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THE ACTUARY AND ENTERPRISE RISK MANAGEMENT: INTEGRATING RESERVE VARIABILITY

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Abstract

Motivation. The development of a wide variety of reserve variability models has been primarily driven by the need to quantify reserve uncertainty. This quantification can be used to enhance the understanding of reserve uncertainty as communicated in Own Risk Solvency Assessment reports, can serve as the basis for satisfying several Solvency II requirements in Europe, and is often used as an input to dynamic financial analysis or dynamic risk models. Moving beyond quantification, the purpose of this monograph is to explore other aspects of reserve variability that allow for a more complete integration of these key risk metrics into the larger enterprise risk management framework.

Method. This monograph will primarily use a case study to discuss and illustrate the process of integrating the output from periodic reserve and reserve variability analysis into the wider enterprise risk management processes. Consequences of this approach include the production of valuable performance indicators and a strengthening of the lines of communication between the actuarial function and other insurance functional departments, both of which are valuable to management.

Results. By expanding the regular reserving process to include regular variability analysis and expanding the associated dialogue with management, actuaries can increase their contributions to the understanding of risks related to claim management within an enterprise.

Conclusions. The value of this process is not limited to reserving, as it can logically and directly be extended into pricing, reinsurance optimization, etc.

Keywords. Reserve variability, enterprise risk management, actual versus expected, back-testing, deviations from expectations, one-year time horizon, validation, reserve distribution testing, assumption consistency, run-off analysis, key performance indicator.

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Foreword

Actuaries are very good at measuring reserve risk. Core parts of enterprise risk management include model validation and governance. To play a bigger role in the management of reserve risk, actuaries need to lead engagement with a wider audience of insurance professionals and expose non-actuaries to the wealth of key performance indicators (KPIs) available from reserve analysis integrated within an enterprise risk management (ERM) framework.

In this monograph, authors Mark Shapland and Jeffrey Courchene aim to link reserve variability to the enterprise risk management process in a practical and useful way for all actuaries, demonstrating that:

- Integration of reserve risk measurement within an ERM framework creates powerful KPIs for managing reserving risk.
- KPIs that provide the direction and significance of deviation from expectation are much more powerful than KPIs that provide the direction and magnitude.
- Proactive engagement with insurance professionals outside of the actuarial silo at the front end of a reserve analysis yields better results.

This monograph offers useful approaches both for the actuary and for leaders of insurance entities. For the practitioner looking for KPIs of managing reserve risk, this monograph should prove to be an invaluable resource.

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1. Introduction

Never has it been more important for actuaries to improve their understanding of reserve variability. Updated International Financial Reporting Standards (IFRS 17) will require all insurance companies to regularly report an independently measured risk adjustment. In Europe, Solvency II directives already require the recognition of a risk margin, and validation standards require the actuarial function to comment on material deviations in the technical provisions compared to prior expectations.

A range of reasonable estimates can be selected based on the results of deterministic methods, some scenario testing, and a few basic rules of thumb. Such a range, together with some heroic assumptions, can provide an unsophisticated aid to management in selecting a risk adjustment. More commonly, however, the calibration of the risk adjustment makes use of modern stochastic modeling techniques, resulting in a distribution of possible outcomes,[1] with the outcomes providing the ability to measure statistical properties such as the mean, mode, percentiles, etc.

The results of stochastic modeling techniques have several uses beyond the calibration of a risk adjustment, many of which can be incorporated into the enterprise risk management (ERM) process such that "new" information can be quickly used to assess the performance of existing models based on prior analyses. For example, the performance of the reserving process can be measured by leveraging the relationship between the modeled statistical distribution of possible outcomes and the actual results, and this can be incorporated into a key performance indicator (KPI).

Back-testing is a validation technique that enables the reserving actuary to assess the "new" information in the loss triangles, relative to "known" information and "future" expectations inherent in the prior analysis. However, without an analysis of reserve variability, an assessment of the ***significance of deviations*** from expectations on both a granular level (individual accident periods) and an aggregate level (by reserving segment, by line of business, or by company) is not quantifiable. Even with an analysis of reserve

[1] A distribution of possible outcomes is an expression of the "full breadth" of the possibilities of the future payouts. Note that the estimation of unpaid claims involves significant uncertainties that cannot be completely estimated, so "full breadth" should be thought of as a reasonable estimate of the distribution to the extent that it can be estimated using historical data (for independent risk) and a subjective adjustment to account for variability attributable to systemic risk. Further, the available historical data may be limited such that an adjustment to account for events not in the data may also be necessary. For this reason, it may not be possible to create a distribution of possible outcomes using the most sophisticated actuarial techniques available.

variability, determining whether significant deviations are a result of mean estimation error, variance estimation error, and/or random error is difficult.

A systematic back-testing process provides management with an early indication of the current period's performance relative to expectations and gives the actuarial team early insight into the quality of prior reserve estimates. Further, a systematic back-testing process allows for the evaluation of the loss distributions relative to the distributional expectations for the current period.

Within a comprehensive ERM process, assumption consistency is an important consideration. When selecting a central estimate[2] for unpaid claims, the practicing actuary commonly weights the results from multiple methods. By assigning weights to multiple methods, the actuary is partially accepting or rejecting the assumptions inherent in each method that contributes to the selection of the central estimate.[3]

Following this logic, the selected central estimate (e.g., average of future payments) for each data element (e.g., incremental paid losses) is a weighted average of the data element across the methods that were accepted (at least partially) as reasonable. Likewise, the selected variance estimate (e.g., variance of future payments) is derived from a weighted average of the underlying model distributions that were accepted as reasonable. A consequence of this approach is that model risk is addressed.[4]

In contrast, an approach that uses a single model (e.g., Mack or an over-dispersed Poisson [ODP] bootstrap of the paid chain ladder method alone) to estimate the uncertainty around a point estimate based on multiple methods uses an assumption set for the variance that at best is partially rejected during the selection of the point estimate and at worst involves assumptions that are completely different from those used for the point estimate.

This monograph will develop and examine a framework for reserve distribution testing and validation and demonstrate its use with real data sets within an ERM framework. It will also illustrate how stochastic results based on a one-year time horizon (as specified in Solvency II) can be used in the subsequent year's process of estimating reserves to obtain an early indication of the expected reserve changes due to the emergence of new information. Testing and validation also help improve related risk measures, such as risk margins, risk adjustments, or risk-based capital, but the mechanics of those measures are well documented and outside the scope of this monograph.

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2 This monograph uses the term "central estimate," consistent with Actuarial Standard of Practice No. 43, "Property/Casualty Unpaid Claim Estimates," promulgated by the Actuarial Standards Board (2011). With respect to Solvency II and IFRS 17, regulations and guidance use the term "best estimate" to effectively mean the same thing.

3 Accepting or rejecting assumptions is a simplification of the entire process and all considerations. For example, not giving weight to a method for a specific year is not rejection of the method or any specific assumption within the method, as the method may be given some weight for another year. Thus, this description of the process of weighting methods to arrive at a central estimate should be interpreted as including all considerations an actuary uses.

4 Weighting deterministic methods is also a way to address model risk. The entire process of weighting multiple models is outside the scope of this monograph, but common issues (such as consistency of variances between models) are assumed to have been resolved when selecting weights.

1.1. Research Context

The importance of assumption consistency should not be underestimated. Paragraph 3.6.2 of Actuarial Standard of Practice No. 43 (2011) states that an actuary "should use assumptions that, in the actuary's professional judgment . . . are not internally inconsistent." Also note that Article 122.2 of the Solvency II Framework Directive (FD) (2009) states that models "used to calculate the probability distribution forecast shall . . . be consistent with the methods used to calculate technical provisions." Finally, Section C of Technical Actuarial Standard M: Modelling (TAS M) (2010) states that assumptions should be consistent in "a model or in a suite of models." TAS M further suggests that different assumptions (i.e., use of multiple methods that use different assumptions) are "not always inconsistent. For example, if several independent models are used in conjunction to provide better estimates than any one model could provide on its own, different assumptions might be chosen deliberately." If, however, inconsistent assumptions are used, TAS M requires a disclosure statement.[5]

Actuarial literature includes many approaches to quantifying the uncertainty of reserve estimates based on the variability observed in the actual historical development of the claims under consideration. In practice, the approaches used most often are statistical approximations to relatively simple regression models. Such approaches have the advantage of being (relatively) straightforward to implement, interpret, and explain. They can be applied equally well to accident or underwriting period data to generate results on the same basis. Two regression models tend to dominate: the Mack (1993) linear regression model and the ODP bootstrap model originally developed by England and Verrall (1999, 2002).[6]

In both cases, the expected values of the reserve estimate are equal to the results of the deterministic paid chain ladder method (using the all-year volume-weighted average development factors), which is rarely the sole basis for the central estimate, especially for immature accident periods. It should be noted that some practitioners overcome this estimation inconsistency by "shifting" the modeled distribution such that the mean of the distribution is equal to the central estimate and the standard deviation from the model is maintained, or by "scaling" the modeled distribution such that the mean of the distribution is equal to the central estimate and the coefficient of variation (CoV) from the model is maintained. The "shifting" is usually implemented in an additive fashion by adding to each iteration the difference between the central estimate and the result of the paid chain ladder method (using the all-year volume-weighted average link ratios) by accident period. Incremental expected payments by development period are often produced by allocating the "shift" in proportion to

[5] TAS M and other TASs (TAS D and TAS R) were replaced in 2017 by the more generic TAS 100: Principles for Technical Actuarial Work and TAS 200: Insurance.

[6] The actuarial literature contains many other models, and we have tried to identify many of them in the reference list. Any of these other models could also be part of the ERM process described in this monograph, and including them would make the modeling process more robust, but the monograph will focus only on these models to keep the scope manageable.

the overall expected average incremental payments before the shift. In contrast, the "scaling" is usually implemented in a multiplicative fashion.

As originally framed, the Mack (1993) model (and, by extension, the Merz and Wüthrich (2008) model) provides a method for estimating a CoV for the reserve estimate. To convert the CoV into an estimate at a specific confidence level, however, it is necessary to select a particular parametric probability distribution whose parameters can be determined by the CoV together with the central estimate.

Alternatively, both models can be bootstrapped, resulting in discrete output that does not require the selection of a particular parametric probability distribution. The ODP bootstrap model originally developed by England and Verrall (1999, 2002) can also be extended to simulate any number of methods without requiring the selection of a particular parametric probability distribution, as described in Shapland (2016). This approach enables the actuary to maximize the assumption consistency between the central estimate of unpaid claims and the calibration of reserve variability.

1.2. Objective

The goal of integrating unpaid claim variability into the ERM process is to improve the management of reserve risk (i.e., the estimation and management of unpaid claims and their uncertainty).

To manage reserve risk, one needs to measure it first. Integrating the measurement and management of reserve risk into a continuously monitored ERM process ensures that assumptions are tracked and validated over time and that changes in assumptions are justified relative to the performance of prior assumptions.

Back-testing is a validation technique that can provide insight to improve a reserving process, in that the actuary is forced to understand inevitable deviations from expectations, and future decision points (i.e., assumptions and expert judgment) can be based on the performance of past decision points.

2. Notation

The notation used is taken from the CAS Working Party on Quantifying Variability in Reserve Estimates summary report (2005).

Many models visualize loss data as a two-dimensional array, (w,d) with accident period or policy period w and development age d (think w = "when" and d = "delay"). For this discussion, it is assumed that the loss information available is an "upper triangular" subset for rows $w = 1, 2, \ldots, n$ and for development ages $d = 1, 2, \ldots, n - w + 1$. The "diagonal" for which $w + d$ equals the constant, k, represents the loss information for each accident period w as of accounting period k.[7]

For purposes of including tail factors, the development beyond the observed data for periods $d = n + 1, n + 2, \ldots, u$, where u is the ultimate time period for which any claim activity occurs—i.e., u is the period in which all claims are final and paid in full—must also be considered.

The monograph uses the following notation for certain important loss statistics:

$c(w,d)$:	cumulative loss from accident year w as of age d[8]
$q(w,d)$:	incremental loss for accident year w from $d - 1$ to d
$c(w,n) = U(w)$:	total loss from accident year w when claims are at ultimate values at time n, or with tail factors[9]
$c(w,u) = U(w)$:	total loss from accident year w when claims are at ultimate values at time u
$R(w)$:	future development after age d for accident year w, i.e., $= U(w) - c(w,d)$
$f(d)$:	factor applied to $c(w,d)$ to estimate $q(w,d + 1)$, or used more generally to indicate any factor relating to age d
$F(d)$:	factor applied to $c(w,d)$ to estimate $c(w,d + 1)$ or $c(w,n)$, or used more generally to indicate any cumulative factor relating to age d
$G(w)$:	factor relating to accident year w—capitalized to designate ultimate loss level

7 For a more complete explanation of this two-dimensional view of the loss information, see the *Foundations of Casualty Actuarial Science* (2001), Chapter 5, particularly pages 210–226.

8 The use of accident year is for ease of discussion. All the discussion and formulas that follow could also apply to underwriting year, policy year, report year, etc. Similarly, year could also be half-year, quarter, or month.

9 This would imply that claims reach their ultimate value without any tail factor. This is generalized by changing n to $n + t = u$, where t is the number of periods in the tail.

$h(k)$: factor relating to the diagonal k along which $w + d$ is constant[10]
$e(w,d)$: a random fluctuation, or error, which occurs at the w,d cell
$E(x)$: the expectation of the random variable x
$Var(x)$: the variance of the random variable x
$Dist(x)$: the distribution of the random variable x
$P_y(x)$: the y percentile of the distribution of the random variable x
$\hat{x}$: an estimate of the parameter x

What are called factors here could also be summands, but if factors and summands are both used, some other notation for the additive terms would be needed. The notation does not distinguish between paid and incurred, but if this is necessary, capitalized subscripts P and I could be used.

[10] Some authors define $d = 0, 1, \ldots, n - 1$, which intuitively allows $k = w$ along the diagonals, but in this case the triangle size is $n \times n - 1$, which is not intuitive. With $d = 1, 2, \ldots, n$ defined as in this monograph, the triangle size $n \times n$ is intuitive, but then $k = w + 1$ along the diagonals is not as intuitive. A way to think about this that helps tie everything together is to assume the variables w are the beginning of the accident periods and the d variables are at the end of the development periods. Thus, if years are used then cell $c(n, 1)$ represents accident year n evaluated at $12/31/n$, or essentially $1/1/n + 1$.

3. Back-Testing

Back-testing compares actual results with the expected results. Among other uses, a basic question being answered is: "Are the actual results better or worse than expected?" This simple question has many important nuances and ramifications, including psychological implications in the sense that people naturally hope results are "better than expected." While intuitively understanding that "worse than expected" results will occur about half of the time (if the selected expected value is unbiased),[11] the tendency to want to beat expectations can lead to bias in the selection of estimates—overestimating the true central estimate of future incremental losses. However, pressure to publish better financial results can result in the opposite bias.

In its simplest form, a back-test can be formulated as in (3.1) for a particular incremental value.

$$q(w,d) - E\left[\hat{q}(w,d)\right] \tag{3.1}$$

By subtracting the expected result from the actual result, a "better than expected" result means that the actual result was less than the expected result. Somewhat counterintuitively, however, this "better than expected" result is a negative number.

The term "run-off" (or "run-off analysis") is often used interchangeably with "back-test," as the goal is to watch how actual results compare to initial expectations. However, the run-off outcome is generally formulated as in (3.2) for a particular incremental value.

$$E\left[\hat{q}(w,d)\right] - q(w,d) \tag{3.2}$$

For the run-off test, a "better than expected" result also means that the actual result was less than the expected result, but in this case the value is positive and perhaps more intuitive. Even as "back-test" and "run-off" can be used interchangeably, formulas (3.1) and (3.2) could also be interchanged between terms. For simplicity, from this point forward the monograph will refer only to "back-testing" and will assume the reader can transition between terms and between formulas (3.1) and (3.2) as preferred.

[11] Technically, "worse than expected" about half the time would be true only for the median, so this is meant merely as a general statement about unconscious biases.

A back-test can be performed at either a granular or an aggregate level. At a granular level, this would involve evaluating a single method or even a specific assumption within a method, with the goal of understanding the efficacy of that method or assumption. At the highest aggregate level, the back-test will provide insight into all the methods and assumptions used to produce a final selected aggregate estimate. Granular-level back-testing tends to be more of an academic or technical review, whereas higher-level back-testing tends to focus on a management level, which is where the remainder of this monograph will focus.

Within the ERM vernacular, the performance of the reserve estimates under back-testing can be considered a KPI. As with other KPIs, information about deviations from expectations (i.e., central estimates) provides valuable information for management—not just senior management, but all levels of management in all departments using the reserve estimates.

3.1. Back-Testing of Deterministic Estimates

For deterministic methods, the point estimate is the sole source of the "expectation" that is compared directly to the actual result.[12] The back-test compares the current actual (incremental) development against the expected (incremental) values from the prior projection (which include all future development periods).

Consider the back-test results in Table 3.1. Actual accruals for accident year (AY) 2021 are shown but expected accruals for AY 2021, and therefore differences, are not shown. This is because the 2021 calendar year (CY) experience includes payments and case reserve changes attributable to AY 2021 and prior. The expectations, in contrast, are based on the reserve analysis as of the prior year-end, in this case for AY 2020 and prior (i.e., as of December 31, 2020). In this monograph, the term "AY < CY" is used to denote the subtotal of all accident years not including the current accident year, and "AY = CY" is used to denote the experience for the most recent accident year that does not have a comparable expectation based on the prior reserve analysis alone.

The "Difference" columns in Table 3.1 are calculated using formula (3.1), but, like all deterministic back-tests, the amounts reveal only the ***direction*** and ***magnitude*** of the outcome. Similar comparisons of actual and "expected" values are not difficult to compile for other data elements (e.g., closed claims, reported claims, etc.), but while the total numbers of positive and negative deviations may be instructive, they do not overcome the lack of a measure of ***significance***.

Calculation of the expected incremental amounts requires care. For this, in the spirit of assumption consistency, each method used can be converted into the incremental value being evaluated (e.g., paid claims) and then weighted together to arrive at an expectation that is consistent with the overall assumption set used to determine the selected estimate by accident period.[13]

[12] For a deterministic analysis, the point estimate does not contain any specific statistical meaning such as a mean, mode, or median, so the term "expectation" likewise does not have any statistical connotation other than being a convenient reference to the central estimate.

[13] The "Results – Deterministic" sheet in the "LOB Backtest.xlsm" file illustrates the process of combining weighted estimates of the incremental values consistently with the overall unpaid estimates by accident year.

Table 3.1. Back-Testing Example: Deterministic Actual vs. Expected

Sample Insurance Company
Consolidation of All Segments
Deterministic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2012	120	3,069	3,701	(632)	1,863	2,158	(295)
2013	108	5,905	7,405	(1,500)	3,145	2,794	351
2014	96	8,986	10,073	(1,087)	3,553	6,142	(2,589)
2015	84	18,992	19,027	(35)	9,872	11,285	(1,413)
2016	72	51,003	47,151	3,852	25,942	26,873	(931)
2017	60	105,067	103,127	1,940	52,012	54,534	(2,522)
2018	48	202,932	194,479	8,453	106,624	106,020	604
2019	36	334,434	325,644	8,790	189,908	192,143	(2,235)
2020	24	841,484	833,793	7,691	454,217	479,073	(24,856)
2021	12	1,798,138			2,528,235		
Totals		3,370,010			3,375,371		
AY < CY		1,571,872	1,544,400	27,471	847,136	881,022	(33,886)

A typical shortcut of multiplying the selected estimate by a selected development pattern could create a disconnection between assumptions at the macro and micro levels and should therefore be avoided, if possible. Since a primary goal of back-testing is to validate assumptions, disconnecting the assumptions from the back-testing will hinder the validation process. ***The significance of the disconnection will depend on the assumptions ignored by using the shortcut***.

A logical extension of this back-test is to check whether the actual outcome falls within the reasonable range that was used to develop and select the central estimate. With a range, the back-test can be formulated as a percentage, with a result between 0% and 100% indicating that the outcome was within the range, a result greater than 100% indicating that the outcome was above the range, and a result less than 0% indicating that the outcome was below the range.

$$\frac{q(w,d) - Min\left[\hat{q}(w,d)\right]}{Max\left[\hat{q}(w,d)\right] - Min\left[\hat{q}(w,d)\right]} \tag{3.3}$$

Continuing the example above, the back-test using a range is illustrated in Table 3.2, with the "Range Percent" columns calculated using formula (3.3).

The range used for this test can vary based on preferences or testing criteria. For example, the range could include only methods given some weight by accident year (the "weighted range"), the range could include all methods given weight for any accident

Table 3.2. Back-Testing Example: Actual to Deterministic Range of Estimates

Sample Insurance Company
Consolidation of All Segments
Deterministic Actual vs. Method Range as of December 31, 2021

AY	Age	Actual Paid	Expected Paid Minimum	Expected Paid Maximum	Percent Range	Actual Incurred	Expected Incurred Minimum	Expected Incurred Maximum	Difference
2012	120	3,069	3,701	3,704	–21075%	1,863	2,158	2,162	–6790%
2013	108	5,905	5,827	8,983	2%	3,145	1,210	4,380	61%
2014	96	8,986	9,887	10,277	–231%	3,553	5,955	6,356	–599%
2015	84	18,992	17,726	20,381	48%	9,872	9,981	12,657	–4%
2016	72	51,003	44,889	49,487	133%	25,942	24,600	29,236	29%
2017	60	105,067	100,495	106,278	79%	52,012	51,856	57,857	3%
2018	48	202,932	191,183	198,745	155%	106,624	102,222	110,845	51%
2019	36	334,434	310,031	338,355	86%	189,908	174,120	205,898	50%
2020	24	841,484	794,706	853,821	79%	454,217	436,298	503,306	27%
2021	12	1,798,138				2,528,235			
Totals		3,370,010				3,375,371			
AY < CY		1,571,872	1,481,602	1,586,896	86%	847,136	811,568	929,564	30%

year (the "method range"), or the range could be expanded to include methods not given any weight or scenario testing (the "possible range").

The relationship between the actual outcome and the range is a bit more instructive than the back-test of actual to "expected" results, but, unfortunately, it still only provides information about the ***direction*** and ***magnitude*** and fails to provide information about ***significance***.[14] Thus, while we could dig further into the systematic testing of the impact of each assumption on the outcome, we will leave that to the reader and move on to measuring the significance of outcomes.

3.2. Back-Testing of Stochastic Estimates

One way to measure the significance of the deviations from expectations is to leverage a reserve variability analysis—i.e., instead of simply reviewing whether the outcomes are better or worse than expected, the question becomes: "Are the outcomes significantly different than expected?" Similar to a deterministic back-test, the calculation of expected values will reflect the models employed during the analysis and used to select the unpaid claim distribution and requires assumption consistency with the methods contributing to the selected central estimate.

[14] Having a range does provide significance in the sense that you can see where the outcome lies within or outside the range. However, here we are referring to measurable statistical significance.

More importantly, to examine the efficacy of the models and assumptions used in a stochastic analysis of unpaid claims, consistency of assumptions for both mean and variance is considered. As noted in Section 1.1, using multiple methods to select a point estimate and then using a single "shifted" model approach may be inconsistent in the sense that the assumption sets for the mean and variance are likely to be completely different.

Assuming that model and assumption consistency is reasonably maintained within a reserve variability analysis, the assessment of the significance or materiality of the resulting differences is a straightforward process using a percentile function. Formula (3.4) uses the Excel PERCENTRANK.INC function, but percentile functions for other software would be similar.[15]

$$P_x\left[q(w,d)\right] = \text{PERCENTRANK.INC}\left\{Dist\left[\hat{q}(w,d)\right], q(w,d)\right\} \tag{3.4}$$

As for the deterministic back-test, the only area in which care needs to be exercised is in the development of the distributions for each incremental value. The output of stochastic models may include only the simulations for the totals by year, but most software will include the simulations of incremental amounts as an output option. Assuming the incremental simulations are available, then the only issue remaining is to ensure that the incremental output has been weighted and shifted/scaled consistently with the overall model assumptions.[16]

For the examples used in this monograph, a reserve variability analysis was completed using four variations of the ODP bootstrap model (i.e., paid chain ladder, incurred chain ladder, paid Bornhuetter-Ferguson, incurred Bornhuetter-Ferguson), including weighting and shifting to match the assumptions and unpaid claim estimates for a deterministic analysis using the same methods in order to estimate the expected distribution of possible outcomes. The approach was used for three sample reserving segments and correlated to derive an aggregate distribution to illustrate the process for a whole company.[17]

Large (small) deviations between actual and expected values are expected when a reserve variability analysis concludes that uncertainty is high (low). The use of an expected distribution of possible outcomes for each accident period and in total (i.e., AY < CY) implies that the use of percentiles automatically adjusts for differences in uncertainty by year or segment, as illustrated in Table 3.3.

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[15] In Excel, the = PERCENTRANK.INC(*Array,X*) function has two required parameters, *Array*, which is the range of values that can be used to determine relative standing within the range, and *X*, which is the value for which you want to determine the rank. The function returns the rank of *X* within the *Array* as a percentage (0, 1, inclusive) of the range of values.

[16] For a useful reference, see Shapland (2016). The "RawSimResults" sheets in the "LOB Backtest.xlsm" file assume that the incremental output by year and by iteration has been weighted and shifted as described in Shapland (2016).

[17] While the terms can be used interchangeably, in this monograph "consolidation" is used to mean a deterministic sum of the parts or segments, whereas "aggregation" is used to mean the stochastic correlation of the parts or segments.

Table 3.3. Back-Test Example: Stochastic Actual vs. Expected

Sample Insurance Company
Aggregation of All Segments
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2012	120	3,069	4,077	31.8%	1,863	2,115	49.8%
2013	108	5,905	6,163	47.9%	3,145	1,819	80.6%
2014	96	8,986	10,176	33.6%	3,553	6,026	20.9%
2015	84	18,992	20,033	39.0%	9,872	10,399	46.3%
2016	72	51,003	48,298	71.6%	25,942	25,562	55.3%
2017	60	105,067	104,415	54.3%	52,012	53,101	44.8%
2018	48	202,932	196,083	74.2%	106,624	104,075	61.7%
2019	36	334,434	331,701	57.1%	189,908	185,173	64.0%
2020	24	841,484	839,689	52.8%	454,217	469,822	29.3%
2021	12	1,798,138			2,528,235		
Totals		3,370,010			3,375,371		
AY < CY		1,571,872	1,560,637	61.2%	847,136	858,093	37.6%

Note that for simplicity the examples and case study do not include an expected distribution of possible outcomes for the most recent accident period (i.e., AY = CY), as this would require modeling that is generally not included in the reserving analysis for the prior period. However, if the reserving analysis is extended to include a distribution for the next accident year (perhaps in a "pricing risk" calibration), then this could be included with the back-test. The only caveat to the inclusion of pricing risk is that it will be based on expectations of future exposures, so any back-test should first adjust the distribution for the actual exposures prior to calculation of percentiles in order to more properly compare these once-future exposures to all the prior years that were based on actual exposures.

Deviations expressed as a percentile provide information about ***direction***, ***magnitude***, and ***significance***. Note that deviations expressed as extreme percentiles do not necessarily indicate a problem with the methodology employed during the prior analysis, as observations at the extreme tails of a distribution of possible outcomes should occur.

3.3. Stochastic Key Performance Indicators

Reviewing a single percentile is instructive but hardly useful. In the greater scheme of determining materiality, the single observation is more about random noise than materiality. Only with a large number of observations can the analyst start to detect material issues by observing patterns or biases in the percentiles. It is in the detection of patterns that the KPIs add value to the stochastic analysis. Consider, for example,

Figure 3.1. Predefined KPI Thresholds

Figure 3.1, which graphically displays predefined thresholds[18] that are used to define stochastic KPI thresholds.

As illustrated in Figure 3.1, the case study in this monograph uses thresholds at the 25th and 75th percentile, the 5th and 95th percentile, and the simulated minimum and maximum of the distribution of possible outcomes to denote material deviations from expectations. Such deviations can be communicated visually using a table of numbers (see Tables 3.3 and 5.12), a chart of individual accident periods (see Figures 3.2a and 3.2b), or a chart of the total calendar year—i.e., all accident years combined (see Figures 3.3a and 3.3b).

Figures 3.2a and 3.2b show the actual incremental paid and actual incremental incurred KPI thresholds by accident year for a single reserving segment; the black, orange, and red points fall within the expected distribution of possible outcomes.[19] Note that the blue color-coded areas represent the areas defined by the predefined thresholds, as illustrated in Figure 3.1.

Figures 3.3a and 3.3b show the actual incremental paid and actual incremental incurred KPI thresholds for the calendar year (i.e., all accident years AY < CY) for a single reserving segment. In this case, one red point falls within the expected distribution of possible outcomes while the other red point exceeds the maximum simulated value. Again, the blue color-coded areas represent the areas defined by the predefined thresholds.

When using tables or charts, the materiality of the deviation can be better understood by using color-coded fonts (see Tables 3.3 and 5.12) or color-coded points representing breaches of predefined thresholds (see Figure 3.1) within the color-coded areas representing the distribution of possible outcomes.

There are caveats to this approach, which include but are not limited to the following:

1. Various additional assumptions (each requiring validation) need to be made to produce a distribution of possible outcomes.
2. Triangle-based models are being used in this monograph, but in some situations alternative models may be needed. In either case, the key is to make sure the model(s) being used reasonably replicate the underlying claims by size of loss.
3. Analysis of industry performance over the past few decades shows that most model distribution estimates, absent adjustment for model weaknesses, may underestimate reserve risk (i.e., the distribution of possible outcomes could be wider).[20]

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[18] The setting of thresholds is not a trivial step in the design of KPIs, as this determines the number of threshold breaches that are expected to occur. Thresholds are discussed further in Section 6 below.

[19] The reserving segment is Commercial Auto and the data used to create Figures 3.2a, 3.2b, 3.3a, and 3.3b is shown in Table 5.12.

[20] Various authors have back-tested the ODP bootstrap and other models and have found a similar tendency toward underestimation of reserve risk. For a recent reference, see Shapland (2019).

Figure 3.2a. Paid KPI Thresholds by Accident Year

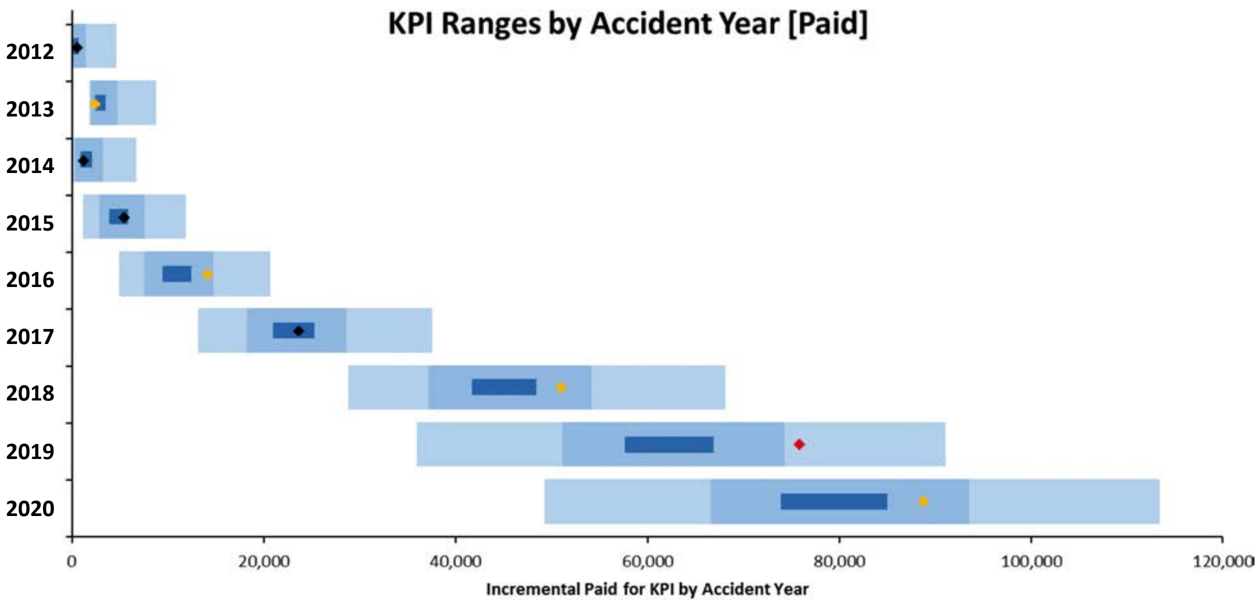


Figure 3.2b. Incurred KPI Thresholds by Accident Year

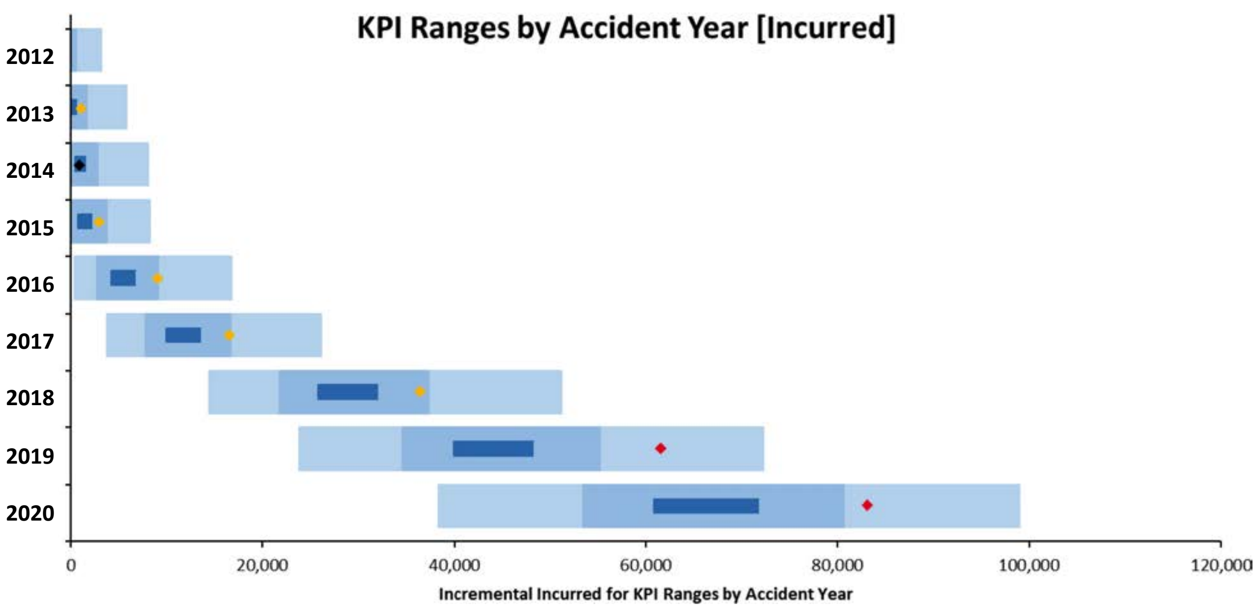


Figure 3.3a. Calendar Year Paid KPI

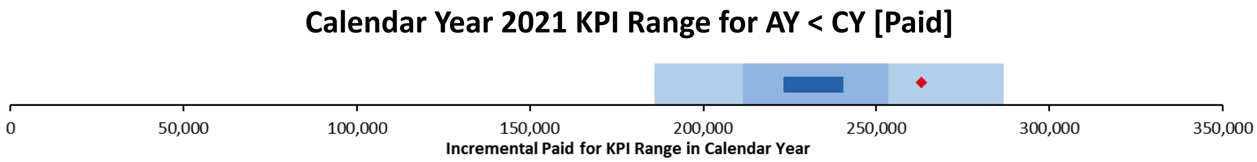
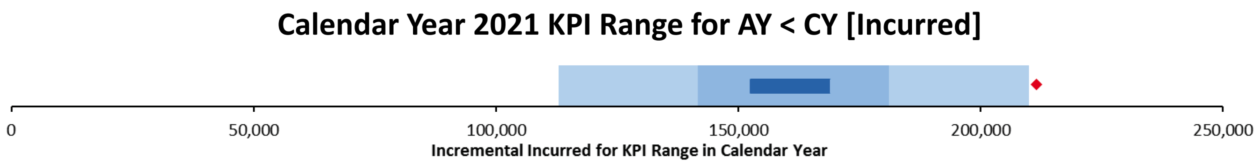


Figure 3.3b. Calendar Year Incurred KPI



4. Reserving Within an ERM Framework

There are numerous definitions of ERM. The common themes and principles that emerge from the various definitions, as summarized by the 2016 International Actuarial Association paper (2016) "Actuarial Aspects of ERM for Insurance Companies," are as follows:

1. ERM is a continuous process.
2. ERM adopts a holistic view of risk and assesses risk from the perspective of the company's aggregate position as well as from a stand-alone perspective.
3. ERM is concerned with all risks, including those that are unquantifiable or difficult to quantify.
4. ERM considers uncertainty from both positive and negative viewpoints.
5. ERM aims to achieve greater value for all stakeholders by assisting in achieving an appropriate risk-reward balance.
6. ERM considers both the short-term and the long-term aspects of risk.

