Pandemics, Wars and More: How to Correct for Singular Events in Mortality Forecasting Models

MSc Actuarial Science
Thom van Rijn
Zürich, 29.09.2023

University of Basel

Singular Event:

A historical event with a large impact on the observed mortality frequencies.

Outline

- Mortality Forecasting Model
  - Nolfi
  - Measuring Accuracy
- Identifying Singular Events
  - Expert Judgement
  - Algorithm
- Correction Methods for Singular Events
  - Correction Step
  - Results
- Conclusion

FIT

SE

COR

Mortality Forecasting Model

Mortality Forecasting Model

Data

- $D_x(t)$: number of deaths within year t for age x
- $E_x(t)$: exposures within year t for age x
- $\bar{q}_x(t) = \frac{D_x(t)}{E_x(t)}$: observed mortality frequency
- Mortality.org

Mortality Forecasting Model

Nolfi

$$q_x(t) = \underbrace{q_x(t_0)}\exp\left(-\frac{\ln(2)}{\underbrace{\beta_x}}(t - t_0)\right)$$

where

… t_0: First fitting year

… $q_x(t_0)$: Starting mortality rate

… β_x: Half-value period

Mortality Forecasting Model
Lee-Carter & Renshaw-Haberman

Lee-Carter:

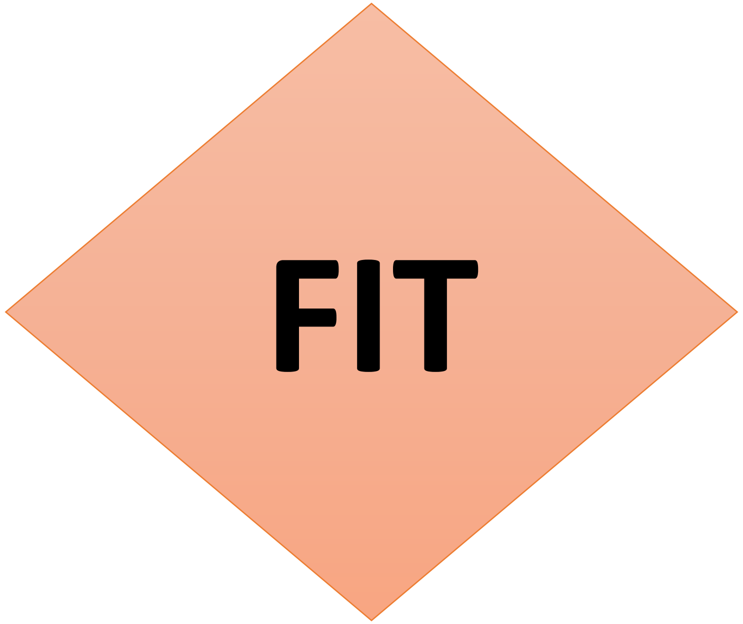
$$q_x(t) = \exp(\underbrace{\alpha_x} + \underbrace{\beta_x k_t})$$

Renshaw-Haberman:

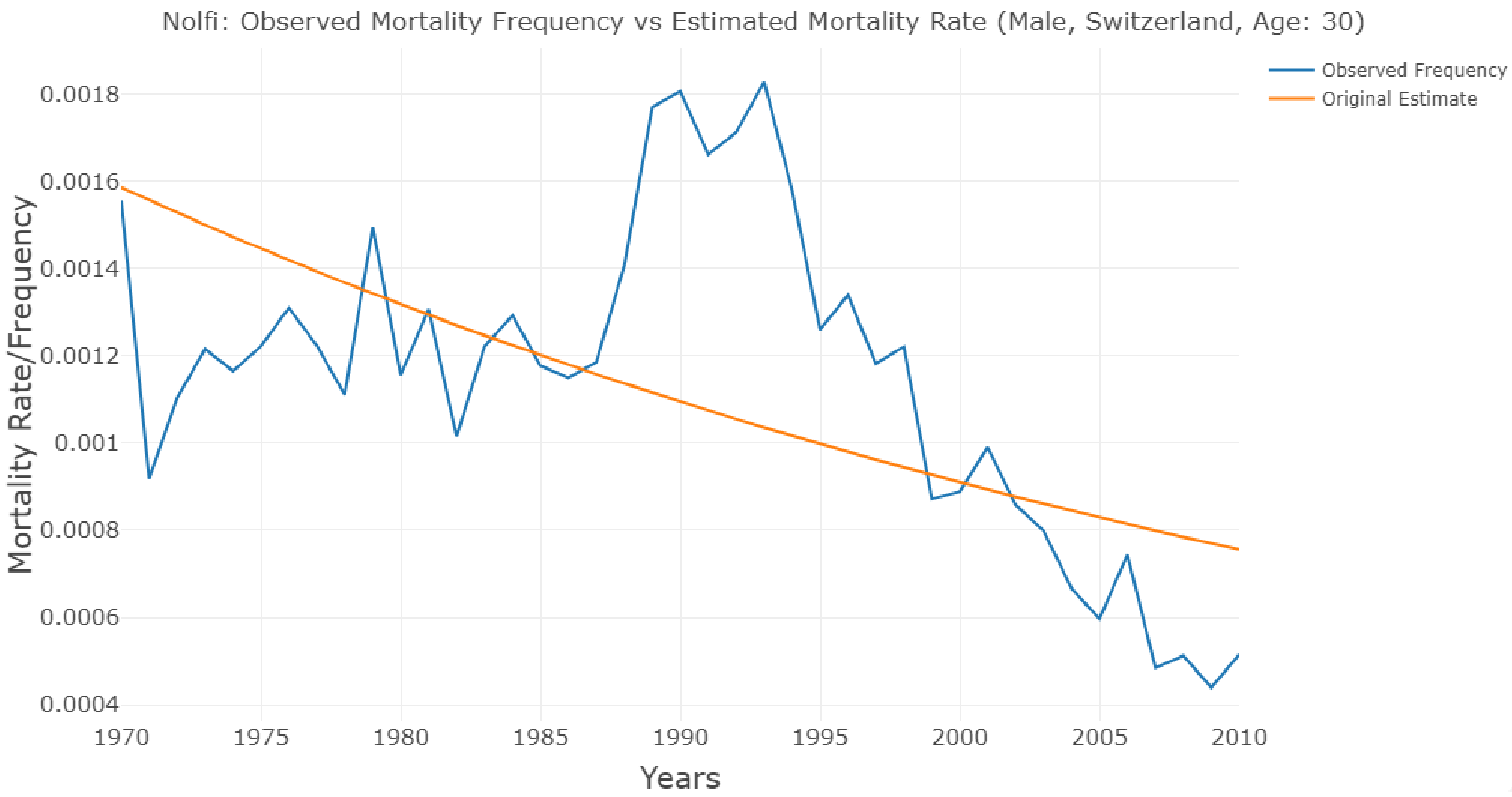
$$q_x(t) = \exp(\underbrace{\alpha_x} + \underbrace{\beta_x k_t} + \underbrace{\beta_x^* \iota_{(t-x)}})$$

Mortality Forecasting Model

Fitting Symbol



Mortality Forecasting Model
Measuring Accuracy



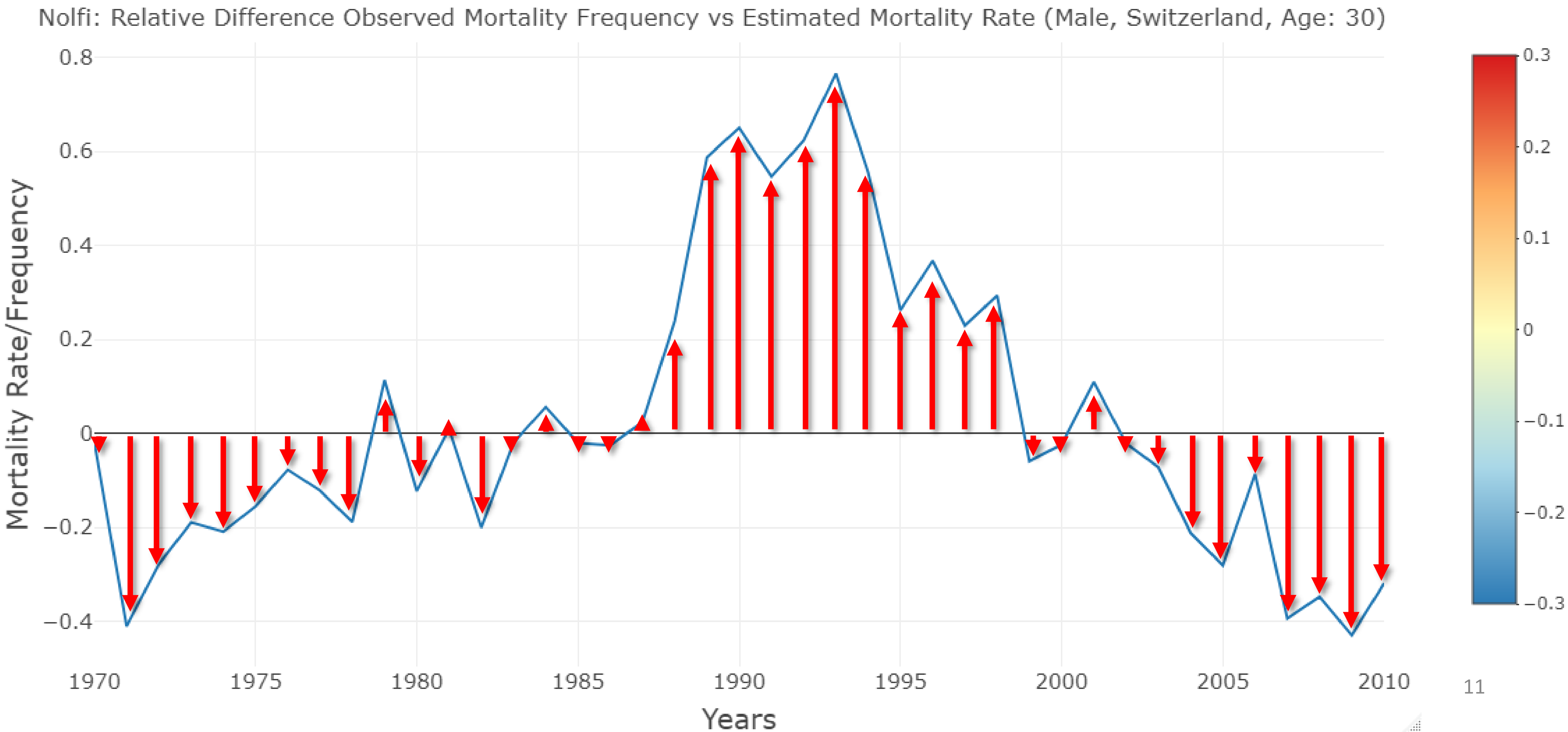
Fitting Mortality Forecasting Model

Measuring Deviations (Errors) - Formula

$$\varepsilon_x(t) = \frac{\bar{q}_x(t)}{\hat{q}_x(t)} - 1$$

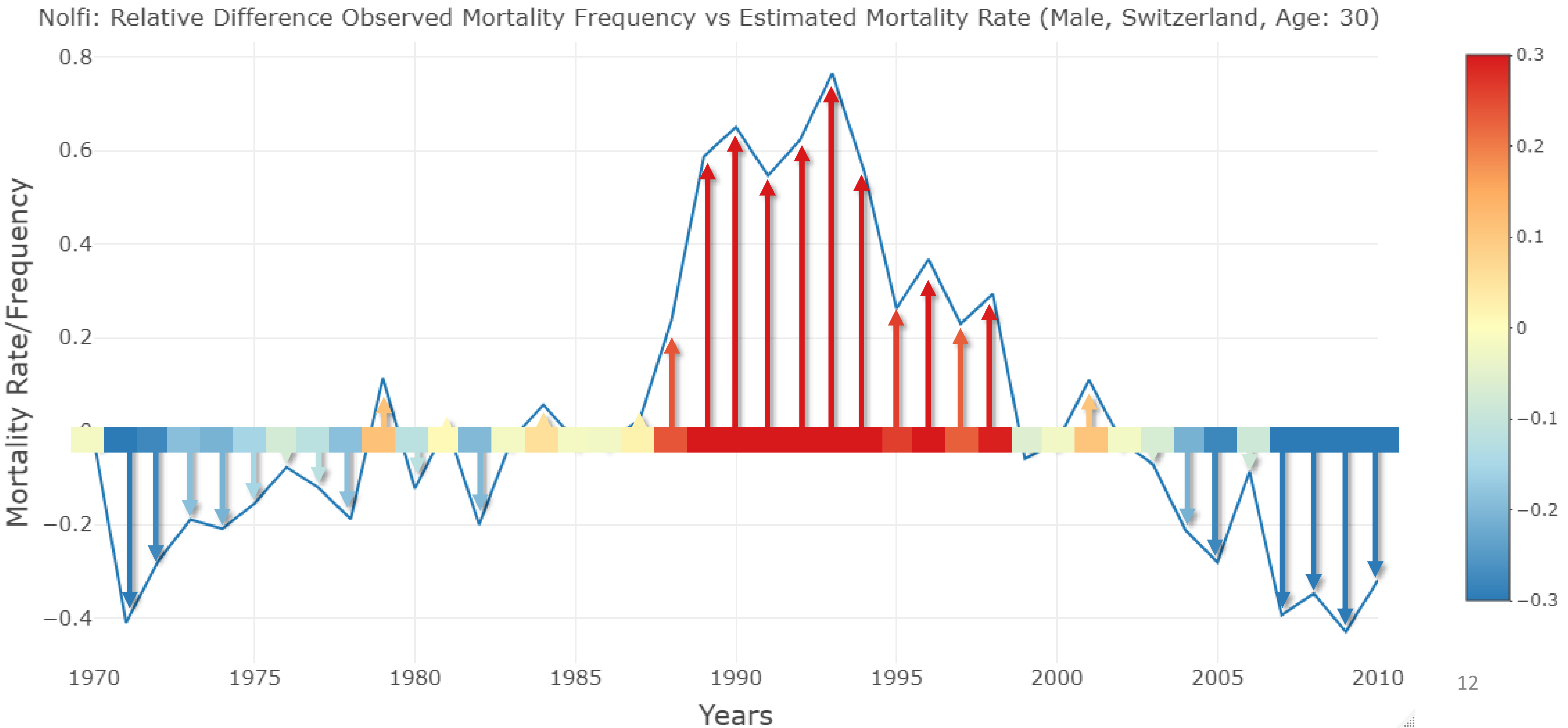
Mortality Forecasting Model

Measuring Accuracy – Graphically (2D)



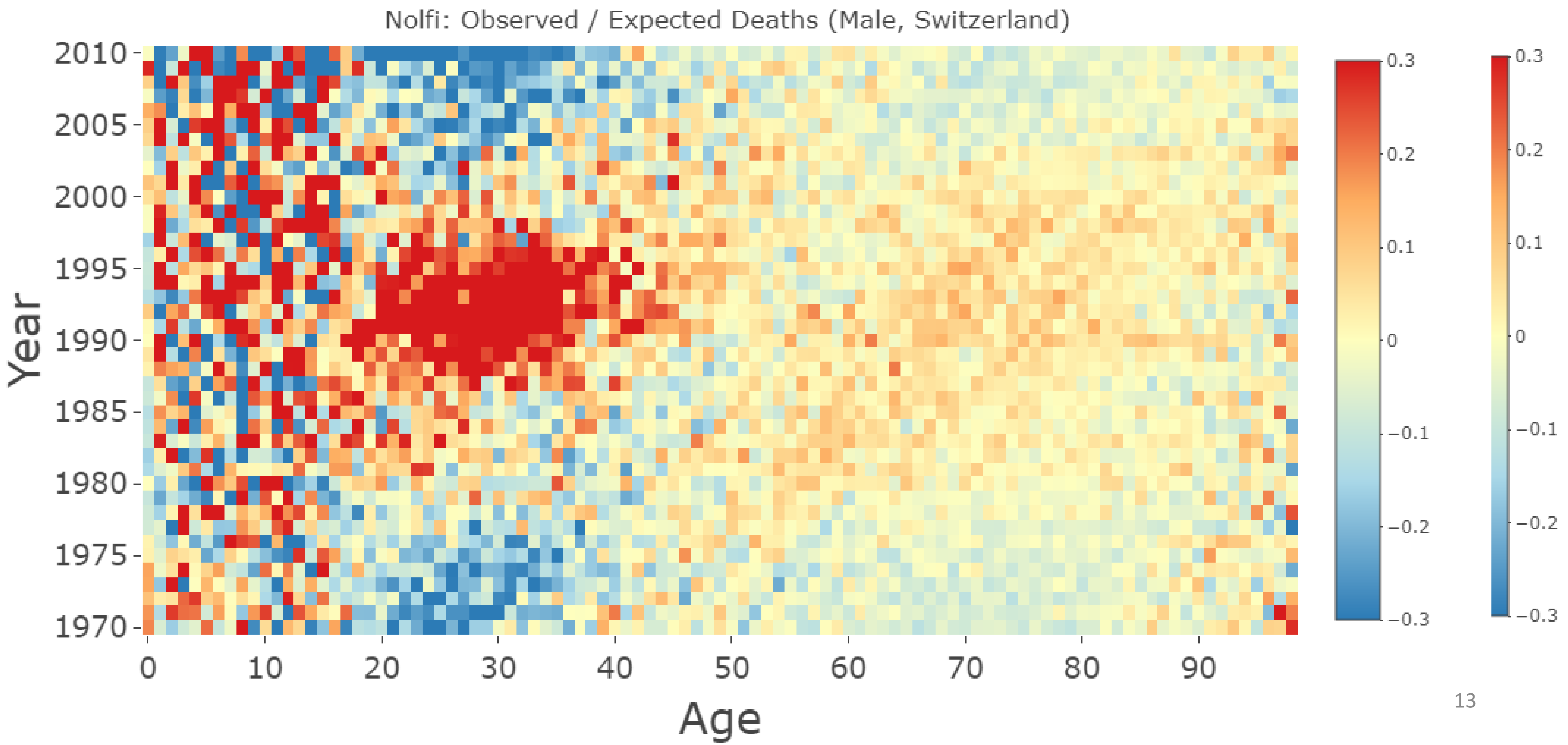
Mortality Forecasting Model

Measuring Accuracy – Graphically (2D)

