BONUS-MALUS SYSTEMS WITH VARYING DEDUCTIBLES

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ABSTRACT

Bonus-malus systems typically lead to high maluses when claims at fault are reported. Such penalties are often difficult to implement in practice. It is shown in this paper that this drawback may be avoided by combining a posteriori premium corrections with a deductible varying according to the level occupied in the scale.

KEY WORDS

Bonus-Malus system, differentiated bonus scale deductibles, Markov chains

1. INTRODUCTION AND MOTIVATION

Rating systems that penalize insureds responsible for one or more accidents by premium surcharges (or *maluses*), and that reward claim-free policyholders by giving them discounts (or *bonuses*) are now in force in many developed countries. Besides encouraging policyholders to drive carefully (i.e. counteracting moral hazard), they aim to better assess individual risks. Such systems are called no-claim discounts, experience rating, merit rating, or bonus-malus systems (BMS, in short). We will adhere to the last terminology in this paper. For a thorough presentation of the techniques relating to BMS, see LEMAIRE (1995).

However, as pointed out by several authors, BMS also suffer from considerable drawbacks:

1. the claim amounts are not used so that a posteriori corrections only rely on the number of claims.
2. policyholders may leave the company after having caused claims, in order to avoid the penalties.

This led HOLTAN (1994) to introduce an alternative approach to BMS that eliminates these disadvantages. This author suggested the use of very high deductibles that may be borrowed by the policyholder to the insurance company. Although technically acceptable, this approach obviously causes considerable practical problems.

While HOLTAN (1994) assumes a high deductible which is constant for all policyholders, and thus independent of the level they occupy in the bonus-malus scale at the claim occurrence time, the present paper lets the deductible vary between the levels of the BMS, and also considers a mixed case setup of both premium and deductible surcharge after a claim. Note that imposing a deductible to the policyholder solves the second drawback mentioned above because the deductible is paid by the policyholder after a claim.

Specifically, the a posteriori premium correction induced by the BMS is replaced with a deductible (in whole or in part). To each level of the BMS is attached an amount of deductible, applied to the claims filed during the coverage period. Although deductibles can be difficult to implement in motor third party liability insurance (companies indemnify the third parties directly and so cannot simply reduce the amount they pay; see HOLTAN (1994) and LEMAIRE & HONGMIN (1994a) for a detailed account of this problem), the system is nevertheless easy to implement for other coverages for which the payments are made directly to the policyholders, like material damage for instance. In the EU, the premium for the material damage cover is either subject to the BMS applying to motor third party liability or to a specific BMS. In the latter case, using the techniques suggested in this paper, one could replace BMS for material damage with deductibles determined by past claim history.

The paper is organised as follows. Section 2 describes the modelling of annual claim amounts. Section 3 recalls the construction of BMS, in the framework of NORBERG (1976). Section 4 explains how the penalties induced by BMS can be replaced with deductibles. Section 5 contains some numerical illustrations. The final Section 6 concludes.

Before proceeding with the results, let us precisely state the assumptions on which this paper is based:

1. the portfolio is homogeneous with respect to claim severities
2. the portfolio is heterogeneous with respect to claim frequencies, following a mixed Poisson distribution
3. there is no a priori risk classification (or we work inside a specified rating cell)
4. the portfolio is closed for ingoing and outgoing policyholders.

We will comment on these restrictions in the final section of this paper.

