Report on

Modeling of Economic Series Coordinated with Interest Rate Scenarios

Research Sponsored by the

Casualty Actuarial Society

and the

Society of Actuaries

Researchers:

Kevin C. Ahlgrim, ASA, MAAA, Ph.D.
Illinois State University

Stephen P. D'Arcy, FCAS, MAAA, Ph.D.
University of Illinois at Urbana-Champaign

Richard W. Gorvett, FCAS, MAAA, ARM, FRM, Ph.D.
University of Illinois at Urbana-Champaign

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Section 1: Introduction and Overview

Introduction

In May, 2001, the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA) jointly issued a request for proposals on the research topic "Modeling of Economic Series Coordinated with Interest Rate Scenarios." The objectives of this request were to develop a research relationship with selected persons to investigate this topic; produce a literature review of work previously done in the area of economic scenario modeling; determine appropriate data sources and methodologies to enhance economic modeling efforts relevant to the actuarial profession; and produce a working model of economic series, coordinated with interest rates, that could be made public and used by actuaries via the CAS / SOA websites to project future economic scenarios. Categories of economic series to be modeled included interest rates, equity price levels, inflation rates, unemployment rates, and real estate price levels.

This topic is of considerable value and importance to the actuarial profession and the broader insurance community, for several reasons. For example, a key aspect of the ***dynamic financial analysis*** (DFA) process, which continues to be an area of substantial development and interest in the actuarial community, is the generation of economic and financial scenarios. These scenarios provide an economic context for the evaluation of an insurer's alternative operating decisions and their potential impact, across a variety of possible future economic conditions, on future corporate value. In other words, such stochastic simulation efforts are predicated upon the ability to probabilistically express possible future economic and financial environments. In addition, an integrated scenario generation capability is critical to recognizing the interdependencies between the various economic and financial series – e.g., consistently modeling the relationships between, say, equity returns and interest rate movements.

Similarly, the generation of scenarios is important for ***regulatory, rating agency, and internal management tests*** of an insurer's potential future operating conditions. An example is ***cash flow testing***; by testing across scenarios, an insurer's cash position and liquidity can be evaluated over a variety of alternative future economic and financial environments.

This document represents the report, produced by the three researchers selected by the CAS / SOA, which summarizes this research and the development of a scenario generation model available for public use. Full descriptions of the project, the research methodology, analytical implications, and the model itself – a spreadsheet-based stochastic simulation model – are provided in this report.

Overview

This report includes the following sections and attachments:

Section 1:	Introduction and Overview
Section 2:	Excerpts from Original CAS / SOA Request for Proposals
Section 3:	Excerpts from Proposal of Selected Researchers

Most of the Section and Appendix titles should be self-explanatory. Brief comments on just a few of these components are provided here.

The *Literature Review* (Section 4) includes brief descriptions of a variety of articles – covering the areas of actuarial science (both life and casualty), finance, and economics – that we believe are relevant, to varying degrees, to this research. (For articles appearing in CAS or SOA publications, hyperlinks to the articles are included.) We appreciate a number of article suggestions made by members of the CAS and SOA oversight committees. Research on the development of financial scenarios, and the analysis of financial and economic time series, is a continually evolving and growing area. We recommend that efforts be made, at least periodically if not continually (e.g., by a formal charge to appropriate CAS / SOA research committees, or by engagement of other interested persons), to provide an ongoing search for and review of relevant new work in the area, in order that the results from this project might be enhanced and updated.

Discussions of Issues (Section 6) describes and comments upon some of the specific issues encountered during the course of this research. In some cases, these issues involved decisions which we as researchers were confronted with; our thought processes and the rationales for selected approaches are included. Often, these issues were either provoked or reinforced by questions or comments from members of the sponsoring actuarial committees. Again, this input was greatly appreciated and valued throughout the project.

Presentations on Research (Appendix B) includes a schedule of presentations which one or more of the researchers have made, or are planning to make, to actuarial, academic, or other organizations regarding this research. Where a presentation has been made, presentation materials, if available, are included. Copies of future presentations will also be provided, on an ongoing basis, to the CAS / SOA, if desired by those organizations.

Simulated Financial Scenario Data (Appendix C) is a spreadsheet database of hundreds of scenarios (i.e., simulation paths) of financial and economic variables, generated as output from the Financial Scenario Model. The intent of this data is to provide an alternative to requiring the @Risk simulation package (an add-on to Excel) in order to run the model. This data can be used directly, in lieu of actually running the model; the "pre-simulated" scenario paths can be used as an input to a DFA or other analytical effort.

The Financial Scenario Model (Appendix D) is an Excel spreadsheet-based program, designed to be run, as mentioned above, through the @Risk simulation add-on. The model includes default values of appropriate parameters – however, these can be changed by the user for purposes of updating for new or additional data, sensitivity testing of parameter values, etc.

<u>Section 2: Excerpts from Original CAS / SOA Request for Proposals</u>

Research Sponsors:	**Casualty Actuarial Society** Committee on Theory of Risk Dynamic Financial Analysis Cte.	**Society of Actuaries** Committee on Finance Research

Research Project: **Modeling of Economic Series Coordinated with Interest Rate Scenarios**

<u>Society of Actuaries</u>

The Society of Actuaries (SOA) is an educational, research, and professional organization dedicated to serving the public and SOA members. Its mission is to advance actuarial knowledge and to enhance the ability of actuaries to provide expert advice and relevant solutions for financial, business, and societal problems involving uncertain future events. The vision of the Society of Actuaries is for actuaries to be recognized as the leading professionals in the modeling and management of financial risk and contingent events.

The SOA was organized in 1949 as a merger of the Actuarial Society of America, founded in 1889, and the American Institute of Actuaries, founded in 1909. The membership of the SOA includes over 16,000 actuaries working in life insurance, retirement systems, health benefit systems, financial and investment management, and other newly emerging practice areas. The SOA Committee on Finance Research (COFR) manages and governs all research activities related to financial and investment management.

<u>Casualty Actuarial Society</u>

The Casualty Actuarial Society (CAS) was organized in 1914 as a professional society with the purpose of advancing the body of knowledge of actuarial science applied to property, casualty and similar risk exposures. This is accomplished through communication with the publics affected by insurance, the presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means. The membership of the CAS includes over 3,000 actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, the federal government and independent consultants.

- *Committee on Theory of Risk*: The Committee on Theory of Risk (COTOR) is charged with developing and demonstrating the utility of specific applications of the theory of risk to various lines of property and casualty insurance.
- *Dynamic Financial Analysis Committee*: The Dynamic Financial Analysis Committee (DFAC) was formed by the CAS in 1998 by merger of the DFA Modeling Task Force of the Valuation, Finance, and Investments Committee and the DFA Task Force on Variables of COTOR. Its charge is to further the CAS initiative on DFA by promoting dissemination of knowledge pertinent to DFA and promoting the use of DFA in practice.

The COTOR and the Dynamic Financial Analysis Committee are jointly sponsoring this research on behalf of the CAS.

CAS and SOA Interest in the Subject

The topic of appropriate modeling techniques for generating economic scenarios in a DFA model or in a cash flow test is extremely important to actuaries. A key part of DFA modeling is the reasonable representation of future economic indices, to model asset and liability risks. In cash flow testing, plausible future scenarios must be created to include or be consistent with plausible values of a variety of economic indices. A standardized approach to this problem would be an important step in providing guidance to practicing actuaries.

Statement of the Problem

DFA is a technique used by actuaries to project the future financial position of an insurance entity. DFA modeling strives to represent the complete distribution of financial values along with a best estimate. DFA techniques have many uses such as solvency testing, capital budgeting and developing reinsurance structures.

A key part of DFA modeling is the representation of future economic conditions and the impact of those conditions on asset, liability and income values. In order to model appropriately changes in financial values the actuary requires a way of projecting future economic indices. It is also important that these indices be projected in such a way as to reflect the appropriate interdependencies between the values, so that, for example, interest rates and inflation rates move together in a reasonable manner.

The goal of this project is to provide actuaries a model for projecting economic indices with realistic interdependencies among the variables. The indices included will ultimately be the decision of the researcher but should include, at a minimum, variables representing:

1) Long Term Interest Rates
2) Short Term Interest Rates
3) Shape of Yield Curve
4) Stock Market Price Levels – Large Cap
5) Stock Market Price Levels – Small Cap
6) General Inflation Rate
7) Medical Inflation Rate
8) Wage Level Inflation
9) Real Estate Price Levels
10) Unemployment Rate (optional)
11) Economic Growth Rate (optional)

Besides being able to do a realistic joint simulation of these variables, the model should also have the capability of imposing a deterministic scenario on one or more variables and simulating the others in a manner consistent with that scenario. This is to accommodate use of the model in cash flow testing.

Project Outline

a) *Literature Survey*: The research should include a comprehensive survey of the literature from actuarial, as well as econometric and any other relevant sources, for any research relating to this issue. The researcher will be expected to compile a bibliography of such research.

b) *Presentation of Model*: The research findings should include a presentation of the model in full detail readily adaptable for use, with parameter estimates and accompanying statistics. Interdependencies or the mechanisms inducing them should be described in full detail. There should also be discussion of how the parameter set can be modified to represent extreme economic conditions. This presentation will become the property of the sponsoring organizations to be exposed to their membership as they see fit.

c) *Estimation and Updating*: The above findings should be supported with a full account of estimation procedures and associated diagnostics sufficient to establish the statistical validity of the model. This should also include a description of data and procedures for updating the model. At its discretion, the CAS may also commission bulletins updating the model for use of the membership.

The primary work product will be a comprehensive report encompassing a), b), and c) above, suitable for posting on the CAS website and for non-refereed publication in the CAS *Forum*. The project will also include writing a paper, suitable for publication in the CAS *Proceedings*, which presents in readable form the matters discussed in b) and c) above. The researchers are also encouraged to publish the research in other journals in order to elicit comment from outside the actuarial community. The project may include a review of the implications of the procedure(s) with respect to the determination of risk-based capital and the actuarial pricing of insurance.

Research Funding

Funding for this research is principally through the CAS and SOA, but may include other research funding organizations. The amount of funds available for this project has not been determined at this time. The final scope of the project will be decided by the committee based on the research costs for various items and the expected results of the project.

Researcher Qualifications

Interested researchers should submit their resumes (if a firm, of the principal individual(s) performing or directing the work), indicating how their background, education, and experience bear on their qualifications to undertake this research. Specifically, researchers should specify their qualifications and expertise to perform research in the application of finance, statistics and actuarial theories to insurance and economic problems. Researchers should indicate their relevant work or research experience and professional accomplishments (e.g. papers published).

In addition, researchers should supply a discussion of their proposed approach to the problem. This discussion should address the project goals presented above. If any of said goals cannot be achieved, the researcher should state that explicitly. The researcher should also address separately each of the requirements listed above indicating intention to comply or any reservation or limitation to compliance.

Finally, researchers should prepare a preliminary cost estimate for the work described.
The CAS and SOA will select the researcher or researchers, who, in the judgment of the selection committee, and on the basis of his or her qualifications and expertise, is best able to perform the research project as outlined above. If COTOR/DFAC/COFR determine that no researcher is suitably qualified to perform this research project, then no contract will be awarded.

Receipt of submissions will be acknowledged. Members of a joint oversight task force of COTOR, DFAC, and COFR will evaluate the researchers' qualifications.

<u>Presentation, Ownership and Publication of Report</u>

The selected researcher(s) will be required to sign the attached (not included here) consulting agreement, which defines the terms and conditions under which the work is performed. If asked, the researcher(s) agrees to be available to present the report at a CAS meeting or seminar. If travel is required, reasonable expenses will be paid in addition to the compensation provided in the research contract.

The CAS and SOA intend to copyright the research and to publish it in appropriate journals. The researcher(s) will also be encouraged to publish the work in a refereed academic journal (*e.g. Journal of Risk and Insurance*) in order to elicit the widest possible comment and recognition. It is intended that the results of the research can be used freely by any interested party. The research will be considered work-for-hire and all rights thereto belong to the CAS and SOA. However, appropriate credit will be given to the researcher(s).

Section 3: Excerpts from Proposal of Selected Researchers

Our Understanding of the Problem

The Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA) are soliciting proposals for research involving the modeling of economic series. This research is important for generating reasonable future economic and financial scenarios, and has critical implications for dynamic financial analysis and cash flow testing. One essential aspect of this research involves the interdependencies between the various economic and financial series, especially, but not exclusively, the interrelationships between the different series and interest rates. In terms of work products, this research would involve surveying and reviewing the relevant literature, presenting an economic scenario generator model, and preparing documentation and one or more papers describing the model and research findings.

Our Proposal to Perform This Research

This project would be a logical extension of the substantial financial and actuarial work and research performed, and being performed, by the three researchers submitting this proposal. We propose the following research plan to address this important and challenging project.

1) ***Literature review***. A comprehensive survey and review of the relevant literature. This review will include a summary of the procedures and findings documented in many significant articles, in each of the following categories:
    a) Actuarial
    b) Financial
    c) Other (e.g., economic, econometric, and/or statistical)
2) ***Development and presentation of an economic scenario generator model***. This includes several items:
    a) *Development of a model to represent economic and financial series*. Specific series include:
        i) <u>Term structure of interest rates</u>. A model for the term structure will include values for *short-*, *medium-*, and *long-term rates*, and will reflect the *shape of the yield curve*. The interest rate model will be a central part of the overall economic scenario generator model, since most other economic and financial series are related to interest rates in some fashion.
        ii) <u>Inflation</u>. An important interdependency in the model will involve the relationship between interest rates and inflation. A variety of different inflation rates will be modeled, including *general*, *medical*, *wage*, and insurance-specific (*line of business*) inflation rates.
        iii) <u>Stock market levels</u>. Based on historical data and patterns, this series can be split into several possible sub-series, for example *large* versus *small cap* stocks.
        iv) <u>Real estate price levels</u>.
        v) <u>Unemployment rates</u>.
        vi) <u>Economic growth rates</u>.

b) *Parameter estimation*. Statistical analyses of relevant historical data will serve to parameterize the model. Interdependencies between variables will be identified through regression-type equations or other means.
c) *Provision for extreme conditions*. There will be a provision for the representation of extreme financial and economic conditions. This provision might involve the selection of appropriate parameter values, or other means (e.g., through a stochastic jump process).

3) ***Creation of software which allows users to model economic and financial series***. This software will follow the model developed above, and will be made available to the sponsoring organizations for general use and comment.
4) ***Documentation***. A comprehensive report describing the literature review, the model, and the parameterization will be written. This report will also include a description of how the parameter estimation can be updated, and a brief discussion of the implications of the model for specific areas of actuarial interest, such as dynamic financial analysis, asset-liability management, risk-based capital, and insurance pricing. In addition, slide presentations summarizing this research will be provided for posting on the sponsoring organizations' websites.
5) ***Additional articles***. The researchers will write a paper discussing this research, with the intention of publishing it in the *CAS Proceedings*. In addition, other articles may be written, possibly for publications such as the *North American Actuarial Journal*, the *Journal of Actuarial Practice*, or the *Journal of Risk and Insurance*.

One of the key aspects of this research involves a model of interest rates. A large number of models of the stochastic term structure of interest rates have been developed, ranging from relatively simple to extremely complex. The models tend to fall into one of two types: equilibrium models that are derived from proposed relationships between supply and demand for funds, and no-arbitrage models that use the current term structure as a starting point and generate changes from the current values. Some term structure models use only one stochastic variable, usually the short-term interest rate, whereas others have two, three, or more stochastic variables, which can include such factors as the long-term interest rate, the volatility factor, and the mean reversion speed. The general consensus is that no single model is best for all applications and the more complex the model, the more sensitive it is to parameter misspecification. Thus, the results of this research may generate more than one interest rate model. Descriptions of these models would include discussion of the strengths and weaknesses of each model and which models would be most appropriate for particular situations. This approach would allow practitioners with different needs and different levels of comfort with term structure models to select the appropriate model for each application.

<u>Sample Reference List</u>

The following sources represent a sample of the works which may be referenced during this project. Many of these sources will form the basis of the literature survey.

Ahlgrim, Kevin C., Stephen P. D'Arcy and Richard W. Gorvett, 1999, Parameterizing Interest Rate Models, *Casualty Actuarial Society Forum*, Summer 1-50.

Açt-Sahalia, Yacine, 1999, Do Interest Rates Really Follow Continuous-Time Markov Diffusions? University of Chicago Working Paper.

Casualty Actuarial Society Financial Analysis Committee, 1989, A Study of the Effects of Asset/Liability Mismatch on Property/Casualty Insurance Companies, *Casualty Actuarial Society Call Paper Program - Valuation Issues*, 1-52.

Chan, K. C., G. Andrew Karolyi, Francis A. Longstaff, and Anthony B. Sanders, 1992, An Empirical Comparison of Alternative Models of the Short-Term Interest Rate, *Journal of Finance*, 47: 1209-1227.

Chapman, David A. and Neil D. Pearson, 2001, What Can Be Learned from Recent Advances in Estimating Models of the Term Structure, forthcoming.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, A Theory of the Term Structure of Interest Rates, *Econometrica*, 53: 385-407.

D'Arcy, Stephen P. and Richard W. Gorvett, 2001, The Effective Duration and Convexity of Liabilities for Property-Liability Insurers Under Stochastic Interest Rates, working paper.

D'Arcy, Stephen P. and Richard W. Gorvett, 2001, Measuring the Interest Rate Sensitivity of Loss Reserves, *Proceedings of the Casualty Actuarial Society* 88.

Fama, Eugene, 1990, Term Structure Forecasts of Interest Rates, Inflation, and Real Returns, *Journal of Monetary Economics*, 25: 59-76

Fama, Eugene, 1984, The Information in the Term Structure, *Journal of Financial Economics*, 13: 509-528

Fama, Eugene and R. Bliss, 1987, The Information in Long-Maturity Forward Rates, *American Economic Review*, ?:680-692

Fisher, Irving, 1930, *The Theory of Interest* (New York: Macmillan), Chapter 19.

Heath, D. , R. Jarrow and A. Morton, 1992, Bond Pricing and the Term Structure of Interest Rates: A New Methodology, *Econometrica*, 60: 77-105.

Ho, T. S. Y. and S. B. Lee, 1986, Term Structure Movements and Pricing Interest Rate Contingent Claims, *Journal of Finance* 41: 1011-1029.

Hodes, Douglas and Sholom Feldblum, 1996, Interest Rate Risk and Capital Requirements for Property/Casualty Insurance Companies, *Proceedings of the Casualty Actuarial Society*, 83:490-562.

Hull, John C., 2000, *Options, Futures, and Other Derivatives* Fourth Edition (Upper Saddle River, NJ: Prentice Hall).

Hull, John C, and Allen White, 1990, "Pricing Interest Rate Derivative Securities," *Review of Financial Studies*, 3:573-592

Litterman, Robert and Josϑ Scheinkman, 1991, Common Factors Affecting Bond Returns, *Journal of Fixed Income* 3:54-61.

Masterson, Norton, 1968, "Economic Factors in Liability and Property Insurance Claims Costs," *Best's Insurance News* (October), pp. 12-18; subsequent updates published periodically in *Best's Review*

Reitano, Robert R., 1996, Non-parallel yield curve shifts and stochastic immunization, *Journal of Portfolio Management*, Winter 1996, p. 71.

Santomero, Anthony and David F. Babbel, 1997, Financial Risk Management: An Analysis of the Process, *Journal of Risk and Insurance* 64:231-270.

Staking, Kim and David Babbel, 1995, The Relation Between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry, *Journal of Risk and Insurance*, 62:690-718.

Tilley, James A., 1988, The Application of Modern Techniques to the Investment of Insurance and Pension Funds (New York: Morgan Stanley & Co.)
Vasicek, O, 1977, An Equilibrium Characterization of the Term Structure, *Journal of Financial Economics*. 5: 177

Researchers and Qualifications

The three researchers submitting this proposal are, we believe, uniquely qualified to perform this research. All three are designated actuaries (two are members of the Casualty Actuarial Society, and one is a member of the Society of Actuaries), each has a Ph.D. in Finance with a significant research background in areas relevant to this project, and – importantly – each has considerable experience as a practicing actuary. This latter characteristic refers to full-time, real-world experience, not merely occasional consulting engagements, and is important to the researchers' understanding of the applications of this project to real-world actuarial considerations. In addition, several prior papers written by these researchers have been awarded prizes by the CAS.

Some of the highlights of work and research performed by the researchers include:

- Development of a public-access dynamic financial analysis model
- Research on historical interest rate movements and stochastic interest rate models
- Research on the interrelationships of interest rates with inflation and other economic variables
- Applications of DFA models to insurance companies
- Considerable teaching, seminar, and presentation experience on the implications of financial issues for life and property-liability insurance

Brief descriptions of each researcher follow, in alphabetical order. Complete resumes of each researcher accompany this proposal.

Kevin Ahlgrim, ASA, MAAA, Ph.D., is an assistant professor in the Department of Finance and Quantitative Methods at Bradley University. He received a BS in actuarial science, and an MS and a Ph.D. in finance, all from the University of Illinois at Urbana-Champaign. Prior to entering academia, he worked as an actuary at CIGNA Corporation and Aon Consulting. Kevin has done extensive research using several interest rate models. His specific research interests lie in the application of fixed income techniques to insurance. Specifically, his dissertation analyzed the impact of different term structure models on the dynamic financial analysis of insurance companies, including both property-liability and life companies. Kevin has also examined other applications of term structure models including pricing Eurodollar futures options and investigating biases of specific models.

Stephen P. D'Arcy, FCAS, MAAA, Ph.D., is a Professor of Finance and the John C. Brogan Faculty Scholar in Risk Management and Insurance at the University of Illinois at Urbana-Champaign. He is a Past-President of the American Risk and Insurance Association and a member of the Board of Directors of the Casualty Actuarial Society. He received his B.A. in applied mathematics from Harvard College and his Ph.D. in finance from the University of Illinois. The courses he teaches include an introduction to insurance, property-liability insurance, casualty actuarial mathematics, advanced corporate finance, employee benefits and financial risk management of insurance enterprises. He teaches a seminar on finance and an on-line course on financial risk management for the Casualty Actuarial Society. Prior to his academic career, he worked as an actuarial student at AEtna Insurance Company and as Actuary at CUMIS Insurance Society. He served on the Governor's Task Force on Medical Malpractice in Illinois. His research interests include dynamic financial analysis, financial pricing models for property-liability insurers, catastrophe insurance futures, pension funding and regulation.

Richard W. Gorvett, FCAS, MAAA, ARM, Ph.D., is an actuarial science professor at the University of Illinois at Urbana-Champaign. He received a BS in mathematics from the University of Illinois at Chicago, an MBA in finance, econometrics, and statistics from the University of Chicago, and a Ph.D. in finance from the University of Illinois at Urbana-Champaign. Prior to entering academia, he worked as a practicing actuary for Allstate, CNA, Tillinghast, and Ernst & Young. He has taught insurance, finance, financial risk management, and actuarial science courses at both the undergraduate and graduate levels. His research activity centers around dynamic financial analysis, insurance securitization, and the application of financial theory to property-liability insurance.

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To update the above personnel descriptions as of July, 2003:

- Mr. Ahlgrim was at Bradley University through the Summer of 2003; effective Fall 2003, he is a finance professor at Illinois State University.
- Mr. Gorvett went to Zurich North America in the Summer of 2001, most recently as Senior Vice President and Director of Internal Audit & Risk Management; effective Fall 2003, he returns to the University of Illinois at Urbana-Champaign as an actuarial science professor.