Key components of a company's ERM system include risk governance, risk strategy, and the steps that make up the core risk management process (consisting of risk identification, risk assessment, risk measurement, risk response, risk monitoring, and risk reporting).

Risk governance generally includes the assignment of roles and responsibilities and the establishment of risk policies and procedures, robust internal control systems, and risk culture. For the assignment of roles and responsibilities, many companies adopt a "three lines of defense" model. The first line is responsible for the regular operations of the business. The second line is responsible for overseeing the operations of the first line. Finally, the third line is responsible for independent review (i.e., audit) and assurance of the operations of the first and second lines.

Once risks are identified, analyzed, and measured, then management is faced with responding to the risks. Responses are often characterized as avoiding, accepting, mitigating, or sharing.

An ERM process does not change the way that an actuarial function manages unpaid claim estimates and the corresponding reserving risk. Rather, the ERM process formalizes the governance around the process and ensures a consistent and continuous approach. The case study below describes one such approach. With or without an ERM process, the actuarial function within an insurance entity is responsible for the reliability and adequacy of the estimation of unpaid claims, including

- promptly reporting major deviations from expectations,
- providing management with relevant information necessary to manage the company, and
- investigating the causes of deviations such that changes to the assumptions and methodologies can be suggested to improve the future central estimates (and estimated distribution) of unpaid claims.

An ERM process includes a change control process. Model changes are restricted, and authorized changes must be evidence based, validated, documented, and processed within approved organizational structures.

The quality of risk measurement and reporting determines how risk is monitored. In the case study below, a high-quality measurement process is described that expands the scope of unpaid claim estimates monitoring to include

- clear links to risk ownership and the establishment of timely automatic reporting mechanisms;
- consistent, accurate, and auditable control of both the deterministic method(s) and methodology supporting the selected central estimate, and the stochastic model(s) supporting the corresponding reserve uncertainty conclusion in the form of an expected distribution of possible outcomes;
- the production of metrics that an actuarial function can use to identify deviations from prior expectations and efficiently allocate analysis resources, prior to commencing with an analysis update;
- allowing for analysis resources to hypothesize and monitor whether deviations from expectations are the result of mean estimation error, variance estimation error, or random error;
- the production of performance indicators that management can use to anticipate the conclusions of the actuarial analyses, based on how the prior assumptions have held up; and
- expansion of the discussion regarding major deviations from expectations to include interested parties outside the actuarial function at the beginning of an analysis update.

Monitoring is scheduled at the appropriate frequency to allow management of the risk in question, meaning sufficiently frequent to allow decisions to be made and action to be taken on an informed basis. The case study below describes a process that uses annual analyses, which is typical, but a more frequent basis can be similarly achieved if the data and processes are adjusted accordingly.

5. Enterprise Risk Management in Action: A Case Study

With the foundation established, the rest of the monograph will use a case study to illustrate the advantages of integrating reserve variability into an ERM system. Summary tables and graphs for each line of business and the aggregate results are shown in Appendices C, D, E, and F, respectively.

In this case study, the back-testing output of the reserve estimation process is the KPIs, i.e., the observed percentiles relative to the expected outcomes. The observed percentiles are assessed individually, with extreme percentiles meriting added scrutiny, and collectively, ensuring that the observed percentiles exhibit the expected tendency to be uniformly distributed. In this case study, it is the actuarial function's performance that is being measured, as it oversees the reserve valuation exercise and has ownership of the models being employed.[21]

In this way, the KPIs allow the actuarial function to assess how well the suite of models and methods performed during the most recent observation period. The KPIs enable the actuarial function to identify and strategically invest more resources in the areas where employed models and methods' projected amounts differ significantly from actual accruals. The sample of observed percentiles, which will be uniformly distributed if the models are reasonably calibrated, can also be reviewed to draw conclusions regarding the individual and aggregate distributions of possible outcomes.

5.1. Introduction

The case study presents the work cycle for an actuarial function within a sophisticated ERM system, including a more robust estimation process for the unpaid claim estimates as of December 31, 2021. To set the stage, a general timeline of activity is established before the details are presented.

[21] This does not imply that the actuarial function operates in a vacuum, nor that only the actuarial department's performance should be reviewed. In practice, the management review of unpaid claim estimates and selection of reserves would be a joint effort involving many departments, but the technical aspects of the case study are focused primarily on the actuarial function's role.

- Prior to year-end 2021: Levels of back-testing granularity are chosen[22] to be entity total, segment total (where entity total = $\sum$ segment), and AY for each segment (where segment total = $\sum$ AY for each segment).[23]
- Prior to year-end 2021: Two levels of thresholds are chosen,[24] and in this case realized claim values (paid and incurred) outside the interquartile range (i.e., 25th to 75th percentile) form the first response level and realized claim values outside the central 90% (i.e., 5th to 95th percentile) form the second response level.[25]
  - Claim values outside the interquartile range receive additional analysis, including testing whether the breach is due to random variation, assumption error, or systematic change in the underlying process (such as new claims handling procedures).
  - Claim values outside the 90th percentile range initiate a process of additional information collection to review data quality, underwriting, claims handling, and other relevant factors.
- Prior to year-end 2021: Elements included in the automatic back-testing system are defined to include paid loss and incurred loss. ***Other elements, such as reported and closed claim counts, could be included in a live system but they are excluded here for simplicity.***
- Prior to year-end 2021: Documentation standards[26] for assumption setting and use of expert judgment are reviewed, revised, and updated for each reserving segment.[27]
- January 3, 2022: The accounting function closes the books such that all data elements as of the December 31, 2021, valuation date are available on an AY and CY basis.
- January 4, 2022: Back-testing of the CY 2021 estimates is completed, and deviations[28] from the estimates for CY 2021 (based on the loss reserve analysis as of December 31, 2020) are known and compared with the action thresholds.
  - Previously identified segments (or previously identified data elements from a segment) are included in the automated back-testing procedure such that a robust validation of the CY 2021 methods and assumptions can be achieved.

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[22] Note that changes in the segmentation and the associated ramifications for the ERM system need to be thoroughly addressed prior to year-end.

[23] Note that it is often more practical to exclude special segments and very mature AYs, such that “entity total = $\sum$ segment + excluded segments” and “segment total = $\sum$ AY for each segment + prior AYs.”

[24] Note that thresholds could be nominal (e.g., differences larger than $1 million), relative (e.g., differences 150% larger than the mean expected), or distributional (e.g., observations above the 95th percentile of possible future outcomes).

[25] Note that the identification of a threshold breach does not imply that an error in the prior calculation has been identified. Rather, a breach brings attention to large deviations, allowing the assumptions and methodology underlying the expectation to be reviewed.

[26] Note that enhanced documentation includes a list of relevant and material assumptions for each segment, the results of sensitivity testing material assumptions, segment-specific diagnostics with qualitative descriptions supporting the conclusions, and justification (if available) for material expert judgment exercised.

[27] Note that documentation together with automated back-testing ensures that a change in employee personnel does not unnecessarily render the historical assumption set (and associated rationale) less transparent or understandable (i.e., the institutional memory stays intact).

[28] The automated back-test identifies areas where the deviations from predictions breach a predefined threshold (for multiple levels of granularity and for multiple data elements).

– AY 2020 and prior incremental accruals (i.e., AY < CY) are compared to the expectations as of December 31, 2020, based on the final distribution of possible outcomes estimated by the actuarial function in the prior reserving analysis. ***The process can be expanded to include specific models, but that is not done here only for simplicity***.
– AY 2021 incremental accruals (i.e., AY = CY) can be compared to the expectations for losses related to the unearned premium as of December 31, 2020, with an adjustment for the actual new business written during 2021. ***For simplicity, these amounts are not included in the details of the case study presented below, although it should be noted that deviations from expectations can be described as a mixture of reserve risk and premium risk.***

- January 4, 2022: The actuarial function determines an efficient allocation of analysis resources so that segments and/or AYs that exhibit a large number of significant deviations receive additional attention.
- January 4, 2022: Breaches in the 5% tail areas initiate an early warning system intended to collect relevant information from other departments (e.g., data quality, underwriting, claims, reinsurance, etc.).
- January 4, 2022: Conditional reserve estimates using the one-year time horizon analysis as of December 31, 2020, are available to management as an early indication of the anticipated reserve changes for the December 31, 2021, evaluation. (See Appendix A for an overview of the one-year time horizon.)
- January 4, 2022: The actuarial function begins with its valuation analysis as of December 31, 2021. The assumptions for the next valuation are reviewed based on considerations from back-testing and the threshold breach reviews.
- January 4–21, 2022: During the analysis, diagnostics and statistical tools are used to review assumptions and calibrate the parameters of each of the methods and models that comprise the segment's methodology. Such diagnostics and tests are kept in a log so that they can be referred to in the actuarial report. Also, interaction with interested parties outside the actuarial function provides a critical sounding board for expert judgment exercised.
- January 24, 2022: The valuation analysis is concluded, and loss reserve recommendations are provided to management. Adjustments to prior methods and assumptions that result in material differences in the valuation are provided, with justifications for the changes.
- February 7, 2022: Further analyses of change are completed and documented, and possible improvements to the process for the December 31, 2022, valuation are documented based on the performance and the findings from the December 31, 2021, analysis.

5.2. Basis of Underlying Data

In producing this case study, real industry data was used.[29] To ensure confidentiality, triangular data for 10 accident years was aggregated from a small number of

[29] The data comes from historical Schedule P triangles, as compiled by SNL Financial.

insurance entities writing commercial auto (CA), private passenger auto (PPA), and homeowners (HO) policies, as of consecutive year-ends. This produced a data set for a fictitious entity.

By performing both deterministic and stochastic analyses of the annual data for this fictitious entity, the case study tries to highlight the wealth of information that is ripe for integration within an ERM framework.[30] In addition, the case study is intended to enhance understanding of the underlying dynamics, including the production of KPIs for reserving risk.

The deterministic analysis was limited to four methods, namely the paid and incurred chain ladder ("Pd CL" and "Inc CL") methods and the paid and incurred Bornhuetter-Ferguson ("Pd BF" and "Inc BF") methods. The selected ultimate loss estimates for each accident year are a weighted average of the four methods. To maximize assumption consistency, four ODP bootstrap models consistent with the four deterministic methods were used. The selected distribution of possible outcomes for each accident year is a weighted average of the four ODP bootstrap models (using the same weights as for the deterministic methods),[31] shifted such that the mean of the distribution for each accident year is equal to the selected unpaid loss.

It is reasonable to expect that the underlying data within the fictitious entity would be available by the first Monday of the year (January 3, 2022) and that the management of the fictitious entity would allow the actuarial department to spend three weeks completing its work. Such a tight schedule emphasizes the importance of activity before year-end, which calibrates the framework such that diagnostics and KPIs are produced as soon as the underlying data is available.

In the case study, the diagnostics and KPIs focus on the performance of the most recent period (i.e., the past CY). The framework and approach can just as easily focus on multiple periods, which would be appropriate for some reserving segments. The multiple-period approach provides insight that could be used to reduce unnecessary adjustments in the underlying actuarial assumptions (i.e., additional volatility caused by overreaction to single-period observations).

5.3. Prior Analysis Model Validation

As noted above, enhanced standards for documentation of assumptions and expert judgment are established for the analysis and validation of each reserving segment. Most of the validations are part of an ongoing process, repeated following a planned schedule as sufficient new data arrives. A nonexhaustive list of assumptions that require validation and examples of enhanced documentation include the following.

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30 Actuarial departments often undertake both deterministic and stochastic analyses at year-end, but only deterministic analysis is performed for interim quarter-ends. To expand the entire ERM process to operate on a quarterly basis, the stochastic elements would also need to be included quarterly.

31 Note that weighting distributions requires that possible outcomes mean the same thing in each model. For example, the unadjusted output for an ODP bootstrap model applied to a paid (an incurred) loss triangle would result in a distribution of possible unpaid loss (incurred but not reported) outcomes. Prior to weighting, the incurred ODP bootstrap models implemented were adjusted such that the outputs were distributions of possible unpaid loss outcomes as described in Shapland (2016).

5.3.1. Selected Loss Development Factors

The Mack (1993) paper introduced three assumptions that underlie the chain ladder method, the first two of which are validated as part of the enhanced documentation for the fictitious entity.

$$E\left[c\left(w,d+1\right)\middle|c\left(w,1\right),\ldots,c\left(w,d\right)\right]=c\left(w,d\right)\times F\left(d\right) \tag{5.1}$$

$$\left\{c(i,1),\ldots,c(i,n)\right\}\ \&\ \left\{c\left(j,1\right),\ldots,c\left(j,n\right)\right\}\text{ are independent for } i\neq j \tag{5.2}$$

$$Var\left[c\left(w,d+1\right)\middle|c\left(w,1\right),\ldots,c\left(w,d\right)\right]=c\left(w,d\right)\times\sigma_d^2 \tag{5.3}$$

Assumption (5.1) says that the all-year loss-weighted average (AYLWA) multiplied by the value in the last diagonal is equivalent to the expected value of the next diagonal given the observations to date. A validation test for this assumption (shown in Figures 5.1 and 5.2) compares the loss development factor (LDF), which is a regression through the origin (red line), to an alternative approach that uses an intercept term (green line).[32] If the regression with an intercept is not significantly different from the regression through the origin (for a given development period), then assumption (5.1) is validated.

For the fictitious entity, the first chain ladder assumption for the development periods shown in Table 5.1 were validated, resulting in a conclusion that the chain ladder methods using the AYLWA are reasonable. Note that each ODP bootstrap model is 100% consistent with using the AYLWA for the deterministic method, so none of the residuals were removed (i.e., no outliers were selected in the calibration of the ODP bootstrap models). The a priori loss ratios and tail factors used in the ODP bootstrap models were also consistent, except that variance assumptions based on expert judgment were added.

Note that the implementation of a "picker approach" (to reflect observable trends) in selecting LDFs would necessitate additional validation of each "pick" and consideration of consistent treatment of the residuals in the calibration of the ODP bootstrap model, but that was not done in the case study in keeping with the theme of simplicity.

5.3.2. Accident Year Independence

Regarding assumption (5.2), the independence of the accident years can be validated using a table of the individual LDFs and color-coding the LDFs that are smaller (green shading) or larger (red shading) than the median LDF for each development period, as illustrated in Table 5.2. This color-coding aids in categorizing each observation as being smaller than (green shading), larger than (red shading), or identical to (no shading) the median LDF for the development age. Thereafter, the counts of smaller and larger LDFs are summed by calendar year.

[32] A more complete exposition of tests that can be used to validate the three Mack assumptions is provided in Venter (1998). The graphs in Figures 5.1 and 5.2 and Tables 5.2 and 5.3 were created using the "Bootstrap Models.xlsm" companion Excel file for Shapland (2016).

Figure 5.1. Commercial Auto: Testing the First Two Paid LDFs

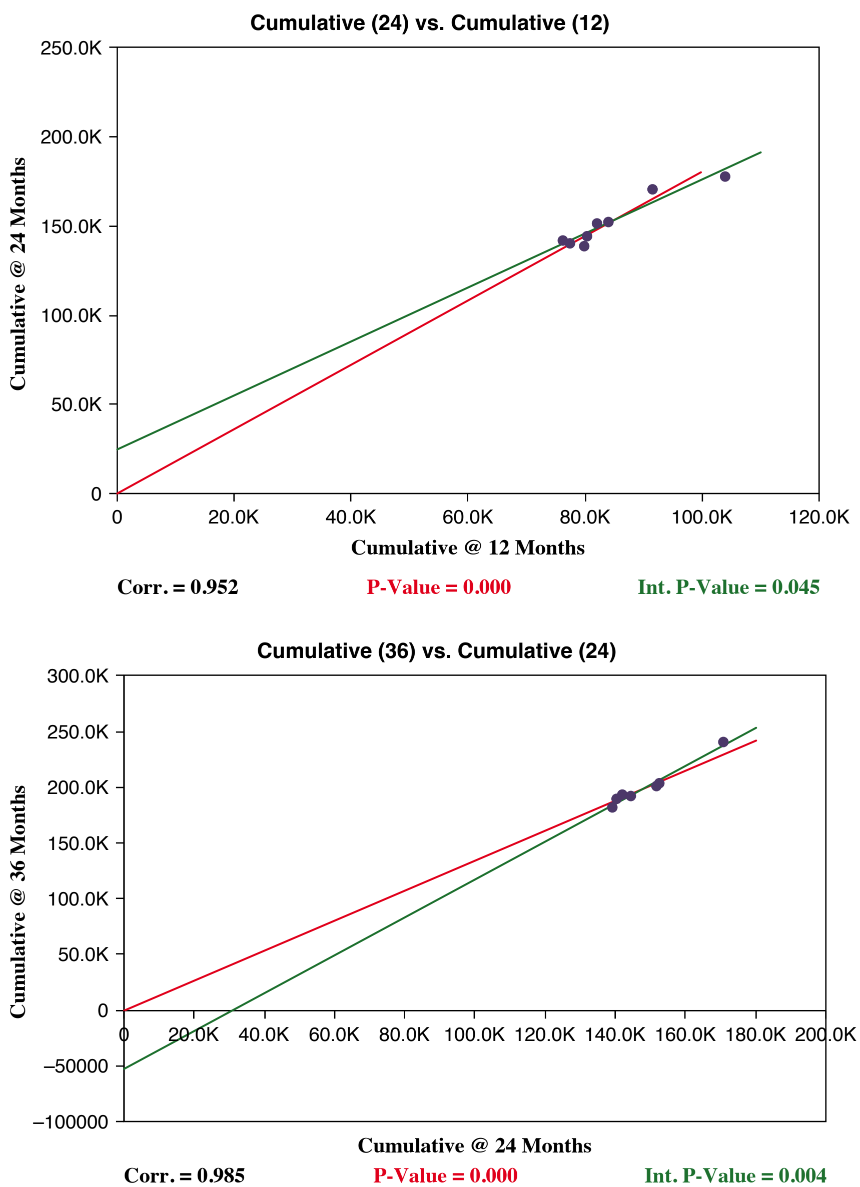
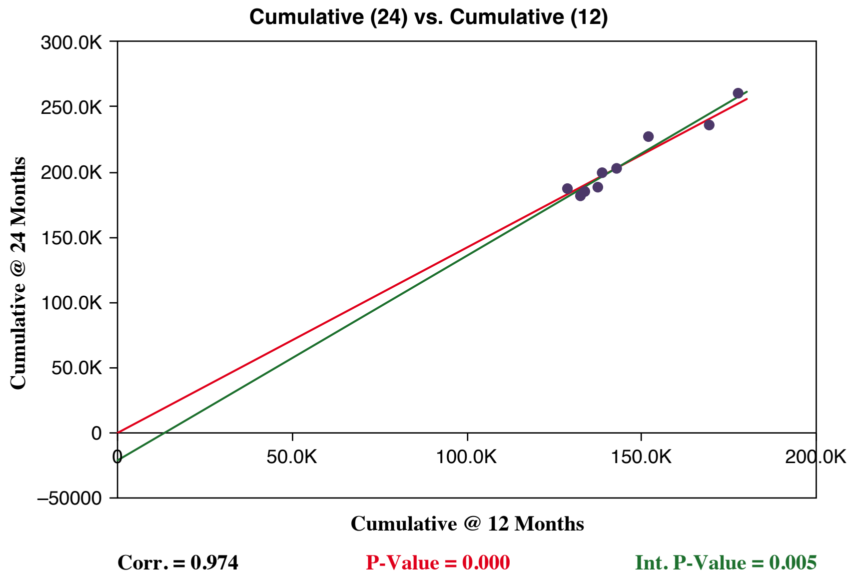


Figure 5.2. Commercial Auto: Testing the First Two Incurred LDFs



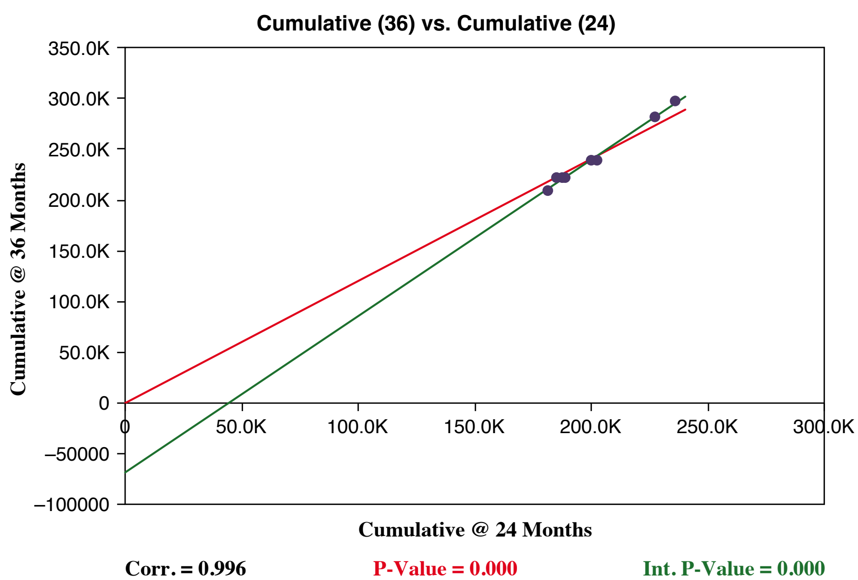


Table 5.1. Commercial Auto: Chain Ladder LDFs (Prior Year Analysis)

Sample Insurance Company
Commercial Auto—Paid Data
Chain Ladder Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120
2012	77,401	140,425	189,316	223,326	243,182	250,182	254,305	256,672	257,689	
2013	76,085	142,122	193,196	224,406	246,220	257,226	263,698	264,871		
2014	79,850	139,041	181,905	209,366	228,012	237,792	240,300			
2015	80,323	144,482	192,134	227,723	249,165	259,339				
2016	83,919	152,487	203,761	245,150	270,525					
2017	82,001	151,768	201,189	245,541						
2018	91,514	170,696	240,652							
2019	103,957	177,709								
2020	105,547									
	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–132
ATA	1.805	1.347	1.184	1.095	1.039	1.018	1.007	1.004	1.002	1.002
CDF	3.385	1.875	1.392	1.176	1.074	1.033	1.015	1.008	1.004	1.002
Unpaid	0.705	0.467	0.282	0.149	0.069	0.032	0.015	0.008	0.004	0.002

Sample Insurance Company
Commercial Auto—Incurred Data
Chain Ladder Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120
2012	133,521	185,161	221,635	241,420	251,646	255,508	256,596	258,041	258,524	
2013	128,727	187,403	222,093	247,345	258,712	265,636	269,558	270,758		
2014	132,567	181,263	209,262	226,237	236,863	241,107	242,171			
2015	137,295	188,962	222,624	247,335	258,856	265,496				
2016	142,862	202,363	239,239	269,940	281,376					
2017	138,650	199,791	239,719	266,101						
2018	151,778	227,353	282,394							
2019	169,171	235,983								
2020	177,611									
	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–132
ATA	1.418	1.193	1.106	1.045	1.022	1.008	1.005	1.002	1.001	1.001
CDF	2.029	1.431	1.200	1.085	1.038	1.016	1.008	1.003	1.001	1.001
Unrptd	0.507	0.301	0.166	0.078	0.037	0.016	0.008	0.003	0.001	0.001

Table 5.2. Commercial Auto: Testing Independence of Accident Years (Prior Year Analysis)

Sample Insurance Company
Commercial Auto—Paid
Test of the Independence Between Accident Years

									Cal Year	
Acc Year	12	24	36	48	60	72	84	96	Small	Large
2012	1.81	1.35	1.18	1.09	1.03	1.02	1.01	1.00	1	0
2013	1.87	1.36	1.16	1.10	1.04	1.03	1.00		0	2
2014	174	1.31	1.15	1.09	1.04	1.01			2	1
2015	180	1.33	1.19	1.09	1.04				4	0
2016	1.82	1.34	1.20	1.10					3	2
2017	1.85	1.33	1.22						1	3
2018	1.87	1.41							1	5
2019	1.71								4	3
Median	1.82	1.34	1.18	1.09	1.04	1.02	1.01	1.00		

Sample Insurance Company
Commercial Auto—Incurred
Test of the Independence Between Accident Years

									Cal Year	
Acc Year	12	24	36	48	60	72	84	96	Small	Large
2012	1.39	1.20	1.09	1.04	1.02	1.00	1.01	1.00	1	0
2013	1.46	1.19	1.11	1.05	1.03	1.01	1.00		0	2
2014	1.37	1.15	1.08	1.05	1.02	1.00			2	0
2015	1.38	1.18	1.11	1.05	1.03				3	1
2016	1.42	1.18	1.13	1.04					3	1
2017	1.44	1.20	1.11						2	4
2018	1.50	1.24							1	6
2019	1.39								4	2
Median	1.41	1.19	1.11	1.05	1.02	1.00	1.01	1.00		

As each development factor has a 50% probability of being smaller or larger than the median, the number of observations for each calendar year and category follows a binominal distribution with $p = 0.5$. Therefore, an expected value and variance for each calendar year and in total can be calculated and the hypothesis can be evaluated, as shown, for example, in Table 5.3.

In practice, the independence of accident years is often a poor assumption, and the results of actuarial methods and models can be distorted by certain calendar year effects such as major changes in the claims handling process or periods of systemic case reserve strengthening.

Table 5.3. Results of a Test of Accident Year Independence (Paid) (Prior Year Analysis)

Sample Insurance Company
Commercial Auto—Paid
Test of the Independence Between Accident Years

Cal Year	Small	Large	Z*	n	E[Z]	Var[Z]	Low^	High^	Result
2012	1	0	0	1	0.00	0.00	0.00	0.00	
2013	0	2	0	2	0.50	0.25	0.00	1.32	Pass
2014	2	1	1	3	0.75	0.19	0.04	1.46	Pass
2015	4	0	0	4	1.25	0.44	0.16	2.34	Fail
2016	3	2	2	5	1.56	0.37	0.56	2.56	Pass
2017	1	3	1	4	125	0.44	0.16	2.34	Pass
2018	1	5	1	6	2.06	0.62	0.77	3.36	Pass
2019	4	3	3	7	2.41	0.55	1.18	3.63	Pass
Total			8		9.78	2.86	7.00	12.56	Pass

Sample Insurance Company
Commercial Auto—Incurred
Test of the independence Between Accident Years

Cal Year	Small	Large	Z*	n	E[Z]	Var[Z]	Low^	High^	Result
2012	1	0	0	1	0.00	0.00	0.00	0.00	
2013	0	2	0	2	0.50	0.25	0.00	1.32	Pass
2014	2	0	0	2	0.50	0.25	0.00	1.32	Pass
2015	3	1	1	4	1.25	0.44	0.16	2.34	Pass
2016	3	1	1	4	125	0.44	0.16	2.34	Pass
2017	2	4	2	6	2.06	0.62	0.77	3.36	Pass
2018	1	6	1	7	2.41	0.55	1.18	3.63	Fail
2019	4	2	2	6	2.06	0.62	0.77	3.36	Pass
Total			7		10.03	3.17	7.10	12.96	Fail

* Z = Min(Small, Large)
^ Alpha = 5.0%

5.3.3. Proportionality of the Variance

Assumption (5.3) asserts that the variance of the cumulative claims amount for a development year is proportional to the cumulative claims amount for the preceding development year, for all accident years. One approach to the testing of this hypothesis is to evaluate the cumulative claim residuals (5.4) against the cumulative claim amounts. The variance assumption cannot be rejected if the residuals are randomly distributed.

$$r(w,d) = [C(w,d+1) - F(w) \times C(w,d)] / sqrt[C(w,d)] \tag{5.4}$$

5.3.4. A Priori Bornhuetter-Ferguson Loss Ratios

In the case study, the a priori or initial expected loss ratios (IELR) used in the Bornhuetter-Ferguson methods were based on published figures (i.e., ultimate loss ratios ("ULR") from Schedule P), expressed as a percentage of premium. As illustrated in Table 5.4, IELRs are an important assumption and an example of expert judgment that requires additional validation.

Validation, in this case, would likely take the form of sensitivity testing the important assumptions underlying the IELR. The common sources of expert judgment in this case would be renewal studies performed by the underwriting department and actuarial analyses summarizing average premium levels achieved compared to the expected premium level.

5.3.5. Weighting Scheme

No single method is perfect, and statistically an average of multiple independent estimates is generally a more robust estimate than any single estimate. For these reasons,

Table 5.4. Commercial Auto: IELRs (Prior Year Analysis)

Sample Insurance Company Commercial Auto				
AY	Paid CL ULR	Inc CL ULR	Management IELR	Selected ULR
2012	73.2%	73.2%	73.3%	73.2%
2013	76.0%	77.3%	77.4%	76.7%
2014	64.5%	64.5%	64.6%	64.5%
2015	62.8%	63.2%	63.2%	63.0%
2016	60.4%	60.7%	60.8%	60.6%
2017	53.2%	53.2%	53.4%	53.2%
2018	57.9%	58.5%	58.5%	58.2%
2019	54.5%	55.3%	54.7%	54.9%
2020	57.3%	57.7%	52.9%	54.7%

it has become best practice for actuaries estimating an insurer's unpaid claim estimate to review and assess the merits of multiple methods for each reserving segment to arrive at a weighted "best" estimate.

Traditional unpaid claim projection methods are generally based on averages that produce an indication of the unpaid claim reserves or a "reasonable estimate" for each accident period and in total. The results of these methods, being based on different data and assumptions, give different estimates. For example, chain ladder approaches applied to aggregate paid losses and aggregate incurred losses will produce different estimates of ultimate losses for each accident period and in total.

Expert judgment supported by supplemental information (e.g., expected loss ratios, severities, and frequencies from underwriting and claims experts) can be helpful in the reconciliation of the results from various methods. The reconciliation of the method results is a process whereby an actuary investigates and rationalizes significant differences at a granular level (i.e., by reserving segment and accident period) in the results from multiple methods.

Although the reconciliation process is generally a source of significant insight, a common outcome is that each of a subset of implemented methods produces different, but reasonable, estimates for a given accident period. In this case, the actuary often chooses to credibility weight the results of the methods that have produced reasonable results, rather than selecting a single method for an accident period.

Estimates for immature accident periods benefit from expert judgment supported by tangential information. For these accident periods, payments are few and case reserves are based on incomplete information, which means that chain ladder methods can be easily distorted by the behavior of a few claims. As accident periods mature, the actuary tends to rely more on period-specific information as found in chain ladder methods. This is because settlement amounts are known for closed claims, and future payments for open claims become more predictable as more claim-specific information is collected (e.g., loss survey, repair estimates, details of injury).

As illustrated in Table 5.5, the selection of a weighting scheme is an example of exercising expert judgment, which should be supported by adequate documentation, including:

- the inputs on which the judgment is based,
- the goals and decision criteria,
- the materiality of the expert judgment made,
- any material limitations and the steps taken to mitigate the effect of these limitations, and
- the validation conducted for the expert judgment.

Other selections based on expert judgment should also be adequately documented.

Article 77 of the Solvency II FD states that the "value of technical provisions shall be equal to the sum of a best estimate and a risk margin." Ignoring discounting and the risk margin for the purposes of this case study, the best estimate is further defined

Table 5.5. Commercial Auto: Weighting Scheme (Prior Year Analysis)

Sample Insurance Company
Commercial Auto
Calculation of Weighted Ultimate as of December 31, 2020

		Ultimate Values by Method				Weights by Method				Weighted Ultimate
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	
2012	108	258,835	258,835	258,837	258,836	50.0%	50.0%	0.0%	0.0%	258,835
2013	96	267,103	271,591	267,143	271,592	50.0%	50.0%	0.0%	0.0%	269,347
2014	84	243,981	244,137	243,991	244,141	50.0%	50.0%	0.0%	0.0%	244,059
2015	72	267,942	269,784	267,999	269,783	50.0%	50.0%	0.0%	0.0%	268,863
2016	60	290,475	292,079	290,608	292,092	50.0%	50.0%	0.0%	0.0%	291,277
2017	48	288,645	288,592	288,785	288,669	50.0%	50.0%	0.0%	0.0%	288,618
2018	36	335,023	338,775	335,956	338,702	25.0%	25.0%	25.0%	25.0%	337,114
2019	24	333,220	337,698	333,662	336,635	0.0%	0.0%	50.0%	50.0%	335,149
2020	12	357,305	360,286	338,097	344,953	0.0%	0.0%	50.0%	50.0%	341,525
Totals		2,642,529	2,661,779	2,625,078	2,645,402					2,634,788

to correspond to the "probability weighted average of future cash flows."[33] Note that Article 122.2 of the FD ensures that models "used to calculate the probability distribution forecast shall . . . be consistent with the methods used to calculate technical provisions." Consistency would include elements of expert judgment exercised by the actuary during the calculation of technical provisions, including the use of shorter-term average development factors, adjustment for trends, etc.

5.3.6. Other Manual Adjustments

Adjustments to the ultimate loss estimate are sometimes implemented based on (i.e., after) the weighting of multiple methods or models. In the case study, the weighting of paid and incurred chain ladder methods for accident year 2013 results in an incurred but not reported (IBNR) value less than 0 for commercial auto. Such a scenario implies that the case reserve may be redundant. A preferred course of action

[33] A strong interpretation of the required correspondence to a probability-weighted average of future cash flows is that a "distribution of possible outcomes" needs to be modeled. Note that deriving such a distribution of possible outcomes may not be possible using even the most sophisticated actuarial techniques available. The best attempt at such, however, would require the consideration of multiple (deterministic) methods and multiple (stochastic) models to calibrate a distribution of possible outcomes. In addition, such a distribution would require consideration of systemic risks that may not have been adequately modeled otherwise.

A weaker interpretation of the required correspondence to a probability-weighted average of future cash flows is that each actuarial method produces future cash flows unique to the assumptions underlying the respective method, as applied to an accident period and reserving segment, and these competing cash flow projections can be weighted together based on the subjective credibility assigned to each accident period of each method.

Table 5.6. Commercial Auto: Manual Adjustment of Accident Year 2013 (Prior Year Analysis)

Sample Insurance Company
Commercial Auto
Total Unpaid Reconciliation as of December 31, 2020

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2012	108	257,689	258,524	258,835	835	311	1,146	258,835	311	1,146
2013	96	264,871	270,758	269,347	5,887	(1,411)	4,476	**271,500**	**742**	**6,629**
2014	84	240,300	242,171	244,059	1,871	1,888	3,759	244,059	1,888	3,759
2015	72	259,339	265,496	268,863	6,157	3,367	9,524	268,863	3,367	9,524
2016	60	270,525	281,376	291,277	10,851	9,901	20,752	291,277	9,901	20,752
2017	48	245,541	266,101	288,618	20,560	22,517	43,077	288,618	22,517	43,077
2018	36	240,652	282,394	337,114	41,742	54,720	96,462	337,114	54,720	96,462
2019	24	177,709	235,983	335,149	58,274	99,166	157,440	335,149	99,166	157,440
2020	12	105,547	177,611	341,525	72,064	163,914	235,978	341,525	163,914	235,978
Totals		2,062,173	2,280,414	2,634,788	218,241	354,374	572,615	2,636,941	356,527	574,768

is to interact directly with the claims team, if possible, to determine the likelihood of this conclusion. For purposes of the case study, a small IBNR has been added and the consequence of this decision is included in the expected values of the subsequent year's back-test, as illustrated in Table 5.6. Throughout the tables in the "LOB Backtest.xlsm" file, deviations from the weighted results are highlighted in green.