2. MODELLING CLAIM COSTS

The framework of credibility theory, with its fundamental notion of randomly distributed risk parameters, was employed in analysis of BMS by Pesonen as early as 1963. To be specific, let us consider a portfolio of motor insurance policies. Let us pick at random a policy from this portfolio. The number of claims N caused by the policyholder is assumed to be mixed Poisson distributed. More precisely, N has conditional discrete probability mass function of the form

$$\Pr[N = k \,|\, \Theta = \theta] = \exp(-\lambda\theta)\frac{(\lambda\theta)^k}{k!}, \quad k = 0, 1, 2, \ldots \qquad (2.1)$$

The random effect Θ, taken with unit mean, represents the risk proneness of the policyholder, i.e. unknown risk characteristics having a significant impact on the occurrence of claims. It is regarded as a random variable. The probability density function of Θ, denoted as $u(\cdot)$, is called the structure function since it describes the composition of the portfolio with respect to expected annual claim frequency. Unconditionally, the discrete probability mass function of N is obtained by averaging the conditional probabilities (2.1) over the domain of Θ, that is

$$\Pr[N = k] = \int_0^{+\infty} \Pr[N = k \mid \Theta = \theta] u(\theta)\, d\theta, \quad k = 0, 1, 2, \ldots$$

In the numerical illustration, we will take Θ distributed according to a Gamma law, so that N will follow a Negative binomial distribution.

Let us denote as $C_1, C_2, \ldots, C_N$ the amounts of the N claims reported by the policyholder. The total claim amount for this policy is

$$S = \sum_{k=1}^{N} C_k,$$

with the convention that the empty sum equals 0. The severities $C_1, C_2, \ldots,$ are assumed to be independent and identically distributed, and independent of the claim frequency N. It is worth mentioning that this assumption has been questioned by several authors. It essentially states that the cost of an accident is for the most part beyond the control of a policyholder. The degree of care exercised by a driver mostly influences the number of accidents, but in a much lesser way the cost of these accidents. Nevertheless, this assumption seems acceptable in practice, at least as an approximation. Note that the severities C_1, $C_2, \ldots,$ are also independent of Θ.

In the numerical illustrations, two different models will be considered for the C_k's: the exponential and the lognormal distributions. The former is light-tailed whereas the latter is longer-tailed. The thickness of the tail will be seen to affect the amount of the deductibles.

Considering the expression (2.2) for the total cost of claims S, the pure premium for a policyholder without claim history is $\lambda \mathbb{E}[C_1]$. This amount will be corrected by a correction factor (called the relativity) according to the level occupied in the BMS, that is, according to the claims reported to the company in the past.

3. Bonus-malus scale

BMS are supposed to have $s + 1$ levels numbered from 0 to s. To each level ℓ corresponds a relative premium r_ℓ. The policyholder at level ℓ will have to pay r_ℓ times the base premium $\lambda \mathbb{E}[C_1]$ to be covered by the company. The position occupied in the scale depends on the past claims history of each policyholder. When the driver causes an accident at fault, he goes up a certain number

of levels in the scale, resulting in an increase of premium: this is the malus. When the driver does not file any claim, he goes down in the scale, resulting in a lower premium: this is the bonus.

In practice, BMS can be represented by Markov chains. Indeed, the knowledge of the current level and of the number of claims of the current year suffice to determine the next level in the scale. Sometimes, fictitious levels have to be introduced to take into account some special transition rules depending on past claims history; see PITREBOIS ET AL. (2003a) for an example with the former compulsory Belgian BMS. Moreover, the Markov chains are generally irreducible, meaning that all states are always accessible in a finite number of steps from all other states. Also, BMS have a bonus level with maximal reward: policyholders being in that level will remain in that level after a claim free year. These two properties ensure that the Markov chain associated to the BMS has a limiting distribution, which is the stationary distribution.

Let $M(\vartheta)$ be the transition matrix of the Markov chain associated to the BMS for a policyholder with annual mean claim frequency ϑ. Then the limiting distribution can be obtained from various formulas. A convenient choice is

$$\boldsymbol{\pi}(\vartheta) = \boldsymbol{e}^t(\boldsymbol{I} - \boldsymbol{M}(\vartheta) + \boldsymbol{E})^{-1}$$

where e is a column vector of 1's and E is a $(s+1) \times (s+1)$ matrix consisting of $s+1$ column vectors e. See ROLSKI ET AL. (1999) for a derivation of this formula. The component $\pi_\ell(\vartheta)$ of $\boldsymbol{\pi}(\vartheta)$ represents the proportion of drivers at level ℓ of the BMS, once the stationary regime has been attained.

