Erratum – Cummins Capital

Yingrui (Elaine) Lu

There is an error in a formula **on page 15** of Cummins "Allocation of Capital in the Insurance Industry" ("Cummins Capital").

**The formula** $\boldsymbol{r_i = -k_i r_F + \beta_i(r_M - r_F)}$ **should be** $\boldsymbol{r_i = -\frac{k_i}{s_i} r_F + \beta_i(r_M - r_F)}$.

The derivation, based on the definitions of parameters from "Cummins Capital," is below.

### The Capital Asset Pricing Model (CAPM)

The CAPM states that the expected return on equity or cost of capital for a firm is determined by the following formula:

$$r_E = r_F + \beta_E(r_M - r_F),$$

where $r_E$ = cost of equity capital,

$r_F$ = default risk-free rate of interest,

$r_M$ = expected return on the "market," and

$\beta_E$ = the firm's beta coefficient = $\mathrm{Cov}(r_E, r_M)/\mathrm{Var}(r_M)$.

where $\mathrm{Cov}(\bullet)$ = the covariance operator and $\mathrm{Var}(\bullet)$ = the variance operator.

$$I = r_A A + r_1 P_1 + r_2 P_2,$$

where $I$ = net income,

$r_A$ = return on assets,

$r_1, r_2$ = rates of return on underwriting from lines 1 and 2,

$A$ = assets, and

$P_1, P_2$ = premiums from lines 1 and 2.

Next introduce the balance sheet identity, which says that assets are equal to equity, plus the liabilities generated by the two lines. Then divide by equity to express the result as a rate of return:

$$r_E = r_A(E + L_1 + L_2) / E + r_1 P_1 / E + r_2 P_2 / E.$$

Then the beta coefficient can be decomposed as follows:

$$\beta_E = \beta_A(1 + k_1 + k_2) + \beta_1 s_1 + \beta_2 s_2,$$

where $\beta_E, \beta_A, \beta_1, \beta_2$ = betas for the firm, assets, and insurance risk of lines 1 and 2,

$k_1, k_2$ = liability leverage ratios for lines 1 and 2, $= L_i/E$, $i = 1, 2$, and

$s_1, s_2$ = premium leverage ratios for lines 1 and 2, $= P_i/E$, $i = 1, 2$.

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$$r_E = \frac{I}{E} = r_A \frac{E + L_1 + L_2}{E} + r_1 \frac{P_1}{E} + r_2 \frac{P_2}{E} = r_A(1 + k_1 + k_2) + r_1 s_1 + r_2 s_2$$

$$Cov(r_E, r_M) = Cov(r_A, r_M)(1 + k_1 + k_2) + Cov(r_1, r_M)s_1 + Cov(r_2, r_M)s_2$$

Divide both sides by $Var(r_M)$:

$$\beta_E = \beta_A(1 + k_1 + k_2) + \beta_1 s_1 + \beta_2 s_2$$

$$r_E = r_F + \beta_E(r_M - r_F) = r_F + [\beta_A(1 + k_1 + k_2) + \beta_1 s_1 + \beta_2 s_2](r_M - r_F) \qquad (1)$$

Replace $r_A$ with $r_F + \beta_A(r_M - r_F)$:

$$r_E = r_A(1 + k_1 + k_2) + r_1 s_1 + r_2 s_2 = [r_F + \beta_A(r_M - r_F)](1 + k_1 + k_2) + r_1 s_1 + r_2 s_2 \qquad (2)$$

Equate (2) and (1):

$$[r_F + \beta_A(r_M - r_F)](1 + k_1 + k_2) + r_1 s_1 + r_2 s_2$$
$$= r_F + [\beta_A(1 + k_1 + k_2) + \beta_1 s_1 + \beta_2 s_2](r_M - r_F)$$

$$r_F(1 + k_1 + k_2) + \beta_A(r_M - r_F)(1 + k_1 + k_2) + r_1 s_1 + r_2 s_2$$
$$= r_F + \beta_A(1 + k_1 + k_2)(r_M - r_F) + \beta_1 s_1(r_M - r_F) + \beta_2 s_2(r_M - r_F)$$

$$r_F(\cancel{1 +} k_1 + k_2) + \cancel{\beta_A(r_M - r_F)(1 + k_1 + k_2)} + r_1 s_1 + r_2 s_2$$
$$= \cancel{r_F} + \cancel{\beta_A(1 + k_1 + k_2)(r_M - r_F)} + \beta_1 s_1(r_M - r_F) + \beta_2 s_2(r_M - r_F)$$

$$r_F k_1 + r_F k_2 + r_1 s_1 + r_2 s_2 = \beta_1 s_1(r_M - r_F) + \beta_2 s_2(r_M - r_F)$$

$$r_1 s_1 + r_2 s_2 = -k_1 r_F - k_2 r_F + \beta_1 s_1(r_M - r_F) + \beta_2 s_2(r_M - r_F)$$

$$r_1 s_1 = -k_1 r_F + \beta_1 s_1(r_M - r_F)$$

$$r_2 s_2 = -k_2 r_F + \beta_2 s_2(r_M - r_F)$$

Divide both sides by $s_i$:

$$r_i = -\frac{k_i}{s_i} r_F + \beta_i(r_M - r_F)$$