5.3.7. Coefficient of Variation of the IELR

The IELRs used in a Bornhuetter-Ferguson model are subject to significant uncertainty, with judgment required in their selection. This uncertainty can be reflected by assuming that each IELR (by year) is a random variable. Each iteration of the ODP bootstrap for Bornhuetter-Ferguson models then has a different set of simulated IELRs. In the case study, the IELR is calibrated to follow a lognormal distribution with a coefficient of variation of 8%.[34]

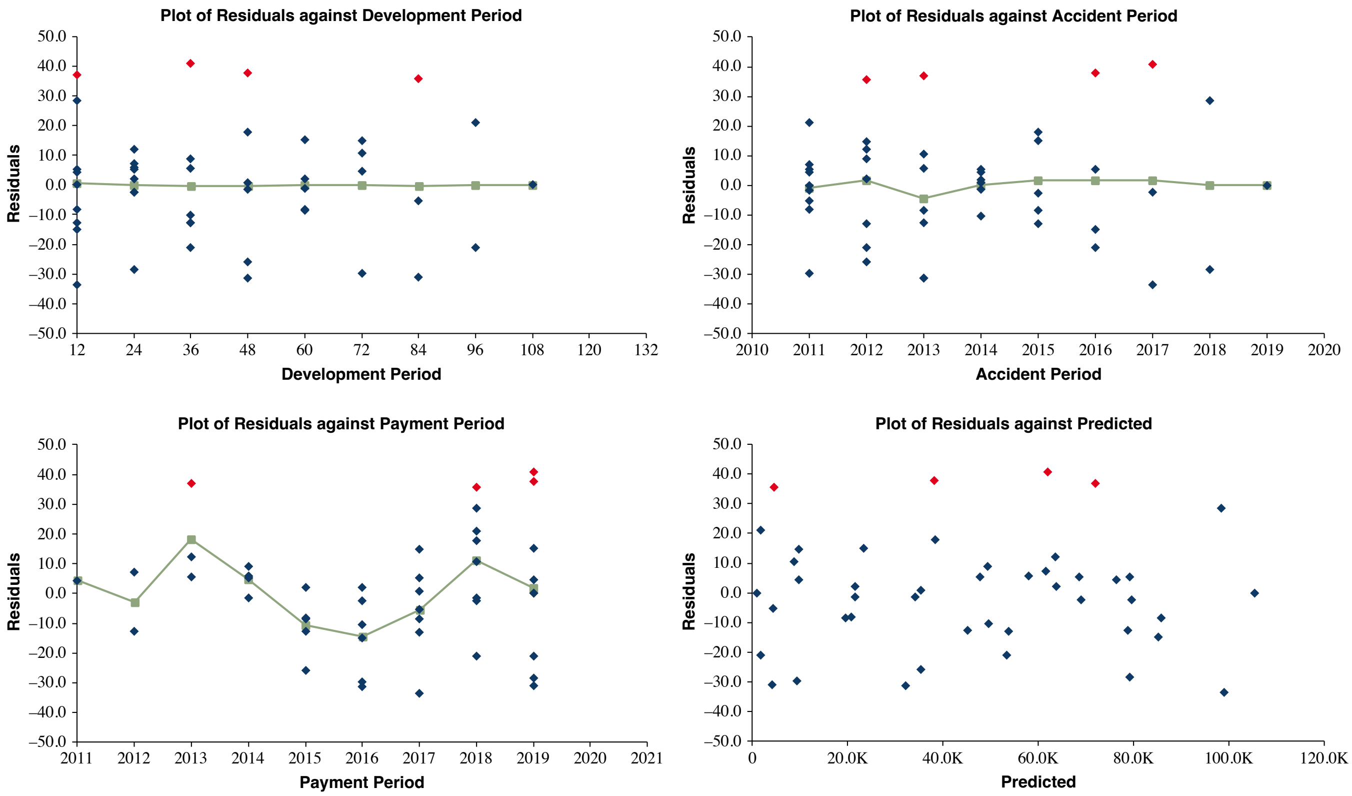
The validation of this assumption would typically be based on reviewing the variability of the ultimate loss ratios from other simulation models for the same data or from external sources such as the benchmarks described in Shapland (2019).

5.3.8. Heteroscedasticity

An analysis of residuals is an example of a validation technique. For the case study, the residuals are analyzed to identify trends or other features in the data that may not be completely modeled by the chain ladder approach.

[34] The mean values are as described in Section 5.3.4. The standard deviations for the IELR are based on 8% of the mean values.

Figure 5.3. Commercial Auto: Plots of Residuals (Paid) (Prior Year Analysis)



Particularly important are the identification of heteroscedasticity and outliers. In the ODP bootstrap model,[35] residuals are resampled with replacement—that is, they are taken from any location in the residual triangle and placed in another random location to form the sample triangle. Therefore, the residuals should all be independent, identically distributed random numbers (i.e., homoscedastic). Heteroscedasticity occurs when the residuals are not identically distributed. Examining the variability of the residuals by period (e.g., by accident year) allows the actuary to visually inspect them to make sure the variability is consistent between periods. If they are not consistent, this is an indication that heteroscedasticity is present in the residuals and additional parameters may be needed to adjust for the different variances by period.[36]

The adjustment for heteroscedasticity is typically made by focusing on the plot of residuals against development period (see top left graph in Figure 5.3) and identifying columns with similar dispersion of residuals. While it is tempting to add hetero groupings to force additional consistency of the residuals (e.g., at 60 months, where the dispersion appears low), this would add more parameters to an already highly parameterized model. This is not to say that trying other hetero groups is never justified, just

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[35] The typical ODP bootstrap model is semiparametric, but conditions could exist for the implementation of a fully parametric ODP bootstrap, which allows for the sampling of residuals from a distribution (a more robust solution).

[36] For a more complete discussion, see Shapland (2016) Section 4.6 and Section 5.

that the ODP bootstrap already has one parameter for every development period and one parameter for every accident period (minus one), so the decision to add parameters for heteroscedasticity must be made carefully.

5.3.9. Process Variance Adjustment to the ODP Bootstrap

One of the last steps in the ODP bootstrap is the use of a distributional assumption to add process variance to the simulated future incremental values. Without this step the projected incremental values would be characterized as point estimates rather than possible outcomes. In the case study, the gamma distribution was used. The normal or lognormal distributions are possible alternative distributions that could be tested to see if they lead to material differences in results, but that is outside the scope of the case study.

5.3.10. Correlation between Segments

Thus far, the list of assumptions that could be tested has been focused on the segment or model level. As the case study is intended to replicate a complete ERM system, correlation to derive an aggregate distribution is also included.

In general, the aggregate distribution of unpaid claims can be materially narrower than the sum of the individual distributions, after considering correlation between the segments. This difference between the correlated aggregate and the sum of the segments would not be as material in cases in which the segments are all strongly positively correlated, in which there is little variability in the individual distributions, or in which one segment is far larger than the rest.

For the case study, correlation was measured using a pairwise approach.[37] A more robust solution, e.g., a maximum likelihood estimation (MLE) copula, could be used to solve for all correlations at once by analyzing all the data at once. However, the MLE copula approach can be less than ideal when data is excluded or missing for one or more segments.[38,39] The measurement of correlation could be performed using paid residuals and/or incurred residuals, both before and after heteroscedasticity adjustments. The resulting correlation matrices for paid loss residuals before heteroscedasticity are shown in Table 5.7.

To aggregate distributions of possible outcomes for the entity, the actuary assesses the inherent correlation for each segment pair. For this, the *p*-values can be reviewed to assess the significance of the correlation between each pair of segments. In this test, the smaller the *p*-value the more significant the calculated correlation, and a larger *p*-value (e.g., greater than 0.05 is a typical threshold) indicates that the correlation is

[37] The pairwise approach is used in the "Aggregation.xlsm" companion file for the Shapland (2016) monograph, which was used to create Tables 5.7 and 5.8.

[38] For example, if you are only using two-year average age-to-age ratios for one segment, then only the data for the last three diagonals can be used in the estimation process. The maximum likelihood copula only uses data points that are common for every segment, so it is possible to have a problematic situation in which there are no common data points for all segments.

[39] It is important to note that any adjustments to the ODP bootstrap model (i.e., anything less than the AYLWA for the link ratios or exclusion of outliers) will result in some of the residuals (that would otherwise be included) being excluded from the correlation matrix calculations.

Table 5.7. Pairwise Rank Correlation of Residuals and *P*-Values—Paid Loss (Prior Year Analysis)

Rank Correlation of Residuals prior to Hetero Adjustment—Paid			
	PPA	CA	HO
PPA	1.000	0.276	–0.142
CA	0.276	1.000	0.027
HO	–0.142	0.027	1.000

P-Value of Rank Correlation of Residuals prior to Hetero Adjustment—Paid			
	PPA	CA	HO
PPA	0.000	0.066	0.352
CA	0.066	0.000	0.860
HO	0.352	0.860	0.000

Table 5.8. Selected Correlation Matrix

	Assumed Correlation Matrix		
	PPA	CA	HO
PPA	1.000	0.276	0.000
CA	0.276	1.000	0.000
HO	0.000	0.000	1.000

not significantly different than zero. Therefore, the *p*-values of 0.352 (HO x PPA) and 0.860 (HO × CA) imply that the measured correlation is not significantly different from zero, while the *p*-value of 0.066 implies 93.4% confidence that the measured correlation is different from zero. The selected correlation in Table 5.8 reflects the consideration of the *p*-values.

The validation of correlation assumptions is a challenge. Monitoring both the measured rank correlation and corresponding *p*-values over time can provide some insight into the stability of the correlation assumptions.[40] Even so, the selected correlation assumption may also consider the impact of issues not in the measured coefficients, such as contagion or lack of prior catastrophe losses.

5.4. Implied Expected Values from Multiple Methods

Future expected incremental values (e.g., paid loss, reported claims, etc.) could be produced in several ways. For example, they could be independently calculated based on an independent analysis, or they could be calculated based on consecutive differences of

[40] Another source would be correlation benchmarks, as described in Shapland (2019).

cumulative estimates that result from a curve fit. Although these practices are common, a continuous ERM process intends to improve the models and methods employed in the estimation process. Therefore, the approach used here is to estimate the future incremental values that arise from the methods (and models) that have received weight and any subsequent adjustments. The idea is that deviations can be traced back to the underlying deterministic calculations, for which validated assumptions with enhanced documentation are available, and subsequent adjustments, for which documentation of decision points is available.

One challenge that immediately arises from this approach is that expected future incremental paid (and incurred) loss values must be gleaned from the expectations inherent in incurred (and paid) methods. In the extreme case in which the incurred chain ladder method receives 100% of the weight for all accident years, expected incremental paid losses still need to be produced even though paid methods received no weight. To address this challenge, at least one paid method and one incurred method requires calibration. The output from both paid and incurred methods is converted into analogous paid and incurred expectations. Continuing the example from the case study (see above for the LDF validation and weighting scheme), formulas (5.5) to (5.8) are used to derive expected cumulative amounts for a particular method, from which incremental amounts follow.[41]

$$E\left[\hat{c}_P\left(w,d\right)\right]_{P-Method} = E\left[\hat{c}_P\left(w,d-1\right)\right]_{P-Method} \times F\left(d-1\right)_{P-Method} \tag{5.5}$$

$$E\left[\hat{c}_P\left(w,d\right)\right]_{I-Method} = E\left[\hat{c}_P\left(w,d\right)\right]_{P-Method} \times \frac{U\left(w\right)_{I-Method}}{U\left(w\right)_{P-Method}} \tag{5.6}$$

$$E\left[\hat{c}_I\left(w,d\right)\right]_{I-Method} = E\left[\hat{c}_I\left(w,d-1\right)\right]_{I-Method} \times F\left(d-1\right)_{I-Method} \tag{5.7}$$

$$E\left[\hat{c}_I\left(w,d\right)\right]_{P-Method} = E\left[\hat{c}_I\left(w,d\right)\right]_{I-Method} \times \frac{U\left(w\right)_{P-Method}}{U\left(w\right)_{I-Method}} \tag{5.8}$$

Note that a consequence of this approach is that any IBNR adjustment made after the weighting of methods will have an impact on both expected paid and incurred amounts. With cumulative paid and incurred amounts by development period so derived for each method, the weighting scheme can be applied to determine the weighted cumulative paid and incurred amounts, from which the incremental amounts can be derived. Examples of the next diagonal of incremental values (i.e., for calendar year 2021 during the year-end 2020 analysis) are shown in Tables 5.9 and 5.10.

[41] Formulas (5.5) and (5.7) may seem redundant in the sense that the expected incremental development for the paid and incurred methods, respectively, is derived directly from the method itself. The formulas are included for completeness of exposition and as a link to the calculations in the "LOB Backtest.xlsm" file.

Table 5.9. Commercial Auto: Implied Expected Paid Losses (Prior Year Analysis)

Sample Insurance Company Commercial Auto Expected Paid Losses during CY 2021						
AY	Paid CL	Inc CL	Paid BF	Inc BF	Weighted	Selected
2012	572	572	573	572	572	572
2013	1,049	5,518	1,068	5,497	3,284	**4,863**
2014	1,642	1,797	1,647	1,796	1,720	1,720
2015	4,560	6,375	4,590	6,348	5,468	5,468
2016	10,624	12,177	10,695	12,130	11,401	11,401
2017	23,280	23,230	23,355	23,247	23,255	23,255
2018	44,341	47,533	44,779	47,112	45,941	45,941
2019	61,648	64,865	61,823	63,957	62,890	62,890
2020	85,007	86,597	78,521	82,254	80,388	80,388
AY < CY	232,723	248,663	227,052	242,913	234,917	**236,497**

Table 5.10. Commercial Auto: Implied Expected Incurred Losses (Prior Year Analysis)

Sample Insurance Company Commercial Auto Expected Incurred Losses during CY 2021						
AY	Paid CL	Inc CL	Paid BF	Inc BF	Weighted	Selected
2012	155	155	157	156	155	155
2013	(3,976)	507	(3,937)	507	(1,735)	**912**
2014	1,062	1,217	1,070	1,220	1,140	1,140
2015	288	2,116	345	2,115	1,202	1,202
2016	4,482	6,061	4,608	6,067	5,271	5,271
2017	11,967	11,915	12,068	11,956	11,941	11,941
2018	26,520	29,980	27,409	29,941	28,462	28,462
2019	41,780	45,513	42,556	45,037	43,797	43,797
2020	72,073	74,156	63,052	67,932	65,492	65,492
AY < CY	154,351	171,620	147,327	164,931	155,725	**158,372**

5.5. Advantages of Using the ODP Bootstrap

In the case study, the ODP bootstrap approach is used to model uncertainty. A main advantage of this approach is that the assumption set in the uncertainty calibration is largely consistent with the assumption set in the point estimate calibration, while areas of inconsistency (or adjustment) are identified, documented, and (to the extent possible) validated for reasonableness. Of course, the uncertainty calibration requires additional assumptions to be made, each of which requires documentation and validation.[42]

Alternatively, the Mack (1993) method could be used for the uncertainty calibration, but this would result in several additional challenges, only some of which can be overcome.[43]

1. The variance assumptions in the Mack closed-form solution would be largely inconsistent with the assumptions used to calibrate a point estimate. Recall that the selected weights imply a full rejection of the chain ladder methods for the most recent accident years.
2. The Mack closed-form solution produces a variance estimate for each accident year and in total, but a distribution needs to be postulated to translate this variance estimate into a distribution of statistical outcomes. The likelihood is low that such a distribution includes all possible outcomes, and validation of such may not be possible.
3. The Mack closed-form solution and resulting variance estimate (on an ultimate basis) would need to be bifurcated such that variance estimates would be available for each development period between the valuation date and the date at which time the losses are fully developed (at ultimate).[44]
4. The practicing actuary learns very little about the data and underlying uncertainty when using a closed-form solution such as Mack. This follows because such solutions require limited calibration to obtain a result and limited diagnostics about the underlying assumptions. Further, the uncertainty is highly dependent on the observable LDFs, compared to the AYLWA, which in the tail area can be limited.
5. The practicing actuary has little ability to adjust the results of the Mack closed-form solution in cases in which the output is inconsistent with expectations.

5.6. ERM Governance Elements and Automatic Alert System

The manipulation and validation of methods and models, while interesting and attractive to actuaries, is only a small part of the case study. The real benefit of a robust ERM process for the actuarial function is the organized prioritization of reserving and underwriting risks, and clear guidance on which deviations are most important to

[42] This does not imply that the ODP bootstrap model is the only model suited for this process. In actual practice, many other models can be considered with their assumptions validated, documented, etc.

[43] This does not imply that the Mack model cannot be used for this purpose. Rather, the shortcomings of a closed-form solution make the process more problematic, and appropriate adjustments are outside the scope of this monograph.

[44] For useful references on this extension, see England, Verrall, and Wüthrich (2018); Merz and Wüthrich (2015); and Shapland (2020).

investigate, leading to better insights into the risks that matter to the organization. In our view, the robust ERM process should include the following five elements, at a minimum:

- Governance
- Automatic alert system
- One-year time horizon as a preliminary monitoring tool
- Allocation of resources
- Additional indicators of performance

5.6.1. Governance

The ERM system used in the case study includes several KPIs to monitor the reserving process. For each KPI, the risk owner and risk reviewer are documented. At the highest level, the KPIs for aggregate (i.e., entity-wide) paid loss and aggregate incurred loss, the risk owner is often the chief actuary, and the risk reviewer is often the chief executive officer (CEO) or chief financial officer (CFO).

In discussing governance, KPIs, and thresholds, it is important to remember that 1% of the observations are expected to fall above the 99th percentile. Incurred claims being significantly different from their expected values does not necessarily mean that the prior estimates and models were calibrated incorrectly. The deviation could be due to

1. a random occurrence,
2. changes in processes or practices that weren't expected at the time of the prior analysis (e.g., a new claims handling team, or a new court interpretation of policy wording), or
3. prior modeling assumptions that may have been inappropriate.

5.6.2. Automatic Alert System

The realized values are subject to thresholds, each with well-defined consequences in the event of a breach. The case study uses thresholds at the 25th and 75th percentiles, the 5th and 95th percentiles, and the minimum and maximum of the simulated distribution of possible outcomes to denote material deviations from the expected results, as illustrated in Figure 3.1.

The CEO (and other members of management) receives an immediate and automatic e-mail from the ERM system as soon as the back-testing analysis is complete (January 4), confirming whether the 5% or 95% thresholds were breached by the aggregate paid loss or aggregate incurred loss.[45]

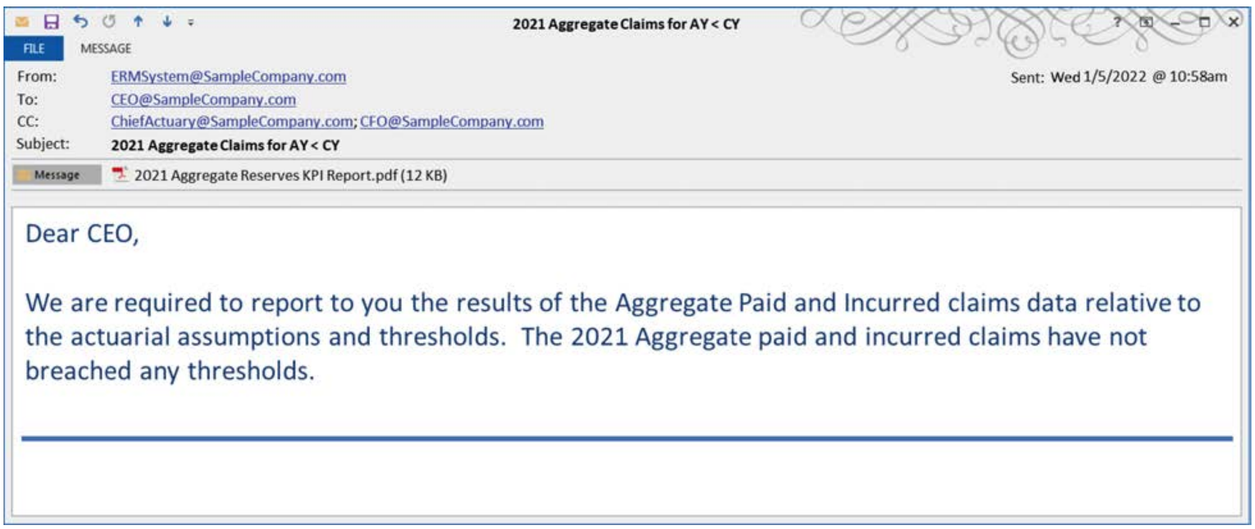
The automated alert system will send as many e-mails as needed, based on the predefined thresholds, to the appropriate risk owners and risk reviewers. For example,

[45] While the automation of the communication of threshold breaches is a critical component to ensure timely reactions, the overall system needs to be fine-tuned to ensure that everyone receives only the communications they need and is not overwhelmed with alerts outside of their responsibilities. That fine-tuning, including how the automated e-mails are phrased, is assumed to have taken place outside the scope of the case study.

while the CEO is the risk reviewer and the chief actuary is the risk owner for the aggregate results, the chief actuary is the risk reviewer, and the reserving actuary is the risk owner for the results by segment.

The illustrated e-mails in Figures 5.4 and 5.5 include reports containing specific results. The reports attached to the e-mails, which also highlight any breached thresholds, are shown in Appendix B. For higher levels of management, a more aggregate view will tend to be the priority, and at lower levels of management a more detailed view will be important, as the automated system will reflect the responsibilities of the individuals.

Figure 5.4. Sample Automated E-Mail #1 to the CEO



2021 Aggregate Claims for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com — Sent: Wed 1/5/2022 @ 10:58am

To: CEO@SampleCompany.com

CC: ChiefActuary@SampleCompany.com; CFO@SampleCompany.com

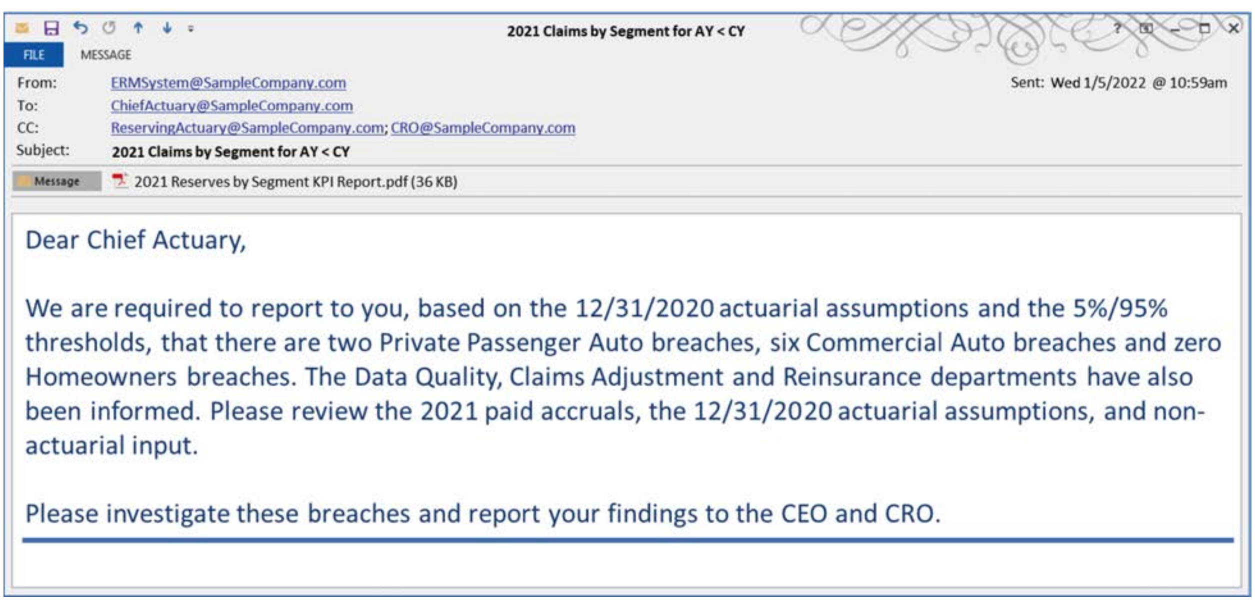
Subject: **2021 Aggregate Claims for AY < CY**

Message — 2021 Aggregate Reserves KPI Report.pdf (12 KB)

Dear CEO,

We are required to report to you the results of the Aggregate Paid and Incurred claims data relative to the actuarial assumptions and thresholds. The 2021 Aggregate paid and incurred claims have not breached any thresholds.

Figure 5.5. Sample Automated E-Mail #2 to the Chief Actuary



2021 Claims by Segment for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com — Sent: Wed 1/5/2022 @ 10:59am

To: ChiefActuary@SampleCompany.com

CC: ReservingActuary@SampleCompany.com; CRO@SampleCompany.com

Subject: **2021 Claims by Segment for AY < CY**

Message — 2021 Reserves by Segment KPI Report.pdf (36 KB)

Dear Chief Actuary,

We are required to report to you, based on the 12/31/2020 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. The Data Quality, Claims Adjustment and Reinsurance departments have also been informed. Please review the 2021 paid accruals, the 12/31/2020 actuarial assumptions, and non-actuarial input.

Please investigate these breaches and report your findings to the CEO and CRO.

5.6.3. One-Year Time Horizon as a Preliminary Monitoring Tool

On the first day of the new analysis (January 4), the actuarial function can use the one-year time horizon reserves from the prior analysis to provide an early indication of both the direction and potential magnitude of aggregate reserve changes. The value comes from estimating the one-year time horizon reserves, which are conditional on the possible outcomes of the ultimate time horizon distribution. Whether the early warning is positive or negative, management can keep an eye on the risk management issues related to reserve changes from the beginning of the reserving analysis process instead of reacting to surprises toward the end of the analysis process, just prior to the publishing of financial results.

The one-year time horizon has been developed and promoted by entities subject to the Solvency II regime in Europe using both an ODP bootstrap approach and a modification to the Mack model developed by Merz and Wüthrich (2008). Essentially, because entities are required to hold sufficient capital to allow them to be 99.5% certain of staying solvent over a one-year time horizon, actuaries have developed techniques that split measures of reserving risk into two pieces, the reserving risk over a single year and the reserving risk over all subsequent years.

The estimation of reserving risk over a one-year time horizon using the ODP bootstrap approach produces a conditional reserve for each simulated first-year diagonal and involves a two-step process:[46]

1. Possible outcomes are simulated as usual but only retained for the simulations of the first calendar year cash flows (the one-year time horizon). These simulated diagonals are used to reparameterize the ODP bootstrap model based on the original data plus the simulated diagonals.
2. Point estimates for the remainder of the unpaid claims after the one-year time horizon are created for each possible outcome of the original triangle plus the simulated one-year diagonal. Note that point estimates in this case have not been adjusted for process variance, as they are intended to represent a reserve estimate that is conditional on the outcome of the one-year time horizon.[47]

By calculating the percentile of the actual calendar year paid within the distribution of the expected calendar year paid using formula (3.4), the conditional reserve would be the same percentile of the distribution of point estimates after the one-year time horizon using formula (5.9).[48] The expected reserve for the new analysis is equal to the expected reserve for the prior analysis less the actual amount paid during the year,

[46] See Appendix A for a graphical overview of the one-year time horizon calculations using the ODP bootstrap model.

[47] Even though the simulation process to generate the one-year diagonal is stochastic, the step to calculate the conditional unpaid claims is deterministic and uses only the calculated age-to-age factors. This second step could also be performed stochastically by "nesting" additional simulations for each iteration of the one-year diagonal to include process variance, but that is outside the scope of this monograph.

[48] For each iteration of the ODP model, a corresponding conditional reserve is calculated based on the simulated "path" of the paid amount over the one-year time horizon. By finding the simulated outcome that is closest to the percentile of the actual result, the actuary would also find the corresponding conditional reserve for the same percentile.

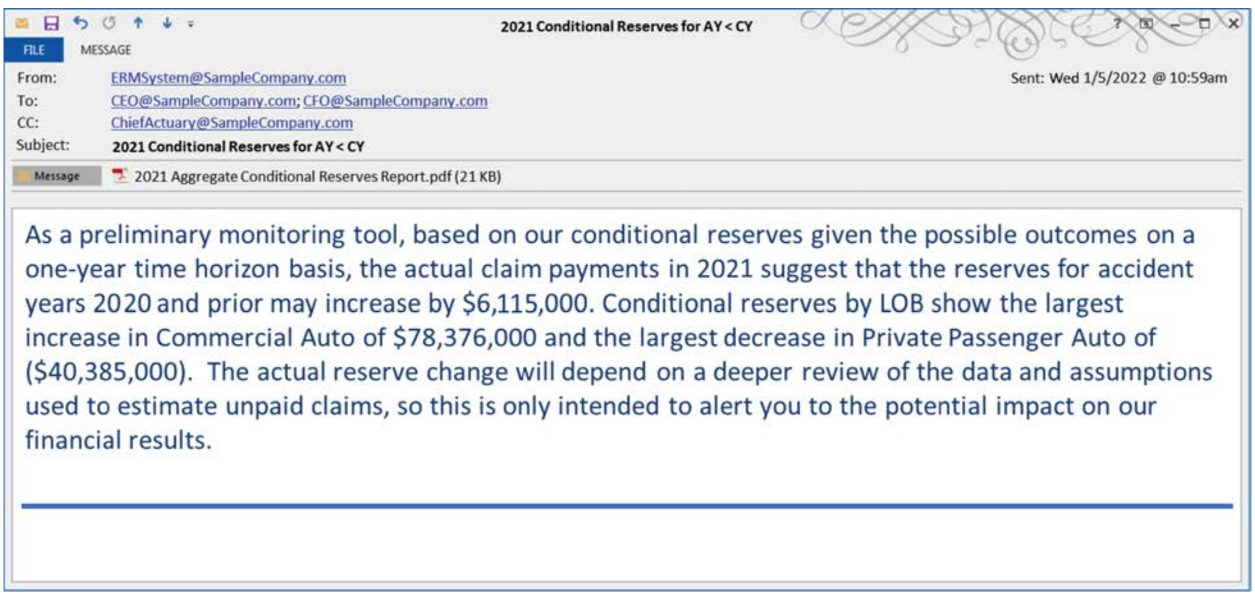
as shown in (5.10). In other words, the new expected reserve is equal to the prior expected reserve if the estimate of ultimate loss did not change at all. The estimated reserve change, therefore, is represented by the difference between the conditional reserve and the expected reserve, i.e., (5.9) minus (5.10), as illustrated in Table 5.11.[49]

$$E\ \hat{R}(w,d+1)\big|x\ = PERCENTILE.INC\left\{Dist\ \sum_{d=t+1}^{u}\hat{q}(w,d)\ ,P_x\left[q(w,d)\right]\right\} \tag{5.9}$$

$$E\ \hat{R}(w,d+1)\ = E\ \hat{R}(w,d)\ - q(w,d) \tag{5.10}$$

The CEO and CFO receive an immediate and automatic e-mail from the ERM system on the first day of the analysis period (January 3) describing a preliminary estimate for the change in reserves, based on the conditional reserves given the possible outcomes under a one-year time horizon and the actual paid loss observed during the most recent calendar year.[50] The report attached to the e-mail is shown in Appendix B. Based on the conditional reserves, the aggregate increase of $6.1 million may not be of immediate concern, but the commercial auto increase of $78.4 million will certainly draw attention.[51]

Figure 5.6. Automated E-Mail #3 to the CEO and CFO



2021 Conditional Reserves for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com — Sent: Wed 1/5/2022 @ 10:59am

To: CEO@SampleCompany.com; CFO@SampleCompany.com

CC: ChiefActuary@SampleCompany.com

Subject: 2021 Conditional Reserves for AY < CY

Message — 2021 Aggregate Conditional Reserves Report.pdf (21 KB)

As a preliminary monitoring tool, based on our conditional reserves given the possible outcomes on a one-year time horizon basis, the actual claim payments in 2021 suggest that the reserves for accident years 2020 and prior may increase by $6,115,000. Conditional reserves by LOB show the largest increase in Commercial Auto of $78,376,000 and the largest decrease in Private Passenger Auto of ($40,385,000). The actual reserve change will depend on a deeper review of the data and assumptions used to estimate unpaid claims, so this is only intended to alert you to the potential impact on our financial results.

[49] Note that the process of calculating conditional reserves by year from the prior period analysis would need to be automated to provide timely feedback to management. Thus, the potential reasons for the deviations (e.g., large claim payment or case reserve change) will not be apparent until further analysis is complete.

[50] In Table 5.11, the AY<CY row is based on the aggregate of all years, whereas the Totals row is the sum of the individual years.

[51] The conditional reserves shown in Appendix B (Figure B.5) and the e-mail in Figure 5.6 are estimated using only paid data. In practice, incurred data could also be used, but it is not shown here for two reasons: (1) the final weighted estimates would include incurred models that have been converted to paid outcomes, and (2) including the incurred results could just muddy the discussion with questions about which to believe and detract from the point of providing an early warning of *possible* reserve changes.

Table 5.11. Differences between Expected and Conditional Reserves

Sample Insurance Company
Aggregation of All Segments
Summary of Conditional Reserves as of December 31, 2021

	Private Passenger Auto			Commercial Auto			Homeowners			Total (Sum)		
AY	Conditional Reserve	Expected Reserve	Change	Conditional Reserve	Expected Reserve	Change	Conditional Reserve	Expected Reserve	Change	Conditional Reserve	Expected Reserve	Change
2012	2,680	2,991	(311)	643	603	40	—	747	(747)	3,323	4,341	(1,018)
2013	7,248	5,498	1,750	3,257	4,242	(985)	164	721	(557)	10,669	10,461	208
2014	8,654	10,061	(1,406)	1,675	2,582	(907)	1,367	1,640	(272)	11,697	14,283	(2,586)
2015	15,635	19,472	(3,836)	5,593	4,121	1,472	(1,153)	1,793	(2,946)	20,075	25,386	(5,311)
2016	31,595	38,066	(6,470)	13,946	6,632	7,313	3,722	340	3,381	49,263	45,039	4,224
2017	73,359	71,302	2,057	20,073	19,441	632	3,979	6,894	(2,915)	97,412	97,638	(227)
2018	151,670	156,061	(4,390)	57,978	45,442	12,536	12,839	9,468	3,370	222,487	210,971	11,516
2019	292,882	322,812	(29,930)	110,701	81,627	29,075	21,590	26,615	(5,024)	425,174	431,054	(5,880)
2020	581,448	574,019	7,430	170,589	147,146	23,442	59,458	80,333	(20,875)	811,496	801,499	9,997
Totals	1,165,174	1,200,281	(35,107)	384,456	311,837	72,619	101,967	128,553	(26,586)	1,651,596	1,640,671	10,926
2021 AY < CY	1,159,897	1,200,281	(40,385)	390,213	311,837	78,376	96,676	128,553	(31,876)	1,646,786	1,640,671	6,115

5.6.4. Allocation of Resources

In addition to the conditional reserves by segment, it is possible to quantify and rank the deviation from prior expectations for each of the outcomes. For the case study, 80 outcomes include 10 paid observations and 10 incurred observations, calculated as 9 AYs and segment total (i.e., AY < CY), for 3 segments and the aggregate (i.e., after correlation).

A ranked list of deviations allows for an alternative approach to managing actuarial resources. Actuarial management often uses an approach that assigns individuals to segments. An advantage of this approach is that an individual develops an area of expertise and relationships with corresponding claims and underwriting professionals. A disadvantage of this approach is that the methodology and corresponding documentation may receive less external challenge, increasing the risk that business will be disrupted in the event that the current expert needs to be replaced.

