Catastrophe Risk Securitization

Insurer and Investor Perspectives

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Abstract

This paper presents a survey of the various instruments used to securitize catastrophe risk listing their advantages and disadvantages. The paper then focuses on the use of catastrophe options, presenting an example of how catastrophe options work from the investor prospective, and demonstrating a method for analyzing the cost of financing catastrophe insurance with the following instruments: (1) insurer capital; (2) reinsurance; and (3) catastrophe options. The procedure first quantifies the cost of financing in terms of the cost of those instruments. The method then permits searching for a mix of instruments that minimizes the cost.

Using a catastrophe model, we create a distribution of simulated losses for each of fifty insurers that report their exposure to ISO. We then create an illustrative catastrophe index based on the combined simulated losses of the fifty insurers. We perform a sample analyses for three insurers.

The analyses show that the best mix of capital, reinsurance, and catastrophe options depends on how well an insurer's losses correlate with the index – that is, on the basis risk. Some insurers can significantly reduce their cost of financing catastrophe insurance by using catastrophe options. To illustrate the effect on premiums of the cost of financing catastrophe insurance, we convert those costs into risk loads.

1. Introduction

Catastrophe Risk - Past, Present, Future

In the nine years from 1989 to 1997, the U.S. property/casualty industry suffered an inflation-adjusted $80.2 billion in catastrophe losses -- 67.9% more than the inflation-adjusted $48.0 billion in catastrophe losses during the 39 years from 1950 to 1988.[1]

Eight of the ten most costly catastrophes (inflation adjusted losses) in U.S. history have occurred in the past decade, and seventeen of the twenty most costly catastrophes have occurred in the past two decades. Three of the twenty most costly catastrophes -- Hurricane Andrew, Hurricane Iniki, and the Los Angeles riots -- all occurred in just one year, 1992. Another two of the twenty most costly catastrophes -- Hurricane Hugo and the Loma Prieta earthquake -- also occurred in one year, 1989.

In a recent study[2], ISO used the Risk Management Solutions, Inc. (RMS) catastrophe model to simulate possible catastrophic events for the insurers who report their exposure to ISO. The study concluded that losses from a severe hurricane along the east cost could exceed $150 billion. Similarly a severe earthquake in California could generate losses of $70 billion or more. A magnitude 8.5 earthquake on the New Madrid Fault in the central U.S. could result in $115 billion in insured losses.

Losses from such a megacatastrophe could severely affect property/casualty insurers and their policyholders. Many insurers could become insolvent or seriously impaired and, therefore, unable to continue insuring the same volume of business. The recognition of this risk has stimulated industry efforts to address the problem of megacatastrophes.

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[1] ISO's Property Claims Services unit currently defines catastrophes as events that cause $25 million or more in direct insured property losses and which affect a significant number of insurers and insureds. Reported direct property losses for each catastrophic event were adjusted to 1997 price levels using Consumer price Indexes obtained from the U.S. Bureau of Labor Statistics.

[2] Insurance Services Office, Inc., *Managing Catastrophe Risk,* May 1996.

Insurance regulators, legislators, government agencies, investment bankers, and others have also contributed to the public policy debate on this critical issue.

Population growth and the increase in insured exposures located in catastrophe-prone areas have contributed to the upward trend in catastrophe losses. Demographic projections indicate that the problem will only grow with time, as the population in coastal areas prone to hurricanes and earthquakes continues to increase.

Catastrophe Management

A property/casualty insurer can measure the extent of its catastrophe risk by conducting a portfolio analysis to determine the expected distribution of losses from possible events such as hurricanes or earthquakes. This distribution of losses is created by analyzing the company's catastrophe exposure with a computer simulation model, which provides an estimate of losses that would result from a representative set of catastrophic events. Where potential catastrophe losses are too high, the insurer might take steps to reduce its concentration of exposures. Some insurers have given up some business in overly exposed areas to reduce their catastrophe risk to a more manageable level. An insurer could also diversify its catastrophe risk by writing more exposures in areas where it has a lower concentration of exposures or in areas not subject to catastrophes. A concern about that strategy is that the insurer could be taking on a different risk by writing new business in areas where it lacks expertise and an effective distribution network.

Many insurers have opted for loss-reduction measures such as increasing deductible sizes, imposing special wind/earthquake deductibles and offering discounts for loss mitigation activities by policyholders (such as the addition of storm shutters).

Property/casualty insurers have pursued many loss mitigation efforts, such as the ISO Building Code Effectiveness Grading Schedule (BCEGS). The BCEGS program evaluates a community's building code and its enforcement. Insurers can offer discounts for structures built in municipalities with good enforcement of an effective loss mitigating building code.

Financing Catastrophe Risk

Insurers have also been looking at ways of financing their catastrophe risk. One approach is adding capital to the balance sheet. Many insurers have benefited from recent stock market gains as a source of additional capital. Because of their improved capital positions, some insurers have elected to retain more catastrophe risk.

The surge in catastrophes that began in 1989 with Hurricane Hugo, resulted in an increased demand for reinsurance. The rising demand, in turn, produced substantial price increases which led to the formation of new catastrophe reinsurers. That increase in reinsurer capital coupled with improved catastrophe experience has led to more plentiful and less expensive catastrophe coverage.

The history of economics in general and the capital markets in particular is one of entrepreneurs devising innovative solutions to pressing social problems. Entrepreneurs devised options and futures contracts for commodities to help farmers and their customers hedge the risk of large swings in the prices and availability of agricultural commodities. And, when mortgage lenders needed additional capital to finance housing and other construction, entrepreneurs created mortgage-backed securities that, in essence, enabled mortgage bankers to bundle individual loans together and to then sell parts of the package to individuals and institutional investors.

The problem of financing catastrophe risk has much in common with previous problems that entrepreneurs solved by turning to the capital markets. Like lenders, insurers need access to additional financial capacity. Like farmers and the buyers of agricultural commodities, insurers need to hedge risk. The capital markets can provide insurers with access to far more financial capacity than has previously been available, and the capital markets can provide insurers with a vehicle for spreading risk far more widely than has been possible. As of the first quarter of 1998, the property/casualty industry’s statutory net worth, or policyholders’ surplus, amounted to $330.5 billion. Adding up the net outstanding value of various kinds of financial instruments (e.g., corporate equities, corporate bonds, government bonds, etc.) as of first-quarter 1998, ISO estimated the size of the U.S. capital market at $25.4 trillion[3] -- more than 75 times the statutory net worth of the property/casualty industry.

Emergence of Capital Market Solutions

Aware of insurers' need for more capacity to finance catastrophe risk, entrepreneurs have been devising solutions that would spread catastrophe risk to investors by "securitizing" it. That is, entrepreneurs have been developing means of packaging insurers' catastrophe risk as securities that could be sold to investors. Such solutions, however, will only be successful if they simultaneously meet insurers' need to spread risk efficiently while offering investors opportunities to improve the performance of their portfolios.

The securitization of catastrophe risk has taken several forms, each with advantages and disadvantages. Primary insurers can use all of the forms of securitizing insurance risk to supplement traditional reinsurance, and reinsurers can use them to supplement traditional retrocessions. Large self-insureds may also be able to use securitization to share their catastrophe risk with investors. To date, the principal forms of securitization include contingent surplus notes, catastrophe or "Act of God" bonds, CatEPuts, and exchange-traded catastrophe options.

2. Catastrophe Risk Securitization

Contingent Surplus Notes

Contingent surplus notes (CSNs) are surplus notes that an insurer has the right to issue to specific intermediaries or investors contingent on certain events taking place. An insurer that wants to use contingent surplus notes to access additional capital in the event of a catastrophe might arrange for a financial intermediary to set up an investment trust. The trust would invest in U.S. government bonds or other liquid securities, and the intermediary would sell shares in the trust to investors. The arrangement would give the

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[3] Insurance Services Office, Inc., *Financing Catastrophe Risk: Capital Market Solutions,* December 1998.

insurer the right, under specified circumstances, to issue surplus notes to the financial intermediary in exchange for cash or liquid assets. The intermediary, in turn, would have the right (but not the obligation) to substitute the surplus notes for the securities held by the trust. The insurer would pay fees to the financial intermediary in exchange for the intermediary's commitment to purchase the insurer's surplus notes. And, the intermediary would pays fees to the trust in exchange for the trust's commitment to purchase the insurer's surplus notes from the intermediary.

When a catastrophe occurs, the insurer can issue surplus notes to the intermediary in exchange for cash or liquid assets, thereby increasing the insurer's surplus and enhancing its ability to pay claims. The intermediary can then exchange the surplus notes for the marketable securities held by the trust, enabling the intermediary to replenish its liquid assets. The insurer would then pay interest and principal on the notes to the trust, and the trust, in turn, would pay the investors. In effect, the process lends investors' capital to the insurer in exchange for the insurer's surplus notes.

The advantages of contingent surplus notes include the following:

- An insurer can tailor a CSN transaction to meet its specific needs, much like an individual reinsurance contract.

- Investors can earn higher returns by investing in contingent surplus note trusts than they can earn by investing directly in Treasury securities. The trusts can pay higher returns as a result of the fees they collect on behalf of investors.

- To the extent that an insurer meets its obligations under its surplus notes, investors receive periodic payments of interest and principal, even after the insurer suffers substantial catastrophe losses.