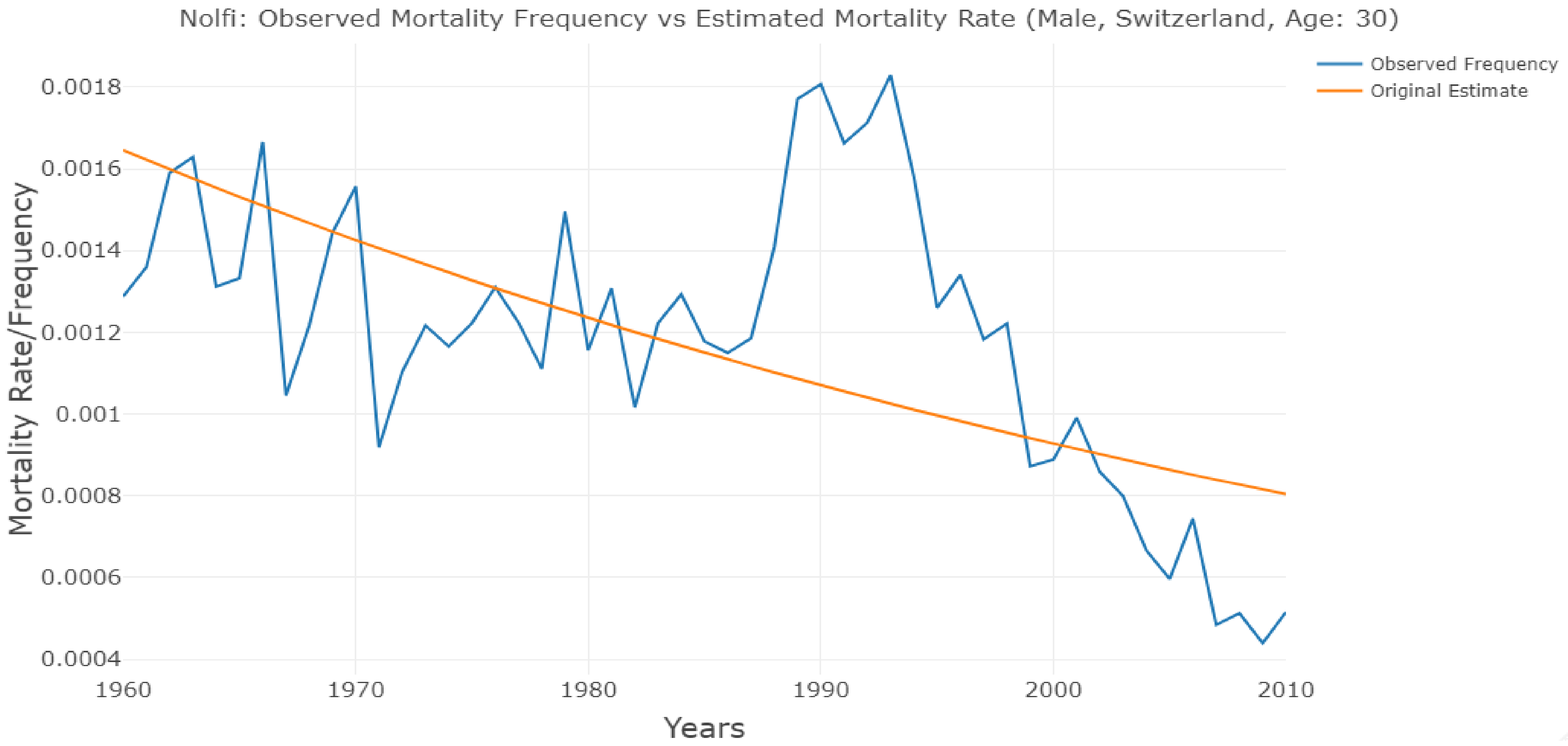


Mortality Forecasting Model

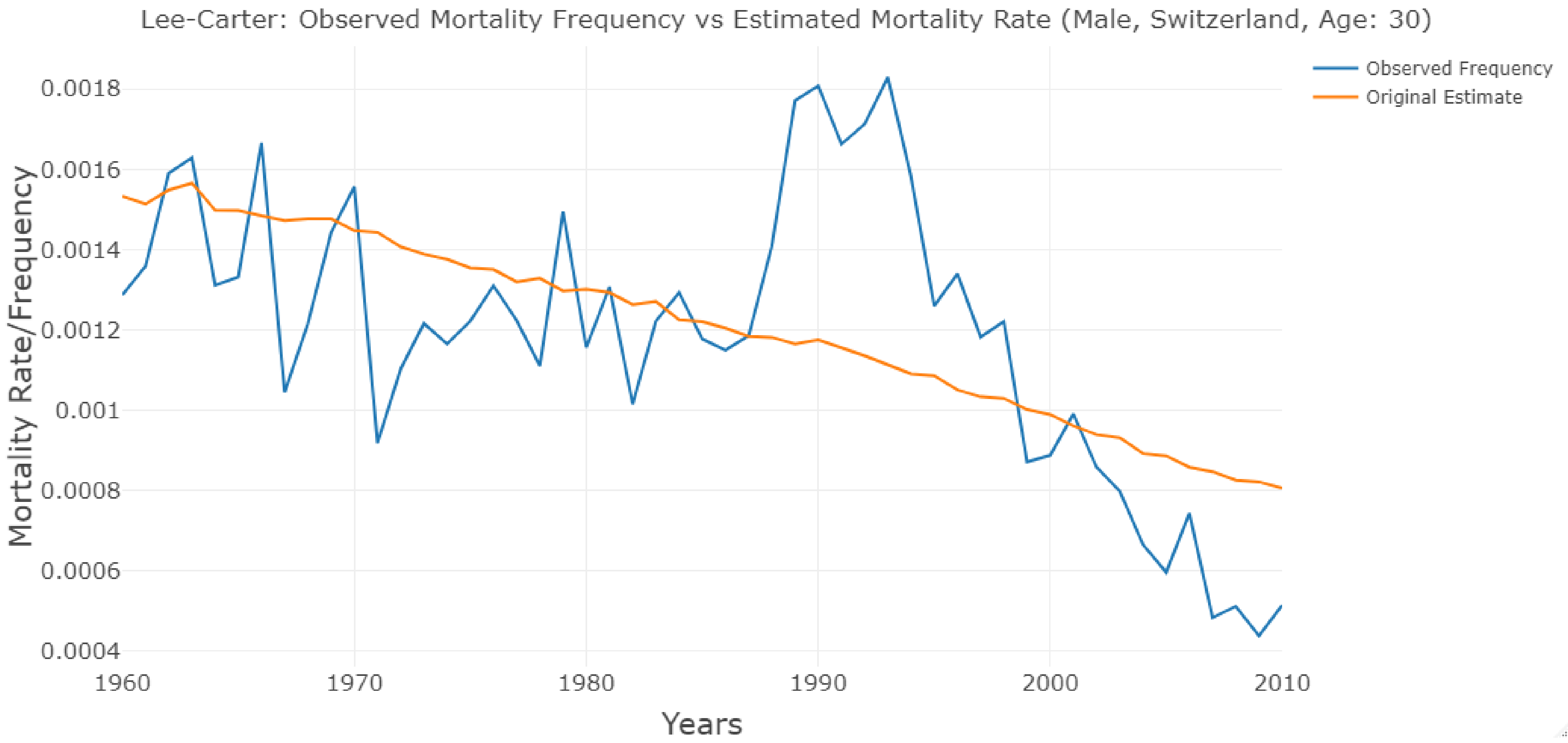
Measuring Accuracy – Graphically (2D)



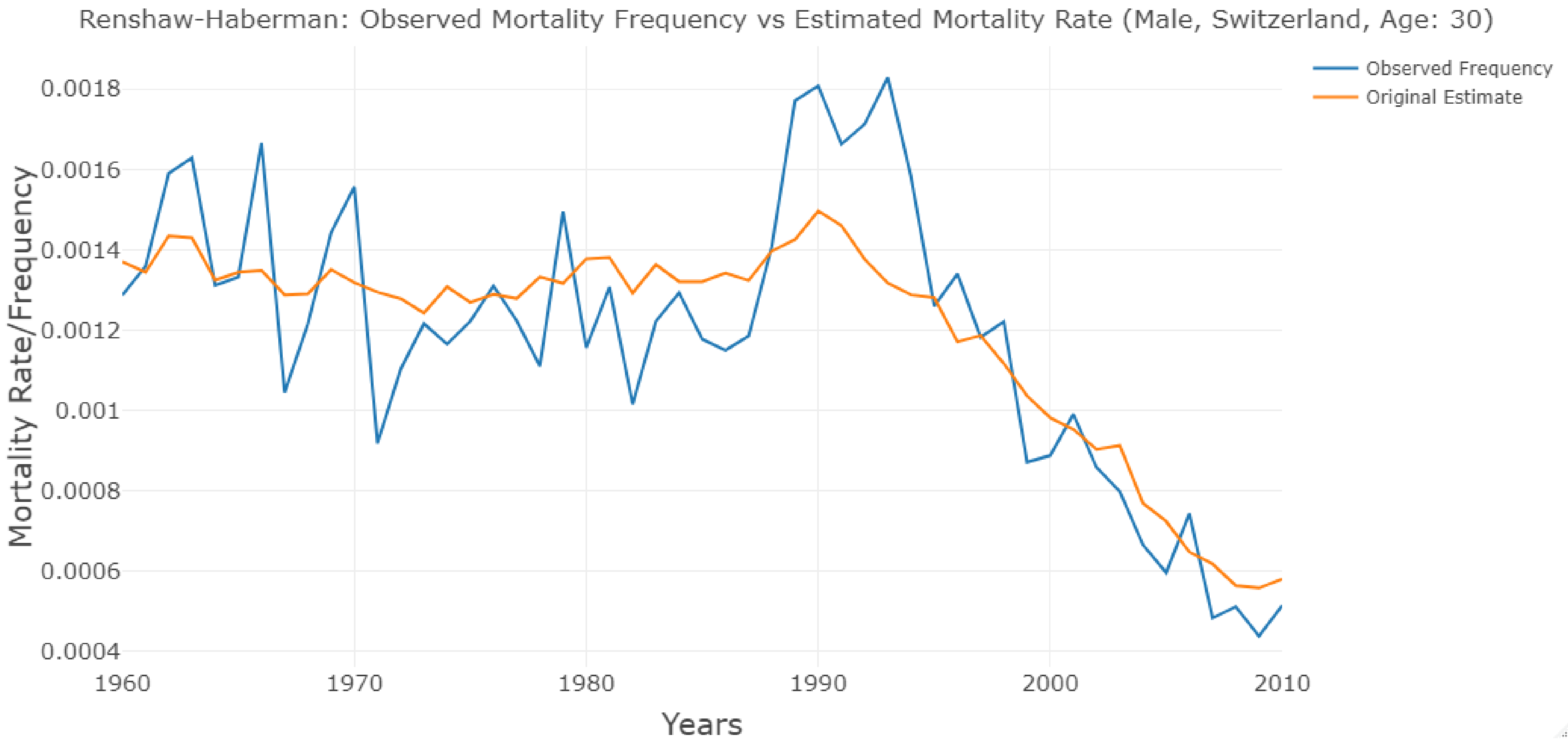
Mortality Forecasting Model
Measuring Accuracy – Why Nolfi



