RESERVES IN SANATORIUM INSURANCE

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Introduction

The object of the Netherlands Foundation for Sanatorium Insurance ("N.S.V.") is to insure the risk of treatment in a sanatorium because of tuberculosis. For insured persons admitted to a sanatorium because of tuberculosis, the N.S.V. pays the insured sum for every day of treatment. At the moment nearly 80 % of the population of the Netherlands is insured directly or indirectly in the N.S.V.

There are three groups of insurances: obligatory insurance, voluntary collective insurances and voluntary individual insurances. The insurances in the first and second group are, strictly speaking, reinsurances of the risk of tuberculosis of a great number of institutions concerned with cost of sickness insurance. The risk of the insurance in these two groups is only administered and pooled by the N.S.V.

This paper concerns the group of the individual insurances only, which group contains about 1.250.000 insured persons. The risk of this group is run by the N.S.V. itself. As the N.S.V. is a non-profit organisation, the premium level is held as low as possible.

From statistical data, derived from the administration of the N.S.V., the "admission frequency" is calculated every year, being the quotient of the number of insured persons, admitted to a sanatorium in that year, and the total number of insured persons. This admission frequency a, which is now about 0,2 $^0/_{00}$, is used as a basis for the calculation of the premium which will be in force during the next year.

Furthermore, a table of discharge probabilities is calculated every year; the discharge probability c_t is the probability that an insured person, who is still treated in a sanatorium at the end of the t^{th} month since his admission in that sanatorium (where, for the month of admission, $t = 1$) will die or be discharged in the $t + 1^{\text{th}}$

month. The statistical data of one year are, however, insufficient to calculate a completely new set of discharge probabilities; therefore the statistical data for the last three years are used, taking into account the trend appearing from the data. Even the statistical data of a period of three years are insufficient to differentiate the discharge probabilities according to age of the patients, but as the age distributions of the insured persons as well as of the insured patients are rather stable, the neglect of the age dependence of the discharge probabilities can still lead to useful results.

From the discharge probabilities c_t the values p_t are calculated, according to the formula

$$p_t = \frac{365}{12}\left(\sum_{u=0}^{\dots}\prod_{s=t}^{t+u}(1-c_s)+\frac{1}{2}\right) \text{ if } t > 0$$

$$p_0 = \frac{365}{12}\left(\sum_{u=1}^{\dots}\prod_{s=0}^{u}(1-c_s)+\frac{3}{4}(1-c_0)+\frac{1}{4}\right)$$

The value p_t represents the expected number of days during which a patient, being in a sanatorium at the end of the t^{th} month after his admission, will still stay in a sanatorium. The calculation of p_t is practically the same as the calculation of the expectation of life in life insurance.

The values c_t and p_t, based upon the data of the year 1961, are given in the appendix to this paper.

The yearly net risk premium per insured person and per unit of insured sum per day is

$$P = a.p_0$$

which, on the basis of data of the year 1961, proves to be about 0.07.

Claim reserve

At the end of every year the claim reserve is calculated, in respect of the persons being treated at that moment in a sanatorium for account of the N.S.V. (individual insurance). This calculation is based on the formula

$$V = \sum_{t=1}^{\dots} N_t b_t p_t$$

in which:

V = the claim reserve

N_t = the number of insured patients, being admitted in the t^{th} month before the balance date, and still being treated in a sanatorium on the balance date

b_t = the average insured sum per day for these patients.

It will be noted that in the calculation of the claim reserve no allowance is made for the possibility of relapse, but very few data are available concerning this probability. About 25 % of the patients admitted in the year 1961 had been treated for tuberculosis before; in about 70 % of these cases the previous period of sickness was five or more years earlier.

However, the probability of a relapse has no significance for the calculation of the claim reserve as the sanatorium treatment is only for account of the N.S.V. if the patient was insured at the moment of his recent admission to a sanatorium.

Making allowance for the probability of relapse would result in an increase of the values p_t, and in a reduction of the admission frequency a, this frequency would then not have to include the admission concerning relapse.

The possibility of a correlation between on the one side the admission frequency and the discharge probabilities and on the other side the insured sum per day is neglected.

Security Reserve

The N.S.V. owns rather large free reserves relating to the individual insurance. Some years ago it proved to be necessary to investigate what part of these reserves should be regarded as a security reserve and what part could be utilised otherwise.

