the delay of expense payments resulting from attorney challenges.

* * * * *

The various claim initiatives and external factors discussed in this section are but a sample of the widespread array of changes affecting the insurance claim environment. To be responsive to these issues, the actuary must be prepared to engage in regular, detailed discussions with the Claims department in order to fully understand the implications of the initiatives. Armed with this knowledge, it then becomes possible to adjust traditional actuarial reserving methodologies to reflect these implications. In the next section, we address a number of these potential adjustments.

Section 4 – Potential Adjustments to Traditional Actuarial Methodologies

The impact of a changing claims environment on traditional actuarial methodologies is not a new topic to the actuarial literature. Berquist and Sherman, as well as Fleming and Mayer[3] described approaches to address overall shifts in claims handling philosophy such as changes to settlement rate or case reserve adequacy. However, it is now clear that the complex interaction of numerous internal claim initiatives as well as environmental forces requires the development of additional actuarial procedures.

We offer a few tentative steps in what is sure to become a marathon of ideas in this area. The suggested approaches are not intended to be ground breaking, but more thought provoking in nature. Undoubtedly, there are far more questions left unanswered than we can even begin to address here.

A. Changes to Settlement Rates by Size of Loss

Berquist and Sherman noted the complexity introduced into the reserve analysis of a shift in claim's department emphasis by size of loss.

> "One problem which is susceptible to the size of loss approach is that of shifts in emphasis by the claims department on priorities in settling large versus small claims. Such a shift can cause major distortions in the loss projections of nearly all reserving methods."

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[3] Fleming, K.G. and Mayer, J.H., "Adjusting Incurred Losses For Simultaneous Shifts In Payment Patterns And Case Reserve Adequacy Levels"

In response to this situation, Berquist and Sherman suggest segmenting the loss experience by size of loss prior to adjustment to equal percentiles of claims closed. As an alternative, we sought to develop an approach that adjusts the results of the Berquist -- Sherman paid loss methodology for a shift in the size of claims being settled.

To illustrate, sample paid loss and closed claim count data is presented in Exhibit 1. The claims disposal (or settlement) rates derived from this information and shown in Exhibit 2 are consistent with an overall speed-up in settlement. Applying the Berquist - Sherman methodology, and adjusting the losses to common closure rates as defined by the latest evaluation produces the adjusted paid loss triangle in Exhibit 2.

As the settlement rate increases, we would generally anticipate an increase in the proportion of larger claims being settled (assuming that larger claims are settled later than smaller claims). If, however, the claims department contends that in addition to settling claims faster, it has focused specifically on reducing its pending large claim case load, an additional adjustment to the Berquist-Sherman methodology may be warranted.

The magnitude of this adjustment would be dependent upon the specific segment of claims being accelerated. In this example, we divide the loss experience into three strata:

- Less than $15,000 per claim
- Greater than $15,000 and less than $50,000
- Greater than $50,000 per claim

Closed claim counts for the greater than $50,000 layer are shown in Exhibit 3. The ratio of these counts to total claim counts reveals a generally increasing trend, supportive of the Claims department contention. To the degree that proportion of large claims settled exceeds that which would be explained by an increase in the overall settlement pattern, an additional adjustment should be made.

We can apply the Berquist-Sherman methodology to the ratios of large claims from Exhibit 3, adjusting these ratios to the current overall disposal rate. We exclude the latest diagonal of ratios, as these are the values we are attempting to project. Adjusted claim count ratios for claims greater than $50,000 are shown in Exhibit 4 as are selected values based on the averages from each disposal period as well as judgment.

Estimated paid claims in the strata (at the current overall disposal rate) may be derived from the product of the selected interpolated ratios and actual total paid claims. The difference between the estimated and actual paid claims in the greater than $50,000 strata (Column 6) suggests acceleration of larger claims beyond that anticipated in the Berquist-Sherman methodology. Relying upon the actual average paid claim for each accident year, adjusted paid losses are produced using the estimated claim count. These adjusted paid losses (Column 9) reflect the losses that would have been expected for the strata given the estimated closed claim count.

This same process is then repeated for the remaining loss strata (not shown in the exhibits). Total estimated claims and adjusted paid losses combining the results of each loss strata analysis are provided in Exhibit 5. We normalized the adjusted paid amounts in order to adjust for any difference between total projected claims and total actual claims.

Traditional Berquist-Sherman paid development factors derived from the adjusted paid loss for all loss layers combined (Exhibit 2) are shown in Column 8. These development factors are used to project the initial ultimate losses in Column 9. However, applying these same development factors to the normalized adjusted losses produces somewhat reduced ultimate estimates for nearly all accident years (Column 11), the result of which would be a lower reserve indication. This result is consistent with an increased acceleration of large claim settlement relative to the change in overall claim settlement.

B. The Use of Claim Metrics in Evaluating the Impact of Claim Initiatives

Drawn from operational management theory, project goals must be supported by specific objectives and processes to maximize the opportunity for success. In turn, quantifiable measurements or metrics must be designed and tracked to support these processes.

Increasingly, actuaries are being called upon to assist in quantifying the impact of various claims initiatives from the standpoint of strategic planning. In ideal situations, the actuary is involved during the design phase of the initiatives and has input into the identification of the metrics that will be used to monitor the program.

In our discussion, metrics are viewed as specific measurements of internal and external cost drivers. Properly constructed claim metric reports provide the actuary with an additional tool to monitor both the implementation and impact of various claim initiatives. Common internal claim metrics include:

- Suits to open claim ratios
- Attorney representation rates
- Third-party contact rates (contact time)
- Average claim settlements
- Ratio of bodily injury to property damage claim counts
- Pending claim counts
- Adjuster workload
- Staff counsel utilization levels

Claim metrics can provide the actuary with the ability to construct regression models to distinguish between the influences of internal claims initiatives and external factors. To highlight the construction of a simplified regression model, we begin with the personal automobile bodily injury data shown in Exhibit 6. Once again, we are faced with an acceleration of settlement rate, which suggests application of the Berquist-Sherman technique. (As a simplifying assumption, no shift in settlement by size of loss is considered.)

The selected age-to-age development factors on Exhibit 7 are based on the average of the latest three incremental link ratios (after adjusting to common closure rates). As an alternative, on Exhibit 8, the selected factors for the first two development periods are based on the latest incremental factors only (in recognition of the apparent declining trend in the respective columns). But should the actuary anticipate that the favorable trend in the link ratios will continue?

In addition to an emphasis on settling claims faster, let us assume that there have been several claims department initiatives aimed at improving the ratio of bodily injury to property damage claims, reducing contact time for third-party claimants, as well as lowering the overall attorney representation rate on pending claims. Further, the Company has benefited from favorable medical inflation trends and increased seat-belt usage. Sample metrics describing these cost drivers (stated in terms of annual change) are shown in Exhibit 9.

Utilizing these metrics, a multiple regression model can be generated with the change in the Berquist-Sherman adjusted 12 to 24 month link ratios as the dependant variable. The resulting model parameters are:

Fitted Change in Development Factor =

	Annual Change in:		
	BI/PD Claim Count Ratio	x	0.08797
+	Attorney Rep. Rate	x	2.68400
+	Contact Time	x	8.64900
+	Med. Inflation	x	0.04777
-	Restraint Use	x	0.61062
+	Constant		0.05177
	R Squared		*0.97976*

In defining a regression model, the actuary must not only be aware of the fit statistics of the model, attention must also be paid to the (reasonability of the) sign of the coefficients. In this model, each of the coefficients suggests movement in the expected direction. For example, an increase in the attorney representation rate results in higher loss development, while an increase in restraint use generates lower loss development. The positive constant term is not surprising in that it suggests that without favorable results from the claims initiatives, loss development (and likely ultimate losses) will be subject to an increasing trend.

Fitted annual changes in the 12 to 24 month development factors derived from this model are shown in Exhibit 10. These fitted results are produced by applying the regression model parameters to the annual change in metrics provided in Exhibit 9. Note that the regression model and projected metrics produce an indicated increase of 3.84% in the dependent variable (12 to 24 month loss development) for the most recent year. Driving this increase are the less than favorable projected results for the medical inflation rate and the bodily injury to property damage claim ratio.

On-level adjustment factors implied by the fitted annual changes are used to adjust the Berquist-Sherman paid 12 to 24 month link ratios to the current metric level (Column 6). These development factors, which have now been adjusted to reflect changes in settlement as well as claim initiatives and external factors, indicate a 12 to 24 month link ratio of factor of 2.181. A similar regression model approach (not shown) developed for the 24 to 36 month period yielded an indicated link ratio factor of 1.334.

The regression model development indications on Exhibit 11 suggest a higher required reserve than would have been produced by simply relying upon the latest link ratios for the first two development periods. Had we extended the declining trend in these link ratios without giving consideration to the underlying metrics, the indicated reserve difference would have been greater.

This example clearly indicates two of the significant benefits derived from the use of regression models in loss development analysis. First, the relative magnitude of the coefficients permits identification of the internal initiatives and external factors with the greatest impact on loss development. Second, the regression model can permit earlier identification of turning points in loss development through leading indicators. However, the parameters of the model should be subjected to frequent re-evaluation and retuning in order to maintain their predictive value.

C. Adjusting for Changes in Legal Expenditures

In adjusting most traditional loss adjustment expense reserving methodologies, data segmentation is critical. Separate classification of expenses such as panel costs by alternative fee arrangement type, staff counsel costs by region, and legal bill auditing fees by claim type, allows the actuary to project future expense costs recognizing the changes implemented by the claims department.

The actuary should be aware of the size and composition of the claims department budget for legal costs. Depending upon their historical accuracy, the budget projections can serve as useful input in the actuary's reserve estimates. For example, the actual ratio of calendar paid expenses to paid losses may be declining as shown below.

	Calendar Year			
	1997	1998	1999	2000
Legal Paid / Loss Paid	24%	22%	21%	19%
Selected Acc. Year 2000 Legal / Loss Ratio				17%

Based upon discussions with the claim department concerning future budgeted legal expenses, the actuary selects an accident year 2000 legal expense to loss ratio of 17%. This ratio assumes continuation of the improvement shown in the chart, as well as a lag between the accident year and actual suit emergence.

Exhibit 12 offers another possible use of budgeted legal expenses. In this example, the company has increased their reliance upon staff counsel attorneys. Further, it is believed that the staff counsel costs for the period 2001 to 2008 will grow by 4% per year and that no additional attorneys will be hired.

The percentage of open suits relating to accident years 2000 and prior can be estimated for each future calendar year based upon historical suit emergence and settlement rates. Applying these percentages to the budgeted staff counsel costs in these future years produces a staff counsel reserve estimate for the combined accident years. The resulting reserve estimate can serve as a reasonability check for the actuary's other projections, or can be allocated to the individual years for reporting purposes.

Data segmentation and detailed discussions with the Claims Department can also assist the actuary in recognizing the impact of alternative fee arrangements on their legal reserve estimates. This approach requires the actuary to project the average cost of legal fees on suits emerging prior to and after the introduction of the fee initiative. In doing so, the actuary must be aware of many issues including:

- Average outside attorney costs by state or region (and percentage of claims affected).
  - average fixed fee or matrix cost, including fees for trial
  - average retainer cost
  - average hourly rate
- Litigation rate by region or state.
- Utilization of Staff Counsel versus Panel Counsel.
- Emergence rate of new lawsuits.

A simplified reserving model based on many of these is shown in Exhibit 13. In this example, the company employs the use of both staff counsel attorneys and outside panel firms. The company has negotiated a series of flat fee and retainer agreements in five out of its six regions of business. Based on conversations with the Claims Department, the average cost of these arrangements is either \$4,000 or \$5,000 per suit, depending upon the region. In the remaining region, staff counsel attorneys are prohibited, and no fee arrangements have been implemented.

Of the approximately 3,500 claims the company anticipates being reported in the coming year, 37% will result in litigation. This rate will of course vary based on the litigousness of the various regions.

359 of the eventual suits will be handled in-house, with the remaining litigated claims distributed to the various panel firms. The weighted average panel cost by region of \$5,729 indicates an average savings due to the alternative fee arrangements of 36% relative to the historical average external legal cost of \$9,000 per litigated claim (\$5,729 / \$9,000 – 1 = 36%). However, this

reduction represents the anticipated savings future for accident periods yet to be filed. The impact on prior accident years may be estimated by weighting this projected average with the average legal cost in place prior to entering into the agreements (Exhibit 14).

Conclusion

We have become convinced that static claims environments have become the exception rather than the rule. In a majority of situations, a combination of internal and external changes will render historical reserving experience of limited value unless one gains a detailed understanding of how this historical data will be affected by the changes. By developing effective communications with the insurer's operating areas, and adjusting the actuarial methodologies as warranted, the resulting reserve analysis is both more meaningful and more valuable in evaluating the benefits of the operational changes.

Cumulative Paid - All Layers ($000's)

Accident Year	12	24	36	48	60	72	84	96
1987	353.0	3,160.4	7,260.9	11,167.4	12,673.5	13,432.6	13,787.2	13,803.5
1988	370.4	3,285.1	8,888.0	14,013.5	16,827.2	17,588.2	18,378.0	19,145.8
1989	509.0	5,967.1	10,409.7	15,074.1	19,139.9	20,110.5	20,751.3	21,313.0
1990	1,016.7	6,368.6	12,502.1	16,891.8	19,992.9	22,408.5	23,359.7	23,362.9
1991	520.9	5,476.7	13,249.0	19,643.6	24,479.8	26,093.5	26,525.3	26,679.8
1992	707.9	6,704.2	15,158.6	19,858.4	22,682.1	24,580.7	25,865.0	26,607.8
1993	695.8	5,201.2	10,750.4	15,170.8	19,566.8	21,141.9	21,735.9	22,601.0
1994	744.8	5,292.2	10,722.8	16,440.3	21,350.1	24,625.1	26,087.9	
1995	1,325.0	6,406.4	15,453.1	22,103.8	26,030.2	28,384.4		
1996	1,298.7	9,210.2	18,938.0	29,172.2	38,053.0			
1997	1,055.9	6,948.3	17,774.2	29,262.8				
1998	1,590.4	9,889.9	25,804.4					
1999	2,212.5	11,071.4						
2000	1,398.4							

Cumulative Paid Counts - All Layers

Accident Year	12	24	36	48	60	72	84	96	Est. Ultimate Count
1987	97	398	572	700	745	766	773	773	776
1988	103	433	672	768	811	827	834	838	841
1989	154	554	771	881	933	952	961	965	971
1990	183	584	783	882	930	966	977	978	989
1991	180	520	715	830	886	906	913	915	927
1992	176	512	668	748	802	825	837	840	847
1993	162	488	647	731	796	815	821	826	833
1994	194	551	708	800	851	879	886		894
1995	209	598	817	916	996	1,020			1,042
1996	237	729	1,002	1,167	1,255				1,312
1997	258	714	991	1,154					1,287
1998	267	784	1,057						1,314
1999	298	774							1,246
2000	319								1,362

Ultimate Claims Disposed Ratios

Accident Yr	12	24	36	48	60	72	84	96
1987	13%	51%	74%	90%	96%	99%	100%	100%
1988	12%	51%	80%	91%	96%	98%	99%	100%
1989	16%	57%	79%	91%	96%	98%	99%	99%
1990	19%	59%	79%	89%	94%	98%	99%	99%
1991	19%	56%	77%	90%	96%	98%	98%	99%
1992	21%	60%	79%	88%	95%	97%	99%	99%
1993	19%	58%	77%	87%	95%	97%	98%	98%
1994	22%	62%	79%	89%	95%	98%	99%	
1995	20%	57%	78%	88%	96%	98%		
1996	18%	56%	76%	89%	96%			
1997	20%	55%	77%	90%				
1998	20%	60%	80%					
1999	24%	62%						
2000	23%							

Cumulative Paid - All Layers ($000's)
All Layers at Equal Percentiles of Ultimate Closed Counts

Accident Year	23%	62%	80%	90%	96%	98%	99%	99%
1987	654.2	5,834.2	9,311.0	11,011.0	12,577.1	13,197.7	13,599.3	13,688.9
1988	689.4	5,940.8	9,048.3	13,118.7	16,365.5	17,407.4	18,320.2	18,363.6
1989	799.5	8,084.8	10,675.5	14,558.3	18,775.5	20,033.1	20,848.3	21,007.9
1990	1,269.7	7,323.7	13,034.7	17,139.8	21,150.7	22,561.0	23,641.9	23,382.8
1991	673.2	8,069.6	15,208.8	19,724.6	24,549.8	26,214.3	26,884.2	27,008.0
1992	822.0	7,377.6	16,235.0	20,642.3	23,144.8	24,938.5	26,134.7	26,578.3
1993	853.2	6,243.6	11,933.3	16,193.9	19,628.9	21,179.0	22,199.3	22,601.0
1994	810.2	5,424.4	11,261.2	16,562.2	21,807.4	24,142.8	26,087.9	
1995	1,526.7	7,827.6	16,808.4	23,615.5	26,067.9	28,384.4		
1996	1,717.5	12,983.8	21,785.5	29,900.3	38,053.0			
1997	1,263.0	9,901.0	20,630.4	29,262.8				
1998	1,836.9	11,103.8	25,804.4					
1999	2,165.6	11,071.4						
2000	1,398.4							

Paid Claim Counts > $50,000

Accident Year	12	24	36	48	60	72	84	96
1987	1.0	10.0	27.0	48.0	56.0	60.0	62.0	62.0
1988	1.0	7.0	27.0	56.0	67.0	72.0	78.0	80.0
1989	1.0	13.0	29.0	53.0	76.0	82.0	85.0	89.0
1990	1.0	12.0	41.0	62.0	81.0	95.0	101.0	101.0
1991	0.5	13.0	45.0	79.0	108.0	114.0	117.0	118.0
1992	0.5	17.0	50.0	74.0	90.0	102.0	109.0	111.0
1993	1.0	14.0	42.0	62.0	81.0	92.0	96.0	98.0
1994	0.5	14.0	33.0	65.0	88.0	104.0	111.0	
1995	3.0	15.0	64.0	95.0	120.0	134.0		
1996	2.0	25.0	63.0	117.0	161.0			
1997	0.5	17.0	75.0	136.0				
1998	3.0	33.0	94.0					
1999	5.0	35.0						
2000	3.0							

Paid Claim Counts > $50,000 / Total Paid Counts

Accident Year	12	24	36	48	60	72	84	96
1987	1.0%	2.5%	4.7%	6.9%	7.5%	7.8%	8.0%	8.0%
1988	1.0%	1.6%	4.0%	7.3%	8.3%	8.7%	9.4%	9.6%
1989	0.6%	2.3%	3.8%	6.0%	8.1%	8.6%	8.8%	9.2%
1990	0.5%	2.1%	5.2%	7.0%	8.7%	9.8%	10.3%	10.3%
1991	0.3%	2.5%	6.3%	9.5%	12.2%	12.6%	12.8%	12.9%
1992	0.3%	3.3%	7.5%	9.9%	11.2%	12.4%	13.0%	13.2%
1993	0.6%	2.9%	6.5%	8.5%	10.2%	11.3%	11.7%	11.9%
1994	0.3%	2.5%	4.7%	8.1%	10.3%	11.8%	12.5%	
1995	1.4%	2.5%	7.8%	10.4%	12.0%	13.1%		
1996	0.8%	3.4%	6.3%	10.0%	12.8%			
1997	0.2%	2.4%	7.6%	11.8%				
1998	1.1%	4.2%	8.9%					
1999	1.7%	4.5%						
2000	0.9%							

Paid Claim Counts > $50,000 / Total Paid Counts at Equal Percentiles of Ultimate Closed Counts (ALL)

Accident Year	23%	62%	80%	90%	96%	98%	99%	99%
1987	1.3%	3.2%	5.7%	6.8%	7.5%	7.7%	7.9%	8.1%
1988	1.1%	1.9%	4.1%	6.7%	8.1%	8.6%	9.3%	9.3%
1989	0.8%	2.7%	3.8%	5.8%	7.9%	8.6%	8.9%	9.0%
1990	0.6%	2.3%	5.5%	7.1%	9.4%	9.9%	10.5%	10.3%
1991	0.4%	3.6%	7.3%	9.6%	12.2%	12.6%	13.0%	13.1%
1992	0.3%	3.7%	8.0%	10.3%	11.4%	12.6%	13.2%	13.2%
1993	0.7%	3.4%	7.5%	9.1%	10.4%	11.7%	12.5%	
1994	0.3%	2.6%	4.9%	8.2%	10.5%	11.6%		
1995	1.5%	2.7%	8.7%	10.9%	12.1%			
1996	1.0%	4.4%	7.1%	10.3%				
1997	0.2%	3.8%	9.1%					
1998	1.2%	4.6%						
1999	1.7%							
Last Diag.	1.7%	4.6%	9.1%	10.3%	12.1%	11.6%	12.5%	
Avg Last 3	1.1%	4.3%	8.3%	9.8%	11.0%	12.0%	12.9%	
Selected	1.0%	4.4%	8.8%	10.3%	12.1%	11.6%	12.5%	

(1) Acc Year	(2) Paid Count (ALL)	(3) Estimated Ratio of Claims > 50K	(4) Estimated Count of Claims > 50K	(5) Actual Claims > 50K	(6) Difference	(7) Actual Paid on Claims > 50K	(8) Average Paid on Claims > 50K	(9) Adj. Paid on Claims > 50K
1994	886	12.5%	110.8	111	0	$17,518	$157.8	$17,493
1995	1020	11.6%	118.5	134	16	$18,277	$136.4	$16,156
1996	1255	12.1%	151.4	161	10	$25,028	$155.5	$23,537
1997	1154	10.3%	118.8	136	17	$17,969	$132.1	$15,699
1998	1057	8.8%	93.0	94	1	$15,970	$169.9	$15,803
1999	774	4.4%	34.1	35	1	$5,287	$151.0	$5,144
2000	319	1.0%	3.2	3	(0)	$448	$149.2	$476
Total	6465		629.8	674	44.2	$100,496		$94,307

(2) Exhibit 1: Cumulative Paid Claim Counts - All Layers
(3) Selected
(4) = (2) x (3)
(6) = (5) - (4)
(8) = (7) / (5)
(9) = (4) x (8)

Cumulative Adjusted Paid - All Layers ($000's)

(1) Acc Year	(2) Actual Claims	(3) Estimated Claims	(4) Adjusted Paid	(5) Normalized Adjusted Paid
1994	886	886	$26,410	$26,398
1995	1,020	1,020	$25,727	$25,732
1996	1,255	1,255	$36,435	$36,434
1997	1,154	1,155	$27,775	$27,751
1998	1,057	1,067	$25,888	$25,644
1999	774	778	$10,821	$10,769
2000	319	322	$1,554	$1,541
Total	6,465	6,483	$154,610	$154,270

(6) Acc Year	(7) Actual Paid	(8) Berquist / Sherman Paid DFU	(9) Initial Ultimate	(10) Adjusted Paid Loss	(11) Adjusted Ultimate	(12) Difference
1994	$26,088	1.022	$26,670	$26,398	$26,987	$317
1995	$28,384	1.073	$30,459	$25,732	$27,613	($2,847)
1996	$38,053	1.132	$43,074	$36,434	$41,241	($1,833)
1997	$29,263	1.420	$41,544	$27,751	$39,398	($2,146)
1998	$25,804	1.976	$50,980	$25,644	$50,664	($316)
1999	$11,071	4.036	$44,680	$10,769	$43,461	($1,219)
2000	$1,398	24.986	$34,941	$1,541	$38,512	$3,571
Total	$160,062		$272,349	$154,270	$267,876	($4,472)
Total Excld 2000	$158,664		$237,408	$152,729	$229,364	($8,043)

(2) Exhibit 1: Cumulative Paid Claim Counts - All Layers
(3) Summation of estimated claim counts from all layers analyzed. (Includes layers not shown in Exhibits)
(4) Summation of adjusted paid losses from all layers analyzed. (Includes layers not shown in Exhibits)
(5) = (4) / (3) x (2)
(7) Exhibit 1: Cumulative Paid Loss - All Layers
(9) = (7) x (8)
(10) = (5)
(11) = (10) x (8)
(12) = (11) - (9)

Exhibit 6

Private Passenger Automobile Liability (000's)
Paid Loss Development

Accident Year	12	24	36	48	60	72	84	96
	Development Month							
1991	1,118	2,712	4,000	4,864	5,384	5,650	5,812	5,892
1992	1,266	2,974	4,281	5,170	5,669	5,960	6,106	6,170
1993	1,251	2,898	4,217	5,070	5,550	5,812	5,939	6,002
1994	1,241	2,848	4,064	4,855	5,331	5,568	5,691	
1995	1,248	2,802	4,030	4,860	5,332	5,559		
1996	1,338	3,018	4,329	5,178	5,684			
1997	1,569	3,407	4,780	5,773				
1998	1,626	3,461	4,800					
1999	1,808	3,796						
2000	1,820							

Link Ratios

Accident Year	12	24	36	48	60	72	84	To Ult
1991	2.427	1.475	1.216	1.107	1.049	1.029	1.014	
1992	2.349	1.439	1.208	1.096	1.051	1.024	1.010	
1993	2.318	1.455	1.202	1.095	1.047	1.022	1.011	
1994	2.295	1.427	1.195	1.098	1.045	1.022		
1995	2.245	1.438	1.206	1.097	1.043			
1996	2.255	1.435	1.196	1.098				
1997	2.171	1.403	1.208					
1998	2.129	1.387						
1999	2.100							
Selected	2.132	1.407	1.203	1.098	1.045	1.023	1.012	1.000
DFU	4.282	2.009	1.428	1.187	1.081	1.035	1.012	1.000

Disposal Rate

Acc Year	12	24	36	48	60	72	84	96
1991	0.650	0.865	0.940	0.970	0.980	0.990	0.995	1.000
1992	0.650	0.865	0.940	0.970	0.980	0.990	0.995	1.000
1993	0.660	0.865	0.940	0.970	0.980	0.990	0.995	1.000
1994	0.660	0.865	0.940	0.970	0.980	0.990	0.995	
1995	0.660	0.865	0.940	0.970	0.980	0.990		
1996	0.660	0.865	0.940	0.975	0.985			
1997	0.660	0.870	0.950	0.975				
1998	0.660	0.870	0.950					
1999	0.670	0.880						
2000	0.670							

Exhibit 7

Paid Loss Development - Berquist Sherman Adjustment ($000's)

Accident Year	Development Month 12	24	36	48	60	72	84	96
1991	1,240	2,957	4,283	5,123	5,517	5,650	5,812	5,892
1992	1,398	3,222	4,572	5,418	5,814	5,960	6,106	6,170
1993	1,317	3,149	4,496	5,309	5,681	5,812	5,939	6,002
1994	1,306	3,079	4,323	5,092	5,449	5,568	5,691	
1995	1,311	3,035	4,302	5,095	5,445	5,559		
1996	1,406	3,266	4,566	5,178	5,684			
1997	1,642	3,569	4,780	5,773				
1998	1,699	3,619	4,800					
1999	1,808	3,796						
2000	1,820							

Link Ratios

Accident Year	12	24	36	48	60	72	84	To Ult
1991	2.384	1.449	1.196	1.077	1.024	1.029	1.014	
1992	2.305	1.419	1.185	1.073	1.025	1.024	1.010	
1993	2.390	1.428	1.181	1.070	1.023	1.022	1.011	
1994	2.357	1.404	1.178	1.070	1.022	1.022		
1995	2.314	1.417	1.184	1.069	1.021			
1996	2.323	1.398	1.134	1.098				
1997	2.173	1.339	1.208					
1998	2.130	1.326						
1999	2.100							
Selected	2.133	1.353	1.176	1.079	1.022	1.023	1.012	1.000
DFU	3.872	1.815	1.341	1.141	1.057	1.035	1.012	1.000

Accident Year	**Paid Loss**	**Paid Dev. Ult**	**Berquist / Sherman Dev. Ult**
1991	5,892	5,892	5,892
1992	6,170	6,170	6,170
1993	6,002	6,002	6,002
1994	5,691	5,757	5,757
1995	5,559	5,752	5,752
1996	5,684	6,144	6,010
1997	5,773	6,850	6,587
1998	4,800	6,853	6,438
1999	3,796	7,627	6,891
2000	1,820	7,794	7,048
Total	**$ 51,187**	**$ 64,840**	**$ 62,546**
Reserve		**$ 13,654**	**$ 11,359**

Exhibit 8

Paid Loss Development - Berquist Sherman Adjustment - Alternative Selection ($000's)

Accident Year	Development Month 12	24	36	48	60	72	84	96
1991	1,240	2,957	4,283	5,123	5,517	5,650	5,812	5,892
1992	1,398	3,222	4,572	5,418	5,814	5,960	6,106	6,170
1993	1,317	3,149	4,496	5,309	5,681	5,812	5,939	6,002
1994	1,306	3,079	4,323	5,092	5,449	5,568	5,691	
1995	1,311	3,035	4,302	5,095	5,445	5,559		
1996	1,406	3,266	4,566	5,178	5,684			
1997	1,642	3,569	4,780	5,773				
1998	1,699	3,619	4,800					
1999	1,808	3,796						
2000	1,820							

Link Ratios

Accident Year	12	24	36	48	60	72	84	To Ult
1991	2.384	1.449	1.196	1.077	1.024	1.029	1.014	
1992	2.305	1.419	1.185	1.073	1.025	1.024	1.010	
1993	2.390	1.428	1.181	1.070	1.023	1.022	1.011	
1994	2.357	1.404	1.178	1.070	1.022	1.022		
1995	2.314	1.417	1.184	1.069	1.021			
1996	2.323	1.398	1.134	1.098				
1997	2.173	1.339	1.208					
1998	2.130	1.326						
1999	2.100							
Selected	**2.100**	**1.326**	1.176	1.079	1.022	1.023	1.012	1.000
DFU	**3.737**	**1.779**	1.341	1.141	1.057	1.035	1.012	1.000

Acc Year	**Paid Loss**	**Paid Dev. Ult**	**Berquist / Sherman Dev. Ult**	**Alternative B/S Dev. Ult**
1991	5,892	5,892	5,892	5,892
1992	6,170	6,170	6,170	6,170
1993	6,002	6,002	6,002	6,002
1994	5,691	5,757	5,757	5,757
1995	5,559	5,752	5,752	5,752
1996	5,684	6,144	6,010	6,010
1997	5,773	6,850	6,587	6,587
1998	4,800	6,853	6,438	6,438
1999	3,796	7,627	6,891	6,755
2000	1,820	7,794	7,048	6,801
Total	**$ 51,187**	**$ 64,840**	**$ 62,546**	**$ 62,163**
Reserve		**$ 13,654**	**$ 11,359**	**$ 10,976**

Exhibit 9

Private Passenger Automobile Liability
Sample Claim Metrics

Acc Year	12 - 24 B/S Dev Factor	Change	BI to PD Ratio	Attorney Rep. Rate	Contact Time	Medical Inflation	Restraint Use
			Annual Change in:				
1991	2.384						
1992	2.305	-3.32%	-1.17%	0.43%	-0.78%	4.33%	4.30%
1993	2.390	3.72%	3.86%	0.65%	-0.65%	17.13%	-2.00%
1994	2.357	-1.39%	-4.86%	0.71%	-0.45%	-3.54%	5.99%
1995	2.314	-1.81%	-3.90%	0.62%	-0.48%	-15.15%	6.55%
1996	2.323	0.35%	1.25%	0.65%	-0.33%	-19.68%	4.75%
1997	2.173	-6.43%	-7.72%	-0.10%	-0.85%	-19.81%	3.60%
1998	2.130	-1.97%	-2.34%	-0.20%	-0.64%	-5.54%	2.06%
1999	2.100	-1.41%	-0.68%	-0.31%	-0.34%	-22.44%	2.10%
*2000**			*0.08%*	*-0.11%*	*-0.12%*	*12.47%*	*1.00%*

*projected

Private Passenger Automobile Liability
Regression Model - Fitted Change in Loss Development Factors

Acc Year	BI/PD Ratio	Attorney	Contact	Med Inflation	Restraint Use	Constant	Fitted Change
	0.088	2.684	8.649	0.048	-0.611	0.052	
1992	-0.103%	1.154%	-6.746%	0.207%	-2.626%	5.177%	-2.937%
1993	0.340%	1.745%	-5.622%	0.818%	1.221%	5.177%	3.679%
1994	-0.428%	1.906%	-3.892%	-0.169%	-3.660%	5.177%	-1.066%
1995	-0.343%	1.664%	-4.152%	-0.724%	-3.998%	5.177%	-2.375%
1996	0.110%	1.745%	-2.854%	-0.940%	-2.900%	5.177%	0.338%
1997	-0.679%	-0.268%	-7.352%	-0.946%	-2.198%	5.177%	-6.267%
1998	-0.206%	-0.537%	-5.535%	-0.265%	-1.257%	5.177%	-2.623%
1999	-0.060%	-0.832%	-2.941%	-1.072%	-1.282%	5.177%	-1.010%
2000	**0.007%**	**-0.295%**	**-1.038%**	**0.596%**	**-0.611%**	**5.177%**	**3.836%**

=coefficent x annual change in metric

Acc Year	(1) 12 - 24 B/S Dev Factor	(2) Change	(3) Fitted Change	(4) Fitted 12 - 24 B/S Dev Factor	(5) Adjust. Factors	(6) On-Level 12 - 24 B/S Dev Factor
1991	2.384			2.384	0.854	2.181
1992	2.305	-3.32%	-2.94%	2.314	0.899	2.173
1993	2.390	3.72%	3.68%	2.399	0.844	2.174
1994	2.357	-1.39%	-1.07%	2.373	0.860	2.167
1995	2.314	-1.81%	-2.38%	2.317	0.897	2.179
1996	2.323	0.35%	0.34%	2.325	0.892	2.179
1997	2.173	-6.43%	-6.27%	2.179	1.002	2.175
1998	2.130	-1.97%	-2.62%	2.122	1.053	2.190
1999	2.100	-1.41%	-1.01%	2.100	1.073	2.181
2000			3.84%	2.181	1.000	
					Selected	**2.181**

(1) = Exhibit 7: 12 - 24 month (67% closed) link ratio
(2) = (1) / (1) prior - 1.00
(3) = regression model result
(4) = (1.00 + (3)) x (4)prior
(5) = ((4)2000 - 1.00) / ((4) - 1.00)
(6) = ((1) - 1.00) x (5)) + 1.00

Exhibit 11

Paid Loss Development - Berquist Sherman Adjustment - Regression Analysis ($000's)

Accident Year	12	24	36	48	60	72	84	96
				Development Month				
1991	1,240	2,957	4,283	5,123	5,517	5,650	5,812	5,892
1992	1,398	3,222	4,572	5,418	5,814	5,960	6,106	6,170
1993	1,317	3,149	4,496	5,309	5,681	5,812	5,939	6,002
1994	1,306	3,079	4,323	5,092	5,449	5,568	5,691	
1995	1,311	3,035	4,302	5,095	5,445	5,559		
1996	1,406	3,266	4,566	5,178	5,684			
1997	1,642	3,569	4,780	5,773				
1998	1,699	3,619	4,800					
1999	1,808	3,796						
2000	1,820							

Link Ratios

Accident Year	12	24	36	48	60	72	84	
1991	**2.181**	**1.334**	1.196	1.077	1.024	1.029	1.014	
1992	**2.173**	**1.331**	1.185	1.073	1.025	1.024	1.010	
1993	**2.174**	**1.337**	1.181	1.070	1.023	1.022	1.011	
1994	**2.167**	**1.324**	1.178	1.070	1.022	1.022		
1995	**2.179**	**1.332**	1.184	1.069	1.021			
1996	**2.179**	**1.334**	1.134	1.098				
1997	**2.175**	**1.338**	1.208					
1998	**2.190**	**1.333**						
1999	**2.181**							
Selected	**2.181**	**1.334**	1.176	1.079	1.022	1.023	1.012	1.000
DFU	**3.902**	**1.789**	1.341	1.141	1.057	1.035	1.012	1.000

Acc Year	**Paid Loss**	**Paid Dev. Ult**	**Berquist / Sherman Dev. Ult**	**Altern. B/S Dev. Ult**	**Regression Adjusted Dev. Ult**
1991	5,892	5,892	5,892	5,892	5,892
1992	6,170	6,170	6,170	6,170	6,170
1993	6,002	6,002	6,002	6,002	6,002
1994	5,691	5,757	5,757	5,757	5,757
1995	5,559	5,752	5,752	5,752	5,752
1996	5,684	6,144	6,010	6,010	6,010
1997	5,773	6,850	6,587	6,587	6,587
1998	4,800	6,853	6,438	6,438	6,438
1999	3,796	7,627	6,891	6,755	6,793
2000	1,820	7,794	7,048	6,801	7,102
Total	**$ 51,187**	**$ 64,840**	**$ 62,546**	**$ 62,163**	**$ 62,502**

Exhibit 12

Estimated Staff Counsel Expense Reserve

(1)	(2)	(3)	(4)
Calendar Year	Budgeted Staff Counsel	Accident Year 2000 and Prior	Implied Reserve
	($000)		($000)
2001	10,000	90%	9,000
2002	10,400	72%	7,488
2003	10,816	58%	6,273
2004	11,249	46%	5,174
2005	11,699	37%	4,328
2006	12,167	29%	3,528
2007	12,653	20%	2,531
2008	13,159	9%	1,184
Total			39,507

Estimated Average Litigation Cost Under Retainer Agreements & Flat Fee Arrangements

(1) Region	(2) Projected Claims	(3) Litigation Rate	(4) Litigated Claims	(5) Staff Rate	(6) Estimated Staff Claims	(7) Estimated External Claims	(8) External Average Cost
A	412	56%	231	60%	138	92	$ 5,000
B	222	39%	87	30%	26	61	$ 5,000
C	132	47%	62	30%	19	43	$ 5,000
D	91	73%	66	75%	50	17	$ 4,000
E	1,221	47%	574	22%	126	448	$ 4,000
Other	1,445	20%	289	0%		289	$ 9,000
Total	3,523	37%	1,309	27%	359	950	$ 5,729

(4) = (2) x (3)
(6) = (4) x (5)
(7) = (4) - (6)
Total (8) = Weighted Average of (8) and (7)

Exhibit 14

Emergence of Savings Under Retainer Agreements & Flat Fee Arrangements

(1)	(2)	(3)	(4)	(5)	(6)
Development Months	Litigation Emergence Rate	Cumulative Litigation Emergence	Historical Average	Projected Average	Weighted Litigation Cost
12	23%	23%	$ 9,000	$ 5,729	6,481
24	34%	57%	$ 9,000	$ 5,729	7,593
36	18%	75%	$ 9,000	$ 5,729	8,182

(3) = Summation of (2)
(5) from Exhibit 13
(6) = ((3) x (4)) + [(1.00 - (3)) x (5)]

A Random Walk Model for Paid Loss Development

Daniel D. Heyer

A Random Walk Model for Paid Loss Development

Daniel D. Heyer

Abstract

Traditional loss development techniques focus on estimating the expected ultimate loss but do not generally indicate the magnitude of possible deviation from this estimate. In a variety of circumstances, however, point reserve estimates are not sufficient. In particular, loss portfolio transfers, commutations, novations, and reserve margin securitization all typically require an estimate of the range of possible loss outcomes.

By adjusting a paid loss model described in Foundations of Casualty Actuarial Science to incorporate a random fluctuation component, a stochastic differential equation model is obtained. This model is analogous to the stock price model used to develop the Black-Scholes option pricing formula. Furthermore, this differential equation has an explicit solution that yields Lognormal distributed development factors similar to the Lognormal link-ratio model published by Roger Hayne.

A slight modification to the model for undiscounted reserves provides a differential equation that accounts for variation in both the amount and timing of loss payments. This equation does not have an explicit solution but can be solved numerically to yield the distribution of the present value reserve.

The opinions expressed in this article are those of the author, not American Re-Insurance Company.

Introduction

Traditional loss development techniques focus on estimating expected ultimate losses but do not generally indicate the magnitude of possible deviation from this estimate. Typically, a reasonable point-estimate reserve is selected after evaluating the range of estimates produced by several projection techniques. Barring significant calendar year effects, this approach is quite effective when reserves from many accident periods are combined into a single aggregate reserve. In this case, the development on any single reserve may be offset by development on the remaining reserves.

In a variety of circumstances, however, reserve point-estimates are insufficient. In particular, loss portfolio transfers, commutations, novations, and reserve margin securitization often involve a single reserve. Furthermore, these contracts are typically priced on an economic basis. Economic pricing requires valuation of the uncertainty arising from both payment amount and timing.

By adjusting a paid loss model described in *Foundations of Casualty Actuarial Science* to incorporate a random fluctuation component, a stochastic differential equation (SDE) for paid loss development is obtained. This model is analogous to the random walk stock price model used to develop the Black-Scholes option pricing formula. This differential equation has an explicit solution that yields Lognormal distributed development factors similar to a loss development model published by Roger Hayne. This distribution may be used to compute prediction intervals for the indicated reserve, and expected adverse deviation from the carried reserve.

A slight modification to the model for undiscounted reserves provides a differential equation for discounted reserves. This equation does not have an explicit solution but may be solved numerically to yield the distribution of the present value reserve.

Historical Motivation for Model Approach

The model developed here is a generalization of two models already familiar to the actuarial profession. The most straightforward model is the Lognormal Age-to-Age Factor model developed by Roger Hayne[1]. This model assumes that age-to-age factors are Lognormal distributed and uses the properties of compounded Lognormal variates to project ultimate losses. As we shall see later, this is an entirely appropriate model for loss development. Implementation of Hayne's model, however, is complicated by several limitations...

- Parameters are estimated for each development age using losses observed at each age. This data becomes sparse at later development ages.
- Tail factors must be estimated.
- Two parameters must be estimated for each development age. This creates a significant potential for over-fitting. (*i.e.* the model has so much flexibility that it is fitting parameters to the noise in the data as well as to the underlying relationship of interest.)

These issues, however, can be addressed by uniting the Hayne model with the Loss Function Model detailed by Ronald Wiser[2]. In this model, Wiser discusses loss rate functions that can be integrated to yield the expected incremental paid losses during any specified period. In general differential equation form...

$$dP = m(t)dt \qquad (1)$$

...where dP is the incremental paid loss over each time dt, P is paid losses and $m(t)$ is the loss rate function. The choice of loss rate function is governed by incurred and reporting patterns, timing of salvage and subrogation recoveries, etc. In general, however, the loss rate function should tend to zero over time. Under this model, age-to-age factors are no longer a practical necessity. Once the parameters have been estimated for the loss rate function, however, age-to-age factors may be computed directly by...

$$\textit{Age-to-Age Factor}(t_1,t_2) = \frac{\int_0^{t_2} m(s)ds}{\int_0^{t_1} m(s)ds} \qquad (2)$$

Typically, $m(t)$ will have far fewer parameters than Hayne's model so there is less opportunity for overfitting. Furthermore, the model already incorporates an implicit tail factor so there is no need to estimate this separately. Note, however, that this tail factor is based solely upon the characteristics of the selected loss rate function. This model does not address the development variability that was the crux of Hayne's model.

The technical question becomes, then, how can we modify Equation (1) to incorporate random variation. The statistical tool for accomplishing this is called stochastic differential equations (SDEs). SDEs allow us to write differential equations with random coefficients or constants. These equations have found application in a variety of engineering, biological and financial systems subject to "noisy growth". In an insurance reserving setting, paid loss development is an example of noisy growth.[3] By assumption, losses follow a "development pattern" and it is the actuary's charge to assess whether deviations from the development pattern are random or systematic. SDEs are one approach for quantifying the paid loss development pattern and statistically testing deviations from that pattern.

Unfortunately, standard Riemann integration techniques cannot be used to solve SDEs. The next section details the basic technical apparatus required to specify and evaluate the equations used in this model. This explanation, however, should not be taken as either a general or complete presentation of the topic.

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[1] Roger Hayne, An Estimate of Statistical Variation in Development Factor Methods, 1985 Proceedings of the Casualty Actuarial Society, Volume LXXII

[2] Ronald Wiser, *Loss Reserving*, Foundations of Casualty Actuarial Science, Third Edition

[3] By contrast, incurred loss development is subject to systematic manipulation by the actuary and does not constitute noisy growth.

Stochastic Differential Equations

The differential equation that forms the basis of this projection method is an extension of Equation (1)...

$$dP = \mu(t)Pdt + \sigma(t)PdB_t \quad \text{...or...} \quad \frac{dP}{P} = \mu(t)dt + \sigma(t)dB_t \tag{3}$$

Here $\mu(t)$ is the loss log-growth rate, dB_t is a Brownian motion noise function (Brownian motion will be discussed in further detail below) and $\sigma(t)$ is a noise scale factor. Solving this equation for $P(t)$ is somewhat problematic as P is a stochastic process rather than a normal function. Was this a Riemann integral we would make the substitution...

$$G(P) = \ln(P) \Rightarrow dG(P) = \frac{dP}{P} \tag{4}$$

This substitution would make the solution of Equation (3) relatively straightforward. When dealing with a stochastic process, however, we cannot so easily use the derivative "chain-rule" to go from $G(P)$ to $dG(P)$. The chain-rule for stochastic processes is given by Îto's lemma.[4] Without proof, a form of this lemma states...

Let X_t be an Îto process given by $dX_t = u(t,x)\cdot dt + v(t,x)\cdot dB_t$. Let $Y_t = g(t, X_t)$ be a twice continuously differentiable transformation of X_t. Then Y_t is also an Ito process and

$$dY_t = \left(\frac{dg(t,x)}{dx}u(t,x) + \frac{dg(t,x)}{dt} + \frac{1}{2}\frac{d^2g(t,x)}{dx^2}v^2(t,x)\right)\cdot dt + v(t,x)\cdot dB_t$$

After applying this lemma, the log-transformation $G(P)$ yields the following solution to Equation (3)...

$$\ln\left(\frac{P_T}{P_0}\right) = \int_0^T\left(\mu(t) - \frac{1}{2}\sigma^2(t)\right)dt + \int_0^T \sigma(t)dB_t \tag{5}$$

This model is called geometric Brownian motion and is frequently used in financial models: a famous example being the Black-Scholes option pricing formula. How do we interpret this result in a loss development context? The left-hand side of the equation may be interpreted as the log link-ratio between two development ages. The log link-ratio is equal to a fixed component given by the integral of $\mu(t) - \frac{1}{2}\sigma^2(t)$ over time, and a random component given by the integral of $\sigma(t)$ over the random noise process. Although not required in theory, the fixed integral $\int_0^\infty\left(\mu(t) - \frac{1}{2}\sigma^2(t)\right)dt$ should generally be finite to ensure a finite ultimate loss.

To understand the random component, we must first understand the basic behaviors of Brownian motion. Brownian motion is a continuous-time random walk process. Conceptually, this is a process that generates Normal random increments for each time increment dt and sums these increments over time. When a function such as $\sigma(t)$ is integrated over a Brownian motion path, we have what is called an Îto integral. Îto integrals have two basic, statistical properties that we will use to understand Equation (5)[5]...

---

[4] For a complete discussion of Ito's lemma see Øksendal, Stochastic Differential Equations, Chapter 4.

[5] These properties only hold for "nice" functions $\sigma(t)$. For a complete discussion of Brownian Motion and its relationship to Îto Integrals see Øksendal, Stochastic Differential Equations, Chapter 3.

$$E\left[\left(\int \sigma(t)dB_t\right)\right] = 0 \quad (6)$$

$$E\left[\left(\int \sigma(t)dB_t\right)^2\right] = \int \sigma(t)^2 dt$$

From these properties we can show that the random noise process is Normal distributed, has an expected value of zero, and a variance of $\int \sigma^2(t)dt$.[6] This yields the following distribution model for Equation (5)...

$$\left(\frac{P_{t_2}}{P_{t_1}}\right) \sim Lognormal\left(\int_{t_1}^{t_2}\left(\mu(t) - \frac{1}{2}\sigma^2(t)\right)dt, \sqrt{\int_{t_1}^{t_2}\sigma^2(t)dt}\right) \quad (7)$$

In other words, the link-ratios between any two ages are Lognormal distributed with the distribution parameters indicated in Equation (7). Using the results of Hayne, this also implies that the paid loss development between any two ages is also Lognormal distributed. A benefit to this approach is that once the model has been fit, development factors for any time interval may be computed regardless of the increment in the underlying data.

Applying the Model

The primary steps in applying the random walk model are verifying that observed age-to-age factors are independently, identically, Lognormal distributed; identifying appropriate functions for $\mu(t)$ and $\sigma^2(t)$; and estimating the parameters for those functions. Paid loss development data representative of non-standard, personal auto, bodily injury liability coverage is used to demonstrate the application of this model.

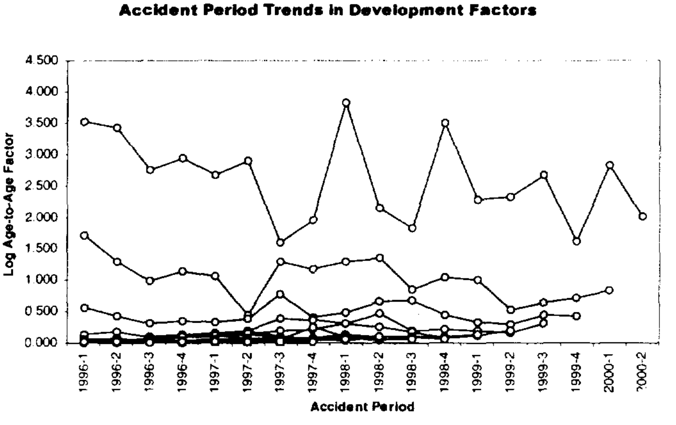
Data Diagnostics - Testing Model Assumptions

This section tests whether the data satisfies the assumptions underlying the random walk model. This is done using the raw data and prior to any model selection or fitting. Note that a violation of the model assumptions does not necessarily imply that the subsequent model fit will be poor. Rather, a violation of the model assumptions means that any statistical tests based upon the model results are biased. The magnitude of that bias depends upon the seriousness of the violation.

The data are shown in Exhibit 1. This data has not been adjusted for any changes in reporting, claim handling, inflation, etc. so the first step is to verify that the age-to-age factors do not show any significant accident year trends. (*i.e.* that within each development age, the age-to-age factors are independently, identically distributed.) This is shown in Figure 1 below...

[6] For the interested reader, this entire derivation is presented in detail in Pliska, Mathematics of Derivative Securities, Chapter 1.

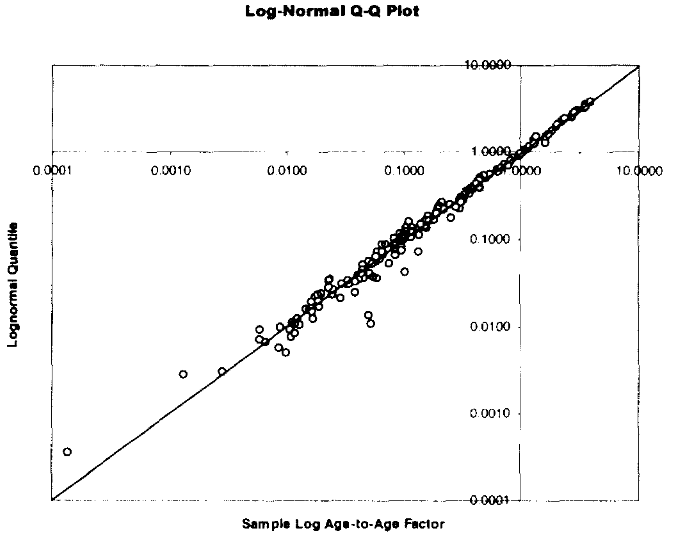
Figure 1



Each line on this plot is the observed log age-to-age factor for a common development age. Although the early development periods (largest development factors) exhibit a slight downward trend in the first few accident periods, this is insignificant given the large, random fluctuations observed in later periods. Accordingly, we can reasonably assume that the development factors at each age are independent. Note, however, that these uncorrected trends will increase the volatility of projections made at early development ages. If these trends could be removed through "data-leveling", the precision of the ultimate loss projections could be greatly improved.

A Q-Q plot was used to verify that the age-to-age factors at each age are Lognormal distributed. This is shown in Figure 2 below...

Figure 2



This plot shows the sample log age-to-age factor and the theoretical sample quantile under the Lognormal distribution; a perfect distribution fit yields a straight line. Although this plot obscures the fit for individual development ages, we can readily see that the Lognormal assumption is quite reasonable. At later development ages (lower, left corner), however, the Lognormal assumption is generally poorer. There are several reasons for this...

- At later ages, the small number of observations makes the data less stable.
- For small samples, the sample quantile is a poor measure of the underlying distribution quantile.
- At later ages, the actual likelihood of favorable development arising from salvage and subrogation recoveries is smaller than predicted by a Lognormal model.

The last point will be particularly important when computing reserve estimates; at later development ages, the lower prediction limit for the required reserve may be negative. In other words, the model recognizes that favorable development could reduce the ultimate loss below the current paid loss. This behavior is probably inconsistent with most lines of business. Fortunately, however, the lower limit is not typically of concern when evaluating reserve estimates.

Curve Family Selection

The next step in the modeling process is to select appropriate families of curves for $\mu(t)$ and $\sigma^2(t)$. This is a non-trivial task: polynomial functions will generally not be appropriate and, consequently, standard sequential model selection techniques cannot be used. The following procedure is presented as a practical approach for streamlining the model selection process. Of course other more theoretically accurate, and computationally more difficult, approaches are possible.

For this data, both $\mu(t)$ and $\sigma^2(t)$ have the same restrictions imposed upon them: they must be positive, decreasing functions that tend to zero over time. This is shown graphically in Figure 3 below. These types of functions are generically referred to as "tail-functions". In this example, three classes of tail function were considered. These functions were...

$$\alpha \cdot e^{-\left(\frac{t}{\beta}\right)^{\gamma}} \tag{8.1}$$

$$\alpha\left[1+\gamma\left(\frac{t}{\beta}\right)\right]^{-\frac{1}{\gamma}} \tag{8.2}$$

$$\alpha \cdot t^{-\beta}+\gamma \tag{8.3}$$

In this example, these specific functions were selected because they encompass a wide range of tail decay rates. In practice, a varied catalogue of tail functions may be obtained by scaling the survival function of various statistical distributions.[7] The tail-functions given above correspond to the scaled tail functions for the Weibull, Generalized Extreme Value, and Power distributions respectively. Also in order, these functions vary from lightest to heaviest tailed. Selecting the most appropriate curve form is complicated by the fact that we cannot directly observe the rate functions $\mu(t)$ and $\sigma^2(t)$. Rather, we can only observe the integrated values of these functions (*i.e.* the log age-to-age factors) as shown by the integrals on the right side of Equation (5). Furthermore, both the rate functions and the resulting log age-to-age factors vary by orders of magnitude. These complications, however, were exploited to develop a model selection procedure.

First, least-squares estimation was used to estimate the parameters of each curve form by fitting each curve's integral to the mean and variance of the observed log age-to-age factors. Typically, the least-squares approach would be inappropriate for this data because the fitted values vary by several orders of magnitude; the least-squares approach fits parameters to the largest values and ignores the smallest values. This characteristic, however, was used to justify the curve family selection. A curve that is fitted to the largest values and coincidentally fits the smallest values, too, is probably capturing the true underlying relationship in the data. By placing the empirical and fitted log age-to-age factors on a log-plot, the curves may be evaluated at both the

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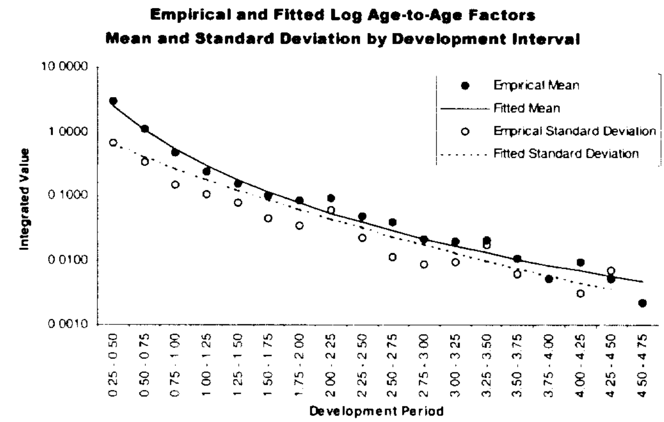
[7] A concise reference for statistical distributions, distribution functions, transformations, etc. is Evans *et al*, Statistical Distributions.

largest and smallest values. This is shown in Exhibit 2. Here the Generalized Extreme Value tail function generally provides the best overall fit for both $\mu(t)$ and $\sigma^2(t)$. In general, however, the same tail function need not be selected for both components. The final parameterization of these curves is shown in Figure 3 below...