Mortality Forecasting Model
Measuring Accuracy – Why Nolfi

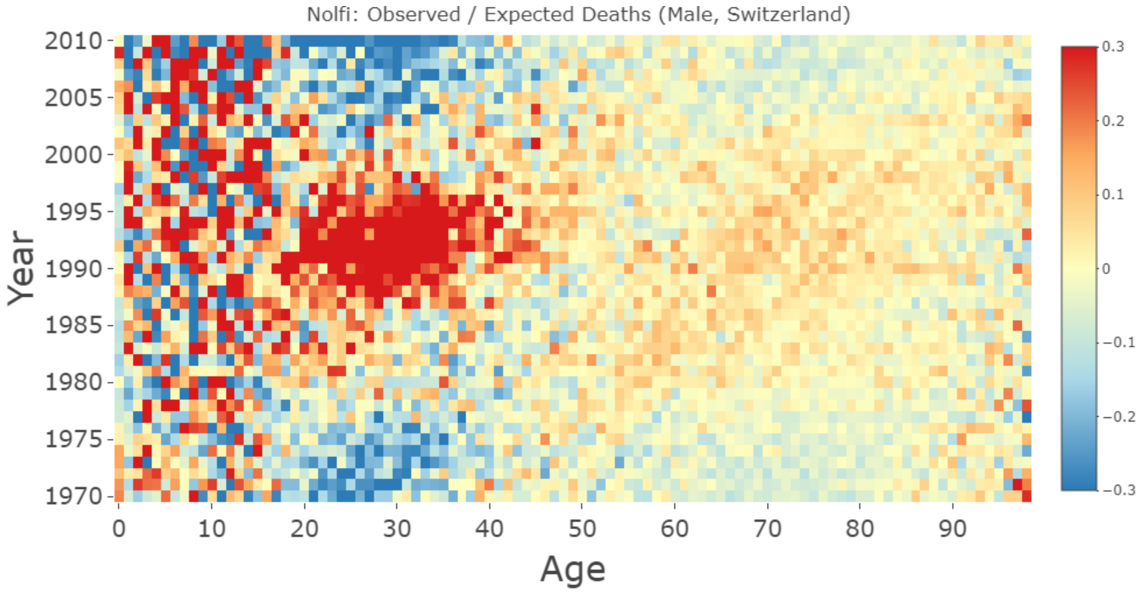


Mortality Forecasting Model
Measuring Accuracy – Why Nolfi

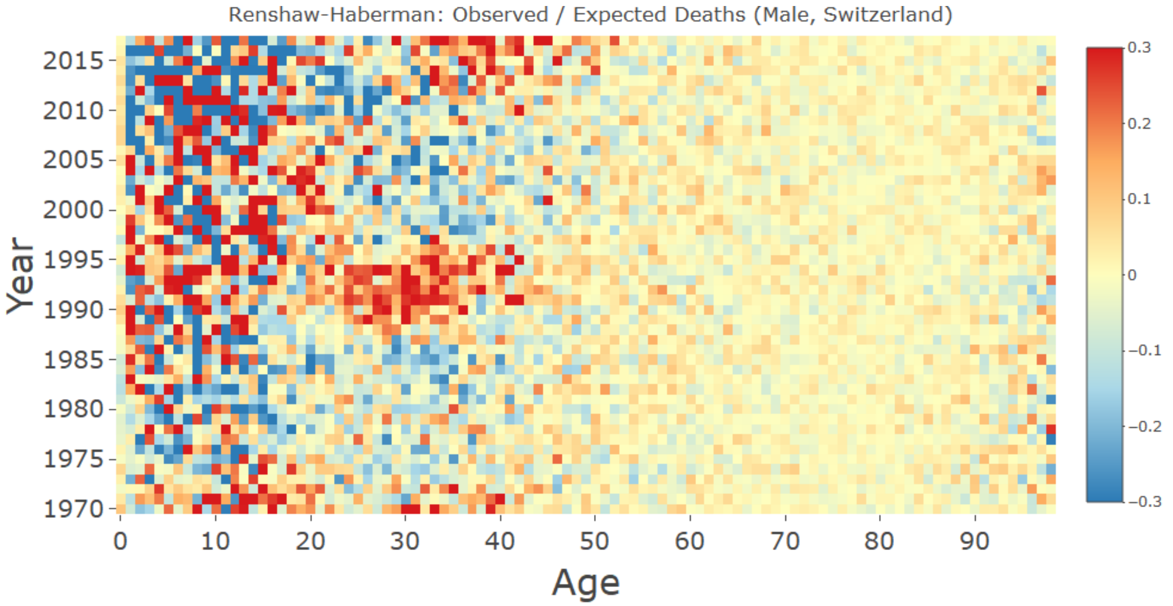


Mortality Forecasting Model
Measuring Accuracy – Why Nolfi

Nolfi



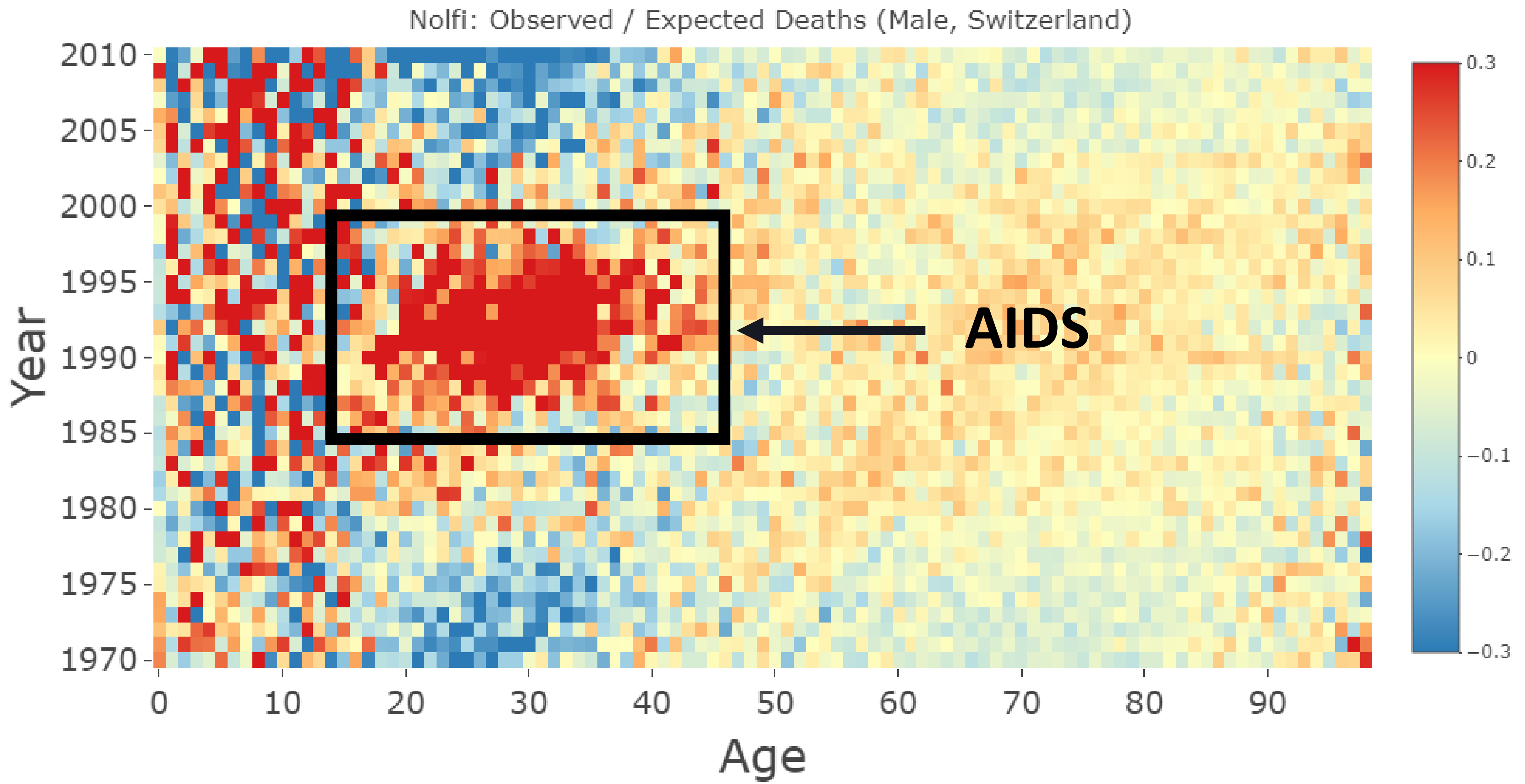
Renshaw-Haberman



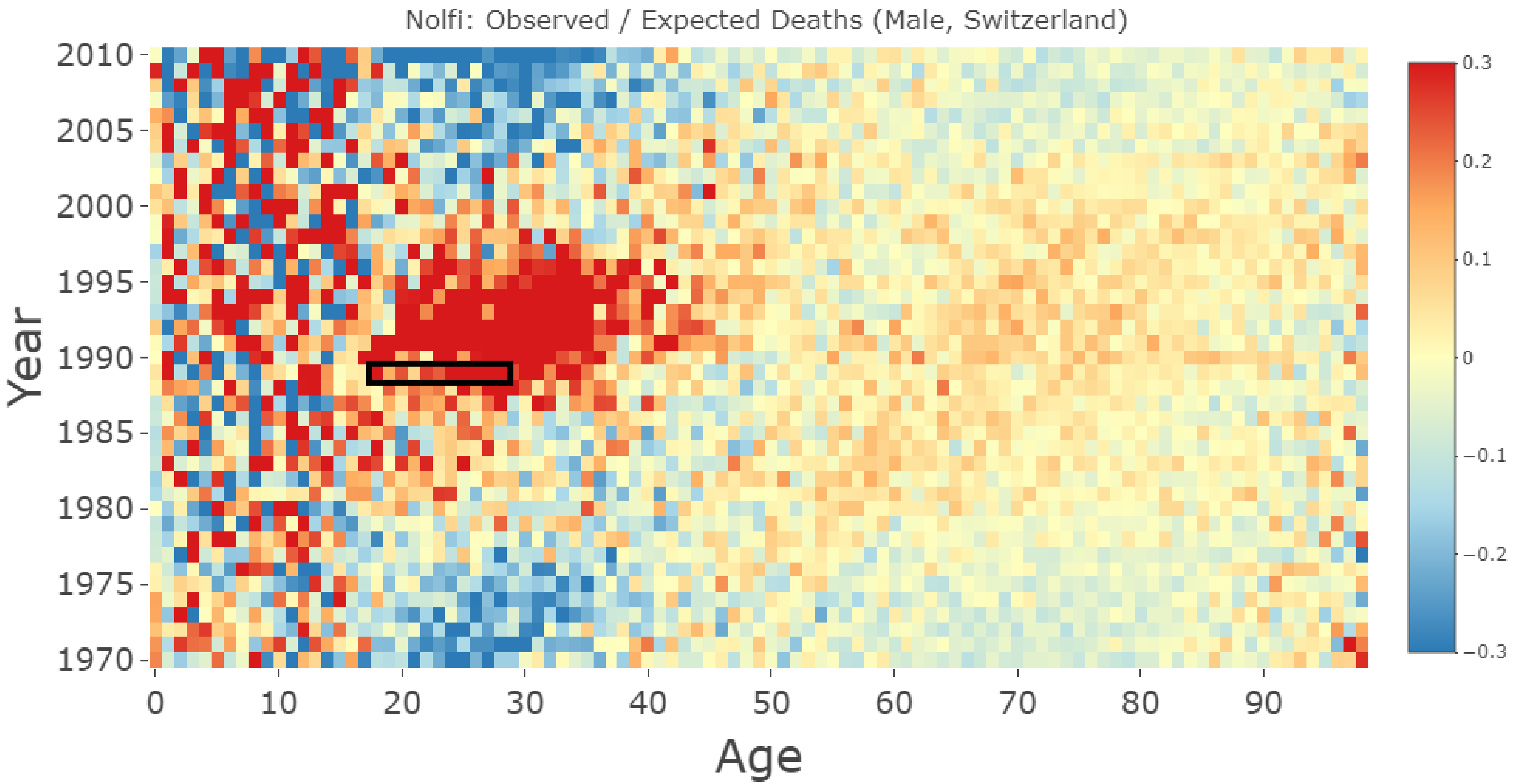
Identifying Singular Events

Identifying Singular Events

Expert Judgement

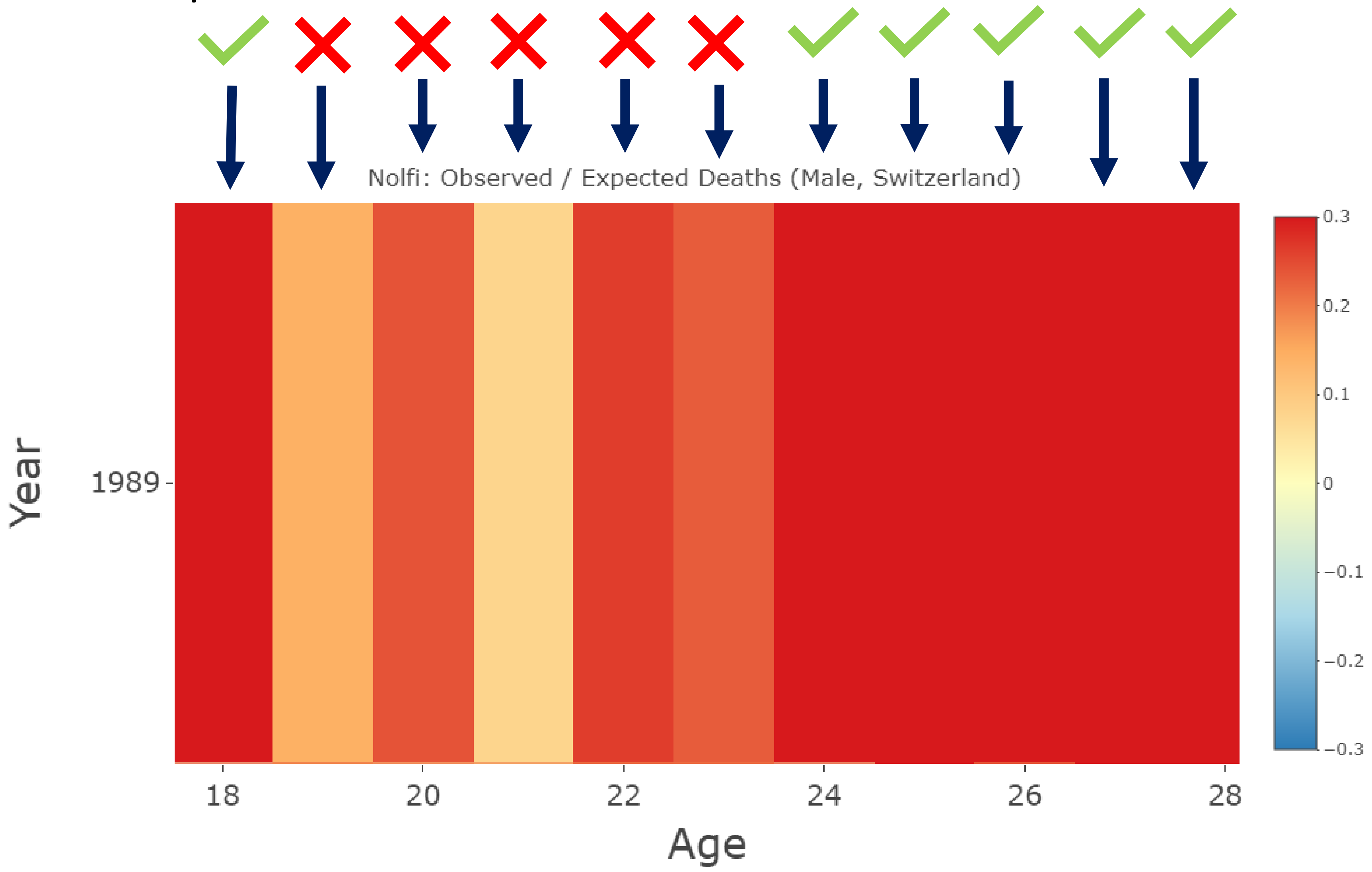


Identifying Singular Events
Algorithm - Epsilon

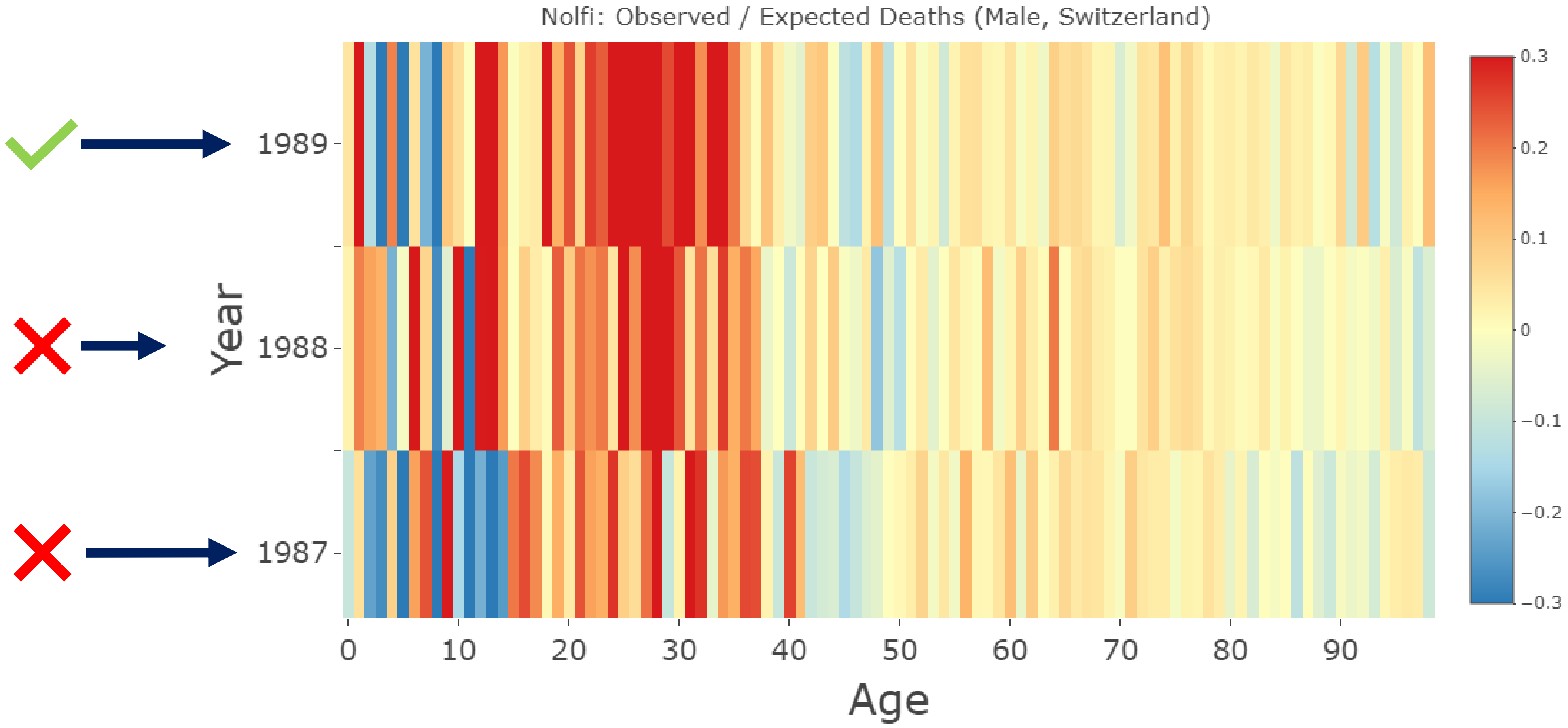


Identifying Singular Events

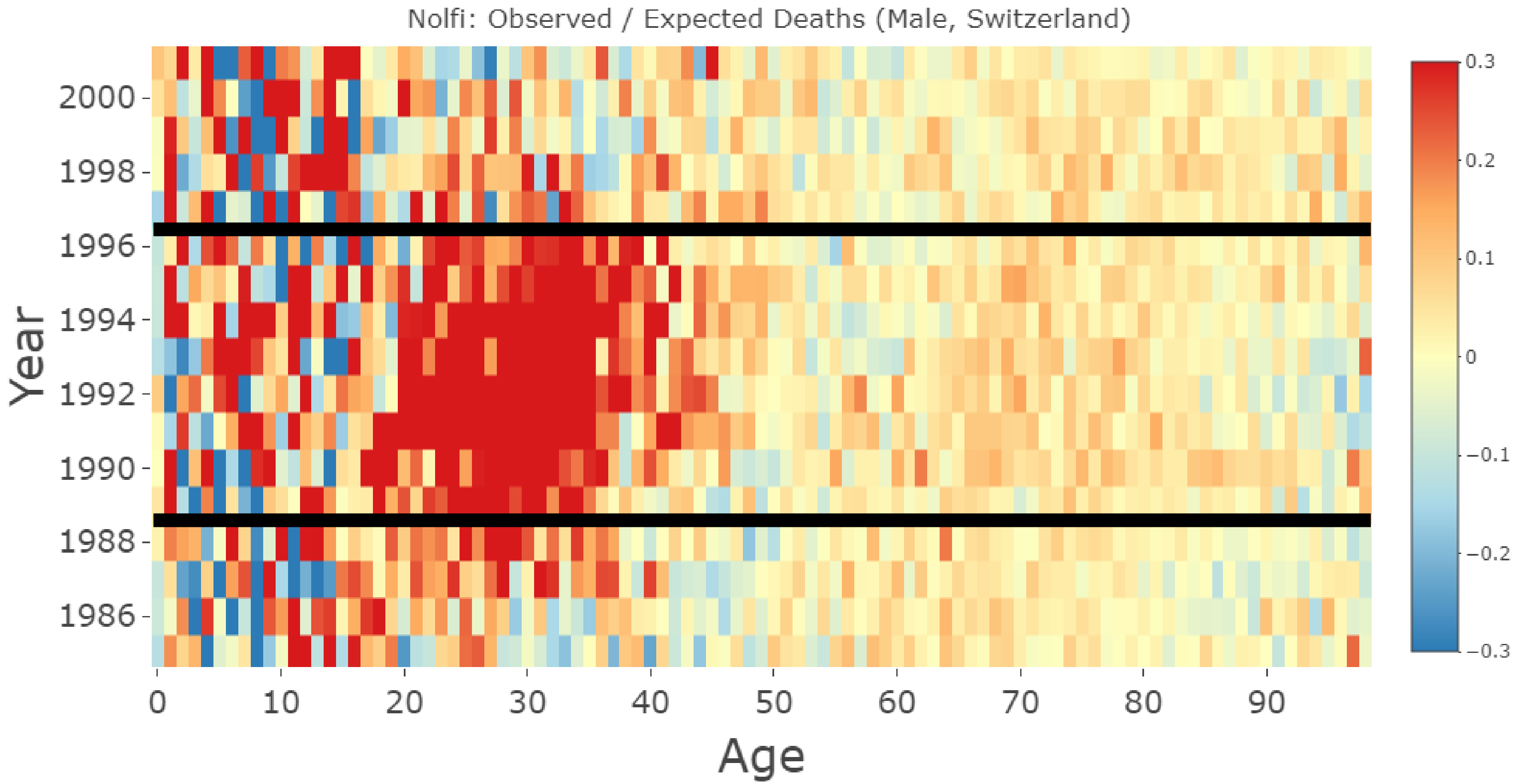
Algorithm - Epsilon



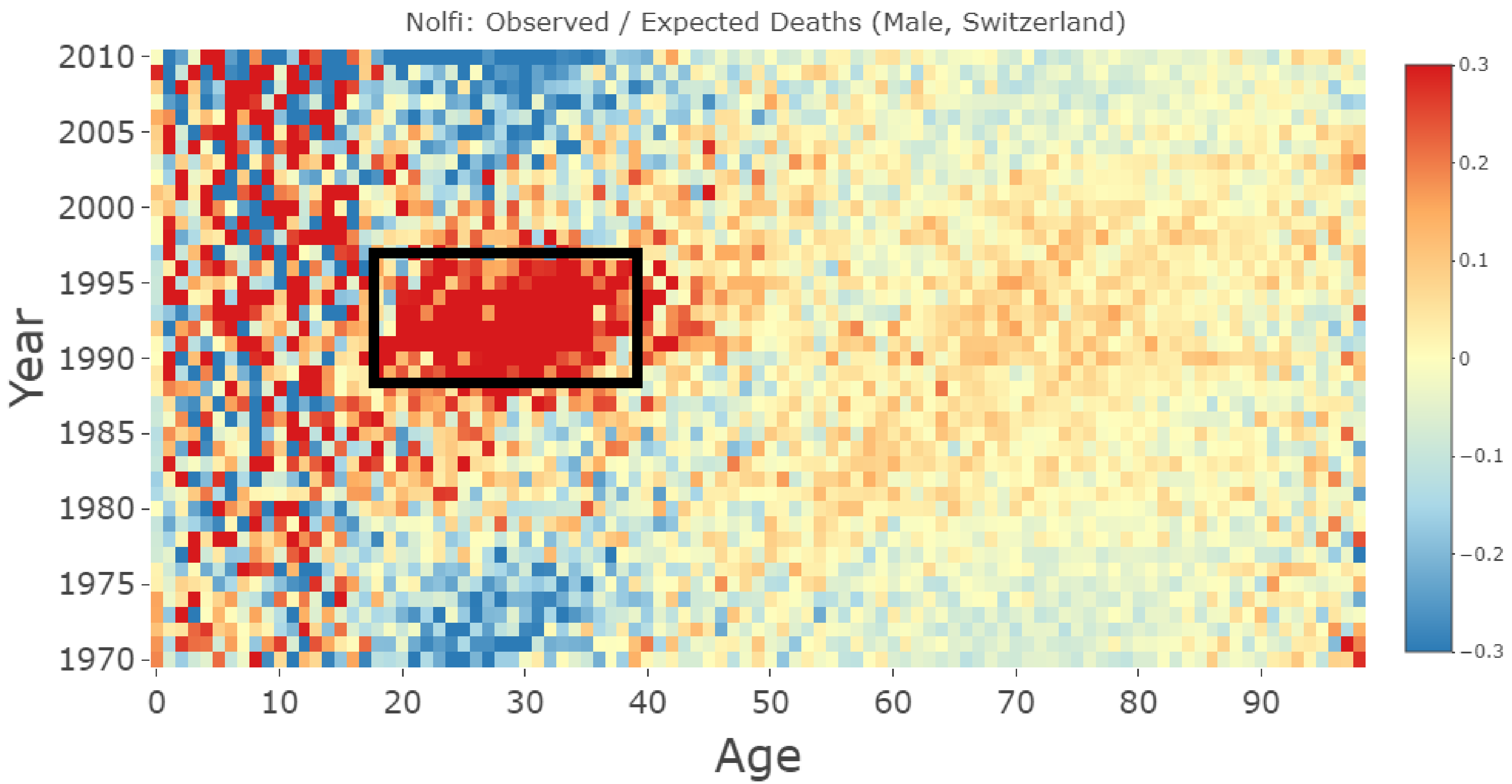
