LOCAL MOMENT MATCHING AND *S*-CONVEX EXTREMA

BY

CINDY COURTOIS AND MICHEL DENUIT

ABSTRACT

The paper is devoted to the local moment matching method and its links with the discrete version of the *s*-convex extremal distributions. It is well-known that the local moment matching method can produce some negative masses. Connecting the local moment matching method to the discrete *s*-convex extrema gives an explicit criterion that explains why (and says when) the local moment matching method gives some negative mass.

KEYWORDS

Discretisation, local moment matching, *s*-convex order.

1. INTRODUCTION

Given any risk X with support included in $[0, +\infty)$, a relevant problem in actuarial applications is to arithmetize it, i.e. to choose a span $\Delta > 0$ and try to determine a risk X_Δ with support points $i\Delta$, $i = 0, 1, 2, \ldots$ that shares some common characteristics with X and that will serve for the numerical computations. This operation is required by the majority of iterative algorithms (like Panjer's algorithm for instance).

In the actuarial literature several methods have been proposed to discretize claim amount distributions, e.g., by GERBER (1982), PANJER & LUTEK (1983), WALHIN & PARIS (1998) and VILAR (2000). The *local moment matching method* (LMM in short) proposed by GERBER (1982) constructs discrete equispaced distributions that matches some moments locally. The idea is to replace the probability mass of F_X over intervals $(a_k, b_k]$ by pointmasses located at $x_i^{(k)} = a_k + i\Delta$ ($i = 0, 1, \ldots, n$), where $\Delta = (b_k - a_k)/n$ and the integer n depends on the number of moments to be matched. The method tries thus to calculate the $n + 1$ masses $p_0^{(k)}, p_1^{(k)}, \ldots p_n^{(k)}$ located at the points $x_0^{(k)}, x_1^{(k)}, \ldots, x_n^{(k)}$ which are solutions of the linear system

$$\sum_{i=0}^{n} (a_k + i\Delta)^r p_i^{(k)} = \int_{a_k}^{b_k} x^r dF_X(x), \quad r = 0, 1, \ldots, n;$$

the unique solution of this system being

$$p_i^{(k)} = \int_{a_k}^{b_k} \left(\prod_{j \neq i} \frac{x - a_k - j\Delta}{(i - j)\Delta} \right) dF_X(x), \quad i = 0, 1, \dots, n.$$

In practice, this system has to be solved in each of the intervals $(0, n\Delta]$, $(n\Delta, 2n\Delta]$, $(2n\Delta, 3n\Delta]$, ... corresponding to $a_0 = 0$, $a_1 = n\Delta$, $a_2 = 2n\Delta$, ... The final probabilities of the discretized distribution are the point masses obtained as solutions of the previous system in each interval, summed for the end points of each interval.

In this paper, moment-based discretisation methods are studied and linked to discrete stochastic extrema. This permits to give simple conditions to verify whether the local moment matching method gives negative masses for a given span $\Delta > 0$. The paper is organized as follows. First, we recall some notions about s-convex extrema and moment spaces. Then these extrema are used in connection with the local moment matching method. In particular, we derive explicit expressions for the admissibility of the local moment matching method. Finally, numerical examples are given.

2. DISCRETE s-CONVEX ORDERINGS AND MOMENT SPACES

DENUIT & LEFÈVRE (1997) introduced the *discrete s-convex orderings* among arithmetic random variables valued in some set $\mathcal{D}_n = \{e_0 + ih : i = 0, \dots, n\}$ of $(n + 1)$ evenly-spaced points, with minimum e_0 and separation parameter $h > 0$ say. Here s is any non-negative integer smaller or equal to n. These orderings are defined as follows. Let Δ_h be the forward difference operator with increment h defined for each function $\phi : \mathcal{D}_n \to \mathbb{R}$ by $\Delta_h \phi(e_0 + ih) = \phi(e_0 + (i + 1)h) - \phi(e_0 + ih)$ for all $i = 0, \dots, n - 1$. Let Δ_h^k, $k \geq 1$, be the k-th forward difference operator defined recursively by $\Delta_h^k \phi(e_0 + ih) = \Delta_h^{k-1}\phi(e_0 + (i + 1)h) - \Delta_h^{k-1}\phi(e_0 + ih)$ for all $i = 0, \dots, n - k$ (by convention, $\Delta_h^1 \phi \equiv \Delta\phi$ and $\Delta_h^0 \phi \equiv \phi$). If X and Y are two random variables valued in $\mathcal{D}_n$, X is said to be smaller than Y with respect to the discrete s-convex order if $\mathbb{E}[\phi(X)] \leq \mathbb{E}[\phi(Y)]$ for all the functions $\phi : \mathcal{D}_n \to \mathbb{R}$ such that $\Delta_h^s \phi(e_0 + ih) \geq 0$ for all $i = 0, \dots, n - s$. In such a case, we write $X \preceq_{s-cx}^{\mathcal{D}_n} Y$. Clearly,

$$X \preceq_{s-cx}^{\mathcal{D}_n} Y \Rightarrow \mathbb{E}[X^k] = \mathbb{E}[Y^k] \quad \text{for } k = 1, 2, \dots, s - 1.$$

Thus, if $X \preceq_{s-cx}^{\mathcal{D}_n} Y$ then the s-1 first moments of X and Y necessarily match. Consequently, the ordering relation $\preceq_{s-cx}^{\mathcal{D}_n}$ can only be used to compare the random variables with the same first $s - 1$ moments. This motivates to introduce the *moment space* $\mathcal{M}_s(\mathcal{D}_n; \mu_1, \mu_2, \dots, \mu_{s-1})$ which contains all random variables valued on $\mathcal{D}_n$ such that the first $s - 1$ moments are fixed to $\mu_k (k = 1, \dots, s - 1; s \geq 1)$.

