# PROPERTY–LIABILITY INSURANCE PRICING MODELS: AN EMPIRICAL EVALUATION

by

Stephen P. D'Arcy and James R. Garven

BIOGRAPHIES:

Mr. D'Arcy is Associate Professor of Finance at the University of Illinois at Urbana–Champaign. Prior to joining the University of Illinois, he was the Actuary at CUMIS Insurance Society and an actuarial student at AEtna Insurance Company. He received a BA in Applied Mathematics from Harvard College in 1972 and a Ph.D. in Finance from the University of Illinois in 1982. He became a Fellow of the CAS in 1975 and a Member of the American Academy of Actuaries in 1978. He is the author of four papers and one comment in the *Proceedings of the Casualty Actuarial Society*, two prior papers in Discussion Paper Programs and two chapters of the text *Foundations of Casualty Actuarial Science*, in addition to papers in the *Journal of Risk and Insurance* and other publications. He served as Chariman of the CAS Task Force on Education and Testing in 1988–89.

Mr. Garven is Assistant Professor of Finance at the University of Texas at Austin. Prior to his current appointment at the University of Texas, Dr. Garven was a member of the finance faculty at The Pennsylvania State University. He received his BS and MBA degrees from Illinois State University in 1977 and 1980, and a Ph.D. in Finance from the University of Illinois in 1984. He has presented papers at the Ratemaking Seminar of the Casualty Actuarial Society, and published articles in journals such as the *Financial Review*, the *Journal of Finance*, and the *Journal of Financial Services Research*.

ABSTRACT:

Over the past two decades, several pricing models that integrate underwriting and investment performance have been proposed or used to determine property–liability insurance rates. In general, these models have been tested separately and only over a relatively limited time horizon. In this article, the major property–liability insurance pricing models are evaluated over the 60–year period from 1926 through 1985 and the results of the various models are compared in terms of the ability to predict actual underwriting profit margins. Differences between model predictions and realized underwriting profit margin series are examined over the entire period as well as various subperiods in order to demonstrate how individual models perform under different conditions. The goal of this research is to assist actuaries and researchers in the application of pricing models and interpretation of results.

## Introduction

Although the standard pricing model of the insurance industry [dating back to the 1921 National Convention of Insurance Commissioners (NCIC) Fire Insurance Committee report] ignores investment income in insurance ratemaking, many insurance pricing models have been proposed that integrate the underwriting and investment income aspects of the insurance contract. These models generally follow one of two paths. Those proposed by insurance practitioners or academics specializing in insurance typically concentrate on the underwriting side of the insurance transaction and select rather arbitrary values for investment income. On the other hand, models developed by financial economists tend to concentrate on the investment aspect of insurance by emphasizing risk–adjusted rates of return on investment while glossing over the specialized characteristics of the underwriting side of the insurance business. More recently, research has been aimed at developing pricing models that adequately address the importance of both underwriting and investment–related issues. The purpose of this article is to compare and contrast the predictive abilities of a number of different insurance pricing models over an extended period in order to demonstrate how various models perform under different economic and competitive conditions. This research will be of assistance to practitioners as well as academics in the application of insurance pricing models and interpretation of results.

Although the minority report of the National Convention of Insurance Commissioners (NCIC) Fire Insurance Committee in 1921 proposed that investment income be considered, this recommendation was defeated (see Webb, 1982). This position began to reemerge within the insurance community during the latter part of the 1960s, when interest rates began to increase and to become more volatile. A landmark study by the NAIC in 1970 effectively reopened the issue of investment income in ratemaking by concluding, "In determining profits, it is submitted that income from all sources should be considered." (see NAIC, 1970, p. 721). Bailey (1976) proposed a method of allocating actual investment income (interest, dividends and realized capital gains) to stockholders and policyholders. Ferrari (1967, 1968) proposed a method of calculating return on equity for insurers based on both underwriting and investment performance and also advanced the investment technique of portfolio theory for use in developing line of business mix strategies. Cooper (1974) extended the use of portfolio theory for insurance applications and developed a model for determining the competitive rate of return on insurance contracts by integrating underwriting and investment performance. In line with the insurance focus, he assumed that the investment return would be the rate earned on "riskless or very low risk investments."

Several other insurance researchers developed sophisticated models of insurance markets that included the effect of investment income, but these studies continued to overlook the complexity of determining a proper value for the investment income. Witt (1973) developed a model of the insurance industry based on monopolistic competition that led to the conclusion that investment income is reflected indirectly in pricing. In another model, Witt (1974) includes investment income as a stochastic variable that is normally distributed with a known mean and standard deviation and is uncorrelated with insurance claims. Kahane and Levy (1975) incorporate an arbitrary investment return in their detailed model of insurance. McCabe and Witt (1980) model insurance behavior on the assumption that the insurer seeks to maximize profit subject to a probability of insolvency constraint. In this model investment performance is estimated by normally distributed random variables for stock and bond returns with means and standard deviations based on historical performance. Spellman, Witt, and Rentz (1975) develop a microeconomic pricing model for insurers, including the effect of investment income, that demonstrates the impact of the elasticity of demand for insurance on the profit maximizing price markup above marginal costs. One key difficulty with the use of these insurance based pricing models is in selecting an appropriate value for investment returns. In general these models do not provide for any risk adjustment for the investment side of the insurance contract. The investment income value tends to be selected from historical values or arbitrarily. The magnitude of investment income for insurers has led to an alternative focus for insurance pricing.

The alternative focus on insurance pricing, brought by financial economists, concentrated on the investment aspect of the insurance transaction and oversimplified the underwriting side. Quirin and Waters (1975) concluded that insurers could, in effect, borrow at a negative interest rate. Walter (1979) observed that the market value of insurer stocks exceeded the book value and concluded that insurance regulation fostered excess profitability. Fairley (1979) concentrated on the historical systematic risk level of underwriting and, based on the Capital Asset Pricing Model (CAPM), proposed a formula for establishing insurance prices that was utilized in Massachusetts for several years before it was shown that the model did not reflect insurance taxation appropriately.

More recent research has brought equal attention to both the financial issues involved in determining investment rates of return and the unique characteristics of the insurance contract. Turner (1987) and Ang and Lai (1987) demonstrate that the CAPM focus on systematic risk ignores an important factor in insurance pricing. Cummins and Harrington (1988) determine that insurer stock returns are consistent with the CAPM model only for part of their tested experience period and that unsystematic risk is a relevant factor in determining rates of return. Witt and Urrutia (1983) develop measures of systematic and unsystematic underwriting risk and compare the impact of rate regulation on the allocation. D'Arcy (1988) demonstrates that the risk–free rate commonly used to discount loss reserves may be excessive given the characteristics of insurance contracts. Derrig (1985) combines applicable tax provisions for insurers with financial pricing models. Following this line of research, this article attempts to provide an equal focus on the underwriting and investment sides of the insurance transaction by testing pricing models that have been derived from both the insurance and financial economics areas over a consistent period.

Both the history of insurance rate regulation and the development of property–liability insurance pricing models have been dealt with effectively in prior literature (e.g., see Cooper, 1974; D'Arcy and Doherty, 1988; and NAIC, 1970). Thus, this material will simply be summarized and referenced here. Prior to the 1970s, the standard pricing practice incorporated an independently selected underwriting profit margin into the ratemaking formula. The selection of a 5 percent underwriting profit margin dates back to 1921, but even then it was not supported on the basis of any numerical calculation. As interest rates rose in the late 1970s, emphasis shifted to the total rate of return, rather than simply the underwriting profit margin (see Biger and Kahane, 1978; D'Arcy, 1983; Haugen and Kroncke, 1971; Plotkin, 1979; Quirin and Waters, 1975; and Venezian, 1983). Some insurers developed a total rate of return model that backed into the appropriate underwriting profit margin by including consideration of the expected investment income. Academics and regulators introduced the CAPM and discounted cash flow analysis into rate hearings (see Cummins and Chang, 1983; Cummins and Harrington, 1985; Fairley, 1979; Hill, 1979; Hill and Modigliani, 1987; and Myers and Cohn, 1987). More recently, the arbitrage pricing model has been applied to insurance pricing (see Kraus and Ross, 1982; and Urrutia, 1987a and 1987b) and the option pricing model has been applied to both pricing and solvency considerations (see Cummins, 1988; Derrig, 1989; and Doherty and Garven, 1986). Although these models differ widely in terms of underlying assumptions, parameter specifications, and methods of calculation, they are generally organized around the basic principle that certain targets must be met so as to justify continued or even further allocation of capital to a particular set of insurance activities.

Insurance prices should not be set according to a given model unless that model accurately represents the pricing mechanism. The only way to determine whether a model is accurate is to test it on actual data for an extended period. Previous empirical tests of property–liability insurance pricing models have generally been restricted to the past decade or two, during which inflation, interest rates and loss payout patterns were at historically high levels, possibly skewing the indications. The purpose of this article will be to provide an empirical evaluation of how well a number of alternative pricing models fare in terms of predicting underwriting profit margins over an extended period of time. The tests contained herein are based on the actual results achieved by all U.S. stock insurers in aggregate from 1926 through 1985. Differences between the model predictions and the actual realized underwriting profit margin series are examined over the entire period as well as various subperiods in order to assess the relative usefulness of the different models under varying conditions. The goal of this research is to assist actuaries and researchers in the application of the various pricing models and interpretation results.

Discovery of a viable pricing model does not imply that regulation should then be enforced requiring insurers to charge the rate indicated by the selected pricing model. This action would make no more sense than requiring stock trades to take place at the price determined by the latest stock pricing model. Competitive markets will tend to drive profit margins toward the theoretically correct level. Regulating the "correct" rate level would then be unnecessary. This research is aimed at providing insight for the participants in the market, both as buyers and sellers of insurance, to increase the availability of information about profitability and the degree of competition in insurance markets.

The insurance pricing models tested in this article include the target underwriting profit margin, target total rate of return, CAPM, discounted cash flow model and option pricing model. Other models have been proposed, but are not included in this study due either to data availability problems or the fact that they have not been widely applied. Models not tested include the National Council on Compensation Insurance ratemaking methodology and the mean–standard deviation model (see Venezian, 1983).

The following section provides a brief overview of the intuition and mathematical structure underlying the alternative ratemaking models tested. Then data and methodology are discussed, followed by empirical results. Conclusions from the study are then drawn and directions for future research are indicated.

## Alternative Insurance Ratemaking Models

### *Target Underwriting Profit Margin*

The target underwriting profit margin ratemaking technique seeks to achieve a predetermined underwriting profit margin without regard to investment income, insurer leverage or level of risk. The standard underwriting profit margins are 2.5 percent for workers' compensation and 5.0 percent for all other lines. These margins evolved from a 1921 National Convention of Insurance Commissioners (NCIC) Fire Insurance Committee report that indicated that "5 percent is the minimum percentage which can be regarded as 'a reasonable underwriting profit'" (see Webb, 1982). No statistical support has ever been provided for the selected level. McCullough (1948) provides an in depth analysis of the 1921 NCIC deliberations and a update of the issue through 1947, with additional insight into the consideration of investment income in ratemaking.

Many researchers dismiss the standard profit formula levels as taken out of thin air and utterly without meaning. However, this may be an excessively harsh view. The members of the NCIC committee included both regulators and industry representatives. They had access to the experience of a large number of insurers. This experience was derived from a period of low interest rates (e.g., long term bond yields were approximately 4 to 5 percent) and fast payouts of insurance losses. Although they were developing a profit standard for fire insurance only, property lines were predominate in the industry. The experience of the time may have indicated that a 5 percent level was appropriate. Selection of a profit margin that was not supported by the available experience simply would not have been approved by a majority of the committee. Thus, this 5 percent level may in fact represent a value derived from a specific era which may or may not be appropriate in other periods or for different lines of insurance. Therefore, although it lacks a coherent theoretical foundation, the target underwriting profit margin method is probably not entirely without merit.

*Target Total Rate of Return*

The first attempts to combine investment income with underwriting profit margins involved developing a target total rate of return for insurers similar to the target total rate of return for utilities (see Cooper, 1974; and Ferrari, 1968). The total return generated from investments and from underwriting combined were set equal to a target. After the investment income is forecasted, the required underwriting profit margin can be calculated.

The formula for the target total rate of return can be represented as follows:

$$\text{TRR} = (\text{IA}/\text{S})(\text{IRR}) + (\text{P}/\text{S})(\text{UPM}), \tag{1}$$

where

TRR = target total rate of return;
IA = investable assets;
S = surplus;
IRR = investment rate of return;
P = premium;
UPM = underwriting profit margin.

The primary problem involved in using this technique is determining the appropriate target for the total rate of return. When applied to public utility firms, a similar problem arises, but this is handled setting the target equal to the weighted average cost of capital. Utilities have two sources of capital: debt and equity. The cost of debt is typically determined by averaging the interest rates on outstanding issues with the expected rate on any new debt issues to be offered. The cost of equity is established by applying an asset pricing model to the firm's security value. If the CAPM is used, then the utility's beta, or systematic risk factor, is determined from past stock price movements and applied in equation (2):

$$E(r_e) = r_f + \beta_e(E(r_m) - r_f), \tag{2}$$

where

$E(r_e)$ = cost of equity capital;
$r_f$ = risk free rate of return;
$\beta_e$ = beta (systematic risk) of equity;
$E(r_m)$ = expected return on the market.

In order to apply the target total rate of return model, TRR is set equal to $E(r_e)$, where $E(r_e)$ is determined as shown in equation (2).[1] Although the authors are not aware of any sources which report property–liability insurer equity betas from 1926 through 1985, Hill (1979) found that the average insurer equity beta from 1951 through 1965 .61. Hill and Modigliani (1987) and Fairley (1979) report equity betas for insurance stocks during the 1970s averaging about unity, or equivalent to the beta of the market as a whole. Both values will be considered in the tests which follow. Combining equations (1) and (2) and rearranging terms leads to:

$$UPM = (S/P)[r_f + \beta_e(E(r_m) - r_f) - (IA/S)IRR]. \tag{3}$$

Annual values for the right hand parameters can be obtained from historical experience and the underwriting profit margin determined from the total rate of return model can then be compared with actual underwriting profit margins.

*Capital Asset Pricing Model*

The CAPM, defined in equation (2), has been applied to insurance by Fairley (1979), Hill (1979) and Hill and Modigliani (1987), among others. The basic form of Fairley's CAPM model is given by equation (4):

$$UPM = -kr_f + \beta_u(E(r_m) - r_f), \tag{4}$$

where

$k$ = funds generating coefficient;
$\beta_u$ = underwriting beta.

Based on this form of the model, the insurer credits the funds generated from the insurance transaction at a risk–free rate which is offset by the required rate of return based on the systematic risk of the underwriting transaction. The funds generated by writing insurance result from the lag between the receipt of premium and the payment of expenses and losses that occurs with many types of insurance. The insurer is expected to credit these balances with the risk–free rate, although the insurer may elect to invest in a more risky asset with a higher expected return. If this is done, any excess return, or below risk–free rate achieved, would be borne by the insurer. On the underwriting component of profitability, in competitive financial markets, insurers would only be rewarded for assuming systematic, or undiversifiable, risk. Thus, the systematic risk of underwriting would be measured and the insurer entitled to offset the risk–free rate payout with the appropriate risk adjusted rate of return of the transaction.

The insurance CAPM model described in equation (4) does not include the effect of taxes. Fairley revised this model to include a provision for taxes, but that model did not account for the differential tax treatment of underwriting income and investment income due to dividend exclusions, tax free investments and capital gains taxation. Hill and Modigliani (1987) developed an insurance CAPM that allows for differential tax rates.[2] This model can be written as follows:

$$\text{UPM} = -kr_f(1-\tau_a)/(1-\tau) + \beta_u(E(r_m)-r_f) + (S/P)r_f(\tau_a/(1-\tau)), \qquad (5)$$

where

$\tau_a$= tax rate on investment income;
$\tau$ = tax rate on underwriting income.

