

*Capital Adequacy and Allocation Using  
Dynamic Financial Analysis*

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# **Capital Adequacy and Allocation Using Dynamic Financial Analysis**

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## **Abstract**

This paper will discuss the use of a Dynamic Financial Analysis (DFA) model to assist a client company in determining the total capital required to support its underwriting activities, and the portion of that total required capital allocated to each operating division. It will discuss issues related to risk measures, capital adequacy standards, and allocation techniques. Most importantly, it will cover the presentation of findings to the Company's Board of Management.

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## 1. Introduction

Dynamic financial analysis or “DFA” models can help insurers with many critical strategic issues and decisions. Examples include:

- ❖ Assessing alternative reinsurance programs;
- ❖ Evaluating capital structure, adequacy and allocation;
- ❖ Determining optimal asset allocation; and
- ❖ Providing a more accurate base for allocation of corporate-level reinsurance costs or investment income to operating divisions.

This paper will discuss the use of a DFA model<sup>1</sup> to assist a client company (the “Company”) in determining the total capital required to support its underwriting activities, and the portion of that total required capital allocated to each operating division. Equally important, it will cover the presentation of findings to the Company’s Board of Management (the “Board”).

The first step in the DFA study was the parameterization of the DFA model for the Company. Their own reserve, planning, and investment information was used to fit loss distributions, expected payment patterns, premium levels, expenses, and reserve runoff distributions. Asset holdings and detailed representations of their reinsurance programs were also input. Once parameterized, the model generated thousands of iterations of Company results, producing as output distributions of the company results.

The next step was for the Company to decide on a risk measure (e.g., probability of ruin) and a standard for that risk measure (e.g., 1 in 100 years or 1%) for determining required capital. There is no industry consensus for risk measure. Therefore, many alternative risk measures were calculated using the detailed output distribution of company results, including: probability of ruin (either on a statutory or GAAP basis); variance or standard deviation of surplus; expected policyholder deficit; and expected annual default loss rate on surplus [6]. **Section 2** covers the evaluation of alternative risk measures, and the determination of required capital.

Given a total required capital amount, the next issue was allocation to the operating divisions. Conceptually, the desire was to allocate based on the relative contribution of each division to the overall risk of the company. Given a selected risk measure, this became an issue of determining each division’s contribution to the total risk measure value. This meant “decomposing” an overall risk measure based on some aggregate distribution for the whole company (e.g., probability of ruin as derived from the distribution of surplus) into the component contributions. Any attempt to decompose an aggregate distribution into its component distributions will quickly run into order dependency issues (see [5] and [9]). To overcome these issues, and arrive at as “fair” an allocation as possible, techniques from game theory were employed. **Section 3** details the allocation approach.

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<sup>1</sup> The model used is ARMS, American Re’s proprietary DFA model. Details of the ARMS system can be found in the **Appendix**.

After the technical analysis was completed, the initial presentation to the Board was prepared. The audience consisted of seasoned professionals with different backgrounds and varying familiarity levels with DFA and probability. The choices made as to what to present and how to present it form the basis of **Section 4**.

As a result of the initial presentation, the Board selected several of its members to take a deeper look into the DFA study. Each of these members met with the DFA study team for individual intensive reviews. These reviews are highlighted in **Section 5**.

Because the material was so new, and the study so exhaustive, a substantial presentation binder was also included, with an executive summary, graphs, financial exhibits, and extensive backup detail. The choice of binder material is discussed in **Section 6**.

## 2. Risk Measures and Required Capital

The choice of risk measure for capital determination is more complex than it may initially appear. There are many valid possibilities, each with its own strengths and weaknesses. The actuarial community has also not converged on a consensus “best” measure, adding to the confusion. To top it off, even if a risk measure is chosen, there is no consensus standard for the “correct” level—should required capital be pegged to a 1% probability of ruin? And over what time horizon—one year?<sup>2</sup>

The actuarial literature describes many viable measures of risks, including:

- ❖ Probability of Ruin
- ❖ Variance or Standard Deviation of Surplus
- ❖ Expected Policyholder Deficit
- ❖ Expected Default Loss Rate on Surplus

Each has its merits and weaknesses.

### *Probability of Ruin*

Probability of ruin (exhaustion of surplus) has several advantages. The concept is readily explainable to non-technical audiences (likelihood of bankruptcy). It is also easy to calculate using the distribution of policyholder surplus. It has support from regulators and rating agencies with their focus on company solvency and claims-paying ability. It also translates fairly well to a capital market framework, being roughly comparable to Value-at-Risk (VaR).