Now, suppose that the moment sequence $(\mu_1, \mu_2, \dots, \mu_{s-1})$ is such that $\mathcal{M}_s(\mathcal{D}_n; \mu_1, \mu_2, \dots, \mu_{s-1})$ is not void. Conditions for such a space to be non void can be

found in KARLIN & STUDDEN (1966): $\mathcal{M}_s(\mathcal{D}_n; \mu_1, \mu_2, ..., \mu_{s-1})$ is non void if, and only if, the point $(\mu_1, ..., \mu_{s-1})$ is an element of the convex hull of $\{(\theta, \theta^2, ..., \theta^{s-1}) | \theta \in \mathcal{D}_n\}$. As in DENUIT, DE VYLDER & LEFÈVRE (1999) and HÜRLIMANN (2005), let us put $\mu_0 \equiv 1$ and for $j = 1, 2, ...,$ and $x, y, z \in \mathbb{R}$, let us define

$$\mu_{j,x} = x\mu_{j-1} - \mu_j, \quad \mu_{j,x,y} = y\mu_{j-1,x} - \mu_{j,x} \quad \text{and} \quad \mu_{j,x,y,z} = z\mu_{j-1,x,y} - \mu_{j,x,y}.$$

With these notations, if $e_i = e_0 + ih$ $(i = 0, 1, ..., n)$, the conditions for $\mathcal{M}_s(\mathcal{D}_n; \mu_1, \mu_2, ..., \mu_{s-1}) \neq \emptyset$, $s = 2, 3, 4$, are

- $s = 2$: $\mu_{1,e_0} \leq 0$ and $\mu_{1,e_n} \geq 0$;
- $s = 3$: $\mu_{2,e_0,e_n} \leq 0$ and $\mu_{2,e_i,e_{i+1}} \geq 0$ for $i = 0, ..., n-1$;
- $s = 4$: $\mu_{3,e_0,e_i,e_{i+1}} \leq 0$ for $i = 1, ..., n-1$, and $\mu_{3,e_i,e_{i+1},e_n} \geq 0$ for $i = 0, ..., n-2$.

Under such conditions, the moment space contains a minimum random variable $X_{\min}^{(s)}$ and a maximum random variable $X_{\max}^{(s)}$ with respect to $\preceq_{s-cx}^{\mathcal{D}_n}$, i.e. such that

$$X_{\min}^{(s)} \preceq_{s-cx}^{\mathcal{D}_n} X \preceq_{s-cx}^{\mathcal{D}_n} X_{\max}^{(s)} \text{ for all } X \in \mathcal{M}_s(\mathcal{D}_n; \mu_1, \mu_2, ..., \mu_{s-1}).$$

Extrema with respect to the discrete version of the s-convex orders have been derived by DENUIT & LEFÈVRE (1997), DENUIT, LEFÈVRE & MESFIOUI (1999) and COURTOIS, DENUIT & VAN BELLEGEM (2006) for $s \geq 1$.

3. ARITHMETIZATION AND DISCRETE s-CONVEX EXTREMA

3.1. One moment known

On each interval $I_k = (k\Delta, (k+1)\Delta]$, $k = 0, 1, 2, ...,$ the first local moment of X is

$$\mu_1^{(k)} = \int_{k\Delta}^{(k+1)\Delta} x \, dF_X(x).$$

We obviously have that

$$k\Delta \mu_0^{(k)} < \mu_1^{(k)} < (k+1)\Delta \mu_0^{(k)}, \quad k = 0, 1, 2, ..., \tag{1}$$

where $\mu_0^{(k)} = \int_{k\Delta}^{(k+1)\Delta} dF_X(x)$ is the probability mass over the interval I_k. Then, for $k = 1, 2, ...,$ the only random variable with support $\{k\Delta, (k+1)\Delta\}$ and mean $\mu_1^{(k)} / \mu_0^{(k)}$ is the 2-convex maximum

$$X_{k,\max}^{(2)} = \begin{cases} k\Delta & \text{with probability } \dfrac{(k+1)\Delta \mu_0^{(k)} - \mu_1^{(k)}}{\Delta \mu_0^{(k)}}, \\[4mm] (k+1)\Delta & \text{with probability } \dfrac{\mu_1^{(k)} - k\Delta \mu_0^{(k)}}{\Delta \mu_0^{(k)}}. \end{cases}$$

The weights associated to $k\Delta$ and $(k+1)\Delta$ are thus respectively

$$w_k^+ = \frac{(k+1)\Delta\mu_0^{(k)} - \mu_1^{(k)}}{\Delta}$$

$$w_k^- = \frac{\mu_1^{(k)} - k\mu_0^{(k)}}{\Delta}.$$

Consequently, if we denote by $X_\Delta^{(2)}$ the arithmetized risk based on the knowledge of one moment of X, we have $\mathbb{P}[X_\Delta^{(2)} = 0] = F_X(0) + w_0^+$ and, for $k = 1, 2, \ldots$, $\mathbb{P}[X_\Delta^{(2)} = k\Delta] = w_k^- + w_k^+$.