Figure 3

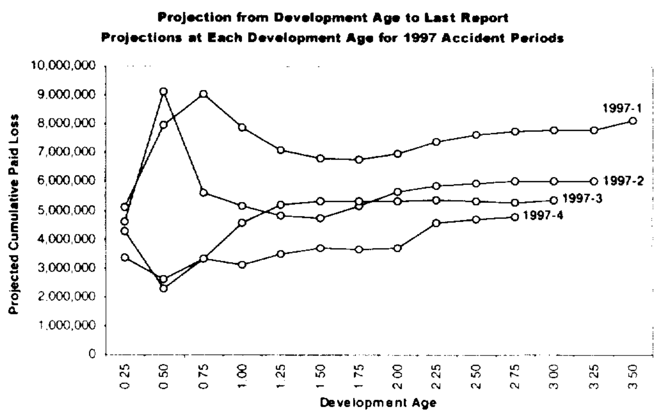


Parameter Estimation

The least-squares parameters used to select the tail functions are not the parameters for paid loss projection; rather maximum likelihood estimation was used to select the parameters for the $\mu(t)$ and $\sigma^2(t)$ tail functions. The maximum likelihood estimation procedure allows the model to be tuned for long-term projections.

With the case study data in triangular form, we can use the model to project the paid losses from each development age to the last reported value (*i.e.* the last diagonal in the development triangle). We can then use the observed value, the projected value, and the projection distribution given by Equation (7) to compute a likelihood statistic for every such projection. The final model parameters, then, are selected to maximize the overall likelihood that the observed losses could be generated by the modeled distribution. The maximum likelihood estimation procedure and the resulting projections are summarized in Exhibit 3 and in Figure 4 below...

Figure 4



In this plot, the losses at each development age are projected to the last diagonal of the development triangle. Each line on the plot shows these projected values for a single accident period. If the model made perfect projections at each development age, this plot would consist of horizontal lines. In reality, however, early projections are relatively inaccurate but quickly converge within a few periods.

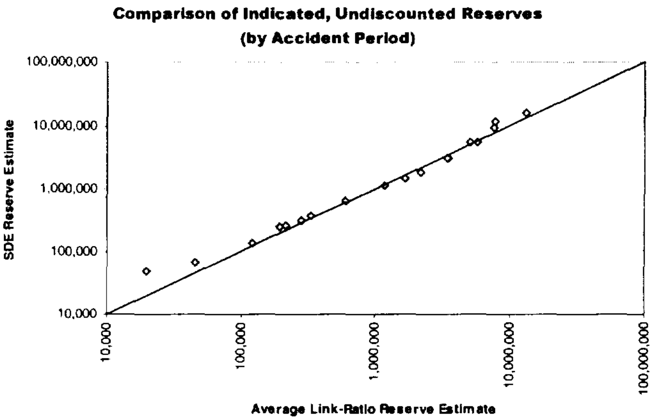
In this application, maximum-likelihood and least-squares estimation differ in one key respect. Least-squares estimation seeks to minimize the volatility of the left-hand side of Figure 4 where the development factors are largest. This creates a large potential for overfitting if there is significant noise in this immature data. Maximum-likelihood estimation does not seek to minimize this volatility *per se*. Rather, maximum-likelihood seeks to ensure that the volatility conforms to an assumed distribution. To the extent that the assumed distribution model is correct, maximum-likelihood will also minimize volatility in the same fashion as least-squares estimation. If the assumed model is incorrect, however, the volatility will be increased due to the bias arising from the model mis-specification.

The parameter estimation technique presented here was chosen for its tractability rather than its statistical properties. In fact, the parameters produced by this procedure will be neither unbiased nor minimum variance. More sophisticated estimation techniques incorporating censored data analysis would rectify these issues.

Model Results

By subtracting the paid-to-date losses from the projected ultimate losses, we have the indicated reserve. A first test for the model is that the expected reserves should be consistent with the reserves indicated by traditional actuarial analysis. These results are shown in Exhibit 4 and in Figure 5 below...

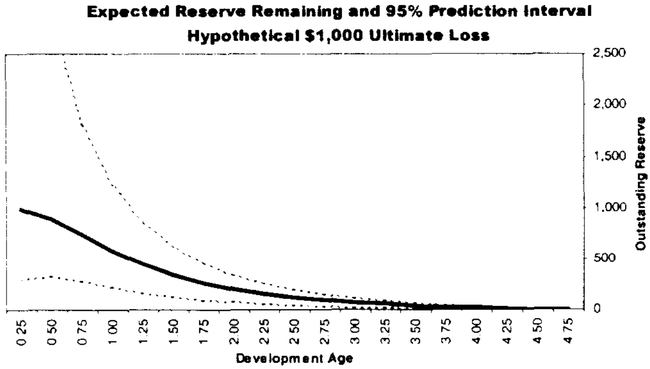
Figure 5



As expected the reserves indicated by traditional and SDE projection methods are similar. Although not readily apparent on the log-log plot above, the largest dollar deviation between the two methods occurs in the largest, least mature reserves. These deviations are consistent with the volatility component of the SDE model. Under the SDE model, large fluctuations are likely during immature development periods. Furthermore, due to the skewness of the Lognormal distribution, these are likely to be large upward fluctuations. This also results in large prediction intervals for the least mature reserves. This is the same effect that C.K. Khury modeled using an arbitrary reserve radius G-function.[8] This is depicted in Figure 6 below...

[8] C.K. Khury, Loss Reserves: Performance Standards, 1980 Proceedings of the Casualty Actuarial Society, Volume LXVII

Figure 6



Here the ultimate loss is $1000 but at the time the reserves are set, this amount is unknown. We can, however, use the model to estimate the probable range of required reserves at each development age. In the plot, this is shown as an expected reserve that declines as losses are paid out, and a prediction interval that contracts as the ultimate loss becomes more certain.

Finally, having a distribution for the required reserve allows calculation of the expected value of future adverse or favorable deviation from the selected reserve amount. The values are computed as tail expected values in the same manner as an excess pure premium or deductible savings is computed. In statistical terms...[9]

$$\text{Favorable Development} = E\left[R_{carried} - R_{req'd} \mid R_{carried} \geq R_{req'd}\right] \cdot P\left[R_{carried} \geq R_{req'd}\right] \quad (9.1)$$

$$\text{Adverse Development} = E\left[R_{req'd} - R_{carried} \mid R_{carried} \leq R_{req'd}\right] \cdot P\left[R_{carried} \leq R_{req'd}\right] \quad (9.2)$$

These results are shown in Exhibit 4 on an undiscounted basis assuming that the carried reserve is set at the average link-ratio reserve.

Discounted Reserves

A small modification to Equation (3) allows similar treatment of discounted (present value) reserves. To motivate this treatment, consider a continuous annuity that pays benefits at a varying rate b_t and force of interest δ...[10]

$$\overline{a}_x = \int e^{-\delta t} \cdot b_t \cdot dt$$
$$d\overline{a}_x = e^{-\delta t} \cdot b_t \cdot dt$$

Discounted loss reserves may be treated analogously if we treat the incremental loss development $\frac{dP}{dt}$ as the "benefit". This is given by...

$$dV = e^{-\delta(t-t_0)} \cdot \frac{dP}{dt} \cdot dt$$
$$dV = e^{-\delta(t-t_0)} \cdot dP \qquad (10)$$
$$dV = \left(\mu(t) - \frac{1}{2}\sigma^2(t)\right)e^{-\delta(t-t_0)}Pdt + \sigma(t)e^{-\delta(t-t_0)}PdB_t$$

...where V is the present value loss reserve and δ is the force of interest used for discounting. Unfortunately, however, this expression does not lend itself to explicit solution in the same manner as Equation (3). Instead, numerical methods must be employed to compute the distribution of present value reserves. These methods can be somewhat difficult to implement.[11] To continue the example from above, the expected present value reserve and reserve volatility computed from Equation (10) are shown in Exhibit 4 and in Figure 7 below...

Figure 7

Implicit Margin in Average Link-Ratio Reserves
(Losses Discounted at 7.0% *per annum* Continuous Compounding)

Accident Period	Average Link-Ratio Reserve (Undiscounted)	Expected Discounted SDE Reserve	Expected Standard Deviation of Discounted SDE Reserve	Expected Margin in Average Link-Ratio Reserve
1996-1	0			
1996-2	19,948	48,252	32,262	-28,304
1996-3	45,365	62,719	30,563	-17,354
1996-4	122,715	128,511	52,979	-5,796
1997-1	194,942	229,769	85,414	-34,828
1997-2	217,319	237,904	82,712	-20,585
1997-3	286,525	281,997	93,894	4,527
1997-4	335,073	338,204	109,595	-3,131
1998-1	611,160	601,721	191,808	9,439
1998-2	1,183,357	1,024,669	323,292	158,688
1998-3	1,666,092	1,362,136	426,181	303,956
1998-4	2,210,746	1,700,183	526,117	510,563
1999-1	3,511,724	2,802,555	851,006	709,169
1999-2	3,426,796	2,805,902	824,190	620,894
1999-3	5,729,009	4,984,688	1,385,324	744,321
1999-4	5,078,453	5,146,741	1,311,692	-68,288
2000-1	7,739,817	8,782,127	1,978,193	-1,042,310
2000-2	7,914,469	11,252,088	2,196,198	-3,337,618
2000-3	13,337,789	14,916,718	2,649,990	-1,578,929
	53,631,298	56,706,885		-3,075,587

[10] Bowers *et al*, Actuarial Mathematics, Chapter 5

[11] For more information on numerical solutions to stochastic integrals see Tavella and Randall, Pricing Financial Instruments.

As this figure makes clear the overall margin is negative, and the positive reserve margins are quite small compared to the volatility of the underlying reserve estimates. Accordingly, there is little practical margin in the average link ratio reserves. This is due largely to the inherent characteristics of the business presented in this example...

- The extreme growth at early development ages makes early reserve estimates highly volatile.
- There is little development at later ages. This decreases the duration of immature reserves and consequently, the magnitude of the implicit margin in the undiscounted reserves.
- Similarly, the magnitude of the discount margin tends to be small at later ages because the indicated reserves are themselves small.

Lines of business characterized by protracted development with significant payments throughout the life of the reserve should contain larger implicit margins.

Conclusions

The model presented here unites common actuarial practice with a basic financial model, and provides concrete justification for the utility of link-ratio techniques. As presented however, this model is relatively crude and there are several areas for enhancement and further research.

- Parameter estimation techniques with more statistically desirable properties (*e.g* unbiased, minimum variance, etc.) should be employed.
- The model treats each accident period separately. Ito's lemma, however, is easily extended to multiple dimensions. This would allow joint modeling of each accident period in the reserve, etc. Significant research, however, would be required to understand the correlation structure between accident periods.
- The model can only be applied to positive, non-zero paid losses. This issue cannot easily be addressed within the geometric Brownian motion framework. For lines with a significant payment lag, additive Brownian motion or Poisson jump (frequency-severity) process may be a more appropriate model.
- Adjusting the model for report lag, calendar-year effects, and other sources of volatility could significantly enhance the precision of reserve estimates made at early development ages.
- Under the geometric Brownian motion model, all random deviations persist. In other words, an increase in the loss payment rate is always due to adverse deviation, never to accelerated claim payment. There are other stochastic differential equations that can accommodate claim payment volatility.
- Having a distribution for the ultimate loss allows common derivative security pricing techniques to be applied to loss portfolio transfers, commutations, and reserve margin securitization. This is an important area for further research if traditional insurance is to remain competitive with the capital markets.

Exhibit 1

Non-Standard Auto Bodily Injury Liability

Cumulative Paid Losses

Development Age

Accident Period	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1996-1	10,996	369,493	2,044,411	3,545,999	4,066,094	4,346,452	4,449,499	4,660,135
1996-2	30,379	923,577	3,358,105	5,149,806	6,065,390	6,434,770	6,940,143	7,217,139
1996-3	63,239	988,450	2,615,749	3,528,112	3,909,767	4,109,248	4,187,078	4,551,604
1996-4	39,093	744,327	2,315,143	3,263,545	3,733,518	4,206,960	4,674,586	5,260,543
1997-1	65,664	942,312	2,713,647	3,734,512	4,354,517	4,891,834	5,384,858	5,910,456
1997-2	59,937	1,091,178	1,699,740	2,473,150	2,992,361	3,446,278	4,138,586	4,846,277
1997-3	56,224	279,527	1,018,077	2,212,843	3,258,599	3,912,537	4,318,166	4,615,991
1997-4	45,392	321,463	1,038,532	1,547,997	2,229,066	2,779,562	3,028,933	3,290,122
1998-1	8,363	387,219	1,398,819	2,267,289	3,059,865	4,156,829	4,772,668	5,265,137
1998-2	47,153	409,959	1,580,925	3,031,628	4,764,035	6,120,361	6,790,407	7,108,747
1998-3	158,143	985,728	2,306,657	4,532,235	5,545,023	6,547,284	7,144,524	7,791,978
1998-4	32,625	1,067,637	3,062,441	4,810,670	5,935,970	6,373,358	7,060,202	7,512,141
1999-1	151,425	1,492,994	4,048,201	5,822,294	6,830,335	7,669,164	8,818,850	
1999-2	169,819	1,937,953	3,227,429	4,280,763	4,987,630	6,027,712		
1999-3	118,032	1,719,811	3,245,182	5,032,184	6,850,662			
1999-4	256,006	1,297,036	2,865,528	4,070,197				
2000-1	85,211	1,442,047	3,297,787					
2000-2	175,762	1,330,312						
2000-3	180,400							

Accident Period	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	Average Link-Ratio Ultimate Loss
1996-1	4,946,500	5,031,364	5,207,964	5,272,084	5,479,258	5,559,845	5,575,454	5,581,591	5,842,002	5,842,787	5,655,215	5,655,215
1996-2	7,710,346	7,937,009	8,272,704	8,558,125	8,718,245	8,742,750	8,800,511	8,852,858	8,953,325	9,042,539		9,082,488
1996-3	4,682,945	4,878,102	5,097,885	5,196,030	5,264,130	5,309,388	5,372,076	5,379,097	5,410,513			5,455,878
1996-4	5,779,476	6,082,782	6,214,310	6,269,460	6,414,850	6,564,550	6,673,776	6,753,290				6,876,005
1997-1	6,592,840	7,039,673	7,358,421	7,541,425	7,667,061	8,072,516	8,204,816					8,399,758
1997-2	5,268,678	5,539,482	5,773,741	5,885,018	5,978,090	6,098,299						6,315,618
1997-3	4,880,118	5,044,104	5,133,113	5,287,851	5,345,448							5,631,970
1997-4	4,239,955	4,492,564	4,692,742	4,813,807								5,148,680
1998-1	5,850,179	6,122,612	6,465,182									7,076,342
1998-2	7,535,256	8,290,525										9,473,882
1998-3	8,217,594											9,883,887
1998-4												9,722,887
1999-1												12,330,574
1999-2												9,454,508
1999-3												12,579,871
1999-4												9,148,650
2000-1												11,037,604
2000-2												9,244,782
2000-3												13,516,188

Log Cumulative Age-to-Age Factors

Development Interval

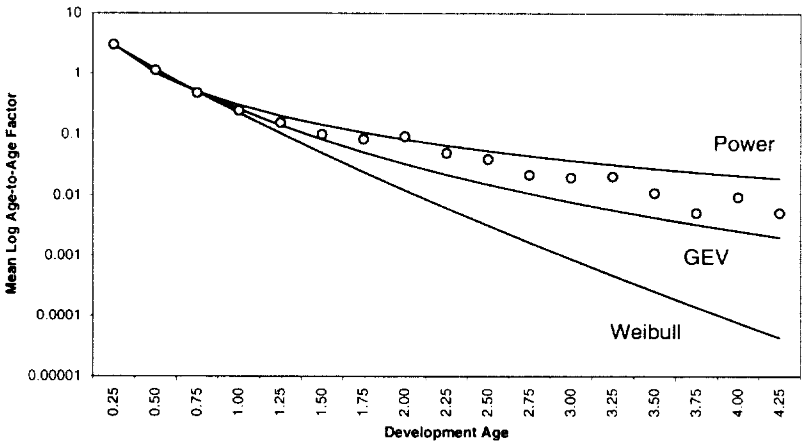
Accident Period	0.25 - 0.50	0.50 - 0.75	0.75 - 1.00	1.00 - 1.25	1.25 - 1.50	1.50 - 1.75	1.75 - 2.00	2.00 - 2.25	2.25 - 2.50	2.50 - 2.75	2.75 - 3.00	3.00 - 3.25	3.25 - 3.50	3.50 - 3.75	3.75 - 4.00	4.00 - 4.25	4.25 - 4.50	4.50 - 4.75
1996-1	3.515	1.711	0.551	0.137	0.067	0.023	0.046	0.060	0.017	0.034	0.012	0.039	0.015	0.003	0.001	0.011	0.000	0.002
1996-2	3.414	1.291	0.428	0.164	0.059	0.076	0.039	0.066	0.029	0.041	0.034	0.019	0.003	0.007	0.006	0.011	0.010	
1996-3	2.749	0.973	0.299	0.103	0.050	0.019	0.083	0.028	0.040	0.044	0.019	0.013	0.009	0.012	0.001	0.006		
1996-4	2.947	1.135	0.343	0.135	0.119	0.105	0.118	0.094	0.048	0.025	0.009	0.023	0.023	0.017	0.012			
1997-1	2.684	1.058	0.319	0.154	0.118	0.096	0.093	0.109	0.066	0.044	0.025	0.017	0.052	0.016				
1997-2	2.902	0.443	0.375	0.191	0.141	0.183	0.158	0.084	0.050	0.041	0.019	0.016	0.020					
1997-3	1.604	1.293	0.778	0.387	0.183	0.099	0.067	0.056	0.033	0.017	0.030	0.011						
1997-4	1.958	1.173	0.399	0.365	0.221	0.086	0.083	0.254	0.058	0.044	0.025							
1998-1	3.835	1.284	0.483	0.300	0.306	0.138	0.098	0.105	0.046	0.054								
1998-2	2.163	1.350	0.651	0.452	0.251	0.104	0.046	0.058	0.096									
1998-3	1.830	0.850	0.675	0.202	0.166	0.087	0.087	0.053										
1998-4	3.507	1.035	0.452	0.210	0.071	0.102	0.062											
1999-1	2.288	0.997	0.329	0.195	0.116	0.140												
1999-2	2.323	0.510	0.282	0.153	0.189													
1999-3	2.679	0.635	0.439	0.309														
1999-4	1.823	0.720	0.423															
2000-1	2.829	0.827																
2000-2	2.024																	
2000-3																		

Exhibit 2

Potential Curve Families for Rate Functions

Least-Squares Fit to Observed Log Age-to-Age Factors

Observed and Fitted Mean Log Age-to-Age Factors



Observed and Fitted Log Age-to-Age Factor Standard Deviations

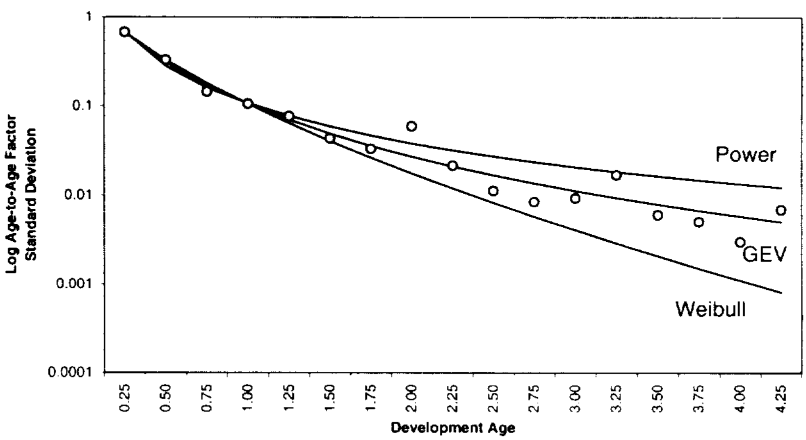


Exhibit 3

Results of Maximum-Liklihood Parameter Estimation

Generalized Extreme Value Tail Function Parameters (for Equation 8.2)
Maximum Likelihood Estimates

	$\mu(t)$	$\sigma^2(t)$
α	58 2410	4 0810
β	0 1550	0 2730
γ	0 2848	0 0678

Fitted Log Age-to-Age Factors (from Integration of Equation 8.2 Over Indicated Time Interval)

Development Interval	0 25 - 0 50	0 50 - 0 75	0.75 - 1 00	1 00 - 1.25	1.25 - 1 50	1 50 - 1.75	1 75 - 2 00	2.00 - 2 25	2.25 - 2.50	2 50 - 2 75	2 75 - 3.00	3 00 - 3.25	3.25 - 3 50	3 50 - 3 75	3 75 - 4 00	4.00 - 4.25	4 25 - 4.50	4.50 - 4 75
Fitted Mean	2 3489	1 0087	0.5104	0 2891	0.1767	0 1148	0 0782	0.0553	0 0403	0.0302	0 0231	0.0180	0.0143	0 0115	0 0094	0.0077	0 0054	0.0054
Fitted Variance	0 2830	0 1249	0.0575	0 0276	0.0137	0 0070	0.0037	0.0020	0 0011	0.0006	0 0004	0 0002	0 0001	0 0001	0.0000	0.0000	0 0000	0.0000

Negative Log-Likelihood Statistics (from Equation 7) for Projection from Development Age to Last Report

Accident Period	Development Age 0.25	0.50	0.75	1 00	1.25	1.50	1 75	2.00	2 25	2.50	2.75	3.00	3.25	3 50	3 75	4 00	4.25	4 50	4.75
1996-1	2.7328	0 4295	0.4443	0 9926	0.3253	-0 4234	-1 2578	-1.6810	-1 8484	-2 2923	-2 4840	-2.8366	-2 2755	-1.9434	-2.6751	-3 5892	-2.8833	-4 3170	
1996-2	1.4637	0.2301	0.5166	0 3426	-0.2511	-1 0140	-1 4209	-1.7582	-2.0495	-2 2928	-2 5900	-2.7116	-2 8560	-3.3861	-3 6166	-3 5307	-4.1843		
1996-3	0.6731	1.1926	1.6963	1 1996	0.2659	-0 6382	-1 4151	-1.6214	-2.0340	-2 2955	-2.2578	-2.4605	-2 7409	-3.1549	-3 1799	-4 2117			
1996-4	0.7653	0 2672	0.2301	-0 3587	-0.8435	-0 9709	-1 0148	-1.5257	-2.0447	-2 3404	-2.5931	-2.4383	-2 7027	-3.3427	-3.9755				
1997-1	0.6052	0 2925	0.1197	-0 4802	-0.7647	-0 6063	-0 1119	0 2347	-0.7680	-1 2579	-1.5438	-1.0425	0 8777	-3.6581					
1997-2	0.6008	1 0559	-0.1433	-0.4831	-0.4046	0 0968	-0.4779	-1.5232	-2.0021	-2.3463	-2.7037	-3.0492	-3 4335						
1997-3	0.6122	0 9721	0.2885	-0.4907	-0.6252	-1 1031	-1 4306	-1 7547	-2.0214	-2.3646	-2.6047	-3.1802							
1997-4	0.5936	0 5042	0.0362	0 4299	0.2466	0.2617	2 0606	4 6896	-1.8996	-2.4125	-3.0300								
1998-1	4.4221	0 7509	0.0756	0 0971	0.2662	-0.6767	-0 8655	-1 0308	-2.0085	-2.2995									
1998-2	0.8801	1 3322	0.5104	0 0442	-0.7341	-1.2131	-1.5261	-1.4234	-1.1127										
1998-3	0.9753	0.2247	-0 1680	-0.4089	-0.8787	-1.2458	-1.6604	-2 1861											
1998-4	1.3941	0.3221	0 1506	-0.1227	-0 5684	-1 3108	-1.8455												
1999-1	0 7295	0.4126	0 2556	-0.4196	-0.9879	-1 5164													
1999-2	1 3357	1.7655	0 3869	-0.4893	-1.2201														
1999-3	0.5747	0.5707	-0 2966	-0 8686															
1999-4	1.8405	0 4546	-0.4428																
2000-1	0.5796	0 0106																	
2000-2	0.4742																		
2000-3																			

Overall Negative Log-Likelihood -142 7721

NOTES

(1) Given a set of parameters and using Equation 7, we can compute the negative log-likelihood of the observed development factors. Above, the likelihoods for the development from each age to the last reported value (i.e. the last diagonal in the development triangle) are tabulated. The values of μ and σ entering Equation 7 are generated by integrating Equation 8.2 over the appropriate time period.

(2) The parameters for Equation 8.2 were generated by maximizing the likelihood (minimizing the negative log-likelihood) of the observed development factors.

Exhibit 4

Indicated Reserves

Discounted and Undiscounted Basis

Undiscounted Reserves (from Equation 7)

Accident Period	Last Recorded Loss	Average Link-Ratio Reserve	95% Lower Prediction Interval	SDE Expected Reserve	95% Upper Prediction Interval	(1) Expected Favorable Deviation	(2) Expected Adverse Deviation
1996-1	5,655,215	0					
1996-2	9,042,539	19,948	-15,082	48,783	113,218	7,578	-36,413
1996-3	5,410,513	45,365	2,972	64,126	126,153	7,344	-26,105
1996-4	6,753,290	122,715	25,719	132,768	241,957	17,640	-27,693
1997-1	8,204,816	194,942	65,692	239,938	418,837	21,902	-66,898
1997-2	6,098,299	217,319	80,566	250,750	426,885	23,272	-56,703
1997-3	5,345,446	286,525	104,864	299,617	503,224	34,374	-47,467
1997-4	4,813,607	335,073	132,621	361,776	604,475	36,570	-63,273
1998-1	6,465,182	611,160	242,665	647,186	1,082,941	68,850	-104,877
1998-2	8,290,525	1,183,357	418,009	1,106,584	1,865,212	189,147	-112,374
1998-3	8,217,594	1,666,092	556,069	1,474,769	2,518,086	317,999	-126,676
1998-4	7,512,141	2,210,746	690,446	1,842,303	3,205,692	508,489	-140,046
1999-1	8,818,850	3,511,724	1,130,708	3,033,491	5,418,658	731,261	-253,028
1999-2	6,027,712	3,426,796	1,128,044	3,026,927	5,607,256	675,099	-275,230
1999-3	6,850,862	5,729,009	2,008,387	5,346,004	10,436,596	985,589	-602,584
1999-4	4,070,197	5,078,453	2,085,288	5,472,294	11,586,239	650,237	-1,044,077
2000-1	3,297,787	7,739,817	3,533,966	9,217,365	22,403,695	901,234	-2,378,782
2000-2	1,330,312	7,914,469	4,290,492	11,674,815	36,888,928	783,212	-4,543,557
2000-3	180,400	13,337,789	4,870,474	15,853,918	85,620,182	2,020,744	-4,536,874
		53,631,298		60,093,412		7,980,541	-14,442,656

Discounted Reserves and Implicit Margin in Average Link-Ratio Reserve (from Numerical Solution of Equation 10)

Accident Period	Last Recorded Loss	Average Link-Ratio Reserve (Undiscounted)	(3) SDE Expected Discounted Reserve	Expected Margin in Average Link-Ratio Reserve
1996-1	5,655,215	0		
1996-2	9,042,539	19,948	48,252	-28,304
1996-3	5,410,513	45,365	62,719	-17,354
1996-4	6,753,290	122,715	128,511	-5,796
1997-1	8,204,816	194,942	229,769	-34,828
1997-2	6,098,299	217,319	237,904	-20,585
1997-3	5,345,446	286,525	281,997	4,527
1997-4	4,813,607	335,073	338,204	-3,131
1998-1	6,465,182	611,160	601,721	9,439
1998-2	8,290,525	1,183,357	1,024,669	158,688
1998-3	8,217,594	1,666,092	1,362,136	303,956
1998-4	7,512,141	2,210,746	1,700,183	510,563
1999-1	8,818,850	3,511,724	2,802,555	709,169
1999-2	6,027,712	3,426,796	2,805,902	620,894
1999-3	6,850,862	5,729,009	4,984,688	744,321
1999-4	4,070,197	5,078,453	5,146,741	-68,288
2000-1	3,297,787	7,739,817	8,782,127	-1,042,310
2000-2	1,330,312	7,914,469	11,252,088	-3,337,618
2000-3	180,400	13,337,789	14,916,718	-1,578,929
		53,631,298	56,706,885	-3,075,587

NOTES

(1) Measures expected favorable development on Average Link-Ratio reserve amount
Similar to a loss elimination value P[Carried Reserve>Required Reserve] * E[(Carried Reserve-Required Reserve)| Carried Reserve>Required Reserve]

(2) Measures expected adverse development on Average Link-Ratio reserve amount
Similar to an excess pure premium. P[Carried Reserve<Required Reserve] * E[(Carried Reserve-Required Reserve)| Carried Reserve<Required Reserve)

(3) Paid losses discounted at 7.0% continuous compounding

Reserving for Financial Guaranty Products

Michael B. McKnight, ACAS, MAAA

Reserving for Financial Guaranty Products

By: Michael McKnight, ACAS, MAAA

Abstract:

This paper provides an overview of the types of financial guaranty products and current market characteristics. It also explores the basics and alternatives of developing reserving procedures for financial guaranty insurance products.

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CAS papers on financial guaranty actuarial methods, either pricing or reserving, are conspicuous by their absence. This lack of published research can be partly explained by the fact that it is a relatively new coverage. Most agree that financial guaranty insurance really began with the coverage of municipal bond obligations. The first such policy was written in 1971 and covered a general obligation bond issued by the city of Juneau, Alaska. Until 1985, financial guaranty information was reported under the surety line of business in the statutory statement. Up until that time, financial guaranty was almost exclusively limited to the municipal bond market. As late as 1998, municipal bonds still accounted for 80% of the premiums for monoline writers; however, there has been a recent explosion in the types of financial products insured by both monoline and multiline insurers.

Before beginning a discussion of the reserving practices of financial guaranty insurers, it is helpful to provide a description of the types of products that fall under this heading. In understanding the types of products, a history of the coverage and current market conditions, the reader will be better prepared to appreciate the various reserving techniques.

What Is / Is Not Financial Guaranty

The National Association of Insurance Commissioners' ("NAIC") Financial Guaranty Insurance Model Act gives the following definition:

"Financial guaranty insurance" means a surety bond, insurance policy or, when issued by an insurer, an indemnity contract and any guaranty similar to the foregoing types, under which loss is payable upon proof of occurrence of financial loss to an insured claimant, obligee or indemnitee as a result of any of the following events:

> (a) failure of any obligor on any debt instrument or other monetary obligation (including common or preferred stock guarantied under a surety bond, insurance policy or indemnity contract) to pay when due principal, interest, premium, dividend or purchase price of or on such instrument or obligation, when such failure is the result of a financial default or insolvency, regardless of whether such obligation is incurred directly or as guarantor by or on behalf of another obligor that has also defaulted;

(b) changes in the levels of interest rates, whether short or long term, or the differential in interest rates between various markets or products;

(c) changes in the rate of exchange of currency;

(d) inconvertibility of one currency into another for any reason, or inability to withdraw funds held in a foreign country resulting from restrictions imposed by a governmental authority;

(e) changes in the value of specific assets or commodities, financial or commodity indices or price levels in general; or

(f) other events which the commissioner determines are substantially similar to any of the foregoing.

The Model Act goes on to list numerous examples of what is not financial guaranty insurance, including various types of bonds, credit insurance, guaranteed investment contracts issued by life insurers, residual value insurance and mortgage guaranty insurance. While these types of insurance are not financial guaranty in the eyes of the NAIC's Model Act, they may be considered financial guaranty in other situations.

Perhaps a more broad definition of the coverage would simply be an insurance contract that guarantees a cash (or cash equivalent) payment from a security, or stream of such payments, at specified points in time.

The NAIC's Model Act led to the creation of the "monoline" company. The NAIC's regulations require monoline companies to write only financial guaranty, surety and, in some states, credit insurance. Conversely, companies that do not write financial guaranty (as defined by the NAIC) are often referred to as "multilines". Some multilines will write various types of financial guaranty coverage. Furthermore, several multiline reinsurers provide protection to the monoline companies.

As previously noted, financial guaranty began with coverage of municipal bond obligations. If the municipality was not able or willing to meet either its principal or interest obligations, the insurance contract would respond in a timely manner. In this case, the insurance contract guarantees the payment of principal and interest at the specified redemption dates. There is no question of fault with a financial guaranty insurance policy – the contract responds just by the fact that the bondholders did not receive the cash payments. Of course, certain subrogation or collateral rights are transferred to the insurance company in the event of a claim.

Not all financial guaranty products are insured via a financial guaranty contract. Many of the multilines that write these types of coverage still do so with a more typical indemnification contract, which allows for the rights of reviewing and challenging claims.

Rationale for Financial Guaranty

In the case of an insured municipal bond, the benefits of financial guaranty insurance to the bondholder are obvious. The benefits to the issuer of the bond are not quite as immediately obvious, but no less real and include a) the fact that the bond is more "liquid", especially in the secondary markets, and b) it has a higher credit rating. It is this second feature that often leads to the use of the term "credit enhancement" when describing financial guaranty products.

The purpose of purchasing credit enhancement insurance is to improve the credit rating on issued debt. Generally, investors will accept lower yields on debt instruments with higher credit ratings. Let's consider "investment grade" bonds. Such bonds have been assigned one of the following credit ratings:

Investment Grade Rating Categories				
Standard & Poor's, Fitch	AAA	AA	A	BBB
Moody's	Aaa	Aa	A	Baa

Within each of these ratings is an implied rate of default. Based on prior experience, it is unlikely that there will be a default on any bonds rated as "investment grade".

Corporate Default Probabilities by Rating Classifications
Average Cumulative Default Rates (%)

Term (yrs):	1	2	3	4	5	7	10	15
AAA	0.00	0.00	0.03	0.06	0.10	0.26	0.51	0.51
AA	0.01	0.04	0.09	0.16	0.25	0.53	0.79	1.07
A	0.04	0.11	0.19	0.32	0.49	0.83	1.41	1.83
BBB	0.22	0.50	0.79	1.30	1.80	2.73	3.68	4.48
BB	0.98	2.97	5.35	7.44	9.22	12.27	15.00	16.36
B	5.30	11.28	15.88	19.10	21.44	24.77	27.88	29.96
CCC	21.94	29.25	34.37	38.24	42.13	44.40	46.53	48.29

Source: S&P CreditWeek, January 31, 2001

As this table indicates, the probability of default is low for all investment grades (i.e., BBB to AAA). However, the probability of default for bonds with a higher rating (e.g., AAA) is smaller than that for bonds with a lower rating (e.g., BBB). Depending on the type of industry and economic conditions, the difference in required yields between any two consecutive rating categories can be anywhere from 15 to 50 basis points ("bps") or more. This difference is known as the **yield spread**.

The yield spread is the additional interest required by investors to compensate for accepting default risk. Historically the yield spread has been more than just the difference in expected defaults; investors demand a premium for accepting this risk. The risk adjusted default probability is typically about three times the historical default probability.

Any corporation or municipality issuing debt would like to minimize the amount of yield required by investors. Moving from one rating category to the next highest has the potential for significant savings in interest rate payments. Credit enhancement improves the rating of a debt instrument by insuring (i.e., guaranteeing) the interest and principal payments. If the corporation or municipality is unable to make interest or principal payments, the financial guaranty insurer makes the payments. The financial guaranty insurer typically has a very high rating. By agreeing to guarantee a debt obligation, the insurer is essentially lending its own rating to the debt issuing corporation or municipality.

The Association of Financial Guaranty Insurors ("AFGI") estimates that bond insurance saved municipalities $3.7 billion in borrowing costs during 1998. This savings is simply the realized yield reduction less the cost of insurance. In turn, the yield reduction is the result of borrowing at the financial guaranty insurer's rating (e.g., AAA) instead of at the entities' own credit rating (e.g., BBB, A-).

Types of Products and Insurers

While insurance for municipal bond obligations has historically been the largest category of financial guaranty insurance, it is not the only category nor is it likely to continue its domination of the coverage. The types of financial products that have been protected by financial guaranty insurance can be broken down as follows:

- Municipalities
  - Revenue Bonds
  - General Obligation Bonds
- Collateralized Debt Obligations ("CDO")
  - Collateralized Bond Obligations ("CBO")
  - Collateralized Loan Obligations ("CLO")
    - Credit Card Receivables
    - Home Equity Loans
    - Automobile Loans
  - Collateralized Mortgage Obligations ("CMO")
- Corporate Debt
  - Corporate Bonds
  - Subordinated Debt
  - Credit Default Swaps
    - Stand-alone
    - Synthetic CLO
- Other
  - Leases
  - Portfolios of Unsecured Loans
  - Emerging Markets
  - Film Production Rights
  - Cruise Ship Construction

As a matter of background, asset backed securities ("ABS") are investments collateralized by loans or leases. For example, they could be a pool of car loans, student loans or equipment leases. An artificial distinction is made in the US capital markets between CMO's and ABS's. So technically, an ABS is an investment collateralized by assets that are not mortgage loans.

There has been a recent trend by the multilines to financially guarantee almost all asset risk categories in the capital markets. In many instances, a very risky asset (e.g., cruise ship construction or future film production receivables) is insured in some way and converted into investment grade bonds.

Monoline companies, on the other hand, typically underwrite to a zero loss ratio ("ZLR"). That is not to say that there are never losses, but the potential for loss is very low. Insured assets have a higher grade debt with minimal chance of default. The limits are typically very large and the premiums are low. With low premium and high potential exposure, monoline insurers must focus on debt instruments that are very solid. A single loss could potentially wipe out several years' worth of premium.

Structured debt products underwritten by the multilines differ from ZLR products only to the extent that losses have a higher probability of occurring. That is not to say that losses on any single insured are expected at the time of underwriting. There is simply a higher frequency associated with the structured debt product. Most of the applications of structured debt are identical to that of ZLR products; namely, increase the credit rating of a debt obligation. However, the structured debt products represent an exposure to loss not in line with the ZLR products and, hence, are not acceptable to many "pure" financial guaranty writers. Furthermore, the monoline insurers' own credit rating is contingent upon minimal exposure (i.e., less than 10% of premiums) from high yield or junk bonds. These writers have been known to participate on some structured debt programs at very high layers, known as **capacity layers**.

Beside bonds, there are other types of exposures associated with structured debt. A classic example is lease obligations. Let's suppose a large corporation owns and then leases out some type of large machinery or real estate. The corporation may like to guarantee the income stream from these leases. Such a program will typically be structured in various layers, or **tranches**, as shown in the following example:

- Equity
- Primary
- Mezzanine
- Capacity

The equity layer is the amount of risk often retained by the insured; in that respect it is similar to a deductible. For example, if we are looking at a portfolio of machinery leases, the insurance does not attach with the first late or defaulting lease payment. The insurance is typically designed to protect against a systematic economic failure in a particular industry. If the leases relate to commercial aircraft, the insurance would protect against a significant recession in the airline industry leading to cancelled leases. The loss of lease income from the failure of a small regional airline would probably be borne entirely by the insured.

Within the primary and mezzanine tranches, there can be several sub-dividing layers. For example, there may be Primary Layer I and Primary Layer II. While the capacity layer could be subdivided, in practice this is usually a very large amount of coverage attaching directly above the last mezzanine layer. As previously mentioned, traditional financial guaranty insurers seem to be more comfortable writing this layer. The lower layers are written by a combination of large commercial insurers and reinsurers.

This concept of layering or "tranching" asset backed securities is not limited to leases. In fact, it is a common feature of many transactions of this nature. Each tier has its own loss probabilities and, in fact, may have a different rating commensurate with the expected loss amounts.

The nomenclature used for identifying the tranches can be different from deal to deal. In some situations there has been an equity layer, a mezzanine layer and then a senior layer. In the most basic transactions, there have been just an equity tier and a senior tier. While in general the equity layer has typically not been insured, there is an increasing trend to insure at least a portion of this tranche.

Market Analysis

Perhaps the dearth of relevant actuarial papers on the subject can be explained by the relatively small size of the credit enhancement market and the few number of companies that dominate it. As previously noted, US companies that write financial guaranty are required by law to be **monoline insurers** (see the following section on regulations). That is to say, a US domiciled company that writes financial guaranty insurance on a direct basis cannot write other lines of business. Having said that, there are some US companies that report premiums for both financial guaranty and other types of insurance in their statutory statements. For example, both Travelers and Fireman's Fund show small amounts of direct financial guaranty premiums written (i.e., \$1 – 2 million) and yet have over \$2 billion of premiums written in other lines. However, the very large US writers of financial guaranty write no other types of business.

For the calendar year 2000, the total financial guaranty premium written by all US insurers is shown below

2000 Financial Guaranty Premiums ***All US Companies Combined***	
Direct Written	\$1.622 billion
Net Written	\$1.396 billion
Source: Thomson Financial Insurance Solutions, May 2001	

Of the \$1.622 billion in direct written premiums for financial guaranty, 94% is produced by only five groups of companies.

2000 Financial Guaranty Direct Premiums Written ***by Group (amounts in millions)***	
Municipal Bond Investors Assurance Company Group ("MBIA")	$623
AMBAC Assurance Corporation ("AMBAC")	$433
Financial Security Assurance Holdings Limited ("FSA")	$326
GE Capital (includes FGIC)	$102
Enhance Financial Group ("Enhance" – note: now part of Radian)	$37
Source: Thomson Financial Insurance Solutions, May 2001.	

Financial guaranty is considered to be very "capital intense"; it requires a significant amount of capital to underwrite this type of exposure. In fact, among the top financial writers there is an average 5:1 ratio of surplus to net premiums written. The table below shows the net written premium and corresponding surplus of the top six individual writers.

Top Financial Guaranty Writers – Surplus
2000 Results (amounts in thousands)

Company Name	S&P Insurance Rating	Group	Financial Guaranty Net Premiums Written	Surplus - Policyholders	Ratio of Surplus to NWP
MBIA Ins Corp	AAA	MBIA	489,242	2,381,669	4.868
AMBAC Assurance Corp	AAA	AMBAC	409,215	1,655,151	4.045
Financial Security Assurance Inc	AAA	FSA	137,238	797,369	5.810
Financial Guaranty Ins Co	AAA	GE Capital	84,141	1,089,826	12.952
Enhance Reinsurance Co	AAA	Radian	78,421	188,632	2.405
Ace Guaranty Re Inc	AAA	Ace	77,898	323,401	4.152
Total			1,276,155	6,436,048	5.043

Source: Thomson Financial Insurance Solutions, May 2001

Note that each of the six companies shown above has a 2000 S&P rating of AAA. Most direct writers of financial guaranty carry a rating of AA- or above. Financial guaranty premiums account for 99% to 100% of the total net written premiums for each of these companies with the exception of Enhance Reinsurance Company and Ace Guaranty Reinsurance Company, for which the percentages are 97% and 98% respectively.

Since its inception in 1971, the US financial guaranty market has been controlled by a relatively small number of companies. The 1980's and 1990's saw a period of consolidation and mergers, reducing the number of companies to those shown above. In the future, there may be a few more additional entrants to this particular market; however the high capital requirements of this sector combined with the limited growth needs of the municipal bond market will undoubtedly serve to restrict the number of traditional financial guaranty writers to the single digits.

One area of potential growth lies with insuring corporate debt. The traditional monoline companies have been focused on municipal exposures (AFGI companies had over 80% of premiums from this sector in 1998), with ABS contributing much of the remainder. Stand-alone corporate debt is seldom insured in isolation – instead, baskets of corporate debt is usually preferred. Banks and other financial institutions are often in search of methods of securitizing debt exposure in a bid to offset regulatory capital and liquidity constraints. To meet the needs of this and other markets, it is possible that a new type of monoline company will emerge to focus exclusively on this type of exposure.

US Government Regulation

A series of bond defaults in the early 1980's led the NAIC and several states to adopt statutes and regulations specific to the financial guaranty insurance industry. The most important of these changes was the creation of the "monoline" company. The NAIC's regulations allow monoline companies to write only financial guaranty, surety and, in some states, credit insurance. Monoline companies cannot write certain exposures that many would consider to be financial guaranty products but are not considered financial guaranty under the Model Act. The minimum surplus and capital requirements for financial guaranty insurers vary from state to state, but in general the minimums are higher than those for any other type of property and casualty insurance company

Current regulations also require that companies writing financial guaranty establish special contingency reserves, shown as a write-in item under aggregate liabilities. The contingency reserves are formula derived and can be considered to be highly punitive. Based on total dollars exposed, the contingency reserve dwarfs any reasonable loss and unearned premium reserves. It is basically a reserve based in proportion to the par value of all in-force policies.

To give an idea of the size of the contingency reserves, the following table compares the contingency reserves with the carried loss reserves for the top six financial guaranty writers:

Top Financial Guaranty Writers – Reserves
2000 Results (amounts in thousands)

Company Name	Group	Loss & LAE Reserves	Contingency Reserves	Ratio of Cont. Res. To Loss Res.
MBIA Ins Corp	MBIA	209,159	2,474,533	11.831
AMBAC Assurance Corp	AMBAC	23,989	1,062,686	44.299
Financial Security Assurance Inc	FSA	19,138	459,361	24.003
Financial Guaranty Ins Co	GE Capital	9,249	823,570	89.044
Enhance Reinsurance Co	Radian	18,743	260,168	13.881
Ace Guaranty Re Inc	ACE	14,972	180,584	12.061
Total		295,250	5,260,902	17.818

Source: Thomson Financial Insurance Solutions, May 2001; reserves are shown on a statutory basis.

In total for these six companies, the contingency reserves are approximately 18 times larger than the carried loss reserves. Note that the contingency reserve amounts were assumed to be the entire amount shown as an aggregate write-in liability item on the companies' statutory balance sheets. In actuality, there are a few other liability items that could show up in this account; however, the vast bulk of the write-in is for contingency reserves. The contingency reserve is a statutory item only; it is not required for GAAP purposes. There will be situations where a company is carrying a bulk loss reserve on a GAAP basis, but is not carrying a similar reserve on a statutory basis because the contingency reserve already serves this purpose.

With the implementation of these regulatory changes in the early 1980's, multiline companies could no longer write financial guaranty insurance, as defined by the NAIC.

The NAIC's definition of financial guaranty is somewhat restrictive and there are other credit enhancement products that the insurance industry would consider financial guaranty but the NAIC would not. This is one of the reasons that multiline insurers will still show premiums in their statutory statement under the financial guaranty line of business. Another reason that financial guaranty premiums still show up for multiline companies is that the premiums relate to long term policies (e.g., 30 year bond obligations) that were written prior to the regulations introduced in the mid 1980's.

Non-US Regulation

Outside of the US, there is little or no special government regulation of financial guaranty insurance. In the absence of government regulation limiting entry to the market, there have been many large multiline insurers entering the financial guaranty arena. However, these insurers are still subject to "market-regulation" by the rating agencies (i.e., S&P, Moody's, and Fitch).

During 2000, S&P recognized that multiline insurers participating in the financial guaranty arena did not always have the same commitment to the timely payment of claims that had been expected of and delivered by the monolines. Investors purchasing assets backed by financial guaranty insurance demand that interest and principal be paid on those dates specified in the financial agreement, whether those payments are made by the issuer or insurer. The monolines have demonstrated the ability and willingness to meet the financial market's expectation of timely, unconditional payments even in the event of fraud. Some multilines, on the other hand, have treated financial guaranty claims in the same manner as other traditional lines of insurance. For example, with a general liability claim the payment mechanisms include the rights of reviewing and challenging claims. With financial guaranty, claims should first be paid and then reviewed.

In recognition of the questionable claims practices of a few multilines participating in financial guaranty transactions, S&P introduced the Insurer Financial Enhancement Ratings ("FER"). While the traditional Insurer Financial Strength Ratings ("FSR") measures the insurers ability to pay claims, the FER provides an indication of the insurer's willingness to pay claims. Investors in financially enhanced instruments expect timely interest and principal payments; the FER rating is an example of the financial markets developing a mechanism to provide oversight in the absence of government regulation.

Reserving

For many years, accountants did not allow monoline companies to establish IBNR reserves, also known as "general" or "unallocated" reserves. The reasons were fairly simple and included the fact that once a bond went into default, the entire financial community would know about the failure and the insurer would then establish a case reserve. There could never be a "pure" IBNR claim, therefore there is no need for an IBNR reserve. There can be future development on known claims, but only when the insurer does not reserve for all future interest and principal payments or anticipates an excessive recovery rate.

Is there really a need for a general or IBNR reserve? We know if we have a large enough block of business, it will produce claims. Obviously the insurer does not know *a priori* which bonds will default or they would not have insured those bonds. However, the insurer has entered into numerous long term agreements (e.g., up to 30 years) during which some bonds will default. Almost immediately after a bond is issued, socio-economic changes begin to occur which might ultimately lead to a default on some bond.

We can be reasonably certain that the insurer has entered into one or more non-cancelable agreements that will produce a claim. It is important that the insurer reflect that liability on the balance sheet either in the unearned premium reserve or loss reserve, or a combination of the two.

The following methods explore reserving techniques currently used by insurers writing financial guaranty products. Some techniques are used by the monolines, while multiline carriers have adopted others. There may be some overlap of the reserve estimates produced by some of these methods and the unearned premium reserve. In each case it is necessary to have a clear understanding of the company's approach to earning premiums. For example, one company may earn the premium for a multi-year contract on a pro rata basis while another company would adopt an earning pattern that more closely matches the probability of loss. In such a situation, the amount of required loss reserves would probably be different for each company due to the fact that one of the companies is carrying more in unearned premium reserves.

Exposure Monitoring

As the name implies, this approach involves tracking each individual bond on a regular (e.g., monthly) basis. Each bond is placed into one of five categories:

1. Clean. These are bonds for "safe" municipalities, or ABSs, where the possibility of default has been judged to be extremely remote.

2. Clean with safety triggers. Certain contracts contain provisions calling for the periodic reporting of key financial data. Should the financial data fail to meet certain thresholds, safety triggers are tripped and the bond is put on a watch. In this case, the contract contains safety triggers but none have been tripped.

   For corporate bonds there may be a sinking-fund provision that requires the issuing company to retire a certain percentage of the debt. Not retiring the complete percentage may activate a safety trigger. For a municipal airport revenue bond, a safety trigger may be the cancellation of certain routes from that airport, which will ultimately result in the loss of landing fees, fueling fees, concession fees, etc.

3. One or two safety triggers are tripped. In this case, some of the safety thresholds have been met, but the bond is not in immediate danger of default. The contract may call for additional reporting requirements and the insurance company will increase the diligence of its watch.

4. More safety triggers are tripped. The bond is still not in default, but the probability of default has increased significantly. The insurance company establishes case reserves based on the amount of principal and interest outstanding. The case reserves can be modified by the probability of default and the anticipated recovery percentage.

5. Bond is in default. The insurance company establishes case reserves based on the amount of principal and interest outstanding. The reserves can be reduced by the anticipated amount of recovery.

Loss Ratio Method

This tried and true method has some applicability within this industry. The monolines have produced the following calendar year loss ratios over the last five years.

Top Financial Guaranty Writers – Calendar Year Loss Ratios
Net Loss and Loss Adjustment Expense Ratios to Earned Premium

Company Name	1996	1997	1998	1999	2000	96-00
MBIA Ins Corp	1.98%	1.45%	54.06%	12.32%	6.20%	17.33%
AMBAC Assurance Corp	-7.11%	1.65%	-7.16%	1.69%	3.64%	-0.65%
Financial Security Assurance	10.34%	5.15%	-6.12%	2.67%	-1.02%	1.22%
Financial Guaranty Ins Co	5.10%	5.55%	-2.91%	-2.53%	-0.39%	1.19%
Enhance Reinsurance Co	2.83%	2.10%	9.26%	4.73%	16.88%	7.77%
Ace Guaranty Re Inc	3.32%	0.51%	46.01%	-5.82%	-0.69%	7.04%
Composite	1.63%	2.52%	24.35%	5.38%	4.35%	8.15%

Source: Thomson Financial Insurance Solutions, May 2001

Ignoring the 1998 blip from MBIA and Ace (due to a single market event), this group of companies has had calendar year loss ratios over a five year period near 5%. While this level of detail is not publicly available for the international multilines, ancillary information suggest that the financial guaranty business produced by this tier of companies runs in the 10% to 20% range.

As the variety of financial guaranty products increases, it becomes more difficult to make rule of thumb comments on the industry's loss ratio. While this line of business is generally characterized as low frequency and high severity, some insurers are dropping down into "working" or equity layers where there is a higher probability of loss and hence a higher absolute premium. There is one reinsurer whose premium on a particular credit enhancement product was 75% of the policy limits, indicating a very high probability of a loss.

Unallocated Reserves as a Percentage of Par Outstanding

This is the most common method of establishing reserves for the monoline companies. Industry studies of bond default using decades of financial results are used to determine appropriate reserve factors (i.e., probable loss amounts expressed as a percentage of par). The following table shows the unallocated reserves held by the monoline companies in relation to the total par outstanding insured.

Top Financial Guaranty Writers – Unallocated Reserves to Par Outstanding
1999 Results (amounts in millions)

	FSA	FGIC	AMBAC	MBIA	Composite
Par Outstanding	129,938	137,358	240,307	384,459	892,062
Unallocated Reserves	55	34	95	232	416
Ratio of Res to Par	0.042%	0.025%	0.039%	0.060%	0.047%

Source: Banc of America Securities, Equity Research, March 2, 2000; reserves are shown on a GAAP basis.

Note that MBIA made an increase to unallocated reserves during 1999 of approximately $153 million. Absent this increase, the industry would have ratios of unallocated reserves to par outstanding in the range of 0.02% to 0.04%.

Unallocated Reserves as a Percentage of Par Written

This is a relatively new method of establishing unallocated reserves. Also based on industry default studies, this method produces reserves as a percent of par written using a rate of between 50 to 200 basis points. As an unusual feature, the reserves are not reduced until a loss occurs or overall reserves have reached a "sufficient" level. As previously mentioned, this is a relatively new technique and companies have not yet reached reserve levels that would offset a "typical" municipal bond default.

Reserves Based on Default Probabilities - Deterministic

In this process, reserves are calculated on a contract-by-contract basis using industry default tables. An example of this approach is shown in the attached Exhibit 1. The required data for this technique includes:

1. Par Value
2. Coupon Rate
3. Expiration Date
4. Default Probability (from industry sources)
5. Anticipated Salvage Recovery Percentage

For each contract, the number of outstanding coupon payments is calculated along with the mean time until default. The mean time until default is the average number of years until default given that there has been a default in the policy period. This amount is calculated using incremental, as opposed to cumulative, default probabilities.

In the event of a default on a bond, the insurer will be able to eventually recover a significant portion of the loss payments. If the bond was a municipality, the city or county will reorganize and resume debt service payments. If it is a corporate bond that defaults, there will be some residual value such as product inventories in the insured company that can be used to offset some, if not all, of the loss payments. Industry studies suggest that a salvage recovery rate of 50% is reasonable, however it could be much more or less depending upon the circumstances. The recovery rate will typically be higher for municipalities than corporates. Whatever the anticipated salvage percentage, it will need to be discounted to reflect the timing difference between the loss payment and the actual recovery. For example, a municipality may default on interest payments and the financial guaranty company responds by making those payments to investors. It is highly likely that the municipality will eventually make the overdue interest payments thereby indemnifying the insurance company for the losses paid. The insurance company can establish an asset for the anticipated recoveries (at least on a GAAP basis), but the asset should be calculated as the present value of the recoveries.