However, probability of ruin has weaknesses as well. It is essentially a binary measure (solvent/insolvent), ignoring what Philbrick calls “gradations of solvency” [10]. It also

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<sup>2</sup> This quandary is not limited to the actuarial and insurance communities. The very same dilemmas exist in capital market risk management—what Value-at-Risk (VaR) threshold should a company manage to, and over what time horizon?

implicitly associates “risk” with a single percentile of the surplus distribution. This can be problematic when considering the marginal impact of changes in the portfolio—changes that do not impact the selected percentile (e.g., 99<sup>th</sup>) have not “added any risk” according to this measure.

#### Variance or Standard Deviation of Surplus

Variance and standard deviation are well-known statistical parameters of distributions. They are well known within the capital market world through the work of Harry Markowitz [7]. They are also convenient as shorthand for characterizing the dispersion of a distribution in a single number.

However, they do not add much beyond probability of ruin<sup>3</sup>. They also can give a distorted notion of variability for skewed distributions.

#### Expected Policyholder Deficit

Expected Policyholder Deficit or “EPD” [2] provides a better indicator of safety for a large organization than probability of ruin, since the measure reflects the whole tail of the distribution rather than a single percentile. It also has rating agency support<sup>4</sup>.

EPD is however more complex to explain to non-technical audiences, and more difficult to calculate. It also uses expected loss as its “base,” expressing the target deficit as a percentage of expected loss. From the policyholder perspective (the original focus of EPD [2]), this is appropriate, since they are concerned with expected insurer “defaults” (deficits) as a percentage of their expected loss payments (their “asset”). However, from a capital adequacy perspective, expected loss may not be the most relevant base. Finally, EPD is difficult to translate to capital market risk measures, although it has a parallel in so-called “Conditional Value-at-Risk” [11].

#### Expected Default Loss Rate on Surplus

Expected Default Loss Rate on Surplus (EDLR), first proposed by Mango [6], takes the severity of ruin focus from EPD one step further by explicitly associating various default percentages with required risk premiums<sup>5</sup>. It also uses the deficit like EPD, but expresses it as a percentage of the surplus itself. This has the advantage of making capital market comparisons very straightforward—see [6]. This ease of comparability also makes explanation to non-technical audiences easy.

EDLR has the disadvantage of not being well known. Also, many are uncomfortable with its utility focus. Even though utility theory is a cornerstone of modern economics, its lack

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<sup>3</sup> In fact, if the functional form of the distribution is known, they add nothing. If the distance between the mean and a given percentile is known for the normal distribution, its variance and standard deviation are also known.

<sup>4</sup> For instance, A.M. Best associates certain Best’s Capital Adequacy Ratio (BCAR) values to EPD measures.

<sup>5</sup> The risk premium standards are based on the company utility profile. See Halliwell [3] for an excellent exposition on the insurance applications of utility theory.

of “units” or other real world ties causes concern among some users. For instance, how would one go about parameterizing one’s company utility curve?

Risk Measure Standards

Even if a risk measure is chosen, the battle is only half over. A standard must be selected for determination of required capital. This apparently straightforward question in fact has several difficult dimensions that must be considered:

- ❖ *On what basis should capital adequacy be assessed—economic, GAAP, statutory?*  
Probability of ruin for example is quite different on an economic versus accounting basis. Economic “ruin”—zero net present value of future payment streams—will be much harder to reach than accounting ruin. Also, a company with positive economic value can be insolvent on an accounting basis.
- ❖ *What is the “right” probability standard?*  
Should it be 1%, 0.4%, 0.1%? Companies face the same issue in catastrophe modeling when trying to define their “capacity” in a given geographical region, and set their reinsurance retention.
- ❖ *What is the “right” time horizon?*  
One year? Two years? Five years? As the time horizon increases, the spread of variability increases, which means the probability of ruin increases, but so does the forecast error.

Framing the Capital Adequacy Question for Presentation

There are really two questions a client can be asking regarding capital adequacy:

- ❖ What is the safety level of my current capital?
- ❖ What is my capital redundancy/(deficiency) for other safety levels?