Identifying Singular Events
Algorithm - Gamma



Identifying Singular Events
Algorithm - Alpha

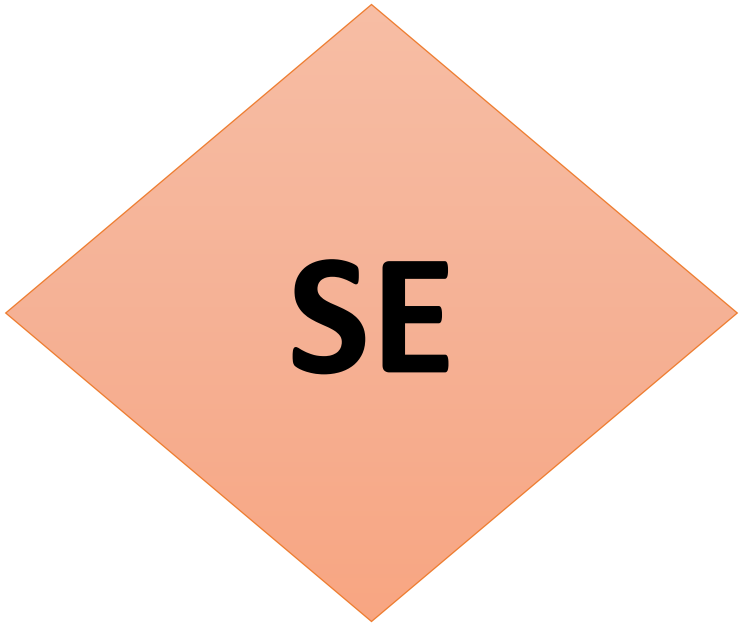


Identifying Singular Events
Algorithm



Identifying Singular Events

Singular Event Diagram



Correction Methods for Singular Events

Correction Methods for Singular Events
Correction

$$\widetilde{D}_x(t) = \begin{cases} z\widehat{D}_x(t) + (1-z)D_x(t) & if \ \ (x,t) \in \Omega^{(se)} \\ D_x(t) & if \ \ (x,t) \notin \Omega^{(se)} \end{cases}$$

where

... z: Credibility Weight,

... $\widehat{D}_x(t)$: Estimated number of deaths,

... $D_x(t)$: Observed number of deaths and

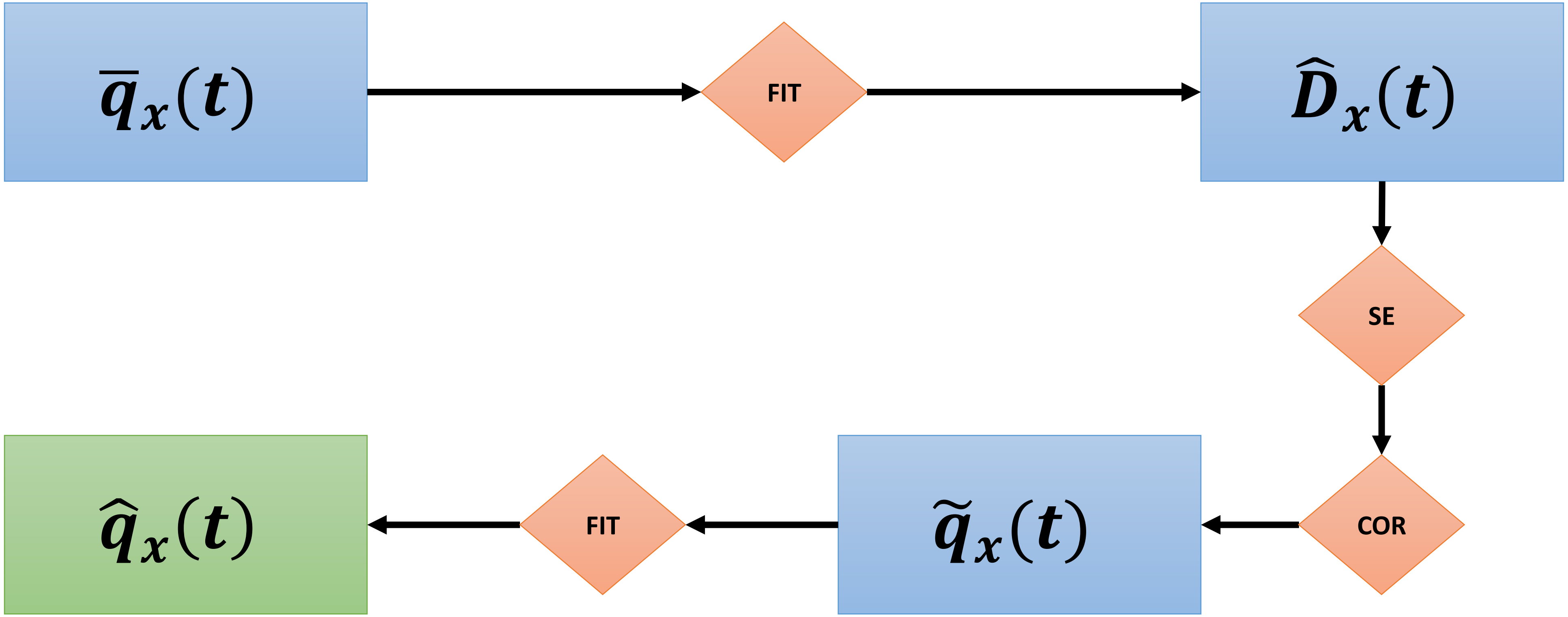
... $\Omega^{(se)}$: Set of tuples (x,t) within singular event.

