An Analysis of Variations in Leverage Ratios Among Insurers

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Biography:

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Abstract:

Insurers accept and manage risk. The insurance market has long sought to measure the operating leverage and risk of various insurers. The premium to surplus ratio and the reserve to surplus ratio are traditional measures of this leverage and risk. This paper examines both the sources of risk to an insurer and how the insurer can reduce that risk. It also examines the effectiveness of various leverage indices. Finally, it proposes an alternative model of aggregate leverage. The parameters of this model are estimated by fitting an econometric model to data representing 115 insurers. This alternative model suggests that conventional indices of leverage fail to identify those insurers whose surplus is highly leveraged by risk.

INTRODUCTION

Insurance is an important economic mechanism in our society. As such, one may view insurance in a variety of ways. For example, one may view insurance as a business involving the transfer of risk from the insured to the insurer. For this transfer to take place, at least one of the following two events must occur. First, society could legislate that a particular transfer of risk be made. Second, the insured could be sufficiently risk averse that he will find his utility increased by more than the risk loading and transactions costs which the insurer requires to accept his risk. During this risk transfer, the combined operations of the law of large numbers and diversification reduce the risk loading required by the insurer.

One may also view insurance as a leveraged trust. Equity owners can pool their assets and earn returns in excess of those generated by their assets. They accomplish this by allowing their equity to be at risk during the insurance transaction. The profits earned through this transaction will augment the returns generated by their invested assets.

One may also view insurance as the management of a portfolio of risk bearing elements. The manager's goals are to minimize the risk and to maximize the returns. The components of the risk portfolio include the following operations of the insurer:

1) Pricing;
2) Underwriting;
3) Marketing;
4) Reserving;
5) Investing;
6) Tax Planning;
7) Management.

Regardless the view, it is clear that the insurer accepts risk. He manages that risk and is in turn in some sense at risk. The risk to the insurer is that his expectations regarding his decisions will not be realized and, as a result, his equity will be diminished.

The insurance market, including its regulators, owners and policyholders, has long sought to measure the operating risk of various insurers. Traditionally, the premium to surplus ratio and the reserve to surplus ratio have served this purpose. These two measures have as their major advantage their ease of understanding and simplicity of calculation. Nevertheless, there is much to detract from their use as measures of leverage or risk. For example, are two insurers with premium to surplus ratios of 2 to 1 equally at risk if one writes exclusively Homeowners' insurance along the Gulf Coast while the other diversifies across all lines and all states? Are two insurers with reserve to surplus ratios of 3.0 to 1 equally at risk if one writes exclusively Earthquake insurance along the San Andreas Fault while the other writes all lines in all states? Finally, are two insurers equally at risk when they are operating identically in all respects, except that one invests his entire portfolio in stocks while the other diversifies between stocks and bonds?

The answers to these questions are obvious. These examples demonstrate the weaknesses inherent in these traditional ratios as measures of aggregate insurer leverage and risk. This paper will examine variations in these ratios and will proceed to develop an alternative index of risk and leverage. The next section will explore some items preliminary to this analysis.

PRELIMINARIES

The insurer assumes risk through the insurance transaction, his investment portfolio and his operations in general. The insurer minimizes risk primarily through two mechanisms: the law of large numbers and diversification.

The law of large numbers works by reducing the variance between actual and expected results as the insurer assumes more individual risk transactions. This variance reduction not only occurs with regard to insurance transactions, but also with regard to investment transactions.

Diversification, in the simplest sense, means not putting all of your eggs in one basket; that is, not writing exclusively Homeowners' insurance along the Gulf Coast. Diversification also includes:

1) Recognizing the inverse relationship between stock prices and bond prices and investing so as to minimize risk.
2) Recognizing the inverse relationship between certain underwriting cycles. For example a downturn in the economy might improve personal auto experience because insureds drive less while degrading workers compensation experience because the recently unemployed are likely to file compensation claims.
3) Recognizing that a kind of financial synergism exists between the insurance and the investment operations such that the longer and more predictable the timing of the loss payments, the less the investment risk since the likelihood of the forced liquidation of a temporarily distressed asset is reduced.

Finally, there exists some relationship or overlap between the law of large numbers and diversification. Buying many shares of the same stock does not involve the law of large numbers. Buying many shares of many different stocks, all of which are equally risky, does involve the law of large numbers.

THE DATA BASE

115 insurers, either unique companies or groups of companies, who were licensed to write in Pennsylvania served as the basis for the subsequent analysis. A variety of current and historic accounting and operational data, downloaded from their 1990 Statutory Annual Statements via the NAIC data base, became the data base. The data included:

1) Detailed asset information;
2) Detailed unearned premium reserve information by line of business;
3) Detailed loss and loss adjustment expense reserve information by line of business;
4) Detailed written premium information by line of business;
5) Detailed written premium information by state;
6) Five year history of aggregate net written premiums.
7) Five year history of aggregate calendar year loss

and loss adjustment expense ratios.

PREMIUM TO SURPLUS RATIO

The premium to surplus ratio is probably the most widely used measure of insurer leverage and risk. It is usually calculated as the ratio of net written premiums during the year to surplus at year end. It is an element of the IRIS tests. It is also an element of most other insurer rating systems. As a rule of thumb, values in excess of 3 to 1 imply excess leverage. Also, this ratio is used often in ratemaking applications to allocate surplus to particular lines of business for the purpose of determining underwriting profit loads.

The premium to surplus ratio's simplicity and ease of use recommend it. Nevertheless, this ratio has significant drawbacks as a measure of aggregate leverage and risk:

1) It assumes that leverage and risk are entirely a function of written premiums;
2) It ignores many other elements of leverage and risk;
3) It assumes that risk does not vary by line of business;
4) It assumes that risk is reduced to zero when the policy expires;
5) It relates a premium flow for an entire year to

a level of surplus which exists at an instant in time.

In order to alleviate the concern raised in item 5 above, net written premiums are related here to the average value of surplus during the year. Exhibits 1 and 2 present in histogram form frequency distributions of these premium to surplus ratios for the 115 insurers in the data base. Exhibit 1 shows the number of insurers whose premium to surplus ratio falls within certain ranges. Exhibit 2 shows the proportion of total industry surplus for which this index of leverage falls within certain ranges.

10 insurers representing 8.2% of the industry-wide surplus are writing at a premium to surplus ratio greater than 3 to 1. 7 of the 115 insurers in this data base have failed 4 or more IRIS tests. Only 2 of these insurers are included in the group of 10. An independent firm rates 5 of these insurers as "Fairly Good" or better.

RESERVE TO SURPLUS RATIO

The reserve to surplus ratio is another widely used measure of insurer leverage and risk. It is usually calculated as the ratio of all loss, loss adjustment expense, and unearned premium reserves at year end to surplus at year end. This ratio also is used often in ratemaking applications to allocate surplus to particular lines of business for the purpose of determining underwriting profit loads. Unlike the premium to surplus ratio, no rule of thumb exists regarding its value. Further, it is not explicitly used in the IRIS tests. However, this ratio is used in at least one insurer rating system.

The reserve to surplus ratio's simplicity and ease of use recommend it. Nevertheless, this ratio has significant drawbacks as a measure of leverage and risk:

1) It assumes that leverage and risk are entirely a function of reserves, implying that leverage and risk are entirely a function of the insurance side of the business;
2) It ignores many other elements of leverage and risk;
3) It assumes that risk does not vary by line of business.

Exhibits 3 and 4 present in histogram form frequency distributions of reserve to surplus ratios for the 115 insurers in this data base. Exhibit 3 shows the number of insurers whose reserve to surplus ratio falls within certain ranges. Exhibit 4 shows the proportion of total industry surplus for which this index of leverage falls within certain ranges.

11 insurers representing 9.5% of industry-wide surplus have reserve to surplus ratios greater than 5 to 1. 5 of these insurers have failed 4 or more IRIS tests. An independent firm rates 8 of these insurers "Fair" or better. This firm rates 7 "Excellent" or better. 3 of the insurers rated "Fair" or better actually failed 4 or more IRIS tests.

AN ALTERNATIVE MODEL OF AGGREGATE LEVERAGE

The analysis above suggests desirable qualities for an alternative model of aggregate leverage. First, such a model should recognize many sources of leverage. It should be able to combine these various sources into one unique index of aggregate leverage. Further, it should reflect law of large numbers and diversification effects. Finally, it should be able to distinguish those insurers who are exceptionally leveraged and at great risk.

One can begin to construct such a model by assuming that surplus serves as risk capital. Further, assume that insurers, at least implicitly, allocate surplus to support the elements of risk in their portfolios when they assume risk and make their risk portfolio management decisions. Finally, assume that insurers generally agree upon the relative riskiness of various transactions.

Given that these assumptions are met, one can use the variations among insurers in certain of their financial data to measure how they allocate surplus to support risk. Then one could estimate how much surplus insurers would allocate, on average, to support any given portfolio of risk. This could serve as an indicator of the risk assumed by the insurer. One could compare an insurer's expected surplus, given his portfolio of risk elements, to his actual surplus. This could serve as an index of the insurer's aggregate leverage.

The first assumption is reasonable. For example, Stephen P. D'Arcy writes in the Foundations of Casualty Actuarial Science:

> Surplus serves as the margin of error for an insurer. Surplus is available to absorb losses generated by inadequate pricing, to offset inadequate loss reserves, or to cover investment losses. If an insurer did not have any surplus, then it would be bankrupt if anything went wrong in its financial statement. Because of the future financial commitments involved in insurance, surplus plays an important role in assuring customers that the commitments can be fulfilled.

