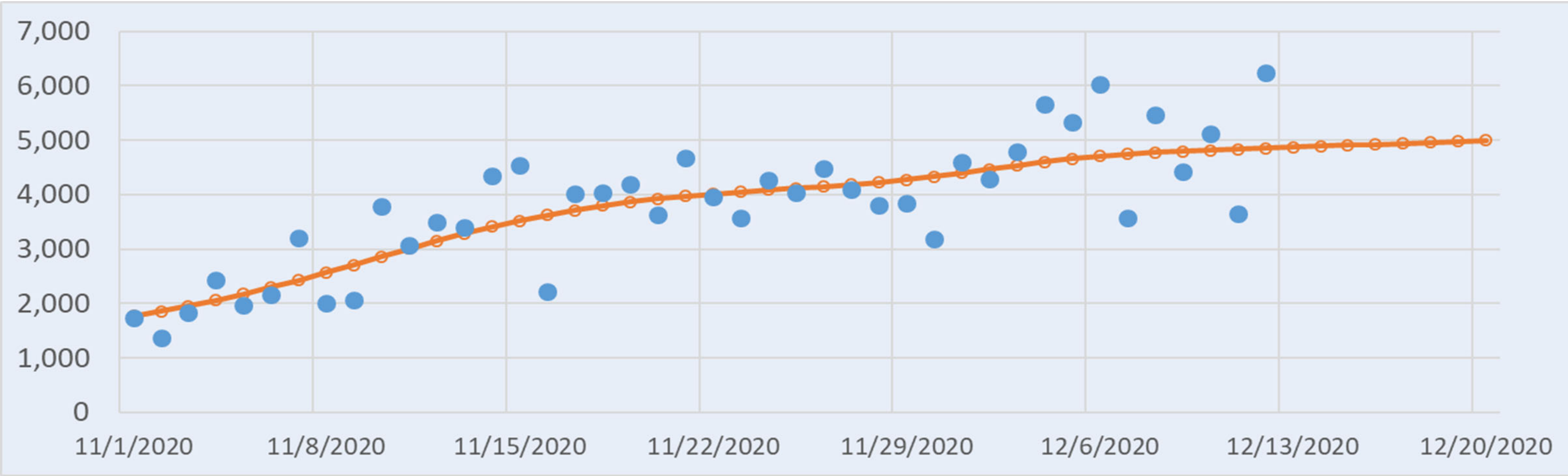

Smoothing Splines for Changing Trends

September 21, 2022
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Agenda

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Business Problem

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Traditionally, trend for frequency or severity has been estimated by fitting log-linear curves to historical data.

This is fine so long as trend is constant across the historical period, but what if trends are changing?

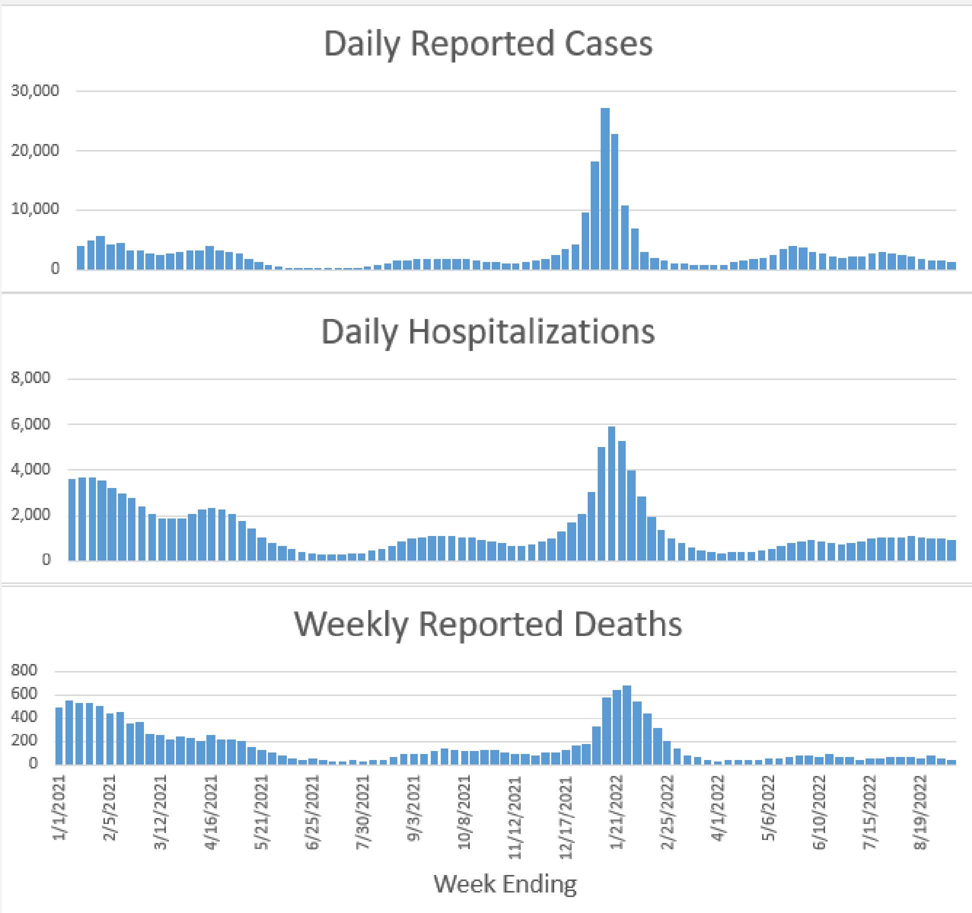
- Impact of economic recessions & recoveries
- Social inflation
- Pandemic stay-at-home