3.2. Two moments known

On each interval $I_k = (k\Delta, (k+2)\Delta]$, $k = 0, 2, 4, \ldots$, the first two local moments of X over I_k are

$$\mu_1^{(k)} = \int_{k\Delta}^{(k+2)\Delta} x\,dF_X(x) \text{ and } \mu_2^{(k)} = \int_{k\Delta}^{(k+2)\Delta} x^2\,dF_X(x).$$

Let us now consider the set of all the probability distributions with support $\{k\Delta, (k+1)\Delta, (k+2)\Delta\}$ and first two moments $\mu_1^{(k)}/\mu_0^{(k)}$ and $\mu_2^{(k)}/\mu_0^{(k)}$. The conditions for that set to be not void are

$$\begin{cases} k\Delta\mu_0^{(k)} < \mu_1^{(k)} < (k+2)\Delta\mu_0^{(k)}, \\ \mu_2^{(k)} < 2\Delta(k+1)\mu_1^{(k)} - k(k+2)\Delta^2\mu_0^{(k)}, \\ \mu_2^{(k)} > \Delta(2k+1)\mu_1^{(k)} - k(k+1)\Delta^2\mu_0^{(k)}, \\ \mu_2^{(k)} > \Delta(2k+3)\mu_1^{(k)} - (k+1)(k+2)\Delta^2\mu_0^{(k)}. \end{cases} \tag{2}$$

As $\int_{k\Delta}^{(k+2)\Delta}(x - k\Delta)\,dF(x) \geq 0$, $\int_{k\Delta}^{(k+2)\Delta}((k+2)\Delta - x)\,dF(x) \geq 0$ and $\int_{k\Delta}^{(k+2)\Delta}(x - k\Delta)((k+2)\Delta - x)\,dF(x) \geq 0$, system (2) reduces to its last two equations, that is

$$\begin{cases} \int_{k\Delta}^{(k+2)\Delta}(x - k\Delta)(x - (k+1)\Delta)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+2)\Delta}(x - (k+1)\Delta)(x - (k+2)\Delta)\,dF(x) > 0. \end{cases}$$

The only random variable with support $\{k\Delta, (k+1)\Delta, (k+2)\Delta\}$ and first moments $\mu_1^{(k)}/\mu_0^{(k)}$ and $\mu_2^{(k)}/\mu_0^{(k)}$ is the 3-convex maximum

$$X_{k,\max}^{(3)} = \begin{cases} k\Delta & \text{with probability } \dfrac{\mu_2^{(k)} - \mu_1^{(k)}(2k+3)\Delta + \mu_0^{(k)}(k+1)(k+2)\Delta^2}{\mu_0^{(k)} 2\Delta^2}, \\[2ex] (k+1)\Delta & \text{with probability } \dfrac{-\mu_2^{(k)} + \mu_1^{(k)}(2k+2)\Delta - \mu_0^{(k)}k(k+2)\Delta^2}{\mu_0^{(k)}\Delta^2}, \\[2ex] (k+2)\Delta & \text{with probability } \dfrac{\mu_2^{(k)} - \mu_1^{(k)}(2k+1)\Delta + \mu_0^{(k)}k(k+1)\Delta^2}{\mu_0^{(k)} 2\Delta^2}. \end{cases}$$

The weights associated to $k\Delta$, $(k+1)\Delta$ and $(k+2)\Delta$ are thus respectively

$$w_k^+ = \frac{\mu_2^{(k)} - \mu_1^{(k)}(2k+3)\Delta + \mu_0^{(k)}(k+1)(k+2)\Delta^2}{2\Delta^2}$$

$$w_{k+1} = \frac{-\mu_2^{(k)} + \mu_1^{(k)}(2k+2)\Delta - \mu_0^{(k)}k(k+2)\Delta^2}{\Delta^2}$$

$$w_{k+2}^- = \frac{\mu_2^{(k)} - \mu_1^{(k)}(2k+1)\Delta + \mu_0^{(k)}k(k+1)\Delta^2}{2\Delta^2}.$$

Consequently, if $X_\Delta^{(3)}$ is the arithmetized risk based on the knowledge of the first two moments of X, we have $\mathbb{P}[X_\Delta^{(3)} = 0] = F_X(0) + w_0^+$ and, for $k = 0, 2, 4, \ldots$, $\mathbb{P}[X_\Delta^{(3)} = (k+1)\Delta] = w_{k+1}$ and $\mathbb{P}[X_\Delta^{(3)} = (k+2)\Delta] = w_{k+2}^- + w_{k+2}^+$.

3.3. Three moments known

On each interval $I_k = (k\Delta, (k+3)\Delta]$, $k = 0, 3, 6, \ldots$, the first local moments of X over I_k are

$$\mu_1^{(k)} = \int_{k\Delta}^{(k+3)\Delta} x\,dF_X(x), \ \mu_2^{(k)} = \int_{k\Delta}^{(k+3)\Delta} x^2\,dF_X(x), \text{ and } \mu_3^{(k)} = \int_{k\Delta}^{(k+3)\Delta} x^3\,dF_X(x).$$