The second assumption is also reasonable. For example, Kneuer in his paper "Allocation of Surplus for a Multi-Line Insurer" poses the question, "Why allocate surplus?" He responds:

> The surplus of an insurer is a finite good. The limitations to surplus prevent the insurer from writing greater volumes of business, or larger risks, or business that has an expectation of higher profits. Thus surplus has a value beyond the insurer's liquidation value. That value is the opportunity to earn additional profits by writing more insurance.

Finally, the third assumption is also reasonable. There are a variety of services and techniques available that rate the quality, or riskiness, of various assets. The actuarial literature is replete with discussions of the measurement of risk engendered by the operations of an insurer.

ESTIMATING THE MODEL

As noted in an earlier section, a variety of calendar year 1990 financial data for 115 insurers licensed to write in Pennsylvania served as the data base. The first test of this model is whether variations in surplus levels among insurers could be explained by variations among those insurers in their stocks of various risk generating elements. One can perform this test by fitting an econometric model to the data. The dependent variable is the actual surplus for each insurer. The independent variables are the various elements from each insurer's financial data base. They include:

1) Unearned premium reserves by line of business;
2) Loss and loss adjustment expense reserves by line of business;
3) Aggregate book values of various assets in the insurer's portfolio;
4) Aggregate bond quality;
5) Aggregate variability of the insurer's loss experience;
6) Five year average premium growth;
7) Maximum geographic concentration.

The model should not include a constant since the insurer who does not accept risk would not need to allocate surplus. The estimated beta coefficients would be a measure of how much surplus an insurer allocates, on average, per unit of each element in his portfolio. A detailed glossary of all variables is included as Appendix 1.

The experienced econometrician will recognize immediately the existence of a serious technical problem for the estimation process: multi-collinearity. Miller and Wichern in their text, Intermediate Business Statistics: Analysis _of Variance, Regression, and Time Series_, write:

> If the independent variables (or some subset of them) are "nearly" linearly dependent, ..., the least squares estimates tend to be unstable and inflated.

Clearly, many of the independent variables are highly correlated. Generally the unearned premium reserves, loss reserves and the assets of the insurer all increase as the size of the insurer increases.

To avoid this problem the surplus, unearned premium reserves by line of business, loss and loss adjustment expense reserves by line of business, and book values of the various assets for each insurer were each ratioed to that insurer's aggregate reserves. This procedure removed size as an influence common to many of the variables. However, it also appeared to remove size as an independent variable from the regression. To reintroduce size, another independent variable, the natural logarithm of each insurer's aggregate net written premiums, was added to the regression equation.

The detailed results of the regression analysis, appear in Appendix 2. The R^2 was 98.6%. The F-Statistic was 211.4. This analysis included 29 independent variables. Only 5 of these variables were not significant at the 80% level as measured by their t-ratio. 18 were significant at the 90% or better level. These values indicate that the model is successful when explaining variations among insurers with regard to their surplus as a function of their risk portfolio.

The results provided one surprise: they indicated that the size variable (LNNWP) was highly correlated with other independent variables. Since the influence of size had ostensibly been eliminated from the other variables by ratioing them to aggregate reserves for each insurer, this suggests that some other behavior measured by the independent variables serves as a proxy for size. Possible candidates include geographic concentration and concentration in a particular line or asset. Subsequent analysis reveals that the size variable is highly correlated with higher concentrations in workers compensation and auto liability. The regression analysis was performed again, but excluding the size variable. The R^2 was again 98.6%. The F-Statistic was 217.7. The results of this regression appear in Appendix 3.

Before leaving this section it is useful to examine how one might use the estimated beta coefficients to allocate surplus. Each dollar of unearned premium reserve or loss and loss adjustment expense reserve is offset by some asset. The combined effect of the asset and the reserve must be considered when allocating surplus. For example, if one refers to Appendix 3, the value of the beta coefficient for bonds (BDA/R) is .7334 and for stocks (STA/R) is .8579. The value of the beta coefficient for workers compensation unearned premium reserves (WCU/R) is -.4256. The value of the beta coefficient for workers compensation loss and loss adjustment expense reserves (WCL/R) is -.6862. An insurer would allocate .4323 (.8579 - .4256) dollars of surplus to support each dollar of workers compensation unearned premium reserve offset by a dollar of holdings in stocks. Similarly, he would allocate ..1717 (.8579 - .6862) dollars of surplus to finance each dollar of his loss reserve. If the insurer invested in bonds, his allocation of surplus would be reduced to .3078 (.7334 - .4256) dollars per dollar of unearned premium reserve and to .0472 (.7334 - .6862) dollars per dollar of loss reserve.

RESULTS

Exhibits 5 and 6 present in histogram form frequency distributions of actual surplus to reserve ratios for the 115 insurers included in this data base. Exhibit 5 shows the number of insurers whose actual ratio falls within certain ranges. Exhibit 6 shows the proportion of total industry surplus for which the actual ratio falls within certain ranges.

Exhibits 7 and 8 present in histogram form frequency distributions of expected surplus to reserve ratios. These expected ratios are an indicator of the aggregate risk assumed by each insurer. Exhibit 7 shows the number of insurers whose expected ratio falls within certain ranges. Exhibit 8 shows the proportion of total industry surplus which is subject to certain levels of risk.

Exhibits 9 and 10 present in histogram form frequency distributions of the aggregate leverage index. The aggregate leverage index is the ratio of the expected surplus to the actual surplus. Exhibit 9 shows the number of insurers whose aggregate leverage index falls within certain ranges. Exhibit 10 shows the proportion of total industry surplus for which the aggregate leverage falls within certain ranges.

27 insurers representing 13.97% of industry-wide surplus have an aggregate leverage index greater than 1.10. Only 3 of the 7 insurers who failed 4 or more IRIS tests are included. An independent firm rates 21 of the 27 insurers as "Good" or better. This firm rates 16 of the 27 insurers as "Excellent" or better.

CONCLUSION

The 3 leverage indices (the premium to surplus ratio, the reserve to surplus ratio and the aggregate leverage index) identify significantly different groups of insurers as highly leveraged. The difference is especially dramatic between the aggregate leverage index and the other two indices. The IRIS tests and the insurer ratings by the independent firm are not always consistent. Finally, the aggregate leverage index identifies as especially leveraged a group of insurers which is not similarly identified as at significant risk by either the premium to surplus ratio, the reserve to surplus ratio, the IRIS tests, or the ratings of the independent firm.

BIBLIOGRAPHY

[1] D'Arcy, Stephen P, et al, Foundations of Casualty Actuarial Science, Chapter 9: "Special Issues".

[2] Kneuer, Paul J, "Allocation of Surplus for a Multiline Insurer", DPP 1987, 191-228.

[3] Miller, Robert B & Wichern, Dean W, Intermediate Business Statistics: Analysis of Variance, Regression, and Time Series".

[4] Willet, Allan H, The Economic Theory of Risk and Insurance, reprinted in the CAS Forum, Winter 1991.