NORBERG (1976) proposed to determine the relativities r_ℓ by minimizing the expected square difference between the true relative premium Θ and the r_ℓ's. This results in

$$r_\ell = \frac{\int_0^\infty \theta \pi_\ell(\lambda\theta) u(\theta) d\theta}{\int_0^\infty \pi_\ell(\lambda\theta) u(\theta) d\theta}, \quad \ell = 0, 1, ..., s. \qquad (3.1)$$

Policyholders in level ℓ of the scale pay $r_\ell \lambda \mathbb{E}[C_1]$ to be covered by the insurance company.

4. INTRODUCING A DEDUCTIBLE WITHIN A POSTERIORI RATEMAKING

Often, the r_ℓ's for high levels ℓ are so large that the system has to be softened before a possible commercial implementation, resulting in financial instability (since the company then faces a progressive decrease of the average premium level because of a clustering of the policyholders in the high-discount classes). To avoid this deficiency, the premium increase that the policyholder has to pay when he goes up in the scale could be (at least partly) replaced by a deductible that would be applied on claims filed by the policyholder during the following year. The company compensates the reduced penalties in the malus zone with the deductibles paid by policyholders who report claims being in the malus

zone. This can be commercially attractive since the policyholders are penalized only if they file claims in the future. The amount of these deductibles depends on the level attained by the policyholder and can be applied either annually or claim by claim.

4.1. Annual deductible

The policyholder occupying level ℓ in the classical BMS should pay $\lambda r_\ell \mathbb{E}[C_1]$ to be covered by the insurance company. If he opts for the scale with deductibles, he would pay only the basis premium, $\lambda \mathbb{E}[C_1]$ but will be subject to an annual deductible d_ℓ.

The amount d_ℓ is found using an indifference principle: on average, the penalties induced by the BMS are equal to the deductibles paid by the policyholders. Let us consider a policyholder occupying level ℓ in the scale. If this policyholder is subject to the a posteriori premium corrections induced by the BMS, he will have to pay $r_\ell \lambda \mathbb{E}[C_1]$ to be covered by the company. If the policyholder is subject to the annual deductible instead, he will have to pay the pure premium $\lambda \mathbb{E}[C_1]$ as well as $\min\{S, d_\ell\}$. The indifference principle is now expressed for level ℓ by the equation

$$r_\ell \lambda \mathbb{E}[C_1] = \lambda \mathbb{E}[C_1] + \mathbb{E}[S|S < d_\ell] \Pr[S < d_\ell] + d_\ell \Pr[S > d_\ell], \qquad (4.1)$$

where S is of the form (2.2) with the counting variable N distributed as a Negative Binomial with mean λr_ℓ (past claims history is used to reevaluate the expected annual claim frequency of the policyholder according to his position in the scale: the annual expected claim frequency equals λr_ℓ for a policyholder occupying level ℓ in the scale). Note that we work with zero interest rate, as it is often the case in nonlife insurance problems, so that we do not use present values but nominal payments.

Equation (4.1) is to be solved for all levels ℓ such that $r_\ell > 100\%$ (that is, for all levels in the malus zone). So, the premium surcharge $(r_\ell - 1) \lambda \mathbb{E}[C_1]$ is replaced with an annual deductible d_ℓ. The relation (4.1) ensures that the substitution is actuarially fair.

In practice, equation (4.1) does not possess an explicit solution so that numerical techniques have to be used. Panjer's algorithm is employed to derive the distribution of S.