The safety level of current capital was expressed using all the available risk measures. This effectively drove home the point that “required capital” is not yet a firm concept with a single, definitive value. It also made clear the effects of the differing focuses and assumptions underlying the various risk measures. Table 1 below shows an example of the Safety Level of Current Capital exhibit (all were done using the same time horizon—e.g., the distribution of surplus one year in the future):

**Table 1**  
**Example Safety Level of Current Capital Table**

Risk Measure	Level Implied by Current Capital
Probability of Ruin	1 in 200 years or 0.5%
EPD	1.2% of Expected Loss
EDLR	2% of Capital

For assessing how redundant or deficient the current capital is when compared against other target values of the risk measures, exhibits like Table 2 below were used (assume current capital = \$1,100):

**Table 2**  
**Example Table for Capital Redundancy/(Deficiency)**

Risk Measure	Capital Need	Excess/ (Deficit) Capital
1 in 100 probability of ruin	\$ 800	\$300
1 in 250 probability of ruin	\$1,000	\$100
1 in 500 probability of ruin	\$1,400	(\$300)
2% EPD	\$ 900	\$200
1% EPD	\$1,200	(\$100)
0.5% EPD	\$1,700	(\$600)
2.0% EDLR	\$1,000	(\$100)
1.0% EDLR	\$2,000	(\$900)
0.5% EDLR	\$3,000	(\$1,900)

**Risk and Safety Trade-off**

If all the company cared about was safety, they could simply increase capital until the required safety level was achieved. In most cases, unfortunately, increasing capital without any change in business activity or the investment asset mix will decrease the Company's profitability. Output from the DFA must show this trade-off in a simple and direct manner. The Board of Management needs to see the impact of increasing or decreasing capital. **Exhibit 1** is an example of the type of graph used to demonstrate this. This graph shows the trade-off between risk and reward for different levels of capital. The graph shows the 50<sup>th</sup> percentile ROE versus the Safety level (here 1-EPD %) for different levels of capital. As expected, removing capital increases the ROE but decreases the safety level. This chart has been found to be an effective tool for communicating the critical trade-off issue for overall capital.

### 3. Capital Allocation

The capital allocation to a division should be based as much as possible on the relative contribution of the divisions to the overall company total risk. The company requires a certain amount of capital to function. That capital is needed because of business written by the divisions. Each division enjoys the benefit of additional underwriting capacity – beyond what it could write as a standalone entity – from its “membership” in the company. However, that combined capital figure needs to be supported with returns. How much of the capital support burden should each division bear? An immediate answer is to allocate capital to division in proportion to the division’s contribution to the total company risk measure.

One way to estimate a division’s contribution to the total risk measure would be to determine its marginal impact – how much does the addition of that division to the rest of the company change the total risk measure? A simple technique to determine the marginal impacts is to “swap in and out” each division—subtract each division in turn from the total company and determine the resulting total risk measure. The marginal impact is the difference between the total company risk measure and the [total company – division] risk measure.

However, for most popular actuarial risk measures – variance, standard deviation, ruin probability, expected policyholder deficit – the sum of these marginal impacts will not equal the total risk measure. Computationally there is no issue; the allocation percentages are relative measures, so each division is allocated in proportion to its marginal impact as a percentage of the sum of the marginal impacts. But is there something else occurring here which merits deeper attention?

The short answer is yes. We must consider **additivity**, **order dependency**, and **stability**. These concepts are known within game theory<sup>6</sup> and the study of “cooperative games with transferable utilities.” Cooperative games with transferable utilities have the following characteristics:

- Participants or “players” have something to share – either a benefit (e.g., bonus pool) or penalty (e.g., taxes);
- The item to be shared is valued the same by all participants (e.g., money);
- The item must be allocated to the players;
- The opportunity to share results from the cooperation of all players;
- Individual players are free to engage in negotiations, bargaining and coalitions; and
- Players have conflicting objectives, each wanting the most benefit or least penalty.

One of the primary goals of the study of a cooperative game is the determination of a fair allocation scheme for dividing the benefit or penalty. Any valid allocation scheme should

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<sup>6</sup> For a fuller discussion of the insurance parallels with game theory, see Lemaire [4] or Mango [5]. An abridged discussion follows here.

first and foremost be **additive**: the sum of all players' allocations must equal the total amount to be allocated. Many popular actuarial risk measures are not additive for purposes of allocation [5]. For example, stand-alone Expected Policyholder Deficit violates this criterion—the sum of the individual capital allocations is greater than the required total.

In many allocation schemes, a player's marginal impact determines the amount of benefit or penalty allocated; however, the marginal impact **depends on the player's order of entry** into the coalition. It is important for an allocation scheme to smooth the effects of order dependency as much as possible<sup>7</sup>.