Let us now consider the set of all the probability distributions with support $\{k\Delta, (k+1)\Delta, (k+2)\Delta, (k+3)\Delta\}$ and first three moments $\mu_1^{(k)}/\mu_0^{(k)}$, $\mu_2^{(k)}/\mu_0^{(k)}$ and $\mu_3^{(k)}/\mu_0^{(k)}$. The conditions for that set to be not void are

$$\begin{cases} k\Delta\mu_0^{(k)} < \mu_1^{(k)} < (k+3)\Delta\mu_0^{(k)}, \\ \mu_2^{(k)} < \Delta(2k+3)\mu_1^{(k)} - k(k+3)\Delta^2\mu_0^{(k)}, \\ \mu_2^{(k)} > \Delta(2k+1)\mu_1^{(k)} - k(k+1)\Delta^2\mu_0^{(k)}, \\ \mu_2^{(k)} > \Delta(2k+3)\mu_1^{(k)} - (k+1)(k+2)\Delta^2\mu_0^{(k)}, \\ \mu_2^{(k)} > \Delta(2k+5)\mu_1^{(k)} - (k+2)(k+3)\Delta^2\mu_0^{(k)}, \\ \mu_3^{(k)} < \Delta(3k+4)\mu_2^{(k)} - \Delta^2\left(3k^2 + 8k + 3\right)\mu_1^{(k)} + k(k+1)(k+3)\Delta^3\mu_0^{(k)}, \\ \mu_3^{(k)} < \Delta(3k+6)\mu_2^{(k)} - \Delta^2\left(3k^2 + 12k + 11\right)\mu_1^{(k)} + (k+1)(k+2)(k+3)\Delta^3\mu_0^{(k)}, \\ \mu_3^{(k)} > 3\Delta(k+1)\mu_2^{(k)} - \Delta^2\left(3k^2 + 6k + 2\right)\mu_1^{(k)} + k(k+1)(k+2)\Delta^3\mu_0^{(k)}, \\ \mu_3^{(k)} > \Delta(3k+5)\mu_2^{(k)} - \Delta^2\left(3k^2 + 10k + 6\right)\mu_1^{(k)} + k(k+2)(k+3)\Delta^3\mu_0^{(k)}. \end{cases} \quad (3)$$

As $\int_{k\Delta}^{(k+3)\Delta}(x - k\Delta)\,dF(x) \geq 0$, $\int_{k\Delta}^{(k+3)\Delta}((k+3)\Delta - x)\,dF(x) \geq 0$ and $\int_{k\Delta}^{(k+3)\Delta}(x - k\Delta)((k+3)\Delta - x)\,dF(x) \geq 0$, system (3) reduces to

$$\begin{cases} \int_{k\Delta}^{(k+3)\Delta}(x - k\Delta)(x - (k+1)\Delta)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+3)\Delta}(x - (k+1)\Delta)(x - (k+2)\Delta)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+3)\Delta}(x - (k+2)\Delta)(x - (k+3)\Delta)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+3)\Delta}(x - k\Delta)(x - (k+1)\Delta)((k+3)\Delta - x)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+3)\Delta}(x - (k+1)\Delta)(x - (k+2)\Delta)((k+3)\Delta - x)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+3)\Delta}(x - k\Delta)(x - (k+1)\Delta)(x - (k+2)\Delta)\,dF(x) > 0, \\ \int_{k\Delta}^{(k+3)\Delta}(x - k\Delta)(x - (k+2)\Delta)(x - (k+3)\Delta)\,dF(x) > 0. \end{cases}$$

The only random variable with support $\{k\Delta, (k+1)\Delta, (k+2)\Delta, (k+3)\Delta\}$ and first moments $\mu_1^{(k)}/\mu_0^{(k)}$, $\mu_2^{(k)}/\mu_0^{(k)}$ and $\mu_3^{(k)}/\mu_0^{(k)}$ is the 4-convex maximum

$$X_{k,\,\max}^{(4)} = \begin{cases} k\Delta & \text{with probability } \dfrac{-\mu_3^{(k)} + \Delta(3k+6)\mu_2^{(k)} - \Delta^2\left(3k^2 + 12k + 11\right)\mu_1^{(k)} + (k+1)(k+2)(k+3)\Delta^3\mu_0^{(k)}}{6\mu_0^{(k)}\Delta^3}, \\[2ex] (k+1)\Delta & \text{with probability } \dfrac{\mu_3^{(k)} - \Delta(3k+5)\mu_2^{(k)} + \Delta^2\left(3k^2 + 10k + 6\right)\mu_1^{(k)} - k(k+2)(k+3)\Delta^3\mu_0^{(k)}}{2\mu_0^{(k)}\Delta^3}, \\[2ex] (k+2)\Delta & \text{with probability } \dfrac{-\mu_3^{(k)} + \Delta(3k+4)\mu_2^{(k)} - \Delta^2\left(3k^2 + 8k + 3\right)\mu_1^{(k)} + k(k+1)(k+3)\Delta^3\mu_0^{(k)}}{2\mu_0^{(k)}\Delta^3}, \\[2ex] (k+3)\Delta & \text{with probability } \dfrac{\mu_3^{(k)} - 3\Delta(k+1)\mu_2^{(k)} + \Delta^2\left(3k^2 + 6k + 2\right)\mu_1^{(k)} - k(k+1)(k+2)\Delta^3\mu_0^{(k)}}{6\mu_{\cdot}^{(k)}\Delta^3}. \end{cases}$$

The weights associated to $k\Delta$, $(k+1)\Delta$, $(k+2)\Delta$ and $(k+3)\Delta$ are thus respectively

$$w_k^+ = \frac{-\mu_3^{(k)} + \Delta(3k+6)\mu_2^{(k)} - \Delta^2\left(3k^2 + 12k + 11\right)\mu_1^{(k)} + (k+1)(k+2)(k+3)\Delta^3\mu_0^{(k)}}{6\mu_0^{(k)}\Delta^3}$$

$$w_{k+1} = \frac{\mu_3^{(k)} - \Delta(3k+5)\mu_2^{(k)} + \Delta^2\left(3k^2 + 10k + 6\right)\mu_1^{(k)} - k(k+2)(k+3)\Delta^3\mu_0^{(k)}}{2\mu_0^{(k)}\Delta^3}$$

$$w_{k+2} = \frac{-\mu_3^{(k)} + \Delta(3k+4)\mu_2^{(k)} - \Delta^2\left(3k^2 + 8k + 3\right)\mu_1^{(k)} + k(k+1)(k+3)\Delta^3\mu_0^{(k)}}{2\mu_0^{(k)}\Delta^3}$$

$$w_{k+3}^- = \frac{\mu_3^{(k)} - 3\Delta(k+1)\mu_2^{(k)} + \Delta^2\left(3k^2 + 6k + 2\right)\mu_1^{(k)} - k(k+1)(k+2)\Delta^3\mu_0^{(k)}}{6\mu_{\cdot}^{(k)}\Delta^3}.$$