*Discounted Cash Flow Model*

The discounted cash flow model was developed by Myers and Cohn (1987) for the 1982 Massachusetts automobile rate hearings as a counterpart to the CAPM approach. Since 1982, this approach has been used as the basis for setting automobile insurance rates in that state (see Derrig, 1987). The basic formulation of the Myers–Cohn model is:

$$P = PV(LE) + PV(UWPT) + PV(IBT), \tag{6}$$

where

PV(·)= present value operator;
P = premiums;
LE = losses, loss adjustment expenses and other expenses;
UWPT = tax generated on underwriting income;
IBT = tax generated on income from the investment balance.

In the Myers–Cohn model, the present value of losses, loss adjustment expenses and other expenses, PV(LE), is found by discounting these cash flows at an appropriate risk–adjusted discount rate.[3] The present value of the taxes on the underwriting profit, PV(UWBT), is calculated by multiplying the tax rate by the present value of underwriting income; viz., PV(UWBT) = $\tau$[PV(P)–PV(LE)].[4] The final component, PV(IBT), is determined by discounting the tax generated on income from the investment balance at the risk–free rate.[5]

Myers and Cohn obtained information from the Massachusetts Rating Bureau on the cash flow patterns by quarter for premiums, losses, and expenses. The premium income and loss and expense outflows were then discounted to a common time, in this case the beginning of the first quarter. The tax on underwriting income was determined by applying a tax rate that approximated the maximum corporate tax rate to the difference between the indicated premiums and the losses (including expenses). As these were undiscounted values, the underwriting income tended to be negative, in which case the technique assumed that the underwriting losses were offsetting other taxable income.

The procedure for determining the tax on investment income involved allocating surplus to the premium income and combining the surplus with the unpaid losses and expenses to determine an aggregate investment level. Technically, the proper surplus allocation is the ratio of surplus to the net present value of the cash flows emanating from the contract, which include losses and loss adjustment expenses, general expenses and the taxes accruing from underwriting and investments. In Massachusetts, this ratio is promulgated by the insurance commissioner. However, since the present value of the premiums is expected to equal the present value of all the cash flows [as shown in equation (5)], the ratio of surplus to net written premiums is often used as an approximation for this allocation.

Several minor adjustments will be made to the Myers–Cohn approach as applied in this article. Some adjustments are made to accommodate a difference in notation to be consistent with the remainder of the article. Others represent a timing difference that occurs based on the different data sources used to obtain the information used in the respective studies. Additionally, all cash flows will be discounted based on the risk–free rate. This would be equivalent to applying an underwriting beta of zero, which is one of the alternatives tested for the CAPM methodology. The discounted cash flow model tested in this study is:

$$\mathrm{P} = \mathrm{PV(E)} + \mathrm{PV(L)} + \mathrm{PV(UWPT)} + \mathrm{PV(IBT)}, \tag{7}$$

where

E = expenses other than loss adjustment expenses;
L = losses and loss adjustment expenses.

By dividing through by the premium P, equation (7) can be rewritten:

$$1 = \mathrm{PV(ER)} + \mathrm{PV(LR)} + \mathrm{PV}(\tau(1\text{–ER–LR})) + \mathrm{PV}(\tau(1\text{+SL})(\mathrm{LR})(\mathrm{LPP})), \tag{8}$$

where

ER = expense ratio;
LR = loss ratio;
$\tau$ = tax rate;

SL = ratio of surplus to premiums;
LPP = loss payout pattern.

The premium payment pattern on the historical data is not known and most likely changes over time. In general, insurers will collect premium income either when the policy is written or in installments over the policy period. In many cases the insurance agent actually collects the premium 30 to 60 days prior to submitting it to the insurer. However, this delay represents a form of compensation to the agent, although the foregone interest is not properly reported as a commission expense. This delay should not affect the proper premium determination. During much of the period studied, policy terms tended to be for either annual or three–year periods, with annual installments. Assuming the premiums were paid at the beginning of the coverage period, or of the year, the premiums would be received by the insurer or its agent one six months prior to the average coverage period. As interest rates rose during the 1970s, insurers adopted shorter policy periods and developed some contracts that delayed the receipt of premiums (such as paid loss retrospective contracts), but these developments would only affect the last few years of the sample period. Thus, it will be assumed that a six–month delay occurs between the receipt of premium and the middle of the coverage period.

Insurers do not maintain expense allocation records that assign expenses, other than loss adjustment expenses, to individual policies. Instead these expenses tend to be allocated to lines of business. The expense ratio for a given line of business is determined by dividing the expenses, other than loss adjustment expenses, for a given line of business by the premiums written in that line during the same period. In many cases, expenses such as commissions, premium taxes, and policy record keeping costs are incurred simultaneously with the writing of the policy. Other expenses, such as those for policy development and ratemaking, would occur before a policy was written. Some other expenses, including policyholder service, would be incurred after the policy is written. Lacking reliable information about the true timing of expenses, it will be assumed that, on average, expenses are incurred when the policy is written. Thus, there is no time lag between collecting the premium and paying expenses.

The other notable feature of expenses for the purpose of this test is the calculation of expenses given the premium level indicated by the discounted cash flow approach. Theoretically, the premium level is determined by discounting the expenses, losses and tax payments, but the level of expenses is based on the premium level. Commissions and premium taxes, which make up approximately one half of the expenses, are directly proportional to the premium income. Other expenses, such as salaries and advertising, could also adjust in line with premium income. Additionally, agent compensation in the form of interest earned on premium balances is not reflected in traditional expense ratios. To avoid adding an additional step and further assumptions in the determination, the actual expense ratio achieved by insurers during the sample period will be used to represent the expenses in this model. To the extent that the indicated premium level diverges from the actual premium level, this will be inaccurate. However, the inaccuracy is tempered by the fact that if the premium level charged had been the indicated, and not the actual level, then expenses would have adjusted partially to compensate for this difference.

The loss ratio is the nominal, undiscounted loss and loss adjustment expense value divided by earned premium. This is the value to be solved for in equation (8). As the expense ratio is assumed to be the historical value, and values for the other parameters can be estimated, then LR is the only unknown in the equation. Once LR is determined, then the indicated underwriting profit margin can be determined by subtracting the sum of the loss ratio and the expense ratio from unity.

### *Option Pricing Model*

The option pricing model has only recently been applied directly to property–liability insurance and to ratemaking in particular. To the authors' knowledge, the main applications of option pricing techniques to the pricing of property–liability insurance are those of Cummins (1988), Derrig (1989) and Doherty and Garven (1986). Cummins derives a continuous time/jump process model for valuing the liabilities of solvency guarantee schemes. Although his model is not explicitly applied to ratemaking, it could be adapted for this purpose. Derrig's objectives are somewhat different. He is concerned with the estimation of solvency probabilities, surplus levels by line, and risk premium loadings. Doherty and Garven are directly concerned with derivation of the competitive or fair insurance price and, for this reason, the following discussion summarizes their approach.

The rationale for applying the option pricing model to the pricing of insurance is that the payoffs received by the insurer's various claimholders are isomorphic to the payoffs on options. For the sake of example, consider the case of a stock insurer in which the three major claimholders include the shareholders, the policyholders, and the tax authorities. To simplify matters, assume that the insurance firm is set up at one point (e.g., at the beginning of the year) and is operated for one period (e.g., one year) at which time all liabilities are discharged or reserved. Doherty and Garven show that the present values of the claims held by shareholders ($V_e$), policyholders (P) (net of expenses), and the tax authorities (T) are as follows:

$$V_e = C[Y_1;L] - \tau C[\theta(Y_1-Y_0)+P;L] = C_1 - \tau C_2, \tag{9}$$

$$P = V(Y_1) - C(Y_1;L), \text{ and} \tag{10}$$

$$T = \tau C[\theta\,(Y_1-Y_0)+P;L], \tag{11}$$

where $V(Y_1)$ is the current market value of the insurer's terminal cash flow $Y_1$, the cash flow $\theta(Y_1-Y_0)+P-L$ represents the amount of $Y_1$ that is subject to taxation, $\theta$ is a tax adjustment parameter ($\theta \in [0,1]$) which accounts for the insurer's tax sheltered investment activities, and C[A;B] is the current market value of an option written on an asset with a terminal value of A and exercise price of B. It is worthwhile to note that the values of the shareholders', policyholders', and tax authorities' claims ($V_e$+P+T) add up to $V(Y_1)$.

Given equations (9), (10) and (11), the objective is to price the insurance policies such that the shareholders receive a fair rate of return on their equity investment in the insurance firm. As in the case of the insurance CAPM, the fair rate of return is that which would be earned in a competitive capital market. Such a return would be made for investors if the present value of their future payoff ($V_e$) were equal to the value of the capital they invested in the firm (S); i.e.,

$$V_e = C[Y_1(P*);L] - \tau C[\ \theta(Y_1(P*)-Y_0(P*))+P*\ ;L] = C_1^* - \tau C_2^* = S. \qquad (12)$$

This is an implicit solution to the fair insurance price. The values of the two call options, $C_1$ and $C_2$ depend, among other things, on the premiums charged to policyholders. The premiums clearly affect the value of the underlying asset against which the two call options are written. Thus the solution requires that a level of premiums P* (net of expenses) be chosen such that equation (12) is satisfied.

The remaining task is to provide an explicit valuation framework for $C_1$ and $C_2$. Doherty and Garven (1986) provide two option pricing models based upon alternative specifications concerning the stochastic characteristics of asset returns and investor risk preferences. One model assumes that asset returns are normally distributed and investor preferences exhibit constant absolute risk aversion (CARA), while the other model assumes lognormally distributed asset returns and constant relative risk aversion (CRRA).[6] Although neither option model provides a closed form solution for P*, P* can be solved for numerically by implementing appropriately parameterized versions of equation (12). Furthermore, P* may be translated into the underwriting profit margin by the routine solution of equation (13):

$$UPM* = \frac{P* - E(L)}{P*}. \qquad (13)$$

The option pricing model has several practical advantages over the CAPM. First, it avoids the need for estimating and using underwriting betas. Secondly, the option pricing model addresses the effects of insolvency and tax shield redundancy on the fair rate of return, while the CAPM assumes that these effects are negligible. The work undertaken here reveals that the tax effects especially have a significant impact on the results.

## Data and Methodology

### *Some Caveats*

Testing property–liability insurance pricing models historically is extremely difficult, which most likely explains why only limited tests have been performed. Data availability problems are significant. Because several of the models require data that are not regularly reported, many assumptions had to be made in testing these models over the sample period. The validity of these assumptions is obviously critical in determining the applicability of the models and valid criticisms of these assumptions can be raised. However, the authors believe that the assumed values represent the best possible estimates of the necessary parameters. While they are recognized as not being completely accurate, no better values are known to be available. If these tests are to be performed, they must necessarily be done with less than perfect input values. For the option pricing model, the indicated underwriting profit margin is a function of individual insurers' premium to surplus ratios, which will not be the same as the aggregate value.

The primary data source for insurance industry information was Best's Aggregates and Averages. Several problems with the use of this data source arise. Industry figures reported are aggregate values for all insurers. As some insurers are owned by others, aggregate figures include some double counting of surplus and other values. Consolidated figures, which avoid this double counting, are only available for the years since 1983. For consistency, aggregate figures were used for all years so that any errors introduced by this assumption would be included over the entire sample period.

Another problem with this data source is that incurred losses constitute calendar year rather than accident year values. Any reserving changes would distort the calendar year results. Numerous researchers have demonstrated that loss reserve adequacy changes over time and the degree of adequacy is correlated with interest rates (see Anderson, 1971; Balcarek, 1972; Forbes, 1984; Smith, 1980; and Weiss, 1985). Unfortunately, accurate accident year experience for the industry is only available for the last seven years of the sample period. Thus, again for consistency, calendar year values are used throughout the period. The only mitigating factor for this assumption is that regulators, the insurance press, and many insurance managers tend to concentrate on calendar year values, increasing the importance of this value for pricing purposes. Over the entire sample period, the changes in loss reserve adequacy will tend to cancel out (the distortions in calendar year values caused by inaccurate reserving will be offset by errors in the opposite direction as the losses are settled in later years) so that the mean values of the actual results and model values will not be distorted by this substitution, but individual years' results will be affected by this problem.

Loss payout patterns for the entire sample period are not available. Over the ten years for which these data are available, the by line results are remarkably stable. Assuming this stability occurs over the entire sample period, loss payout patterns are generated for earlier years based on line of business distributions.

Predictions generated by several of the models tend to be quite sensitive to tax–related assumptions. No useful information on the marginal tax rate of the industry is published. This study assumes that the applicable tax rate is the maximum corporate income tax rate excluding excess profits taxes. However, the sensitivity of these models to this parameter and the significant impact of the Tax Reform Act of 1986 to insurance industry taxation indicate the importance of more research on the effective tax rate for the insurance industry.

In addition to the problems involved in obtaining usable data, several potential econometric problems may exist. The goodness of fit tests rely on the assumption that forecast errors are uncorrelated with economic variables such as interest rates. Some research suggests that reserve adequacy and other factors that affect insurance profitability are correlated with economic variables (see Fields and Venezian (1989; Venezian (1988a and 1988b), and Weiss (1985)). To the extent that this correlation occurs, the evaluations of the different models may be subject to error. Furthermore, underwriting profit margins for stock insurers for all lines combined are used in this study. However, insurers set rates for individual coverages or lines of business. If an aggregation bias (i.e., a systematic distortion caused by combining different lines of business together) exists, then the results of this research could be misleading. This problem is discussed more fully in a recent article by Fields and Venezian (1989) and suggests that the results of this research must be interpreted with caution. Neither of the above listed econometric problems affects the mean levels of the actual results or the model forecasts. However by affecting the mean square errors, they may influence the time series tests.

*Description of the Data*

Actual underwriting profit margins for all lines of business combined are used as a basis of comparison for evaluating the various models. The experience for the 60 years from 1926 through 1985 is used since this era spans nine underwriting cycles over a variety of economic conditions.

The underwriting profit or loss for each year is determined by subtracting the sum of the loss and loss adjustment expense ratio (calendar year incurred loss and loss adjustment expenses divided by earned premium) and the expense ratio (total expenses excluding loss adjustment expenses, investment expenses and federal income taxes divided by net written premium) from one. The aggregate values for all stock insurers, as reported in Best's Aggregates and Averages, are used to determine the actual underwriting profit or loss.

The target underwriting profit margin for all lines combined is determined by calculating the weighted average of the 2.5 percent workers' compensation margin and the 5.0 percent margin for all other lines. This weighted average is calculated based on the premium distribution for all stock insurers as reported in Best's Aggregates and Averages.

The total rate of return values for the appropriate underwriting profit margin are based on the surplus to premium ratios, the risk–free rate of return, the insurance industry systematic risk level, the excess market return, the ratio of investable assets to surplus and the investment rate of return earned by the insurance industry. The net written premium and surplus values for all stock insurers combined are taken from A. M. Best's Aggregates and Averages. The risk–free rate and the market return is taken from Ibbotson and Sinquefield (1986). The value of investable assets can also be derived from Best's, but measurement of the investment rate of return presents some difficulties. Best's reports two investment return values for all stock insurers combined. The first, net investment income, is the total, net of expenses, of all interest and dividend income received. The second investment income value reported is the investment profit or loss which adds the net realized capital gains or losses to the net investment income value. This does not include unrealized capital gains and losses, which a true market rate of return should. Gains that are realized in one period could have occurred in either the current or prior periods. Tax effects dependent on insurer profitability may have led to the selling of assets to realize gains or losses, so the carryover effect of unrealized gains and losses cannot be assumed to be neutral. However, despite these problems, each of the reported investment income values is used separately to generate the UPM value. As the reported values represent total dollars of investment income achieved, these values are equal to the product (IA)(IRR) [see equation (1)].