An alternative approach, using the ranked list of deviations, includes the allocation of resources based on the quantitative deviation from expectations. The ranked list of deviations highlights areas where there is new data that may be materially different from prior data and requires a greater level of investigation. Either focusing existing staff on these areas or assigning additional staff will enable the provision of a timely and meaningful response to management as to whether the deviation is due to randomness or another effect that requires management input. This approach presupposes that the actuarial department managers have a strong sense of the strengths and weaknesses of their team.

5.6.5. Additional Indicators of Performance

In the case of the commercial auto segment, the back-testing of stochastic elements observed on day one of the analysis is quite poor, so immediately digging into the drivers will be important. As shown in Table 5.12, two of the incurred observations (highlighted with gray shading) have breached the minimums and maximums defined by the prior models. A further two incurred and two paid observations have breached the 5%/95% threshold (highlighted with red font), and 5 incurred and 4 paid observations have breached the 25%/75% threshold (highlighted with orange font). Only 5 observations sit comfortably in the core 50%, from 25% to 75% of the distribution of possible outcomes. Absent changes in the methodology and modeling, the one-year time horizon exercise implies a deterioration of more than 13% (equal to 78,376 / [262,931 + 311,837], referring to values found in Tables 5.11 and 5.12).

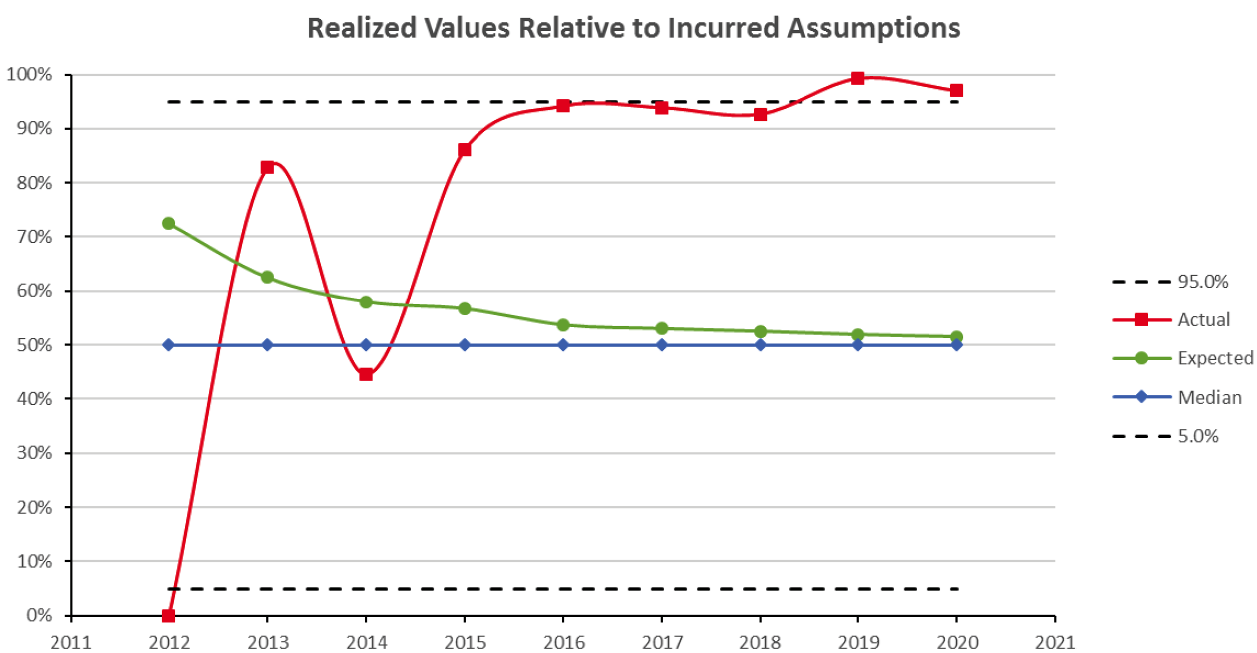
A closer examination of the incurred observations in Table 5.12 and Figure 5.7 reveals that immature AYs appear to have been significantly underestimated. Though not conclusive, the realized values imply that there may have been a problem with the deterministic methods underlying the prior analysis. Although the minimum and maximum have been breached, the prior uncertainty estimates may have been

Table 5.12. Assessing the 20 Observations for Commercial Auto

Sample Insurance Company
Commercial Auto
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2012	120	543	571	57.9%	(47)	154	**0.0%**
2013	108	2,387	3,131	**21.8%**	1,040	448	**82.8%**
2014	96	1,177	1,665	33.5%	851	1,167	44.5%
2015	84	5,403	5,044	63.1%	2,954	1,669	**86.1%**
2016	72	14,120	11,061	**91.1%**	9,035	5,606	**94.2%**
2017	60	23,636	23,276	56.1%	16,524	11,960	**93.9%**
2018	48	51,020	45,272	**86.7%**	36,454	29,103	**92.7%**
2019	36	75,813	62,481	**96.5%**	61,541	44,392	**99.3%**
2020	24	88,832	79,698	**86.1%**	83,154	66,555	**97.0%**
2021	12	99,123			178,539		
Totals		362,054			390,045		
AY < CY		262,931	232,199	**98.9%**	211,506	161,054	**100.0%**

Figure 5.7. Assessing the Incurred AY Observations for Commercial Auto



too narrow or the mean may have been too low, or a combination of both, as 8 of the 10 realizations are above the 75th percentile of the distribution.

A closer look at the paid observations in Table 5.12 and Figure 5.8 indicates that immature AYs appear to have again been significantly underestimated. Though not conclusive, the realized values imply again that there may have been a problem with the deterministic methods underlying the prior analysis. Again, the prior uncertainty estimates may have been too narrow or the means too low or both (but to a lesser extent than observed in the incurred KPIs).

The skewness across AYs in the models underlying both the incurred and paid expectations is highlighted by the differences between the expected values or means (the green line) and median values (the blue line) in Figures 5.7 and 5.8.

An ERM system also has predefined actions that are conditional on the breaching of the 95th percentile threshold. For commercial auto, these actions include immediate and automatic e-mails from the ERM system to the data quality officer, claims officer, and reinsurance officer, among others, as illustrated in Figures 5.9 to 5.11. This presupposes some training of nonactuarial professionals so that they understand that 5 of the 100 observations should breach the 95th percentile and that a breach does not necessarily indicate that the methods and models were calibrated incorrectly. However, as part of the risk management collaboration that is being cultivated, these e-mails move all concerned to action.

Attached to the e-mails illustrated in Figures 5.9, 5.10, and 5.11 are reports that the recipients can open to review the specific results. The reports attached to the e-mails, which also highlight any breached thresholds, are shown in Appendix B.

Figure 5.8. Assessing the Paid AY Observations for Commercial Auto

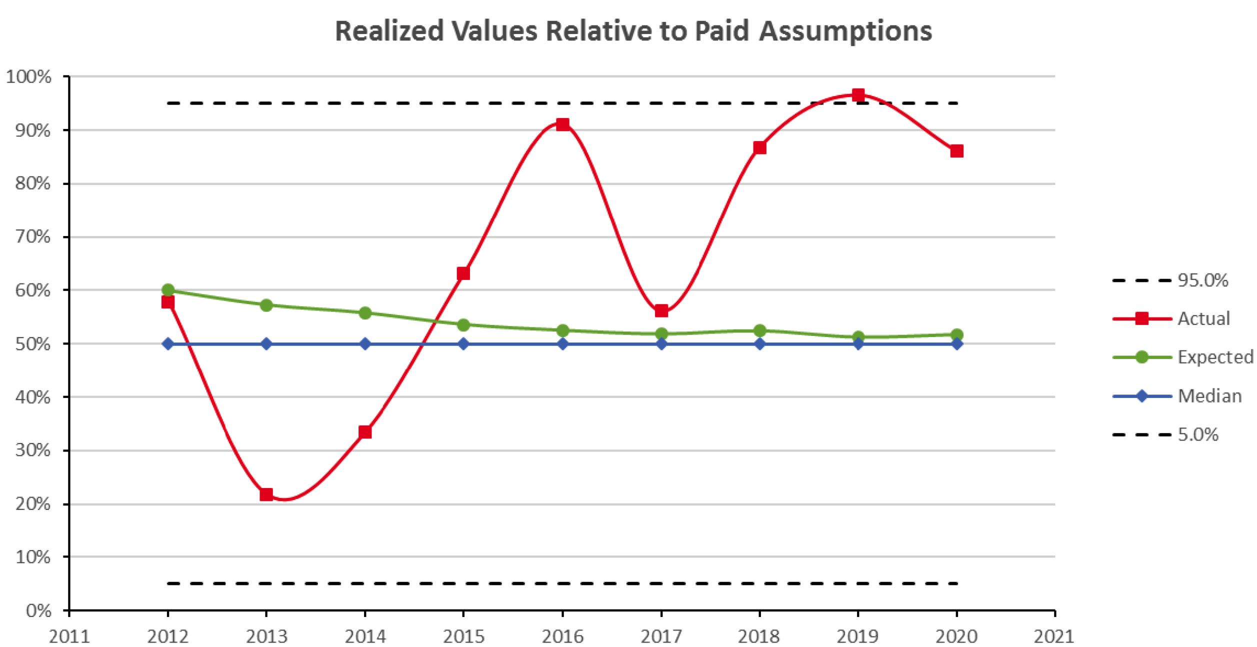
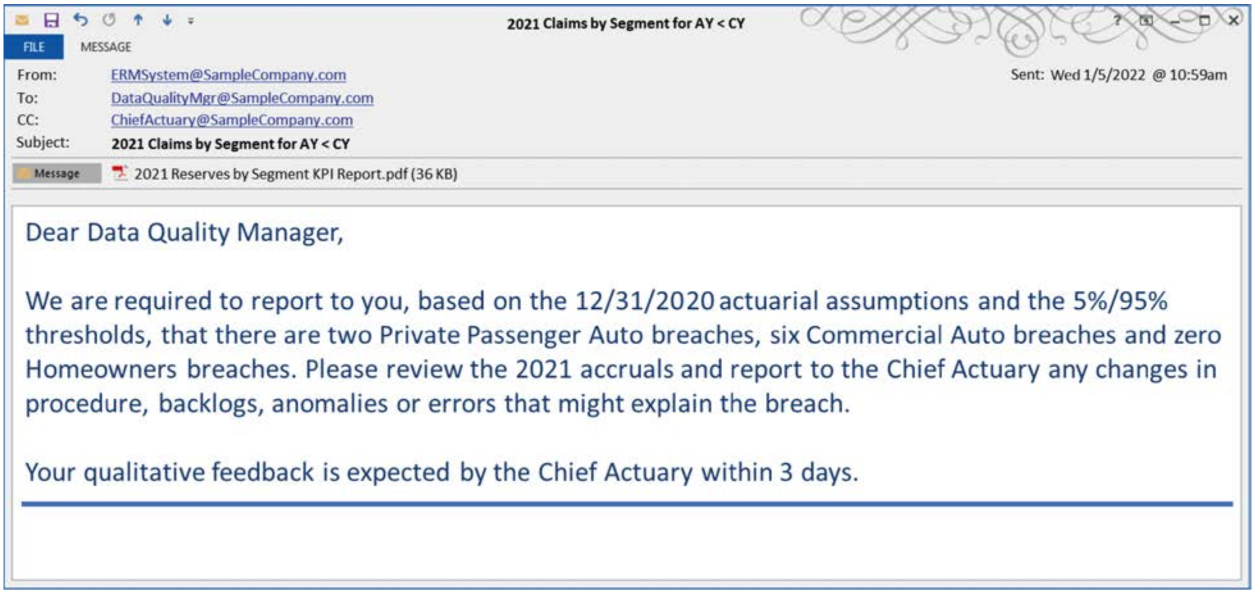


Figure 5.9. Automated E-Mail #4 to the Data Quality Officer



2021 Claims by Segment for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com

To: DataQualityMgr@SampleCompany.com

CC: ChiefActuary@SampleCompany.com

Subject: 2021 Claims by Segment for AY < CY

Sent: Wed 1/5/2022 @ 10:59am

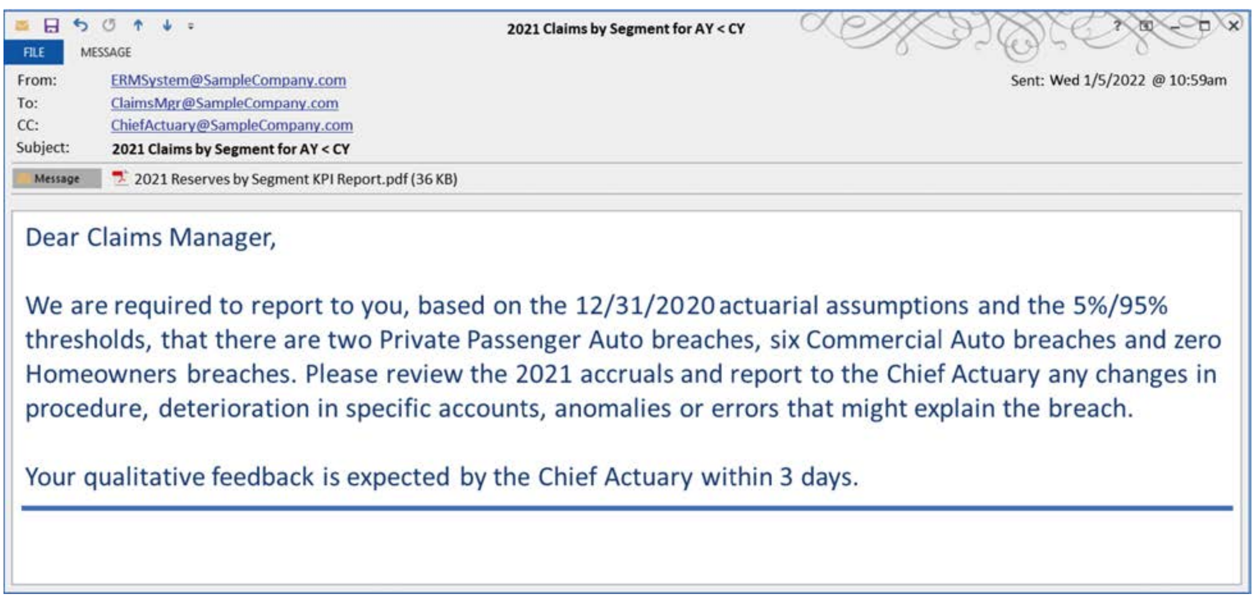
Message | 2021 Reserves by Segment KPI Report.pdf (36 KB)

Dear Data Quality Manager,

We are required to report to you, based on the 12/31/2020 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. Please review the 2021 accruals and report to the Chief Actuary any changes in procedure, backlogs, anomalies or errors that might explain the breach.

Your qualitative feedback is expected by the Chief Actuary within 3 days.

Figure 5.10. Automated E-Mail #5 to the Claims Officer



2021 Claims by Segment for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com

To: ClaimsMgr@SampleCompany.com

CC: ChiefActuary@SampleCompany.com

Subject: 2021 Claims by Segment for AY < CY

Sent: Wed 1/5/2022 @ 10:59am

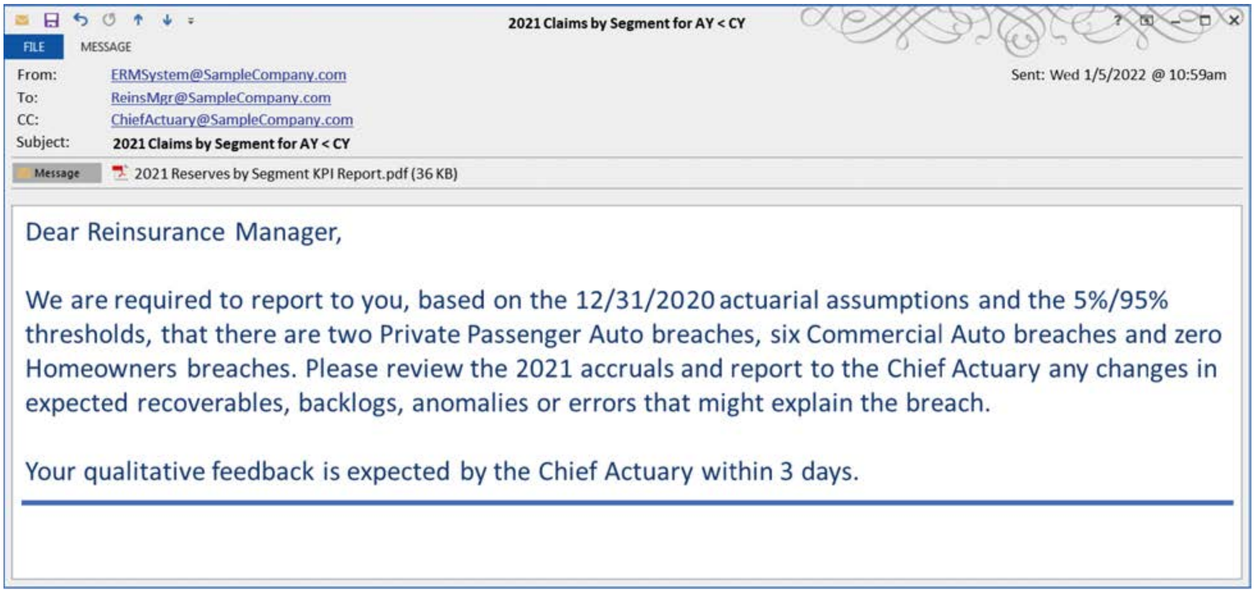
Message | 2021 Reserves by Segment KPI Report.pdf (36 KB)

Dear Claims Manager,

We are required to report to you, based on the 12/31/2020 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. Please review the 2021 accruals and report to the Chief Actuary any changes in procedure, deterioration in specific accounts, anomalies or errors that might explain the breach.

Your qualitative feedback is expected by the Chief Actuary within 3 days.

Figure 5.11. Automated E-Mail #6 to the Reinsurance Officer


2021 Claims by Segment for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com Sent: Wed 1/5/2022 @ 10:59am
To: ReinsMgr@SampleCompany.com
CC: ChiefActuary@SampleCompany.com
Subject: 2021 Claims by Segment for AY < CY

Message 2021 Reserves by Segment KPI Report.pdf (36 KB)

Dear Reinsurance Manager,

We are required to report to you, based on the 12/31/2020 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. Please review the 2021 accruals and report to the Chief Actuary any changes in expected recoverables, backlogs, anomalies or errors that might explain the breach.

Your qualitative feedback is expected by the Chief Actuary within 3 days.

5.7. Using Back-Testing Diagnostics to Assess Uncertainty

As noted above, a single back-test observation has limited value in terms of assessing the overall quality of the variability estimates. However, reviewing a large number of observed percentiles compared to the expectations can be a value-added exercise. For the example in Table 5.13, 50% of the observations are expected to manifest within the 25th to 75th percentile. Likewise, 90% of the observations are expected to manifest within the 5th to 95th percentile, and 10% of the observations are expected to manifest either below the 5th or above the 95th percentile.

Based solely on the 80 observations, the commercial auto line of business appears to need attention (which is consistent with the values for conditional reserves). Further, the homeowners and private passenger auto lines of business appear to be behaving with less uncertainty than expected. While not definitive, this process provides clues as to where the ODP bootstrap models may have been underestimating or overestimating the inherent uncertainty. While it is tempting to draw conclusions, restraint is required, as random noise can easily result in a larger or smaller number of extreme observations than witnessed in Table 5.13. Nevertheless, evidence is mounting that commercial auto deserves the most attention.

5.8. The Feedback Loop

A critical and common component of reserving and ERM is the feedback loop. Reviewing and reevaluating models and assumptions is a healthy part of any reserve analysis, and an open discussion of risks within the ERM framework naturally leads back to the original assumptions. In the case study, all assumptions discussed in Section 5.3 were systematically reviewed and alternative assumptions evaluated to determine if, with the benefit of hindsight, there was a material difference in the back-test.

Table 5.13. Assessing Uncertainty in the 80 Observations

Sample Insurance Company
Summary of Theshold Activity by Segment as of December 31, 2021

	Number						Percentage					
	25% < X < 75%		5% < X < 95%		5% < X < 95%		25% < X < 75%		5% < X < 95%		5% < X < 95%	
	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual
PPA	10	14	18	18	2	2	50.0%	70.0%	90.0%	90.0%	10.0%	10.0%
CA	10	**5**	18	**14**	2	**6**	50.0%	**25.0%**	90.0%	**70.0%**	10.0%	**30.0%**
HO	10	12	18	20	2	0	50.0%	60.0%	90.0%	100.0%	10.0%	0.0%
AGG	10	18	18	20	2	0	50.0%	90.0%	90.0%	100.0%	10.0%	0.0%
Total	40	49	72	72	8	8	50.0%	61.3%	90.0%	90.0%	10.0%	10.0%

The only assumption that proved to have more than an insignificant impact on the back-test was the a priori loss ratio assumption for the Bornhuetter-Ferguson models. As shown in Table 5.4, the management IELR of 52.9% for 2020 is a bit low compared to the projected loss ratios from the Pd CL and Inc CL models, so the 2020 IELR was changed to 57.5%. Comparing Table 5.14 with Table 5.12, the back-test of this assumption has a significant impact on the paid results for 2020, but

Table 5.14. Revised Observations for Commercial Auto after A Priori Adjustment for 2020

Sample Insurance Company
Commercial Auto
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2012	120	543	571	57.9%	(47)	154	**0.0%**
2013	108	2,387	3,131	**21.8%**	1,040	448	**82.8%**
2014	96	1,177	1,665	33.5%	851	1,167	44.5%
2015	84	5,403	5,044	63.1%	2,954	1,669	**86.1%**
2016	72	14,120	11,061	**91.1%**	9,035	5,606	**94.2%**
2017	60	23,636	23,276	56.1%	16,524	11,960	**93.9%**
2018	48	51,020	45,272	**86.7%**	36,454	29,103	**92.7%**
2019	36	75,813	62,481	**96.5%**	61,541	44,392	**99.3%**
2020	24	88,832	85,603	65.4%	83,154	73,782	**85.3%**
2021	12	99,123			178,539		
Totals		362,054			390,045		
AY < CY		262,931	238,104	**96.7%**	211,506	168,281	**99.9%**

the incurred results for 2020 are not as significant and the impact on the AY < CY results was insignificant.

While the assumed loss ratios over the past few years have been decreasing, in light of the back-testing it seems more likely that the loss ratios have remained constant at best or have been increasing.

The benefit of hindsight led to an observation that a calendar year trend was evident yet overlooked (see bottom left graph in Figure 5.12). It is important here to pause and contemplate how frequently such trends are observed and disregarded (or considered immaterial). The point here is that the enhanced documentation provides an evidence trail that confirms that the trend was not addressed. With the benefit of hindsight, however, more attention is given to such diagnostics as a material driver of performance.

After identification of this possible explanation, a new model as of the previous valuation date can be calibrated. In this case, the relationship between the ODP bootstrap model and the generalized linear model (GLM) it is based on became useful. The ODP bootstrap model uses one parameter for every development year and one parameter for every accident year (minus one). Therefore, the ODP bootstrap model is unable to add parameters to account for calendar year effects without removing corresponding accident year or development year parameters.

New GLM bootstrap models based on paid and incurred data were calibrated with calendar year parameters, which were able to model the calendar year effect

Figure 5.12. Commercial Auto: Plots of Residuals (Paid)

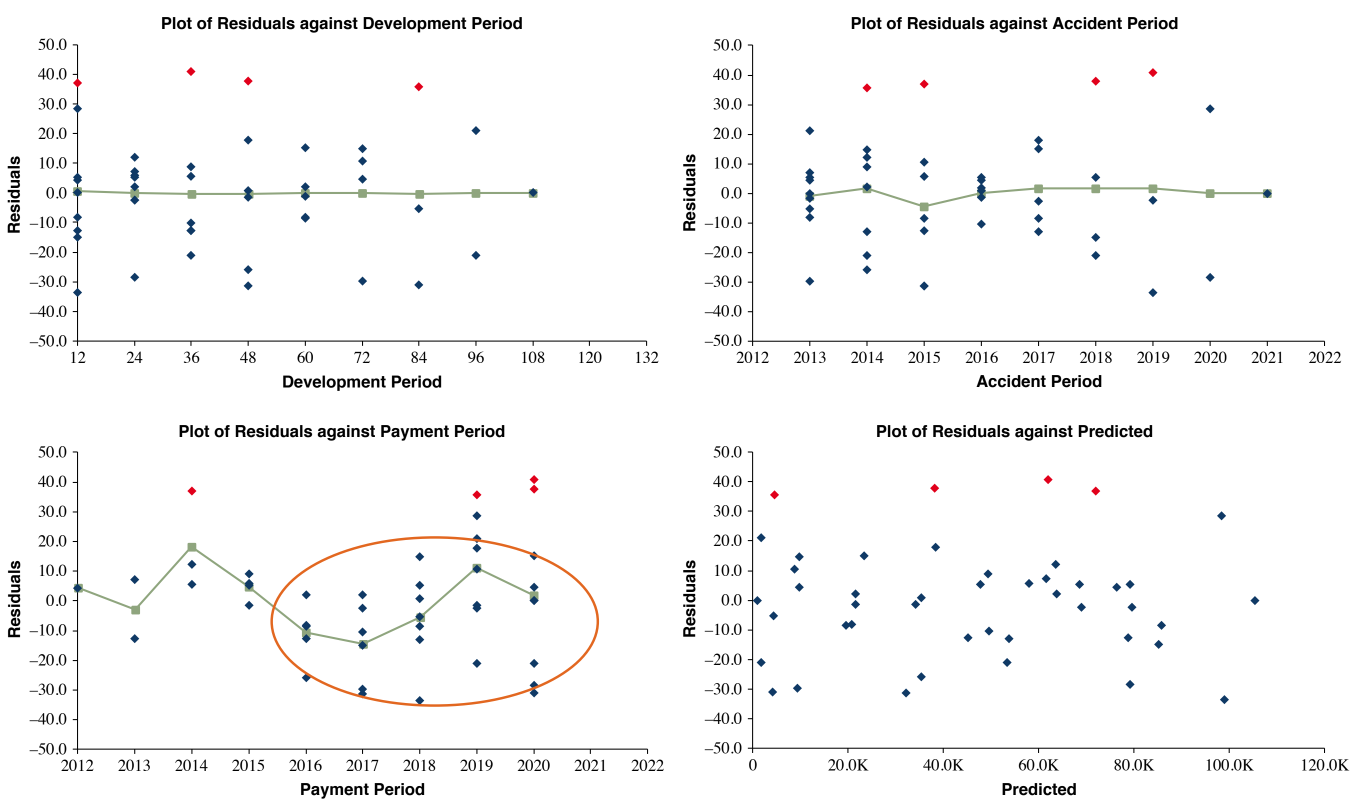
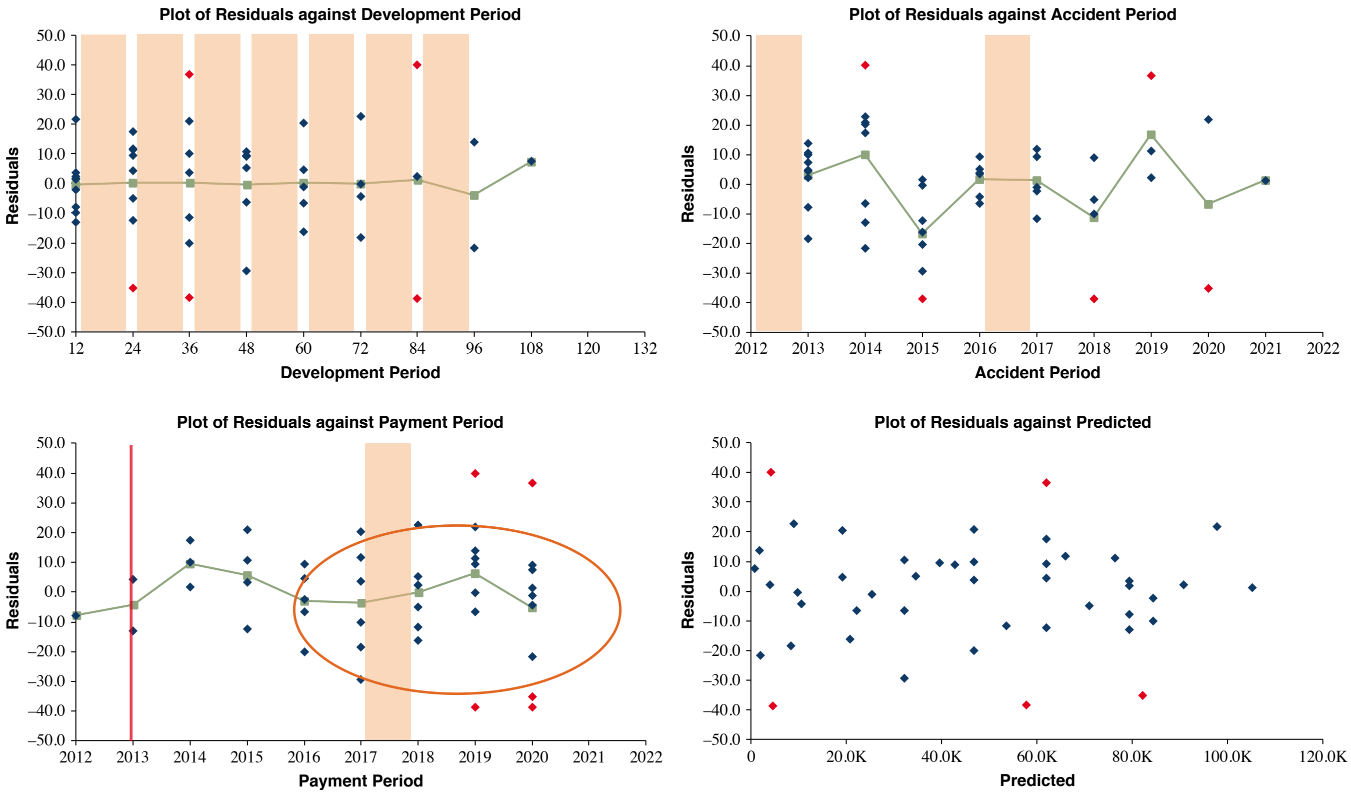


Figure 5.13. Commercial Auto: Plots of Residuals (Paid) for GLM Bootstrap Model



(see Figure 5.13, where shading refers to the parameters being used). The underlying calendar year trends inherent in the new GLM bootstrap models imply no trend from 2013 until 2017, but an annual trend of 7.3% for years 2017 and subsequent using the paid data and a trend of 6.4% using the incurred data.

The new GLM bootstrap models based on paid and incurred data performed better than the prior selected models, as seen in Table 5.15, and many of the model statistics are better.

At first glance, Table 5.15 does not appear to be significantly better than Table 5.12. However, a review of Figures 5.14 and 5.15 (for the GLM bootstrap) reveals that adding the calendar year trend to the models counteracts the upward trend in Figures 5.7 and 5.8 (prior to the GLM bootstrap) to a significant degree (more for paid than incurred), which provides a rationale (or evidence) for the increasing loss ratios over the last few years. This corroborates the earlier back-test of the Bornhuetter-Ferguson a priori loss ratios. The resulting variations in Figures 5.14 and 5.15 also indicate that the variability of the potential outcomes may still be too narrow (e.g., the a priori variance assumption associated with the ODP based on the Bornhuetter-Ferguson method could be larger), but this is just a preliminary review.[52]

[52] For the Bornhuetter-Ferguson method used with the ODP bootstrap model, the typical deterministic a priori mean loss ratio assumption can incorporate uncertainty by including a variance assumption. Back-testing can then help assess whether the a priori loss ratio variance assumption is too low or too high.