Exhibit 1: Premium to Surplus Ratios

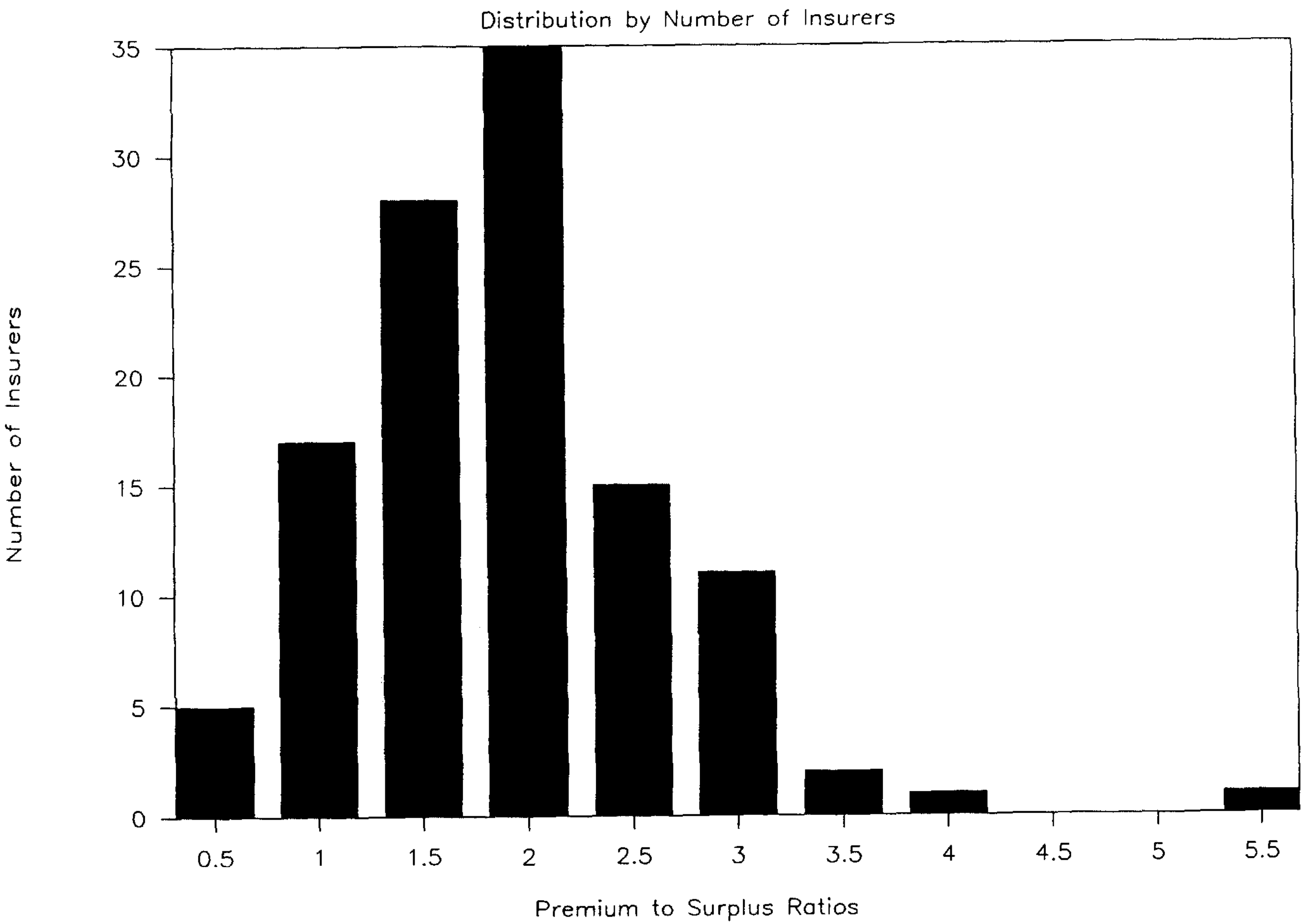


Exhibit 2: Premium to Surplus Ratios

Distribution by Proportion of Surplus

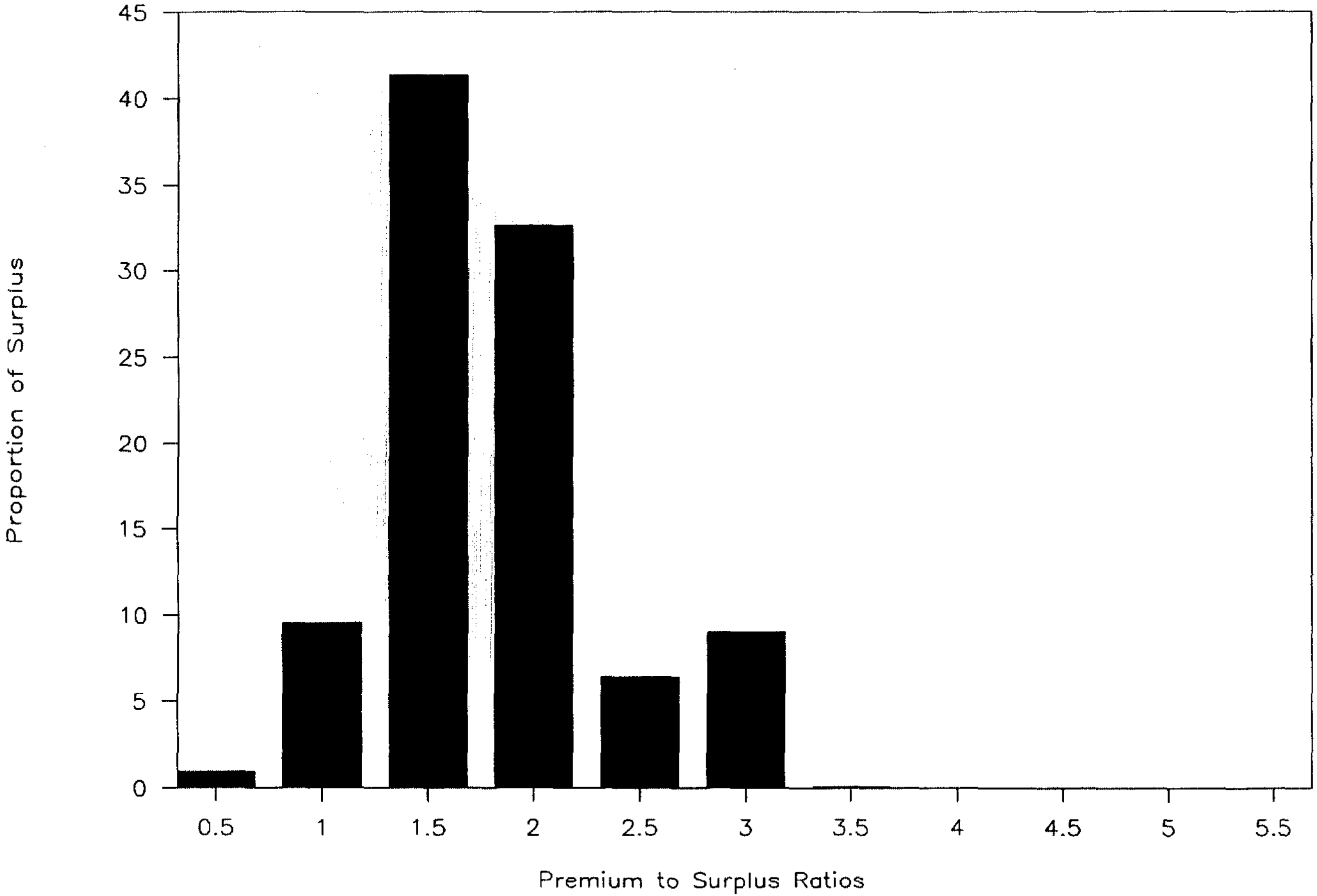
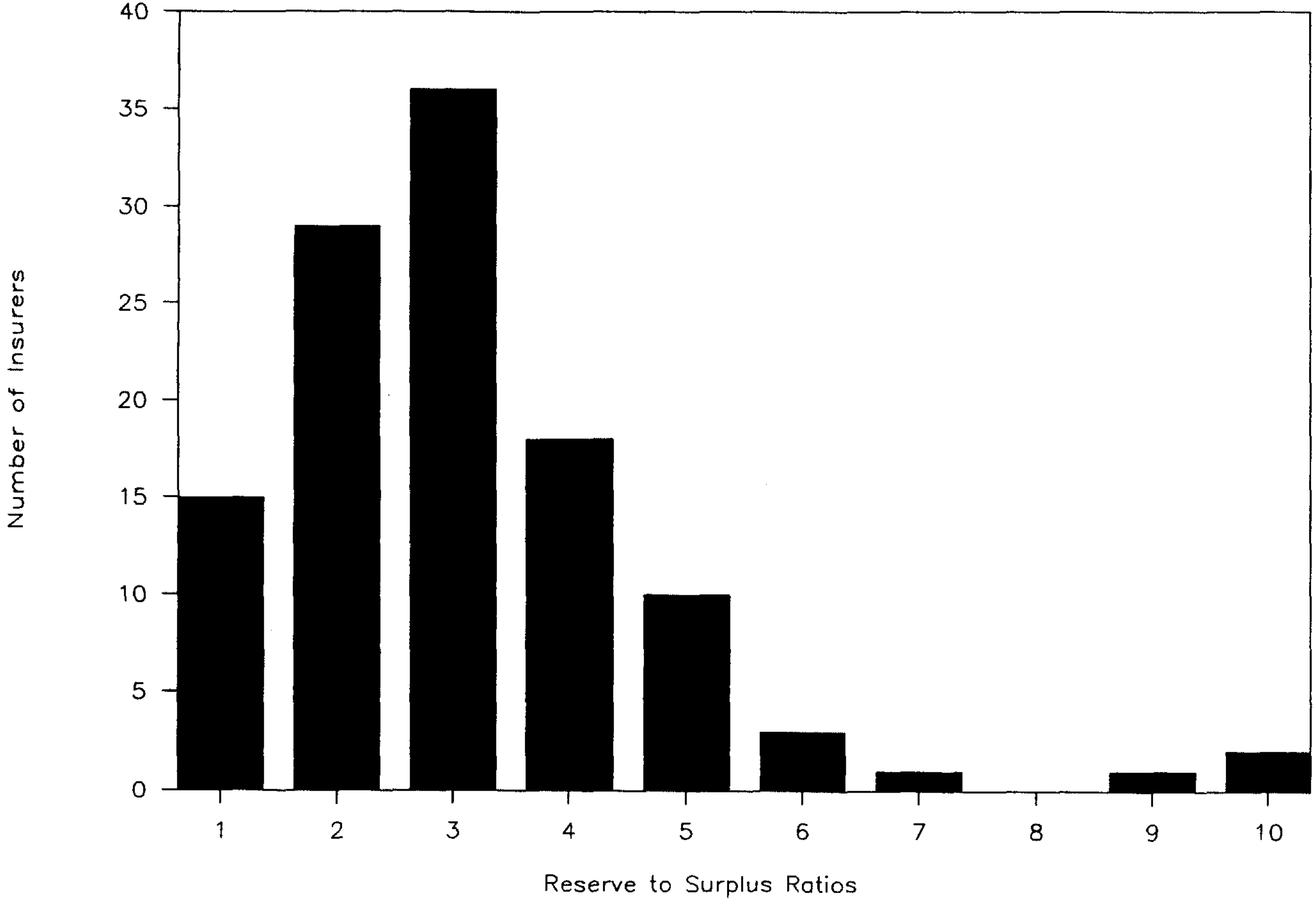