In addition to the industry reported investment returns, a market value for IRR is included. The rate used is the long term government bond yield (not total rate of return) reported in Ibbotson and Sinquefield, as this represents the most common insurer investment. The yields represent the expected rate of return for insurers investing in bonds, since changes in market values of bonds resulting from interest rate level changes are difficult to predict. One problem that must be recognized in this approach is the discrepancy between the reported values for insurer assets and market values. Insurers value bonds at their amortized value. Thus, any difference between the purchase price and maturity value is assumed to be reduced proportionally as the time to maturity elapses. Any market value fluctuations are ignored. The direction and magnitude of this distortion in reported values from market values depends on past interest rate changes and portfolio turnover.

The CAPM including taxes, as shown in equation (5), is used to derive predictions for underwriting profit margins. In order to estimate CAPM-based underwriting profit margins, values for the funds generating coefficient, k, the tax rate on investments, $\tau_a$, the tax rate on underwriting income, $\tau$, the systematic risk of underwriting, $\beta_u$, and the expected return on the market as a whole, $E(r_m)$ must first be estimated, along with the previously mentioned risk-free rate and surplus to premium ratio. The funds-generating coefficient was determined by dividing the loss and loss adjustment expense reserves by the net written premium for stock property-liability insurers, as provided by Best's Aggregates and Averages, from 1939 through 1985. Prior to 1939, the loss and loss adjustment expense value was not provided. For the years 1926 through 1938, the value of k was determined by applying regression coefficients to the percentage of premium written in Schedule P lines of business for the current and two preceding years. The regression coefficients were obtained by fitting the 1939 through 1985 values of k to current and lagged values of the percentage of premium written in Schedule P lines of business.

The tax rate applied to underwriting income is the maximum corporate tax rate each year, including excess profits taxes, as reported by Seater (1980). The tax rate on investment income used in this study is 50 percent of the tax rate on underwriting income. Although the tax rate on investments would vary depending on the portfolio mix and realization of gains and losses, information necessary to determine the investment tax rate is not available for most of the period analyzed. Beginning in 1983, a consolidated industry annual statement was calculated by the A. M. Best organization, and starting in 1984, included in Best's Aggregates and Averages. Based on this information, the ratio of taxable investment income to total investment income was 45.5 percent in 1983, 50.2 percent in 1984 and 52.7 percent in 1985.[7] From these values, the assumption that 50 percent of investment income would be taxable was derived.

The expected return on the market is determined year–by–year by adding the average market risk premium, or difference between common stock returns and Treasury bill returns, for 1926 through 1985. During this period, common stock returns were 8.55 percentage points higher on average than Treasury bill returns.

Two values are used for the underwriting betas in this study. Hill (1979) calculated a value of –0.23, with a standard error of 0.24 (based on a pooled regression) for the beta of liabilities for the period 1951 through 1965. Fairley (1979) determined a value of –0.21 for the beta of liabilities. Although the beta for liabilities has been shown to have a negative sign, in determining underwriting profits liabilities are subtracted. The negative systematic risk for liabilities then becomes positive systematic risk for profitability. Thus, one value of the underwriting beta used in this study is 0.20. Other research indicates that the underwriting beta is not significantly different from zero. Cummins and Harrington (1985) use quarterly data to determine underwriting betas and found that they were neither significant nor stable. Regressing actual underwriting profit margins against market returns for the period 1926 through 1985 produces a value of +0.02, insignificantly different from zero. Thus, the alternative measure of underwriting beta used in this study is zero.

For the discounted cash flow model, the present value factor for the loss ratio is determined from loss payout patterns reported in Woll (1987). His study covers the period 1981 through 1985 for Schedule O lines and 1977 through 1985 for Schedule P lines, the only periods for which industry data are available, and finds the loss payout patterns by line over this period are quite stable. Assuming this stability exists over the full 60–year sample period, accounting for the changing mix of business over this period and changes in interest rates is accomplished as follows:[8]

$$\mathrm{PVFLR} = (\mathrm{SPA}_i)(\mathrm{SPD}_i)+(1-\mathrm{SPA}_i)(\mathrm{SOD}_i), \tag{14}$$

where

PVFLR = present value factor for the loss ratio;
SPA = percent of net premiums written in Schedule P lines (based on the 1985 definition of Schedule P);
SPD = Schedule P discount factor;
SOD = Schedule O discount factor;
i = year indicator.

The Schedule P and O discount factors are determined as follows:

$$\mathrm{SPD}_i = \sum_{j=1}^{n} \mathrm{SPR}_j/(1+r_f)^{j-.5}, \tag{15}$$

where $\mathrm{SPR}_j$ = percent of Schedule P losses paid in j th year of development, and

$$\mathrm{SOD}_i = \sum_{j=1}^{n} \mathrm{SOR}_j/(1+r_f)^{j-.5}, \tag{16}$$

where $\mathrm{SOR}_j$ = percent of Schedule O losses paid in the $j^{th}$ year of development.

The tax generated on underwriting income is determined by multiplying the maximum corporate tax rate for each year, excluding excess profits taxes due to both the assumed temporary nature of these taxes and their extremely high levels, by the underwriting profit margin. As it is assumed that written premium precedes earned premium by six months, and the taxes are incurred when the underwriting profits are earned, then the underwriting taxes are discounted by the risk–free interest rate for six months.

The tax generated on income from the investment balance is also determined based on the loss payout pattern described by Woll (1987). For each year the investable loss reserves as a percentage of the loss ratio are determined by averaging the beginning and ending values for the proportion of losses unpaid, calculated separately for Schedule O and P lines and then combined. This represents the average investment balance derived from loss (and loss adjustment expense) reserves. Additionally, surplus that supports these reserves is also invested and incurs taxation.[9] To account for this additional investment, the loss reserve investment balance is multiplied by one plus the ratio of surplus to net written premium for the industry, determined from Best's Aggregates and Averages for all stock property–liability insurers for the year the policies are written. The entire investment balance is multiplied by the risk–free interest rate for the year the policies were written. The investment income is assumed to accrue equally over the year (actually it would be more heavily weighted toward the beginning of the year when the largest amount is available to invest; instead assume a level amount invested over the entire year), thus the tax would be incurred halfway through the year. Thus, the present value of the tax is determined by discounting the tax by the runoff year number less one half.

Combining the steps described above into equation (8) leaves an equation with one unknown, LR, which can then be solved. The indicated underwriting profit margin is determined by subtracting the actual expense ratio and the indicated loss ratio from unity.

The values used in the option pricing model are as follows:

| | |
|---|---|
| Initial Equity | 1.000 |
| Standard Deviation of Claims Costs | .2958 |
| Correlation Between Investment Returns and Claims Costs | .0763 |
| Tax Adjustment Parameter | .5000 |
| Market Risk Premium | .0855 |
| Standard Deviation of Market Return | .2137 |

The values for the funds generating coefficient, risk–free rate of interest and statutory tax rate all vary by year and are determined as described previously. The expected claims cost, beta of the investment portfolio, and standard deviation of the investment portfolio also vary by year. These values, and the parameter values not used in prior models, are determined as follows:

1) The expected claims costs is the ratio of incurred loss and loss adjustment expenses to surplus for stock insurers for each year.

2) The standard deviation of expected claim costs is the standard deviation of the ratio described above over the period 1926 through 1985.

4) The correlation between the investment returns and claim costs is the correlation for the period 1926 through 1985.

5) The beta of the investment portfolio is the actual beta for investment profit or loss based on the 1926 through 1985 period. The alternative value is the weighted average beta for stock insurers based on their portfolio distribution among bonds, preferred stock and common stock each year. The bond beta was determined from the Ibbotson and Sinquefield (1986) data to be 3.64 percent over the entire 1926 through 1985 period. The preferred stock beta was determined from S&P data to be 14.78 percent over the period 1931 through 1981, the only years the necessary information to determine rates of return was available.

6) From portfolio theory, it is well known that the standard deviation of a well diversified portfolio is proportional to the standard deviation of the market, the factor of proportionality being beta. Therefore, the standard deviation of the investment portfolio is determined in this manner on a yearly basis.

7) The standard deviation of the market is the standard deviation of common stock returns for the period 1926 through 1985.

*Methodology*

Various criteria exist for evaluating the accuracy of the forecasts generated by a predictive model. This study uses the mean square error criterion as developed by Theil (1966).

The mean square error (MSE) of a prediction is calculated as follows:

$$\mathrm{MSE} = \frac{1}{n}\sum_{t=1}^{n}(P_t - A_t)^2, \tag{17}$$

where $A_t$ is the actual and $P_t$ is the predicted value of a variable in period t. Since this measure is zero in the case of perfect forecasts, predictive models which yield low mean square errors have greater forecast accuracy than do models with high mean square errors.

A related measure of forecast accuracy is given by Theil's U statistic:

$$U = \sqrt{\frac{MSE}{\sum_{t=1}^{n} A_t^2/n}} . \tag{18}$$

As is the case with mean square error, Theil's U assumes a value of zero in the case of perfect forecasts. Also, Theil's U standardizes mean square error as a function of both the level and variability of the actual underwriting profit margin series, thereby facilitating comparison of predictive models over time as well as within various subperiods.[10] Since the objective is to evaluate the forecast accuracy of the various property–liability insurance pricing models in this manner, this study reports Theil's U as well as mean square error.

## Empirical Results

### *Analysis of Raw Data and Summary Statistics*

Next, raw data and summary statistics for realized underwriting profits and model predictions are evaluated on a model–by–model basis. This is followed by an analysis of mean square error and Theil's U statistics.

*Target Underwriting Profit Margin:* The actual and predicted underwriting profit margin series are listed in Table 1 for 1926 through 1985 and presented graphically in Figures 1–9. Also, summary statistics are reported in Table 2 and presented graphically in Figure 10. In Table 1, the first and second columns list the year and actual (ACT) underwriting profit margin series. The third column lists target underwriting profit margin (TARG) predictions obtained by weighting the 2.5 percent and 5.0 percent profit standards according to the historical evolution of the industry's premium distribution. A cursory examination of these data indicates that the target underwriting profit margin is (not surprisingly) insensitive to changes in economic conditions and, since 1956, does not even fall within the range of actual underwriting profit margins. Interestingly, in spite of the obvious theoretical shortcomings of this method, the average actual underwriting profit margin from 1926 through 1955 was 4.65 percent, just .10 percentage points lower than the average target underwriting profit margin. Over the entire sample period, the average target underwriting profit margin was 4.73 percent, 3.41 percentage points higher than the average actual underwriting profit margin of 1.32 percent. Thus, although this method may have once had some validity, it is obviously not realistic under the market conditions which have prevailed throughout most of the postwar period.

*Target Total Rate of Return:* The predicted underwriting profit margins for the total rate of return methodology, based upon various parameter assumptions, are listed in columns 4 through 9 of Table 1 and graphed in Figures 3 through 5. Column 4 lists values for TRR1. This model is based upon an insurer equity beta of .61 and the use of net investment income in the calculation of the insurer's investment return. The TRR2 series in column 5 differs from TRR1 in that investment return is defined by the net profit or loss value rather than net investment income. In column 6, the market yield on bonds is substituted for the investment return in order to produce the TRR3 series. The models represented in columns seven through nine (TRR4–TRR6) are similar to TRR1–TRR3, the only difference being that the insurer equity beta in those models is assumed to be equal to unity rather than .61.

The TRR1 model tends to underestimate the actual return series prior to 1956 and overestimate afterwards. The TRR1 series is much less volatile than the actual underwriting profit margin series, with a standard deviation of 2.36 percent compared to 5.74 percent. In contrast, the predictions generated by the TRR2 model are extremely volatile, with a standard deviation of 8.92 percent, 3.18 percentage points higher than the standard deviation of the actual underwriting profit margin series. This volatility most likely results from the discretionary nature of realized capital gains and losses. The insurer can, to a significant extent, time the realization of capital gains and losses so as to derive maximum tax–related benefits. Thus, the reported investment income including realized capital gains and losses will generally tend to be biased as a function of the tax position of the insurer, and these effects will not tend to cancel out in aggregate due to the systematic nature of insurance profitability. The results for this model clearly illustrate the pitfalls associated with basing prices upon a definition of investment income which includes realized capital gains and losses.

The TRR3 series tracks the actual return series fairly well during some years (such as 1926 through 1932 and 1983 through 1985), but in general this series underestimates actual returns (especially prior to 1956 and after 1965). The mean of the TRR3 series is –2.41 percent, 3.73 percentage points below the average actual underwriting profit margin.

TRR4 and TRR5 appear to suffer from problems similar to those mentioned for TRR1 and TRR2, as could be expected since the values for investment income are the same with only the insurer equity beta changing from .61 to unity. However, this adjustment appears to have a more favorable impact upon the TRR6 series. For this model, the mean of 0.57 percent is only 0.75 percentage points below the mean of the actual return series, while the standard deviation of 5.43 percent is only 0.31 percentage points below the standard deviation of the actual series. The predicted returns track the actual returns fairly closely over the entire sample period. Consequently, the use of current long term bond yields and an insurer equity beta of unity appears to produce the most accurate predictions under the total rate of return methodology.

*Capital Asset Pricing Model and Discounted Cash Flow Model*: The predicted underwriting profit margins based on the CAPM are listed in columns 10 and 11 of Table 1 and graphed in Figure 6. The values in column 10 for CAP1 are based upon an underwriting beta of zero, whereas the values in column 11 for CAP2 are based upon an underwriting beta of 0.2. As the only difference between these two models is the value of beta multiplied by the excess return on the market, which is estimated to be 8.5 percent, then each of the CAP2 values is simply 1.70 percentage points higher than the CAP1 values. The CAPM models predict returns that generally fall substantially below the actual returns for most years in the sample, and this shortfall is particularly noticeable during the period 1933 through 1955. The means of –2.81 percent for CAP1 and –1.11 percent for CAP2 are 4.13 and 2.43 percentage points below the average actual underwriting profit margin as measured over the entire sample period. The only years during which these models made accurate predictions were during the mid–1970s, which may have, coincidentally, bolstered their assumed applicability during that period.

The predicted values for the discounted cash flow (DCF) model are presented in column 12 of Table 1 and graphed in Figure 7. This model generates a mean return estimate that is close to the mean of the CAP2 return series, (–1.27 percent vs. –1.11 percent), but with an even lower standard deviation. The DCF model also performs well during the mid–1970s and early 1980s, but not particularly well during other subperiods.