Many bonds are retired early, which terminates exposure to the insurer but does not result in a return of any premiums to the insured. Shortening the exposure period reduces the probability of default. The method described above could be modified to reflect the "expected" maturity date instead of the actual maturity date. Of course, this would result in lower reserve estimates.

Reserves Based on Default Probabilities - Stochastic

This technique is essentially the same as the previous method with the exception that several key variables are allowed to be stochastically determined. For example, the probability of default is a simple binomial experiment and the recovery rate can be based on the normal distribution (with appropriate limits in place to keep the simulated value from going above one or less than zero). On an expected basis, the deterministic and stochastic methods should produce identical results. The value of the stochastic approach is that it can produce ranges of reserve estimates at various confidence levels. In fact, this type of method can be used determine appropriate capital requirements if, for example, the company wants to set aside a capital amount sufficient to respond to a 1 in 1,000 event (i.e., 99.9% confidence level).

One area that deserves special attention with the simulation approach is that of correlation. While the probability of default is so minimal for municipals that correlation may not be a significant issue, correlation between corporate debt exposures should be factored into the stochastic model. One method suggested for measuring the debt correlation between two corporate counterparties is to study the correlation between their equity prices. Incorporating correlation into the stochastic model will not change the expected value but will increase the variance.

Moody's Binomial Expansion Technique ("BET")

The rating agency, Moody's, promotes the use of the BET to calculate the expected losses of CBOs and CLOs. Underlying this technique, as used by Moody's, is the **diversity score concept**. The diversity score, *D*, represents a fictitious pool of D homogenous and uncorrelated bonds (or loans) that mimics the behavior of the original portfolio. In this hypothetical pool, all bonds have the same probability of default, *p*, which is the weighted average probability of default of the original pool. Furthermore, each asset has the same par value, which is calculated as the total collateral value divided by D. The calculation of the diversity score is beyond the scope of this paper, but the technique is mentioned for completeness.

The expected loss is calculated as follows:

$$\sum_{j=1}^{D} P_j E_j$$

Where: Pj is the probability of j defaults; and

Ej is the present value of the outstanding assets (bonds or loans).

The probability of j defaults is calculated simply by the binomial formula as:

$$P_j = \frac{D!}{j!(D-j)!} p^j (1-p)^{D-j}$$

Reinsurance – Quota Share

There are a handful of specialized reinsurance companies that provide protection to the monoline companies, much of which is written on a quota share basis. The most basic approach to reserving in this situation is to use the reserves (or proxies thereof) of the underlying monoline carrier. Either through direct communication with insurance company or via market research, ratios of unallocated reserves to outstanding par are computed by industry group (e.g., domestic municipal, domestic non-municipal, international, etc.,). These ratios are then applied to the appropriate assumed par by industry group for each of the insureds. In this manner, the reinsurer maintains reserve levels that are consistent with the underlying insurer.

Reinsurance – Tranches

As previously noted, ABS instruments are often layered or tranched with different (re)insurers participating on different layers. In some circumstances, one insurer will essentially "front" the deal and then cede various layers. In contrast with the traditional insurance market, the ceded layers may actually be the lower layers – those tranches with a higher probability of loss. In such situations, the rating agencies will often assign a rating to each layer commensurate with the expected loss amount. Given that the layer on which a (re)insurer is participating is rated (or a rating can be implied), techniques based on default tables can be used to estimate the reserve requirements.

Summary

The number of financial guaranty deals underwritten is growing at a fantastic rate, as are the different types of such products. In fact, the term "financial guaranty" is often dropped in favor of other more comprehensive terms such as "capital market products". The lines between insurance and the capital markets are becoming more and more blurred. While the nomenclature in the capital markets is very different from that of the insurance industry, many of the underlying concepts will be familiar to actuaries. The need to evaluate the current financial implications of future contingent events is a common concern in both the capital and insurance markets. The actuary is ideally trained to measure these risks.

Historically, financial guaranty had been a line of business with an extremely low frequency and the potential for a very high severity. The need for a "general" loss reserve was often questioned. As the types and volume of transactions increase, "do we need a reserve" is being replaced by "how do we establish a reserve".

The best reserving techniques can be selected for a given situation only after an analysis of the underlying exposure is completed. What triggers a loss? What is the frequency of claims? Is there any potential for salvage recoveries? How does the company earn premiums? How are the loss reserve and the unearned premium reserve related? What is the exposure period? Can insurance contracts be cancelled and, if so, by which party? In these respects, reserving for financial guaranty products is very similar to reserving for other lines of business. The best approach is determined only after an understanding of the risks is gained.

Reserves Based on Default Probabilities

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Counterparty	Rating	Coupon	Payable	Par	Maturity	Exposure	Prob of Default	Mean Time to Default	Interest Payments Outstanding	NPV of Salvage %	Reserve Amount
Apple County Sewage Plant	A	4.6%	Semi-annually	120,000,000	12/31/2008	7.0	0.83%	4.8	2.50	85.0%	166,581
Cameron City General Obligation	BBB	5.2%	Semi-annually	15,000,000	6/30/2014	12.5	4.05%	6.1	6.50	85.0%	121,925
Delphi Municipality	AA	4.9%	Annually	100,000,000	12/31/2015	14.0	1.01%	7.8	7.00	85.0%	203,465
Waynestown Electric	BBB	5.1%	Semi-annually	5,000,000	12/15/2012	11.0	3.91%	5.9	5.50	85.0%	37,551
Sub-Total (Municipals)				240,000,000							529,521
								Ratio of Reserves to Par Outstanding			0.22%
Celston Apparel Co.	BBB	7.8%	Semi-annually	50,000,000	3/31/2008	6.2	2.29%	4.0	2.50	40.0%	820,965
Fiberboard Inc.	A	7.4%	Semi-annually	75,000,000	5/30/2007	5.4	0.49%	3.7	2.00	40.0%	253,134
Lakeland Industries	AA	7.1%	Semi-annually	28,500,000	12/31/2011	10.0	0.79%	6.5	3.50	40.0%	168,660
Metalurgy Amalgamated Ltd.	BB	8.3%	Semi-annually	140,000,000	1/15/2005	3.0	5.35%	2.3	1.00	40.0%	4,867,002
Quiet Comforters Inc.	A	7.6%	Semi-annually	10,000,000	12/31/2014	13.0	1.70%	7.4	6.00	40.0%	148,512
Sub-Total (Corporates)				303,500,000							6,258,273
								Ratio of Reserves to Par Outstanding			2.06%
Grand-Total				543,500,000							6,787,794
								Ratio of Reserves to Par Outstanding			1.25%

Notes:

Evaluation Date 12/31/2001

(3) Simple Interest

(7) Number of years from evaluation date to maturity

(8) From Exhibit 1, Page 2

(9) In years; based on Exhibit 1, Page 2

(10) Number of *annualized* interest payments outstanding between maturity date and mean time to default. For example, if the mean time to default occurs 19 months before the maturity of the bond, it is assumed that there are 3 semi-annual coupon payments remaining, which translates to 1.5 *annualized* coupon payments. In this same example, a bond paying interest annually intstead of semi-annually would have 2 annualized coupon payments remaining.

(11) In the event of default, there is a potential for a significant recovery of the loss payments. In the case of municipalities, a defaulting city or county has no choice but to reorganize and resume debt service payments. A corporation will have assets that can be liquidated. In each case, there is an issue of the time value of money from the point at which loss payments are made and salvage recoveries are received. The values shown are for demonstrative purposes only.

(12) = (8) x [(3) x (5) x (10) + (5)] x [1 - (11)]

Reserves Based on Default Probabilities

Average Cumulative Default Rates

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00%	0.00%	0.03%	0.06%	0.10%	0.18%	0.26%	0.40%	0.45%	0.51%	0.51%	0.51%	0.51%	0.51%	0.51%
AA	0.01%	0.04%	0.09%	0.16%	0.25%	0.37%	0.53%	0.63%	0.70%	0.79%	0.85%	0.92%	0.96%	1.01%	1.07%
A	0.04%	0.11%	0.19%	0.32%	0.49%	0.65%	0.83%	1.01%	1.21%	1.41%	1.56%	1.65%	1.70%	1.73%	1.83%
BBB	0.22%	0.50%	0.79%	1.30%	1.80%	2.29%	2.73%	3.10%	3.39%	3.68%	3.91%	4.05%	4.22%	4.37%	4.48%
BB	0.98%	2.97%	5.35%	7.44%	9.22%	11.11%	12.27%	13.35%	14.29%	15.00%	15.65%	16.00%	16.29%	16.36%	16.36%
B	5.30%	11.28%	15.88%	19.10%	21.44%	23.20%	24.77%	26.01%	26.99%	27.88%	28.48%	28.96%	29.34%	29.68%	29.96%

Average Incremental Default Rates

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00%	0.00%	0.03%	0.03%	0.04%	0.08%	0.08%	0.14%	0.05%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.01%	0.03%	0.05%	0.07%	0.09%	0.12%	0.16%	0.10%	0.07%	0.09%	0.06%	0.07%	0.04%	0.05%	0.06%
A	0.04%	0.07%	0.08%	0.13%	0.17%	0.16%	0.18%	0.18%	0.20%	0.20%	0.15%	0.09%	0.05%	0.03%	0.10%
BBB	0.22%	0.28%	0.29%	0.51%	0.50%	0.49%	0.44%	0.37%	0.29%	0.29%	0.23%	0.14%	0.17%	0.15%	0.11%
BB	0.98%	1.99%	2.38%	2.09%	1.78%	1.89%	1.16%	1.08%	0.94%	0.71%	0.65%	0.35%	0.29%	0.07%	0.00%
B	5.30%	5.98%	4.60%	3.22%	2.34%	1.76%	1.57%	1.24%	0.98%	0.89%	0.60%	0.48%	0.38%	0.34%	0.28%

Note: These default probabilities are from S&P's CreditWeek January 31, 2001 and are based on corporate debt. Studies by both S&P and J.J.Kenny Co. Inc. indicate that the frequency of default for a domenstic investment-grade corporation is greater than that of a similarly rated municipality. Therefore, in real world applications, this default table would not be appropriate for use with municipal bonds.

The Impact of Catastrophic Cases on Workers Compensation Medical Loss Reserves

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ABSTRACT

The Impact of Catastrophic Cases on Workers Compensation Medical Loss Reserves

Catastrophic claims (defined as burn injuries, acquired head injuries, spinal cord injuries and multiple trauma injuries) account for less than 1% of all Workers Compensation claims but as much as 20% of total Workers Compensation losses. The ultimate value of a catastrophic claim can be very difficult to predict, with significant increases in case reserves many years after the injury occurred being not uncommon. These claims introduce a high amount of variability to the ultimate medical loss reserve projections when using standard loss development triangle techniques.

This paper focuses on the distorting impact catastrophic claims can have on workers compensation ultimate medical reserve projections and introduces techniques for eliminating this distortion. The issue of the impact of catastrophic claims on ultimate medical loss reserve projections is one that has received relatively little attention explicitly in the actuarial literature, but is one that is important to accurate reserve estimation by accident year.

The Impact of Catastrophic Cases on Workers Compensation Medical Loss Reserves

Introduction

Catastrophic claims account for less than 1% of all Workers Compensation claims but as much as 20% of total Workers Compensation losses. For the purpose of this paper, the definition of a catastrophic claim follows common industry practice; burn injuries, acquired head injuries, spinal cord injuries and multiple trauma injuries. Catastrophic claims can cost millions of dollars in medical costs and can extend over several decades or more.

The ultimate value of a catastrophic claim can be very difficult to predict early in the life of the claim and often even after many years have passed. As a result, these claims account for a high percentage of the late medical reported as well as paid loss development and a great deal of the variability in the medical loss development triangle and in ultimate loss projections.

Within a company's claims department, these claims call for and receive special case reserving treatment. This was not always so. Over the last 15 or more years, the approaches for managing and case reserving these claims have changed and become more sophisticated. This paper discusses the distortion in medical incurred loss development triangles and ultimate loss projections caused by catastrophic claims and by changes in their case reserve adequacy resulting from industry practices in managing and reserving these claims. It then discusses how this may be affecting the accuracy of loss projections based on incurred loss development and suggests an alternative tool for dealing with the actuarial issues created by these claims which involves excluding the catastrophic claims entirely from the loss development triangles.

Background

Since the ultimate values of catastrophic claims are more unpredictable than non-catastrophic claims, catastrophic claims cause a great deal of the volatility in incurred and paid loss development factors. There are many factors contributing to the relatively higher unpredictability of catastrophic claims. Difficulties arise in anticipating the impact of medical inflation; foreseeing changes in the condition of the claimant or his or her home care giver(s) combined with the impact any change may have on the future stream of payments; foreseeing future medical advances that may be utilized for the claimant's care and their rising costs; and predicting whether the life expectancy is impaired and, if so, to what extent.

Annual medical payments can exceed $100,000 on these cases, and anticipating future medical inflation can be extremely difficult. Also, the future introduction and utilization of costly medical procedures, apparatuses and drugs may affect future medical payments on catastrophic claims. Regarding life expectancies for the catastrophically injured population, the experts interviewed for this paper did not reach a consensus as to whether these life expectancies are materially lower than the total population. There was a common theme that it depends on the specifics of the case and that the variability of the life expectancies is greater than for the total population.

The injured person's response to and recovery from a severe injury and its treatment are variable and unpredictable, as are subsequent treatment needs and lifespan. Psychosocial factors like the support of and relationship with the spouse and family are important in determining the likely degree of long term institutionalization and the likelihood of any return to home and an independent care situation. The difference between the initial expected and actual lifetime medical, rehabilitation and maintenance costs can be in the millions of dollars for some claims.

The state of the catastrophic claims handling and reserving "art" has evolved significantly since the 70's. That changing state of the "art" is reflected in high medical incurred tail loss development factors in the current observed loss development factors as compared to historical levels. Insurer claim departments and third party claims handling administrators (TPA's) are far more focused on early and proactive intervention and case management of catastrophic medical cases than they were twenty years ago. In addition, they are far more adept at understanding the complex factors that affect the cost of these claims and anticipating their impact on the ultimate cost. Given the greater focus on early accurate measurement of the ultimate cost of catastrophic claims today than in the past, it is reasonable to hypothesize that the paid and reported losses for catastrophic cases will not develop in the same fashion as they did in the past, and the differences may be dramatic.

Case reserves for catastrophic claims were in many cases stair-stepped in the 70's and 80's. That is, often no meaningful attempt was made to project the ultimate cost of catastrophic claims. The impact of this tendency to stair-step catastrophic case reserves is embedded in the loss development factors we rely on today to predict future loss development. Today, however, insurers, their claims administrators, managed care providers and reinsurers are far more proactive in not only managing catastrophic claims but also in determining realistic projected ultimate values of each catastrophic claim and regularly reviewing their estimates. Many companies and TPA's have claims adjusters or nurse case managers that specialize in catastrophic cases. Third party vendors now exist that deal exclusively with these types of claims. As a result, catastrophic claims are more adequately reserved today than is implied by the historical medical incurred loss development factors. Not only does the inclusion of catastrophic claims cause volatility in the observed development patterns, a significant portion of the historical incurred development caused by catastrophic claims may not be repeated on today's claims.

An Alternative Reserving Tool

One goal of this paper is to increase the awareness of actuaries to the existence and potential impact of catastrophic claims in the historical losses and development patterns. For example, when projecting the ultimate losses for a particular accident year, one needs to be aware of whether there are any catastrophic claims in that year. This should affect the magnitude of the incurred loss development factor applied. The presence of catastrophic claims tends to increase the variability of the ultimate reserve and the risk of material adverse deviation. If a catastrophic claim is present, it is valuable to understand the details of its case reserve derivation: the level of effort put into estimating the case reserve, the life expectancy and medical inflation assumptions used, the catastrophic claim experience of the individual who developed the reserve, the time elapsed since the last review, and in general the likely upsides and downsides from the case manager's perspective. From this review, the actuary should be able to gain a sense of the variability in the cost estimate, and where it falls in the range of potential outcomes.

Another goal of this paper is to suggest an alternative tool for projecting workers compensation losses that can help identify the distorting impact catastrophic claims are having on reserve projections and that can in many cases provide more accurate projections. The approach is to isolate and restate the loss development patterns to exclude the catastrophic claims, to then develop the non-catastrophic claim losses separately, and to rely on separate existing case specific techniques to estimate the ultimate value on the catastrophic claims.

Excluding the catastrophic claims produces much more stable development patterns and much more stable and smaller medical tail factors. The loss triangle of data exclusive of the catastrophic claims will have most if not all remaining claims with little or no ongoing medical payments after 15 years. Given this greater stability and shorter tail, more accurate projections of ultimate loss for the non-catastrophic claims can be made.

For the remaining catastrophic claims, qualified nurse case managers can perform detailed evaluations of the future cost of these claims called Life Care Plans. These are the best way to estimate the ultimate cost of these claims. The ultimate value of each claim is best estimated individually (as is the payout pattern, which will be needed for cash flow and discounting purposes).

This tool is most valuable in reviewing accident years that are at least two or three years old. Given the nature and severity of catastrophic injuries, they are identified early. However, it is usually not until the early acute phase of the treatment is completed that Life Care Plans are prepared and that the actuary can rely upon individual case reviews.

A claim nurse case manager or claims adjuster with extensive experience with catastrophic claims best develops Life Care Plans. These evaluations consider many factors such as psychosocial and other factors as well as physical factors in making projections of the length of acute care, the likelihood and expected point at which the injured person will be able to return to the home and then to non-supervised status, the point at which medical costs will stabilize, if ever, the maintenance costs once a level of stabilization is reached, the life expectancy, etc.

Because of the difficulty of managing these catastrophic claims, Life Care Plans are frequently created today (although not 10 or 15 years ago). This careful review helps manage the claim more effectively and provide the proper care without spending excessively. It does this by developing a long term plan for the victim's care and treatment, one that often involves frequent communication with the victim's family. Given the detail that goes into a Life Care Plan, inaccuracies in the individual estimates can be identified quickly after a significant change in conditions or treatment plan occurs. Also, these inaccuracies are not contaminating your non-catastrophic claim loss development triangles. Moreover an actuary can work with the developer of a Life Care Plan to develop the high end of the range and low end of the range of reserve estimates for each of these claims to help in setting ultimate reserves for these claims within the context of setting the aggregate reserves.

Highly experienced catastrophic nurse case managers and claims adjusters are uniquely qualified to put together Life Care Plans, and annual lifetime care cost projections for each catastrophic claim. In putting a Life Care Plan together, the experts consider many factors, including those mentioned above, as well as how people tend to react in these difficult situations and how all these factors interact.

How Is This Different from Limited Loss Development Patterns?

The approach of excluding catastrophic claims from the loss development triangles and separately analyzing the individual catastrophic claims is in some ways similar to projecting losses on a limited per occurrence basis, but it has certain advantages over that approach. It is true that some of the volatility introduced by the inclusion of catastrophic claims in the development patterns can be eliminated using limited loss development patterns, especially in the tail. This is not an adequate solution, however. Using limited loss development factors leaves the concern of projecting losses by year in excess of the limit, and the presence or absence of catastrophic claims, and their volatility, greatly influences the excess losses. Also, the limited loss triangles will still contain the distortion caused by the case reserve strengthening that has occurred on catastrophic claims over the last 25 or so years.

Data Challenges

Obviously, in order to perform this approach it is necessary to identify catastrophic claims and remove them from the entire loss development triangle. Research done in preparing this paper indicates that each actuary may have to rely on different approaches depending upon how the company's data is coded. For some, the system may have a unique catastrophic claim identifier, in which case this approach is relatively easy to do. If this is not the case, catastrophic claims may have unique claim descriptions such that the claims can be culled out by searching the claim description. The number of these claims is typically small and manageable, even in the largest companies, and each has the attention of the claims department so that a manual process of identifying and removing these claims may be appropriate. Narrowing the search by starting with only claims over, say $250,000 in medical loss can save time in identifying these catastrophic claims, particularly on the older years.

An Example

An example will now be presented to demonstrate the concept of isolating and excluding catastrophic claims from the incurred losses and loss development patterns. This example will demonstrate the increased stability in the development triangles when the catastrophic claims are removed. It demonstrates that more accurate ultimate reserves are derived. It demonstrates that, given that catastrophic claims are reserved far more adequately today than during the time period reflected in the loss development triangles, traditional methods tend to create an upward bias in the loss projections. There may still be years in which the ultimate projections are understated by the traditional approach, namely years where catastrophic claims occurred and there is still potential for significant development on them. In total, however, the traditional approach may be resulting in an overstatement of ultimate losses.

The alternative approach involves separating catastrophic claims from the medical losses and loss development triangles. In the attached exhibits displaying hypothetical reported workers compensation loss development triangles, Appendix 2 represents the loss triangles including the catastrophic claims. Appendix 3 shows the triangles for just the catastrophic claims. Appendix 4 displays the triangles restated to exclude the catastrophic claims.

Once catastrophic claims are excluded, the ultimate losses for the non-catastrophic medical losses can be projected using standard actuarial techniques: loss development, frequency/severity analysis, etc. The actuaries must then review each of the catastrophic claims with the case managers to estimate the probable range of outcomes. This multi-disciplined approach can be valuable not only in informing the actuary of the range of potential costs of the catastrophic claims, but also in educating the case manager of the potential impact of future medical inflation on the cost of the claim.

For the latest few accident years, this alternative approach may not work without adjustment because of the potential for late emerging catastrophic claims, and should be supplemented or modified. Because these catastrophic claims tend to arise from sudden and severe accidents they are usually known relatively quickly, they tend to generate a small pure IBNR component. Nonetheless, there are examples of cases that start out as moderately serious cases and later deteriorate into catastrophic claims. Also, there can be IBNR catastrophic claims due to reporting lags. Finally, for recently occurring catastrophic injuries, enough time may not have passed to do a Life Care Plan or to reasonably evaluate the ultimate cost of the case.

In order to address this IBNR concern the more recent few accident years can be projected through the traditional method of applying including catastrophic claims loss development factors to including catastrophic claims losses. These loss development factors reflect an average of years with high and low frequency and severity of catastrophic claims.

Another approach is to derive a catastrophic claim emergence pattern so as to measure the expected number of pure IBNR catastrophic claims. These expected claim counts are then multiplied by a catastrophic claim projected average severity to derive an estimate of unreported ultimate catastrophic losses. This average severity should be based on a long term history of catastrophic claim severity. Given the volatility in average severity for these infrequent claims, each year's average severity should be trended to the cost level for the year being estimated, and an average severity should be selected based on a review of the results over a long period of time. The unreported ultimate catastrophic losses are then added to the reported ultimate catastrophic losses (assuming Life Care Plans have been performed on the reported catastrophic claims) and the ultimate losses for non-catastrophic losses.

An example of this approach is shown in Appendices 5, 6, and 7. Appendix 7 shows the catastrophic claim emergence pattern, which indicates that well under one catastrophic claim per accident year is expected to emerge after the end of the first year. Appendix 6 shows the derivation of the trended average medical costs per catastrophic claim. Appendix 5 combines the expected claim count and severity to determine ultimate loss projections for IBNR claims.

When trending catastrophic medical claim severities, a higher trend rate than the average workers compensation medical trend rate should be used. These claims tend to have a high percentage of ongoing medical cost from long term care and pharmaceuticals, both of which are experiencing (and are expected to continue to experience) higher inflation rates than medical costs on average.

This paper has described an alternative approach to estimating ultimate medical reserves for workers compensation that treats catastrophic claims separately. The results from this alternative approach should be considered relative to results based on traditional methods in light of a number of factors. For example, if the volume of catastrophic claims is relatively consistent from year to year, traditional methods may not work too badly unless case reserve adequacy has changed. If the claims department procedures for handling catastrophic claims have changed over the years (for example if they previously tended to stair step the case reserves), this alternative approach is important to avoid distorted results. If the case managers performing the Life Care Plans lack expertise on catastrophic claims, the accuracy of the alternative approach may be threatened. At a minimum, this alternative approach is useful in sensitivity testing the impact of catastrophic claims on loss development patterns.

Appendix 1 shows the derivation of the results for the standard and alternative approaches. The Summary exhibit compares the results of this alternative approach compared to the standard approach. The overall redundancy in reserves is significant. Again this is caused by the impact of significant case reserve strengthening on catastrophic claims in the standard loss development method. The alternative approach indicates that the significant strengthening that occurred on catastrophic claims in the past will not occur to nearly the same extent and properly removes the distorting impact from the projections.

This example also illustrates that, even if the standard loss development factors were not distorted by non-repeating case reserve strengthening, the development factors, while accurate on average, are not accurate for any year. The years with the catastrophic claims will be understated and the years without the catastrophic claims will be overstated. In practice, there is no reason to think these overages and underages will perfectly "balance" out overall, so this approach improves the overall accuracy in addition to the by-year accuracy.

Summary

This paper is intended to increase the awareness of actuaries of the important role catastrophic claims play in workers compensation reserving. Changes in case management and reserving techniques for catastrophic claims are discussed in the context of the potential for distortion these changes have on ultimate medical loss projections. An alternative approach to developing workers compensation medical losses that deals with this distortion is illustrated. While many other factors have affected workers compensation loss development factors over time, this approach attempts to isolate and adjust for one important factor.

Summary

Comparison of Results From Alternative Methods
All Figures in Thousands

Acc Yr (1)	Standard (2)	Catastrophic (3)	Difference (4)
1988	21,789	21,912	123
1989	37,638	37,028	-611
1990	31,898	31,255	-643
1991	30,337	30,278	-59
1992	25,470	25,724	254
1993	35,395	35,550	155
1994	27,313	27,134	-179
1995	25,014	24,933	-81
1996	26,102	27,047	945
1997	32,006	29,036	-2,969
1998	35,991	33,055	-2,936
Total ex 97,98	260,957	260,861	-96
Total	328,953	322,952	-6,001

Appendix 1

Derivation of Ultimate Loss Projections From Alternative Methods
All Figures in Thousands

	Standard Method				Catastrophic Claims			Excluding Catastrophic Claims		
Acc Yr	Reported Medical Losses as of 12/31/98	Selected Loss Develop-ment Factors	Selected Ultimate Medical Losses	Acc Yr	Reported Medical Losses as of 12/31/98	Selected Ultimate Medical Losses	Acc Yr	Reported Medical Losses as of 12/31/98	Selected Loss Develop-ment Factors	Selected Ultimate Medical Losses
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1988	20,525	1.062	21,789	1988	0	0	1988	20,525	1.068	21,912
1989	35,278	1.067	37,638	1989	6,000	5,500	1989	29,278	1.077	31,528
1990	29,749	1.072	31,898	1990	4,000	3,500	1990	25,749	1.078	27,755
1991	28,152	1.078	30,337	1991	0	0	1991	28,152	1.076	30,278
1992	23,518	1.083	25,470	1992	3,000	3,500	1992	20,518	1.083	22,224
1993	32,359	1.094	35,395	1993	7,000	8,000	1993	25,359	1.086	27,550
1994	24,481	1.116	27,313	1994	0	0	1994	24,481	1.108	27,134
1995	21,916	1.141	25,014	1995	5,000	6,000	1995	16,916	1.119	18,933
1996	22,096	1.181	26,102	1996	5,000	7,500	1996	17,096	1.143	19,547
1997	25,086	1.276	32,006	1997	0	571	1997	25,086	1.135	28,465
1998	22,568	1.595	35,991	1998	0	2,058	1998	22,568	1.373	30,996

Notes:
(2), (3) from Appendix 2.
(4) = (2) x (3).
(6), (7) selected judgmentally based on author's experience with catastrophic claims and catastrophic claim development.
For 1997 and 1998, see Appendix 5.
(9), (10) from Appendix 4.

Appendix 2

Workers Compensation Reported Medical Losses and Loss Development Factors Including Catastrophic Claims
All Figures in Thousands

Accident	Evaluation Age in Months											
Year	12	24	36	48	60	72	84	96	108	120	132	144
1976												
1977												
1978												
1979												
1980												
1981												
1982												7,725
1983											8,572	8,623
1984										9,516	9,535	9,525
1985									11,939	12,130	12,312	16,166
1986								15,352	15,337	15,367	15,506	16,002
1987							19,479	19,713	19,575	19,183	19,375	19,666
1988						20,697	21,028	20,671	20,609	20,382	20,525	
1989					29,419	29,654	32,234	32,653	32,817	35,278		
1990				27,851	28,241	28,467	28,438	28,495	29,749			
1991			27,580	28,049	27,516	27,571	28,040	28,152				
1992		20,638	21,051	20,819	21,277	23,660	23,518					
1993	25,965	30,743	30,404	32,168	32,007	32,359						
1994	19,713	23,951	23,903	24,190	24,481							
1995	17,962	21,698	21,807	21,916								
1996	17,786	20,845	22,096									
1997	20,069	25,086										

Accident	Age Interval in Months											
Year	12 to 24	24 to 36	36 to 48	48 to 60	60 to 72	72 to 84	84 to 96	96 to 108	108 to 120	120 to 132	132 to 144	144 to 156
1976												
1977												
1978												
1979												
1980												
1981												
1982												1.006
1983											0.999	0.997
1984										1.015	1.313	1.002
1985									1.002	1.009	1.032	0.998
1986								0.993	0.980	1.010	1.015	
1987							0.983	0.997	0.989	1.007		
1988						1.067	1.013	1.005	1.075			
1989					1.008	0.999	1.002	1.044				
1990				0.981	1.002	1.017	1.004					
1991			0.989	1.022	1.112	0.994						
1992		0.989	1.058	0.995	1.011							
1993	1.215	0.998	1.012	1.012								
1994	1.208	1.005	1.005									
1995	1.172	1.060										
1996	1.250											
1997												
	Simple Average of Latest 4											
[1]	1.211	1.013	1.016	1.003	1.033	1.024	1.001	1.010	1.012	1.010	1.090	1.001
	Simple Average of Latest 3											
[3]	1.210	1.021	1.025	1.010	1.042	1.003	1.006	1.015	1.015	1.009	1.120	0.999
	Simple Average of Latest 2											
[4]	1.211	1.033	1.009	1.004	1.062	1.006	1.003	1.025	1.032	1.009	1.024	1.000
	Volume Weighted Average of Latest 5											
[5]	1.213	1.059	1.029	1.022	1.013	1.014	1.011	1.004	1.002	1.001	1.002	1.007
	Selected											
Selected	1.250	1.080	1.035	1.023	1.020	1.010	1.005	1.005	1.005	1.005	1.005	1.005
Dev to Ult	1.595	1.276	1.181	1.141	1.116	1.094	1.083	1.078	1.072	1.067	1.062	1.056

Appendix 2

Workers Compensation Reported Medical Losses and Loss Development Factors Including Catastrophic Claims
All Figures in Thousands

Accident Year	156	168	180	192	204	216	228	240	252	264
1976										
1977										
1978										
1979			4,977	4,992	5,012	5,012	5,268			
1980		6,874	6,956	7,464	7,502	7,464	7,763			
1981	8,197	8,238	8,205	8,386	8,335	8,319				
1982	7,949	8,165	8,661	8,670	8,418					
1983	8,675	8,693	8,736	8,806						
1984	9,497	9,535	9,678							
1985	16,196	16,214								
1986	15,970									
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										

Accident Year	156 to 168	168 to 180	180 to 192	192 to 204	204 to 216	216 to 228	228 to 240	240 to 252	252 to 264	264 to Ult
1976										
1977										
1978				1.004	1.000	1.055				
1979			1.073	1.005	0.995	1.040				
1980		0.996	1.022	0.994	0.998					
1981	1.153	0.945	1.001	0.971						
1982	1.002	1.005	1.008							
1983	1.004	1.015								
1984	1.001									
1985										
1986										
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										
[1]	1.040	0.990	1.026	0.994	0.998	1.046	1.002	1.000	1.000	1.000
[3]	1.002	0.988	1.010	1.003	1.010	1.014	1.002	1.000	1.000	1.000
[4]	1.003	1.010	1.005	1.001	1.008	1.020	1.003	1.000	1.000	1.000
[6]	1.008	0.999	1.009	1.004	1.010	1.013	1.002	1.000	1.000	1.000
Selected	1.005	1.005	1.006	1.005	1.006	1.005	1.006	1.004	1.011	1.000
Dev to Ult	1.051	1.046	1.041	1.035	1.030	1.025	1.020	1.015	1.011	1.035

Appendix 3

Workers Compensation Reported Medical Losses Catastrophic Claims Only
All Figures in Thousands

Accident	Evaluation Age in Months											
Year	12	24	36	48	60	72	84	96	108	120	132	144
1976												
1977												
1978												
1979												
1980												
1981												
1982												1,000
1983											0	0
1984										0	0	0
1985									1,000	1,000	1,000	4,500
1986								0	0	0	0	0
1987							0	0	0	0	0	0
1988						0	0	0	0	0	0	
1989					3,000	3,000	4,500	4,500	4,500	6,000		
1990				2,500	2,500	2,500	2,500	2,500	4,000			
1991			0	0	0	0	0	0				
1992		1,500	1,500	1,500	1,500	3,000	3,000					
1993	4,000	7,000	7,000	7,000	7,000	7,000						
1994	0	0	0	0	0							
1995	4,000	4,750	4,500	5,000								
1996	2,500	3,000	5,000									
1997	0	0										

Appendix 3

Workers Compensation Reported Medical Losses Catastrophic Claims Only
All Figures in Thousands

Accident Year	156	168	180	192	204	216	228	240	252	264
1976										
1977										
1978										
1979			1,200	1,200	1,200	1,200	1,400			
1980		1,000	1,000	1,500	1,500	1,500	1,800			
1981	0	0	0	0	0	0				
1982	1,000	1,800	1,600	1,600	1,500					
1983	0	0	0	0						
1984	0	0	0							
1985	4,500	4,500								
1986	0									
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										

Appendix 4

Workers Compensation Medical Reported Losses and Loss Development Factors Excluding Catastrophic Claims
All Figures in Thousands

Accident Year	Evaluation Age in Months											
	12	24	36	48	60	72	84	96	108	120	132	144
1976												
1977												
1978												
1979												
1980												
1981												
1982												6,725
1983											8,572	8,623
1984										9,516	9,535	9,525
1985									10,939	11,130	11,312	11,666
1986								15,352	15,337	15,367	15,506	16,002
1987							19,479	19,713	19,575	19,183	19,375	19,666
1988						20,697	21,028	20,671	20,609	20,382	20,525	
1989					26,419	26,654	27,734	28,153	28,317	29,276		
1990				25,351	25,741	25,967	25,938	25,995	25,749			
1991			27,580	28,049	27,516	27,571	28,040	28,152				
1992		19,138	19,551	19,319	19,777	20,660	20,518					
1993	21,965	23,743	23,404	25,168	25,007	25,359						
1994	19,713	23,951	23,903	24,190	24,481							
1995	13,962	16,948	17,307	16,916								
1996	15,286	17,845	17,096									
1997	20,069	25,086										

Accident Year	Age Interval in Months											
	12 to 24	24 to 36	36 to 48	48 to 60	60 to 72	72 to 84	84 to 96	96 to 108	108 to 120	120 to 132	132 to 144	144 to 156
1976												
1977												
1978												
1979												
1980												
1981												
1982												1.006
1983											0.999	0.997
1984										1.016	1.031	1.003
1985									1.002	1.009	1.032	0.996
1986								0.993	0.980	1.010	1.015	
1987							0.983	0.997	0.989	1.007		
1988						1.041	1.015	1.006	1.034			
1989					1.009	0.999	1.002	0.991				
1990				0.981	1.002	1.017	1.004					
1991			0.988	1.024	1.045	0.993						
1992		0.986	1.075	0.994	1.014							
1993	1.215	0.998	1.012	1.012								
1994	1.214	1.021	0.977									
1995	1.167	0.958										
1996	1.250											
1997												
	Simple Average of Latest 4											
[1]	1.212	0.991	1.013	1.003	1.017	1.012	1.001	0.997	1.001	1.011	1.019	1.001
	Simple Average of Latest 3											
[3]	1.210	0.992	1.022	1.010	1.020	1.003	1.007	0.998	1.001	1.009	1.026	0.999
	Simple Average of Latest 2											
[4]	1.209	0.990	0.995	1.003	1.029	1.005	1.003	0.998	1.011	1.009	1.024	1.000
	Volume Weighted Average of Latest 5											
[5]	1.213	1.059	1.029	1.022	1.013	1.014	1.011	1.004	1.002	1.001	1.002	1.007
	Selected											
Selected	1.210	0.992	1.022	1.010	1.020	1.003	1.007	0.998	1.001	1.009	1.026	0.999
Dev to Ult	1.373	1.135	1.143	1.119	1.108	1.086	1.083	1.076	1.078	1.077	1.068	1.040

Appendix 4

Workers Compensation Medical Reported Losses and Loss Development Factors Excluding Catastrophic Claims
All Figures in Thousands

Accident Year	156	168	180	192	204	216	228	240	252	264
1976										
1977										
1978										
1979			3,777	3,792	3,812	3,812	3,888			
1980		5,874	5,956	5,964	6,002	5,964	5,963			
1981	8,197	8,238	8,205	8,386	8,335	8,319				
1982	6,949	7,365	7,061	7,070	6,918					
1983	8,675	8,693	8,736	8,806						
1984	9,497	9,535	9,678							
1985	11,698	11,714								
1986	15,970									
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										

Accident Year	156 to 168	168 to 180	180 to 192	192 to 204	204 to 216	216 to 228	228 to 240	240 to 252	252 to 264	264 to Ult
1976										
1977										
1978				1.005	1.000	1.020				
1979			1.001	1.006	0.994	1.000				
1980		0.996	1.022	0.994	0.998					
1981	1.060	0.959	1.001	0.979						
1982	1.002	1.005	1.008							
1983	1.004	1.015								
1984	1.001									
1985										
1986										
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										
[1]	1.017	0.994	1.008	0.996	0.997	1.010	1.002	1.000	1.000	1.000
[3]	1.002	0.993	1.010	1.003	1.010	1.014	1.002	1.000	1.000	1.000
[4]	1.003	1.010	1.005	1.001	1.008	1.020	1.003	1.000	1.000	1.000
[5]	1.008	0.999	1.009	1.004	1.010	1.013	1.002	1.000	1.000	1.000
Selected	1.002	0.993	1.010	1.003	1.010	1.014	1.002	1.000	1.005	1.000
Dev to Ult	1.041	1.039	1.046	1.035	1.032	1.021	1.007	1.005	1.005	1.000

Appendix 5

Derivation of Ultimate Catastrophic Medical Losses for Accident Years 1997 and 1998

Acc Yr	Expected Number of Catastrophic Claims	Average Severity (000)	Selected Ultimate Catastrophic Medical Losses (000)
(1)	(2)	(3)	(4)
1997	0.111	5,140	571
1998	0.374	5,500	2,058

Notes:
(2) from Appendix 7, Cumulative Row
(3) from Appendix 6
(4) = (2) x (3)

Appendix 6

Derivation of Average Medical Cost per Catastrophic Case

Acc Yr	Ultimate Number of Catastrophic Claims	Ultimate Medical Loss on Catastrophic Claims (000)	Catastrophic Medical Severity Index	Trended Medical Loss on Catastrophic Claims (000)	Average Cost per Catastrophic Claim Trended To 1998
(1)	(2)	(3)	(4)	(5)	(6)
1978	0	0	1.000	0	0
1979	0	0	1.110	0	0
1980	2	1,800	1.232	8,782	4,391
1981	0	0	1.368	0	0
1982	1	1,500	1.518	5,939	5,939
1983	0	0	1.685	0	0
1984	0	0	1.870	0	0
1985	2	4,500	2.076	13,029	6,514
1986	0	0	2.305	0	0
1987	0	0	2.558	0	0
1988	0	0	2.839	0	0
1989	2	5,500	3.152	10,490	5,245
1990	1	3,500	3.498	6,014	6,014
1991	0	0	3.743	0	0
1992	1	3,500	4.005	5,253	5,253
1993	2	8,000	4.286	11,220	5,610
1994	0	0	4.586	0	0
1995	2	6,000	4.907	7,350	3,675
1996	1	7,500	5.250	8,587	8,587
1997			5.618		
1998			6.011		
Total	14	41,800		76,663	5,476
Selected					5,500

Medical Inflation Rate from 1982 to 1990	11.0%
Medical Inflation Rate from 1990 to 1998	7.0%

Appendix 7

Workers Compensation Reported Catastrophic Claim Count Emergence

Accident Year	Evaluation Age in Months											
	12	24	36	48	60	72	84	96	108	120	132	144
1976												
1977												
1978												
1979	1	1	2	2	2	2	2	2	2	2	2	2
1980	1	2	2	2	2	2	2	2	2	2	2	2
1981	0	0	0	0	0	0	0	0	0	0	0	0
1982	1	1	1	1	1	1	1	1	1	1	1	1
1983	0	0	0	0	0	0	0	0	0	0	0	0
1984	0	0	0	0	0	0	0	0	0	0	0	0
1985	1	2	2	2	2	2	2	2	2	2	2	2
1986	0	0	0	0	0	0	0	0	0	0	0	0
1987	0	0	0	0	0	0	0	0	0	0	0	0
1988	0	0	0	0	0	0	0	0	0	0	0	
1989	1	2	2	2	2	2	2	2	2	2		
1990	0	0	1	1	1	1	1	1	1			
1991	0	0	0	0	0	0	0	0				
1992	1	1	1	1	1	1	1					
1993	1	2	2	2	2	2						
1994	0	0	0	0	0							
1995	1	2	2	2								
1996	1	1	1									
1997	0	0										

Accident Year	Age Interval in Months											
	12 to 24	24 to 36	36 to 48	48 to 60	60 to 72	72 to 84	84 to 96	96 to 108	108 to 120	120 to 132	132 to 144	144 to 156
1976												
1977												
1978	0	1	0	0	0	0	0	0	0	0	0	0
1979	1	0	0	0	0	0	0	0	0	0	0	0
1980	0	0	0	0	0	0	0	0	0	0	0	0
1981	0	0	0	0	0	0	0	0	0	0	0	0
1982	0	0	0	0	0	0	0	0	0	0	0	0
1983	0	0	0	0	0	0	0	0	0	0	0	0
1984	1	0	0	0	0	0	0	0	0	0	0	0
1985	0	0	0	0	0	0	0	0	0	0	0	0
1986	0	0	0	0	0	0	0	0	0	0	0	
1987	0	0	0	0	0	0	0	0	0	0		
1988	1	0	0	0	0	0	0	0	0			
1989	0	1	0	0	0	0	0	0				
1990	0	0	0	0	0	0	0					
1991	0	0	0	0	0	0						
1992	1	0	0	0	0							
1993	0	0	0	0								
1994	1	0	0									
1995	0	0										
1996	0											
1997												
	All Year Average											
[1]	0.263	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Selected											
Selected	0.263	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Cumulative	0.374	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Appendix 7

Workers Compensation Reported Catastrophic Claim Count Emergence

Accident Year	156	168	180	192	204	216	228	240	252	264
1976										
1977										
1978										
1979	2	2	2	2	2	2	2			
1980	2	2	2	2	2	2	2			
1981	0	0	0	0	0	0				
1982	1	1	1	1	1					
1983	0	0	0	0						
1984	0	0	0							
1985	2	2								
1986	0									
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										

Accident Year	156 to 168	168 to 180	180 to 192	192 to 204	204 to 216	216 to 228	228 to 240	240 to 252	252 to 264	264 to Ult
1976										
1977										
1978	0	0	0	0	0	0				
1979	0	0	0	0	0	0				
1980	0	0	0	0	0					
1981	0	0	0	0						
1982	0	0	0							
1983	0	0								
1984	0									
1985										
1986										
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996										
1997										
[1]	0.000	0.000	0.000	0.000	0.000	0.000				
Selected	0.000	0.000	0.000	0.000	0.000	0.000				
Cumulative	0.000	0.000	0.000	0.000	0.000	0.000				

Markovian Annuities and Insurances

Thomas Struppeck, FCAS, MAAA, ASA

Markovian Annuities and Insurances

Abstract

Traditionally, property and casualty products have been thought of as "short duration contracts", while life insurance products have been thought of as "long duration contracts". Many modern property and casualty products have risk profiles and cash flow characteristics that are more akin to life insurance than to traditional property and casualty lines. In this paper, using bond insurance as a primary example, we show how such products can be priced and reserved using techniques from the capital markets and from life insurance.

The "life reserves" held by life companies are essentially premium deficiency reserves in that they are required not to pay losses that have occurred, but rather to make up the shortfall in future premium collections. Since bond insurance is so similar to life insurance, it is no surprise that the appropriate reserves for bond insurers are also premium deficiency reserves.

Introduction

Many insurance pricing and reserving problems can be phrased as questions about the value of a contingent annuity. This annuity might represent an anticipated stream of premium payments or a stream of loss payments. Typically, the stream of payments will terminate when a certain event occurs. This paper describes how to price and reserve for what we will call "Markovian annuities" and insurance products associated with them. Our main example will be bond insurance.

Markovian annuities are in some sense generalizations of level premium life insurance and, also for example, catastrophe reinsurance from the property and casualty side. As we will see, traditional life insurance pricing and reserving techniques suggest methods for valuing certain property and casualty reserves. These generalized methods in turn may be useful to life actuaries evaluating business priced with select and ultimate tables.

The paper is broken into thirteen sections. The first is this introduction, followed by a section describing what we will call the "risk-neutral world". Then Markov processes are discussed and a simple example is given. We then digress a little bit to discuss rating agencies. We then return to the topic of perpetuities and tie the first part of the paper together by introducing the notion of bond insurance.

We begin the second half of the paper by seeing how insurance can be used to turn risky assets into risky liabilities on an insurer's balance sheet. Valuing these liabilities is one of the central topics of this paper. To accomplish this, we first review the notion of a replicating portfolio, an idea that has its origins in the capital markets. Having built this machinery, we are finally ready to analyze bonds. The next two sections contain some remarks on accounting considerations and a detailed example. Finally, we make some concluding remarks and have a short bibliography.

The author would like to thank the Committee on Reserves for sponsoring the call, and to thank in particular the colleagues who read early drafts of this paper for their many helpful comments.

Perpetuities and the Risk-Neutral World

For ease of exposition, we will make several simplifying assumptions. None of these is necessary for what follows, but relaxing them introduces unnecessary complications that might mask what is really going on. Here and throughout the paper we will assume:

1) A flat, constant yield curve with an interest rate of 8%.
2) An unlimited supply of risk neutral investors willing to purchase or sell any stream of future cash flows, contingent or certain, at its expected present value.

3) No reporting lag.

4) Losses are paid at the end of the year.

5) Finally, assume that all losses occur at the end of the year.

Initially at least, we will examine perpetuities and contingent perpetuities. By a **contingent perpetuity** we mean a stream of payments of \$1.00 at the end of each year that terminates when a certain event occurs. The occurrence of this event we will call a **default**. A contingent perpetuity that cannot default we will call a **risk-free perpetuity**.

Contingent perpetuities are quite general; for example a life annuity payable to a 40-year old could be considered as a contingent perpetuity, the terminating event in this case being the annuitant's death.

As a first example, let's compute the market price in our risk-neutral world of a risk-free perpetuity. Denote by $\mathbf{a}_{rf}$ the market price of this perpetuity in our risk neutral world and let $v = 1/(1+i) = 1/1.08 = .926$ be the discount rate. We have:

$$\mathbf{a}_{rf} = v\,(1 + \mathbf{a}_{rf}).$$

That is, an investor is ambivalent between having the perpetuity today and having the present value of a portfolio consisting of the dollar that the perpetuity will pay in one year and another perpetuity one year from today. Equivalently, in the language of interest theory, an investor is ambivalent between a perpetuity-immediate and the present value of a perpetuity-due.

Solving, we obtain the familiar:

$\mathbf{a}_{\mathbf{rf}} = v/(1 - v) = 1/i = 1/0.08 = 12.5$

Remark: If we had been evaluating an annuity that had a fixed number of payments, the annuity that we have after one year would not be identical to our initial annuity (it would have one less year remaining). In essence, perpetuities do not age, and this fact makes them easier to handle. This is an example where evaluating an infinite sum is easier than evaluating the corresponding finite sum.

Next we will evaluate a contingent perpetuity with a terminating event, but first we need a definition.

Markov Processes

A **(discrete) Markov process** is a stochastic process where the state at time t+1 depends only on the state at time t. Formally, it is a triple ($\mathbf{S}$, $\mathbf{p}$, $\mathbf{s}_0$) where:

$\mathbf{S}$ is the set of "states"

$\mathbf{p}$ is a function that given an element of $\mathbf{S}$ returns a probability measure on $\mathbf{S}$ and

$\mathbf{s_0}$ is an element of $\mathbf{S}$ called the **initial state**.

For our purposes, the set of states will be finite with, say, n elements. In this case, the mapping $\mathbf{p}$ can be expressed as an nxn matrix, called the **transition matrix**. The entries in the matrix will be real numbers between 0 and 1, inclusive. Also, each row of the matrix will sum to 1; such a matrix is called a **stochastic matrix**[1].

An Example

Suppose that every year there is a 10% chance of an earthquake of a certain magnitude. Our set of states will consist of two states: "no quake yet" (or NQY) and "had quake" (or HQ). Our transition matrix is 2x2 and looks like this:

[1] There is a vast literature on Markov processes; a good introduction is [R].

	NQY	HQ
NQY	0.90	0.10
HQ	0.00	1.00

The first row says, if we haven't had a quake yet, then there is a 90% chance that we won't have one this year and a 10% chance that we will. The second row simply says, if we have already had a quake, then we have already had a quake! Also, we will suppose that the initial state is NQY.

We now have the three ingredients needed to have a Markov process, namely, the set of states, the transition matrix, and the initial state. We will return to this example after a final definition.

Suppose that we have a Markov process, $(\mathbf{S}, \mathbf{p}, \mathbf{s_0})$. From the set of possible states, $\mathbf{S}$, we select a subset $\mathbf{T}$ and call these **terminating states**. Consider now a contingent perpetuity that pays $1.00 at the end of each period until the Markov process enters one of the states in $\mathbf{T}$ at which point it permanently stops paying and becomes worthless. Such a contingent perpetuity we will call a **Markovian annuity**.

As an example, consider a life annuity on a 40 year-old. Let the set of states be his possible ages ("40", "41", "42", ...) along with a special state, "Dead". And let the transition probabilities be given by the life table (i.e., for each age N, state "N" goes to state "N+1" with probability $\mathbf{p}_N$ and to state "Dead" with probability $\mathbf{q}_N$). If we define "Dead" to be the terminating state, then this life annuity is a Markovian annuity.

Casualty actuaries reserving for certain worker's compensation claims, such as "permanent totals" and "permanent partials" already use similar techniques. In fact, in some jurisdictions, these are the only reserves that insurers can discount. This is the so-called "tabular discount" in statutory accounting.

Returning to our earthquake example from above, if we let the state HQ ("had quake") be the terminating state we can value the Markovian annuity that pays $1.00 at the end of each year until there is a quake. Denote this perpetuity by $\mathbf{a}_{eq}$. We have:

$$\mathbf{a}_{eq} = v\,(1+\mathbf{a}_{eq})(.90)$$

This says that in our risk neutral world an investor is ambivalent between owning this annuity today and having the discounted value of a portfolio consisting of $1.00 and the annuity, a year from now, if he gets it. The difference between this formula and the formula for a risk-free perpetuity is the final factor of .90, which is the annual probability that the perpetuity does not default. Using the fact that the interest rate is 8% and solving, we obtain:

$$\mathbf{a}_{eq} = 5.$$

Observe that this is only 40% of the value of the risk-free perpetuity, $\mathbf{a}_{rf}$, which we earlier showed has value 12.5, even though the only difference between the two is a 10% annual default probability.

Suppose that an investor has $5,000 to invest. He could buy 400 risk-free perpetuities ("the risk-free portfolio") or 1,000 of these earthquake perpetuities ("the risky portfolio"). Assume for the moment that the default events are all independent. After one year, with the risk-free portfolio he will have on average the 400 perpetuities that he started with (no defaults) and $400 in cash. The market value of this portfolio is $5,400. With the risky portfolio at the end of one year, he will have (on average) 900 non-defaulted perpetuities and each of them will have paid him $1.00, so he will have $900 in cash. The market value of this portfolio is 900*5 + 900 = 5,400 --- the same as the risk-free portfolio.

The (expected) return of the risk-free portfolio consisted of interest of 400 (the cash) and capital gains of 0 (no defaults). The (expected) return of the risky portfolio consisted of interest of 900 (the cash) and capital gains of –500 (the value of the 100 defaulted perpetuities which are now worthless). This must be so, because in the risk neutral world all investments have the same expected returns (8%).

This example had only two states, defaulted and non-defaulted. In the next section we will consider an example that has four states and is considerably more interesting. To motivate it, we will briefly discuss rating agencies.

Rating Agencies

In our risk-neutral world securities are priced at their expected present values. In order to compute these expectations, investors need to know what the probabilities are that various cash flows will actually occur. In our earthquake example, all investors knew that the annual probability of an earthquake (default) was 10%. How do they obtain this information?

In our simplified risk neutral world (and in the real world) there are entities called rating agencies. Rating agencies evaluate investments and estimate the probabilities that various payments will be made. In our simple world, the rating agencies classify all risky perpetuities into one of four classes named A, B, C, and D.

Securities rated B by the rating agency are considered more risky (likely to default) than those rated A; those rated C are even more risky than those rated B; those rated D have already defaulted and are now worthless[2]. Each year the rating agency reevaluates each security and reclassifies it. Movements between the various non-defaulted classes are described as follows: if a security is now less risky than it was before (i.e. its rating has gone from B to A, C to B, or C to A) we say that the security has been **upgraded**; securities that are now riskier than before (A to B, B to C, or A to C) are said to have

[2] Real world defaulted securities may not be worthless. Estimating the amount of recovery available from a defaulted security is generally a difficult problem on which much research has been done. For simplicity, we will assume that the recovery is zero.

been **downgraded** and finally, securities that are left at their previous risk levels are said to have had their ratings **reaffirmed**.

The movements between rating classes in our simple world is given by the following transition matrix:

	A	B	C	D
A	0.90	0.05	0.04	0.01
B	0.09	0.81	0.05	0.05
C	0.01	0.14	0.75	0.10
D	0.00	0.00	0.00	1.00

Suppose that we wish to determine the price of an A-rated perpetuity, $\mathbf{a_A}$. Under the transition matrix, we have a Markovian annuity. To price this, we proceed as before:

$$\mathbf{a_A} = v\,(.90\,\mathbf{a_A} + .05\,\mathbf{a_B} + .04\,\mathbf{a_C} + (1-.01)\,(1))$$

where $\mathbf{a_B}$ and $\mathbf{a_C}$ are B-rated and C-rated perpetuities, respectively.

This comes directly from the first row of the transition matrix. An investor is ambivalent between an A-rated perpetuity today and the present value of a portfolio which contains an A-rated perpetuity 90% of the time, a B-rated perpetuity 5% of the time, a C-rated perpetuity 4% of the time, and $1.00 that is paid unless the original perpetuity has defaulted (non-default = 99%).

Before, we had one equation in one unknown. Now it appears that we have one equation in three unknowns. Fortunately, there are more rows of the transition matrix and these supply us with more equations, namely:

$\mathbf{a_B} = v\,(.09\,\mathbf{a_A} + .81\,\mathbf{a_B} + .05\,\mathbf{a_C} + (1 - .05)\,(1))$ and

$\mathbf{a_C} = v\,(.01\,\mathbf{a_A} + .14\,\mathbf{a_B} + .75\,\mathbf{a_C} + (1 - .10)\,(1))$

Now we have three linear equations in three unknowns. Solving we obtain:

$\mathbf{a_A} = 9.027$

$\mathbf{a_B} = 7.687$ and

$\mathbf{a_C} = 6.262$

These are the market prices for risky perpetuities in our risk-neutral world; we will use these prices in the following sections.

