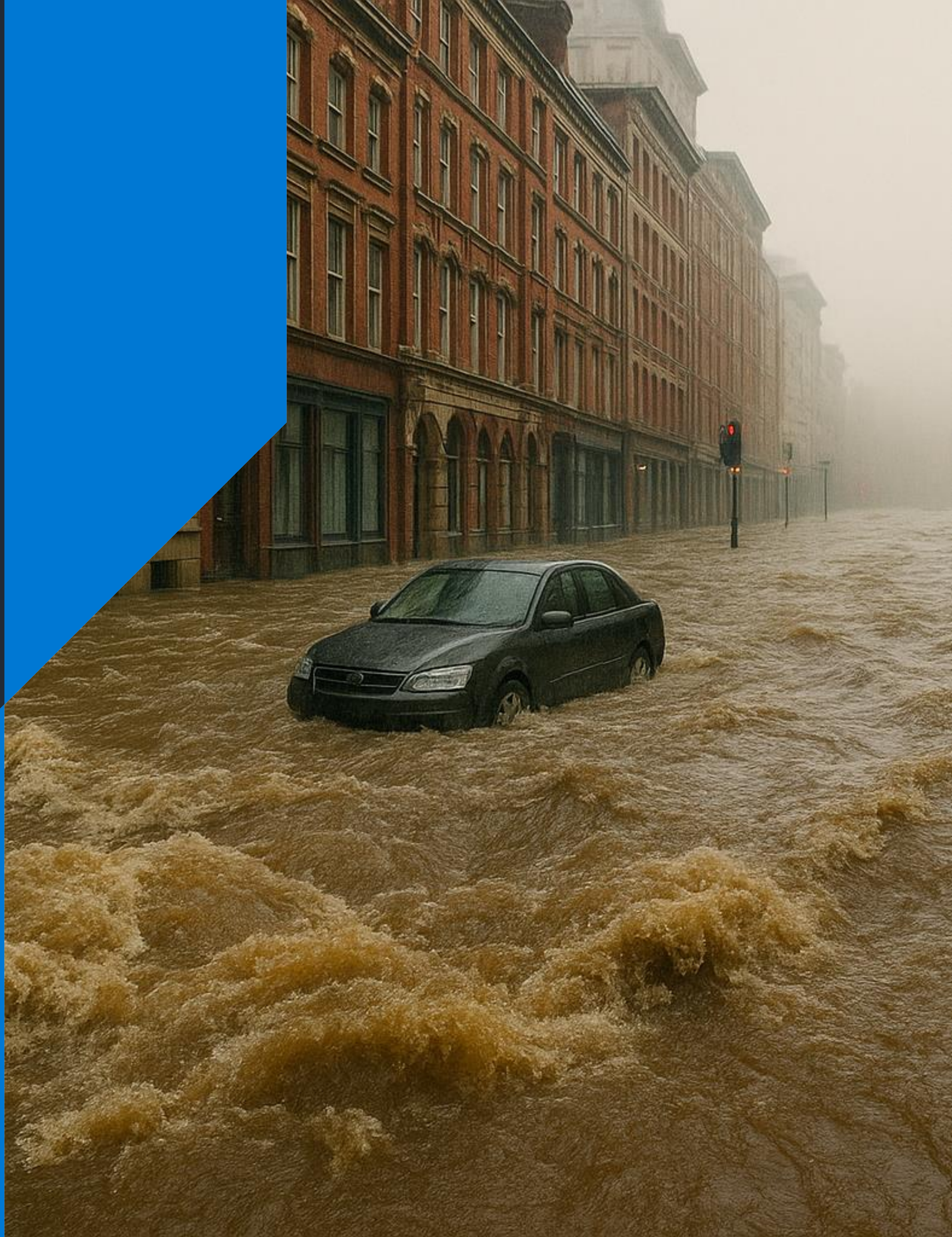
Actuarial Challenges in Developing NatCat Tariffs

under Italy’s 2024 Budget Law



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Agenda

1. Context
2. Next-Generation Actuarial Approach for Flood Modeling

1. Context

- ☛ **Why a Law on Nat Cat Insurance is Essential**
- ➢ Law Highlights
- ➢ Key impacts for Insurers
- ➢ Developing a Pricing Model

1. Context

Why a Law on Nat Cat Insurance is Essential (1/3)

2.7 Mn

Nat Cat insurance contracts 2018-2022

5% in high-risk areas

€160 K average mortgages loan

↑ **Default rates**

↓ **Property values**

43%

Low insurance Nat Cat coverages for high-risk properties

AAL = €3.4 Bn

2008-2022
(peaks over €10 Bn)