*Option Pricing Model*: The predicted underwriting profit margins based upon the option pricing model are listed in columns 13 through 18 of Table 1 and graphed in Figures 8 and 9. The O1R series listed in column 13 is based upon the normal option pricing model, while the O2R series in column 16 makes use of the lognormal option pricing model. The standard deviations for these models are quite close to the standard deviation of the actual return series; i.e., 5.30 percent and 5.51 percent for O1R and O2R compared with 5.74 percent for the actual return series. However, unlike the previous models, O1R and

O2R tend to systematically overestimate the actual return series. This propensity toward overestimation is especially evident from 1950 through 1976, when all deviations between predicted and actual returns were positive for both models. This result largely derives from the fact that these models implicitly assume that tax shields can be fully utilized by the insurer only when the terminal (next period) value of its assets exceeds the value of its tax shields. In states of the world where the terminal value of the insurer's assets is less than the value of its tax shields, these option models assume that those tax shields which are not utilized expire worthless. Since wasted tax shields effectively increase the burden of the corporate tax upon the insurer, higher underwriting profit margin estimates are produced during this period by O1R and O2R than by alternative financial pricing models such as CAP1, CAP2, and DCF which implicitly assume either that tax shields are always fully utilized or that the tax system provides for the contemporaneous realization of tax rebates as well as liabilities.[11]

Since the O1R and O2R models determine upper bounds for the effect of underutilized tax shields on the underwriting profit margin, two alternative option models, O1N and O2N, were devised so as to produce option pricing estimates under the same tax assumptions as are implicitly assumed by the CAP1, CAP2, and DCF models; viz., assume that tax shields are never underutilized. Consequently, the estimates generated by O1N and O2N represent lower bounds for the effect of underutilized tax shields on the underwriting profit margin.[12] The primary difference between the O1N and O2N models and the CAP1 and CAP2 models is due to the fact that the option models endogenize the probability of insolvency, whereas the CAPM models implicitly assume that either the probability of insolvency is negligible or that shareholders have unlimited liability. Interestingly, the O1N and O2N series are very close to being perfectly positively correlated with the CAP1 and CAP2 series.[13] Furthermore, these two models also share the tendency of the CAP1 and CAP2 models to underestimate the true return series. Since the only structural difference between the O1R/O2R and O1N/O2N models is due to tax assumptions, a third set of option–related models, O1C and O2C, was devised by forming linear combinations of the O1R/O1N models and O2R/O2N models which minimize mean square error as measured over the sample period.[14]

*Analysis of Mean Square Error and Theil's U*

The mean square errors and Theil's U statistics for the various models described in the previous section are listed in Tables 3 and 4 for the entire sample period and various subperiods, and Theil's U statistics are graphed for the entire sample period in Figure 11. Based on these data, the relative performance of the different models can be assessed. Over the entire period, the TRR6 model produced the lowest mean square error and Theil's U statistics. Other relatively low values for these statistics are generated by O1C, O2C, TRR4 and CAP2. Counter to original expectations, the financial pricing models do not perform as well as a model that ignores taxation and bases the total rate of return on current long–term bond yields.

Breaking the sample into two 30–year subperiods indicates that TRR6 has the lowest mean square error for the period 1926 through 1955, while TRR1 has the lowest mean square error for the period 1956 through 1985. However, for the second period the advantage of the total rate of return models over the option pricing models is diminished, with TRR1 having only a marginally lower mean square error than O2N.

A similar result is apparent from the ten–year subperiods. The lowest mean square errors are generated by the following models: TRR4 during 1926 through 1935, TRR6 during 1936 through 1945, TARG during 1946 through 1955, CAP1 during 1956 through 1965, and O1C during 1966 through 1975 and 1976 through 1985. For the last decade, the mean square errors of O2C and TRR6 are almost as low as the O1C value. Thus, during the periods of higher and more volatile interest rates and longer loss payout patterns, the financial pricing models apparently are capable of producing more accurate estimates of the underwriting profit margin than the simpler total rate of return models.

Evaluating the mean square errors of the different models across the ten year periods illustrates the relative strengths and weaknesses of each model. The TARG model produced relatively low mean square errors only during 1936 through 1945 and 1946 through 1955, periods during which interest rates and loss payout patterns approximated the period during which the target was established. TRR1, which calculates the investment income based upon the portfolio rate of return rather than current rates, performed best during the decade 1976 through 1985, a period marked by relatively high and volatile interest rates. TRR2 and TRR5 generated very high mean square error values, but also produced their lowest mean square errors during 1976 through 1985. TRR3 generated relatively stable mean square errors, except for the periods 1956 through 1965 and 1966 through 1975, periods during which interest rates were rising. TRR4 generated its lowest mean square errors during the 1926 through 1935 and 1936 through 1945 periods, with short payout patterns but very unstable economic conditions. TRR6 tended to generate relatively low mean square errors, except during the 1956 through 1965 period when interest rates began to rise.

Shifting attention to the financial pricing models shows that CAP1 generated its lowest relative mean square error in the 1956 through 1965 period, exactly when the previously discussed models were becoming less accurate. Interestingly, the mean square errors for CAP2 were lower than those of CAP1 in every period except 1956 through 1965. The DCF model never produced a mean square error below the lower of the CAPM values. The lowest mean square error for DCF occurred in the 1976 through 1985 period.

O1R generated relatively low mean square errors for each period except 1956 through 1965 and 1966 through 1975. Interest rates began to rise substantially during these periods. O1N generated an mean square error below the value for O1R only during the 1956 through 1965 period, indicating that tax shields appeared to be most useful during that period. O1C generated low mean square errors during the periods 1936 through 1945, 1946 through 1955 and 1976 through 1985. The construction of this model weighting O1R and O1N to produce the lowest mean square error is such that these may have been the most typical of the periods covered in the sample. O2R never generates a mean square error below that of O1R, indicating that the normal option pricing model is superior to the lognormal model if tax shields are redundant. However, the mean square errors of O2N are below the comparable values for O1N indicating that the lognormal model is superior if tax shields are not redundant. For O2C, the lowest mean square errors are generated in 1946 through 1955 and 1976 through 1985 and in both cases these values are almost the same as for O1C. The tax rate volatility during the former of these two periods and the interest rate volatility during the latter indicate that these models are well suited for periods with volatile input parameters.

## Conclusion

The purpose of this article has been to provide an empirical evaluation of how well a number of alternative rate of return models fare in terms of predicting underwriting profit margins. The tests contained herein are based on the actual results achieved by all U.S. stock insurers in aggregate from 1926 through 1985.

The results of these tests indicate that the higher rankings usually tend to go to the total rate of return and option pricing models. This is especially true during the second half of the sample period as nominal interest rates began to rise significantly. Unfortunately, relative rankings of the alternative models are not very stable over time. Also, in spite of the fact that capital asset, discounted cash flow and option pricing models use more variables than the target and total rate of return models, this does not always translate into greater predictive accuracy. It may be that the estimation process itself has introduced error of a sufficiently high order of magnitude so as to turn the theoretical advantages of these models into disadvantages. For example, the predictions generated by the target and total rate of return models are not at all sensitive to assumptions concerning values taken on by tax–related parameters. However, the capital asset, discounted cash flow and the option pricing models require explicit recognition of taxes and are therefore sensitive to errors in the estimation of these parameters. Furthermore, the option models are particularly sensitive to changes in the tax related parameters. Thus, further testing of the relative merits of the various pricing models may need to await a more extensive development of an historical database on the taxation of insurers.

Overall, the CAPM approaches do not predict the actual returns well until the period from 1956 through 1965, and even then they do not outperform the option pricing models by much. Subsequent to that period, the option pricing model and even the total rate of return models outperform the CAPM approaches. The discounted cash flow model seems to be in line with the CAPM values and never performs any better than the CAPM. Over the entire period from 1926 through 1985 period, TRR6, O1C and O2C have the lowest mean square errors. When the ten year periods are analyzed, the CAPM, discounted cash flow and option pricing models tend to have lower mean square errors. However, for the 1976 through 1985 period, the mean square error for TRR6 is practically the same as for the O1C and O2C models. In general, the authors believe that this research provides the strongest support for the TRR6 model, which ignores taxation, and for the option pricing models in which taxation is an important factor.

This research suggests a number of possible avenues for future research. The favorable performance of the option pricing model, which is based on total variability rather than simply systematic risk as suggested by the CAPM, indicates that additional parameter values must be obtained. This model highlights the importance of taxation, for which only limited data are available. Research on the investment mix between taxable and nontaxable securities for the property–liability insurance industry over an extended period would provide more accurate values for the tax adjustment parameter. The role of tax loss carryforwards and carrybacks could be better understood if an analysis of insurer positions over time, particularly in relation to the underwriting cycle, were documented. The effect of the Tax Reform Act of 1986, with its significant changes for insurers, is another important issue. Additional studies focusing on loss reserves, both on changes in the loss payout pattern and the effect of patterns of under and overreserving over time, could provide useful additional information about the value of the different pricing models. Additional long–term testing of the various pricing models for individual lines of business or for individual coverages would avoid the problems caused by the aggregation bias to which this study is susceptible. Such studies would require additional assumptions about surplus allocation and the tax effects by line, but reasonable assumptions could be made to facilitate this research.

Finally, the analysis in this article is based on data for stock insurers only. Although stock companies dominate the property–liability insurance industry, there are also many firms which employ alternative ownership structures (e.g., mutuals, reciprocals, Lloyds, and captives). Future research aimed at these market segments and the interaction among them would be helpful if data limitations could be overcome.

## Notes

1. Although the CAPM formula is used to calculate $E(r_e)$, the total rate of return method does not depend on the CAPM, as other techniques could be used to obtain $E(r_e)$.

2. Although the arbitrage pricing model is not tested in this study, Urrutia (1987a and 1987b) also applied a differential tax rate in his tests of that model.

3. As in the case of the total rate of return model, while the appropriate discount rate may be calculated from the CAPM formula, the Myers–Cohn model does not preclude the choice of other techniques.

4. Since P is received at the beginning of the period and is therefore known with certainty, PV(P) is determined by discounting P at the risk–free rate.

5. Myers showed in the 1985 Massachusetts automobile rate hearings that the risk adjusted present value of the investment tax liability for a fixed effective tax rate is the same for all asset portfolios. Consequently, the correct value for PV(IBT) is obtained by discounting the tax liability at the risk–free rate. See Derrig (1985) for a review of Myers' proof.

6. The reader is referred to Doherty and Garven (1986) for details concerning the derivation and mathematical structure of their models.

7. The reader is referred to the appendix for details on how these values were calculated.

8. Insurers are required to report loss development by line of business each year in Schedules O and P of the Annual Statement. Although the definitions have changed somewhat over the years, the intent of the division is to report the fast closing lines in Schedule O and the slower closing lines in Schedule P. In general, property lines are included in Schedule O and the exhibit focuses on the losses paid and salvage and subrogation received in the same or subsequent years. As losses in these lines tend to be settled quickly, the exhibit displays details of loss development for only three years. Schedule P details loss development for a longer period, originally seven years, now being extended to ten years. The adequacy of reserve levels for losses and loss adjustment expenses are shown for each of the prior periods, allowing for a longer check of reserve adequacy. The lines of business currently included in Schedule P are: automobile liability, other liability, medical malpractice, workers' compensation, farmowners multiple peril, homeowners multiple peril, commercial multiple peril, ocean marine, aircraft and boiler and machinery. All other lines are included in Schedule O.

9. Myers and Cohn point out that this taxation should be borne by the policyholders since the providers of insurance capital could alternatively invest this capital directly in the capital markets and avoid this additional layer of corporate taxation.

10. By dividing MSE through by the mean of the square of the actual return series, Theil's U effectively renders MSE scale–free. Essentially, Theil's U provides an indication of relative as opposed to absolute error. This method of standardization facilitates the comparison of predictive models over time by taking out the effects of intertemporal changes in the level as well as variability of the actual return series.

11. Closer inspection of applicable tax rates and nominal rates of interest during 1950 through 1976 and the years immediately preceding and following this period provides some clues as to why the O1R and O2R estimates were so high. Prior to 1950 (specifically, during 1926 through 1949), the average tax rate was 23.7 percent. However, during 1950 through 1976, tax rates averaged 50.13 percent. This substantial increase in tax rates increased the value of the hypothetically wasted tax shields, thereby causing the prediction error to be too high. Although tax rates remained relatively high during 1977 through 1985 (averaging 47.11 percent), substantial increases in nominal rates of interest during that period lowered the probabilities of tax shield underutilization for the two option models, thereby lessening the impact of wasted tax shields on the predictions generated by these models.

12. As a practical matter, tax shields which are not utilized in one period can be carried back and forward if necessary. Under federal tax law, losses only produce contemporaneous tax rebates under the condition that income from previous years is adequate to fully offset loss carrybacks. When this is not the case, losses are carried forward at a zero rate of interest, and beyond a certain point in time, carryforwards that are not utilized expire. Consequently, the present value of a tax rebate from a loss carryforward is worth less than a contemporaneous tax rebate for two reasons: 1) carryforwards do not earn interest, and 2) the insurer may not produce enough income in future periods to use the carryforwards before they expire. By representing the limiting case where tax loss carrybacks or carryforwards are disallowed, the O1R and O2R models determine upper bounds for the effect of underutilized tax shields on the underwriting profit margin. Although the effect of the carryback–carryforward provision is not estimated in this article due to data limitations as well as the lack of a viable multiperiod option model of insurance pricing which directly incorporates this provision, its effect can nevertheless be qualitatively inferred. Since the effect of the carryback–carryforward provision is to reduce the burden of wasted tax shields, lower underwriting profit margins would be implied than are predicted by either O1R or O2R. However, one can also be sure for the reasons given above that the implied margins would exceed those predicted by O1N and O2N.

13. Because the implied probabilities of insolvency averaged .01 percent for normal option model and .12 percent for the lognormal option model during the entire sample period, the CAPM and O1N/O2N models are structurally very close to being the same models. The correlations between the CAPM and option models are .9847 in the case of the O1N series and .9684 for the O2N series.

14. The weighting scheme described here involves redefining $P_t$ in equation 17 for both the normal and lognormal option models in the following manner:

$$P_t = XP_{1t} + (1-X)P_{2t},$$

where X corresponds to the weight applied to prediction $P_{1t}$, $P_{1t}$ is the period t prediction generated by the option models which assume the possibility of redundant tax shields (i.e., O1R and O2R), and $P_{2t}$ corresponds to the period t prediction generated by the option models which assume nonredundant tax shields (i.e., O1N and O2N). Substituting this definition for $P_t$ into equation (17), differentiating with respect to X, and solving for X when the derivative assumes a value of zero yields the following equation for X:

$$X = \frac{\sum_{t=1}^{n}(A_t - P_{2t})}{\sum_{t=1}^{n}(P_{1t} - P_{2t})}.$$

Applying this formula to the O1R/O1N and O2R/O2N data for 1926 through 1985 yields X values of .6272 and .4019 respectively. These weights were then applied to these data in order to generate the O1C and O2C data series. One important consequence associated with applying this minimum mean square error formula is that the means of the O1C and O2C series are identical to the mean of the actual underwriting profit margin series.

## References


Anderson, Dan R., 1971, Effects of Under and Overevaluations in Loss Reserves, *Journal of Risk and Insurance* 38:585–600.

Ang, James. A. and Tsong–Yue Lai, 1987, Insurance Premium Pricing and Ratemaking in Competitive Insurance and Capital Markets, *Journal of Risk and Insurance* 54:767–779.

Bailey, Robert A., 1976, Underwriting Profit from Investments, *Proceedings of the Casualty Actuarial Society* 54:1–8.

Balcarek, Robert J., 1972, Loss Reserving in the Sixties, *Proceedings of the Casualty Actuarial Society* 59:18–25.

Berquist, John R. and R. E. Sherman, 1977, Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach, *Proceedings of the Casualty Actuarial Society* 64:123–184.

Biger, Niger and Yehuda Kahane, 1978, Risk Considerations in Insurance Ratemaking, *Journal of Risk and Insurance* 45:121–132.

Cooper, Robert W., 1974, *Investment Return and Property–Liability Insurance RatemakinG*. Homewood, Illinois: Richard D. Irwin.

Cummins, J. David, 1988, Risk Based Premiums for Insurance Guaranty Funds, *Journal of Finance* 43:823–839.

Cummins, J. David and Lena Chang, 1983, An Analysis of the New Jersey Formula for Including Investment Income in Property–Liability Insurance Ratemaking, *Journal of Insurance Regulation* 1:555–573.

Cummins, J. David and Scott E. Harrington, 1985, Property–Liability Insurance Rate Regulation: Estimation of Underwriting Betas Using Quarterly Profit Data, *Journal of Risk and Insurance* 52:16–43.

Cummins, J. David and Scott E. Harrington, 1987, *Fair Rate of Return in Property–Liability Insurance*, Boston: Kluwer–Nijhoff Publishing.