<u>Section 4: Literature Review</u>

Brief Summaries of Articles Reviewed and Relevant to the Project Involving
Modeling of Economic Series Coordinated with Interest Rate Scenarios

- Ahlgrim, D'Arcy and Gorvett, 1999, "Parameterizing Interest Rate Models," *Casualty Actuarial Society Forum*, Summer 1-50.
  http://www.casact.org/pubs/forum/99sforum/99sf001.pdf
  - ***Uses simulation to develop future scenarios for various applications. Wilkie's Provides a review of historical interest rate movements from 1953-1999, summarizes the key elements of several interest rate models and describes how to select parameters of the models to fit historical movements.***

- Ait-Sahalia, 1999, "Do Interest Rates Really Follow Continuous-Time Markov Diffusions?" University of Chicago Working Paper
  - ***Examines whether interest rates follow a diffusion process (continuous time Markov process), given that only discrete-time interest rates are available. Based on the extended period 1857 to 1995, this work finds that neither short-term interest rates nor long-term interest rates follow Markov processes, but the slope of the yield curve is a univariate Markov process and a diffusion process.***

- Bernstein, 1996, *Against the Gods: The Remarkable Story of Risk*, New York: John Wiley & Sons, Inc.
  - ***Provides an excellent long term perspective of risk and the development of methods to measure and deal with risk. Explains the shift from a focus on hazard risk to financial risk over the last few decades.***

- Casualty Actuarial Society Financial Analysis Committee (CASFAC), 1989, "A Study of the Effects of Asset/Liability Mismatch on Property/Casualty Insurance Companies," *Valuation Issues*, 1-52. http://www.casact.org/pubs/dpp/dpp89/89dpp001.pdf
  - ***Discusses the potential impact of an asset-liability mismatch for property-liability insurers. By "mismatch," this article means that anticipated cash flows from existing assets and liabilities will not precisely offset each other. Several mismatch scenarios are evaluated, and it is found that both potential risk and reward are greater, the greater the mismatch.***

- Chan, Karolyi, Longstaff, and Schwartz, 1992, "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate, *Journal of Finance*, 47: 1209-1227.
  - ***CKLS estimate the parameters of a class of term structure models using the generalized method of moments technique and the time series of monthly interest rate data from 1964-1989. They find that the volatility of interest rates is extremely sensitive to the level of the rate.***

- Chapman and Pearson, 2001, "What Can Be Learned from Recent Advances in Estimating Models of the Term Structure," forthcoming.

- ***Provides a comprehensive review of term structure models. They conclude that volatility increases with the level of the short term interest rate and, within normal interest rate ranges, mean reversion is weak. They also point out that the appropriate measure for volatility depends on whether the period 1979-1982 (when the Federal Reserve shifted policy from focusing on interest rates to inflation rates) is treated as an aberration or included in the sample period. They also conclude that more research is needed to determine which interest rate model is best.***

- Chapman and Pearson, 2001, "Recent Advances in Estimating Term-Structure Models," Financial Analysts Journal (July/August), 77-95.
  - ***Provides a summary of term structure literature and contrasts the issues that have been resolved with those areas that require further research. They point out that mean reversion of interest rates is weak and that absolute volatility appears to be related to rate levels. Unfortunately, the specific nature of volatility is currently unresolved.***

- Cox, Ingersoll, and Ross, 1985, "A Theory of the Term Structure of Interest Rates," Econometrica, 53: 385-407.
  - ***Using a general equilibrium framework, CIR develop a process for the short-term interest rate. The CIR term structure model is:***

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}\,dB_t$$

- Fama, 1984, "The Information in the Term Structure," *Journal of Financial Economics* 13, 509-528
  - ***Examines the ability of forward rates to forecast future spot rates. Based on data for 1974 and subsequent, he finds evidence that very short-term (one-month) forward rates can forecast spot rates one month ahead. Data prior to 1974 indicate that this predictive power extends five months into the future.***

- Fama, 1990, Term Structure Forecasts of Interest Rates, Inflation, and Real Returns," *Journal of Monetary Economic*s 25, 59-76
  - ***Examines the ability to forecast one-year spot interest rates in the context of forecasting its components: the one-year inflation rate, and the real return on one-year bonds. It is found that the expected values of those two components move opposite to one another. This results in a situation where the five-year yield spread (the yield on five-year bonds over the one-year spot rate) is unable to forecast near-term spot rates (one or two years ahead); while the spread has power to forecast the inflation and real return components of the spot rate, those components tend to offset somewhat. As the time horizon is extended, the ability to forecast the spot rate improves. Fama also finds that forecasts of these variables are related to the business cycle.***

- Fama and Bliss, 1987, "The Information in Long-Maturity Forward Rates," *American Economic Review* 77, 680-692
  - ***Examines expected returns on U.S. Treasury securities with maturities of up to five years. They find that the one-year interest rate has a mean-reverting tendency,***

which results in one-year forward rates having some forecasting power two to four years ahead. Thus, the paper provides evidence that, while forward rates are not good forecasters of very near-term changes in interest rates, they are better at forecasting long-term changes.

- Fisher, 1930, *Theory of Interest*, New York: The Macmillan Company.
  - ***Summarizes the economic intuition behind adjusting nominal interest rates based on expected future inflation.***

- Hardy, 2001, "A Regime-Switching Model of Long-Term Stock Returns," *North American Actuarial Journal*, 5 (2), 41-53. http://www.soa.org/library/naaj/1997-09/naaj0104_4.pdf
  - ***Using monthly data from the S&P 500 and the Toronto Stock Exchange, a regime-switching lognormal model is parameterized and compared with other models. The author finds the performance of the regime-switching model to be favorable.***

- Heath, Jarrow, and Morton, 1992, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation," *Econometrica*, 60: 77-105.
  - ***Rather than developing a process for the short rate, HJM model the movements of the entire term structure through a family of forward rate processes.***

    $$df(t,T) = \mu(t,T,f(t,T))dt + \sigma(t,T,f(t,T))dB_t \quad \textit{where}$$

    $$f(t,T) = -\frac{\partial \ln P(t,T)}{\partial T}$$

    HJM find that the drift in forward rates can be restated in terms of the volatilities, implying that the market price of risk is unimportant in contingent claims valuation.

- Hibbert, Mowbray, and Turnbull, 2001, "A Stochastic Asset Model & Calibration for Long-Term Financial Planning Purposes," Technical Report, Barrie & Hibbert Limited
  - ***This paper describes a model that generates consistent values for the term structure of interest rates, both real and nominal, inflation rates, equity returns and dividend payouts. The model can be used to generate multiple potential paths for each of these variables for use in financial modeling. The paper provides an excellent review of interest rates, inflation rates and equity returns over the last 100 years, or longer, as well as for more recent periods.***

    The real interest rate model is a 2-factor Hull-White model. The short-term rate reverts to a mean reversion value that is itself a random variable which reverts to the long term mean value. Inflation is also a 2-factor model, with a double mean reversion process. The equity model determines the equity return in excess of the nominal interest rate as a Markov regime-switching model with one regime having a higher expected return and lower variance, and the other regime a slightly lower expected return but much larger variance. The equity dividend yield model is a one factor first order autoregressive process.

    The paper does an excellent job describing the calibration process for this model. Interest rates in the United Kingdom since 1725 are illustrated, and the question of

which time period should be used to determine the value is clearly demonstrated by the use of different time periods. Similarly, the average inflation rate in England for periods ranging from the last 700 years down to the last 10 years are shown. For example, the average annual inflation rate over the last 700 years is slightly less than 1%. Over the last 100 years the inflation rate averaged 4%, over the last 50 years 6%, over the last 30 years 8%, over the last 20 years 6% and over the last 10 years 4%.

Two sets of parameter values are illustrated, one that allows negative interest rates and does not include a risk premium for long term interest rates and another that limits nominal interest rates to positive values. The model is used to simulate 1000 scenarios over a 30 year horizon based on monthly steps. The resulting means, standard deviations and distributions plotted several ways are then compared to illustrate the impact of this change in calibration. A graph termed the funnel of doubt is used to illustrate the range of outcomes for interest rates and inflation rates over the 30 year period.

Finally, the paper compares the results this model generates to the output from the Wilkie model. The Wilkie model is shown to generate inconsistent relationships among inflation, bank interest rates and the yield on consols. The autoregressive feature of equity returns included in the Wilkie model generates a distribution over a long term horizon that is much more compact than historical experience would indicate.

Their model is presented on www.barrhibb.com.

- Ho and Lee, 1986, "Term Structure Movements and Pricing Interest Rate Contingent Claims," *Journal of Finance*, 41: 1011-1029.
    - ***Taking the existing term structure as an input, Ho and Lee develop an arbitrage-free term structure model. The original model is presented as a binomial tree, where the short-term rate is related to forward rates plus a random factor. It has since been shown that the continuous-time equivalent of their model is:***

$$dr_t = \theta(t)dt + \sigma dB_t$$

- Hodes and Feldblum, 1996, "Interest Rate Risk and Capital Requirements for Property/Casualty Insurance Companies," *Proceedings of the Casualty Actuarial Society*, 83:490-562. http://www.casact.org/pubs/proceed/proceed96/96490.pdf
    - ***Examines the effect of interest rate risk on the assets and liabilities of a property-liability insurer. Indicates a need for greater detail on cash flows from assets.***

- Hull and White, 1990, "Pricing Interest-Rate-Derivative Securities," *Review of Financial Studies*, 3: 573-592.
    - ***Extend the models models of Vasicek and CIR to be arbitrage free. By introducing a time-dependent drift, the resulting term structure of the Hull and White model is consistent with current market prices of bonds. The paper goes on to compare***

option prices under the model to Vasicek and CIR term structure models. The one-factor Hull-White model is:

$$dr_t = \kappa(\theta(t) - r_t)dt + \sigma dB_t$$

- Hull and White, 1994, "Numerical Procedures for Implementing Term Structure Models II: Two-Factor Models," *Journal of Derivatives* (Winter), 37-48.
    - ***Extend the one-factor Hull-White model (1990) to include a stochastic mean reversion level. The two-factor model allows for more flexible yield curve dynamics. The paper also reviews a numerical procedure (trinomial trees) for implementing the models. The two-factor Hull-White model is:***

$$dr_t = (\theta(t) + u_t - ar_t)dt + \sigma_1 dB_{1t}$$
$$du_t = -bu_t dt + \sigma_2 dB_{2t}$$

- Litterman and Scheinkman, 1991, "Common Factors Affecting Bond Returns," *Journal of Fixed Income* (June), 54-61.
    - ***Use principal components analysis to determine the important factors that affect term structure movements. They found that only three specific shifts (level, steepness, and curvature) explain almost 99% of the variance in interest rates. In fact, the first factor, which represented level shifts in the term structure, explained almost 90% of the total variation in rates.***

- Masterson, 1968, "Economic Factors in Liability and Property Insurance Claims Costs, 1935-1967," *Proceedings of the Casualty Actuarial Society* 55, 61-89; subsequent updates published periodically in *Best's Review*.
http://www.casact.org/pubs/proceed/proceed68/68061.pdf
    - ***Analyzes property-liability insurance claims costs in the context of economic factors. A variety of external economic series are considered.***

- Pennacchi, 1991, "Identifying the Dynamics of Real Interest Rates and Inflation: Evidence Using Survey Data," *Review of Financial Studies*, 4: 53-86.
    - ***Over the period 1968-1988, there is evidence that the instantaneous real interest rates and expected inflation are significantly negatively correlated. The inflation expectations are based on surveys of professional economic forecasters, which may not necessarily correspond with market expectations.***

- Redington, 1952, "Review of the principles of life office valuations", *Journal of the Institute of Actuaries*, 78: 1-40.
    - ***Represents a path-breaking approach to dealing with risk for insurers. Introduces the "funnel of doubt" terminology and explains a strategy for immunizing an insurer from interest rate risk.***

- Reitano, 1992, "Non-parallel Yield Curve Shifts and Immunization," *Journal of Portfolio Management*, 18: 36-43.
    - ***Demonstrates the danger of immunizing against only parallel shifts in the yield curve. Using key rate durations (partial durations), Reitano shows that even small***

non-parallel shifts in the yield curve may cause extreme changes in asset values. Reitano suggests a specific remedy for immunizing against specific shifts in the yield curve.

- Risa, 2001, "Nominal and Inflation Indexed Yields: Separating Expected Inflation and Inflation Risk Premia," Columbia University Working Paper.
  - ***This paper provides an excellent review of the literature on the relationship between inflation and interest rates. Based on nominal and inflation indexed bonds from the United Kingdom from 1983-1999, the nominal and inflation indexed interest rates are derived. The inflation risk premium is determined based on a four factor pricing model.***

- Santomero and Babbel, 1997, "Financial Risk Management: An Analysis of the Process," *Journal of Risk and Insurance* 64:231-270.
  - ***Provides an extensive analysis of financial risk management as performed by insurers. Based on a series of interviews with management at a variety of insurers, reports on the current state of this process in the insurance industry. Concludes that significant improvements in financial risk management are necessary and that even the most advanced insurers are not doing an effective enough job managing these risks.***

- Shiller, 2000, *Irrational Exuberance*, Princeton, NJ: Princeton University Press.
  - ***Analyses long term patterns in stock returns. Presents the case that the stock market was significanly overvalued by the beginning of 2000. Compares recent market valuation to similar situations over the prior 130 years.***

- Sornette, 2003, *Why Stock Markets Crash: Critical Events in Complex Financial Systems*, Princeton, NJ: Princeton University Press.
  - ***Examines the behavior of stock markets, and proposes a simple theory, built upon the author's expertise in non-linear processes and complex systems, for why markets crash.***

- Staking and Babbel, 1995, "The Relation Between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry," *Journal of Risk and Insurance*, 62:690-718.
  - ***Utilizes a modification of the Taylor Separation Method to project the total cash flows from claim payments, rather than focusing solely on loss severity. This approach incorporates the volume and type of business written and historical loss development. This method assumes that inflation in a given year affects all unpaid losses for a given line equally, regardless of the accident year. The relationship between the market value of the firm and its leverage and surplus duration is then measured. The results of these relationships calculated for 25 insurers over 7 years are displayed graphically by a saddle-shaped curve representing the relationship among leverage, surplus duration, and the Tobin's Q value (which measures the ratio of market to book value). These results suggest the need for further study on the duration measure. While the mean value of leverage for this sample, 3.47, lies***

along the crest of the saddle, suggesting that on average insurers adopt a leverage ratio that maximizes the market value of the firm, the mean value of surplus duration, 9.68, lies near the minimum values of the curve. If surplus duration were any lower or higher than the average value, then the market value of the firm would increase. Since the distribution of surplus duration values was not bimodal, this suggests that insurers were operating at a surplus duration level that minimized the firm's value. This finding suggests either that surplus duration is not measured accurately, or that insurers need to look at duration much more closely.

- Tilley, 1988, "The Application of Modern Techniques to the Investment of Insurance and Pension Funds" (New York: Morgan Stanley & Co.)
  - ***Provides an excellent, non-mathematical discussion regarding the problem of immunization for insurance companies and pension plans. This paper explains the classical measures of interest sensitivity and then describes the effects of interest-sensitive cash flows and how duration measures need to be adapted for these instruments.***

- Tuckman, 1996, *Fixed Income Securities*, New York: John Wiley & Sons, Inc.
  - ***Chapter 9 of the book discusses some differences between arbitrage-free and equilibrium term structure models. While providing an overview of the approaches of both types of models, Tuckman also summarizes the advantages and disadvantages of the model classes.***

- Vasicek, 1977, "An Equilibrium Characterization of the Term Structure," *Journal of Financial Economics*, 5: 177-188.
  - ***Derives a general form for the term structure of interest rates. The Vasicek term structure model is:***

    $$dr_t = \kappa(\theta - r_t)dt + \sigma dB_t$$

    Vasicek also uses arbitrage arguments between two generic bonds to show how the market price of risk is constant.

- Wilkie, 1986, "A Stochastic Investment Model for Actuarial Use," *Transactions of the Faculty of Actuaries*, 39: 341-403.
  - ***Uses simulation to develop future scenarios for various applications. Wilkie's model postulates that inflation is the independent variable – the "driving force" – in the model and uses a "cascade" approach to model other variables, including (1) dividends, (2) dividend yields, and (3) interest rates (more specifically, the yield on long-term Consols). Wilkie uses a first-order autoregressive model for inflation and the other variables are linked to the realization of inflation.***

- Wilkie, 1995, "More on a Stochastic Model for Actuarial Use," *British Actuarial Journal*, pp. 777-964
  - ***Wilkie updates his 1986 model by spelling out many of the time series issues involved in the generation of economic scenarios. Wilkie presents the methodology for selecting the structural form of a process that will be used to represent key***

variables in his "stochastic investment model." The paper includes several appendices that fully develops the time series tools used throughout the presentation including cointegration, simultaneity, vector autoregression (VAR), autoregressive conditional heteroscedasticity (ARCH), and forecasting.

In some cases, Wilkie provides an economic argument for the structural form of an assumption. He then estimates parameters for each equation of the model by looking at data from 1923-1994 and performs tests on competing models for fit. The subsequent analysis in many circumstances is inconclusive. In many circumstances, these tests flatly reject the realism of the proposed models or the analysis indicates some violation of the underlying statistical assumptions. Wilkie ignores many of these issues and simply resolves conflicts through personal preference without formal guidelines. Wilkie also comments on using annual vs. monthly data and presents some parameter estimates for the equations based on data from several different countries.

The following presents an overview of the variables in Wilkie's stochastic investment model:

(1) Inflation -Wilkie's primary variable in the entire model is inflation. He uses a first-order autoregressive process to capture the dynamics of inflation. However, the distribution of actual inflation is more positively skewed and more fat-tailed than implied by the AR(1) process. Using an ARCH process for inflation reduced these problems.

(2) Wages are analyzed in several ways to test for the different potential relationships with inflation:
a. Cointegration with inflation – real wages may follow a deterministic drift or they follow their own distinct autoregressive process.
b. Simultaneous determination where wages and inflation are based on past wages and past inflation (VAR).

(3) Dividend Yields – Yields on common stock follow an AR(1) process, but also depend on inflation. Wilkie verifies that looking at yields is the correct approach since dividends and share prices are indeed cointegrated.

(4) Dividends are driven by current and past levels of inflation, plus a carryover effect from previous yields and dividends.

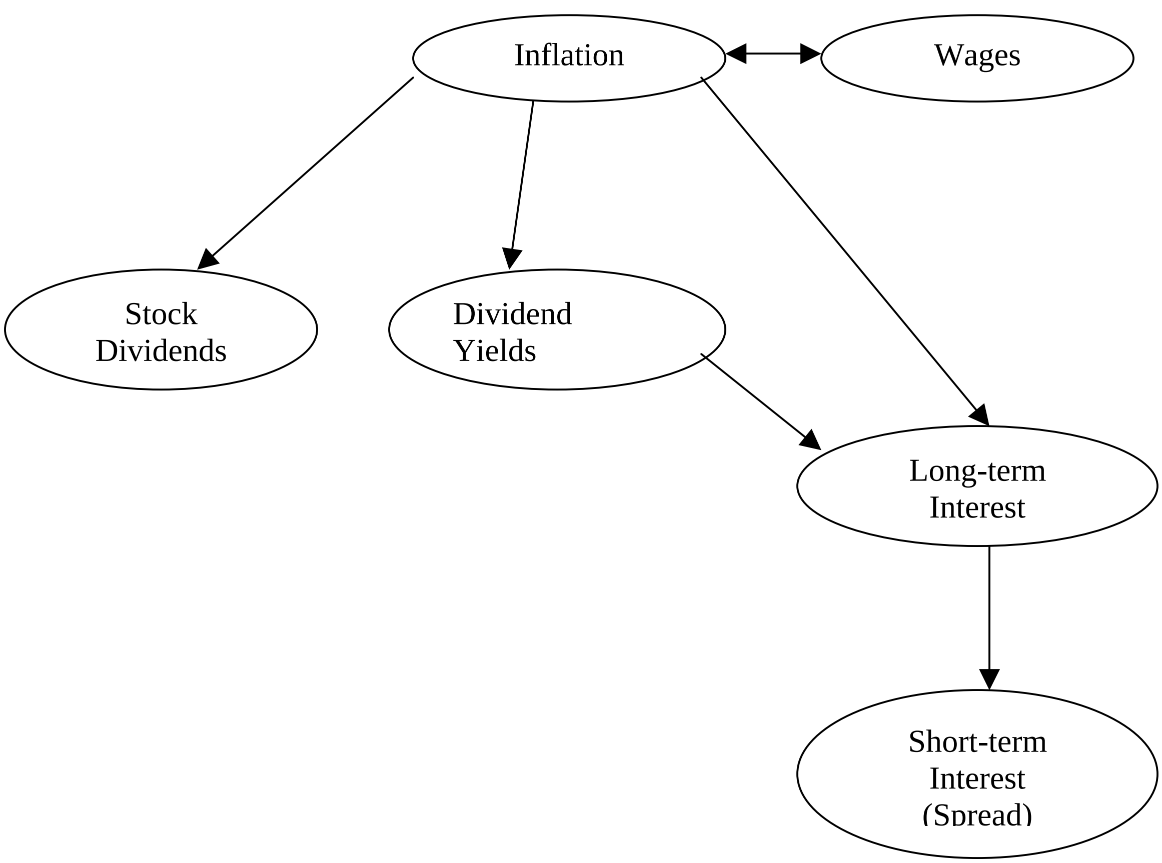
(5) Long-term Interest Rates – Yields on fixed-interest consols are driven by an exponentially weighted average of past levels of inflation, plus an AR process for the real interest rate. In addition, the unpredictable component of the dividend yield has an influence on the consol yield.

(6) Short-term Interest Rates - Wilkie models the spread between short- and long-term rates as an AR(1) process.

(7) Real Estate – Analogous to dividends and dividend yields on stocks, Wilkie uses a process for property yields and property income.

(8) Foreign exchange – Wilkie also looks at a model of exchange rates as fluctuations around those implied by purchasing power parity.

WILKIE MODEL
RELATIONSHIPS OF VARIABLES



- Wilmott, 1998, *Derivatives: The Theory and Practice of Financial Engineering*, John Wiley & Sons: Chichester, England.
  - ***A large book which provides a thorough discussion of derivatives theory and pricing.***

- Yan, 2001, "Dynamic Models of the Term Structure," *Financial Analysts Journal,* July/August: 60-74.
  - ***Discusses the application of dynamic term structure models for the pricing of interest rate derivatives.***

Section 5: Descriptions of Data and Approach

In this section, detailed descriptions are provided of our analytical methodology associated with each of the economic time series[1]. Embedded in these descriptions are references to the sources of historical time series data; where relevant, we have provided a hyperlink to these sources.

Inflation model

Inflation (denoted by *q*) is assumed to follow an Ornstein-Uhlenbeck process of the form (in continuous time):

$$dq_t = \kappa \ (\mu_q - q_t) dt + \sigma dB_q \tag{1}$$

The simulation model samples the discrete form equivalent of this process as:

$$\begin{aligned} \Delta q_t &= q_{t+1} - q_t = \kappa_q (\mu_q - q_t) \Delta t + \varepsilon_q \sigma_q \sqrt{\Delta t} \\ q_{t+1} &= q_t + \kappa_q (\mu_q - q_t) \Delta t + \varepsilon_q \sigma_q \sqrt{\Delta t} \\ &= \kappa_q \Delta t \cdot \mu_q + (1 - \kappa_q \Delta t) \cdot q_t + \varepsilon_q \sigma_q \sqrt{\Delta t} \end{aligned}$$

From this last equation, we can see that the expected level of future inflation is a weighted average between the most recent value of inflation (q_t) and a mean reversion level of inflation, μ_q. The speed of reversion is determined by the parameter κ_q. In the continuous model, mean reversion can be seen by considering the first term on the right-hand side of (1) (which is called the drift of the process). If the current level of inflation (q_t) is above the average, the first term is negative. Therefore, equation (1) predicts that the expected change in inflation will be negative; that is, inflation is expected to fall. The second term on the right-hand side of (1) represents the uncertainty in the process. One can think of the Brownian motion term (B_t) as representing a draw from a standardized normal random variable (represented by ε_q in the discrete form of the model). Uncertainty also includes the parameter σ_q, which scales the magnitude of the volatility associated with the inflation process.

