RESERVES IN SANATORIUM INSURANCE

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INTRODUCTION

The object of the Netherlands Foundation for Sanatorium Insurance ("N.S.V.") is to insure the risk of treatment in a sanatorium because of tuberculosis. For insured persons admitted to a sanatorium because of tuberculosis, the N.S.V. pays the insured sum for every day of treatment. At the moment nearly 80 % of the population of the Netherlands is insured directly or indirectly in the N.S.V.

There are three groups of insurances: obligatory insurance, voluntary collective insurances and voluntary individual insurances. The insurances in the first and second group are, strictly speaking, reinsurances of the risk of tuberculosis of a great number of institutions concerned with cost of sickness insurance. The risk of the insurance in these two groups is only administered and pooled by the N.S.V.

This paper concerns the group of the individual insurances only, which group contains about 1.250.000 insured persons. The risk of this group is run by the N.S.V. itself. As the N.S.V. is a non-profit organisation, the premium level is held as low as possible.

From statistical data, derived from the administration of the N.S.V., the "admission frequency" is calculated every year, being the quotient of the number of insured persons, admitted to a sanatorium in that year, and the total number of insured persons. This admission frequency $a$, which is now about 0.2 %, is used as a basis for the calculation of the premium which will be in force during the next year.

Furthermore, a table of discharge probabilities is calculated every year; the discharge probability $c_t$ is the probability that an insured person, who is still treated in a sanatorium at the end of the $t^{th}$ month since his admission in that sanatorium (where, for the month of admission, $t = 1$) will die or be discharged in the $t + 1^{th}$
month. The statistical data of one year are, however, insufficient to calculate a completely new set of discharge probabilities; therefore the statistical data for the last three years are used, taking into account the trend appearing from the data. Even the statistical data of a period of three years are insufficient to differentiate the discharge probabilities according to age of the patients, but as the age distributions of the insured persons as well as of the insured patients are rather stable, the neglect of the age dependence of the discharge probabilities can still lead to useful results.

From the discharge probabilities \( c_t \) the values \( p_t \) are calculated, according to the formula

\[
p_t = \frac{365}{12} \left( \sum_{t=1}^{\infty} \prod_{s=1}^{t} (1 - c_s) + \frac{1}{2} \right) \text{ if } t > 0
\]

\[
p_0 = \frac{365}{12} \left( \sum_{t=1}^{\infty} \prod_{s=1}^{t} (1 - c_s) + \frac{3}{4} (1 - c_0) + \frac{1}{4} \right)
\]

The value \( p_t \) represents the expected number of days during which a patient, being in a sanatorium at the end of the \( t \)th month after his admission, will still stay in a sanatorium. The calculation of \( p_t \) is practically the same as the calculation of the expectation of life in life insurance.

The values \( c_t \) and \( p_t \), based upon the data of the year 1961, are given in the appendix to this paper.

The yearly net risk premium per insured person and per unit of insured sum per day is

\[
P = a_p_0
\]

which, on the basis of data of the year 1961, proves to be about 0.07.

Claim reserve

At the end of every year the claim reserve is calculated, in respect of the persons being treated at that moment in a sanatorium for account of the N.S.V. (individual insurance). This calculation is based on the formula

\[
V = \sum_{t=1}^{\infty} N_t b_t p_t
\]
in which:

\[ V = \text{the claim reserve} \]
\[ N_t = \text{the number of insured patients, being admitted in the } t^{th} \text{ month before the balance date, and still being treated in a sanatorium on the balance date} \]
\[ b_t = \text{the average insured sum per day for these patients}. \]

It will be noted that in the calculation of the claim reserve no allowance is made for the possibility of relapse, but very few data are available concerning this probability. About 25% of the patients admitted in the year 1961 had been treated for tuberculosis before; in about 70% of these cases the previous period of sickness was five or more years earlier.

However, the probability of a relapse has no significance for the calculation of the claim reserve as the sanatorium treatment is only for account of the N.S.V. if the patient was insured at the moment of his recent admission to a sanatorium.

Making allowance for the probability of relapse would result in an increase of the values \( p_t \), and in a reduction of the admission frequency \( a \), this frequency would then not have to include the admission concerning relapse.

The possibility of a correlation between on the one side the admission frequency and the discharge probabilities and on the other side the insured sum per day is neglected.

