TITLE: THE EFFECT OF INFLATION OF LOSSES AND PREMIUMS FOR PROPERTY-LIABILITY INSURERS

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INTRODUCTION

During the past few years, the topic of inflation has dominated economic news. Prices have been rising steadily, and interest rates have been gyrating in an effort to control inflation. Businesses, including property-liability insurers, are seeking new means of measuring and coping with the effects of inflation. The purpose of this paper is to determine how inflation affects the important element of losses, and consequently how premiums are influenced.

We start with a simple model of an individual policy, which is the basic unit comprising the property-liability insurer. The key element of the single-policy model is its stream of future claim payments. By applying inflationary adjustments to these anticipated amounts, the dynamic effects of inflation-rate changes through time are determined. These results allow us to calculate the effects of inflation upon incurred losses and loss reserves. This analysis shows that, when claim costs are related to prices at the time of settlement, incurred losses may rise faster than the inflation rate at the time policies are sold.

By introducing investment income directly into the pricing calculation, we further show how inflation in claim costs is related to interest rates and how the combination of these two elements influences the competitive price to charge for the policy. This development illustrates why insurers are, to a large extent, insulated against unanticipated changes in rates of inflation.

Finally, the theoretical results, which are somewhat at odds with traditional concepts, are summarized into specific areas of practical application.

INFLATION AND CLAIM COSTS

An Individual Policy Model

In order to develop a model relating inflation and claim costs, we begin with the smallest practical unit, the individual policy. Also, because claim costs will change through time we need a reference point for the claim payments being made for this hypothetical single policy. Thus, for a policy written at time t, denote the series of expected claim payments as:

Time	t	$t+1$	$t+2$	. . .	$t+n$
Claim Payment	0	a_1	a_2	. . .	a_n

We further define the pure premium for this policy as the sum of all claim payments:

(1) $P = a_1 + a_2 + . . . + a_n$,

where n is the maximum duration of claim payment. Note that the above formulation implies that payments are made at equal intervals, with the first payment occurring at the end of the first period. If we take the time intervals to be sufficiently small, this approach can approximate reality to any required degree (some of the expected payments may be zero).

At this point we can make some additional assumptions which are not essential, but will make the subsequent analysis easier to follow:

a) for a policy written at time t, the policy contract is effective for only exact time t, and covered losses can only occur at time t. A policy whose duration spans more than one time period can be considered as a series of separate policies, each with a different effective date and its own associated payment stream.

b) although at the time a policy is sold, the series of claim payments may not be precisely known due to random variation, the expected value of the payments is known. For purposes of discussion we can further assume that the actual payments will equal the expected payments.

c) claim frequency per policy is constant through time. We assume that the expected number of incurred claims per policy is independent of the accident date. This implies that the pure premium P is proportional to incurred claim severity alone.

d) the claim payments also include loss adjustment expenses. For the balance of this paper we will use the terms "claim" and "loss" interchangeably, in line with common usage.

As a numerical example of the individual policy model, suppose that at the end of the first year (time t+1) a payment of $10 is made, and subsequent payments of $20 and $10 are made at the end of the second and third year (t+2 and t+3), respectively. Then the total losses incurred at time t are $40, which, of course, equals the pure premium. We will extend this example throughout the remainder of our analysis.

Inflation and Claim Costs - Two Models

Having established a model of claim payments for a single policy, we now determine a relationship between claim payments at various points in time and corresponding costs of goods and services which determine the value of the claim payments. Let us define a claim cost index as follows:

Time	t	t+1	t+2	. . .	t+n
Claim Cost Index	I_0	I_1	I_2		I_n

We denote the kth payment for a claim incurred at time t+j as $a_k(j)$, with $a_k(0) = a_k$, for simplicity. Consider the case where the value of loss payment is related only to costs at the time when losses are incurred. For losses occurring at time t, and for losses occurring at time t+j, their respective kth payments are related as follows:

(2) $a_k(j)/a_k(0) = I_j/I_0, \qquad k = 1,2,\ldots, n.$

Hence, for losses incurred j periods apart, the value of their kth payments is proportional to the relative level of the claim cost indices at the time each loss occurred.

Returning to the earlier numerical example, suppose the claim cost index at time 0 is 100, and the index at time 3 is 120. For a policy inforce at time 3, all claim payments will be 120/100 times greater than the respective payments for a policy inforce at time 0. Thus the series of payments (10, 20, 10) originating from losses incurred at time 0, would rise to (12, 24, 12) if that same policy were in effect at time 3 instead.

Notice that this model preserves the proportional structure between each series of claim payments arising from different policy periods. In other words $a_1(i)/a_2(i) = a_1(j)/a_2(j)$, $a_2(i)/a_3(i) = a_2(j)/a_3(j)$, etc., for all values of i and j.

We can refer to the relationship in Equation (2) as the accident-date (AD) claim inflation model. If we denote the pure premium for a policy written at time t+j as $P(j)$, the AD assumption gives

$$(3)\quad P(j) = \sum_{k=1}^{n} a_k(j) \quad = (I_j/I_0) \sum_{k=1}^{n} a_k \quad = (I_j/I_0)P,$$

since $P(0) = P$. This relationship is implicit in standard ratemaking methodology, where the anticipated pure premium at the policy effective date (equal to accident date in the model) is proportional to current pure premium (see references [13] and [14]).

Examples of coverages for which the AD assumption would generally apply are Workers' Compensation indemnity (non-escalating) benefits and Automobile Physical Damage.

A different model, relating claim payments and inflation, determines the value of loss payments by <u>costs at the time when losses are paid</u>. In this case we define the kth payment for a claim incurred at time $t+j$ in terms of the original $a_k = a_k(0)$ and the relevant claim cost indices:

$$(4)\quad a_k(j)/a_k(0) = I_{k+j}/I_k, \qquad k = 1,2,\ldots, n.$$

Notice that the payment a_k is made at time $t+k$ and $a_k(j)$ is made at time $t+k+j$, so that this formula does in fact relate the value of claim payments to costs when they are paid. We shall call this relationship the <u>payment-date</u> (PD) claim inflation model.

From (2) and (4) we see that the PD assumption is equivalent to AD if and only if $I_{k+j}/I_k = I_j/I_0$ for all k. As shown later, this situation occurs only if the claim inflation rate is constant.

Examples of coverages for which the PD assumption is reasonable include Workers' Compensation medical benefits and certain types of General Liability, such as medical malpractice and products liability. Although technically these policy contracts specify indemnification according to costs at the accident date, the actual paid value can be largely determined by both social sentiment at the time of settlement (e.g. "pain and suffering") and by an upward assessment of the actual incurred costs (e.g. wage replacement) at the time of settlement.