Real world rating agencies such as Standard & Poor's (S&P) and Moody's Investors Service (Moody's) have much more refined class plans than we have shown here. Not only are there generally more rating classes, but also rating agencies will sometimes indicate that a rating is "on watch". This frequently means that a rating change is being considered or that new news is expected. Rating agencies serve an important role in financial markets by reducing information asymmetries between issuers and investors. Rating agencies are discussed more fully in [F], [M], and [W].

A final comment on transition matrices, the transition matrix describes the migration over time among the various rating classes. A portfolio initially consisting only of A-rated securities will, over time, become more risky as some of the securities get downgraded. On the other hand, a portfolio that consists of only C-rated securities will, over time, get less risky as securities get upgraded. Here we are only looking at the surviving (non-defaulted) securities. Is there a portfolio that maintains its riskyness over time?

It turns out that the answer is yes. This "eigenportfolio" for lack of a better name, is related to the dominant (left) eigenvector of a certain submatrix of the transition matrix. The corresponding eigenvalue turns out to be one minus the average default rate for the "eigenportfolio". As the reader may check, for the transition matrix given earlier, a portfolio consisting of 50.32% A-rated securities, 32.49% B-rated securities, and 17.19% C-rated securities will (in expectation) maintain its proportions over time, the eigenvalue in this case being 0.96153 and the average default rate being 0.03847.

Transition matrices appear in many fields of study. For example, they are used to study population dynamics in mathematical ecology where they are called "Leslie matrices".

Leslie matrices are named after P.H. Leslie who introduced them into biology in the mid-forties. See [L].

Perpetuities

Suppose that a company wishes to raise funds in our risk neutral world. The company wants to borrow \$1,000. In exchange for \$1,000 today the company will pay annual interest until it defaults. Further suppose that our company is rated "B" by our rating agency. Recall from our previous calculations that $\mathbf{a_B}$ a B-rated perpetuity paying \$1.00 each year has a value of \$7.687. We wish to find the amount of the coupon, K, that must be paid so that the market price of the security will be exactly \$1,000.00. In symbols:

$$1{,}000 = K\ \mathbf{a_B}$$

That is, an investor is ambivalent between keeping his \$1,000 today and getting the present value of a perpetual stream of payments of \$K annually until default. Replacing $\mathbf{a_B}$ with its value, \$7.687, and dividing we obtain:

$$K = \$130.09$$

Recall that in the risk neutral world, all investments are expected to yield 8%. The investor has only invested $1,000.00, so his expected yield must be 8% of this, namely $80.00. The "extra" $50.09 (=130.09 – 80.00) is compensation for the expected change in the market price of the perpetuity (a capital gain or loss). There are four possible outcomes. It is possible that the perpetuity had defaulted; in this case the investor gets no coupon payment and owns a worthless security. The other possibilities are that the perpetuity has been downgraded, upgraded, or has had its rating affirmed.

Notice that the coupon amount is fixed when the security is issued, and that subsequent upgrades or downgrades do not change the amount of the coupon. Suppose that the perpetuity has been downgraded, so it now is rated "C". The investor will still receive $130.09 per year until default, but now default is expected sooner. We previously computed the value of a stream of $1.00 payments from a C-rated security when we learned that $\mathbf{a}_C$ had a value of $6.262. Using this fact, we can find the market value of the downgraded security. It pays $130.09 per year, so its market value must be:

$$130.09\,\mathbf{a}_C = \$814.62$$

On a mark-to-market basis, the investor has suffered a loss, even though no cash payment has been late or missed. It is generally believed that investors like to get their principal back (although in the risk neutral world they really don't care provided that the coupon is adequate). Real world bonds have maturity dates when the principal is paid back.

Modeling this adds no real obstacles, and adds some interesting twists. To appreciate these subtleties, we will first examine perpetuities in more detail.

Bond Insurance

Suppose that our investor wants to purchase an insurance policy that will pay him $1 when his B-rated perpetuity defaults. Assuming that the insurance company cannot itself default[3], what is a fair premium for this insurance?

Denote by $\mathbf{A_B}$ the one-time premium that the insurer would charge for this insurance. In the risk neutral world, this premium is the expected present value of the benefit, so there will be no ambiguity in denoting the benefit by this same symbol. We have the tools to price this at our fingertips.

$$1.00 = 0.08\ \mathbf{a_B} + 1.08\ \mathbf{A_B}$$

What this says is: an investor is ambivalent between having $1.00 today and receiving the interest on the $1.00 ($0.08) every year until a default occurs. When the default occurs, he gets back his dollar and the final year's interest.

[3] One of the most contentious issues addressed by the white paper on fair value liabilities was related to how the fair value of a liability should depend on the creditworthiness of the parties. See [T], in particular item 15 of the Executive Summary.

This identity should look very familiar to students of life contingencies; it is the fundamental identity relating annuity values and insurance prices. The more traditional version involves annuities-due and discount rates (instead of annuities-immediate and interest rates), because life insurance premiums are paid in advance while bond interest is received in arrears. In this example we can solve and learn that the market price of this insurance is 0.3565 (recall that we computed that $\mathbf{a_B}$ = 7.687 in an earlier section).

Suppose that an investor has $1,000 to invest. He elects to purchase a B rated perpetuity that will pay him $80/year (at a cost of 80 * 7.687 = 614.97) and he uses the rest to purchase an insurance policy that will pay him $1,080 when this perpetuity defaults (at a cost of 1,080 * .3565 = 385.03). He has now spent his $1,000 and he has created a synthetic risk-free bond. This bond will pay him $80/year until a default occurs at which point the insurance pays at the end of the year the final interest payment and the principal.

Suppose that a second investor purchases for $1,000 a B-rated perpetuity (which we learned earlier pays annual coupons of 130.09). If he now insures the perpetuity (for his principal plus the risk-free interest on it, i.e. $1,080), but arranges to pay premiums annually in arrears while the perpetuity has not defaulted, what will his annual premium be?

Well, he too has, in effect, turned his risky perpetuity into a risk-free perpetuity. His investment is $1,000, so he is entitled to exactly $80 per year (8%). The difference between the promised coupon, $130.09, and the risk-free coupon, $80.00, must be the insurance premium charged (if not an arbitrage would result)[4]. Bond traders call this difference the **spread**.

There is an interesting relationship between the spread and the default rate. To see it, consider a one year bond which will either default and be worthless (probability = 20%) or will mature and will pay $1,350 in one year (probability = 80%). What would an investor in the risk-neutral world pay for this bond?

The expected present value (at 8%) of this investment is $1,000. So the spread is 27%[5] The default probability is only 20%. The extra 7% is needed because only non-defaulted bonds pay the coupon. The 27% can be thought of as an assessment on the surviving bonds (80%) to pay the principal (100%) and the risk free interest on it (8%) for the defaulting ones (20%). We have:

Spread = 1/(1 – default) * (1 + risk-free) * (default)

0.27 = 1/(1 – 0.20) * (1 + 0.08) * (.20)

[4] Arbitrage opportunities are discussed in a subsequent section.
[5] Spreads are normally quoted in hundredths of a percent, called **basis points**; so, a 27% spread would be said to be a 2,700 basis point spread.

It is interesting to note that the above formula suggests that spreads should widen with increases in the risk-free rate, and that this effect should be more pronounced for worse credits.

Turning Assets Into Liabilities

By using bond insurance as described in the previous section, an investor can take a risky asset portfolio and turn it into a risk-free portfolio. The risk gets transferred to an insurance company where it resides on the liability side of the balance sheet. How should an insurance company account for contracts of this type? What constitutes a loss? How should reserves be valued?

Suppose that an entity purchases a risky perpetuity for $1,000 and insures it. We have seen that the premium paid will be the spread above the risk-free rate and that the insured amount will be $1,080, which is the $1,000 face amount plus the risk-free return (8%).

How does this look from the insurer's point of view? The insurer expects to receive the spread income until the year of the default. At the end of that year, the insurer will pay the $1,080 claim. A moment's thought reveals that the premium stream that the insurer expects to receive is, in fact, a Markovian annuity.

Suppose that we were insuring a B-rated perpetuity. At the end of the year, there are four possible states:

1) It has been upgraded (now rated A).
2) It has had its rating reaffirmed (now rated B).
3) It has been downgraded (now rated C).
4) It had defaulted (now rated D).

In the fourth case, we have paid the loss and there is no reserve. In the second case (rating has been affirmed), we will be receiving as premium the spread on a B-rated bond for insuring a B-rated bond. This premium is, of course, exactly adequate.

If the bond has been downgraded, however, the future spread income is no longer adequate. The expected future premium after the downgrade is $\mathbf{S_B}\ \mathbf{a_C}$, where $\mathbf{S_B}$ denotes the spread on a B-rated perpetuity. The required future premium becomes $\mathbf{S_C}\ \mathbf{a_C}$, where $\mathbf{S_C}$ denotes the spread on a C-rated perpetuity. The shortfall is $(\mathbf{S_C} - \mathbf{S_B})\ \mathbf{a_C}$.

Notice that increase in the bond's mortality contributes in two distinct ways to the shortfall. Not only has the expected future premium income decreased by $\mathbf{S_B}\ (\mathbf{a_C} - \mathbf{a_B})$, but also the required premium has increased from $\mathbf{S_B}\ \mathbf{a_B}$ to $\mathbf{S_C}\ \mathbf{a_C}$. Effectively, fewer premium payments are expected and, additionally, the expected loss payment has been accelerated.

The total shortfall in future premium should be recognized on the balance sheet (and the income statement) as an increase in the premium deficiency reserve. The appropriate accounting treatment of such changes in value is discussed briefly in the Accounting Considerations section.

The fourth possibility is an upgrade. In this case, the future premium income is excessive and under fair-value accounting this too would be reflected in the reserve for unexpired risks. Under codification, it appears that the negative premium deficiency could be used to offset premium deficiencies from other insured perpetuities (ones that had been downgraded), provided that management groups these together for internal reporting. Again, this will be discussed in more detail in the later section.

Remark: There is an important principle here. Memoryless ➔ No Reserve.

This is the case for constant mortality in whole life insurance and it is true here as well. Recall that for a whole life policy the reserve is really a premium deficiency reserve. Typically, premiums are level, but at most ages human mortality is increasing, so early on the premium is more than is needed for current mortality (the difference going into the reserve). Later on the premium is inadequate for the current mortality (but the reserve is there to fund the shortfall). In the constant mortality case, the level (constant) premium exactly matches the current (constant) mortality at all ages, hence there is no need for a reserve. In the same way for perpetuities, if at the end of the year there has been no change in rating (i.e. mortality has stayed constant), then there will be no change in the reserve[6].

While this holds for perpetuities, it does not in general hold for bonds. The difference is that over time bonds approach maturity, when the principal becomes due. A (non-defaulted) maturing bond pays its principal payment regardless of its rating. A risky bond one year from maturity and a risky bond two years from maturity may have very different prices. The life insurance analog of this phenomenon is that an endowment policy even with constant mortality still will build up a reserve (to pay the endowment amount at maturity). We will see how the prices of risky bonds change over time in a following section, but first we will examine a technique from the capital markets used for pricing risky cash flows.

Replicating Portfolios

Reserving frequently involves estimating the value of a collection of future cash flows. A very elegant technique for valuing such flows comes from modern finance theory. The crux of the idea is extremely simple: if two collections of cash flows are identical, then they must have identical prices.

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[6] These are premium deficiency reserves and, as such, should be carried at discounted value. The annual unwind in the reserve is exactly enough to make up for the annual deficiency in premium.

Suppose that we have a collection of (contingent) cash flows that we wish to value. We try and find a second collection of securities that taken together have cash flows identical with our collection in all states of the world. For example, if the first one pays a dollar when there is a particular earthquake, the second one must also pay a dollar for the same earthquake. Such a collection is called a **replicating portfolio** for the first collection. Generally, it will be difficult to find such a portfolio because it must match exactly in all cases. However, if you are lucky enough to find one and the securities have market prices, then you have found the market value of your set of cash flows.

Let's look at some simple examples. Suppose that available in the market are three securities, all newly issued, risk-free annuities-immediate with terms of 3, 5, and 10 years, respectively. The market prices in our risk-neutral world for these annuities are given in the next table. (What is especially nice about this approach is that if you have real-world prices for these securities, you get the real-world price of your liability!)

$\mathbf{a}_3 = 2.577$

$\mathbf{a}_5 = 3.993$ and

$\mathbf{a}_{12} = 7.536$

Suppose that we wish to reserve for a stream of payments of \$8 for three years followed by \$2 for nine more years. A moment's thought reveals that this stream of payments can be obtained by buying 6 of $\mathbf{a}_3$ and 2 of $\mathbf{a}_{12}$. (Both of these types of annuities pay during the first three years yielding eight dollars per year; for the last nine years only the second type pays, yielding the required two dollars per year.) The cost of this portfolio is \$30.535 (= 6(2.577) + 2(7.536)) and, since it matches our payment stream exactly, is the market price of our liability.

As a second example, consider an obligation to pay \$1 per year for seven years starting in five years. We would like to reserve for this stream of payments by finding the market value of this liability. This is a 5-year deferred, seven-year annuity. It can be replicated as follows: purchase an $\mathbf{a}_{12}$ and sell an $\mathbf{a}_5$. You may wonder how we can sell something that we don't own, but for the moment, assume that this transaction can be done. What are the cash flows from the resulting portfolio? Well, in years one through five, we receive a dollar from the $\mathbf{a}_{12}$. The investor that purchased the $\mathbf{a}_5$ from us expects to receive a dollar. We take the dollar that we get from the $\mathbf{a}_{12}$ and give it to the purchaser of the $\mathbf{a}_5$. The investor is happy because he does not care which dollar he gets, he just wants a dollar to be paid to him at the end of each of five years. At the end of year five, the $\mathbf{a}_5$ makes its last payment and expires worthless. In years six through twelve we receive one dollar from the original $\mathbf{a}_{12}$. This exactly matches the payments that we will make on the deferred annuity, so this is a replicating portfolio. How much does this portfolio cost? Well, we know that we can buy an $\mathbf{a}_{12}$ for 7.536, since that is its market price. We can also sell an $\mathbf{a}_5$ for 3.993, since that is its market price, so the net cost of the portfolio is 3.543 (= 7.536 – 3.993). This is the market price for our liability.

Something interesting has happened; we have been able to compute the exact market price for this liability even though no market for it (directly) exists.

In the last example, we bought one annuity and sold another; practitioners would describe this as a **long position** in the $\mathbf{a}_{12}$ and a **short position** in the $\mathbf{a}_5$. We will use this terminology in what follows. We need to define one more term.

A portfolio with some positive cash flows, no negative cash flows, and zero net cost is called a **risk-free arbitrage opportunity**. Such a portfolio would also be a tremendous bargain! So much so, that there would be unlimited demand for it. This demand would be so great that it would cause market prices to shift to eliminate the opportunity. There are no risk-free arbitrage opportunities in the risk-neutral world, and it is generally believed that there are none in the real world either.

Suppose that two portfolios have identical cash flows, then they must have identical prices. Here is why. Suppose that the prices were different, then we would short the more expensive one (sell it) and go long the cheaper one (buy it). The resulting portfolio would have a positive cash flow at time zero (the difference in the prices), have no net cost, and would have no negative cash flows, so it would be a risk-free arbitrage opportunity. There would be unlimited selling pressure on the more expensive one, pushing its price down, while there would be unlimited buying pressure on the cheaper one, driving its price up. This process would continue until the two prices were equal.

The reader may have come across replicating portfolios before in studying the Black-Scholes solution to the call option-pricing problem. See for instance, [B].

Bonds

As previously noted, in the real world investors like to get their principal returned to them. A newly issued bond may have a maturity of thirty years. Such a bond will pay annual interest at the end of each of the first twenty-nine years and then will pay back the principal amount and the final year's interest at the end of year thirty. Of course, along the way, the bond may default.

Issuers tend to set the coupon so that their bonds will sell "at par". That is, they generally adjust the spread that they offer to pay, so that a bond with $1,000 in principal will sell for $1,000 at issue. Table 1, below, shows the annuity values and required coupon amounts for newly issued C-rated bonds to trade at par.

A comment on how the annuity values are computed is in order. The annuity values are computed recursively from the transition matrix. One year from maturity, the bond either defaults (probability = 10%) or it matures (probability = 90%). With i = 8%, we find the value of a one year C-rated annuity to be 0.9(1/1.08) = 0.83333. The values of A-rated and B-rated one-year annuities are found similarly. Once these values are in hand, we can value two year annuities using the transition matrix as we did above for perpetuities, then recursively we can compute the values for longer term annuities. The results are shown in Table 1.

TABLE 1

Newly issued C-rated bonds

Years to Maturity (N)	Actuarial PV of Principal	Annuity Value	"Required" Coupon	Actuarial PV of Coupons
1	833.33	0.83333	200.00	166.67
2	701.22	1.53455	194.70	298.78
3	595.12	2.12967	190.12	404.88
4	508.84	2.63851	186.15	491.16
5	437.88	3.07639	182.72	562.12
6	378.90	3.45529	179.75	621.10
7	329.39	3.78468	177.19	670.61
8	287.48	4.07216	174.97	712.52
9	**251.74**	**4.32389**	173.05	748.26
10	221.05	4.54494	**171.39**	778.95

Suppose that a firm issues for $1,000 a 10-year C-rated bond and that one year later the bond is still C-rated. What is the market price of the bond now?

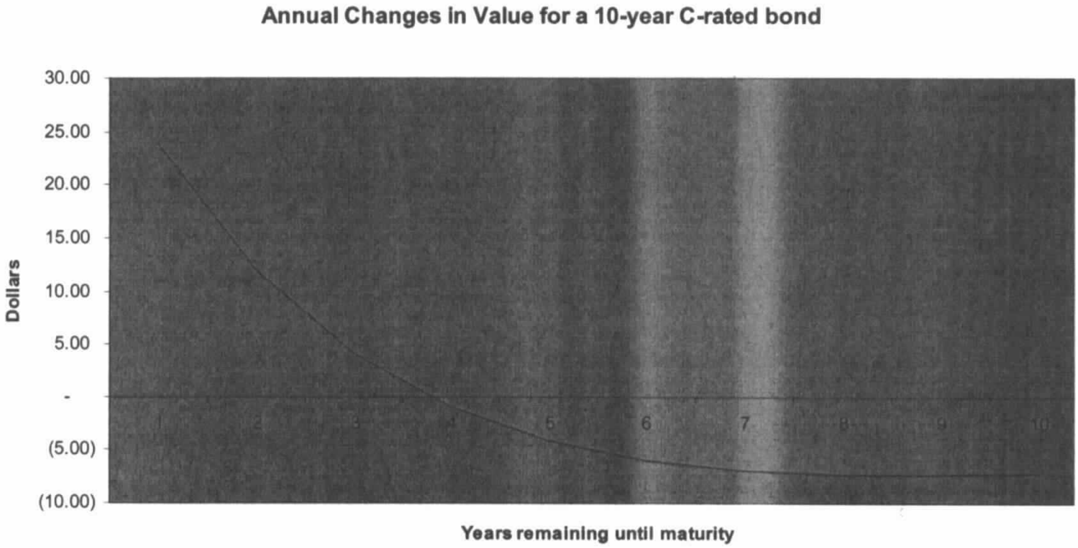
The bond when issued was a 10-year bond and one year has passed, so it is now a 9-year bond. It is still rated "C", so Table 1 contains all of the information that we need. From column 2 we learn that the principal amount has an actuarial present value of $251.74.

From column 3 we see that each dollar of coupon has an actuarial present value of \$4.32389. Now the coupon gets set when the bond is issued, so it is still 171.39 (from column 4, row 10). Combining all of the information we see that the market price is

251.74 + 171.39(4.32389) = 992.80

The bond was worth \$1,000.00 at issue, but now it is worth only \$992.80. There has been no default nor has there been a downgrade, but the owner of the bond still lost \$7.20 (in market value). Figure 1 shows the annual change in the market price of this bond assuming that its rating never changes over its life.

Figure 1



There are two competing forces affecting the bond price. Reviewing Table 1, we see that the required coupon increases as the time to maturity decreases, since the coupon is fixed at the 10-year value as the bond approaches maturity the coupons become more and more inadequate, pushing the price down. On the other hand, the actuarial present value of the principal payment rapidly increases as maturity nears. The combined effect is shown in Figure 1, where we can see that the coupon effect dominates when there are many years left to maturity, but when the bond is close to maturity the value of the principal starts to dominate.

Suppose that you are an insurer and that you have insured a 10-year C-rated bond against default. If one year has passed and the bond is still rated "C", you should put up a reserve. In particular, you should carry a premium deficiency reserve sufficient to allow you to reinsure your risk[7]. A loss reserve is not appropriate, because the covered event is default and default has not occurred. On the other hand, even in the risk-neutral world a reinsurer would require compensation in order to take over your current position. The amount that the reinsurer would require is exactly the difference between the current market price of the bond and the principal amount.

To see this we will create a replicating portfolio that exactly duplicates the cash flows of that the insurer will have to pay out. The cost of this perfect reinsurance will be the cost of the replicating portfolio. The required portfolio is a short position in the risky bond

[7] In the risk-neutral world, reinsurers will assume risks for the difference between their expected future discounted premiums and their expected future discounted losses.

(principal amount = $1,000) and a long position in a risk-free security (principal amount = $1,000). We will check the cash flows in each possible scenario.

During years when the bond does not mature and does not default, we receive premium equal to the spread, and investment income from the risk-free bond. The sum of these is exactly the coupon payment that we need to make on our short position, so we have no net flows. In the year that the bond matures if there is no default, things are exactly as in the previous case except that we need to pay the principal on our short position, we do this with the principal from the risk-free security. The short position is now closed, and the insurance has expired without a claim: no net cash flow, no outstanding liabilities (nor assets) remain. Finally, if there is a default, we sell the risk-free security (for $1,080); this is exactly the insured amount of the bond (recall that the policyholder insures the bond for principal and risk-free interest). In all three cases there are no net cash flows. That is, the portfolio exactly hedges the insurance policy and the cost of the portfolio is exactly what a reinsurer would charge (in the risk-neutral world) to take this risk from your books.

The cost of this portfolio is the difference between the cost of the risk-free bond ($1,000.00) and the market price of the risky bond which we earlier calculated to be $992.80 (the value of a 9-year C-rated bond, paying a 10-year C-rated coupon).

One might wonder why a premium deficiency arises in this case. We started with a C-rated bond and one year later we still had a C-rated bond --- no default, yet it appears that we have a loss. The reason is that in some sense you have had bad luck. While nothing explicitly bad has happened (a default), nothing good has happened either (an upgrade). The market had already priced the possibility of an upgrade into the required coupon. When the upgrade did not occur, the market price reflected the lack of good news.

Accounting Considerations

The NAIC's statutory accounting codification project now requires an estimation of the premium deficiency reserve for all property/casualty companies. Because of our simplifying assumptions (no reporting lag, losses and payments occurring only at the end of a year) the types of insurance products described here do not generate loss reserves, but they will generate premium deficiency reserves.

Accounting practice seems to be to earn spread income as it is received. Assuming that the spread income is treated as written when received, the insurer will carry no unearned premium reserve for these products. We have seen that earning the spread as received is exactly correct for perpetuities because of their memoryless feature. However for bonds, a premium deficiency could arise.

Should contracts such as bond insurance be treated as insurance at all? Guidance on this point under International Accounting Standards (IAS) rules can be found in [S]. Subissue 1-G states the Steering Committee's view that a contract is to be treated as insurance (and would come under IAS 37) if the triggering event is a failure "to make payment when due". However, if the triggering event were a downgrade, it would be treated as a financial instrument (and would come under IAS 39).

Under US GAAP, the line of demarcation seems less clear. FAS 133 covers derivatives and FAS 60 covers insurance. FAS 133 explicitly excludes "insurance" from its scope. I would presume then that bond insurance would be insurance, however it is not clear to me how a policy that protected against a rating agency downgrade would be treated under US GAAP. Anecdotally, I have heard that in the past "downgrade insurance" has been treated as insurance by some auditors, but I do not know if this is standard practice.

Assuming that these contracts are appropriately accounted for as insurance, they will generate premium deficiency reserves. Some contracts will generate positive premium deficiencies and others may generate negative premium deficiencies. Under codification, to the extent that management groups these contracts together for internal reporting they should be offset against one another for statutory accounting purposes, with only a net premium deficiency, if any, being reported.

Reserving the World Series

In this final example we will illustrate how an arbitrage argument can be used to evaluate the value of a wager on the outcome of a series when only partial information is available.

Suppose that you have wagered $100 that team A will beat Team B in a best 4 out of 7 series. You believe that the probability that either team will win any given game is 50%. Your team (Team A) loses the first game. What is the value of your wager, given the first game result? In other words, what reserve should you be holding against the potential $100 loss?

In the risk free world, answering this question is equivalent to determining what an investor would pay you (or demand that you pay him) to take over your position. This last question we can answer through an arbitrage argument. Let R(a,b) be the amount that the investor would be willing to pay you (or that he would demand) when Team A has won "a" games, and Team B has won "b" games. The possible states of the series are pairs (x,y) where "x" and "y" are each between zero and four (but they cannot both be four). Transitions between states occur based on the outcome of the next game, state (x,y) being equally likely to go to state (x+1,y) or state (x,y+1). The initial state was (0,0). We have a Markov process.

Since the series ends when either team has won 4 games, we have:

$R(0,4) = R(1,4) = R(2,4) = R(3,4) = -100$ and

$R(4,0) = R(4,1) = R(4,2) = R(4,3) = 100$

From this we conclude that $R(3,3) = .5(-100) + .5(100) = 0$. This follows because when you have a 3-3 tie the final game is decisive.

As we continue to back-solve we learn that:

$R(2,3) = .5\ R(3,3) + .5\ R(2,4) = 0 - 50 = -50$

$R(3,2) = .5\ R(4,2) + .5\ R(3,3) = 50 + 0 = 50$

$R(2,2) = .5\ R(3,2) + .5\ R(2,3) = 50 - 50 = 0$

$R(3,1) = .5\ R(4,1) + .5\ R(3,2) = 50 + 25 = 75$

$R(1,3) = .5\ R(2,3) + .5\ R(1,4) = -25 - 50 = -75$

$R(2,1) = .5\ R(3,1) + .5\ R(2,2) = 37.5 - 0 = 37.5$

$R(1,2) = .5\ R(2,2) + .5\ R(1,3) = 0 - 37.5 = -37.5$

$R(3,0) = .5\ R(4,0) + .5\ R(3,1) = 50 + 37.5 = 87.5$

$R(0,3) = .5\ R(1,3) + .5\ R(0,4) = -37.5 - 50 = -87.5$

$R(1,1) = .5\ R(2,1) + .5\ R(1,2) = 37.5 - 37.5 = 0$

$R(2,0) = .5\ R(3,0) + .5\ R(2,1) = 43.75 + 18.75 = 62.5$

$R(0,2) = .5\ R(1,2) + .5\ R(0,3) = -18.75 + -43.75 = -62.5$

$R(1,0) = .5\ R(2,0) + .5\ R(1,1) = 31.25 + 0 = 31.25$

$R(0,1) = .5\ R(1,1) + .5\ R(0,2) = 0\ \ -31.25 = -31.25$

So, the investor would take over your position for a payment of \$31.25. This is the reserve that you should carry for this wager. Note that it is a premium deficiency reserve, since the wager isn't lost yet, but your odds of winning have diminished.

It is interesting to note that the above calculation gives an explicit **defeasance strategy** for the wager from any point in time. A defeasance strategy is a set of explicit instructions on what bets to place and for how much to insure that the net cash flows from all of the bets exactly match the cash flows of the liability. In effect, we have explicitly exhibited a replicating portfolio of single game, even money bets that have a cumulative payoff of precisely \$100 if Team A wins the series and -\$100 if Team B wins the series.

This example is not as artificial as it might appear. A reinsurer negotiating a commutation of an inforce treaty could easily find itself in a comparable position. Determining the value of a reinsurance treaty midterm is generally a difficult problem, but if a replicating portfolio with market prices can be found, then the problem is solved.

Conclusion

Reserving actuaries need to opine on the adequacy of the unearned premium reserve for certain lines of business. Determining the existence of a premium deficiency or estimating its size can be difficult. For certain types of risks we have shown how it is possible to estimate the required premium deficiency reserve by using market prices and an arbitrage argument.

Spread income is traditionally earned as received. This is exactly correct for perpetuities that have not had their ratings changed. For bonds though, a premium deficiency can arise even if there is no change in rating.

In order to compute the premium deficiency future premium flows need to be estimated. Viewing these as Markovian annuities can facilitate this estimation. Life contingency techniques and notation, turn out to be quite convenient for this.

Life contingency texts have many formulas and identities that life reserves satisfy. Most of these have analogs for Markovian annuities and insurances. This is not surprising since such annuities are generalizations of level premium life insurance.

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Projecting Workers Compensation Losses Using Open Claim Count and Average Loss Payment, and Application to Analysis of California Workers Compensation Loss Development

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PROJECTING WORKERS COMPENSATION LOSSES USING OPEN CLAIM COUNT AND AVERAGE LOSS PAYMENT, AND APPLICATION TO ANALYSIS OF CALIFORNIA WORKERS COMPENSATION LOSS DEVELOPMENT

Michael Teng

Abstract

This paper presents a model for projecting Workers Compensation losses based on the number of open claims and the average payment on open claims. In California, where the loss trend is growing and the claim closure rate appears to have slowed down, one can put different trend and claim closure assumptions into the model to study their impact on ultimate losses.

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PROJECTING WORKERS COMPENSATION LOSSES USING OPEN CLAIM COUNT AND AVERAGE LOSS PAYMENT, AND APPLICATION TO ANALYSIS OF CALIFORNIA WORKERS COMPENSATION LOSS DEVELOPMENT

INTRODUCTION

In recent years Workers Compensation results have deteriorated significantly for a number of California carriers, resulting in earning hits, rating downgrades, stock price depreciation, and even bankruptcies. In their synopsis of the California WC market, Moody's Investors Service pointed out three forces driving the bad results in California: Low price, "inexpensive, naïve reinsurance capital", and adverse loss development [1]. The situation improved somewhat in 2000. Most carriers increased rates substantially because of profitability concerns and the disappearance of reinsurance capital. Loss development, on the other hand, remained an area of great uncertainty.

One major reason for the loss development is claim severity trend, which has grown from less than 1% per year in the early 90's to about 12% in the late 90's [2]. Since benefit changes were relatively modest during this period, this large trend was primarily driven by a changing pattern of benefit utilization in California, which impacts calendar year claim cost across claims of all ages.

This presents a challenge to actuarial loss projection models that are based on accident year age-to-age link ratios. When loss trend is growing on a calendar year basis across all accident years, the link ratios will likely increase. This may explain the increasing medical loss link ratios in the California Workers' Compensation Insurance Rating Bureau's (WCIRB) analysis [3]. In projecting losses, actuaries have to select link ratios that represent future loss development. Unfortunately, in the case of California WC, the actual link ratios have consistently trended beyond the actuarial selections, resulting in adverse development in the loss ratio estimates. For example, the estimate for the 1999 loss ratio increased from 0.996 to 1.148 in just six months [3].

This paper presents an alternative loss projection model that is based on the number of claims staying open over time and the average payment made on open claims. Different claim closure and inflation assumptions can be put into the model to test their impact on link ratios and ultimate losses. So, rather than using judgment to select link ratios, one can explicitly account for trend and claim closure rate in projecting losses.

LOSS PROJECTION

Historical claims data are used to project the number of open claims for each accident year at each future valuation period. Exhibits 1, 2, and 3 show how this can be done. First, one projects reported claims at future valuation points, using age-to-age reported claim link ratios. Next, one projects the closed claim counts using claim closure ratios. The difference of the two is the open claim count. Exhibit 3 shows the average open claim count for each future valuation period. Average open claim count can be interpreted as the number of claims for which loss payments are made during that period.

Ideally, one would use total claim count in this analysis. But sometimes only the indemnity claim count is available, as is the case for some rating bureaus. In this instance, using just indemnity claim count will probably suffice, since medical-only claims are usually closed quickly, which means they do not significantly impact open claim volume. Moreover, medical-only claims account for only about 6% of total losses [4], so their impact on average payment is small as well.

The next step is to estimate average payment per open claim. One can look at average loss payment per open claim during historical periods, and project these payments forward. Average loss payments are calculated separately for indemnity and medical losses. Average indemnity payments are shown in Exhibit 4, where payment in each period is divided by the average open claim count in that period to arrive at average loss payment.

To project future average loss payments, one can look at how historical average payments have developed over time. This is shown in Exhibit 5, Page 1. Ratios of average payment from one period to the next are also shown. A pattern is selected at the bottom of the exhibit.

Historical average payment development factors may be unstable. One way to validate whether the selections are reasonable is to successively multiply the selected development factors to get "cumulative" factors, and compare these against historical cumulative factors for each accident year. The chart on Exhibit 5, Page 2 shows that the selected cumulative factors are in line with the historical cumulative factors, which validates the selections.

The next step is to project future average payments for each accident year. For each accident year, future average payments are based on historical average payments projected forward using the selected development factors in Exhibit 5, Page 1. For example, for accident year 1997, the next payment period to be forecasted is the 24-36 month period (see Exhibit 5, Page 1). To estimate the average payment for the 24-36 month period, one can develop the average payments in the 0-12 and 12-24 month periods. The average payment for the 0-12 month, $7,156, is multiplied by the development factor from 0-12 to 12-24 month period, 1.281, and again by the development factor from 12-24 to 24-36 month period, 1.579. This product comes to $14,483, which represents an estimate for the 24-36 month average payment based on data for the 0-12 month period. This is shown in Exhibit 6 in the 0-12 month column for 1997. Throughout this paper, some rounding errors may develop in certain calculations, as in this case. This should not distract the reader from the intent of the calculations.

Likewise, the average payment for the 12-24 month period is projected forward to the 24-36 month period to provide another estimate. The average payment during the 12-24 month period is $10,834. To project this to the 24-36 month period, one multiplies $10,834 by the 1.579 development factor to get $17,111, shown in Exhibit 6 in the 12-24 month column for 1997. So for accident year 1997, there are two estimates for the 24-36 month payment period: $14,483 and $17,111. The selected payment is $15,797 based on the average of two estimates. Exhibit 6 shows the results of this process for all accident years. Note that the top portion of Exhibit 6 represents estimates for future average payments. For example, the 12-24 month period data are the future payment estimates based on payments made during this period, and not actual payments during the 12-24 month period.

The next step is to project average payments for all future payment periods using the selected development factors in Exhibit 5, Page 1. For example, for accident year 1998, the average payment for the 12-24 month period is selected at $10,634. For the 24-36 month period, the average payment is $10,634 x 1.579, or $16,795. For the 36-48 month period, the average payment is $16,795 x 1.050, or $17,634. Projected average payments for all future periods are shown at the bottom of Exhibit 6.

Finally, the forecasted average payments in Exhibit 6 are multiplied by the average open claim counts in Exhibit 3 to arrive at the projected payments for all future payment periods. This is shown in Exhibit 7. For example, for accident year 1996 at the 36-48 month period, the projected number of open claims is 544 (Exhibit 3), and the projected average payment per open claim is $13,884 (Exhibit 6), so the total payment is 544 x $13,884 = $7,556,000 (Exhibit 7). Payments for all future periods are aggregated for each accident year and added to losses already paid to arrive at projected loss payments through 120 months. Finally, a tail factor is applied to losses at 120 months to get ultimate losses.

Exhibits 8 through 11 perform the same calculation for medical losses.

AN ALTERNATIVE METHOD FOR CALCULATING AVERAGE PAYMENTS

An alternative method for calculating average loss payments is by trending historical payments for each payment period. Exhibit 12 shows the average payment trend by accident year by payment period. This data shows that in a real world scenario, trends can be quite erratic, and one often needs to select a smooth trend factor. In this example, a 5.0% trend is selected for all payment periods.

Next, for each payment period, all historical average payments are trended to the first year for which a projection is to be made (see Exhibit 13). For example, for the 24-36 month payment period, the first average payment forecast is for accident year 1997. So all historical average payments for the 24-36 month payment period are trended to 1997. The trended average for accident year 1996 is \$3,753 (Exhibit 12) x (1+5.0%), or \$3,940. The trended average for 1995 is \$3,881 x $(1+5.0\%)^2$, or \$4,279. This calculation is repeated for all accident years, and \$3,808 is selected for 1997 at the 24-36 month period. \$3,808 is also used as the baseline from which the average payments for all subsequent years are calculated. For instance, the projected average payment for 1998 at 24-36 month period is \$3,808 x (1+5.0%), or \$3,999.

In Exhibit 14, average payments are multiplied by average open claim counts to produce total payments for all future payment periods. The ultimate losses are calculated as the sum of losses already paid and all future loss payments, times a tail factor.

CONSIDERATIONS IN PROJECTING AVERAGE PAYMENTS

Selecting the appropriate method to project average payments involves a number of considerations. First, claim trends may follow either an accident year or calendar year pattern. General medical inflation tends to impact loss payments on a calendar year basis, while benefit changes may impact losses on either an accident year or calendar year basis (see Scott [5]). The best approach may be to forecast future average payments on a blended calendar / accident year basis.

Exhibits 15 through 17 demonstrate a blended calendar / accident year approach. In Exhibit 15, the medical cost indices are plotted for the entire data triangle. Calendar year cost indices are placed diagonally along the calendar year periods, which may reflect cost drivers such as general medical inflation and changes in utilization. Accident year indices may also be used to reflect trends that are not part of calendar year indices. These are shown at the right hand side of Exhibit 15, and may reflect accident year benefit changes. Indices used in Exhibit 15 are based on the WCIRB's pure premium filing [6]. Other publications such as the NCCI Annual Statistical Bulletin [7] also contain information that can be used to develop cost indices. The blended indices are the product of calendar and accident year cost indices.

The top part of Exhibit 16 shows the historical average payments trended to the next payment diagonal, and the bottom part of the exhibit shows the forecasted average payments for all future payment periods. The following formula is used to trend historical average payments to the next payment diagonal.

Average Payment for the Next Payment Diagonal =

(Historical Average Payment x Blended Index for the Next Payment Diagonal) / Blended Index for the Historical Period

For example, for the 24-36 month period, the next payment to be projected is for accident year 1997. So all historical averages for the 24-36 month payment period are trended to 1997. The trended average payment for accident year 1996 is $3,753, which is the actual average payment per Exhibit 15, times 1.000 (blended index for 1997 at the 24-36 period), divided by 0.989 (blended index for 1996 at the 24-36 period). This comes to $3,796. As another example, the trended average for 1993 is $2,643 x 1.000 / 0.951, or $2,778.

Future average payments are selected based on these trended historical average payments. For the 24-36 month period, the selected average payment for accident year 1997 is $3,536. This is the baseline average payment for the 24-36 month payment period. Average payments for subsequent years can be calculated as follows:

Average Payment =

(Baseline Average Payment x Subsequent Year's Blended Index) / Blended Index for the Baseline Average Payment.

Take 1998 for example. The projected average payment is $3,536 x 1.012 (blended index for 1998 at the 24-36 period) / 1.000 (blended index for 1997 at the 24-36 period), or $3,577. Loss projections using these forecasted average payments are shown in Exhibit 17.

In this example, it is assumed that medical trends are the same regardless of the age of payment. But one can vary trend by age. Medical services rendered at later ages are usually follow-up visits and routine medical evaluations that are far less costly than the initial medical treatments, which may involve hospitalizations and surgeries. One can do a special study to quantify the trends for different categories of medical services, and use this information to refine the trend assumptions in the model.

In doing the analysis, one may notice aberrations in historical average payment data. Distortions may be caused by catastrophe claims or structured settlements. One way to mitigate these distortions is to select average payments based on multiple years of data, as is done in this paper. An alternative would be to remove large claims from the data, project losses based on "normal" losses, and then use a loading factor for large losses.

Another area to consider is change in claim settlement practices, which may alter future claim closure rates and average payments. If, for instance, the management decides to aggressively settle claims instead of keeping them open, one can speed up the claim closure rates in the model. One may also consider increasing some interim average payment assumptions to reflect the impact of lump sum settlements on average payments. Raising closure rates will increase losses paid in the earlier periods because more claims are settled early at higher cost, but will reduce payments later because there will be fewer claims remaining open. Exhibit 18 provides an example. Here the claim closure rates are accelerated to reflect aggressive claim settlement. This reduces the number of open claims at later periods and hence ultimate losses (see Exhibit 19).

TESTING THE MODEL

The critical assumptions underlying this model are the open claim counts and the average payments. As actual data emerge over time, one can validate the claim count and average payment assumptions. This is shown in Exhibit 20. Column (5) compares actual open claims (Column (4)) at mid-year to projected open claims at the beginning and end of the year (Columns (2) and (3)). The actual claim volume appears to be halfway between the beginning and ending claim counts, which validates the model's claim count assumptions.

The average payment is a different story. Column (6) shows the average payment assumptions, and Column (9) shows actual average payments halfway through the year. One would expect the actual average payments to be about half of the targeted full year payments. But for accident years 1996-1998, the actual average payments have far exceeded the halfway mark (see Column (10)), which indicates the model may have understated average loss payments for those years.

To study the variance between actual and expected average payments, it may be helpful to break down average payments by benefit type. For example, historical data indicates that medical payments made during the 12-24 month period are split evenly between physician and non-physician payments. The expected 12-24 month average medical payment for accident year 1998 is $5,467 (see Exhibit 20, Column (6)). This implies that the benchmark for physician payments is $5,467 x 50% = $2,734 and the same number for other types of medical payments. The actual payment, halfway through the year, was $5,848. A further drill down of the data reveals that $4,500 comes from physician payments. At this rate the annualized physician payment will be $9,000, or over three times the expected average of $2,734. On the other hand, the non-physician portion of the actual payment, halfway through the year, is $5,848 - $4,500 = $1,348. This annualizes to $2,696, which is close to the expected payment of $2,734. This points to possible deterioration in the physician payment trend and should be studied further. This type of analysis not only helps the actuaries set appropriate trend assumptions, but also helps the claims department detect and mitigate areas of leakage.

APPLICATION TO CALIFORNIA WC

In California, a common explanation for the growing cost trend is the presumption of correctness of the primary treating physician. The California WC system gives the primary treating physician the rebuttable presumption of correctness in prescribing medical services and determining the claimant's disability rating, and at the same time limits a payor's ability to question the treating physician's opinions [8]. There is some evidence that physicians may be stepping up medical treatments because of this feature, which may explain why California's WC medical cost trend has consistently exceeded general medical inflation by over 10 points each year.

One can use this model to test how sensitive the losses are to different inflation assumptions. Exhibit 21 shows the projected medical loss payments and link ratios using a 5% inflation assumption (see Exhibit 14). Exhibit 22 uses 10% inflation instead of 5%, and one can see a steeper increase in the link ratios and higher future loss payments.

One can also vary the assumptions in the claim closure pattern. The June, 2000 WCIRB study [3] showed that claim closure rates may be slowing down. Slower claim closure extends the claim payment duration, which increases the amount of losses paid and makes the ultimate losses more sensitive to inflation. Exhibit 23 shows a scenario where future claim closure ratios are reduced to reflect slower claim settlement. Exhibit 24 applies average payments with 10% inflation to the open claim counts in Exhibit 23. The resulting increases in the link ratios and future loss payments (Exhibit 23, Page 2) are even more pronounced than those shown in Exhibit 22.

CONCLUSION

In actuarial models that project losses using aggregate loss development triangles, it may be difficult to account for variables such as inflation and claim closure pattern. The model presented in this paper provides a tool to explicitly analyze the impact of inflation and claim closure pattern on ultimate losses. This model is useful for a line like WC where claims are reported quickly and losses are generally paid out over the lifetime of a claim. By putting different inflation and claim closure assumptions into the model, one can see the impact on the link ratios and the ultimate losses. This type of sensitivity analysis is particularly useful in a situation like California WC, where recent cost trends and claim closure rates have not been stable.

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WC Reported Claims

Exhibit 1

Accident Year	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	2,735	2,833	2,860	2,876	2,889	2,896	2,896	2,898	2,898	2,901
1990	3,019	3,133	3,172	3,191	3,206	3,210	3,216	3,218	3,220	
1991	3,534	3,736	3,790	3,810	3,825	3,831	3,836	3,839		
1992	4,873	5,061	5,119	5,145	5,160	5,174	5,178			
1993	6,711	6,917	6,961	6,987	7,006	7,011				
1994	8,241	8,479	8,549	8,584	8,601					
1995	8,113	8,349	8,410	8,446						
1996	9,748	9,974	10,031							
1997	10,687	10,958								
1998	6,944									

Selected Age-to-age Development Factor (Based on historical claims development)

Factor:	1.026	1.007	1.004	1.002	1.002	1.001	1.001	1.000	1.000	---

Projected Future Reported Claims (Applying selected development factors to claim count data)

1989										2,901
1990									3,220	3,220
1991								3,839	3,839	3,839
1992							5,178	5,183	5,183	5,183
1993						7,011	7,018	7,025	7,025	7,025
1994					8,601	8,618	8,627	8,635	8,635	8,635
1995				8,446	8,463	8,480	8,488	8,497	8,497	8,497
1996			10,031	10,071	10,091	10,111	10,122	10,132	10,132	10,132
1997		10,958	11,035	11,079	11,101	11,123	11,134	11,145	11,145	11,145
1998	6,944	7,125	7,174	7,203	7,218	7,232	7,239	7,246	7,246	7,246

WC Closed Claims

Exhibit 2

Accident Year	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	2,158	2,423	2,637	2,733	2,813	2,851	2,872	2,885	2,886	2,894
1990	2,325	2,666	2,855	3,023	3,094	3,146	3,178	3,194	3,197	
1991	2,648	2,939	3,312	3,518	3,687	3,751	3,786	3,800		
1992	3,737	4,254	4,535	4,831	5,017	5,079	5,129			
1993	5,318	5,867	6,319	6,673	6,818	6,891				
1994	6,510	7,309	7,923	8,213	8,387					
1995	6,206	7,276	7,850	8,126						
1996	7,731	8,814	9,364							
1997	8,491	9,660								
1998	5,449									

Weighted Average Closure Ratio (Ratio of closed claims to reported claims)

	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
Avg of 3	79.2%	87.9%	93.1%	95.8%	97.4%	98.2%	98.9%	99.2%	99.4%	99.8%
Avg of 5	78.6%	87.1%	92.1%	95.1%	97.1%	98.2%	98.9%	99.2%	99.4%	99.8%
Selected	79.2%	87.9%	93.1%	95.8%	97.4%	98.2%	98.9%	99.2%	99.4%	99.8%

Projected Future Closed Claims (Applying selected closure ratio to future reported claims)

	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989										2,894
1990									3,197	3,212
1991								3,800	3,817	3,830
1992							5,129	5,144	5,154	5,171
1993						6,891	6,939	6,971	6,985	7,008
1994					8,387	8,459	8,530	8,570	8,586	8,615
1995				8,126	8,241	8,324	8,393	8,432	8,448	8,476
1996			9,364	9,650	9,826	9,925	10,008	10,054	10,074	10,107
1997		9,660	10,277	10,615	10,810	10,918	11,010	11,060	11,082	11,119
1998	5,449	6,265	6,682	6,902	7,028	7,099	7,158	7,191	7,205	7,229

Projected Average Open Claim

Exhibit 3

Accident	Projected Number of Claims Open (Reported claim in Exhibit 1 minus closed claim in Exhibit 2)									
Year	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989										7
1990									23	8
1991								39	22	9
1992							49	40	30	13
1993						120	79	54	40	17
1994					214	159	97	66	49	21
1995				320	222	156	95	65	49	21
1996			667	421	265	186	113	77	58	24
1997		1,298	758	464	291	205	125	85	64	27
1998	1,495	859	493	301	189	133	81	55	41	17

Accident	Average Number of Claims Open During Each Period								
Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									15
1991								30	16
1992							44	35	21
1993						99	66	47	29
1994					186	128	81	58	35
1995				271	189	126	80	57	35
1996			544	343	226	150	95	68	41
1997		1,028	611	377	248	165	105	74	45
1998	1,177	676	397	245	161	107	68	48	29

Average Indemnity Loss Payment Per Open Claim

Exhibit 4
Page 1

Accident Year	Indemnity Losses Paid in Each Period ($000) 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	1,050	1,472	2,518	1,301	725	400	181	195	34	38
1990	1,468	2,987	2,657	1,821	993	695	572	287	132	
1991	2,129	3,855	4,069	3,457	1,778	770	452	308		
1992	2,492	4,113	5,580	3,792	2,155	1,335	647			
1993	3,492	6,410	7,067	5,135	2,624	1,755				
1994	4,339	8,787	8,524	5,727	2,663					
1995	4,876	10,227	9,234	5,178						
1996	6,917	13,299	11,917							
1997	7,857	18,927								
1998	6,203									

Accident Year	Number of Claims Open 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	577	410	223	143	76	45	24	13	12	7
1990	694	467	317	168	112	64	38	24	23	
1991	886	797	478	292	138	80	50	39		
1992	1,136	807	584	314	143	95	49			
1993	1,393	1,050	642	314	188	120				
1994	1,731	1,170	626	371	214					
1995	1,907	1,073	560	320						
1996	2,017	1,160	667							
1997	2,196	1,298								
1998	1,495									

Average Indemnity Loss Payment Per Open Claim

Exhibit 4
Page 2

Accident Year	Average Number of Open Claim During Each Period 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	289	494	317	183	110	61	35	19	13	10
1990	347	581	392	243	140	88	51	31	24	
1991	443	842	638	385	215	109	65	45		
1992	568	972	696	449	229	119	72			
1993	697	1,222	846	478	251	154				
1994	866	1,451	898	499	293					
1995	954	1,490	817	440						
1996	1,009	1,589	914							
1997	1,098	1,747								
1998	748									

Accident Year	Average Indemnity Loss Payment per Open Claim (Losses paid divided by average open claim) 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	3,640	2,983	7,956	7,109	6,618	6,614	5,243	10,560	2,698	4,034
1990	4,231	5,146	6,778	7,509	7,092	7,895	11,215	9,252	5,611	
1991	4,806	4,581	6,382	8,978	8,270	7,065	6,954	6,930		
1992	4,387	4,234	8,023	8,444	9,429	11,221	8,988			
1993	5,014	5,248	8,354	10,743	10,453	11,396				
1994	5,014	6,058	9,492	11,489	9,104					
1995	5,114	6,864	11,309	11,768						
1996	6,859	8,372	13,046							
1997	7,156	10,834								
1998	8,298									

Development in Average Indemnity Loss Payment

Exhibit 5
Page 1

Accident Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
	Average Indemnity Loss Payment per Open Claim (From Exhibit 4)									
1989	3,640	2,983	7,956	7,109	6,618	6,614	5,243	10,560	2,698	4,034
1990	4,231	5,146	6,778	7,509	7,092	7,895	11,215	9,252	5,611	
1991	4,806	4,581	6,382	8,978	8,270	7,065	6,954	6,930		
1992	4,387	4,234	8,023	8,444	9,429	11,221	8,988			
1993	5,014	5,248	8,354	10,743	10,453	11,396				
1994	5,014	6,058	9,492	11,489	9,104					
1995	5,114	6,864	11,309	11,768						
1996	6,859	8,372	13,046							
1997	7,156	10,834								
1998	8,298									
	Change in Average Indemnity Payment from Period to Period									
1989		0.820	2.667	0.894	0.931	0.999	0.793	2.014	0.255	1.495
1990		1.216	1.317	1.108	0.945	1.113	1.420	0.825	0.607	
1991		0.953	1.393	1.407	0.921	0.854	0.984	0.997		
1992		0.965	1.895	1.052	1.117	1.190	0.801			
1993		1.047	1.592	1.286	0.973	1.090				
1994		1.208	1.567	1.210	0.792					
1995		1.342	1.648	1.041						
1996		1.221	1.558							
1997		1.514								
Averages										
Avg of 3		1.359	1.591	1.179	0.961	1.045	1.069	1.279	0.431	1.495
4 x Hi/Lo		1.281	1.579	1.131	0.947	1.102	0.893	0.997	0.431	1.495
Selected		1.281	1.579	1.050	1.050	1.050	1.050	1.000	0.600	0.600

Analysis of Selected Average Payment Development Pattern

Exhibit 5
Page 2

Accident Year	Successive Multiplication of Average Payment Development Factors (From Exhibit 5, Page 1) 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	1.000	0.820	2.186	1.953	1.818	1.817	1.441	2.902	0.741	1.108
1990	1.000	1.216	1.602	1.775	1.676	1.866	2.651	2.187	1.326	
1991	1.000	0.953	1.328	1.868	1.721	1.470	1.447	1.442		
1992	1.000	0.965	1.829	1.925	2.149	2.557	2.049			
1993	1.000	1.047	1.666	2.142	2.085	2.273				
1994	1.000	1.208	1.893	2.292	1.816					
1995	1.000	1.342	2.211	2.301						
1996	1.000	1.221	1.902							
1997	1.000	1.514								
Selected	1.000	1.281	2.024	2.125	2.231	2.343	2.460	2.460	1.476	0.886

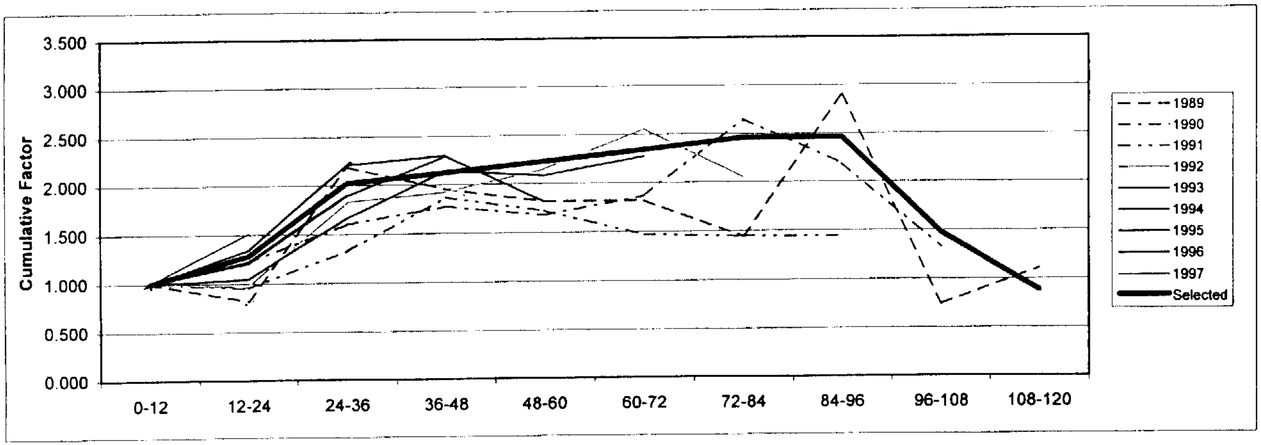

Exhibit 6

Selected Future Average Indemnity Payment

Accident Year	Historical Average Indemnity Payment Developed to Subsequent Payment Period 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Avg of Last 3	Avg of Last 5	Avg of 5 ex Hi/Lo	Selected Avg Pmt
1990					2,815	2,984	4,037	3,331	3,367		3,578	3,307	3,227	3,227
1991				6,236	5,471	4,451	4,172	4,158			4,261	4,898	4,698	4,200
1992			9,752	9,775	10,396	11,782	8,988				10,389	10,139	9,974	9,974
1993		10,074	10,154	12,436	11,524	11,965					11,975	11,231	11,215	11,215
1994	11,746	11,076	10,988	12,666	9,559						11,071	11,207	11,270	11,270
1995	11,411	11,952	12,469	12,357							12,259	12,047	12,154	12,154
1996	14,575	13,884	13,698								14,052	14,052	13,884	13,884
1997	14,483	17,111									15,797	15,797	15,797	15,797
1998	10,634										10,634	10,634	10,634	10,634