Consequently, if $X_\Delta^{(4)}$ is the arithmetized risk based on the knowledge of the first three moments of X, we have $\mathbb{P}[X_\Delta^{(4)} = 0] = F_X(0) + w_0^+$ and, for $k = 0, 3, 6, \ldots$, $\mathbb{P}[X_\Delta^{(4)} = (k+1)\Delta] = w_{k+1}$, $\mathbb{P}[X_\Delta^{(4)} = (k+2)\Delta] = w_{k+2}$, and $\mathbb{P}[X_\Delta^{(4)} = (k+3)\Delta] = w_{k+3}^- + w_{k+3}^+$.

3.4. General case

Given any risk X with support included in $[0, +\infty)$ and probability distribution function denoted by F_X, the problem addressed in this section is to choose a span $\Delta > 0$ and try to determine the arithmetic distribution with support points $i\Delta$, $i = 0, 1, 2, \ldots$, that has the property of equating $s - 1$ moments with F_X.

To that end, we will proceed as follows. On each of the intervals $I_k = (k(s-1)\Delta, (k+1)(s-1)\Delta]$ ($k = 0, 1, 2, \ldots$), let us denote by $\mu_j^{(k)}$, $j = 0, 1, \ldots, s-1$, the local moment of X on I_k, that is

$$\mu_j^{(k)} = \int_{k(s-1)\Delta}^{(k+1)(s-1)\Delta} x^j dF_X(x).$$

For $j = 0$, $\mu_0^{(k)}$ is the probability mass for the interval I_k so that $\mu_j^{(k)}/\mu_0^{(k)}$ are the conditional moments. Furthermore, we suppose that the moment space $\mathcal{M}_s(\mathcal{E}_s^{(k)}; \frac{\mu_1^{(k)}}{\mu_0^{(k)}}, \frac{\mu_2^{(k)}}{\mu_0^{(k)}}, \ldots, \frac{\mu_{s-1}^{(k)}}{\mu_0^{(k)}})$ of all the random variables valued in $\mathcal{E}_s^{(k)} = k(s-1)\Delta + \{0, \Delta, \ldots, (s-1)\Delta\}$ and with prescribed first $s-1$ moments $\frac{\mu_j^{(k)}}{\mu_0^{(k)}}$ ($j = 1, 2, \ldots, s-1$) is non void. Such conditions are recalled in Section 2. Then, the unique element of $\mathcal{M}_s(\mathcal{E}_s^{(k)}; \frac{\mu_1^{(k)}}{\mu_0^{(k)}}, \frac{\mu_2^{(k)}}{\mu_0^{(k)}}, \ldots, \frac{\mu_{s-1}^{(k)}}{\mu_0^{(k)}})$ is the s-convex maximum. The moment-based discretisation method thus amounts to choose, on each interval I_k, the extremal random variable with support points $k(s-1)\Delta, (k(s-1)+1)\Delta, \ldots, (k+1)(s-1)\Delta$.

4. NUMERICAL APPLICATIONS

In this section, we will present numerical applications of the moment-based discretisation method presented in Section 3. Specifically, we study the admissibility of the local moment matching method when discretizing Exponential, Gamma, Pareto and Log-Normal distributions for various values of their parameters. These parametric models have the following moments as functions of their parameters

- $X \sim \mathcal{E}xp(\theta) \Rightarrow \mathbb{E}[X^k] = \frac{k!}{\theta^k}$, $k = 1, 2, \ldots$;

- $X \sim \mathcal{G}am(a, \tau) \Rightarrow \mathbb{E}[X^k] = \frac{1}{\tau^k} \prod_{i=0}^{k-1}(\alpha + i)$, $k = 1, 2, \ldots$;

- $X \sim \mathcal{P}ar(a, \theta) \Rightarrow \mathbb{E}[X^k] = \frac{k! \alpha \theta^k (\alpha - k - 1)!}{\alpha!}$ if $\alpha > k$, $k = 1, 2, \ldots$;

- $X \sim \mathcal{L}\mathcal{N}or(\mu, \sigma) \Rightarrow \mathbb{E}[X^k] = \exp\left(\mu k + \frac{\sigma^2 k^2}{2}\right)$, $k = 1, 2, \ldots$.