We can rearrange the last equation above to show that this process is an autoregressive process.

$$\begin{aligned} q_{t+1} - \mu_q &= \mu_q \kappa_q \Delta t - \mu_q + (1 - \kappa_q \Delta t) \cdot q_t + \varepsilon_q \sigma_q \sqrt{\Delta t} \\ &= (1 - \kappa_q \Delta t) \cdot q_t - (1 - \kappa_q \Delta t) \cdot \mu_t + \varepsilon_q \sigma_q \sqrt{\Delta t} \\ &= \phi_1 (q_t - \mu_q) + \varepsilon_q \sigma_q \sqrt{\Delta t} \end{aligned}$$

In order to estimate the parameters of the inflation model, we run the following regression:

$$q_{t+1} = \alpha + \beta q_t + \varepsilon'_{qt} \tag{2}$$

Note that we have not run the regression using the change in inflation as the dependent variable since this would not allow us to simultaneously derive the mean reversion speed (κ_q) and the mean reversion level (μ_q). To derive the parameters of the inflation process, we transform the regression coefficients in (2):

$$\beta = (1 - \kappa_q \Delta t)$$
$$\kappa_q = \frac{1-\beta}{\Delta t} \qquad (3)$$

and

$$\alpha = \kappa_q \mu_q \Delta t = \frac{1-\beta}{\Delta t} \mu_q \Delta t$$
$$\mu_q = \frac{\alpha}{1-\beta}$$

We gathered inflation data from the Bureau of Labor statistics website (www.bls.gov) and ran several regressions of this type to estimate κ_q and μ_q. We considered the frequency of the observations when performing regressions analysis. One concern was that individual monthly CPI levels might contain errors that would bias the regression coefficients. For example, if the CPI level of May 2003 was overstated, then inflation in May would appear "high" while the subsequent inference of inflation would appear "low". If the time series of CPI contained any errors of this type, the mean reversion strength may become overstated.

Given the noisy fluctuations in monthly data, we selected the parameters for the inflation process by looking at annual regressions. By calculating the change in CPI over the course of a year, we have a more stable and accurate depiction of the inflation rate. The popularly reported time series of CPI uses a base period (i.e., resets the index value at 100) between the years 1982 and 1984. Given the fact that the CPI level is only reported to the first decimal place, using the current base does not lend itself to capturing minor changes in inflation in the first half of the 20th-century; a small change in CPI may lead to large swings in inflation when the level of the index is low. The only publicly available series reported on the old base level (1967 = 100) is: Not seasonally adjusted, alternate base (1967), U.S. city averages, all items.

The annual rate of inflation was measured as:

$$q_t = \ln \frac{CPI_t}{CPI_{t-1}}$$

Where CPI_t is the reported index value for year t and CPI_{t-1} is the prior year's reported index value of the same month. We ran two annual regressions: (1) all available data and (2) the years after World War II.

Time Period	κ_q	μ_q	σ_q
1913-2001	0.37	3.3%	4.0%
1946-2001	0.47	4.8%	3.0%

For use in our projections, we selected κ_q to be 0.4 and the mean reversion level to be 4.8% to capture the post war economic period. Although it might appear that the speed of mean reversion over the second half of the 20th-century has increased, it should be noted that the standard error of the estimate of κ_q is higher than over the larger time period.

Instead of being concerned with the annualized, instantaneous level of inflation, bond investors are more concerned with the expected level of inflation over the life of their investment. Given the existing level of inflation (q_t) and the parameters of the assumed process in (1), we can derive expectations of future inflation over various horizons. Vasicek (1977) uses an Ornstein-Uhlenbeck process assumes to model interest rates and provides a closed-form solution for long-term bond yields as a function of the current interest-rate and the model parameters. According to Vasicek (1977), the time t price of a bond, $P(t,T)$ that matures at time T is:

$$P(t,T) = A(t,T)e^{-rB(t,T)} \tag{4}$$

where $A(t,T)$ and $B(t,T)$ are functions of the parameters of the assumed process for interest rate movements.

Our process for inflation follows the same Ornstein-Uhlenbeck process, so we can develop a "term structure" of inflation analogous to Vasicek (1977). This term structure posits an expected, inflation rate over various horizons. A term structure of inflation is needed to generate nominal interest rates, since investors are concerned not only about the time value of money, but also about the expected level of inflation over the life of bonds.

Real Interest rates

A significant amount of work has been done in the area of interest rate modeling. The role of the financial scenario generator is not to explain past movements in interest rates, but rather to develop a model that posits plausible projections of future interest-rate levels. (It might also be noted that trying to develop a model that mimics past movements may be a futile exercise since, despite the volume of research in the area, no tractable model has yet been shown to be satisfactory in accurately explaining historical interest-rate movements.)

In selecting a term structure model, we were concerned about two specific issues. First, we considered the number of stochastic factors to incorporate that generates term structure movements. Choosing the number of stochastic factors for a term structure model represents a balance between (1) having a large number of factors to adequately emulate empirical rate movements and (2) limiting the number of factors so the resulting model is simple enough to be tractable. With one-factor term structure models, the dynamics of the entire yield curve are completely driven by the single source of uncertainty. Resulting yield curve movements are subsequently constrained: yields of all maturities are perfectly correlated to the one stochastic factor and the range of potential yield curves is limited. Introducing additional sources of uncertainty (such as allowing the long end of the curve to fluctuate and/or introducing stochastic volatility) provide for a fuller variety of yield curve shapes. The downside is that introducing multiple dimensions of yield curve movements increases the complexity of the model quickly.

Our second concern was in choosing a term structure model that has closed form solutions for bond prices so that the entire term structure can be quickly and easily retrieved from the existing the values of the stochastic factors. When closed form solutions for bond yields are available, this allows users of the term structure model to track various points on the yield curve during a simulation. For example, users of a term structure model who are interested mortgage prepayment rates will be interested in the refinancing rate, which may be closely related to bond yields of specific maturities (such as 10 years). Without some explicit closed form solution, the modeler has no foundation to imply yields of different maturities from a limited set of stochastic factors.

To derive real interest rates, we selected the two-factor Vasicek term structure model, which is a simple case of the two-factor Hull-White model. In the two-factor Vasicek model, the short-term rate (denoted by *r*) reverts to a long-term rate (denoted by *l*) that is itself stochastic.

$$\begin{aligned} dr_t &= \kappa_1 (l_t - r_t) dt + \sigma_1 dB_1 \\ dl_t &= \kappa_2 (\mu - l_t) dt + \sigma_2 dB_2 \end{aligned}$$

In order to estimate the parameters of the model, we look at the discrete analog of the model:

$$\begin{aligned} \Delta r_t &= \kappa_1 (l_t - r_t) \Delta t + \sigma_1 \varepsilon_{1t} \\ \Delta l_t &= \kappa_2 (\mu - l_t) \Delta t + \sigma_2 \varepsilon_{2t} \end{aligned}$$

$$\begin{aligned} r_{t+1} - r_t &= \kappa_1 (l_t - r_t) \Delta t + \sigma_1 \varepsilon_{1t} = (\kappa_1 l_t + \kappa_1 r_t) \Delta t + \sigma_1 \varepsilon_{1t} \\ l_{t+1} - l_t &= \kappa_2 (\mu - l_t) \Delta t + \sigma_2 \varepsilon_{2t} = (\kappa_2 \mu - \kappa_2 l_t) \Delta t + \sigma_2 \varepsilon_{2t} \end{aligned}$$

We can also see how each of the stochastic factors is updated over time, analogous to the situation for inflation presented above. First, we rearrange the discrete form of the two-factor Vasicek term structure model:

$$\begin{aligned} r_{t+1} &= r_t + (\kappa_1 l_t + \kappa_1 r_t) \Delta t + \sigma_1 \varepsilon_{1t} \\ &= \kappa_1 \Delta t \cdot l_t + (1 - \kappa_1 \Delta t) \cdot r_t + \sigma_1 \varepsilon_{1t} \\ l_{t+1} &= l_t + (\kappa_2 \mu - \kappa_2 l_t) \Delta t + \sigma_2 \varepsilon_{2t} \\ &= \kappa_2 \Delta t \cdot \mu_l + (1 - \kappa_2 \Delta t) \cdot l_t + \sigma_2 \varepsilon_{2t} \end{aligned} \tag{5}$$

From these equations, we can see that the short rate is again a weighted average between the current level r_t and the mean reversion factor l_t. The mean reversion factor is itself a weighted average of some long-term mean and its current value.

Hibbert, Mowbray, and Turnbull (2001) (HMT) also use this process for real interest rates. They derive closed form solutions for bond prices (and therefore yields), which are considerable more complicated than the one-factor Ornstein-Uhlenbeck process for inflation:

$$P^r(t,T) = A^r(t,T)e^{-r_t B_1(t,T) - l_t B_2(t,T)} \quad (6)$$

where r_t and l_t are the values for the short and long real interest rate and A^r, B_1, and B_2 are functions of underlying parameters in the two-factor Vasicek specification.

Estimating the equations in (5) is a difficult procedure since real interest rates are not directly observable in the market. We compute *ex post* real interest rates based on the difference between nominal rates observed in the market less the monthly (annualized) inflation rate. We use the three-month Constant Maturity Treasury (CMT) as a proxy for the instantaneous short rate and the 10-year CMT yield as a proxy for the long rate. (We also looked at longer Treasury yields as a proxy for the long rate. Results were not sensitive to the choice of maturity.) Nominal interest rates are from the Federal Reserve's historical database (http://www.federalreserve.gov/releases/).

There are several issues with the Federal Reserve's interest rate data. First, at the long end of the yield curve, there are significant gaps in many of the series. For example, the 20-year CMT was discontinued in 1987; yields in 20-year securities after this date would have to be interpolated from other yields. Also, given the decision of the Treasury to stop issuing 30-year bonds, the future of 30-year rate data is uncertain (in fact, the Fed's data stops reporting 30-year CMT data in the early 2002). At the short end of the yield curve, it was difficult to determine a proxy for the short rate. Ideally, one would want an interest-rate that most closely resembles an instantaneous rate. While the one-month CMT is reported back only to 2001, the 3-month rate is available beginning in 1982. While we could have reverted to a private, proprietary source of data to create a longer time series, we restricted ourselves to only publicly available data sources that would be available to any user of the model.

We use the following regressions on monthly data from 1982 to 2001:

$$r_{t+1} = \alpha_1 + \alpha_2 l_t + \alpha_3 r_t + \varepsilon'_{rt}$$
$$l_{t+1} = \beta_1 + \beta_2 l_t + \varepsilon'_{lt}$$

Traditional OLS regressions are not possible since the short rate process is dependent upon the long rate; these are simultaneous equations. Instead, we use two-stage least squares estimation. In order to estimate the short-rate equation, we first obtain estimates for the long-rate $\hat{l}_t$.

$$\text{Stage 1: } l_{t+1} = \beta_1 + \beta_2 l_t + \varepsilon'_{2t}$$
$$\text{Stage 2: } \Delta r_{t+1} = \alpha_1(\hat{l}_t - r_t) + \varepsilon'_{1t}$$

The resulting parameters were selected from the regression results.

Real Interest Rate Process
Estimated from 1982 - 2001

κ_r	μ_r	σ_r	κ_l	σ_l
6.1	2.8%	10.0%	5.1	10.0%

These parameters indicate a high level of volatility that is tempered by strong levels of mean reversion.

Nominal interest rates

Fisher (1930) provides a thorough presentation of the interaction of real interest rates and inflation and their effects on nominal interest rates. He argues that nominal interest rates compensate investors not only for the time value of money, but also for the erosion of purchasing power that results from inflation. In the model presented here, the underlying movements in inflation and real interest rates generate the process for nominal interest rates. If bonds are priced using expectations of inflation and real interest rates until the bond's maturity, then nominal interest rates are implied by combining the term structure of inflation (eq. (4)) and the term structure of real interest rates (eq. 6). Therefore:

$$P^i(t,T) = P^r(t,T) \times P^q(t,T)$$

where *i* refers to nominal interest rates and the superscripts on the bond prices correspond to the underlying stochastic variables.

Unfortunately, the parameters for the real interest rate process shown above generate a distribution that severely restricts the range of potential future nominal interest rates. For example, the 1st percentile of the distribution for the 20-year nominal rate is 5.9% and the 99th percentile is 8.2%. There are several candidates for problems with real interest rates that may lead to this seemingly unrealistic distribution of future nominal rates: (1) the use of *ex post* real interest rate measures is unsuitable, (2) monthly measurement of real interest rates contains mean reverting errors which exaggerate mean reversion speed, or (3) the time period used to measure real interest rates is too short.

As a result, the parameters for real interest rates were altered to allow nominal interest rates to more accurately reflect historical volatility. Specifically, mean reversion speed was dramatically reduced. Given that mean reversion speed and volatility work together to affect the range of interest rate projections, volatility was also reduced. The following parameters are used as the "base case" in the model. These parameters are in line with what was used in Hull (2001).

κ_r	μ_r	σ_r	κ_l	σ_l
1.0	2.8%	1.00%	0.1	1.65%

Equity Model – Regime Switching

In order to capture the fat tails of the equity return distribution that have been observed historically, we use a regime-switching model to capture equity returns. To motivate the rationale for the model, consider October 1987. This single observation may appear to be too "extreme" and very unlikely given a single variance assumption. Instead, suppose that equity returns at any point in time are generated from two distinct distributions, a "high volatility" regime or a "low volatility" regime. The chance of switching from one regime to the other over the next time step is dictated by transition probabilities. During times of economic instability, the returns on equities may be more uncertain, representing a transition to the high volatility regime. Thus, the observation from October 1987 may simply be a draw from the high volatility regime.

In the financial scenario model, equity returns are based on the risk-free nominal interest rate ($q + r$) and a risk premium or the excess equity return attributable to capital appreciation (x):

$$s_t = q_t + r_t + x_t$$

To estimate the parameters of the regime switching equity return model, we used the approach of Hardy (2001). In her model, Hardy assumes that stock prices are lognormally distributed under each regime. But while Hardy looks at total equity returns, including dividends and the underlying compensation from the risk free rate, we use the excess equity returns x. Following the procedure outlined in Hardy (2001), we then maximize the likelihood function implied from the regime switching process.

We allow the returns of small stocks and large stocks to be generated independently and estimate each equity process separately. Numerous web sites are available to capture the time series of capital appreciation of these indices (see for example, finance.yahoo.com). We used the longest time series available for large stocks (1871-2002), available at Robert Shiller's web site (http://www.econ.yale.edu/~shiller/data/ie_data.htm). To look at small stocks, we used Ibbotson data captured from 1926-1999. As expected, the risk and return of small stocks appears higher than large stocks under both regimes. Using the procedure of Hardy (2001), we developed the following parameter estimates:

Excess Monthly Returns

	Large Stocks (1871-2002)		***Small Stocks (1926-1999)***	
	Low Volatility Regime	***High Volatility Regime***	***Low Volatility Regime***	***High Volatility Regime***
Mean	0.8%	-1.1%	1.0%	0.3%
Variance	3.9%	11.3%	5.2%	16.6%
Probability of Switching	1.1%	5.9%	2.4%	10.0%

Note that while the expected return in the high volatility regime is lower, it is more likely that if the high volatility regime is ever reached, the equity market will revert back to the low volatility regime since the probability of switching is higher.

Equity Dividend Yields

Similar to the process used by HMT and Wilkie (1986), we assume that (the log of) the dividend yield follows an autoregressive process.

$$d(\ln y_t) = \kappa_y(\mu_y - \ln y_t)dt + \sigma_y dB_{yt}$$

Estimation of this process is analogous to the inflation process described above. One source of difficulty for estimating the dividend yield process is in obtaining data. There is no long time series of dividend yields and that is publicly available for equity indices. To obtain this information, we used a proprietary source of financial data (Telerate). However, one may be able to estimate the dividend yield of indices that contained a limited number of stocks (such as the Dow Jones industrial average).

Real Estate (Property)

Given the that the real estate portfolios of insurers are dominated by commercial properties, we use the National Council of Real Estate Investment Fiduciaries (NCREIF) pricing index to capture the quarterly returns on commercial properties (see www.ncreif.com). The NCREIF data is generated from market appraisals of various property types including apartment, industrial, office, and retail. While the use of appraisal data may only approximate sharp fluctuations in market valuation, publicly obtainable transaction-based real estate data was not available.

Using *quarterly* return data from NCREIF from 1978 to 2001, we estimated two separate Ornstein-Uhlenbeck models for real estate: the first model included inflation while the second model did not. While we expected inflation to be a driver of real estate returns, the results were not significant.

Unemployment

There are many plausible ways to link unemployment rates to other economic variables. One approach to estimating unemployment is based on the well-known Phillips curve. The Phillips curve illustrates a common inverse relationship between unemployment and inflation. (It should be noted that since the original publication of the Phillips curve in 1958, recent data has not fit the inverse relationship as well.) The approach taken by Phillips seems plausible: As the economy picks up, inflation increases to help temper the demand driven economy. At the same time, unemployment falls as firms hire to meet the increasing demand. When the economy slows down, unemployment rises, and inflationary pressures subside.

We include a first-order autoregressive process in the Phillips curve:

$$du_t = \kappa_u(\mu_u - u_t)dt + \alpha_u dq_t + \sigma_u \varepsilon_{ut}$$

It is expected that when inflation increases (dq_t >0), unemployment decreases (the coefficient on inflation changes (α_u) is negative). One may argue that there is a lag between inflation and unemployment. To keep the model simple, we did not pursue any distributed lag approach.

The discrete form of the unemployment model:

$$\begin{aligned} u_{t+1} &= u_t + \kappa_u \mu_u - \kappa_u \Delta t \cdot u_t + \alpha_u(q_{t+1} - q_t) + \sigma_u \varepsilon_{ut} \\ &= \kappa_u \mu_u + (1 - \kappa_u \Delta t) \cdot u_t + \alpha_u(q_{t+1} - q_t) + \sigma_u \varepsilon_{ut} \end{aligned}$$

Which suggests the following regression:

$$u_{t+1} = \beta_1 + \beta_2 u_t + \beta_3(q_{t+1} - q_t) + \sigma_u \varepsilon_{ut}$$

We use inflation data as described above and retrieve monthly unemployment data from the Bureau of Labor Statistics (www.bls.gov). Using data from 1948 to 2001 and transforming the regression coefficients as in (3), we get:

$$du_t = 0.13 \times (6.1\% - u_t)dt - 0.72 dq_t + 0.76\% \times dB_{ut}$$

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[1] In the Financial Scenario Model software, there exist options for the user to override the stochastically simulated variables in certain circumstances. Specifically:

- The user may limit the following variables to non-negative values only: nominal interest rates, real interest rates, and inflation.
- The user may input selected specific scenarios for the following variables: nominal interest rates, inflation, and equity returns.

Additional discussion of these issues is provided in *Section 6 – Discussion of Issues* and in *Appendix A – User's Guide* of this report.

Section 6: Discussions of Issues

In this section, some of the important issues associated with our analysis are discussed. These issues generally deal with specific questions or decisions which were faced in performing the analysis and developing the model.

Equilibrium vs. Arbitrage Free Models

One of the primary processes in a financial scenario generator is a term structure model. A tremendous variety of models is available for both practitioners and researchers. (For a discussion of many of the available models see Yan (2001)). No single term structure model has yet proven itself worthy for all possible applications (see the discussion in Chapman and Pearson (2001)). In virtually all cases, the user of a term structure model has a tradeoff to consider: complexity of the model vs. accuracy. This tradeoff depends on the specific application of the term structure model.

There are two important issues to consider when choosing among term structure models. The first consideration is related to the theoretical background of the model. Specifically, there are equilibrium models and arbitrage-free models. Equilibrium models typically begin with an assumption for short-term interest rates, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond's maturity. One of the primary advantages of equilibrium models is that the prices of many popular securities have closed-form analytic solutions. Another advantage is that equilibrium models are fairly easy to use. On the negative side, equilibrium term structure models generate yield curves which are inconsistent with current market prices. While the parameters of these models may be selected carefully, there is no guarantee that the resulting term structure will generate observed market prices.

Contrary to equilibrium models, arbitrage-free term structure models project future interest rate paths that emanate from the existing yield curve. For applications using arbitrage-free term structure models, resulting prices will be based on the concept of arbitrage. Unfortunately, arbitrage-free term structure models are frequently more difficult to use than their equilibrium counterparts.

Outside of the previously mentioned items, there are other benefits and costs associated with equilibrium and arbitrage-free term structure models.

- Pricing accuracy
    - Arbitrage-free models are more useful for pricing derivatives. Since derivatives are priced against the underlying assets, a model that explicitly captures the market prices of those underlying assets is superior to models that more or less ignore market values. Jegadeesh (1998) looks at the pricing of interest rate caps and determines that arbitrage-free models price interest rate caps more accurately than equilibrium models. Hull (2000) and Tuckman (1994) also comment that given that arbitrage-free

term structure models are founded upon the absence of arbitrage, using these models to price derivative contracts is more plausible then equilibrium approaches.
  - Unfortunately, comments revolving around the pricing accuracy of arbitrage-free term structure models are based on a short pricing horizon. There have been no formal long-term tests of accuracy.
- Internal consistency
  - Exploding models – Arbitrage-free models can "explode" over long periods of time. With many arbitrage-free models, the forward rate plays a central role in the expected path of interest rates. Forward rates are related to the slope of the yield curve. Depending on the existing slope of the yield curve, forward rates may exhibit strange behavior impacting projections of interest rate paths in arbitrage-free term structure models. For steeply sloped yield curves, the forward rate may become very large. For inverted yield curves, the forward rate may even become negative. Especially for long-term projections, simulation paths may become extreme.
  - Arbitrage-free models also suffer from inconsistency across time. (see Wilmott (1998) and Tuckman (1994)). The underlying assumption of many arbitrage-free term structure models is that the risk-free rate is closely related to the forward rate curve. If the model were correct, the forward rates would be the perfect predictors of future spot rates. At time 0, the known term structure implies future spot rates and volatilities throughout the projection period; all of the mean reversion levels and volatilities in the future are known on the projection date, without any regard to any risk premium that may be contained in these values. Equilibrium models provide more consistent statements about interest rates over time.
- Data issues
  - Isolating the term structure – Determining the "true" term structure for input into an arbitrage-free model is difficult. One usually considers risk-free securities such as U.S. Treasuries. There are several difficulties in looking at U.S. Treasury data. First, market data gathered from strip data is noisy; term structures that are created from this data are not smooth. An alternative source for long-term interest rate data is to look at long-term U.S. Treasury coupon bonds. Even when there was a regular history of issuing 30-year bonds, liquidity affected long-term rates. When on-the-run Treasuries were issued, the securities typically had higher liquidity and therefore higher price, forcing down long-term yields. The result is a forward rate curve that initially increased until liquidity issues dominated. The end of the forward rate curve dipped, leading to a strange forward rate curve. Aside from liquidity concerns, the future of 30-year bonds is uncertain, given the federal government's termination of 30-year bond issues.
  - Noisy data and interpolation - When there is sparse data available (which is typically the case for long-term Treasuries), there are fewer points to interpolate the term structure. This makes arbitrage-free models very sensitive to the market data or any inefficiencies in market quotes that is due to noisy data.
  - Market price of risk – While these issues affect the input for arbitrage-free models, equilibrium models require some measure of the market price of risk. This information can be harder to obtain than spot rates.