The allocation scheme must also not systematically punish or reward certain players on a basis not reflected in the risk measure. In short, they should be fair and impartial. Otherwise, there would be incentives for the punished player or players to break apart from the group and form a faction. In such an instance, the coalition is referred to as unstable. A fair allocation scheme will result in a **stable coalition**.

These desirable characteristics of additivity, order independence and stability can all be found in an allocation scheme based on the **Shapley value**. It is named after Lloyd Shapley, one of the early leaders in the field of game theory. The Shapley value is an allocation scheme that is:

- Additive;
- Order independent; and
- Stable.

The Shapley value is the average of marginal impacts taken over all possible entrance orders. For example, consider a company with three divisions A, B, and C. The Shapley value for division A would be:

$$\begin{aligned} & [ \text{Marginal impact of A being added to an empty company} + \\ & \text{Marginal impact of A being added to division B} + \\ & \text{Marginal impact of A being added to division C} + \\ & \text{Marginal impact of A being added to divisions B \& C} ] / 4 \end{aligned}$$

For a small number of divisions, this calculation is not too burdensome. However, as the number of divisions increases, the number of permutations grows geometrically. Is there any way the process can be simplified?

It turns out that for the risk measure of variance (applied to any variable such as net income, losses or other), the Shapley value reduces to

$$\text{Shapley value} = \text{Var}[\text{division}] + \text{Cov}[\text{Rest of Company, division}],$$

When compared to the formula for marginal variance,

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<sup>7</sup> See Philbrick [5] or Mango [2] for discussion of this phenomenon.

$$\text{Marginal variance} = \text{Var}[\text{division}] + 2 \times \text{Cov}[\text{Rest of Company, division}],$$

the Shapley value splits the co-variance evenly among divisions.

Using the Shapley value and a risk measure of variance makes the calculation manageable. Each division's Shapley value is the division's variance plus the co-variance with the remaining divisions. This is an extremely desirable quality, as we can now get all the information we need from only one run.

Specifically, the allocated capital for the Company was based on each division's variance of statutory net income.

#### **4. The Initial Board Presentation**

The original results were presented during a two-hour meeting with the Board of Management. The DFA team focused first on capital adequacy, then capital allocation. An exhibit similar to Table 1 showed the implied safety level of current capital using the different risk measures (see Section 2). An exhibit similar to Table 2 showed the additional capital needed to achieve various target safety levels.

Next came the simulated GAAP and SAP financial statements. **Exhibit 2** shows the layout of the GAAP Balance sheet and Income Statement. Median values are shown, along with standard deviations. Standard deviation was selected as a simple measure of variability. With so many figures on the page, it was important to convey variability in the simplest manner possible. As mentioned before, standard deviation is effective at conveying variability in a single number. This audience was not particularly statistically inclined, so very little was lost in making this simplifying decision.

The balance of the presentation was spent on the allocation of capital among the major divisions of the Company – see **Exhibit 3**. Allocation output should be displayed not only as absolute amounts of allocated capital, but also as percentages of the total. These percentages will often draw a great deal of attention. In this case, some of the Board members represented individual divisions. One cannot expect to present allocation percentages representing relative risk contributions without digging more deeply into the basis of risk measurement. This presentation was no exception, and issues raised in Section 3 were discussed in some detail, including variance of net income as a risk measure, order dependency, covariance, and fairness of allocations.

Dialogue at the Board level of this nature is one of the real benefits of a DFA study. By framing the implications of these issues, DFA facilitates the discussion by grounding it in measurable quantities. Without the DFA study, the discussions would be anecdotal at best.

In addition to the allocated capital, expected return on that capital was also displayed. This Return on Risk-Adjusted Capital or “RORAC” raised still more engaging discussion. Here, not only are divisional differences in risk reflected, but also market reward. Few more politically sensitive measurements can be conceived.

Presenters must always be cognizant of the familiarity level of their audience with the material. When presenting new material, it is critical to provide comparable context with more familiar terminology or concepts. In the case of the risk measures, this meant providing familiar counterparts such as Premium to Surplus ratio. Risk is a multi-dimensional phenomenon that can only be appreciated and understood in pieces. The right side of Exhibit 3 shows all of these more familiar risk measures:

**Asset Needed Ratio** = [ Allocated Capital + Premium ] / Expected Losses

**Premium to Surplus Ratio** = Premium / Allocated Capital

**Loss Percentage** = Divisional share of Total Expected Loss

**Loss Ratio** = Expected Losses / Premium

## 5. Follow-Up Meetings

Subsequent to this were several one-on-one follow-up meetings with selected Board members (representing different operating divisions of the Company) whose charges were to:

- Increase their understanding of the DFA model, its parameterization and output;
- Dig more deeply into certain issues raised in the initial presentation; and
- Address certain division-specific concerns.