In Tables 1, 2, 3, 4, 5, 6, 7, 8 and 9, we study the admissibility (i.e. when all the derived masses are positive) of the local moment matching method in term of the number of moments used and the chosen span. Since the local moment matching method is applicable only if the random variable to be discretized has a bounded support, the admissibility of the method is tested with respect to arbitrary fixed upper limits of the supports, denoted by N (see Tables 1, 2, 3, 4). We see from these examples that there is a maximum span $\Delta_{\max} > 0$ for which, for all given span $\Delta > 0$ such that $(s-1)\Delta < N$ (i.e. there is more than one interval I_k in Section 3) and $\Delta \leq \Delta_{\max}$, the local moment matching method leads to a discretisation of the distribution of interest with positive masses. In view of Tables 1, 2, 3 and 4, it appears that the chosen value for N does not seem to have any impact on the admissibility of the span Δ. Therefore, we work with $N = 5$ in the subsequent tables. We see from Tables 5-9 that the admissibility pattern is always the same: the discretization becomes possible provided Δ becomes small enough. Moreover, a close examination of Tables 5-9 reveals that it seems to be easier to discretize these distributions as they become "larger", that is, as their expected value increases. In the Negative Exponential

TABLE 1

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD WITH TWO OR THREE MOMENTS KNOWN, $\theta_{EXP} = 0.6$, $\alpha_{GAM} = 2$, $\tau_{GAM} = 1$, $\alpha_{PAR} = 4$, $\theta_{PAR} = 5$, $\mu_{LN} = 0$, $\sigma_{LN} = 1.5$ and $N = 2.5$.

Upper limit	Span	Exponential	Gamma	Pareto	Log-Normal
$s = 3$	2	Yes	Yes	No	No
	1.75	Yes	Yes	Yes	Yes
	1.5	Yes	Yes	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes
$s = 4$	2	Yes	Yes	No	No
	1.75	Yes	Yes	No	No
	1.5	Yes	Yes	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes

case, for instance, we see from Table 5 that smaller spans are admissible as θ decreases.

All the examples discussed so far consist in discretizing a continuous distribution. Let us now apply this approach to arithmetize a discrete distribution with an irregularly spaced support. To this end, we consider the hypothetical claim severity distribution given in WALHIN & PARIS (1998):

Support points	0	7	12	17	21	23	28	39	46	53	67
Probabilities	0.05	0.1	0.15	0.05	0.05	0.05	0.1	0.1	0.1	0.15	0.1

TABLE 2

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD WITH TWO OR THREE MOMENTS KNOWN,
$\theta_{EXP} = 0.6$, $\alpha_{GAM} = 2$, $\tau_{GAM} = 1$, $\alpha_{PAR} = 4$, $\theta_{PAR} = 5$, $\mu_{LN} = 0$, $\sigma_{LN} = 1.5$ and $N = 5$.

Upper limit	Span	Exponential	Gamma	Pareto	Log-Normal
$s = 3$	3	No	No	No	No
	2.75	No	No	No	No
	2.5	No	Yes	No	No
	2.25	No	No	Yes	Yes
	2	Yes	No	Yes	Yes
	1.75	Yes	No	Yes	Yes
	1.5	Yes	Yes	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes
$s = 4$	3	No	No	No	No
	2.75	No	No	No	No
	2.5	No	No	No	No
	2.25	No	Yes	No	No
	2	Yes	Yes	No	No
	1.75	Yes	Yes	No	No
	1.5	Yes	No	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes

TABLE 3

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD WITH TWO OR THREE MOMENTS KNOWN, $\theta_{EXP} = 0.6$, $\alpha_{GAM} = 2$, $\tau_{GAM} = 1$, $\alpha_{PAR} = 4$, $\theta_{PAR} = 5$, $\mu_{LN} = 0$, $\sigma_{LN} = 1.5$ and $N = 7.5$.

Upper limit	Span	Exponential	Gamma	Pareto	Log-Normal
$s = 3$	4	No	No	No	No
	3.75	No	No	No	No
	3.5	No	No	No	No
	3.25	No	No	No	No
	3	No	No	No	Yes
	2.75	No	No	No	Yes
	2.5	No	No	No	Yes
	2.25	No	No	Yes	Yes
	2	Yes	No	Yes	Yes
	1.75	Yes	No	Yes	Yes
	1.5	Yes	Yes	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes
$s = 4$	4	No	No	No	No
	3.75	No	No	No	No
	3.5	No	No	No	No
	3.25	No	No	No	No
	3	No	No	No	No
	2.75	No	No	No	No
	2.5	No	No	No	No
	2.25	No	No	No	No
	2	Yes	No	No	No
	1.75	Yes	No	No	No
	1.5	Yes	No	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes

TABLE 4

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD WITH TWO OR THREE MOMENTS KNOWN, $\theta_{EXP} = 0.6$, $\alpha_{GAM} = 2$, $\tau_{GAM} = 1$, $\alpha_{PAR} = 4$, $\theta_{PAR} = 5$, $\mu_{LN} = 0$, $\sigma_{LN} = 1.5$ and $N = 10$.

Upper limit	Span	Exponential	Gamma	Pareto	Log-Normal
$s = 3$	5	No	No	No	No
	4.75	No	No	No	No
	4.5	No	No	No	No
	4.25	No	No	No	No
	4	No	No	No	No
	3.75	No	No	No	No
	3.5	No	No	No	No
	3.25	No	No	No	No
	3	No	No	No	Yes
	2.75	No	No	No	Yes
	2.5	No	No	No	Yes
	2.25	No	No	Yes	Yes
	2	Yes	No	Yes	Yes
	1.75	Yes	No	Yes	Yes
	1.5	Yes	No	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes
$s = 4$	5	No	No	No	No
	4.75	No	No	No	No
	4.5	No	No	No	No
	4.25	No	No	No	No
	4	No	No	No	No
	3.75	No	No	No	No
	3.5	No	No	No	No
	3.25	No	No	No	No
	3	No	No	No	No
	2.75	No	No	No	No
	2.5	No	No	No	No
	2.25	No	No	No	No
	2	Yes	No	No	No
	1.75	Yes	No	No	No
	1.5	Yes	No	Yes	Yes
	1.25	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes

TABLE 5

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD ON EXPONENTIALLY DISTRIBUTED RISKS
WITH PARAMETER θ, TWO OR THREE MOMENTS KNOWN AND $N = 5$.