90% of calamities

Climate-related disasters
Flood & Landslide

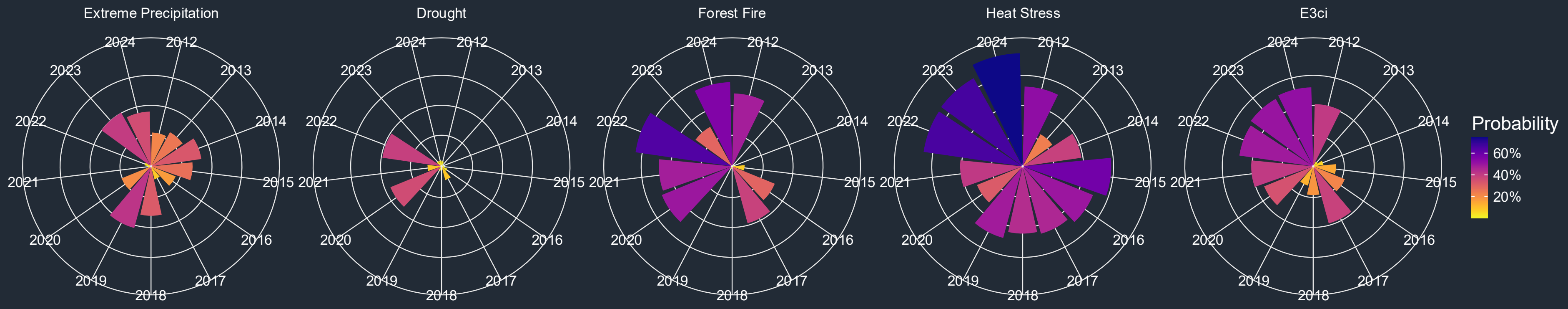
EAAL = €5+ Bn

Next 10 Yrs

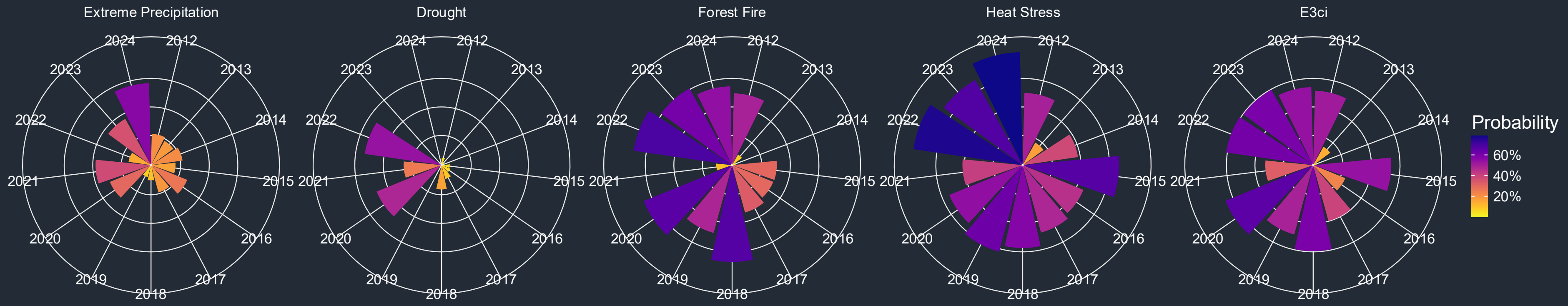
1. Context

Why a Law on Nat Cat Insurance is Essential (2/3)

ITALY



EU



- Over the past 15 years Italy has experienced a critical increase in heatwaves and wildfires, as well as a rise in extreme precipitation events. In general, also in other European countries very similar trends are observed for various extreme events.
- The charts illustrate the trends of selected extreme natural events in the Italian and European areas.
- A probability greater than 5% indicates an intensification in the frequency of extreme events compared to the reference period 1981–2010.

1. Context

Why a Law on Nat Cat Insurance is Essential (3/3)

- Italian territory is **highly exposed** to a wide range of natural hazards, including hydrogeological, seismic and extreme weather events; in particular, more than **90%** of cities are **exposed to landslides, floods, and/or coastal erosion**.
- Moreover, climate change has made floods and landslides more frequent, creating growing risks for the business sector.

Business Impact

+7.3% of exit probability in firms located in cities hit by floods or landslides.

An average of -4.9% in revenues and -2.2% in employment in the three years after the Nat Cat event among firms that survived.

The negative impacts were concentrated among **small and micro companies** and **younger firms**.

Insurance Gap

97% of **large companies** insured (**~0.1%** of the overall companies in the territory).

72% of **medium companies** insured (**~0.6%** of the overall companies in the territory).

5% of **small and micro companies** insured (**~99.3%**, of the overall companies in the territory).

- For the reasons above, accurate pricing of Nat Cat risks **is essential** to ensure **financial stability** and **market sustainability**. Inadequate or imprecise pricing approaches may expose insurers to severe economic losses and undermine long-term resilience.

1. Context

- Why a Law on Nat Cat Insurance is Essential
- **Law Highlights**
- Key impacts for Insurers
- Developing a Pricing Model

1. Context

Law Highlights

- The 2024 Italian Budget Law introduced the obligation for all companies in Italy to protect themselves against natural catastrophes through specific insurance coverage. The obligation to insure is bilateral – it applies both to companies, which must obtain coverage, and to insurance undertakings, which are required to provide it. In particular, the law requires companies to purchase insurance policies covering:
  - Floods.
  - Landslides.
  - Earthquakes.
- The law introduces the obligation gradually, with different deadlines depending on company size:

Large Companies	Medium Companies	Small and Micro Companies
By the end of March 2025	*By the end of September 2025*	*By the end of December 2025*

- SACE, the Italian state-owned export credit agency, plays a key role in the framework. It acts as a reinsurer and guarantor of last resort, supporting private insurance companies in managing catastrophic risk exposure. This ensures sufficient market capacity, stabilizes premiums, and fosters a more resilient national insurance system.

1. Context

- Why a Law on Nat Cat Insurance is Essential
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- **Key impacts for Insurers**
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1. Context

Key impacts for Insurers

- The introduction of the mandatory law on Nat Cat insurance represents a structural shift for the insurance industry. The decree not only broadens the scope of coverage and redefines the allocation of risk, but also requires insurers to **adapt their internal processes**, governance, and technical capabilities across **multiple business functions**.

Key Functions Impacted

Underwriting & Product Development

Insurers must design and issue Nat Cat coverage as mandated by law, with limited discretion in risk selection and the need to adapt existing product portfolios.

Reinsurance & Risk Management

Companies will need to rely more heavily on reinsurance to manage catastrophic accumulations, while defining and reviewing annual risk tolerance limits.

Pricing & Actuarial

Premium calculation requires robust actuarial models and frequent updates to ensure alignment with regulatory expectations and market sustainability.

Claims Management

Processes and resources must be strengthened to handle potentially large volumes of claims arising from catastrophic events, ensuring timely settlement and customer trust.

- Beyond these four core areas, other functions will also be impacted, such as **Compliance & Legal**, **Finance & Capital Management** (e.g. for Solvency II implications), and **Distribution & Client Relations** in explaining new obligations to clients.