Accident Year	Projected Future Average Indemnity Payment per Open Claim 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									3,227
1991								4,200	2,520
1992							9,974	5,985	3,591
1993						11,215	11,215	6,729	4,037
1994					11,270	11,834	11,834	7,100	4,260
1995				12,154	12,762	13,400	13,400	8,040	4,824
1996			13,884	14,578	15,307	16,072	16,072	9,643	5,786
1997		15,797	16,587	17,416	18,287	19,201	19,201	11,521	6,912
1998	10,634	16,795	17,634	18,516	19,442	20,414	20,414	12,248	7,349

Projected Future Indemnity Losses Paid

Exhibit 7

Accident Year	Future Paid Indemnity Losses (In $000, equals average payment in Exhibit 6 times average open claim in Exhibit 3) 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Total
1989										
1990									50	50
1991								128	39	167
1992							442	207	76	725
1993						1,114	742	316	115	2,286
1994					2,100	1,511	962	409	150	5,132
1995				3,294	2,414	1,684	1,072	456	167	9,086
1996			7,556	5,002	3,452	2,408	1,533	652	238	20,842
1997		16,236	10,128	6,574	4,537	3,164	2,014	857	313	43,824
1998	12,517	11,351	7,001	4,544	3,136	2,187	1,392	593	217	42,937

Accident Year (1)	Losses Already Paid (2)	Projected Payments Through 120 Mos. (3)	Projected Total Paid Thru 120 (2)+(3) (4)	Develop-ment Beyond 120 Mos. (5)	Projected Ultimate Losses (4)x(5) (6)
1989	7,914		7,914	1.020	8,072
1990	11,611	50	11,661	1.020	11,894
1991	16,818	167	16,985	1.020	17,325
1992	20,114	725	20,839	1.020	21,256
1993	26,483	2,286	28,769	1.020	29,345
1994	30,040	5,132	35,173	1.020	35,876
1995	29,515	9,086	38,601	1.020	39,373
1996	32,133	20,842	52,975	1.020	54,035
1997	26,784	43,824	70,609	1.020	72,021
1998	6,203	42,937	49,140	1.020	50,123

Average Medical Loss Payment Per Open Claim

Exhibit 8
Page 1

Accident Year	Medical Losses Paid in Each Period ($000) 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	2,636	1,977	908	363	183	66	48	40	32	38
1990	3,245	3,579	1,258	393	338	307	121	114	82	
1991	4,515	4,365	1,813	603	488	318	185	243		
1992	6,532	5,156	1,598	942	682	548	178			
1993	8,486	4,870	2,236	1,573	1,032	754				
1994	9,644	6,262	2,487	1,591	826					
1995	9,836	5,992	3,169	1,846						
1996	10,999	8,226	3,428							
1997	14,834	8,185								
1998	10,514									

Accident Year	Number of Claims Open 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	577	410	223	143	76	45	24	13	12	7
1990	694	467	317	168	112	64	38	24	23	
1991	886	797	478	292	138	80	50	39		
1992	1,136	807	584	314	143	95	49			
1993	1,393	1,050	642	314	188	120				
1994	1,731	1,170	626	371	214					
1995	1,907	1,073	560	320						
1996	2,017	1,160	667							
1997	2,196	1,298								
1998	1,495									

Average Medical Loss Payment Per Open Claim

Exhibit 8
Page 2

Accident Year	Average Number of Open Claim During Each Period 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	289	494	317	183	110	61	35	19	13	10
1990	347	581	392	243	140	88	51	31	24	
1991	443	842	638	385	215	109	65	45		
1992	568	972	696	449	229	119	72			
1993	697	1,222	846	478	251	154				
1994	866	1,451	898	499	293					
1995	954	1,490	817	440						
1996	1,009	1,589	914							
1997	1,098	1,747								
1998	748									

Accident Year	Average Medical Loss Payment per Open Claim (Losses paid divided by average open claim) 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	9,137	4,006	2,869	1,984	1,673	1,085	1,386	2,166	2,525	4,037
1990	9,352	6,165	3,209	1,619	2,412	3,493	2,369	3,671	3,483	
1991	10,192	5,187	2,844	1,566	2,269	2,919	2,850	5,461		
1992	11,500	5,307	2,297	2,099	2,984	4,603	2,469			
1993	12,184	3,987	2,643	3,292	4,110	4,896				
1994	11,143	4,317	2,770	3,192	2,824					
1995	10,315	4,022	3,881	4,195						
1996	10,907	5,178	3,753							
1997	13,510	4,685								
1998	14,066									

Development in Average Medical Loss Payment

Exhibit 9

Accident Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
	Average Medical Loss Payment per Open Claim (From Exhibit 8)									
1989	9,137	4,006	2,869	1,984	1,673	1,085	1,386	2,166	2,525	4,037
1990	9,352	6,165	3,209	1,619	2,412	3,493	2,369	3,671	3,483	
1991	10,192	5,187	2,844	1,566	2,269	2,919	2,850	5,461		
1992	11,500	5,307	2,297	2,099	2,984	4,603	2,469			
1993	12,184	3,987	2,643	3,292	4,110	4,896				
1994	11,143	4,317	2,770	3,192	2,824					
1995	10,315	4,022	3,881	4,195						
1996	10,907	5,178	3,753							
1997	13,510	4,685								
1998	14,066									
	Change in Average Medical Payment from Period to Period									
1989		0.438	0.716	0.691	0.844	0.648	1.278	1.563	1.166	1.599
1990		0.659	0.521	0.505	1.490	1.449	0.678	1.549	0.949	
1991		0.509	0.548	0.551	1.449	1.286	0.976	1.916		
1992		0.462	0.433	0.914	1.422	1.543	0.536			
1993		0.327	0.663	1.246	1.249	1.191				
1994		0.387	0.642	1.153	0.885					
1995		0.390	0.965	1.081						
1996		0.475	0.725							
1997		0.347								
Averages										
Avg of 3		0.404	0.777	1.160	1.185	1.340	0.730	1.676	1.057	1.599
4 x Hi/Lo		0.389	0.694	1.117	1.335	1.367	0.827	1.563	1.057	1.599
Selected		0.389	0.694	1.160	1.185	1.340	0.730	1.676	1.057	1.050

Selected Future Average Medical Payment

Exhibit 10

Accident Year	Historical Average Medical Payment Developed to Subsequent Payment Period 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Avg of Last 3	Avg of Last 5	Avg of 5 ex Hi/Lo	Selected Avg Pmt
1990					4,392	4,747	4,409	4,075	3,657		4,047	4,256	4,292	4,292
1991				3,218	3,936	3,778	5,050	5,774			4,867	4,351	4,255	4,255
1992			5,179	4,080	4,895	5,635	4,138				4,889	4,785	4,737	4,737
1993		3,720	3,554	3,817	4,022	3,575					3,805	3,737	3,704	3,704
1994	5,533	5,515	5,100	5,069	3,785						4,651	5,000	5,228	5,228
1995	3,822	3,834	5,333	4,971							4,713	4,490	4,402	4,402
1996	3,410	4,166	4,352								3,976	3,976	4,166	4,166
1997	3,642	3,250									3,446	3,446	3,446	3,446
1998	5,467										5,467	5,467	5,467	5,467

Accident Year	Projected Future Average Medical Payment per Open Claim 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									4,292
1991								4,255	4,467
1992							4,737	5,009	5,259
1993						3,704	6,209	6,564	6,893
1994					5,228	3,818	6,400	6,766	7,105
1995				4,402	5,899	4,308	7,222	7,635	8,017
1996			4,166	4,936	6,615	4,831	8,098	8,562	8,990
1997		3,446	3,997	4,736	6,347	4,635	7,769	8,214	8,625
1998	5,467	3,792	4,398	5,211	6,984	5,100	8,549	9,039	9,490

Projected Future Medical Losses Paid

Exhibit 11

Accident Year	Future Paid Medical Losses (In $000, equals average payment in Exhibit 10 times average open claim in Exhibit 3) 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Total
1989										
1990									66	66
1991								130	70	199
1992							210	173	111	494
1993						368	411	308	197	1,283
1994					974	488	520	390	250	2,622
1995				1,193	1,116	541	578	433	277	4,138
1996			2,267	1,694	1,492	724	772	579	370	7,899
1997		3,542	2,440	1,788	1,575	764	815	611	391	11,926
1998	6,435	2,563	1,746	1,279	1,127	546	583	437	280	14,996

Accident Year (1)	Losses Already Paid (2)	Projected Payments Through 120 Mos. (3)	Projected Total Paid Thru 120 (2)+(3) (4)	Development Beyond 120 Mos. (5)	Projected Ultimate Losses (4)x(5) (6)
1989	6,291		6,291	1.040	6,542
1990	9,436	66	9,502	1.040	9,882
1991	12,530	199	12,730	1.040	13,239
1992	15,636	494	16,130	1.040	16,775
1993	18,951	1,283	20,234	1.040	21,043
1994	20,811	2,622	23,433	1.040	24,370
1995	20,843	4,138	24,981	1.040	25,980
1996	22,653	7,899	30,552	1.040	31,774
1997	23,019	11,926	34,945	1.040	36,343
1998	10,514	14,996	25,510	1.040	26,531

Exhibit 12

Trending of Average Medical Loss Payment

Accident Year	Average Medical Loss Payment per Open Claim (From Exhibit 8) 0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	9,137	4,006	2,869	1,984	1,673	1,085	1,386	2,166	2,525	4,037
1990	9,352	6,165	3,209	1,619	2,412	3,493	2,369	3,671	3,483	
1991	10,192	5,187	2,844	1,566	2,269	2,919	2,850	5,461		
1992	11,500	5,307	2,297	2,099	2,984	4,603	2,469			
1993	12,184	3,987	2,643	3,292	4,110	4,896				
1994	11,143	4,317	2,770	3,192	2,824					
1995	10,315	4,022	3,881	4,195						
1996	10,907	5,178	3,753							
1997	13,510	4,685								
1998	14,066									

	Trend in Average Medical Loss Payment								
1989-90	2.3%	53.9%	11.9%	-18.4%	44.1%	222.1%	71.0%	69.5%	38.0%
1990-91	9.0%	-15.9%	-11.4%	-3.3%	-5.9%	-16.4%	20.3%	48.8%	
1991-92	12.8%	2.3%	-19.2%	34.0%	31.5%	57.7%	-13.4%		
1992-93	5.9%	-24.9%	15.0%	56.8%	37.7%	6.4%			
1993-94	-8.5%	8.3%	4.8%	-3.0%	-31.3%				
1994-95	-7.4%	-6.8%	40.1%	31.4%					
1995-96	5.7%	28.8%	-3.3%						
1996-97	23.9%	-9.5%							
1997-98	4.1%								

Average Trend										
Avg of all	6.6%	5.2%	14.2%	29.8%	8.0%	67.4%	26.0%	59.1%	38.0%	
Ex. Hi/Lo	4.9%	0.7%	9.9%	32.7%	12.8%	32.0%	20.3%	48.8%	0.0%	
Selected	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%

Selecting Future Average Medical Loss Payment

Exhibit 13

Accident Year	Historical Average Medical Payment Trended to Subsequent Payment Period 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989					1,384	1,684	2,507	2,784	4,239
1990				3,078	4,246	2,743	4,047	3,657	
1991			1,999	2,758	3,379	3,142	5,734		
1992		2,932	2,551	3,454	5,075	2,592			
1993	5,089	3,212	3,810	4,531	5,141				
1994	5,247	3,206	3,520	2,965					
1995	4,656	4,279	4,405						
1996	5,709	3,940							
1997	4,920								
1998									
Average of Last 3	5,095	3,808	3,912	3,650	4,532	2,826	4,096	3,221	4,239
Average of Last 5	5,124	3,514	3,257	3,357	3,845	2,540	4,096	3,221	4,239
Avg of 5 ex Hi/Lo	5,085	3,453	3,294	3,166	4,233	2,667	4,047	3,221	4,239
Selected Avg Pmt	5,095	3,808	3,912	3,650	4,532	2,826	4,096	3,221	4,239

Accident Year	Projected Future Average Medical Payment per Open Claim 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									4,239
1991								3,221	4,451
1992							4,096	3,382	4,673
1993						2,826	4,301	3,551	4,907
1994					4,532	2,967	4,516	3,728	5,152
1995				3,650	4,758	3,115	4,742	3,915	5,410
1996			3,912	3,833	4,996	3,271	4,979	4,110	5,680
1997		3,808	4,107	4,024	5,246	3,434	5,228	4,316	5,964
1998	5,095	3,999	4,312	4,226	5,508	3,606	5,489	4,532	6,263

Projected Future Medical Losses Paid (Using Trend Method) **Exhibit 14**

Accident Year	Future Paid Medical Losses (In $000, equals average payment in Exhibit 13 times average open claim in Exhibit 3) 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Total
1989										
1990									65	65
1991								98	69	168
1992							181	117	99	397
1993						281	284	167	140	872
1994					845	379	367	215	181	1,986
1995				989	900	391	379	222	187	3,069
1996			2,129	1,315	1,127	490	475	278	234	6,048
1997		3,914	2,508	1,519	1,302	566	548	321	270	10,949
1998	5,997	2,703	1,712	1,037	889	386	374	219	185	13,502

Accident Year (1)	Losses Already Paid (2)	Projected Payments Through 120 Mos. (3)	Projected Total Paid Thru 120 (2)+(3) (4)	Develop-ment Beyond 120 Mos. (5)	Projected Ultimate Losses (4)x(5) (6)
1989	6,291		6,291	1.040	6,542
1990	9,436	65	9,501	1.040	9,881
1991	12,530	168	12,698	1.040	13,206
1992	15,636	397	16,033	1.040	16,674
1993	18,951	872	19,823	1.040	20,615
1994	20,811	1,986	22,797	1.040	23,709
1995	20,843	3,069	23,912	1.040	24,868
1996	22,653	6,048	28,701	1.040	29,849
1997	23,019	10,949	33,968	1.040	35,326
1998	10,514	13,502	24,016	1.040	24,977

Trending of Average Medical Loss Payment Using Calendar and Accident Year Approach

Exhibit 15

Average Medical Loss Payment per Open Claim (Exhibit 8, Page 2)

Accident Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989	9,137	4,006	2,869	1,984	1,673	1,085	1,386	2,166	2,525	4,037
1990	9,352	6,165	3,209	1,619	2,412	3,493	2,369	3,671	3,483	
1991	10,192	5,187	2,844	1,566	2,269	2,919	2,850	5,461		
1992	11,500	5,307	2,297	2,099	2,984	4,603	2,469			
1993	12,184	3,987	2,643	3,292	4,110	4,896				
1994	11,143	4,317	2,770	3,192	2,824					
1995	10,315	4,022	3,881	4,195						
1996	10,907	5,178	3,753							
1997	13,510	4,685								
1998	14,066									

Cost Index on Calendar Year Basis

Accident Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Cost Index on Acc. Yr Basis
1989						0.948	0.960	0.971	0.980	0.989	0.991
1990					0.948	0.960	0.971	0.980	0.989	1.000	0.991
1991				0.948	0.960	0.971	0.980	0.989	1.000	1.012	0.991
1992			0.948	0.960	0.971	0.980	0.989	1.000	1.012	1.023	0.991
1993		0.948	0.960	0.971	0.980	0.989	1.000	1.012	1.023	1.035	0.991
1994	0.948	0.960	0.971	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.000
1995	0.960	0.971	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.000
1996	0.971	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.072	1.000
1997	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.072	1.084	1.000
1998	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.072	1.084	1.097	1.000

Blended Calendar Year / Accident Year Cost Index

Accident Year	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989						0.940	0.951	0.963	0.971	0.980
1990					0.940	0.951	0.963	0.971	0.980	0.991
1991				0.940	0.951	0.963	0.971	0.980	0.991	1.003
1992			0.940	0.951	0.963	0.971	0.980	0.991	1.003	1.014
1993		0.940	0.951	0.963	0.971	0.980	0.991	1.003	1.014	1.026
1994	0.948	0.960	0.971	0.980	0.989	1.000	1.012	1.023	1.035	1.047
1995	0.960	0.971	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.059
1996	0.971	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.072
1997	0.980	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.072	1.084
1998	0.989	1.000	1.012	1.023	1.035	1.047	1.059	1.072	1.084	1.097

Selecting Future Average Medical Loss Payment

Exhibit 16

Accident Year	Historical Average Medical Payment Trended to Subsequent Payment Period 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989					1,154	1,444	2,230	2,577	4,084
1990				2,566	3,672	2,439	3,746	3,524	
1991			1,666	2,386	3,032	2,908	5,524		
1992		2,444	2,206	3,099	4,740	2,497			
1993	4,242	2,778	3,419	4,232	4,997				
1994	4,498	2,851	3,258	2,857					
1995	4,140	3,961	4,243						
1996	5,284	3,796							
1997	4,740								
1998									
Average of Last 3	4,722	3,536	3,640	3,396	4,256	2,615	3,834	3,050	4,084
Average of Last 5	4,581	3,166	2,959	3,028	3,519	2,322	3,834	3,050	4,084
Avg of 5 ex Hi/Lo	4,493	3,142	2,961	2,841	3,815	2,468	3,746	3,050	4,084
Selected Avg Pmt	4,722	3,536	3,640	3,396	4,256	2,615	3,834	3,050	4,084

Accident Year	Projected Future Average Medical Payment per Open Claim 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									4,084
1991								3,050	4,131
1992							3,834	3,086	4,179
1993						2,615	3,878	3,121	4,228
1994					4,256	2,669	3,958	3,186	4,315
1995				3,396	4,306	2,700	4,004	3,223	4,365
1996			3,640	3,435	4,356	2,731	4,051	3,260	4,416
1997		3,536	3,682	3,475	4,406	2,763	4,098	3,298	4,467
1998	4,722	3,577	3,725	3,516	4,457	2,795	4,145	3,336	4,519

Projected Future Medical Losses Paid (Using Calendar / Accident Year Trend Approach) Exhibit 17

Accident Year	Future Paid Medical Losses (In $000, equals average payment in Exhibit 16 times average open claim in Exhibit 3) 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Total
1989										
1990									63	63
1991								93	64	157
1992							170	107	88	365
1993						260	256	146	121	783
1994					793	341	322	184	152	1,791
1995				920	814	339	320	183	151	2,728
1996			1,981	1,179	982	409	386	221	182	5,340
1997		3,634	2,248	1,312	1,093	455	430	245	202	9,621
1998	5,558	2,418	1,479	863	719	299	283	161	133	11,913

Accident Year (1)	Losses Already Paid (2)	Projected Payments Through 120 Mos. (3)	Projected Total Paid Thru 120 (2)+(3) (4)	Development Beyond 120 Mos. (5)	Projected Ultimate Losses (4)x(5) (6)
1989	6,291		6,291	1.040	6,542
1990	9,436	63	9,499	1.040	9,879
1991	12,530	157	12,688	1.040	13,195
1992	15,636	365	16,000	1.040	16,640
1993	18,951	783	19,734	1.040	20,523
1994	20,811	1,791	22,602	1.040	23,506
1995	20,843	2,728	23,571	1.040	24,514
1996	22,653	5,340	27,993	1.040	29,113
1997	23,019	9,621	32,640	1.040	33,945
1998	10,514	11,913	22,427	1.040	23,324

Claim Closure Pattern Reflecting Earlier Claim Settlement

Exhibit 18

Accident Year	Accelerated Claim Closure Rate 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989					97.4%	98.4%	99.2%	99.6%	99.6%	99.8%
1990				94.7%	96.5%	98.0%	98.8%	99.3%	99.3%	99.8%
1991			87.4%	92.3%	96.4%	97.9%	98.7%	99.0%	99.4%	99.8%
1992		84.1%	88.6%	93.9%	97.2%	98.2%	99.1%	99.2%	99.4%	99.8%
1993	79.2%	84.8%	90.8%	95.5%	97.3%	98.3%	98.9%	99.2%	99.4%	99.8%
1994	79.0%	86.2%	92.7%	95.7%	97.5%	98.2%	98.9%	99.2%	99.4%	99.8%
1995	76.5%	87.1%	93.3%	96.2%	97.4%	98.2%	98.9%	99.2%	99.4%	99.8%
1996	79.3%	88.4%	93.4%	96.2%	97.4%	98.2%	98.9%	99.2%	99.4%	99.8%
1997	79.5%	88.2%	93.4%	96.2%	97.4%	98.2%	98.9%	99.2%	99.4%	99.8%
1998	78.5%	88.4%	93.4%	96.2%	97.4%	98.2%	98.9%	99.2%	99.4%	99.8%

Projected Number of Claims Open (Projected claims reported times the complement of closure rate)

	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989										7
1990									23	8
1991								39	22	9
1992							49	40	30	13
1993						120	79	54	40	17
1994					214	159	97	66	49	21
1995				320	222	156	95	65	49	21
1996			667	383	265	186	113	77	58	24
1997		1,298	728	421	291	205	125	85	64	27
1998	1,495	826	474	274	189	133	81	55	41	17

Average Number of Claims Open During Each Period

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									15
1991								30	16
1992							44	35	21
1993						99	66	47	29
1994					186	128	81	58	35
1995				271	189	126	80	57	35
1996			525	324	226	150	95	68	41
1997		1,013	575	356	248	165	105	74	45
1998	1,161	650	374	232	161	107	68	48	29

Projected Future Indemnity Losses Paid, Reflecting Earlier Claim Settlement

Exhibit 19

Accident Year	Future Paid Indemnity Losses (In $000, equals average payment in Exhibit 6 times average open claim in Exhibit 18)									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Total
1989										
1990									50	50
1991								128	39	167
1992							442	207	76	725
1993						1,114	742	316	115	2,286
1994					2,100	1,511	962	409	150	5,132
1995				3,294	2,414	1,684	1,072	456	167	9,086
1996			7,287	4,720	3,452	2,408	1,533	652	238	20,291
1997		16,005	9,532	6,203	4,537	3,164	2,014	857	313	42,626
1998	12,343	10,916	6,588	4,288	3,136	2,187	1,392	593	217	41,660

Accident Year	Losses Already Paid	Projected Payments Through 120 Mos.	Projected Total Paid Thru 120 (2)+(3)	Development Beyond 120 Mos.	Projected Ultimate Losses (4)x(5)
(1)	(2)	(3)	(4)	(5)	(6)
1989	7,914		7,914	1.020	8,072
1990	11,611	50	11,661	1.020	11,894
1991	16,818	167	16,985	1.020	17,325
1992	20,114	725	20,839	1.020	21,256
1993	26,483	2,286	28,769	1.020	29,345
1994	30,040	5,132	35,173	1.020	35,876
1995	29,515	9,086	38,601	1.020	39,373
1996	32,133	20,291	52,424	1.020	53,473
1997	26,784	42,626	69,410	1.020	70,798
1998	6,203	41,660	47,863	1.020	48,821

Testing the Model at June 30, 1999 Exhibit 20

Accident Year	Open Claim Inventory @ 12/98	Projected Open Inventory @ 12/99	Actual Open Inventory @ 6/99	% Toward 12/99	Projected Average Med Paid In 1999	Actual Med Paid Thru 6/99 ($000)	Average Open Claim Thru 6/99	Actual Average Payment	Ratio to Target
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1990	23	8	23	0%	4,292	51	23	2,217	52%
1991	39	22	30	53%	4,255	182	35	5,275	124%
1992	49	40	44	53%	4,737	86	47	1,849	39%
1993	120	79	101	46%	3,704	414	111	3,747	101%
1994	214	159	177	67%	5,228	486	196	2,486	48%
1995	320	222	258	63%	4,402	670	289	2,318	53%
1996	667	421	506	66%	4,166	1,551	587	2,645	63%
1997	1,298	758	989	57%	3,446	3,540	1,144	3,096	90%
1998	1,495	859	1,194	47%	5,467	5,848	1,345	4,350	80%
Total	4,225	2,567	3,322	54%					

Notes:

(2),(3) From Exhibit 3.
(4),(7) Actual data through 6/99.
(5) =[(2)-(4)] / [(2)-(3)].
(6) *From Exhibit 10.*
(8) Average of (2) and (4).
(9) = (7)x1000 / (8).
(10) = (9) / (6).

Paid Medical Losses

Exhibit 21

Accident Year	Cumulative Medical Losses Paid ($000) 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	2,636	4,613	5,521	5,884	6,067	6,133	6,181	6,221	6,252	6,291
1990	3,245	6,824	8,082	8,475	8,812	9,120	9,240	9,354	9,436	9,501
1991	4,515	8,880	10,693	11,296	11,784	12,102	12,287	12,530	12,629	12,698
1992	6,532	11,688	13,286	14,228	14,910	15,458	15,636	15,817	15,934	16,033
1993	8,486	13,356	15,592	17,165	18,197	18,951	19,231	19,516	19,682	19,823
1994	9,644	15,906	18,393	19,985	20,811	21,655	22,034	22,401	22,616	22,797
1995	9,836	15,828	18,997	20,843	21,832	22,732	23,124	23,503	23,725	23,912
1996	10,999	19,225	22,653	24,782	26,097	27,224	27,714	28,189	28,467	28,701
1997	14,834	23,019	26,933	29,441	30,960	32,262	32,828	33,376	33,697	33,968
1998	10,514	16,511	19,214	20,926	21,963	22,852	23,238	23,612	23,832	24,016

Age-to-Age Development Factor

1989	1.750	1.197	1.066	1.031	1.011	1.008	1.006	1.005	1.006
1990	2.103	1.184	1.049	1.040	1.035	1.013	1.012	1.009	1.007
1991	1.967	1.204	1.056	1.043	1.027	1.015	1.020	1.008	1.006
1992	1.789	1.137	1.071	1.048	1.037	1.011	1.012	1.007	1.006
1993	1.574	1.167	1.101	1.060	1.041	1.015	1.015	1.009	1.007
1994	1.649	1.156	1.087	1.041	1.041	1.017	1.017	1.010	1.008
1995	1.609	1.200	1.097	1.047	1.041	1.017	1.016	1.009	1.008
1996	1.748	1.178	1.094	1.053	1.043	1.018	1.017	1.010	1.008
1997	1.552	1.170	1.093	1.052	1.042	1.018	1.017	1.010	1.008

Note: Numbers below the line show projected losses and loss development based on Exhibit 14.

Future Average Medical Loss Payment at 10% Inflation Rate

Exhibit 22
Page 1

Accident Year	Projected Future Average Medical Payment per Open Claim (Using Exhibit 13 as Base)								
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									4,239
1991								3,221	4,663
1992							4,096	3,543	5,129
1993						2,826	4,506	3,897	5,642
1994					4,532	3,108	4,957	4,287	6,206
1995				3,650	4,985	3,419	5,452	4,715	6,827
1996			3,912	4,015	5,483	3,761	5,997	5,187	7,509
1997		3,808	4,303	4,417	6,032	4,137	6,597	5,705	8,260
1998	5,095	4,189	4,733	4,858	6,635	4,551	7,257	6,276	9,086

	Future Paid Medical Losses (In $000, equals average payment in this Exhibit times open claim in Exh 3)								
1989									
1990									65
1991								98	73
1992							181	123	108
1993						281	298	183	161
1994					845	397	403	247	218
1995				989	943	430	436	268	236
1996			2,129	1,378	1,237	563	572	351	309
1997		3,914	2,627	1,667	1,496	682	692	425	374
1998	5,997	2,831	1,879	1,192	1,070	488	495	304	268

Paid Medical Losses at 10% Inflation Rate

Exhibit 22
Page 2

Accident Year	Cumulative Medical Losses Paid ($000) 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	2,636	4,613	5,521	5,884	6,067	6,133	6,181	6,221	6,252	6,291
1990	3,245	6,824	8,082	8,475	8,812	9,120	9,240	9,354	9,436	9,501
1991	4,515	8,880	10,693	11,296	11,784	12,102	12,287	12,530	12,629	12,701
1992	6,532	11,688	13,286	14,228	14,910	15,458	15,636	15,817	15,940	16,048
1993	8,486	13,356	15,592	17,165	18,197	18,951	19,231	19,529	19,712	19,873
1994	9,644	15,906	18,393	19,985	20,811	21,655	22,052	22,455	22,702	22,920
1995	9,836	15,828	18,997	20,843	21,832	22,775	23,205	23,641	23,908	24,144
1996	10,999	19,225	22,653	24,782	26,159	27,396	27,960	28,531	28,882	29,192
1997	14,834	23,019	26,933	29,560	31,228	32,724	33,406	34,098	34,523	34,897
1998	10,514	16,511	19,343	21,222	22,414	23,484	23,972	24,467	24,771	25,038

Age-to-Age Development Factor

1989	1.750	1.197	1.066	1.031	1.011	1.008	1.006	1.005	1.006
1990	2.103	1.184	1.049	1.040	1.035	1.013	1.012	1.009	1.007
1991	1.967	1.204	1.056	1.043	1.027	1.015	1.020	1.008	1.006
1992	1.789	1.137	1.071	1.048	1.037	1.011	1.012	1.008	1.007
1993	1.574	1.167	1.101	1.060	1.041	1.015	1.015	1.009	1.008
1994	1.649	1.156	1.087	1.041	1.041	1.018	1.018	1.011	1.010
1995	1.609	1.200	1.097	1.047	1.043	1.019	1.019	1.011	1.010
1996	1.748	1.178	1.094	1.056	1.047	1.021	1.020	1.012	1.011
1997	1.552	1.170	1.098	1.056	1.048	1.021	1.021	1.012	1.011

Note: Numbers below the line show projected losses and loss development based on Exhibit 22, Page 1.

Claim Closure Pattern Reflecting Slower Claim Settlement

Exhibit 23

Accident Year	Projected Claim Closure Rate 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989					97.4%	98.4%	99.2%	99.6%	99.6%	99.8%
1990				94.7%	96.5%	98.0%	98.8%	99.3%	99.3%	99.8%
1991			87.4%	92.3%	96.4%	97.9%	98.7%	99.0%	99.4%	99.8%
1992		84.1%	88.6%	93.9%	97.2%	98.2%	99.1%	99.2%	99.4%	99.8%
1993	79.2%	84.8%	90.8%	95.5%	97.3%	98.3%	98.5%	99.2%	99.4%	99.8%
1994	79.0%	86.2%	92.7%	95.7%	97.5%	98.0%	98.5%	99.2%	99.4%	99.8%
1995	76.5%	87.1%	93.3%	96.2%	96.5%	98.0%	98.5%	99.2%	99.4%	99.8%
1996	79.3%	88.4%	93.4%	95.0%	96.5%	98.0%	98.5%	99.2%	99.4%	99.8%
1997	79.5%	88.2%	91.0%	95.0%	96.5%	98.0%	98.5%	99.2%	99.4%	99.8%
1998	78.5%	86.0%	91.0%	95.0%	96.5%	98.0%	98.5%	99.2%	99.4%	99.8%

Projected Number of Claims Open (Projected claims reported times the complement of closure rate)

Accident Year	12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989										7
1990									23	8
1991								39	22	9
1992							49	40	30	13
1993						120	105	54	40	17
1994					214	172	129	66	49	21
1995				320	296	170	127	65	49	21
1996			667	504	353	202	152	77	58	24
1997		1,298	993	554	389	222	167	85	64	27
1998	1,495	997	646	360	253	145	109	55	41	17

Average Number of Claims Open During Each Period

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									15
1991								30	16
1992							44	35	21
1993						113	79	47	29
1994					193	151	98	58	35
1995				308	233	148	96	57	35
1996			585	428	278	177	115	68	41
1997		1,146	774	471	305	195	126	74	45
1998	1,246	822	503	306	199	127	82	48	29

Future Average Medical Loss Payment at 10% Inflation Rate and Slower Claim Closure

Exhibit 24
Page 1

Accident Year	Projected Future Average Medical Payment per Open Claim (Exhibit 22, Page 1) 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1989									
1990									4,239
1991								3,221	4,663
1992							4,096	3,543	5,129
1993						2,826	4,506	3,897	5,642
1994					4,532	3,108	4,957	4,287	6,206
1995				3,650	4,985	3,419	5,452	4,715	6,827
1996			3,912	4,015	5,483	3,761	5,997	5,187	7,509
1997		3,808	4,303	4,417	6,032	4,137	6,597	5,705	8,260
1998	5,095	4,189	4,733	4,858	6,635	4,551	7,257	6,276	9,086

	Future Paid Medical Losses (In $000, equals average payment in this Exhibit times open claim in Exh 23)								
1989									
1990									65
1991								98	73
1992							181	123	108
1993						318	358	183	161
1994					875	469	484	247	218
1995				1,125	1,161	508	524	268	236
1996			2,289	1,720	1,523	666	687	351	309
1997		4,363	3,328	2,081	1,843	806	832	425	374
1998	6,349	3,442	2,380	1,489	1,318	576	595	304	268

Paid Medical Losses at 10% Inflation Rate and Slower Claim Closure

Exhibit 24
Page 2

Accident Year	Cumulative Medical Losses Paid ($000) 12 mos.	24 mos.	36 mos.	48 mos.	60 mos.	72 mos.	84 mos.	96 mos.	108 mos.	120 mos.
1989	2,636	4,613	5,521	5,884	6,067	6,133	6,181	6,221	6,252	6,291
1990	3,245	6,824	8,082	8,475	8,812	9,120	9,240	9,354	9,436	9,501
1991	4,515	8,880	10,693	11,296	11,784	12,102	12,287	12,530	12,629	12,701
1992	6,532	11,688	13,286	14,228	14,910	15,458	15,636	15,817	15,940	16,048
1993	8,486	13,356	15,592	17,165	18,197	18,951	19,269	19,627	19,810	19,971
1994	9,644	15,906	18,393	19,985	20,811	21,686	22,155	22,639	22,886	23,104
1995	9,836	15,828	18,997	20,843	21,967	23,128	23,636	24,160	24,427	24,663
1996	10,999	19,225	22,653	24,942	26,662	28,185	28,851	29,538	29,889	30,198
1997	14,834	23,019	27,382	30,710	32,791	34,634	35,440	36,271	36,696	37,070
1998	10,514	16,864	20,305	22,686	24,174	25,492	26,068	26,663	26,967	27,235

Age-to-Age Development Factor

1989	1.750	1.197	1.066	1.031	1.011	1.008	1.006	1.005	1.006
1990	2.103	1.184	1.049	1.040	1.035	1.013	1.012	1.009	1.007
1991	1.967	1.204	1.056	1.043	1.027	1.015	1.020	1.008	1.006
1992	1.789	1.137	1.071	1.048	1.037	1.011	1.012	1.008	1.007
1993	1.574	1.167	1.101	1.060	1.041	1.017	1.019	1.009	1.008
1994	1.649	1.156	1.087	1.041	1.042	1.022	1.022	1.011	1.010
1995	1.609	1.200	1.097	1.054	1.053	1.022	1.022	1.011	1.010
1996	1.748	1.178	1.101	1.069	1.057	1.024	1.024	1.012	1.010
1997	1.552	1.190	1.122	1.068	1.056	1.023	1.023	1.012	1.010

Note: Numbers below the line show projected losses and loss development based on Exhibit 24, Page 1.

Survey of Loss Reserving Actuaries Report

CAS Committee on Reserves

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EXECUTIVE SUMMARY

Effective January 1, 1998, the NAIC adopted a change in how loss adjustment expense (LAE) is split into categories within Schedule P of the property and casualty statutory Annual Statement. The purpose of the Survey of Loss Reserving Actuaries was to solicit input from loss reserve practitioners on how these changes impacted loss reserving since 1998, and how they may impact future years and other aspects of actuarial work. The following are the key findings of the survey:

- Nearly two-thirds (63.5%) of the respondents reported that they were company reserving actuaries, while one-quarter (25.7%) reported that they were consulting reserving actuaries.
- When asked to describe how their company classified ALAE vs. ULAE prior to the change on January 1, 1998, nearly six in ten (58.1%) respondents reported using claim specific / non-claim specific as their criteria.
- When asked to describe the major expense reclassification for their company, over one-half (56.8%) of the respondents reported that External Claim Adjusters were reclassified from ALAE to A&O.
- Over three-fourths (82.4%) of the respondents reported that they implemented changes with the 1998 Annual Statement.
- Over half (55.4%) of the respondents reported that their company selected the Calendar Year (all accident years for calendar year 1998 and beyond) method to implement the new LAE split.
- When asked what they used to classify expenses, over one-half (54.1%) reported using an Expense Tracking System, while nearly a one-quarter (23.0%) used Formula Allocations.
- A majority (55.4%) of the respondents reported that their company is currently maintaining internal expense reporting under the former categorization while adopting the new categorization for statutory reporting.
- Over half (58.1%) of the respondents reported that their company was not using the new expense categorization for any purposes other than Annual Statement reporting.
- When respondents were asked to indicate areas they believed that further research was needed regarding the impact of the new LAE categories, the most popular responses were Reinsurance Contracts (18.9%) and Ratemaking Practices (13.5%).

SURVEY METHODOLOGY

Designing the Questionnaire

A four-page, 17-item self-administered questionnaire (see Appendix) was developed by the CAS Committee on Reserves and approved by the CAS Executive Council.

Conducting the Survey

A total of 3,239 questionnaires were mailed to Fellows and Associates of the CAS the week of March 1, 2000. In addition, the survey could be completed online through the CAS Web Site. Respondents were asked to complete the survey by May 1, 2000.

Data Analysis

A total of 74 (2.3%) completed questionnaires were returned to the CAS Office. Close to a third (29.7%) of the surveys were completed electronically. Responses to survey questions were compiled, coded, and entered into a database. The responses were then analyzed using a statistical analysis software package (SPSS).

Responses to Open-ended Questions

The survey contained several open-ended questions that asked respondents to write-in their responses. Where responses to open-ended questions are summarized in the report, a number precedes each response. This identification number represents the specific survey on which the comments were written. This allows those reading the report to track the written comments of a particular respondent, if desired.

RESULTS

Question 1:
Please indicate your type of employment.

Response	Frequency	Percent
Company Reserving Actuary	47	63.5
Consulting Reserving Actuary	19	25.7
Insurance Department Actuary	0	0.0
Other	6	8.1
Blank	2	2.7
Total	74	100.0

Nearly two-thirds (63.5%) of the respondents reported that they were company reserving actuaries, while one-quarter (25.7%) reported that they were consulting reserving actuaries.

Written responses to "Other":
-Company reserve management
- Company Life/Health Actuary
- Accountant
- Accountant
- CFO
- CFO

Question 2:
Prior to the change on January 1, 1998, how did your company classify ALAE vs. ULAE? You may want to refer to the background information provided at the front of the survey.

Response	Frequency	Percent
Claim Specific / Non-claim specific	43	58.1
External versus Internal	10	13.5
Combination of A and B	17	23.0
Neither	1	1.4
Blank	3	4.1
Total	74	100.0

When asked to describe how their company classified ALAE vs. ULAE prior to the change on January 1, 1998, nearly six in ten (58.1%) respondents reported using claim specific / non-claim specific as their criteria.

Written comments to Question 2:
- External versus Internal, but all legal rep has been outside and no independent claims adjustments are used.

Question 3:
Which choice would most closely approximate the major expense reclassification for your company?

	Prior to January 1, 1998	After January 1, 1998
1. Internal Defense Costs	ULAE	DCC
2. External Claim Adjusters	ALAE	A&O

Response	Frequency	Percent
#1	11	14.9
#2	42	56.8
No material changes	10	13.5
Other	8	10.8
Blank	3	4.1
Total	74	100.0

When asked to describe the major expense reclassification for their company, a majority (56.8%) of the respondents reported that External Claim Adjusters were reclassified from ALAE to A&O.

Written responses to "Other":
- #1 expected to be greater ultimate impact, #2 greater paid-to-date.
- Clients are confused. Data is contaminated.
- Coverage defense costs from general or ALAE to ULAE.
- External Defense Costs.
- Both 1 and 2.
- I don't think the choices are listed properly.
- The change was not adopted.
- Both 1 and 2.

Question 4:
Did your company implement these changes in their 1998 Annual Statement?

Response	Frequency	Percent
Yes	61	82.4
No	9	12.2
Do not know	0	0.0
Blank	4	5.4
Total	74	100.0

Over three-fourths (82.4%) of the respondents reported that they implemented changes with the 1998 Annual Statement.

Written comments to Question 4:
- One of our companies assumed there was no limit.
- Yes, but not very accurately.
- Also, reserve adjustments at 12/31/97 in anticipation of changes.
- Yes, for reserves only. Paid reclassified beginning 1999.

Question 5:
Which method did your company select to implement the new LAE split?

Response	Frequency	Percent
Accident Year (Accident year 1998 and beyond)	25	33.8
Calendar Year (All accident years for calendar year 1998 and beyond)	41	55.4
Do not know	3	4.1
Blank	5	6.8
Total	74	100.0

Over half (55.4%) of the respondents reported that their company selected the Calendar Year (all accident years for calendar year 1998 and beyond) method to implement the new LAE split.

Question 6:
Which of the following were used to classify expenses?

Response	Frequency	Percent
Expense Tracking System	40	54.1
Formula Allocations	17	23.0
Special "time/expense" studies	12	16.2
Other	11	14.9
Do not know	11	14.9

When respondents were asked what they used to classify expenses, over one-half (54.1%) reported using an Expense Tracking System, while nearly a one-quarter (23.0%) used Formula Allocations.

Written responses to "Other":
- Outside Adjuster expenses are reported as A&O, the only change. No special efforts are required as this data is claim specific and identified by a unique code.
- Used expense tracking system for paid expenses and formula allocations for expense reserves.
- All external expenses are assigned a new statistical code which indicates if the expense is DCC or A&O.
- Questionnaire to MGA's.
- Special coding for payments to external adjusters.
- Clients and auditors selected criteria.
- Systems in place reflected ISO stat plan definitions of ALAE to ULAE.
- Adjusters, in-house legal.
- Bulk reclass of internal legal operation.
- Reports from TPA.
- By claim (external).
- Coded in the claims system.

Question 7:

If your company used formula allocations to reclassify expenses, what allocation base was used?

Responses:
- Claim counts.
- To split ALAE reserves into DCC and A&O, I reviewed historical paid ALAE split into DCC and A&O to develop a percentage split for each line and AY. All of our ULAE is A&O.
- Several.
- Expense reserves were allocated based on expense payments.
- Claim counts, paid external AE, paid loss.
- Paid expenses.
- Formula allocations only used for reserves. A % of ALAE reserves classified as A&O. The % varied by accident year (maturity of accident year).
- Square 1 = outage, headcount, etc.
- Premium and Loss dollar allocations.
- Result of Time Expense Study.
- Salary.
- ALAE payments.
- Expense tracking system for one of our companies was used to prorate for our other company.
- Claim counts, claim dollars.

Question 8:
Is your company currently maintaining internal expense reporting under the former categorization while adopting the new categorization for statutory reporting?

Response	Frequency	Percent
Yes	41	55.4
No	21	28.4
Do not know	6	8.1
Blank	6	8.1
Total	74	100.0

A majority (55.4%) of the respondents reported that their company is currently maintaining internal expense reporting under the former categorization while adopting the new categorization for statutory reporting.

Written comments for question 8:
- Aware of new categorization and will incorporate if we use services that make a difference.
- No, however, during much of 1999 company retained old definitions.
- Yes, for some purposes.
- Some companies do, others do not.

Question 9:

Is your company using the new expense categorization for any purposes other than Annual Statement reporting?

Response	Frequency	Percent
Yes	17	23.0
No	43	58.1
Do not know	7	9.5
Blank	7	9.5
Total	74	100.0

Over half (58.1%) of the respondents reported that their company was not using the new expense categorization for any purposes other than Annual Statement reporting.

Written responses to "If Yes, Explain":
- Only one they use.
- Budget, expense tracking, management reports, tax reports.
- Internal expense reporting.
- Using same categorization for internal reporting.
- Internal reporting.
- Excess profits reports.
- Internal reserve studies are done and communicated using the former categorization. Internal profit and loss statements are done using the new categorization.
- We do track components of ALAE payments for internal reasons but we reserve ALAE by total ALAE.
- Functional categorization always used for reserve, expense analysis.
- All internal statistics and financial reporting.
- Internal reporting, LAE reserve calculation.
- Internal reporting.
- Loss & LAE sensitive rating plans.
- Internal reporting.
- Internal reporting.
- All financial reporting (internal and external).

Question 10:
Explain how your companies accomplished a reclassification of expenses from categories where claim detail was not maintained (for example, internal defense attorney costs, formerly categorized as ULAE) to categories such as DCC, where at least some detail (i.e. accident year) would be required.

- Didn't reclassify paid expenses. Applied definition on CY basis.
- This was not a major issue. The main change was for external adjusters, moving from ALAE to A&O.
- Recoding of expense activity through ledger coordinated from claim payment system.

Question 10 (cont.):

Explain how your companies accomplished a reclassification of expenses from categories where claim detail was not maintained (for example, internal defense attorney costs, formerly categorized as ULAE) to categories such as DCC, where at least some detail (i.e. accident year) would be required.

– We didn't have internal expenses that would be classified as DCC. If we did, it would be assigned based on WTS.

– Claims staff estimated their time between the categories DCC and A&O – not revisited in 1999. Loss payments used to allocate between accident years.

– Internal defense attorney costs were negligible so no reallocation done.

– Estimate Total Paid ULE/DCC as a % of Total Paid ULE using Salaries plus Overhead. Allocate Paid ULE/DCC to Line of Business using judgment %'s. Allocate Paid ULE/DCC to Acc Year using Calendar Year Closed Claim Costs + Open Counts.

– N/A. Claim detail was already being captured for internal defense attorney.

– Did not apply for my company.

– Nothing changed except column headings in 1999. The company's operations are such that nothing needs to be shifted.

– Detail on internal DCC was always maintained so shifting was easy.

– Internal expense code was available.

– Nothing to reclassify.

– We obtained as much detail as possible and used interviewing of claims personnel and gut feeling to make projections.

– Varies. On one extreme a company may decide that expenses go in the same categories as before. On the other hand, a company may decide that DCCP amounts to no more than attorney fees.

– My company has no internal legal staff.

– For internal defense costs, tracking of costs to the claim level was instituted in 1997 for calendar year 1997 expenses. Since we have a high volume of such costs, the 18 months of data available by 6/30/98 gave us adequate information to use techniques based on incremental development.

– Most likely by claim distribution.

– Wild guessing. Make data look like what it should look like or what they want it to look like.

– Since we have insignificant internal expenses that could be characterized as DCC, we are calling all internal expenses A&O. We are continuing to use the old ULAE accident year allocation rule of 45/5.

– Assumes reinsurance. Contracts written since 1/1/96 for ALAE (DCC) as one component of LAE, all other expenses defined as an other component.

Question 10 (cont.):

Explain how your companies accomplished a reclassification of expenses from categories where claim detail was not maintained (for example, internal defense attorney costs, formerly categorized as ULAE) to categories such as DCC, where at least some detail (i.e. accident year) would be required.

– For treaty reinsurance, an arbitrary formula reallocation was used, varying by subject treaty. The treaties follow the old definition!! For MGA's, we surveyed them. If they responded, we used what they gave us. For those who didn't respond, we prorated following the pattern of those who did.

– We did not encounter this situation. All of our expenses that were reclassified had coding on them that allowed us to accomplish the reclassification.

– ALAE reserves were reclassed as O&A based on ALAE payments being reclassed as O&A. ALAE reserves are not kept at a detailed level, but ALAE payments are. We then allocated these reclassed ALAE reserves to AY using judgment.

– Used department-specific expenses, allocated bases on claim counts, price losses, and price external LAE, as appropriate.

– A constant average cost per claim was applied to each claim handled by internal defense units. (This was not a large expense item at my company).

– No internal defense costs.

– No material change.

– Clients used a variety of arbitrary criteria. Few clients fully comprehend the revision. Virtually all view the revision as a regulatory item which does not impact management information.

– For these three relatively small companies, old ULAE is still A&O so there was NO reclassification from ULAE to DCC.

– We don't have the detail, so we just use allocation procedures to put the new DCC dollars somewhere. There is nothing in the regulation that says what type of detail you have to maintain on the new expenses.

– Since no actual data was available, expenses were booked to Personal Auto Bodily Injury. Amounts were considered immaterial.

– Claim detail was generally always maintained.

– Not sure.

– No internal defense attorney costs.

– This particular company is unusual and doesn't involve 3rd party litigation.

– Detail was maintained.

– Our company did not need to make any changes to comply with the new categories.

– Rudimentary formula allocations (guesses).

– We had all needed detail.

– We calculate ULAE reserves under the old definition by coverage and accident year, then estimate the percentage attributable to internal defense attorney costs, based on input/claims data from our Law Department. These percentages are mainly based on actuarial judgment.

Question 10 (cont.):
Explain how your companies accomplished a reclassification of expenses from categories where claim detail was not maintained (for example, internal defense attorney costs, formerly categorized as ULAE) to categories such as DCC, where at least some detail (i.e. accident year) would be required.

- No internal attorneys.
- Internal legal per-hour rates and internal medical cost containment per-transaction rates were determined. Costs are assigned to individual claims based on these rates.
- Since 1989, the company has utilized both time tracking and flat fee accounting methods to charge claims files for internal defense costs. The company historically carries a claim level code to identify external adjuster expense which is easily classified as adjuster expense on Schedule "P".
- I do not know. Generally, I accept a company/client's data as valid if the results look reasonable.
- We first allocated the calendar year internal defense costs into DCC. The calendar year payments were spread to accident years using a claim count process involving numbers of claim payments and numbers of open claims. Reserves were computed using runoffs of claim counts.
- We spread the reclass based on the "old" ALAE data still captured by our systems.
- A reclassification of expenses between categories would not have a significant impact given our volume of expenses in current classification we use.
- Claim detail was maintained prior to this change on expenses that were reclassified, therefore the reclassification was not difficult.
- Expense tracking system.
- Did not affect us.
- We do not have internal defense attorneys.
- Estimation based on discussions with claims management.
- Didn't need to. Internal litigation was coded to claim files.
- The change was not adopted.
- The company uses no internal defense attorneys, and so this major change item did not apply. Payments to independent adjusters are given a unique transaction code, and so were easily recategorized. Other items were immaterial.
- Primarily moved independent adjusters fees to A&O.

Question 11:
On a calendar year basis, the new categorizations apply to the incremental calendar year change across all accident years beginning January 1, 1998. From a Schedule P standpoint, this means for accident years 1997 and prior, the 12/31/98 evaluation of ALAE (i.e. the current column) and all future evaluations (or columns) will reflect a mixed definition. Accident year 1998 and future accident years will be under the new DCC definition. On an accident year basis, the new categorizations will apply to only accident year 1998 and future accident years. Prior accident years will continue to run-off under the old definition of ALAE.

What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

– CY Basis: Compare total LAE projections - new vs. old definition - to benchmarks that have not changed (premium, loss reserves, paid LAE, etc.). We've found that % of total LAE which is A&O is greater than that which was ULAE. We apply "pd to pd" method to determine A&O/Loss ratio as the basis of projecting A&O reserves. We have tried to establish "pd to pd" DCC/Loss factors as well.

– The company has maintained internal expense reports that utilize the old ALAE/ULAE segregation. They will continue to do this until sufficient experience has been gathered using the new categories.

– Company reserves per LAE are not overly significant.

– The only challenge is the need to refine our database and make a few special calculations. Just a nuisance.

– Where expense was shifted to A&O have moved it back to DCC to be consistent with former ALAE definition. Have still used ratio for A&O based on ULAE.

– Internal reserving continues to use ALAE & ULAE. Opining actuary uses ALAE & ULAE. For Schedule P analysis, we had accounting staff restate 1998 & 1999 accident years in terms of ALAE & ULAE.

– Biggest change is outside adjusters but reserves for outside adjusters have been estimated independent of other ALAE reserves for many years. Definition change was easy to handle from a reserving perspective.

– Significant judgments required in the selection of projection factors. We calculate total LAE reserve needs using historical triangle of ULE & ALE (DCC + A/O) and make sure that the judgment calls we're making for DCC and A/O individually, yield an overall LAE reserve similar to what we develop in total.

– Internally, we have not changed our reserving practices and still review ALAE and ULAE reserves separately. There is just an extra step required to split ALAE into DCC and A&O for statutory reporting.

– Under the circumstances described, nothing different needs to be done.

– Need to track development separately. Will use combined LAE as well.

– Reserving practices not changed. Still analyze ALAE separately from ULAE. The differences between ALAE & DCC and ULAE and A&O are dealt with in the data reconciliation of the actuarial report.

Question 11 (cont.):
What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

– Duplication of history in internal statistics and Schedule P data. Worked on breaking out these adjustments.

– For older years we did not have the detail to construct expense triangles under the new definition therefore we had to combine all expenses to form a LAE triangle to determine expense development.

– We used the old ALAE/ULAE definition to group data for reserving purposes then reallocated the bulk reserve for annual statement purposes according to the new categorization.

– N/A for this client. In general, though, I would probably propose a mapping of ULAE into accidents and build a hypothetical "DCC" triangle.

– We continue to capture the old definition of ALAE & ULAE to estimate reserves. We then allocate to the DCC and A&O based on an allocation system using internal expense code data.

– We applied the change on an accident year basis as this seemed cleaner to me. The challenges are from lack a historical data under the new definition. Also, reinsurance contracts have not changed the definition so detail must be kept in both fashions.

– The challenge is to find for each company individually procedures and methods which give reasonable results. Reasonability is about all one can probably hope for, at least for a while. With respect to DCCP, if one is using paid to paid factors by accident year which are developed to their ultimate values and multiplied by estimates of ultimate claims, it is possible to presume that the pattern of development will be the same with the new data as for the old except for the discontinuity as of January 1, 1998. Calculations can then be based upon this method being careful in the application of the ultimate losses to take account of the fact that payments prior to January 1, 1998 are of a different nature than those thereafter. With respect to AOP, if calendar year paid to paid ratios are utilized, it makes sense to examine the data separately for calendar years prior to 1998 and years 1998 and later.

– Looking at historical triangles will obviously be skewed (since we took the calendar year approach). Since my company has no internal legal staff handling DCC, that poses no problem, but independent adjuster costs are shifting from ULAE to DCC.

– We are able to keep separate the data for internal defense costs, external adjuster costs, and other ALAE costs. Currently we estimate "old ALAE definition" amounts, then subtract estimated external adjuster costs and add estimated internal defense costs. During a transition period, which will vary by line of business, this additional analysis will be required. Once we have adequate "new definition" data, we can revert to a simpler analysis.

– If they do it by calendar year basis your prior data will not be consistent w/ current year data thus leaving tests like the IRIS Ratios w/ no value on an Accident Year basis. I think we avoid the above problem.

– Easy for those who don't change. We will let you know when they move to new definitions. On CY basis, restating screws up triangles.

Question 11 (cont.):
What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

– Won't run-off calendar year 98 and subsequent be under the new DCC definition? Our reserving practices have not changed at all. We are still developing our ALAE/external and ULAE/internal reserves the same way we have in the past. What has changed is that we now have to allocate our developed ALAE/external reserve to the new DCC and A&O categories. We are allocating to these categories based on paid DCC and A&O expenses collected for calendar year 98 and subsequent. We consider all ULAE/internal reserves to be A&O. We are also collecting and building historical triangles of external DCC and A&O paid expenses for AY 1998 and subsequent. As soon as sufficient history is available we will use triangular analysis to develop our external DCC and A&O reserves.

– Expenses defined by reinsurance contracts.

– The challenge of Accident Year (selected) is to keep the pre-1988 Accident years on the old basis. The challenge for Treaties is to get anything like the new definitions into the contracts. There is also the problem of availability of UW years on some treaties.

– Internally, we have recast our triangles to be consistent with the current definitions.

– The definitional change only affects our statutory reports; internal reserving data was left unchanged. Therefore, our reserving practices haven't changed.

– Lack of data for internal DCC on a historical basis. Use of new definitions (for data-gathering and reserve analysis only) for all accident years.

– Currently, we continue to project ultimates using data under the former categorization, and then allocate the resulting IBNR needs to the new categories for AYs 1998 and subsequent. In the future, we will likely try to obtain restated (according to the new categorization) historical data to directly project the new category amounts for AYs 1998 and subsequent.

– The biggest issue is separating the A&O component of ALAE reserves. We can separate the historical payments of ALAE by component. We looked at historical A&O payments as % of total ALAE payments by accident year at different evaluation points (12, 24, 36 uses, etc.) From that, we could derive the % of total bulk ALAE reserves for A&O by accident years at different evaluation points.