Adequacy of a Two-Factor Interest Rate / Inflation Model

The number of factors to use in modeling interest rates is a decision which frequently elicits passionate debate. Here, it is important to keep in mind the purpose toward which this research is working: to produce reasonable distributions of future variable values. Our work is NOT intended for security-trading purposes. This is a hugely important context to keep in mind – it has implications for the type of interest rate model used, the number of parameters employed, etc. Furthermore, there is often a misunderstanding as to the types and movements of yield curves that are available from two-factor (and with respect to some issues, even one-factor) models. For example, humped curves are indeed possible. (A good paper for considering the types of yield curve movements that predominate historically is an article by Litterman and Schenkman.) We believe that the two-factor model we have employed is a reasonable selection in view of both historical and parsimony considerations.

Building a Model which Uses @Risk Software

@Risk, the stochastic simulation software which we use (a product of Palisades Corp.), provides a variety of useful output statistics relative to simulated interest rate scenarios (as well as many other useful metrics). However, during our project, concern was expressed about our use of this software, as not all potential users might have access to it. While we appreciate this concern, we believe that leveraging off an existing and widely available simulation spreadsheet add-on package is the most effective approach. The programming required to do the same things in Excel alone, with respect to both modeling capabilities and output metrics, would be considerable. Furthermore, we sincerely believe that any organization truly interested in generating economic and financial scenarios, presumably to enhance strategic planning and operational decision-making, should commit the resources to purchasing and understanding such a package.

Nevertheless, a potential alternative is also provided. We have created a database which contains scenario values, predicated upon default model parameters, for a variety of economic variables for use by others. In other words, for example, for a particular set of input assumptions, we have created a spreadsheet containing hundreds of scenario simulations, or iterations, for each economic series. (See Appendix C: *Simulated Financial Scenario Data*.)

Model Parameters

All of the default parameters in the model are identified, and changeable by the user. This includes the ability to input certain specific economic / financial scenarios. Thus, for example, the model has the ability to accept, as inputs, specifically identified nominal interest rate scenarios (for example, as specified in NY Regulation 126).

Degree of "Fit" of Model Relative to Historical Data

This is certainly a large and challenging area, relating to all aspects of modeling and the underlying data – e.g., the possibility that there might be measurement errors in some data series, the underlying reasons for the possible presence of "mean reversion," etc. In general, we believe our theoretical framework provides a parsimonious approach to closed-form solutions of particular variables of interest. Having said that, investigation into historical correlations among variables, and the resulting potential impact upon model parameter selection, should continue as new series data emerges over time.

Regime-Shifting Values for Equity Returns

Examination of historical equity data (for the S&P 500) shows that a normal distribution does not adequately reflect the outliers in historical S&P 500 equity returns. A better fit has been found to involve a regime-switching model (incorporated in our Financial Scenario Model spreadsheet) in which one regime has a relatively low standard deviation and the other a relatively high standard deviation. The key point is that no one knows which regime they are in at any given time, and they cannot elect to invest only when the higher expected values are occurring.

Stochastic Simulation versus User-Specified Scenarios

In general, the financial scenario model provides for stochastic simulation of future economic variables, based upon user-specified parameters for the various economic processes. However, there are instances – particularly relating to regulatory tests and to sensitivity testing – where it is desirable to allow the user to input specific scenarios for the future values of certain processes. In particular, it was decided to provide a scenario specification option for three economic variables in the model: nominal interest rates, inflation, and equity returns. For example, with respect to nominal interest rates, each of the "New York 7" regulatory interest rate tests are pre-programmed into the model and may be selected by the user; the user may also specify a scenario of her/his own creation for any of the three economic processes. More specific information regarding each of these scenario specification options is provided in *Appendix A – User's Guide*.

Exclusion of Negative Values

Much discussion occurred regarding the need for, and the most appropriate approach to, limiting the future simulation of certain variables to non-negative values. Most of the concern centered around nominal and real interest rates, but inflation was also discussed within this context. The issue was further complicated by the fact that nominal interest rates are modeled as a function of *both* real interest rates and inflation. Ultimately, it was decided to provide the user with two options:

- *Placing lower bounds on the levels of inflation and real interest rates*. The model then simulates these processes "normally" (i.e., as if there were no lower bound), but selects the maximum of the lower bound and the normal simulated value as the final simulated value of the process.
- *Eliminating the potential for negative nominal interest rates.* In this case, the model uses the standard inflation simulation, but effectively places a lower bound on the real interest rates such that the resulting nominal interest rate is non-negative.

Section 7: Results of Model Simulations

Regardless of the mathematical sophistication of the variables incorporated in a model, the accuracy of the calibration process used to determine the parameters, and the timeliness of the values on which the calibration is based, the most important test of the validity of any model is the reasonability of the results. This section will examine the results of a representative run of the model and compare these results with historical values for interest rates, inflation, stock and real estate returns, dividend yields and unemployment rates.

The model was run to generate 5,000 iterations using the base parameters and with the box labeled "Do Not Allow Negative Nominal Interest Rates" checked. The results are presented in several different ways. Table 1 lists the mean levels for the first (one month or one year) and last (50 year) values for key variables, and the 1st and 99th percentile levels for a mid time period (10 years). The figures show "funnel of doubt" plots for different variables that indicate the level of uncertainty that surrounds the output. These "funnel of doubt" graphs (a very descriptive term coined by Redington in 1952) are referred to as "summary graphs" in @RISK. These graphs can be generated in @RISK for any array of a single variable over time. The x axis indicates the time period, with larger values representing times further in the future. The y axis indicates the value assumed by the variable. The "funnel of doubt" graphs show the mean value for the 5,000 iterations (solid line) the 25th and 75th percentile values (dark shaded section) and the 1st and 99th percentile values (lighter shaded section). Expanding funnels indicate that the values become more uncertain the longer the time horizon over which the projection occurs. Narrowing funnels indicate that the variables become more predictable when forecasting the long-term movements of a particular variable.

Next, histograms are used to illustrate the full probability distribution of the values for a particular variable at one point in time. In addition, the distribution of historical values, where appropriate, is plotted on the histograms to show how the model results compare with the actual values for that variable. Finally, correlation matrices are shown for the simulation values and for historical values.

Table 1 lists the results for selected key variables. These include the Real and Nominal Interest Rates, the Inflation Rate, the Unemployment Rate, Large Stock Returns, Small Stock Returns, Dividend Yields and Real Estate Returns. For Interest Rates and Inflation, annualized values for 1 Month, 1 Year and 10 Year periods are shown. The first column indicates the first value the model generates for these variables. For both Interest Rates and Inflation, this is the value assumed after one month. For the other variables, the results are shown after the first year. (Since the stock and real estate returns are annualized, the 1st and 99th percentile values of the initial monthly returns for these variables represent unrealistic annual values that are not really indicative of the results the model would predict over a full year.)

Real Interest Rates

The results for Real Interest Rates indicate that the mean value for the first month is 0 percent. By the end of the 50 year period modeled, this value has moved to 3.0 percent. This result is entirely in line with the specifications of the model. The one month value would be closely aligned with the initial short-term interest rate (rinit1), which is 0% in the base parameters. The value would just begin to revert to the long-term mean after one month. The mean of the final value in the results, after 50 years, is around the mean reversion level for the long rate (rm2), which is 2.8 percent.

To provide an idea about the range of values this variable assumes, the 1st and 99th percentile values of the 1 month real interest rate after 10 years are displayed in columns 3 and 4. In 1 percent of the iterations, the 1 month real interest rate, on an annualized basis, is negative 5.3 percent. On first observation, this result seems nonsensical. Why would an investor be willing to lose money, in real terms, by investing at a negative real interest rate? Instead, an investor would just hold cash rather than lose 5.3% per year, after adjusting for inflation. However, this may not be as unrealistic as it seems. First, this is only the 1 month rate, and the annualized rate over the month represents a negative 0.4%. Second, this return may represent the best return available. If inflation is high, then holding cash would generate an even larger loss. In times of high inflation, the best real return an investor can receive may be negative. Finally, real interest rates are not observable. The true real interest rate is the return required, over and above expected inflation, for the specific interval. However, no one knows the expected inflation rate, as determined by the financial market, at any point in time.

In practice, two approaches have been used for estimating the expected inflation rate. In one, economists' forecasts of inflation have been used to measure the expected inflation rate. Economists, though, do not represent investors. By training and occupation, the economists included in the surveys are not at all representative of the general financial market participants. Investors may consider some economists' forecasts in making their own determination of what to expect regarding future economic conditions, but many other factors, including their own experience, the counsel of other participants, and recent historical experience, are used to determine their inflation expectations. There is no survey of representative market participants to determine what they truly anticipate for the inflation rate.

The second approach has been to examine actual inflation rates that have occurred, and then subtract those from prior interest rates. This approach is termed an ex-post analysis, since it examines what actually occurred over a particular time interval. This approach is also flawed for several reasons. First, there is no reason to believe that the market is prescient regarding inflation expectations. Especially in the case of an unexpected shock to the system, such as oil price increases during the 1970s, the market does not know what will happen in the future. It cannot even be assumed that errors in forecasting will cancel out over time, since the market could be biased to under, or over, estimate future inflation. Second, actual inflation cannot be measured. The Consumer Price Index and other values commonly used to determine inflation are widely recognized as being inaccurate. These indices track the prices of specific goods and services that are not completely representative of the entire economy. Also, these indices cannot recognize the substitution effect that consumers engage in continually, such as buying more chicken than usual when beef prices rise more rapidly than chicken prices, or driving less when gasoline prices soar. Due to these problems, it is not possible to claim that real interest rates cannot be negative, so a small negative value over a short time interval does not necessarily represent a problem.

On the opposite side, the 99th percentile value for 1 month real interest rates after 10 years is 10%. The same limitations described above also apply to this value.

For 1 year real interest rates, the mean of the first value, one month into the future, is 0.3%. This reflects regression from the initial value of 0 % to the long-term mean of 2.8%. The mean of the last value, after 50 years, is, in line with these parameters, 2.9%. The 1st – 99th percentile range after 10 years is a negative 5.1% to 9.7%, reflecting a similar distribution for the full year as was observed for the monthly values. For the 10 year real interest rates, the means of both the first value is 1.1% and the final value is 2.6%, reflecting the strength of the mean reversion over this long a period of time. The 1st – 99th percentile range after 10 years is negative 3.3% to 7.6%, reflecting the more compact distribution for long-term (10 year) real interest rates, compared to shorter time horizons.

The Funnel of Doubt graphs of 1 month, 1 year and 10 year real interest rates, Figures 1-3, all reflect the same shape, although the scaling differs. These graphs do indeed resemble a funnel with a narrow neck. This pattern occurs because the first 12 points represent monthly intervals, which have small changes in values, and the latter steps are annual or larger intervals, which lead to correspondingly larger changes. The level of uncertainty increases over the entire 50 year time frame, but the shifts toward the end of the simulation period are less pronounced. This shape occurs due to the structure and parameterization of the model. The stochastic term generates the initial spread of the distribution, but the mean reversion impact offsets this tendency, keeping the "funnel of doubt" from expanding further extensively.

Inflation

The next variable of interest is the inflation rate, which is added to the real interest rate to determine the nominal, or observed, interest rate. As shown in Table 1, the mean value of the 1 month inflation rate is 1.1% after only one month, and 4.8% after 50 years. This is in line with the base parameters of 1.0% for the initial inflation rate (qint1) and 4.8% for the long-term mean (qm2). The mean reversion speed for inflation is much lower than the mean reversion speed for real interest rates (0.4 compared to 1.0). After 50 years the monthly inflation rate averages out to be the long-term mean. The 1st – 99th percentile range after 10 years is -5.3% to 14.5%. Inflation can be negative (which would be termed deflation) and small negative monthly values have occurred in recent years. Monthly inflation values in excess of 14.5% did occur during the 1970s.

The mean 1 year inflation rate begins at 1.6% and moves to 4.8% by the end of 50 years, again both in line with the model parameters. The 1st – 99th percentile range of the 1 year inflation rate after 10 years is –3.7% to 12.9%. Although the United States has not experienced deflation over an entire year since 1954, any realistic model must assign a significant probability to this occurring again.

The mean 10 year inflation rate begins at 3.6% and moves to 4.5% by the end of 50 years. These are closer to the long-term mean parameter of 4.8%, but fall below due to the issue of geometric means discussed in the Users Guide. The 1st – 99th percentile range of the 10 year inflation rate after 10 years is 2.0% to 6.9%, demonstrating that, for longer time horizons, the geometric average of inflation is less variable.

The Funnel of Doubt graphs of 1 month, 1 year and 10 year inflation rates are shown on Figures 4-6. The uncertainty of the 10 year inflation rate is much smaller than it is for 1 month and 1 year, reflecting the strength of the mean reversion term for this single factor model. Although inflation varies widely over shorter time horizons, in this model the long-term inflation rate is much less variable. This pattern can be altered by increasing the volatility of the inflation process (qs1) or reducing the mean reversion speed (qk1).

The histograms for the 1 year projected inflation rates and of actual 1 year inflation rates from 1913-2003 (from January to January) are shown on Figure 7. It is readily apparent that the modeled inflation rates generate a nice bell shaped curve, whereas the actual inflation rates are much less smooth. One reason for this difference is that the model results are is based on 5,000 iterations, while the actual data contain only 90 data points. More importantly, though, the projected values are derived from a concise mathematical expression that will produce a smooth distribution of results, but the actual inflation rates depend on the interactions of an almost unlimited number of variables. The key question, though, is whether the model adequately expresses the probability distribution of potential inflation rates. The actual inflation rates are more leptokurtic (fatter in the tails than a normal distribution) than the modeled values, but reflect the central portion of the graph fairly well. All of the large negative inflation rates occurred prior to 1950. Many of the positive outliers are from years prior to 1980, when monetary policy was less focused on controlling inflation. If the user wishes to obtain a distribution for inflation that reflects the long-term experience, then the inflation parameters should be revised to generate a more diffuse distribution.

Nominal Interest Rates

Nominal interest rates are the sum of the real interest rate and the inflation rate. The mean values for 1 month nominal interest rates were 1.1% for the first month and 7.8% for the 50th year. The initial nominal interest rate indicated in the model (1.1%) is in line with the current (June, 2004) level of 1.1%. Since inflation is running in the range of 2.5% (2.3% for the entire year 2003, and at the 2.1% level for the first five months of 2004), the current nominal interest rate reflects either an expected reduction in inflation (or even deflation), a low (or negative) real interest rate, or some combination of these two effects. The 1st – 99th percentile range for the 1 month nominal interest rate after 10 years is 0% to 19.4%. In this simulation, nominal interest rates were restricted to be not negative to generate results in line with economic values.

There has been significant debate over the proper way to deal with negative nominal interest rates in interest rate models. Some modelers have set boundary conditions that prevent nominal interest rates from becoming negative. This is one of the options available in running this model. Other modelers have not been concerned over negative interest rates, either because the mathematical characteristics of the model are more important than the practical applications, or the incidence of negative nominal interest rates is too infrequent to require significant attention.

The occurrence of negative nominal interest rates can be problematical, since the purpose of this research is to develop values that can be used in applications for insurance companies and others. However, artificial constraints on the values do reduce the connection of this model to the research underlying the structure of the interest rate model. The model allows users to set a minimum value of zero for the nominal interest rate, or to accept negative nominal interest rates, depending on the application.

The mean 1 year nominal interest begins at 1.9% and moves to 7.7% by the end of 50 years. The initial value is again in line with the current level of interest rates. The 1^{st} – 99^{th} percentile range of the 1 year nominal interest rate after 10 years is 0% to 18.3%.

The mean 10 year nominal interest begins at 4.6% and moves to 7.1% by the end of 50 years. The initial value is in line with the current level (July 2004) of interest rates for long-term bonds, as 10 year U. S. Treasury bonds have a yield to maturity of 4.4%. The 1^{st} – 99^{th} percentile range of the 1 year nominal interest rate after 10 years is 0.6% to 12.7%.

The Funnel of Doubt graphs of 1 month, 1 year and 10 year nominal interest rates, Figures 8-10, are similar to the real interest rate and inflation graphs, but have a barrier at zero since the restriction that nominal interest rates not be negative is applied in this case. This restriction is illustrated by the 1^{st} percentile line on Figures 8 and 9, but not for the 25^{th} percentile line. The effect of the restriction is not apparent for the 10 year nominal interest rates. The level of uncertainty increases over the 50 year time period used in the forecast. The further into the future the forecast of nominal interest rates is made, the more uncertainty surrounds the estimates. Since the nominal interest rate is determined by adding the real interest rate to the inflation rate, the increasing uncertainty reflected by real interest rates and the inflation rate generates the same behavior for nominal interest rates.

The histograms for the 3 month, 1 year and 10 year model nominal interest rates and the actual 3 month, 1 year and 10 year nominal interest rates are displayed in Figures 11-13. (The 1 month values are not available for historical data consistently over a long enough time period to be relevant. The model can generate 3 month interest rates, which were used for this graph.) The modeled interest rates are influenced by the initial parameter values, particularly the initial real interest rate (rinit1), the initial mean reversion level for the real interest rate (rinit2), and the initial inflation level (qinit1). The influence of these initial values diminishes over time, so histograms of these values selected further into the future would differ from those based on more contemporaneous values. Based on the base parameters selected, the values will increase over time. In this case, the model distributions in Figures 11-13 are based on values after one year.

Significant differences do exist between the modeled and historical distributions for interest rates. In Figure 11, the modeled 3 month nominal interest rates are 0 in almost 20% of the cases, whereas actual 3 month interest rates have never been below 0.5 percent (the column reflecting the 1% bin represents values between 0.5 and 1.5 percent). However, combining the model values for 0 and 1 percent indicates a total very much in line with actual values. In addition, the model distributions are smoother than the actual values, which is natural since the model results are based on 5,000 iterations whereas the actual results, even though derived from 845 (monthly) or 614 (1 and 10 year) observations, are not at nearly as smooth, indicating that the system that generates interest rates is not as straightforward as the model. Modeled interest rates are generally lower than the historical rates, which results from starting the model based on the current level of interest rates, which are lower than historical averages.

For the 1 year interest rates, shown in Figure 12, the modeled rates are notably lower than historical rates. This difference is due to setting the initial values at the current level of interest rates, which are much lower than historical average values. Since the model starts with current interest rates, it projects interest rates that, even one year from now, tend to be lower than historical values.

The comparison between the 10 year modeled rates and the 10 year historical rates, Figure 13, indicates a few differences. The modeled interest rates are more compact than actual 10 year interest rates have been. If the user feels that the variance of the model values should be closer to the historical distribution, then the strength of the mean reversion factor in the interest rate model can be reduced, but this would increase the incidence of negative interest rates unless the user selects to avoid negative nominal interest rates. The other significant difference is the skewness. The historical rates exhibit positive skewness, but the modeled rates have a slight negative skewness. Finally, the model rates are lower than historical values, again due to starting with the current low levels of interest rates.

Stock Returns and Dividends

The values for large and small stock returns indicate, as expected, higher average returns and greater variability for small stocks. As shown on Table 1, the mean of the initial values (after one year) of large stocks is 8.7% and of small stocks is 13.4%. The mean of the large stock values increases to 11.6% at the end of 50 years and for small stocks increases to 13.6%. The 1st – 99th percentile range after 10 years is –15.9% to 29.6% for large stocks and –15.9% to 39.7% for small stocks.

The Funnel of Doubt graphs, Figures 14 and 15, indicate an inverted funnel, compared to the displays of interest rates and inflation. This means that uncertainty reduces over time and is due to the way the values are calculated. The values listed for large and small stock returns are those projected to be earned over the relevant time periods, shown in annual terms. The 1 year values are returns over a one year period. The 10 year values are those earned over a ten year period, etc. Since the individual draws are independent, the variance reduces over longer time periods. Thus, the average annual returns expected over a 50 year period are much more predictable than those for a 1 year period.

Stock returns are determined based on a random draw from one of two distributions. For both large and small stocks, two regimes exist. For large stocks, the mean values in each regime are 0.8% monthly return for regime 1 and -1.1% monthly return for regime 2 with volatilities of 3.9% for regime 1 and 11.3% for regime 2. For small stocks the mean values are 1.0% and 0.3% and the volatilities are 5.2% and 16.6% for the two regimes. Which regime applies during any month is determined randomly, based on indicated switching probabilities. The regimes switches are correlated, so if large stocks are in the low volatility regime, then small stocks are more likely to be in the low volatility regime as well.

Histograms of the 1 year returns for the large (Figure 16) and small (Figure 17) stock returns as generated by the model are displayed, along with actual 1 year returns for 500 large stocks for 1871-2004 and small stock returns over the period 1926-2003. The large stocks are based on the S&P 500 (or a sample chosen to behave similarly for the years prior to the construction of the S&P 500). The data are available online at a website generated by Robert Shiller, author of *Irrational Exuberance* (http://www.econ.yale.edu/~shiller/data/ie_data.htm). The small stock values are based on Ibbotson's *Stocks, Bonds and Bills*. The graphs for large stocks (Figure 16) are relatively similar, although, as would be expected, the results of the 5,000 iterations of the model produce a smoother distribution. The histograms for small stocks (Figure 17) show that historical values have been more variable, with a notable outlier at 190% return, which represents a single observation. The model values also have single observations around that level, but no one bin produces as large a proportion of the outcomes as the one occurrence out of 78 years of the historical experience to be as obvious on the graph.

The dividend yield for equities is 1.5% for the first year and 2.3% for the last year values. The 1^{st} – 99^{th} percentile range after 10 years is 0.6% to 3.9%. The Funnel of Doubt graph of the dividend yield, Figure 18, increases over time as interest rates and inflation do. Figure 19 displays the histogram of the modeled dividend yields and the actual dividend yields over the period 1871-2003, based on data available from Robert Shiller. Historically, dividend yields have varied more widely than the model predicts and have been centered at a higher level. This is the result, in part, of a long-term decline in dividend yields. According to Peter Bernstein (*Against the Gods*, p. 183) prior to the late 1950s dividends tended to be higher than interest rates on corporate bonds. This was based on the understanding that stocks were riskier than bonds and therefore should pay a higher return. Since 1959, though, dividend yields have tended to be lower than interest rates, ranging from 1.1% to 5.4%, which is more in line with the model.

Unemployment and Real Estate Returns

The mean value of the unemployment rate, as shown on Table 1, begins at 6.0% and increases to 6.1% (which is the long-run mean value) for the end of 50 years. The 1^{st} – 99^{th} percentile range after 10 years is 3.5% to 8.7%. Figure 20 shows the Funnel of Doubt graph, neither increases over time (as interest rates and inflation do) nor decreases (as stock returns do). The unemployment rate is modeled as a one factor mean reverting value, with the initial unemployment rate (uinit) set at 5.0%, the mean reversion level (um) set at 6.1% and the monthly mean reversion speed (uk) set at .132 in the base parameters. By the end of 1 year (the starting point for this graph), the mean value has attained the 6.0% level and the spread has reached the long-run level as well, resulting in a stable funnel. The unemployment rate also has a negative correlation with changes in inflation. The histogram of modeled unemployment rates, along with the distribution of actual values over the period 1948-2003 are shown in Figure 21. The historical values represent the unemployment rate each January from 1948-2004. By selecting only a single unemployment rate from each year, the frequency of the historical values corresponds with that of the model values, which are the unemployment rates indicated after the first year of the model run. Although the actual unemployment rates have varied a bit more than the model results do, the distributions are relatively close.