The disadvantages of contingent surplus notes include the following:

- An insurer needs to get a state insurance department's approval to actually issue surplus notes, even after it has made all of the necessary arrangements with financial intermediaries and investors.

- Insurers using CSNs may incur high transaction costs. In addition to paying various fees, insurers would incur costs providing intermediaries and investors with the information those parties need to evaluate the risk they are assuming and their potential returns. Financial intermediaries and investors may require information about the probability that an insurer's catastrophe losses will be high enough to result in the insurer issuing contingent surplus notes and information about the likelihood that the insurer will be able to repay the notes once they are issued.

- Evaluating the probability that surplus notes would be repaid can be difficult, because the notes would be subordinate to other claims on the insurer and because the relevant state insurance department would have to grant permission for the insurer to repay the notes.

- Investors that do buy shares in a CSN trust may have difficulty reselling those shares to other investors who lack information about the catastrophe exposure and financial condition of insurer benefiting from the trust. The trust agreement may place restrictions on investors' ability to transfer shares in the trust to other investors.

- When an insurer uses contingent surplus notes to raise capital, the insurer takes on a debt that must be repaid. Under Generally Accepted Accounting Principles (GAAP), that debt would appear on the insurer's balance sheet as a liability. If the insurer instead used traditional reinsurance, its reinsurer would reimburse its catastrophe losses, and its GAAP balance sheet might remain free of additional debt.

Catastrophe Bonds

A catastrophe or "act of God" bond is a corporate bond with special language that requires investors to forgive some or all principal or interest in the event that catastrophe losses surpass the "trigger" specified in the bond. The trigger can be based on:

- the catastrophe losses of a particular insurer
- the catastrophe losses of the property/casualty industry overall
- the level of a particular catastrophe index
- the parameters of particular events (such as wind speeds for hurricanes or Richter scale magnitudes for earthquakes)

Some catastrophe bonds have dual triggers (for example, both a specified dollar loss for an individual insurer and a specified dollar loss for the industry overall).

The triggers in catastrophe bonds can also specify particular geographic areas (such as the entire country, regions, collections of states, or finer levels of geography), and particular lines of insurance (such as homeowners or automobile physical damage).

Some catastrophe bonds have multiple classes of risk (or tranches). Splitting bonds into different tranches can make the bonds appealing to diverse investors. One tranche may offer a higher yield in exchange for the risk that investors will have to forgive repayment of principal in the wake of catastrophe losses, appealing to investors who will accept greater risk in exchange for a higher return. Another tranche may be "principal protected," appealing to investors willing to accept a lower return in exchange for lower risk.

Principal-protected tranches may also qualify for higher ratings from credit rating companies, expanding the market for such tranches to institutional investors limited to investing in only higher rated forms of debt.

From an investor's perspective, three risks of catastrophe bonds and some other forms of securitization are that protection against catastrophe losses may lead an insurer to:

- relax its underwriting standards;
- manage the geographic concentration of its exposures less carefully;
- settle claims more liberally.

Some catastrophe bonds have features that mitigate these risks in much the same way as reinsurers manage the corresponding risks in standard reinsurance agreements. First, some catastrophe bonds have high triggers, which act like high attachment points in excess-of-loss reinsurance contracts. High triggers provide an incentive for insurers to maintain underwriting discipline and practice prudent risk management. Second, some catastrophe bonds require that insurers share in losses above the trigger, much as proportional reinsurance requires insurers to share in losses. Proportional sharing of excess losses provides an incentive for insurers to underwrite carefully, manage their exposure to catastrophe risk prudently and not settle claims too liberally.

The advantages of catastrophe bonds include the following:

- An insurer can tailor the triggers and other provisions of catastrophe bonds to meet its specific needs, much like individual reinsurance contracts.

- When catastrophes trigger the provisions in catastrophe bonds that require investors to forgive repayment of principal, the insurer can immediately write down its liability for the bonds. Writing down the liability increases the insurer's surplus, or net worth.

- Catastrophe bonds offer investors higher yields than otherwise comparable bonds that do not contain provisions forgiving principal or interest in the event of catastrophic losses. According to the Hogue Insurance Stock Report, catastrophe bonds have been priced to yield three to four percentage points more than comparably rated (comparably risky) corporate bonds.[4]

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[4] The Hogue Insurance Stock Report, *Insurance Advocate*, December 13, 1997.

- Catastrophe bonds provide investors with an opportunity to reduce portfolio risk through diversification. The returns on most stocks and bonds depend to some extent on economic conditions. Therefore, those returns tend to rise and fall together, making it difficult for investors to fully hedge portfolio risk by diversifying their investments. The return on a catastrophe bond depends on the occurrence of a catastrophic event fulfilling the terms of the trigger. The occurrence of a qualifying catastrophic event will result in the issuer defaulting on the interest and/or principle. Since the occurrence of catastrophes is independent of economic conditions, the default risk on catastrophe bonds is not correlated with the default risk on other bonds and stocks. Adding catastrophe bonds to a portfolio of traditional investments can improve overall investment results.

The disadvantages of catastrophe bonds include the following:

- Insurers issuing catastrophe bonds may face high transaction costs because of the need to provide significant amounts of information to investors. Investors (like reinsurers) may require substantial amounts of information about an insurer's exposure to catastrophe losses to evaluate the level of risk they are assuming. Their expected rate of return for assuming that risk, especially when the trigger in a catastrophe bond is based on an individual insurer's loss experience.

- Investors' need for information about the catastrophe exposure of a specific insurer may also make catastrophe bonds less liquid than similar investments without the special features of catastrophe bonds.

- When an insurer issues catastrophe bonds, the insurer takes on debt. That debt may make the insurer appear less financially sound than it would if it instead bought traditional reinsurance.

- Catastrophe bonds do not have the same beneficial effect on an insurer's reported financial leverage as traditional reinsurance. For example, when calculating the ratio of net written premiums to surplus -- one widely used measure of the amount of risk supported by each dollar of surplus -- an insurer can deduct traditional reinsurance premiums from direct premiums written. Deducting reinsurance premiums reduces the premium-to-surplus ratio, making the insurer appear more financially sound. But, when using catastrophe bonds, an insurer does not pay reinsurance premiums and, consequently, cannot deduct those premiums from its direct written premiums. Therefore, the insurer's premium-to-surplus ratio will be higher than it would be if the insurer instead used traditional reinsurance.

Entrepreneurs have, however, developed a means of offsetting catastrophe bonds' disadvantages relative to traditional reinsurance. The solution involves using "special purpose reinsurers" or "special purpose vehicles."

A special purpose reinsurer is a business entity formed specifically to issue catastrophe bonds and to then sell traditional reinsurance to a particular insurer. The use of a special purpose reinsurer eliminates the need for the insurer to carry debt on its balance sheet and also enables the insurer to deduct a reinsurance premium when calculating its net-premium-to-surplus ratio. And, if the special purpose reinsurer is offshore, it may be exempt from U.S. taxes, ultimately reducing the reinsurance premiums it must charge.

A special purpose reinsurer can also protect investors from other credit risk inherent in the operations of the insurer using the special purpose reinsurer. If the insurer were to become insolvent for reasons having nothing to do with catastrophe losses, the special purpose reinsurer would still have an obligation to repay the catastrophe bonds it sold to investors. On the other hand, if a special purpose reinsurer encounters financial difficulty, the insurer using the special purpose reinsurer would not have to make its resources available.

Catastrophe Equity Puts

Catastrophe equity puts, or CatEPuts℠,[5] are a form of option that stock insurers can buy from investors. Those options give an insurer the right to sell a specified amount of its stock to investors at a predetermined price if catastrophe losses surpass a specified trigger. Thus, catastrophe equity puts can provide insurers with additional equity capital precisely when they need funds to cover catastrophe losses.

An insurer that uses catastrophe equity puts faces a counter-party risk -- the risk that the sellers of the catastrophe equity puts will not have enough cash available to purchase the insurer's stock. Insurers can minimize this risk by buying catastrophe equity puts only from investors with superior credit ratings. These puts can also contain language that requires investors to collateralize the options if their credit ratings deteriorate.

An insurer that uses catastrophe equity puts also faces a risk that exercising its options will trigger a change in control of the company. This risk can be eliminated by basing the catastrophe equity puts on nonvoting shares, such as preferred stock.

Investors selling catastrophe equity puts face the risk that they will end up owning shares of an insurer that is no longer viable. Investors can minimize this by including in the catastrophe equity puts language that prevents insurers from exercising their puts when they suffer losses so severe that they would still be impaired even after exercising their options and receiving the new capital. This would provide less protection to insurers and their policyholders.

Investors that sell catastrophe equity puts face the same stock market risk as investors that sell traditional put options on stocks -- the risk that unanticipated downward movement in the price of a stock will make the predetermined price specified in a put option less attractive than it was when the put was sold. An insurer that buys catastrophe equity puts does not face a corresponding risk that unanticipated upward movement in the price of its

[5] CatEPut is a service mark of Aon Corporation.

stock would reduce the attractiveness of the predetermined price, because the insurer is free to decide not to exercise its catastrophe equity puts and to instead raise capital using other means.