Exhibit 3: Reserve to Surplus Ratios

Distribution by Number of Insurers



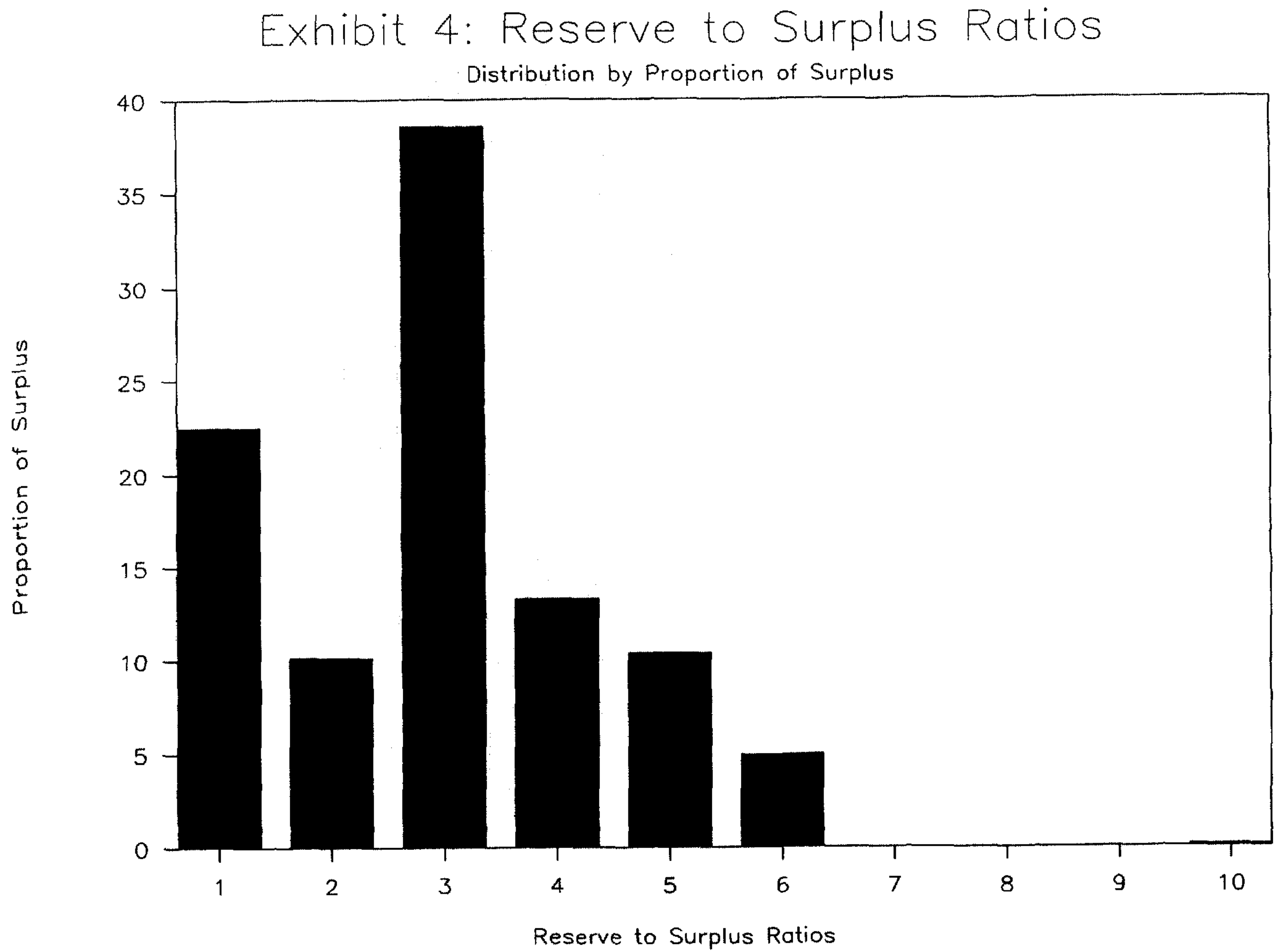


Exhibit 4: Reserve to Surplus Ratios
Distribution by Proportion of Surplus
Proportion of Surplus
40
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Reserve to Surplus Ratios

Exhibit 5: Actual Surp. to Res. Ratios

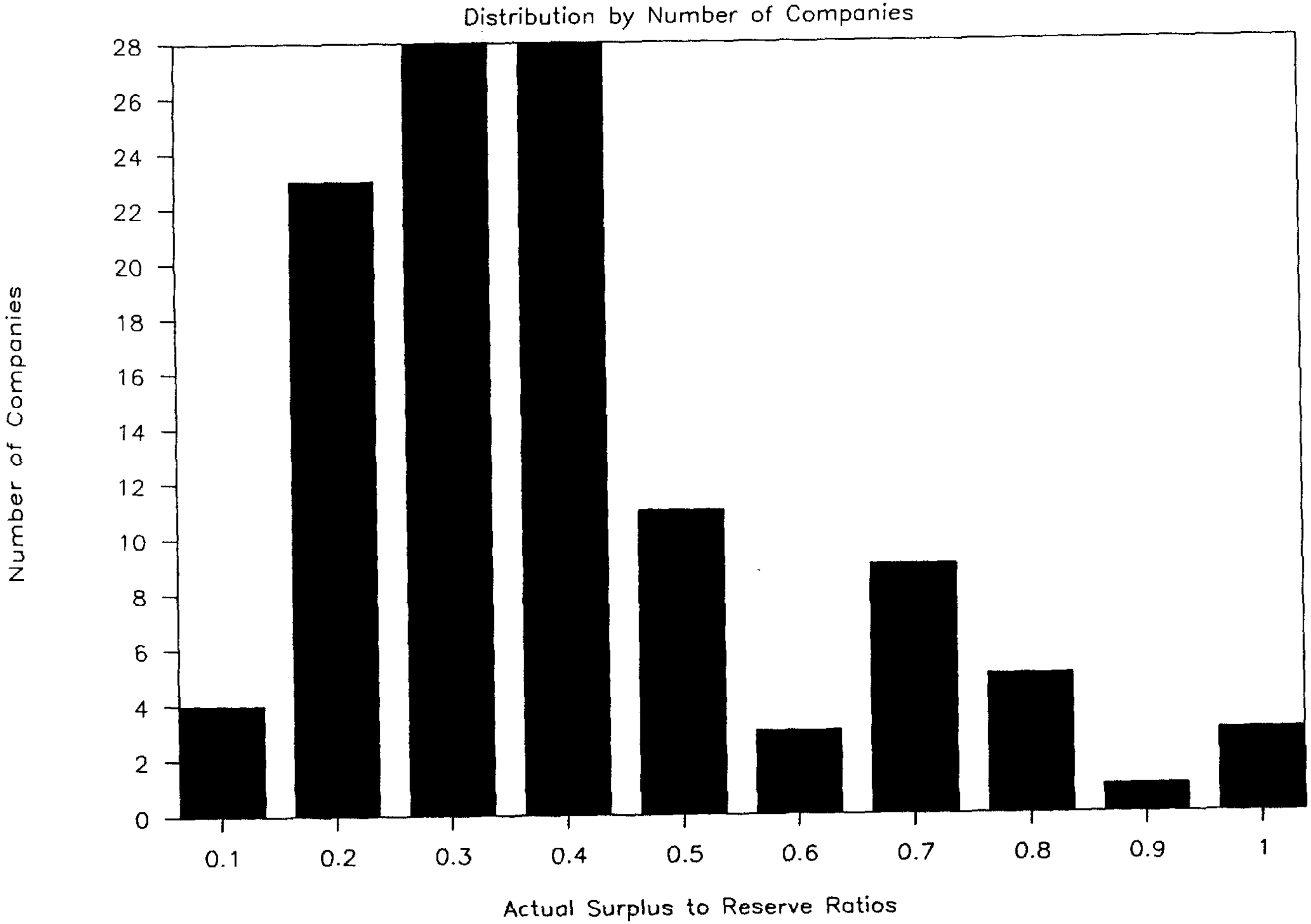
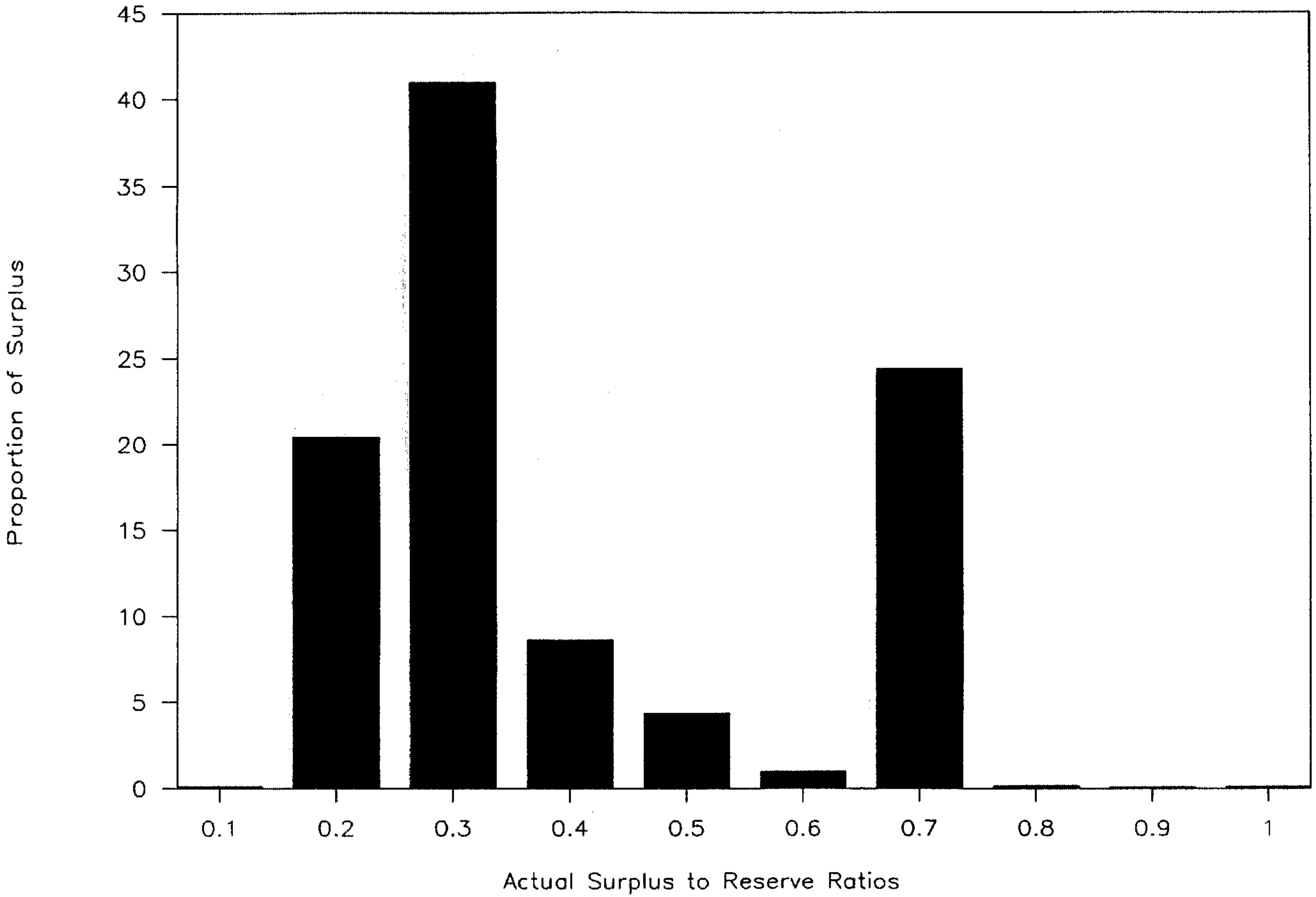