Trend selections try to balance between using one fitted trend for all years versus using the [noisy] actual year-to-year historical changes.

This is the trade-off between bias and variance.

Smoothing methods can help!

Smoothing Splines – COVID example



<u>Time-series related to COVID19 pandemic</u>

Evaluating how the numbers are changing is useful for decision-making.

Should we return to the office?

Should I avoid crowded stores or restaurants?

But the numbers are “noisy” and not always easy to identify a trend.

Numbers from New Jersey

Source: https://www.nj.gov/health/cd/topics/covid2019_dashboard.shtml

Smoothing Splines – COVID example

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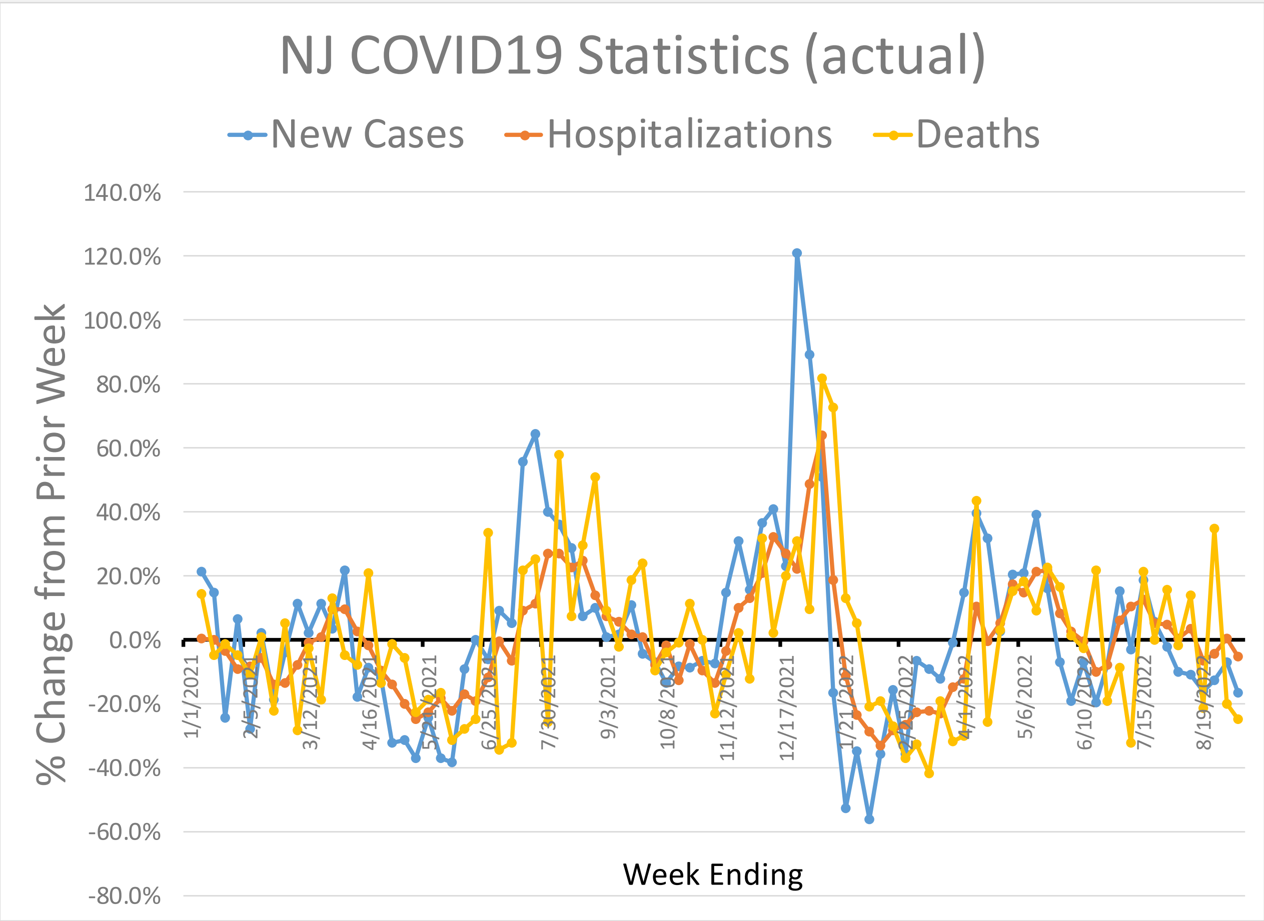