Real estate returns are the final variable included in this model. From Table 1, the mean value of real estate returns is 8.1% in the first year and 9.4% after 50 years. The 1st – 99th percentile range after 10 years is 3.0% to 16.1%. The Funnel of Doubt graph, Figure 22, is similar to the returns on stocks, for the same reasons. The histograms of modeled results and the actual returns based on the National Index from National Council of Real Estate Investment Fiduciaries (NCREIF) for 1978-2003 are shown on Figure 23. The model values show a smooth distribution that is centered about the historical returns. Unfortunately, only 26 years of annual returns are available, so it is difficult to draw any conclusions on the fit.

Correlations

Table 2 displays the correlation matrix for all the output variables discussed above 1 year into the simulation (row 16). Table 3 displays the correlations matrix for large and small stock returns, 3 month, 1 year and 10 year treasury interest rates and inflation over the period April 1953 – December 2001, based on the Ibbotson data (for stocks) and St. Louis Federal Reserve Data for interest rates and inflation. Several conclusions can be drawn about the validity of the model based on a comparison of the two correlation matrices. First, the historical correlation between large and small stocks is .744. The correlation between the model values of large and small stocks is .698, which looks quite reasonable.

The correlation between inflation and T-bills has been .593 historically. This correlation is also clearly reflected in the model values, with a correlation of .906 between the one month inflation rate and the 1 month nominal interest rate, .892 between the 1 year inflation rate and the 1 year nominal interest rate, and .617 between the 10 year inflation rate and the 10 year nominal interest rate. Since the nominal interest is the sum of the real interest rate and the inflation rate, and the real interest rate is constrained to be no less than the negative of the inflation rate, this correlation is built into the model.

Historically, T-bill rates and stock returns have been negatively correlated (-.078 for large stocks and -.065 for small stocks). In the model, there was a slight positive correlation between the 1 year nominal interest rate and stock returns (.099 for large stocks and .087 for small stocks). Also, the historical correlation between inflation and stock returns has been negative (-.138 for large stocks and -.100 for small stocks). The correlation in the model values between the 1 year inflation rate and large stocks was .089 and .076 for small stocks. Although the signs of the correlations differ, these small values are not particularly significant.

Summary

This model provides an integrated framework for sampling future financial scenarios which represent a reasonable approximation to historical values. The model should prove useful for a variety of applications, including dynamic financial analysis, dynamic financial condition analysis, pricing embedded options in policies, solvency testing and operational planning. The model produces output values for interest rates, inflation, stock and real estate returns, dividends and unemployment. However, there are several areas that may be a cause of concern to some users. The model inflation rates have fewer outliers than long-term historical experience. Users should decide if the recent inflation experience is considered to be the best predictor of future inflation, or if the parameters should be changed to reflect these expectations.

During the Depression, unemployment rates in the United States were around the 25 percent level. Since 1948, rates have been much lower, and the model values correspond fairly closely with this recent experience. However, users of the model may want to establish parameters for the unemployment model that reflect earlier, and more volatile, experience.

The base parameters provide one feasible set of values to use in modeling future economic conditions. These should be viewed as a starting point in these applications. However, users should develop an understanding of the impact of the different parameters and then adjust these parameters as necessary to generate distributions that are suitable for the particular applications of the model.

Section 8: Conclusion and Acknowledgements

The key elements and deliverables of this research, as documented in this report, include:

- A **literature review** (Section 4), which summarizes relevant literature from the fields of actuarial science, economics, and finance.
- A **summary of data sources and methodology** (Section 5), which provides the analytical underpinnings of our work, and provides the foundation for updating this research with respect to future data emergence.
- A **Financial Scenario Model** (Appendix D), programmed in Excel and @Risk, and designed to be used by actuaries, after downloading from the CAS or SOA website, to simulate integrated future economic and financial time series for DFA, cash flow, or other purposes.
- **Simulated financial scenario data** (Appendix C), based on default model parameter assumptions, which can be used in lieu of the model if @Risk software is not available.
- A **User's Guide** (Appendix A) to the model, to make it as consumer-friendly as possible.
- A **discussion of results** (Section 7) from sample implementation of the model.

We wish to acknowledge and thank the Casualty Actuarial Society and the Society of Actuaries for sponsoring this research, and for allowing us to participate in this worthy endeavor. In particular, guidance and comments from members of the CAS Committee on Theory of Risk and the DFA Committee, and the SOA Committee on Finance Research, were greatly appreciated, and served to significantly enhance our analysis and the deliverables from this project. Special thanks go to Phil Heckman and Steve Siegel, who acted as our key contacts with the CAS and SOA, respectively.

Appendix A: User's Guide to the Financial Scenario Model

The purpose of the Financial Scenario Model is to develop projections for future economic conditions and related financial data on an integrated basis, explicitly accounting for relationships between key variables. The model is intended to be a useful tool, general enough to pertain to a variety of actuarial applications including, but not limited to: dynamic financial analysis, cash flow testing, solvency testing, stress testing, reserving, and pricing.

This user guide documents the use of the Financial Scenario Model and includes a description of (1) the layout of the model, (2) the steps required to perform a simulation, and (3) a sensitivity analysis of the parameters.

PLATFORM: EXCEL AND @RISK

The Financial Scenario Model is an Excel spreadsheet that benefits from the use of a simulation software package called @RISK available through Palisade Corporation (www.palisade.com). @RISK leverages the simplicity of spreadsheets and integrates powerful analysis tools that help to randomly select future scenarios and examine risk in a stochastic financial environment. @RISK allows users to define variables as a distribution, then randomly select from these assumptions. Sampling from the distribution allows a modeler to understand the impact of risk on key financial results.

There are several benefits that @RISK provides over a stand alone spreadsheet. First, @RISK allows input variables to have explicit, user-defined distributions. The creator of an @RISK simulation project can choose from a variety of distributional assumptions, from common alternatives such as the normal and uniform distribution, to more complex distributions, which are difficult to simulate using only standard Excel functions, such as the beta, exponential, and Weibull distributions. @RISK also allows the modeler to define his own distribution and even select the distribution to best fit available empirical data.

A second benefit of @RISK is that the software can more easily capture the correlation of dependent variables. While correlating multi-normal distributions can be easily accomplished through direct means, programming correlations of other distributions can be more difficult. @RISK includes an explicit correlation matrix that allows the user to input key relationships, regardless of the distribution of the dependent variables. After the user defines a correlation matrix for use in a simulation, @RISK also checks to be sure that joint correlations are internally consistent.

Finally, a significant benefit of @RISK is the ability to capture, study, and report simulation results. @RISK allows users to use familiar spreadsheet models and makes it easy to track key output cells. However, @RISK's power is in the ability to analyze output and create a variety of reports for effective communication of any risk analysis.

NAVIGATING THE MODEL

As mentioned in the main report, the following variables are included in the Financial Scenario Model and projected forward for 50 years:

- Inflation
- Real interest rates
- Nominal interest rates, which are implied from the processes for inflation and real interest rates
- Large and small stock returns
- Equity dividends
- Returns from real estate investments
- Rate of unemployment

In projecting these series, the Financial Scenario Model is set up in eight worksheets. Each of the worksheets is described here.

(1) *@RISK Correlations* – The correlation matrix on this sheet determines the joint dependency of the underlying modeled variables. See the discussion in the sensitivity analysis section below to see the effects of changing the default correlation values.

(2) *Scenarios* – Users of the financial scenario model can define specific scenarios in several of the variables including interest rates, inflation, and equity returns. Creating user-defined scenarios begins on the *Scenarios* worksheet. Specific guidelines on inputting scenarios are described later in this user guide.

(3) *ModelInput* – This sheet is the main area for user input. Users must also enter information about the current environment (interest rates and inflation) and make any desired changes to the default parameters of the model before beginning the simulation.

(4) *StochProcs* – This sheet provides the details behind the month-by-month projections of each of the financial variables included in the model. The parameters from the *ModelInput* directly impact the time series shown on *StochProcs*. These details provide the basis for the model's output shown on the output sheets (see (5) and (6) below).

(5) *OutputIntRates* – Given the central importance of interest rates in the Financial Scenario Model, output for interest rates are shown on a separate sheet. Nominal interest rates are determined by the combination of inflation and real interest rates; all three of these variables are shown on *OutputIntRates*. Each row represents a projection date and shows the resulting term structure (e.g., nominal interest rate by maturity).

(6) *OtherOutput* – In addition to projections of interest rates as shown on *OutputIntRates*, returns on stocks (small and large), dividend yields, real estate returns, and unemployment are shown on the *OtherOutput* sheet (by projection date).

(7) and (8) *IntRateChart* and *InitTermStructure* – After the user initializes the Financial Scenario Model on the *ModelInput* page, these two sheets show the

implied starting term structure graphically (*IntRateChart)* and in tabular form (*InitTermStructure)*. From this initial (implied) term structure, the model begins to project future scenarios.

PERFORMING PROJECTIONS

To simulate outcomes under the Financial Scenario Generator, there are four basic steps:

1. Initializing the model, adjusting any parameters, and specifying any scenarios of relevance to the modeler,
2. Indicating the important variables that form the basis for analysis (choosing output variables),
3. Selecting the simulation settings and performing the simulation, and
4. Viewing the output reports.

Each of these steps is described below.

Step 1: Parameter Selection

First, on the *ModelInput* worksheet, the user must initialize the model to the current interest rate environment. Users must include a recent measure of inflation, as well as current yield information on default free securities (Treasury yields). Some sources of this information include the Wall Street Journal, Yahoo! Finance (bonds.yahoo.com/rates.html), or the CNN/Money Magazine web site (money.cnn.com/markets/bondcenter/). This initialization determines the starting values when projecting nominal interest rates, inflation, and real interest rates.

The default parameter values included on the rest of the *ModelInput* sheet were selected based on an analysis of historical data. Where possible, the use of public information accessible through the World Wide Web was chosen to parameterize these processes (see Section 5 of the report for a description of this data, their sources, and the methodologies used to isolate the default parameters). If the user wishes to alter any of these assumptions, he can put the new parameter value in the *ModelInput* worksheet. To better understand the relationship between individual parameters and the resulting output, see the sensitivity analysis section below.

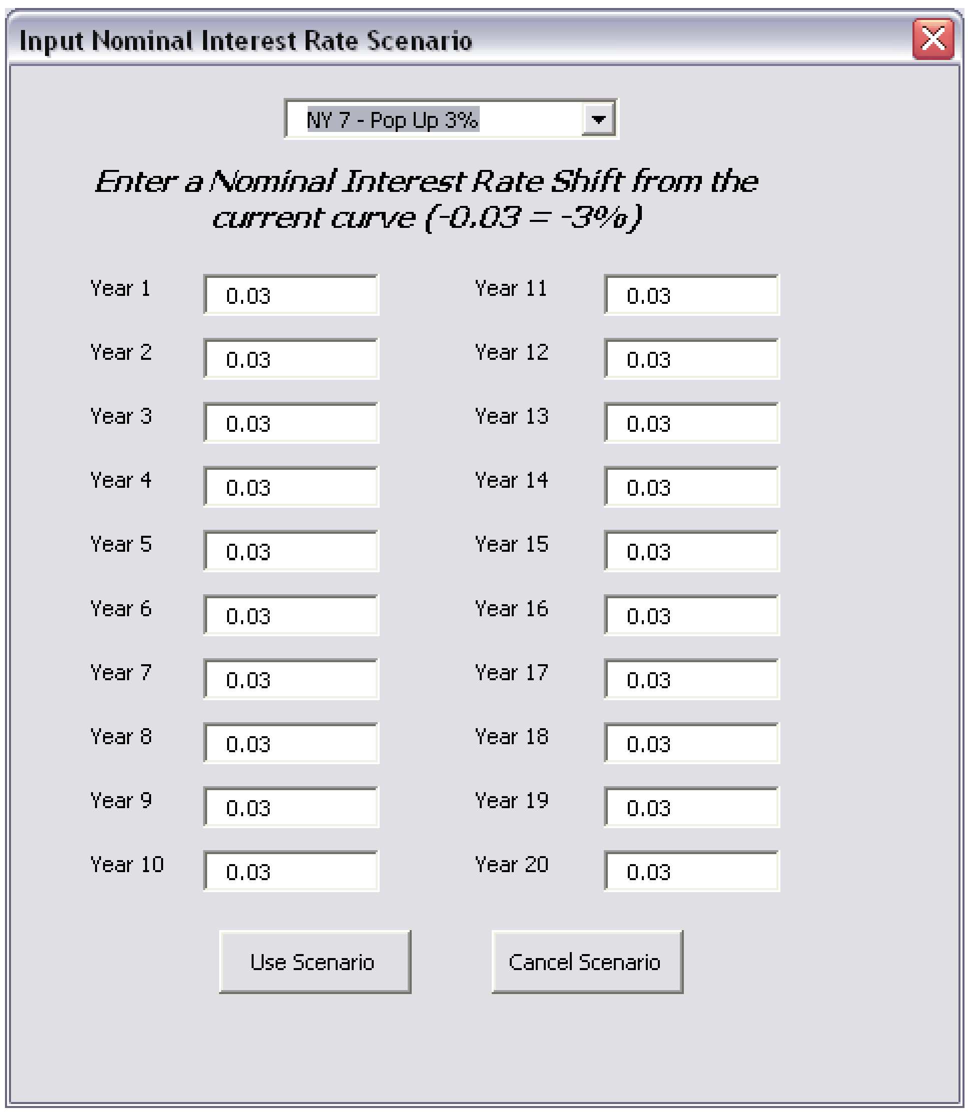
In addition to changing the parameters of the individual processes (if desired), the "Model Input" sheet includes check boxes for the following options:

- *Placing lower bounds on the level of inflation and real interest rates*. When placing lower bonds on inflation or real interest rates, the model projects these processes normally, and simply chooses the maximum of the lower bound and the resulting simulated value.
- *Eliminating the potential for negative nominal interest rates.* Recall that the realized nominal interest rate is the combination of inflation and real interest

rates. If negative nominal interest rates are not allowed, the process for inflation is not affected, but the model effectively puts a lower bound on resulting real interest rates so the resulting nominal interest rate is not negative.

In addition to the assumptions of lower bounds, users may be interested in isolating the effects of specified patterns of financial variables. For example, the user may be interested in specifying a path for interest rates over the next 30 years while allowing all other variables to remain stochastic. The *Scenarios* sheet provides the opportunity for these situations.

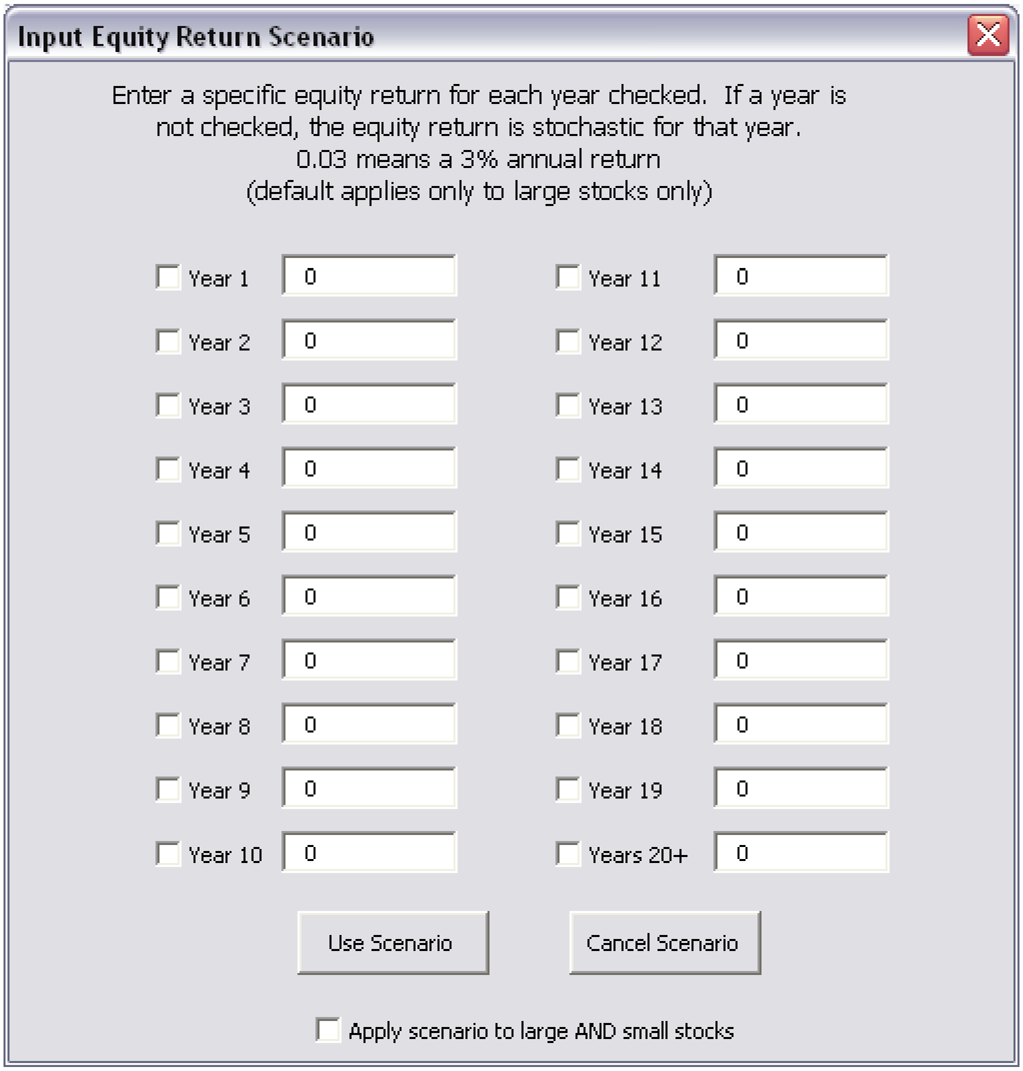
When the interest rate scenario box is checked, the following popup form allows the user to specify the path based on changes from the existing term structure. (NOTE: The interest rate scenario popup form specifies the percentage point change, in each of the subsequent simulated years, relative to the *initial* term structure implied by the model. If the user wishes to view the initial term structure, see the *IntRateChart* sheet.)



Note that the drop down box at the top of the form includes seven scenarios related to the New York Regulation 126. In low interest rate environments, the model automatically adjusts NY 126's decreasing interest rate scenarios by incorporating lower bounds. "Custom Scenarios" are any other interest rate movements desired by the user.

When specifying interest rate scenarios, the inflation process remains stochastic (similar to the case where negative nominal interest rates are prohibited). The model then backs into the real interest rate, based on the user-defined nominal interest rate less the modeled value for inflation.

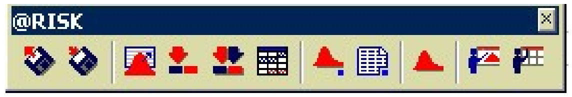
For equity return scenarios, users select the years for which they would like to specify equity returns (see the popup form below). The check box in front of each year indicates a user-supplied equity return. For those years that are not checked, equity returns are stochastic based on the model's parameters. As an example, one may be interested in the impact of a drop in the stock market of 50% in the tenth projection year. In this case, the user will check the box in front of "Year 10", and put -0.50 in the accompanying text box. Note that, by default, a user-selected return scenario applies only to large stocks; should the user be interested in applying the scenario to the entire stock portfolio (large *and* small stocks), s/he can check the box at the bottom of the popup form.

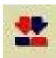


Inflation scenarios specify a rate of inflation over the next 50 years. A word of caution when using inflation scenarios: the user should be sure to understand the relationship between inflation scenarios and the assumed mean reversion level of inflation on *ModelInput*. Section 5 of the report discussed the link between long-term interest rates and long-term expectations of inflation. The latter is based on a term structure of inflation which is derived from the model's assumption of the long-term rate of inflation. The inflation scenario specifies the path of the *short-term* inflation rate, but long-term inflation expectations are a function of the current (short-term) inflation rate and the mean reversion level. The user is cautioned about potential inconsistencies between scenario-specific inflation rates and the assumed level of mean reversion.

Step 2: Choosing the Output Variables and @RISK Settings

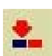
There are several @RISK settings that need to be adjusted before running a simulation. The @RISK Toolbar shown below contains shortcuts to all of the important functions performed in a simulation.



*List input and output*

Input cells are spreadsheet cells that contain specialized @RISK functions such as distribution definitions. In the financial scenario model, there are thousands of defined distributions; given that the model does projections for seven distinct financial series, each month for over 50 years, there are over 4,200 input cells in the model (12 months x 50 years x 7 variables). Output cells are user defined cells that become the focus for risk analysis. Any cell that the user wishes to study is an output cell. See the instructions immediately below to define output cells.

The input and output listing allows the user to view the input and output cells that will be tracked during a simulation. Users may also define names for individual input or output cells, or groups of cells, to make reports generated by @RISK easier to read. For example, in the 5th projection year, the nominal term structure from the model (1-month, 3-month, 1-year, 3-year, 5-year, 10-year, and 20-year spot rates) may be named “5yr projected nominal”.

 *Adding an output cell*

Adding an extra output cell is simply a matter of locating the cell (or group of cells) of interest in the model’s spreadsheet and adding it to the output list by clicking the “Add Output” button (illustrated above).

Users should not feel constrained to the cells already shown in the worksheet. Additional variables may be defined on the spreadsheet using standard Excel functions (or @RISK functions) of variables that are already tracked by the Financial Scenario Model. For example, the slope of the yield curve can be defined in any number of ways (e.g., the difference between the 1- and 10-year bond yields or the difference between the 1-month and 5-year yields). Users can define the variable of interest using the formula in Excel and then add the defined variable to the output list. In addition, if users incorporate the Financial Scenario Model as an engine for other models (such as a dynamic financial analysis model), one may define the output cell as the scenario-specific insurer surplus.

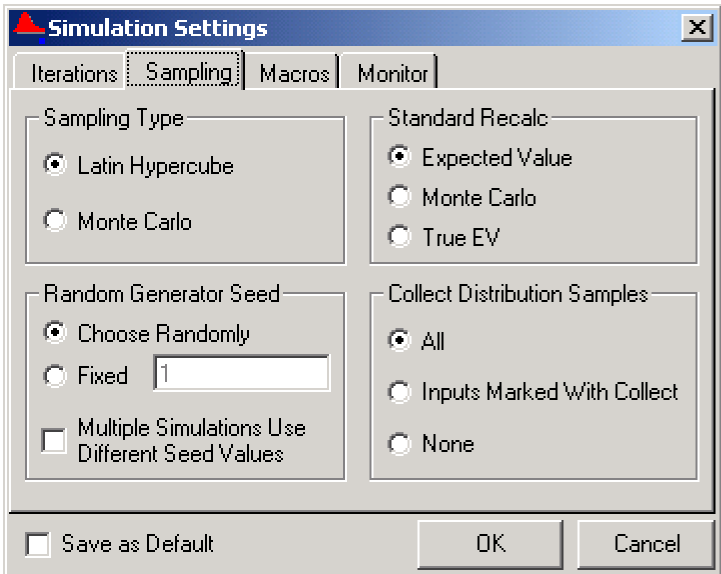
NOTE: When working with any user-defined scenarios described above (including lower bounds), @RISK may delete previously captured output variables. After the defining or canceling scenarios, it is best to list the input and output to be sure the relevant variables are still in the output list. Users may need to redefine the output cells if @RISK has eliminated it from the list. In fact, before running any simulation, it is good practice to review the output list to be sure the simulation captures all desired output cells. Nothing is more frustrating than running an hour long simulation, only to discover that @RISK has inadvertently deleted one of the variables in which you were interested!