Exhibit 6: Actual Surp. to Res. Ratios

Distribution by Proportion of Surplus



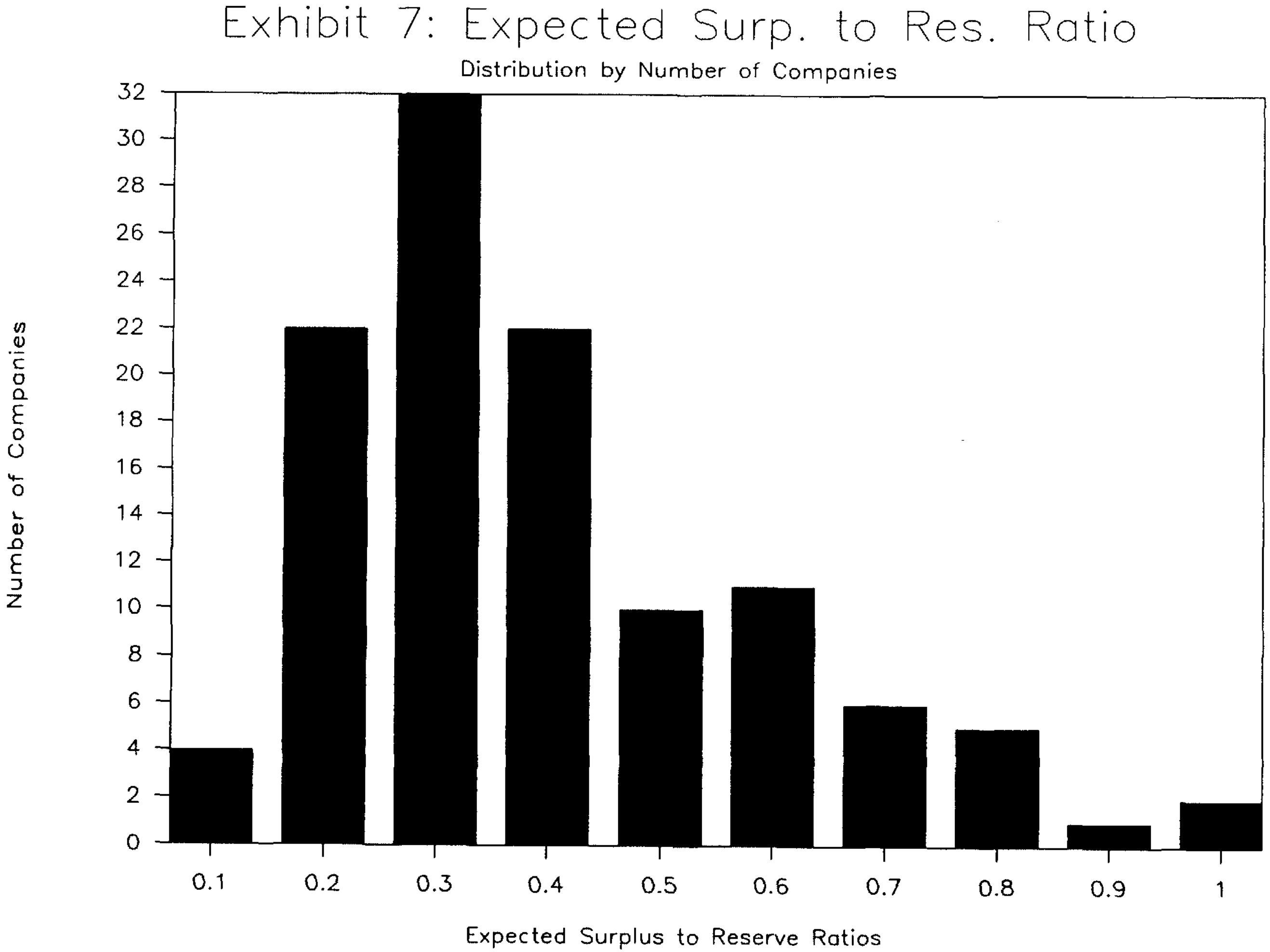


Exhibit 7: Expected Surp. to Res. Ratio
Distribution by Number of Companies
Number of Companies
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Expected Surplus to Reserve Ratios