	Span	$\theta = 0.2$	$\theta = 0.5$	$\theta = 1$	$\theta = 2$
$s = 3$	3	Yes	No	No	No
	2.75	Yes	No	No	No
	2.5	Yes	Yes	No	No
	2.25	Yes	Yes	No	No
	2	Yes	Yes	No	No
	1.75	Yes	Yes	No	No
	1.5	Yes	Yes	No	No
	1.25	Yes	Yes	Yes	No
	1	Yes	Yes	Yes	No
	0.75	Yes	Yes	Yes	No
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes
$s = 4$	3	Yes	No	No	No
	2.75	Yes	No	No	No
	2.5	Yes	No	No	No
	2.25	Yes	Yes	No	No
	2	Yes	Yes	No	No
	1.75	Yes	Yes	No	No
	1.5	Yes	Yes	No	No
	1.25	Yes	Yes	No	No
	1	Yes	Yes	Yes	No
	0.75	Yes	Yes	Yes	No
	0.5	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes

Table 10 displays the results obtained in this case. Contrarily to the continuous case, there seems now to exist "extremal" spans $\Delta_{\min} > 0$ and $\Delta_{\max} > 0$ for which, for all given span $\Delta > 0$ (such that there is more than one interval I_k in Section 3) and $\Delta_{\min} \leq \Delta \leq \Delta_{\max}$, the local moment matching method leads to a discretisation of the distribution of interest with positive masses. When discretisation is possible, we also give the first three moments of the approached discrete distributions (see Table 10). When using $s - 1$ moments, the more Δ tends to $\Delta_{\min}$, the more the sth moment tends to the true one. This was

TABLE 6

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD ON GAMMA DISTRIBUTED RISKS
WITH PARAMETERS (α, τ), TWO OR THREE MOMENTS KNOWN AND $N = 5$.

	Span	$\alpha = 0.5$			$\alpha = 1$		
		$\tau = 0.25$	$\tau = 0.5$	$\tau = 1$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 1$
$s = 3$	3	No	No	No	Yes	No	No
	2.75	Yes	No	No	Yes	No	No
	2.5	Yes	No	No	Yes	Yes	No
	2.25	Yes	No	No	Yes	Yes	No
	2	Yes	No	No	Yes	Yes	No
	1.75	Yes	Yes	No	Yes	Yes	No
	1.5	Yes	Yes	No	Yes	Yes	No
	1.25	Yes	Yes	No	Yes	Yes	Yes
	1	Yes	Yes	No	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes	Yes	Yes
$s = 4$	3	No	No	No	Yes	No	No
	2.75	No	No	No	Yes	No	No
	2.5	Yes	No	No	Yes	No	No
	2.25	Yes	No	No	Yes	Yes	No
	2	Yes	No	No	Yes	Yes	No
	1.75	Yes	No	No	Yes	Yes	No
	1.5	Yes	No	No	Yes	Yes	No
	1.25	Yes	Yes	No	Yes	Yes	No
	1	Yes	Yes	No	Yes	Yes	Yes
	0.75	Yes	Yes	No	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes	Yes	Yes

expected because, as the span decreases, the approached distribution tends to the true one.

5. DISCUSSION

In this paper, it appears that arithmetizing a risk by the local moment matching method with $s-1$ moments is equivalent to locally consider the discrete

TABLE 7

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD ON GAMMA DISTRIBUTED RISKS
WITH PARAMETERS (α, τ), TWO OR THREE MOMENTS KNOWN AND $N = 5$.

	Span	$\alpha = 1.5$			$\alpha = 2$		
		$\tau = 0.25$	$\tau = 0.5$	$\tau = 1$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 1$
$s = 3$	3	Yes	Yes	No	Yes	Yes	No
	2.75	Yes	Yes	No	Yes	Yes	No
	2.5	Yes	Yes	No	Yes	Yes	Yes
	2.25	Yes	Yes	No	Yes	Yes	No
	2	Yes	Yes	No	Yes	Yes	No
	1.75	Yes	Yes	No	Yes	Yes	No
	1.5	Yes	Yes	Yes	Yes	Yes	Yes
	1.25	Yes	Yes	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes	Yes	Yes
$s = 4$	3	Yes	Yes	No	Yes	Yes	No
	2.75	Yes	Yes	No	Yes	Yes	No
	2.5	Yes	Yes	No	Yes	Yes	No
	2.25	Yes	Yes	No	Yes	Yes	No
	2	Yes	Yes	No	Yes	Yes	No
	1.75	Yes	Yes	No	Yes	Yes	No
	1.5	Yes	Yes	No	Yes	Yes	No
	1.25	Yes	Yes	Yes	Yes	Yes	Yes
	1	Yes	Yes	Yes	Yes	Yes	Yes
	0.75	Yes	Yes	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes	Yes	Yes

s-convex extremal risks with appropriate moments. This allows us to give simple and explicit criteria to find quickly when the local moment matching method will fail to produce a bona fide discrete probability distribution. Explicit conditions to be fulfilled have been derived here for the 1-, 2-, and 3-moment case.

To end with, let us mention that, in case a certain local moment sequence is not admissible, VILAR (2000) proposed a complementary approach based

TABLE 8

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD ON PARETO DISTRIBUTED RISKS
WITH PARAMETERS (α, θ), TWO OR THREE MOMENTS KNOWN AND $N = 5$.