The advantages of catastrophe equity puts are:

- An insurer can tailor triggers to meet its needs, much like individual reinsurance contracts.
- Catastrophe equity puts provide investors with an equity interest in the insurer in exchange for the capital that they provide.
- Catastrophe equity puts provide investors with an opportunity to reduce portfolio risk through diversification. The execution of these puts requires the occurrence of a catastrophic event fulfilling the terms of the trigger and not on changes in economic conditions.
- The requirement that the insurer be viable for it to exercise these puts limits the risk for the investor.

The disadvantages of catastrophe equity puts are:

- As with catastrophe bonds and other customized approaches to securitizing insurance risk, customization may mean that investors will need large amounts of information to evaluate the amount of risk that they are assuming and their potential rates of return. Thus, insurers may find that they face relatively high transaction costs.

- Investors need for information and the associated transaction costs can be reduced by basing the triggers in catastrophe equity puts or other customized approaches to securitizing risk on aggregate industry losses or the parameters of catastrophic events. But doing so creates basis risk for the insurer securitizing risk.

- Catastrophe equity puts do not have the same beneficial effect on an insurer’s reported financial leverage as traditional reinsurance.

- Exercising catastrophe equity puts after a catastrophe may dilute the value of an insurer's outstanding shares.

- The requirement that the insurer be viable for it to exercise these puts reduces their value when the insurer is subject to large catastrophe losses.

Exchange-Traded Catastrophe Options

Insurers that want protection against catastrophe losses can buy exchange-traded catastrophe options from investors. Exchange-traded catastrophe options are standardized contracts based on catastrophe indices. The indices reflect the catastrophe experience of large sets of insurers or the entire property/casualty insurance industry. The contracts entitle the buyer of the option to a cash payment from the seller if catastrophes cause the index used in the options to rise above a strike price, or trigger, specified in the options. Such cash payments can help an insurer bolster its surplus and pay claims in the wake of catastrophe losses. Investors' incentive to sell catastrophe options is the payment they receive from insurers for doing so. If catastrophe losses are too low to cause the index used in a catastrophe option to rise to the specified strike price, the option expires worthless and the investor who sold the option keeps the funds received for selling the option.

Insurers and investors can trade options based on catastrophe indices compiled by ISO's Property Claim Services (PCS) unit on the Chicago Board of Trade (CBOT). Insurers and investors can trade catastrophe options based on the Guy Carpenter Catastrophe Indices (GCCI) on the Bermuda Commodities Exchange (BCOE).

The advantages of exchange-traded catastrophe options include:

- Because exchange-traded catastrophe options are standardized contracts based on catastrophe indices, an insurer (purchaser) does not have to provide a wealth of new information to investors (sellers) each time it wants to attract additional risk capital.

This lowers transaction costs compared to those incurred issuing catastrophe bonds or other customized securities.

- Both investors and insurers have ready access to the specifications for exchange-traded catastrophe options and the historical performance of the catastrophe indices used in settling those contracts. Thus, investors do not face the risk that individual insurers' knowledge of their own exposure to catastrophe losses or the individual insurer's loss experience will place investors at a disadvantage when trading catastrophe options (low counterparty risk).

- The use of organized exchanges and standardized, index-based contracts makes it easier for investors and insurers to liquidate positions in catastrophe options than positions in catastrophe bonds or other insurance-linked securities. To liquidate a position in catastrophe options, an investor need only buy options with the same strike price as the options he sold. This offsets the financial effects of the having sold options. Similarly, an insurer need only sell options with the same strike price as the options he purchased.
- As a rule, option exchanges use clearinghouses to settle trades. The clearinghouses guaranty that investors selling exchange-traded options will be paid. And, the clearinghouses collect margin deposits from investors selling options, enabling the clearinghouses to guaranty that insurers will receive payment when their catastrophe options "finish in the money". That is actual catastrophe losses cause the catastrophe index used in settling the option to rise above the strike price for the option, resulting in a payment to the insurer that bought the option.

- Because investors' returns on catastrophe options depend on catastrophe losses, and less on economic conditions, returns on catastrophe options are not closely correlated with the returns on other investments. Thus, investors can use catastrophe options to improve the performance of investment portfolios, much like the way investors can use catastrophe bonds.

The disadvantages of exchange traded catastrophe options include:

- An individual insurer's loss experience may not closely match the loss experience underlying the catastrophe index used in a particular option. Thus, an insurer may suffer high catastrophe losses but find that its catastrophe options expire worthless because the index did not reach the strike prices for the options. At other times, an insurer may suffer only minor catastrophe losses but nonetheless collect on its catastrophe options. The possibility of a poor correlation between an insurer's loss experience and the performance of catastrophe options -- called basis risk -- can reduce options' effectiveness as a substitute for reinsurance. The amount of basis risk varies by insurer, depending on how its mix and distribution of exposures compares to that underlying the catastrophe index used in settling specific catastrophe options. Basis risk would probably be reduced by the purchase of varying numbers of catastrophe options by region or state.
- To minimize basis risk, it is necessary that the insurer's geographic distribution match the geographic distribution underlying the index as closely as possible in catastrophe-prone areas. Construction other building factors are not as important. For example, a commercial property insurer should be able to construct an effective hedge using a personal property index.

Option Opportunities for Reinsurers

An individual insurer may find that its loss experience correlates poorly with the indices used in valuing catastrophe options, because the distribution of the insurer's exposure differs from that used in the indices. By assuming risks from several insurers, a reinsurer could make its distribution of exposures more like that used in the indices. This would reduce the basis risk the reinsurer faces when using catastrophe options, protecting its surplus and making it possible for the reinsurer to write additional business.

If insurers using catastrophe options want protection against basis risk, reinsurers may be able to provide complementary forms of reinsurance coverage. For example, if an insurer buys catastrophe options to obtain coverage for a particular layer of loss, a reinsurer might be able to write coverage for losses in the layer that are not covered by the options because of basis risk.

Market Limitations on Use of Options

So far, trading in catastrophe options has created relatively little insurance capacity. This could reflect a number of factors. Some insurers may not understand how to use options. Others may not have sufficient confidence to rely on catastrophe options. The absence of an Andrew-sized catastrophe in recent years and the current softness in reinsurance markets may have led to a lack of interest on the part of some insurers. The light trading in catastrophe options may reflect a lack of interest on the part of investors. With the passage of time, insurers and investors may become more comfortable with catastrophe options, which could become a major source of insurance capacity.

Some Further Thoughts

Rating agencies' evaluation of an insurer's financial strength is a critical element in attracting and retaining business. If rating agencies do not view an insurer's securitization measures as financially sound, the insurer may receive a poor rating and, therefore, suffer a loss of business. Consequently, rating agencies' acceptance of a catastrophe securitization approach may be important to its success.

The capital markets can bring an immense amount of financing into the insurance industry, and perhaps significantly lower the cost of catastrophe risk financing for the long term. The challenge is to figure out how to efficiently bring these resources into the insurance industry.

Cost Considerations

A key factor for delivering an efficient mix of risk financing instruments is the cost of the individual instruments. The cost will be sensitive to the variation in results – many years with small catastrophe losses and occasional years with very large catastrophe losses. The cost of this variation is reflected in the risk load, which is a provision to fund the various instruments available to finance catastrophe insurance. This cost ultimately become part of the price of insurance.

The intense competitive forces in the marketplace may cause insurers to focus on short-term operating results at the expense of long-term solidity. This amounts to insurers ignoring the possibility of rare large catastrophes in their decision making. Insurers may not adequately reflect risk load in pricing thereby not collecting the funds needed to provide catastrophe risk financing.

3. Traditional Approaches to Financing Catastrophe Risk

Raising Insurer Capital

An insurer always has the option of raising sufficient capital to cover its potential losses, but to raise capital, the insurer must increase its net income to justify this capital. There is also the lost opportunity since the capital committed to an insurer is not available for another venture.

Compared with other industries, most property/casualty insurers have not generally achieved high historic returns. Competition from the large number of suppliers has been a major contributing factor. Furthermore, regulation has in some cases also acted to keep insurance rates below actuarially indicated levels.

If an insurer has a heavy concentration of exposures in catastrophe-prone areas, the amount of capital needed can be relatively large compared with the insurer's existing surplus. Furthermore, the additional capital may only be needed occasionally when catastrophe losses are unusually large – perhaps every 100 or 250 years. Committing a large amount of additional capital to cover infrequent losses is extremely inefficient and virtually impossible to sustain in a highly competitive marketplace.

Those considerations drive an insurer to seek alternatives to raising capital.

Reinsurance

The capital of US reinsurers was $13.2 billion in 1992. It grew to $26.2 billion by the end of 1997. With the increased demand for reinsurance following the catastrophes in the early 1990s, new offshore reinsurers provided additional capacity. But that capacity is also relatively small compared with the size of potential catastrophe losses.

Reinsurers provide modest layers of coverage which are usually sufficient to protect small insurers but not larger insurers.

The availability of reinsurance varies considerably over the life of an insurance cycle. The price may also vary substantially depending on supply and demand as well as recent experience.

Reinsurance pays for the primary insurer's losses that exceed certain amounts, or on a quota share basis. The reinsurance coverage follows the fortunes of the primary insurer. On the other hand, reinsurance can also have high and variable transaction costs for the customized coverage provided.

It is important to remember that a reinsurer may not be able to meet its obligations if a large catastrophe occurs. A reinsurer failure could result in the insolvency of some insurers, assessment of surviving insurers, and economic hardship to policyholders.