Exhibit 8: Expected Surp. to Res. Ratio

Distribution by Proportion of Surplus

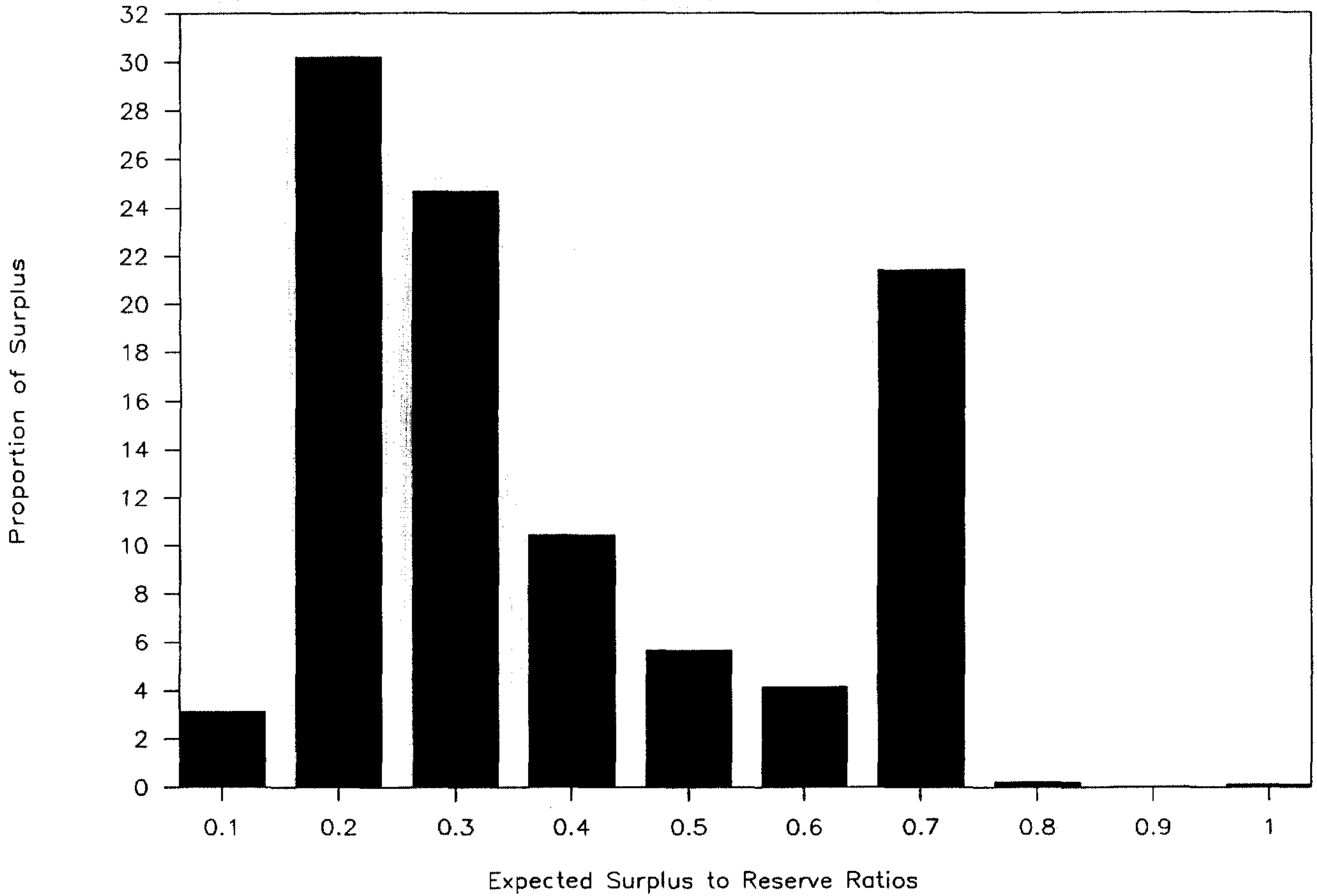


Exhibit 9: Aggregate Leverage Index

Distribution by Number of Companies

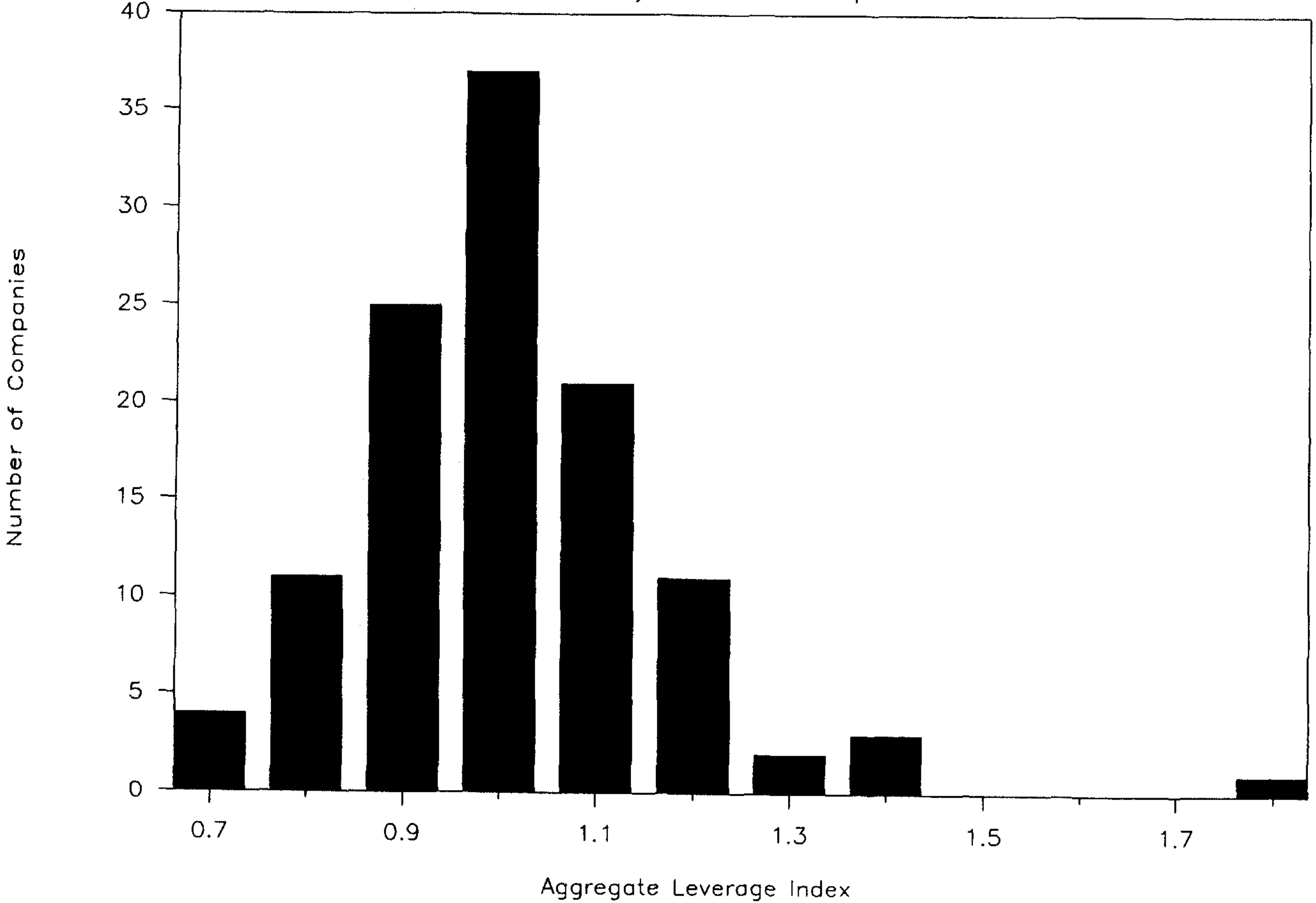


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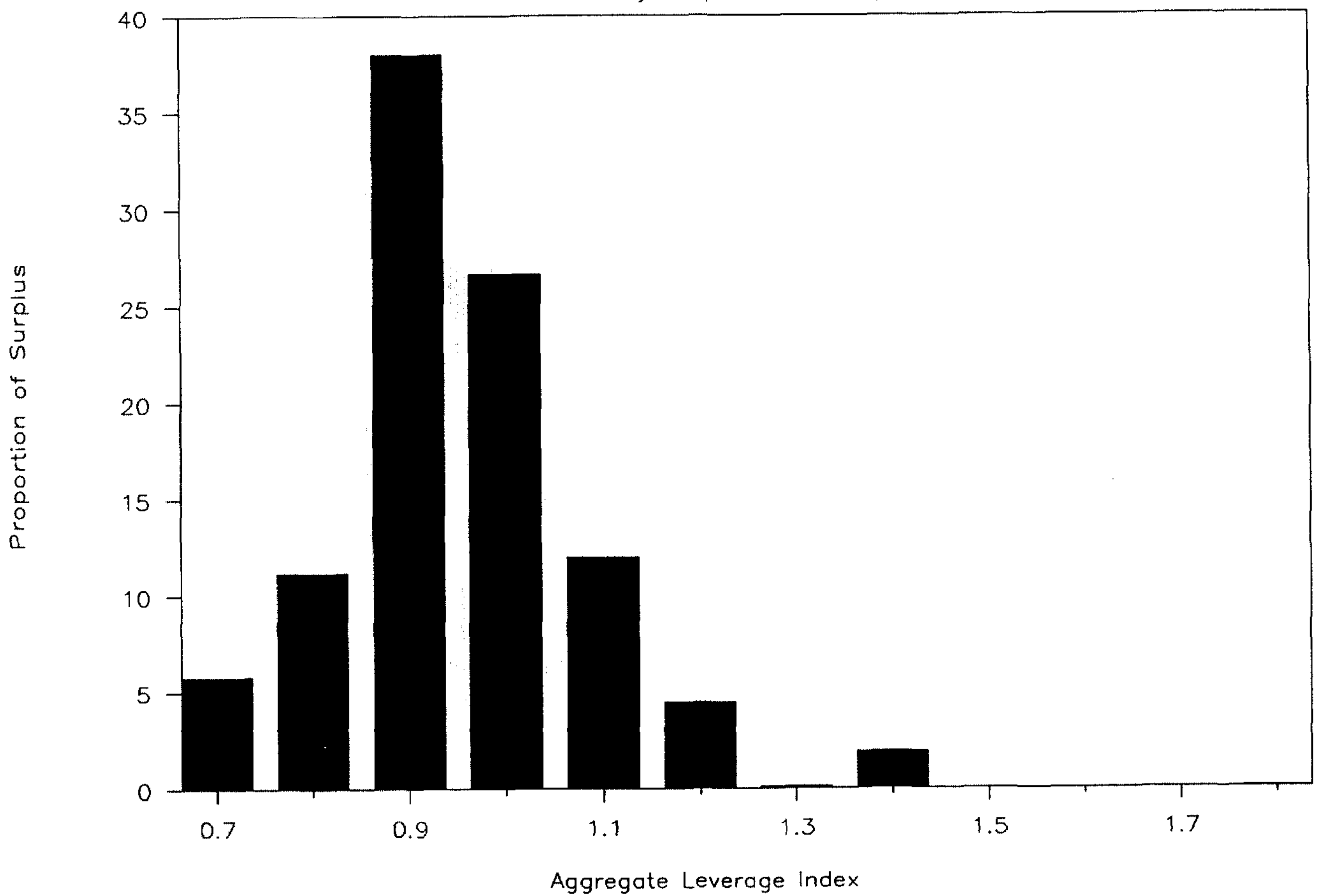
Exhibit 10: Aggregate Leverage Index

Distribution by Proportion of Surplus



Appendix 1

GLOSSARY OF VARIABLES

Unearned Premium Reserves:

PRU/R;	Annual Statement Lines 1, 2, 3, 4, 5, 9, 12, 25, 26 & 27.
WCU/R;	Annual Statement Line 16.
AULU/R;	Annual Statement Line 19.
AUPU/R;	Annual Statement Line 21.
LIU/R;	Annual Statement Lines 11 & 17.
AHU/R;	Annual Statement Lines 13, 14, 15 & 28.
BNU/R;	Annual Statement Lines 23 & 24.
FGU/R;	Annual Statement Line 10.
REU/R;	Annual Statement Line 30.
OTU/R;	Annual Statement Lines 8, 22, 29 & 31.

Loss and Loss Adjustment Expense Reserves:

PRL/R;	Annual Statement Lines 1, 2, 3, 4, 5, 9, 12, 25, 26, & 27.
WCL/R;	Annual Statement Line 16.
AULL/R;	Annual Statement Line 19.
AUPL/R;	Annual Statement Line 21.
LIL/R;	Annual Statement Lines 11 & 17.
AHL/R;	Annual Statement Lines 13, 14, 15 & 28.
BNL/R;	Annual Statement Lines 23 & 24.
FGL/R;	Annual Statement Line 10.
REL/R;	Annual Statement Line 30.
OTL/R;	Annual Statement Lines 8, 22, 29 & 31.

Balance Sheet Assets:

STA/R;	Stocks, item 2.
REA/R;	Real Estate, item 4.
ABA/R;	Agents Balances, item 9.
BDA/R;	Bonds, item 1.
OTA/R;	Other Assets, items 3, 5, 6, 7 & 8.

Miscellaneous Variables:

PGWTH;	Average annual premium growth for last 5 years expressed as a percent.
BQ;	Average bond quality as ranked by the NAIC.
MAXSW;	Maximum concentration within any one state.
LNNWP;	Natural logarithm of aggregate net written premiums.

NOTE: Variables whose name includes the elements "/R" have been ratioed to individual insurer aggregate reserves.