Correction Methods for Singular Events

Correction Symbol

Correction Methods for Singular Events

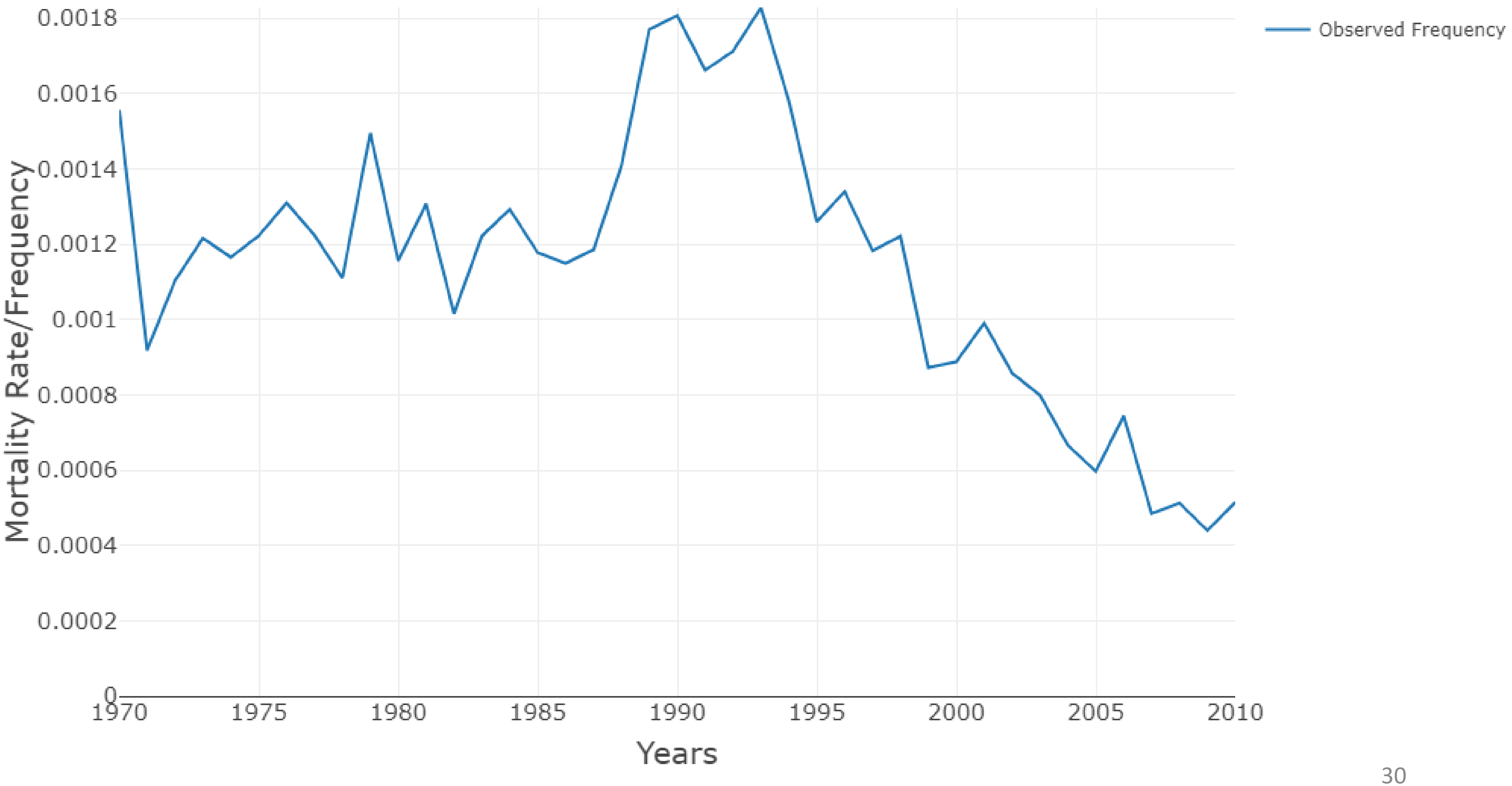
Single Correction Method



Correction Methods for Singular Events

Single Correction Method - Steps

$\bar{q}_x(t)$

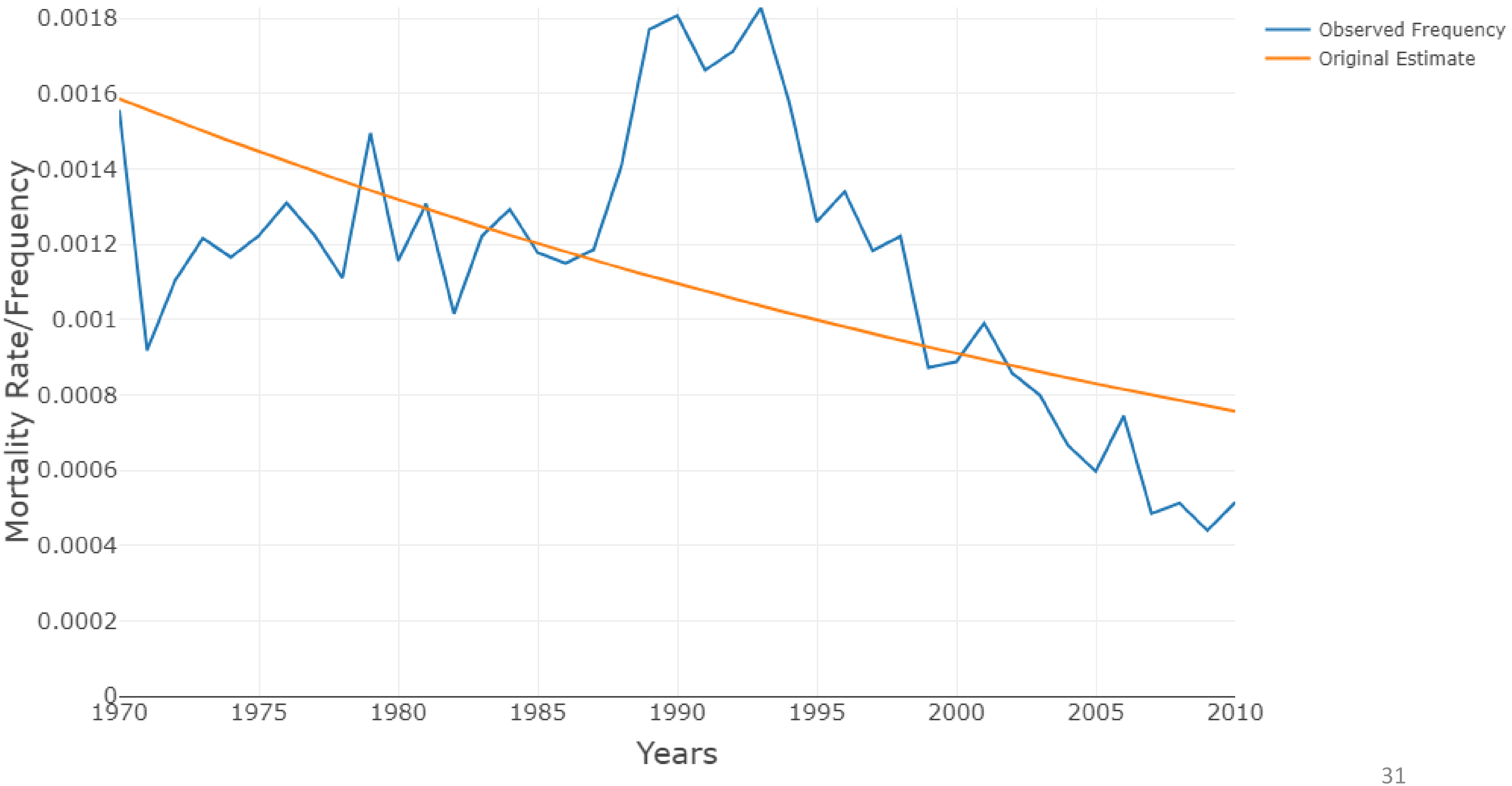


Correction Methods for Singular Events

Single Correction Method - Steps

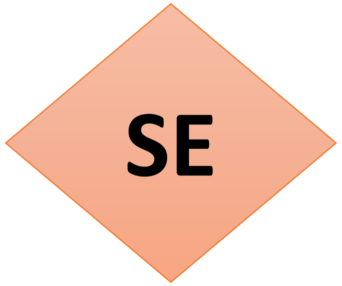
FIT

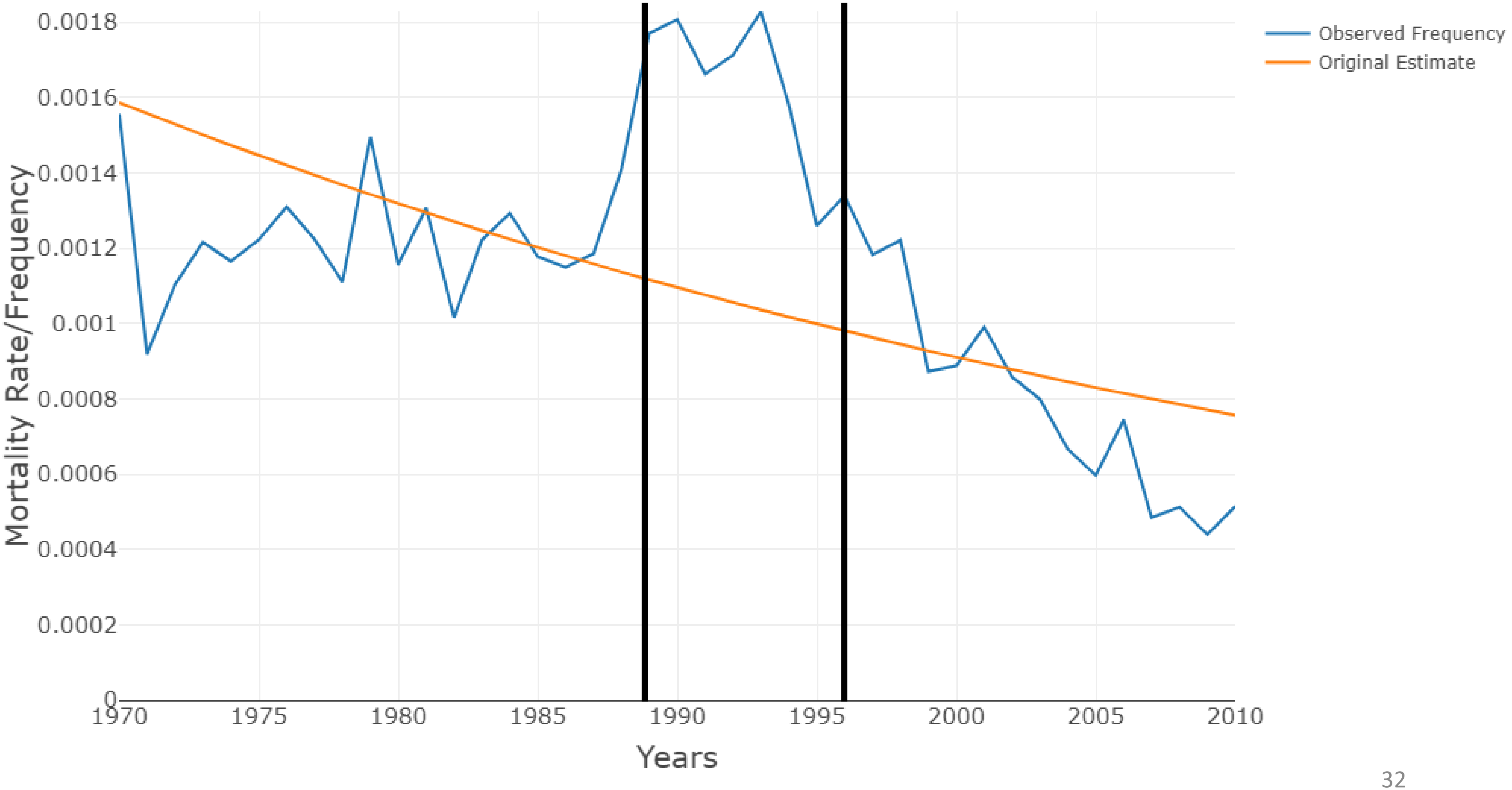
$$\widehat{D}_x(t)$$



Correction Methods for Singular Events

Single Correction Method - Steps



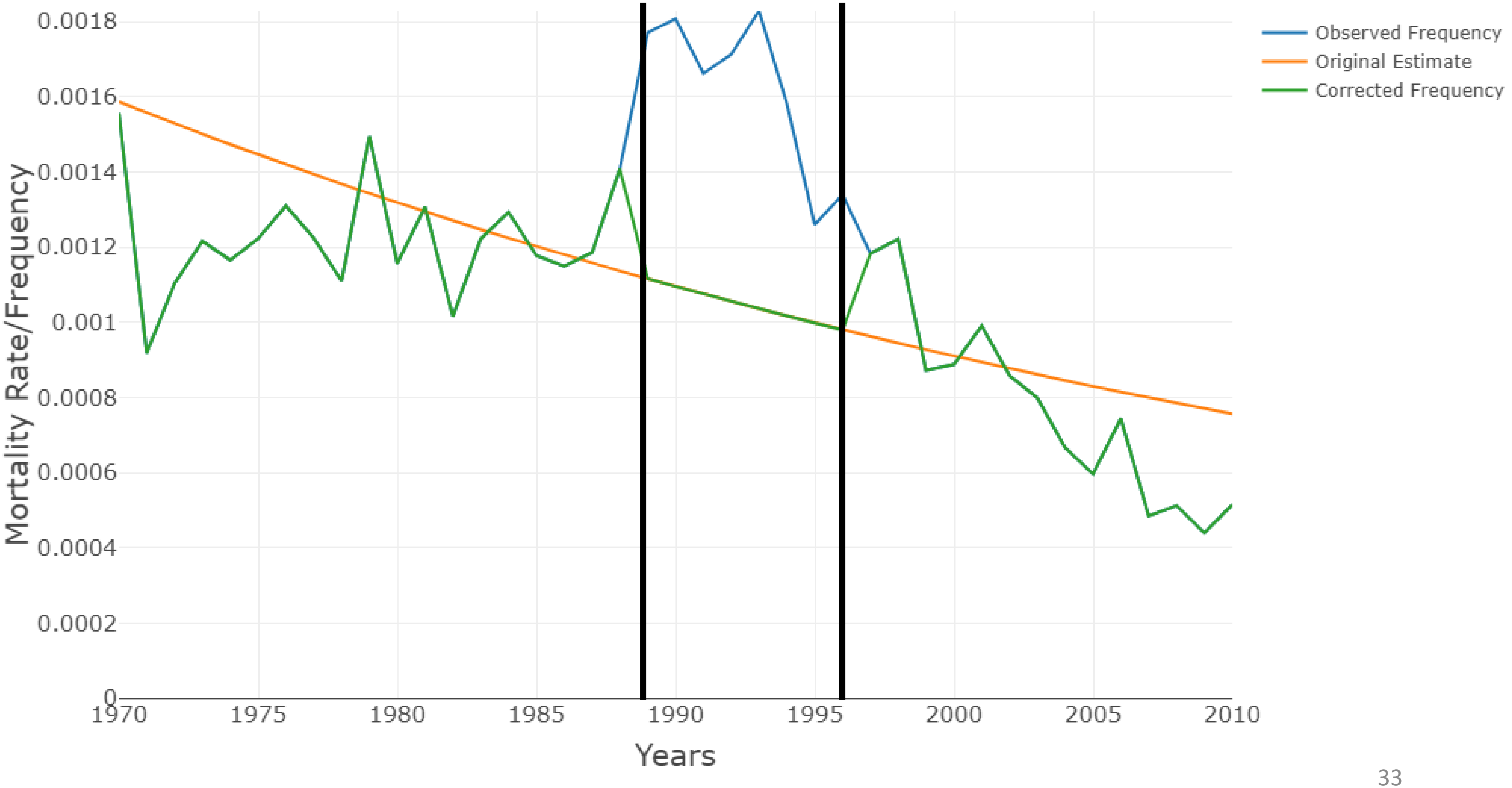


Correction Methods for Singular Events

Single Correction Method - Steps

COR

$$\tilde{q}_x(t)$$

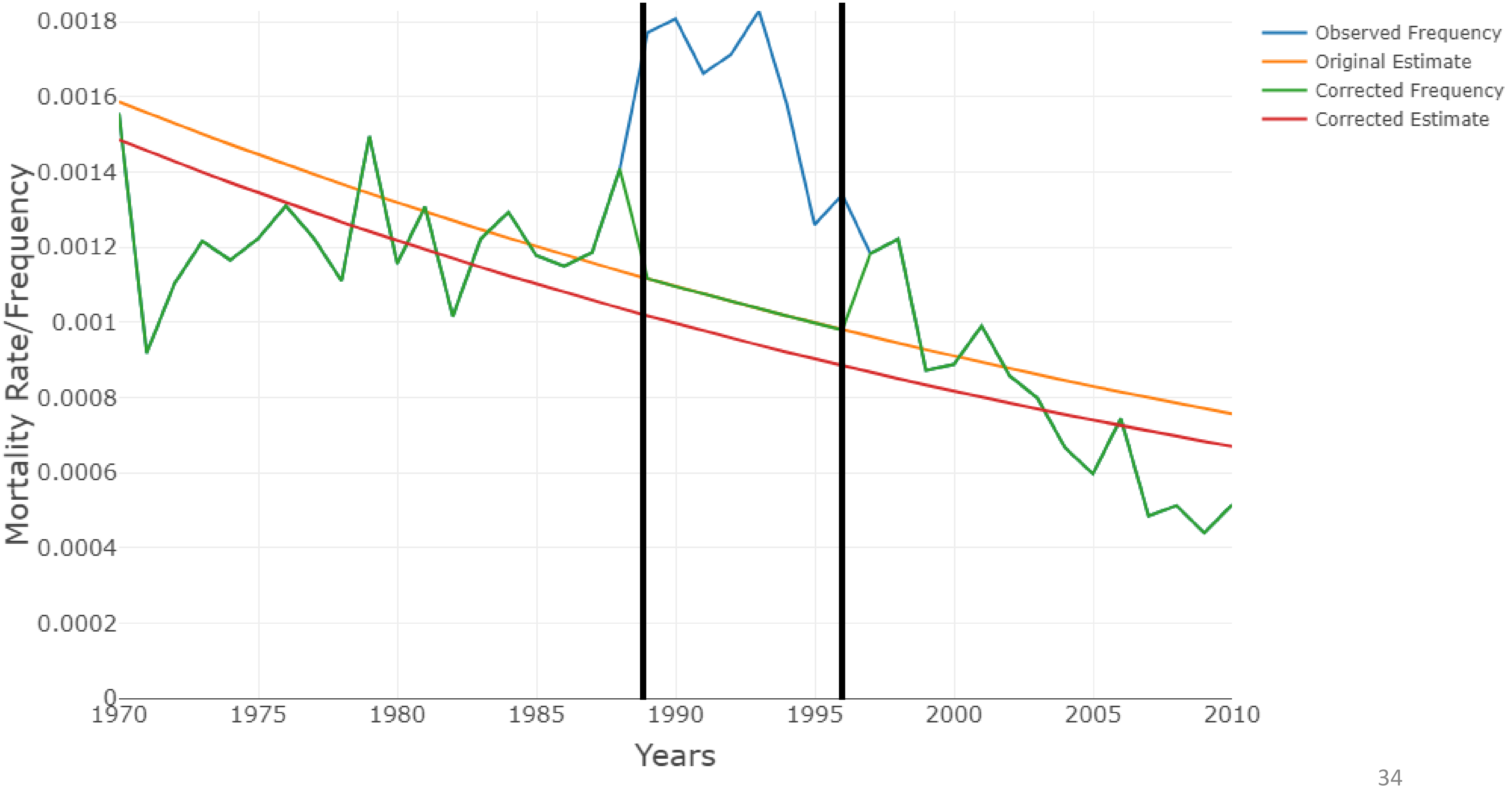