4. The Cost of the Instruments Used in Financing Insurance

The remainder of the paper will focus on one promising form of securitization – options on a catastrophe index – and see how insurers can combine them with capital and reinsurance to finance catastrophe risk.

We classify the various instruments for financing catastrophe insurance into the following elements:

1. Insurer Capital – This is money put up by investors in the insurance company. The company can use its capital to pay losses if current income is insufficient.
2. Reinsurance – This is money provided by outside entities that agree to pay losses in accordance with a predetermined function of the insurer's loss. Some securitization deals fall into this category.
3. Catastrophe Options – This is money provided by outside entities that agree to pay money contingent on the occurrence of a catastrophic event recorded on an index. That payment may or may not correspond with the insurer's loss. That is, catastrophe options do present basis risk.

Each instrument has a cost and a benefit. The insurer's problem is to find the combination of instruments that provides adequate financing for the least cost.

We define:

The cost of financing insurance =

the expected loss (net of reinsurance recoveries and recoveries from catastrophe options)

\+ the cost of capital

\+ the cost of reinsurance

\+ the cost of catastrophe options

Our purpose in using reinsurance and catastrophe options is to reduce the expected loss and the cost of capital – and ultimately the cost of financing insurance.

Although this definition covers the insurer's entire operation, we will focus on catastrophes. Thus, our discussion of the cost of financing insurance will reflect *only the catastrophe losses*, with one exception – the cost of capital. The insurer's other assets and liabilities affect that cost. This discussion will ignore the remaining elements of the insurer's operation.

Quantifying the Cost of Financing Insurance

To perform this analysis, we will need to quantify the cost of financing insurance in terms of the probability of a catastrophic loss. We give some sample costing formulas below. The formulas have the advantage of being simple, but they are by no means unique or necessary to the examples given below.

For any random variable, Z, we define:

$\mu_Z =$ the expected value of Z

$\sigma_Z =$ the standard deviation of Z.

See Appendix B for the formulas for the various means and standard deviations used below.

Quantifying the Cost of Capital

We employ a probabilistic capital requirements formula as the starting point for this methodology. In the United States, insurers are not subject to an official probabilistic capital requirements formula. However, most actuaries believe that capital requirements should have probabilistic input. Actuaries generally accept the idea of a formula, but any particular formula will spark a debate. While we use one such formula here, an insurer can use another formula that suits the needs and perceptions of its management.

Let X be a random variable representing the insurer's total loss, net of recoveries from reinsurance and catastrophe options. Our formula for the cost of capital is:

$$\text{Cost of Capital} = K \times T \times \sigma_X$$

where:

T is a factor reflecting the insurer's risk aversion; and

K is the required return needed to attract sufficient capital.

We can link T to the insurer's probability of insolvency. For example, if we assume the insurer's losses follow a normal distribution, a choice of T = 2.32 corresponds to a one-in-one-hundred chance of insolvency. If the insurer is more risk averse, or if the distribution of insurer results is unusually skewed, the insurer can select a higher value of T.

The insurer will select K so that its rate of return is close to that obtained by other investments with similar risk. K will vary with market conditions.

In the examples below, we will let

$$X = X_O + X_C$$

where:

$X_C =$ All catastrophe losses net of recoveries from reinsurance and index contracts; and

$X_O =$ All other net losses.

When we partition X is this manner, the formula for the cost of capital becomes

$$\text{Cost of Capital} = K \times T \times \sqrt{\sigma^2_{X_O} + \sigma^2_{X_C}}$$

under the assumption that X_O and X_C are independent[6].

[6] We have elected to simplify our analysis by assuming independence. A more comprehensive analysis would allow for common factors affecting the insurer's entire book of business.

Quantifying the Cost of Reinsurance

The cost of catastrophe reinsurance depends upon market conditions. After a large catastrophe, the demand for reinsurance usually rises and reinsurer capital falls. Therefore, catastrophe insurance is in short supply and the reinsurance available fetches a high price. High prices attract new capital to reinsurers, and prices generally fall until the next catastrophe occurs.

The benefit of the reinsurance treaty is to reduce the insurer's cost of capital by reducing its expected loss, μ_{X_C} , and its standard deviation of loss, σ_{X_C} .

To develop a strategy for using reinsurance, an insurer needs to know its reinsurance costs. Those costs depend upon the retention and the limit of the reinsurance treaty, and each reinsurer has its own prices.

Let X_R be a random variable representing the reinsurance recovery. We will use the following formula for the cost of reinsurance in the examples below:

$$\text{Reinsurance Cost} = (\mu_{X_R} + \lambda \cdot \sigma^2_{X_R}) \times (1 + e)$$

where λ is a risk load multiplier, and e is an acquisition expense factor.

Quantifying the Cost of Catastrophe Options

In this paper, we will work with binary options on a catastrophe index. The holders of those options exercise them for a fixed amount, such as $1,000, when the index exceeds a predetermined strike price. Otherwise the options expire worthless.

To the seller of such options, the expected return should be competitive with other available investments of comparable risk. One way of gauging comparable risk is the analysis of bond defaults. For example, Moody's Investors Service has a web site that publishes bond default rates and interest rate spreads. In browsing Moody's web pages one finds the following statements about default rates:

- "Moody's trailing 12-month default rate for speculative-grade issuers ended 1997 at 1.82% -- up from last year's 1.64%, but well below its average since 1970 of 3.38%."
- "Moody's expects its speculative-grade 12-month default rate to rise toward the 2.5% level in 1998." [7]

With respect to interest rate spreads, Moody's states the following:

- "The spread of the median yield-to-maturity of intermediate-term speculative-grade bonds over seven-year US Treasuries climbed just 3 basis points to 267 basis points -- 92 basis points below its January 1993 to January 1997 average of 359 basis points." [8]

When comparing speculative-grade bonds to catastrophe options, the investor might consider the following:

- The projected 12-month default rate of speculative-grade bonds is 2.5%.
- We can estimate the probability of exercising the catastrophe options (as we will show below). We can compare that probability with estimated default rates for bonds.
- Catastrophe options can require posting a 100% margin at the time of sale. The money in the margin account earns a risk-free rate of return. Thus, the price of the option should be comparable to the interest rate spread for a bond of comparable risk over risk-free investments.
- The average spread of speculative-grade bonds over intermediate-term risk-free investments is about 3.5%. The spread could be lower over a 12-month term, but it should not be lower than the projected default rate.

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[7] The web site URL is http://www.moodys.com/defaultstudy/index.html. We obtained this quote on April 3, 1998.

[8] The web site URL is http://www.moodys.com/economic/1QDFLT97.htm. We obtained this quote on April 3, 1998.

- The exercise of a catastrophe option is not correlated with the other economic risks. That fact makes the catastrophe options more attractive to investors and should lower their price.

With all this information, one can compare the posted price of catastrophe options with bonds of equivalent risk. Investors will have varying interpretations of the information, but our point is that information relevant to the pricing of catastrophe options is publicly available.

A more sophisticated way of using public information to help price catastrophe options is included in Appendix A.

5. An Illustrative Example

As an illustration of the kind of analysis investors can do, we developed a catastrophe index to quantify the expected payout an investor would have to make as a result of selling options on that index. For an insurer we used a catastrophe model to quantify the cost of financing insurance in terms of the costs of attracting capital, buying reinsurance, and buying catastrophe options. We compared the insurer's losses – generated by the catastrophe model – to the benefits provided by the various instruments.

To do the analysis, we took a sample of fifty insurers that report their personal lines exposure to ISO. We then analyzed the personal lines exposure for each of the fifty insurers using a hurricane model provided by Risk Management Solutions, Inc.[9] The analysis provided loss estimates and annual rates of occurrence for about 9,000 events for the insurers in the sample. We created "index" events by summing the losses for each event over all the insurers. We then multiplied the loss for each event by a factor that set the largest event equal to 100.

We then produced Table 5.1 below. The table contains the illustrative index values and the model-generated losses for one of the fifty insurers from the sample. We produced a similar exhibit for each of the fifty insurers.

With information like that provided in the exhibit, we can adjust insurer losses for any recoveries from a reinsurance contract or from catastrophe options. Since the model gives us the probability[10] of any loss and/or recovery, we can calculate any summary

[9] All hurricane loss estimates incorporated in this paper were developed by ISO's use of Risk Management Solutions' (RMS) proprietary IRAS hurricane technology. However, development of the individual company exposure data and the analyses were performed by ISO. Therefore the loss projections and conclusions presented in this paper are the responsibility of ISO.

[10] Event probabilities can be calculated from the RMS model output. The RMS model provides annual rates of occurrence for individual events.

statistics needed to determine the cost and benefits of the various instruments used in financing insurance[11].