**Security Reserve**

The N.S.V. owns rather large free reserves relating to the individual insurance. Some years ago it proved to be necessary to investigate what part of these reserves should be regarded as a security reserve and what part could be utilised otherwise.

For that purpose an investigation has been made of the fluctuations to be expected in the yearly result of the insurance (profit or loss). The method used for this investigation, in so far as random fluctuations are concerned, will be described in the following.

The result \( R \) of the insurance of a certain year can be divided into a part \( R_1 \), concerning the liquidation of the claim reserve at
the beginning of the year, and a part \( R_2 \), concerning the new admissions in the course of the year, i.e.

\[ R = R_1 + R_2. \]

In this formula \( R_1 \) can be defined as:

\[ R_1 = V - 365 \, tA\, b - (365 + v) \, (N\bar{b} - A\bar{b}), \]

where

\( V = \) claim reserve at the beginning of the year.

\( A = \) number of discharged and deceased patients in the year (shortly: the discharges).

\( b = \) average insured sum per day of the discharges in the year.

\( t = \) average moment of discharge in the year.

\( N = \sum \limits_{i=1}^{\frac{b}{2}} N_t = \) number of patients at the beginning of the year.

\( v = \) average claim reserve at the end of the year per unit of insured sum per day, and per insured patient at that time.

\( \bar{b} = \) average insured sum per day of the patients at the beginning of the year;

\[ = \frac{\sum \limits_{i=1}^{\frac{b}{2}} N_t \, b_i}{\sum \limits_{i=1}^{\frac{b}{2}} N_t}. \]

In this formula \( V, N, v \) and \( \bar{b} \) are constants: \( A, b \) and \( t \) are variables.

The variable \( A \) has the expected value

\[ \bar{A} = \sum \limits_{i=1}^{\frac{b}{2}} N_t \, \left(1 - \prod \limits_{i=0}^{n} (1 - c_{t+b})\right) \]

and the variance \( \sigma_A^2 \). A further investigation, referred to at the end of this paper, has shown that \( A \) has approximately a Poisson-distribution; so that \( \sigma_A^2 = A \).

The variable \( b \) has the expected value \( \bar{b} \) and the variance \( \frac{\sigma_b^2}{A} \).

The variable \( t \) has the expected value \( \bar{t} \) and the variance \( \frac{\sigma_t^2}{A} \).

It can be assumed further that, for one discharge, \( t \) is distributed according to the “trapezium” distribution:

\[ f(t) = (12\bar{t} - 6)t + (4 - 6\bar{t}) \, (0 \leq t \leq 1) \]
which has the mean \( \bar{t} \)—by definition—and the variance

\[
\sigma_t^2 = \bar{t} - \frac{1}{6} \bar{t}^2
\]

Since the claim reserve \( V \) does not contain a security loading, the values \( V, \bar{t}, \bar{A}, \bar{b}, N \) and \( v \) have to be consistent with each other in the sense that

\[
\bar{R}_1 = V - 365 \bar{t} \bar{A} \bar{b} - (365 + v) (N \bar{b} - \bar{A} \bar{b}) = 0.
\]

From this it follows that

\[
\sigma^2(R_1) = E(R_1^2),
\]

which can be reduced to

\[
\sigma^2(R_1) = (365 + v - 365 \bar{t})^2 \cdot (\bar{b}^2 \sigma_A^2 + \sigma_b^2 \cdot \bar{A}) + 365^2 \sigma_t^2 (\bar{b}^2 \cdot \bar{A} + \sigma_b^2)
\]

With regard to the year 1962 the expected values are as follows:

\[
\bar{A} = 186 \\
\bar{b} = 12.66 \quad \sigma_b = 9.02 \\
\bar{t} = 0.433 \quad \sigma_t = 0.281
\]

Further: \( v = 116 \).

This leads to

\[
\sigma(R_1) = 70.740.
\]

The influence of \( \sigma_t \) on \( \sigma(R_1) \) is small; if \( \sigma_t \) were zero, \( \sigma(R_1) \) would be only 3% lower than the actual value.