	Span	$\theta = 2$			$\theta = 4$			$\theta = 6$		
		$\alpha = 4$	$\alpha = 5$	$\alpha = 6$	$\alpha = 4$	$\alpha = 5$	$\alpha = 6$	$\alpha = 4$	$\alpha = 5$	$\alpha = 6$
$s = 3$	3	No	No	No	No	No	No	No	No	No
	2.75	No	No	No	No	No	No	No	No	No
	2.5	No	No	No	No	No	No	No	No	No
	2.25	No	No	No	No	No	No	Yes	No	No
	2	No	No	No	No	No	No	Yes	Yes	No
	1.75	No	No	No	Yes	No	No	Yes	Yes	Yes
	1.5	No	No	No	Yes	No	No	Yes	Yes	Yes
	1.25	No	No	No	Yes	Yes	No	Yes	Yes	Yes
	1	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
	0.75	Yes	No	No	Yes	Yes	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$s = 4$	3	No	No	No	No	No	No	No	No	No
	2.75	No	No	No	No	No	No	No	No	No
	2.5	No	No	No	No	No	No	No	No	No
	2.25	No	No	No	No	No	No	No	No	No
	2	No	No	No	No	No	No	Yes	No	No
	1.75	No	No	No	No	No	No	Yes	No	No
	1.5	No	No	No	No	No	No	Yes	Yes	No
	1.25	No	No	No	Yes	No	No	Yes	Yes	Yes
	1	No	No	No	Yes	Yes	No	Yes	Yes	Yes
	0.75	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
	0.5	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
	0.25	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	0.1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	0.05	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

on the nearness of local moment built in terms of linear programming. More precisely, if a certain sequence of n local moments is not admissible, then the author proposes to determine the arithmetic distribution which conserves the $n-1$ first moments and has the nearest nth moment to the true one. Obviously, if the sequence of n moments is admissible, then the solution coincides with the one obtained from the local moment matching method with n moments.

TABLE 9

ADMISSIBILITY OF THE LOCAL MOMENT MATCHING METHOD ON LOG-NORMALLY DISTRIBUTED RISKS
WITH PARAMETERS $\mu = 0$ AND σ, TWO OR THREE MOMENTS KNOWN AND $N = 5$.

	Span	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$
$s = 3$	3	No	No	No
	2.75	No	No	No
	2.5	No	No	No
	2.25	No	No	Yes
	2	No	Yes	Yes
	1.75	No	Yes	Yes
	1.5	No	Yes	Yes
	1.25	No	Yes	Yes
	1	No	Yes	Yes
	0.75	Yes	Yes	Yes
	0.5	No	Yes	Yes
	0.25	No	Yes	Yes
	0.1	No	No	Yes
	0.05	No	No	Yes
$s = 4$	3	No	No	No
	2.75	No	No	No
	2.5	No	No	No
	2.25	No	No	No
	2	No	No	No
	1.75	No	No	No
	1.5	No	No	Yes
	1.25	No	Yes	Yes
	1	No	Yes	Yes
	0.75	No	Yes	Yes
	0.5	No	Yes	Yes
	0.25	No	Yes	Yes
	0.1	No	Yes	Yes
	0.05	No	Yes	Yes

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TABLE 10

LOCAL MOMENTS AND ADMISSIBILITY WITH THE HYPOTHETICAL CLAIM SEVERITY DISTRIBUTION
GIVEN IN WALHIN & PARIS (1998)

		Admissibility	μ_1	μ_2	μ_3
True moments			31.5	1401.8	71879.1
Local moments $s = 2$	$\Delta = 1$	Yes	31.5	1401.8	71879.1
	$\Delta = 5$	Yes	31.5	1407	72390
	$\Delta = 10$	Yes	31.5	1419	73650
	$\Delta = 15$	Yes	31.5	1443	76072.5
	$\Delta = 20$	Yes	31.5	1474	79200
	$\Delta = 25$	Yes	31.5	1505	81562.5
Local moments $s = 3$	$\Delta = 1$	No			
	$\Delta = 5$	No			
	$\Delta = 10$	No			
	$\Delta = 15$	No			
	$\Delta = 16$	No			
	$\Delta = 16.7499$	No			
	$\Delta = 16.7500$	Yes	31.5	1401.8	71650.22
	$\Delta = 17$	Yes	31.5	1401.8	71639.7
	$\Delta = 18$	Yes	31.5	1401.8	71523
	$\Delta = 19$	Yes	31.5	1401.8	71335.5
	$\Delta = 19.7532$	Yes	31.5	1401.8	71183.75
	$\Delta = 19.7533$	No			
	$\Delta = 20$	No			
	$\Delta = 25$	No			
Local moments $s = 4$	$\Delta = 1$	No			
	$\Delta = 5$	No			
	$\Delta = 7.4514$	Yes	31.5	1401.8	71879.1
	$\Delta = 8.6402$	Yes	31.5	1401.8	71879.1
	$\Delta = 10$	No			
	$\Delta = 15$	No			
	$\Delta = 16$	No			
	$\Delta = 17$	No			
	$\Delta = 18$	No			
	$\Delta = 19$	No			
	$\Delta = 20$	No			

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CINDY COURTOIS
Institut des Sciences Actuarielles
Université Catholique de Louvain
rue des Wallons 6
B-1348 Louvain-la-Neuve
Belgium

MICHEL DENUIT
Institut de Statistique et Institut des Sciences Actuarielles
Université Catholique de Louvain
rue des Wallons 6
B-1348 Louvain-la-Neuve
Belgium