These meetings provided a more focused and interactive forum for the DFA study team to provide details behind the study. Among the items raised in these sessions:

- **Possible Error in Risk Measure Calculation**  
The capital adequacy results for one of the risk measures “did not feel right” to some of the Board members. Their intuitions turned out to be correct, and a calculation error was uncovered as a result of further review. This kind of fresh perspective can often uncover anomalous results that those performing the study miss due to their intimate involvement<sup>8</sup>.

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<sup>8</sup> Actuaries in general are so technically focused they often underestimate the value of input from those less technically inclined. However, what these others may lack in technical expertise can be more than made up for in business sense. This business sense is most often expressed intuitively. Such hunches and feelings are to be ignored at one’s own peril.

- **Details Behind the 20 Worst Scenarios**  
The Board was also interested in the drivers behind the 20 worst scenarios. Subsequent research revealed (not surprisingly) that the most severe scenarios resulted from the compounded effect of two or more of the following occurring in the same time period:
  - Major natural catastrophe
  - Adverse reserve development
  - Casualty line loss ratio deterioration
  - Asbestos and Environmental reserve deterioration
  - Unusually low investment returns
  
- **Concern over the Probability of Achieving a Target ROE**  
Board members were also uncomfortable with the estimated probability of achieving a target ROE (they felt the probability was too high!). Further review revealed another calculation error. This sort of feedback cycle is critical to properly evaluating the results of a complex study like this.
  
- **Splitting Runoff Capital from Ongoing Capital**  
This issue was raised as part of a discussion of the practical implications of capital allocation. Should ongoing business be allocated all the investment returns (from reserves as well as premium funds), but also all the capital? Or should separate “Runoff” versus “Ongoing” capital amounts (and asset pools) be maintained? In response to the request, a new allocation was generated with the divisional capital amounts for ongoing business only. All input reserve categories were aggregated into the “Runoff” division. The resulting familiar risk measures (e.g., Premium to Surplus ratios) were more in line with expectations.

## **6. The Reference Binder**

The presentation of results for a study of this magnitude requires significant backup material, in addition to that covered in the presentation itself. Typically, senior management members will have varying levels of familiarity with DFA, probability, simulation, and correlation. It is important to provide supporting material in one location where attendees can make notes, seek more detail, and refer back in the coming weeks. To support those needs, a detailed reference binder (300 pages of detailed exhibits and explanations) was prepared.

The binder had the following sections:

1. Executive Summary
2. Introduction to the DFA model
3. Overview of Findings
4. Economic Modeling
5. Asset Modeling

6. Liability Modeling
7. Reinsurance Modeling
8. Risk Measures and Capital Adequacy

The Executive Summary section covered the actual presentation material discussed in Section 4. The other portions of the binder will be discussed here.

## **2. Introduction to the DFA Model**

Comfort comes with familiarity. For senior management of an insurance company today, many of the concepts underlying a typical DFA model may be unfamiliar. The results of such a model can therefore make management uncomfortable, and rightly so. Comfort will come slowly over time, as they grow conversant in the new terminology, and become confident the model is accurately modeling the behavior of their company.

The DFA model introduction (see the **Appendix**) pictorially displays the flow of information through the model. This is followed by brief, bullet point descriptions of each major model component. It was important to build knowledge and comfort slowly, in stages, starting from high-level overview descriptions like this. The role of pictures cannot be underestimated. Pictures can provide a structural framework around which the detailed flesh of the model is later built.

## **3. Overview of Findings**

Attempts at distilling the voluminous output of the study to a limited, manageable number of exhibits proved extremely difficult. This section contained:

- GAAP and SAP Balance Sheets and Income Statements
- Plots of the projected distributions over the next three years of Stockholder's Equity, ROE and Net Income
- Profitability vs. Safety plots (similar to Exhibit 1)
- Summaries of important input statistics

## **4. Economic Modeling**

This provides detailed background on the technical foundation and parameterization of the Global Economic Module—the economic scenario generation portion of the DFA model. The economic scenarios provide a consistent integrated framework that drives both asset valuation and liability trends.

## **5. – 7. Asset Modeling, Liability Modeling, Reinsurance Modeling**

These sections discussed the parameterization of the Company, including the data issues and shortcuts that an ambitious timeframe necessitated.