1. Context

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1. Context

Developing a Pricing Model (1/2)

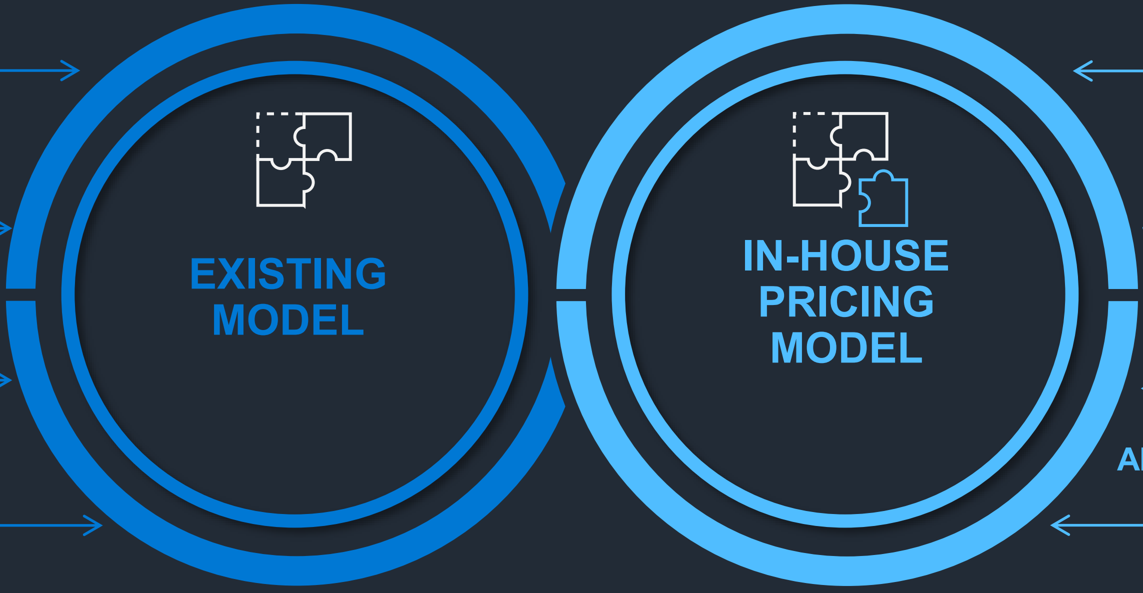
- Current models used by insurers are often black-box solutions relying on reinsurer-provided rates. As a result, portfolio optimization using own data is limited, highlighting the added value of an in-house model.

Outsourced tariffs provided by reinsurers or other **external providers**

Black-box CAT models with low transparency on risk drivers

Lack of customization and/or **optimization** for the company's own portfolio

Full dependence on external player and limited ability to actively challenge



Direct control over models and geographic/sectoral **customization** of pricing

Ability to perform **scenario analysis** and **sensitivity** testing on key parameters

Enhanced negotiation with reinsurers and development of internal capabilities

Alignment with regulatory expectations and **risk management guidelines**

1. Context

Developing a Pricing Model (2/2)

FREQUENCY

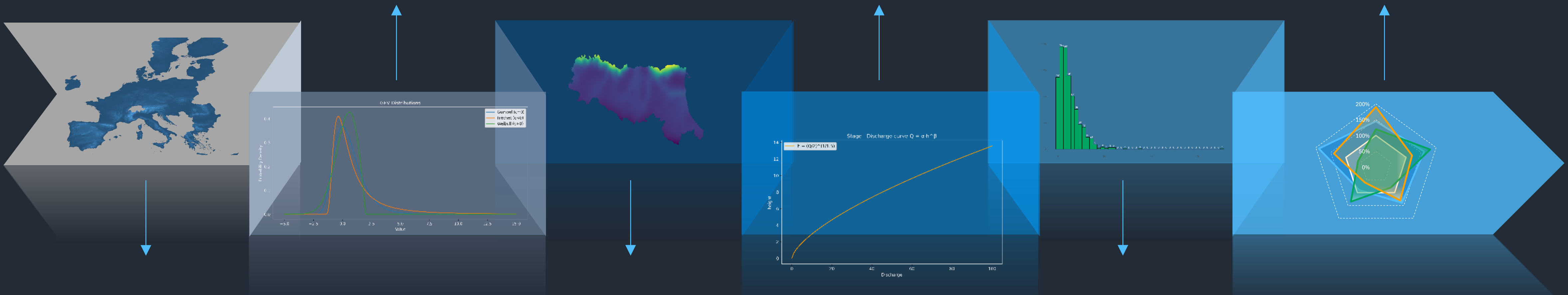
- Extreme distribution calibration on historical data.
- Return periods estimation.

EXPECTED LOSS

- Portfolio geolocalisation and average loss estimation.

IMPACT ANALYSIS

- Stress test and multiple scenarios.
- Backtesting and comparison with actual tariff.



DATA COLLECTION

- Sources: national and global data
- Target: flood height using empirical formula and downscaling data.

HAZARD

- Definition of event intensity clusters based on the generalized frequency model.
- Development of a GIS data model.

MODELS CONVOLUTION

- Frequency - Severity models convolution.
- AAL (Average Annual Loss) estimation and tariff implementation.

2. Next-Generation Actuarial Approach for Flood Modeling

- Spatial Data
- Downscaling
- Stage-Discharge Rating curve
- Extreme Modeling

2. Next-Generation Actuarial Approach for Flood Modeling

Spatial Data - Introducing spatial data

- Modern technologies provide access to remote sensing data with a resolution up to 5 meters.
- Trade off between detail (resolution), extension (global or local) and spatio-temporal detail (better low resolution and higher spatio-temporal or higher resolution and lower spatio-temporal?).
- How can we get high spatio-temporal detail at least in the region we are modeling?

2. Next-Generation Actuarial Approach for Flood Modeling

Spatial Data - Sources and distortions (1/3)

- **Distortion** refers to the alteration of the true shape, area, distance, or direction of features in remote sensing images due to various factors such as sensor geometry, Earth's curvature, and topography.
- Due to distortion, tall structures like the Eiffel Tower appear shifted or skewed from their actual geographic location.
- The base is at the correct location, but the top point appears further out, creating a "shadow" or "plate" effect that does not correspond to the true footprint.

2. Next-Generation Actuarial Approach for Flood Modeling

Spatial Data - Sources and distortions (2/3)

Relief Displacement

- Relief displacement refers to the apparent shift of elevated objects (like the top of the Eiffel Tower) from their true ground position in aerial or satellite images.
- The formula for relief displacement (d) is:

$$d = \frac{h \cdot r}{H}$$

Where:

- d: radial displacement of the elevated point (in mm or pixels on the image).
- h: height of the object above the reference plane (e.g., above ground level).
- r: radial distance from the center of the image to the base of the object (on the photo, in mm or pixels).
- H: flying height of the sensor/platform above the reference plane.