Table 5.15. Assessing the Commercial Auto Observations for the GLM Bootstrap Models

Sample Insurance Company
Commercial Auto
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2012	120	543	432	69.4%	(47)	228	**2.0%**
2013	108	2,387	942	**96.6%**	1,040	516	**86.8%**
2014	96	1,177	2,117	**14.0%**	851	1,181	37.9%
2015	84	5,403	5,001	64.1%	2,954	2,665	64.7%
2016	72	14,120	12,100	**82.3%**	9,035	6,659	**89.8%**
2017	60	23,636	27,514	**11.8%**	16,524	13,869	**84.2%**
2018	48	51,020	46,010	**87.6%**	36,454	31,896	**87.7%**
2019	36	75,813	66,910	**94.6%**	61,541	50,020	**98.5%**
2020	24	88,832	88,362	54.1%	83,154	78,184	**77.8%**
2021	12	99,123			178,539		
Totals		362,054			390,045		
AY < CY		262,931	249,388	**86.0%**	211,506	185,218	**98.7%**

Figure 5.14. Assessing the Incurred AY Observations for Commercial Auto (GLM Bootstrap Model)

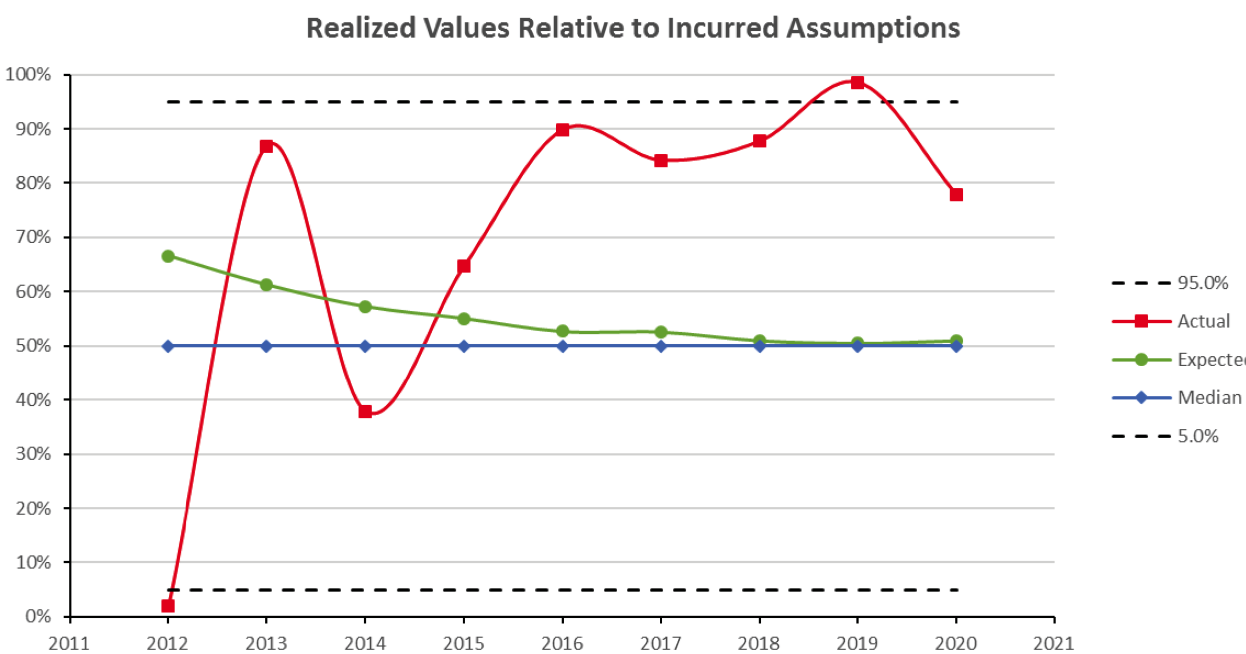
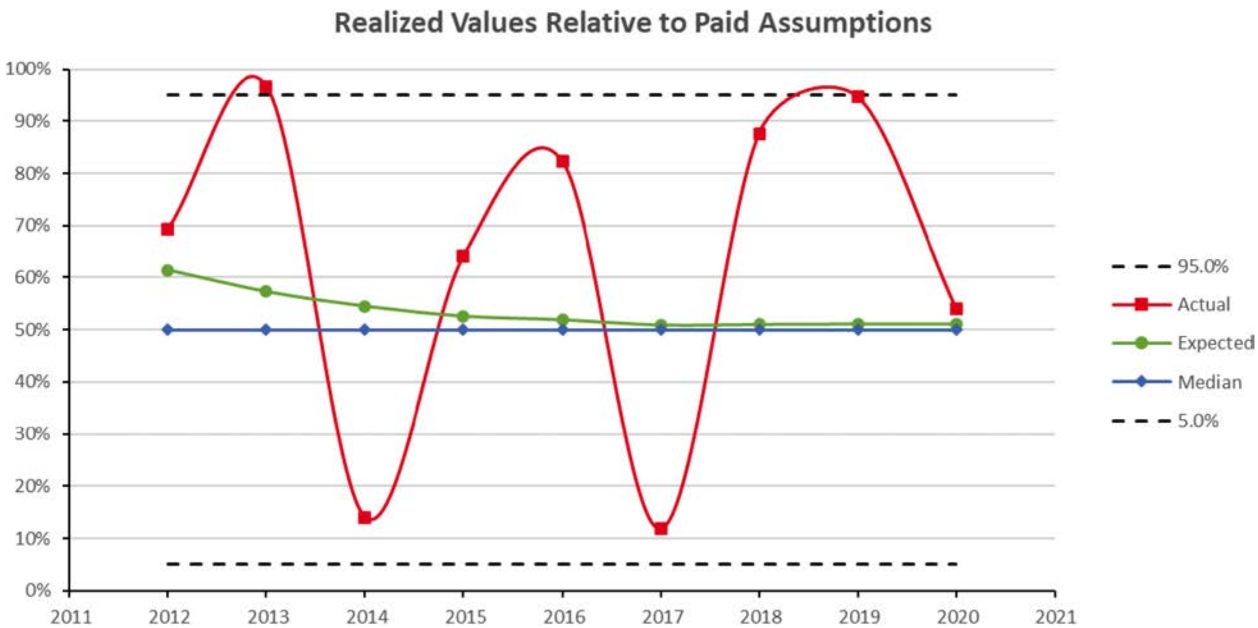


Figure 5.15. Assessing the Paid AY Observations for Commercial Auto (GLM Bootstrap Model)

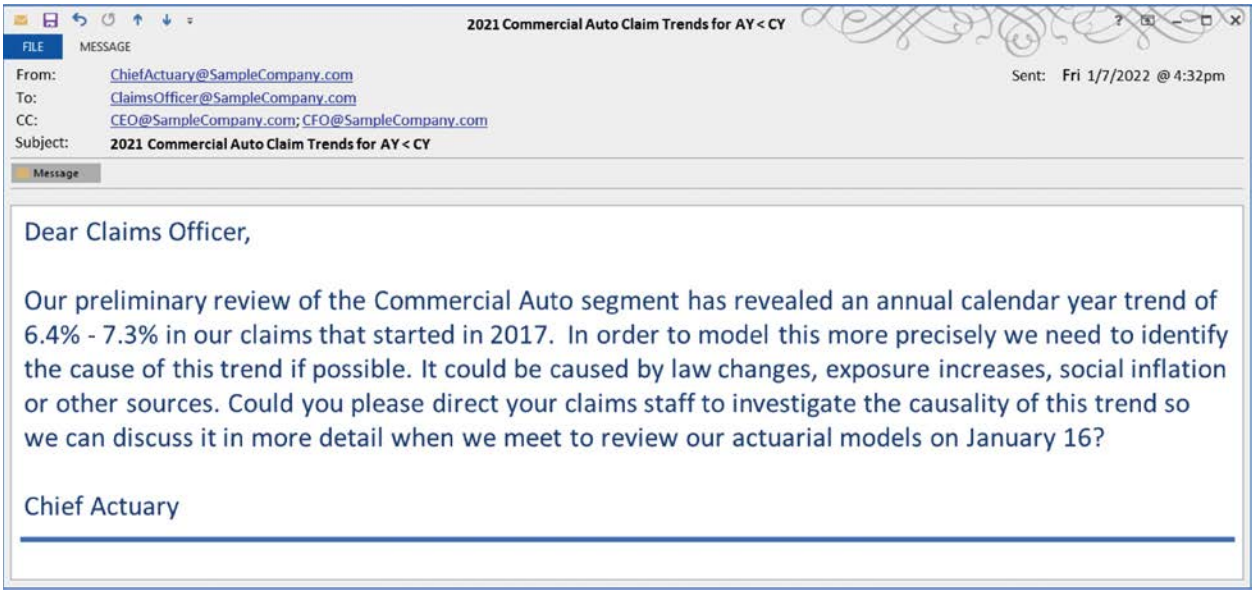


In this case study, the ERM process has provided the information to identify the problem segment, and the enhanced documentation has allowed quick testing of the prior assumptions to provide an alternative model that can be considered and implemented by the actuarial resources for the current valuation. Additionally, the GLM approach has both identified when the positive calendar year trend begins (i.e., the break point) and quantified the trend rates, which allows the actuary to engage more directly with the claims department, where deeper knowledge may exist to improve the modeling process.

A manually created e-mail from the chief actuary to the relevant claims officer, as illustrated in Figure 5.16, is the logical next step in the process so that communication around this issue can begin. Note that the process allows the actuary to speak to the claims officer in the language the claims officer understands: no mention of triangles, IBNR, accident years, or any other actuarial concepts that may be unfamiliar.

The value of this active feedback loop on reserving risk within the ERM process cannot be overstated. Not only does it naturally expand the actuarial conversation about risk drivers to the entire firm, but it also flows into other risks such as claims management and pricing risk. Indeed, consider the impact that identifying this trend will have on future pricing discussions for commercial auto policies.

Figure 5.16. Manual E-Mail to the Claims Officer



2021 Commercial Auto Claim Trends for AY < CY

FILE MESSAGE

From: ChiefActuary@SampleCompany.com

Sent: Fri 1/7/2022 @ 4:32pm

To: ClaimsOfficer@SampleCompany.com

CC: CEO@SampleCompany.com; CFO@SampleCompany.com

Subject: 2021 Commercial Auto Claim Trends for AY < CY

Message

Dear Claims Officer,

Our preliminary review of the Commercial Auto segment has revealed an annual calendar year trend of 6.4% - 7.3% in our claims that started in 2017. In order to model this more precisely we need to identify the cause of this trend if possible. It could be caused by law changes, exposure increases, social inflation or other sources. Could you please direct your claims staff to investigate the causality of this trend so we can discuss it in more detail when we meet to review our actuarial models on January 16?

Chief Actuary

6. Managing Reserve Risk in Practice

By design, to keep the focus on the key concepts and processes, this monograph simplifies some of the processes that would be more rigorous in real life. In practice, some parts of the process can be more or less complex, or at least more or less thorough, so some of the potential variations in approach are highlighted in the following non-exhaustive list.

Benchmarking – As noted in the discussion above, it is not always clear if excess deviations are due to misestimation of the mean or variance. More importantly, back-testing the common unpaid claim estimation models has shown a tendency by the models to underestimate the width of the distribution. Incorporating benchmarks, as suggested in Shapland (2019), into the ERM process could simplify the processes and improve the usefulness.

Separate treatment of catastrophes and large losses – Selecting the limit for segregation of large claims is an important consideration when designing the ERM framework, and, depending on expected frequencies and severities, the monitoring process may be more or less sophisticated.

Use of uncertainty models for estimates of ultimate claim count – Using closed claim and reported claim triangles as inputs, the ERM framework could be expanded to include claim counts in the monitoring system. The claim count data could be useful in monitoring specific types of claims (e.g., large claims) or types of events (e.g., catastrophes), but it would require additional modeling effort.

Handling "all-prior" data – Some data triangles include an "all-prior" years row to keep the triangle size within a limit. Assuming it is not practical to expand the data triangle and given the age of this data, it may be more pragmatic to simply monitor changes in a more deterministic fashion instead of adding a stochastic element.[53]

More frequent reviews – The analysis described in the monograph is based on annual data, but in practice the monitoring process is likely to be more effective on a quarterly or perhaps even monthly basis. This would require balancing resources with effectiveness and would certainly increase many aspects of the ERM framework, so a pragmatic solution might be to keep some segments at less frequent intervals and restrict the more frequent intervals to the more critical segments.

[52] For a useful reference on the analysis and estimation of "all-prior" data, see Shapland (2014).

Roll forward – One way to manage more frequent reviews, when using quarterly or monthly data, would be to adjust expectations, thresholds, etc., for exposures. Using this approach slightly increases the risk that trend changes or other important issues may be missed, so it may be better to use this after a period of time to calibrate the models, thresholds, etc.

Including future pricing – As discussed in Section 3.2, if the reserving analysis is extended to include a distribution for the next accident year (perhaps in a "pricing risk" calibration), then this could be included with the back-test. A caveat to the inclusion of pricing risk is that it will be based on expectations of future exposures, so any back-test should first adjust the distribution for the actual exposures prior to calculation of percentiles to more properly compare these once-future exposures to all the prior years that were based on actual exposures.

Thresholds – In footnote 18, we mentioned that the setting of thresholds is not a trivial step in the design of KPIs, as this determines the number of threshold breaches that are expected to occur. Setting the thresholds too low can result in too many outliers and a "Chicken Little" syndrome and setting thresholds too high risks having too few outliers and possibly missing a critical trend or other parameterization issue. In footnote 23, we mentioned that thresholds could be nominal (e.g., differences larger than $1 million), relative (e.g., differences 150% larger than the mean expected), or distributional (e.g., observations above the 95th percentile of possible future outcomes). Using different types of thresholds could allow for layers of thresholds to distinguish significant events from trivial events.

Anticipated segmentation changes (i.e., bifurcation or combination of segments) – In footnote 21, we mentioned that changes in the segmentation and the associated ramifications for the ERM system need to be thoroughly addressed prior to year-end. The segmentation could evolve naturally as management responsibility changes or as the feedback process leads to a better understanding of the data or issues that need to be monitored.

7. Conclusions

While the value of regularly including reserve variability estimates as part of the "normal" reserving cycle processes is questioned by some, and the estimation process is perhaps feared by others, the purpose of this monograph is to show how making reserve variability estimates a routine part of the analysis can greatly benefit the risk management process. Keeping these estimates in the "back room" or "hidden until needed" does not benefit anyone. If casualty actuaries intend to truly embrace enterprise risk management, then deep discussions of reserving risk must become part of the actuarial lexicon.

Acknowledgments

The authors gratefully acknowledge the many authors listed in the References (and others not listed) who contributed to the foundation of stochastic reserving and enterprise risk management, without which this research would not have been possible. The authors thank Wayne Blackburn for his thorough review and insightful comments. The authors are also grateful to the participants in various seminars and sessions at the General Insurance Research Organising Committee (GIRO) conferences, the Casualty Loss Reserve Seminar, and the European Actuarial Academy stochastic modeling seminars where the concepts in the monograph were first presented and discussed. Finally, the authors thank the CAS Committee on Reserves and the Monograph Committee for their comments, which also greatly improved the quality of the monograph.

Supplementary Material

To give the reader a deeper understanding of the concepts discussed in the monograph, two companion files are provided in the "Actuary & ERM.zip" file:

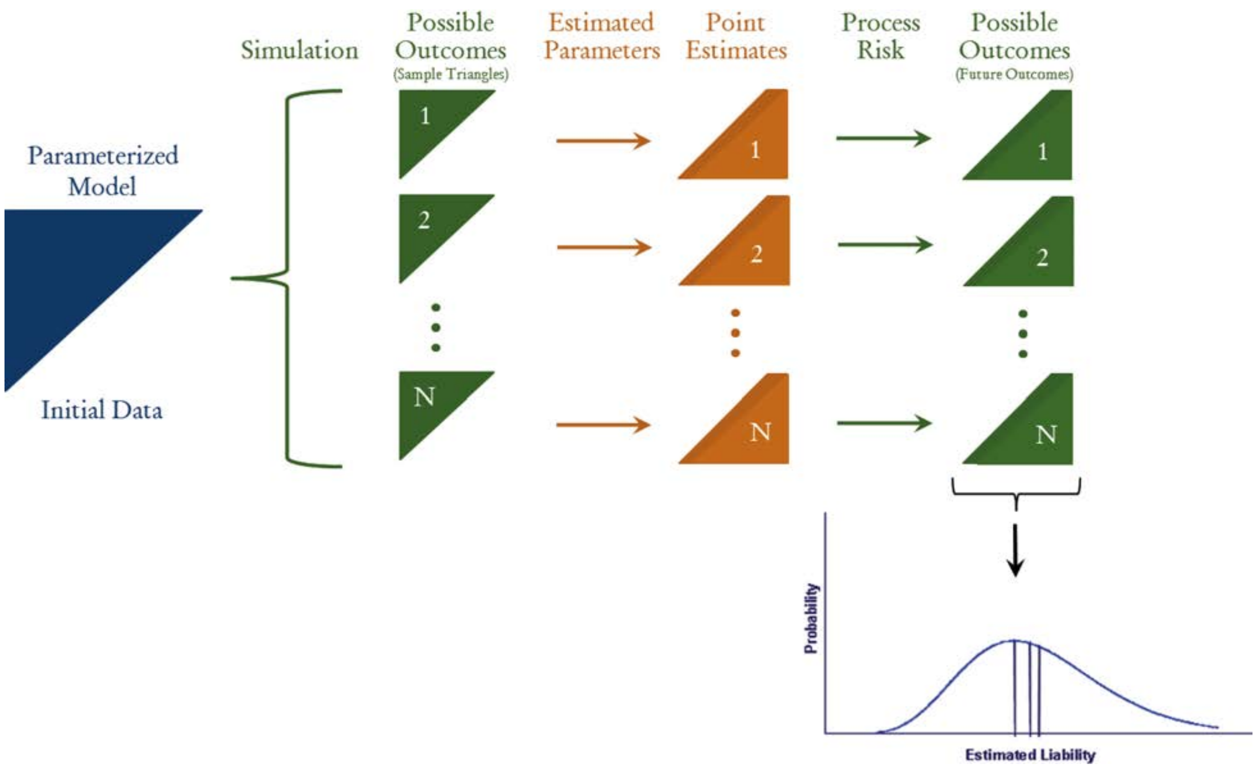
LOB Backtest.xlsm – This file provides the detailed calculations described in this monograph for a single segment or line of business. Data can be entered, and simulation output can be added for the calculation of both expected and actual outcomes, along with various statistical measures and results. Deterministic calculations and results are also included for comparison to stochastic results.

AGG Backtest.xlsm – This file can be used to summarize the deterministic and stochastic results from the "LOB Backtest.xlsm" file (selected results need to be copied to this file) for three lines of business. Aggregate simulation output can be added for the calculation of both expected and actual outcomes, along with various statistical measures and results.

APPENDICES

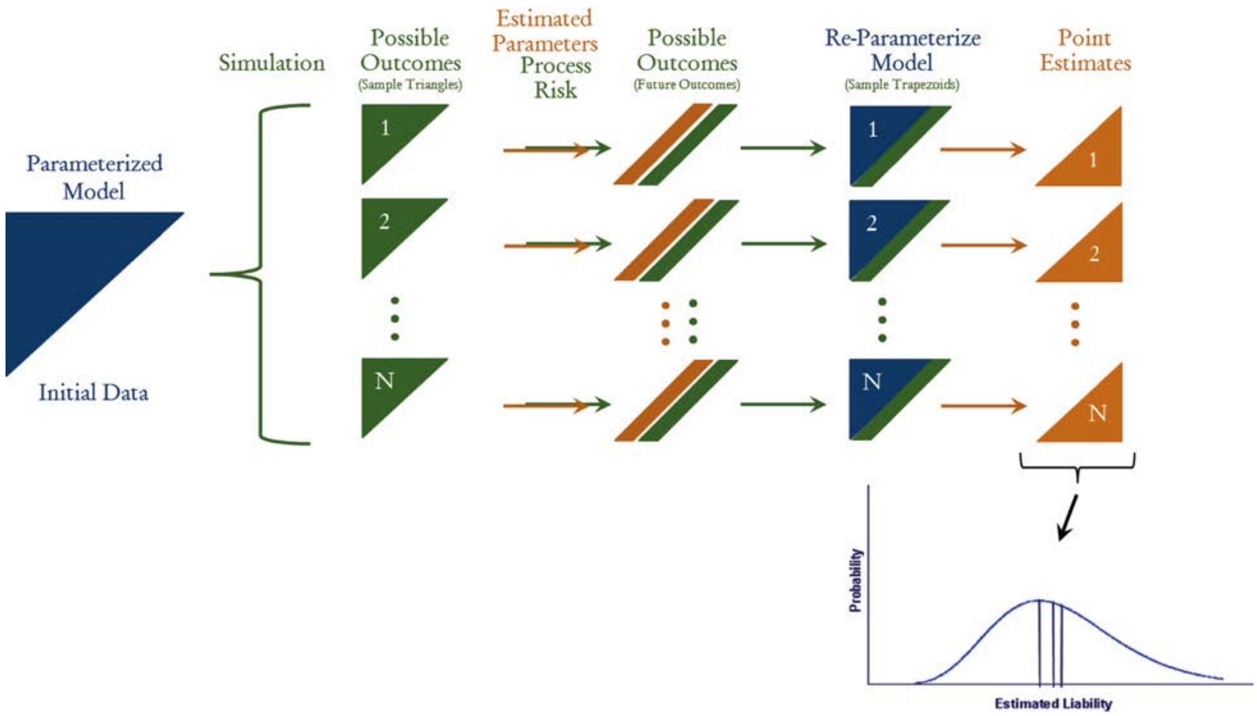
Appendix A—Overview of One-Year Time Horizon

A “standard” ODP bootstrap model can be represented graphically as follows:



- The “standard” model is based on paid data, but incurred data can also be used to reflect information in case reserves and converted to a random payment stream.
- The standard model is based on the chain ladder methodology, but other methods such as Bornhuetter-Ferguson and Cape Cod can also be included.
- Multiple models can also be “weighted” and “shifted” to reconcile with the deterministic “best estimate.”
- The segment results can be aggregated to derive a consolidated corporate result, even though these graphs are for one segment.

By using the first diagonal of the possible future outcomes and then calculating a point estimate for the remaining unpaid claims, the one-year time horizon can be represented graphically as follows:



- The "one-year" model is based on paid data, but incurred data can also be used to reflect information in case reserves and converted to a random payment stream for the first diagonal and expected payments for the remaining diagonals.
- The one-year model is based on the chain ladder methodology, but other methods such as Bornhuetter-Ferguson and Cape Cod can also be included. For internal consistency, all the assumptions for the standard model should apply unchanged for the one-year model.
- Multiple models can also be "weighted" and "shifted" to reconcile with the deterministic "best estimate." The weights should be the same as for the standard model and shifting should be consistent with the standard model so that the first diagonal after shifting is identical.
- Distributions of conditional point estimates can also be created for each accident year even though the total of all accident years combined is shown in the graphs.
- The segment results can be aggregated to derive a consolidated corporate result, even though these graphs are for one segment.

Appendix B—Reports Attached to E-mails

Figure B.1. Report on 2021 Aggregate Exposures

Stochastic Model Results

2021 Aggregation of All Segments Exposure

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Stochastic Model Detail

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Model Name	2021 Aggregation of All Segments Exposure	Assumption Owner	Chief Actuary
Description	Expected Aggregation of All Segments claim payments during 2021 for exposure periods prior to 2021 based on data generated by claims system as of 12/31/2021 relative to the 12/31/2020 actuarial assumptions.	Reports To	Chief Executive Officer
Assumption Value	Expected Value	Assumption Value Date	12/31/2020
Assumption Minimum	5.0%	Next Update Due	12/31/2021
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	1,571,872	Incurred Actual	847,136
Paid Expected	1,560,637	Incurred Expected	858,093
Paid Percentile	61.2%	Incurred Percentile	37.6%

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Stochastic Values

New Value

Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0001	12/31/2012	120	3,069	4,077	31.8%	1,863	2,115	49.8%
Edit | Del	0002	12/31/2013	108	5,905	6,163	47.9%	3,145	1,819	80.6%
Edit | Del	0003	12/31/2014	96	8,986	10,176	33.6%	3,553	6,026	20.9%
Edit | Del	0004	12/31/2015	84	18,992	20,033	39.0%	9,872	10,399	46.3%
Edit | Del	0005	12/31/2016	72	51,003	48,298	71.6%	25,942	25,562	55.3%
Edit | Del	0006	12/31/2017	60	105,067	104,415	54.3%	52,012	53,101	44.8%
Edit | Del	0007	12/31/2018	48	202,932	196,083	74.2%	106,624	104,075	61.7%
Edit | Del	0008	12/31/2019	36	334,434	331,701	57.1%	189,908	185,173	64.0%
Edit | Del	0009	12/31/2020	24	841,484	839,689	52.8%	454,217	469,822	29.3%
Edit | Del	0010	12/31/2021	12	1,798,138	0		2,528,235	0	

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Figure B.2. Report on 2021 Private Passenger Auto Exposures

Stochastic Model Results

2021 Private Passenger Auto Exposure

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Stochastic Model Detail Edit Delete Clone

Model Name	2021 Private Passenger Auto Exposure	Assumption Owner	Reserving Actuary
Description	Expected Private Passenger Auto claim payments during 2021 for exposure periods prior to 2021 based on data generated by claims system as of 12/31/2021 relative to the 12/31/2020 actuarial assumptions.	Reports To	Chief Actuary
Assumption Value	Expected Value	Assumption Value Date	12/31/2020
Assumption Minimum	5.0%	Next Update Due	12/31/2021
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	1,071,854	Incurred Actual	571,794
Paid Expected	1,076,388	Incurred Expected	631,511
Paid Percentile	44.9%	Incurred Percentile	0.6%

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Stochastic Values New Value Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0011	12/31/2012	120	2,500	2,733	48.2%	2,042	2,056	56.7%
Edit | Del	0012	12/31/2013	108	3,485	2,908	69.4%	2,261	1,312	81.0%
Edit | Del	0013	12/31/2014	96	7,582	8,098	43.4%	4,061	5,207	33.2%
Edit | Del	0014	12/31/2015	84	13,765	14,773	37.5%	8,076	8,835	41.7%
Edit | Del	0015	12/31/2016	72	33,083	35,326	30.5%	16,495	20,439	15.6%
Edit | Del	0016	12/31/2017	60	75,969	74,381	61.4%	35,496	40,022	21.2%
Edit | Del	0017	12/31/2018	48	139,715	140,849	45.5%	68,886	74,159	25.6%
Edit | Del	0018	12/31/2019	36	234,781	243,390	26.5%	119,582	128,507	20.2%
Edit | Del	0019	12/31/2020	24	560,974	553,931	62.3%	314,895	350,974	2.9%
Edit | Del	0020	12/31/2021	12	764,210	0		1,205,957	0	

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Figure B.3. Report on 2021 Commercial Auto Exposures

Stochastic Model Results

2021 Commercial Auto Exposure

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Stochastic Model Detail

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Model Name	2021 Commercial Auto Exposure	Assumption Owner	Reserving Actuary
Description	Expected Commercial Auto claim payments during 2021 for exposure periods prior to 2021 based on data generated by claims system as of 12/31/2021 relative to the 12/31/2020 actuarial assumptions.	Reports To	Chief Actuary
Assumption Value	Expected Value	Assumption Value Date	12/31/2020
Assumption Minimum	5.0%	Next Update Due	12/31/2021
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	262,931	Incurred Actual	211,506
Paid Expected	232,199	Incurred Expected	161,054
Paid Percentile	98.9%	Incurred Percentile	100.0%

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Stochastic Values New Value Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0021	12/31/2012	120	543	571	57.9%	(47)	154	0.0%
Edit | Del	0022	12/31/2013	108	2,387	3,131	21.8%	1,040	448	82.8%
Edit | Del	0023	12/31/2014	96	1,177	1,665	33.5%	851	1,167	44.5%
Edit | Del	0024	12/31/2015	84	5,403	5,044	63.1%	2,954	1,669	86.1%
Edit | Del	0025	12/31/2016	72	14,120	11,061	91.1%	9,035	5,606	94.2%
Edit | Del	0026	12/31/2017	60	23,636	23,276	56.1%	16,524	11,960	93.9%
Edit | Del	0027	12/31/2018	48	51,020	45,272	86.7%	36,454	29,103	92.7%
Edit | Del	0028	12/31/2019	36	75,813	62,481	96.5%	61,541	44,392	99.3%
Edit | Del	0029	12/31/2020	24	88,832	79,698	86.1%	83,154	66,555	97.0%
Edit | Del	0030	12/31/2021	12	99,123	0		178,539	0	

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Figure B.4. Report on 2021 Homeowners Exposures

Stochastic Model Results

2021 Homeowners Exposure

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Stochastic Model Detail Edit Delete Clone

Model Name	2021 Homeowners Exposure	Assumption Owner	Reserving Actuary
Description	Expected Homeowners claim payments during 2021 for exposure periods prior to 2021 based on data generated by claims system as of 12/31/2021 relative to the 12/31/2020 actuarial assumptions.	Reports To	Chief Actuary
Assumption Value	Expected Value	Assumption Value Date	12/31/2020
Assumption Minimum	5.0%	Next Update Due	12/31/2021
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	237,087	Incurred Actual	63,836
Paid Expected	252,049	Incurred Expected	65,528
Paid Percentile	28.4%	Incurred Percentile	50.2%

Edit Delete Clone

Stochastic Values New Value Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0031	12/31/2012	120	26	773	13.9%	(132)	(95)	83.5%
Edit | Del	0032	12/31/2013	108	33	125	61.9%	(156)	59	31.4%
Edit | Del	0033	12/31/2014	96	227	414	57.2%	(1,359)	(349)	23.5%
Edit | Del	0034	12/31/2015	84	(176)	217	14.1%	(1,158)	(105)	18.5%
Edit | Del	0035	12/31/2016	72	3,800	1,911	85.6%	412	(482)	67.2%
Edit | Del	0036	12/31/2017	60	5,462	6,758	37.5%	(8)	1,119	12.2%
Edit | Del	0037	12/31/2018	48	12,197	9,961	74.9%	1,284	813	81.4%
Edit | Del	0038	12/31/2019	36	23,840	25,830	40.5%	8,785	12,274	37.9%
Edit | Del	0039	12/31/2020	24	191,678	206,060	28.0%	56,168	52,293	62.7%
Edit | Del	0040	12/31/2021	12	934,805	0		1,143,739	0	

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Figure B.5. Report on 2021 Conditional Reserves

Stochastic Model Results

2021 Conditional Reserves by Segment

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Stochastic Model Detail Edit Delete Clone

Model Name	2021 Conditional Reserves by Segment	Assumption Owner	Chief Actuary
Description	Expected conditional reserves as of 12/31/2021 for exposure periods prior to 2021 based on data generated by claims system during CY 2021 relative to the 12/31/2020 actuarial assumptions.	Reports To	Chief Executive Officer
Assumption Value	Percentile of One-Year Horizon	Assumption Value Date	12/31/2020
Output Value	One-Year Reserve Estimate	Next Update Due	12/31/2021

▼ **Realized Value**

Sum of Yrs	(2,154)	Sum of Yrs	10,926	Sum of Yrs	72,619	Sum of Yrs	(35,107)
CY 2021	(2,086)	CY 2021	6,115	CY 2021	78,376	CY 2021	(40,385)
Overall Change:	Aggregation of All Segme	Overall Change:	Sum of Segments	Largest Increase:	CA	Largest Decrease:	PPA

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Stochastic Values New Value Help

Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
					Aggregation of All Segments				
Edit | Del	0001	12/31/2012	120	7,410	3,069	4,341	31.8%	2,539	(1,802)
Edit | Del	0002	12/31/2013	108	16,366	5,905	10,461	47.9%	11,349	888
Edit | Del	0003	12/31/2014	96	23,269	8,986	14,283	33.6%	10,961	(3,322)
Edit | Del	0004	12/31/2015	84	44,378	18,992	25,386	39.0%	21,615	(3,771)
Edit | Del	0005	12/31/2016	72	96,042	51,003	45,039	71.6%	49,308	4,269
Edit | Del	0006	12/31/2017	60	202,705	105,067	97,638	54.3%	97,157	(481)
Edit | Del	0007	12/31/2018	48	413,903	202,932	210,971	74.2%	222,250	11,279
Edit | Del	0008	12/31/2019	36	765,488	334,434	431,054	57.1%	427,667	(3,387)
Edit | Del	0009	12/31/2020	24	1,642,982	841,484	801,499	52.8%	795,671	(5,828)
Edit | Del	0010	SUM OF YRS		3,212,543	1,571,872	1,640,671		1,638,516	(2,154)
Edit | Del	0011	CY 2021		3,212,543	1,571,872	1,640,671	61.2%	1,638,584	(2,086)

Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
					Sum of All Segments				
Edit | Del	0012	12/31/2012	120	7,410	3,069	4,341	N/A	3,323	(1,018)
Edit | Del	0013	12/31/2013	108	16,366	5,905	10,461	N/A	10,669	208
Edit | Del	0014	12/31/2014	96	23,269	8,986	14,283	N/A	11,697	(2,586)
Edit | Del	0015	12/31/2015	84	44,378	18,992	25,386	N/A	20,075	(5,311)
Edit | Del	0016	12/31/2016	72	96,042	51,003	45,039	N/A	49,263	4,224
Edit | Del	0017	12/31/2017	60	202,705	105,067	97,638	N/A	97,412	(227)
Edit | Del	0018	12/31/2018	48	413,903	202,932	210,971	N/A	222,487	11,516
Edit | Del	0019	12/31/2019	36	765,488	334,434	431,054	N/A	425,174	(5,880)
Edit | Del	0020	12/31/2020	24	1,642,982	841,484	801,499	N/A	811,496	9,997
Edit | Del	0021	SUM OF YRS		3,212,543	1,571,872	1,640,671		1,651,596	10,926
Edit | Del	0022	CY 2021		3,212,543	1,571,872	1,640,671	N/A	1,646,786	6,115

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Figure B.5. Report on 2021 Conditional Reserves (*Continued*)

Stochastic Model Results

2021 Conditional Reserves by Segment

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Stochastic Model Detail Edit Delete Clone

Model Name	2021 Conditional Reserves by Segment	**Assumption Owner**	Chief Actuary
Description	Expected conditional reserves as of 12/31/2021 for exposure periods prior to 2021 based on data generated by claims system during CY 2021 relative to the 12/31/2020 actuarial assumptions.	**Reports To**	Chief Executive Officer
Assumption Value	Percentile of One-Year Horizon	**Assumption Value Date**	12/31/2020
Output Value	One-Year Reserve Estimate	**Next Update Due**	12/31/2021

▼ **Realized Value**

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Stochastic Values New Value Help

Private Passenger Auto (PPA)									
Action	**Number**	**Exposure Period**	**Age**	**Original**	**Actual Paid**	**Current**	**Paid Percentile**	**Conditional**	**Change**
Edit | Del	0023	12/31/2012	120	5,491	2,500	2,991	48.2%	2,680	(311)
Edit | Del	0024	12/31/2013	108	8,983	3,485	5,498	69.4%	7,248	1,750
Edit | Del	0025	12/31/2014	96	17,643	7,582	10,061	43.4%	8,654	(1,406)
Edit | Del	0026	12/31/2015	84	33,237	13,765	19,472	37.5%	15,635	(3,836)
Edit | Del	0027	12/31/2016	72	71,149	33,083	38,066	30.5%	31,595	(6,470)
Edit | Del	0028	12/31/2017	60	147,271	75,969	71,302	61.4%	73,359	2,057
Edit | Del	0029	12/31/2018	48	295,776	139,715	156,061	45.5%	151,670	(4,390)
Edit | Del	0030	12/31/2019	36	557,593	234,781	322,812	26.5%	292,882	(29,930)
Edit | Del	0031	12/31/2020	24	1,134,993	560,974	574,019	62.3%	581,448	7,430
Edit | Del	0032	SUM OF YRS		2,272,135	1,071,854	1,200,281		1,165,174	(35,107)
Edit | Del	0033	CY 2021		2,272,135	1,071,854	1,200,281	44.9%	1,159,897	(40,385)

Commercial Auto (CA)									
Action	**Number**	**Exposure Period**	**Age**	**Original**	**Actual Paid**	**Current**	**Paid Percentile**	**Conditional**	**Change**
Edit | Del	0034	12/31/2012	120	1,146	543	603	57.9%	643	40
Edit | Del	0035	12/31/2013	108	6,629	2,387	4,242	21.8%	3,257	(985)
Edit | Del	0036	12/31/2014	96	3,759	1,177	2,582	33.5%	1,675	(907)
Edit | Del	0037	12/31/2015	84	9,524	5,403	4,121	63.1%	5,593	1,472
Edit | Del	0038	12/31/2016	72	20,752	14,120	6,632	91.1%	13,946	7,313
Edit | Del	0039	12/31/2017	60	43,077	23,636	19,441	56.1%	20,073	632
Edit | Del	0040	12/31/2018	48	96,462	51,020	45,442	86.7%	57,978	12,536
Edit | Del	0041	12/31/2019	36	157,440	75,813	81,627	96.5%	110,701	29,075
Edit | Del	0042	12/31/2020	24	235,978	88,832	147,146	86.1%	170,589	23,442
Edit | Del	0043	SUM OF YRS		574,768	262,931	311,837		384,456	72,619
Edit | Del	0044	CY 2021		574,768	262,931	311,837	98.9%	390,213	78,376

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(*continued on next page*)

Figure B.5. Report on 2021 Conditional Reserves (*Continued*)

Stochastic Model Results

2021 Conditional Reserves by Segment

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Model Name	2021 Conditional Reserves by Segment	**Assumption Owner**	Chief Actuary
Description	Expected conditional reserves as of 12/31/2021 for exposure periods prior to 2021 based on data generated by claims system during CY 2021 relative to the 12/31/2020 actuarial assumptions.	**Reports To**	Chief Executive Officer
Assumption Value	Percentile of One-Year Horizon	**Assumption Value Date**	12/31/2020
Output Value	One-Year Reserve Estimate	**Next Update Due**	12/31/2021

▼ **Realized Value**

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				Homeowners (HO)					
Action	**Number**	**Exposure Period**	**Age**	**Original**	**Actual Paid**	**Current**	**Paid Percentile**	**Conditional**	**Change**
Edit | Del	0045	12/31/2012	120	773	26	747	13.9%	0	(747)
Edit | Del	0046	12/31/2013	108	754	33	721	61.9%	164	(557)
Edit | Del	0047	12/31/2014	96	1,867	227	1,640	57.2%	1,367	(272)
Edit | Del	0048	12/31/2015	84	1,617	(176)	1,793	14.1%	(1,153)	(2,946)
Edit | Del	0049	12/31/2016	72	4,140	3,800	340	85.6%	3,722	3,381
Edit | Del	0050	12/31/2017	60	12,356	5,462	6,894	37.5%	3,979	(2,915)
Edit | Del	0051	12/31/2018	48	21,665	12,197	9,468	74.9%	12,839	3,370
Edit | Del	0052	12/31/2019	36	50,455	23,840	26,615	40.5%	21,590	(5,024)
Edit | Del	0053	12/31/2020	24	272,011	191,678	80,333	28.0%	59,458	(20,875)
Edit | Del	0054	SUM OF YRS		365,640	237,087	128,553		101,967	(26,586)
Edit | Del	0055	CY 2021		365,640	237,087	128,553	28.4%	96,676	(31,876)

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Appendix C—Back-Testing Results for Private Passenger Auto

Table C.1. Calculation of Weighted Ultimate (Deterministic)

Sample Insurance Company
Private Passenger Auto
Calculation of Weighted Ultimate as of December 31, 2020

