Errata

Practical Mixed Models for Actuaries

Date Approved: 12/14/2025
In the corrections below, the **boldface text** indicates the corrected wording.

Page 6

2nd paragraph; the term "multicollinearity" is removed from this sentence.
"During these ages, children grow significantly, and Age and Ht should be strongly related to each other; in fact, their linear correlation coefficient is equal to 0.79. Thus, including both of these variables in a linear model may pose some estimation problems ~~(multicollinearity)~~."

Page 24

1st paragraph
"For risk class $j = 2$ (triangle symbol) with claim cost $X_{21} = 750$ at time $t = 1$, its long-term average might be close to $\bar{X}_2 = 750$; and for risk class $j = 1$ (circle symbol), starting with $X_{\mathbf{11}} = 625$, that long-term average may equal $\bar{X}_1 = 650$."

Page 29

5th paragraph
"Under these conditions, the homogeneous linear combination $g_{11} X_{11} + \ldots + g_{JT} X_{JT}$ that is the best unbiased predictor of $X_{\boldsymbol{j},T+1}$ in the sense of minimal mean squared error

$$\mathrm{E}[(X_{\boldsymbol{j},T+1} - g_{11} X_{11} - \ldots - g_{JT} X_{JT})^2] \qquad (3.4)"$$

Page 34

3rd paragraph
"One shortcoming is that in the decomposition of the experience X_{jt},

$$X_{jt} = m + \Xi_j + \epsilon_{jt},$$

we have assumed that the deviation Ξ_j from the overall mean m for each risk class j has the same variance, namely, $\mathrm{Var}[\Xi_j] = \boldsymbol{\tau^2}$."

Page 36

1st full paragraph
"**Theorem 3.2 (Bühlmann–Straub model).** *The mean squared error best homogeneous unbiased predictor* $\sum_{it} g_{it} X_{\boldsymbol{it}}$ *of the risk premium* $\mathrm{m} + \Xi_j$ *in model Equation 3.13, that is, the solution to the following restricted minimization problem,"*

Page 41

The shaded boxes are incorrectly placed on this page. The image below shows how the shading should appear.

First, we create a data frame with the data we have

```
D <- cbind(risk.class = 1:J,
           as.data.frame(matrix(X.jt,
                                nrow = J,
                                ncol = Tm,
                                byrow = TRUE)),
           as.data.frame(matrix(w.jt,
                                nrow = J,
                                ncol = Tm,
                                byrow = TRUE)))
```

and then we estimate the model via

```
(BS <- cm(~ risk.class,
          data = D,
          ratios = 2:6,
          weights = 7:11))
```

Page 42

The shading and spacing are incorrect for the code blocks. The image below shows how the shading and spacing should appear.

For the first five risk classes, our by-hand calculations match those from the `cm()` function.

```
rbind("   cm():" = predict(BS)[1:5],
      "by-hand:" = P.hat[1:5])
```

```
                1        2        3        4        5
   cm(): 67.4199 74.65462 82.37383 81.77987 75.53623
by-hand: 67.4199 74.65462 82.37383 81.77987 75.53623
```

And they match across all risk classes:

```
all(round(abs(predict(BS) - P.hat), 10) == 0)
```

```
[1] TRUE
```

Page 45

2nd paragraph

"To this end, we can restate Equation 3.24 as follows

$$X_j = Q_j \beta(\Theta_j) + \epsilon_j, \tag{3.25}$$

where X_j is a $T \times 1$ column vector of responses for risk **class** $j=1, 2, \ldots, J$ and T is the number of time periods we have observed."

Page 46

2nd paragraph

"For our example, we have $t_{j1}=1$, $t_{j2}=2, \ldots$, for all states j."

4th paragraph

"**Definition 3.1 (Hachemeister model assumptions).** The risk **class** j is characterized by **a class** risk profile ϑ_j, which is itself the realization of a random variable Θ_j. We make the following assumptions:"

Page 49–50

The shaded boxes are incorrectly placed starting at the bottom of page 49 and through page 50. The image below shows how the shading should appear.