A Numerical Example

The two models can be compared using our simple numerical example, along with an increasing rate of inflation:

Comparison of Accident-Date vs. Payment-Date Inflation Models

Time	0	1	2	3	4	5
Claim Cost Index	100	105	110	120	130	150

Accident-Date Model

Relative Indices (I_j/I_0):	1	2	3	4	5	Average
Incurred at Time 0	100/100	100/100	100/100			1.00
Time 1		105/100	105/100	105/100		1.05
Time 2			110/100	110/100	110/100	1.10

Payments for Losses:	1	2	3	4	5	Total
Incurred at Time 0	10	20	10			40
Time 1		10.5	21	10.5		42
Time 2			11	22	11	44

Payment-Date Model

Relative Indices (I_{k+j}/I_k):	1	2	3	4	5	Average
Incurred at Time 0	105/105	110/110	120/120			1.000
Time 1		110/105	120/110	130/120		1.078
Time 2			120/105	130/110	150/120	1.189

Payments for Losses:	1	2	3	4	5	Total
Incurred at Time 0	10	20	10			40.00
Time 1		10.48	21.82	10.83		43.13
Time 2			11.43	23.64	12.50	47.57

Notice how the progression of loss payments and total incurred losses (pure premium) varies according to the two different inflation models. For AD, the incurred loss per policy through time is dependent only on <u>one claim cost index</u>, at the time of accident. But for PD, the incurred loss per policy is a function of all future cost indices which extend over the duration for which claims will be paid. This property can cause incurred losses to grow faster than the claim inflation rate at the time losses occur. In the preceding example, under AD, incurred losses/policy increase by 5%, from 40 at time 0 to 42 at time 1. This matches the increase in the claim cost index. But under PD, the corresponding change is 7.8%, a weighted average of changes in future claim indices.

Also, this example shows that the PD model does not (except when the inflation rate is constant) preserve the proportional structure of the claim payment series. For instance, under PD, $a_2(1)/a_1(1) = 21.82/10.48 = 2.082$, while $a_2(2)/a_1(2) = 23.64/11.43 = 2.068$. With the AD model, the ratio of the second to first payments is always 2, regardless of the accident date.

A Unified Inflation Model

Because both the AD and PD models may be applicable according to the particular type of policy, it would be convenient to have a model which combines them. We can do this by using a simple exponential weighting factor:

$$(5) \quad a_k(j) = (I_j/I_0)^{1-\alpha}(I_{k+j}/I_k)^{\alpha}a_k, \qquad 0 \leq \alpha \leq 1; \qquad k = 1,2,\ldots,n.$$

Here if $\alpha=0$, the AD model applies, and if $\alpha=1$, the PD model holds. For intermediate values, the inflationary effect is a geometric average of the AD and PD effects.

Now assume that $I_{j+1}/I_j = 1+c$ for all j. In other words, the claim inflation rate has a constant value of c per period. Thus $I_j/I_0 = (1+c)^j = I_{k+j}/I_k$, and from (5) we get

(6) $a_k(j) = (1+c)^{j(1-\alpha)}(1+c)^{j\alpha}a_k = (1+c)^j a_k$, and

(7) $P(j) = \sum_{k=1}^{n} a_k(j) = P(1+c)^j$, which is independent of α.

Therefore, in order to determine incurred losses, it does not matter which model AD or PD applies, as long as inflation remains constant. However, as we shall see next, when the rate of inflation _varies_, there is a difference.

EFFECT OF CHANGES IN THE INFLATION RATE

We now examine the behavior of incurred losses as claim inflation changes. Assume that a policy is written at time t and that inflation has remained at a constant rate of c per period. If we are at time t, under PD the payments a_k are indexed to future claim costs. Thus the pure premium P is a function of forecasted or implicit claim indices I_k, where $k = 1,2,\ldots, n$. Under AD, the a_k are a function only of I_0, which is known at time t.

As shown previously, if inflation remains constant through time, then the set of payments $\{a_k(j)\}$ will increase uniformly by a factor of 1+c per period, for either the AD or PD models. This means that the implicit claim cost indices are

$$(8) \quad I_k = I_0(1+c)^k, \qquad k = 1,\ldots, n.$$

Now if inflation changes to a rate of c' per period, beginning at time t, there is a new set of implicit claim cost indices $I'_k = I_0(1+c')^k$. Since the a_k are proportional to the claim cost index at the time of payment (t+k), the new loss payments will be

$$(9) \quad a'_k = a_k(I'_k/I_k) = a_k[(1+c')/(1+c)]^k, \quad k = 1,\ldots, n.$$

Under AD, however, $a'_k = a_k$ since the payments are proportional to the claim cost index I_0, which remains unchanged (i.e. $I'_0 = I_0$). As in equation (5), we can again determine a unified AD/PD model for the effect of inflation changes upon future loss payments:

$$(10) \quad a'_k = (I'_0/I_0)^{1-\alpha}(I'_k/I_k)^{\alpha}a_k = [(1+c')/(1+c)]^{\alpha k}a_k,$$

where $0 \leq \alpha \leq 1$. Defining δ such that $1+\delta = [(1+c')/(1+c)]^{\alpha}$, we get

(11) $a'_k = (1+\delta)^k a_k$, for $k = 1, \ldots, n$.

Notice that if $\alpha = 0$, then $\delta = 0$ and $a'_k = a_k$. Thus the AD model applies fully. If $\alpha = 1$, then $1+\delta = [(1+c')/(1+c)]$, or δ equals the change in inflation rate ($\delta \simeq c'-c$). If $0 \leq \alpha \leq 1$, then δ is equivalent to a reduced change in the inflation rate, or $\delta \simeq \alpha(c'-c)$. A numerical example will help illustrate the preceding results.

Let c = 10% and c' = (1.1)(1.02) - 1 = 12.2%, i.e. a 2% increase in annual claim costs. Suppose that initially we have:

Time	0	1	2	3	Total
Claim Cost Index	100.0	110.0	121.0	133.1	
Claim Payment		10	20	10	40

When inflation increases by 2%, the claim payments depend upon α and the new claim cost indices:

Time	0	1	2	3	Total
New Claim Cost Index	100.0	112.2	125.9	141.2	
Claim Payment: $\alpha = 0$		10	20	10	40
$\alpha = .5$		10.10	20.40	10.30	40.80
$\alpha = 1$		10.20	20.81	10.61	41.62

Notice that when $\alpha = 1$ (full PD model) the claim payments increase in proportion to the change in the claim cost index. For example, the second payment becomes $a'_2 = 20(125.9/121.0) = 20.81$. Alternatively, equation (11) gives $a'_2 = 20(1.02)^2 = 20.81$.

The mitigating effect of the α weight is also illustrated. With $\alpha = .5$, $\delta = (1.122/1.1)^{.5} - 1 = 0.995\%$, or about one-half of the 2%

full PD effect.

Again, it is important to notice that if δ is greater than zero, then incurred losses will increase, if inflation increases, even though claims have already been incurred prior to the inflation change. We can quantify this relationship more explicitly. From (11) we derive a post-inflation-change pure premium (dropping the limits of summation for clarity):

(12) $P' = \sum a'_k = \sum (1+\delta)^k a_k.$

Assuming that the inflation change is positive ($\delta > 0$), we have $(1+\delta)^k \geq 1 + \delta k$ for $k \geq 1$. Hence

(13) $P' \geq \sum (1+\delta k) a_k = P + \delta \sum k a_k.$

The rate of change in the total incurred losses (pure premium) can be calculated as

(14) $[(P'-P)/P] \geq (P + [\delta \sum k a_k] - P)/[\sum a_k] = \delta(\sum k a_k)/(\sum a_k).$

Let $m = (\sum k a_k)/(\sum a_k)$, or the average duration of claim payment (weighted by amount of payment). Then (14) simplifies to

(15) $(\Delta P)/P \geq \delta m.$

This important result establishes the powerful leverage effect of changes in claim inflation. If there exists at least a partial PD inflation effect ($\delta > 0$) and the average claim duration is long enough, then a small underestimate of future inflation levels could be magnified into a large underestimate of incurred losses. The converse is also true: a drop in the inflation rate can produce a greater reduction to incurred losses. Because δ is so important we shall give it the name <u>claim cost accelerator</u>.

A Geometric Model

A further illustration of inflation leverage can be seen by using a specific relationship for the claim payment pattern:

(16) $a_k = aq^{k-1}$, for $k \geqslant 1$ and $0 < q < 1$.