4.2. Per claim deductible

Of course, the deductible could also be applied to each claim filed by the policyholder. The indifference principle invoked above will again be used to determine the amount of the deductible. Considering a policyholder in level ℓ, he will have to pay $\lambda r_\ell \mathbb{E}[C_1]$ if he is subject to the a posteriori corrections induced by the BMS. On the contrary, if a fixed deductible d_ℓ is applied per claim, he will have to pay $\lambda \mathbb{E}[C_1]$ as well as $\min\{C_k, d_\ell\}$ for each of the claim C_k reported to

the company. Note that the expected number of claims is now $r_\ell \lambda$ because past claims history is used to update the claim frequency distribution. According to the indifference principle, the amount of deductible d_ℓ for a policyholder in level ℓ is the solution to the equation

$$\lambda r_\ell \mathbb{E}[C_1] = \lambda \mathbb{E}[C_1] + \lambda r_\ell \big(\mathbb{E}[C_1|C_1 < d_\ell]\Pr[C_1 < d_\ell] + d_\ell \Pr[C_1 > d_\ell]\big).$$

This equation can be simplified as

$$r_\ell \mathbb{E}[C_1] = \mathbb{E}[C_1] + r_\ell \big(\mathbb{E}[C_1|C_1 < d_\ell]\Pr[C_1 < d_\ell] + d_\ell \Pr[C_1 > d_\ell]\big). \quad (4.2)$$

Again, (4.2) has to be solved (numerically, in most cases) for all the levels ℓ for which $r_\ell > 100\%$.

4.3. Mixed case

We could also mix both types of penalties. Specifically, the a posteriori corrections r_ℓ are softened and the policyholder is also subject to a deductible (either annual or per claim). This combination allows the actuary to get acceptable r_ℓ's and to achieve financial stability thanks to the deductibles, as it will be seen from the numerical illustration proposed in the next section.

In the mixed case, the bonuses (i.e., the r_ℓ's less than 1) are kept unchanged but the maluses (i.e., the r_ℓ's larger than 1) are reduced by a fixed percentage: instead of r_ℓ, the policyholder will be subject to the penalty $r_\ell \times (1 - \alpha)$ for some specified $0 < \alpha < 1$. To compensate these reduced penalties, the policyholder is subject to deductibles d_ℓ varying according to the level ℓ occupied in the malus zone. The parameter α may be selected in order to achieve a good balance between premium increase and amounts of deductibles. The mixed system, combining relativities and deductibles, is expected to be the most relevant in practice. This will become clear from the numerical illustrations carried out in the next section.

Let us now give the equations providing the d_ℓ's. In case of an annual deductible, the indifference principle allows us to write

$$\lambda r_\ell \mathbb{E}[C_1] = (1 - \alpha)\lambda r_\ell \mathbb{E}[C_1] + \mathbb{E}[\min\{S, d_\ell\}]$$
$$= (1 - \alpha)\lambda r_\ell \mathbb{E}[C_1] + \mathbb{E}[S|S < d_\ell]\Pr[S < d_\ell] + d_\ell \Pr[S > d_\ell] \quad (4.3)$$

for the r_ℓ's larger than 1 (i.e. in the malus zone). The left-hand side of this equation is the amount of premium paid by the policyholder when the standard BMS is in force, while the right-hand side is the average amount paid by the policyholder in the mixed BMS-deductible system (that is, a reduced penalty plus the expected value of $\min\{S, d_\ell\}$). The solution cannot be obtained explicitly.

Let us now turn to the case where deductibles are applied per claim. The same reasoning yields

$$\lambda r_\ell \mathbb{E}[C_1] = (1 - \alpha)\lambda r_\ell \mathbb{E}[C_1] + \lambda r_\ell \mathbb{E}[\min\{C_1, d_\ell\}]$$

so that the d_ℓ's are the solution of

$$\alpha \mathbb{E}[C_1] = \mathbb{E}[\min\{C_1, d_\ell\}] = \mathbb{E}[C_1 | C_1 < d_\ell] \Pr[C_1 < d_\ell] + d_\ell \Pr[C_1 > d_\ell].$$

A noteworthy feature of this case is that the d_ℓ's do not depend on ℓ. The same deductible applies to all the levels in the malus zone. Again, there is no explicit expression for the d_ℓ's in general, and numerical techniques have to be used to solve this equation.