The percent change in the weekly numbers shows a cyclical pattern but not obvious how the different series are related.

Too messy to read ?

Smoothing Splines – COVID example

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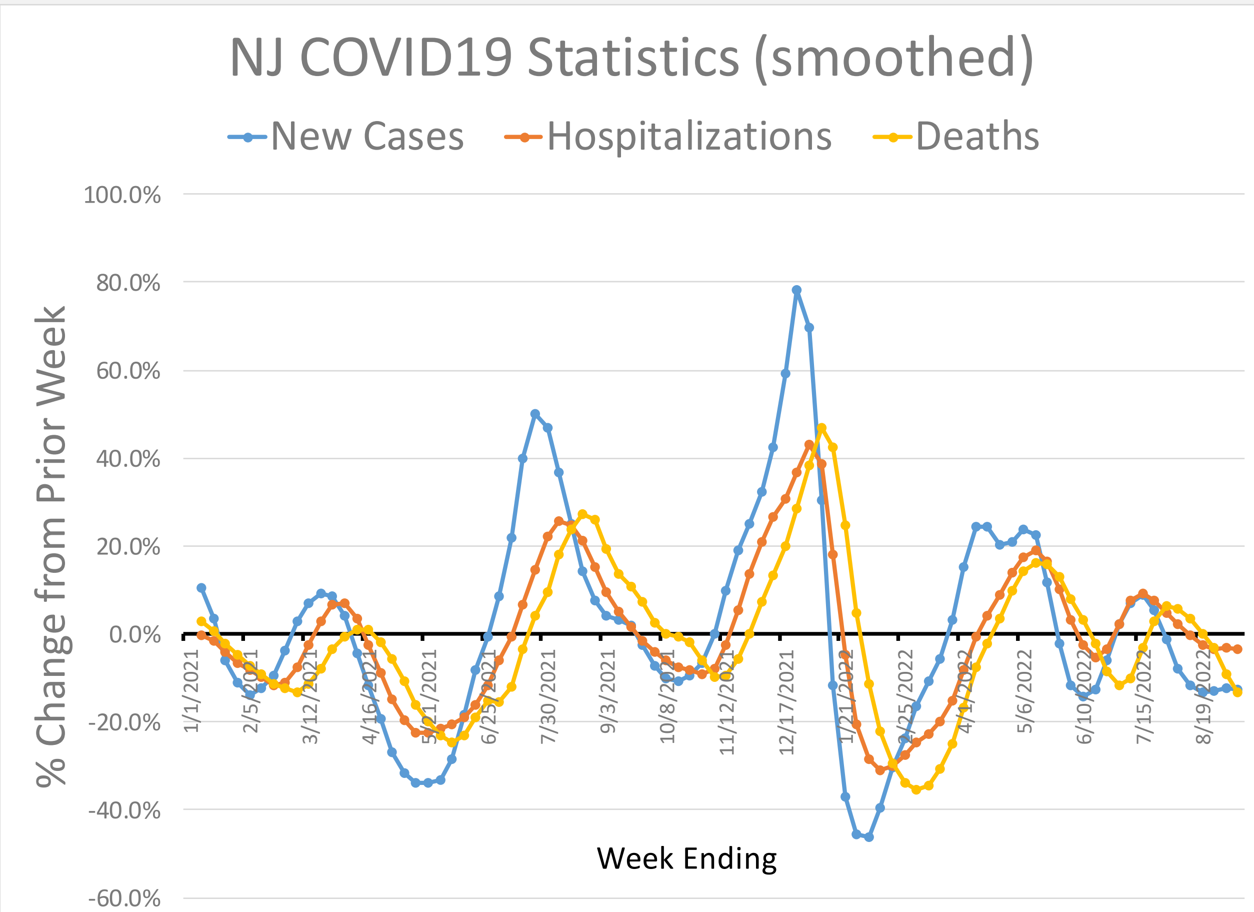