Here we see that each successive payment is proportional to the preceding value by a constant factor q. This geometric model of claim payments has been used by others (e.g. McClenahan [11]) due to its simplicity and reasonableness (it fits empirical loss payment data well, especially at longer durations). The pure premium can be readily computed as a function of the two parameters a and q:

(17) $P = \sum_{k=1}^{\infty} a_k = a \sum_{k=1}^{\infty} q^{k-1} = a/(1-q)$.

The average duration can also be derived as

(18) $m = (\Sigma ka_k)/(\Sigma a_k) = 1/(1-q)$.

Thus $P = am$. When the claim inflation rate increases, equation (11) gives

$$
\begin{aligned}
(19) \quad P' = \Sigma a'_k &= \Sigma(1+\delta)^k a_k = \Sigma(1+\delta)^k aq^{k-1} \\
&= (1+\delta)\Sigma\, a(1+\delta)^{k-1}q^{k-1} = (1+\delta)\Sigma\, a(q')^{k-1} \\
&= (1+\delta)am'.
\end{aligned}
$$

Here $q' = (1+\delta)q$, indicating that under the new inflation rate c', the ratio of successive payments increases by a factor of $1+\delta$. The average duration of claim payment is now

$$(20) \quad m' = 1/(1-q') = 1/[1 - (1+\delta)q],$$

which is greater than m, for $\delta > 0$. The rate of increase in pure premium can be determined as

$$(21) \quad (\Delta P)/P = \delta m/[1 - (m-1)\delta] \geqslant \delta m.$$

As a numerical illustration, suppose c = 1% per month, δ = .2% per month and q = .9. Thus the average duration is 10 months when inflation is 10%, but rises to 1/(1 - .9(1.002)) = 10.183 months when inflation increases. Since P'/P = 1.002(10.18/10) = 1.0204, the pure premium increases by 2.04%, which in this case is less than the annual inflation rate increase of $(1.002)^{12} - 1 = 2.43\%$. The following table gives, for the geometric model, the percentage increase in pure premium as a function of various pre-inflation-change average durations and claim cost accelerators (δ):

Percentage Change in P for Various (δ,m) values

δ (annual %)	m (months) 1	3	6	12	24	60	120
0	0	0	0	0	0	0	0
.2	0.0	0.0	0.1	0.2	0.4	1.0	2.0
.5	0.0	0.1	0.2	0.5	1.0	2.6	5.2
1.0	0.1	0.3	0.5	1.0	2.0	5.2	11.0
2.0	0.2	0.5	1.0	2.0	4.1	11.0	24.7
5.0	0.4	1.2	2.5	5.1	10.8	32.2	94.9

Notice that if the product δm is small, (less than 12 in this table) then the approximation $(\Delta P)/P$ = m can be used. If δm is large, this approximation understates the incurred loss change.

We have already seen that if the claim cost accelerator is greater than zero, then a rise in the claim inflation rate (measured by the change in implicit claim cost indices) will increase losses on claims already incurred. We now determine what the inflationary effect is for policies written at various durations prior to the change in claim inflation rate.

Let F(i) be the number of policies inforce at time t-i. Then the loss reserve at time t for the policy period t-i is

$$(22)\quad R(i) = F(i) \sum_{k=i+1}^{n} a_k(-i)$$

Here the index i in $a_k(-i)$ is negative because the losses are incurred prior to time t. Our previous formulation had losses occurring at or after time t. For policies inforce at time t, we have R(0) = F(0)P, since all of the claim payments have yet to be made.

As an example, consider the loss reserve for policies inforce at t-2, with a four-period (n = 4) payment stream. For each policy, an amount of $a_1(-2)$ is paid at time (t-2) + 1 = t-1, and $a_2(-2)$ is paid at time t. Thus, at time t, the only remaining payments are $a_3(-2)$ and $a_4(-2)$, so the loss reserve for each policy is $a_3(-2) + a_4(-2)$, and the total reserve is $F(2)[a_3(-2) + a_4(-2)]$.

Now assume as before that the claim inflation rate is constant at c per period, until time t, when it changes to c'. In other words $I'_0 = I_0$ and $I'_k = (1+c')^k$. Because, for a loss incurred at t-i, the kth payment occurs at time t+k-i, from Equation (9) we get

$$(23)\quad a'_k(-i) = a_k(-i)[I'_{k-i}/I_{k-i}],$$

when the full PD model applies. Following our earlier development of the unified AD/PD model, we get a more general version of Equation (11):

$$(24)\quad a'_k(-i) = (1+\delta)^{k-i} a_k(-i), \text{ for } k > i.$$

This produces a post-inflation-change loss reserve for policy period t-i, equal to

$$(25)\quad R'(i) = F(i) \sum_{k=i+1}^{n} a'_k(-i) = F(i) \sum_{k=i+1}^{n} (1+\delta)^{k-i} a_k(-i).$$

The ratio of the new reserve to its pre-inflation-change value is therefore independent of the number of policies written:

$$(26)\quad R'(i)/R(i) = [\sum_{k=i+1}^{n} (1+\delta)^{k-i} a_k(-i)]/[\sum_{k=i+1}^{n} a_k(-i)].$$

If $\delta > 0$, then $R'(i) > R(i)$, so the loss reserve for prior policy periods will increase with the claim inflation rate.

Now let m_i be the average (dollar-weighted) duration of <u>remaining</u> claim payments for policy period t-i, evaluated at time t:

$$(27)\quad m_i = [\sum_{k=i+1}^{n} (k-i) a_k(-i)]/[\sum_{k=i+1}^{n} a_k(-i)].$$

Following the development of Equations (12) through (15), we arrive at a result similar to that of (15):

$$(28)\quad [\Delta R(i)]/R(i) \gtrsim \delta m_i.$$

Here m_i, the average remaining duration of claim payment, compares to m, which is the average total duration of claim payment. Notice that $m_0 = m$ and $m_i \leq m$, for $i > 0$.

A Numerical Illustration

Let the basic claim inflation rate be 10% per year with $a_1 = 10$, $a_2 = 20$ and $a_3 = 10$. Thus $a_1(-1) = 10/1.1$, $a_1(-2) = 10/1.21$, and so forth. Also, suppose that only one policy is written per year. For policies written at time t and prior, we get the following relevant payments and loss reserves:

t-i: Loss Date	Losses Paid at: t+1	t+2	t+3	R(i): Reserve at Time t	Total Incurred Loss	m_i: Average Remaining Duration
t	10	20	10	40	40	2.00
t-1	20/1.1	10/1.1		30/1.1	40/1.1	1.33
t-2	10/1.21			10/1.21	40/1.21	1.00
Total	36.45	29.09	10	75.54	109.42	

Notice that the total incurred losses increase by 10% per year, the same as the inflation rate. Also, the loss payments comprising the loss reserve form the familiar "triangle", the components of which usually are unknown when we are at time t.

Now suppose that starting at time t, claim inflation increases to 12.21% per year, and that the AD/PD weight α is 50%. Then $\delta = (1.1221/1.1)^{.5} - 1 = .01$. The new single policy payments and reserves (rounded to nearest .001) can be compared to their previous values:

Loss Date	Losses Paid at: t+1	t+2	t+3	R'(i)	R(i)	$[\Delta R(i)]/R(i)$	δm_i
t	10.100	20.402	10.303	40.805	40.000	.0201	.0200
t-1	18.364	9.274		27.638	27.273	.0134	.0133
t-2	8.347			8.347	8.264	.0100	.0100
Total	36.811	29.676	10.303	76.790	75.537	.0166	

Here the aggregate loss reserve increases by 1.66%. Notice also that in this example, the approximation of Equation (28) is quite close.