Appendix 2

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MTB > REGRESS 'S/R' 29 'PRU/R' 'WCU/R' 'AULU/R' 'AUPU/R' 'LIU/R' 'AHU/R'&
MTB > 'BNU/R' 'FGU/R' 'REU/R' 'OTU/R'&
MTB > 'PRL/R' 'WCL/R' 'AULL/R' 'AUPL/R' 'LIL/R' 'AHL/R' 'BNL/R' 'FGL/R' 'REL/R'
MTB > &
MTB > 'OTL/R' 'STA/R' 'REA/R' 'ABA/R' 'BDA/R' 'OTA/R'&
MTB > 'PGWTH' 'BQ' 'MAXSW' 'LNNWP';
SUBC> NOCONSTANT.
* NOTE *    LNNWP is highly correlated with other  predictor variables

The regression equation is
S/R = - 0.510 PRU/R - 0.507 WCU/R - 0.294 AULU/R - 0.834 AUPU/R - 0.793 LIU/R
           + 0.79 AHU/R - 0.427 BNU/R + 1.43 FGU/R - 3.80 REU/R - 0.668 OTU/R
           - 0.726 PRL/R - 0.814 WCL/R - 0.929 AULL/R - 0.15 AUPL/R
           - 0.726 LIL/R - 0.639 AHL/R - 1.17 BNL/R - 5.58 FGL/R - 0.333 REL/R
           + 0.108 OTL/R + 0.858 STA/R + 0.462 REA/R + 0.550 ABA/R
           + 0.749 BDA/R + 0.760 OTA/R + 0.0326 PGWTH - 0.0675 BQ
           + 0.0443 MAXSW + 0.00574 LNNWP
```

Predictor	Coefficient Independent Variable	Standard Deviation of Coeff	T-ratio
Noconstant			
PRU/R	-0.5104	0.1611	-3.17
WCU/R	-0.5072	0.1776	-2.86
AULU/R	-0.2942	0.2906	-1.01
AUPU/R	-0.8345	0.3229	-2.58
LIU/R	-0.7932	0.1377	-5.76
AHU/R	0.7920	1.4800	0.54
BNU/R	-0.4269	0.3809	-1.12
FGU/R	1.4260	4.5580	0.31
REU/R	-3.8030	1.8450	-2.06
OTU/R	-0.6679	0.8023	-0.83
PRL/R	-0.7260	0.1333	-5.44
WCL/R	-0.8138	0.1275	-6.38
AULL/R	-0.9289	0.1728	-5.38
AUPL/R	-0.1520	1.0490	-0.15
LIL/R	-0.7263	0.1377	-5.28
AHL/R	-0.6389	0.4522	-1.41
BNL/R	-1.1715	0.4495	-2.61
FGL/R	-5.5830	5.0640	-1.10
REL/R	-0.3331	0.2843	-1.17
OTL/R	0.1076	0.2815	0.38
STA/R	0.8578	0.0471	18.20
REA/R	0.4624	0.2350	1.97
ABA/R	0.5503	0.0931	5.91
BDA/R	0.7490	0.0346	21.68
OTA/R	0.7604	0.0821	9.26
PGWTH	0.0326	0.0262	1.25
BQ	-0.0675	0.0388	-1.74
MAXSW	0.0443	0.0268	1.65
LNNWP	0.0057	0.0047	1.21

Standard Deviation of the regression = 0.06090
R-Squared value = 0.986171

Analysis of Variance

Source	Degrees Freedom	Sequential Sum Squares	Mean Sum Squares
Regression	29	22.74705	0.78438
Error	86	0.31895	0.00371
Total	115	23.06601	

SOURCE	DF	SEQ SS
PRU/R	1	13.91088
WCU/R	1	2.05169
AULU/R	1	2.04133
AUPU/R	1	0.09465
LIU/R	1	0.34214
AHU/R	1	0.22691
BNU/R	1	0.26153
FGU/R	1	0.05433
REU/R	1	0.00000
OTU/R	1	0.00261
PRL/R	1	0.03132
WCL/R	1	0.37610
AULL/R	1	0.06977
AUPL/R	1	0.02433
LIL/R	1	0.54413
AHL/R	1	0.04886
BNL/R	1	0.00002
FGL/R	1	0.01217
REL/R	1	0.00164
OTL/R	1	0.00459
STA/R	1	0.29120
REA/R	1	0.00037
ABA/R	1	0.00312
BDA/R	1	1.99058
OTA/R	1	0.33609
PGWTH	1	0.00270
BQ	1	0.01214
MAXSW	1	0.00644
LNNWP	1	0.00543

Obs.	PRU/R	S/R	Fitted S/R	Standard Deviation Fitted S/R	Residual	Standard Deviation Residuals
5	0.076	0.36718	0.37305	0.05881	-0.00587	-0.37 X
14	0.032	0.61825	0.59037	0.05494	0.02788	1.06 X
24	0.053	0.36658	0.35741	0.06044	0.00917	1.23 X
25	0.034	0.24207	0.43475	0.03416	-0.19269	-3.82R
34	0.081	0.72313	0.72015	0.06052	0.00298	0.44 X
36	0.140	0.50788	0.63020	0.02904	-0.12232	-2.29R
44	0.016	0.43416	0.42735	0.05523	0.00682	0.27 X
55	0.000	0.78309	0.68280	0.03945	0.10030	2.16R
64	0.000	0.49744	0.49435	0.05649	0.00310	0.14 X
66	0.000	0.31748	0.33600	0.05967	-0.01852	-1.52 X
75	0.106	1.11822	0.97979	0.03157	0.13843	2.66R
76	0.080	0.41918	0.41832	0.05443	0.00086	0.03 X
79	0.370	0.39812	0.56182	0.02268	-0.16370	-2.90R
80	0.015	0.35194	0.34800	0.05904	0.00394	0.26 X
82	0.088	0.69395	0.56830	0.02691	0.12565	2.30R
90	0.094	0.44986	0.33194	0.02276	0.11791	2.09R

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

Appendix 3

```
MTB > BRIEF 3
MTB > REGRESS 'S/R' 28 'PRU/R' 'WCU/R' 'AULU/R' 'AUPU/R' 'LIU/R' 'AHU/R'&
MTB > 'BNU/R' 'FGU/R' 'REU/R' 'OTU/R'&
MTB > 'PRL/R' 'WCL/R' 'AULL/R' 'AUPL/R' 'LIL/R' 'AHL/R' 'BNL/R' 'FGL/R' 'REL/R'
MTB > &
MTB > 'OTL/R' 'STA/R' 'REA/R' 'ABA/R' 'BDA/R' 'OTA/R'&
MTB > 'PGWTH' 'BQ' 'MAXSW';
SUBC> NOCONSTANT.
```

The regression equation is