Correction Methods for Singular Events

Single Correction Method - Steps

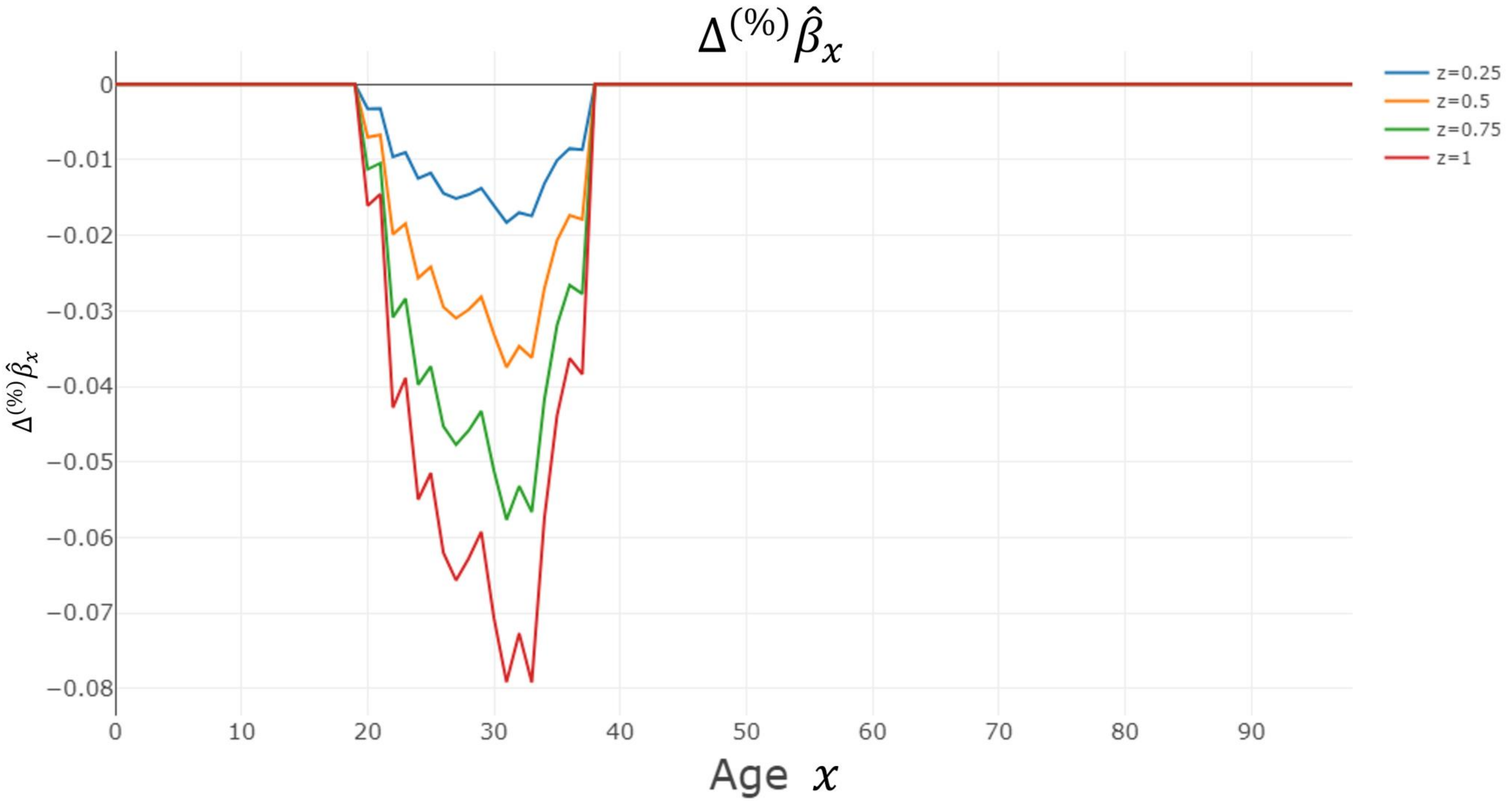
FIT

$$\hat{q}_x(t)$$



Correction Methods for Singular Events

Single Correction Method – Parameter Differences

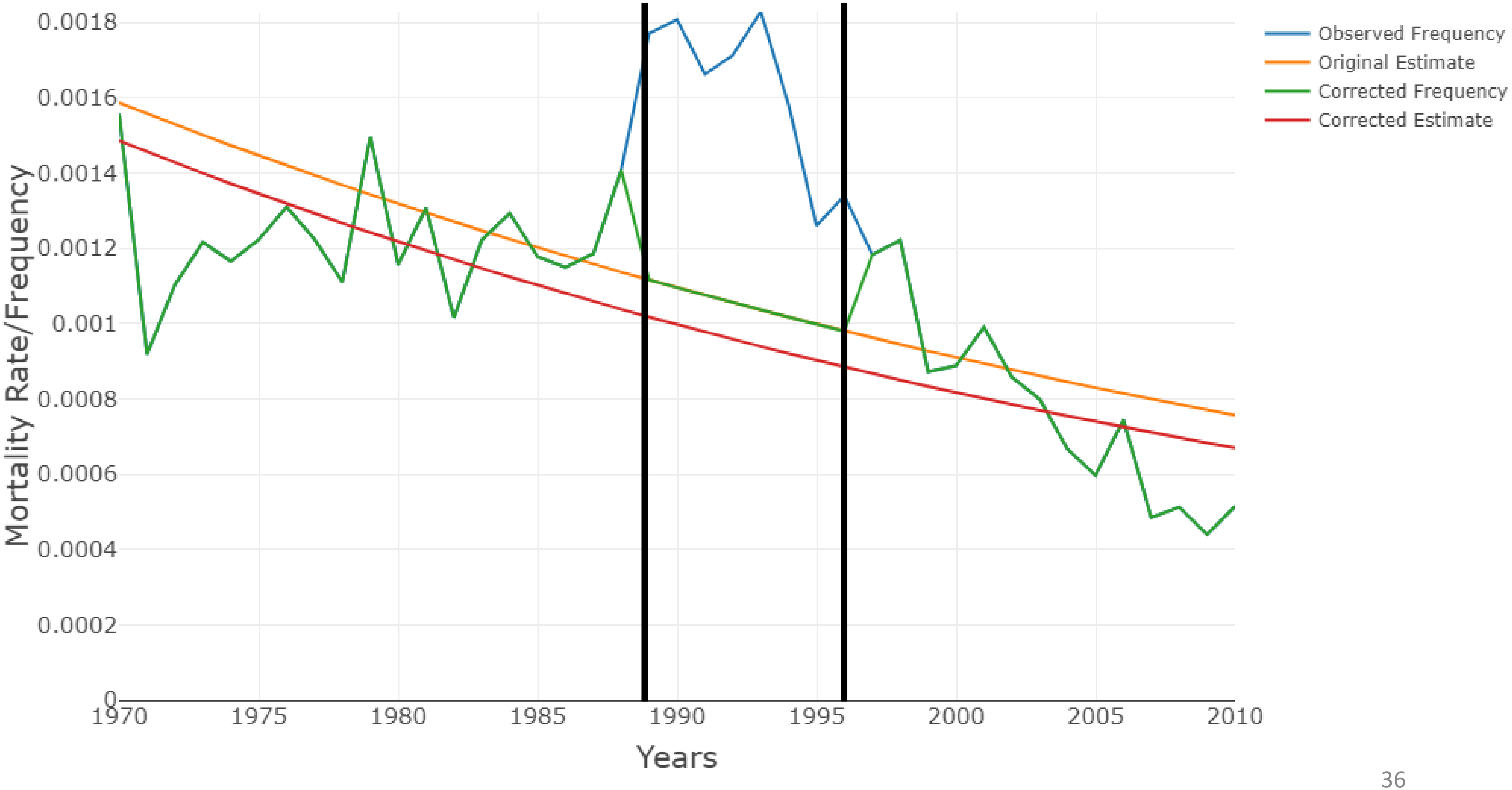


Correction Methods for Singular Events

Single Correction Method - Steps

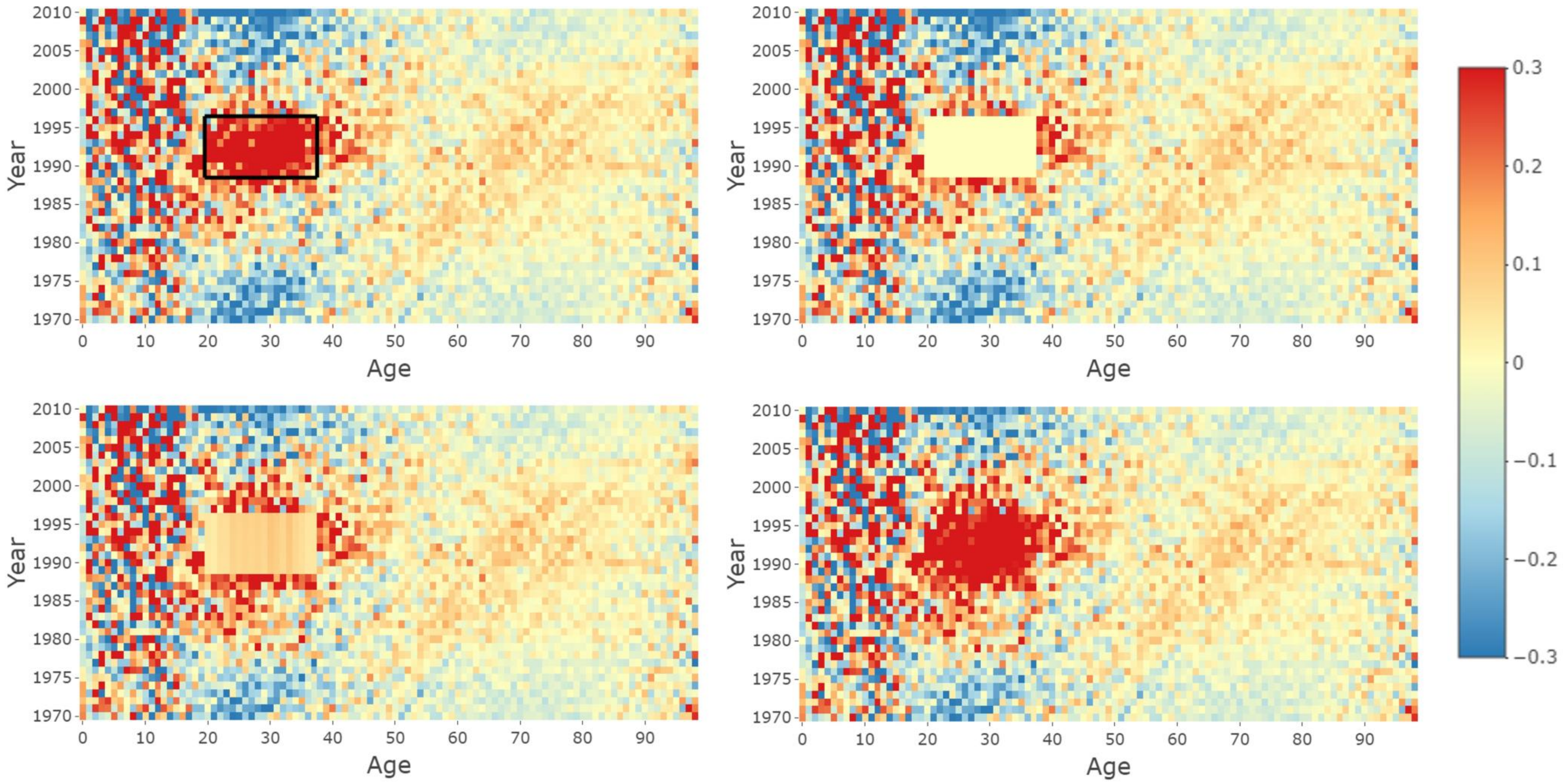
FIT

$\hat{q}_x(t)$



Correction Methods for Singular Events

Single Correction Method – Steps 3D



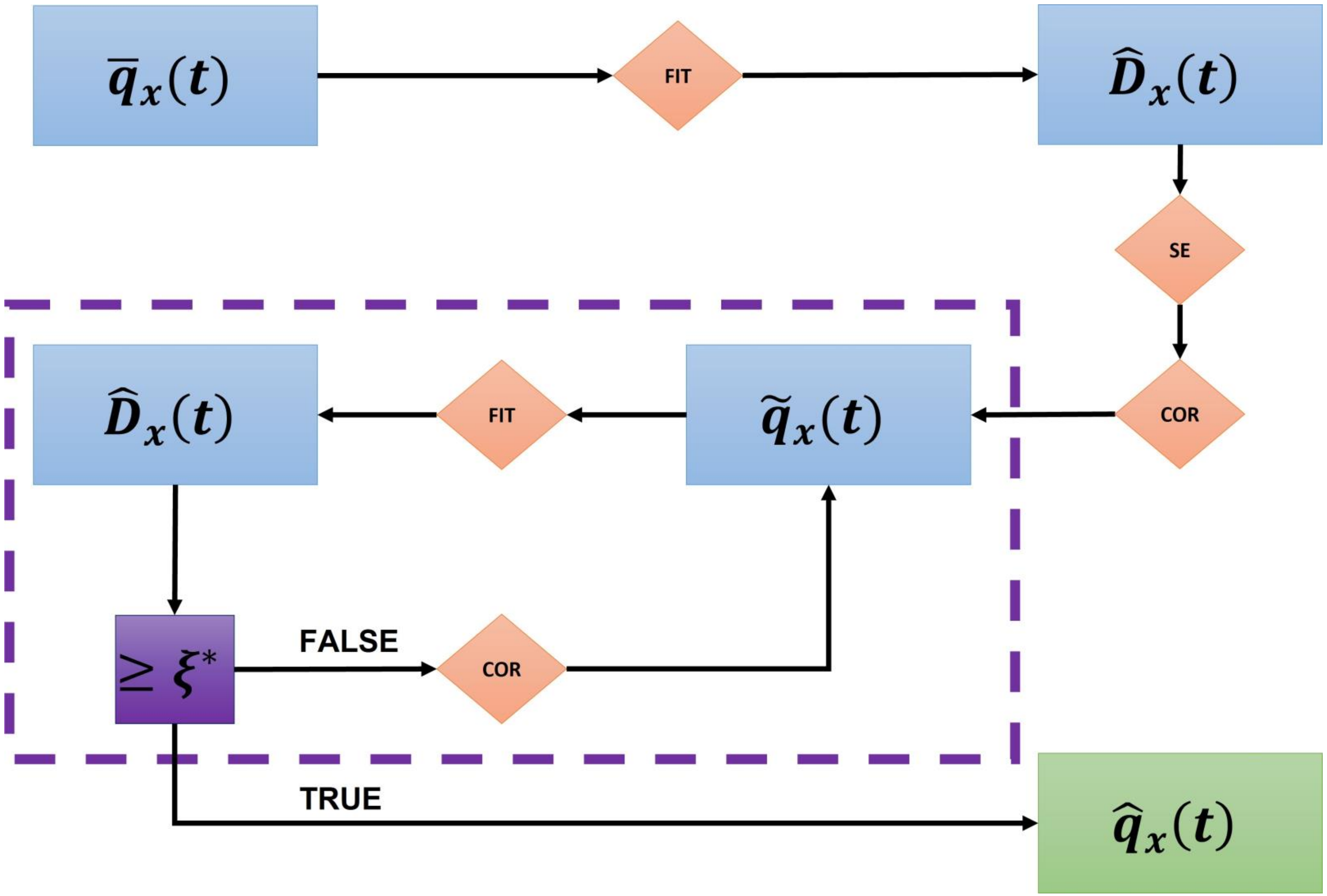
Correction Methods for Singular Events

Single Correction Method – Error Improvement

After
Before

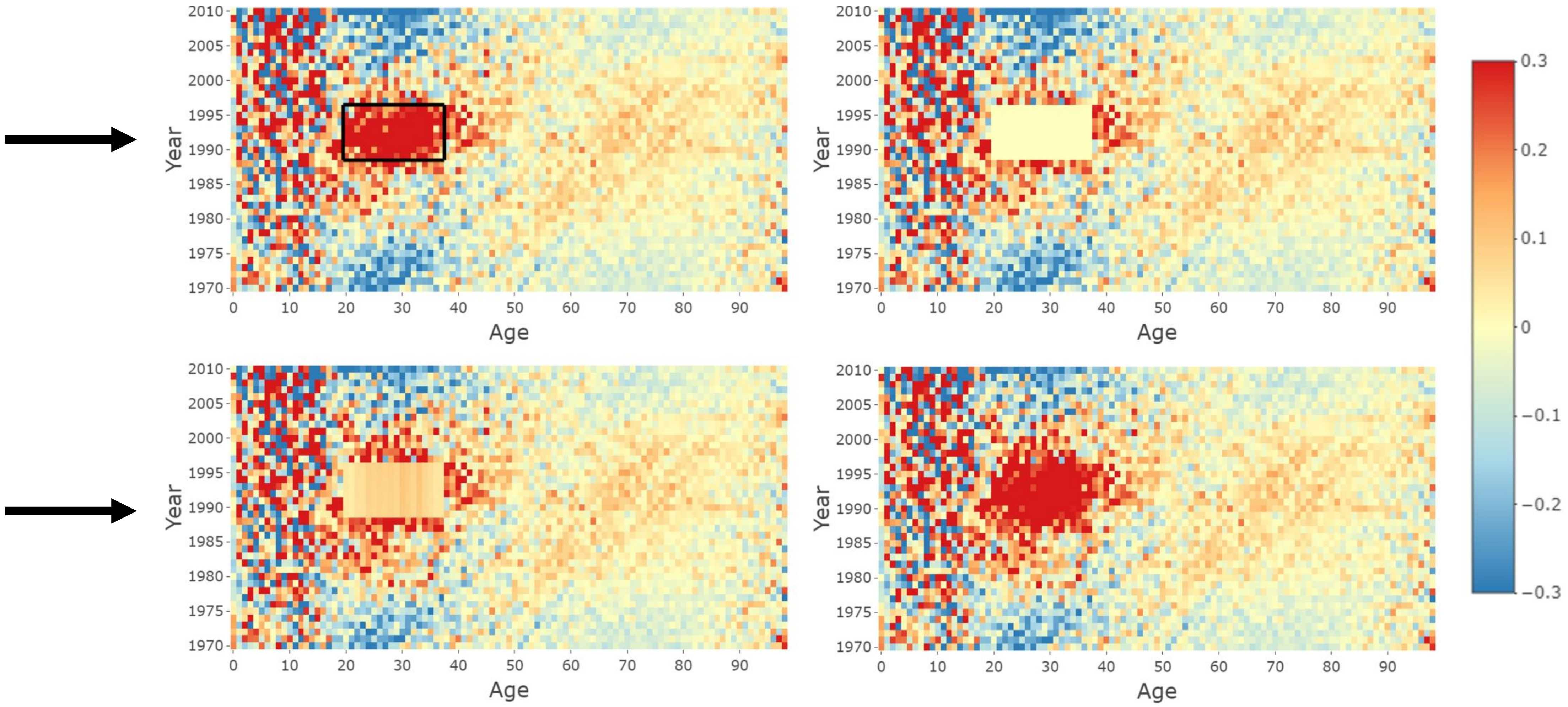
Correction Methods for Singular Events