The Geometric Model

As with the incurred loss model, we can determine an exact parametric relationship between inflation changes and loss reserves if we specify the distribution of claim payments. Using the geometric model of (16), we define a more general relationship

$$(29)\quad a_k(-i) = a_i q^{k-1}\ ,\ k = 1,2,\ldots,$$

where a_i is analogous to the constant a in (16). The average remaining duration is computed from (27):

$$(30)\quad m_i = [\sum_{k=i+1}^{\infty}(k-i)a_i q^{k-1}]/[\sum_{k=i+1}^{\infty}a_i q^{k-1}]$$

$$= [q^i a_i \sum_{k=i+1}^{\infty}(k-i)q^{k-i-1}]/[q^i a_i \sum_{k=i+1}^{\infty}q^{k-i-1}]$$

$$= 1/(1-q) = m.$$

This remarkable property of the geometric distribution, where the average remaining duration equals the average total duration, is often called "lack of memory". The reserve at time t for policy period t-i can be found in terms of the average claim duration:

$$(31)\quad R(i) = F(i)\sum_{k=i+1}^{\infty}a_i q^{k-1} = F(i)a_i q^i \sum_{k=i+1}^{\infty}q^{k-i-1}$$

$$= F(i)q^i a_i/(1-q) = F(i)q^i a_i m.$$

When the rate of inflation changes, we get a new reserve value

$$(32)\quad R'(i) = F(i)\sum_{k=i+1}^{\infty}a'_k(-i) = F(i)\sum_{k=i+1}^{\infty}(1+\delta)^{k-i}a_i q^{k-1}$$

$$= F(i)(1+\delta)q^i a_i/(1-q') = F(i)(1+\delta)q^i a_i m',$$

where $q' = (1+\delta)q$. From (31) and (32), the rate of change in the loss reserve now becomes

$$(33) \quad [\Delta R(i)]/R(i) = [(1+\delta)m'/m]-1 = \delta m[1-(m-1)\delta] \geq \delta m.$$

This result is the same as (21), and is due to the fact that $m_i = m$ for all i. Consequently, since the inflationary effect of each policy-year component is identical, the <u>aggregate</u> <u>reserve</u> $R(0) + R(1) + \ldots + R(n)$ will change by the same percentage, regardless of the inforce levels (exposure) in prior periods. To the extent that the geometric model applies to loss payments, the approximation $[\Delta R(i)]/R(i) \simeq \delta m$ provides a good rule of thumb for assessing the impact of inflation changes on aggregate loss reserves.

For example, suppose that for a particular product line, the claim inflation rate has been at a constant 9% per year, the average claim duration is two years, and the accident-date inflationary effect (α) is 75%. If claim inflation increases to 13%, then $\delta \simeq .75(.13-.09) = 3\%$. Thus the aggregate loss reserve will increase by about $.03 \times 2 = 6\%$.

<u>An Empirical Method</u>

The analysis thus far has assumed that we know α and the implicit claim cost indices. Appendix I develops an empirical method for <u>estimating</u> these, and thus determining incurred losses from paid loss data.

INFLATION AND PRICE DETERMINATION

Basic Price Model

Using the dynamic claim payment model, we have determined the pure premium for an individual policy. This information is sufficient for traditional ratemaking, but lacks the key ingredient of investment income. We now expand our single policy model to include the effects of investment income.

According to basic financial principles (see [4] and [7]), the value of a project (i.e., the policy contract) is equal to the present value of all cash expenditures for the project. Let W be the premium written (and collected) at time t for this individual policy, and let r be the applicable interest rate. Assume that the loss payments include all expenses, or that the premium is net of them. Further assume that there are no income taxes. The present value of the policy is therefore

$$(34) \quad V = W - a_1(1+r)^{-1} - a_2(1+r)^{-2} - \ldots - a_n(1+r)^{-n}$$

$$= W - P_d,$$

where we define P_d as the discounted pure premium.

The amount of premium W to be charged will generally be a function of the uncertainty of the payment stream and competitive nature of the insurance market. References [1] and [3] discuss how such a market price can be established for a hypothetical policy, and what the relevant interest rate should be. (These are important topics, but outside the scope of this paper.) In this model we assume, as before, that the payment stream is certain.

This means that an insurer is indifferent to the immediate amount of cash equal to V, or to the stream of future cash flows represented by the right-hand side of (34). The underwriting gain for this policy is the premium minus losses, or

$$(35) \quad U = W - P$$

Denote the underwriting gain per unit of premium, or underwriting margin, as $u = U/W$. The total value V of this policy can be expressed as the sum of underwriting gain plus discounted investment income (J):

$$(36) \quad V = (W - P) + (P - P_d) = U + J.$$

Notice that the discounted investment income equals the difference between the pure premium and the discounted pure premium. Appendix II shows that J in fact equals the present value of future investment income from cash flows. Since V combines both underwriting and investment income in a single value, we shall term the quantity $v = V/W$ as the total profit margin. This is an important quantity which will appear repeatedly in the subsequent analysis.

Assume now that W is the competitive, or market price for this policy. In other words, all insurers know the expected future claim payments and have equivalent expenses. Also assume that the claim inflation and interest rates remain constant through time. What happens to the market price next period? The dynamics of the policy model can be readily established.

Let V_1, W_1, and $P_d(1)$ be the total value, premium and discounted pure premium for the policy written at time t+1. Each a_k will increase by the claim inflation rate c, with the interest rate r remaining the same. Thus

$$(37) \quad V_1 = W_1 - \sum_{k=1}^{n}(1+c)a_k(1+r)^{-k} = W_1 - (1+c)P_d = W_1 - P_d(1).$$

However, the value V_1 must increase over V by a factor of 1+r. Since the two policies are identical contracts, the cash equivalent V can be invested to yield V(1+r) at time t+1. This amount is therefore equal to the present value, or cash equivalent, of a new policy written at time t+1. Thus the real value of the period t+1 policy denominated in period t dollars, is $V_1/(1+r) = V$.

Now since $V_1 = (1+r)V$, equation (37) allows us to determine the market price W_1 of the policy at time t+1.

$$(38)\quad W_1 - (1+c)P_d = V_1 = (1+r)V = (1+r)vW.$$

Since, from (34), $P_d = (1-v)W$, we get

$$(39)\quad W_1 = [(1+c)(1-v) + (1+r)v]W = [1 + c + v(r-c)]W.$$

This result indicates that if $r \neq c$, then the market price will not increase at the same rate as the pure premium. This also means that the underwriting margin will change. The difference in underwriting margin is

$$(40)\quad \Delta u = u_1 - u = [1 - (P_1/W_1)] - [1 - (P/W)]$$

$$= (1-u)\frac{v(r-c)}{1+c+v(r-c)}.$$

Here we see that if $r = c$, the competitive underwriting margin remains constant. But if $r > c$, it increases, and it decreases if $r < c$.

To illustrate the preceding concepts, we return to our earlier numerical example with $a_1 = 10$, $a_2 = 20$, and $a_3 = 10$. Let $r = 10\%$, $c = 12\%$ and $W = 42$. We get $P = 40$ and $P_d = 33.13$. Hence $U = 2$, $u = 4.8\%$, $J = 6.87$, $V = 8.87$, and $v = 21.1\%$.