Table 5.1
Illustrative Index and Insurer Information

Event	Event Probability	Index Total Loss	Illustrative Index Value	Direct Insurer Loss
1	0.000001210	9,383,371,976	100.000	1,212,550,269
2	0.000001210	8,355,070,420	89.041	1,509,161,589
3	0.000001810	8,215,939,065	87.558	1,303,694,653
4	0.000007020	7,833,207,664	83.480	761,956,629
5	0.000007020	7,806,652,657	83.197	734,137,782
6	0.000004660	7,708,720,644	82.153	735,660,852
7	0.000007910	7,595,628,983	80.948	1,004,861,128
8	0.000050600	7,558,164,289	80.548	1,071,076,934
9	0.000007020	7,430,446,811	79.187	688,269,904
10	0.000001810	7,270,327,316	77.481	1,652,933,116
11	0.000002590	7,151,707,629	76.217	741,327,246
12	0.000005760	7,088,876,652	75.547	654,930,780
13	0.000009060	7,053,981,070	75.175	1,450,085,508
14	0.000022900	7,047,690,340	75.108	1,148,344,417
15	0.000001210	7,041,865,077	75.046	1,003,713,967
16	0.000007020	6,957,052,342	74.142	718,320,849
17	0.000000460	6,912,766,871	73.670	612,322,934
18	0.000002590	6,846,487,556	72.964	607,625,092
19	0.000000767	6,784,428,830	72.303	1,035,338,915
20	0.000000460	6,772,931,882	72.180	564,886,456
21	0.000001810	6,760,672,693	72.050	1,269,991,504
22	0.000021000	6,713,497,690	71.547	921,203,300
23	0.000000738	6,707,044,084	71.478	582,199,078
24	0.000018700	6,685,296,288	71.246	757,962,586
25	0.000000202	6,630,347,892	70.661	1,078,827,927
⇓	⇓	⇓	⇓	⇓

[11] The loss statistics calculated in this paper assume that the events are independent.

Illustrative Catastrophe Options

Using the illustrative catastrophe index, we set up illustrative catastrophe options that pay $1,000 if the largest single event loss in the year exceeds a specified strike price. If no single event exceeds the strike price, the option is not exercised and the buyer receives $0. In the examples that follow, we consider trades on options with strike prices of 5, 10, 15, . . . , 95, 100. The following table gives the probabilities that each option will be exercised. See the Appendix B for the formula for calculating those probabilities.

Table 5.2

Strike Price	Exercise Probability
0	1.00000000
5	0.16313724
10	0.07855957
15	0.04006306
20	0.02321354
25	0.01387626
30	0.00816229
35	0.00440132
40	0.00296168
45	0.00187601
50	0.00100615
55	0.00070126
60	0.00040197
65	0.00028771
70	0.00018975
75	0.00013880
80	0.00008846
85	0.00001125
90	0.00000121
95	0.00000121
100	0.00000121

The catastrophe options used in this example have a structure similar to those traded on the Guy Carpenter Catastrophe Index (GCCI),[12] with four important differences:

1. The scale of the indices is different. The illustrative index has 100 as its highest value whereas the GCCI has 700 as its highest value.
2. The sets of insurers that make up the indices are different.
3. The illustrative index simply sums the losses for each insurer, whereas the GCCI uses a complex set of rules designed to keep a single insurer from having too much influence at the ZIP-code level.
4. The illustrative index is an annual index, whereas the GCCI is semiannual and overlaps with the normal hurricane season in either one or five months.

The following table gives the costs used in the examples below. To calculate the price of the option, we added 0.035% of the variance of the contract payoff to the expected payoff. We arrived at the 0.035% figure by comparing the exercise probability of an option with a strike price of 20, against the price of a speculative-grade bond, as discussed above.

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[12] For information about the options traded on the Guy Carpenter Catastrophe Index, visit the Bermuda Commodities Exchange web site at http://www.bcoe.bm

Table 5.3

Strike Price	Index Total Loss	Exercise Probability	Expected Payout	Contract Price
0	0	1.00000000	1000.000	1000.000
5	469,168,599	0.16313724	163.137	210.920
10	938,337,198	0.07855957	78.560	103.895
15	1,407,505,796	0.04006306	40.063	53.523
20	1,876,674,395	0.02321354	23.214	31.150
25	2,345,842,994	0.01387626	13.876	18.666
30	2,815,011,593	0.00816229	8.162	10.996
35	3,284,180,192	0.00440132	4.401	5.935
40	3,753,348,790	0.00296168	2.962	3.995
45	4,222,517,389	0.00187601	1.876	2.531
50	4,691,685,988	0.00100615	1.006	1.358
55	5,160,854,587	0.00070126	0.701	0.947
60	5,630,023,186	0.00040197	0.402	0.543
65	6,099,191,784	0.00028771	0.288	0.388
70	6,568,360,383	0.00018975	0.190	0.256
75	7,037,528,982	0.00013880	0.139	0.187
80	7,506,697,581	0.00008846	0.088	0.119
85	7,975,866,180	0.00001125	0.011	0.015
90	8,445,034,778	0.00000121	0.001	0.002
95	8,914,203,377	0.00000121	0.001	0.002
100	9,383,371,976	0.00000121	0.001	0.002

Investor Examples

While this example is illustrative, it has a structure similar to the Guy Carpenter Catastrophe Index (GCCI) and the catastrophe options traded on that index. The existence of exposures in zip code detail allow the investor to use a catastrophe model to determine the likelihood that an option with a given strike price will be exercised. The expected payout is simply the product of the exercise probability of an option with the given price and $1,000 - the amount the option will pay if the index exceeds the strike price. While the market will establish the contract price based on the supply and demand for funds relative to other investment opportunities, we developed an illustrative price by adding a portion of the variance of the contract payoff to the expected payoff, as discussed above.

If an investor sells an option with a strike price of 25 as set forth in Table 5.3, then there is a 1.4% probability of losses exceeding the trigger with an average annual expected payout of $13.88. The investor would receive $18.67 for selling this option if the illustrative price applied.

If the actual index losses were $2,345,842,994 or more, then the index would exceed the strike price of 25 and the investor would pay $1,000 to the purchaser of the option. If the losses were below this amount, then the option would expire with no value. This would occur 98.6% of the time.

This same approach would work for the catastrophe options based on the Property Claims Service Index (PCSI) traded on the Chicago Board of Trade (CBOT) although the exposure distribution underlying the index would have to be estimated. There are some differences in the operation of this index, e.g., payouts vary with the index value over the strike price.

Insurer Examples

The following analysis of three insurers shows how those insurers can reduce the cost of financing insurance through the proper use of reinsurance and catastrophe options. The insurers are three members of the sample of fifty insurers that we selected above. We randomly adjusted the losses of each insurer to protect their anonymity.

- Insurer #1 is a medium sized national insurer with exposure that tracks relatively well with the exposure underlying the illustrative index.
- Insurer #2 is a large national insurer with exposure that tracks less well with the exposure underlying the index than Insurer #1.
- Insurer #3 is a regional insurer with exposure that does not track well with that of the index.

We provide summary statistics for the insurers' catastrophe losses.

Table 5.4

	Insurer #1	Insurer #2	Insurer #3
Expected Catastrophe Loss	34,839,348	95,417,229	2,385,629
Std. Dev. Of Catastrophe Loss	81,044,318	196,767,192	18,098,024
Coef. of Correlation with Index	0.93	0.75	0.35

We now provide the economic assumptions underlying our estimate of the cost of financing insurance. The assumptions made here are not specific to the particular insurer, but we could modify the assumptions and/or make them specific after a discussion with an insurer's management.

The Cost of Financing Insurance

As discussed above, we use the following formula for the cost of insurer capital:

$$\text{Cost of Capital} = K \times T \times \sqrt{\sigma_{X_O}^2 + \sigma_{X_C}^2}$$

with the required return K = 20%; the risk aversion factor T = 3.00, and the standard deviation of the insurer's non-catastrophe losses σ_{X_O} = the insurer's initial σ_{X_C}. In a real case, we would estimate σ_{X_O} by analyzing the insurer's other losses.

In the examples that follow, we use the following formula for the cost of reinsurance:

$$\text{Reinsurance Cost} = (\mu_{X_R} + \lambda \cdot \sigma_{X_R}^2) \times (1 + e)$$

with the risk load multiplier $\lambda = 1.5 \times 10^{-7}$ and expense factor e = 10%. The selected value of λ is close to what ISO uses in its risk load formula for increased limits ratemaking.

If the insurer buys N_S contracts for strike price S at cost C_S, the total cost of the index contracts is:

$$\sum_S N_S \cdot C_S$$

Table 5.3 gives the values of C_S for each strike price, S.

The insurer's management has to make three key decisions to minimize the cost of financing insurance:

1. How much capital should the insurer retain?
2. What layer of reinsurance does the insurer buy?
3. How many index contracts, N_S, does the insurer buy at a given strike price, S?

Now, for a given reinsurance layer and a given set of index contracts, we can calculate the quantities μ_{X_R}, $\sigma^2_{X_R}$, μ_{X_C}, and $\sigma^2_{X_C}$ using formulas given in Appendix B.

Thus our expression for the cost of financing insurance becomes

$$\mu_{X_C} + K \times T \times \sqrt{\sigma^2_{X_O} + \sigma^2_{X_C}} + (\mu_{X_R} + \lambda \cdot \sigma^2_{X_R}) \times (1 + e) + \sum_S N_S \cdot C_S$$

We seek to minimize this expression by choosing the right layer of reinsurance and the right numbers, N_S, of catastrophe options.

We do not now have an analytic solution to this minimizing problem. That is because of the effort involved in deriving one and because we do not feel that the assumptions we made in calculating the cost of financing insurance are final.[13] Instead, we used a numerical search algorithm, Excel Solver™. As it is difficult to ascertain that the

[13] For an analytic solution to a simpler problem, see "A Buyer's Guide to Options on a Catastrophe Index" by Glenn Meyers. The paper has been accepted for publication in the *Proceedings of the Casualty Actuarial Society*.

numerical search solution is indeed the optimum, we should characterize the results as "the best solution we could find."