For Asset modeling, the Company's actual asset portfolio was input in asset class detail. The DFA model has advanced asset capabilities, making it worthwhile to enter the assets in such detail. Sophisticated asset modeling adds to the total risk picture by "setting in motion" pieces that are static in many other models.

Liabilities were modeled at the detail level dictated by many constraints, including available supporting data (e.g., reserve studies) and per-risk reinsurance covers requiring individual claim level simulation. The binder covered category definitions, data gathering, development and trend factor selection, loss curve fitting, and reconciliation with the Company's business plan.

The Reinsurance portion lists the in-force covers that were modeled, and shows the "reinsurance map"—the graphical reinsurance coverage depiction tool. An example map is shown in **Exhibit 4**. Reinsurance covers were modeled in extensive detail, including ceded premium. Results were produced on gross, ceded and net bases.

### **8. Risk Measures and Capital Adequacy**

This section is similar to Section 2 of the paper, discussing many possible risk measures, their relative advantages and disadvantages, and issues related to selecting a risk measure.

## **7. Conclusion**

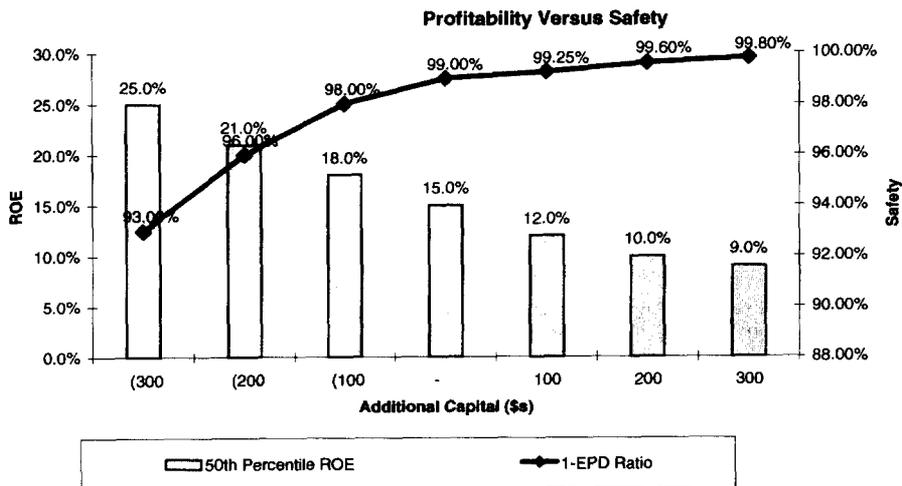
DFA models are the actuarial equivalent of advanced experimental apparatus. Like our counterparts in physics (though on a lesser scale), actuaries can use DFA models to pose and answer hypothetical questions that previously could not even have been asked. This paper addresses many such questions. It is therefore not surprising that many of these issues have yet to be fully and satisfactorily resolved. We must be careful when presenting our DFA studies not to oversell it. Focus on the strengths of the models, the questions they allow us to answer, but be open to criticisms, because there is much that is unanswered. Breakthroughs can come from unexpected places. It is because of this that we must strive in our communications to simplify our results, and translate them so they may reach the widest possible audience.

DFA *system* development has progressed fairly rapidly within the industry. What is lagging behind is widespread understanding and comfort with the issues DFA raises. DFA systems can produce so many answers, there may not be enough people who know the right questions to ask. The business leaders of our industry are looking to the CAS membership to be the bridge between the science of DFA and the art of business management and strategy. They need guidance on the best risk measures, the interpretation of the levels of those risk measures, the incorporation into planning, the practical meaning of using distributions in place of static values, and the details behind the challenges of parameterization. Clearly continued sharing of all aspects of our research efforts – such as this Call Paper Program – will lead us all closer to those goals.

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**Exhibit I**  
**Example of 50<sup>th</sup> Percentile ROE and Safety Trade-off Graph**



This exhibit shows an example of the ROE versus Safety trade-off graph. The 50<sup>th</sup> percentile of ROE (left y-axis) and the Safety measure (right y-axis) are shown for different levels of additional capital (x-axis).

When capital is removed, the ROE improves but the Safety score deteriorates.

When capital is added, the ROE deteriorates but the Safety score improves.