For this policy written at time t+1, we get $P_1 = 40\ (1.12) = 44.8$ and $P_d(1) = 33.13\ (1.12) = 37.11$. The new competitive premium becomes $W_1 = [1.12 + .211\ (.1 - .12)]42 = 1.116(42) = 46.86$. Thus $V_1 = 46.86 - 37.11 = 9.75 = 1.1V$. Here $W_1 > 1.1W$, because the claim inflation rate exceeds the interest rate. Also, we get $U_1 = 46.86 - 44.8 = 2.06$ and $u_1 = 2.06/46.86 = 4.4\%$, a decrease from the prior period value of 4.8%. The total profit margin also drops slightly to $v_1 = 20.8\%$.

Effect of Inflation Changes

We have determined earlier how changes in the claim inflation rate will affect incurred losses. The preceding section has shown how the policy price W will vary through time with a constant interest and claim inflation rate. To determine the effect of inflationary movements on the price, we need to know how inflation and interest rates are related. The classic Fisher effect (see[6] and [7]) states that under conditions of economic equilibrium ,

(41) $1+r = (1+c)(1+b)$,

where b is a positive constant equal to the real rate of interest. Most economists agree that over the long run this concept is valid (see[8] for details). However, for the short run we may have $b \leq 0$, i.e., a negative real interest rate.

When inflation changes, we define the new interest rate r' such that $1+r' = (1+\rho)(1+r)$. Here ρ represents the change in the interest rate. The new inflation rate c' is denoted as before. For the Fisher effect we can therefore determine the claim cost accelerator in terms of the change in interest rate. Equation (41) and the definition of δ give

(42) $1 + \delta = [(1+c')/(1+c)]^{\alpha} = \left[\frac{1+r'}{1+b}\Big/\frac{1+r}{1+b}\right]^{\alpha}$

$$= [(1+r)(1+\rho)/(1+r)]^{\alpha} = (1+\rho)^{\alpha}.$$

Thus $\delta \leqslant \rho$ if ρ is positive, and $\delta \geqslant \rho$ if ρ is negative, since $0 \leqslant \alpha \leqslant 1$. Note that this is true even if $c > r$.

Now let the interest rate change to r' starting at time t, with the claim inflation rate becoming c'. Denoting the new values of the relevant variables with primes, we have from (34):

(43) $V' = W' - P'_d = W' - \sum a'_k(1+r')^{-k}$

$$= W' - \sum a_k(1+\delta)^k[(1+r)(1+\rho)]^{-k}$$

$$= W' - \sum a_k(1+r)^{-k}(1+e)^{-k},$$

where $1+e = (1+\rho)/(1+\delta)$. In other words, e can be considered as the <u>change in the real interest rate</u> affecting the policy.

In a competitive market, the new total value V' of the policy will equal the pre-inflation-change value V. This is because the present value of the stream of interest payments from a dollar equals that dollar, regardless of the interest rate when the dollar is invested. Consequently, we can find the new market price W':

(44) $W' - P'_d = V$. Dividing by W, we get

(45) $W'/W = v + (P'_d/W) = v + (1-v)P'_d/P_d$,

since $W = P_d/(1-v)$. The rate of change in the market price is then

(46) $(\Delta W)/W = (W'/W) - 1 = -1 + v + (1-v)P'_d/P_d$

$$= (1-v)[(P'_d/P_d) - 1] = (1-v)[(\Delta P_d)/P_d].$$

Assuming the Fisher effect and that the full payment-date claim inflation model applies, we have $\alpha = 1$ and $\delta = \rho$. Thus the real policy interest rate change e is 0, giving $P'_d = P_d$ and hence $W' = W$. In other words, the effect of the claim inflation rate change is exactly offset by an equivalent change in the interest rate at which the new payments are discounted. In this case the insurer is perfectly insulated against unanticipated inflation.

To determine the effect of inflation rate changes when $e \neq 0$, we use the approximation $(1+e)^{-k} \gtrsim 1 - ek$, for $k \gtrsim 1$ and for all e. Hence

$$(47) \quad P'_d/P_d = [\sum a_k(1+r)^{-k}(1+e)^{-k}]/P_d \gtrsim (\sum[a_k(1+r)^{-k}(1-ek)])/P_d$$

$$\gtrsim [P_d - e\sum ka_k(1+r)^{-k}]/P_d$$

$$= 1 - e[\sum ka_k(1+r)^{-k}]/[\sum(a_k(1+r)^{-k}]$$

$$\gtrsim 1 - em_d,$$

where m_d is the average duration of discounted claim payments, analogous to the average undiscounted value m from equation (15). For any positive interest rate, $m_d < m$. From (46) we now have

$$(48) \quad (\Delta W)/W \gtrsim (1-v)[(1 - em_d) - 1] = -(1-v)em_d$$

$$\gtrsim (\delta - \rho)(1-v)m_d.$$

Thus if inflation increases and $\delta < \rho$, then $W' < W$. In other words, the market price of the policy actually decreases. This means that, once the policy is written, if inflation rises, the insurer will be better off. The price W actually charged will exceed the indicated amount W' because (discounted) investment income will rise faster than incurred claim costs.

If inflation decreases ($\rho < 0$), we get opposite results. Here $\Delta W > 0$ since $\rho < \delta$. Thus an unanticipated drop in the inflation rate will cause the insurer to sustain a greater loss of investment income than the gain from reduced claim costs.

We can also determine the effect of inflation changes on the underwriting margin. Since $[(\Delta W)/W] + 1 = W'/W$, we use (48) to get

$$(49) \quad \Delta u = (U'/W') - (U/W) = (P/W) - (P'/W')$$

$$\simeq (P/W) - [P(1+\delta m)]/[1+(\delta-\rho)(1-v)m_d]W,$$

since $P' \simeq (1+\delta m)P$, from (15). With the further approximation $1/(1+x) \simeq 1-x$, for small x, (49) reduces to

$$(50) \quad \Delta u \simeq (1-u)(1-[1+\delta m][1-(\delta-\rho)(1-v)m_d])$$

If the change in claim inflation rate equals the change in interest rate, or $\delta = \rho$, as might occur if the payment-date model fully applies, then this simplifies to

$$(51) \quad \Delta u \simeq -\rho m(1-u).$$

In other words, if an inflation increase is fully reflected in claim payments by the same amount as it raises interest rates, then (1) the competitive underwriting margin must drop, and (2) the decrease is proportional to the average claim duration.

This conclusion is similar to that obtained by others (e.g., D'Arcy[2]) using a calendar-year approach equating real return on surplus to levered return on assets and underwriting margin. As shown in Appendix III, this calendar approach will give $\Delta u \simeq -\rho(R/W)$, where R is the aggregate loss and unearned premium reserve. The reserve/premium ratio R/W will be proportional to m for a stable (no-growth) company. However, this result does not consider the possibility that if $\delta \neq \rho$ the ratio R'/W' (after the inflation change) will not equal R/W. Also, the reserve/premium ratio is a function of the prior inforce growth and would therefore not be a fundamental characteristic of the individual policy. The approach taken in our model has the advantage that it is totally prospective, being independent of prior exposures (unlike an ongoing calendar-period model).

On the other hand, if the accident-date claim inflation model applies fully, we have $\delta = 0$, and (50) gives

$$(52) \quad \Delta u \simeq -\rho(1-v)m_d(1-u).$$

In this case, the reduction in underwriting margin is similar to when $\delta = \rho$, but not as great, since $m_d < m$ and $(1-v) < 1$. The reason why the change in underwriting margin is insensitive to the mode of claim payment inflation can be seen in the following: if $\delta = 0$ (accident-date model), then P' = P, and W' < W. Thus the incurred losses remains the same, but the indicated premium decreases. On the other hand, if $\delta = \rho$ (payment-date model), then P' > P, while W' = W. Thus the incurred losses increase, but the indicated premium remains constant. The losses/premium ratio becomes nearly the same in each instance, creating a similar change in underwriting margin.