Cummins, J. David and Scott E. Harrington, 1988, The Relationship Between Risk and Return: Evidence for Property–Liability Insurance Stocks, *Journal of Risk and Insurance* 53:15–31.

D'Arcy, Stephen P., 1983, The Impact of Investment Income on Property–Liability Insurance Underwriting Profit Margins, *Journal of Insurance Regulation* 2:204–220.

D'Arcy, Stephen P., 1988, Use of the CAPM to Discount Property–Liability Loss Reserves, *Journal of Risk and Insurance* 55:481–491.

D'Arcy, Stephen P. and N. A. Doherty, 1988, *The Financial Theory of Pricing Property–Liability Insurance Contracts*, Huebner Foundation Monograph Number 15. Homewood. Ill: Richard D. Irwin.

Derrig, Richard A., 1985, The Effect of Federal Income Taxes on Investment Income in Property–Liability Ratemaking, Actuarial Working Paper, Massachusetts Rating Bureaus.

Derrig, Richard A., 1989, Solvency Levels and Risk Loadings Appropriate for Fully Guaranteed Insurance Contracts, in *Financial Models of Insurance Solvency*, Boston: Kluwer–Nijhoff Publishing Co.

Derrig, Richard A., 1987, The Use of Investment Income in Massachusetts Private Passenger Automobile and Workers' Compensation Ratemaking, Chapter 6 in Cummins and Harrington (1987).

Doherty, Neil A. and James R. Garven, 1986, Price Regulation in Property–Liability Insurance: A Contingent Claims Approach, *Journal of Finance* 41:1031–1050.

Fairley, William B., 1979, Investment Income and Profit Margins in Property Liability Insurance, *Bell Journal of Economics* 10:192–210.

Ferrari, J. Robert, 1968, The Relationship of Underwriting, Investments, Leverage, and Exposure to Total Return on Owners' Equity, *Proceedings of the Casualty Actuarial*

*Society* 55:295–302.
Ferrari, J. Robert, 1967, A Theoretical Portfolio Selection Approach for Insuring Property and Liability Lines, *Proceedings of the Casualty Actuarial Society* 54:33–54.
Fields, Joseph A. and Emelio C. Venezian, 1989, Interest Rates and Profit Cycles: A Disaggregated Approach, *Journal of Risk and Insurance* 56:312–319.
Forbes, Stephen W., 1984, Loss Reserving Performance Within the Regulatory Framework, *Journal of Risk and Insurance* 51:527–538.
Haugen, Robert A. and Charles O. Kronke, 1971, Rate Regulation and the Cost of Capital in the Insurance Industry, *Journal of Financial and Quantitative Analysis* 6:1283–1305.
Hill, Raymond D., 1979, Profit Regulation in Property Liability Insurance. *Bell Journal of Economics* 10:172 through 191.
Hill, Raymond D. and Franco Modigliani, 1987, The Massachusetts Model of Profit Regulation in Non–Life Insurance: An Appraisal and Extensions, Chapter 2 in Cummins and Harrington, (1987).
Ibbotson, Roger G. and Rex A. Sinquefield, 1986, *Stocks, Bonds, Bills and Inflation (SBBI)*, updated in *SBBI 1986 Yearbook*, Chicago: Ibbotson Associates.
Kahane, Yehuda and Haim Levy, 1975, Regulation in the Insurance Industry: Determination of Premiums in Automobile Insurance, *Journal of Risk and Insurance* 42:117–132.
Kraus, Alan and Stephen A. Ross, 1982, The Determination of Fair Profits for the Property–Liability Insurance Firm, *Journal of Finance* 37:1015–1028.
McCabe, George M. and Robert C. Witt, 1980, Insurance Pricing and Regulation Under Uncertainty: A Chance Constrained Approach, *Journal of Risk and Insurance* 47:607–635.
McCullough, Robert C. 1948, Second Report of the Special Sub–Committee of the Fire and Marine Committee, National Association of Insurance Commissioners, RE Underwriting Profit or Loss and the Commissioners' 1921 Standard Profit Formula, *Proceedings of the National Association of Insurance Commissioners*, Seventy–Ninth Session, 1948), 73–159.
Myers, Stewart C. and Richard Cohn, 1987, Insurance Rate Regulation and the Capital Asset Pricing Model, Chapter 3 in Cummins and Harrington (1987).
NAIC (National Association of Insurance Commissioners), 1970, Measurement of Profitability and Treatment of Investment Income in Property and Liability Insurance, *Proceedings of the National Association of Insurance Commissioners*, 1970 Annual Meeting, IIA:719–893.
Plotkin, Irving H., 1979, Total Rate of Return and the Regulation of Insurance Profits, *Casualty Actuarial Society Call Paper Program*, 206–263.
Quirin, David G. and William R. Waters, 1975, Market Efficiency and the Cost of Capital: The Strange Case of Fire and Casualty Insurance Companies, *Journal of Finance* 30:427–445.
Seater, James J., 1980, Marginal Federal Personal and Corporate Income Tax Rates in the U.S., 1909–1975. Federal Reserve Bank of Philadelphia, Research Paper No. 57.
Smith, Barry D., 1980, An Analysis of Auto Liability Loss Reserves and Underwriting Results, *Journal of Risk and Insurance* 47:305–320.
Spellman, Lewis J., Witt, Robert C. and William F. Rentz, 1975, Investment Income and Non–Life Insurance Pricing, *Journal of Risk and Insurance* 42:567–577.
Theil, Henri, 1966, *Applied Economic Forecasting*, Amsterdam: North Holland.
Turner, Andrew L., 1987, Insurance in an Equilibrium Asset–Pricing Model, Chapter 4 in Cummins and Harrington (1987).
Urrutia, Jorge L., 1987a, Determination of Competitive Underwriting Profit Margins Using the Arbitrage Pricing Theory Model, *Journal of Insurance Issues and Practice* 10.
Urrutia, Jorge L., 1987b, A Financial Model for Pricing Insurance, *CPCU Journal* 40:153–157.

Venezian, Emelio C., 1988a, Effect of Serially Autocorrelated Profit Margins on the Solvency of Insurers, Unpublished Manuscript.

Venezian, Emelio C., 1983, Risk, Return, and Rate Regulation in Property and Liability Insurance, *Journal of Insurance Regulation* 2:98–124.

Venezian, Emelio C., 1988b, Statistical Theory and Empirical Estimation of the Difference Between Target and Observed Underwriting Profit Margins With Application to Private Passenger Automobile Insurance in the Commonwealth of Massachusetts, Unpublished Manuscript.

Walter, James E., 1981, Regulated Firms Under Uncertain Price Change: The Case of Property and Liability Insurance Companies, *Journal of Risk and Insurance* 46:5–21.

Webb, Bernard L., 1982, Investment Income in Insurance Ratemaking, *Journal of Insurance Regulation* 1:46–76.

Weiss, Mary, 1985, A Multivariate Analysis of Loss Reserving Estimates in Property–Liability Insurers, *Journal of Risk and Insurance* 52:199–221.

Witt, Robert C., 1973, Pricing Problems in Auto Insurance: An Econometric Analysis, *Journal of Risk and Insurance* 40:75–93.

Witt, Robert C., 1974, Pricing, Investment Income, and Underwriting Risk: A Stochastic View, *Journal of Risk and Insurance* 41:109–134.

Witt, Robert C. and J. Urrutia, 1983, Pricing Competition, Regulation, and Systematic Underwriting Risk in Automobile Insurance, *Geneva Papers on Risk and Insurance* 8:403–429.

Woll, Richard G., 1987, Insurance Profits: Keeping Score, *Casualty Actuarial Society Discussion Paper Program*, 506–507.

*Appendix*

The taxable portion of investment income is determined by subtracting from the total investment income the portion of investment income that is exempt from Federal income taxation. Based on the consolidated industry Annual Statement blanks calculated by A. M. Best Company, the following information was derived for the period 1983 through 1985:

| | **1983** | **1984** | **1985** |
|---|---|---|---|
| (A) Bond Interest Exempt from U.S. Tax | 6,449 | 6,436 | 6,347 |
| (B) Preferred Stock Dividends | | | |
| Unaffiliated | 997 | 996 | 875 |
| Affiliated | 13 | 11 | 13 |
| (C) Common Stock Dividends | | | |
| Unaffiliated | 1,330 | 1,403 | 1,322 |
| Affiliated | 1,361 | 736 | 1,057 |
| (D) Net Realized Capital Gains | 2,112 | 3,063 | 5,483 |
| (E) Total Investment Income (Not net of expenses) | 19,940 | 21,967 | 26,255 |
| (F) Tax Exempt Income (A) + .85[(B)+(C)] + .6(D) | 10,863 | 10,949 | 12,415 |
| (G) Taxable Investment Income (E) – (F) | 9,077 | 11,018 | 13,840 |
| (H) Tax Adjustment Parameter (G)/(E) | .455 | .502 | .527 |