Radial Distortion

- Radial distortion is a geometric distortion that causes points to be shifted radially with respect to the optical center of the image.
- A common formula for radial distortion (Δ_r) is:

$$\Delta r = k_1 r^3 + k_2 r^5 + k_3 r^7 + \ldots$$

Where:

- Δ_r: radial shift due to distortion.
- r: distance from the optical center.
- k_1, k_2, k_3: distortion coefficients (calibrated for the sensor).

2. Next-Generation Actuarial Approach for Flood Modeling

Spatial Data - Sources and distortions (3/3)

Practical Example (Eiffel Tower)

- Suppose:
  - Eiffel Tower height (h): 330 m.
  - Flying height (H): 2000 m.
  - Radial distance from image center to base (r): 50 mm (on the photo negative).
- Displacement of the top:
  - d=330×50 / 2000=8.25 mm.
- So, the top of the Eiffel Tower will appear shifted by 8.25 mm from its base on the image.

Satellite	Altitude (m)	R (mm)	Displacement (mm)	Pixel Size (mm)	Displacement (pixels)
WV-3	617,000	10	0.00535	0.005	1.07
WV-3	617,000	50	0.02675	0.005	5.35
WV-3	617,000	50	0.02675	0.03	0.89
Landsat 8	705,000	50	0.0234	0.005	4.68
Landsat 8	705,000	50	0.0234	0.03	0.78

2. Next-Generation Actuarial Approach for Flood Modeling

Spatial Data - Mapping rivers and streams

Mapping rivers and streams

- Get high quality spatial data (which however require processing steps).
- Breach data: remove errors that block natural flow.
- Fill data: correct depressions and sinks that do not represent actual terrain features.
- Measure flow accumulation: amount of water that would flow into each cell (identify drainage patterns and potential flooded areas).
- Extract rivers: setting a threshold for flow accumulation, we can delineate rivers and streams within the landscape.

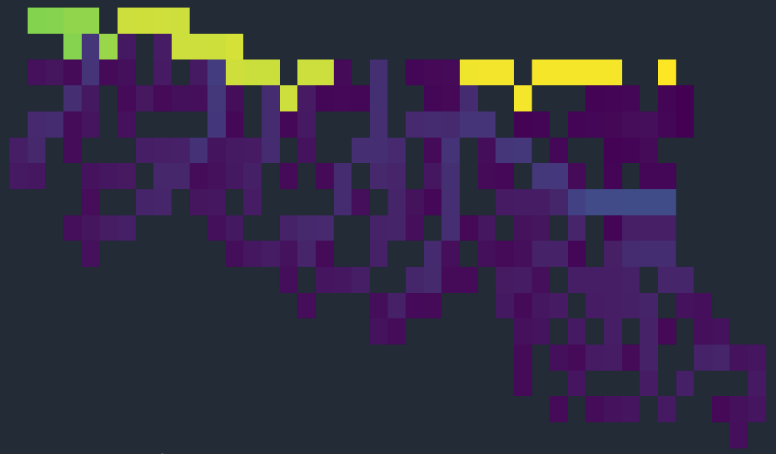
2. Next-Generation Actuarial Approach for Flood Modeling

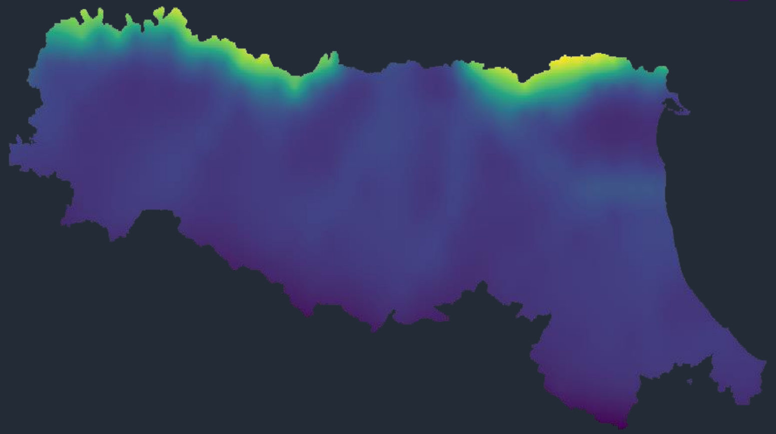
- Spatial Data
- **Downscaling**
- Stage-Discharge Rating curve
- Extreme Modeling

2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - A method to rule them all

- Even with modern technologies, spatial data can show some limitations due to sensors or budget:
  - Usually, we only have some points in space, however, the risk influences total space, how can we estimate risk in missing points?
  - Spatial data can show holes or missing values or errors, which need to be estimated to correctly extract information (remember breaching and filling data before extracting rivers).
  - We are interested in augmenting spatial resolution because initial resolution is too coarse to provide the information we need with the needed detail.
  - All of above can be made using downscaling methods.





2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - Geostatistical techniques (1/2)

- Spatial autocorrelation describes the extent to which a variable is correlated with itself through space.
- Tobler's First Law of Geography:

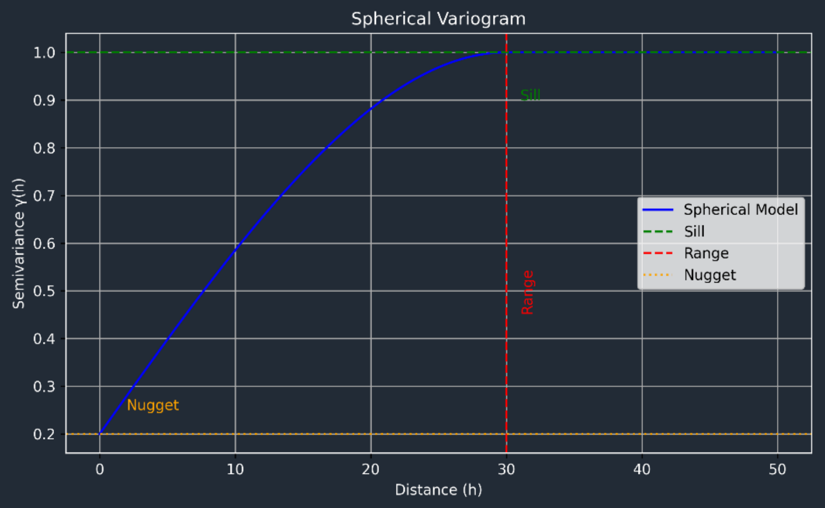
  "Everything is related to everything else but near things are more related than distant things".
- In order to progress towards spatial predictions, we need a variogram model for (potentially) all distances. If we would connect these estimates with straight lines, it would lead to statistical models with non-positive definite covariance matrices, which would block using them in prediction.
- To avoid this, we fit parametric models to the estimates , where we take as the mean value of all the values involved in estimating.