5. Numerical illustrations

5.1. Bonus malus scale

For the numerical illustration, we choose to work with the soft BMS proposed by TAYLOR (1997). There are 9 bonus-malus levels. The starting level is level 6. A higher level number indicates a higher premium. The transition rules are as follows. If no claims have been reported by the policyholder then he moves one level down. If $n > 0$ claims are reported during the year then the policyholder moves $2n$ levels up. The transition rules are described in Table 5.1. The transition matrix $M(\vartheta)$ associated to this BMS is given by

$$M(\vartheta) = \begin{vmatrix} \exp(-\vartheta) & 0 & \vartheta \exp(-\vartheta) & 0 & \frac{\vartheta^2}{2}\exp(-\vartheta) & 0 & \frac{\vartheta^3}{3!}\exp(-\vartheta) & 0 & 1-\Sigma \\ \exp(-\vartheta) & 0 & 0 & \vartheta \exp(-\vartheta) & 0 & \frac{\vartheta^2}{2}\exp(-\vartheta) & 0 & \frac{\vartheta^3}{3!}\exp(-\vartheta) & 1-\Sigma \\ 0 & \exp(-\vartheta) & 0 & 0 & \vartheta \exp(-\vartheta) & 0 & \frac{\vartheta^2}{2}\exp(-\vartheta) & 0 & 1-\Sigma \\ 0 & 0 & \exp(-\vartheta) & 0 & 0 & \vartheta \exp(-\vartheta) & 0 & \frac{\vartheta^2}{2}\exp(-\vartheta) & 1-\Sigma \\ 0 & 0 & 0 & \exp(-\vartheta) & 0 & 0 & \vartheta \exp(-\vartheta) & 0 & 1-\Sigma \\ 0 & 0 & 0 & 0 & \exp(-\vartheta) & 0 & 0 & \vartheta \exp(-\vartheta) & 1-\Sigma \\ 0 & 0 & 0 & 0 & 0 & \exp(-\vartheta) & 0 & 0 & -\exp(-\vartheta) \\ 0 & 0 & 0 & 0 & 0 & 0 & \exp(-\vartheta) & 0 & -\exp(-\vartheta) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \exp(-\vartheta) & 1-\exp(-\vartheta) \end{vmatrix}.$$

where Σ represents the sum of the elements in columns 1 to 8 in the same row.

5.2. Claim frequencies

In this section, the structure function is taken to be a gamma probability density function with unit mean, i.e.

$$u(\theta) = \frac{1}{\Gamma(a)} a^a \theta^{a-1} \exp(-a\theta), \quad \theta > 0,$$

for some $a > 0$. We take the values of λ and a from PITREBOIS ET AL. (2003b), that is $\lambda = 0.1474$ and $a = 0.8888$.

TABLE 5.1.

TRANSITION RULES FOR THE SOFT BMS IN TAYLOR (1997)

Starting level	Level occupied if				
	0	1	2	3	≥ 4
	claim(s) is/are reported				
8	7	8	8	8	8
7	6	8	8	8	8
6	5	8	8	8	8
5	4	7	8	8	8
4	3	6	8	8	8
3	2	5	7	8	8
2	1	4	6	8	8
1	0	3	5	7	8
0	0	2	4	6	8

5.3. Claim severities

For the claim amounts, we work with two distributions with different tails (to investigate the influence of the claim sizes on the deductibles). Considering the inflated 1989 Taiwanese property damage loss distribution used by LEMAIRE & HONGMIN (1994b), we take the C_k's lognormally distributed with parameters $\mu = 9.2576$ and $\sigma^2 = 1.3569$. We also work with exponentially distributed C_k's with the same mean, equal to $\exp(9.2576 + \frac{1.3569}{2}) = 20,662$ for the sake of comparison. The distribution of S is then determined by the Panjer's algorithm. Note that the pure premium amounts to $0.1474 \times 20,662 = 3,045.6$ monetary units.