Although the underwriting margin will shift with inflation, the total profit margin v is more stable. When $\rho = \delta$, which is a reasonable long-run situation, we have W' = W. Since V' = V, the total profit per unit of premium also remains unchanged at v' = v. If $\delta = 0$, however, we have $v' = (W/W')v > v$, since the market price W' drops.

Using our earlier numerical example, we had $u = .048$, $v = .211$ and $m = 2$. With $r = 10\%$, we get $m_d = 1.952$. Suppose that $\rho = 2\%$. If δ also is 2%, we get $\Delta u \cong -.02(2)(.952) = -.038$. So $u' = .010$. But if $\delta = 0$, we have $\Delta u \cong -.02(.789)(1.952)(.952) = -.029$, so $u' \cong .019$. This example also shows that the underwriting margin shift is not sensitive to the accident or payment-date mode of claim inflation.

The Geometric Model

Again we can apply the geometric claim payment model to illustrate the preceding results concisely, without resorting to approximation. Letting $a_k = aq^{k-1}$ in equation (43), we have

(53) $P'_d = \sum_{k=1}^{\infty} aq^{k-1}(1+r)^{-k}(1+\rho)^{-k}(1+\delta)^k$, which reduces to

(54) $P'_d = [a(1+\delta)]/[(1+r)(1+\rho) - (1+\delta)q] = a(1+\delta)/(1+r'-q')$.

Without the inflation change, $\delta = \rho = 0$ and thus

(55) $P_d = a/(1+r-q)$.

The average duration of discounted claim payments can be found in a similar manner:

(56) $m_d = (1+r)/(1+r-q)$, and

$m'_d = (1+r')/(1+r'-q')$.

The change to the indicated price can also be determined. From (46) and some algebraic manipulation, we get

(57) $(\Delta W)/W = [(1-v)(\delta-\rho)(1+r)]/(1+r'-q')$

For $\delta = 0$, this produces

$$(58)\quad (\Delta W)/W = -\rho(1+r)(1-v)/[(1+r)(1+\rho) - q] .$$

The change in underwriting margin can also be explicitly determined as a function of the average claim payment duration:

$$(59)\quad \Delta u = -\rho(1-u)/[1-(1+\rho)q] = -\frac{\rho m(1-u)}{1-\rho(m-1)} \leq -\rho m(1-u),$$

when $\delta = \rho$. For $\delta = 0$, we get a result in terms of the average discounted claim payment duration:

$$(60)\quad \Delta u = -\rho\frac{(1-v)(1+r)(1-u)}{(1+r)(1+\rho v) - q} = -\rho\frac{(1-v)m_d(1-u)}{1+\rho v m_d}$$

$$\leq -\rho(1-v)m_d(1-u).$$

APPLICATIONS

The preceding sections have discussed how inflation influences losses and premiums for a property-liability insurer. The results have important consequences for various areas of application, as outlined briefly in the following:

Loss Reserve Methods

Because of the way in which claim payments may be affected by future inflation levels, a loss reserve method based upon historical loss payments should be preferred over one which uses case estimates (i.e., claim adjuster evaluations). The payment method would be most important for long-tail coverages, where costs at the time of settlement may heavily influence the claim amount, and where it would be difficult for various individual claim adjusters to predict future inflation levels consistently. However, by modifying the projected series of aggregate claim payments using the estimated claim cost accelerator (δ), future inflation levels can be directly applied to assessment of loss reserves.

Appendix I outlines a method for finding the claim inflation rate from loss payment data, and for estimating the α weight for accident vs. payment-date inflation sensitivity. Note that it would not be possible to determine the relative AD/PD inflation effects using a case incurred loss method.

Because loss reserves can depend (the degree varies by line of business) upon future levels of inflation, the claim inflation rate must be predicted for applicable types of coverage. Note that standard methods, both paid and incurred-loss, must implicitly forecast future inflation. By explicitly forecasting future inflation levels and applying them to reserve calculation, we are able to provide reserve estimates which are conditional upon economic forecasts. Thus the dependency of reserve estimates upon economic conditions can be directly shown. Consequently the subjective element of implicit inflation assumptions, which are inherent in traditional reserve methods, can be quantified. To the extent that future inflation can be predicted, this explicit method will improve the accuracy of loss reserve estimates. And when inflation cannot be predicted, the method will isolate a major source of error in the reserve-setting process. This error source can be further incorporated into reserve confidence interval methods.

Pricing Methods

For price-setting in an inflationary environment, the loss payment approach will also be preferred to a incurred loss method. However, the traditional ratemaking methods suffer severely when inflation is high, because of the Fisher effect. For many lines of business, when investment income is ignored, the difference between underwriting gain and total profits is so great that a price based upon an anticipated underwriting gain becomes ficticious. Competition will drive the price down to the point where total profits are equivalent to those from other lines. Consequently, it makes sense to explicitly include investment income in price calculations, as done in life insurance.

Once the inflation-adjusted anticipated payment stream is calculated for a particular type of policy, then the appropriate interest rate can be applied to produce the discounted payments (here we would include all expenses and deferred premium collections, in addition to the losses). The price is now determined by selecting the desired total profit margin.

The profit margin typically includes a charge for the element of risk. The following section discusses the risk which is due to the uncertainty of inflation, apart from the uncertainty of pure loss fluctuations.

<u>Risk of Inflation</u>

We would normally expect the Fisher effect to apply to property-liability insurers, particularly in the long run. When claim inflation and interest rates maintain a stable relationship, there is a mutually offsetting effect as inflation rises. However, suppose that claim inflation goes up faster than the change in the interest rate. An extreme case of this situation might occur as a result of so-called "social" inflation, where claim payments would increase as a result of <u>non-economic</u> influences (e.g., large verdicts in liability suits). Here $\delta > \rho$, resulting in a premium inadequacy which can be substantial for large average-duration liability coverages such as automobile bodily injury.

Normally, however, we expect that $\delta \le \rho$, particularly if there is at least a partial accident-date claim inflation effect. Consequently, insurers are <u>insulated</u> to a large extent against an increase in unanticipated inflation. In fact, an unanticipated inflation increase can actually <u>help</u> insurers in this instance, since the additional (discounted) investment income will exceed the increase in claim costs.

In general, the pricing decision must be based upon an <u>estimate</u> of the gap between interest and payment-date claim inflation. The risk to the insurer is that, when inflation changes, this gap may prove to be other than predicted. Hence an additional element should be included in the total profit margin to compensate for this risk. Note that if investment income is not included in the price, then traditional pricing would have to include a risk charge for the full difference between the anticipated and actual δ. Since δ and ρ should be strongly correlated, when investment income is included in pricing the variation in $\rho - \delta$ will be less than the variation in δ alone -- hence less of a risk charge.

Inflation Accounting

For a typical multi-line property-liability insurance company, we may expect the payment-date claim inflation effect to be present to some degree. This means that when inflation is rising, loss reserves can expand at an even greater rate. Assets, on the other hand, will tend to maintain their nominal value. Bonds valued at amortized cost will be unaffected by inflation/interest movements, and long-run stock values will trend with inflation (see [8]). Thus at a given evaluation date, an unanticipated rise in the inflation rate will raise loss reserves, with the asset side of the balance sheet remaining constant. Hence under statutory accounting, an unanticipated rise in inflation will reduce surplus.