- Assuming isotropy (same relationship across every direction), the variogram becomes:

$$2\,\hat{\gamma}(h) = \frac{1}{|N(h)|}\sum_{N(h)}\left((Z(s_i) - Z(s_j)\right)^2$$

- Where $|N(h)|$ denotes the number of distinct pairs in.

$$N(h) = \{(s_i, s_j) : \|s_i - s_j\| = h, i, j = 1, \ldots, n\}$$



2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - Geostatistical techniques (2/2)

- In geostatistics and spatial analysis, variograms describe how spatial correlation changes with distance.

Spherical Model

$$\gamma(h) = \begin{cases} C_0 + C\left[\frac{3h}{2a} - \frac{h^3}{2a^3}\right], & 0 < h \le a \\ C_0 + C, & h > a \end{cases}$$

- C_0: nugget (variance at zero distance).
- C: sill (total variance minus nugget).
- a: range (distance where the model flattens).

Exponential Model

$$\gamma(h) = \left\{C_0 + C\left[1 - \exp\left(-\frac{h}{a}\right)\right]\right.$$

- Approaches the sill asymptotically, never truly reaches it.
- a: practical range (distance at which γ(h) reaches about 95% of the sill).

Gaussian Model

$$\gamma(h) = \left\{C_0 + C\left[1 - \exp\left(-\frac{h^2}{a^2}\right)\right]\right.$$

- Smoother near the origin than the exponential model.
- a: range parameter.

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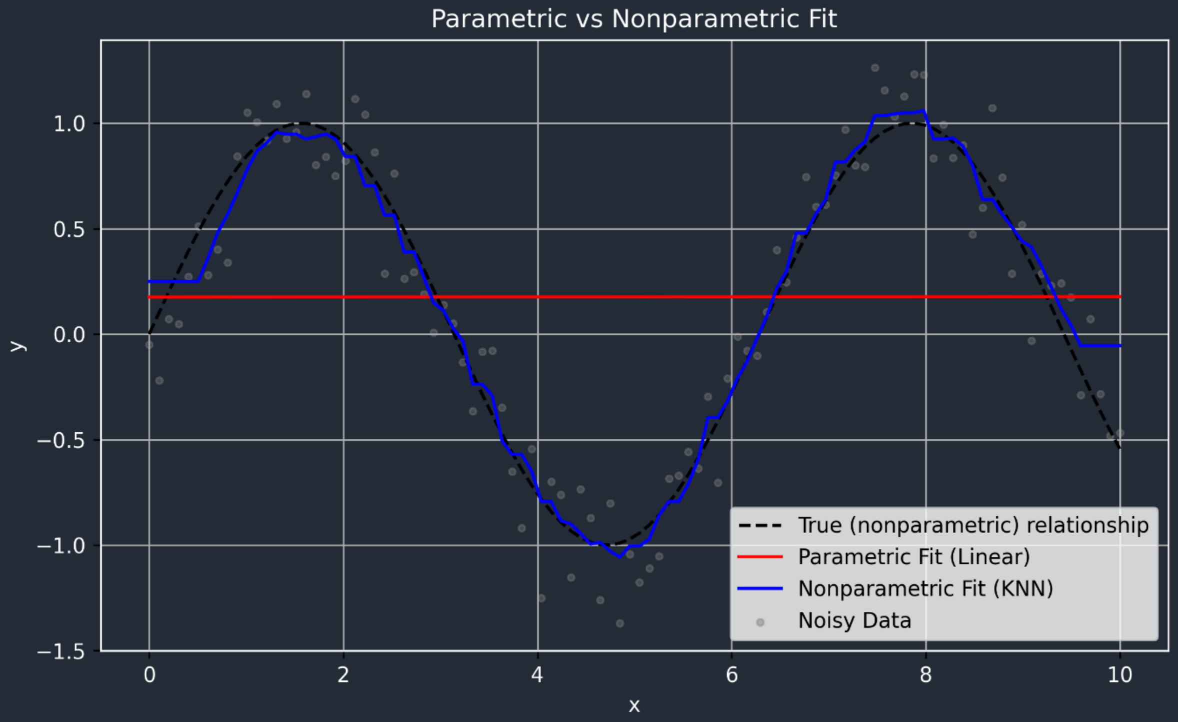
2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - Nonparametric methods

- No prior assumptions about the relationship between x and y.
- Better solutions (in case the parametric function is wrong) than parametric models (which could be the case, often parametric shapes are far from the true one).
- We are interested in high detailed risk prevision (of course if we could explain risk parametrization that would be better but the main objective is to be precise).
- Be careful with extrapolation (model choice in downscaling must take in consideration if we want to extrapolate or not).



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2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - Neural Network

Universal Approximation Theorem

- A feedforward neural network with a single hidden layer containing a sufficient number of neurons and a suitable non-linear activation function (such as the sigmoid) can approximate any continuous function on a compact subset of $\mathbb{R}^n$ to any desired degree of accuracy.
- The formula expressing this theorem (in its classical form, Cybenko, 1989) is:
  - For every continuous function $f : \mathbb{R}^n \to \mathbb{R}$ defined on a compact set $K \subset \mathbb{R}^n$, and for every $\varepsilon > 0$, there exist coefficients a_i, weights w_i, and biases b_i $(\,for\ i = 1, \ldots, N)$, such that:

$$\left| f(x) - \sum_{i=1}^{n} a_i \sigma(w_i{}^T x + b_i) \right| < \varepsilon \qquad \forall\, x \in K$$

Where:

- σ is a non-constant, continuous function (such as the sigmoid).
- N is a sufficiently large number of neurons in the hidden layer.
- a_i, b_i and w_i are real parameters (output weights, weight vectors, and biases, respectively).