Stationary Iterative Correction Method



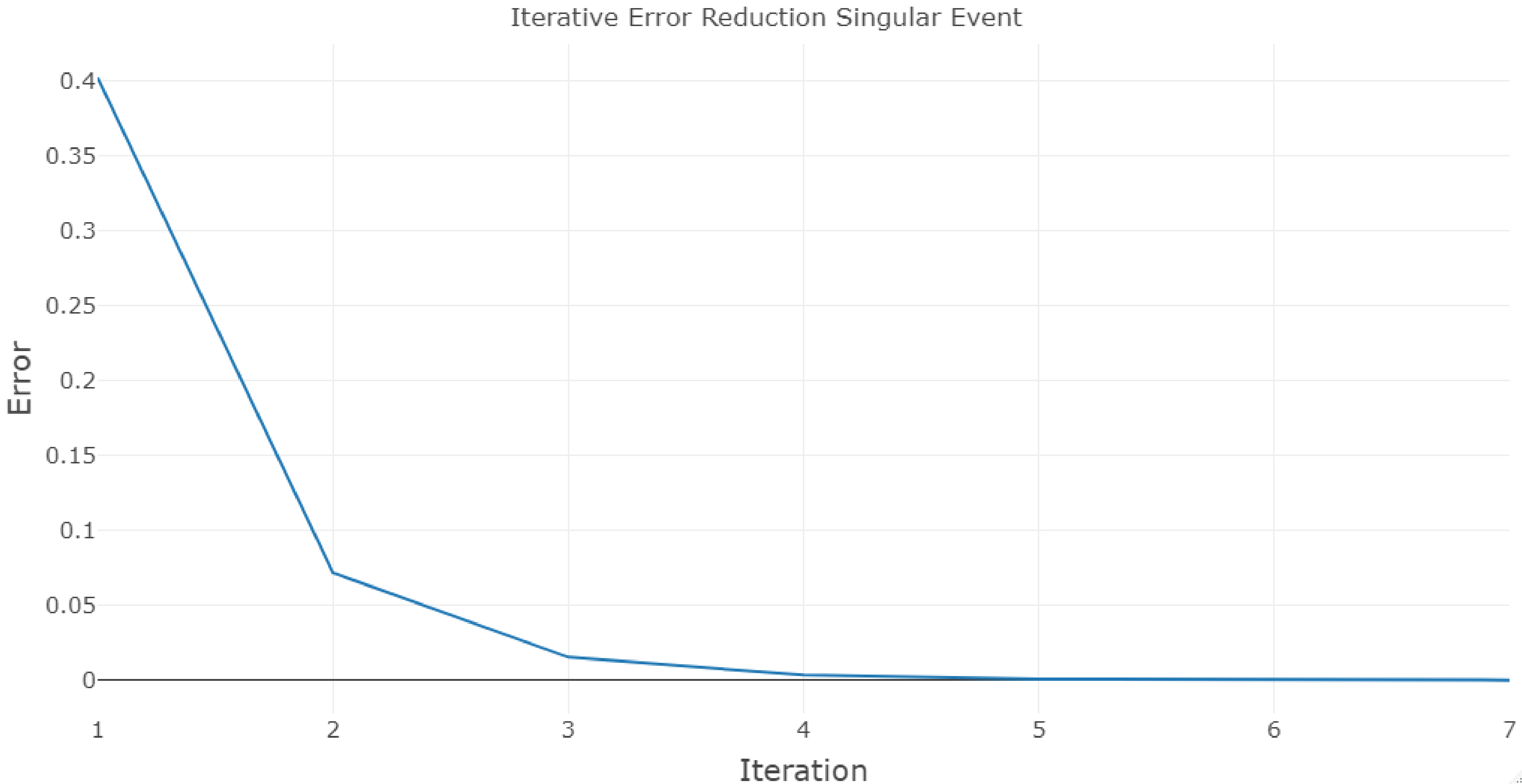
Correction Methods for Singular Events

Stationary Iterative Correction Method – SE Error



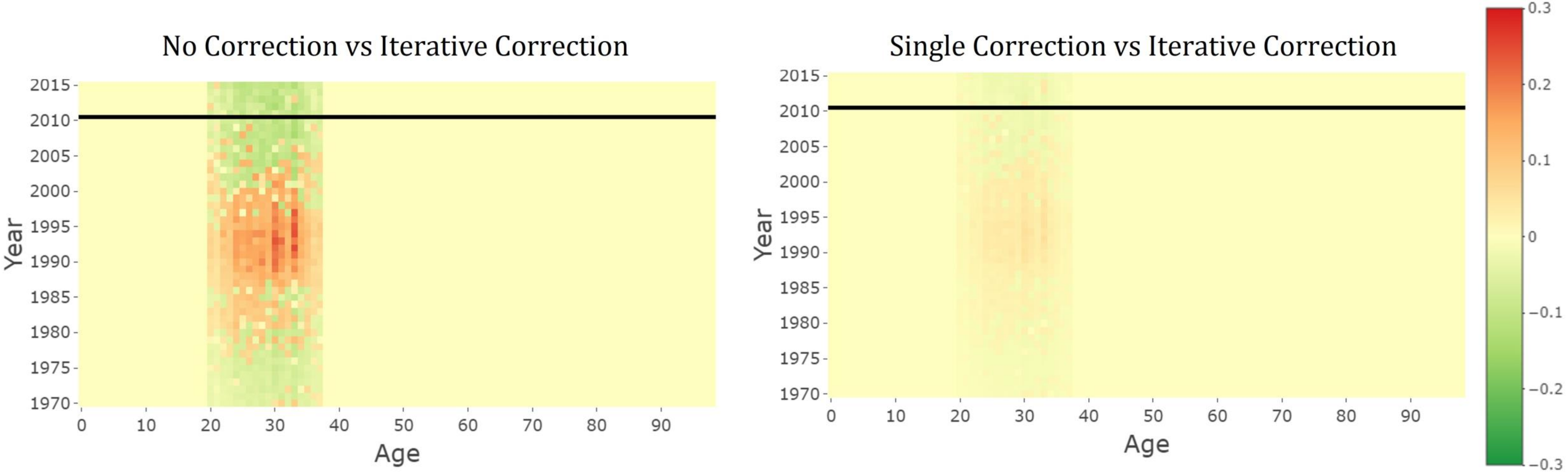
Correction Methods for Singular Events

Stationary Iterative Correction Method – SE Error



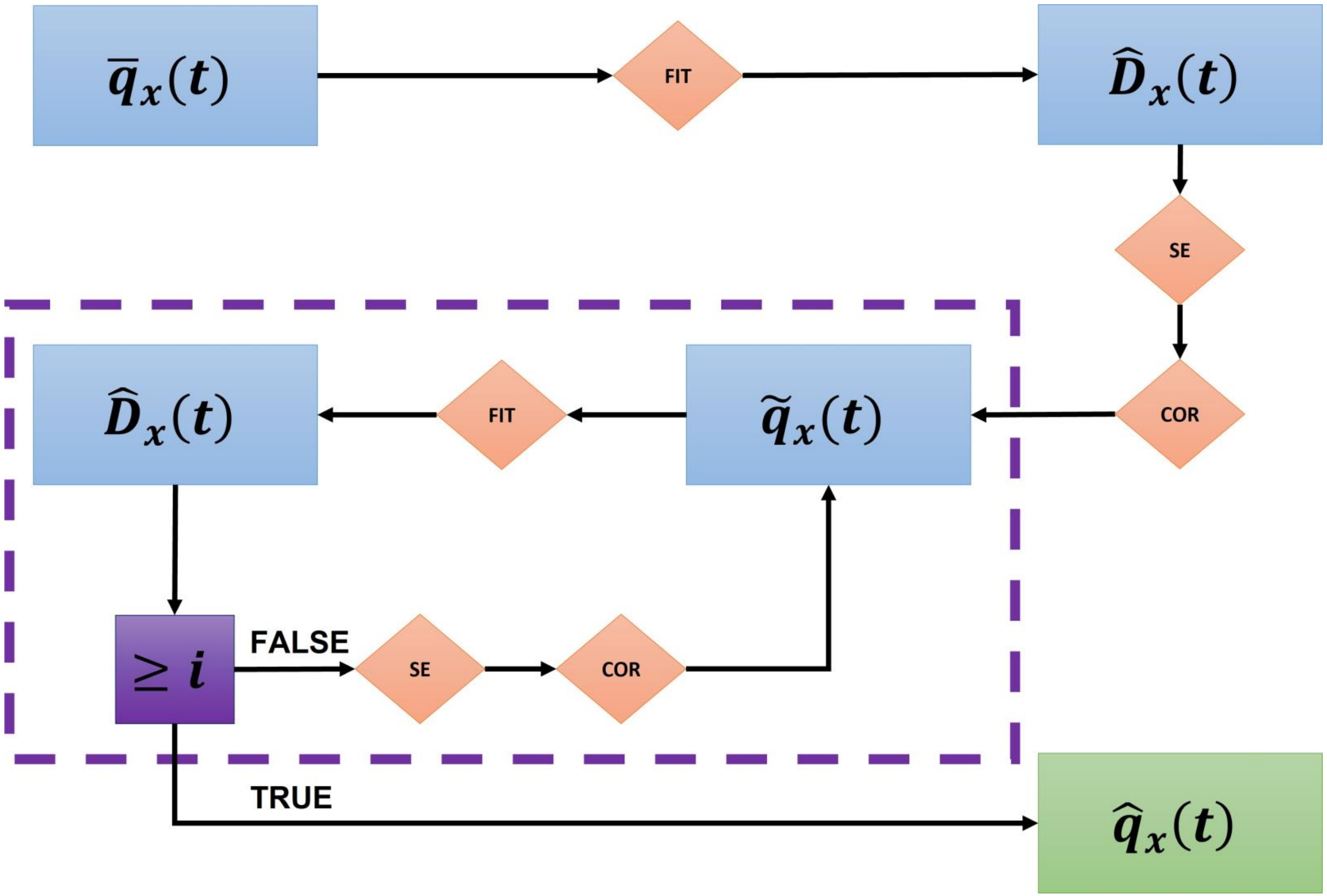
Correction Methods for Singular Events

Stationary Iterative Correction Method – Improvement



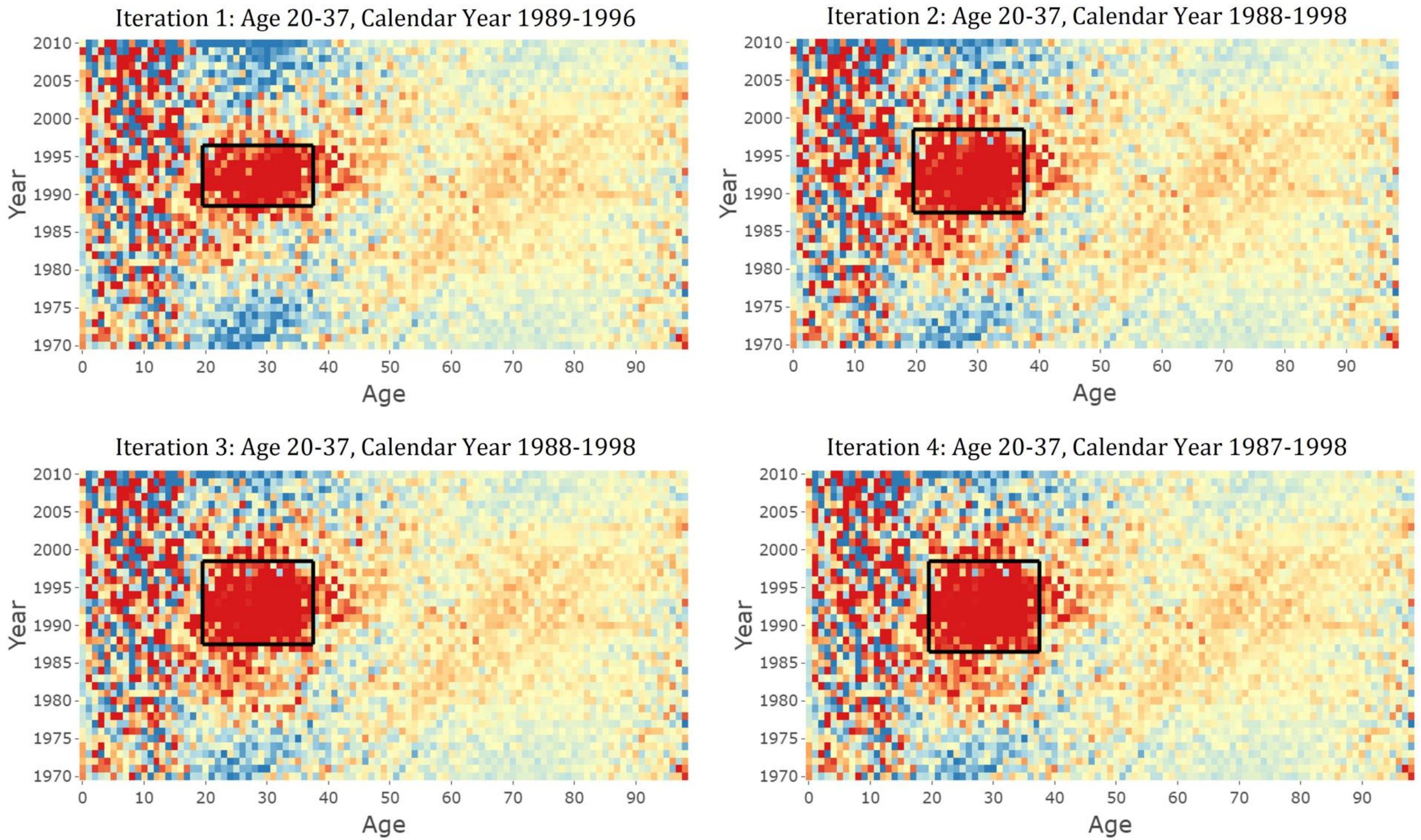
Correction Methods for Singular Events

Moving Iterative Correction Method

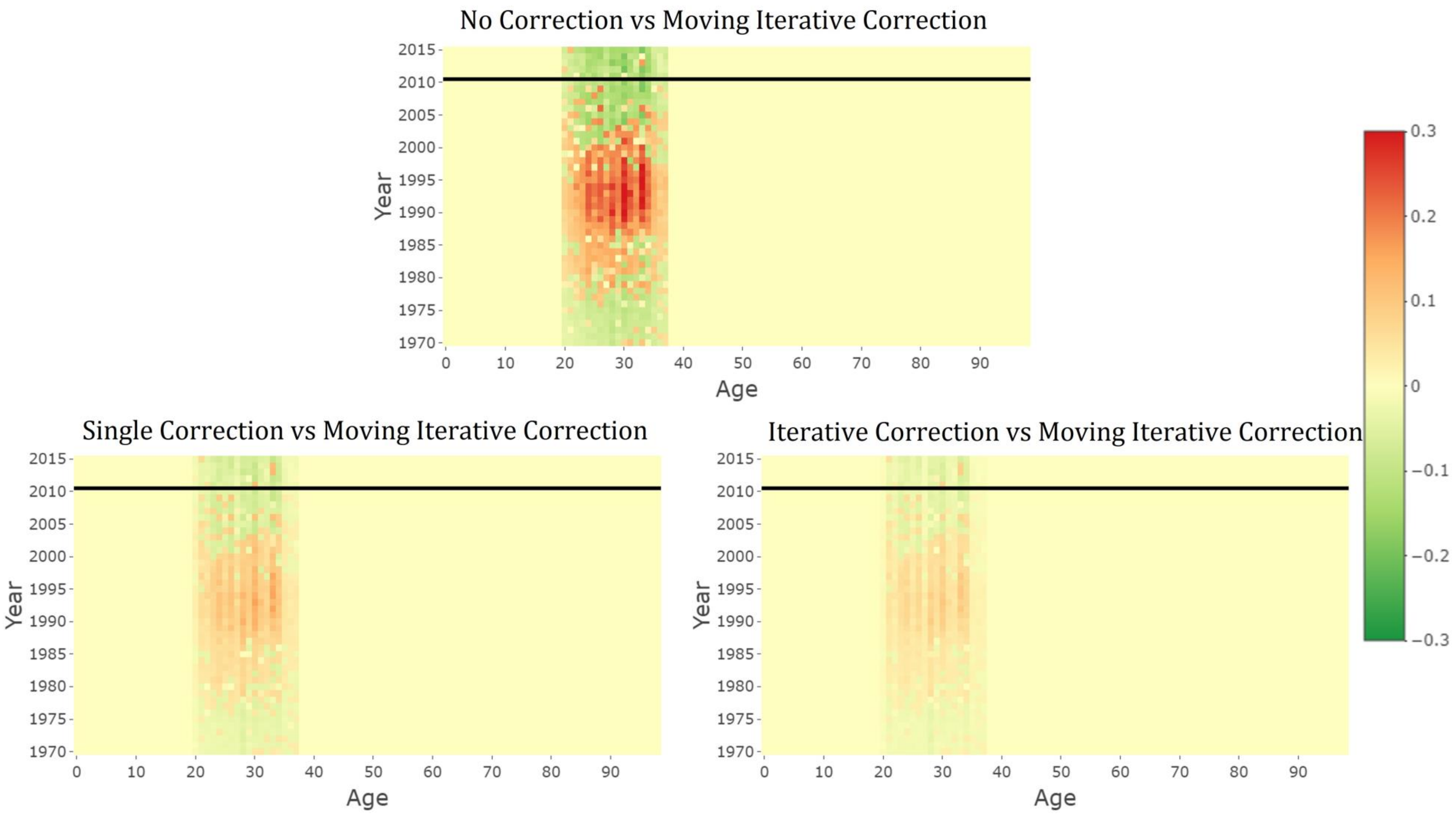


Correction Methods for Singular Events

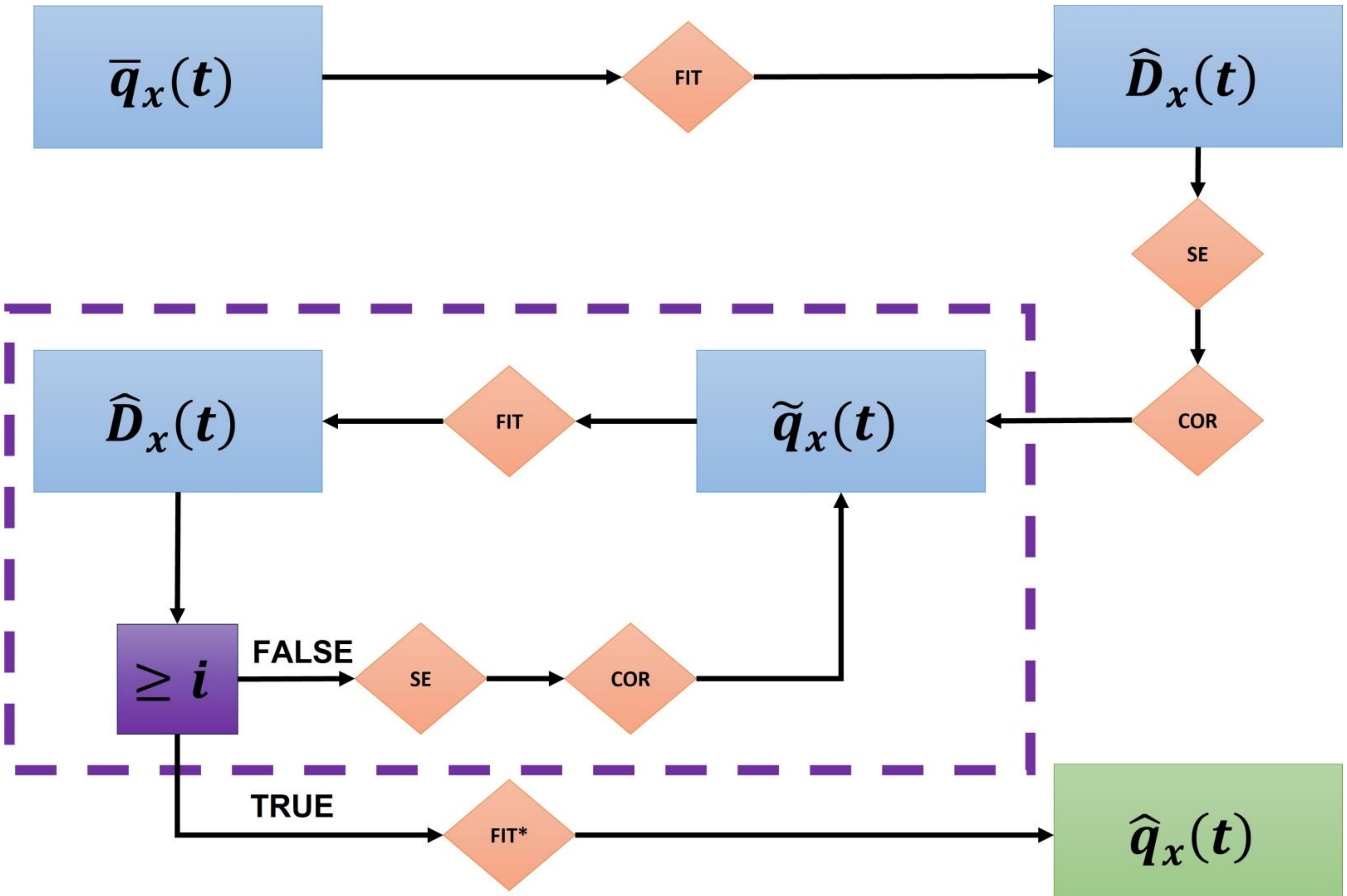
Final Fit Correction Method – Growing SE