However, we have seen how the Fisher effect will tend to maintain a stable discounted pure premium (P'_d) when inflation increases. This also means that if loss reserves are discounted, their value will not suffer the full effects of unanticipated inflation. Consequently, insurers may obtain more stable estimate of their true liabilities if loss reserves are discounted at an interest rate which parallels market interest rates. This approach could be adopted at least for internal accounting purposes.

Profitability Measures

Because increases to the claim inflation rate will cause the underwriting margin to drop, trade or combined ratios do not serve as useful profitability measures when inflation levels become high. This is particularly true for long-duration lines such as Worker's Compensation or General Liability. By discounting loss payments, the underwriting margin will approximate the total profit margin, which is much less sensitive to inflationary movements. This modified measure of underwriting profit directly includes the effect of investment income and has the further advantage that it produces combined ratios which show an "underwriting" profit. Hence a comparison of results between various property and liability lines is more valid.

SUMMARY

The analysis developed in this paper has demonstrated some important relationships between inflation, losses and premiums. In the traditional sense, rising inflation can appear to be detrimental to property-liability insurance operations, due to the accelerating effect of future inflation on claim payments. But if we bring investment income into the picture, we find that inflationary trends may not be so damaging. If inflation rates become high enough in the future, however, the insurance industry must consider modifying its current accounting measures of profitability.

The results presented here are, of course, dependent upon the validity of the assumptions used, particularly the treatment of accident vs. payment-date claim inflation sensitivity. Also the choice of an applicable interest rate is important. Thus, in order to apply the model results developed here, much data analysis must be done. The effort should be worthwhile.

APPENDIX I

Estimating Incurred Losses Under Inflation Using Payment Data

Using claim payment data, it is possible to estimate the relative effect, α, between accident-date and payment-date claim inflation. From this, we may determine incurred loss values based upon predicted future claim cost indices. The following method is one of several which can be derived from the model assumptions in the text.

For a claim incurred at time t+j, define $b_k(j)$ as the payment made at time t+k+j. (This is analogous to the definition of the payment per policy $a_k(j)$.) For a given period t+j, the sum of these $b_k(j)$ over all k will equal the incurred claim severity. Thus it will be useful to think of these payments as partial severities.

To avoid extra notation, we illustrate the α-estimating procedure with a brief example. Suppose there are only three periods (n = 3) over which payments are made, and there are also three periods of actual payment data. We wish to predict the remaining partial severities for losses already incurred. The following table summarizes the available data and the remaining unknown partial severities:

	Claim Inflation Index					
Accident Period	I_0	I_1	I_2	I_3	$\hat{I}_4$	$\hat{I}_5$
t		$b_1(0)$	$b_2(0)$	$b_3(0)$		
t+1			$b_1(1)$	$b_2(1)$	$\hat{b}_3(1)$	
t+2				$b_1(2)$	$\hat{b}_2(2)$	$\hat{b}_3(2)$

Here, the estimated values are indicated by carats.

The model in the text uses a <u>true</u> claim cost index, where claim payments are perfectly correlated with the appropriate indices. Since in practice the true index might not be obtainable, we need to allow for systematic errors in applying an empirical claim cost index. Denoting the true claim cost indices by I^*_k, this adjustment is made by $I_{k+1}/I_k = \beta(I^*_{k+1}/I^*_k)$.

If $\beta = 1$, then the true claim cost index will equal the empirical index, and severity growth will tend to equal the α-weighted change in the index. But if $\beta < 1$, then actual claim severity growth will <u>exceed</u> the weighted change in the empirical index; the converse is true if $\beta > 1$.

Assuming that the claim frequency per policy is constant, we have $b_k(j)/b_k(i) = a_k(j)/a_k(i)$ for all i and j. From equation (5) in the text, therefore, we get the following relationships:

$$\text{(A)}\quad b_1(1)/b_1(0) \;=\; \beta(I_1/I_0)^{1-\alpha}\;(I_2/I_1)^{\alpha}$$

$$\text{(B)}\quad b_1(2)/b_1(1) \;=\; \beta(I_2/I_1)^{1-\alpha}\;(I_3/I_2)^{\alpha}$$

$$\text{(C)}\quad b_2(1)/b_2(0) \;=\; \beta(I_1/I_0)^{1-\alpha}\;(I_3/I_2)^{\alpha}$$

Here we have formed the ratios of <u>all successive partial severities having the same duration</u>. These are then transformed into linear equations by taking logarithms. Equation (A) becomes, for example:

$$\text{(A')}\quad \alpha[\ln(I_2/I_1) - \ln(I_1/I_0)] + \beta' \;=\; \ln[b_1(1)/b_1(0)] - \ln(I_1/I_0),$$

where $\beta' = \ln(\beta)$. Now the set of equations (A'), (B') and (C') can be solved by least-squares regression to yield the estimates $\hat{\alpha}$ and $\hat{\beta}$ for the unknown parameters α and β.

Using these estimates, along with forecasts of the future claim cost indices I_4 and I_5, we can readily obtain the remaining partial severities. For example,

$$\hat{b}_3(1) = \hat{\beta}(I_1/I_0)^{1-\hat{\alpha}} (\hat{I}_4/I_3)^{\hat{\alpha}} b_3(0).$$

By summing the actual and estimated partial severities for each loss period, we get the total incurred severity. Total incurred losses for a given policy or accident period are determined by multiplying the incurred severity estimate by the number of incurred claims (this also may need to be estimated due to IBNR losses).

As a numerical illustration of the procedure, suppose that we have the following data:

Time		0	1	2	3	4	5	Sum of Actual Partial Severities
Claim Cost Index		100	110	115	130	(140) (estimated)	(155)	
Partial Severity For Claims Incurred at:	Time 0		10.1	20.0	9.9			40.0
	Time 1			10.6	22.2			32.8
	Time 2				11.6			11.6

Solving the three regression equations, we get $\hat{\alpha} = .72$ and $\hat{\beta} = .99$. Using the forecasted values $\hat{I}_4 = 140$ and $\hat{I}_5 = 155$, the estimated remaining partial severities are $\hat{b}_3(1) = 10.6$, $\hat{b}_2(2) = 23.5$, and $\hat{b}_3(2) = 11.4$. Thus the incurred severity at time 1 is $32.8 + 10.6 = 43.4$, and the incurred severity at time 2 is $11.6 + 23.5 + 11.4 = 46.5$.

In general, if there are n partial severities per claim, then $n(n-1)/2$ remaining partial severities must be estimated in order to obtain incurred loss estimates for all prior accident periods. If m periods of claim payment data are available ($m \geq n$), then $n(m-n) + n(n+1)/2$ equations need to be solved for $\hat{\alpha}$ and $\hat{\beta}$.

To apply the preceding method successfully, future claim cost indices must also be estimated. Econometric techniques are well-suited for this purpose (see references [9] and [12] for details), since claim cost components can be directly related to general price levels, whose values are routinely predicted by economists. Masterson [10] has compiled extensive series of claim cost indices for various property-liability coverages.

It should be emphasized that the $\hat{\alpha}$ and $\hat{\beta}$ estimates are subject to <u>statistical</u> <u>error</u>. Consequently, a large amount of claim data may be necessary in order to obtain stable estimates of these parameters. Because large claims may distort the partial severity values, <u>limiting</u> the size of individual claims in the data will tend to reduce the variance of these parameter estimates. With sparse data, the estimate of $\hat{\alpha}$ could even lie outside the range of 0 to 1. In this case, an estimation technique which forces $\hat{\alpha}$ into this range would be useful.