– No material difference.

– Have focused on total L & LAE reserve. Further, several clients advise that claim service contracts, with third parties, obviate the need for A&O reserves or A&O payments.

– Most coverage defense issues relate to mass torts, which are concentrated in the "prior" AY. Also, AY triangle analysis typically is not performed for mass tort business, and these expenses were always analyzed separately, anyway, hence no new challenges, no new issues.

Question 11 (cont.):
What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

– For these three relatively small companies 1) The shift to DCC and A&O was implemented piecemeal throughout AY 1998 so even AY 1998 is a mixture of old and new. Only AY 1999 has pure DCC and A&O available. There won't be any DCC and A&O patterns for a few years yet. 2) For AY 1997 and prior, no company chose or could restate history. So there are no historical DCC or A&O patterns. 3) Since the three companies have to maintain old ALAE and ULAE for reinsurance purposes, they are tracking ALAE by DCC and A&O components. 4) The reserving practices have changed as follows: The preliminary reserving methodology continues to address ALAE and ULAE like before. Then ALAE is allocated to DCC and A&O based upon payments made since 7/98 by LOB/AY categories. ULAE is all assigned to A&O.

– There are no reserving challenges for us, except for doing the final allocation for annual statement purposes. There is no reason for us to throw out our historical data or historical way of setting reserves just because of this change. It is our opinion that we have to establish the right overall level of reserves. Which category they ultimately end in is immaterial.

– We analyze our ALE and ULE reserves separately using an accident year change in Paid ALE to Incurred L/R estimate for ALE and a calendar year Paid to Paid and Paid to Paid plus O/S as an estimate of the relationship of the ULE O/S to Loss O/S for ULE. In all but one Reserve Analysis there appeared to be no distortion in the rate of ALE to ULE. In the one we used the latest year diagonal which effectively eliminated the distortion. We will probably change to a Paid to Paid method and analyze the reserve in total.

– We kept enough detail on ALAE: Internal vs External & ULAE: Internal vs External so that the change over was not cumbersome.

– We reserve at the old level, and the financial area reallocates the result to the new classifications.

– No change to reserving practices except to recognize lower DCC costs in the calculations.

– This company had minimal ALAE under the old definition and it was more appropriate to redefine all years to a consistent basis using the new definitions.

– Instead of attempting to restate history, reserving is being done based on the old ALAE/ULAE definitions.

– Since the company's expense classifications have historically been consistent with the new categories, we did not need to do anything different.

– We use the old ALAE/ULAE and internal reports for LAE reserve adequacy testing. Reconciliations to the annual statement for actuarial reports is ugly. I still haven't run across a case where old ALAE+ULAE = new DCC + A&O.

– We are analyzing using the "old" definition and then re-allocating the final reserve.

– None, we maintained ALAE and ULAE definitions for reserving and make an adjustment to reflect the change in Schedule P.

– Internally, we have maintained the old definitions, so we have not changed our basic reserving practices. Our challenge is in estimating how much "old definition" ULAE to move to DCC for statutory purposes. Our system can capture ALAE (old definition) moving to A&O, so that hasn't been as difficult.

Question 11 (cont.):
What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

– Biggest impact was on auto physical damage. This line is so short tailed that I simply applied a little actuarial judgment.

– 1) Initially evaluate ALAE & ULAE with data segregated under the old definition. 2) Estimate independent adjuster expense reserves separately using development patterns for that expense. 3) Transfer indicated independent adjuster reserve from ALAE to ULAE. (Internal legal and medical cost containment reserves are not material for us).

– The DCC reserving changes are not a problem for us since we have accounted for internal defense cost as such since 1989. The change for IA's is not a problem because of short tail nature of that expense.

– We use data summarized by the old definition to determine required LAE reserves.

– Most client/companies have tracked data under both definitions. (In most cases, the recoverability of loss adjustment data is based on the pre-1998 definition, meaning they have to capture the data anyway). For those that don't, I can develop ratios regarding category shift from averages of other clients. Given the data in both formats, I have not found it necessary to change my reserving practices. The "mixed definition" is problematic, because the column will be mislabeled through the 2008 Annual Statement.

– We ignore previous ALAE and ULAE payments. Our methods rely on recent calendar years only and use claim counts to spread the calendar year LAE payments and claim count runoff patterns to get LAE reserves by accident year.

– No change to reserving practices. I don't pay any attention to the #s shown in the statement. I can continue to compare long term ratios based on the old definitions.

– Our reserving practices have historically been based on Bulk IBNR and ultimate losses including loss adjustment expenses; accordingly, we follow the same reserving practices given the minimal impact the changes would have on the financial presentation.

– Our Company handled the new categorizations on a calendar year basis. Our reserving practice has not changed; we simply added another level of detail to our analysis when evaluating our reserves.

– I asked all companies to give me the LAE under the old definition so that projections are possible. Without this, I don't have any way of doing it.

– Use outside actuary. No major problems noted.

– We continue our reserving practices based on old definitions and allocations.

– We were able to restate our ALAE triangles (not Schedule P data) and ULAE formula to be on the new definition.

– The change was not adopted.

– At this time we are evaluating expense reserves based on data accumulated per the prior definitions. The estimated reserve is then allocated using claim counts and claim $ according to the new definitions.

Question 11 (cont.):
What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

– The company has no internal defense attorneys, and so no recategorization took place. Independent adjuster costs are minimal (less than 0.5% of all loss adjustment expense). The amounts involved for the company are insignificant. No attempt has been made to change reserving practices.

– I have attempted to get my clients to break out the portion of A&O expenses that reflect "old" ALAE, i.e., expenses that can be allocated to an accident year. We analyze these expenses as in the past. The remainder, or "old" ULAE, is then analyzed separately as in the past.

– We analyze and select ALAE reserves based on the old definition as a starting point. We then have outside adjuster expense factors (which vary by line and acc year) which are applied to the old definition ALAE reserves to determine the outside adjuster expense reserve portion. These outside adjuster reserves are then subtracted from the old definition ALAE reserves and added to the old definition ULAE reserves. Since we do not use any in-house attorneys, there were no issues for us with the definition change as it related to this piece. The outside adjuster expense factors are analyzed and selected annually using paid outside adjuster expense triangles.

Question 12:
In your opinion, what impact will the categorization change have on industry Schedule P data as individual companies make different choices on how they will handle the change?

– Only has integrity in the aggregate (DCC + A&O combined). Should expand Parts 2-4 to include both DCC and A&O.

– Industry Schedule P data for all companies combined will be distorted by the change and by different ways of handling the change. Could impact companies that use industry data for benchmarking.

– I envision no major benefit to anyone. ALAE was always assigned to an AY. Internal expenses were assigned by allocation. I see no change occurring, just column change.

– Less expense under DCC than ALAE. More expense under A&O than ULAE.

– A big mess.

– Will only be able to analyze total LAE expenses and reserves for accident years prior to 1998.

– I expect that the allocations which companies will be doing in the future to get to DCC + A/O splits will yield every bit as much inconsistency from company to company as the old ALE and ULE split did.

– Minimal impact.

– Little to none.

– It will be different but not necessarily more consistent. My clients emphasize LAE to reduce losses to varying degrees.

– Industry Schedule P data will be a mish-mash of various company definitions.

– Large companies with internal legal staffs will have reclassification going both ways. Small companies probably will have reclassification going only one way.

Question 12 (cont.):

In your opinion, what impact will the categorization change have on industry Schedule P data as individual companies make different choices on how they will handle the change?

– I feel it gives us an inconsistent look when comparing companies. It would be more accurate to classify all expenses as LAE. Then we would have an accurate comparison of expenses.

– Allocation methods may vary widely. I could understand the inclusion of ALAE in Schedule P triangles. I am not sure how to interpret the inclusion of cost containment.

– We will not be able to trust the Schedule P data for years prior to 1998. While it may not have a huge impact for personal lines, I would be worried about some of the other liability and company lines because of the potential for long ALAE.

- I don't think it will improve reliability.

– You will not be able to use industry data as readily as in the past. Should have had all companies handle the change the same way.

– I am not optimistic that the data will be any more homogeneous between companies using the new definitions than it was using the old ones. The Schedule P data will be of much less value for a number of years than it has been in the past. It is not clear it will be more valuable at any time in the future because of this change than it has in the past.

– Should make for a clearer comparison of defense costs among various companies.

– The inconsistency will obviously cause Schedule P distortions.

– Industry conglomerate data could be rendered useless for 10 years especially for small companies who employ outside adjusters.

– It will defeat the whole purpose of having a standardized format for Schedule P. There should not be different choices on handling the change. For 10 years –> data=garbage. Especially bad for small companies, lots of outside adjusters.

– Schedule P data will be distorted. The reliability of any triangular analysis based on Schedule P data during the 10-year phase in period must be questioned.

– Schedule P has not been useful for reserve testing due to limit differences, reinsurance changes, statutory coverage differences for multistate writers. Schedule P combines various coverages that should be reserved separately.

– I will mainly follow combined as I cannot trust separations (especially pre-AY 1998).

– This change was not all that material. Given the limitations of Schedule P for reserve analysis, I don't think this will materially affect the quality of any industry analysis that uses Schedule P.

– ALAE and ULAE (or DCC and O&A) ratios will be inconsistent across companies if Schedule P is relied on for comparisons. This could lead to market analysts making incorrect conclusions. Also, Schedule P, Part 2 (Runoff) could be distorted since only DCC is included but O&A is not.

– No consistent basis across industry. Also depends on prior treatment of internal DCC.

– The data will be more volatile.

– It will take 10 years before any form of consistency is gained.

Question 12 (cont.):

In your opinion, what impact will the categorization change have on industry Schedule P data as individual companies make different choices on how they will handle the change?

– Increased emphasis on aggregate LAE data; reserve developments are now "minimums"; reinsurance treaty definitions may not follow annual statement.

– Most commercial lines companies were reporting expenses for the liability lines using the ISO stat plan, which used a functional definition for ALAE. Hence these companies were completing the annual statement consistent with new definition, inconsistent with old. Little impact expected for commercial lines. Most companies not using the ISO stat plan were personal lines NAII members, so personal lines industry data may be impacted (e.g. Our Company piece probably impacted).

– 1) Parts 2 and 3 are useless and will be for a number of years; 2) Part 1 is useless for DCC and A&O, and will be for a number of years. Only combined DCC+A&O (old LAE) has some usefulness; 3) Regulators aren't going to have any useful industry loss expense data for a few years.

– Overall it won't have any impact to us. Industry Schedule P data was of limited use to us before, and this will only make it worse. The regulation doesn't force companies to change to claims practices, so it is just a reporting issue. Any actuary with common sense knows not to place a lot of faith in data that comes from another company that doesn't operate the same way you do.

– This change in practice makes absolutely no sense whatsoever. I have been told that the reason for the change is that ALE is not comparable between companies (some companies utilize outside adjusters more than others). Well, companies are different and though old rule measured that difference, the proper place to break out legal and adjusting was in Part 4 (A/S) expense class. Break out line veto, direct legal and direct adjusters.

– Depends how much past practices would conflict with present practices on a company by company basis.

– It will make P a mixed bag with respect to LAE.

– In the long run, it is an improvement, but more dependence will be placed on evaluating the combined LAE as a cross check of reasonableness for the next few years.

– More inconsistencies will exist now than prior to the change.

– I expect some inconsistency for a few years until definitions are refined and companies fully adapt to the reclassification.

– Schedule P data is less useful, both in looking at the industry and at individual companies.

– The industry Schedule P will continue to be ambiguous. But, in the future the standardization will be beneficial.

– Impossible to tell – some companies have ULAE going into DCC and others have ULAE going into A&O. Could be a wash, probably is not.

– Industry Schedule P data will be even more difficult to decipher than ever before—I get the impression that most industry analysts don't have any idea how to interpret the impact. We'll need several full years of data under the new definitions to have any idea what the impact is.

– No opinion.

Question 12 (cont.):

In your opinion, what impact will the categorization change have on industry Schedule P data as individual companies make different choices on how they will handle the change?

– Some distortion on aggregate industry ULAE & ALAE development patterns. Uncertainty about appropriate adjustments due to variety of company treatments.

– Company comparisons will be more meaningful.

– It will make Schedule P even less useful than it already is.

– I have always believed that industry data must be handled cautiously – for reasons such as this.

– Can't use Schedule P to determine the adequacy of LAE reserves.

– Should be better once everybody implements it. But the next couple of years will be a transition period.

– Anyone using Schedule P will now have to ask questions regarding different assumptions that companies make than they did before. There is still consistency from company to company as to what is allocated between categories.

– As a result of the categorization changes, the impact to Schedule P will be that the data provided in the "Defense and Cost Containment" and "Adjusting and Other Payments" categories will lose its creditability. However, the overall impact would be minimal since these costs would be included in the total losses and loss expense.

– If more companies within the industry choose the calendar year method, the industry will produce a more favorable loss development on accident year 1997 and prior.

– At my personal lines only company, the shift was very insignificant.

– Schedule P is worthless for any comparison.

– I prefer the old split of internal versus external expenses. This new definition only makes Schedule P less useful.

– Incurred development (Part 2) will differ in its meaning by company so industry aggregate will be a mixed bag. This is because Part 2 (as well as Parts 3 and 4) only considers loss & DCC. I would suggest changing these parts to include DCC and A&O for AY 1998.

– Will probably be less useful for a period of time until companies have been on the new definition for several years. See #16 for additional comments.

– There will be less consistency in industry data going forward. It was my understanding that the primary reason that this change was adopted was to allow improved direct comparisons between companies. I do not support that reason as being more important than ratemaking, pricing, reserving, underwriting and reinsurance reasons for continuing allocating as many claim-specific dollars as possible to individual claims. Regulations should benefit policyholders and not simply add to the expense dollars policyholders should pay. One additional reason this change was not implemented here is that I/T resources were not available to do this work at a time when they were already overburdened doing Y2K remediation work. See further comments in response to number 15.

– Schedule P data will be a mixture of categorizations. Looking at expense data will be difficult and less meaningful for several years.

Question 12 (cont.):
In your opinion, what impact will the categorization change have on industry Schedule P data as individual companies make different choices on how they will handle the change?

– We do not see this as a significant issue. A review of the annual statement for different companies shows that companies have historically varied substantially when completing the annual statement. Thus, the history is hardly consistent prior to this change. We hope that more uniformity will result in the future because of this change.

– Data will be less reliable, less useful since companies will be inconsistent. Also, no one will be able to figure out how to analyze A&O category.

– Since each company will handle this differently, it may make it difficult to compare DCC and A/O results across companies.

Question 13:
How will users of Schedule P adjust for possible distortions in the data?

– Lots of uncertainty. Use interrogations to ascertain how company has implemented the new definition.

– It may be possible to make broad assumptions concerning what portion of ALAE is now recorded as A&O, etc. A statistical study could be undertaken.

– View change with possible factors to reflect distortions.

– They probably will make no adjustment.

– Restate in terms of ALAE & ULAE. Add ALAE/ULAE and DCC/A&O together prior to analysis.

– Will have to rely on accident years 1998 and subsequent to estimate the distribution of LAE for accident years prior to 1998.

– Ignore DCC and A/O and rely on total LAE.

– They probably won't adjust.

– Don't know, but it's not anywhere near or significant a "distortion" as that of shifting policy limit, attachment points, and "ultimate net loss" – type arrangement.

– I think most will treat ALAE w/ DCC and ULAE w/ A&O.

– How can they? Schedule P has become even more useless.

– Add everything together to get proper view.

– They will have to look at the combined DCC and A&O to get an accurate view of expenses.

– May need to combine ALAE & ULAE for several years. Could use individual company info but must find out what each company did.

– Combine all LAE.

– No need for adjustment.

– Without knowledge of company specific changes it will be extremely difficult to utilize Schedule P.

– The same way you raise teenagers -- any way you can.

– I have not given this much thought since we do not use Schedule P much.

Question 13 (cont.):
How will users of Schedule P adjust for possible distortions in the data?

– Analysts could use loss data only or loss & LAE data but loss & ALAE data will be screwed.

- Not use Schedule P! Not believe the data if they do use it. Combine the prices – current & historical and compare total to total. Look at losses alone.

- Not sure.

- Not use Schedule P. Ask for actuarial report.

- Combine ULAE and ALAE (or A&O & DCC) or even combine Loss and all LAE. (I don't trust case ALAE either, even as the old definition).

- I think they should simply acknowledge that there might be some small distortion and proceed.

- When possible, look at Loss & LAE instead of just Loss & DCC or O&A separately.

- Don't know; perhaps they will focus on Loss + Total LAE data, or just Loss Only data.

- I would look at total Loss and total LAE and not bother with the components. Not sure whey the switch was necessary.

- Increased focus on aggregate LAE data, if informed. Won't justify, if uninformed.

- Who uses Schedule P data? No impact on commercial lines in general, so no adjustment needed. State Farm and other big NAII members probably don't use Schedule P, so no harm, no foul. Small personal lines companies that have no other reserving data, or rely (unadvisably) on industry Schedule P data may be impacted. Not obvious for me (who is not impacted) why the impact can't be treated like a distortion from a cat.

- 1) Can't adjust on an industry basis with any assurance; 2) On company by company basis, it will depend upon the company reclassification approach and any supporting data. So there are no generalities.

- Who knows. Given the low intelligence level of the people who pushed for this change, who can guess what they will do. Since fewer people will be able to use the data, it probably doesn't matter what they do. And since I won't use their analyses, I don't care what they do.

- They can't. Schedule P has been rendered useless for separate analysis of ALE and ULE reserves.

- I'm not sure there is a clear-cut way to adjust for these types of distortions.

- Not sure.

- It may require using calendar year ratios on the last year or two or using combined LAE for calculations.

- Look at LAE in total and not the subsets.

- One simple method would be to compare historical expense/indemnity prior to the change and after the change. Apply a factor to adjust all years to a common standard.

- Like always, they will make the most of available information, with necessary qualifications.

Question 13 (cont.):
How will users of Schedule P adjust for possible distortions in the data?

- Use other sources of company data as/if they become available. Adjust individual companies to the industry average -- which is contrary to the purpose of looking at an individual company.

- Unknown.

– On a company basis, you need a disclosure in Schedule P Interrogations of some sort. Without a disclosure, it would be impossible to adjust for it. Trying to look at total LAE is not, in my opinion, an adequate approach. On an industry-wide basis -- impossible w/o more info.

– Ignore 1998/1999 Schedule P's for the development of ULAE/ALAE ratios. We've already seen this with a 1998 financial exam -- the auditors ignored the 1998 Schedule P and used 1997 P's to develop ULAE ratios. We were told that this was how they were handling the problem.

– No opinion.

– Combine ALAE & ULAE for analyses purposes.

– Users will require additional data from the company to evaluate adjustments by expense type and line of business.

- ????

– (I assume this question refers to individual Schedule P and not industry Schedule P). There is never a good substitute for knowing the company under evaluation, and having access to key personnel who can interpret the data for you as you analyze. If I were to analyze a company's Schedule P without that company's knowledge, I would likely combine both categories of expenses and evaluated them as a whole.

– Each company will have to be dealt with on its own merits.

– Will either have to use total LAE (many actuaries have been doing this anyway with industry data) or pick certain companies you know have implemented the new guidelines properly.

– Depends on the company being analyzed. As far as trying to review industry totals is concerned, I would think that only total LAE could be reviewed with any confidence.

– Users will adjust for the possible distortions in the data by applying more weight to the total losses and loss expenses and to adjust the individual analyses based on the data.

– They will have to include activity on Adjusting and Other Expenses when evaluating the loss data.

– I don't know how anyone can.

– I thing w/o providing detail of data under the old method, distortions will be impossible to quantify.

– They probably won't and may reach distorted conclusions.

– Do not know.

Question 13 (cont.):
How will users of Schedule P adjust for possible distortions in the data?

– This is a good question. Historically, Parts 2 and 3 have been prepared on a loss plus allocated basis given the fact that, by their very nature, unallocated loss adjustment expenses could only be assigned by accident year on a judgment basis. I am not clear what is expected to show up in Parts 2 and 3 now. To the extent any ULAE-type losses get into Part 2, loss development measures will be distorted. To the extent they get into Part 3, paid loss development patterns will be distorted. Who knows how users will adjust for these distortions, or if they even can.

– Look at all LAE combined. There still may be distortions because the distribution of expenses to accident year may have changed with the change in categorization.

– We do not use Schedule P very often. We assume that we would examine Schedule P in order to determine whether an apparent shift exists around calendar year 1998. If so, we would restate older years on a basis consistent with the most recent years.

– See #11. However, this cannot be done on industry data. No one will have a clue what's in accident years 1997 and prior.

– So far, outside users of Schedule P data do not seem concerned enough to adjust for the distortion created by implementing this change on a calendar year basis. No one has asked us to provide information to help them adjust the data.

Question 14:
As a result of the revised expense categories, Schedule P data subsequent to January 1, 1998 is on a different basis than that of prior years. What impact has this had on reports, for example IRIS tests, that are based on Schedule P data?

– IRIS tests involving ALAE are impacted (Ratio 10 & 11). IRIS tests 10 & 11 probably understated.

– Not a significant issue for my company since expenses are a relatively small portion of loss and expense.

– Minor impact.

– No idea.

– Probably very little.

– No apparent impacts.

– False indications of downward development in ALAE for accident years prior to 1998.

– For a company which has more $'s shift from ALE to A/O than from ULE to DCC and implemented the change on a calendar year basis, IRIS ratios are easier to pass at 12/31/98 and 12/31/99. This is because there are 12/31/97 reserves for ALE (now A/O) which will not have any subsequent payments in Schedule P Part 2.

– Minimal impact.

– No perceptible difference.

– Depends on how big the change was and what LOB. For many lines, LAE is smaller portion. Also, total LAE should not be affected.

Question 14 (cont.):
As a result of the revised expense categories, Schedule P data subsequent to January 1, 1998 is on a different basis than that of prior years. What impact has this had on reports, for example IRIS tests, that are based on Schedule P data?

– More leeway in reserve developments because most adjustments went from ALAE to ULAE for my company.

– I would imagine that the shifts of dollars into DCC was bigger than the shift of dollars out of ALAE. Resulting one to two year development is probably worse than otherwise.

– None really since our company implemented on an accident year basis. It would cause either redundancies or deficiencies to show up if implemented on a CY basis depending on the amount of outside adjusting or internal defense utilized.

– Varies by company. There may be an effect which is not all that significant. Both theoretical research - i.e., examining scenarios to better understand the effects - and compilation of actual results will be helpful. The latter will be particularly useful. As to area, reinsurance and ratemaking may be the most significant for some time to come. Retrospectively rated policies will be a sensitive area worth attention.

– For our company, impact was not a serious issue.

– If companies adopt the definition on a CY basis, the test can be useless since reserve development can't be calculated on a consistent basis.

– It makes them wrong. They are screwed up and I think not very useful.

– IRIS tests 9, 10, and 11 are distorted. The distortions will be favorable for us since by definition, DCC is a smaller reserve than ALAE. The distortions should go away next year.

– Probably show deficiencies as ALAE will be smaller than previous year.

– Probably not a big enough difference to matter. Fairer anyway as previously staff versus adjuster companies were treated differently.

– Small.

– For our company, the impact has not been significant.

– No material impact.

– Don't know.

– Haven't thought through that. Our IRIS tests not an issue.

– No material impact.

– IRIS ratios 9, 10 & 11 are now minimum development because some expenses are A&O and A&O does not wind up in "development" columns of Schedule P.

– No impact on IRIS for most commercial lines companies. No impact expected for Schedule P - Part 1. No impact for ISO stat plan companies. Not obvious that issue is big enough for those impacted (personal lines, non ISO companies).

– It didn't trigger unacceptable IRIS test values for my companies.

– It hasn't had an impact yet.

– I guess this would be a reasonable topic for *Proceedings* or *Forum* Paper.

– For us, it did not produce any exceptional IRIS values.

Question 14 (cont.):
As a result of the revised expense categories, Schedule P data subsequent to January 1, 1998 is on a different basis than that of prior years. What impact has this had on reports, for example IRIS tests, that are based on Schedule P data?

– Not sure.

– There is an effect for this company but it is minor.

– None for this company.

– Appears to be minimal for our companies.

– None for this company.

– It didn't ruin any of our IRIS ratios. The main problem we've had is reconciling our work (old LAE definitions) to the annual statements.

– Unknown.

– No material impact.

– I haven't noticed any impact on our reports. One area that has impacted us -- statutory reporting other than Schedule P, for example, NJ Excess Profits. These reports are supposed to tie to the Annual Statement, yet they include historical loss development factors as part of the calculation. This is a problem -- using "old definition" Loss and ALAE historical LDF's and applying them to "new definition" Loss & ALAE. We've kept this report on the "old definition" basis, subject to DOI approval. I'm sure there are other examples of this.

– If appropriately reserved at 12/31/97, there would be some distortion in one-and-two year developments at 12/31/97 and two-year development at 12/31/98. Independent adjuster expense has a short tail, so it should not have a large impact. There could be more distortion for companies that use internal legal staffs extensively for defense of claims.

– For our company, the impact has been immaterial.

– For our company it lowered the development of prior accident years because we implemented it on a calendar year basis.

– IRIS tests 9, 10 and 11 become meaningless. For small companies, such as most of my clients, this generally works in their favor -- as dollars were reserved, or under-reserved, and paid as ULAE (or A&O), beyond the scope of the Schedule P - Part 2 test. Generally, I do not go through any exercise to determine if a favorable Schedule P value would have become unfavorable with an adjustment.

– Not sure.

– None.

– The revised basis of Schedule P has had a nominal impact to our reports such as the IRIS test, etc.

– This will artificially improve the 1 and 2 year reserve development ratios.

– There is an obvious impact on the loss development tests.

– There are clear distortions without any attempt as an industry to quantify the problem.

– Good question. If I have time, I'll look into that.

– Probably minor impacts.

Question 14 (cont.):
As a result of the revised expense categories, Schedule P data subsequent to January 1, 1998 is on a different basis than that of prior years. What impact has this had on reports, for example IRIS tests, that are based on Schedule P data?

– I can only speculate what affect the distortions referenced in item 13 will have on various reports and tests.

– We have observed no impact at our company because the amounts are so small (in our case).

– Development will be distorted, depending on how companies implement the change.

Question 15:
Please indicate if you believe that further research is needed regarding the impact of the new LAE categories in the following areas.

Response	Frequency	Percent
a) Federal Income Taxes	7	9.5
b) Commission Agreements	4	5.4
c) Case Reserving Practices	6	8.1
d) Retrospectively-rated Policies	6	8.1
e) Ratemaking Practices	10	13.5
f) Reinsurance Contracts	14	18.9

When respondents were asked to indicate areas they believed that further research was needed regarding the impact of the new LAE categories, the most popular responses were reinsurance contracts (18.9%) and ratemaking practices (13.5%).

If you have experience with the change in any of the following areas, the Committee would appreciate your input.

– I think a Practice Note would be helpful for reserving.

– Yes, future information should be developed to ascertain possible distortions.

– I believe there has not been any impact as we treat the change as regulatory reporting required only.

– Our reinsurance contracts continue to require ALAE & ULAE. Ratemaking continues to use ALAE & ULAE.

– Can be confusion/problems because at least some reinsurance have not changed the definition of ALAE in their contracts.

– No further research.

– (Reinsurance Contracts) Many cover L & ALAE, no Loss & DCC!

– (Case Reserving Practices) Currently we are using old categories and old methods, and perhaps that is the best way to continue until someone comes up with a better method.

– (Reinsurance Contracts) Reinsurance contracts are always ambiguous on the treatment of LAE. I would say that very little effect will be felt.

Question 15 (cont.):
Please indicate if you believe that further research is needed regarding the impact of the new LAE categories. If you have experience with the change in any of the following areas, the Committee would appreciate your input.

– No thoughts, since not impacted.

– (Retrospectively-rated Policies and Ratemaking Practices) Information about actual ractices and effects will be helpful.

– No. I try not to use Schedule P for reserve testing.

– (Case Reserving Practices) In the real world, claims personnel will move to new definitions at different speeds within (same) and among (a lot) companies. (Reinsurance Contracts) If a reinsurance contract says it will pay for outside adjusters (as most do) there is no way they will report only DCC until contract terms and pricing (for outside adjuster companies) are changed.

– Another area needing research -- Excess Profits Reports.

– I would hope reinsurance contracts don't change. As a company our desire long-term is to meet both definitions A&O vs DCC and ALAE & ULAE. There is value to both.

– Let's bring Schedule P to actual dollars like rest of statement. Let's require/request full and complete claim count information in the process of these other modifications. Ten year Schedule P data for all lines.

– This is a broad question. Further research seems to always be needed, and refinements are nearly always possible.

– (Ratemaking Practices and Reinsurance Contracts) My experience is they seem to reflect "old definitions" not new.

– We don't see any need for research on above.

– (Reinsurance Contracts) Evaluation of new reinsurance contracts is impossible since you can't distinguish from Schedule P which expenses will be subject to the contracts.

– (All checked) Probably all of these areas. But since my company is NOT changing our internal reporting, I do not see an immediate impact other than statutory reporting.

– I believe that policyholders are best served when as many loss adjustment expense dollars as possible are assigned to specific claims -- whether those claims are in litigation or not. From a ratemaking perspective, class relativity factors, territorial rates, and state indications will be based primarily on claim-specific expenses. From an underwriting perspective, underwriting decisions (including pricing retrospectively-rated policies) will be made based on primarily claim-specific expenses. Reinsurance payments will be made based on primarily claim-specific expenses. To the extent contingent commissions are paid based on loss experience including LAE, they will be based on primarily claim-specific expenses. The more that is directly allocated to individual claims, the better. The argument that a company can keep two sets of books (one set for the above business reasons and another for the regulators) is ludicrous – it ignores the fact that policyholders would have to bear the cost of a second set of books.

– (Federal Income Taxes and Reinsurance Contracts) Most reinsurance contracts make reference to loss and "allocated adjustment expense." From the reinsurer's perspective, being claim-specific is more important than the DCC and A&O definition.

Question 15 (cont.):
Please indicate if you believe that further research is needed regarding the impact of the new LAE categories. If you have experience with the change in any of the following areas, the Committee would appreciate your input.

– We do not foresee any significant change for our company. The changes appear to be rather straightforward for companies with larger amounts.

– Loss Reserving -- Significant impact on doing loss reserve analysis.

Question 16:
Other comments, suggestions, or issues affecting your work due to these changes.

– NAIC should require restatement of all AY's to the new definition on a historical basis, even if allocation and estimates are necessary.

– This change seemed unnecessary. In general, loss adjustment expenses are smaller than pure loss costs, so why the separation into two categories? Just have one category – LAE.

– I believe the transition problems created by this change outweigh any potential benefit from the change.

– I think the change has created additional confusion. Schedule P can no longer be used to obtain ALAE and ULAE separately and there is increasing need to complete reconciliation from internal data to Schedule P data.

– The advantages of any such change as this should clearly outweigh the disadvantages. We are not convinced that criteria has been met.

– When changing a standard of practice only one option should be afforded else you leave yourself open to being inconsistent.

– Make changes standardized, shouldn't be choices as to how to report data. The whole point of an annual statement format is so everyone's data is in the same format. You can pick up any statement and understand the numbers.

– It is an awful experience and Schedule P is not currently a satisfying product worthy of our pride (in my opinion).

– The new definitions also moved legal expense incurred on declaratory judgment (DJ) actions from ALE to A&O. This was a significant rebucketing for environmental and asbestos expenses.

– Seems like a lot of work is required to satisfy this definitional change. I'm not sure the benefit outweighs the extra work.

– Difficulty of explaining to non-actuaries why change in definition causes change in companies reserve.

– The DCC includes cost containment expenses. I am surprised they are not in A&O. Don't adjusters try to contain costs? In any event, cost containment should be defined with some examples given.

– We will have to continue to use the "old" ALAE and ULAE approach with allocation to DCC and A&O for a few more years. Then DCC and A&O patterns might be useful in their own right.

Question 16 (cont.):
Other comments, suggestions, or issues affecting your work due to these changes.

– This was a stupid change, and I am sure glad that the CAS pushed for it It would be nice to see the CAS try to explain to our accountants and claims people the benefit of this extra work. In case you hadn't notice, Schedule P is perceived to be for actuaries use only. Any changes to it are deemed to be at our request. The question is why we haven't justified making this change. As stated above, this change makes industry Schedule P less useful.

– See number 14

– In my opinion, this change was poorly thought-out and serves no purpose at all. I am unaware of any beneficial purpose that will ultimately be served.

– I think it's crazy to have four categories: Losses, DCC, A&O and General expenses – especially since DCC and A&O include overhead. Simpler would be two categories: losses (including direct loss expenses) and expenses.

– I think the whole change was pointless.

– This resulted in a greatly increased amount of work on my part and on company personnel parts for no discernable benefit. It also caused great confusion as there is no clear definitions of what is A&O and what is DCC.

– It causes headaches for me regarding statutory reporting but since my company management has decided against making internal reporting changes the burden of the change pretty much falls on me.

– It is possible that companies that have to reclassify internal litigation expenses will find their reserves less adequate and companies reclassifying adjuster expenses will find their reserves more adequate.

– New definitions such as these, where limited historical information is available, present real reserving challenges. 'The information systems challenges were also significant given the change was made in the midst of Y2K preparation.

– What is the impact on statistical reporting? Will there be further changes in that area or will it continue to use the ALAE and ULAE categories?

– It has made loss reserve analysis significantly more difficult. I do not see any benefits whatsoever to the changes. Only problems.

Question 17:
Please let us know how the CAS Committee on Reserves may provide assistance to you as a loss reserve practitioner.

– Practice Note on reserving for these changes.

– Information about actual practices and effects will be helpful.

– By providing practical suggestions on how to deal with this issue.

– Eliminate Schedule P, Parts 2, 3, etc. It is too simple minded to be of any use in financial analysis. Only the actuarial report is useful.

– Guidelines for the allocations absent sound data!! Help!!

Question 17 (cont.):
Please let us know how the CAS Committee on Reserves may provide assistance to you as a loss reserve practitioner.

– Encourage the NAIC to not change definitions going forward.

– Any info on how former ALAE reclassed as O&A might develop over time (to assist in AY splits) would be helpful.

– Possible studies of how hypothetical shifts impact reserves.

– Sessions at CAS meetings or the CLRS covering reserve projection techniques to account for/recognize the change in categorization.

– Sessions at CLRS on how to address changes short-term and long-term. What are acceptable/reasonable approaches when historical triangles do not have separate components.

– What should reinsurance companies do with former ALAE reported to them by clients. Should they ask for it to be split between DCC and A&O?

– Lobby for Schedule P to be actual dollars – no 000 omitted. Lobby for better claim count data. Lobby for 10-year Schedule P data for all lines.

– These issues were analyzed by a multi-disciplinary task force on the issue years ago. (I can't remember the exact time, but our 2nd to last meeting finished hours before the World Trade Center bombing). Many of the same issues, and new ones, were raised several years ago during NAIC CATF discussions. Where were you then? Why the fuss now? You need to keep more up-to-speed of NAIC happenings. You should monitor COPLFR issues for items to work on.

– Provide information about methodologies for DCC and A&O used and seem to work for companies that were able to change past history (if there are any such companies). That will give us a starting point for figuring out how and when to adjust methodologies for companies that were unable to reclassify history.

– Encourage the NAIC to return to the definitions of ALE and ULE that make sense. Separate legal and adjusting as an expense category.

– Develop guidelines for A&O expense allocation and reserving.

– Best choice is to go back to old definition. Second choice is to have 4 LAE categories – allocated DCC, unallocated DCC, allocated A&O, and unallocated A&O. This would allow using LAE under either old or new definitions.

– Reverse the decision to change the definition.

– I would be interested in how other reserving actuaries are handling it and any recommended changes in methodology.

– Please publish the results of this survey.

Additional Written Comments

– The answers provided are for a small regional mutual company that uses some outside adjusters and does not have inside legal staff for claims litigation. This description represents over 50% of my consulting practice. These clients tend to have unsophisticated expense allocation systems and have tended to implement this change on a calendar year basis.

– I do reserving work with many small companies and the LAE issue has been and remains a hot – and sore – subject. This decision has resulted in much confusion and many hours of work for no obvious benefit. There is no more uniformity of reporting now than there was before; maybe even less so. I distributed copies of the survey to the companies on my mailing list (not all clients) and asked them to return either to me, to you directly, or to complete it online. I don't know how may responded to you online, but I am including 3 responses that I received as well as my own. If I can be of help, I am willing to discuss this issue with the committee.

– My responses represent approximately 15-20 companies I work with.

APPENDIX

Part I

(1) Please indicate your type of employment:

a. _____ Company Reserving Actuary
b. _____ Consulting Reserving Actuary
c. _____ Insurance Department Actuary
d. _____ Other__________________

(2) Prior to the change on January 1, 1998, how did your company classify ALAE vs. ULAE? You may want to refer to the background information provided at the front of the survey.

a. _____ Claim Specific / Non-claim Specific
b. _____ External versus Internal
c. _____ Combination of A and B
d. _____ Neither

(3) Which choice would most closely approximate the major expense reclassification for your company?

	Prior to January 1, 1998	After January 1, 1998
1. Internal Defense Costs	ULAE	DCC
2. External Claim Adjusters	ALAE	A&O

a. _____ #1
b. _____ #2
c. _____ No material changes
d. _____ Other____

(4) Did your company implement these changes in their 1998 Annual Statement?

a. _____ Yes
b. _____ No
c. _____ Do not know

(5) Which method did your company select to implement the new LAE split?

a. _____ Accident Year (Accident year 1998 and beyond)
b. _____ Calendar Year (All accident years for calendar year 1998 and beyond)
c. _____ Do not know

(6) Which of the following were used to classify expenses?

a. _____ Expense Tracking System
b. _____ Formula Allocations
c. _____ Special "time/expense" studies
d. _____ Other _________________________
e. _____ Do not know

(7) If your company used formula allocations to reclassify expenses, what allocation base was used?

a. ____

(8) Is your company currently maintaining internal expense reporting under the former categorization while adopting the new categorization for statutory reporting?

a. _____ Yes
b. _____ No
c. _____ Do not know

(9) Is your company using the new expense categorization for any purposes other than Annual Statement reporting?

a. _____ Yes; Explain__________________________
b. _____ No
c. _____ Do not know

Part II - Please use additional paper if necessary.

(10) Explain how your companies accomplished a reclassification of expenses from categories where claim detail was not maintained (for example, internal defense attorney costs, formerly categorized as ULAE) to categories such as DCC, where at least some detail (i.e. accident year) would be required.

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(11) On a calendar year basis, the new categorizations apply to the incremental calendar year change across all accident years beginning January 1, 1998. From a Schedule P standpoint, this means for accident years 1997 and prior, the 12/31/98 evaluation of ALAE (i.e. the current column) and all future evaluations (or columns) will reflect a mixed definition. Accident year 1998 and future accident years will be under the new DCC definition. On an accident year basis, the new categorizations will apply to only accident year 1998 and future accident years. Prior accident years will continue to run-off under the old definition of ALAE.

What are the reserving challenges of dealing with this and what solutions have you found? How have you changed your reserving practices?

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(12) In your opinion, what impact will the categorization change have on industry Schedule P data as individual companies make different choices on how they will handle the change?

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(13) How will users of Schedule P adjust for possible distortions in the data?

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(14) As a result of the revised expense categories, Schedule P data subsequent to January 1, 1998 is on a different basis than that of prior years. What impact has this had on reports, for example IRIS tests, that are based on Schedule P data?

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(15) Please indicate if you believe that further research is needed regarding the impact of the new LAE categories in the following areas. If you have experience with the change in any of the following areas, the Committee would appreciate your input.

_____ Federal Income Taxes _____ Retrospectively-rated Policies
_____ Commission Agreements _____ Ratemaking Practices
_____ Case Reserving Practices _____ Reinsurance Contracts

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(16) Other comments, suggestions, or issues affecting your work due to these changes.

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(17) Please let us know how the CAS Committee on Reserves may provide assistance to you as a loss reserve practitioner.

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Optional

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Name

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Title

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Company

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Address

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Phone

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Fax

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E-mail

Please return this survey by May 1, 2000 to:

Casualty Actuarial Society
Attn: Committee on Reserves
1100 North Glebe Road, Suite 600
Arlington, VA 22201

Charting the Path for Workers Compensation Claim Management

Dan Corro

Abstract:

With so much discussion about claim benchmarking, treatment protocols and the like, did you ever wish someone could just point you in the right direction? This analysis of the detailed workers compensation [WC] claim data now becoming available to researchers leads to a picture that resembles a simplified navigational chart. As described in the paper, that map--together with a few rules--provides powerful and potentially valuable guidance in administering WC claims.

Cost analyses are often issue driven. Consequently they tend to be focussed on a single cost liability. Medical costs, wage replacement benefits and loss adjustment expenses are the major categories in WC insurance. The focus is usually on determining their individual, ultimate cost liability. This paper describes some findings based on a new way to model claim costs that puts as much emphasis on their timing and interaction as on the costs themselves. As an illustration, back strain cases are looked at taking note of the mix between medical and lost time benefits. The major finding is hardly a surprise: mix matters. What might surprise you are the prospects for translating esoteric theory into practical guidance.

Introduction

Actuaries, especially life and pension actuaries, have always made use of mortality tables and the stick-man annuity formulas they seem to inspire. Nowadays, that type of analysis is more broadly applied. Engineers, for example, use it to evaluate the mean time to failure of a machine part while medical researchers use it to analyze drug trials and to evaluate treatment protocols. With these applications has come a major facelift. The study of "life contingencies" has been significantly advanced, especially through the incorporation of regression models and statistical theory, and is now called "survival analysis" (see [1] for a succinct, hands-on presentation).

At the same time, advances in data processing have yielded new and different crops of insurance data. Claim information files include a wealth of information never before captured in a readily accessible way. While the driving force was automated claims handling, the information collected may provide researchers with the raw materials needed for more refined statistical analyses. New WC industry-wide claim databases are being built that capture unprecedented detail on individual claims. In some instances there is even the ability to "drill down" to individual payment transactions. The work discussed here is the result of jury-rigging together a methodology to make greater use of that information.

This paper presents some early findings based on this new approach. Back injury cases are studied with an eye toward the interaction of medical and indemnity costs. While the theory is immature and the results only preliminary, hopefully they provide a taste of the fare we expect this new harvest of WC data to bring.

Background

We studied the interaction of medical and indemnity costs for a sample of back strain cases. The claim data used is from the NCCI Detailed Claim Information [DCI] database. The DCI is accurately described as precursor of the newer and more ambitious claim data marts now coming on-line. It is the natural "legacy system" and remains a good test bed for research. The DCI sample used in this study includes lost time claims from the states of Connecticut, New Jersey, New York and Pennsylvania. The study is restricted to injuries from 1983 to 1999 with medical and indemnity benefits each capped at $1,000,000.

Chart 1 groups the claims by indemnity and medical cost quartile, producing 16 [=4x4] separate buckets. Not surprisingly, the saddle shape confirms the strong correlation between indemnity and medical costs, especially at the high and low end cost cases. Because that relationship is so strong, understanding it better should lead to better claims management tools.

Consider, for example, the ongoing debate over the "sports medicine" approach to claims management. Recall the basic argument in its favor: aggressive medical care results in a faster return to work, thereby lowering the wage replacement cost liability. From a simplistic bean counter mentality, the challenge is to identify those cases for which the indemnity savings outweigh the added medical cost. Simplistic as that formulation may be, it poses a difficult question that remains to be resolved. A model that accurately captures the medical-indemnity cost interaction could contribute to that discussion, perhaps leading eventually to a definitive result.

So the goal is to model claims keeping track of the timing, itemization and interaction of claim payments. Individual payment transactions enable us to chart the progress of a claim as a function of time. With a little imagination, we can visualize this as a continuous path. This is a major departure from the traditional way of capturing claim data as a series of discrete snapshots (1^{st} report, 2^{nd} report, ...etc.). Tracing a continuous movement suggests a problem in Newtonian physics. On the other hand, it is more natural to think of a claim as exhibiting survival-oriented behavior, rather than the mindless motion of a "body of mass". This point of view suggests the use of survival analysis techniques, since much of that theory deals with behavioral responses. The model we are investigating is a hybrid, using techniques from survival analysis to organize and process the empirical data and then exploiting some ideas from mathematical physics to do the calculations and derive conclusions.

WC Cartography 101

While it is not really necessary to understand how such a "map" is derived to make use of one, it is helpful to have some basic understanding in order to avoid over-reading and misinterpretation. The discussion here is very general, the mathematical development is presented in [4], albeit without the word "map" (see also [3] and [5]).

A claim is represented as a trip or path on the map, beginning at the lower left-hand corner. Movement to the right, or due eastward, corresponds to paying medical benefits and movement upward, or due north, to paying indemnity benefits. As the model does not allow for recoveries (negative payments), claims progress in a northeasterly direction with no ability to backtrack.

In conventional survival analysis you observe "lives" and typically only take note of their "births" and "deaths" (and whether they hung around long enough to actually be observed to die). Much of its language has normative content, which can be bothersome. It is usually not good to "die" and often the kinder and gentler terms of "start" and "failure" are used. In our application, however, a life is a WC claim with "birth" corresponding to opening the claim file and "death" to claim closure. In this context, a quick death is not necessarily a bad outcome.

When constructing mortality tables, actuaries make use of the "force of mortality". That term is a bit old-fashioned. Survival analysis uses the more contemporary term "hazard rate function". Either one refers to the (instantaneous) rate of failure, expressed as a positive number. Taking its cue from the older "force" language, a key innovation of the claim model used here is to give hazard both magnitude and direction. Hazard is captured as a vector concept.

To continue with the terminology lesson, the proper name for this mathematical gadget is "vector field". In fact, we visualize the hazard literally as a field (or grassy meadow, if you prefer—the point is that blades of grass look like "vectors", since they have both length and direction). Claims cut out paths through this field from birth in the lower left to their eventual closure[1]—see Chart 2.

Unlike conventional survival analysis, we want to focus on more than just the birth and death of a claim and this is where the physics comes in. We model each observed claim by its entire path through the hazard field. Chart 2 shows two claims, C and D, that both close at the same cost (a,b) in medical and indemnity benefits, respectively. The two claims, however, took different routes in getting to that same end result. Conventional survival analysis is one-dimensional. Think of an infinitesimal bug on a time line. The bug can go fast or slow but not backward in time and has no opportunity to choose the path less traveled by. It is hoped that the use of multi-dimensional models to capture path choices will make all, or at least some of, the difference (with apologies to Robert Frost).

We have discussed two new ideas:

- Modeling a WC claim via its complete payment history and
- Capturing hazard as a vector field.

The two concepts work together: we visualize a claim as a trek over hazardous terrain and we look to our theory for guidance, presented here in the form of a "map".

We will not discuss here the task of constructing the hazard vector field, except to note that this is where 99.9% of the difficulty lies and that this part of the theory remains quite immature. For this study, we used ad hoc regression models to smooth out the discrete survival patterns produced from the empirical data.[2] It is hoped that with further study we will identify some functional forms that provide good analytical representations of WC claim survival data.

The remainder of this paper discusses the implication of a mathematical result known as "Green's Theorem in the Plane", a classical result discussed in most courses on advanced calculus.. It is certainly not necessary to understand Green's theorem to appreciate those

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[1] In Cartesian coordinates, claims naturally enough originate at the origin (0,0). The x-coordinate tracks the cumulative medical payments while the y-coordinate the cumulative indemnity payments.

[2] For those interested in the methodology, we note that the claim data was fit to a survival vector field, rather than directly to a hazard vector field. More precisely, the steps taken were: (1) produce a lattice of survival vectors from the claim data (2) "invert" that survival lattice into a "gauntlet" hazard vector field and finally (3) use OLS regression models to smooth the gauntlet. (See [3] and [4]). For the last step, the x-component and the log of the y-component of the hazard vector were fit to a list of rational functions in x and y of degree 2 or less ($1, x, y, xy, x^2, y^2, 1/(1+x), 1/(1+y), 1/(1+x^2), 1/(1+y^2), 1/(1+xy)$). Both regressions had R^2 values of 0.95.

implications. Those implications are translated into a simple set of "navigational rules" in the next section. For those readers who are interested, the remainder of this section describes in a non-technical way what Green's theorem says and how it applies here. The truly math-phobic have permission to skip to the next section.

Take a deep breath: Green's Theorem tells us that the difference, *C-D*, in the work done going along two life paths to a common point equals the integral of the "rotation" of the hazard over the area between the paths (whew—and that's the simplest case). That is, the difference can be found by integration over the region *R* in Chart 2. This means that if the rotation is positive (counter-clockwise) on *R*, then more work is accomplished toward claim resolution by taking the lower path *C*. Conversely, if the rotation is negative (clockwise) on *R*, then more work is accomplished by taking the high road *D*. Moreover, while the paths must start and end together, the starting point need not be the origin.

The navigation map is just a plot of where the rotation is positive and negative. To express this in familiar terms, areas where the rotation is positive are called "land" and areas where the rotation is negative are called "water". Boundaries, where the rotation is zero, are (you guessed it) "coastlines". The navigation map produced in the back strain case study is shown in Chart 3. A coastline is "eastern" ("western", "southern", "northern", etc.) when you move east to reach the coast from inland. New York City, for example, is on the eastern US coastline, irrespective of whether it happens to fall on the left or right hand side of any particular map you are reading. In Chart 3, for example, the coastline on the left is an eastern coastline while the land area on the right is bordered by a western and by a southern coastline.

Rules to Die For

It is easy to use a claim navigation map like Chart 3, prepared from the back strain case study, provided you keep a few simple rules in mind. These rules apply when you have pre-allocated amounts of medical and indemnity dollars to spend. This is because the life paths must start and end together in order for what Green's theorem says about work to work. Remember that this simple model does not provide for subrogation or other recoveries, and so you can only go north or east. There are four cases, depending upon your current circumstances.

- *You are on water with no land in sight.* Head north then east to make more progress toward resolving the claim.

- *You are on land with no water in sight.* Head east then north to make more progress toward resolving the claim.

- *You are near a western coastline.* Avoid the coast to make more progress toward resolving the claim. (Western coastlines are paths of least resistance and so following them minimizes the work accomplished toward closing the claim).

- *You are near an eastern coastline.* Follow the coastline to make more progress toward resolving the claim. (Eastern coastlines are paths of maximum resistance).

As in so many adventure novels, it all comes down to finding the right map.

It is important to understand that the map and rules discussed here do not reveal any "best" course toward resolving a claim, they are only helpful in deciding between two ways of getting to the same place. It is clear from the theory that questions about the existence, uniqueness and determination of optimal paths are much harder. See [4] for an illustration of how the theory rhymes with fixed asset allocation and benefit cost minimization (What, too many syllables?).

Of course, this simple, two-color map can be refined into a "contour map" that warns of particularly rough terrain and especially turbulent waters. Also, while the bean counters would certainly urge you to shorten your trips, distance traveled ($\sqrt{\Delta x^2 + \Delta y^2}$) does not equal the money paid getting there ($\Delta x + \Delta y$); suggesting maybe using an alternative scale. Hopefully, advances will be made on these and on related issues as the theory is applied.

There are two basic problems to be addressed by a mature theory:

- First, assess the "work" remaining to resolve a claim
- And then, determine an efficient path for completing that work.

The next section presents a case study with more and less efficient paths and so the path choice does matter. The extent to which these problems can be solved remains to be found

Back Strains: A Case Study

We are finally at the fun part. Refer to Chart 3 which shows the map for resolving back strain claims. When there is no rotation, the path does not matter[3]. The basic finding of this study is that timing matters and that there is both positive and negative rotation out there influencing the resolution of the claim sample.

For example, what does the map suggest as regards the sports medicine debate? First, note that we are only considering **dollars** of medical and indemnity benefits. Other such

[3] Vector fields with rotation identically 0 are called *conservative*. These are the vector field that have a potential function and are characterized by the fact that the amount of work done moving from one point to another is independent of the path taken. For example, the potential energy a rock loses when moved from the top to the bottom of a hill will be the same whether you throw it, kick it or carry it in your shoe. The astute reader will note that the map discussion conveniently ignores the possibility of "conservative coastal areas". While perhaps politically odd, such areas can occur. The smoothing functions used here reduce them to (lower dimensional, measure zero) subsets that can be ignored. In any event, where the rotation vanishes identically, progress toward claim resolution is independent of the path and the only guidance Green has to offer is to the limits of indifference. Short form: it would have messed up the rules without adding anything.

models may incorporate better medical utilization metrics; here we make do with medical dollar costs as a surrogate for medical utilization.

With that limitation in mind, though, the experience of the lower cost cases (near the origin) does not support the sports medicine approach. There, it is best to follow the eastern coastline which allocates the lion's share of dollars to replacing lost income. This has some common sense appeal. Note that this observation applies only so far because further north that coastline veers west along an impossible track

Look next toward the right but still along the bottom. That part of the map pictures a danger inherent in the sports medicine approach. There you run the risk of becoming trapped within an inlet and being forced aground on a western shore. Recall the rule to avoid this because expending resources along a western coastline achieves minimal progress toward resolving claims.

For higher cost claims, the upper right region tells yet another story. There we see a western coastline just below the line y = x and nearly parallel with it. There is an identifiable path of least resistance along which medical benefits and income replacement benefits continue to be paid out at about equal rates. The spine and especially the "saddle horn" in Chart 1 suggest that this is a popular route. Since western coastlines are to be avoided, this advises against such a middle of the road course. While it is not clear which is better in any given case, the suggestion here is to either adopt or clearly reject the sports medicine model in any given case. And further, sticking by that decision whenever possible. It warns of maintaining a level of palliative care inadequate to bring the injured worker back to work. Of course, in practice there may be little recourse away from that track.

Combining the map (Chart 2) with the claim distribution (Chart 1) highlights the value of making a determination early on and breaking away from the pack. This observation again has some common sense appeal. At this stage, the map offers little but an "I told you so" in the event of a bad call. Consider how much more valuable the theory would become if it could lead to identifying the "correct" choice on a case by case basis. By investigating how certain claim characteristics impact the geography, the approach provides a blueprint for resolving the debate over the sports medicine model.

The skeptic may view the upper right of the chart as just a graphical representation of a known and rather obvious pitfall to avoid when managing a back strain injury claim. Nevertheless, this picture was drawn from "hard" empirical data, not anecdotes. At such an early stage, the theory is unable to assess the degree that this picture is the reflection of intelligent versus blind choices.

Suppose you are confronted with a fairly serious back strain injury. You recognize that there is much "work" needed to resolve this case and so you look toward the upper right as your likely final destination. You believe you would do better ending up on land and so you decide to use the sports medicine approach. You must make an important strategic decision and decide upon a landing point along that dreaded western coastline.

You also need to avoid being sucked along the coast, as that path offers less resistance to having claim payments just continue on. That western coast is especially dangerous since it offers an optimal course for those seeking to maximize their take from the WC system. Naively, then, the map suggests landing on the south shore, since that avoids being drawn into the "y=x" pitfall for at least the near term. Nevertheless, you must still be wary of medical costs looming near due east along that same coastline. However, more specific information would be needed together with some number crunching to determine whether that is actually what the model indicates in any particular case scenario.

Summary

A confluence of factors has combined to produce a new generation of computerized WC insurance claim information. This paper describes, in a mostly non-technical way, a new mathematical model for WC claims. The model was developed to take advantage of that claim data. It combines ingredients from contemporary survival analysis with classical physics. A case study of back strains was done to determine whether the theory could be applied to real world data, and if so whether anything of interest would come from it. To illustrate the potential applications, the theory is used to construct a "map" to help navigate the resolution of WC claims. That simple picture is a "surface map" in more than one sense. Hopefully it represents only the surface of what this theory may potentially yield. There is the chance we may strike gold by digging deeper into the theory and mining the data.