For \( R_2 \) the following formula has been used:

\[
R_2 = \rho_0 \bar{D} \bar{b} - D \bar{d} (365 (1 - \bar{u}) + \bar{w}) + E \bar{e} (365 (1 - \bar{t}) + \bar{w})
\]

where:

\[
D = \text{number of admissions in the year} \\
\bar{d} = \text{average insured sum per day of the admissions in the year} \\
\bar{u} = \text{average moment of admission in the year} \\
E = \text{number of discharges in the year from the admissions in the year} \\
\bar{e} = \text{average insured sum per day of the discharges} \\
\bar{t} = \text{average moment of discharge}
\]
$w =$ average claim reserve at the end of the year per insured patient, being admitted in the year.

The variables $D$, $d$, $E$ and $e$ have the expected values $\bar{D}$, $\bar{d}$, $\alpha \bar{D}$ and $\bar{e}$ respectively and the variances $\sigma^2_D$, $\frac{\sigma^2_D}{\bar{D}}$, $\alpha \sigma^2_D$ and $\frac{\sigma^2_e}{\bar{E}}$ respectively.

Because of the small influence of $\sigma_t$ on $\sigma(R_1)$, the values $u$ and $t$ in the formula of $R_2$ are, for the sake of simplicity, considered as constants.

The variance of $R_2$ can be written as:

$$
\sigma^2(R_2) = g_1^2 (\bar{D}^2 \sigma^2_D) - 2 g_1 g_2 (\alpha \bar{D}^2 \sigma^2_D) + g_2^2 (\bar{D}^2 \sigma^2_D) + o^2_R
$$

where $g_1 = 365 (1 - t) + w$
and $g_2 = 365 (1 - u) + w$

For the year 1962:

$\bar{b} = 12.66$

$\bar{D} = 250$

$\sigma^2_D = 250$, assuming $D$ to be Poisson distributed.

$\alpha = 0.207$

$u = 0.5$

$t = 0.714$

$w = 240$

$\phi_0 = 352$ (see appendix).

From these figures, with a loading of 3 % to compensate for the neglect of the variances of $u$ and $t$, $\sigma(R_2)$ proved to be $\sigma(R_2) = 102.40$

Assuming independence between $R_1$ and $R_2$:

$$
\sigma^2(R) = \sigma^2(R_1) + \sigma^2(R_2)
$$

from which follows

$\sigma(R) = \text{about } 125.000$.

This value has to be considered in relation to the claim reserve.
of the year, namely 676.000, and the net risk premium of about 1.114.000.

It is clear that the result of the calculations is influenced to a great extent by the assumptions regarding the distributions of $A$, $D$ and $E$, and, in relation with these distributions, the assumption of independence of $R_1$ and $R_2$. Concerning these assumptions the following remarks can be made.

During the last ten years the admission frequency showed a steady decrease; in the period from 1952 up to and including 1961 the admission frequency is decreased from 0.7 $\%$ to about 0.2 $\%$. In addition the discharge frequencies have grown; the mean duration of the sanatorium treatment has consequently decreased. These trends in the admission frequency and in the discharge frequencies are still continuing.

The claim reserve as well as the net risk premium are calculated on the basis of recent data, but not an extrapolation of those data. This means, that the expected value of $R$ is not zero but is, in fact, positive.

Assuming a second degree trend in the admission probabilities, the actual number of admissions in the ten years 1952-1961 do not lead to rejection of the hypothesis that these numbers are Poisson-distributed.

That does not alter the fact that the possibility of an epidemic of some importance can not be ignored. During recent years there have been some epidemics, but, as a result of the frequent periodical X-ray examination of nearly the entire population of the Netherlands, these epidemics have never been important. This suggests that it might be assumed that the number of admissions has a negative binomial distribution. A further calculation shows that a rise in $\sigma_A$ and $\sigma_D$ of $q$ $\%$ results (within certain limits of $q$) in a rise of $\sigma(R)$ of about 0.6 $q$ $\%$.

On account of the above-mentioned reflections the N.S.V. has decided to establish a security reserve of about four times the standard deviation of $R$; in addition a slightly smaller is set up to cover the possibility that the decrease of the sanatorium risk will turn into an increase, in which case it will need some time to adjust the gross premium. Because of these large reserves there was no need to make a better calculation of $\sigma(R)$. 
### RESERVES IN SANATORIUM INSURANCE

**Appendix**

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