2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - Gradient Boosting (1/2)

- **Extreme Gradient Boosting (XGBoost)** is a highly efficient and scalable implementation of gradient boosting for decision trees. It builds an ensemble of trees sequentially, each one correcting errors from the previous ones.
- Gradient boosting builds an additive model in the form:

$$\hat{y}_i^t = \sum_{k=1}^{t} f_k \, x_i$$

Where:

- $\hat{y}_i^t$: prediction for sample *i* after *t* trees.
- f_k: the *k-th* tree (decision function).
- x_i: feature vector of sample *i*.

- XGBoost optimizes a regularized objective at each iteration:

$$\mathcal{L}^{t} = \sum_{i=1}^{n} \ell \left[y_i, y_i^{(t-1)} + f_t(x_i) + \Omega\,(f_t) \right]$$

2. Next-Generation Actuarial Approach for Flood Modeling

Downscaling - Gradient Boosting (2/2)

- To efficiently optimize, XGBoost uses a second-order Taylor expansion of the loss for the new tree:

$$\mathcal{L}^{t} \approx \sum_{i=1}^{n} \left[g_i f_t(x_i) + \frac{1}{2} h_i f_t^2(x_i) + \Omega(f_t) \right]$$

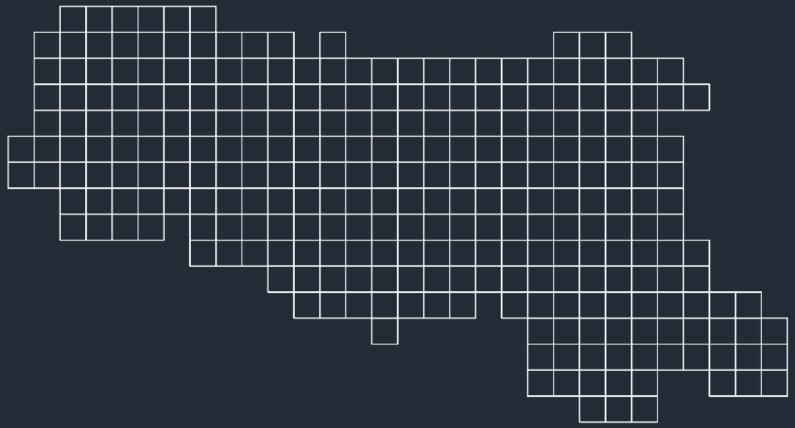
- Summary of XGBoost steps:
  - Initialize predictions (e.g., mean for regression).
  - For each iteration:
    - Compute gradients (g_i) and Hessians (h_i) for all data points.
    - Build a new tree f_t to fit the gradients.
    - Compute optimal weights and split points.
    - Update predictions: $\hat{y}_i(t) = \hat{y}_i(t-1) + f_t(x_i)$
  - Repeat until desired number of trees is reached or loss stops improving.

2. Next-Generation Actuarial Approach for Flood Modeling

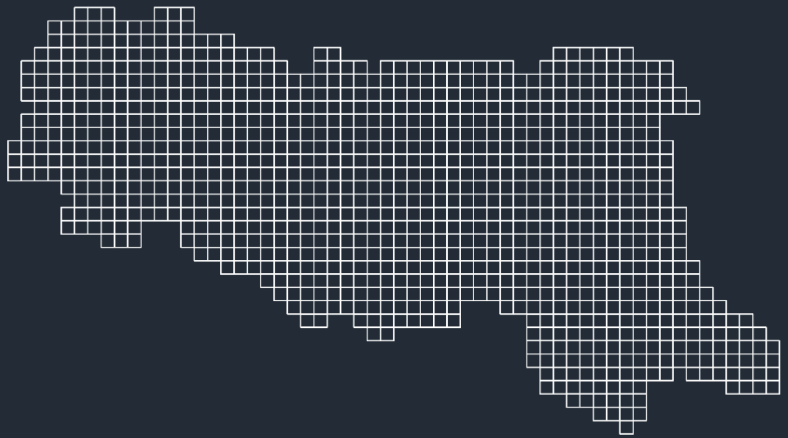
Downscaling - Results

- **Using downscaling,** we can improve spatial detail about the information we are interested in. Improved accuracy leads to better decision making and more accurate tariff.

10 km grid over Emilia-Romagna



5 km grid over Emilia-Romagna



1 km grid over Emilia-Romagna

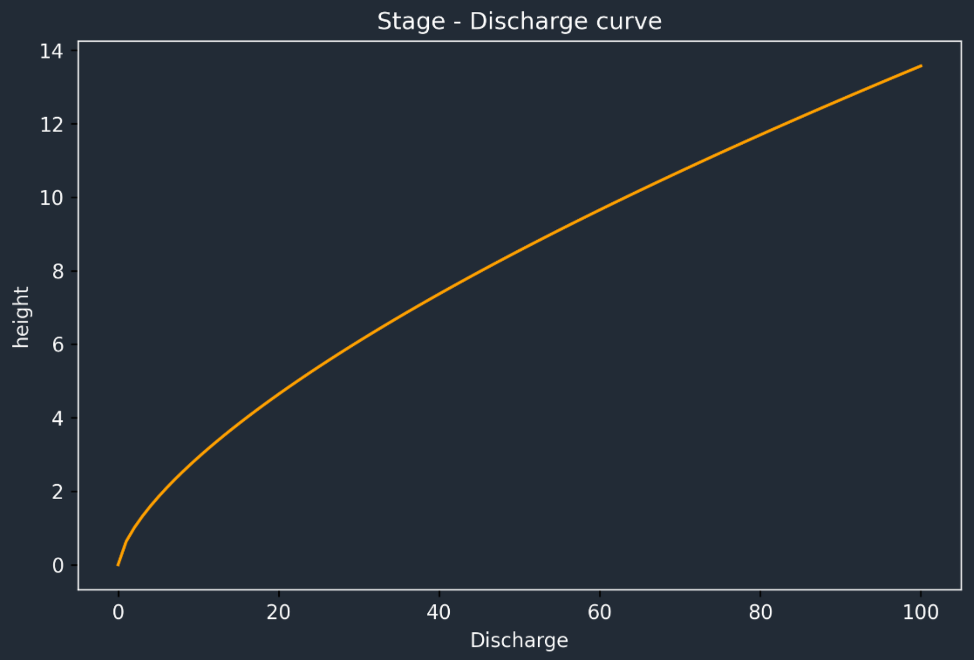
2. Next-Generation Actuarial Approach for Flood Modeling