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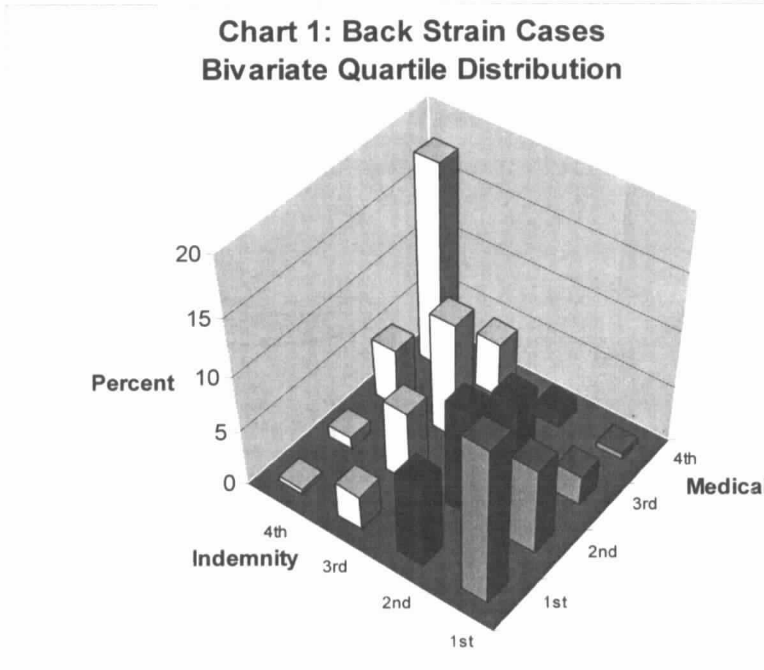
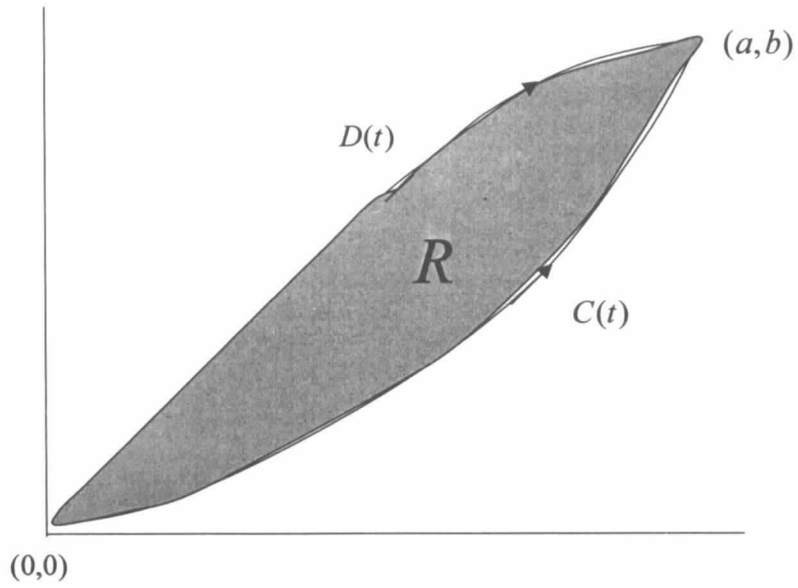


Chart 1: Back Strain Cases
Bivariate Quartile Distribution
20
15
Percent
10
5
0
4th
Indemnity
3rd
2nd
1st
4th
Medical
3rd
2nd
1st

Chart 2: Claim Paths



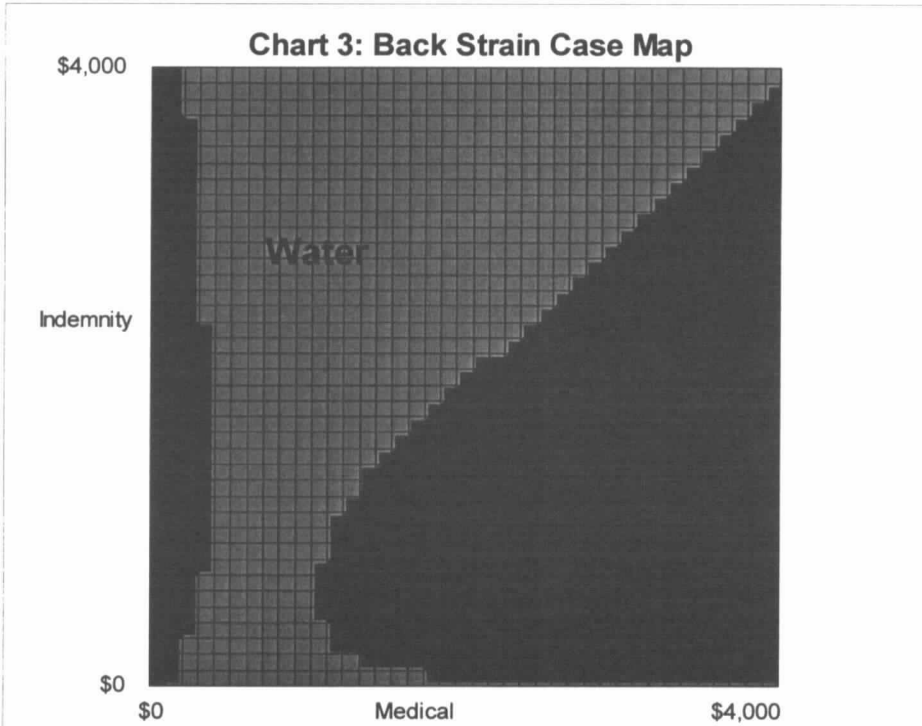


Chart 3: Back Strain Case Map
$4,000
Indemnity
$0
$0
Medical
$4,000
Water

Monitoring Cost Changes with Log-Linear Cost Models: Lessons from a Case Study

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Monitoring Cost Changes with Log-Linear Cost Models: Lessons from a Case Study

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Abstract:

When studying Worker's Compensation (WC) claim cost experience, researchers often prefer models that relate claim characteristics and other cost drivers to the logarithm of the claim cost, rather than to the dollar cost itself. Linear models based directly on dollars, however, are better suited to decomposing the differences in costs observed over time or between claim populations. Reconciling the two methods within one analysis can be awkward. This led us to a new perspective: one that enables the two approaches to work together while preserving the most desirable features of each.

The paper presents a general method for analyzing cost differences. It also illustrates the method in the context from whence it came: monitoring the post-reform experience of WC claim costs.

Keywords: Workers' Compensation Insurance, reform, Oxacca decomposition, log-linear model, log-log model, exponential weight.

Introduction

Analysts are often asked to interpret the economic landscape and assess the influence of several exogenous or predetermined factors on one endogenous variable. An example is workers' compensation [WC] claim cost taken as the endogenous variable to be studied in reference to a list of exogenous claim characteristics and cost drivers. Models are associated with some sort of mathematical representation such as linear, nonlinear, logarithmic linear function form, etc. From the structural perspective, the coefficients (or derivatives, or elasticities) from the different models correspond to different interpretations. From the standpoint of statistical considerations, there are reasons to opt for one structural model over another if it enhances our ability to interpret the data. That model choice, however, may not prove convenient when those cost relationships are only a part of a larger investigation. For example, it may be required to analyze how the average cost per case—not its logarithm-- has changed post-reform. This may demand some contortion to incorporate the model results into a picture suitable for decision-making. The need to fit a "round" cost model into a "square" hole within a summary report may lower the confidence level of those findings and raise the concern whether the methodology is internally consistent.

It is standard practice to use log-linear and log-log regression models in the analysis of WC claim costs. While useful for the investigation of proportional cost relationships, those transformed models are not well suited for predicting individual or even average dollar claim costs. Those models focus on the "geometric" mean cost while interest centers on the "arithmetic" average cost per case.

On the other hand, regression equations provide a powerful computational device for benchmarking select sets of claim costs and for analyzing dollar cost differences into components associated with cost drivers. This technique, based Oxacca style decompositions, exploits the fact that regression equations relate the "arithmetic" mean cost with average levels of the cost drivers.

This paper describes a method for changing the assigned weights of observations in the determination of the logged cost model. That "exponential weight" refinement is designed to improve the performance of the model after conversion back to a dollar scale. The derivation of a specific reweighting formula is motivated from the basic data fitting geometry of OLS regression (see [1] where the technique is tested on a large database of WC lost time claims). The idea is just to shift the log-linear regression model from its "geometric" to an "arithmetic" perspective that makes it consistent with the decomposition formula.

The next three sections provide technical background material: (1) the use logged cost models, (2) Oxacca style difference equations and (3) the exponential weight. The next section outlines a general methodology for putting the three pieces together. This is illustrated in the final section that presents a case study. The case study deals with monitoring WC claim costs post reform and is the context from which this work evolved. An Appendix provides additional detail on regressions discussed in that case study.

The Use of Logged Cost Models

The use of log-linear and or log-log regression models is the preferred practice for the analysis of workers compensation insurance claim costs. For simplicity, we refer to regression equations in which the dependent variable is the logarithm of a dollar cost as "logged cost models". The use of a logarithmic scale generally renders the cost distribution pattern more symmetric and less influenced by large "outlier" claims. It has the additional advantage of not predicting negative costs. While this typically results in better fits and higher R^2 values, it is well known that the attempt to reverse the transformation by exponentiation usually fails to yield very useful dollar cost estimates. Indeed, on average the figures that result are smaller--sometimes spectacularly smaller--than the original costs used to construct the model. As explained in the paper, this is a formal consequence of the geometric mean cost being less than the arithmetic mean. While the transformed models provide useful information on cost relationships, that transformation renders them of little value for directly predicting dollar cost estimates.

The common sense explanation for this is that the high cost claims are effectively given less weight in a logged cost model. This is viewed as one of the prices to be paid for mitigating the influence of outlier claims. We pursue this from a simple geometric point of view rather than from the more challenging perspective of model specification error. We begin with the observation that cost data is typically presented with a "natural weight". This may simply be one claim one vote within a claim population or, as is often the case, a weight inferred from claim sampling procedures or other information on the probability of claim occurrence. It is key that this "natural" quality in dollar terms need not be preserved under transformation of the data. In particular, this typically occurs when costs are recalibrated via the log function. This suggests reweighting the data to offset that effect. Reweighting observations is a common practice in constructing regression models to temper the effect of outliers or more generally to deal with heteroscedasticity. In a subsequent section we introduce a reweighting scheme that shifts the focal point of a logged cost model so as to make it better suited to producing dollar cost estimates. We will show that from this weight's perspective, the advantages of the logged cost models can be essentially retained while generating figures more readily broken down into cost components.

Let X represent an observation, $Z = Z_X$ the corresponding claim cost and $\{X_i\}$ the values of a set of explanatory variables. This note considers logged cost models of the form:

$$Y = \ln(Z) = \sum \beta_i X_i + \varepsilon$$

where ε represents the error term. The X_i may be categorical or continuous and, if continuous, be expressed in their original scale (log-linear cost model) or transformed to a logarithmic scale (log-log cost model).

On the continuous side, pre-injury wage and rate of compensation are important examples. Typically, dollar amounts like the pre-injury wage would be logged while that need not be the case for other continuous variables, such as the rate of compensation (periodic lost time compensation expressed as a percentage of the wage). Observe that the model parameter β_i does not vary with claim cost Z, referred to as an assumption of constant elasticity (for X_i in logged form). For example, it is common to use the full wage (or log thereof) so as to capture utilization effects related with total income. This is done even though workers compensation benefit statutes impose maximum wage replacement levels. Their presence, it has been argued, compromises the assumption of constant elasticity. There are, however, important considerations that challenge or at least mitigate that criticism. The point here is not to debate the issue but to simply point out that it is worth considering the implications on the use of the regression equation when $\{\beta_i\}$ is observed to vary with Z.

The appeal of a logged cost model in this context is best seen in the case of categorical variables. In the simplest case, suppose that the explanatory variable X_i corresponds to a {yes,no} condition, taking on the respective values $\{1,0\}$. In terms of the original cost z, the model associates an adjustment factor of $\alpha_i = e^{\beta_i}$. Most claim characteristics are better associated with such a proportional shift than to a particular dollar amount, as would occur if the logarithm were not used to transform the dependent variable of the cost model. While researchers may cite a litany of more technical considerations, it is primarily this observation together with the desire to avoid negative cost estimates which provides the strongest motivation for using logarithms to model workers compensation claim costs.

As with continuous variables, there is the issue as to whether the adjustment factor α_i associated with a characteristic variable changes with Z. Consider, for example, the characteristic indicating whether an attorney represents the claimant. For most purposes it is clearly preferable to model the associated cost impact as a proportional rather than as a flat loading. Again there are countervailing considerations: some state statutes regulate attorney fees by imposing maximums or sliding scales relative to the settlement amount.

The expense of collecting and storing detailed information on every claim may be prohibitively high, so oftentimes cost analyses resort to using claim samples. The efficiency of the claim sampling process may be further improved through stratification. In the case of the Detailed Claim Information (DCI) database used in the case study discussed later, state specific sampling ratios are used. Also, DCI sampling rules require that the claims be stratified so that the relatively simple and quickly resolved cases--for which many of the claim characteristics are missing or inapplicable--do not bog down the collection, storage and processing tasks. In this situation, a weight variable would be applied in deriving a cost model. In this study we abuse the notation $\omega_x (= \omega_{y_x} = \omega_{z_x})$ to denote the weight assigned to the claim x based upon the sampling rules. In the case of the DCI, ω_x is determined as the inverse of the applicable state sampling ratio, selectively increased by a factor to account for stratification. Let Γ denote a claim sample set. The set of weights $\{\omega_x \mid x \in \Gamma\}$ (which is really a function $\omega : \Gamma \to [0,\infty)$, but we ignore that nicety here) has the very desirable feature that, assuming the sampling is done correctly, the corresponding weighted arithmetic mean is an unbiased estimator of the average cost per case of lost time claims. Although not necessarily an integer, the value ω_x can be interpreted as the number of claims represented by the sampled claim x. When the set $\{\omega_x\}$ is this sampling weight, the sum $W = \sum \omega_x$ provides an estimate of the size of the lost time claim population. Making the normalization $p_x = \frac{\omega_x}{W}$ converts the weights into a probability density with the weighted mean coinciding with the expected claim cost:

$$E(Z) = \sum p_x z_x = \frac{\sum \omega_x z_x}{\sum \omega_x} = \frac{1}{W} \sum \omega_x z_x .$$

Oxacca Style Decompositions

Suppose the claim sample is divided into n mutually disjoint subsets:

$$\Gamma=\bigcup_{i=1}^{n}\Gamma_i \quad i\neq j \Rightarrow \Gamma_i \cap \Gamma_j = \phi$$

and consider a (weighted) ordinary least squares (OLS) linear model on the claim sample of the form:

$$Y=\sum_{i=1}^{n}\alpha_i\delta_i+\sum_j\beta_j X_j+\varepsilon \text{ where } \delta_i(x)=\begin{cases}0 & x\notin\Gamma_i\\ 1 & x\in\Gamma_i\end{cases}$$

We are interested in analyzing the differences of Y among these subsets akin to the Oxacca decomposition of mean differences from linear models. Let horizontal and vertical bars denote, respectively, taking a (weighted) mean and restriction to a subset. In this context, we may express the error term as:

$$\varepsilon=\varepsilon\cdot 1=\varepsilon\cdot(\sum_i\delta_i)=\sum_i\varepsilon_i \quad \text{where } \varepsilon_i=\varepsilon\cdot\delta_i$$

and a property of OLS regression implies that:

$$0=\bar{\varepsilon}=\overline{\varepsilon_i}=\overline{\varepsilon_{|\Gamma_i}}, \quad 1\leq i\leq n$$

This leads us to Oxacca style decompositions of differences of means over the various subsets. Indeed, the differences can be itemized into "base" and "mix" components.

$$\overline{y_{|\Gamma_i}}-\overline{y_{|\Gamma_k}}=\underbrace{(\alpha_i-\alpha_k)}_{\text{base}}+\underbrace{\sum_j\beta_j\left(\overline{x_{j|\Gamma_i}}-\overline{x_{j|\Gamma_k}}\right)}_{\text{mix}}$$

It is important to keep in mind that these means are determined using the same weights as are used to determine the regression equation.

The base difference can be interpreted as "unexplained" in the sense that the cost model does not associate it with any claim characteristic other belonging to a particular subset.

Alternatively, it can be interpreted as the result of selecting a common "baseline claim", specified as a set of assumed values for the explanatory variables, and then using the cost model to generate two predicted costs for that same claim. The first assumes that the claim belongs to the first subset of the comparison and the second assumes it belongs to the second subset, all else equal. Subtracting the first predicted cost from the second determines the "difference in base cost" component.

It may be useful to further itemize the mix component, since its summands are related with the explanatory variables of the model. For example, we have referred to some of the explanatory variables as "claim characteristics" and to others as "cost drivers". The decomposition can effectively group together the set of marginal cost impacts associated with the covariates of the cost model.

The Exponential Weight

As was noted above, the translation to logarithms compresses costs and has the effect of making claims more "equal". In particular, the high cost claims have less influence in the mean. A natural correction to this is a scheme that assigns more weight to higher cost claims when evaluating the regression model. For example, you could make the weight of an observation proportional to its dollar cost. It turns out, however, that such a weight overcompensates (c.f. [1])..

As before, let Z denote claim cost and begin with a set $\{\omega_z \mid z \in \Gamma\}$ of weighted costs from a claim sample of size N. We want to determine another set of N weights $\{\gamma_z \mid z \in \Gamma\}$ for that same cost data that behaves better under taking logs. It turns out that there is an essentially unique way to do this—refer to [1] for details. The first step is to sort the data by size of cost $\Gamma = \{z_i \mid z_i \le z_{i+1}; 1 \le i \le N-1\}$. Simplify the notation by letting $\omega_i = \omega_{z_i}$ and $\gamma_i = \gamma_{z_i}$ denote the corresponding weights. There is an ordered set $\{\gamma_i \mid 1 \le i \le N\}$ called the *corresponding exponential weight* that is uniquely determined from the conditions:

$$\left(\prod_{i=1}^{k} z_i^{\gamma_i}\right)^{\frac{1}{\sum_{i=1}^{k}\gamma_i}} = \frac{\sum_{i=1}^{k}\omega_i z_i}{\sum_{i=1}^{k}\varpi_i}; 1 \le k \le N \quad \text{and} \quad \sum_{i=1}^{N}\gamma_i = \sum_{i=1}^{N}\omega_i$$

This just means that the exponentially weighted geometric mean equals the weighted arithmetic mean determined using the original weight.

Putting the Pieces Together

This section presents the basic methodology in a simple but generic setting. All that is involved is putting the pieces together from the previous three sections. As above, we begin with a weight $\{\omega_z \mid z \in \Gamma\}$ and a decomposition

$$\Gamma = \bigcup_{i=1}^{n} \Gamma_i \quad i \ne j \Rightarrow \Gamma_i \cap \Gamma_j = \phi$$

Let γ_i be the exponential weight corresponding to the weight $\omega_{|\Gamma_i}$ on the sub-sample Γ_i. Combine the γ_i into a weight γ on Γ so that $\gamma_{|\Gamma_i} = \gamma_i$. Note that both weights ω and γ assign the same weight $W_i = \sum_{z \in \Gamma_i}\omega_z = \sum_{z \in \Gamma_i}\gamma_z$ to each sub-sample Γ_i

The weight γ provides the perspective that enables logged cost models to itemize differences among the sub-samples. To see this, we let Y=log(Z) as above. Also let a bar indicate the (weighted arithmetic) mean using the weight ω and a double bar the (weighted arithmetic) mean using the weight γ.

We are interested in how the cost Z changes over the Γ_i, as measured by the average cost per case that we denote by $\overline{z_i} = \overline{z_{|\Gamma_i}}$. Letting $r_{i,j} = \frac{\overline{z_i}}{\overline{z_j}}$ the idea is to decompose those relative differences in terms of explanatory variables.

So construct an OLS log-linear model using the weight γ:

$$\log(Z) = Y = \sum_{i=1}^{n} \alpha_i \delta_i + \sum_k \beta_k X_k + \varepsilon$$

We have arranged things so that

$$\log(\overline{z_i}) = \log\left(\frac{\sum_{z \in \Gamma_i} \omega_z z}{W_i}\right) = \log\left(\prod_{z \in \Gamma_i} z^{\frac{\gamma_z}{W_i}}\right) = \frac{\sum_{z \in \Gamma_i} \gamma_z \log(z)}{W_i} = \overline{\overline{y_{|\Gamma_i}}} = \alpha_i + \sum_k \beta_k \overline{\overline{x_{k|\Gamma_i}}}$$

and, as above, there is an Oxacca style decomposition:

$$r_{ij} = e^{\alpha_i - \alpha_j} \prod_{k} e^{\beta_k \left(\overline{x_{k|\Gamma_i}} - \overline{x_{k|\Gamma_j}}\right)}$$

$e^{\alpha_i - \alpha_j}$ = base cost compoent factor

$e^{\beta_k \left(\overline{x_{k|\Gamma_i}} - \overline{x_{k|\Gamma_j}}\right)}$ = factor associated with covariate X_k

This shows how to itemize the relative cost differences, expressed in dollar terms, using elasticities from a logged cost model.

The next section applies this when the claim sample is divided into four disjoint subsets.

Γ_1=TB, experience of a reform (Test) state pre-reform (Before)

Γ_2=CB, experience of a group of non-reform (Control) states pre-reform (Before)

Γ_3=TA, experience of a reform (Test) state post-reform (After)

Γ_4=CA, experience of a group of non-reform (Control) states post-reform (After).

As noted before, in that case study the covariates were grouped into two general categories: "claim characteristics" and "cost drivers". Those categories used to determine component factors associated with the explanatory variables of the log-linear cost model.

A Case Study: Monitoring Post Reform Claim Severity

Much of the previous discussion makes reference to this example. This final section illustrates the concepts discussed above. Along with revisiting the methodology, it discusses findings of some independent interest.

Background: NCCI post-reform monitoring (PRM) reports analyze losses in states those enacted major legislative reforms of their WC systems over the last decade. The reports attempt to gain an understanding of the effects of the reforms on the system outcomes, and evaluate the consistency of the outcomes with the reforms' objectives. With the availability of the necessary data, the post-reform monitoring reports compare the actual claim frequency and severity before the enactment of the laws with outcomes after. This section illustrates the analysis for a group of seven states (Arkansas, Connecticut, Florida, Georgia, Kansas, and Kentucky, Montana). These states enacted major legislative reforms from June 1, 1993 through July 1, 1994 and each was the focus of a post-reform study by NCCI during 1998. The paper *NCCI Post Reform Monitoring Reports* [2] provides background and presents findings for the same group of seven states within the context of post-reform cost analyses.

Data Source: The comparison of lost-time claim severity uses data from the NCCI Detailed Claim Information (DCI) database. The DCI is primarily used for research, and contains detailed information on a stratified random sample of lost time claims. In addition to incurred and paid claim costs, the DCI includes many claim characteristics, such as the part of body injured, the nature of the injury and its cause. It also includes indicators for attorney involvement, vocational rehabilitation; claim milestones such as date of injury, date of first disability payment, return to work or claim closure; as well as claimant demographics like age, gender, and pre-injury wage. The post reform monitoring studies use multivariate cost models to control the mix of injuries, claim characteristics and claimant demographics and to evaluate average claim costs in the pre- and post-reform periods. Indices for medical costs and wages are used to hold purchasing power constant over the two time periods.

General Approach: The analysis compares average claim costs in the pre- and post-reform periods in the reform states with outcomes from a group of jurisdictions that did not enact major systemic reforms.[1] Workers compensation experience improved significantly during the time period considered here and that improvement was not confined only to states instituting statutory reforms.

While it is impossible to exactly isolate the effectiveness of reforms from the general turnaround in experience, it is important to evaluate reform within that broader context. A simple comparison of experience before and after reform cannot achieve this. To that end, the analysis incorporates the experience of a "control" group of states that did not enact major reforms. In comparing case severity of the "test" reform states to the non-

[1] Those states are : Alaska, Arizona, the District of Columbia, Idaho, Illinois, Indiana, Iowa, Louisiana, Maryland, Michigan, Mississippi, Missouri, South Carolina, Utah, Vermont, Virginia and Wisconsin.

reform states' experience, it is equally important to account for the fact that the respective mix of injuries can significantly influence the result.

Average claim costs are compared between the two time periods for the reform and control states. For the reform states, pre- and post-reform time periods were selected based on the effective date of the reform law (typically, the pre-reform period ran from 18 to 6 months before while the post-reform period ran from 6 to 18 months after). For the control group states, the pre-reform period used is June, 1992 to May, 1993 and the post-reform period is May, 1994 to April, 1995. Those periods were selected so that, on average, the injury dates would be aligned with the before and after periods in the reform states. Comparison of outcomes in the reform states with the non-reform states provides a reference to the industry trends, while still differentiating the reform and non-reform state experience.

Linear and Logged Cost Models: As discussed above, it is standard practice for researchers to model the logarithm of cost, log(Z), when building models of claim costs. It is however, comparatively rare to find a justification for this beyond an exercise in hand waving. Chart 1 below shows the actual incurred costs for the DCI claim sample, arranged by increasing cost. Each "actual" point represents one percentile of the cost. More precisely, the data is sequenced by increasing size of claim z and then collected into 100 subsets of approximately equal weight. Chart 1 also shows the corresponding mean of $\hat{z}$, the predicted cost using a linear cost model and a second fit using an analogous logged cost model.

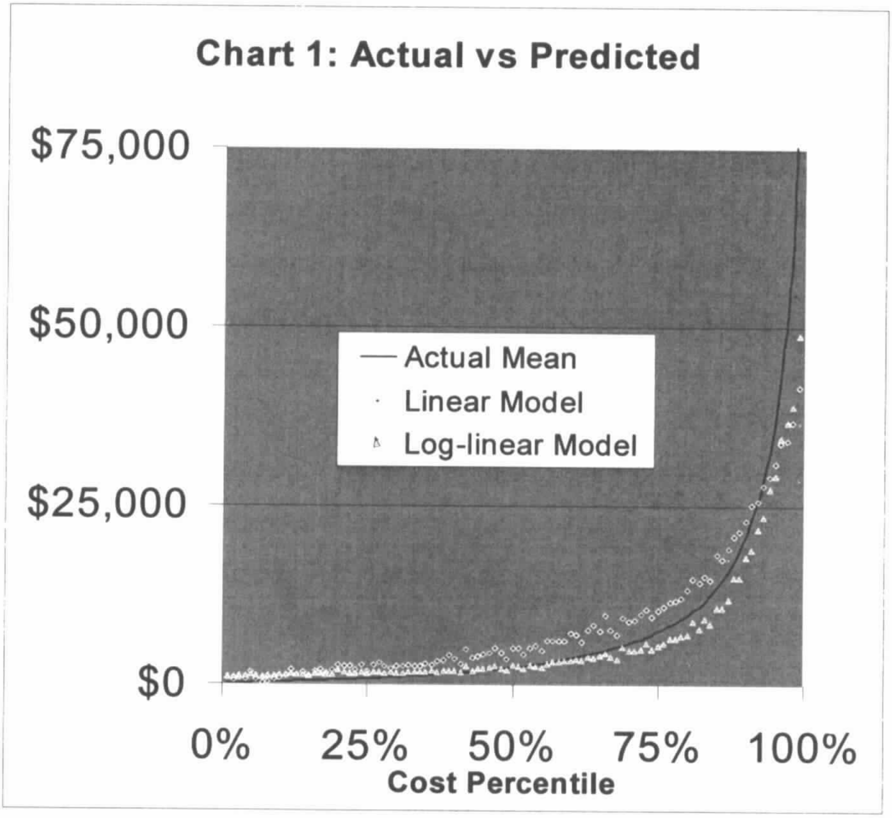
Chart 1: Actual vs Predicted
$75,000
$50,000
$25,000
$0
Actual Mean
Linear Model
Log-linear Model
0%
25%
50%
75%
100%
Cost Percentile

Predicted costs reflect regression toward the mean. Moreover, many of the explanatory values used in the cost models are $\{0,1\}$-indicator variables, which limits the range of predicted values. As a result, the fitted values show less variation than the actual costs. In particular, predicted costs understate the cost of the most expensive cases, a phenomenon that accounts for much of the error of the regressions. Chart 1 illustrates that while this is true for both linear and logged cost models, it is especially apparent for the linear model. Logged cost models typically exhibit a better fit. In this case, the adjusted R^2 is 0.983 for the logged cost model, more than double that of the linear model, at 0.427.

The graph of any (perhaps weighted) OLS linear model $z = f(x) + \varepsilon$ has a natural "center of gravity" at the point $\langle \bar{x}, \bar{z} \rangle = \langle \bar{x}, f(\bar{x}) \rangle$. When the same weight is used to construct a logged cost model $\log(z) = g_1(x) + \varepsilon$, however, the center of gravity of the regression, when transformed via exponentiation back to the original dollar scale, is moved to the point $\langle \bar{x}, \tilde{z} \rangle = \langle \bar{x}, \exp(g_1(\bar{x})) \rangle = \langle \bar{x}, \exp(\overline{\log(z)}) \rangle$ where $\tilde{z}$ is recognized as the (weighted) geometric mean of z. From the above remarks, we see that the sample weight can be "exponentially adjusted" in such a way that, when that new weight is used, the focal point of the logged cost model is shifted back to the (arithmetic) average cost per case. In this study, the exponential weight adjustment was applied to each of the four subsets $\{\text{CB}, \text{CA}, \text{TB}, \text{TA}\}$ identified above. Chart 2 compares the logged cost model fit using the sample and its corresponding exponential weight (refer to the Appendix for the logged cost model parameters using the exponentially adjusted weight).

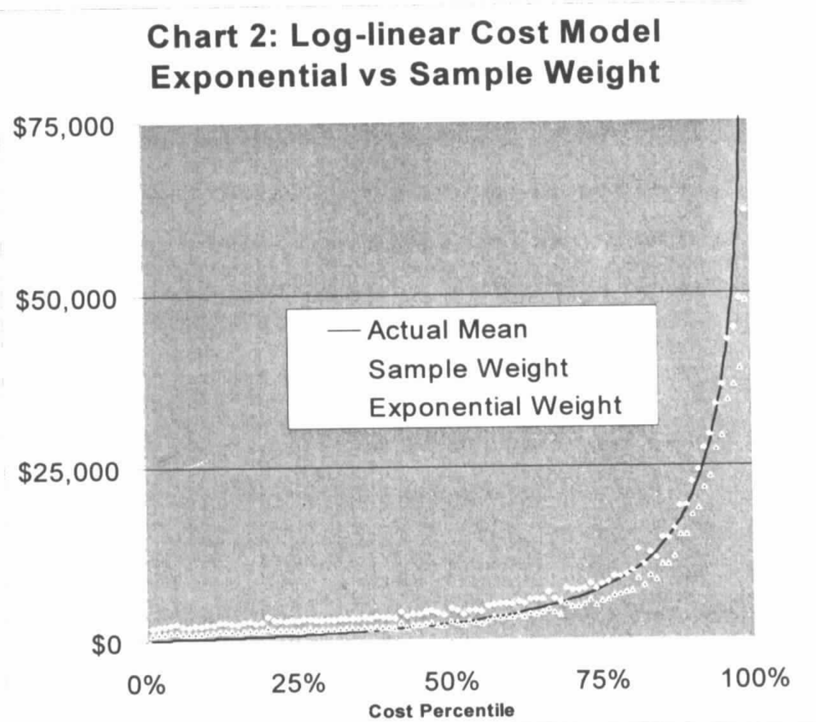
Chart 2: Log-linear Cost Model
Exponential vs Sample Weight
$75,000
$50,000
$25,000
$0
Actual Mean
Sample Weight
Exponential Weight
0%
25%
50%
75%
100%
Cost Percentile

Again, when weight is held constant, the effect of the logarithmic scale renders high cost z claims less influential in an OLS model for log(*Z*) than in an analogous model for *Z*. The exponential weight offsets that—whence its name—by assigning greater weight to the higher cost claims. This, in effect, shifts the center of gravity of the regression equation. Chart 2 illustrates this: while the sample weight log-linear fit is quite good from over 40-60th percentile range (the geometric mean of lost time costs is typically tracks with the median); the exponentially adjusted weight model fits best in the 70-90th percentile range (as is typical, the arithmetic mean of lost time costs—here about $10,000—is near the 80th percentile). The exponentially adjusted weight provides a better fit for high cost claims and optimizes the model fit near the value used to measure case severity. In this instance, the overall effect on the goodness of fit is small: use of the exponentially adjusted weight increases the adjusted R^2 slightly, to 0.988.

In light of the many {0,1}-indicator explanatory variables used in the cost models, it is worth recalling another advantage of logged cost models over simple linear models: most claim characteristics are more naturally associated with a proportional cost shift rather than a flat dollar loading. It should also be noted that continuous explanatory variables were converted to logarithmic scale in determining the logged cost models (log-log model form).

A more technical problem is that of heteroscedasticity. An important assumption of the classical OLS regression model $z = f(x) + \varepsilon$ is that the ε all have the same variance. As with much cross-sectional data, this is problematic in the case of WC case severity. Indeed, more expensive cases show greater cost variability and it is likely that this affects the variability of the residuals. The presence of heteroscedasticity has important implications for the interpretation and application of the cost model, especially as regards predictions and their confidence intervals (its presence does not, however, invalidate the model coefficients used here to decompose cost differences). Although few would believe that lost time costs actually conform to any simple linear (or log-linear) functional form, in the classical OLS regression sense, this is relevant in light of the use the model to decompose cost differences. Indeed, the conceptual basis of the decomposition comes from interpreting the regression equation as the tangent hyperplane to the graph of the cost function at the center of gravity. The model coefficients regarded as partial derivatives that measure the slope at that point along the axis of the corresponding explanatory variable. The better the choice for the functional form of the cost model, therefore, the more credible the decomposition. By the same token, when using regression models to analyze case severity, it is advantageous to optimize the fit at a center of gravity which conforms to the severity measure being used—in this case the (sample weighted arithmetic) average cost per case.

Heteroscedasticity is also among the justifications cited for the use of the log transformation. The simplest approach to dealing with heteroscedasticity is to divide the observations into groups and examine the residuals for any pattern. Given the concern expressed above that higher cost cases are also the more variable, it is natural to again consider cost percentiles. Recall that in preparing Charts 1 and 2, claims were collected, according to size, into 100 groups of roughly equal weight. The idea here is to normalize the cost of each group to a common (weighted) mean of 1. The lowest quartile is excluded in order to avoid erratic results due, at least in part, to division by comparatively small numbers. This generates 75 subsets of similar size and scale for which we can compare the model residuals. Chart 3 shows the standard deviation of the residuals for the linear and logged cost models, determined using the sample and exponentially adjusted weights, respectively (the pattern for the log-linear cost model derived using the original sample weight is quite similar to that using the exponential adjusted weight). Observe that, for both models, not only does the regression equation consistently under-predict the highest z values, it does so in such a way as to yield relatively little variation in the error, as compared with the size of z. While both models show a pattern of decline with increasing cost, that decline is less pronounced for the log-linear cost model. Indeed, while the log-linear variation measure remains mostly in the interval [1,2], the values from the linear model decline from 5 to nearly 0. From this simple picture, then, the log-linear cost model shows less evidence of heteroscedasticity.

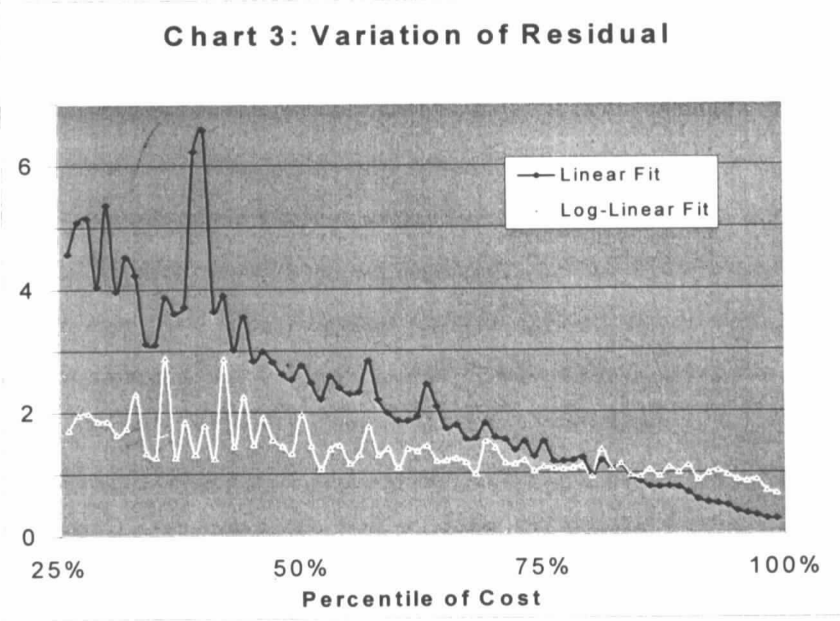
Chart 3: Variation of Residual
Linear Fit
Log-Linear Fit
6
4
2
0
25%
50%
75%
100%
Percentile of Cost

To summarize, the case study illustrates the primary reason for using logged cost models is a much better fit to the data. Also, proportional cost effects are generally preferred to flat dollar loadings. Among the other motivations for using the log transformation is the need to counter heteroscedasticity and outliers by making higher cost cases less influential in the model. While the exponential weight adjustment runs somewhat counter to that by shifting weight to higher cost cases, it still improves the situation as regards heteroscedasticity and outliers and has the major advantage of optimizing the fit at the point measure of case severity.

Cost Decomposition: The previous two sections illustrate how convenient linear models are for decomposing dollar differences but that log-linear cost models generally provide a better fit to the data and have other conceptual advantages. This purpose of this section is again to put the pieces together. Applying the logarithm in conjunction with an "exponential" transformation of the sample weight, the mean values of the logged cost model invert back to the original (weighted) arithmetic mean. This enables a decomposition of the relative difference in case severity very similar to the Oxacca style dollar decomposition derived using linear cost models.

As above, the post-reform relative difference in mean cost per case among the non-reform states can therefore be expressed as:

$$\log(\overline{z\,|_{CA}}) - \log(\overline{z\,|_{CB}}) = \underbrace{(\alpha_{CA} - \alpha_{CB})}_{\text{base cost}} + \underbrace{\sum_j \beta_j \left(\overline{\overline{x_j\,|_{CA}}} - \overline{\overline{x_j\,|_{CB}}}\right)}_{\text{case mix}} + \underbrace{\sum_k \gamma_k \left(\overline{\overline{x_k\,|_{CA}}} - \overline{\overline{x_k\,|_{CB}}}\right)}_{\text{targeted cost drivers}}$$

This is the itemization of the relative difference in lost time case severity presented in the PRM studies. The results for the DCI claim data is shown in Tables 1a and 1b.

Table 1a: Components of Relative Difference: Post- vs Pre-Reform				
Comparison Group	***Relative Difference *#***	***Components***		
		Base Cost	***Claim Mix***	***Cost Drivers***
Control Group	-4.3%*	-13.3%	2.1%	6.9%
Test Group	-19.4%	-18.5%	2.6%	-3.6%

* Statistically different from 0 with 95% confidence, based on a 2-tailed T-Test.
\# Relative difference of x Vs. y is determined as natural log(x/y), expressed as a percentage.
SOURCE: NCCI DCI, claims evaluated 18-months after report of injury.

Observe that for the reform states test group the cost drivers contributed to the decline in case severity, while those factors worked to increase costs in the non-reform states.

Table 1b: Components of Relative Difference: Test vs Control				
Time Period	***Relative Difference *#***	***Components***		
		Base Cost	***Claim Mix***	***Cost Drivers***
Pre-Reform	30.8%*	14.8%	-0.1%	16.1%
Post-Reform	15.7%*	9.6%	0.5%	5.6%

* Statistically different from 0 with 95% confidence, based on a 2-tailed T-Test.
\# Relative difference of x Vs. y is determined as natural log(x/y), expressed as a percentage.
SOURCE: NCCI DCI, claims evaluated 18-months after report of injury.

The claim mix component is small in comparison with the other two components. This decomposition indicates that pre-reform cost drivers contributed a larger share to the higher severity of the reform states. The higher cost differential was cut in half post-reform and under this decomposition, targeted cost drivers account for a smaller share of that smaller difference.

Conclusions: A number of states enacted major reforms of their workers compensation systems in the last decade to control rapidly increasing claim frequency and costs. The most common tools to address these problems were the introduction of managed care provisions, the imposition of stricter compensability standards and fewer incentives for attorney involvement. NCCI post-reform monitoring reports analyze claim frequency and severity in these states before and after the enactment of reforms, comparing the outcomes to trends in a group of non-reform states. This paper describes the method used to analyze the severity of lost time cases using DCI claim data.

Factors other than the reforms, including the influence of economic cycles and secular trends, may have affected the outcomes. These factors may have countered the effects of the reforms where the observed improvements were modest. In addition, the analysis did not evaluate the impact of each reform provision on lost time case severity. It is likely that some reform measures may have greater impact than the others. For these reasons, a comparison of outcomes, such as a simple T-test of means, between the two periods with a reference to the countrywide trend provides only a limited understanding of the effects of the reforms on the system costs. As described here, multivariate cost models address this by decomposing the difference into components. A customized logged cost model is described and shown to possess some important technical features. That is the method used to prepare the PRM studies. The DCI results presented to illustrate the methodology indicate that cost drivers targeted by reform indeed play a different role in the reform states than in the non-reform control group of states. Still, those findings confirm the view that factors other than those associated with claim characteristics captured in the DCI—like economic cycles and secular trends--may significantly influence costs.

From the reform versus non-reform state perspective, simple cost comparisons indicate that the reform states maintain a significantly higher case severity. That cost differential, however, was halved post-reform and the multivariate analysis assigns much of that relative improvement in claim severity to cost drivers targeted by reform

References

[1] Corro, Dan, 1999, A Practical Suggestion for Log-Linear Workers Compensation Cost Models. *Casualty Actuarial Society Forum. Spring 1999:* pp. 363-393.

[2] Corro, Dan and Helvacian, N. Mike, 1999, NCCI Post Reform Monitoring Reports. *NCCI 1999 Workers Compensation Mid-Year Issues Report:* pp. 11-15.

APPENDIX: Regressions Discussed in the Case Study

Dependent Variable: INCURRED COST

Table 1. Analysis of Variance

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	49	1.5142475E14	3.0903009E12	581.754	0.0001
Error	38145	2.0262767E14	5312037561.2		
U Total	38194	3.5405242E14			

Root MSE	72883.72631	R-square	0.4277
Dep Mean	8557.63163	Adj R-sq	0.4270
C.V.	851.68104		

Table 2. Parameter Estimates

Variable Description	DF	Parameter Estimate	Standadard Error	T for H0: Parameter=0	Prob > |T|
TEST BEFORE SUBGROUP	1	-476.835672	729.14915362	-0.654	0.5131
TEST AFTER SUBGROUP	1	-2294.367470	747.57602381	-3.069	0.0021
CONTROL BEFORE SUBGROUP	1	-2080.279861	689.14790226	-3.019	0.0025
CONTROL AFTER SUBGROUP	1	-3001.570552	697.96546494	-4.300	0.0001
EMPLOYER PAYROLL SIZE $0	1	1469.861287	431.95451854	3.403	0.0007
EMPLOYER PAYROLL SIZE $1-$100K	1	557.942641	311.42257706	1.792	0.0732
EMPLOYER PAYROLL SIZE $100K-$1M	1	-6.767765	270.72389656	-0.025	0.9801
EMPLOYER PAYROLL SIZE $1M-$10M	1	240.760277	262.07125972	0.919	0.3583
CLASS IN SCHEDULE GROUP 05[2]	1	654.826799	570.71037800	1.147	0.2512
CLASS IN SCHEDULE GROUP 07	1	909.315255	1023.9962096	0.888	0.3745
CLASS IN SCHEDULE GROUP 10	1	105.300221	674.23379474	0.156	0.8759
CLASS IN SCHEDULE GROUP 12	1	-462.269804	674.63152253	-0.685	0.4932
CLASS IN SCHEDULE GROUP 14	1	93.132284	713.15373065	0.131	0.8961
CLASS IN SCHEDULE GROUP 17	1	185.409428	426.85143513	0.434	0.6640
CLASS IN SCHEDULE GROUP 18	1	-337.910290	492.17394439	-0.687	0.4924
CLASS IN SCHEDULE GROUP 20	1	-569.888682	957.46087475	-0.595	0.5517
CLASS IN SCHEDULE GROUP 21	1	572.977130	1546.8026243	0.370	0.7111
CLASS IN SCHEDULE GROUP 24	1	-102.002622	1141.1471662	-0.089	0.9288
CLASS IN SCHEDULE GROUP 25	1	1433.375913	1085.8342328	1.320	0.1868
CLASS IN SCHEDULE GROUP 26	1	801.569830	654.71935752	1.224	0.2208
CLASS IN SCHEDULE GROUP 27	1	1290.795104	379.54043241	3.401	0.0007
CLASS IN SCHEDULE GROUP 33	1	623.471859	1173.2744182	0.531	0.5951
CLASS IN SCHEDULE GROUP 34	1	-446.411085	299.33417378	-1.491	0.1359
CLASS IN SCHEDULE GROUP 35	1	432.100881	340.52812259	1.269	0.2045
CLASS IN SCHEDULE GROUP 36	1	-654.397301	343.07361011	-1.907	0.0565
TRAUMATIC INJURY	1	1034.054233	471.84163281	2.192	0.0284
PRE-INJURED WEEKLY WAGE	1	8.369938	0.52696165	15.883	0.0001
INJURY AGE	1	52.989464	7.92956518	6.683	0.0001
MALE CLAIMANT	1	1545.291364	223.90733173	6.901	0.0001
INJURED PART OF BODY - INTERNAL ORGANS	1	-4979.403718	564.72830329	-8.817	0.0001
INJURED PART OF BODY - HEAD	1	-373.060091	574.04135302	-0.650	0.5158
INJURED PART OF BODY - NECK	1	3235.641607	724.93327242	4.463	0.0001
INJURED PART OF BODY - LOWER BACK	1	-795.568308	335.14611281	-2.374	0.0176
INJURED PART OF BODY - UPPER BACK	1	-1479.748378	601.39009721	-2.461	0.0139
INJURED PART OF BODY - LOWER EXTREMITY	1	-2697.935157	337.36867599	-7.997	0.0001
INJURED PART OF BODY - UPPER EXTREMITY	1	-3309.790946	319.73928384	-10.352	0.0001
FATAL CLAIM	1	110398	3559.8311671	31.012	0.0001
STATUS OF CLAIM IS OPEN	1	24269	305.71884613	79.380	0.0001
WEEKLY BENEFIT	1	0.176502	0.06403396	2.756	0.0058
HOSPITALIZATION INDICATOR	1	3362.047743	199.67950657	16.837	0.0001
SURGERY INDICATOR	1	7044.530354	266.84389548	26.399	0.0001
VOCATIONAL REHABILITATION BENEFITS	1	25215	760.93884532	33.136	0.0001
CLAIMANT REPRESENTED BY AN ATTORNEY	1	3530.486859	305.37792234	11.561	0.0001
RETURN TO WORK INDICATOR	1	-3675.050427	204.25644599	-17.992	0.0001
PERMANENT TOTAL AWARD	1	75476	2277.5335650	33.139	0.0001
SCHEDULED PERMANENT PARTIAL AWARD	1	4859.151359	380.87552656	12.758	0.0001
NON-SCHEDULED PERMANENT PARTIAL AWARD	1	7546.803456	499.52747703	15.108	0.0001
DISFIGUREMENT AWARD INDICATOR	1	4956.894055	870.70708865	5.693	0.0001
LUMP SUM PAYMENT INDICATOR	1	10976	661.99166040	16.580	0.0001

[2] The classifications have been arranged into general industry divisions, designated "Schedules," and further subdivided into smaller "Groups" of classifications having similar or related characteristics. Source: Classification Codes & Statistical Codes for Workers' Compensation & Employers Liability Insurance, National Council on Compensation Insurance, Inc., 1997 Edition.

Dependent Variable: LOG OF INCURRED COST

Table 3. Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	49	51997071.56	1061164.7257	65684.008	0.0001
Error	38145	616255.46055	16.15560		
U Total	38194	52613327.021			

Root MSE	4.01940	R-square	0.9883
Dep Mean	9.04757	Adj R-sq	0.9883
C.V.	44.42522		

Table 4. Parameter Estimates

Variable Description	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
TEST BEFORE SUBGROUP	1	4.930303	0.07994791	61.669	0.0001
TEST AFTER SUBGROUP	1	4.745682	0.08107615	58.534	0.0001
CONTROL BEFORE SUBGROUP	1	4.782400	0.07902394	60.518	0.0001
CONTROL AFTER SUBGROUP	1	4.649294	0.07995381	58.150	0.0001
EMPLOYER PAYROLL SIZE $0	1	0.146117	0.02352213	6.212	0.0001
EMPLOYER PAYROLL SIZE $1-$100K	1	0.071917	0.01710374	4.205	0.0001
EMPLOYER PAYROLL SIZE $100K-$1M	1	-0.027990	0.01514450	-1.848	0.0646
EMPLOYER PAYROLL SIZE $1M-$10M	1	-0.028072	0.01459912	-1.923	0.0545
CLASS IN SCHEDULE GROUP 05	1	0.111370	0.03093370	3.600	0.0003
CLASS IN SCHEDULE GROUP 07	1	-0.005698	0.06112679	-0.093	0.9257
CLASS IN SCHEDULE GROUP 10	1	0.010150	0.03737508	0.272	0.7860
CLASS IN SCHEDULE GROUP 12	1	0.031146	0.03685414	0.845	0.3980
CLASS IN SCHEDULE GROUP 14	1	0.078072	0.04032587	1.936	0.0529
CLASS IN SCHEDULE GROUP 17	1	0.074217	0.02269132	3.271	0.0011
CLASS IN SCHEDULE GROUP 18	1	0.048409	0.02663034	1.818	0.0691
CLASS IN SCHEDULE GROUP 20	1	0.091989	0.05046347	1.823	0.0683
CLASS IN SCHEDULE GROUP 21	1	0.061793	0.08601793	0.718	0.4725
CLASS IN SCHEDULE GROUP 24	1	0.020143	0.06040751	0.333	0.7388
CLASS IN SCHEDULE GROUP 25	1	0.331156	0.05723375	5.786	0.0001
CLASS IN SCHEDULE GROUP 26	1	0.123215	0.03439618	3.582	0.0003
CLASS IN SCHEDULE GROUP 27	1	0.094100	0.01950895	4.823	0.0001
CLASS IN SCHEDULE GROUP 33	1	0.067074	0.05952280	1.127	0.2598
CLASS IN SCHEDULE GROUP 34	1	-0.033787	0.01687428	-2.002	0.0453
CLASS IN SCHEDULE GROUP 35	1	0.074462	0.01862132	3.999	0.0001
CLASS IN SCHEDULE GROUP 36	1	-0.037061	0.02049899	-1.808	0.0706
TRAUMATIC INJURY	1	-0.107969	0.02383974	-4.529	0.0001
PRE-INJURED WEEKLY WAGE	1	0.129590	0.00814714	15.906	0.0001
INJURY AGE	1	0.309847	0.01681119	18.431	0.0001
MALE CLAIMANT	1	0.105804	0.01270655	8.327	0.0001
INJURED PART OF BODY - INTERNAL ORGANS	1	-0.404079	0.03242312	-12.463	0.0001
INJURED PART OF BODY - HEAD	1	-0.044888	0.03109403	-1.444	0.1489
INJURED PART OF BODY - NECK	1	0.159659	0.03573244	4.468	0.0001
INJURED PART OF BODY - LOWER BACK	1	-0.058334	0.01772142	-3.292	0.0010
INJURED PART OF BODY - UPPER BACK	1	-0.108579	0.03453432	-3.144	0.0017
INJURED PART OF BODY - LOWER EXTREMITY	1	-0.250969	0.01801989	-13.927	0.0001
INJURED PART OF BODY - UPER EXTREMITY	1	-0.261498	0.01691327	-15.461	0.0001
FATAL CLAIM	1	2.050513	0.08586028	23.882	0.0001
STATUS OF CLAIM IS OPEN	1	1.487664	0.01239017	120.068	0.0001
WEEKLY BENEFIT	1	0.260116	0.00969333	26.834	0.0001
HOSPITALIZATION INDICATOR	1	0.708042	0.01237411	57.220	0.0001
SURGERY INDICATOR	1	0.586466	0.01220345	48.057	0.0001
VOCATIONAL REHABILITATION BENEFITS	1	0.808863	0.02547978	31.745	0.0001
CLAIMANT REPRESENTED BY AN ATTORNEY	1	0.373587	0.01376672	27.137	0.0001
RETURN TO WORK INDICATOR	1	-0.397740	0.01090690	-36.467	0.0001
PERMANENT TOTAL AWARD	1	1.415209	0.06972410	20.297	0.0001
SCHEDULED PERMANENT PARTIAL AWARD	1	0.644809	0.01624483	39.693	0.0001
NON-SCHEDULED PERMANENT PARTIAL AWARD	1	0.783743	0.02121763	36.938	0.0001
DISFIGUREMENT AWARD INDICATOR	1	0.597890	0.03679728	16.248	0.0001
LUMP SUM PAYMENT INDICATOR	1	1.138939	0.02717348	41.914	0.0001

A Note on the Inverse Relationship Between Hazard and Life Expectancy

Dan Corro

Abstract:

Intuitively, life expectancy and hazard rate should be inversely related to each other. Whereas life expectancy, or mean time to failure, is determinable as a simple descriptive statistic, the concept of hazard is defined as an instantaneous failure rate and involves taking limits. This note investigates "inverting" life expectancy as a method for estimating the hazard rate. The main result is that given any finite collection of (internally consistent) pairs of age and associated life expectancy values, there is a uniquely determined step function that determines a "gauntlet" survival model with the given life expectancies at their respective ages. The Appendix provides a simple computer algorithm for implementing this model in practice.

I. Introduction

In general, life expectancy is determinable as a simple descriptive statistic. It is both easier to interpret and to estimate than the hazard rate, which is defined as an instantaneous failure rate and involves taking limits. When working with insurance data, "claim life expectancy" is often available as a reserve (c.f. [3]). In practice, reserves may be related with claim survival data only to the extent that closed, i.e. "dead", cases are characterized by having no reserves. On the other hand, knowledge of the hazard rate function is useful for many insurance applications (c.f. [6]). It might be very useful, therefore, to be able to go directly from life expectancy to the hazard rate.

In the exponential decay survival model, for example, life expectancy and hazard are both constant and inverse to each other. If you were confronted with survival data, you might observe the expectation of life early on to get an intuitive feel. If the life expectation were fairly constant, you would naturally gravitate to the exponential decay model and you would already know to assign the reciprocal of the mean time to failure as the constant hazard. This note suggests a generalization of this simple approach, detailing how to approximate hazard with a step function directly from information on life expectancy.

While this approach is just an alternative organization to the usual way of empirically calculating hazard, it has some technical and conceptual advantages. In particular, the approach is simple to explain and amenable to implementation on a computer. Censored observations are handled in a transparent fashion. Moreover, the technique can be extended to higher dimensions (c.f. [4]). As noted, in the case of insurance applications, reserves can be regarded as life expectancies and so the method provides a direct way of incorporating reserves into hazard models.