## Table 1
## Actual versus Predicted Underwriting Profit Margins, 1926 through 1985

| (1) Year | (2) ACT | (3) TARG | (4) TRR1 | (5) TRR2 | (6) TRR3 | (7) TRR4 | (8) TRR5 | (9) TRR6 | (10) CAP1 | (11) CAP2 | (12) DCF | (13) O1R | (14) O1N | (15) O1C | (16) O2R | (17) O2N | (18) O2C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1926 | -0.60 | 4.77 | -0.84 | -3.63 | -0.45 | 1.49 | -1.30 | 1.88 | -0.84 | 0.86 | -2.58 | 0.18 | -1.84 | -0.57 | 2.01 | 0.03 | 0.83 |
| 1927 | 3.40 | 4.76 | -0.34 | -9.84 | 0.60 | 2.45 | -7.05 | 3.39 | -0.80 | 0.90 | -2.41 | 0.47 | -2.06 | -0.47 | 2.15 | -0.28 | 0.70 |
| 1928 | 4.80 | 4.77 | 0.78 | -5.11 | 1.75 | 4.27 | -1.62 | 5.24 | -0.85 | 0.85 | -2.46 | 0.37 | -2.49 | -0.70 | 1.81 | -0.83 | 0.23 |
| 1929 | 3.70 | 4.76 | 2.75 | 6.46 | 3.11 | 6.43 | 10.14 | 6.80 | -1.32 | 0.32 | -3.56 | -0.28 | -2.99 | -1.29 | 1.16 | -1.30 | -0.31 |
| 1930 | -1.90 | 4.76 | -1.50 | 16.89 | 0.84 | 2.06 | 20.46 | 4.40 | -0.68 | 1.02 | -1.83 | 0.39 | -2.21 | -0.58 | 1.93 | -0.51 | 0.47 |
| 1931 | -2.70 | 4.79 | -4.09 | 2.08 | -1.94 | -0.90 | 5.27 | 1.25 | -0.32 | 1.38 | -0.84 | 0.59 | -1.70 | -0.26 | 2.21 | 0.03 | 0.91 |
| 1932 | -4.90 | 4.82 | -3.76 | 6.60 | -3.07 | -0.57 | 9.79 | 0.12 | -0.30 | 1.40 | -0.71 | 0.90 | -1.60 | -0.03 | 2.55 | 0.14 | 1.11 |
| 1933 | 3.20 | 4.81 | -2.98 | -3.07 | -1.72 | 0.62 | 0.54 | 1.88 | -0.09 | 1.61 | -0.23 | 1.53 | -1.88 | 0.26 | 2.83 | -0.35 | 0.93 |
| 1934 | 4.70 | 4.78 | -2.60 | 4.19 | -0.82 | 1.21 | 8.00 | 2.99 | -0.05 | 1.65 | -0.12 | 1.78 | -2.09 | 0.34 | 2.85 | -0.68 | 0.74 |
| 1935 | 7.60 | 4.77 | -0.93 | -17.67 | 0.33 | 3.49 | -13.25 | 4.75 | -0.05 | 1.65 | -0.13 | 2.22 | -2.61 | 0.42 | 2.68 | -1.52 | 0.17 |
| 1936 | 6.70 | 4.74 | -0.58 | -17.00 | 0.69 | 4.15 | -12.27 | 5.43 | -0.05 | 1.65 | -0.12 | 3.69 | -2.76 | 1.29 | 3.76 | -1.85 | 0.40 |
| 1937 | 7.70 | 4.73 | -1.63 | 24.58 | 0.19 | 2.23 | 28.44 | 4.04 | -0.10 | 1.60 | -0.23 | 3.16 | -2.32 | 1.12 | 3.91 | -1.08 | 0.93 |
| 1938 | 6.50 | 4.73 | -0.88 | -8.75 | 0.34 | 3.48 | -4.39 | 4.70 | 0.01 | 1.71 | 0.01 | 3.99 | -2.44 | 1.59 | 4.43 | -1.38 | 0.95 |
| 1939 | 6.90 | 4.75 | -0.16 | -1.57 | 1.52 | 4.44 | 3.04 | 6.13 | -0.01 | 1.69 | -0.01 | 4.43 | -2.74 | 1.76 | 4.41 | -1.89 | 0.64 |
| 1940 | 6.50 | 4.76 | -0.43 | 3.28 | 1.55 | 3.82 | 7.53 | 5.80 | 0.00 | 1.70 | 0.00 | 5.77 | -2.45 | 2.71 | 6.13 | -1.50 | 1.57 |
| 1941 | 6.20 | 4.74 | -0.73 | 3.74 | 1.65 | 2.88 | 7.35 | 5.26 | -0.01 | 1.69 | -0.03 | 7.14 | -2.05 | 3.71 | 8.16 | -0.94 | 2.72 |
| 1942 | 3.90 | 4.67 | -0.08 | 1.73 | 0.68 | 3.33 | 5.14 | 4.10 | -0.05 | 1.65 | -0.09 | 8.60 | -1.68 | 4.77 | 10.39 | -0.42 | 3.92 |
| 1943 | 7.90 | 4.61 | 0.24 | -9.26 | 0.94 | 4.19 | -5.31 | 4.89 | -0.09 | 1.61 | -0.08 | 10.34 | -2.06 | 5.72 | 11.68 | -1.19 | 3.98 |
| 1944 | 5.60 | 4.65 | 0.42 | -7.97 | 0.87 | 4.41 | -3.97 | 4.87 | -0.09 | 1.61 | -0.07 | 10.54 | -2.11 | 5.82 | 11.82 | -1.29 | 3.98 |
| 1945 | 4.20 | 4.68 | 1.10 | -14.15 | 1.59 | 5.41 | -9.84 | 5.89 | -0.08 | 1.62 | -0.04 | 11.01 | -2.21 | 6.08 | 12.13 | -1.52 | 3.97 |
| 1946 | 1.20 | 4.73 | 0.18 | 5.61 | 1.04 | 3.30 | 8.74 | 4.16 | -0.10 | 1.60 | -0.15 | 7.95 | -1.71 | 4.35 | 9.65 | -0.41 | 3.63 |
| 1947 | 3.70 | 4.73 | -0.18 | 1.45 | 0.46 | 2.32 | 3.95 | 2.95 | -0.16 | 1.54 | -0.28 | 6.36 | -1.40 | 3.47 | 8.30 | 0.16 | 3.43 |
| 1948 | 8.80 | 4.73 | -0.10 | 0.72 | 0.02 | 2.21 | 3.02 | 2.32 | -0.26 | 1.44 | -0.49 | 6.11 | -1.38 | 3.32 | 8.08 | 0.22 | 3.38 |
| 1949 | 12.40 | 4.77 | 0.38 | -6.21 | 0.82 | 2.97 | -3.62 | 3.41 | -0.31 | 1.39 | -0.59 | 7.05 | -1.54 | 3.85 | 8.92 | -0.08 | 3.54 |
| 1950 | 7.00 | 4.79 | 0.32 | -6.43 | 1.13 | 3.03 | -3.71 | 3.85 | -0.32 | 1.38 | -0.49 | 7.77 | -1.43 | 4.34 | 9.87 | 0.03 | 3.98 |
| 1951 | 2.90 | 4.78 | 0.52 | -4.19 | 0.71 | 3.13 | -1.58 | 3.32 | -0.33 | 1.37 | -0.27 | 9.18 | -1.13 | 5.34 | 11.85 | 0.37 | 4.98 |
| 1952 | 5.60 | 4.78 | 0.72 | -3.27 | 0.40 | 3.28 | -0.70 | 2.97 | -0.36 | 1.34 | -0.25 | 9.50 | -1.07 | 5.56 | 12.27 | 0.41 | 5.18 |
| 1953 | 6.90 | 4.77 | 0.54 | 1.38 | -0.14 | 2.99 | 3.84 | 2.32 | -0.44 | 1.26 | -0.36 | 9.20 | -1.06 | 5.37 | 11.96 | 0.45 | 5.08 |
| 1954 | 6.40 | 4.77 | 0.58 | -16.44 | 0.10 | 3.68 | -13.34 | 3.20 | -0.15 | 1.55 | 0.01 | 11.44 | -1.35 | 6.67 | 14.25 | -0.16 | 5.63 |
| 1955 | 5.10 | 4.78 | 1.63 | -8.15 | 0.79 | 4.95 | -4.84 | 4.10 | -0.22 | 1.48 | 0.16 | 12.11 | -1.18 | 7.15 | 14.96 | -0.10 | 5.95 |
| 1956 | -0.50 | 4.77 | 2.09 | 0.21 | 1.12 | 5.34 | 3.46 | 4.37 | -0.45 | 1.25 | 0.22 | 10.44 | -1.07 | 6.15 | 13.30 | 0.27 | 5.51 |
| 1957 | -2.90 | 4.77 | 1.48 | 8.75 | 0.21 | 4.20 | 11.46 | 2.93 | -0.93 | 0.77 | -0.28 | 8.14 | -1.30 | 4.62 | 10.86 | 0.34 | 4.57 |
| 1958 | 0.05 | 4.78 | 1.00 | -16.51 | -0.53 | 4.16 | -13.36 | 2.63 | -0.39 | 1.31 | 0.07 | 10.26 | -1.30 | 5.95 | 13.05 | 0.06 | 5.28 |
| 1959 | 2.20 | 4.78 | 2.30 | -2.59 | -0.64 | 5.43 | 0.53 | 2.49 | -0.67 | 1.03 | 0.14 | 10.30 | -1.07 | 6.06 | 13.16 | 0.29 | 5.46 |
| 1960 | 1.60 | 4.78 | 1.45 | 0.85 | -0.99 | 4.44 | 3.84 | 1.99 | -0.82 | 0.88 | 0.00 | 9.33 | -1.34 | 5.35 | 12.11 | 0.14 | 4.95 |
| 1961 | 0.60 | 4.77 | 2.19 | -15.39 | -0.53 | 5.79 | -11.78 | 3.08 | -0.53 | 1.17 | 0.49 | 11.06 | -1.36 | 6.43 | 13.87 | -0.13 | 5.50 |
| 1962 | 1.00 | 4.77 | 1.80 | 9.59 | -0.41 | 4.99 | 12.78 | 2.77 | -0.84 | 0.86 | 0.20 | 9.86 | -1.37 | 5.67 | 12.67 | 0.03 | 5.11 |
| 1963 | -1.00 | 4.76 | 2.80 | -8.23 | 0.29 | 6.36 | -4.66 | 3.86 | -0.86 | 0.84 | 0.71 | 10.37 | -1.38 | 5.99 | 13.21 | -0.05 | 5.28 |
| 1964 | -1.90 | 4.75 | 3.28 | -4.97 | 0.33 | 6.88 | -1.37 | 3.93 | -1.11 | 0.59 | 0.49 | 9.53 | -1.60 | 5.38 | 12.19 | -0.21 | 4.77 |
| 1965 | -1.90 | 4.75 | 2.85 | -1.60 | 0.29 | 6.13 | 1.68 | 3.58 | -1.48 | 0.22 | -0.25 | 7.83 | -1.90 | 4.20 | 10.32 | -0.34 | 3.94 |
| 1966 | 1.90 | 4.73 | 1.95 | 11.45 | -0.76 | 4.57 | 14.06 | 1.85 | -2.24 | -0.54 | -1.34 | 5.92 | -2.32 | 2.85 | 8.37 | -0.56 | 3.03 |
| 1967 | 1.10 | 4.73 | 1.77 | -6.30 | -1.44 | 4.54 | -3.53 | 1.32 | -2.01 | -0.31 | -1.04 | 6.28 | -2.27 | 3.09 | 8.71 | -0.55 | 3.17 |
| 1968 | -0.02 | 4.72 | 2.50 | -4.09 | -1.65 | 5.26 | -1.33 | 1.11 | -2.53 | -0.83 | -0.16 | 6.85 | -2.39 | 3.40 | 9.55 | -0.66 | 3.44 |
| 1969 | -0.60 | 4.72 | 1.28 | 9.96 | -3.08 | 3.40 | 12.07 | -0.96 | -4.01 | -2.31 | -1.78 | 3.67 | -3.64 | 0.94 | 6.21 | -1.60 | 1.54 |
| 1970 | 0.70 | 4.72 | 0.90 | 1.75 | -4.04 | 2.98 | 3.82 | -1.97 | -4.05 | -2.35 | -2.73 | 2.81 | -3.84 | 0.33 | 5.24 | -1.77 | 1.05 |
| 1971 | 4.20 | 4.73 | -0.52 | -7.11 | -4.20 | 1.80 | -4.79 | -1.89 | -2.77 | -1.07 | -1.84 | 4.59 | -2.93 | 1.79 | 6.96 | -1.05 | 2.17 |
| 1972 | 4.60 | 4.73 | -0.50 | -10.12 | -4.22 | 2.07 | -7.55 | -1.65 | -2.52 | -0.82 | -1.26 | 5.60 | -2.81 | 2.46 | 7.97 | -1.04 | 2.58 |
| 1973 | 1.80 | 4.70 | -0.20 | 12.87 | -4.45 | 2.01 | 15.08 | -2.24 | -5.31 | -3.61 | -2.83 | 1.98 | -5.03 | -0.63 | 4.31 | -2.98 | -0.05 |
| 1974 | -5.00 | 4.68 | -2.89 | 16.22 | -6.57 | -1.36 | 17.75 | -5.03 | -7.58 | -5.88 | -5.18 | -2.19 | -7.40 | -4.13 | 0.46 | -4.72 | -2.64 |
| 1975 | -7.50 | 4.67 | -3.13 | -12.76 | -8.19 | -1.42 | -11.04 | -6.47 | -5.58 | -3.88 | -3.68 | -0.26 | -5.70 | -2.29 | 2.25 | -3.24 | -1.03 |
| 1976 | -2.00 | 4.67 | -3.04 | -10.66 | -8.05 | -1.27 | -8.88 | -6.28 | -4.78 | -3.08 | -3.31 | 0.81 | -4.92 | -1.33 | 3.24 | -2.61 | -0.26 |
| 1977 | 3.00 | 4.66 | -3.64 | -3.78 | -7.69 | -1.88 | -2.02 | -5.93 | -4.94 | -3.24 | -3.38 | 1.07 | -4.98 | -1.19 | 3.45 | -2.74 | -0.25 |
| 1978 | 3.40 | 4.64 | -2.96 | -5.29 | -8.30 | -1.09 | -3.41 | -6.43 | -7.50 | -5.80 | -3.98 | -0.66 | -7.19 | -3.09 | 1.64 | -4.92 | -2.28 |
| 1979 | 0.40 | 4.61 | -2.39 | -7.47 | -7.99 | -0.33 | -5.41 | -5.93 | -11.59 | -9.89 | -4.47 | -3.66 | -10.69 | -6.28 | -1.47 | -8.33 | -5.57 |
| 1980 | -2.40 | 4.60 | -1.54 | -8.93 | -10.82 | 0.81 | -6.58 | -8.47 | -13.32 | -11.62 | -3.45 | -4.65 | -12.28 | -7.49 | -2.58 | -9.95 | -6.99 |
| 1981 | -4.90 | 4.60 | -1.21 | 1.17 | -13.73 | 1.07 | 3.44 | -11.46 | -19.17 | -17.47 | -4.25 | -9.65 | -17.42 | -12.55 | -7.68 | -14.84 | -11.96 |
| 1982 | -8.70 | 4.63 | -4.80 | -8.44 | -16.97 | -2.35 | -5.98 | -14.51 | -14.27 | -12.57 | -3.62 | -6.12 | -13.67 | -8.94 | -4.15 | -11.29 | -8.42 |
| 1983 | -11.80 | 4.64 | -6.04 | -9.27 | -14.82 | -3.53 | -6.76 | -12.31 | -12.50 | -10.80 | -3.04 | -4.76 | -12.30 | -7.57 | -2.76 | -9.97 | -7.07 |
| 1984 | -19.00 | 4.64 | -7.02 | -6.00 | -16.67 | -4.84 | -3.82 | -14.49 | -14.67 | -12.97 | -4.35 | -7.76 | -14.57 | -10.30 | -5.55 | -11.92 | -9.36 |
| 1985 | -16.50 | 4.68 | -6.62 | -14.04 | -14.36 | -4.50 | -11.92 | -12.24 | -10.64 | -8.94 | -4.03 | -4.23 | -10.68 | -6.63 | -2.00 | -8.22 | -5.72 |

Note: The model abbreviations given in the column headings are defined in the following manner: ACT = actual return series; TARG = target underwriting profit margin; TRR1–TRR6 = various parameterizations of the total rate of return model; CAP1–CAP2 = various parameterizations of the capital asset pricing model; DCF = discounted cash flow model; O1R, O1N, O1C, O2R, O2N, O2C = various parameterizations of the option pricing model.

**Table 2**
**Summary Statistics of Underwriting Profit Margin Models, 1926–1985**

| Model | Mean | Standard Deviation | Skewness | Minimum | Maximum |
|---|---|---|---|---|---|
| ACT | 1.32 | 5.74 | –1.34 | –19.00 | 12.40 |
| TARG | 4.73 | 0.06 | –0.80 | 4.60 | 4.82 |
| TRR1 | –0.48 | 2.36 | –0.86 | –7.02 | 3.28 |
| TRR2 | –2.58 | 8.92 | 0.66 | –17.67 | 24.58 |
| TRR3 | –2.41 | 4.92 | –1.64 | –16.97 | 3.11 |
| TRR4 | 2.51 | 2.72 | –0.86 | –4.84 | 6.88 |
| TRR5 | 0.40 | 8.89 | 0.72 | –13.36 | 28.44 |
| TRR6 | 0.57 | 5.43 | –1.43 | –14.51 | 6.80 |
| CAP1 | –2.81 | 4.45 | –2.03 | –19.17 | 0.01 |
| CAP2 | –1.11 | 4.45 | –2.03 | –17.47 | 1.71 |
| DCF | –1.27 | 1.60 | –0.85 | –5.18 | 0.71 |
| O1R | 4.27 | 5.30 | –0.59 | –9.65 | 12.11 |
| O1N | –3.64 | 3.77 | –2.15 | –17.42 | –1.06 |
| O1C | 1.32 | 4.56 | –1.12 | –12.55 | 7.15 |
| O2R | 6.27 | 5.51 | –0.41 | –7.68 | 14.96 |
| O2N | –2.00 | 3.42 | –2.23 | –14.84 | 0.45 |
| O2C | 1.32 | 4.01 | –1.44 | –11.96 | 5.95 |

# Table 3
## Mean Square Error (%)

| Model | 1926-85 | 1926-55 | 1956-85 | 1926-45 | 1946-65 | 1966-85 | 1926-35 | 1936-45 | 1946-55 | 1956-65 | 1966-75 | 1976-85 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Time Period | | | | | | | | | | | |
| TARG | 0.44 | 0.13 | 0.76 | 0.14 | 0.19 | 1.00 | 0.24 | 0.04 | 0.11 | 0.28 | 0.34 | 1.86 |
| TRR1 | 0.27 | 0.35 | 0.19 | 0.33 | 0.25 | 0.24 | 0.20 | 0.45 | 0.40 | 0.09 | 0.09 | 0.40 |
| TRR2 | 1.25 | 1.65 | 0.86 | 1.73 | 1.15 | 0.87 | 1.48 | 1.99 | 1.47 | 0.83 | 1.22 | 0.52 |
| TRR3 | 0.28 | 0.27 | 0.29 | 0.22 | 0.22 | 0.42 | 0.14 | 0.29 | 0.40 | 0.04 | 0.24 | 0.60 |
| TRR4 | 0.23 | 0.12 | 0.34 | 0.09 | 0.27 | 0.34 | 0.09 | 0.09 | 0.18 | 0.36 | 0.13 | 0.54 |
| TRR5 | 1.06 | 1.28 | 0.84 | 1.41 | 0.87 | 0.90 | 1.42 | 1.39 | 1.02 | 0.73 | 1.28 | 0.52 |
| TRR6 | 0.16 | 0.11 | 0.22 | 0.07 | 0.17 | 0.26 | 0.11 | 0.03 | 0.18 | 0.16 | 0.10 | 0.39 |
| CAP1 | 0.35 | 0.36 | 0.33 | 0.30 | 0.25 | 0.48 | 0.19 | 0.41 | 0.49 | 0.02 | 0.23 | 0.74 |
| CAP2 | 0.24 | 0.22 | 0.26 | 0.19 | 0.17 | 0.35 | 0.15 | 0.22 | 0.30 | 0.03 | 0.12 | 0.58 |
| DCF | 0.32 | 0.39 | 0.26 | 0.33 | 0.26 | 0.37 | 0.26 | 0.41 | 0.49 | 0.03 | 0.14 | 0.60 |
| O1R | 0.35 | 0.17 | 0.53 | 0.14 | 0.62 | 0.29 | 0.14 | 0.14 | 0.22 | 1.01 | 0.17 | 0.40 |
| O1N | 0.43 | 0.56 | 0.30 | 0.53 | 0.33 | 0.43 | 0.32 | 0.74 | 0.63 | 0.03 | 0.23 | 0.64 |
| O1C | 0.21 | 0.16 | 0.27 | 0.17 | 0.25 | 0.22 | 0.18 | 0.15 | 0.13 | 0.36 | 0.06 | 0.38 |
| O2R | 0.56 | 0.26 | 0.86 | 0.17 | 1.03 | 0.47 | 0.15 | 0.19 | 0.42 | 1.64 | 0.41 | 0.64 |
| O2N | 0.31 | 0.42 | 0.20 | 0.41 | 0.23 | 0.29 | 0.23 | 0.59 | 0.44 | 0.02 | 0.12 | 0.46 |
| O2C | 0.21 | 0.17 | 0.26 | 0.19 | 0.22 | 0.23 | 0.18 | 0.21 | 0.13 | 0.30 | 0.08 | 0.38 |

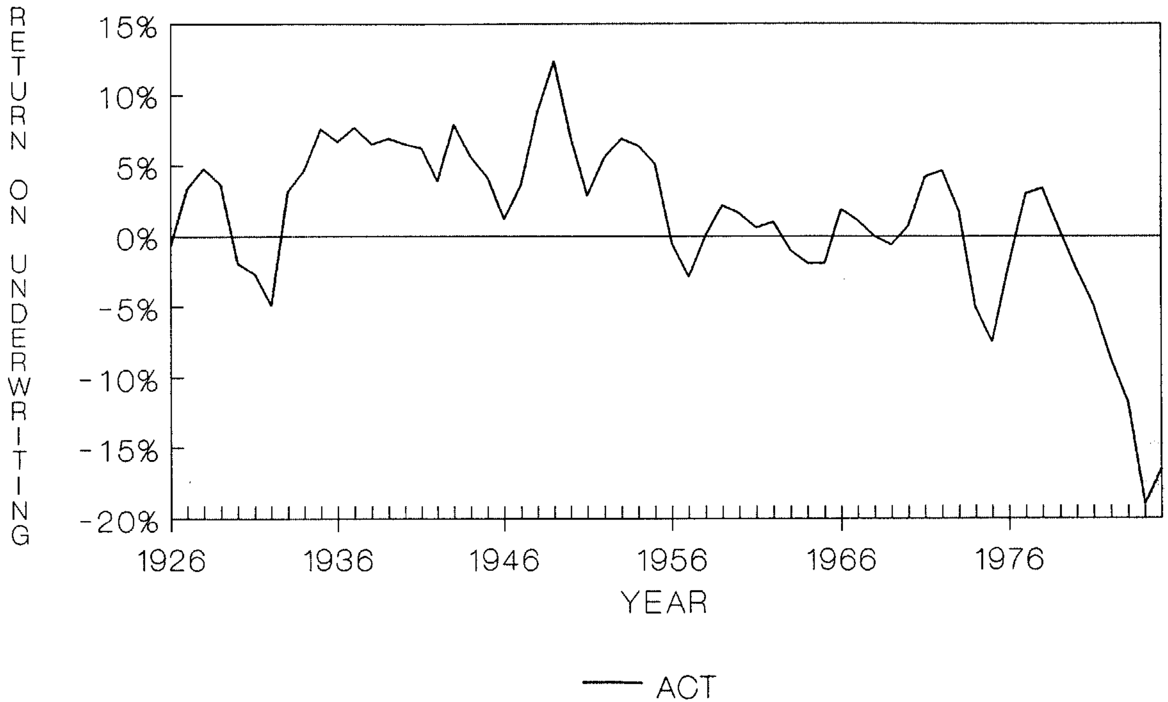
Note: MSE $= \frac{1}{n}\sum_{t=1}^{n}(P_t - A_t)^2$, where $A_t$ and $P_t$ are the actual and predicted period t values.