- Spatial Data
- Downscaling
- **Stage-Discharge Rating curve**
- Extreme Modeling

2. Next-Generation Actuarial Approach for Flood Modeling

Stage-Discharge Rating curve - Obtaining a Stage-Discharge rating curve

- The purpose of continuous collection of stage data and discharge measurements is to establish a relationship between water level and volumetric flow over a wide range of conditions.
- Stage-Discharge Relationship:
  - Essential for design to assess flow characteristics like depth and discharge.
  - Allows evaluation of various flow conditions beyond a single design flow rate.
  - Measuring river discharge continuously is costly, time-consuming, and impractical, especially during floods.



2. Next-Generation Actuarial Approach for Flood Modeling

Stage-Discharge Rating curve - Empirical equations

Manning formula

$$Q = \frac{1}{n} AR^{2/3}S^{1/2}$$

Where:

- Q: flow rate (m^3/s or cfs).
- n: manning's roughness coefficient (unitless).
- A: cross-sectional area of flow (m^2 or ft^2).
- R: hydraulic radius (m or ft) = A / P.
- P: wetted perimeter (m or ft).
- S: channel slope (m/m or ft/ft).
- *This equation gives engineers the ability to predict how fast and how much water will move through a particular section of a channel, pipe, or stream under steady, uniform flow conditions.*

- Once we have observed discharge and height values, finding α and β becomes an optimization problem.

2. Next-Generation Actuarial Approach for Flood Modeling

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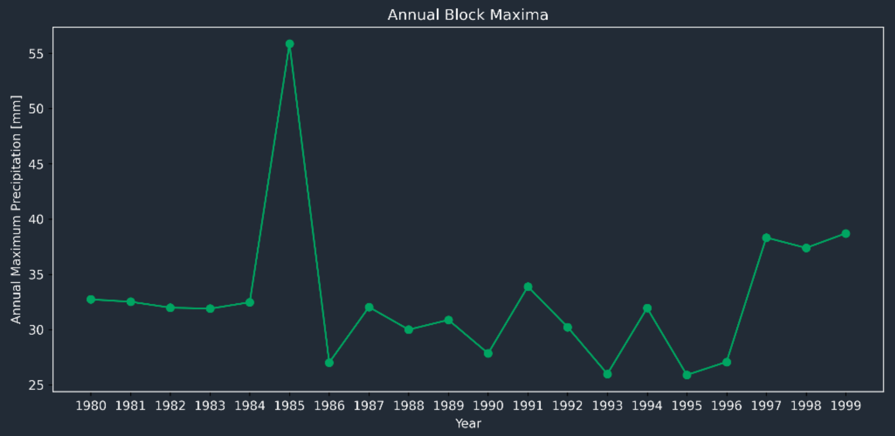
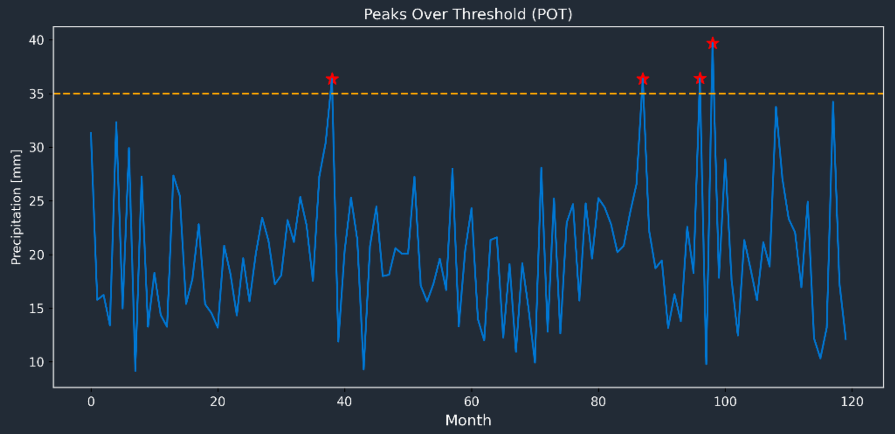
2. Next-Generation Actuarial Approach for Flood Modeling

Extreme Modeling - Extreme modeling approaches

- Extreme value analysis plays a crucial role in understanding the behavior of rare, high-impact natural hazards such as floods and storms.
- In this context, extreme events are typically defined as those observations which exceed a high threshold value. The choice of this threshold is central to the analysis and is often determined indirectly by specifying a desired return period—the average time between occurrences of events of a certain magnitude. For instance, a "centennial storm" refers to an event expected to occur, on average, once every 100 years, while a "millennial flood" is one expected every 1,000 years.
- The second approach is the Peaks Over Threshold (POT) method, which utilizes all data points exceeding a chosen threshold rather than only the maximum per block. Under appropriate conditions, the exceedances above the threshold are well described by the Generalized Pareto Distribution (GPD).