<u>Step 3: Simulation Settings and Running a Simulation</u>

Simulation settings

Simulation settings provide information about how projections will be performed. After selecting the simulation setting button, the number of iterations (sample paths) must be selected. If the number of iterations selected is 1,000, the model projects 1,000 different paths for each of the financial variables defined. Depending on the speed of the computer and the size of the model, the user may be interested in more or fewer paths.

The screenshot below shows the Sampling tab of the Simulation Settings window in @RISK.



Sampling Type

For more efficient sampling of the input cells, the @RISK sampling type should be set to Latin Hypercube. Monte Carlo sampling often requires a larger number of iterations for the resulting sample distribution to converge to the true distributional assumption. The poorer performance of Monte Carlo sampling is especially pronounced if the distribution is slightly skewed or there are outcomes with very low probability. Latin Hypercube sampling, a type of stratified sampling technology, more accurately recreates true distributions with fewer iterations. The importance of sampling approaches for insurers selecting interest rate scenarios is fully discussed in Chueh (2002).

Random Generator Seed

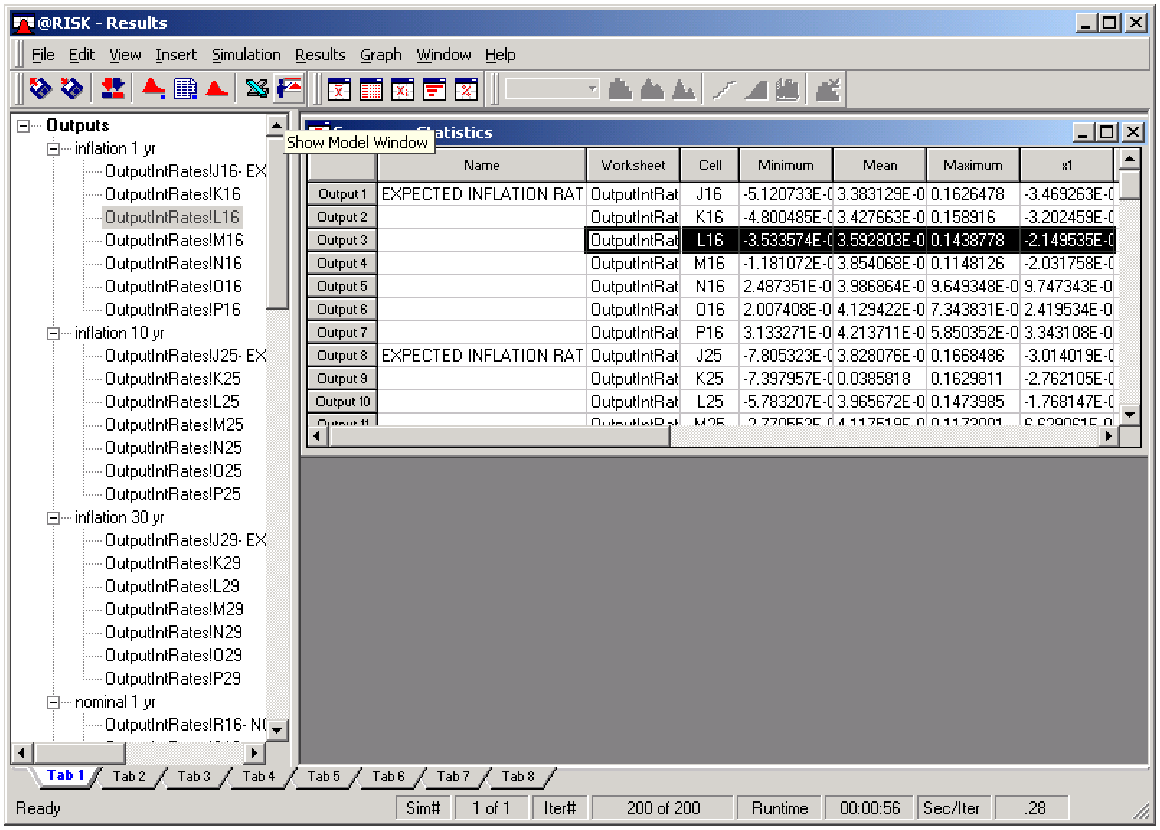
The seed value for the random number generator is useful when looking at the sensitivity of projections to initial values and model parameters. The seed assures the same set of random numbers will be used for each simulation tested in order to isolate the effects of alternative initial conditions. If the user wants a set of independent results, the seed should be changed or chosen randomly.

Collect Distribution Samples
Most users will be most interested in analyzing only the output cells; the random draws and distributions of the input cells are trivial. To reduce the amount of data that is stored and analyzed by @RISK (and correspondingly, to reduce processing time), one can bypass the accumulation of the input data. Selecting "None" will speed up the generation of output reports and statistics.

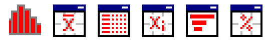
Start the simulation
Starting the simulation will perform the number of iterations defined in the Simulation Settings. After initializing all of the input cells, @RISK will provide an update of the timing of this simulation based on the number of iterations that have been performed. Including the time required for the projections, as well as the processing time required to display output statistics on the screen, 1,000 iterations takes about 10 minutes on a 1GHz PC. The time increases as the number of output cells increases.

Step 4: Reviewing Output and Creating Reports

Results window
After @RISK completes a simulation, users will be shown the results window displaying summary statistics for all of the output cells (and input cells if chosen in the simulation settings as mentioned above). If the user returns to Excel, he can get back to the @RISK results window by clicking on the button as illustrated above.



@RISK's Results Toolbar

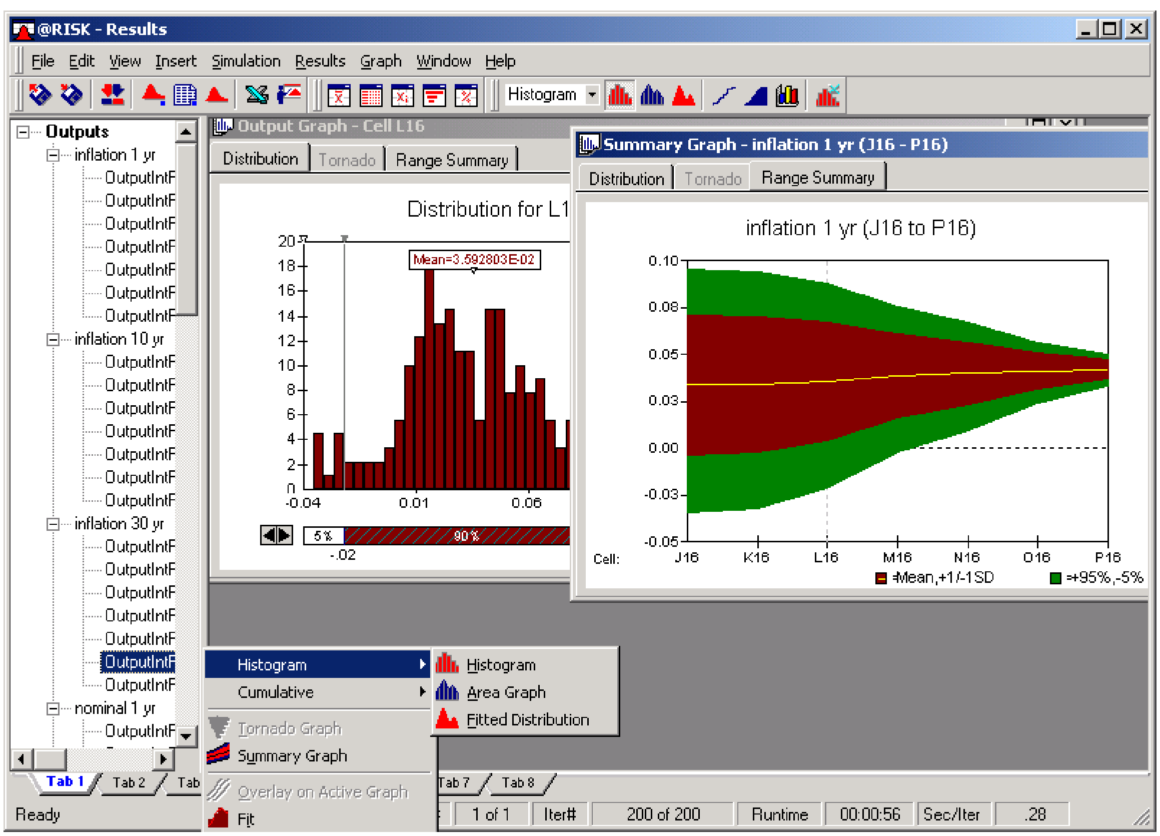


The results window then allows users to review detailed output statistics, illustrate results using @RISK's graphing capabilities, and create reports. The following discussion is not meant to exhaust the abundant and powerful reporting capabilities of @RISK. Rather, we only try to highlight some of the basic reporting features.

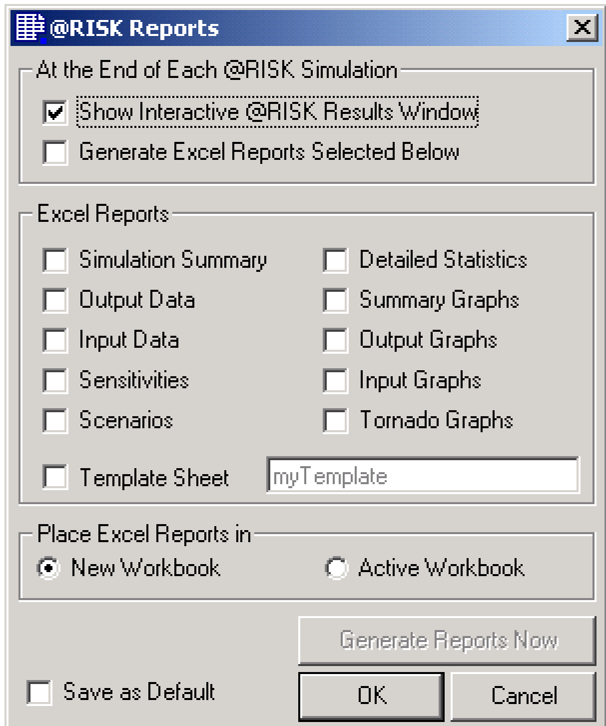
 *Summary statistics*. The default screen for the @RISK results window is summary statistics. Basic statistics for all of the output cells are shown to give the user a simple snapshot of the simulation results.

 *Detailed statistics*. These sample statistics provide more information than in the summary window including standard deviation, variances, skewness, kurtosis, and the distribution percentiles. In addition, @RISK allows the user to ask scenario-based questions like what percentage of the time is a particular output cell negative.

 *Creating Graphs*. Two standard graphs in @RISK are histograms and summary graphs. Summary graphs have been referred to as "funnel of doubt" graphs in the results section. The following screen shot shows the results window with these two types of graphs.



*Generating output reports*
By creating output reports, the user of the model can dump useful information directly into Excel for additional analysis, formatting, printing, or saving. The reports can dump the summary or detailed statistics in Excel, generate graphs in Excel format, or even dump all of the output for additional analysis in Excel. The following screen shot shows the @RISK Reports window:



*Saving simulations (Icons from the @RISK Toolbar or Results Window)*
Users have two options when trying to save a simulation for future reference: dump the data into an Excel file or save the simulation in @RISK's file format (.rsk files). When a simulation is saved, all of the input and output cells, data, and simulation settings are retained for later use.

SENSITIVITY ANALYSIS

An important aspect of any projection of financial results is potential model risk. The Financial Scenario Generator is not purported to be a perfect predictor of future economic conditions. To the contrary, the model explicitly recognizes the lack of predictability of future economic conditions by generating a distribution of alternate business environments. Since the magnitude of uncertainty is based on specific assumptions, actuaries need to be aware of the dependency of projections on these assumptions.

This section looks at sensitivity analysis for some of the key financial variables to give users of the model some guidance on selecting parameters and understanding their influence on simulation results. For each variable, projections are presented similar to the results and tables section of the main document. However, this user guide will focus its attention on choosing alternative values of the model's parameters to illustrate the changes. These sensitivity results are provided to gain insight into the workings of the model. It is hoped that this documentation will make the model more usable and friendly, increasing the model's ability to be customized and provide a platform for a wide variety of user-defined applications.

It should be noted that the values in this simulation do not (exactly) match the results presented in the main document since the results presented here are based on fewer iterations than were presented in the main document.

Inflation

Inflation is allowed to fluctuate around some average level of inflation based on the following mean-reverting process (an Ornstein-Uhlenbeck process):

$$dq_t = \kappa_q(\mu_q - q_t)dt + \sigma_q dB_q$$

The rate of inflation reverts toward a long-term mean μ_q. When the current level of inflation (q_t) exceeds this long-term average, the model predicts that, on average, inflation will fall in the future. This is represented in the first term of the above equation, called the drift; if inflation is high, the first term is negative. How quickly inflation moves back to its reversion level depends on the speed parameter κ_q.

While the inflation process presented above is in continuous time, the model simulates this process in monthly time steps according to the following:

$$\Delta q_t = \kappa_q(\mu_q - q_t)\frac{1}{12} + \sigma_q\sqrt{\frac{1}{12}}\varepsilon_q$$

The default parameters of the process are show below:

Base Case Inflation Parameters

0.400	mean reversion speed (κ_q)
0.040	volatility of inflation (σ_q)
0.048	long-term mean reversion level (μ_q)
-0.020	lower bound for short-rate inflation
0.025	initial inflation level (q_0)

Instead of looking only at current level of inflation, the financial scenario model develops a term structure of inflation. The term structure of inflation is important since investors pay prices for long-term bonds based on their expectations of inflation over the bond's lifetime. To determine the term structure of nominal interest rates, (expected) long-term inflation is required for determining the inflation premium on long-term bonds, which is an important part of nominal interest rates.

Using the default parameters, a simulation was performed using 200 iterations and the results of the projections are shown below:

Base Case – Inflation Statistics

Value	*Mean*	*Std. Dev.*	*5th Percentile*	*95th Percentile*
1st Projection Year				
1-Year Inflation	3.59%	3.15%	-2.15%	8.79%
10-Year Inflation	4.13%	0.94%	2.42%	5.68%
10th Projection Year				
1-Year Inflation	3.97%	3.74%	-1.77%	10.50%
10-Year Inflation	4.24%	1.11%	2.53%	6.19%

As can be seen from the statistics, the one-year inflation rate does tend toward the mean reversion level of 4.8%. The initial inflation level is 2.5%, so after one year (the start of the first projection year) the mean projected inflation rate is 3.59%, and in the 10th projection year the average level of inflation increases to 3.97%. The simulation results also show that the uncertainty in inflation increases slightly over time, since the standard deviation in the first year is 3.15% vs. the tenth projection year, where the standard deviation is 3.74%. The range in the one-year inflation rate, or the difference in the 5th and 95th percentiles, is also higher in the 10th projection year.

The 10-year inflation rate included in the table above is determined from the short-term level of inflation combined with the assumed underlying process for future inflation movements. Because the parameters of the inflation process exactly determine the expected path of inflation over the next 10 years, we can use the results of Vasicek (1977) to derive the term structure of inflation and determine long-term rates of inflation as a function of the current value and the parameters of the process.

From the above table, it is seen that the 10-year inflation rate also exhibits the tendency toward the mean reversion level of 4.8%; after the first projection year, the 10-year rate of inflation is 4.13% and in the tenth projection year, the rate is 4.24%. But because the 10-year rate at any point in time is based on an *expected* path of inflation of the next 10-year period, the volatility of the 10-year rate is lower than the 1-year rate of inflation.

The shape of the term structure of inflation is influenced by the parameters in the following ways:

- The short end of the term structure of inflation is the current inflation rate (q_t)
- Since inflation tends toward some mean reversion level, the end of the curve is closely related to this mean reversion level (but see how volatility affects the long end of the curve below)

- If mean reversion speed is high, the term structure of inflation quickly approaches the mean reversion level. If mean reversion speed is low, the term structure of inflation is much flatter.

To understand the sensitivity of results to the selected parameters, projections were also performed under several alternative scenarios. Increasing the mean reversion speed (κ_q) forces inflation back to its reversion level much faster. The following analysis is based on a simulation where the mean reversion speed was increased from 0.4 to 0.8:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
1-Year Inflation				
κ_q = 0.4 (Base case)	3.59%	3.15%	-2.15%	8.79%
κ_q = 0.8	4.15%	2.29%	0.26%	7.90%
10-Year Inflation				
κ_q = 0.4 (Base case)	4.13%	0.94%	2.42%	5.68%
κ_q = 0.8	4.58%	0.42%	3.88%	5.26%
10th Projection Year				
1-Year Inflation				
κ_q = 0.4 (Base case)	3.97%	3.74%	-1.77%	10.50%
κ_q = 0.8	4.49%	2.46%	0.56%	8.79%
10-Year Inflation				
κ_q = 0.4 (Base case)	4.24%	1.11%	2.53%	6.19%
κ_q = 0.8	4.64%	0.45%	3.93%	5.43%

Because inflation is pulled toward the mean reversion level faster than in the base case, each of the projections of future inflation is closer to 4.8%. In addition, the volatility exhibited in projected inflation is lower than under the base case. Given the presumption of faster mean reversion, the standard deviation and the measure of range is significantly smaller than under the original parameters. In effect, when the mean reversion speed is higher, any deviation from the mean reversion level will be shorter-lived than the case where mean reversion speed is slower. Since disturbances from 4.8% tend to die out more quickly, the volatility of inflation also declines.

The second parameter of the inflation process represents the randomness of movements or *the volatility* from expected. The volatility parameter (σ_q) controls the size of the deviations from the expected movement toward the mean reversion level. The following simulation shows the effects of increasing the volatility parameter from 0.04 to 0.08:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
1-Year Inflation				
σ_q = 0.04 (Base case)	3.59%	3.15%	-2.15%	8.79%
σ_q = 0.08	3.61%	6.30%	-7.88%	14.00%
10-Year Inflation				
σ_q = 0.04 (Base case)	4.13%	0.94%	2.42%	5.68%
σ_q = 0.08	3.20%	1.87%	-0.22%	6.30%
10th Projection Year				
1-Year Inflation				
σ_q = 0.04 (Base case)	3.97%	3.74%	-1.77%	10.50%
σ_q = 0.08	4.02%	7.49%	-7.45%	17.09%
10-Year Inflation				
σ_q = 0.04 (Base case)	4.24%	1.11%	2.53%	6.19%
σ_q = 0.08	3.32%	2.23%	-0.09%	7.22%

All the measures of volatility including standard deviation and the range of inflation are considerably higher than under the base case. Yet the lower value of the projected mean of the 10-year inflation rate (3.32%) may be a bit of a surprise. The projected one-year rate of inflation has approximately the same mean as before (with higher uncertainty), but the projection of the mean 10-year rate appears lower than before. To explain these results, let's look at the development of the long-term rate.

Recall that long-term inflation rates are derived from the (expected) path of inflation over the investment horizon. The longer horizon inflation rates assume frequent compounding. As a result, rates of inflation over long periods (like the 10-year rate) are implied by the geometric average of inflation along the projected path.

To illustrate the effects of increasing volatility on geometric average inflation, consider the following example that calculates the two-year (geometric average) inflation rate under three assumptions of volatility.

Year 1 Inflation	***Year 2 Inflation***	***Two-Year Inflation***
4.8%	4.8%	4.80%
2.8%	6.8%	4.78%
0.8%	8.8%	4.72%

Under the first path, inflation is assumed to be constant (no volatility), so the two-year inflation rate is equal to the same realized rate over each of the two years. Under the second scenario volatility is introduced, but each year, the (arithmetic) mean inflation rate of the distribution is unaffected and equal to 4.8%. The result is that when inflation measured over two years (using geometric average), the multi-year rate is lower than the no volatility case. When volatility is increased further in the third scenario, two-year inflation rate appears even lower. The implication is that by increasing the volatility of inflation, long-term rates, which are compounded over time, appear to decline.

The next simulation decreases the mean reversion level (μ_q) from 4.8% to 3.0%:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
1-Year Inflation				
μ_q = 0.048 (Base case)	3.59%	3.15%	-2.15%	8.79%
μ_q = 0.030	2.78%	3.15%	-2.96%	7.98%
10-Year Inflation				
μ_q = 0.048 (Base case)	4.13%	0.94%	2.42%	5.68%
μ_q = 0.030	2.62%	0.94%	0.91%	4.17%
10th Projection Year				
1-Year Inflation				
μ_q = 0.048 (Base case)	3.97%	3.74%	-1.77%	10.50%
μ_q = 0.030	2.89%	3.74%	-2.84%	9.43%
10-Year Inflation				
μ_q = 0.048 (Base case)	4.24%	1.11%	2.53%	6.19%
μ_q = 0.030	2.66%	1.11%	0.95%	4.60%

As expected, the mean of the projected inflation rates approaches 3%. However, another interesting aspect of lowering the mean reversion level is introduced. In the base case simulation, it appeared that the term structure of inflation was upward sloping. When the mean reversion level was decreased to 3.0%, the term structure of inflation is inverted. The latter case is a result of relatively higher volatility when the mean reversion level is reduced. As explained earlier, higher volatility leads to lower geometric averages for long-term rates and inflation. In the base case, the effects of volatility on the geometric average for long-term inflation was masked by the increasing tendency of inflation. But when the mean reversion level is reduced, volatility effects dominate and the long-term inflation is lower than short rates.

Finally, one last sensitivity analysis on the interest rate process shows how the mean reversion speed (κ_q) and the volatility (σ_q) can work together. The following simulation simultaneously increases κ_q to 0.8 and σ_q to 0.08:

Value	*Mean*	*Std. Dev.*	*5th Percentile*	*95th Percentile*
1st Projection Year				
1-Year Inflation				
$\kappa_q = 0.4, \sigma_q = 0.04$ (Base case)	3.59%	3.15%	-2.15%	8.79%
$\kappa_q = 0.8, \sigma_q = 0.08$	4.15%	4.58%	-3.62%	11.67%
10-Year Inflation				
$\kappa_q = 0.4, \sigma_q = 0.04$ (Base case)	4.13%	0.94%	2.42%	5.68%
$\kappa_q = 0.8, \sigma_q = 0.08$	4.29%	0.83%	2.88%	5.65%
10th Projection Year				
1-Year Inflation				
$\kappa_q = 0.4, \sigma_q = 0.04$ (Base case)	3.97%	3.74%	-1.77%	10.50%
$\kappa_q = 0.8, \sigma_q = 0.08$	4.51%	4.92%	-3.34%	13.12%
10-Year Inflation				
$\kappa_q = 0.4, \sigma_q = 0.04$ (Base case)	4.24%	1.11%	2.53%	6.19%
$\kappa_q = 0.8, \sigma_q = 0.08$	4.35%	0.89%	2.93%	5.92%

From the previous analysis, we saw that increasing mean reversion speed forces the inflation rate more quickly to the mean reversion level of 4.8%. This stronger attraction to the mean reversion level would reduce the standard deviation and the range of the projected inflation rates. However, in this simulation, the volatility parameter is also increased. The increased volatility more than offsets the decrease in uncertainty driven by stronger mean reversion. Viewed another way, the volatility parameter increases the randomness of projected inflation, but higher mean reversion dampens the effects of heightened volatility. The end result is that mean reversion speed and volatility have similar (but opposite) effects on projected inflation.