**Exhibit 2**  
**Summary Financials**

**XYZ Corporation**  
**GAAP Income Statement**

Line Item	50th Percentile				Standard Deviation		
	1998	1999	2000	2001	1998	2000	2001
0 Gross Premiums Written							
1 Ceded Premiums Written							
2 Net Premiums Written							
3 Change in Unearned Premiums							
4 Premiums Earned - Net							
5 Losses Incurred							
6 Commissions							
7 ULAE							
8 Overhead							
9 Brokerage							
10 Total Losses and Expenses							
11 Underwriting Gain/ (Loss)							
12 Investment Income - Taxable							
13 Investment Income - Non-Taxable							
14 Capital Gains (Losses)							
15 Investment Expenses							
16 Income From Subsidiaries							
17 Total Investment Income							
18 Total Misc.							
19 EBIT							
20 Interest Expenses							
21 EBT							
22 Taxes - Cap Gains							
23 Taxes - Ordinary							
24 Taxes - Total							
25 Operating Earnings							
26 Div. #1							
27 Div. #2							
28 Net Income							

**XYZ Corporation**  
**GAAP Balance Sheet**

Line Item	50th Percentile				Standard Deviation		
	1998	1999	2000	2001	1998	2000	2001
0 Investments - Tradeable Bonds							
1 Investments - Non-Tradeable Bonds							
2 Investments - Preferred Stock							
3 Investments - Tradeable Equities							
4 Investments - Non-Tradeable Equities							
5 Investments - Other							
6 Cash							
7 Total Investments And Cash							
8 Accrued Investment Income							
9 Premium and Other Receivables							
10 Deferred Acquisition Costs							
11 Reinsurance Recoverables							
12 Other Assets							
13 Total Assets							
14 Loss and ALAE Reserves							
15 Unearned Premium Reserves							
16 Total Reserves							
17 Loss Balances Payable							
18 Funds held under reins treaties							
19 Senior debt							
20 Senior Notes							
21 Other Liabilities							
22 Total Liabilities							
23 Special Debt							
24 Common Stock							
25 Paid-in Capital							
26 Retained Earnings							
27 Other Income							
28 Total Stockholder's Equity							
29 Preliminary Liab Etc.							
30 Misc.							
31 Total Liabilities							

The financials display each accounting line item's median and standard deviation by year. For less statistically sophisticated audiences, standard deviation is a familiar measure that adequately conveys differences in variability for presentation purposes.

**Exhibit 3**  
**Capital Allocation**

**Capital Allocation**

Economic Basis / Excluding Runoff / Inv Inc Allocated / 3 Year Run  
Major Divisions

Division Label	Capital		Reward			Risk			Other Statistics		
	% of Total Company Capital	Capital Allocation	Div. ROE	Contribution to Company Bottom Line	Contribution to Company Bottom Line	Asset Needed Ratio	Premium/ Surplus Ratio	Loss Percentage	Losses	Premiums	Loss Ratio
Div. #1	16.4%	\$ 16	12%	\$ 2.5	25%	3.1	0.8	15%	\$ 11	\$ 14	78%
Div. #2	1.0%	1	24%	0.3	3%	2.0	4.7	4%	3	5	66%
Div. #3	55.8%	56	4%	3.0	30%	3.5	1.1	51%	36	60	60%
Div. #4	16.2%	16	5%	1.0	10%	3.3	1.0	15%	11	16	66%
Div. #5	10.5%	11	24%	3.1	31%	2.9	1.5	14%	10	16	62%
	\$	100	\$	10.0	100%			100.0%			

Capital allocations and Reward measures are displayed as both absolute amounts and percentages of the total. Some interesting risk measures are then shown:

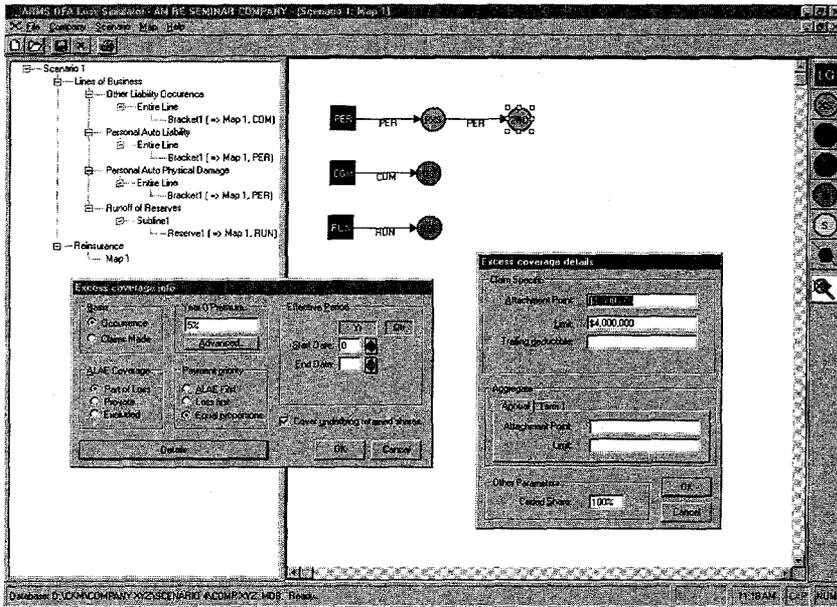
**Asset Needed Ratio** = [ Allocated Capital + Premium ] / Expected Losses