Correction Methods for Singular Events
Final Fit Correction Method – Improvement

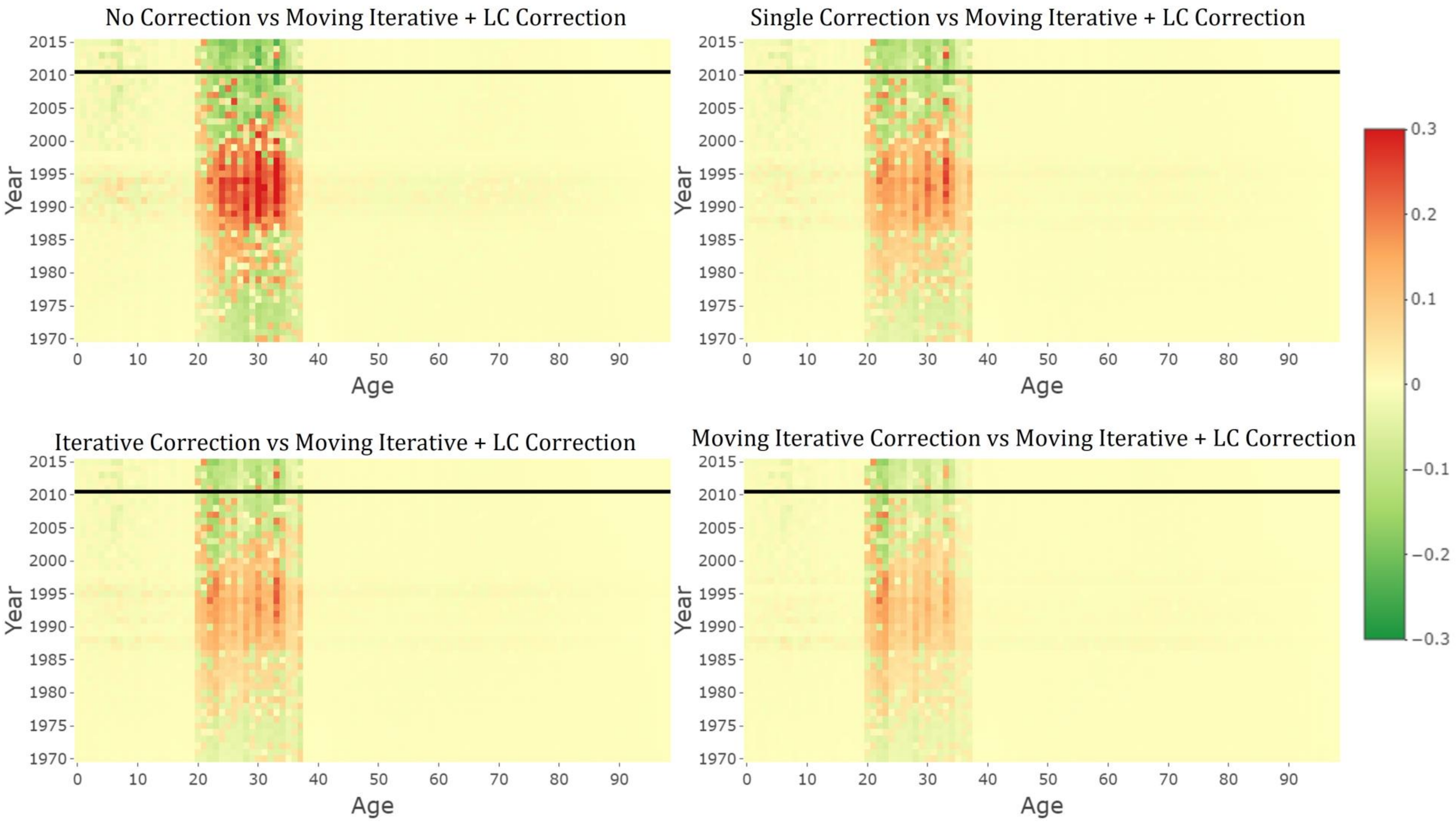


Correction Methods for Singular Events
Final Fit Correction Method



Correction Methods for Singular Events

Final Fit Correction Method – Error Improvement



Conclusion

- Singular events distort parameters

- Correcting for the singular event reduces distortion
  - Improves estimation quality
  - Improves projection quality

- Showed improvements for:
  - AIDS
  - Spanish Flu (see Appendix)
  - WW I (see Appendix)
  - WW II (see Appendix)

- COVID-19

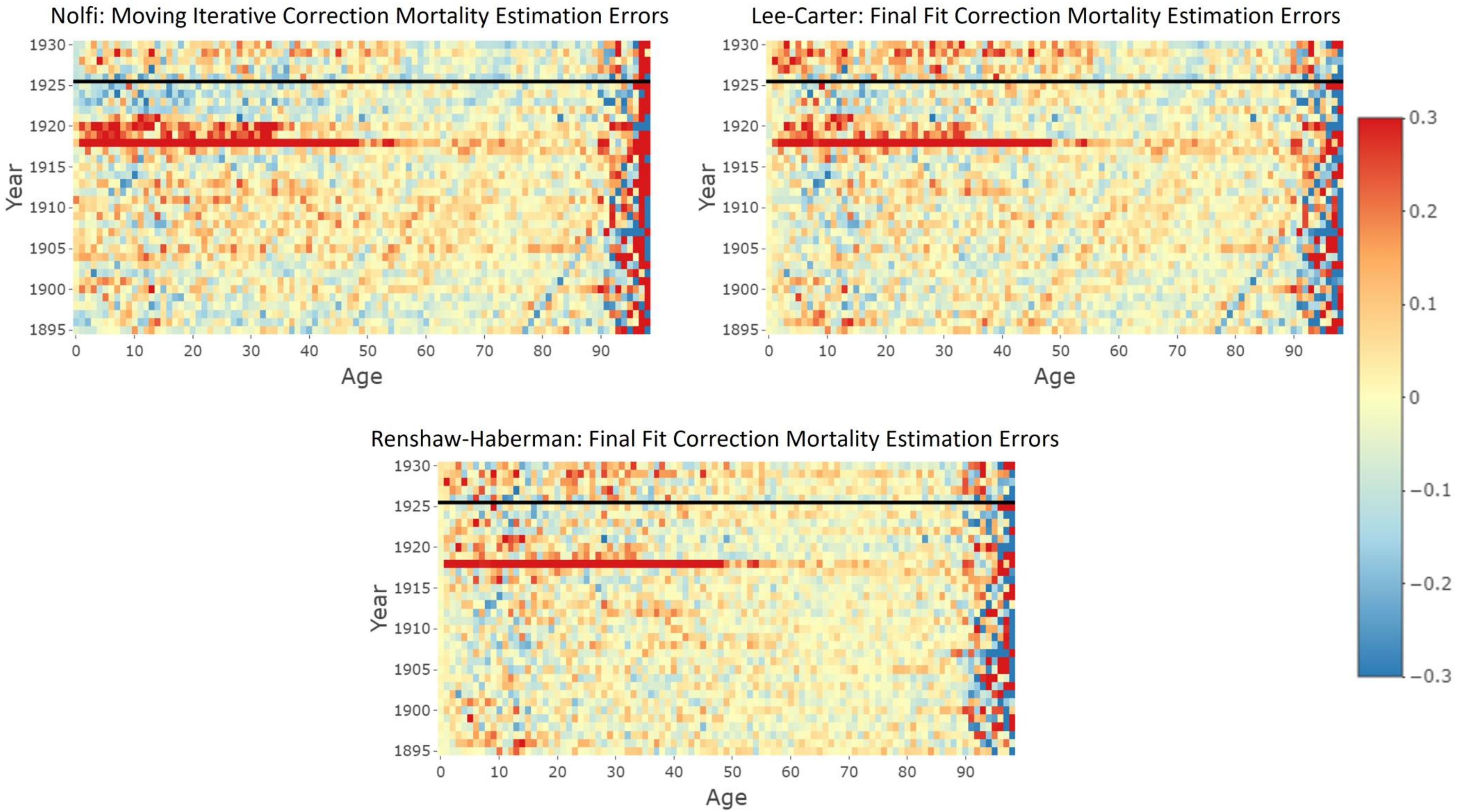
?

References

- Spanish Flu: By Otis Historical Archives, National Museum of Health and Medicine - Emergency hospital during influenza epidemic (NCP 1603), National Museum of Health and Medicine.https://www.buckscountycouriertimes.com/news/20190923/mxfctter-museum-to-mark-historic-influenza-pandemic/1, Public Domain, https://commons.wikimedia.org/w/index.php?curid=25513204
- Human Mortality Database: mortality.org

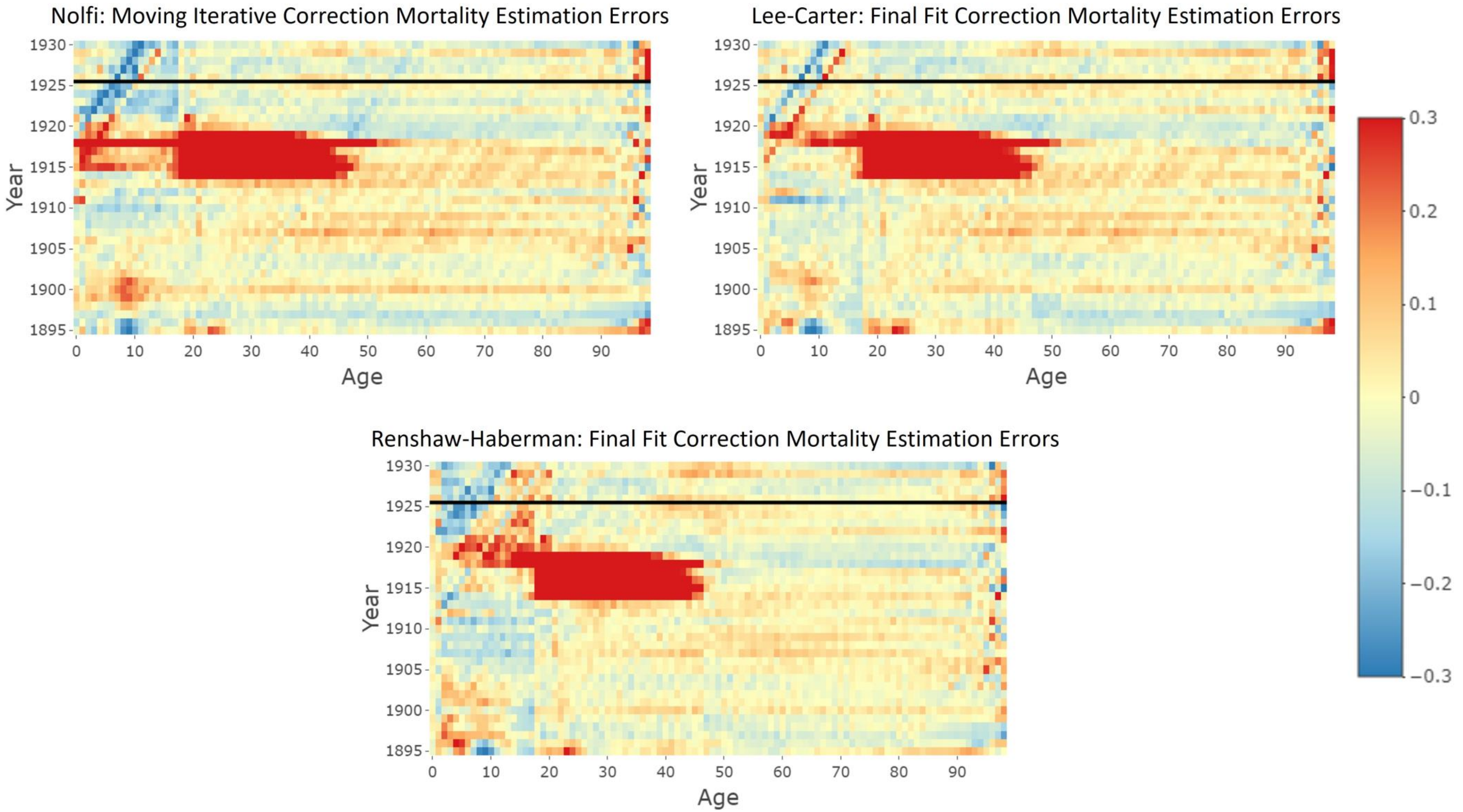
Appendix

Spanish Flu

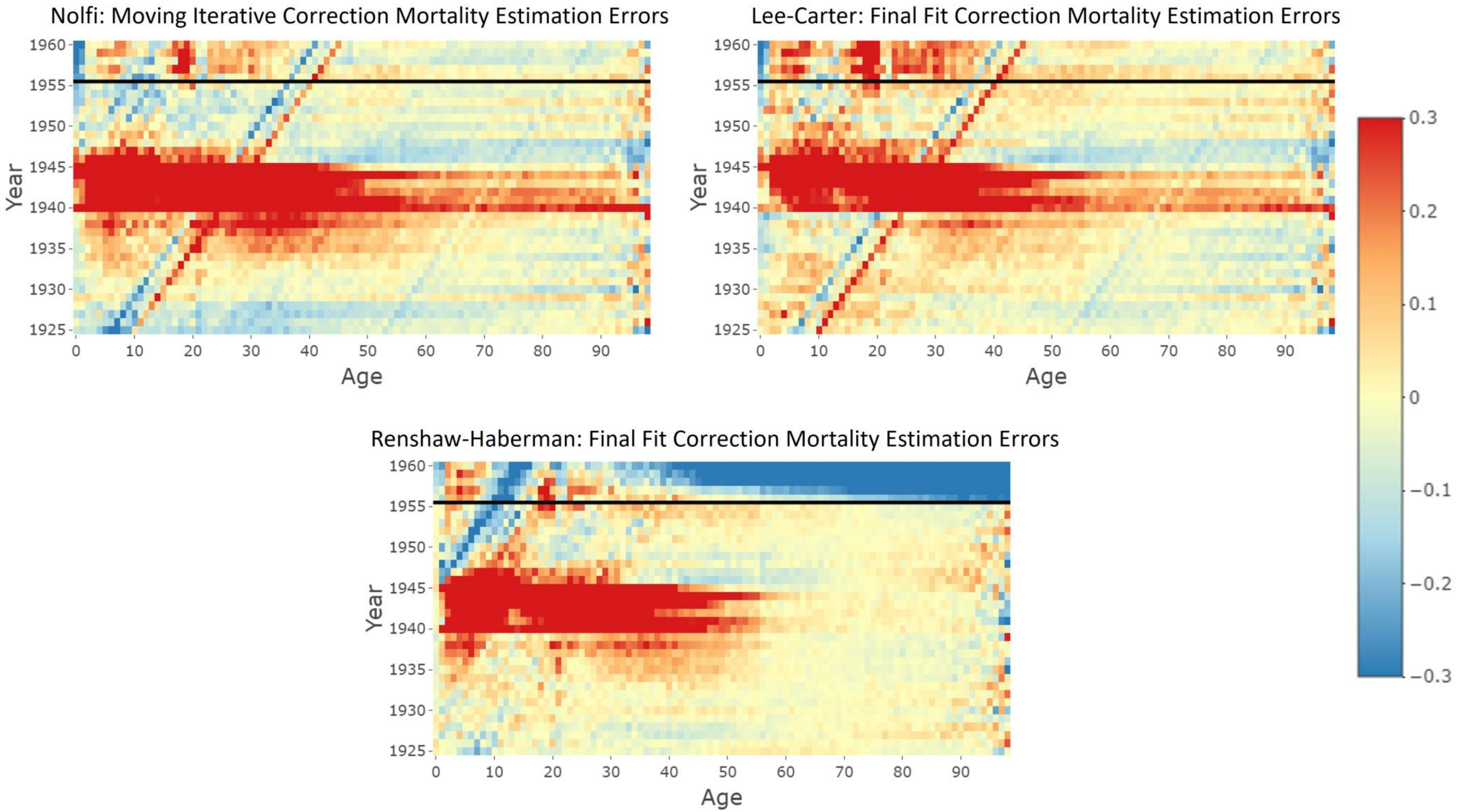


Nolfi: Moving Iterative Correction Mortality Estimation Errors
Lee-Carter: Final Fit Correction Mortality Estimation Errors
Renshaw-Haberman: Final Fit Correction Mortality Estimation Errors
Year
1930
1925
1920
1915
1910
1905
1900
1895
Age
0
10
20
30
40
50
60
70
80
90
0.3
0.2
0.1
0
−0.1
−0.2
−0.3

World War I


Nolfi: Moving Iterative Correction Mortality Estimation Errors
Lee-Carter: Final Fit Correction Mortality Estimation Errors
Renshaw-Haberman: Final Fit Correction Mortality Estimation Errors
Year
1930
1925
1920
1915
1910
1905
1900
1895
Age
0
10
20
30
40
50
60
70
80
90
0.3
0.2
0.1
0
−0.1
−0.2
−0.3

World War II


Nolfi: Moving Iterative Correction Mortality Estimation Errors
Lee-Carter: Final Fit Correction Mortality Estimation Errors
Renshaw-Haberman: Final Fit Correction Mortality Estimation Errors
Year
1960
1955
1950
1945
1940
1935
1930
1925
Age
0
10
20
30
40
50
60
70
80
90
0.3
0.2
0.1
0
−0.1
−0.2
−0.3