5.4. Annual deductible

Let us compute the relativities $r_0, r_1, \ldots, r_s$ according to formula (3.1). They are displayed in the second column of Table 5.2. In the pure BMS case, it is thus clear that the r_ℓ's associated to the upper levels are considerable (more than 350% for level 8). Now, let us replace the r_ℓ's in the malus zone (i.e. levels 1 to 8) with an annual deductible d_ℓ. In order to obtain the d_ℓ's from equation (4.1), we first discretize the claim sizes using the one moment matching method. Panjer's algorithm is then used to derive the distribution of S. Finally, (4.1) is solved numerically.

The third column of Table 5.2 displays the new relativities. In this case, the maluses disappear ($r_\ell = 100\%$ for $\ell = 1, \ldots, 8$) and are compensated by the deductibles listed in the two last columns. The fourth column of Table 5.2 shows the deductible to be applied if the loss amounts are exponentially distributed and the last column shows the deductible to be applied if the loss amounts are lognormally distributed. Since the lognormal distribution has a

TABLE 5.2.

RESULTS FOR AN ANNUAL DEDUCTIBLE VARYING
ACCORDING TO THE LEVEL OCCUPIED IN THE MALUS ZONE

Level ℓ	r_ℓ	r_ℓ with deductible	d_ℓ if $C_1 \sim$ Expo	d_ℓ if $C_1 \sim$ LogN
8	353.7%	100%	40,526	63,139
7	306.7%	100%	34,245	50,548
6	262.3%	100%	28,095	45,534
5	231.1%	100%	23,561	41,176
4	189.2%	100%	17,071	33,581
3	170.2%	100%	13,906	29,119
2	122.8%	100%	5,072	12,849
1	114.6%	100%	3,322	8,769
0	58.0%	58.0%	0	0

thicker tail than the exponential one, we expect larger amounts of deductible for the former. This is indeed the case, as it can be seen from Table 5.2.

The very high r_ℓ's in the second column induce high amounts of deductible, even in the exponential case (more than ten times the annual base premiums in the highest levels with exponentially distributed claim amounts and more than fifteen times with lognormally distributed claim amounts). Therefore, this solution seems to be difficult (if not impossible) to implement in practice.

5.5. Per claim deductible

In this case, there is an analytical solution when the claims are exponentially distributed: the deductible d_ℓ involved in (4.2) is simply given by $\ln r_\ell$ times the expected claim cost. No explicit solution is available when claim amounts are lognormally distributed, and numerical procedures have to be used in this case to find the deductibles d_ℓ.

Table 5.3 displays the results obtained for a deductible per claim, when the premium paid by the policyholder is held constant whatever the claim history. As it was the case for the annual deductible, the second column gives the relative premium (computed with the help of (3.1)) associated to each level of the scale in case of a classical BMS. The third column gives the relative premium in case of a scale with the deductible system. The fourth column shows the deductible to be applied if the loss amounts are exponentially distributed and the last column shows the deductible to be applied if the loss amounts are lognormally distributed.

Again, the amounts of deductible displayed in the last two columns of Table 5.3 are very high compared to the annual premium. This results from the severe r_ℓ's listed in column 2. In order to get acceptable amounts of deductible keeping the financial stability of the system, we will combine in the next section softened penalties in the malus zone with moderate deductibles.

TABLE 5.3.

RESULTS FOR A DEDUCTIBLE PER CLAIM VARYING ACCORDING TO THE LEVEL OCCUPIED IN THE MALUS ZONE.

Level ℓ	r_ℓ	r_ℓ with deductible	d_ℓ if $C_1 \sim$ Expo	d_ℓ if $C_1 \sim$ LogN
8	353.7%	100%	26,099	37,034
7	306.7%	100%	23,152	31,284
6	262.3%	100%	19,928	25,504
5	231.1%	100%	17,311	21,191
4	189.2%	100%	13,176	15,031
3	170.2%	100%	10,986	12,077
2	122.8%	100%	4,251	4,228
1	114.6%	100%	2,816	2,766
0	58.0%	58.0%	0	0

TABLE 5.4.