For that purpose an investigation has been made of the fluctuations to be expected in the yearly result of the insurance (profit or loss). The method used for this investigation, in so far as random fluctuations are concerned, will be described in the following.

The result R of the insurance of a certain year can be divided into a part R_1, concerning the liquidation of the claim reserve at the beginning of the year, and a part R_2, concerning the new admissions in the course of the year, i.e.

$$R = R_1 + R_2.$$

In this formula R_1 can be defined as:

$$R_1 = V - 365\, tAb - (365 + v).\ (N\bar{b} - Ab),$$

where

V = claim reserve at the beginning of the year.
A = number of discharged and deceased patients in the year (shortly: the discharges).
b = average insured sum per day of the discharges in the year.
t = average moment of discharge in the year.
$N = \sum\limits_{t=1}^{\cdots} N_t$ = number of patients at the beginning of the year.
v = average claim reserve at the end of the year per unit of insured sum per day, and per insured patient at that time.
$\bar{b}$ = average insured sum per day of the patients at the beginning of the year;

$$= \frac{\sum\limits_{t=1}^{\cdots} N_t\, b_t}{\Sigma\, N_t}$$

In this formula V, N, v and $\bar{b}$ are constants: A, b and t are variables.

The variable A has the expected value

$$\bar{A} = \sum_{t=1}^{\cdots} N_t \left(1 - \prod_{s=0}^{11} (1 - c_{t+s})\right)$$

and the variance σ_A^2. A further investigation, referred to at the end of this paper, has shown that A has approximately a Poisson-distribution; so that $\sigma_A^2 = A$.

The variable b has the expected value $\bar{b}$ and the variance $\dfrac{\sigma_b^2}{A}$

The variable t has the expected value $\bar{t}$ and the variance $\dfrac{\sigma_t^2}{A}$

It can be assumed further that, for one discharge, t is distributed according to the "trapezium" distribution:

$$f(t) = (12\bar{t} - 6)t + (4 - 6\bar{t}) \quad (0 \leqslant t \leqslant 1)$$

which has the mean $\bar{t}$—by definition—and the variance

$$\sigma_t^2 = \bar{t} - \frac{1}{6} - \bar{t}^2$$

Since the claim reserve V does not contain a security loading, the values V, $\bar{t}$, $\bar{A}$, $\bar{b}$, N and v have to be consistent with each other in the sense that

$$\bar{R}_1 = V - 365\,\overline{tAb} - (365 + v)\,(N\bar{b} - \overline{Ab}) = 0.$$

From this it follows that

$\sigma^2(R_1) = E(R_1^2)$, which can be reduced to

$\sigma^2(R_1) = (365 + v - 365\,\bar{t})^2.\ (\bar{b}^2\,\sigma_A^2 + \sigma_b^2\,.\,\bar{A}) + 365^2\,\sigma_t^2\,(\bar{b}^2.\bar{A} + \sigma_b^2)$

With regard to the year 1962 the expected values are as follows:

$$\bar{A} = 186$$
$$\bar{b} = 12{,}66 \qquad \sigma_b = 9{,}02$$
$$\bar{t} = 0{,}433 \qquad \sigma_t = 0{,}281$$

Further: $v = 116$.

This leads to

$$\sigma(R_1) = 70.740.$$

The influence of σ_t on $\sigma(R_1)$ is small; if σ_t were zero, $\sigma(R_1)$ would be only 3 % lower than the actual value.

For R_2 the following formula has been used:

$R_2 = p_0\,\overline{Db} - Dd\ (365\,(1 - u) + w) + E\,e\ (365\,(1 - t) + w)$

where:

D = number of admissions in the year

d = average insured sum per day of the admissions in the year

u = average moment of admission in the year

E = number of discharges in the year from the admissions in the year

e = average insured sum per day of the discharges

t = average moment of discharge

w = average claim reserve at the end of the year per insured patient, being admitted in the year.

The variables D, d, E and e have the expected values $\bar{D}$, $\bar{b}$, $\alpha\,\bar{D}$ and $\bar{b}$ respectively and the variances σ^2_D, $\frac{\sigma^2_b}{D}$, $\alpha\,\sigma^2_D$ and $\frac{\sigma^2_b}{E}$ respectively.