II. Notation and Background

Let $f(t)$ denote a continuous function on the nonnegative real numbers $\Re_+ = [0,\infty)$ satisfying:

$$\int_0^\infty f(t)dt = 1$$

Regard $f(t)$ as a probability density of failure times and define the function:

$$S(t) = 1 - \int_0^t f(s)ds = \int_t^\infty f(s)ds$$

As is customary, we refer to $S(t)$ as the *survival function*, $f(t)$ as the *probability density function [PDF]* and t as "time". We also let T denote the random variable for the distribution of survival times and $\mu = E(T)$ the mean duration, which we assume throughout to be finite. Survival analysis refers to the following function:

$$h(t) = \frac{f(t)}{S(t)}$$

as the *hazard rate function* or sometimes as the *force of mortality*. The hazard rate function measures the instantaneous rate of failure at time t and can be expressed as a limit of conditional probabilities:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr\{t \le T < t + \Delta t \mid T \ge t\}}{\Delta t}$$

There are many well-known relationships and interpretations of these functions—refer to Allison[1] for a particularly succinct discussion;. It is convenient to recall that setting

$$g(t) = \int_0^t h(s)ds \text{ then } S(t) = e^{-g(t)}.$$

Fix t and restrict attention to values of time $w > t$. The conditional probability of survival to w, given survival to t, is $S_t(w) = \frac{S(w)}{S(t)}$. In this context (see [3]), the *expectation of life at time t*, given survival to time t, is just:

$$\rho(t) = \frac{\int_t^\infty (w-t)f(w)dw}{\int_t^\infty f(w)dw} = \int_t^\infty S_t(w)dw = \int_t^\infty \frac{S(w)}{S(t)}dw$$

Observe that under our assumptions, $\rho(0) = \mu$ and the function $\rho(t)$ is well defined for all $t>0$. . Observe too that for any $a<b$ with $S(a) > 0$ we have the relation:

$$\rho(a)S(a) = \int_a^\infty S(t)dt = \int_a^b S(t)dt + \int_b^\infty S(t)dt$$

$$\le \int_a^b S(a)dt + \int_b^\infty S(t)dt = S(a)(b-a) + \rho(b)S(b)$$

$$\Rightarrow \quad a + \rho(a) \le b + \frac{\rho(b)S(b)}{S(a)} \le b + \rho(b)$$

with strict inequality exactly when $S(b) < S(a)$.

This paper concerns itself with how the two functions $h(t), \rho(t)$ relate to each other. While we might expect an inverse relationship of some sort, note that the two are conceptually quite different: h is local while ρ is global. Still, it is reasonable to expect that the average values of h over an appropriate interval might relate with the values of ρ over that interval.

Example: Suppose the expectation of life (mean time to failure) is constant on the interval $[a,b)$, $\rho(t) = \alpha, a \le t < b$, including the case $b = \infty$. Then

$$\alpha S(t) = \rho(t)S(t) = \int_t^\infty S(w)dw$$

$$\Rightarrow \quad \alpha\frac{dS}{dt} = -S(t) \quad \Rightarrow S(t) = e^{-\frac{t}{\alpha}}$$

$$\Rightarrow g(t) = \frac{t}{\alpha} \Rightarrow h(t) = \frac{dg}{dt} = \frac{1}{\alpha} \quad a \le t < b$$

The following proposition generalizes this:

Proposition 1: For any real numbers $a<b$ with $S(a)>0$, there exists a $\varsigma \in [a,b]$ with:

$$h(\varsigma) = \frac{S(a) - S(b)}{S(a)\rho(a) - S(b)\rho(b)}$$

Proof: Consider the integral $\int_a^b S(t)h(t)dt$. Because $S(t)$ is nonnegative, the intermediate value theorem for integrals implies there is $\varsigma \in [a,b]$ with:

$$\int_a^b S(t)h(t)dt = h(\varsigma)\int_a^b S(t)dt = h(\varsigma)\left(\int_a^\infty S(t)dt - \int_b^\infty S(t)dt\right) = h(\varsigma)(S(a)\rho(a) - S(b)\rho(b))$$

On the other hand, taking $u(t) = -g(t) = -\int_0^t h(w)dw, \quad \frac{du}{dt} = -h(t)$ and we have:

$$\int_a^b S(t)h(t)dt = -\int_{-g(a)}^{-g(b)} e^u du = e^{-g(a)} - e^{-g(b)} = S(a) - S(b)$$

and the result follows.

Not surprisingly, there are formal relationships between hazard $h(t)$ and life expectancy $\rho(t)$, as in:

Proposition 2:

$$i) \quad 1+\frac{d\rho}{dt}=h(t)\rho(t)$$

$$ii) \quad \rho(t)>0 \Rightarrow h(t)=\frac{1}{\rho(t)}+\frac{d(\ln\rho)}{dt}$$

$$iii) \quad \rho(t)>0 \Rightarrow \frac{1}{\rho(t)}=-\frac{d(\ln\rho S)}{dt}$$

$$iv) \quad \lim_{t\to\infty}\rho(t)=\lim_{t\to\infty}\frac{1}{h(t)}$$

Proof: The verification is straightforward: from the definition of $\rho(t)$ and the formula for differentiating a ratio:

$$\frac{d\rho}{dt}=\frac{S(t)(-S(t))-\int_t^\infty S(w)dw\left(\frac{dS}{dt}\right)}{S(t)^2}=\frac{f(t)\int_t^\infty S(w)dw-S(t)^2}{S(t)^2}$$

$$=\frac{f(t)}{S(t)}\frac{\int_t^\infty S(w)dw}{S(t)}-1=h(t)\rho(t)-1$$

$$\Rightarrow 1+\frac{d\rho}{dt}=h(t)\rho(t)$$

establishing i); ii) is immediate from i):

$$\rho(t)>0 \Rightarrow h(t)=\frac{h(t)\rho(t)}{\rho(t)}=\frac{1+\frac{d\rho}{dt}}{\rho(t)}=\frac{1}{\rho(t)}+\frac{\frac{d\rho}{dt}}{\rho(t)}=\frac{1}{\rho(t)}+\frac{d\ln(\rho)}{dt}$$

And iii) can be readily derived from ii):

$$\rho(t)>0 \Rightarrow -\frac{d\ln(S)}{dt}=h(t)=\frac{1}{\rho(t)}+\frac{d\ln(\rho)}{dt}$$

$$\Rightarrow \frac{1}{\rho(t)}=-\frac{d\ln(S)}{dt}-\frac{d\ln(\rho)}{dt}=-\frac{d(\ln(S)+\ln(\rho))}{dt}=-\frac{d\ln(\rho S)}{dt}$$

Finally iv) is a straightforward application of L'Hôpital's rule (see [5] p.90): indeed, under our assumptions we have:

$$\mu = \int_0^\infty S(t)dt = \lim_{t\to\infty} \int_0^\infty S(w)dw = \lim_{t\to\infty} \int_0^t S(w)dw + \int_t^\infty S(w)dw$$

$$= \int_0^\infty S(t)dt + \lim_{t\to\infty} \int_t^\infty S(w)dw = \mu + \lim_{t\to\infty} \int_t^\infty S(w)dw$$

$$\Rightarrow 0 = \lim_{t\to\infty} \int_t^\infty S(w)dw = \lim_{t\to\infty} S(t)$$

So invoking L'Hôpital's rule:

$$\lim_{t\to\infty} \rho(t) = \lim_{t\to\infty} \frac{\int_t^\infty S(w)dw}{S(t)} = \lim_{t\to\infty} \frac{-S(t)}{-f(t)} = \lim_{t\to\infty} \frac{1}{h(t)}$$

completing the proof.

It is easy to see that the expectation of life function uniquely determines the survival model. Indeed, Proposition 2 shows that the function $\rho(t)$ determines the hazard function $h(t)$ and whence specifies the complete survivorship model. Proposition 2 also generalizes the inverse relationship between survival and hazard noted for the exponential decay model. Indeed, it shows that in general hazard and life expectancy do *not* follow a simple inverse relationship. Indeed, $h(t)$ is the sum of *two* components, one inversely related and the other directly related to $\rho(t)$. More precisely, hazard consists of a "first order" component in fact being the inverse of $\rho(t)$ and a "second order" component responding to the proportional change in $\rho(t)$ as captured by the latter's logarithmic derivative.

Our interest is in finding a more "elementary" relationship between $h(t)$ and $\rho(t)$ -- preferably one amenable to calculation from empirical discrete data and, in particular, one that avoids derivatives.

The following technical lemma is the key result needed to invert life expectancy to hazard and its proof blueprints an algorithm for the calculation.

Lemma: For any triplet of positive real numbers $\alpha, \beta, \gamma > 0$ with $\gamma > 1 - \frac{\beta}{\alpha}$, there exists a *unique* $\eta > 0$ such that:

$$\alpha\eta = \frac{e^{\beta\eta} - 1}{e^{\beta\eta} - \gamma}$$

Proof: Consider the function

$$\psi(x) = \psi(\alpha,\beta,\gamma;x) = \alpha x - \frac{e^{\beta x}-1}{e^{\beta x}-\gamma}$$

the lemma asserts that $\psi(x)$ has exactly 1 positive real root. Define

$$\varphi(x) = \frac{d\psi}{dx} = \alpha - \frac{(e^{\beta x}-\gamma)(\beta e^{\beta x}) - (e^{\beta x}-1)(\beta e^{\beta x})}{\left(e^{\beta x}-\gamma\right)^2}$$
$$= \alpha + \frac{\beta e^{\beta x}(\gamma-1)}{\left(e^{\beta x}-\gamma\right)^2}$$

We consider three cases:

Case $\gamma = 1$: Here $\psi(x) = \alpha x - 1$ clearly has unique positive root $\frac{1}{\alpha}$.

Case $\gamma < 1$: In this case, we first verify that $\varphi(x)$ has a unique positive root. Indeed, noting that for $x>0$, $e^{\beta x} > 1 > \gamma \Rightarrow e^{\beta x} - \gamma > 0$, we find that:

$$\varphi(x) = 0$$
$$\Leftrightarrow \alpha(e^{\beta x}-\gamma)^2 = \beta e^{\beta x}(1-\gamma)$$
$$\Leftrightarrow e^{\beta x} - \gamma = \sqrt{\frac{\beta e^{\beta x}(1-\gamma)}{\alpha}} = \sqrt{e^{\beta x}}\sqrt{\frac{\beta(1-\gamma)}{\alpha}}$$

Letting $y = \sqrt{e^{\beta x}}$ this equation becomes:

$$y^2 - \sqrt{\frac{\beta(1-\gamma)}{\alpha}}y - \gamma = 0$$

which has roots:

$$\sqrt{\frac{\beta(1-\gamma)}{4\alpha}} \pm \sqrt{\frac{\beta(1-\gamma)}{4\alpha} + \gamma}$$

only one of which is >0, and so

$$y = \sqrt{\frac{\beta(1-\gamma)}{4\alpha}} + \sqrt{\frac{\beta(1-\gamma)}{4\alpha} + \gamma}$$

$$\Rightarrow e^{\beta x} = y^2 = \gamma + y\sqrt{\frac{\beta(1-\gamma)}{4\alpha}}$$

$$= \gamma + \left(\sqrt{\frac{\beta(1-\gamma)}{4\alpha}} + \sqrt{\frac{\beta(1-\gamma)}{4\alpha} + \gamma}\right)\sqrt{\frac{\beta(1-\gamma)}{4\alpha}}$$

$$= \gamma + \frac{\beta(1-\gamma)}{2\alpha} + \sqrt{\frac{\beta(1-\gamma)}{2\alpha}\left(\frac{\beta(1-\gamma)}{2\alpha} + 2\gamma\right)}$$

It follows that setting

$$\tau = \frac{\ln\left(\gamma + \frac{\beta(1-\gamma)}{2\alpha} + \sqrt{\frac{\beta(1-\gamma)}{2\alpha}\left(\frac{\beta(1-\gamma)}{2\alpha} + 2\gamma\right)}\right)}{\beta}$$

then τ is the unique positive root of $\varphi(x) = \frac{d\psi}{dx}$. Note that

$$\gamma > 1 - \frac{\beta}{\alpha} \Rightarrow \frac{\beta}{\alpha} > 1 - \gamma > 0 \Rightarrow \frac{\beta}{1-\gamma} > \alpha$$

$$\Rightarrow \varphi(0) = \alpha + \frac{\beta(\gamma-1)}{(\gamma-1)^2} = \alpha - \frac{\beta}{1-\gamma} < 0$$

and it follows that $\psi(x)$ is decreasing on $(0,\tau)$. The next claim is that $\psi(x) < 0$ for x positive and near 0. To verify this, consider:

$$\lambda(x) = \frac{\alpha x e^{\beta x} - \alpha\gamma x}{e^{\beta x} - 1}$$

Combining the assumption that $\gamma > 1 - \frac{\beta}{\alpha}$ with L'Hospital's rule, we find that:

$$\lim_{x\to 0} \lambda(x) = \lim_{x\to 0} \frac{\alpha\beta x e^{\beta x} + \alpha e^{\beta x} - \alpha\gamma}{\beta e^{\beta x}} = \frac{\alpha}{\beta}(1-\gamma) < 1$$

This means there exists $\varepsilon > 0$ such that $\lambda(x) < 1$ for $0 < x < \varepsilon$. Since $e^{\beta x} > 1 > \gamma$, we have that

$$1 > \lambda(x) = \frac{\alpha x e^{\beta x} - \alpha \gamma x}{e^{\beta x} - 1}$$

$$\Leftrightarrow e^{\beta x} - 1 > \alpha x e^{\beta x} - \alpha \gamma x = \alpha x (e^{\beta x} - \gamma) > 0$$

$$\Leftrightarrow \frac{e^{\beta x} - 1}{e^{\beta x} - \gamma} > \alpha x \Leftrightarrow \psi(x) = \alpha x - \frac{e^{\beta x} - 1}{e^{\beta x} - \gamma} < 0$$

proving the claim. It follows that $\psi(x)$, which is negative near 0, remains negative and can have no root in $(0, \tau)$ since $\psi(x)$ is decreasing over that interval. On the other hand, observe that

$$1 > \gamma \Rightarrow 0 < e^{\frac{\beta}{\alpha}} - 1 < e^{\frac{\beta}{\alpha}} - \gamma$$

$$\Rightarrow \psi(\frac{1}{\alpha}) = \frac{\alpha}{\alpha} - \frac{e^{\frac{\beta}{\alpha}} - 1}{e^{\frac{\beta}{\alpha}} - \gamma} = 1 - \frac{e^{\frac{\beta}{\alpha}} - 1}{e^{\frac{\beta}{\alpha}} - \gamma} > 0$$

Which means that $\psi(x)$ increases from negative to positive with a unique root on $[\tau, \frac{1}{\alpha}]$ and remains positive and increasing on $(\frac{1}{\alpha}, \infty)$. In particular, $\psi(x)$ has a unique positive root and the lemma is established for the case $\gamma < 1$. This leaves only the remaining:

Case $\gamma > 1$: In this case $\gamma - 1 > 0$ clearly implies that:

$$\varphi(x) = \frac{d\psi}{dx} = \alpha + \frac{\beta e^{\beta x}(\gamma - 1)}{\left(e^{\beta x} - \gamma\right)^2} > 0$$

and so $\psi(x)$ is monotonic increasing and can therefore have at most one root in any interval in its domain. We therefore need to investigate the behavior of $\psi(x)$ at 0 and $\delta = \frac{\ln(\gamma)}{\beta}$. We evidently have the following one-sided limits:

$$\lim_{x>0, x\to 0} \psi(x) = \lim_{x>0, x\to 0} \alpha x - \frac{e^x - 1}{e^x - \gamma} = 0 - \frac{0}{1-\gamma} = 0$$

$$\lim_{x>\delta, x\to\delta} \psi(x) = \frac{\alpha \ln(\gamma)}{\beta} - (\gamma - 1)\left(\lim_{e^t > \gamma, t\to \ln(\gamma)} \frac{1}{e^{xt} - \gamma} \right) = \frac{\alpha \ln(\gamma)}{\beta} - (+\infty) = -\infty$$

$$\lim_{x<\delta, x\to\delta} \psi(x) = \frac{\alpha \ln(\gamma)}{\beta} - (\gamma-1)\left(\lim_{e^t<\gamma, t\to\ln(\gamma)} \frac{1}{e^{xt}-\gamma} \right) = \frac{\alpha \ln(\gamma)}{\beta} - (-\infty) = +\infty$$

Let $\omega = 3\gamma + e^{\frac{2\beta}{\alpha}}$ and $\varepsilon = \frac{\ln(\omega)}{\beta} > \frac{\ln(\gamma)}{\beta} = \delta$, the claim is that $\psi(\varepsilon) > 0$. To verify this, note that

$$\omega > 3\gamma \Rightarrow \omega - \gamma > 2\gamma > 2\gamma - 2$$
$$\Rightarrow \frac{\omega - \gamma}{2} > \gamma - 1 > 0$$
$$\Rightarrow \frac{1}{2} > \frac{\gamma - 1}{\omega - \gamma} > 0$$

Similarly:

$$\omega > e^{\frac{2\beta}{\alpha}} \Rightarrow \ln(\omega) > \frac{2\beta}{\alpha}$$
$$\Rightarrow \frac{\alpha}{\beta}\ln(\omega) > 2 > \frac{3}{2} > 1 + \frac{\gamma - 1}{\omega - \gamma} = \frac{\omega - 1}{\omega - \gamma}$$

From the definitions we find that:

$$\psi(\varepsilon) = \frac{\alpha}{\beta}\ln(\omega) - \frac{\omega - 1}{\omega - \gamma} > 0,$$

which establishes the claim. We have shown that $\psi(x)$ is positive, in fact is monotonic increasing from 0 upward on $(0,\delta)$, that $\psi(x)$ increases monotonically from negative to positive with a unique root in $(\delta,\varepsilon]$, and $\psi(x)$ is positive and monotonic increasing on (ε,∞). This proves the assertion in the case $\gamma > 1$ and completes the proof of the lemma.

Now consider a positive interval $[a,b)$ on which the hazard is flat:

$$h(t) = \eta, \quad a \le t < b$$
$$\Rightarrow g(b) - g(a) = \int_a^b h(t)dt = \eta(b-a)$$
$$\Rightarrow \frac{S(a)}{S(b)} = e^{\eta(b-a)}$$

Clearly $\eta = 0 \Leftrightarrow S(a) = S(b)$ so consider the case $\eta > 0$. Proposition 1 implies that:

$$\eta = \frac{S(a)-S(b)}{S(a)\rho(a)-S(b)\rho(b)}$$

$$\Leftrightarrow \rho(a)\eta = \frac{S(a)-S(b)}{S(a)-S(b)\left(\frac{\rho(b)}{\rho(a)}\right)} = \frac{\frac{S(a)}{S(b)}-1}{\frac{S(a)}{S(b)}-\left(\frac{\rho(b)}{\rho(a)}\right)} = \frac{e^{\eta(b-a)}-1}{e^{\eta(b-a)}-\left(\frac{\rho(b)}{\rho(a)}\right)}$$

$$\Leftrightarrow \psi(\rho(a), b-a, \frac{\rho(b)}{\rho(a)}; \eta) = 0$$

Note too that since $\eta > 0$:

$$a + \rho(a) < b + \rho(b) \Leftrightarrow 1 - \frac{b-a}{\rho(a)} < \frac{\rho(b)}{\rho(a)}$$

In [2], a survivorship model whose hazard is a step function is quite naturally described as a *gauntlet* survivorship model. The main result of this note is that any collection of life expectations that is finite and satisfies the above inequality can be approximated by a gauntlet survivorship model. In fact, the associated gauntlet is essentially a canonical form hazard approximation and the Appendix provides a computer algorithm for determining it.

Theorem: Given an ordered sequence of pairs of real numbers $\{(a_i, \alpha_i) \mid 1 \le i \le n\}$ such that:

i) $0 = a_1 < \ldots < a_i < a_{i+1} < \ldots < a_n$

ii) $\alpha_i > 0, 1 \le i \le n$

iii) $1 - \frac{a_{i+1}-a_i}{\alpha_i} \le \frac{\alpha_{i+1}}{\alpha_i}, 1 \le i \le n$

And with the function ψ as in the lemma, define the step function $h : \Re_+ \to \Re_+$ as follows:

$$h(t) = \frac{1}{\alpha_n}, \quad t \ge a_n$$

$$h(t) = \begin{cases} 0 & 1 - \frac{a_{i+1}-a_i}{\alpha_i} = \frac{\alpha_{i+1}}{\alpha_i} \\ \psi^{-1}(\alpha_i, a_{i+1}-a_i, \frac{\alpha_{i+1}}{\alpha_i}; \{0\}) & 1 - \frac{a_{i+1}-a_i}{\alpha_i} < \frac{\alpha_{i+1}}{\alpha_i} \end{cases} \quad a_i \le t < a_{i+1}, \quad 1 \le i < n$$

Then the survivorship model determined by the hazard function $h(t)$ has expectation of life function $\rho(t)$ satisfying $\rho(a_i) = \alpha_i, 1 \le i \le n$.

Proof. The lemma guarantees that the function $h : \Re_+ \to \Re_+$ is well defined and the above example shows that:

$$\rho(t) = \frac{1}{\frac{1}{\alpha_n}} = \alpha_n, \quad t \geq a_n$$

The proof is by contradiction. Assuming the result false means that there is an $i<n$ such that:

$$\rho(a_j) = \alpha_j, i+1 \leq j \leq n \qquad \rho(a_i) \neq \alpha_i$$

Set

$$\beta = a_{i+1} - a_i, \quad \gamma = \frac{\alpha_{i+1}}{\alpha_i} = \frac{\rho(a_{i+1})}{\alpha_i}, \quad h(t) \equiv \eta \text{ on } [a_i, a_{i+1})$$

Suppose first that

$$1 - \frac{a_{i+1} - a_i}{\alpha_i} < \frac{\alpha_{i+1}}{\alpha_i}$$

By definition of h, this implies that

$$\psi(\alpha_i, \beta, \gamma; \eta) = 0$$
$$\Rightarrow \eta\alpha_i = \frac{e^{\beta\eta} - 1}{e^{\beta\eta} - \gamma}$$

The comments just proceeding the statement of the theorem applied to the hazard function $h(t) \equiv \eta$ on $[a_i, a_{i+1})$ show that:

$$\psi(\rho(a_i), \beta, \frac{\rho(a_{i+1})}{\rho(a_i)}; \eta) = 0$$
$$\Rightarrow \eta\rho(a_i) = \frac{e^{\beta\eta} - 1}{e^{\beta\eta} - \frac{\rho(a_{i+1})}{\rho(a_i)}} = \frac{e^{\beta\eta} - 1}{e^{\beta\eta} - \gamma\left(\frac{\alpha_i}{\rho(a_i)}\right)}$$

It follows that:

$$\eta\rho(a_i)\left(e^{\beta\eta} - \gamma\left(\frac{\alpha_i}{\rho(a_i)}\right)\right) = e^{\beta\eta} - 1 = \eta\alpha_i\left(e^{\beta\eta} - \gamma\right)$$
$$\Rightarrow \rho(a_i)e^{\beta\eta} - \alpha_i\gamma = \alpha_i e^{\beta\eta} - \alpha_i\gamma$$
$$\Rightarrow \rho(a_i)e^{\beta\eta} = \alpha_i e^{\beta\eta} \Rightarrow \rho(a_i) = \alpha_i$$

which contradicts the choice of i. We must therefore have:

$$1-\frac{a_{i+1}-a_i}{\alpha_i}=\frac{\alpha_{i+1}}{\alpha_i}$$
$$\Leftrightarrow \alpha_i-(a_{i+1}-a_i)=\alpha_{i+1}$$
$$\Leftrightarrow \alpha_i=a_{i+1}-a_i+\alpha_{i+1}$$

However, from our earlier observations on the hazard function $h(t)\equiv 0$ on $[a_i,a_{i+1})$, in this event:

$$S(a_i)=S(a_{i+1})$$
$$\Rightarrow a_i+\rho(a_i)=a_{i+1}+\rho(a_{i+1})=a_{i+1}+\alpha_{i+1}$$
$$\Rightarrow \rho(a_i)=a_{i+1}+\alpha_{i+1}-a_i=\alpha_i$$

This contradiction completes the proof of the theorem.

Remark: Compare the definition

$$h(t)=\frac{1}{\alpha_n},\quad t\geq a_n$$

of the Theorem with Proposition 2 (iv).

Remark: The discussion in [5; pp148-156] points out some shortcomings in the state of the art as regards the application of bivariate loss distributions. In [4] the survival model structure is generalized to higher dimensions using the concept of a hazard vector field $\eta:\Im\to\Im$ and its associated survival vector field $\rho:\Im\to\Im$, using the notation of that paper. Among the observations in that paper is the relationship:

$$b\in\Im_a\Rightarrow b+\rho(b)\in\Im_{a+\rho(a)}$$

Given any assignment of survival vectors to a finite discrete rectangular lattice $L\subset\Im$ that satisfies this consistency condition, the methods derived here can be applied to determine a "gauntlet" hazard vector field whose associated survival vector field coincides with the original assignment of survival vectors on L. Indeed, the primary motivation for this note was to seek a way of determining hazard that was amenable to vector arithmetic.

References

[1] Allison, Paul D., *Survival Analysis Using the SAS® System: A Practical Guide,* The SAS Institute, Inc., 1995.

[2] Corro, Dan, *Determining the Change in Mean Duration Due to a Shift in the Hazard Rate Function*, CAS Forum, Winter 2001.

[3] Corro, Dan, *Modeling Loss Development with Micro Data*, CAS Forum, Fall 2000.

[4] Corro, Dan, *Modeling Multi-Dimensional Survival with Hazard Vector Fields*, CAS Forum, Winter 2001.

[5] Klugman, Stuart A., Panjer, Harry H., Gordon E. Willmot, *Loss Models,* Wiley Series in Probability and Statistics, John Wiley & Sons, Inc.,1998..

[6] Wang, Shaun, *"Implementation of Proportional Hazards Transforms in Ratemaking"*, PCAS LXXXV, pp. 940-979.

APPENDIX

The SAS LOG includes both source code and annotations of a sample run whose output is in the SAS LISTING that follows the log. The SAS syntax is readily adapted to any programming context that supports conditional loop processing.

```
SAS LOG:
 NOTE: The initialization phase used 0.07 CPU seconds and 6068K.
 1          *************************************************************;
 2          ***        INVERSTING MEAN FAILURE TIME    ***********************;
 3          *************************************************************;
 4          OPTIONS MPRINT LS=131 PS=59 NOCENTER;
 5          TITLE 'INVERTING MEAN FALURE TIME';
 6          DATA ONE;
 7          INPUT A ALPHA;
 8          CARDS;

 NOTE: The data set WORK.ONE has 6 observations and 2 variables.
 NOTE: The DATA statement used 0.01 CPU seconds and 6952K.

 15         ;
 16         PROC SORT  DATA=ONE;
 17         BY DESCENDING A;

  NOTE: HOST sort chosen, but SAS sort recommended.
 NOTE: There were 6 observations read from the dataset WORK.ONE.
 NOTE: The data set WORK.ONE has 6 observations and 2 variables.
 NOTE: The PROCEDURE SORT used 0.11 CPU seconds and 7044K.

 18         DATA ONE;
 19         SET ONE;
 20         KEEP A ALPHA BETA GAMMA ERROR;
 21         BETA = LAG(A) - A;
 22         GAMMA = LAG(ALPHA)/ALPHA;
 23         IF GAMMA < 1 - (ALPHA/BETA) THEN ERROR = 1;
 24                                          ELSE ERROR = 0;

 NOTE: Missing values were generated as a result of performing an operation on missing
values.
      Each place is given by: (Number of times) at (Line):(Column).
      1 at 21:15   1 at 22:19   1 at 23:14   1 at 23:22
 NOTE: There were 6 observations read from the dataset WORK.ONE.
 NOTE: The data set WORK.ONE has 6 observations and 5 variables.
 NOTE: The DATA statement used 0.01 CPU seconds and 7044K.

 25         PROC SORT  DATA=ONE;
 26         BY A;

 NOTE: HOST sort chosen, but SAS sort recommended.
 NOTE: There were 6 observations read from the dataset WORK.ONE.
 NOTE: The data set WORK.ONE has 6 observations and 5 variables.
 NOTE: The PROCEDURE SORT used 0.02 CPU seconds and 7044K.
```

```
DATA ONE;SET ONE;
KEEP A ALPHA ETA ERROR;
IF BETA = . THEN DO;
   ETA = 1/ALPHA;
   END;
ELSE IF (ABS(GAMMA - 1 + (ALPHA/BETA)) < 0.00005) THEN DO;*TOLERANCE;
   ETA = 0;
   END;
ELSE DO;
   IF (ABS(GAMMA - 1) < 0.00005) THEN DO;*TOLERANCE;
      ETA = 1/ALPHA;
      END;
   ELSE DO;
      IF GAMMA < 1 THEN DO;
         TEMP = (BETA*(1 - GAMMA))/(2*ALPHA);
         LHS  = LOG(GAMMA + TEMP + SQRT(TEMP*(TEMP + 2*GAMMA)))/BETA;
         RHS  = 1/ALPHA;
         END;
      ELSE DO;
         LHS  = LOG(GAMMA)/BETA;
         TEMP = 3*GAMMA + EXP((2*BETA)/ALPHA);
         RHS  = LOG(TEMP)/BETA;
         END;
     ETA  = (RHS + LHS)/2;
     DO WHILE (RHS - LHS > 0.00005);*ADJUST TO DESIRED TOLERANCE;
        TEMP = EXP(BETA*ETA);
        PSI_ETA = ALPHA*ETA - (TEMP - 1)/(TEMP - GAMMA);
        IF PSI_ETA > 0 THEN RHS = ETA;
                       ELSE LHS = ETA;
        ETA  = (RHS + LHS)/2;
        END;
     END;
  END;

NOTE: There were 6 observations read from the dataset WORK.ONE.
NOTE: The data set WORK.ONE has 6 observations and 4 variables.
NOTE: The DATA statement used 0.03 CPU seconds and 7054K.

PROC PRINT DATA=ONE;
NOTE: There were 6 observations read from the dataset WORK.ONE.
NOTE: The PROCEDURE PRINT printed page 1.
NOTE: The PROCEDURE PRINT used 0.02 CPU seconds and 8062K.

NOTE: The SAS session used 0.30 CPU seconds and 8062K.
NOTE: SAS Institute Inc., SAS Campus Drive, Cary, NC USA 27513-2414
```

SAS LISTING:

INVERTING MEAN FALURE TIME

Obs	A	ALPHA	ERROR	ETA
1	0	9.0	0	0.16227
2	1	9.5	0	0.05405
3	2	9.0	0	0.05712
4	3	8.5	0	0.06059
5	4	8.0	0	0.06451
6	5	7.5	0	0.13333

Exposure Rating Loss Layers: Unifying the Property Perspective of Severity with the Liability Perspective of Frequency

Jonathan Evans, FCAS, MAAA

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Abstract

For problems such as rating excess of loss reinsurance and estimating deductible credits, actuaries frequently employ exposure rating factors. In the context of property insurance this takes the form of loss tables such as the Lloyds scale or Salzmann tables. These tables display the fraction of loss cost retained for layers expressed as fractions of insured value, or policy limit. In the liability insurance context, Increased Limits Factors (ILFs) or Excess Loss Factors (ELFs) tables are expressed in terms of actual dollar amounts for attachment points and limits. Implicit in the property tables is the assumption that an increase in policy limit or insured value corresponds to a proportional scale factor increase in the claim severity random variable, but other than the change in scale the distribution of claim sizes remains the same and any increase or decrease in loss cost per exposure is frequency based. Without a special adjustment to the loss cost or premium rate, the implied loss frequency is the same for the larger policy. Implicit in the liability tables is the assumption that larger policies produce the same distribution of claim severity. In summary, the property perspective generally assumes that all the extra exposure shows up as larger claims, and the liability perspective generally assumes that all the extra exposure shows up as more claims. This paper shows how both perspectives for claim severity, and additional considerations of frequency changes may easily be incorporated into a unified model. Additionally, such a unified approach allows for a compromise where increasing exposure for a given policy or risk may be partially reflected in the scale of claim size and partially in the frequency.

A Generic Example of Property Exposure Rating of a Loss Layer

A typical property exposure rating scale might look like:

Fraction of Policy Limit	Fraction of Retained Loss Cost
200%	100%
100%	99%
90%	98%
75%	95%
50%	80%
10%	40%
5%	25%
0%	0%

Note: Losses in excess of the main policy limit occur due to multiple coverage limits, such as personal property and business interruption, or extra contractual obligations, etc.

Suppose an actuary is reviewing two new property risks for reinsurance cost. One risk is a small store covered by a business owners policy (BOP), with a property limit of \$300,000. The other is a large industrial warehouse structure with extensive sprinklers and other loss control devices, which is covered by a general commercial fire policy valued at \$2 million. The actuary's company has a property per risk reinsurance treaty for its BOP exposures which covers losses of \$850,000 excess of \$150,000. The company also requires that facultative reinsurance certificates be purchased for all property risks in excess of \$1 million.

To estimate the loss cost ceded to the BOP per risk treaty, for the newly insured store, an actuary performs the following exposure rating analysis. The attachment point for the treaty is 50% of the policy limit. This means that the company expects to retain 80% of ground up expected losses (due to the first \$150,000 retained layer). The reinsurance limit plus attachment point of the treaty exceeds the maximum loss level of 200% of policy limit. So the reinsurance layer and primary layer together cover 100% of the loss cost. The expected percentage of losses ceded to the reinsurance layer is 100% - 80% = 20%. The company premium rate for BOP policies is \$2 per \$1,000 of limit. Ignoring expense adjustments and ceding commissions, \$120 of the \$600 of direct premium are ceded to the treaty.

Now consider the case of the facultative coverage on the warehouse. Since \$1 million is also 50% of the limit for the warehouse, the actuary gets the same cession percentage of 20%. If the base rate is the same, \$800 of the \$4,000 of direct premium on the warehouse will be ceded to the facultative certificate.

Is this reasonable? Probably not. Whereas a fire or other peril might easily destroy the store, it is unlikely that the entire warehouse would be destroyed in a single event. If the reduced loss cost per exposure unit for the larger building is reflected entirely in the rate, this is equivalent to reducing the frequency. This is also probably not reasonable. The warehouse likely has constant movement of stock by small vehicles and cranes. It probably experiences more frequent small to medium size losses.

A Generic Example of Liability Exposure Rating of a Loss Layer

A typical table of liability increased limits factors might look like:

Occurence Limit	Increased Limits Factor
50,000,000	6.125
10,000,000	3.625
5,000,000	3.000
2,000,000	2.250
500,000	1.500
200,000	1.200
100,000	1.000

Suppose an actuary is reviewing two new general liability policies for reinsurance cost. One policy covers a small 1,500 square foot "mom and pop" corner store with $200,000 of sales per year. The other covers a 150,000 square foot discount retail superstore with $20,000,000 of sales per year. Each policy has an occurrence limit of $2 million dollars.

The actuary's company has a reinsurance treaty covering occurrence losses of $1.8 million excess of $200k. From the table above we can see that the rate for $2 million limits is 2.25 times the base rate and the rate for $200,000 is 1.2 times the base rate. So the ceded rate for the reinsurance layer should be 2.25 – 1.2 = 1.05 times the base rate. If the company has a base rate of $1 per $1,000 of sales, then the small store policy should cede $210 of the $450 of direct premium, and the superstore policy should cede $21,000 of the $45,000 of direct premium.

Is this reasonable? Probably not. The larger store will almost certainly experience a higher frequency of claims. However, it is also very likely to experience larger claims, i.e. a different severity distribution. Potential plaintiffs and their lawyers will probably view the larger store as a deep pockets defendant. As such, they will be more willing to pursue larger claims, and less likely to settle for smaller amounts. Juries are also more willing to award larger claims against such a defendant.

A Unified Model

Assume that loss cost per exposure is constant for policies with different magnitudes of exposure. Let E1, S1, and F1 be the exposure, average severity, and average frequency for a policy. Similarly E2, S2, and F2 are the same parameters for a larger risk of like kind. The property perspective is:

S2 = S1 x (E2 / E1) and F2 = F1.

The liability perspective is:

S2 = S1 and F2 = F1 x (E2 / E1).

Now introduce a new parameter, A, and suppose that:

S2 = S1 x (E2/E1) ^ A , and

F2 = F1 x (E2/E1) ^ (1 – A) .

Notice that the property perspective corresponds to $A = 1$, and the liability perspective corresponds to $A = 0$. Values for A between 0 and 1 represent a compromise between these two perspectives.

Now, suppose that we relax the assumption that loss costs per exposure are constant for policies with different magnitudes of exposure. We can do this by introducing a second parameter, B, and restating our equations as:

$S2 = S1 \times (E2/E1) \wedge A$, and

$F2 = F1 \times (E2/E1) \wedge B$

Note that $A + B$ is not necessarily equal to 1. When $A + B = 1$, loss costs per exposure are constant and
$B = 1 \quad A$.

If L1 and L2 are the expected losses for each policy, then:

$L2 = L1 \times (E2/E1) \wedge (A + B)$.

Generic Examples of Exposure Rating a Loss Layer Using the Unified Model

First, we reconsider the property example. Instead of $A = 1$ and $B = 0$, we believe $A = 0.8$ and $B = 0.1$ are more appropriate values. Thus:

$E2 / E1 =$ \$2 million / \$300k $= 6.67$,

$S2 / S1 = (6.67) \wedge .8 = 4.56$, and

$F2 / F1 = (6.67) \wedge .1 = 1.21$.

Assume our calculation for the BOP policy on the small store was correct, but the calculation for the warehouse policy must be modified. We need to adjust the loss scale table by multiplying the percentages of policy limit by the factor $4.56 / 6.67 = 0.68$.

This produces an adjusted table of:

Fraction of Policy Limit	Fraction of Retained Loss Cost
135%	100%
68%	99%
61%	98%
51%	95%
34%	80%
7%	40%
3%	25%
0%	0%

This new table suggests a premium cession rate of only 5%. Now consider the situation for total expected losses. Since (6.67) ^ (0.8 + 0.1) = 5.52, we should adjust our direct premium by a factor of 5.52 / 6.67 = 0.83. So the direct premium should be $3,320 instead of $4,000, and the ceded premium should be $166 instead of $800. Notice that in this case our rate for the policy holder has dropped 17%, and almost all of this lower rate is compensated for by decreased reinsurance costs!

Now we reconsider the liability example. Instead of A = 0 and B = 1, we believe A = 0.15 and B = 0.9 are more appropriate values. Thus,

E2 / E1 = $20 million / $200k = 100, and

S2 / S1 = (100) ^ 0.15 = 2.00, and

F2 / F1 = (100) ^ 0.9 = 63.10 .

Assume our calculation for the general liability policy on the corner store was correct, but the calculation for the superstore policy must be modified. We should adjust the increased limits factor table by multiplying the occurrence limits by the factor 2.00 from above.

This produces an adjusted table of:

Occurence Limit	Increased Limits Factor
100,000,000	6.125
20,000,000	3.625
10,000,000	3.000
4,000,000	2.250
1,000,000	1.500
400,000	1.200
200,000	1.000

We can interpolate (using Factor1 + (Z ^ ½) x (Factor2 – Factor1), where Z = ($2,000,000 - $1,000,000)/($4,000,000-$1,000,000), and the square root interpolation is just a rough estimate.) to get a factor of 1.930 for a $2 million occurrence limit. Now our cession rate should be:

0.93 / 1.93 = 48.2 %.

Whereas before our cession rate was:

$21,000/$45,000 = 46.7 %.

Our total expected losses should be adjusted by a factor of :

100 ^ (0.9 + 0.15) = 125.9 .

So the direct premium should be $56,655 instead of $45,000, and the ceded premium should be $27,308 instead of $21,000.

Estimating Parameters

Taking logarithms allows the equations for frequency and severity to be restated in a linear form:

$$\text{Ln}(S2) - \text{Ln}(S1) = A\,(\text{Ln}(E2) - \text{Ln}(E1))$$

$$\text{Ln}(F2) - \text{Ln}(F1) = B\,(\text{Ln}(E2) - \text{Ln}(E1))$$

Data may be collected for both claim severity and frequency by exposure size of policy. A and B can then be estimated as the slope estimates from regressions of the logarithm of claim severity and the logarithm of claim frequency, respectively, against the logarithm of exposure by policy. This also automatically generates the scaling factor for expected losses per unit of exposure for a policy as A + B, without any other special data analysis.

Conclusion

Both the standard property and liability methods of exposure rating loss layers correspond to special cases of a more general exposure rating method. The difference is whether higher exposure for a policy is assumed to reflect increased claim severity, as in the property case, or increased claim frequency, as in the liability case. The parameters of the general method encompass additional cases, which may more accurately fit actual loss exposure for different layers of losses. Estimation of the parameters is easily accomplished by regression of logarithms of historical data for claim severity, claim frequency, and exposure by policy. An additional benefit is that once these parameters are estimated they also reflect an estimate of the way in which expected losses per exposure change for policies of different exposure sizes. The parameters may be used in a fairly straightforward way to adjust ILF's, ELF's, loss scales, and rates per exposure for different policies by exposure size.

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Pitfalls in the Probability of Ruin Type Risk Management

Jonathan Evans, FCAS, MAAA

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Abstract

Funding levels for many insurance and financial risk entities are often set to achieve a certain low probability of ruin. Specific real world examples which utilize the same essential methodology include: funding self insurance at a certain percentile of aggregate losses, Value at Risk (VAR) funding of investment banks, return period or PML funding of property catastrophe exposures, and probability of ruin through stochastic modeling commonly used in Europe as in Daykin (1994). We use the concepts of probability of ruin, return period, and percentile interchangeably in this paper. Butsic (1992) has pointed out that these analyses neglect to consider the severity of insolvency. This paper addresses a somewhat related issue. Probability of ruin may often be inconsistent with many other reasonable risk management criteria. For example, combining two independent risks may produce a required funding level at a 1% probability of ruin which is actually higher than the sum of the separate 1% probability of ruin funding levels for each of the risks. Use of this criterion for risk management may lead to the nonsensical result of discouraging risk sharing between independent risks. We examine several examples of this phenomenon and how it may lead to undesirable risk management strategies.

Homeowners Insurance, a Trivial Real World Example

A single house generally has a 90^{th} percentile loss in a given year of 0. However, a portfolio of 1,000,000 houses will invariably have a 90^{th} percentile loss in a given year much greater than 0. So the 90^{th} percentile of the combined risks is greater than the sum of the 90^{th} percentiles of the separate risks. If a homeowner wishes to minimize his 90^{th} percentile loss (or perhaps even 99^{th} percentile loss) he should buy no insurance at all, since the premium itself guarantees a 90^{th} percentile loss greater than 0. Equivalently, a large insurance group should form a separate member company for each policy, so as to keep the 90^{th} percentile losses at 0.

We can find trivial examples of this phenomenon at arbitrarily high percentiles less than 100%, or equivalently arbitrarily small probabilities of ruin greater than 0% (See Appendix – Theorem 1).

How Can This Happen ?

Many people are stunned by this result. They are properly taught to think of pooling or sharing of risk as a way of reducing or managing risk. This is always true if risk is measured by standard deviation. Two separate risks, whatever their correlation, will always have a total standard deviation less than or equal to the sum of their separate standard deviations. However, certain percentile type measurements may be greater for a combination than the sum of the separate parts, even for very high percentiles. It is important to note that the Normal distribution does not exhibit this phenomenon (See Appendix – Theorem 2).

A Symmetric Example

This phenomenon is not just a characteristic of skewed distributions. It can also happen for some symmetric distributions. Consider the sum or convolution, X1 + X2 of two identical and independent copies of the random variables X, as follows:

X	Probability	X1 + X2	Probability
1	20%	2	4%
0	60%	1	24%
-1	20%	0	44%
		-1	24%
		-2	4%

The 75th percentile of X1 and X2 separately is clearly 0, but the 75th percentile of X1 + X2 is 1.

Lognormal Example

It can also happen for smooth continuous distributions with only one local maximum. For an example using continuous loss distributions, consider two independent risks with simulated (65,000 iterations) lognormal distributions X1 and X2:

		X1	X2	X1 + X 2	Difference Between Percentile of Sum and Sum of Percentiles
	mean	100,000	300,000	400,000	
	CV	300%	200%	168%	
	sigma	1.51743	1.26864	NA	
	mu	10.36163	11.80682	NA	
Percentiles	99%	1,085,317	2,550,976	2,896,718	-739,575
	95%	389,469	1,066,725	1,313,245	-142,950
	90%	225,804	678,361	871,090	-33,076
	85%	154,968	499,047	664,510	10,495
	80%	113,881	388,787	537,483	34,815
	75%	88,751	315,419	445,164	40,994
	70%	70,837	260,696	379,135	47,602
	65%	57,266	218,513	326,772	50,994
	60%	46,875	185,408	284,290	52,007
	55%	38,724	157,213	247,084	51,147
	52%	34,419	143,000	228,476	51,057

Although at the 90th percentile we see a combined percentile less than the sum of the separate percentiles, as high as the 85th percentile the combined value is larger.

Self Insured Workers Compensation, Frequency/Severity Example

We can extend the lognormal example to a real world frequency/severity process. Consider two large factories whose workers compensation risks are independent. The factories are considering pooling their self insured workers compensation. State law requires that self insured workers compensation be funded at the 75^{th} percentile of gross loss before required per occurrence excess coverage. Let Factory 1 have a claim severity distribution equal to the first lognormal from the previous example and a Poisson claim frequency distribution with a mean of 2.1. Let Factory 2 have the second lognormal from the previous example for its severity and a Poisson claim frequency with mean of 1.2. A typical result from 65,000 simulations is:

		Factory 1	Factory 2	Factory 1 + Factory2	Difference Between Percentile of Sum and Sum of Percentiles
	Poisson Frequency	2.1	1.2	3.3	
	Severity mean	100,000	300,000	172,727	
	Severity CV	300%	200%	257.2%	
	sigma	1.51743	1.26864	NA	
	mu	10.36163	11.80682	NA	
Aggregate Loss Percentiles	99%	1,822,088	3,226,341	3,804,187	-1,244,242
	95%	786,388	1,465,536	1,906,977	-344,947
	90%	499,407	947,544	1,322,473	-124,478
	85%	368,077	693,410	1,035,558	-25,929
	80%	286,534	533,505	842,276	22,236
	75%	230,766	417,824	700,176	51,585
	70%	187,598	330,133	592,035	74,304
	65%	154,029	260,452	503,291	88,810
	60%	126,295	205,706	429,314	97,313
	55%	104,266	159,913	367,566	103,387
	52%	92,662	135,263	334,483	106,558

The factories choose not to pool their risk, since doing so would require a net additional contribution of $51,585 to their self insurance fund, even though the higher percentiles for the pooled risk are much less than the sum of the parts.

Property Catastrophe Example

The phenomenon can also happen with portfolios of property catastrophe exposures. Consider two such portfolios. One is for risks exposed to California earthquakes and the other is exposed to Atlantic Hurricanes. Catastrophe modelers typically calculate Poisson frequencies for loss events of different sizes. These events are sorted in descending order and frequencies are accumulated to give a Poisson frequency of an event of a given size or greater. The return period of these losses is defined as the inverse of this cumulative frequency. Portfolios are then evaluated by the size of loss for a given return period, or "PML". Since these two perils are independent and Poisson we can add the separate frequencies to get frequencies for a combined Poisson distributed portfolio.

	Incremental Frequency			Cumulative Frequency at Level and Above			Approximate Return Periods		
Size of Loss Event	Atlantic Hurricane	California Earthquake	Combined	Atlantic Hurricane	California Earthquake	Combined	Atlantic Hurricane	California Earthquake	Combined
100,000,000	0.0100	0.0100	0.0200	0.0100	0.0100	0.0200	100	100	50
60,000,000	0.0100	0.0100	0.0200	0.0200	0.0200	0.0400	50	50	25
20,000,000	0.0200	0.0200	0.0400	0.0400	0.0400	0.0800	25	25	13
10,000,000	0.0600	0.0600	0.1200	0.1000	0.1000	0.2000	10	10	5

Although there are differences between the meanings of frequency and the probability of one or more events in a year, for low frequencies these numbers are essentially the same. So a 100 year return period event has approximately a 1% probability of occuring one or more times in a year. By combining the portfolio we get a 25 year return period loss which is greater than the sum of the 2 separate 25 year loss events. However at the 50 year return period we get a combined loss less than the sum of the separate losses. If credit rating agencies, catastrophe reinsurers, and regulators evaluate companies based on the 25 year return period it does not make sense to combine these risks.

A Related Example: "The Reinsurance Broker's Gimmick"

A reinsurance salesman may propose the following scheme:

"Randomly select half of your property catastrophe policies. Cede 100% of these. You will be ceding half of your premiums and losses, but my assistant – a world renowned statistician and catastrophe management expert - will show you that your 100 year PML will decrease by 60% or more. This is an excellent, cost effective way to manage your Cat risk."

Policies spread throughout Florida or California overall may have a low average correlation for a given hurricane or earthquake event. This is because a given event in either state is relatively localized inside of the state. When viewed from the perspective of two randomly split portfolios recombined this situation may exhibit a similar pattern to the previous example which used a Florida portfolio and a California portfolio. So in exchange for 50% of the premium the 100 year loss may come down by 60% or more, but what the salesman and his brilliant assistant neglect to mention is that the 200 year loss may come down by only 40% or less.

Stochastic Simulation Example

A European investor spends 200 million German Marks to capitalize an insurance company to underwrite maintenance, warranty, and recall insurance for a large European auto manufacturer over a 5 year period. Expected annual losses for routine claims are 1 billion German Marks, with a coefficient of variation of 10%. Investment income exactly offsets underwriting expenses, the risk load is 5% (reduced to 4% after the first year) of routine expected losses, premium is collected and losses are paid annually, and only autos sold and owned in Europe are covered. The investor runs into difficulty after an actuary working for European Union officials models 10,000 stochastic simulations of the company with a Gamma distribution (Billions of Marks are Gamma distributed with Alpha =100, Beta = 0.01) for routine annual claims and a 1% chance in any year that there will be a large model recall costing 2 billion Marks. The actuary discovers that the company has a 12.1% chance of bankruptcy over the course of its 5 year operation. European Union officials state that the absolute maximum probability of bankruptcy they will accept is 10%.

Fortunately, the European investor has a cousin who works as an investment banker on Wall Street in New York and is quite expert at engineering financial derivatives. The cousin proposes to offer annual aggregate loss reinsurance coverage for 400 million Marks xs 1,100 million Marks. In exchange the investor will cede 2.4% of premium and agree to assume the costs for North American owned autos also in the event of a recall, which his cousin had previously agreed to insure. The cost of the North American autos covered in the event of a recall will be another 2 billion Marks. When the actuary adjusts his model for the new reinsurance derivative, he generates a ruin probability of 9.4% The officials concede and the deal is finalized. Some key simulation results are:

Percentiles	Liquidation Loss Before Financial Engineering Deal	Net Liquidation Loss After Financial Engineering Deal
99%	-1.779	-3.828
95%	-0.162	-0.053
90%	-0.020	0.000
80%	0.000	0.000
70%	0.000	0.000

Sample Simulation (Billions of Marks):

	Year 1	Year 2	Year 3	Year 4	Year 5
Beginning Surplus	0.200	0.107	0.059	0.135	0.203
Premium	1.050	1.040	1.040	1.040	1.040
Losses	1.143	1.088	0.964	0.972	0.999
Cat Loss	0	0	0	0	0
Ending Surplus	0.107	0.059	0.135	0.203	0.244
Liquidation Loss Before Financial Engineering Deal	0.000				
Net Beginning Surplus	0.200	0.125	0.052	0.103	0.146
Ceded Premium	0.025	0.025	0.025	0.025	0.025
Ceded Losses	0.043	0.000	0.000	0.000	0.000
Assumed Cat Loss	0	0	0	0	0
Net Ending Surplus	0.125	0.052	0.103	0.146	0.162
Net Liquidation Loss After Financial Engineering Deal	0.000				

What the officials did not consider was that the expected policyholder deficit or expected value of insolvency, which the actuary's model generated, was 84 million Marks before the reinsurance derivative and 167 million Marks after the reinsurance derivative. This is the expected cost to the auto manufacturer (or government guarantor) due to the insurer's default. The default cost has doubled because even though the probability of default has decreased modestly the average cost of default has risen dramatically.

Probability of ruin simulations and analyses, which do not include other risk measurements, are particularly likely to miss the dangers of exotic reinsurance agreements or financial derivatives. With the growing use of Value at Risk (VAR) by investment bankers to analyze derivatives this danger may also be present in banking. A somewhat mitigating factor is that many VAR calculations estimate a variance and then use a Normal distribution to get a percentile. The Normal distribution is not in itself vulnerable to this inconsistency with regard to percentiles versus standard deviations (See Appendix – Theorem 2). The Normal distribution is also generally not vulnerable to inconsistencies between percentile type measures and expected policyholder deficit type measures, see Butsic (1992).

Two Possible Defenses of Probability of Ruin Type Methods

There is a strong case for a minimum probability threshold for risk management. A reasonable value judgement may be that events which have less than a 1 in 1,000,000 chance of happening should simply be ignored. It may be ridiculous for routine decisions to be based on worst possible outcomes. Similarly, perhaps some people would say that we can allow the 1 in 50 chance event to be worse in exchange for lowering the 1 in 25 chance event.

A second related argument arises if real world entities such as regulators, reinsurance markets, or credit rating agencies are fixed on a certain percentile level for things like pricing reinsurance, setting capital requirements, and assigning credit ratings. If this is the case, then a risk manager or insurance executive may still find the optimal strategy to be based on percentile type measures. That is to say, a single player in the market place may not be wise to ignore existing standards.

Possible Solution

Fixing either a certain tolerable probability of ruin or minimizing the probability of a loss of a certain magnitude allows for undesirable results, primarily because it ignores other levels of probability or time horizon. The optimal strategy may change dramatically for different levels of probability or time horizon. Standard Deviation considers all levels of probability but may give unreasonable weight to large rare events. A possible compromise is to introduce another measure which covers many or all levels of probability/loss size. For example, a utility function with decreasing weight for less probable levels of loss could be used to weight the magnitude of ruin at various levels of probability.

Conclusion

Probability of ruin type calculations are pervasive throughout insurance and finance. However, their use as a standard for setting risk based capital requirements or as a selection criterion for comparing different risk management strategies may lead to nonsensical and undesirable consequences. In some cases this is obvious, such as when it implies that homeowners should not buy any insurance since doing so would increase their 90^{th} percentile losses. Other cases are more subtle, such as the case where randomly ceding half of a portfolio of catastrophe exposed property risks reduces a 100 year PML by more than half, even though this reduces the 250 year and higher PMLs by less than half. Any application of probability of ruin type methods to risk management should be accompanied by consideration of alternative measurements of risk.

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Appendix

Theorem 1

For any "percentile" f such that $0 < f < 1$ there exist 2 independent nonnegative random variables such that the f percentile of their sum is greater than the sum of the f percentiles of each of the random variables.

Proof ("Flipping 2 weighted Coins"):

Let X1 be a random variable with a probability of being 0 equal to f and a probability of being 1 equal to $1 - f$. Let X2 be a random variable identical to and independent of X1.

The f percentiles of X1 and X2 are both equal to 0.

Prob(X1 + X2 > 0) =
Prob (X1 > 0 OR X2 > 0) =
Prob(X1 > 0) + Prob(X2 > 0) – Prob(X1 > 0 AND X2 >0).

Independence implies

Prob(X1 > 0 AND X2 > 0) = Prob(X1 > 0) *Prob(X2 > 0).

So, Prob(X1 + X2 > 0) = 2 * (1 - f) – (1 - f)^2

Since $0 < f < 1$ we also know $0 < 1- f < 1$

Therefore (1 – f) ^ 2 < 1 - f and 2 * (1-f) – (1-f)^2 > 1 - f

So the Prob(X1 + X2 > 0) > 1 – f

QED

Theorem 2

The Normal distribution does not demonstrate the phenomenon in Theorem 1.

Proof:

Consider two independent normal distributions:

X1 = Normal(Mean1, Sigma1) and
X2 = Normal(Mean2, Sigma2)

It immediately follows that

X1 + X2 = Normal(Mean1 + Mean2, SigmaTotal).

For any percentile there exists a unique constant k, such that for any normal distribution the value of that percentile is equal to mean + k Sigma. So we have the following percentile values:

Risk	Value at Percentile
X1	Mean1 + k Sigma1
X2	Mean2 + k Sigma2
X1 + X2	Mean1 + Mean2 + k SigmaTotal

SigmaTotal is always less than or equal to Sigma1 + Sigma2. For percentiles greater than the 50^{th}, $k > 0$. So for percentiles greater than the 50^{th} percentile the value of X1 + X2 is always less than or equal to the sum of the corresponding percentile values for X1 and X2.