In order to reduce the computing time, we restricted the reinsurance retention and limit to multiples of $1,000,000 and the number of catastrophe options to multiples of 100. In addition we forced the number of catastrophe options to be the same for each of the following groups of strike prices: 5, 10, 15, and 20; 25, 30, 35 and 40; 45,50, and 55; 60, 65, and 70; 75, 80, and 85; and 90, 95, and 100. These restrictions seem reasonable in light of the other uncertainties in the problem.

The search for the minimum cost of financing insurance produced the following results:

Table 5.5

Contract	**Number of Index Contracts**		
Range	**Insurer #1**	**Insurer #2**	**Insurer #3**
5-20	47,400	93,100	0
25-40	74,400	118,100	6,300
45-55	59,500	67,900	0
60-70	47,600	28,600	0
75-85	81,400	545,100	0
90-100	37,200	634,800	0
	Reinsurance		
Retention	73,000,000	457,000,000	54,000,000
Limit	13,000,000	36,000,000	105,000,000

The elements of the cost of financing insurance are as follows:

Table 5.6

Best Solution Obtained for the Cost of Financing Insurance

	Insurer #1	**Insurer #2**	**Insurer #3**
Expected Net Loss	16,315,629	62,086,995	1,464,410
Cost of Capital	47,905,407	143,662,761	12,914,922
Cost of Reinsurance	2,132,070	1,848,530	1,726,342
Cost of Index Contracts	22,252,015	42,409,101	249,427
Cost of Financing Insurance	88,605,121	250,007,387	16,355,100

We compared the "best solution" with two alternative solutions:

Table 5.7

Cost of Financing Insurance without Reinsurance or Index Contracts

	Insurer #1	Insurer #2	Insurer #3
Expected Net Loss	34,839,348	95,417,229	2,385,629
Cost of Capital	62,095,747	166,962,499	15,356,683
Cost of Reinsurance	0	0	0
Cost of Index Contracts	0	0	0
Cost of Financing Insurance	96,935,095	262,379,728	17,742,312

Table 5.8

Cost of Financing Insurance after
Dropping the Smallest Element from the Best Solution

	Insurer #1	Insurer #2	Insurer #3
Expected Net Loss	17,945,994	63,198,145	1,648,555
Cost of Capital	48,508,962	145,045,517	13,023,441
Cost of Reinsurance	0	0	1,726,342
Cost of Index Contracts	22,252,015	42,409,101	0
Cost of Financing Insurance	88,706,971	250,652,763	16,398,337

We can make two observations:

- The introduction of catastrophe options and reinsurance can significantly reduce the cost of financing insurance. In the examples the cost was reduced by 8.6 % for Insurer #1, 4.7% for Insurer #2, and 7.8% for Insurer #3.
- The role of catastrophe options was more significant for the insurers whose catastrophe losses were better correlated with the index. Conversely the role of reinsurance was more significant for the insurer whose catastrophe losses were poorly correlated with the index.

The Marginal Cost of Financing Catastrophe Insurance

The examples illustrate that reinsurance and catastrophe options can significantly reduce the cost of financing insurance. However the analysis does not address the question of how much the insurer needs to build the cost of financing into its premiums. Actuaries usually refer to that cost as the risk load.[14]

To answer the question, we calculate the cost of financing insurance, with and without the catastrophe lines. We call the difference between those costs the marginal cost of financing catastrophe insurance. If the insurer can recover that cost in the premiums it charges, it should write the insurance.

Continuing our example, the cost of financing insurance without catastrophe insurance[15] is: $K \times T \times \sigma_{X_o}$. Thus the marginal cost of financing catastrophe insurance becomes

$$\mu_{X_C} + K \times T \times \left(\sqrt{\sigma_{X_O}^2 + \sigma_{X_C}^2} - \sigma_{X_O}\right) + (\mu_{X_R} + \lambda \cdot \sigma_{X_R}^2) \times (1 + e) + \sum_S N_S \cdot C_S$$

We summarize the results for the three insurers in our illustrative example:

Table 5.9

The Marginal Cost of Financing Catastrophe Insurance Using the Best Solution

	Insurer #1	Insurer #2	Insurer #3
Cost of Financing without Cats	43,908,324	103,258,865	10,764,807
Cost of Financing with Cats	88,605,121	250,007,387	16,355,100
Marginal Cost of Cats	44,696,797	146,748,522	5,590,293
Marginal Cost/Expected Loss	1.283	1.538	2.343

We do a similar calculation without considering reinsurance or contracts on a catastrophe index.

[14] See "The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking" by Glenn Meyers, *Proceedings of the Casualty Actuarial Society LXXXIII*, 1997, for background on risk loads for catastrophe ratemaking. That paper goes beyond the current paper by allocating the risk load to individual insureds. However it accounts only for the cost of capital, and does not account for reinsurance and catastrophe options.

[15] Technically, we should include the expected value of the losses without the catastrophe insurance. But the focus of this paper is on catastrophes, and the expected loss for the noncatastrophe exposure will cancel out when we compute the marginal cost of financing catastrophe insurance.

Table 5.10

The Marginal Cost of Financing Catastrophe Insurance Without Reinsurance or Index Contracts

	Insurer #1	Insurer #2	Insurer #3
Cost of Financing without Cats	43,908,324	103,258,865	10,764,807
Cost of Financing with Cats	96,935,095	262,379,728	17,742,312
Marginal Cost of Cats	53,026,771	159,120,863	6,977,505
Marginal Cost/Expected Loss	1.522	1.668	2.925

Here we see that the proper use of reinsurance and catastrophe options can have a significant effect on premiums, as the marginal cost of financing catastrophe insurance is substantially lower for each insurer using a mix of reinsurance and catastrophe options.

6. The Next Steps

This paper has taken a first step beyond the insurer capital and reinsurance paradigm, by showing how to incorporate instruments with basis risk to reduce the cost of financing catastrophe insurance. Having taken this first step, there are a number of directions that can be taken. We list a few.

- The insurer could consider buying catastrophe options on a regional or state index, as well as a national index. The additional flexibility could decrease the cost of providing insurance for some insurers – such as Insurer #3 above.
- Returns from catastrophe options could be imbedded within the reinsurance. That is, the reinsurance would cover the difference between the insurer's actual loss and the index recovery.
- We could create a customized index to form the basis of settlement between the insurer and a reinsurer. Such an index would be based on the industry data, but with a customized set of ZIP-codes. With such an arrangement, adverse selection by the primary insurer would no longer be an issue.
- A reinsurer could use the catastrophe options as a hedge for its combined exposure. To do this, the reinsurer would have to combine the exposure of all its treaties and do an analysis similar to that done above. The options could give the reinsurer increased capacity to write more catastrophe coverage.

Appendix A

An Exercise in Comparing the Price of Catastrophe Options with Corporate Bonds

Investors in catastrophe options must be compensated for placing their money at risk. Their compensation must be comparable to what they can obtain from other investments of comparable risk. This appendix makes such a comparison with publicly available data. This analysis suffers from the fact that current spreads, reflecting current economic conditions are compared to long-term default probabilities. A better analysis would compare spreads and default probabilities over a long time period.

Ultimately, the marketplace will decide on the price of such instruments.

Studies such as this increase investor familiarity with catastrophe options.

While the details of investing in catastrophe options will vary by exchange, in this analysis we assume that the investor posts $1,000 with the exchange, which in turn invests the money in risk-free treasury bills. If no qualifying catastrophe occurs, this money is then returned to the investor with interest. If a qualifying catastrophe does occur, the money is paid to the insurer. The probability that $1,000 is paid to an insurer is given by Table 5.2.

This situation is similar to that of an investor who buys a $1,000 corporate bond. The investor loans the corporation $1,000 and is repaid at the end of the bond's term if the corporation does not default. An investor, facing the risk of default, will only loan the corporation money if he receives an interest rate over and above that paid by risk-free treasury bills. The difference between the corporate bond rate, and the otherwise equivalent treasury bill is called a "spread".

The performance of corporate bonds is well documented. Bond rating agencies publish default statistics for various credit rating categories. Other services publish spreads. By making the assumption that the spread is a charge for the risk of default, one can come up with an equivalent charge for catastrophe options. This appendix describes one way to do this.

Graph A.1 shows first year default probabilities that were obtained from Standard and Poor's[16] for the time period 1981-1997. Table A.1 gives the average default probabilities during this period.