RESULTS FOR AN ANNUAL DEDUCTIBLE VARYING ACCORDING TO THE LEVEL OCCUPIED IN THE MALUS ZONE, COMBINED WITH REDUCED RELATIVITIES r_ℓ.

Level ℓ	r_ℓ	r_ℓ with deductible	d_ℓ if $C_1 \sim$ Expo	d_ℓ if $C_1 \sim$ LogN
8	353.7%	282.9%	7,150	15,304
7	306.7%	245.4%	6,815	15,001
6	262.3%	209.8%	6,495	14,776
5	231.1%	184.9%	6,274	14,515
4	189.2%	151.4%	5,976	14,245
3	170.2%	136.2%	5,840	14,122
2	122.8%	98.2%	5,498	13,816
1	114.6%	91.7%	5,437	13,763
0	58.0%	58.0%	0	0

TABLE 5.5.

RESULTS FOR A DEDUCTIBLE PER CLAIM VARYING ACCORDING TO THE LEVEL OCCUPIED IN THE MALUS ZONE, COMBINED WITH REDUCED RELATIVITIES r_ℓ.

Level ℓ	r_ℓ	r_ℓ with deductible	d_ℓ if $C_1 \sim$ Expo	d_ℓ if $C_1 \sim$ LogN
8	353.7%	282.9%	4,611	4,604
7	306.7%	245.4%	4,611	4,604
6	262.3%	209.8%	4,611	4,604
5	231.1%	184.9%	4,611	4,604
4	189.2%	151.4%	4,611	4,604
3	170.2%	136.2%	4,611	4,604
2	122.8%	98.2%	4,611	4,604
1	114.6%	91.7%	4,611	4,604
0	58.0%	58.0%	0	0

5.6. Annual deductible in the mixed case

Another solution would be to keep the system of maluses but to reduce the penalties r_ℓ applied to the policyholders in the malus zone, for example by $\alpha = 20\%$. In exchange of these reduced r_ℓ's, the policyholders are subject to an annual deductible on the claims they will eventually file.

Equation (4.3) used to compute the annual deductible becomes

$$\lambda r_\ell \mathbb{E}[C_1] = 80\% \, r_\ell \lambda \mathbb{E}[C_1] + \mathbb{E}[S \,|\, S < d_\ell] \Pr[S < d_\ell] + d_\ell \Pr[S > d_\ell] \qquad (5.1.)$$

to be solved for each level ℓ such that $r_\ell > 100\%$.

Table 5.4 displays the numerical results. The r_ℓ's displayed in column 3 are equal to 80% of those in column 2, except for the bonus level 0. Note that the reduced level 1 enters the bonus zone. The annual deductibles in the exponential case (column 4) are now reasonable (about twice the annual premium for most levels) but those in the lognormal case remain considerable (up to five times the pure premium).

5.7. Per claim deductible in the mixed case

If we combine softened penalties for the BMS with deductibles per claim, equation (4.4) becomes

$$20\% \, \mathbb{E}[C_1] = \mathbb{E}[C_1 \,|\, C_1 < d_\ell] \Pr[C_1 < d_\ell] + d_\ell \Pr[C_1 > d_\ell]. \qquad (5.2)$$

As already mentioned, since this equation does not depend on the level ℓ, the amount of deductible will be the same for each level of the scale.

Table 5.5 displays the numerical results. The third column gathers the relativities: those in the malus zone have been reduced by 20% compared to column 2. The last two columns display the amounts of deductible. In this case, the amounts of deductible are reasonable and can be implemented in practice (about 150% of the annual pure premium). Quite surprisingly, the lognormal distribution now produces smaller deductibles than its exponential counterpart.