Real Interest Rates

The process for real interest rates is more complex than for inflation. Not only do short-term rates revert to some mean reversion level, the reversion level itself is stochastic. The (continuous time equivalent) model for real interest rates is as follows:

$$dr_t = \kappa_r(l_t - r_t)dt + \sigma_r dB_1$$
$$dl_t = \kappa_l(\mu_l - l_t)dt + \sigma_l dB_2$$

Short-term real interest rates (r_t) are mean-reverting to a time-dependent level l_t. Although the reversion level is random, it fluctuates around some mean value μ_l. Like the inflation process, the speed of mean reversion of the short- and long-term real interest rates are affected by speed parameters (κ_r and κ_l) and the magnitude of randomness is determined by volatility parameters (σ_r and σ_l). One alternative view of this two-factor model allows the opposite ends of the (real interest rate) term structure to move simultaneously, with some correlation between these two factors.

Base Case Real Interest Rate Parameters

1.0000	mean reversion speed for short rate process (κ_r)
0.0100	volatility of short rate process (σ_r)
0.1000	mean reversion speed for long rate process (κ_l)
0.0165	volatility of long rate process (σ_l)
0.0280	long-term mean reversion level for long rate (μ_l)
0.0100	initial short-term real interest rate (r_0)
0.0250	initial mean reversion level for real interest rate (l_0)
0.5000	correlation between long and short processes

Based on 200 iterations, here are the results of real interest rates under the base case parameters:

Value	***Mean***	***Std. Dev.***	***5^{th} Percentile***	***95^{th} Percentile***
1^{st} Projection Year				
1-Year Real Rate	2.22%	1.18%	0.24%	4.28%
10-Year Real Rate	2.40%	1.02%	0.78%	4.06%
10^{th} Projection Year				
1-Year Real Rate	2.96%	3.42%	-3.13%	8.17%
10-Year Real Rate	2.74%	2.40%	-1.53%	6.37%

Similar to the discussion of the inflation process above, the real rate of interest is pulled toward the mean reversion level. Since the initial short-term real interest rate is 1%, it tends to increase toward the initial mean reversion level of 2.5%. In subsequent projection years, the mean reversion level tends toward 2.8%. The distribution of short-term real interest rates is wider than the distribution for long-term real rates, as measured by the standard deviation and the two percentiles noted.

The sensitivity results presented here isolate individual parameters in the real interest rate process. The simulations are shown to give some introduction to the impact of the various parameters on the resulting simulation results. It should be noted that altering several parameters at once may not have the additive effects of individual parameter changes.

The following table illustrates the projection results when the reversion speed of the short process (κ_r) is increased from 1.0 to 1.5:

Value	*Mean*	*Std. Dev.*	*5th Percentile*	*95th Percentile*
1st Projection Year				
1-Year Real Rate				
κ_r = 1.0 (Base case)	2.22%	1.18%	0.24%	4.28%
κ_r = 1.5	2.40%	1.28%	0.36%	4.55%
10-Year Real Rate				
κ_r = 1.0 (Base case)	2.40%	1.02%	0.78%	4.06%
κ_r = 1.5	2.43%	1.02%	0.78%	4.03%
10th Projection Year				
1-Year Real Rate				
κ_r = 1.0 (Base case)	2.96%	3.42%	-3.13%	8.17%
κ_r = 1.5	2.96%	3.41%	-2.94%	8.09%
10-Year Real Rate				
κ_r = 1.0 (Base case)	2.74%	2.40%	-1.53%	6.37%
κ_r = 1.5	2.72%	2.34%	-1.45%	6.28%

The results show that real interest rates are driven more quickly to the mean reversion levels; note that in the first projection year, the average 1-year interest rate is 2.40% vs. 2.22% under the base case. However, perhaps surprisingly, the volatilities of the different interest rates are not dramatically affected. As with the inflation process, one may have expected that increasing the mean reversion speed would pull real rates more quickly back to their mean reversion levels and reduce volatility.

With the real interest rate process, there are two differences from the inflation process that explains the apparent lack of sensitivity between reversion speed and volatility. First, real interest rates are based on two processes (short *and* long rates), while inflation is based on only one process. Therefore, changing the mean reversion speed of the short rate process will have less of an impact given that the long rate process is unchanged. The relative impact of any parameter changes is a function of existing parameter values. (For example, note the larger impact of the mean reversion speed of the long rate process illustrated below.) Second, the short rate mean reversion speed is initially higher under the real interest rate process than it is for inflation. The impact of reversion speed on volatility is dampened as κ_r increases, especially as κ_r exceeds 1.0.

Increasing the volatility of the short real interest rate process (σ_r) from 0.01 to 0.02 yields the following simulation results:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
1-Year Real Rate				
σ_r = 0.01 (Base case)	2.22%	1.18%	0.24%	4.28%
σ_r = 0.02	2.22%	1.52%	-0.54%	4.83%
10-Year Real Rate				
σ_r = 0.01 (Base case)	2.40%	1.02%	0.78%	4.06%
σ_r = 0.02	2.39%	1.05%	0.63%	4.11%
10th Projection Year				
1-Year Real Rate				
σ_r = 0.01 (Base case)	2.96%	3.42%	-3.13%	8.17%
σ_r = 0.02	2.98%	3.58%	-3.01%	8.64%
10-Year Real Rate				
σ_r = 0.01 (Base case)	2.74%	2.40%	-1.53%	6.37%
σ_r = 0.02	2.73%	2.42%	-1.50%	6.44%

In this case, the measures of volatility all increase for each of the real interest rate projections. But the geometric average effects that were illustrated with inflation are less prevalent when adjusting the short real interest rate process. We will see below that when adjusting the long rate volatility parameters, these effects are more evident.

When adjusting the ***long*** rate reversion speed (κ_l) from 0.1 to 0.2:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
1-Year Real Rate				
κ_l = 0.1 (Base case)	2.22%	1.18%	0.24%	4.28%
κ_l = 0.2	2.23%	1.12%	0.31%	4.21%
10-Year Real Rate				
κ_l = 0.1 (Base case)	2.40%	1.02%	0.78%	4.06%
κ_l = 0.2	2.53%	0.71%	1.35%	3.69%
10th Projection Year				
1-Year Real Rate				
κ_l = 0.1 (Base case)	2.96%	3.42%	-3.13%	8.17%
κ_l = 0.2	2.96%	2.52%	-1.24%	6.73%
10-Year Real Rate				
κ_l = 0.1 (Base case)	2.74%	2.40%	-1.53%	6.37%
κ_l = 0.2	2.77%	1.32%	0.60%	4.85%

In this projection, the long rate moves more quickly toward its mean of 2.8%. This simulation shows that the mean of the 1-year real rate is not significantly different from the base case projections, yet the 10-year rate is closer to the long-term mean reversion level of 2.8%. More strikingly, increasing the mean reversion strength in the long rate process has a considerable effect on the volatility, particularly in the longer projections. In the 10th projection year, the standard deviation of the 1-year rate is 26% lower than the base case (2.52% vs. 3.42% in the base case) and the 10-year rate is 45% lower (1.32% vs. 2.40%).

As another illustration of the sensitivity of real interest rates to the long rate process, the following simulation statistics are calculated by raising the long rate volatility (σ_l) from 0.0165 to 0.03:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
1-Year Real Rate				
σ_l = 0.0165 (Base case)	2.22%	1.18%	0.24%	4.28%
σ_l = 0.03	2.24%	1.90%	-0.79%	5.53%
10-Year Real Rate				
σ_l = 0.0165 (Base case)	2.40%	1.02%	0.78%	4.06%
σ_l = 0.03	2.02%	1.82%	-0.94%	4.94%
10th Projection Year				
1-Year Real Rate				
σ_l = 0.0165 (Base case)	2.96%	3.42%	-3.13%	8.17%
σ_l = 0.03	3.18%	6.12%	-8.06%	12.32%
10-Year Real Rate				
σ_l = 0.0165 (Base case)	2.74%	2.40%	-1.53%	6.37%
σ_l = 0.03	2.48%	4.35%	-5.29%	9.05%

When compared to the simulation where short rate volatility is increased, the result here shows that changes in the long rate process significantly impacts the volatilities of real interest rates. The standard deviation of each of the real interest rates is significantly higher than the base case and the range in percentiles is substantially wider. In addition, the geometric average effects that were noted when discussing the inflation are magnified. When the volatility of the short rate process was increased these effects were not as evident.

Finally, the last simulation adjusts the long-term mean reversion level for real interest rates (μ_l), from 2.8% to 1%.

Value	*Mean*	*Std. Dev.*	*5th Percentile*	*95th Percentile*
1st Projection Year				
1-Year Real Rate				
μ_l = 0.028 (Base case)	2.22%	1.18%	0.24%	4.28%
μ_l = 0.01	2.09%	1.18%	0.12%	4.16%
10-Year Real Rate				
μ_l = 0.028 (Base case)	2.40%	1.02%	0.78%	4.06%
μ_l = 0.01	1.74%	1.02%	0.12%	3.39%
10th Projection Year				
1-Year Real Rate				
μ_l = 0.028 (Base case)	2.96%	3.42%	-3.13%	8.17%
μ_l = 0.01	1.86%	3.42%	-4.23%	7.07%
10-Year Real Rate				
μ_l = 0.028 (Base case)	2.74%	2.40%	-1.53%	6.37%
μ_l = 0.01	1.40%	2.40%	-2.86%	5.03%

Each of the projected real interest rates tends toward the lower mean reversion level. The mean of the real rates is below the base case scenario, but the measures of volatility are similar. In essence, we're simply shifting the center of the distribution of the projected real rate.

Nominal interest rates

Investors demand compensation for the time value of money as well as any erosion of purchasing power. Therefore, projected nominal interest rates in the financial scenario generator are based on the combination of real interest rates and inflation. For each month during the next 50 years, the model produces nominal interest rates for seven key maturities: 1-month, 3-months, 1-year, 3-years, 5-years, 10-years, and 20-years.

Given the illustrations and sensitivities presented above for inflation and real interest rates, the following table shows how the effects of each of the changes in the parameters on nominal interest rate projections compared to the base case.

Summary of Parameter Sensitivities on Nominal Interest Rates

Value	*Mean*	*Std. Dev.*	*5th Percentile*	*95th Percentile*
1st Projection Year				
1-Year Nominal Rate				
Base case	5.81%	3.22%	0.52%	10.81%
$\kappa_q = 0.8$	6.36	2.44	2.28	10.33
$\sigma_q = 0.08$	5.83	6.26	-5.70	15.74
$\mu_q = 0.030$	5.00	3.22	-0.29	9.99
$\kappa_r = 1.5$	5.99	3.30	0.64	11.41
$\sigma_r = 0.02$	5.82	3.23	0.57	8.87
$\kappa_l = 0.2$	5.83	3.21	0.49	10.77
$\sigma_l = 0.03$	5.83	3.55	-0.09	11.78
$\mu_l = 0.01$	5.69	3.22	0.40	10.68
10-Year Nominal Rate				
Base case	6.53	1.38	4.30	8.86
$\kappa_q = 0.8$	6.99	1.09	5.29	8.66
$\sigma_q = 0.08$	5.60	2.12	2.04	9.22
$\mu_q = 0.030$	5.03	1.38	2.79	7.35
$\kappa_r = 1.5$	6.56	1.38	4.34	8.88
$\sigma_r = 0.02$	6.52	1.39	4.23	8.87
$\kappa_l = 0.2$	6.66	1.17	4.66	8.61
$\sigma_l = 0.03$	6.15	2.05	2.83	9.45
$\mu_l = 0.01$	5.87	1.38	3.64	8.20
10th Projection Year				
1-Year Nominal Rate				
Base case	6.93	5.09	-0.80	15.63
$\kappa_q = 0.8$	7.45	4.24	0.64	14.86
$\sigma_q = 0.08$	6.99	8.25	-5.77	21.28
$\mu_q = 0.030$	5.86	5.09	-1.87	14.56
$\kappa_r = 1.5$	6.93	5.07	-0.92	15.71
$\sigma_r = 0.02$	6.95	5.23	-1.13	16.15
$\kappa_l = 0.2$	6.92	4.53	-0.32	13.96
$\sigma_l = 0.03$	7.14	7.18	-4.70	19.10
$\mu_l = 0.01$	5.83	5.09	-1.90	14.53
10-Year Nominal Rate				
Base case	6.98	2.65	2.56	11.20
$\kappa_q = 0.8$	7.38	2.45	3.21	11.20
$\sigma_q = 0.08$	6.06	3.28	0.95	11.88
$\mu_q = 0.030$	5.39	2.65	0.98	9.61
$\kappa_r = 1.5$	6.96	2.59	2.65	11.12
$\sigma_r = 0.02$	6.97	2.67	2.51	11.38
$\kappa_l = 0.2$	7.01	1.73	4.13	9.78
$\sigma_l = 0.03$	6.73	4.49	-1.01	13.77
$\mu_l = 0.01$	5.64	2.65	1.23	9.86

Through casual observation of the financial markets, it is easy to recognize that interest rates of different time horizons are correlated. In the financial scenario generator, the parameter that controls this correlation is part of the real interest rate process. In the base case, the correlation between the long and short real interest rate processes is 0.5. The following table selects two nominal interest rates (proxies for the short and long nominal rates) and analyzes the sensitivity of the nominal rate correlation to the real interest rate correlation parameter.

Real Interest Correlation	***Nominal rate 3-mo / 20-yr correlation***
0.5	0.81
0.0	0.77
-0.5	0.73

Note that the effects of changing the real interest rate correlation do not have dramatic effects on the resulting nominal rate correlation. For example, when the correlation between the long and short real interest rate is dropped from 0.5 to 0.0, the nominal rate correlation between long and short rates only falls from 0.81 to 0.77. This is because nominal interest rates are also related to inflation. Given that inflation is a one-factor process, movements in the inflation rate affect all nominal rates predictably, which establish a certain level of correlation among all maturities. Therefore, only the real interest rate portion of the nominal rate is altered in the correlation shifts noted above, which is why the effects on nominal rate correlation appear muted.

Equity Returns

Movements in equity returns are composed of three separate pieces: the risk-free rate, excess equity returns (risk premia), and returns from dividends. The risk-free rate is based on the short-term nominal interest rate, which was discussed above. Returns from dividends are modeled separately, based on the same structural process for inflation (a mean-reverting Ornstein-Uhlenbeck process). This section discusses the excess return component of equity returns.

Excess equity returns are based on a regime-switching model, similar to Hardy (2001). In the financial scenario model, there are two states of the world: a low volatility regime and a high volatility regime. The return generating process is allowed to switch regimes based on a 2×2 transition probability matrix. At any moment in time, excess equity returns follow a normal distribution that is dependent upon the state of the world.

The following parameters were selected based on an analysis of excess equity returns in the U.S., shown separately for large and small stocks.

Large Stocks

0.008	Mean monthly excess return in low volatility regime
0.039	Volatility of monthly return in low volatility regime
-0.011	Mean monthly excess return in high volatility regime
0.113	Volatility of monthly return in high volatility regime
0.011	Switch from low to high regime ($P_{L,H}$)
0.059	Switch from high to low regime ($P_{H,L}$)

The annual excess return for large stocks in the low volatility regime is around 10%. In each month, there is a 1.1% probability of changing from low volatility regime to the high volatility regime. The distribution in the high volatility regime is significantly wider, but it is more likely to switch back to low regime.

Small Stocks

0.010	Mean monthly excess return in low volatility regime
0.052	Volatility of monthly return in low volatility regime
0.003	Mean monthly excess return in high volatility regime
0.166	Volatility of monthly return in high volatility regime
0.024	Switch from low to high regime ($p_{l,h}$)
0.100	Switch from high to low regime ($p_{h,l}$)

As expected, the risk and returns for small stocks tend to be higher on average than for large stocks. In addition, small stocks tend to move back and forth between regimes more readily than large stocks.

Using the base case parameters above for inflation, real interest rates, and excess equity returns, the annually compounded, geometric average equity returns are shown below:

Value	***Mean***	***Std. Dev.***	***5th Percentile***	***95th Percentile***
1st Projection Year				
Avg. Ann. Return - Large Stocks	11.71%	21.52%	-28.93%	41.79%
Avg. Ann. Return - Small Stocks	15.93%	34.46%	-40.32%	70.93%
10th Projection Year				
Avg. Ann. Return - Large Stocks	12.19%	9.32%	-5.18%	26.26%
Avg. Ann. Return - Small Stocks	14.68%	11.81%	-7.93%	31.65%

Instead of illustrating the effects of changing each of the parameters listed above, consider the unconditional probabilities of being in each of the two states. For large stocks, since it is more likely to switch regimes if you are in the high volatility regime, it follows that it is more likely that we would be in the low volatility regime. In fact, one can calculate the probability of being in low or high regime (π_L, π_H) for large stocks as:

$$\pi_L = \frac{P_{H,L}}{P_{L,H} + P_{H,L}} \approx 84\%$$

$$\pi_H = \frac{P_{L,H}}{P_{L,H} + P_{H,L}} \approx 16\%$$

Therefore, 84% of the draws for large stocks come from the high return, low volatility regime. For small stocks, the same calculation shows $\pi_l = 81\%$ and $\pi_h = 19\%$. Thus, small stocks also have a higher probability of deriving their returns from the high volatility regime.

After determining the unconditional probabilities, the user can then estimate the effects of parameter changes on the distribution of equity returns.

The correlation between small and large equity returns is driven by two separate parameters. The first correlation controls the dependency of regime switches. If both small and large stocks have a tendency to move from one regime to another simultaneously, the correlation of regime switching would be high. However, since small stocks have a higher probability of switching regimes than large stocks, even if the regime switching correlation is close to 1.0, it does *not* follow that the two classes of stocks will always switch regimes at the same time.

The second correlation between small and large stocks is the regime independent, stochastic excess return. Regardless of the existing regimes for each class of stock, there may be a tendency for all stock returns to move together. Therefore, a correlation is introduced to connect the random component of stock returns. The net result of these two correlations is illustrated in the following table.

Regime Switch Correlation	***Stochastic Excess Return***	***Correlation***
0.90	0.95	0.75
0.00	0.00	0.00
0.50	0.95	0.73
0.90	0.50	0.39
0.50	0.50	0.38

The first example in the table shows the base case scenario, where the correlation controlling for regime switches at the same time is 0.90 and the regime independent return correlation is 0.95. This produces a overall correlation between small and large stocks of 0.75 based on 200 iterations. The additional examples help illustrate that the excess return correlation is more important than the correlation for regime switches. This can be seen in the third and fourth examples. When the regime switching correlation is reduced from 0.90 to 0.50, the correlation of monthly returns is not markedly affected (from 0.75 to 0.73). But when the correlation within regimes is dropped from 0.95 to 0.50, the resulting correlation of monthly returns falls by almost 50 percent (from 0.75 to 0.39).

Appendix B: Presentations on This Research

New Zealand University
(Presentation given; details to be added.)

CAS Basic DFA Limited Attendance Seminar, Washington, DC, July 2003
A brief reference to the project, along with a description of research highlights, was provided.

Actuarial Research Conference, Ann Arbor, MI, August 2003
The PowerPoint presentation is included in this report as Appendix B-1. An abstract of presentation that was made follows:

> The Casualty Actuarial Society and the Society of Actuaries are co-sponsoring research involving the modeling of economic series. This research is important for generating reasonable future economic and financial scenarios, and has critical implications for dynamic financial analysis and cash flow testing. An essential aspect of this research involves determining the interdependencies between the various economic and financial series, especially, but not exclusively, the interrelationships between the different series and interest rates. This session will present a summary of the results and findings of this research, and will provide a description of how the results can be utilized by academics and actuarial practitioners.

American Risk & Insurance Association Annual Meeting, Denver, CO, August 2003
The following session proposal was accepted for presentation at this meeting:

Financial Scenario Generator Project

Introduction

In April, 2001, the Casualty Actuarial Society (CAS) and the Society of Actuaries (SoA) solicited proposals for research involving the modeling of economic series, including inflation, interest rates, equity returns, dividend yields, real estate returns and unemployment rates. We propose to present the results of our work on this project. This research is important for generating reasonable future economic and financial scenarios, and has critical implications for dynamic financial analysis and cash flow testing. One essential aspect of this research involves the interdependencies between the various economic and financial series, especially the interrelationships between the different economic series and interest rates. We will summarize the key work in this area, present the financial scenario generator model, explain the process for determining the model parameters and describe the results of running the model with the selected parameters.

Literature Review

The key prior work in the area of actuarial modeling of financial variables was performed by Wilkie (1986, 1995) and Hibbert, Mowbray and Trunbull (2001). This project builds on their work, and develops a financial scenario generator model that actuaries and researchers can use in dynamic financial analysis, cash flow testing and regulatory applications. The works of Hull-White (1990, 1994), Vasicek (1977) and Hardy (2001) are incorporated into the model.

Financial Scenario Generator

In this model, inflation and real interest rates are each modeled separately, and then combined to determine the nominal interest rate. Equity returns, real estate returns and the unemployment rate are also modeled separately, but are dependent on the nominal interest rate levels determined previously. Inflation is modeled as a one-factor mean reverting process. Real interest rates are modeled as a two factor mean reverting process. Equity returns are modeled by a regime switching process, with two different lognormal distributions used to generate returns. Real estate returns are modeled based on a one factor mean reverting process. Unemployment is modeled as a one factor mean reverting process, with an additional factor representing an inverse relationship to nominal interest rates.

Results

Regardless of the mathematical sophistication of the variables incorporated in a model, the accuracy of the calibration process used to determine the parameters, and the timeliness of the values on which the calibration is based, the most important test of the validity of any model is the reasonability of the results. This section will examine the results of a representative run of the model and compare these results with historical values for interest rates, inflation, stock and real estate returns, dividend yields and unemployment rates.

The model was run using the base parameters to generate 5000 iterations. The results are presented in a variety of different ways to allow individuals wanting to apply the model to understand the usefulness of this approach. Summary statistics, correlation matrices, funnel of doubt graphs, comparison graphs showing model and historical values are all presented to show the reasonability and accuracy the model can provide.

Conclusion

Insurance companies and other financial institutions require models that can generate realistic values for key variables that affect operating results, assets and liabilities. The values must be able to be projected over a long time horizon, be internally consistent and be fully understood by the individuals responsible for applying them within organizations. This research represents a step in this process by providing a model of financial variables that are relevant for insurance companies. This model can be calibrated to any historical data set of inflation, interest rates, equity returns, real estate returns or unemployment patterns, representing different countries or different time horizons. Additional variables can be added to the model as needed. The goal is to foster more accurate financial planning for insurers.

CAS Asset-Liability Management / Finance Limited Attendance Seminar, Chicago, IL, September 2003
A brief reference to the project, along with a description of research highlights, was provided.

Western Risk and Insurance Association, January 2004
The PowerPoint presentation is included in this report as Appendix B-2.

Enterprise Risk Management Symposium, Chicago, IL, April 2004
The PowerPoint presentation is included in this report as Appendix B-3.

Actuarial Research Clearing House 2004
The paper is included in this report as Appendix B-4.