		Ultimate Values by Method				Weights by Method				Weighted Ultimate
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	
2012	108	1,218,574	1,218,574	1,218,578	1,218,577	50.0%	50.0%	0.0%	0.0%	1,218,574
2013	96	1,376,278	1,375,860	1,376,284	1,375,866	50.0%	50.0%	0.0%	0.0%	1,376,069
2014	84	1,439,598	1,439,241	1,439,624	1,439,261	50.0%	50.0%	0.0%	0.0%	1,439,420
2015	72	1,561,673	1,558,592	1,561,726	1,558,664	50.0%	50.0%	0.0%	0.0%	1,560,133
2016	60	1,649,696	1,645,907	1,649,700	1,646,004	50.0%	50.0%	0.0%	0.0%	1,647,802
2017	48	1,669,252	1,665,339	1,670,112	1,665,994	50.0%	50.0%	0.0%	0.0%	1,667,295
2018	36	1,746,970	1,739,396	1,750,509	1,741,935	25.0%	25.0%	25.0%	25.0%	1,744,703
2019	24	1,841,516	1,816,296	1,855,755	1,827,462	0.0%	0.0%	50.0%	50.0%	1,841,608
2020	12	1,897,487	1,829,829	1,944,009	1,877,128	0.0%	0.0%	50.0%	50.0%	1,910,569
Totals		14,401,045	14,289,034	14,466,298	14,350,890					14,406,172

Table C.2. Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Private Passenger Auto
Total Unpaid Reconciliation as of December 31, 2020

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2012	108	1,213,083	1,214,471	1,218,574	1,388	4,103	5,491	1,218,574	4,103	5,491
2013	96	1,367,086	1,369,955	1,376,069	2,869	6,114	8,983	1,376,069	6,114	8,983
2014	84	1,421,777	1,427,920	1,439,420	6,143	11,500	17,643	1,439,420	11,500	17,643
2015	72	1,526,896	1,538,117	1,560,133	11,221	22,016	33,237	1,560,133	22,016	33,237
2016	60	1,576,653	1,604,722	1,647,802	28,069	43,080	71,149	1,647,802	43,080	71,149
2017	48	1,520,024	1,584,626	1,667,295	64,602	82,669	147,271	1,667,295	82,669	147,271
2018	36	1,448,927	1,583,503	1,744,703	134,576	161,200	295,776	1,744,703	161,200	295,776
2019	24	1,284,015	1,535,603	1,841,608	251,588	306,005	557,593	1,841,608	306,005	557,593
2020	12	775,576	1,238,406	1,910,569	462,830	672,163	1,134,993	1,910,569	672,163	1,134,993
Totals		12,134,037	13,097,323	14,406,172	963,286	1,308,849	2,272,135	14,406,172	1,308,849	2,272,135

Table C.3. Expected Incremental Development—Paid (Deterministic)

Sample Insurance Company
Private Passenger Auto—Paid Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										2,742	2,749	5,491
2013									2,783	3,097	3,104	8,983
2014								8,029	3,128	3,239	3,247	17,643
2015							13,923	8,893	3,390	3,511	3,519	33,237
2016						34,453	16,297	9,393	3,581	3,708	3,717	71,149
2017					73,449	36,693	16,490	9,504	3,623	3,752	3,761	147,271
2018				139,035	79,111	38,585	17,340	9,994	3,810	3,946	3,955	295,776
2019			237,853	152,195	84,565	41,245	18,536	10,683	4,073	4,218	4,227	557,593
2020		547,018	256,629	157,719	87,634	42,742	19,208	11,071	4,220	4,371	4,381	1,134,993

Table C.4. Expected Incremental Development—Incurred (Deterministic)

Sample Insurance Company
Private Passenger Auto—Incurred Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										2,050	2,053	4,103
2013									1,481	2,315	2,319	6,114
2014								5,322	1,331	2,421	2,425	11,500
2015							9,743	5,576	1,443	2,624	2,629	22,016
2016						21,433	8,685	5,890	1,524	2,772	2,776	43,080
2017					40,949	19,818	8,788	5,959	1,542	2,805	2,809	82,669
2018				76,014	41,204	20,892	9,264	6,282	1,626	2,957	2,962	161,200
2019			135,434	78,332	44,616	22,622	10,031	6,802	1,760	3,201	3,207	306,005
2020		361,322	130,571	82,786	47,153	23,908	10,601	7,189	1,860	3,383	3,389	672,163

Table C.5. Actual vs. Expected Back-Test (Deterministic)

Sample Insurance Company
Private Passenger Auto
Deterministic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2012	120	2,500	2,742	(242)	2,042	2,050	(8)
2013	108	3,485	2,783	702	2,261	1,481	780
2014	96	7,582	8,029	(447)	4,061	5,322	(1,261)
2015	84	13,765	13,923	(158)	8,076	9,743	(1,667)
2016	72	33,083	34,453	(1,370)	16,495	21,433	(4,938)
2017	60	75,969	73,449	2,520	35,496	40,949	(5,453)
2018	48	139,715	139,035	680	68,886	76,014	(7,128)
2019	36	234,781	237,853	(3,072)	119,582	135,434	(15,852)
2020	24	560,974	547,018	13,956	314,895	361,322	(46,427)
2021	12	764,210			1,205,957		
Totals		1,836,064			1,777,751		
AY < CY		1,071,854	1,059,284	12,569	571,794	653,748	(81,954)

Table C.6. Actual to Range of Estimates Back-Test (Deterministic)

Sample Insurance Company
Private Passenger Auto
Deterministic Actual vs. Method Range as of December 31, 2021

AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2012	120	2,500	2,742	2,744	–12977.0%	2,042	2,050	2,052	–332.1%
2013	108	3,485	2,574	2,993	217.7%	2,261	1,272	1,691	236.3%
2014	96	7,582	7,851	8,218	–73.5%	4,061	5,144	5,515	–291.9%
2015	84	13,765	12,402	15,469	44.5%	8,076	8,215	11,282	–4.5%
2016	72	33,083	32,601	36,307	13.0%	16,495	19,564	23,302	–82.1%
2017	60	75,969	71,579	75,753	105.2%	35,496	39,041	43,372	–81.8%
2018	48	139,715	134,970	143,551	55.3%	68,886	71,591	80,910	–29.0%
2019	36	234,781	222,411	249,543	45.6%	119,582	117,907	148,270	5.5%
2020	24	560,974	500,290	570,167	86.8%	314,895	308,639	389,322	7.8%
2021	12	764,210				1,205,957			
Totals		1,836,064				1,777,751			
AY < CY		1,071,854	987,421	1,104,745	72.0%	571,794	573,423	705,671	–1.2%

Table C.7. Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company
Private Passenger Auto
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	5,491	2,751	50.1%	19	16,929	1,188	3,538	5,318	19	7,256	10,281
2013	8,983	3,423	38.1%	(395)	27,201	3,633	6,557	8,844	13,467	11,195	14,917
2014	17,643	4,155	23.6%	5,353	34,375	11,018	14,771	17,448	14,798	20,330	24,790
2015	33,237	5,245	15.8%	15,269	60,704	24,910	29,619	33,085	32,036	36,639	42,225
2016	71,149	6,902	9.7%	48,314	99,369	60,123	66,324	71,033	72,699	75,783	82,763
2017	147,271	9,088	6.2%	114,275	187,688	132,806	141,043	147,027	142,651	153,290	162,219
2018	295,776	14,568	4.9%	244,570	348,069	272,495	285,945	295,225	281,357	305,146	320,628
2019	557,593	25,394	4.6%	457,369	651,838	516,980	540,414	556,720	552,490	574,475	599,860
2020	1,134,993	46,822	4.1%	973,312	1,337,053	1,062,388	1,102,616	1,132,386	1,181,722	1,165,441	1,216,110
Total	2,272,135	59,102	2.6%	2,064,755	2,479,344	2,177,063	2,231,575	2,270,627	2,295,340	2,311,669	2,371,532

Table C.8. Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company
Private Passenger Auto
Stochastic Estimates as of December 31, 2020
Estimated Paid Claims by Calendar Year

CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2021	1,076,388	31,344	2.9%	949,483	1,213,672	1,025,966	1,054,657	1,075,871	1,048,875	1,096,712	1,129,462
2022	551,046	19,390	3.5%	479,596	631,486	519,806	537,516	550,967	553,695	564,102	582,949
2023	311,957	13,916	4.5%	259,341	367,185	289,477	302,543	311,686	316,778	321,297	335,118
2024	163,631	9,937	6.1%	130,776	200,970	147,538	156,774	163,477	162,064	170,225	180,340
2025	80,988	7,270	9.0%	52,760	116,518	69,328	76,043	80,859	84,649	85,870	93,146
2026	40,653	5,645	13.9%	20,217	62,342	31,712	36,714	40,478	39,787	44,381	50,138
2027	22,548	4,548	20.2%	7,784	40,869	15,431	19,416	22,362	21,178	25,499	30,348
2028	12,196	3,877	31.8%	(166)	29,026	6,142	9,531	12,012	8,133	14,672	18,808
2029	8,412	3,700	44.0%	(121)	27,344	2,614	5,876	8,238	(121)	10,742	14,779
2030	4,316	2,311	53.6%	(50)	15,575	764	2,652	4,155	(50)	5,756	8,407
Total	2,272,135	59,102	2.6%	2,064,755	2,479,344	2,177,063	2,231,575	2,270,627	2,295,340	2,311,669	2,371,532

Table C.9. Mean Future Incremental—Paid (Stochastic)

Sample Insurance Company
Private Passenger Auto—Paid
Mean Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										2,733	2,758	5,491
2013									2,908	3,022	3,053	8,983
2014								8,098	3,080	3,226	3,239	17,643
2015							14,773	8,493	3,216	3,363	3,392	33,237
2016						35,326	15,895	9,164	3,479	3,614	3,670	71,149
2017					74,381	36,251	16,246	9,369	3,594	3,713	3,719	147,271
2018				140,849	78,253	38,124	17,114	9,886	3,733	3,891	3,925	295,776
2019			243,390	149,664	83,084	40,493	18,186	10,534	3,985	4,107	4,150	557,593
2020		553,931	253,630	155,843	86,574	42,317	19,004	10,953	4,164	4,262	4,316	1,134,993

Table C.10. Standard Deviation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Private Passenger Auto—Paid
Standard Deviation Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										1,534	1,543	2,751
2013									1,496	1,721	1,722	3,423
2014								2,135	1,567	1,785	1,763	4,155
2015							2,748	2,262	1,679	1,864	1,895	5,245
2016						4,154	2,887	2,321	1,745	1,952	1,988	6,902
2017					5,827	4,105	2,892	2,358	1,770	1,987	2,013	9,088
2018				8,864	6,479	4,403	3,076	2,516	1,860	2,084	2,091	14,568
2019			13,598	9,804	6,879	4,728	3,270	2,652	1,990	2,215	2,225	25,394
2020		25,362	14,095	10,125	7,121	4,866	3,297	2,703	2,032	2,275	2,311	46,822

Table C.11. Coefficient of Variation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Private Passenger Auto—Paid
CoV Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										56.1%	55.9%	50.1%
2013									51.4%	57.0%	56.4%	38.1%
2014								26.4%	50.9%	55.3%	54.4%	23.6%
2015							18.6%	26.6%	52.2%	55.4%	55.9%	15.8%
2016						11.8%	18.2%	25.3%	50.2%	54.0%	54.2%	9.7%
2017					7.8%	11.3%	17.8%	25.2%	49.3%	53.5%	54.1%	6.2%
2018				6.3%	8.3%	11.5%	18.0%	25.5%	49.8%	53.5%	53.3%	4.9%
2019			5.6%	6.6%	8.3%	11.7%	18.0%	25.2%	49.9%	53.9%	53.6%	4.6%
2020		4.6%	5.6%	6.5%	8.2%	11.5%	17.3%	24.7%	48.8%	53.4%	53.6%	4.1%

Table C.12. Estimated Unpaid Claims by Accident Year in 2021 (Stochastic)

Sample Insurance Company
Private Passenger Auto—Paid
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year, Calendar Year 2021 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	2,733	1,534	56.1%	9	9,689	444	1,629	2,563	9	3,697	5,509
2013	2,908	1,496	51.4%	(269)	10,441	750	1,873	2,750	(252)	3,766	5,640
2014	8,098	2,135	26.4%	1,608	20,022	4,867	6,616	7,934	8,649	9,413	11,850
2015	14,773	2,748	18.6%	6,175	26,858	10,506	12,878	14,607	13,421	16,523	19,567
2016	35,326	4,154	11.8%	19,713	52,817	28,828	32,396	35,169	36,788	38,033	42,514
2017	74,381	5,827	7.8%	52,662	98,238	65,082	70,380	74,239	70,540	78,233	84,209
2018	140,849	8,864	6.3%	105,135	178,702	126,665	134,837	140,706	140,360	146,614	155,792
2019	243,390	13,598	5.6%	189,263	302,308	221,056	234,122	243,174	238,506	252,536	266,186
2020	553,931	25,362	4.6%	462,086	667,072	513,991	536,419	553,004	547,742	570,306	597,839
Total	1,076,388	31,344	2.9%	949,483	1,213,672	1,025,966	1,054,657	1,075,871	1,048,875	1,096,712	1,129,462

Table C.13. Actual vs. Expected Back-Test and Conditional Reserve (Stochastic)

Sample Insurance Company
Private Passenger Auto
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2012	120	2,500	2,733	48.2%	2,042	2,056	56.7%	2,680	2,991	(311)
2013	108	3,485	2,908	69.4%	2,261	1,312	81.0%	7,248	5,498	1,750
2014	96	7,582	8,098	43.4%	4,061	5,207	33.2%	8,654	10,061	(1,406)
2015	84	13,765	14,773	37.5%	8,076	8,835	41.7%	15,635	19,472	(3,836)
2016	72	33,083	35,326	30.5%	16,495	20,439	15.6%	31,595	38,066	(6,470)
2017	60	75,969	74,381	61.4%	35,496	40,022	21.2%	73,359	71,302	2,057
2018	48	139,715	140,849	45.5%	68,886	74,159	25.6%	151,670	156,061	(4,390)
2019	36	234,781	243,390	26.5%	119,582	128,507	20.2%	292,882	322,812	(29,930)
2020	24	560,974	553,931	62.3%	314,895	350,974	2.9%	581,448	574,019	7,430
2021	12	764,210			1,205,957					
Totals		1,836,064			1,777,751			1,165,174	1,200,281	(35,107)
AY < CY		1,071,854	1,076,388	44.9%	571,794	631,511	0.6%	1,159,897	1,200,281	(40,385)

Figure C.1. KPI Thresholds by Accident Year—Paid (Stochastic)

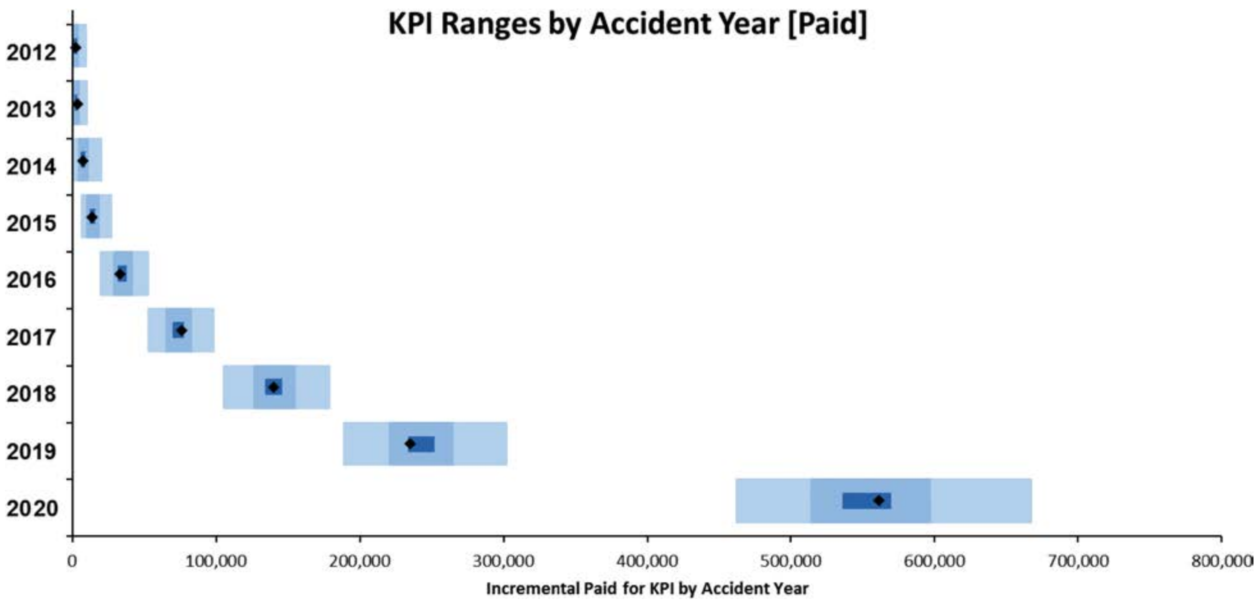


Figure C.2. KPI Thresholds by Calendar Year—Paid (Stochastic)

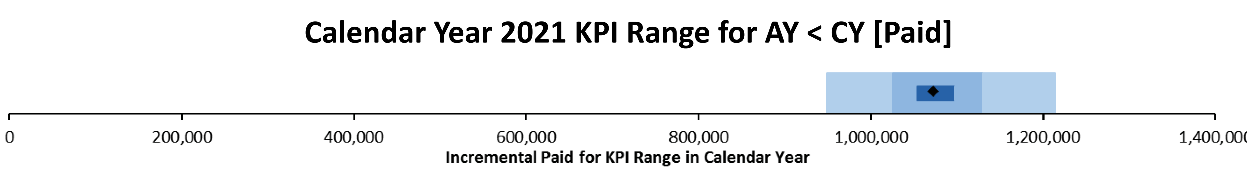


Figure C.3. KPI Thresholds by Accident Year—Incurred (Stochastic)

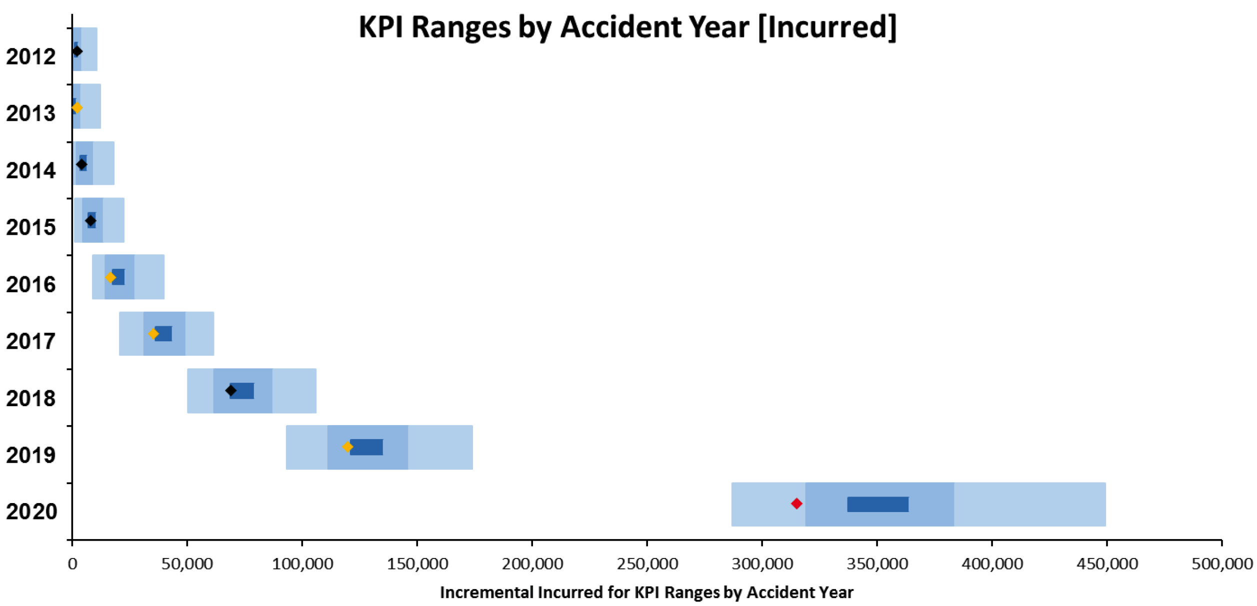


Figure C.4. KPI Thresholds by Calendar Year—Incurred (Stochastic)

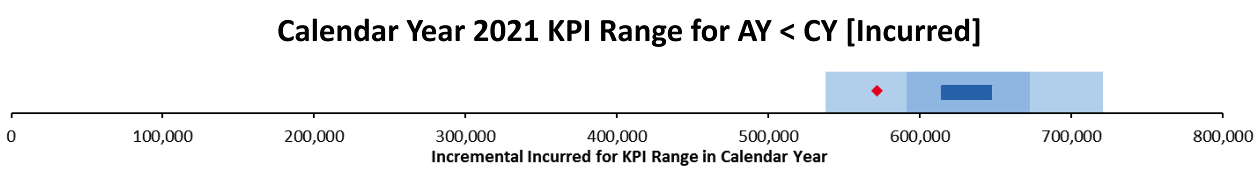


Figure C.5. Realized Values vs. Assumptions—Paid (Stochastic)

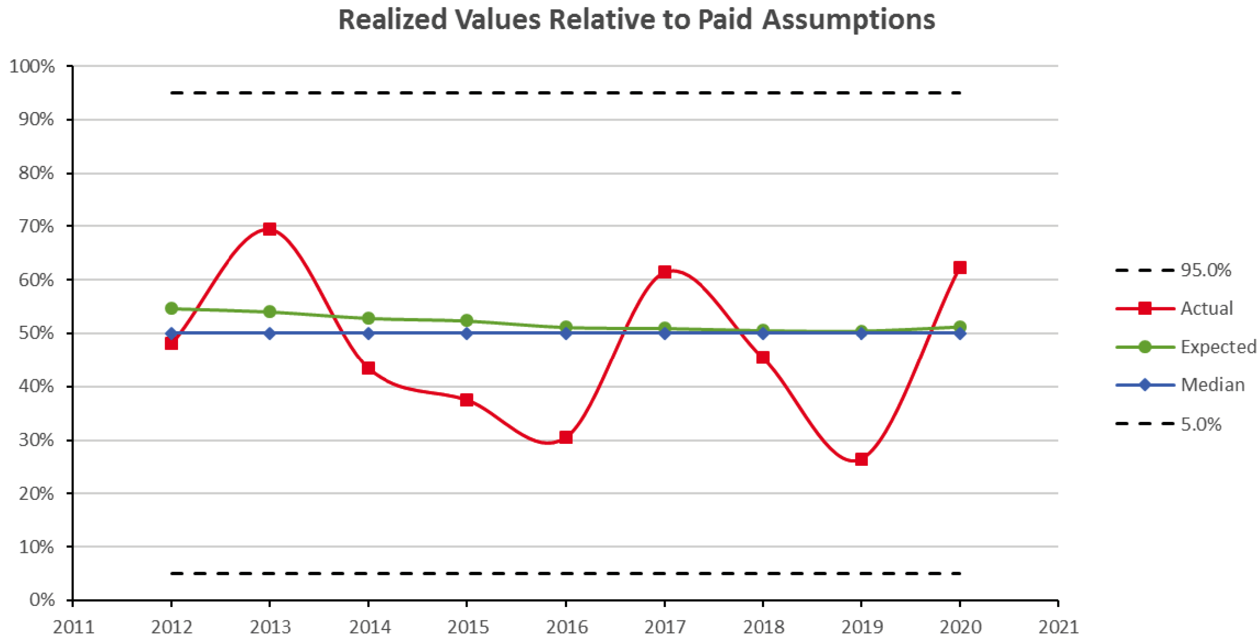
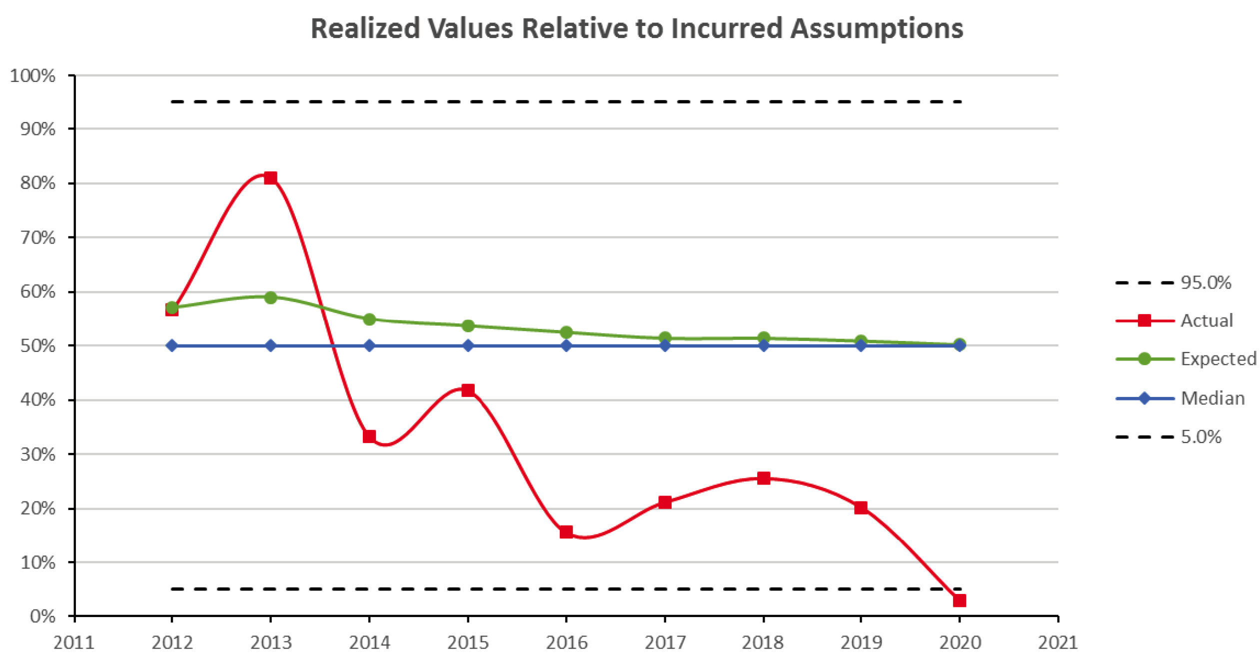


Figure C.6. Realized Values vs. Assumptions—Incurred (Stochastic)



Appendix D—Back-Testing Results for Commercial Auto

Table D.1. Calculation of Weighted Ultimate (Deterministic)

Sample Insurance Company
Commercial Auto
Calculation of Weighted Ultimate as of December 31, 2020

		Ultimate Values by Method				Weighted by Method				
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	Weighted Ultimate
2012	108	258,835	258,835	258,837	258,836	50.0%	50.0%	0.0%	0.0%	258,835
2013	96	267,103	271,591	267,143	271,592	50.0%	50.0%	0.0%	0.0%	269,347
2014	84	243,981	244,137	243,991	244,141	50.0%	50.0%	0.0%	0.0%	244,059
2015	72	267,942	269,784	267,999	269,783	50.0%	50.0%	0.0%	0.0%	268,863
2016	60	290,475	292,079	290,608	292,092	50.0%	50.0%	0.0%	0.0%	291,277
2017	48	288,645	288,592	288,785	288,669	50.0%	50.0%	0.0%	0.0%	288,618
2018	36	335,023	338,775	335,956	338,702	25.0%	25.0%	25.0%	25.0%	337,114
2019	24	333,220	337,698	333,662	336,635	0.0%	0.0%	50.0%	50.0%	335,149
2020	12	357,305	360,286	338,097	344,953	0.0%	0.0%	50.0%	50.0%	341,525
Totals		2,642,529	2,661,779	2,625,078	2,645,402					2,634,788

Table D.2. Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Commercial Auto
Total Unpaid Reconciliation as of December 31, 2020

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2012	108	257,689	258,524	258,835	835	311	1,146	258,835	311	1,146
2013	96	264,871	270,758	269,347	5,887	(1,411)	4,476	**271,500**	**742**	**6,629**
2014	84	240,300	242,171	244,059	1,871	1,888	3,759	244,059	1,888	3,759
2015	72	259,339	265,496	268,863	6,157	3,367	9,524	268,863	3,367	9,524
2016	60	270,525	281,376	291,277	10,851	9,901	20,752	291,277	9,901	20,752
2017	48	245,541	266,101	288,618	20,560	22,517	43,077	288,618	22,517	43,077
2018	36	240,652	282,394	337,114	41,742	54,720	96,462	337,114	54,720	96,462
2019	24	177,709	235,983	335,149	58,274	99,166	157,440	335,149	99,166	157,440
2020	12	105,547	177,611	341,525	72,064	163,914	235,978	341,525	163,914	235,978
Totals		2,062,173	2,280,414	2,634,788	218,241	354,374	572,615	2,636,941	356,527	574,768

Table D.3. Expected Incremental Development—Paid (Deterministic)

Sample Insurance Company
Commercial Auto—Paid Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										572	574	1,146
2013									4,863	882	884	6,629
2014								1,720	959	540	541	3,759
2015							5,468	1,810	1,056	595	596	9,524
2016						11,401	4,957	1,961	1,144	644	646	20,752
2017					23,255	10,556	4,912	1,943	1,134	638	640	43,077
2018				45,941	27,285	12,374	5,758	2,277	1,329	748	750	96,462
2019			62,890	44,425	27,071	12,277	5,712	2,259	1,319	742	744	157,440
2020		80,388	61,679	44,125	26,889	12,194	5,674	2,244	1,310	737	739	235,978

Table D.4. Expected Incremental Development – Incurred (Deterministic)

Sample Insurance Company
Commercial Auto — Incurred Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										155	156	311
2013									912	(85)	(85)	742
2014								1,140	455	147	147	1,888
2015							1,202	1,341	502	161	162	3,367
2016						5,271	2,284	1,452	544	175	175	9,901
2017					11,941	5,989	2,263	1,439	539	173	173	22,517
2018				28,462	13,911	6,991	2,642	1,680	629	202	202	54,720
2019			43,797	29,442	13,736	6,903	2,609	1,659	621	200	200	99,166
2020		65,492	44,040	28,917	13,491	6,780	2,562	1,629	610	196	196	163,914

Table D.5. Actual vs. Expected Back-Test (Deterministic)

Sample Insurance Company
Commercial Auto
Deterministic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2012	120	543	572	(29)	(47)	155	(202)
2013	108	2,387	4,863	(2,476)	1,040	912	128
2014	96	1,177	1,720	(543)	851	1,140	(289)
2015	84	5,403	5,468	(65)	2,954	1,202	1,752
2016	72	14,120	11,401	2,719	9,035	5,271	3,764
2017	60	23,636	23,255	381	16,524	11,941	4,583
2018	48	51,020	45,941	5,079	36,454	28,462	7,992
2019	36	75,813	62,890	12,923	61,541	43,797	17,744
2020	24	88,832	80,388	8,444	83,154	65,492	17,662
2021	12	99,123			178,539		
Totals		362,054			390,045		
AY < CY		262,931	236,497	26,434	211,506	158,372	53,134

Table D.6. Actual to Range of Estimates Back-Test (Deterministic)

Sample Insurance Company
Commercial Auto
Deterministic Actual vs. Method Range as of December 31, 2021

AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2012	120	543	572	573	–1947.6%	(47)	155	157	–11482.4%
2013	108	2,387	2,629	7,097	–5.4%	1,040	(1,329)	3,154	52.8%
2014	96	1,177	1,642	1,797	–300.2%	851	1,062	1,220	–133.1%
2015	84	5,403	4,560	6,375	46.4%	2,954	288	2,116	145.9%
2016	72	14,120	10,624	12,177	225.1%	9,035	4,482	6,067	287.2%
2017	60	23,636	23,230	23,355	323.6%	16,524	11,915	12,068	3013.1%
2018	48	51,020	44,341	47,533	209.3%	36,454	26,520	29,980	287.1%
2019	36	75,813	61,648	64,865	440.3%	61,541	41,780	45,513	529.3%
2020	24	88,832	78,521	86,597	127.7%	83,154	63,052	74,156	181.0%
2021	12	99,123				178,539			
Totals		362,054				390,045			
AY < CY		262,931	228,631	250,242	158.7%	211,506	149,974	174,267	253.3%

Table D.7. Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company
Commercial Auto
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	1,146	814	71.0%	(10)	5,794	78	535	1,001	(10)	1,614	2,674
2013	6,629	1,224	18.5%	4,226	12,888	4,900	5,718	6,480	5,217	7,369	8,901
2014	3,759	1,453	38.6%	301	11,438	1,635	2,703	3,633	2,931	4,649	6,345
2015	9,524	2,142	22.5%	3,182	20,485	6,275	8,015	9,377	10,379	10,869	13,349
2016	20,752	3,200	15.4%	10,281	35,184	15,708	18,540	20,585	18,785	22,831	26,235
2017	43,077	4,575	10.6%	26,937	64,990	35,935	39,920	42,912	45,008	46,064	50,902
2018	96,462	8,635	9.0%	64,159	131,809	82,929	90,631	96,052	94,959	101,869	111,214
2019	157,440	14,252	9.1%	106,918	218,146	134,900	147,693	157,063	161,109	166,699	181,556
2020	235,978	20,115	8.5%	165,204	320,049	204,296	222,059	235,235	228,038	249,252	269,810
Total	574,768	27,218	4.7%	472,897	687,879	530,792	556,111	574,426	558,264	592,649	620,040

Table D.8. Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company
Commercial Auto
Stochastic Estimates as of December 31, 2020
Estimated Paid Claims by Calendar Year

CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2021	232,199	12,743	5.5%	186,133	286,448	211,733	223,345	231,854	239,707	240,793	253,653
2022	155,214	10,078	6.5%	123,220	202,461	138,975	148,466	154,950	152,408	161,829	172,239
2023	94,488	7,627	8.1%	67,914	124,583	82,240	89,213	94,253	97,115	99,485	107,381
2024	49,452	5,311	10.7%	33,520	73,129	40,823	45,820	49,320	49,423	52,929	58,355
2025	22,776	3,557	15.6%	10,658	37,548	17,087	20,273	22,624	21,106	25,137	28,853
2026	10,624	2,554	24.0%	2,401	21,272	6,697	8,827	10,460	11,167	12,231	15,060
2027	4,974	1,804	36.3%	522	13,768	2,328	3,680	4,783	5,419	6,057	8,218
2028	2,823	1,412	50.0%	(123)	11,759	872	1,773	2,649	2,360	3,651	5,416
2029	1,476	950	64.4%	8	7,844	222	771	1,325	8	2,002	3,244
2030	741	621	83.8%	4	4,737	28	275	596	4	1,045	1,956
Total	574,768	27,218	4.7%	472,897	687,879	530,792	556,111	574,426	558,264	592,649	620,040

Table D.9. Mean Future Incremental—Paid (Stochastic)

Sample Insurance Company
Commercial Auto—Paid
Mean Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										571	575	1,146
2013									3,131	1,735	1,763	6,629
2014								1,665	983	557	555	3,759
2015							5,044	1,988	1,170	657	666	9,524
2016						11,061	5,146	2,028	1,189	658	672	20,752
2017					23,276	10,564	4,895	1,925	1,135	636	646	43,077
2018				45,272	27,668	12,508	5,837	2,304	1,348	757	768	96,462
2019			62,481	44,600	27,194	12,354	5,746	2,265	1,308	744	746	157,440
2020		79,698	61,955	44,373	26,936	12,267	5,703	2,264	1,311	730	741	235,978

Table D.10. Standard Deviation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Commercial Auto—Paid
Standard Deviation Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										515	519	814
2013									881	534	538	1,224
2014								908	826	500	500	1,453
2015							1,465	990	879	523	533	2,142
2016						2,208	1,565	1,042	912	547	559	3,200
2017					3,189	2,197	1,559	1,027	908	563	556	4,575
2018				5,203	3,869	2,573	1,795	1,181	1,062	626	625	8,635
2019			7,006	5,566	4,081	2,625	1,792	1,197	1,056	629	634	14,252
2020		8,276	6,947	5,516	4,013	2,599	1,783	1,182	1,064	623	621	20,115