Because of the small influence of σ_t on $\sigma(R_1)$, the values u and t in the formula of R_2 are, for the sake of simplicity, considered as constants.

The variance of R_2 can be written as:

$$\sigma^2(R_2) = g_1^2\,(\alpha\bar{b}^2\,\sigma^2_D + \alpha\sigma^2_b\,\bar{D}) - 2\,g_1g_2\,(\alpha\,\bar{b}^2\,\sigma^2_D\,) + g_2^2\,(\bar{b}^2\,\sigma^2_D + \sigma^2_b\,\bar{D})$$

where $g_1 = 365\ (1 - t) + w$

and $g_2 = 365\ (1 - u) + w$

For the year 1962:

$\bar{b}$ = 12,66

$\bar{D}$ = 250

σ^2_D = 250, assuming D to be Poisson distributed.

α = 0,207

u = 0,5

t = 0,714

w = 240

p_0 = 352 (see appendix).

From these figures, with a loading of 3 % to compensate for the neglect of the variances of u and t, $\sigma(R_2)$ proved to be $\sigma(R_2) = 102.40$

Assuming independence between R_1 and R_2:

$$\sigma^2(R) = \sigma^2(R_1) + \sigma^2(R_2)$$

from which follows

$\sigma(R)$ = about 125.000.

This value has to be considered in relation to the claim reserve of the year, namely 676.000, and the net risk premium of about 1.114.000.

It is clear that the result of the calculations is influenced to a great extent by the assumptions regarding the distributions of A, D and E, and, in relation with these distributions, the assumption of independence of R_1 and R_2. Concerning these assumptions the following remarks can be made.

During the last ten years the admission frequency showed a steady decrease; in the period from 1952 up to and including 1961 the admission frequency is decreased from 0,7 $^0/_{00}$ to about 0,2 $^0/_{00}$. In addition the discharge frequencies have grown; the mean duration of the sanatorium treatment has consequently decreased. These trends in the admission frequency and in the discharge frequencies are still continuing.

The claim reserve as well as the net risk premium are calculated on the basis of recent data, but not an extrapolation of those data. This means, that the expected value of R is not zero but is, in fact, positive.

Assuming a second degree trend in the admission probabilities, the actual number of admissions in the ten years 1952-1961 do not lead to rejection of the hypothesis that these numbers are Poisson-distributed.

That does not alter the fact that the possibility of an epidemic of some importance can not be ignored. During recent years there have been some epidemics, but, as a result of the frequent periodical X-ray examination of nearly the entire population of the Netherlands, these epidemics have never been important. This suggests that it might be assumed that the number of admissions has a negative binomial distribution. A further calculation shows that a rise in σ_A and σ_D of q % results (within certain limits of q) in a rise of $\sigma(R)$ of about 0.6 q %.

On account of the above-mentioned reflections the N.S.V. has decided to establish a security reserve of about four times the standard deviation of R; in addition a slightly smaller is set up to cover the possibility that the decrease of the sanatorium risk will turn into an increase, in which case it will need some time to adjust the gross premium. Because of these large reserves there was no need to make a better calculation of $\sigma(R)$.

Appendix

t	c_t	p_t	t	c_t	p_t
0	0,015	352	28	0,161	226
1	0,019	342	29	0,155	236
2	0,022	318	30	0,149	245
3	0,028	294	31	0,144	255
4	0,033	272	32	0,139	265
5	0,041	250	33	0,134	275
6	0,050	230	34	0,130	285
7	0,060	210	35	0,126	295
8	0,076	192	36	0,122	305
9	0,095	177	37	0,119	314
10	0,113	163	38	0,116	324
11	0,135	151	39	0,113	334
12	0,160	142	40	0,109	345
13	0,182	136	41	0,106	354
14	0,200	130	42	0,102	364
15	0,216	129	43	0,098	373
16	0,222	130	44	0,095	382
17	0,226	132	45	0,091	390
18	0,230	135	46	0,089	397
19	0,225	141	47	0,085	404
20	0,223	147	48	0,082	409
21	0,220	154	49	0,078	414
22	0,218	162	50	0,074	417
23	0,207	173	51	0,072	419
24	0,196	184	52	0,070	420
25	0,186	195	53 etc.	0,069	420
26	0,176	205			
27	0,168	216			