Graph A.1
1st Year Default Probabilities

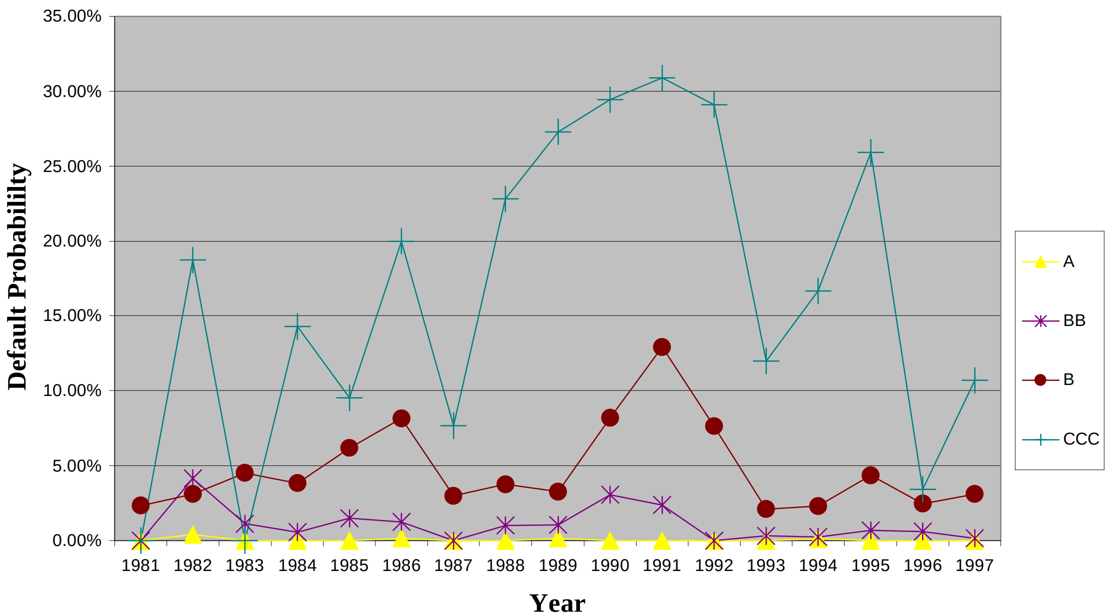

Table A.1
Default Probabilities for Corporate Bonds

Rating Category	1st Year Default Probability
AAA	0.00%
AA	0.00%
A	0.05%
BBB	0.18%
BB	0.90%
B	4.72%
CCC	19.09%

[16] The bond default information used in this appendix came from a demonstration package, CreditPro™, which is available from Standard and Poor's.

Standard and Poor's also did an analysis of recoveries after default and found that the bonds lost, on average, 56% of their value. The standard deviation of the loss was 26%.

We now combine the probability of default information with the recovery information to obtain the expected loss and the variance of the loss. Let p be the probability of default.

$$E[\text{Loss}] = 0.56 \cdot p$$

$$\begin{aligned} \text{Var}[\text{Loss}] &= E_{\text{Default}}\left[\text{Var}[\text{Loss}|\text{Default}]\right] + \text{Var}_{\text{Default}}\left[E[\text{Loss}|\text{Default}]\right] \\ &= p \cdot 0.26^2 + p \cdot (1-p) \cdot 0.56^2 \end{aligned}$$

The expected loss and the standard deviation of the loss for each credit rating category are given in Table A.2.

Table A.2
Loss Statistics for Corporate Bonds

Rating Category	1st Year Default Probability	E[Loss]	Std[Loss]
AAA	0.00%	0.00%	0.00%
AA	0.00%	0.00%	0.00%
A	0.05%	0.03%	1.38%
BBB	0.18%	0.10%	2.62%
BB	0.90%	0.50%	5.84%
B	4.72%	2.64%	13.15%
CCC	19.09%	10.69%	24.77%

We now quantify the risk for the catastrophe options. Here the act of exercising the option is analogous to the default of a corporate bond. The expected loss, as a percentage of the principal, is given by the probability, p, of exercising the option, and the standard deviation of the loss is given by $\sqrt{p \cdot (1-p)}$. The expected loss and the standard deviation of the loss for each option is given in Table A.3.

Table A.3
Loss Statistics for Catastrophe Options

Strike Price	Pr{Exercise} i.e. E[Loss]	Std[Loss]
5	16.313724%	36.95%
10	7.855957%	26.91%
15	4.006306%	19.61%
20	2.321354%	15.06%
25	1.387626%	11.70%
30	0.816229%	9.00%
35	0.440132%	6.62%
40	0.296168%	5.43%
45	0.187601%	4.33%
50	0.100615%	3.17%
55	0.070126%	2.65%
60	0.040197%	2.00%
65	0.028771%	1.70%
70	0.018975%	1.38%
75	0.013880%	1.18%
80	0.008846%	0.94%
85	0.001125%	0.34%
90	0.000121%	0.11%
95	0.000121%	0.11%
100	0.000121%	0.11%

Having quantified the risk for corporate bonds, we now find the price the market places associated with this risk. We obtained the following information about spreads from Bridge Information Systems[17]. The spreads apply to current market conditions and they change daily. The spreads are quoted in basis points, where 1% = 100 basis points.

Table A.4
Spreads of One Year Corporate Bonds Over One Year Treasury Bonds

Rating Category	Financials	Banks	Industrials	Utilities	Trans.
AAA	65	60	45	48	55
AA+	68	77	50	52	60
AA	70	82	55	54	65
AA-	73	84	60	56	70
A+	92	85	65	58	75
A	95	88	70	60	80
A-	98	90	76	62	90
BBB+	105	105	90	69	100
BBB	110	110	103	72	110
BBB-	115	115	118	76	125
BB+	240	275	175	130	190
BB	265	280	210	140	240
BB-	285	285	260	145	300
B+	480	430	310	150	350
B	505	455	385	225	425
B-	530	480	460	300	475
CCC	580	580	500	350	550

[17] The URL on the Worldwide Web is http://www.bonds-online.com/corpindex.html. We used the "Bridge Evaluator" spreads which "are estimated 'new issue' bullet levels."

We combined the results in the above tables to produce the following table[18].

Table A.5
Estimated Spreads for Standard and Poor's Rated Bonds

Rating Category	E[Loss]	Std[Loss]	Spread
AA	0.00%	0.00%	0.57%
AA	0.00%	0.00%	0.69%
A	0.03%	1.38%	0.84%
BBB	0.10%	2.62%	1.08%
BB	0.50%	5.84%	2.53%
B	2.64%	13.15%	4.48%
CCC	10.69%	24.77%	5.53%

Here are some points to ponder.

- There is a positive spread for AAA and AA bonds even though there has been no default, in the first year after rating was made, in the last seventeen years. In addition to possible differences in state and local income taxes, investors are probably seeking some spread to justify purchasing a corporate bond which is defined to have more risk than "zero-risk" US Treasuries.
- The current spread for CCC bonds is less than the expected loss, as calculated over the last seventeen years. One might speculate that the current spread is indicative of the current expected loss, but if this is so, why is the spread less than the expected loss for only the CCC bonds? Investors may feel that they can identify the better risks among the CCC rated bond issuers.

Our problem is to find spreads, or risk charges, for catastrophe options that are competitive with the spreads for corporate bonds. More specifically, we want to use the spread information in Table A.5 to estimate equivalent spreads for Table A.3. Since there are twenty different strike prices, it would be helpful to express these spreads in a formula.

[18] For example, to calculate the BBB spread, we took the average of the BBB+, BBB, and BBB- for each of the Bridge Information Systems data categories. The final spread in Table A.5 numbers for expected differences in the future is the average of the Financials, the Banks and the Industrials Bridge catagories.

We propose a formula of the following form:

$$\text{Catastrophe Option Spread} = K_\alpha \cdot \text{Std[Loss]}^\alpha \qquad \text{(A.1)}$$

If α = 1, we have the familiar standard deviation principle, and if α = 2 we have the familiar variance principle.

We would like to use the data in Table A.5 to estimate the parameters of Equation A.1. However the spreads for AAA, AA and CCC bonds cause problems, so for this exercise we used only the data for the remaining A, BBB, BB and B bonds. We estimated K_α for α = 1 and 2. We also found that α = 0.35 provided a better fit to the data. The results are in Table A.6.

Table A.6
Projected Catastrophe Options Spreads Using A, BBB, BB and B Rated Bonds

Rating Category	**Actual** Spread	**Formula Spreads**[19]		
		α = 0.35	**α = 1**	**α = 2**
A	0.84%	0.96%	0.28%	0.05%
BBB	1.08%	1.26%	0.58%	0.19%
BB	2.53%	2.04%	1.58%	0.94%
B	4.48%	4.69%	5.06%	4.84%
CCC	***5.53%***	***13.24%***	***15.25%***	***18.48%***

Here we see that α = 0.35 provides a better fit than the traditional actuarial formulas. Discussion about why this may be the case is certainly appropriate.

[19] $K_{0.35} = .0416$, $K_1 = .1840$ and $K_2 = 1.2706$.

We now apply Equation A.1 with $\alpha = 0.35$ to get projected spreads to the catastrophe options described in Table A.2. For comparison purposes, we also provide the spreads used in Table 5.3, which were based on the variance principle. The spreads for Equation A.1 above a strike price of 5 (and the resulting prices) are greater than those from Table 5.3.

Table A.7
Projected Spreads for Catastrophe Options

Strike	**Spreads**	
Price	**Eq. A.1**	**Table 5.3**
5	2.93829%	4.77832%
10	2.62951%	2.53358%
15	2.35400%	1.34603%
20	2.14611%	0.79361%
25	1.96457%	0.47893%
30	1.79216%	0.28335%
35	1.60962%	0.15337%
40	1.50220%	0.10335%
45	1.38710%	0.06554%
50	1.24401%	0.03518%
55	1.16791%	0.02453%
60	1.05959%	0.01406%
65	0.99937%	0.01007%
70	0.92918%	0.00664%
75	0.87971%	0.00486%
80	0.81303%	0.00310%
85	0.56674%	0.00039%
90	0.38364%	0.00004%
95	0.38364%	0.00004%
100	0.38364%	0.00004%

We then calculated the cost of financing catastrophe insurance for Insurer #1 using the spreads implied by Equation A.1. The results are in Table A.8 and A.9.