Table D.11. Coefficient of Variation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Commercial Auto—Paid
CoV Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										90.1%	90.2%	71.0%
2013									28.2%	30.8%	30.5%	18.5%
2014								54.6%	84.0%	89.8%	90.1%	38.6%
2015							29.0%	49.8%	75.2%	79.6%	80.0%	22.5%
2016						20.0%	30.4%	51.4%	76.7%	83.2%	83.2%	15.4%
2017					13.7%	20.8%	31.8%	53.4%	80.0%	88.5%	86.1%	10.6%
2018				11.5%	14.0%	20.6%	30.7%	51.3%	78.8%	82.7%	81.3%	9.0%
2019			11.2%	12.5%	15.0%	21.2%	31.2%	52.8%	80.8%	84.5%	84.9%	9.1%
2020		10.4%	11.2%	12.4%	14.9%	21.2%	31.3%	52.2%	81.2%	85.4%	83.8%	8.5%

Table D.12. Estimated Unpaid Claims by Accident Year in 2021 (Stochastic)

Sample Insurance Company
Commercial Auto—Paid
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year, Calendar Year 2021 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	571	515	90.1%	(5)	4,550	7	182	441	(5)	813	1,573
2013	3,131	881	28.2%	1,923	8,619	2,052	2,457	2,966	2,052	3,634	4,804
2014	1,665	908	54.6%	47	6,639	440	990	1,522	1,421	2,191	3,355
2015	5,044	1,465	29.0%	1,265	11,797	2,893	3,975	4,902	5,069	5,945	7,666
2016	11,061	2,208	20.0%	4,960	20,538	7,667	9,509	10,915	10,312	12,486	14,886
2017	23,276	3,189	13.7%	13,209	37,472	18,316	21,040	23,131	21,086	25,331	28,725
2018	45,272	5,203	11.5%	28,879	68,025	37,212	41,731	44,991	42,206	48,538	54,277
2019	62,481	7,006	11.2%	36,066	90,980	51,265	57,668	62,265	61,583	67,022	74,418
2020	79,698	8,276	10.4%	49,321	113,281	66,688	74,012	79,329	73,977	85,090	93,641
Total	232,199	12,743	5.5%	186,133	286,448	211,733	223,345	231,854	239,707	240,793	253,653

Table D.13. Actual vs. Expected Back-Test and Conditional Reserve (Stochastic)

Sample Insurance Company
Commercial Auto
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2012	120	543	571	57.9%	(47)	154	0.0%	643	603	40
2013	108	2,387	3,131	21.8%	1,040	448	82.8%	3,257	4,242	(985)
2014	96	1,177	1,665	33.5%	851	1,167	44.5%	1,675	2,582	(907)
2015	84	5,403	5,044	63.1%	2,954	1,669	86.1%	5,593	4,121	1,472
2016	72	14,120	11,061	91.1%	9,035	5,606	94.2%	13,946	6,632	7,313
2017	60	23,636	23,276	56.1%	16,524	11,960	93.9%	20,073	19,441	632
2018	48	51,020	45,272	86.7%	36,454	29,103	92.7%	57,978	45,442	12,536
2019	36	75,813	62,481	96.5%	61,541	44,392	99.3%	110,701	81,627	29,075
2020	24	88,832	79,698	86.1%	83,154	66,555	97.0%	170,589	147,146	23,442
2021	12	99,123			178,539					
Totals		362,054			390,045			384,456	311,837	72,619
AY < CY		262,931	232,199	98.9%	211,506	161,054	100.0%	390,213	311,837	78,376

Figure D.1. Graph of KPI Thresholds by Accident Year—Paid (Stochastic)

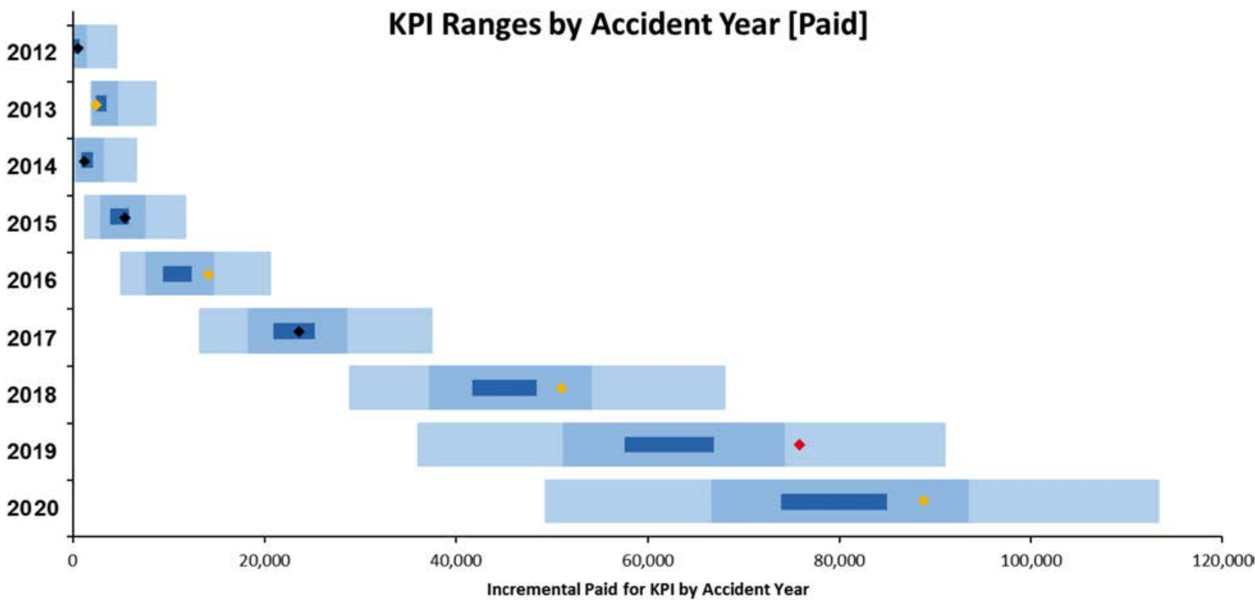


Figure D.2. Graph of KPI Thresholds by Calendar Year—Paid (Stochastic)

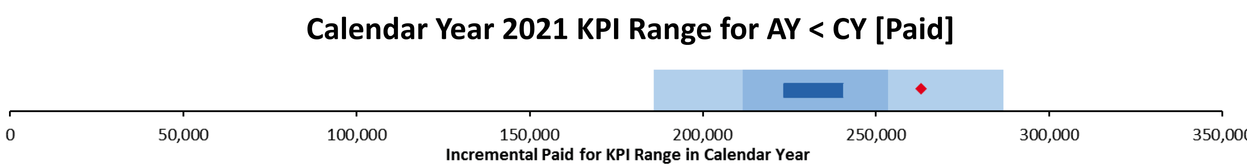


Figure D.3. Graph of KPI Thresholds by Accident Year—Incurred (Stochastic)

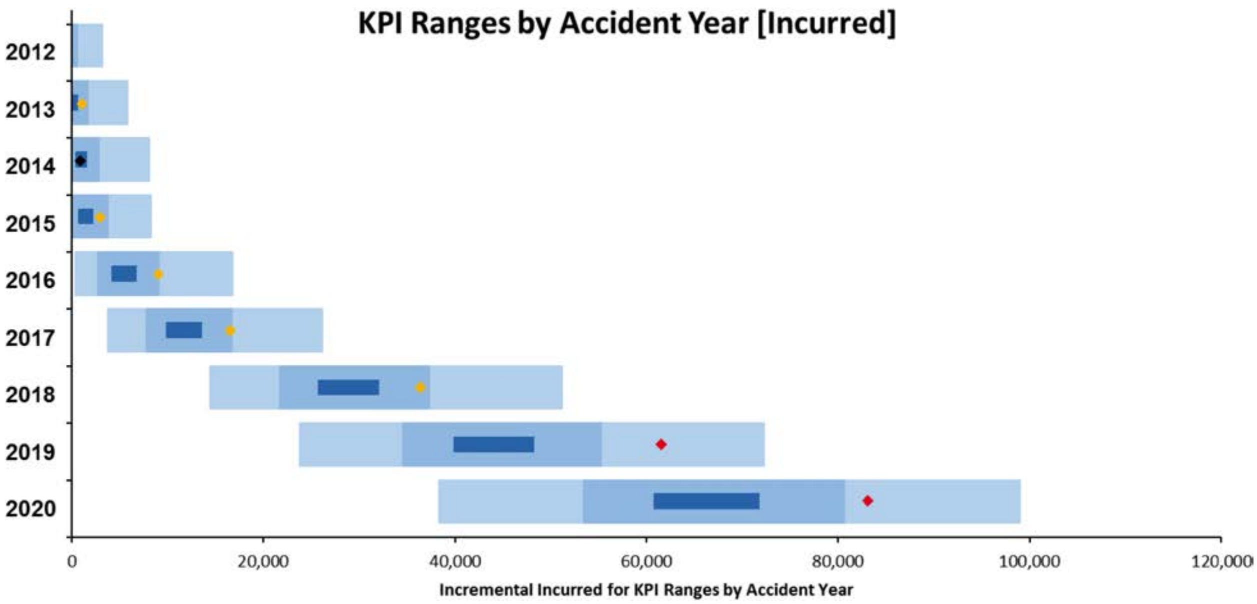


Figure D.4. Graph of KPI Thresholds by Calendar Year—Incurred (Stochastic)

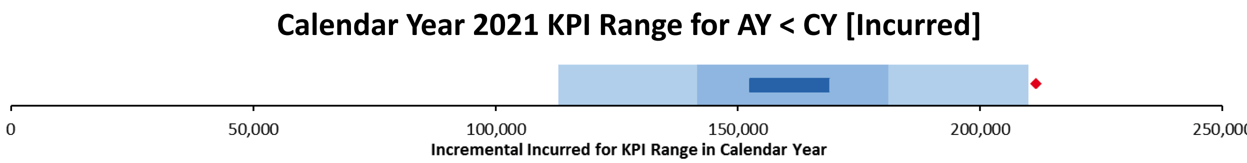


Figure D.5. Graph of Realized Values vs. Assumptions—Paid (Stochastic)

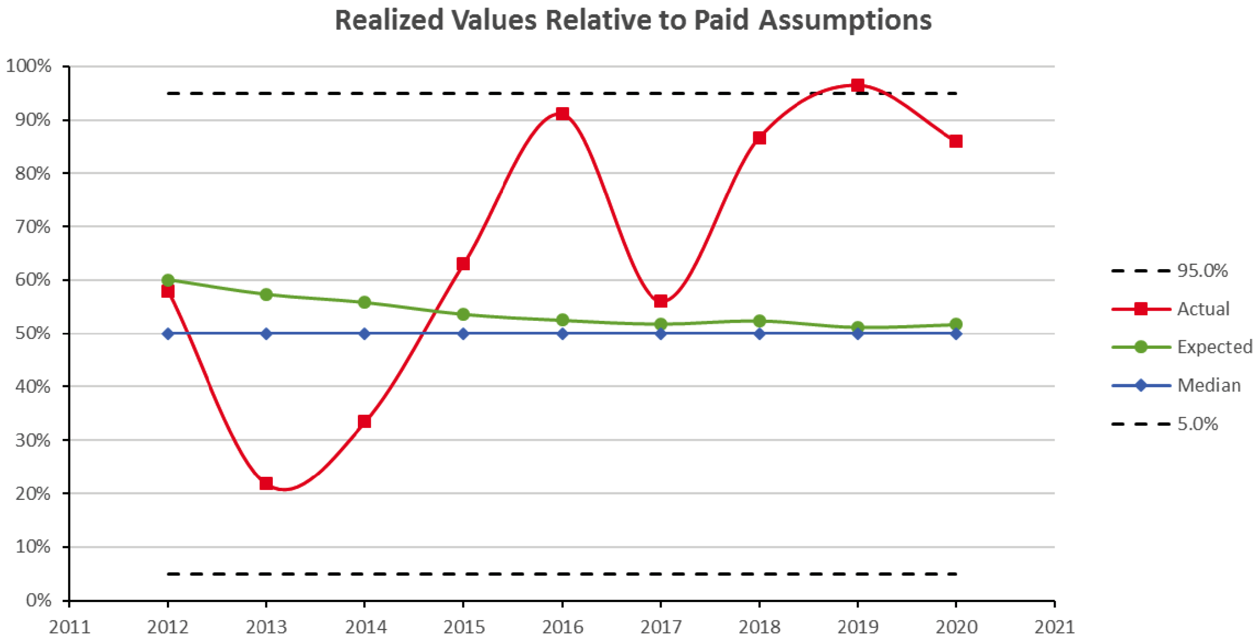
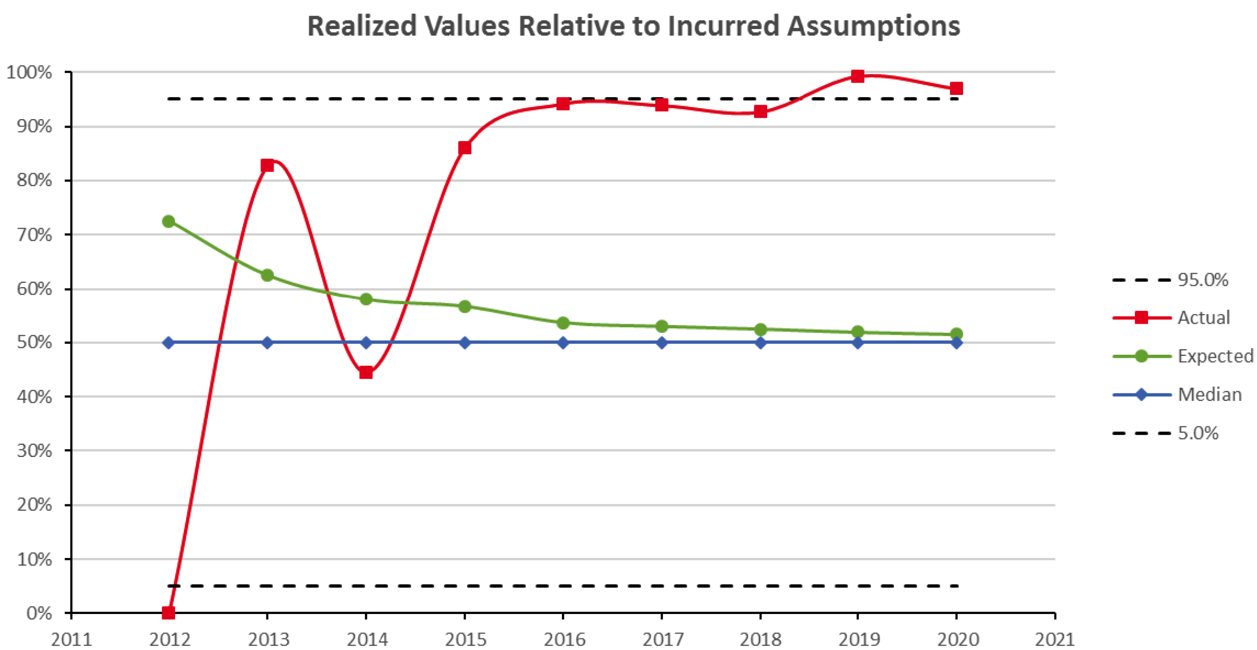


Figure D.6. Graph of Realized Values vs. Assumptions—Incurred (Stochastic)



Appendix E—Back-Testing Results for Homeowners

Table E.1. Calculation of Weighted Ultimate (Deterministic)

Sample Insurance Company
Homeowners
Calculation of Weighted Ultimate as of December 31, 2020

		Ultimate Values by Method				Weights by Method				Weighted
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	Ultimate
2012	108	328,806	328,806	328,806	328,806	50.0%	50.0%	0.0%	0.0%	328,806
2013	96	423,382	422,484	423,380	422,484	50.0%	50.0%	0.0%	0.0%	422,933
2014	84	542,749	542,575	542,751	542,574	50.0%	50.0%	0.0%	0.0%	542,662
2015	72	551,124	549,747	551,123	549,745	50.0%	50.0%	0.0%	0.0%	550,435
2016	60	680,803	678,422	680,808	678,412	50.0%	50.0%	0.0%	0.0%	679,612
2017	48	758,487	757,002	758,506	756,997	50.0%	50.0%	0.0%	0.0%	757,744
2018	36	702,481	700,796	702,653	700,788	25.0%	25.0%	25.0%	25.0%	701,679
2019	24	801,498	797,111	801,473	797,161	0.0%	0.0%	50.0%	50.0%	799,317
2020	12	992,257	996,379	993,794	996,481	0.0%	0.0%	50.0%	50.0%	995,137
Totals		5,781,585	5,773,322	5,783,294	5,773,446					5,778,327

Table E.2. Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Homeowners
Total Unpaid Reconciliation as of December 31, 2020

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2012	108	328,033	328,901	328,806	868	(95)	773	328,806	(95)	773
2013	96	422,179	422,654	422,933	475	279	754	422,933	279	754
2014	84	540,795	543,199	542,662	2,404	(537)	1,867	542,662	(537)	1,867
2015	72	548,818	550,729	550,435	1,911	(294)	1,617	550,435	(294)	1,617
2016	60	675,472	680,658	679,612	5,186	(1,046)	4,140	679,612	(1,046)	4,140
2017	48	745,388	758,597	757,744	13,209	(853)	12,356	757,744	(853)	12,356
2018	36	680,014	701,622	701,679	21,608	57	21,665	701,679	57	21,665
2019	24	748,862	787,351	799,317	38,489	11,966	50,455	799,317	11,966	50,455
2020	12	723,126	930,676	995,137	207,550	64,461	272,011	995,137	64,461	272,011
Totals		5,412,687	5,704,387	5,778,327	291,700	73,940	365,640	5,778,327	73,940	365,640

Table E.3. Expected Incremental Development—Paid (Deterministic)

Sample Insurance Company
Homeowners—Paid Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										386	387	773
2013									(240)	497	497	754
2014								325	266	638	638	1,867
2015							(364)	418	270	647	647	1,617
2016						1,297	397	516	333	798	799	4,140
2017					6,423	2,763	443	575	371	890	891	12,356
2018				9,503	6,648	2,568	412	535	345	827	828	21,665
2019			24,902	11,755	7,541	2,913	467	607	391	939	940	50,455
2020		206,388	33,665	14,702	9,432	3,643	584	759	489	1,174	1,175	272,011

Table E.4. Expected Incremental Development—Incurred (Deterministic)

Sample Insurance Company
Homeowners—Incurred Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										(48)	(47)	(95)
2013									401	(61)	(61)	279
2014								(319)	(61)	(78)	(78)	(537)
2015							340	(412)	(62)	(80)	(80)	(294)
2016						169	(432)	(509)	(76)	(98)	(98)	(1,046)
2017					1,645	(1,143)	(482)	(568)	(85)	(109)	(109)	(853)
2018				1,543	839	(1,064)	(449)	(528)	(79)	(102)	(102)	57
2019			12,913	745	955	(1,212)	(511)	(602)	(90)	(116)	(116)	11,966
2020		52,259	13,378	925	1,185	(1,504)	(634)	(747)	(112)	(144)	(144)	64,461

Table E.5. Actual vs. Expected Back-Test (Deterministic)

Sample Insurance Company
Homeowners
Deterministic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2012	120	26	386	(360)	(132)	(48)	(84)
2013	108	33	(240)	273	(156)	401	(557)
2014	96	227	325	(98)	(1,359)	(319)	(1,040)
2015	84	(176)	(364)	188	(1,158)	340	(1,498)
2016	72	3,800	1,297	2,503	412	169	243
2017	60	5,462	6,423	(961)	(8)	1,645	(1,653)
2018	48	12,197	9,503	2,694	1,284	1,543	(259)
2019	36	23,840	24,902	(1,062)	8,785	12,913	(4,128)
2020	24	191,678	206,388	(14,710)	56,168	52,259	3,909
2021	12	934,805			1,143,739		
Totals		1,171,892			1,207,575		
AY < CY		237,087	248,619	(11,532)	63,836	68,902	(5,066)

Table E.6. Actual to Range of Estimates Back-Test (Deterministic)

Sample Insurance Company
Homeowners
Deterministic Actual vs. Method Range as of December 31, 2021

AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2012	120	26	386	386	–143771.0%	(132)	(48)	(47)	–33682.3%
2013	108	33	(688)	207	80.5%	(156)	(48)	850	–12.1%
2014	96	227	235	413	–4.6%	(1,359)	(407)	(229)	–534.5%
2015	84	(176)	(1,051)	322	63.7%	(1,158)	(350)	1,030	–58.5%
2016	72	3,800	99	2,485	155.1%	412	(1,028)	1,372	60.0%
2017	60	5,462	5,673	7,170	–14.1%	(8)	900	2,417	–59.9%
2018	48	12,197	8,582	10,415	197.2%	1,284	650	2,526	33.8%
2019	36	23,840	22,756	27,002	25.5%	8,785	10,700	15,091	–43.6%
2020	24	191,678	203,968	207,819	–319.1%	56,168	49,431	53,586	162.1%
2021	12	934,805				1,143,739			
Totals		1,171,892				1,207,575			
AY < CY		237,087	243,694	253,519	–67.2%	63,836	63,878	73,919	–0.4%

Table E.7. Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company
Homeowners
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	773	920	119.1%	(18)	7,510	(16)	121	459	(18)	1,101	2,668
2013	754	1,334	176.9%	(2,345)	11,715	(831)	(164)	445	(446)	1,359	3,384
2014	1,867	1,847	98.9%	(2,791)	15,138	(541)	573	1,534	1,422	2,847	5,402
2015	1,617	1,975	122.1%	(4,363)	14,310	(989)	206	1,315	921	2,700	5,238
2016	4,140	2,932	70.8%	(4,812)	24,814	9	2,020	3,791	1,561	5,885	9,480
2017	12,356	4,435	35.9%	404	35,123	5,775	9,158	11,996	12,056	15,160	20,191
2018	21,665	5,686	26.2%	5,673	46,724	13,069	17,642	21,254	23,445	25,267	31,717
2019	50,455	9,708	19.2%	23,208	98,051	35,582	43,515	49,808	41,265	56,737	67,307
2020	272,011	30,285	11.1%	176,947	402,593	224,048	250,890	271,241	293,093	291,855	323,755
Total	365,640	33,369	9.1%	247,985	505,728	312,138	342,419	364,523	360,985	387,991	421,695

Table E.8. Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company
Homeowners
Stochastic Estimates as of December 31, 2020
Estimated Paid Claims by Calendar Year

CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2021	252,049	25,430	10.1%	171,900	348,486	211,598	234,404	251,252	261,859	269,070	294,959
2022	55,570	9,158	16.5%	29,368	103,028	41,386	49,232	55,076	52,236	61,369	71,445
2023	26,772	6,387	23.9%	7,593	56,696	17,092	22,144	26,470	27,827	30,888	37,890
2024	14,401	4,923	34.2%	333	38,744	7,102	10,932	13,965	13,221	17,409	23,173
2025	6,241	3,422	54.8%	(2,952)	24,140	1,334	3,813	5,881	5,630	8,306	12,436
2026	3,212	2,583	80.4%	(4,367)	18,449	(318)	1,383	2,867	2,281	4,693	7,986
2027	2,735	2,471	90.3%	(5,722)	17,438	(656)	1,006	2,423	770	4,070	7,339
2028	2,318	2,271	98.0%	(3,834)	15,984	(819)	769	1,965	1,163	3,562	6,552
2029	2,340	1,852	79.1%	0	18,642	155	940	1,938	—	3,281	5,981
Total	365,640	33,369	9.1%	247,985	505,728	312,138	342,419	364,523	360,985	387,991	421,695

Table E.9. Mean Future Incremental—Paid (Stochastic)

Sample Insurance Company
Homeowners—Paid
Mean Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	Total
2012										773	773
2013									125	629	754
2014								414	237	1,215	1,867
2015							217	293	205	903	1,617
2016						1,911	319	403	259	1,248	4,140
2017					6,758	2,604	416	545	348	1,685	12,356
2018				9,961	6,391	2,487	402	503	333	1,588	21,665
2019			25,830	11,299	7,304	2,814	459	585	373	1,792	50,455
2020		206,060	33,797	14,743	9,478	3,682	608	775	527	2,340	272,011

Table E.10. Standard Deviation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Homeowners—Paid
Standard Deviation Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	Total
2012										920	920
2013									831	1,054	1,334
2014								952	995	1,243	1,847
2015							704	934	1,030	1,236	1,975
2016						1,805	844	1,062	1,187	1,397	2,932
2017					3,045	1,966	892	1,170	1,287	1,508	4,435
2018				3,658	2,927	1,919	867	1,092	1,236	1,419	5,686
2019			6,340	4,080	3,298	2,086	951	1,234	1,378	1,574	9,708
2020		24,137	7,203	4,746	3,852	2,459	1,138	1,508	1,636	1,852	30,285

Table E.11. Coefficient of Variation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Homeowners—Paid
CoV Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	Total
2012										119.1%	119.1%
2013									665.2%	167.5%	176.9%
2014								229.9%	419.4%	102.3%	98.9%
2015							324.5%	318.6%	503.5%	136.9%	122.1%
2016						94.4%	264.4%	263.5%	458.1%	112.0%	70.8%
2017					45.1%	75.5%	214.7%	214.7%	369.8%	89.5%	35.9%
2018				36.7%	45.8%	77.2%	215.6%	217.1%	370.6%	89.4%	26.2%
2019			24.5%	36.1%	45.2%	74.1%	207.1%	210.9%	370.0%	87.9%	19.2%
2020		11.7%	21.3%	32.2%	40.6%	66.8%	187.1%	194.6%	310.6%	79.1%	11.1%

Table E.12. Estimated Unpaid Claims by Accident Year in 2021 (Stochastic)

Sample Insurance Company
Homeowners—Paid
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year, Calendar Year 2021 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	773	920	119.1%	(18)	7,510	(16)	121	459	(18)	1,101	2,668
2013	125	831	665.2%	(1,973)	6,958	(1,083)	(157)	(63)	(74)	294	1,701
2014	414	952	229.9%	(2,175)	9,496	(742)	(26)	118	(26)	693	2,285
2015	217	704	324.5%	(1,892)	9,688	(523)	(96)	(27)	(96)	360	1,645
2016	1,911	1,805	94.4%	(2,885)	14,491	(317)	565	1,550	(564)	2,884	5,331
2017	6,758	3,045	45.1%	47	22,789	2,482	4,544	6,378	4,282	8,579	12,327
2018	9,961	3,658	36.7%	1,207	28,737	4,701	7,304	9,587	9,740	12,199	16,585
2019	25,830	6,340	24.5%	8,694	52,980	16,319	21,257	25,371	19,688	29,857	37,189
2020	206,060	24,137	11.7%	132,533	295,967	167,429	189,609	205,307	200,574	221,714	247,353
Total	252,049	25,430	10.1%	171,900	348,486	211,598	234,404	251,252	261,859	269,070	294,959

Table E.13. Actual vs. Expected Back-Test and Conditional Reserve (Stochastic)

Sample Insurance Company
Homeowners
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2012	120	26	773	13.9%	(132)	(95)	83.5%	—	747	(747)
2013	108	33	125	61.9%	(156)	59	31.4%	164	721	(557)
2014	96	227	414	57.2%	(1,359)	(349)	23.5%	1,367	1,640	(272)
2015	84	(176)	217	14.1%	(1,158)	(105)	18.5%	(1,153)	1,793	(2,946)
2016	72	3,800	1,911	85.6%	412	(482)	67.2%	3,722	340	3,381
2017	60	5,462	6,758	37.5%	(8)	1,119	12.2%	3,979	6,894	(2,915)
2018	48	12,197	9,961	74.9%	1,284	813	81.4%	12,839	9,468	3,370
2019	36	23,840	25,830	40.5%	8,785	12,274	37.9%	21,590	26,615	(5,024)
2020	24	191,678	206,060	28.0%	56,168	52,293	62.7%	59,458	80,333	(20,875)
2021	12	934,805			1,143,739					
Totals		1,171,892			1,207,575			101,967	128,553	(26,586)
AY < CY		237,087	252,049	28.4%	63,836	65,528	50.2%	96,676	128,553	(31,876)

Figure E.1. KPI Thresholds by Accident Year—Paid (Stochastic)

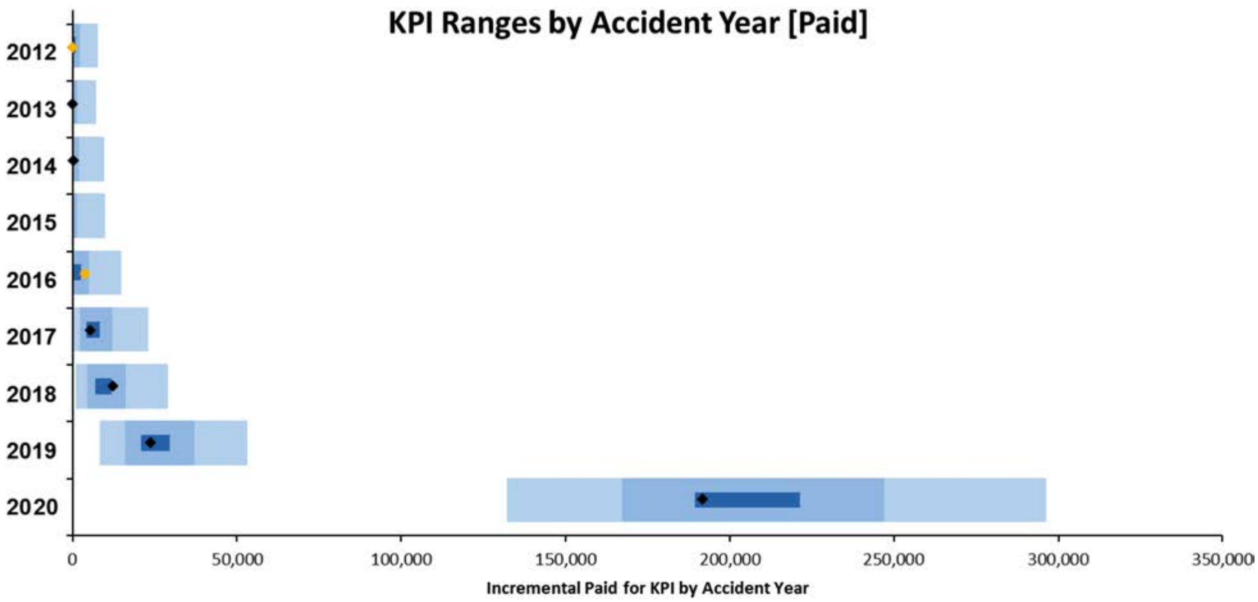


Figure E.2. KPI Thresholds by Calendar Year—Paid (Stochastic)

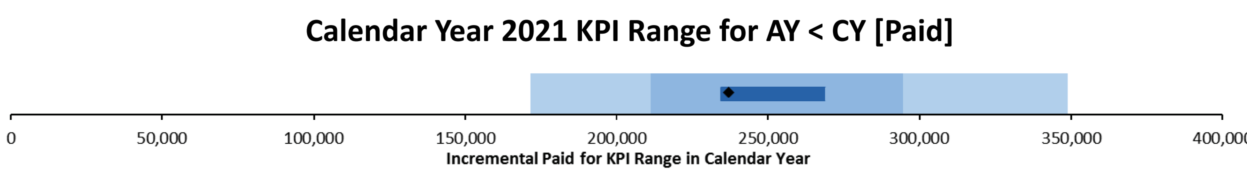


Figure E.3. KPI Thresholds by Accident Year—Incurred (Stochastic)

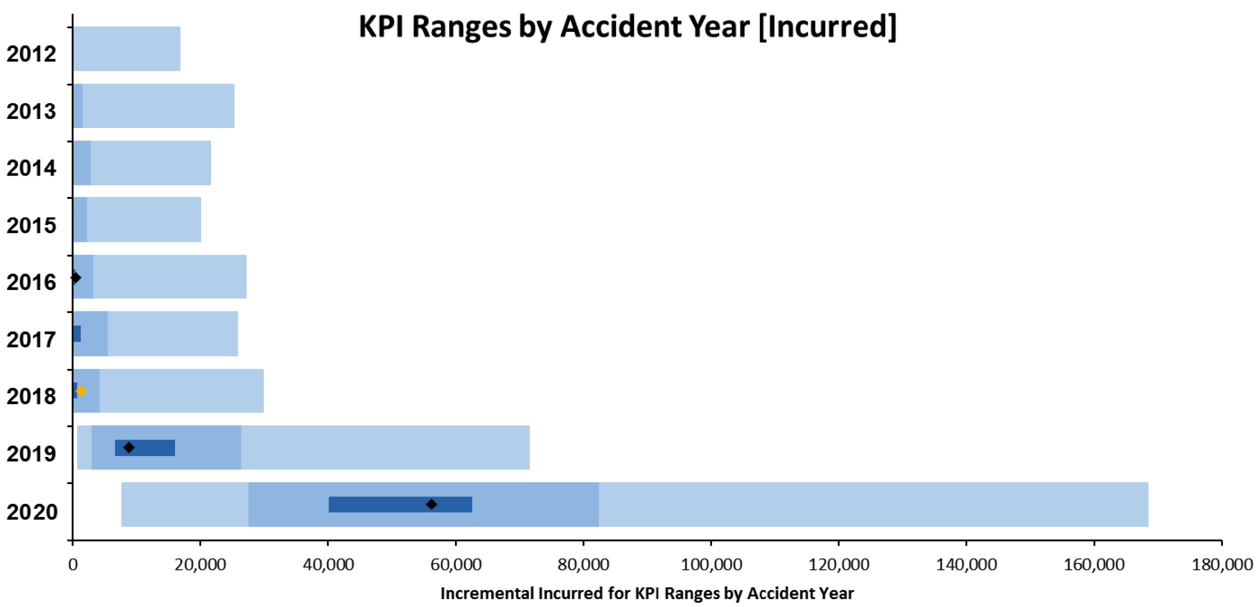


Figure E.4. KPI Thresholds by Calendar Year—Incurred (Stochastic)

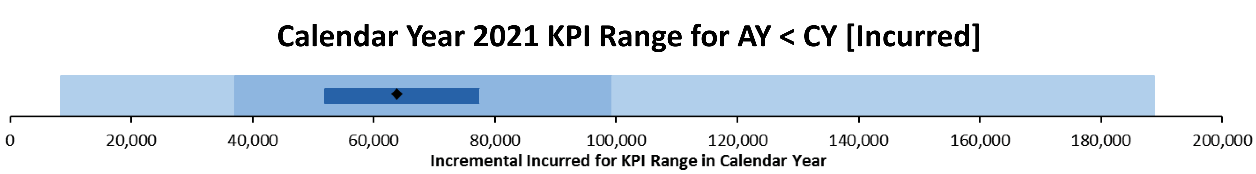


Figure E.5. Realized Values vs. Assumptions—Paid (Stochastic)