Modeling of Economic Series Coordinated with Interest Rate Scenarios:

A progress report on research sponsored by the Casualty Actuarial Society and the Society of Actuaries[1]

Researchers and Authors:

Kevin C. Ahlgrim, ASA, MAAA, Ph.D.
Illinois State University

Stephen P. D'Arcy, FCAS, MAAA, Ph.D.
University of Illinois at Urbana-Champaign

Richard W. Gorvett, FCAS, MAAA, ARM, FRM, Ph.D.
University of Illinois at Urbana-Champaign

Abstract

This paper represents a brief progress report on research, being sponsored by the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA), in the area of modeling of economic series. In particular, research is being undertaken to provide a foundation for the actuarial community's modeling of a variety of economic variables, including interest rates, equity returns, inflation, unemployment rates, and real estate price levels. The specific objectives of this research are to provide a summary of the literature in the area of economic scenario modeling, identify data sources and methodologies in this area useful to the actuarial profession, and produce working software for economic modeling, including appropriate documentation, to be made available to the general actuarial community in order to allow actuaries to use and build upon this research.

Section 1: Introduction

Consider the following activities, each of which is actively engaged in by the 21st-century actuary:

[1] The investigators wish to thank the Casualty Actuarial Society and the Society of Actuaries for providing financial support for this research, as well as guidance and feedback on the subject matter.

- A property-liability insurance company uses dynamic financial analysis (DFA) to compare alternative strategic and operational decisions, and to identify areas of potential opportunity relative to competitors within the insurance industry.
- A life insurer, both for regulatory and internal decision-making purposes, quantifies the impact on its future solvency and profitability of several alternative financial scenarios.
- An insurer evaluates different asset-liability management (ALM) techniques across a variety of possible future economic environments.

All of these activities – activities which every year are the purview of a greater number of actuaries – have at least one component in common: the necessity of systematically and efficiently modeling future values of, and the interrelationships between, economic and financial variables. These and other activities, critical to strategic decision-making and optimization of insurer performance, should be built upon a solid conceptual and practical foundation with respect to modeling the financial and economic environment, which is the broad context within which insurers operate.

Recognizing this, the Casualty Actuarial Society and the Society of Actuaries, in May 2001, issued a Request for Proposals for research in the area of "Modeling of Economic Series Coordinated with Interest Rate Scenarios." The goal of this solicited research was to provide actuaries with a floor, or foundation, for future work in the modeling of financial scenarios, and to prepare a model for projecting economic and financial indices, incorporating realistic interdependencies among the variables. Specific deliverables from the research project included:

1) A *literature review* of work previously done in this area;
2) Identification of appropriate *data sources and methodologies* to enhance economic modeling efforts relevant to the actuarial profession; and
3) Production of a *working model* of economic series, coordinated with interest rates, that could be made public and used by actuaries via the CAS and SOA websites to project future economic scenarios. Categories of economic series to be modeled included interest rates, equity price levels, inflation rates, unemployment rates, equity dividend yields, and real estate price levels.

As alluded to above, this topic is of considerable value and importance to the actuarial profession and the broader insurance community. For example, a key aspect of the dynamic financial analysis process, which continues to be an area of substantial development and interest among actuaries, is the generation of economic and financial scenarios. These scenarios provide an economic context for the evaluation of an insurer's alternative operating decisions and their potential impact, across a variety of possible future economic conditions, on future corporate value. Such stochastic simulation efforts are predicated upon the ability to probabilistically express possible future economic and financial environments. In addition, an integrated scenario generation capability is critical to recognizing the interdependencies between the various economic and financial series –

e.g., consistently modeling the relationships between, say, equity returns and interest rate movements.

Similarly, the generation of scenarios is important for regulatory, rating agency, and internal management tests of an insurer's potential future operating conditions. An example is cash flow testing. By testing across scenarios, an insurer's cash position and liquidity can be evaluated over a variety of alternative future economic and financial environments.

This paper represents a status report, as of August 2003, summarizing the authors' progress on this research and the development of a scenario model. The materials underlying this paper, along with a substantial written report, have recently been provided to committees of the CAS and the SOA for comment. These materials represent a draft of the proposed final report and model. It is anticipated that some enhancements, either necessary and/or desirable, will be made in response to forthcoming comments from these committees.

The remainder of this paper is organized as follows. Section 2 provides an overview of the draft report, including the specific deliverables which comprise the report and our research results. Section 3 discusses the specific modeling approaches currently being taken with respect to the various economic series being modeled. Section 4 describes several specific issues that are illustrative of the kinds of important questions asked, and decisions made, in the process of economic modeling. Section 5 concludes, and Section 6 provides a bibliography.

Section 2: Overview of Draft Report

In its current incarnation, the draft report summarizing progress to-date on this research project includes the following sections and attachments:

Section 1:	Introduction and Overview
Section 2:	Excerpts from Original CAS / SOA Request for Proposals
Section 3:	Excerpts from Proposal of Selected Researchers
Section 4:	Literature Review
Section 5:	Descriptions of Data and Approach
Section 6:	Discussions of Issues
Section 7:	Results of Model Simulations
Section 8:	Conclusions and Acknowledgements
Appendix A:	User's Guide to the Financial Scenario Model
Appendix B:	Presentations on This Research
Appendix C:	Simulated Financial Scenario Data
Appendix D:	The Financial Scenario Model

Most of the Section and Appendix titles should be self-explanatory. A few very brief comments on just a few of these components are warranted here.

The *Literature Review* (Section 4) includes brief descriptions of a variety of articles – covering the fields of actuarial science (both life and casualty), finance, and economics – that the researchers believe are relevant, to varying degrees, to this research.[2] Research on the development of financial scenarios, and the analysis of financial and economic time series, is a continually evolving and growing area. We recommend that efforts be made, at least periodically if not continually (e.g., by a formal charge to appropriate CAS / SOA research committees, or by engagement of other interested persons), to provide an ongoing search for and review of relevant new work in this area, in order that the results from this project might be enhanced and kept current.

Discussions of Issues (Section 6) describes and comments upon some of the specific issues encountered during the course of this research. In some cases, these issues involved decisions with which we as researchers were confronted; our thought processes and the rationales for selected approaches are included here.[3]

Simulated Financial Scenario Data (Appendix C) is a spreadsheet database of hundreds of scenarios (i.e., simulation paths) of financial and economic variables, generated as output from the Financial Scenario Model. The intent of this data is to provide an alternative to requiring the @Risk simulation package (an add-on to Excel)[4] in order to run the model. This data can be used directly, in lieu of actually running the model; the "pre-simulated" scenario paths can be used as an input to a DFA model or other analytical program.

The Financial Scenario Model (Appendix D) is an Excel spreadsheet-based program, designed to be run, as mentioned above, through the @Risk simulation add-on. The (ultimately) downloadable version of the model will contain default values of appropriate parameters – however, these can be changed by the user for purposes of updating for new or additional data, sensitivity testing of parameter values, etc.

Please note that our intention is ultimately to post the completed report, along with the financial scenario model and hyperlinks to various presentations and articles emerging from this research, on the websites of both the CAS and the SOA.

Section 3: Progress Report on Research

After an extensive review of the literature in a number of relevant fields, it was found that three articles in particular provided a strong historical foundation for this economic scenario research. Wilkie (1986) used simulation to model future economic scenarios, for a variety of applications. In his article, inflation was modeled as the "driving" variable, with

[2] Various CAS and SOA members made several suggestions for articles to be included in this literature review. These suggestions are gratefully acknowledged.

[3] Often, these issues were either provoked or reinforced by questions or comments from members of the sponsoring actuarial committees. Again, this input was greatly appreciated and valued throughout the project.

[4] "@Risk" is a software package produced by the Palisade Corporation. For additional information, please see the company's website at www.palisade.com.

several other economic processes being driven off of inflation in a "cascade"-type fashion. A first-order autoregressive (AR) process was used for inflation, along with certain other variables. Wilkie (1995) followed up on his earlier paper, expanding upon and enhancing the modeling and econometrics. Finally, Hibbert, Mowbray, and Turnbull (2001) examined issues in modeling a number of economic variables. Included in this paper is a comparison of outputs with the results from the Wilkie model. Taken together, these three articles provide an excellent background for understanding our current research.

For this research, the economic series modeled included the following:

- Inflation
- Real interest rates
- Nominal interest rates
- Equity returns
  - Large stocks
  - Small stocks
- Equity dividend yields
- Real estate returns
- Unemployment

The general approach used for each of these processes will be briefly summarized below[5]. In the formulas which follow, standard Brownian motion processes are represented by dB, and subscripts refer either to time (t) or to the relevant economic variable. The descriptions are all provided in a continuous-time framework, as this is the framework in which most financial research of this nature is done. Of course, parameterization and estimation are performed relative to real, actual data, which is discrete in nature. Mathematically, the discrete forms of these processes are analogous to the continuous-time versions provided below.

Inflation

We model inflation (q) as an Ornstein-Uhlenbeck process. Specifically, inflation is assumed to be a one-factor, mean-reverting process of the form

$$dq_t = \kappa_q (\mu_q - q_t)\, dt + \sigma_q\, dB_q$$

where q is the short-term inflation rate, κ is the speed of mean reversion, μ is the long-run rate to which the process tends to revert, and σ is a volatility parameter. In discrete format, this amounts to an autoregressive process. The model was parameterized using U.S. Bureau of Labor Statistics Consumer Price Index (CPI) data. Two time periods were examined, in order to test the sensitivity of the parameters to different economic epochs: 1913 to 2001, and 1946 to 2001. Because of the noise in monthly CPI series, annual CPI values and regressions were employed. Using an approach similar to Vasicek (1977) for

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[5] Much more detail regarding the modeling, mathematics, parameterizations, etc., is provided in the forthcoming formal report of this research, as well as in one or more planned articles.

interest rates, we then produced a term structure of inflation rates (necessary for modeling nominal interest rates – see below).

Real Interest Rates

Real interest rates (***r***) are modeled according to a two-factor Vasicek model[6]. This structure is similar to the one-factor Vasicek model (Vasicek, 1977), in that it is a mean-reverting process with the short-term real interest rate being a stochastic variable. However, in addition, there is a second stochastic factor: the long-run mean (***l***, below) to which the short-term rate tends to revert. The specific formulas used are

$$dr_t = \kappa_r (l_t - r_t)\, dt + \sigma_r\, dB_r$$
$$dl_t = \kappa_l (\mu_l - r_t)\, dt + \sigma_l\, dB_l$$

These equations were parameterized with monthly Federal Reserve data from 1982 to 2001, using two-stage least-squares estimation techniques. Real interest rates, while not directly observable, were estimated *ex post* by determining the differences between nominal interest rates and inflation rates.

Nominal Interest Rates

Once inflation and real interest rates have been modeled, nominal interest rates (***i***) are determined from them, based on the standard Fisher (1930) relationship:

$$i = \{(1 + q) \times (1 + r)\} - 1$$

The model software developed in coordination with this research will allow the user, if s/he so desires, to toggle a "non-negativity" switch, thereby preventing future modeled nominal interest rates from falling below zero.

Equity Returns

A great deal of attention has been paid to modeling equity returns in the financial, economic, and even physics (under the guise of "econophysics") literatures[7]. One of the empirical observations that is frequently noted regarding historical equity returns is the "fat tails" issue – that actual equity returns, when examined as a historical distribution of returns, tend to have fatter tails than typical theoretical distributions would suggest. To that end, we have used (along the lines of Hardy (2001)) a "regime-switching" model for equity returns, with the two regimes having, respectively, low and high volatility. A Markov Chain framework represents the probabilities of switching between regimes from month to month.

We have modeled equity returns (s_t) by specifying an excess return (x_t) over and above the modeled nominal interest rate:

$$s_t = q_t + r_t + x_t$$

[6] This is also a simple case of the two-factor Hull-White interest rate model. See Hull and White (1994).
[7] For econophysics, see, for example, Sornette (2003).

The excess return represents a risk premium attributable to capital appreciation (since the dividend yield of equities is modeled separately – see below). We estimated the processes underlying small stocks and large stocks separately.

Equity Dividend Yields

The equity dividend yield (***y***) is modeled such that the natural logarithm of the dividend yield follows an AR process:

$$d(\ln y_t) = \kappa_y (\mu_y - \ln y_t)\, dt + \sigma_y\, dB_{yt}$$

The parameterization process for this model is similar to that involved for the inflation model, described above.

Real Estate Returns

We estimated two versions of an Ornstein-Uhlenbeck model for real estate returns: including and excluding inflation.[8] We used quarterly National Council of Real Estate Investment Fiduciaries (NCREIF) pricing indices to capture returns on commercial properties. The NCREIF data is generated from market appraisals of various property types, including apartment, industrial, office, and retail.

Unemployment

With respect to the unemployment process, a well-known tool, the Phillip's Curve, posits an inverse relationship between the unemployment rate (u) and inflation (q).[9] We have chosen to build upon that relationship, but to include an AR(1) process:

$$du_t = \kappa_u (\mu_u - u_t)\, dt + \alpha_u\, dq_t + \sigma_u\, \varepsilon_{ut}$$

Unemployment data, from 1948 to 2001, from the Bureau of Labor Statistics was utilized.

Section 4: Discussion of Several Issues

This section highlights several of the key issues and questions encountered during this research project. This section is not meant to be a comprehensive survey of all critical issues, but rather illustrative of the kinds of problems encountered, and the thought process of the researchers with regard to their resolution.

Inflation versus Interest Rates

In any effort to model key economic variables, a question which must be resolved early in the modeling process involves which variables come "first" – i.e., which variable(s) is(are) key and causal, and which other variables are functions of those. This is a typical question when dealing with interest rates and inflation. Interest rates are probably more frequently

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[8] While we expected inflation to be a driver of real estate returns, the results to-date have not indicated significance.

[9] While this relationship seems plausible in many ways, it should be noted that more recent data (since the original publication of the Phillips Curve in 1958) has not fit this inverse relationship as well.

directly modeled, and a potential disadvantage of varying from that approach can be an inability to take advantage of, and build upon, extensive prior work of numerous authors in interest rate modeling. On the other hand, in the U.S., the Federal Reserve has a large role in impacting the levels of interest rates, in response to inflationary pressures and general economic conditions. This decision, therefore, is not a trivial one.

We chose to model both inflation and real interest rates as the "driver" variables. This decision led to the modeling structures for those two variables, along with nominal interest rates, mentioned in the preceding section. Under our approach, is it possible for the user of the model to correlate the shock (***dB***) terms of the inflation and real interest rate processes, allowing for a direct, partial connection between these series.

Equilibrium vs. Arbitrage Free Models

One of the primary processes in a financial scenario model is a term structure process. A tremendous variety of term structure models is available for both practitioners and researchers. (For a discussion of many of the available models, see Yan (2001)). No single term structure model has yet proven itself worthy for all possible applications (see the discussion in Chapman and Pearson (2001)). In virtually all cases, the user of a term structure model has one or more tradeoffs to consider – e.g., complexity of the model vs. accuracy. The nature of these tradeoffs depend on the specific application of the term structure model.

There are several important issues to consider when choosing among term structure models. One consideration is related to the theoretical background of the model. Specifically, term structure models are typically categorized as "equilibrium" models and "arbitrage-free" models. Equilibrium models typically begin with an assumption for short-term interest rates, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond's maturity. One of the primary advantages of equilibrium models is that the prices of many popular securities have closed-form analytic solutions. Another advantage is that equilibrium models are fairly easy to use. On the negative side, equilibrium term structure models generate yield curves which are inconsistent with current market prices. While the parameters of these models may be selected carefully, there is no guarantee that the resulting term structure will generate observed market prices.

Contrary to equilibrium models, arbitrage-free term structure models project future interest rate paths that emanate from the existing yield curve. For applications using arbitrage-free term structure models, resulting prices will be based on the concept of arbitrage. Unfortunately, arbitrage-free term structure models are frequently more difficult to use than their equilibrium counterparts.

Although we will not elaborate here, other considerations in selecting a term structure model include pricing accuracy, internal consistency, data issues, intended use of the model, and the time horizon over which simulations will be performed. With regard to the last two considerations mentioned above, it is important to bear in mind that insurance and actuarial applications of such models, such as in DFA models, generally involve long time horizons (e.g., five years of simulation projections), with generally fairly course time intervals (monthly, quarterly, or even annual). This is a very different framework from a short-term (hourly, daily, or weekly) trading horizon. It may well be that different models are justifiable for such different analytical frameworks.

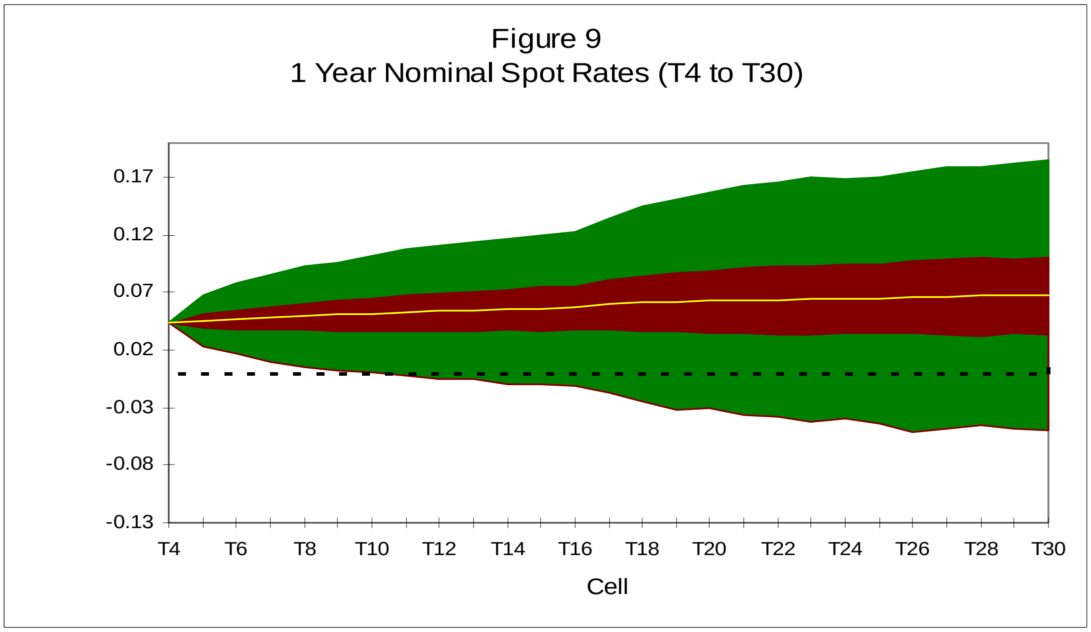
Adequacy of a Two-Factor Interest Rate / Inflation Model
The number of factors to use in modeling interest rates is a decision which frequently elicits passionate debate. Again, as mentioned above, it is important to keep in mind the purpose toward which this research is working: to produce reasonable distributions of future economic values. Our work is ***not*** intended for security-trading purposes. This is a hugely important context to keep in mind – it has implications for the type of interest rate model used, the number of parameters employed, etc. Furthermore, there is often a misunderstanding as to the types and movements of yield curves that are available from two-factor (and with respect to some issues, even one-factor) models. For example, humped curves are indeed possible. (A good paper for considering the types of yield curve movements that predominate historically is Litterman and Schenkman (1991).) We believe that the two-factor model we have employed is a reasonable selection in view of both historical and parsimony considerations.

Section 5: Types of Results

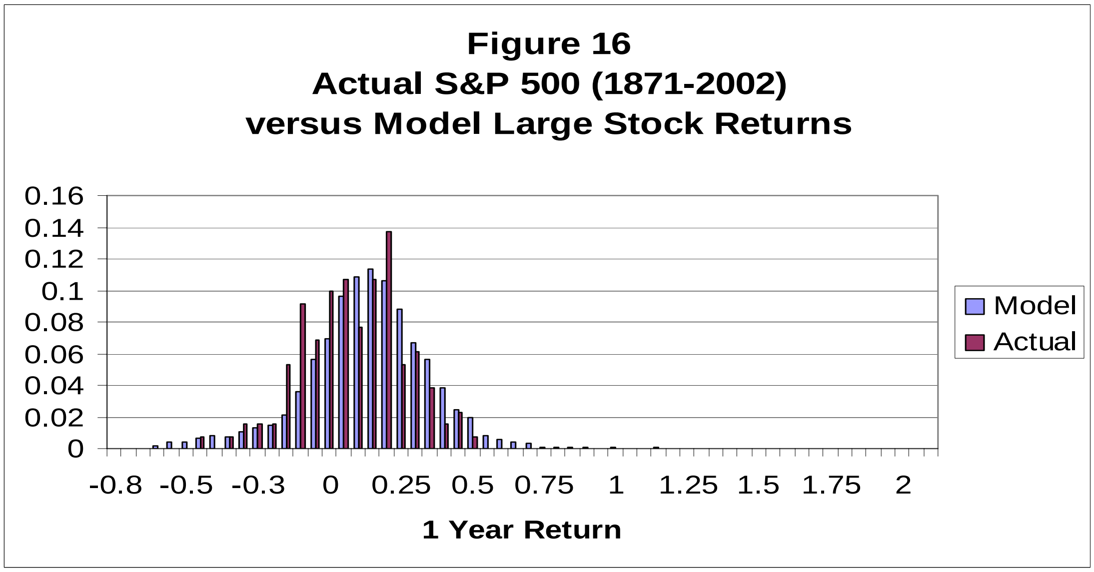
In addition to documentation regarding the analytical infrastructure of this research, our work has provided a number of results in terms of

- *Parameters* for the various economic processes;
- Indications of future *volatilities* for the modeled processes; and
- *Comparisons* of model fits with historical data.

Since our formal report is, as of this writing, currently being reviewed by the sponsoring societies, we have refrained from providing specific quantitative results. These will be provided on the CAS and SOA websites, as part of our formal report, upon completion of this project. However, below are two samples of exhibits from preliminary work. These are included for illustrative purposes only. The first exhibit, immediately below, is a "funnel of doubt" chart, which shows the volatility of simulated future values, over time, of a stochastic variable (in this case, the 1-year nominal spot interest rate). The solid relatively-horizontal line in the middle shows the mean value over time of the (in this case, 5,000) simulations of that stochastic variable. The two shaded areas surrounding the mean line are the 25th to 75th percentile values (the dark shaded section), and the 1st to 99th percentile values (lighter shaded section).



The second illustrative exhibit, immediately below, needs little explanation. This type of exhibit graphically represents the nature of the fit between actual and modeled values (in this case, of the distribution of large stock returns).



Section 6: Conclusion

At the time of writing this article, and the presentation of this material at the Actuarial Research Conference (August 2003, in Ann Arbor, MI), an initial draft of the final report of this research had been submitted to the oversight committees of the CAS and the SOA. While comments from these committees will undoubtedly improve and enhance this research, the current key elements and deliverables of this research include:

- A *literature review* summarizing the relevant literature from several fields, including actuarial science, economics, and finance;
- A *summary of data sources and methodology* providing the analytical underpinnings of our work;
- A *Financial Scenario Model,* programmed in Excel and @Risk, and designed to be available for use by actuaries by downloading from the CAS or SOA website;
- *Simulated financial scenario data,* based on default model parameter assumptions, which can be used in lieu of the model if @Risk software is not available;
- A *User's Guide* to the model; and
- A *discussion of results* from sample implementation of the model.

This paper has presented a survey and highlights of our progress on this actuarial research. Hopefully, it has provided at least a flavor of this work. Significantly more detail, along with specific parameter values and quantitative results, is included in our formal report. The formal report will be available through the CAS and the SOA websites in the near future.

This research, and the accompanying model, should be of wide interest, including to such insurance and financial professionals as insurers, regulators, and pension funds. Specific applications of this research might include parts of dynamic financial analysis, cash flow testing, financial planning, investment analysis, capital budgeting, and analysis of alternative financial risk management solutions.

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