Table A.8
Solution to Minimize the Cost of Financing Catastrophe Insurance

	Number of Index Contracts	
Range	**Eq. A.2**	**Table 5.2**
5-20	52,900	47,400
25-40	0	74,400
45-55	0	59,500
60-70	0	47,600
75-85	0	81,400
90-100	0	37,200
Retention	348,000,000	73,000,000
Limit	54,000,000	13,000,000

Table A.9
The Cost of Financing Catastrophe Insurance

	Insurer #1 with Spreads Given By:	
	Eq. A.2	**Table 5.3**
Expected Net Loss	18,206,502	16,315,629
Cost of Capital	49,871,242	47,905,407
Cost of Reinsurance	967,821	2,132,070
Cost of Index Contracts	21,459,013	22,252,015
Cost of Financing Insurance	90,504,578	88,605,121

Noteworthy is the fact that when you significantly increase the price of contracts with the higher strike prices, the insurer will not buy any.

Appendix B

The Calculation of the Statistics for a Maximum Event Index Contract

This appendix gives the formulas for the statistics used in calculating the cost of financing insurance. The calculations are complicated by the fact that the catastrophe index recovery for an event depends upon whether or not the event was the largest event. We solve this by calculating conditional statistics based on the event being the largest – and then calculate global statistics by summing over the conditional probabilities.

We are given n (about 9000) events from the catastrophe model and the index values associated with each event. We assume that the events are independent and that they can only happen once in a year[20]. The events are sorted in decreasing order of the index value. Table B.3 gives the first 30 rows of the calculation. The following table gives the formulas used in this exhibit.

Table B.1
Formulas for Table B.3

ith Row of Column	Description and Formula
Event	The ith event specified by the catastrophe model
Index Value	The value of the index if the ith event is the largest
Event Probability, p_i	The probability of the ith event as specified by the catastrophe model
Max Event Probability, ${}_M p_i$	The probability that the ith event happens and all larger events do not happen $${}_M p_i = p_i \cdot \prod_{j=1}^{i-1}(1-p_j)$$
Contract Value, v_i	The amount paid by the insurer's portfolio of catastrophe options *given that the ith event is the maximum event*
Direct Insurer Loss, x_i	The loss generated by catastrophe model for the ith event on the insurer's exposure

[20] The RMS model provides annual rates of occurrence for events. Because rates are so small, making the assumption that each event can only happen once per year is reasonable.

Table B.1 – Continued

ith Row of Column	Description and Formula
Reinsurance Recovery, r_i	The amount recovered from the reinsurance contract for the ith event
Event Loss Given Max, e_i	$e_i = x_i - v_i - r_i$
E[Loss | Event is the Max], E_i	$E_i = e_i + \sum_{j=i+1}^{n} E\left[(x_j - r_j)\right]$ $= e_i + \sum_{j=i+1}^{n} (x_j - r_j) \cdot p_j$ $= e_i + E_{i+1} - e_{i+1} + (x_{i+1} - r_{i+1}) \cdot p_{i+1}$
E[Loss^2 | Event is the Max], $_2E_i$	$_2E_i = E_i^2 + \sum_{j=i+1}^{n} Var\left[(x_j - r_j)\right]$ $= E_i^2 + \sum_{j=i+1}^{n} (x_j - r_j)^2 \cdot p_j \cdot (1 - p_j)$ $= E_i^2 + {}_2E_{i+1} - E_{i+1}^2 + (x_{i+1} - r_{i+1})^2 \cdot p_{i+1} \cdot (1 - p_{i+1})$

Table B.2
Cost of Financing Insurance Statistics

Overall Statistic	Formula
E[Reinsurance Recovery], μ_{X_R}	$\mu_{X_R} = \sum_{i=1}^{n} p_i \cdot r_i$
Var[Reinsurance Recovery], $\sigma_{X_R}^2$	$\sigma_{X_R}^2 = \sum_{i=1}^{n} r_i^2 \cdot p_i \cdot (1 - p_i)$
E[Net Catastrophe Loss], μ_{X_C}	$\mu_{X_C} = \sum_{i=1}^{n} {}_M p_i \cdot E_i$
Var[Net Catastrophe Loss], $\sigma_{X_C}^2$	$\sigma_{X_C}^2 = \sum_{i=1}^{n} {}_M p_i \cdot {}_2E_i - \mu_{X_c}^2$

Exercise Probabilities

Let PE_i denote the probability that maximum event catastrophe option at the level of event i will be exercised. The option will be exercised if either the ith or a lower numbered (higher loss) event happens. That is: $PE_1 = p_1$, $PE_i = p_i + PE_{i-1} \cdot (1 - p_i)$

Table B.3 Preliminary Calculations for the Cost of Financing Insurance Statistics

Event	Index Value	Event Probability	Max Event Probability	Contract Value	Direct Insurer Loss	Reinsurance Recovery	Event Loss Given Max	E[Loss|Max]	E[Loss^2|Max]
1	100.00	0.000001210	0.000001210	1,125,200,000	1,212,550,269	16,000,000	71,350,269	105,039,888	1.06712E+16
2	89.04	0.000001210	0.000001210	1,021,700,000	1,509,161,589	16,000,000	471,461,589	505,149,400	2.33194E+17
3	87.56	0.000001810	0.000001810	1,021,700,000	1,303,694,653	16,000,000	265,994,653	299,680,134	7.95274E+16
4	83.48	0.000007020	0.000007020	939,300,000	761,956,629	16,000,000	(193,343,371)	(159,663,127)	4.17510E+16
5	83.20	0.000007020	0.000007020	939,300,000	734,137,782	16,000,000	(221,162,218)	(187,487,015)	5.30470E+16
6	82.15	0.000004660	0.000004660	939,300,000	735,660,852	16,000,000	(219,639,148)	(185,967,298)	5.23874E+16
7	80.95	0.000007910	0.000007910	939,300,000	1,004,861,128	16,000,000	49,561,128	83,225,155	8.84949E+15
8	80.55	0.000050600	0.000050598	939,300,000	1,071,076,934	16,000,000	115,776,934	149,387,575	2.02818E+16
9	79.19	0.000007020	0.000007019	856,900,000	688,269,904	16,000,000	(184,630,096)	(151,024,174)	3.88460E+16
10	77.48	0.000001810	0.000001810	856,900,000	1,652,933,116	16,000,000	780,033,116	813,636,074	6.19226E+17
11	76.22	0.000002590	0.000002590	856,900,000	741,327,246	16,000,000	(131,572,754)	(97,971,674)	2.23955E+16
12	75.55	0.000005760	0.000005759	856,900,000	654,930,780	16,000,000	(217,969,220)	(184,371,820)	5.20551E+16
13	75.18	0.000009060	0.000009059	856,900,000	1,450,085,508	16,000,000	577,185,508	610,769,915	3.42608E+17
14	75.11	0.000022900	0.000022898	856,900,000	1,148,344,417	16,000,000	275,444,417	309,002,893	8.34181E+16
15	75.05	0.000001210	0.000001210	856,900,000	1,003,713,967	16,000,000	130,813,967	164,371,248	2.37695E+16
16	74.14	0.000007020	0.000007019	774,500,000	718,320,849	16,000,000	(72,179,151)	(38,626,801)	1.07551E+16
17	73.67	0.000000460	0.000000460	774,500,000	612,322,934	16,000,000	(178,177,066)	(144,624,990)	3.68535E+16
18	72.96	0.000002590	0.000002590	774,500,000	607,625,092	16,000,000	(182,874,908)	(149,324,364)	3.85299E+16
19	72.30	0.000000767	0.000000767	774,500,000	1,035,338,915	16,000,000	244,838,915	278,388,677	6.68006E+16
20	72.18	0.000000460	0.000000460	774,500,000	564,886,456	16,000,000	(225,613,544)	(192,064,034)	5.58109E+16
21	72.05	0.000001810	0.000001810	774,500,000	1,269,991,504	16,000,000	479,491,504	513,038,744	2.37731E+17
22	71.55	0.000021000	0.000020997	774,500,000	921,203,300	16,000,000	130,703,300	164,231,531	2.34399E+16
23	71.48	0.000000738	0.000000738	774,500,000	582,199,078	16,000,000	(208,300,922)	(174,773,109)	4.83588E+16
24	71.25	0.000018700	0.000018697	774,500,000	757,962,586	16,000,000	(32,537,414)	976,524	6.73762E+15
25	70.66	0.000000202	0.000000202	774,500,000	1,078,827,927	16,000,000	288,327,927	321,841,651	9.01151E+16
26	70.57	0.000001210	0.000001210	774,500,000	1,017,469,903	16,000,000	226,969,903	260,482,415	5.82464E+16
27	70.29	0.000001210	0.000001210	774,500,000	1,162,380,661	16,000,000	371,880,661	405,391,786	1.45612E+17
28	68.99	0.000001810	0.000001810	726,900,000	1,273,618,722	16,000,000	530,718,722	564,227,570	2.89618E+17
29	68.73	0.000007250	0.000007249	726,900,000	966,395,280	16,000,000	223,495,280	256,997,239	5.66513E+16
30	68.64	0.000007020	0.000007019	726,900,000	598,955,192	16,000,000	(143,944,808)	(110,446,942)	2.59361E+16