Also note that, as with any incurred loss estimation technique, the data should be separated into homogeneous groups, processing errors corrected, and other adjustments made. This will tend to eliminate non-random sources of error in the claim data and should allow more precise parameter estimation.

APPENDIX II

Present Value of Cash Flow and Discounted Investment Income

Although it may not be intuitively obvious, the present value of cash flows from an individual policy equals the sum of underwriting gain plus discounted investment income from the same policy. Here investment income is defined as being generated by the loss reserve (normally, investment income is derived from other liabilities as well, such as the unearned premium reserve, but these are absent from the model).

Using the notation of equation (34), assets equal to the reserve $a_1 + \ldots a_n$ are immediately available for investment at time t, so the investment income received at time t+1 is $(a_1 + \ldots + a_n)r$. At time t+1, also, a_1 is paid, reducing the reserve amount to $a_2 + \ldots + a_n$. Thus the investment income received at time t+2 is $(a_2 + \ldots + a_n)r$.

With each successive period another payment is made, reducing the loss reserve and consequent investment income. Finally, at time t+n the last amount of investment income, $(a_n)r$, is earned. The following table displays the sequence of investment income values, along with their respective discount factors:

(1) Time	(2) Investment Income	(3) Discount Factor
t+1	$(a_1 +\ldots+ a_n)r$	$(1+r)^{-1}$
t+2	$(a_2 +\ldots+ a_n)r$	$(1+r)^{-2}$
.	.	.
.	.	.
.	.	.
t+n-1	$(a_{n-1} + a_n)r$	$(1+r)^{-n+1}$
t+n	$(a_n)r$	$(1+r)^{-n}$

The total discounted investment income, J, is the sum of column (2) times column (3). This sum can be rearranged by grouping all terms containing a_1, then all terms with a_2, and so forth:

$$J = a_1 r \sum_{j=1}^{1}(1+r)^{-j} + a_2 r \sum_{j=1}^{2}(1+r)^{-j} + \ldots + a_n r \sum_{j=1}^{n}(1+r)^{-j} .$$

Since $\sum_{j=1}^{k}(1+r)^{-j} = [1 - (1+r)^{-k}]/r$, we get

$$J = a_1 r[1 - (1+r)^{-1}]/r + a_2 r[1 - (1+r)^{-2}]/r + \ldots + a_n r[1 - (1+r)^{-n}]/r$$

$$= (a_1 + \ldots + a_n) - a_1(1+r)^{-1} - \ldots - a_n(1+r)^{-n}$$

$$= (W - U) - a_1(1+r)^{-1} \ldots - a_n(1+r)^{-n} = V - U .$$

Thus $V = U + J$. In other words, the present value of policy cash flows does in fact equal the sum of underwriting gain plus discounted investment income.

Notice that if we define investment income as being generated by investable assets (accumulated premium minus losses), a similar analysis shows that this "investment income" equals V, the present value of the policy. Consequently, this "investment income" measure really is a total income measure, since underwriting income is implicitly included. In order to split total return into investment and underwriting components, we must define investment income as being generated by loss (plus unearned premium) reserves.

APPENDIX III

Inflation and the Competitive Underwriting Margin: Calendar-Period Model

Here we will apply the Fisher effect to both the return on surplus and the interest rate on invested assets to determine how a change in the inflation rate will affect the competitive underwriting margin. We will also consider the effect of income taxes.

As in the individual policy model, assume no transactions occur between t and t+1. Let S, A, and R denote the respective value of surplus, invested assets and total reserves at time t. Following the assumptions of the policy model, earned premium equals written premium and there is thus no unearned premium reserve. Hence R is the loss reserve in this case. The change in surplus (see [1] or [5] for a full development) is

$$\Delta S = [Ar + Wu](1-T),$$

where T is the income tax rate and the interest rate r is fully taxable. The return on owners' equity (or return on surplus) is

$$r_E = (\Delta S)/S = [(S + R)r/S + (Wu/S)]\ (1-T),$$

since $A = R + S$, or assets equal reserves plus surplus. This equation can be simplified to

$$r_E = (1-T)[(1 + yz)r + zu],$$

where $y = R/W$ is the reserve/premium ratio and $z = W/S$ is the premium/surplus ratio. Suppose that inflation changes and that interest rates move accordingly, so that $1+r' = (1+r)(1+\rho)$. Now assuming that investors in the insurance firm demand the same real return as before, we need $1+r'_E = (1+r_E)(1+\rho)$. What is the required new underwriting margin u' such that the real return on equity will be maintained?

Since $r'_E = (1-T)[(1+yz)r' + zu']$, we get

$r'_E - r_E = (1-T)[(1+yz)(r'-r) + z(u'-u)]$. Using the definitions of

r'_E and r', the preceding equation reduces (after some manipulation) to

$$\Delta u = -\rho(y-u) + (\rho/z)[T/(1-T)].$$

For a zero tax rate, the change in underwriting margin becomes

$$\Delta u = -\rho(y-u) \cong -\rho y = -\rho(R/W).$$

As a numerical example, let the reserve/premium ratio be 1, the premium/surplus ratio 3, and the underwriting margin 0. Also assume a 46% income tax rate. These are reasonable values for a typical property-liabilty company today. The change in underwriting margin is $\Delta u = -\rho(1-0) + (\rho/3)(.46/.54) = -.716\rho$. Thus for each point of interest rate increase, the competitive underwriting margin will drop by about 0.7 points.

GLOSSARY OF NOTATION

Because numerous concepts are introduced in the text, the following glossary of notation may assist the reader. Where a letter appears on the same line both with and without a subscript or index, the omitted subscript or index is understood to be zero. Symbols denoted with an asterisk are introduced in the Appendix.

Symbol	Definition
a	First claim payment under geometric distribution
a_k, $a_k(j)$	For policy written at t+j, the kth claim payment
* A	Total assets for P-L insurer
b	Real interest rate under Fisher effect
* $b_k(j)$	Average payment for claim incurred at t+j, paid at t+j+k
c	Rate of claim inflation
e	Real interest rate change for policy
$F(i)$	Number of policies inforce at time t-i
$I(k)$	Claim inflation index at time t+k
J	Discounted investment income for individual policy
m, m_i	Average remaining duration of claim payment after i payments are made
m_d	Average duration of discounted claim payment
n	Number of periods over which claims are paid
p, p_j	Pure premium for policy written at time t+j
P_d, $P_d(i)$	Discounted pure premium for policy written at time t+i
q	Parameter of geometric loss payment model
r	Interest rate at which claim payments are discounted
* r_E	Return on owners' equity for P-L insurer
* R	Total loss and unearned premium reserve
$R(i)$	Loss reserve at time t for all policies written at time t-i
* S	Surplus, or owners' equity
t	Reference point for time in policy model
* T	Income tax rate
u	Underwriting margin per unit of premium written at t+i
U, U_i	Underwriting gain for single policy written at t+i
v, v_i	Total profit margin per unit of premium written at t+i
V, V_i	Total profit for policy written at t+i
W, W_i	Premium for policy written at t+i
* y	Reserve/Premium ratio (R/W)
* z	Premium/Surplus ratio (W/S)
α	Exponential weight for accident vs. payment-date claim inflation
* β	Constant which adjusts for true vs. empirical claim inflation index
δ	Claim cost accelerator; effective rate of post-incurred claim inflation
ρ	Change in interest rate

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