6. CONCLUSION

Combining BMS with varying deductibles presents a number of advantages:

1. according to signal theory, policyholders choosing varying deductible should be good drivers;
2. even if the policyholder leaves the company after a claim, he has to pay for the deductible (but in motor third party liability insurance, there may be difficulties linked to the collection of the deductible);
3. in the mixed system, the r_ℓ's and the severity of the deductibles may be tuned in an optimal way in order to attract the policyholders.

The numerical illustrations have shown that, provided appropriate values for the parameters are selected, the amounts of deductible are moderate in the mixed case (reduced BMS relativities combined with deductibles per claim).

Even if deductibles are difficultly applicable in motor third party liability insurance, they are particularly appealing in material damage as well as in other motor insurance products. Note that in this case, coinsurance can also be considered: instead of applying different deductibles according to the level occupied by the policyholder, the company could also cover a fixed percentage of the claim amount, and this percentage could be allowed to vary according to the level. The results derived in this paper are easily extended to the determination of such percentages.

As pointed out by one of the referees, another application of the techniques developed in this paper is as follows. For commercial reasons, the company may want to set the relativities at some prescribed percentages, keeping the financial equilibrium. This can be done in the framework of the mixed BMS-deductible system. It suffices to use different α's determined to get the desired relativities. The d_ℓ's will then be determined to ensure financial equilibrium via the indifference principle.

European directives have introduced complete rating freedom since July 2004. Insurance companies operating in most EU countries are now free to set up their own rates, select their own classification variables and design their own BMS. In most European countries, companies have taken advantage of this freedom by introducing more rating variables. It can be expected that they will start to compete on the basis of BMS. In that respect, this paper offers an alternative approach to a posteriori premium corrections.

We have adopted in this paper the point of view of the insurer: the relativities are transformed in such a way that it is actuarially equivalent to apply classical relativities or to apply adapted relativities together with a deductible. The financial equilibrium of the system is ensured in both cases.

A referee raised another point of view, corresponding to the vision of the policyholder. It is clear that the deductible that is defined by the insurer does not necessarily correspond to the optimal retention of the policyholder aiming at avoiding future premium increases. For small claims, it is indeed often more interesting for the policyholder to defray himself the third party than to report the claim and suffer future penalties. This phenomenon is called the hunger for bonus and has been introduced by LEMAIRE (1977).

LEMAIRE (1977) proposed an algorithm providing the optimal retention for the policyholder in order to avoid future premium increases. In our setting we can make the following comments :

– when the deductible applies per claim, it becomes clear that the optimal retention of the policyholder will not be smaller than the deductible. Indeed it would be suboptimal to claim for a loss amount smaller than the deductible because there would be no compensation from the insurer and a malus would be applied.

– when the deductible applies on a yearly basis, all calculations become much more complicated. A policyholder causing an accident with loss smaller than

the deductible may be tempted not to report the claim to the insurer in order to avoid future maluses. However if he does so, then he will be in the same situation in case of another accident in the same year, implying that he may pay more than the value of the deductible in a single year. This implies that the optimal retention in the case of a yearly deductible may be smaller than the yearly deductible.

In practice, it would not be possible to propose deductibles that correspond to the optimal retentions suggested by the hunger for bonus. Indeed the behaviour of the policyholders is a function of their expectations for the future, which are not identical for all policyholders. On top of that rationality of the policyholders must be assumed. We conclude that we cannot use these optimal retentions due to the subjective elements that would be used in the calculations.

It is clear that the insurer should work with the true claim and amount frequency of the policyholders. In practice, introducing deductibles may have an influence on the claiming policy of the policyholders, affecting their anticipated behaviour in the scale and, as such, the financial equilibrium of the scale. This problem is extremely difficult to handle. See WALHIN & PARIS (2000, 2001) for more details.

The analysis conducted in this paper is only a first step towards an efficient solution. The homogeneity of the portfolio with respect to claim severities as well as the absence of a priori risk classification constitute restrictive assumptions that should be relaxed. This will be the topic of future works. Also, the degree of bonus hunger induced by the mixed system will be studied in a forthcoming work.

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