# Table 4
## Theil's U Statistic

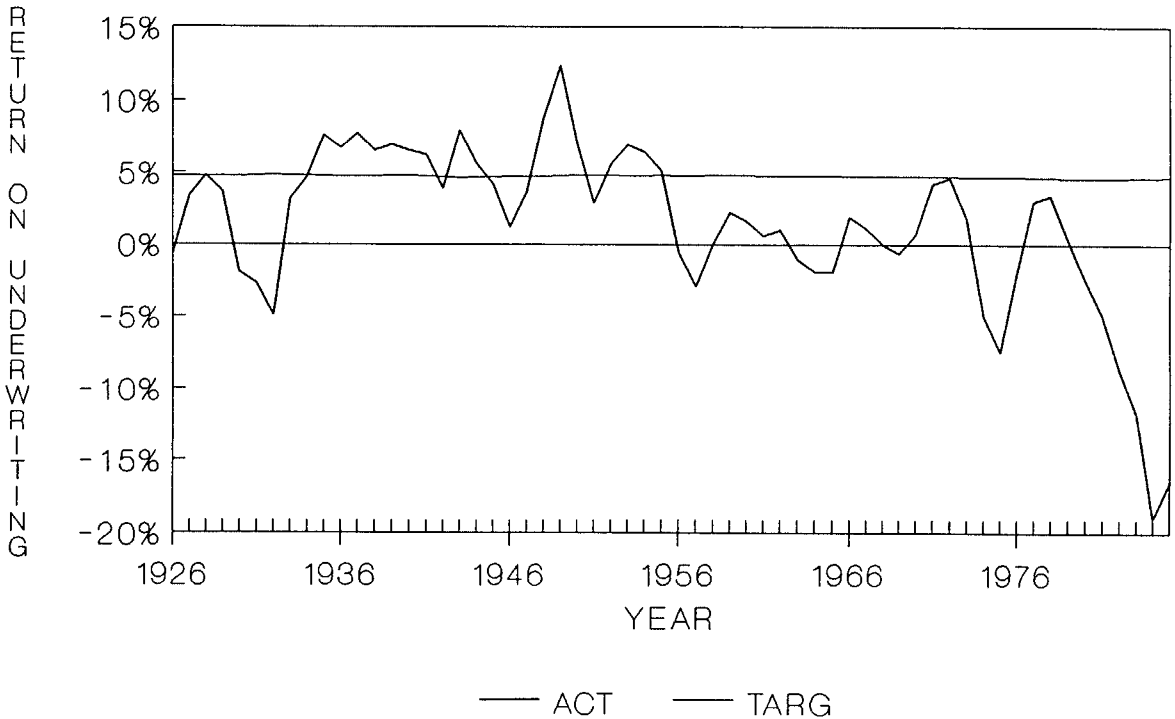
| Model | Time Period 1926-85 | 1926-55 | 1956-85 | 1926-45 | 1946-65 | 1966-85 | 1926-35 | 1936-45 | 1946-55 | 1956-65 | 1966-75 | 1976-85 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TARG | 113.01 | 60.91 | 146.81 | 69.22 | 89.84 | 139.23 | 116.91 | 30.82 | 48.25 | 329.83 | 162.05 | 135.66 |
| TRR1 | 88.42 | 101.25 | 73.90 | 106.33 | 101.95 | 68.46 | 106.98 | 106.05 | 94.34 | 191.29 | 81.76 | 66.34 |
| TRR2 | 189.83 | 219.50 | 155.75 | 245.45 | 219.89 | 129.86 | 291.61 | 222.53 | 181.00 | 567.29 | 307.61 | 75.86 |
| TRR3 | 90.28 | 89.72 | 90.81 | 86.39 | 95.99 | 89.68 | 88.65 | 85.39 | 93.82 | 128.14 | 135.04 | 81.16 |
| TRR4 | 81.68 | 58.92 | 98.85 | 54.88 | 106.69 | 80.87 | 70.54 | 46.50 | 63.72 | 373.91 | 100.58 | 77.65 |
| TRR5 | 174.81 | 193.44 | 154.63 | 221.26 | 191.73 | 132.09 | 286.40 | 186.14 | 150.55 | 532.85 | 314.58 | 76.18 |
| TRR6 | 68.40 | 56.17 | 78.46 | 49.68 | 84.93 | 68.87 | 79.31 | 28.55 | 63.51 | 250.98 | 88.99 | 65.49 |
| CAP1 | 99.93 | 103.00 | 96.86 | 102.34 | 103.43 | 96.88 | 105.98 | 100.73 | 103.84 | 96.04 | 132.81 | 90.59 |
| CAP2 | 82.30 | 80.96 | 83.59 | 80.31 | 83.97 | 82.62 | 92.17 | 74.60 | 81.78 | 115.87 | 96.91 | 80.37 |
| DCF | 95.93 | 106.40 | 84.57 | 107.86 | 104.31 | 84.12 | 122.09 | 101.08 | 104.50 | 101.07 | 102.37 | 81.17 |
| O1R | 100.29 | 69.89 | 122.76 | 69.40 | 161.34 | 74.96 | 88.05 | 59.55 | 70.51 | 628.74 | 116.28 | 67.01 |
| O1N | 111.45 | 128.28 | 92.26 | 135.72 | 117.91 | 91.63 | 135.19 | 135.95 | 118.09 | 114.70 | 132.39 | 84.21 |
| O1C | 78.08 | 67.46 | 87.14 | 76.16 | 101.76 | 65.57 | 102.02 | 61.69 | 54.33 | 374.38 | 70.04 | 64.90 |
| O2R | 126.83 | 86.33 | 156.38 | 77.04 | 208.38 | 95.66 | 91.63 | 69.78 | 96.93 | 800.07 | 177.98 | 77.03 |
| O2N | 94.49 | 110.84 | 75.62 | 119.14 | 98.65 | 74.98 | 115.12 | 120.83 | 96.70 | 97.82 | 95.14 | 71.64 |
| O2C | 78.55 | 71.18 | 85.08 | 82.05 | 95.35 | 67.16 | 100.99 | 72.33 | 54.20 | 342.08 | 79.39 | 65.22 |

**Note:** $U = \sqrt{\dfrac{MSE}{\sum_{t=1}^{n} A_t^2/n}}$. See Theil (1966) for more information concerning this statistic.

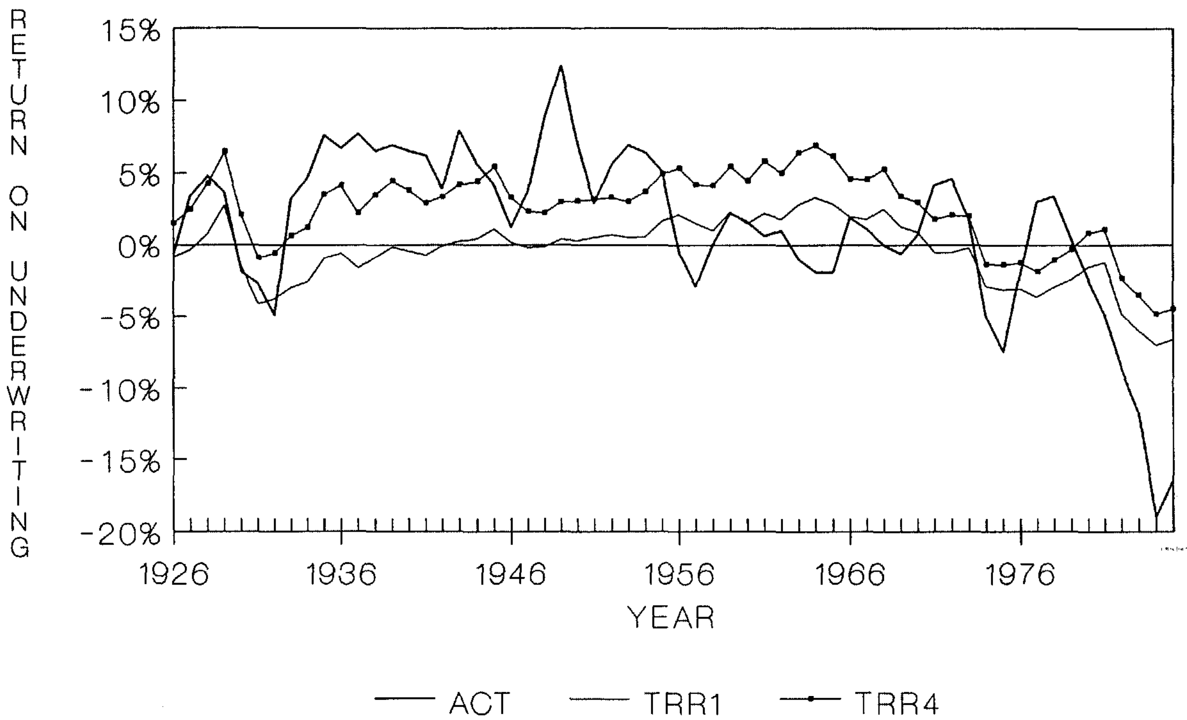
## Figure 1
### Actual Underwriting Profit Margin Series
### 1926-1985