Let us implement these calculations using Hachemeister's data. First, let's define some quantities: the weights `W.jt`, the time points `T.jt`, the observed severities `X.jt`, and the vector `S`, which tells us which state these observations belong to.

```
W.jt <- db$claims
T.jt <- db$time
X.jt <- db$severity
S <- db$state
N <- length(unique(T.jt))
J <- length(unique(S))
```

Next, we calculate the summary statistics that we will need. Table 3.3 shows the correspondence between our programming variable names and the written notation used in the text.

Table 3.3. Correspondence between programming variable and written notation in the text.

Variable	Written Notation	Variable	Written Notation
`W.jb`	$w_{j\bullet}$	`Ej.tX`	$E_j^{(s)}[tX_j]$
`W.bb`	$\sum_{j=1}^{J} w_{j\bullet}$	`Vj.t`	$\mathrm{Var}_j^{(s)}[t]$
`Ej.t`	$E_j^{(s)}[t]$	`Ws.jb`	$\mathrm{Var}_j^{(s)}[t]w_{j\bullet}$
`Ej.t2`	$E_j^{(s)}[t^2]$	`Ws.bb`	$\sum_{j=1}^{J} \mathrm{Var}_j^{(s)}[t]w_{j\bullet}$
`Ej.X`	$E_j^{(s)}[X_j]$		

```
W.jb <- tapply(W.jt, S, sum)
W.bb <- sum(W.jb)
Ej.t <- tapply(W.jt * T.jt, S, sum) / W.jb
Ej.t2 <- tapply(W.jt * T.jt^2, S, sum) / W.jb
Ej.X <- tapply(W.jt * X.jt, S, sum) / W.jb
Ej.tX <- tapply(W.jt * T.jt * X.jt, S, sum) / W.jb
Vj.t <- Ej.t2 - Ej.t^2
Ws.jb <- Vj.t * W.jb
Ws.bb <- sum(Ws.jb)
```

With these definitions, we can calculate the intercept and slope for each state using

Page 55

3rd paragraph

"These calculations yield

$$\hat{\sigma}^2 = 4.9870187 \times 10^7, \quad \hat{\tau}_0^2 = 1.8029435 \times 10^4, \quad \hat{\tau}_1^2 = 665.5618.$$

Page 58

Solution 3.4

"**Solution 3.4** For state 1, **the credibility slope is outside, and the intercept is inside** the intervals defined by the stand-alone and collective estimates, and for state 2, only the slope is not between the stand-alone and collective estimates."

Page 63

2nd full paragraph, bottom of page

"With these extensions, the credibility premium is of the same form as in the balanced Bühlmann model, namely,

$$Z_j X_{j\circ} + (1 - Z_j)\, X_z \qquad Z_j = \frac{w_{j\cdot}}{w_{j\cdot} + \sigma^2/\tau^2}."$$

Page 75

4th paragraph, bottom of page

"The fitted values from the mixed model are the credibility-weighted values given by

$$\hat{y}_j = Z_j \bar{y}_j + (1 - Z_j)\, \bar{y},$$

Page 98

4th paragraph

"And, finally, we have the double hierarchical generalized linear model (DHGLM), where we take an HGLM and allow random effects in the dispersion model.*"

[Add new footnote to this sentence.]

***One can also introduce explanatory variables via a link function and a linear predictor into the variance of the random effects, but we shall not explore this extension in this monograph."**

Page 120

Item 5, bottom of page

"Double hierarchical generalized linear model (DHGLM). This extends the HGLM model by allowing random effects in the model for the dispersion parameter.*"

[Add new footnote to this sentence.]

***DHGLMs can also further model the variance of random effects, as mentioned earlier in the footnote on p. 98, but we do not explore this extension in this monograph.**