```
S/R = - 0.399 PRU/R - 0.426 WCU/R - 0.251 AULU/R - 0.757 AUPU/R - 0.682 LIU/R
        + 0.71 AHU/R - 0.357 BNU/R + 1.72 FGU/R - 3.47 REU/R - 0.588 OTU/R
        - 0.605 PRL/R - 0.686 WCL/R - 0.767 AULL/R + 0.12 AUPL/R
        - 0.590 LIL/R - 0.491 AHL/R - 0.989 BNL/R - 4.50 FGL/R
        - 0.202 REL/R + 0.204 OTL/R + 0.858 STA/R + 0.412 REA/R
        + 0.571 ABA/R + 0.733 BDA/R + 0.721 OTA/R + 0.0311 PGWTH
        - 0.0612 BQ + 0.0332 MAXSW
```

Predictor	Coefficient Independent Variable	Standard Deviation of Coeff	T-ratio
Noconstant			
PRU/R	-0.3991	0.1326	-3.01
WCU/R	-0.4256	0.1647	-2.58
AULU/R	-0.2507	0.2891	-0.87
AUPU/R	-0.7565	0.3173	-2.38
LIU/R	-0.6816	0.1025	-6.65
AHU/R	0.7100	1.4820	0.48
BNU/R	-0.3571	0.3775	-0.95
FGU/R	1.7160	4.5630	0.38
REU/R	-3.4710	1.8290	-1.90
OTU/R	-0.5876	0.8017	-0.73
PRL/R	-0.6054	0.0887	-6.82
WCL/R	-0.6862	0.0717	-9.56
AULL/R	-0.7672	0.1097	-6.99
AUPL/R	0.1160	1.0280	0.11
LIL/R	-0.5901	0.0795	-7.43
AHL/R	-0.4912	0.4366	-1.13
BNL/R	-0.9886	0.4244	-2.33
FGL/R	-4.4970	4.9970	-0.90
REL/R	-0.2016	0.2634	-0.77
OTL/R	0.2043	0.2706	0.76
STA/R	0.8579	0.0473	18.15
REA/R	0.4125	0.2319	1.78
ABA/R	0.5714	0.0917	6.23
BDA/R	0.7334	0.0321	22.82
OTA/R	0.7208	0.0755	9.55
PGWTH	0.0311	0.0262	1.19
BQ	-0.0612	0.0385	-1.59
MAXSW	0.0332	0.0253	1.31

Standard Deviation of the regression = 0.06106

R-Squared value = 0.985936

Analysis of Variance

Source	Degrees Freedom	Sequential Sum Squares	Mean Sum Squares
Regression	28	22.74163	0.81220
Error	87	0.32438	0.00373
Total	115	23.06601	

Source	Degrees Freedom	Sequential Sum Squares
PRU/R	1	13.91088
WCU/R	1	2.05169
AULU/R	1	2.04133
AUPU/R	1	0.09465
LIU/R	1	0.34214
AHU/R	1	0.22691
BNU/R	1	0.26153
FGU/R	1	0.05433
REU/R	1	0.00000
OTU/R	1	0.00261
PRL/R	1	0.03132
WCL/R	1	0.37610
AULL/R	1	0.06977
AUPL/R	1	0.02433
LIL/R	1	0.54413
AHL/R	1	0.04886
BNL/R	1	0.00002
FGL/R	1	0.01217
REL/R	1	0.00164
OTL/R	1	0.00459
STA/R	1	0.29120
REA/R	1	0.00037
ABA/R	1	0.00312
BDA/R	1	1.99058
OTA/R	1	0.33609
PGWTH	1	0.00270
BQ	1	0.01214
MAXSW	1	0.00644

Obs.	PRU/R	S/R	Fitted S/R	Standard Deviation Fitted S/R	Residual	Standard Deviation Residuals	
1	0.052	0.33227	0.37061	0.02481	-0.03835	-0.69	
2	0.045	0.30856	0.32743	0.01326	-0.01887	-0.32	
3	0.105	0.29429	0.29499	0.01716	-0.00070	-0.01	
4	0.102	0.34262	0.32243	0.03780	0.02018	0.42	
5	0.076	0.36718	0.37758	0.05885	-0.01040	-0.64	X
6	0.026	0.28611	0.24389	0.02601	0.04223	0.76	
7	0.200	0.36005	0.39248	0.01523	-0.03242	-0.55	
8	0.056	0.25909	0.21990	0.01459	0.03919	0.66	
9	0.011	0.28368	0.25721	0.01970	0.02646	0.46	
10	0.130	0.24204	0.28068	0.01560	-0.03864	-0.65	
11	0.439	0.74657	0.77704	0.03475	-0.03046	-0.61	
12	0.105	0.26338	0.31924	0.02126	-0.05586	-0.98	
13	0.455	0.81664	0.74894	0.03858	0.06770	1.43	
14	0.032	0.61825	0.59943	0.05457	0.01882	0.69	X
15	0.104	0.35633	0.35482	0.01747	0.00151	0.03	
16	0.263	0.31985	0.33904	0.02260	-0.01919	-0.34	
17	0.072	0.16889	0.24470	0.01744	-0.07582	-1.30	
18	0.099	0.47169	0.52380	0.02415	-0.05211	-0.93	
19	0.051	0.77267	0.81772	0.02920	-0.04505	-0.84	
20	0.107	0.30830	0.25608	0.01324	0.05222	0.88	
21	0.033	0.20052	0.20124	0.02034	-0.00072	-0.01	
22	0.092	0.24629	0.20618	0.01885	0.04011	0.69	
23	0.035	0.19241	0.19021	0.05176	0.00220	0.07	
24	0.053	0.36658	0.35837	0.06060	0.00821	1.09	X
25	0.034	0.24207	0.43171	0.03416	-0.18964	-3.75R	
26	0.148	0.42707	0.44780	0.02340	-0.02073	-0.37	
27	0.007	0.44786	0.44603	0.02236	0.00183	0.03	
28	0.028	0.09444	0.08130	0.02090	0.01314	0.23	
29	0.072	0.38886	0.37758	0.02408	0.01128	0.20	
30	0.052	0.51719	0.48236	0.02090	0.03483	0.61	
31	0.081	0.25509	0.18040	0.02032	0.07469	1.30	
32	0.040	0.35233	0.31907	0.02273	0.03326	0.59	
33	0.040	0.24169	0.23475	0.03288	0.00694	0.13	
34	0.081	0.72313	0.72079	0.06068	0.00234	0.34	X
35	0.077	0.28739	0.27181	0.01524	0.01557	0.26	
36	0.140	0.50788	0.63676	0.02860	-0.12888	-2.39R	
37	0.000	0.28971	0.28862	0.03210	0.00108	0.02	
38	0.020	0.25669	0.28643	0.03288	-0.02974	-0.58	
39	0.104	0.65577	0.61516	0.02921	0.04061	0.76	
40	0.068	0.42592	0.41044	0.03044	0.01548	0.29	
41	0.080	0.35126	0.32503	0.02963	0.02624	0.49	
42	0.006	0.20638	0.20902	0.03417	-0.00264	-0.05	
43	0.111	0.36162	0.33981	0.01254	0.02180	0.36	
44	0.016	0.43416	0.43263	0.05520	0.00153	0.06	X
45	0.261	0.97654	0.97497	0.03019	0.00157	0.03	
46	0.106	0.29185	0.29953	0.01479	-0.00769	-0.13	
47	0.062	0.26306	0.18226	0.01378	0.08080	1.36	
48	0.021	0.42733	0.48582	0.02867	-0.05849	-1.08	
49	0.042	0.23628	0.21873	0.01567	0.01755	0.30	
50	0.146	0.37888	0.44978	0.02296	-0.07090	-1.25	

51	0.067	0.41885	0.40369	0.02725	0.01516	0.28
52	0.135	0.50723	0.56523	0.04732	-0.05800	-1.50
53	0.054	0.29681	0.25060	0.02139	0.04621	0.81
54	0.000	0.14773	0.13317	0.03461	0.01456	0.29
55	0.000	0.78309	0.67857	0.03940	0.10452	2.24R
56	0.150	0.30438	0.30770	0.01975	-0.00332	-0.06
57	0.547	0.98319	0.89900	0.03681	0.08419	1.73
58	0.000	0.76549	0.84386	0.03694	-0.07837	-1.61
59	0.016	0.17415	0.14160	0.02379	0.03255	0.58
60	0.124	0.43187	0.45435	0.01471	-0.02248	-0.38
61	0.246	0.41994	0.48605	0.03139	-0.06611	-1.26
62	0.054	0.27426	0.21995	0.02173	0.05431	0.95
63	0.092	0.20709	0.27849	0.01675	-0.07140	-1.22
64	0.000	0.49744	0.48618	0.05623	0.01127	0.47 X
65	0.331	0.79082	0.79211	0.02980	-0.00129	-0.02
66	0.000	0.31748	0.33246	0.05976	-0.01498	-1.19 X
67	0.127	0.56938	0.58970	0.02698	-0.02033	-0.37
68	0.010	0.51358	0.60407	0.04498	-0.09049	-2.19R
69	0.093	0.35415	0.32821	0.02826	0.02593	0.48
70	0.087	0.39880	0.36556	0.01259	0.03324	0.56
71	0.057	0.32765	0.33507	0.01481	-0.00742	-0.13
72	0.148	0.35277	0.32956	0.01791	0.02321	0.40
73	0.018	0.70245	0.59581	0.02324	0.10664	1.89
74	0.211	0.39205	0.38158	0.02407	0.01047	0.19
75	0.106	1.11822	0.97358	0.03123	0.14465	2.76R
76	0.080	0.41918	0.42015	0.05456	-0.00097	-0.04 X
77	0.092	0.22549	0.22571	0.01956	-0.00021	-0.00
78	0.140	0.48552	0.53071	0.01710	-0.04518	-0.77
79	0.370	0.39812	0.55863	0.02259	-0.16051	-2.83R
80	0.015	0.35194	0.35021	0.05917	0.00173	0.11 X
81	0.012	0.25459	0.28437	0.02510	-0.02978	-0.54
82	0.088	0.69395	0.58304	0.02406	0.11090	1.98
83	0.358	0.65165	0.73222	0.02340	-0.08057	-1.43
84	0.173	0.86125	0.77338	0.02043	0.08788	1.53
85	0.049	0.23720	0.22790	0.01203	0.00930	0.16
86	0.000	0.23959	0.26657	0.03130	-0.02698	-0.51
87	0.200	0.71139	0.70744	0.03238	0.00395	0.08
88	0.008	0.21100	0.20151	0.01694	0.00948	0.16
89	0.110	0.65207	0.57660	0.02118	0.07547	1.32
90	0.094	0.44986	0.33110	0.02281	0.11876	2.10R
91	0.003	0.47340	0.54054	0.03130	-0.06714	-1.28
92	0.254	0.31535	0.26062	0.02777	0.05474	1.01
93	0.096	0.23397	0.28231	0.01834	-0.04835	-0.83
94	0.181	0.47466	0.37443	0.02275	0.10023	1.77
95	0.069	0.35141	0.41477	0.03575	-0.06337	-1.28
96	0.056	0.48681	0.53114	0.02901	-0.04432	-0.82
97	0.087	0.19227	0.17316	0.01589	0.01910	0.32
98	0.067	0.22055	0.26428	0.01388	-0.04373	-0.74
99	0.021	0.55629	0.57130	0.05147	-0.01500	-0.46
100	0.133	0.40090	0.37598	0.01679	0.02493	0.42
101	0.037	0.19978	0.24608	0.02002	-0.04630	-0.80
102	0.139	0.52612	0.51341	0.01736	0.01271	0.22
103	0.119	0.72346	0.65464	0.02522	0.06882	1.24
104	0.163	0.41143	0.40590	0.02778	0.00553	0.10

105	0.038	0.31221	0.33274	0.01153	-0.02053	-0.34
106	0.081	0.29748	0.31788	0.04554	-0.02041	-0.50
107	0.048	0.34765	0.33386	0.02984	0.01380	0.26
108	0.076	0.20875	0.24076	0.01669	-0.03201	-0.54
109	0.062	0.18895	0.23262	0.01274	-0.04367	-0.73
110	0.000	0.11405	0.16463	0.02960	-0.05058	-0.95
111	0.000	0.32781	0.29826	0.03980	0.02955	0.64
112	0.041	0.02183	0.00671	0.03610	0.01512	0.31
113	0.058	0.32859	0.35362	0.03075	-0.02503	-0.47
114	0.033	0.23374	0.18601	0.01756	0.04772	0.82
115	0.087	0.17491	0.18413	0.02020	-0.00921	-0.16

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.