# Figure 2
## Actual and Target Underwriting Profit Margin Series, 1926-1985

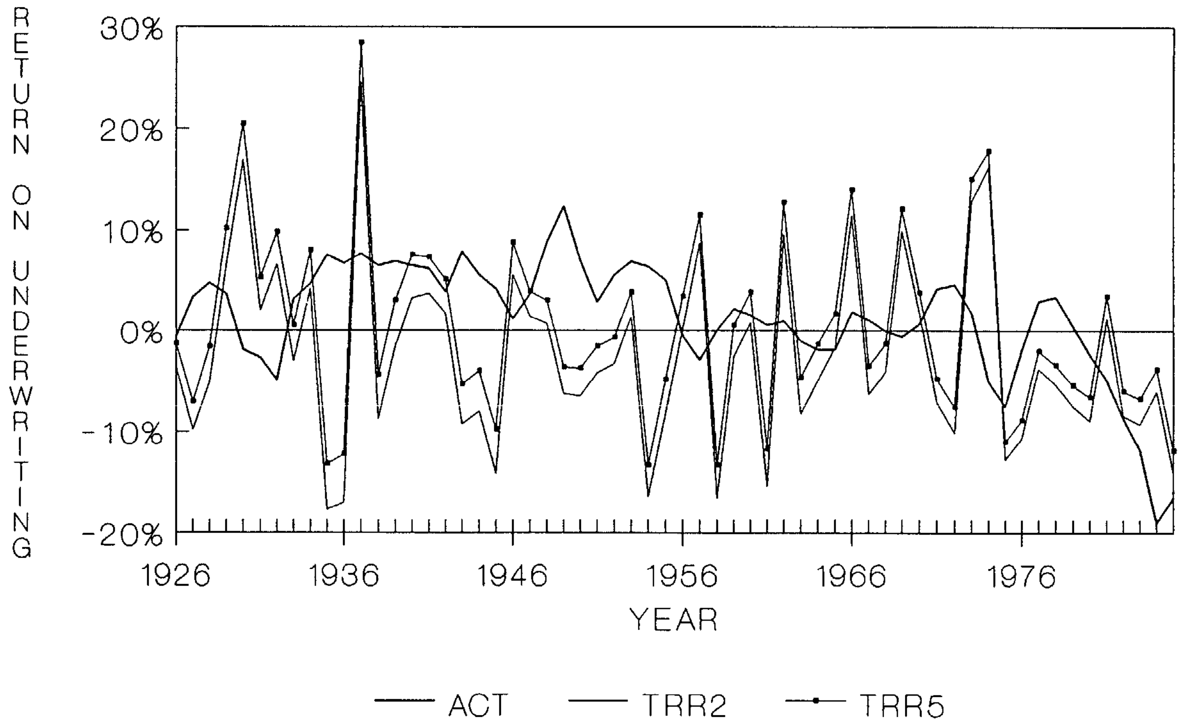


# Figure 3
## Actual and TRR1/TRR4 Underwriting Profit Margin Series, 1926-1985



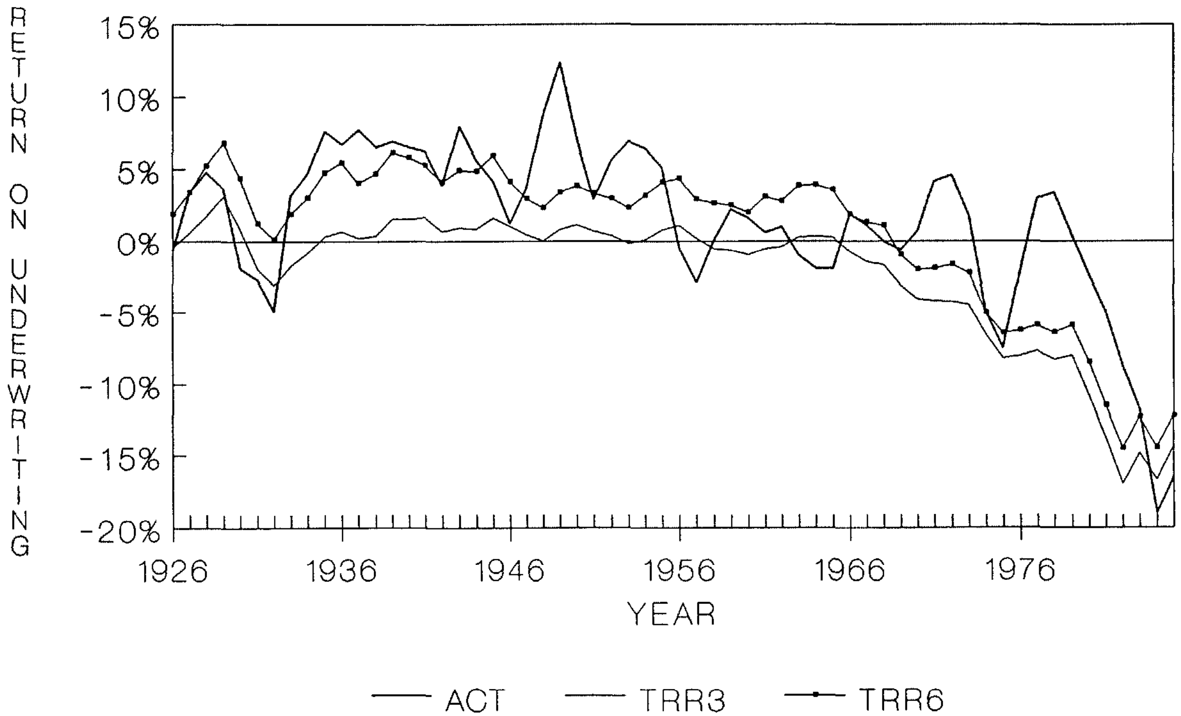

**TRR = Total Rate of Return**

# Figure 4
## Actual and TRR2/TRR5 Underwriting Profit Margin Series, 1926-1985



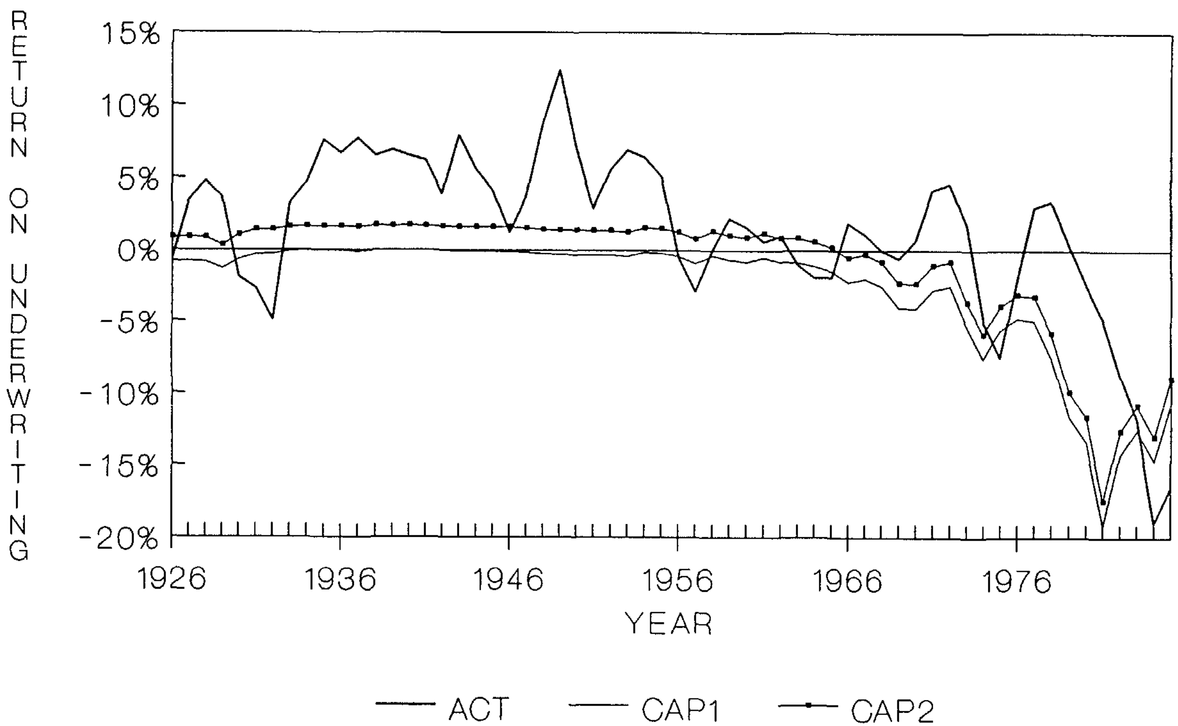

**TRR = Total Rate of Return**

## Figure 5
**Actual and TRR3/TRR6 Underwriting Profit Margin Series, 1926-1985**



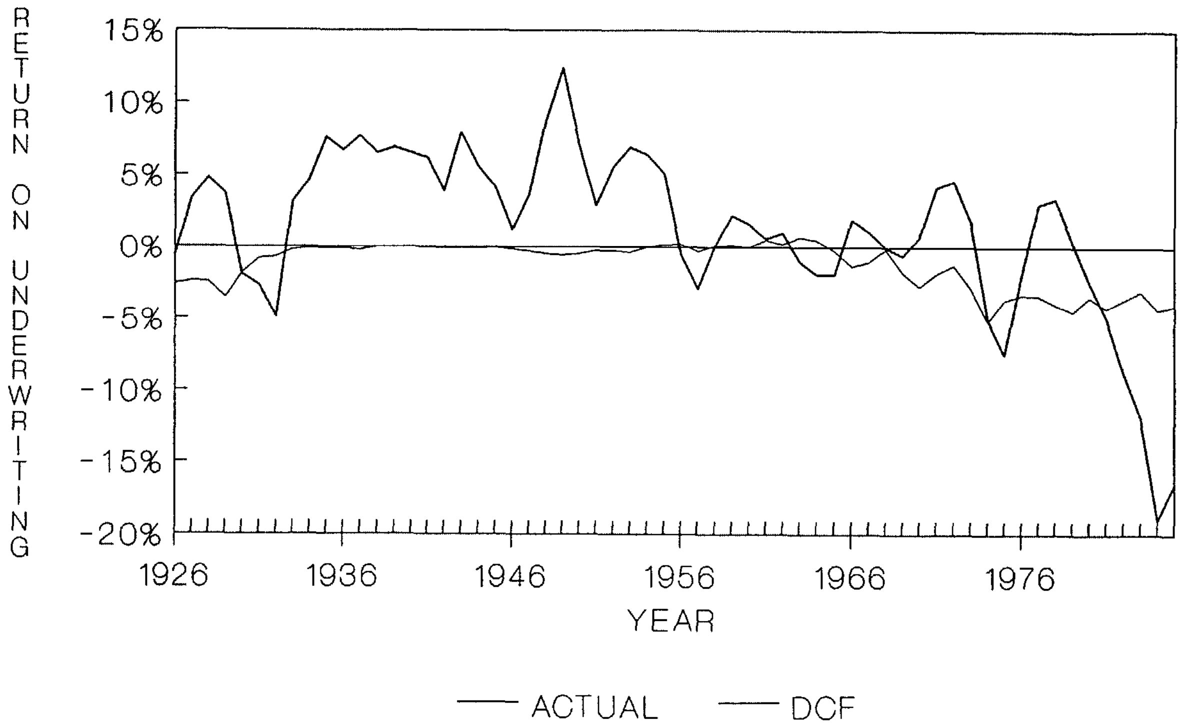

**TRR = Total Rate of Return**

# Figure 6
## Actual and CAPM Underwriting Profit Margin Series, 1926-1985



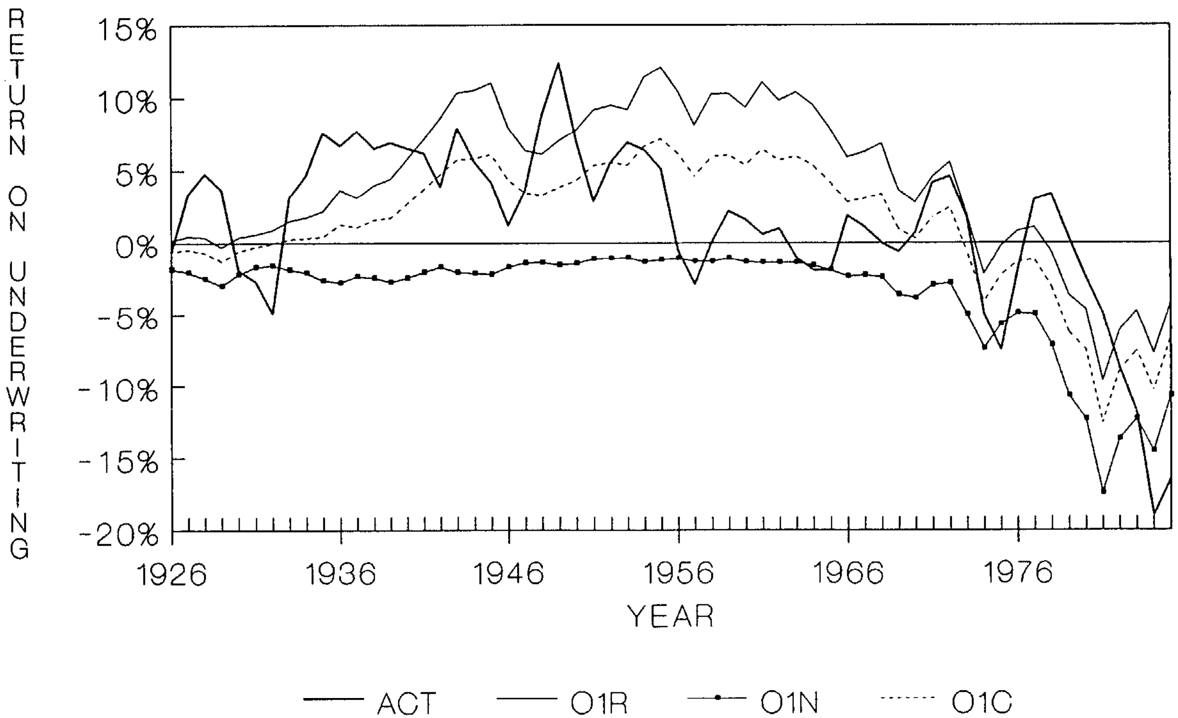

CAPM = Capital Asset Pricing Model

# Figure 7
## Actual and DCF Underwriting Profit Margin Series, 1926-1985



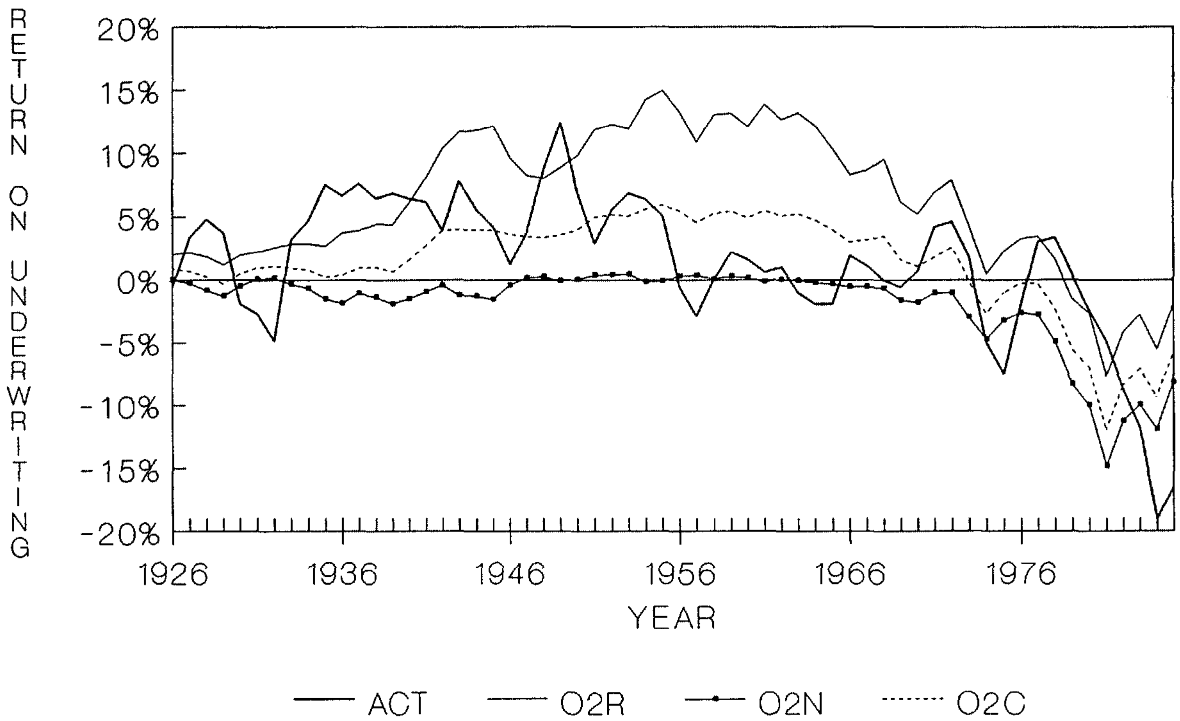

**DCF = Discounted Cash Flow**

# Figure 8
## Actual and OPM (O1) Underwriting Profit Margin Series, 1926-1985




OPM = Option Pricing Model

# Figure 9
## Actual and OPM (O2) Underwriting Profit Series, 1926-1985




OPM = Option Pricing Model

Figure 10
**Actual and Predicted Underwriting Profit Margin Summary Statistics, 1926-1985**

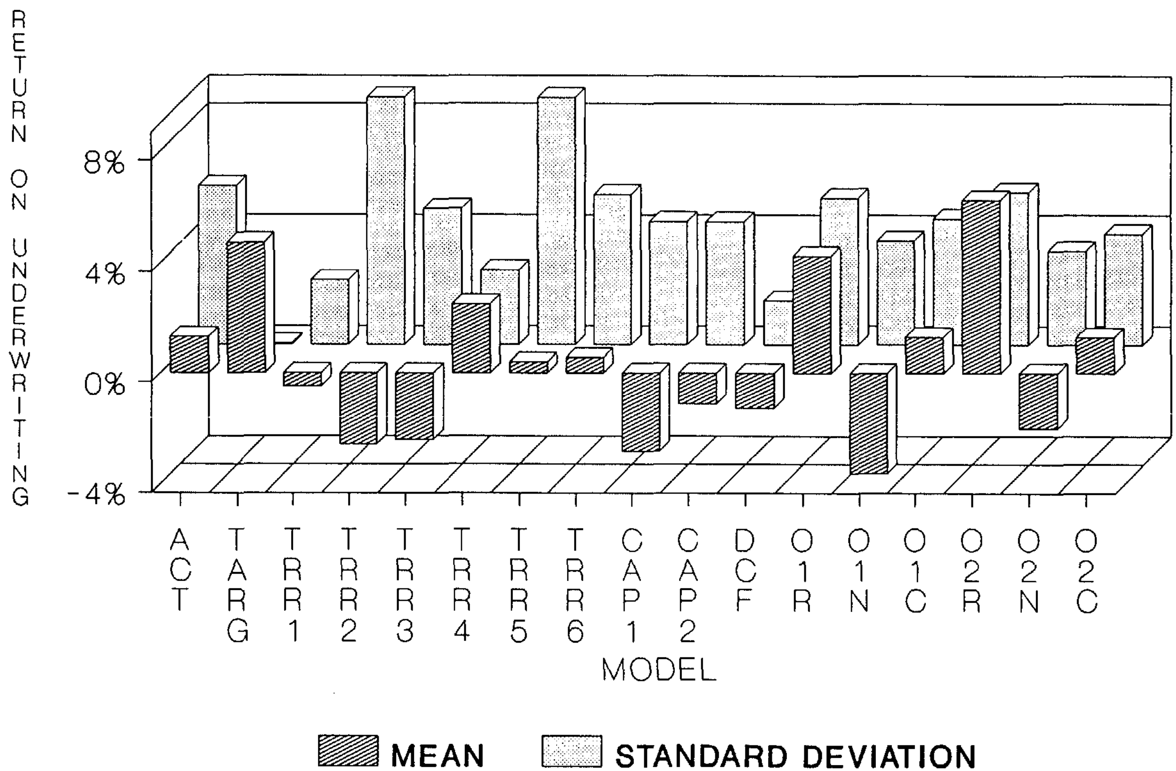


**Figure 11**
**Theil's U Statistics, 1926-1985**