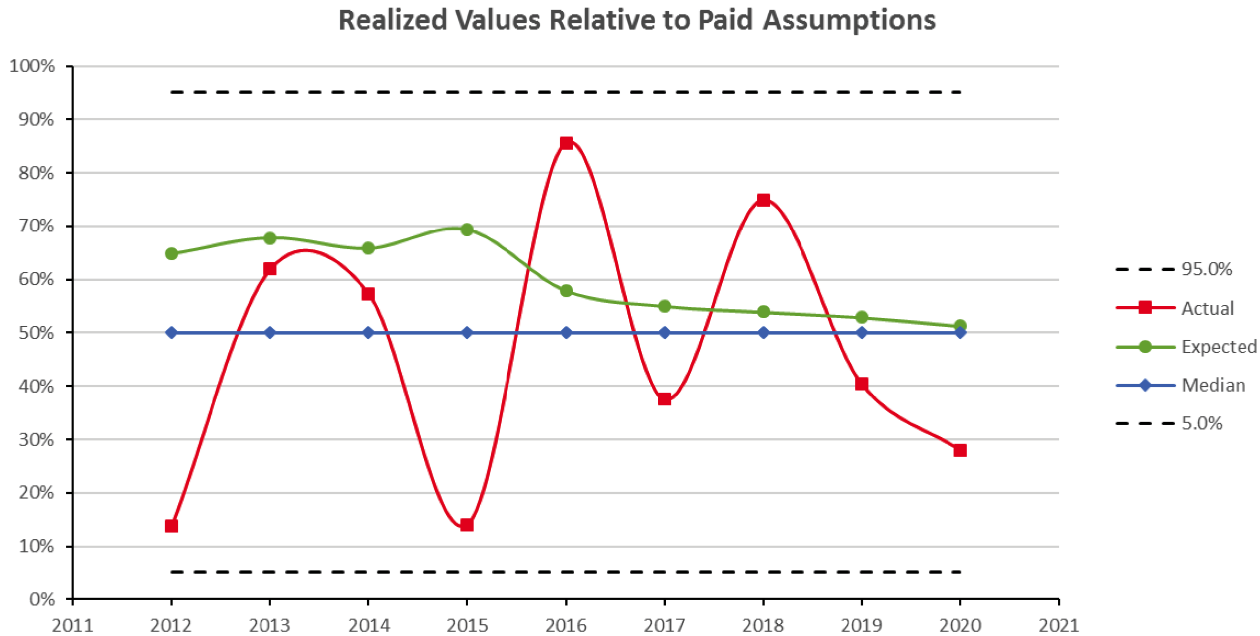
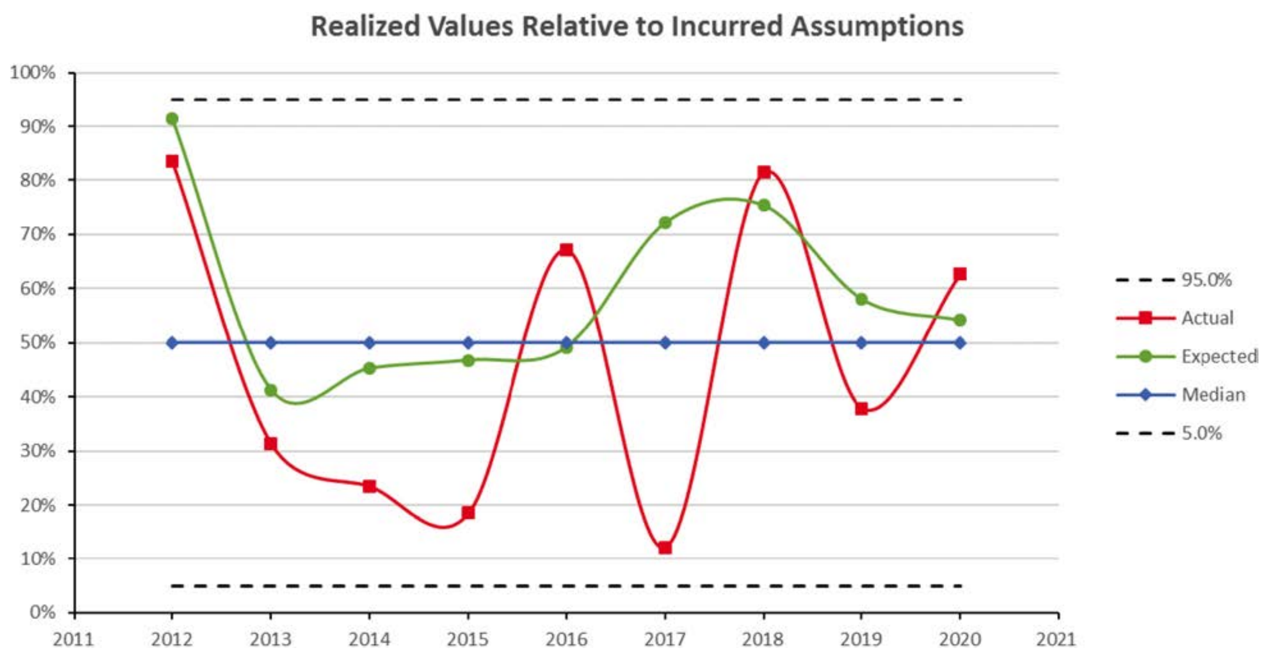


Figure E.6. Realized Values vs. Assumptions—Incurred (Stochastic)



Appendix F—Back-Testing Aggregate Results

Table F.1. Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Consolidation of All Segments
Total Unpaid Reconciliation as of December 31, 2020

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2012	108	1,798,805	1,801,896	1,806,215	3,091	4,319	7,410	1,806,215	4,319	7,410
2013	96	2,054,136	2,063,367	2,068,349	9,231	4,982	14,213	**2,070,502**	**7,135**	**16,366**
2014	84	2,202,872	2,213,290	2,226,141	10,418	12,851	23,269	2,226,141	12,851	23,269
2015	72	2,335,053	2,354,342	2,379,431	19,289	25,089	44,378	2,379,431	25,089	44,378
2016	60	2,522,650	2,566,756	2,618,692	44,106	51,936	96,042	2,618,692	51,936	96,042
2017	48	2,510,953	2,609,324	2,713,658	98,371	104,334	202,705	2,713,658	104,334	202,705
2018	36	2,369,593	2,567,519	2,783,496	197,926	215,977	413,903	2,783,496	215,977	413,903
2019	24	2,210,586	2,558,937	2,976,074	348,351	417,137	765,488	2,976,074	417,137	765,488
2020	12	1,604,249	2,346,693	3,247,231	742,444	900,538	1,642,982	3,247,231	900,538	1,642,982
Totals		19,608,897	21,082,124	22,819,287	1,473,227	1,737,163	3,210,390	22,821,440	1,739,316	3,212,543

Table F.2. Expected Incremental Development—Paid (Deterministic)

Sample Insurance Company
Consolidation of All Segments—Paid Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										3,701	3,709	7,410
2013									7,405	4,476	4,485	16,366
2014								10,073	4,353	4,417	4,426	23,269
2015							19,027	11,120	4,716	4,752	4,762	44,378
2016						47,151	21,651	11,869	5,058	5,151	5,162	96,042
2017					103,127	50,012	21,845	12,022	5,128	5,281	5,292	202,705
2018				194,479	113,044	53,527	23,509	12,806	5,484	5,521	5,533	413,903
2019			325,644	208,375	119,178	56,435	24,715	13,549	5,783	5,899	5,911	765,488
2020		833,793	351,973	216,546	123,955	58,580	25,466	14,073	6,020	6,282	6,295	1,642,982

Table F.3. Expected Incremental Development—Incurred (Deterministic)

Sample Insurance Company
Consolidation of All Segments—Incurred Data
Expected Incremental Future Development as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										2,158	2,161	4,319
2013									2,794	2,169	2,172	7,135
2014								6,142	1,726	2,489	2,494	12,851
2015							11,285	6,504	1,883	2,706	2,711	25,089
2016						26,873	10,537	6,833	1,991	2,849	2,853	51,936
2017					54,534	24,663	10,569	6,831	1,995	2,868	2,873	104,334
2018				106,020	55,954	26,819	11,457	7,434	2,175	3,057	3,062	215,977
2019			192,143	108,519	59,307	28,313	12,129	7,859	2,291	3,285	3,291	417,137
2020		479,073	187,988	112,628	61,829	29,184	12,530	8,072	2,358	3,436	3,441	900,538

Table F.4. Actual vs. Expected Back-Test (Deterministic)

Sample Insurance Company
Consolidation of All Segments
Deterministic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2012	120	3,069	3,701	(632)	1,863	2,158	(295)
2013	108	5,905	7,405	(1,500)	3,145	2,794	351
2014	96	8,986	10,073	(1,087)	3,553	6,142	(2,589)
2015	84	18,992	19,027	(35)	9,872	11,285	(1,413)
2016	72	51,003	47,151	3,852	25,942	26,873	(931)
2017	60	105,067	103,127	1,940	52,012	54,534	(2,522)
2018	48	202,932	194,479	8,453	106,624	106,020	604
2019	36	334,434	325,644	8,790	189,908	192,143	(2,235)
2020	24	841,484	833,793	7,691	454,217	479,073	(24,856)
2021	12	1,798,138			2,528,235		
Totals		3,370,010			3,375,371		
AY < CY		1,571,872	1,544,400	27,471	847,136	881,022	(33,886)

Table F.5. Actual to Range of Estimates Back-Test (Deterministic)

Sample Insurance Company
Consolidation of All Segments
Deterministic Actual vs. Method Range as of December 31, 2021

AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2012	120	3,069	3,701	3,704	–21075%	1,863	2,158	2,162	–6790%
2013	108	5,905	5,827	8,983	2%	3,145	1,210	4,380	61%
2014	96	8,986	9,887	10,277	–231%	3,553	5,955	6,356	–599%
2015	84	18,992	17,726	20,381	48%	9,872	9,981	12,657	–4%
2016	72	51,003	44,889	49,487	133%	25,942	24,600	29,236	29%
2017	60	105,067	100,495	106,278	79%	52,012	51,856	57,857	3%
2018	48	202,932	191,183	198,745	155%	106,624	102,222	110,845	51%
2019	36	334,434	310,031	338,355	86%	189,908	174,120	205,898	50%
2020	24	841,484	794,706	853,821	79%	454,217	436,298	503,306	27%
2021	12	1,798,138				2,528,235			
Totals		3,370,010				3,375,371			
AY < CY		1,571,872	1,481,602	1,586,896	86%	847,136	811,568	929,564	30%

Table F.6. Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company
Aggregation of All Segments
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	7,410	3,000	40.5%	209	20,930	2,762	5,258	7,230	7,126	9,376	12,584
2013	16,366	3,857	23.6%	4,326	35,971	10,293	13,681	16,160	13,955	18,874	23,025
2014	23,269	4,798	20.6%	7,340	41,630	15,697	19,961	23,038	24,448	26,387	31,552
2015	44,378	6,012	13.5%	23,290	73,490	34,774	40,249	44,172	43,645	48,324	54,552
2016	96,042	8,137	8.5%	68,354	129,130	82,986	90,380	95,868	97,281	101,523	109,899
2017	202,705	11,141	5.5%	162,433	245,913	184,872	195,065	202,429	213,672	210,093	221,392
2018	413,903	18,019	4.4%	348,396	495,863	385,145	401,826	413,324	431,386	425,535	444,597
2019	765,488	31,256	4.1%	643,540	893,747	714,958	744,538	764,726	758,282	786,020	818,610
2020	1,642,982	62,139	3.8%	1,378,415	1,972,517	1,544,716	1,602,194	1,641,001	1,633,958	1,682,508	1,746,787
Total	3,212,543	79,355	2.5%	2,811,937	3,596,084	3,084,602	3,161,789	3,211,505	3,295,980	3,261,725	3,343,252

Table F.7. Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company
Aggregation of All Segments
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Calendar Year

CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2021	1,560,637	43,888	2.8%	1,326,487	1,761,442	1,490,151	1,531,594	1,560,068	1,569,675	1,589,323	1,634,164
2022	761,830	24,692	3.2%	671,495	861,974	721,379	745,435	761,974	778,026	778,144	802,553
2023	433,217	17,767	4.1%	368,636	499,640	404,462	420,952	433,003	430,492	445,020	463,153
2024	227,484	12,686	5.6%	180,708	277,701	206,908	218,837	227,342	231,979	235,833	248,870
2025	110,005	8,936	8.1%	81,148	145,658	95,506	104,003	109,870	108,106	115,810	124,858
2026	54,489	6,783	12.4%	30,217	81,348	43,677	49,928	54,233	53,345	58,990	65,976
2027	30,258	5,508	18.2%	11,536	54,292	21,555	26,490	30,113	31,602	33,792	39,599
2028	17,338	4,694	27.1%	1,748	38,761	9,925	14,127	17,132	15,736	20,273	25,447
2029	12,228	4,234	34.6%	351	31,873	5,612	9,261	12,025	15,750	14,892	19,631
2030	5,057	2,388	47.2%	(46)	15,791	1,427	3,333	4,900	4,363	6,546	9,313
Total	3,212,543	79,355	2.5%	2,811,937	3,596,084	3,084,602	3,161,789	3,211,505	3,295,980	3,261,725	3,343,252

Table F.8. Mean Future Incremental—Paid (Stochastic)

Sample Insurance Company
Aggregation of All Segments—Paid
Mean Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										4,077	3,333	7,410
2013									6,163	5,387	4,816	16,366
2014								10,176	4,300	4,998	3,794	23,269
2015							20,033	10,774	4,591	4,922	4,058	44,378
2016						48,298	21,360	11,595	4,927	5,520	4,342	96,042
2017					104,415	49,419	21,556	11,839	5,077	6,033	4,365	202,705
2018				196,083	112,311	53,119	23,353	12,692	5,415	6,236	4,693	413,903
2019			331,701	205,564	117,582	55,662	24,391	13,384	5,665	6,643	4,896	765,488
2020		839,689	349,382	214,959	122,988	58,266	25,315	13,992	6,001	7,332	5,057	1,642,982

Table F.9. Standard Deviation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Aggregation of All Segments—Paid
Standard Deviation Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										1,851	1,623	3,000
2013									1,927	2,080	1,809	3,857
2014								2,494	2,030	2,244	1,833	4,798
2015							3,202	2,660	2,162	2,280	1,974	6,012
2016						5,017	3,331	2,742	2,331	2,477	2,065	8,137
2017					7,305	5,065	3,417	2,795	2,369	2,568	2,101	11,141
2018				10,921	8,101	5,518	3,644	3,008	2,443	2,580	2,185	18,019
2019			16,733	12,067	8,683	5,833	3,853	3,164	2,615	2,786	2,312	31,256
2020		36,658	17,799	12,858	9,241	6,087	3,943	3,330	2,814	2,992	2,388	62,139

Table F.10. Coefficient of Variation of Future Incremental—Paid (Stochastic)

Sample Insurance Company
Aggregation of All Segments—Paid
CoV Future Incremental as of December 31, 2020

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2012										45.4%	48.7%	40.5%
2013									31.3%	38.6%	37.6%	23.6%
2014								24.5%	47.2%	44.9%	48.3%	20.6%
2015							16.0%	24.7%	47.1%	46.3%	48.6%	13.5%
2016						10.4%	15.6%	23.6%	47.3%	44.9%	47.6%	8.5%
2017					7.0%	10.2%	15.8%	23.6%	46.7%	42.6%	48.1%	5.5%
2018				5.6%	7.2%	10.4%	15.6%	23.7%	45.1%	41.4%	46.6%	4.4%
2019			5.0%	5.9%	7.4%	10.5%	15.8%	23.6%	46.2%	41.9%	47.2%	4.1%
2020		4.4%	5.1%	6.0%	7.5%	10.4%	15.6%	23.8%	46.9%	40.8%	47.2%	3.8%

Table F.11. Estimated Unpaid Claims by Accident Year in 2021 (Stochastic)

Sample Insurance Company
Aggregation of All Segments—Paid
Stochastic Estimates as of December 31, 2020
Estimated Unpaid Claims by Accident Year, Calendar Year 2021 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2012	4,077	1,851	45.4%	4	12,459	1,386	2,758	3,891	3,545	5,211	7,424
2013	6,163	1,927	31.3%	92	14,962	3,317	4,823	5,994	6,136	7,317	9,584
2014	10,176	2,494	24.5%	2,955	24,018	6,391	8,444	9,987	8,710	11,747	14,546
2015	20,033	3,202	16.0%	9,752	35,160	15,071	17,795	19,882	19,530	22,094	25,607
2016	48,298	5,017	10.4%	27,691	69,353	40,292	44,825	48,117	49,900	51,560	56,893
2017	104,415	7,305	7.0%	76,379	135,132	92,822	99,305	104,299	105,433	109,283	116,607
2018	196,083	10,921	5.6%	157,181	242,812	178,556	188,588	195,828	193,134	203,222	214,311
2019	331,701	16,733	5.0%	257,765	396,823	304,516	320,387	331,465	315,168	342,845	359,464
2020	839,689	36,658	4.4%	679,077	1,011 508	781,489	815,305	839,033	862,142	862,844	900,811
Total	1,560,637	43,888	2.8%	1,326,487	1,761,442	1,490,151	1,531,594	1,560,068	1,569,675	1,589,323	1,634,164

Table F.12. Actual vs. Expected Back-Test and Conditional Reserve (Stochastic)

Sample Insurance Company
Aggregation of All Segments
Stochastic Actual vs. Expected as of December 31, 2021

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2012	120	3,069	4,077	31.8%	1,863	2,115	49.8%	2,539	4,341	(1,802)
2013	108	5,905	6,163	47.9%	3,145	1,819	80.6%	11,349	10,461	888
2014	96	8,986	10,176	33.6%	3,553	6,026	20.9%	10,961	14,283	(3,322)
2015	84	18,992	20,033	39.0%	9,872	10,399	46.3%	21,615	25,386	(3,771)
2016	72	51,003	48,298	71.6%	25,942	25,562	55.3%	49,308	45,039	4,269
2017	60	105,067	104,415	54.3%	52,012	53,101	44.8%	97,157	97,638	(481)
2018	48	202,932	196,083	74.2%	106,624	104,075	61.7%	222,250	210,971	11,279
2019	36	334,434	331,701	57.1%	189,908	185,173	64.0%	427,667	431,054	(3,387)
2020	24	841,484	839,689	52.8%	454,217	469,822	29.3%	795,671	801,499	(5,828)
2021	12	1,798,138			2,528,235					
Totals		3,370,010			3,375,371			1,638,516	1,640,671	(2,154)
AY < CY		1,571,872	1,560,637	61.2%	847,136	858,093	37.6%	1,638,584	1,640,671	(2,086)

Figure F.1. KPI Thresholds by Accident Year—Paid (Stochastic)

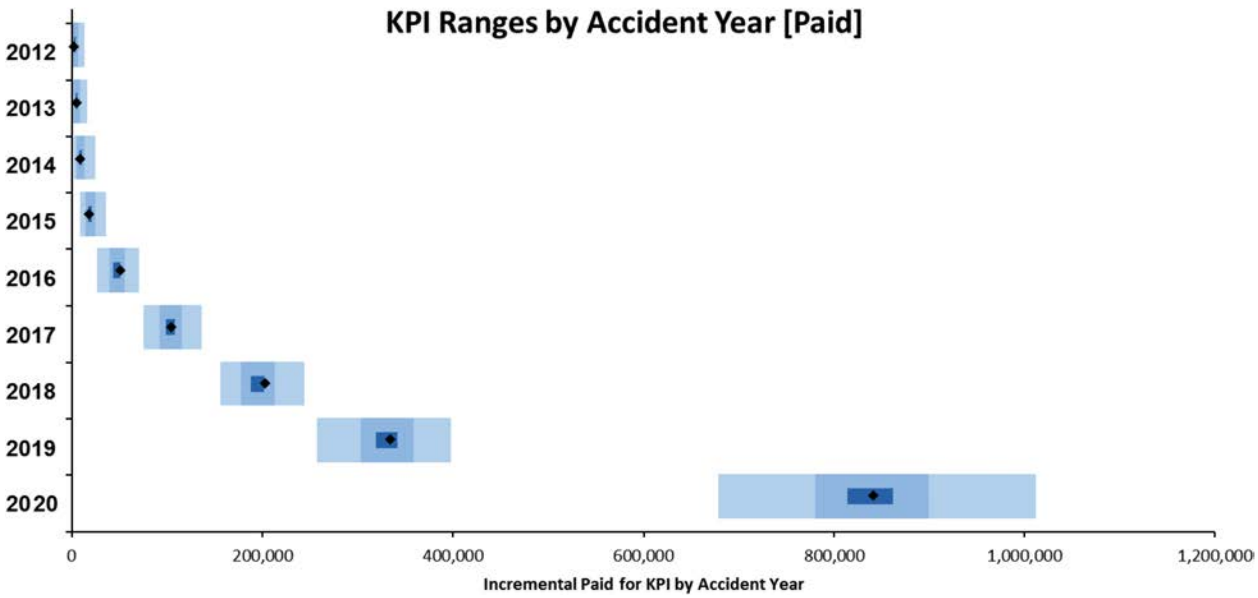


Figure F.2. KPI Thresholds by Calendar Year—Paid (Stochastic)

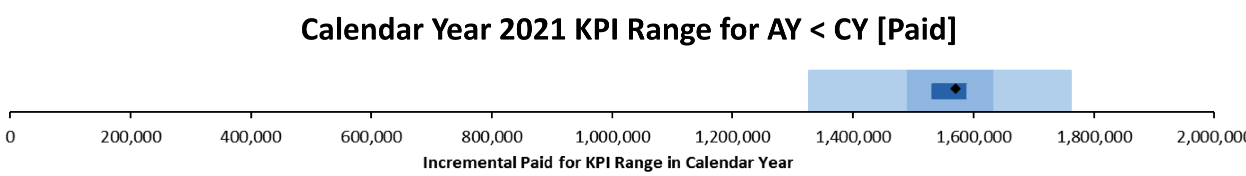


Figure F.3. Graph of KPI Thresholds by Accident Year—Incurred (Stochastic)

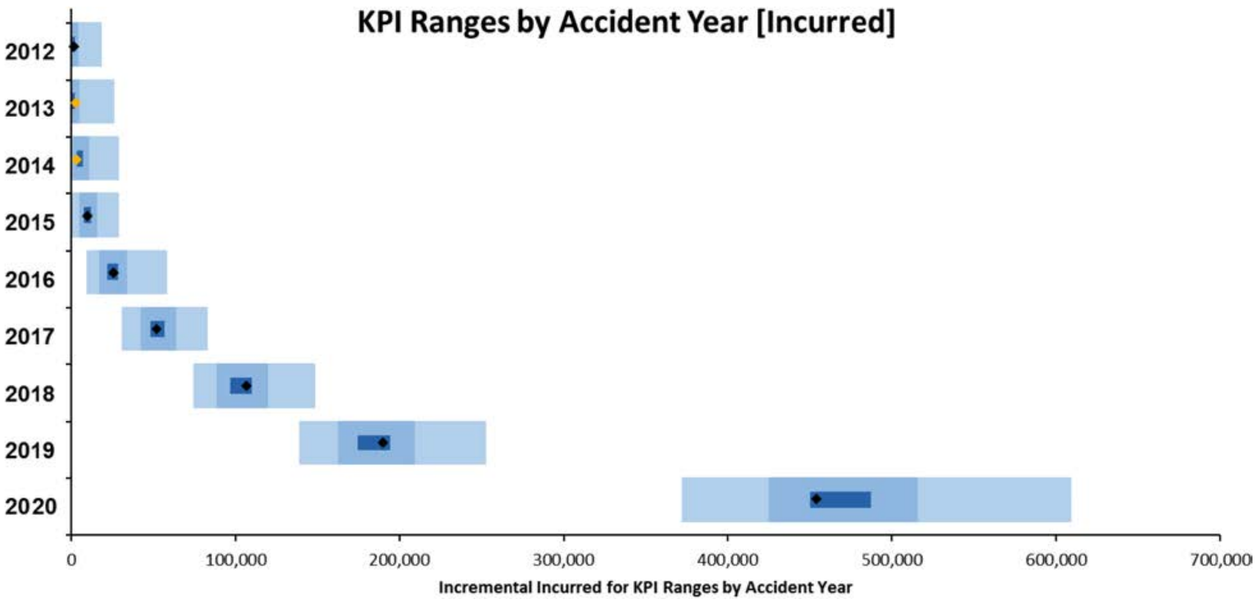


Figure F.4. KPI Thresholds by Calendar Year—Incurred (Stochastic)

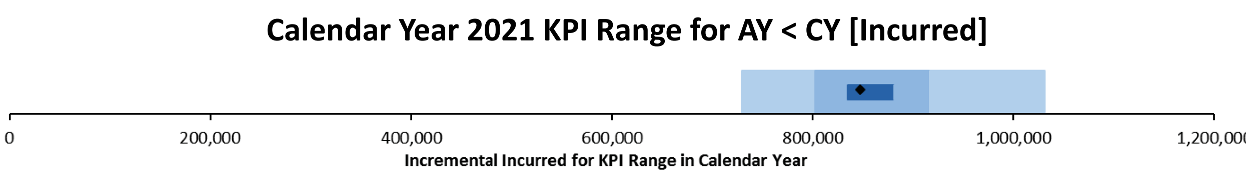


Figure F.5. Realized Values vs. Assumptions—Paid (Stochastic)

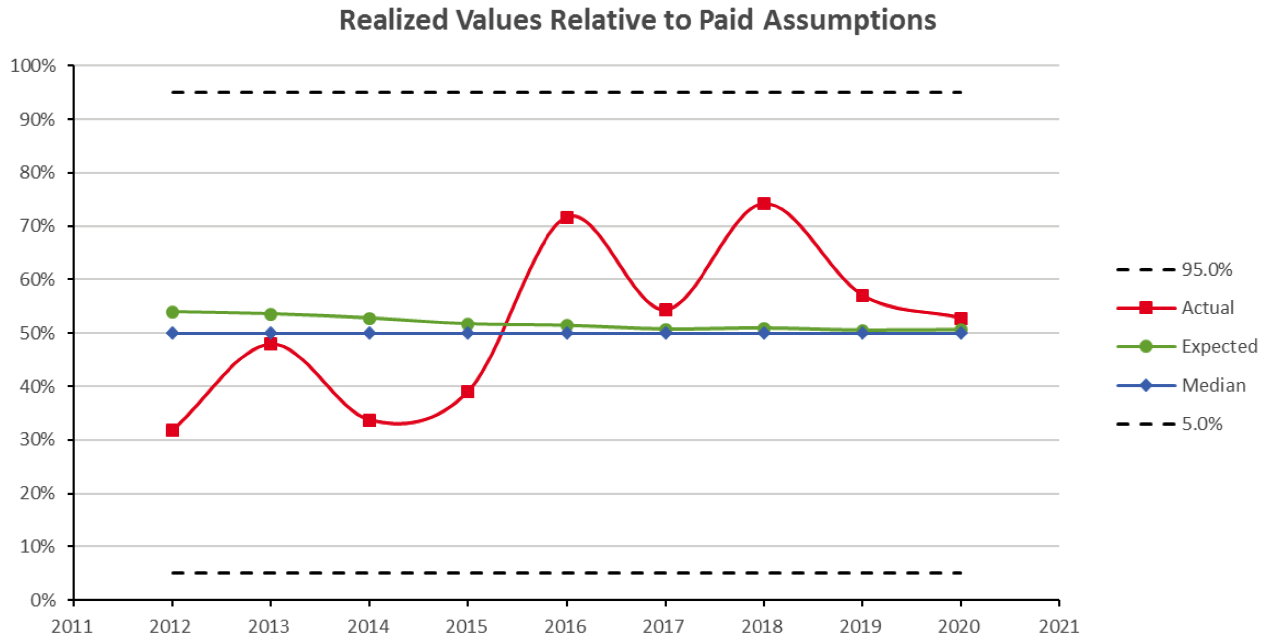


Figure F.6. Realized Values vs. Assumptions—Paid (Stochastic)

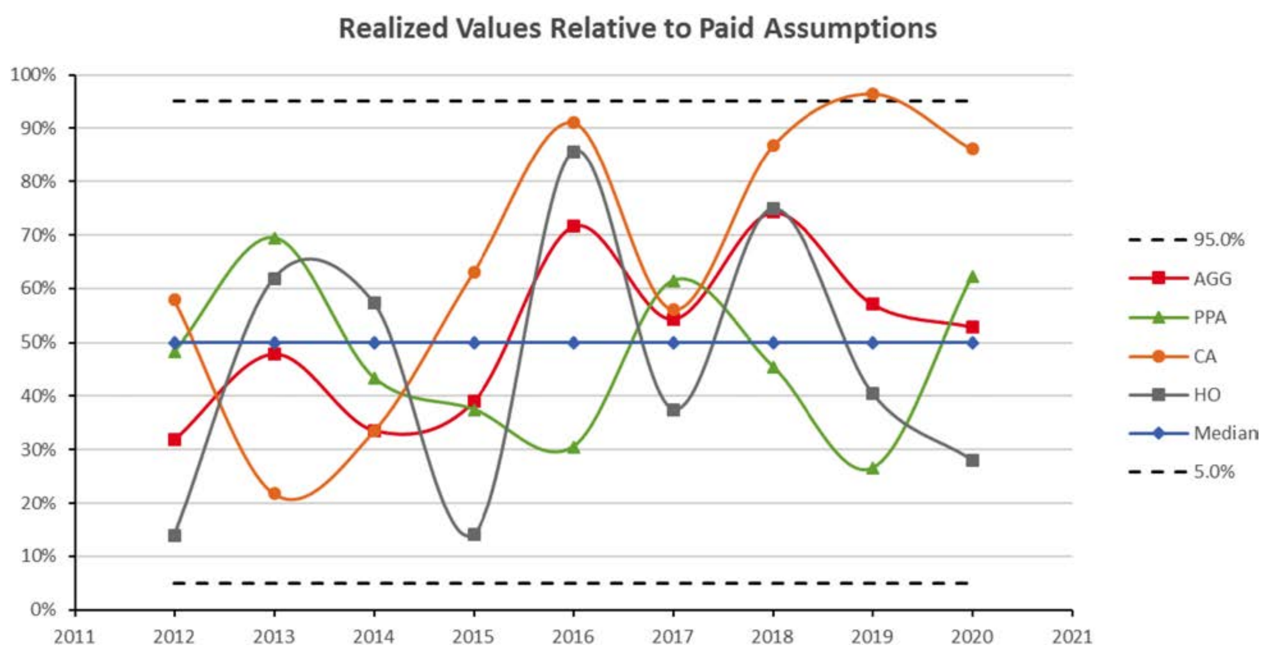


Figure F.7. Realized Values vs. Assumptions—Incurred (Stochastic)

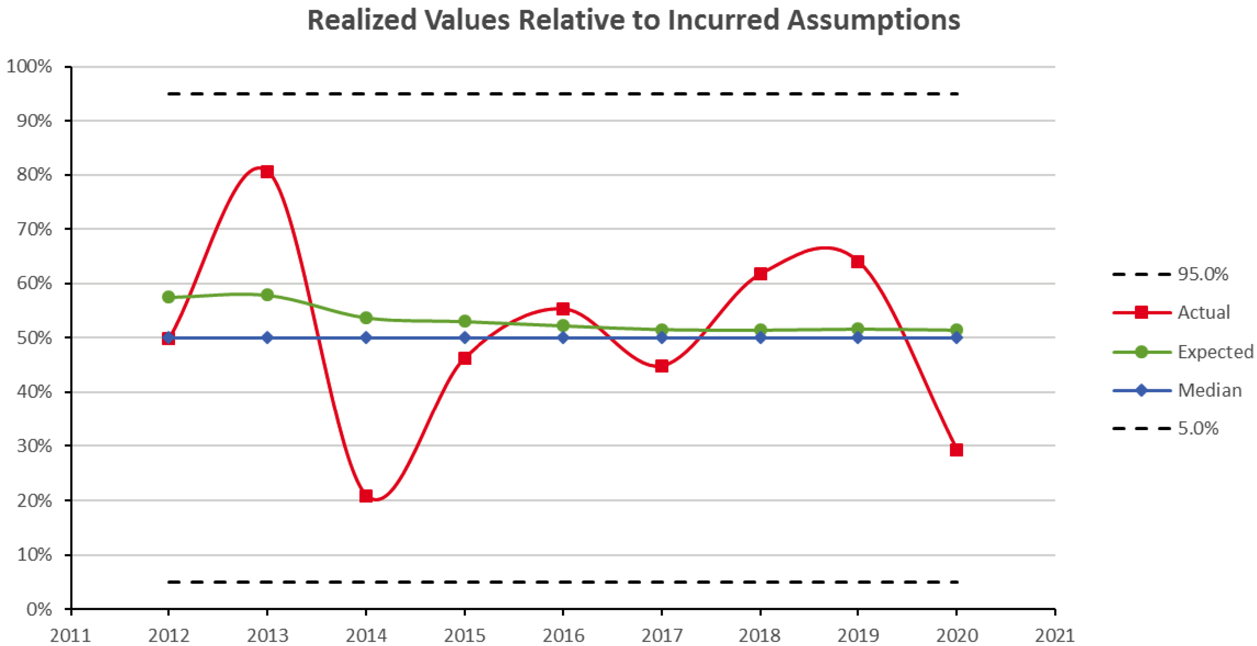
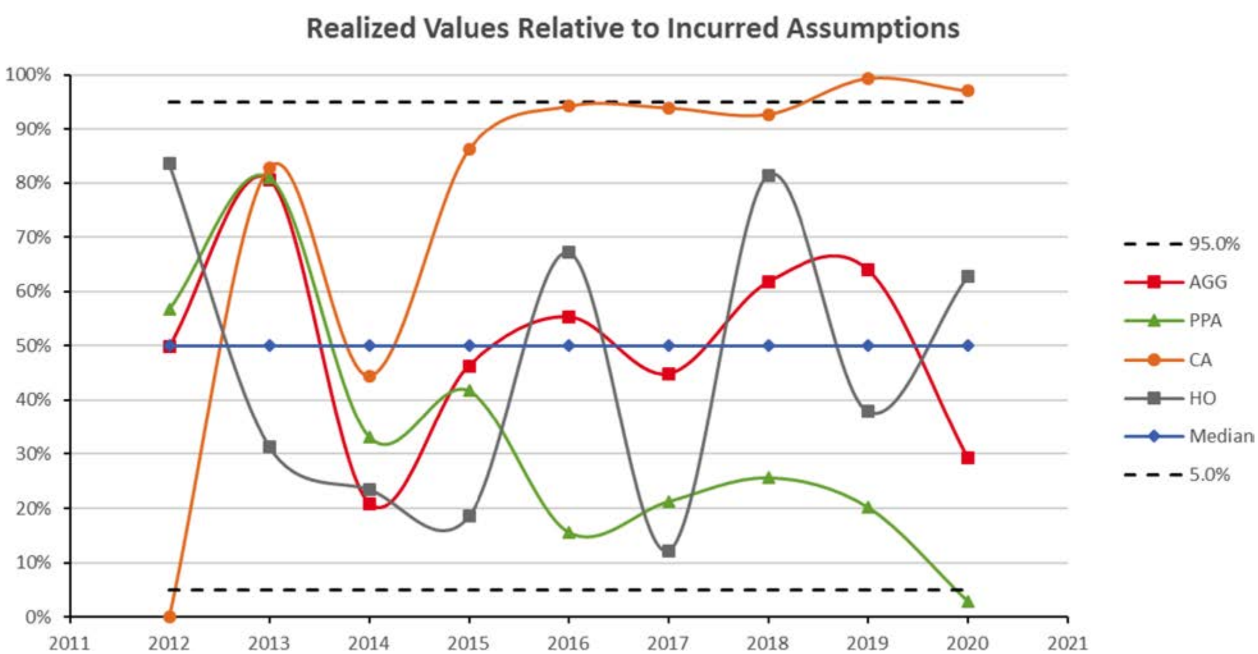


Figure F.8. Realized Values vs. Assumptions—Incurred (Stochastic)



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Abbreviations and Notations

The following abbreviations and notations are used in the monograph.

AY	accident year
AY = CY	the latest AY for which there is no comparable expectation based on the prior annual reserve analysis
AY < CY	all AYs except the latest AY for which there is a comparable expectation based on the prior annual reserve analysis
AYLWA	all-year loss-weighted average
BF	Bornhuetter-Ferguson
CA	commercial automobile
CEO	Chief Executive Officer
CFO	Chief Financial Officer
CL	chain ladder
CoV	coefficient of variation
CY	calendar year
ERM	enterprise risk management
FD	Framework Directive
GLM	generalized linear model
HO	homeowners
IBNR	incurred but not reported
IELR	initial expected loss ratio
IFRS	International Financial Reporting Standards
Inc BF	incurred Bornhuetter-Ferguson method
Inc CL	incurred chain ladder method
KPI	key performance indicator
LDF	loss development factor
MLE	maximum likelihood estimation
ODP	over-dispersed Poisson
Pd BF	paid Bornhuetter-Ferguson method
Pd CL	paid chain ladder method
PPA	private passenger automobile
TAS M	Technical Actuarial Standard: Modelling
ULR	ultimate loss ratio

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