**Premium to Surplus Ratio** = Premium / Allocated Capital

**Loss Percentage** = Divisional share of Total Expected Loss

**Loss Ratio** = Expected Losses / Premium

***Exhibit 4***  
***Example of Reinsurance Structuring***



The DFA model uses a graphical “coverage map” to depict reinsurance programs. The palette on the right has objects representing subject losses (squares) and various types of covers. In the above illustration, an excess cover and two quota shares have been added. A second excess cover is in the process of being added, and a few of the screens “behind” the excess cover are displayed. The graphical map, once completed, serves as very effective documentation that the reinsurance program has been correctly depicted in the model.

## Appendix

### Introduction to ARMS, American Re's DFA Model

**ARMS** is American Re-Insurance Company's DFA model. It integrates assets and liabilities across economic scenarios. It also provides detailed modeling capabilities for insurance liabilities and reinsurance. The system is also used to assist both Munich Re<sup>9</sup> and American Re-Insurance Company clients in evaluating and setting up efficient re-insurance and investment structures. The structure of the system is laid out in Figure 1.

## ARMS Structure

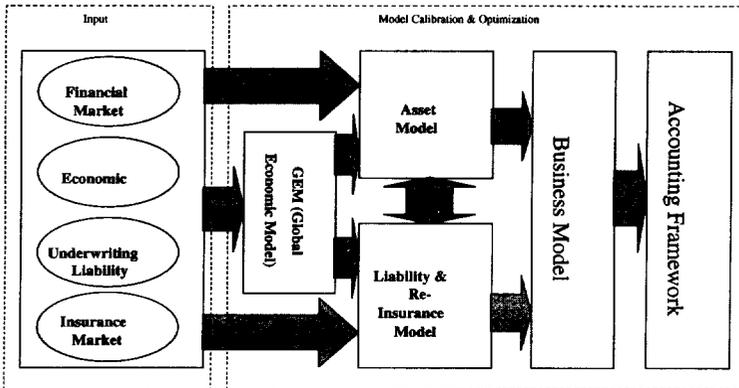


Figure 1. American Re-Insurance Company's Risk Management System (ARMS) is an integrated compilation of models. Historical data from financial and economic markets, underwriting decision processes, and insurance market trends are inputs to the system (left). Output includes balance sheet and income statements, and illustrative charts and reports.

ARMS is composed of several integrated modules which handle different aspects of the simulation.

The **Global Economic Module** or **GEM** generates plausible time series outcomes of future economies based on user specifications and parameter settings. The user specifications are inputs reflecting the current economic environment and expectations for long-term median trends. The parameter settings are referred to as calibration parameters and those are set via the **Constraint Evaluator System**<sup>10</sup>.

<sup>9</sup> American Re-Insurance Company is a member of the Munich Re Group.

<sup>10</sup> See Berger and Madsen [1] for details behind the calibration of the GEM.

Each of the economic time series scenarios are fed to the **Asset Module** as well as the **Liability and Re-insurance Module**. Economic scenarios integrate the simulation of liabilities and assets, ensuring internally consistent simulations. For example, inflation parameters from the economic model influence the trend in the prospective loss severity distributions. Similarly, the prospective premium trend can also be tied to inflation. Any discounting for future pricing purposes is based on output from the economic model.

The **Accounting Framework** refers not only to accounting but also to tax implications. There are several advantages to separating this functionality. They include the facilitation of operating in a multi-country (and therefor multi-regulatory) environment.

Wrapped around all this functionality is a non-convex optimization engine – the driving force behind the **Constraint Evaluator System**. Since each of these models must be calibrated in one form or another, access to a non-convex optimization system minimizes traditional trial and error attempts to ensure the reasonability of results. Ideally, we want to back-test the models with historical data and ensure optimal performance before we start modeling prospectively.