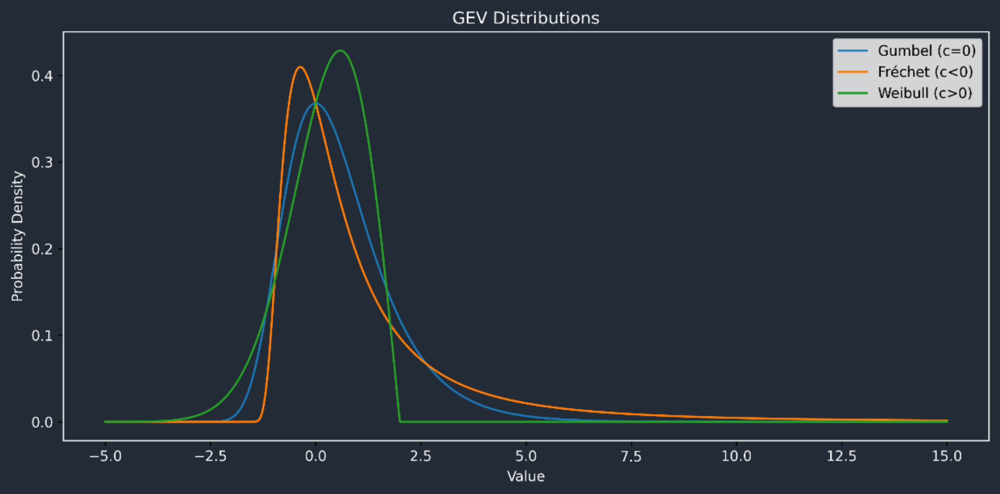




2. Next-Generation Actuarial Approach for Flood Modeling

Extreme Modeling - What does extreme mean for natural hazard

- In extreme value analysis, we often focus on the annual maxima of a stationary process X, typically recorded during specific seasons when extreme events are most likely to occur (such as summer for heatwaves or winter for floods). For each year, the maximum value observed is denoted by Mn, and the collection of these maxima over multiple years forms the basis for statistical modeling.



- Extreme value theory shows that, under certain conditions, the distribution of maxima converges to a Generalized Extreme Value (GEV) distribution. The GEV family has three types, depending on the shape parameter ξ: Gumbel (ξ=0), Fréchet (ξ>0), and Weibull (ξ<0). In many practical cases, especially for environmental data, the Gumbel distribution is an appropriate model.

GEV: Given x such that $1 + \xi \frac{(x-\mu)}{\sigma} > 0$:

$$f(x) = \frac{1}{\sigma}\left[1 + \xi \frac{(x-\mu)}{\sigma}\right]^{-1/\xi - 1} \exp\left\{-\left[1 + \xi \frac{(x-\mu)}{\sigma}\right]^{-1/\xi}\right\}$$

If $\xi \to 0$ then GEV reduces to Gumbel and we have:

$$f(x) = \frac{1}{\sigma}\, exp\left(-\frac{x-\mu}{\sigma}\right) exp\left\{-\, exp\left(-\frac{x-\mu}{\sigma}\right)\right\}$$

2. Next-Generation Actuarial Approach for Flood Modeling

Extreme Modeling - Estimating extreme distribution's parameters

- To estimate the parameters μ and σ of the Gumbel distribution from a sample of annual maxima $\mathrm{y}_1, \ldots, \mathrm{y}_\mathrm{m}$, the method of maximum likelihood is commonly used.
- The likelihood function is constructed from the probability density function of the Gumbel distribution:

$$f(x) = \frac{1}{\sigma} exp\left(-\frac{\mathrm{y} - \mu}{\sigma}\right) exp\left\{- exp\left(-\frac{\mathrm{y} - \mu}{\sigma}\right)\right\}$$

- The log-likelihood function for a sample of maxima is:

$$\ell(\mu, \sigma) = \left[- log(\sigma) - \frac{\mathrm{y}_1 - \mu}{\sigma} - exp\left(-\frac{\mathrm{y}_1 - \mu}{\sigma}\right)\right] + \cdots + \left[- log(\sigma) - \frac{\mathrm{y}_\mathrm{m} - \mu}{\sigma} - exp\left(-\frac{\mathrm{y}_\mathrm{m} - \mu}{\sigma}\right)\right]$$

- To find the maximum likelihood estimates of μ and σ, we maximize this log-likelihood function with respect to both parameters.
- Accurate estimation of the Gumbel parameters using MLE allows for robust modeling of extreme events and reliable calculation of return levels, which are vital for forecasting and risk management in environmental and engineering applications.
- As a closed-form solution does not exist for both parameters simultaneously, optimization can be done using optimization algorithms.

2. Next-Generation Actuarial Approach for Flood Modeling

Extreme Modeling - Model selection and limitations

- Model selection is based on two criteria: AIC and BIC.
- Both are based on the maximized log-likelihood ($\hat{\ell}$) of the model and penalize the inclusion of additional parameters to avoid overfitting.
- For a model with p parameters and a sample size n, these criteria are defined as:

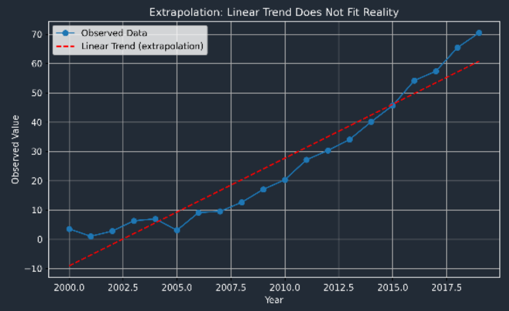
$$\mathrm{AIC} = -2\,\hat{\ell} + 2\,\mathrm{p} \qquad \mathrm{BIC} = -2\,\hat{\ell} + \mathrm{p}\log(\mathrm{n})$$

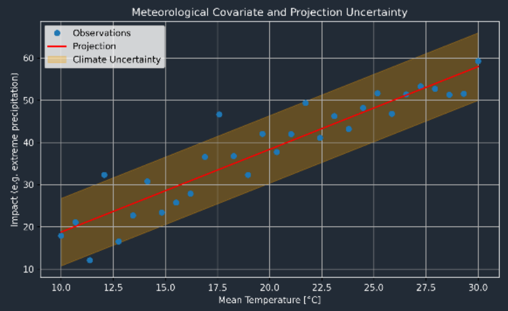
  – AIC is designed to minimize the information loss and is effective in selecting the "least bad" model from a set that may not contain the true model. It achieves a balance between bias and variance.

  – BIC tends to favor simpler, more parsimonious models and is consistent in the sense that it will select the true model (if it is among the candidates and unique) as the sample size grows. BIC often chooses models with fewer parameters than AIC.

Limitations and Recommendations

- Extrapolation assumes a monotonic (often linear) trend which may not reflect the true evolution of the phenomenon.
- Not advisable to extrapolate far into the future (conditions may differ from past observations).
- Add meteorological variables as covariates: represent future evolution but add uncertainties related to climate modeling.





Thank you for your attention and participation.

Should you have any questions or require further information on any of the topics discussed, please do not hesitate to contact us.
We remain at your disposal and would be pleased to provide any additional details or clarification you may need.

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