The smoothing spline helps remove the "noise" from the time series data.

We can then see the logical relationship between the series.

- Hospitalizations lag reported cases by 1 to 2 weeks.
- Deaths lag hospitalizations by 1 to 2 weeks.

Business Problem

Insurance trends share similar problems with the questions around COVID.

- Data can be very noisy
- Trends are not constant over time

Social Inflation is one area where changes may emerge slowly over time rather than as a shock event.

What is “Social Inflation?”

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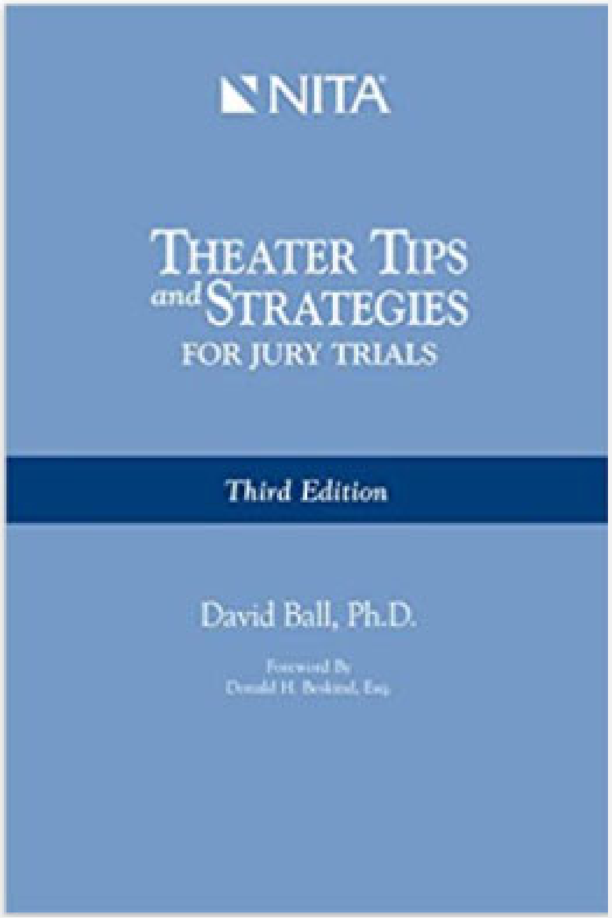

Social Inflation includes evolving legal strategies.

But how do we quantify something like this ?

Amazon description:

In this new, third edition of Theater Tips and Strategies for Jury Trials, David Ball updates his methods and approaches to jury persuasion. This practical step-by-step guide helps you navigate the changes that occur in jury trials instead of being blindsided by them.

Based on both research and the experience of lawyers and trial consultants across the country, Theater Tips and Strategies for Jury Trials, Third Edition, presents techniques of the stage and screen you can use to win in the courtroom. Ball tells how to use theater concepts to persuade and motivate jurors. He tells attorneys how to look, talk, and act naturally, and to communicate the truth clearly and memorably, so they gain trust and credibility from judges and jurors.

Ball provides practical guidance for voir dire, openings and closings, testimony, and focus groups. He describes what practitioners can learn from actors about their manner, voice projection, and behavior. He explains how to grab the jury from the beginning--just as a good movie opening captures the audience. He details how to prepare your "cast" of witnesses so they testify clearly, credibly, and memorably. He offers advice on telling the story so that it commands attention and motivates jurors to argue for your side.

The Geneva Association

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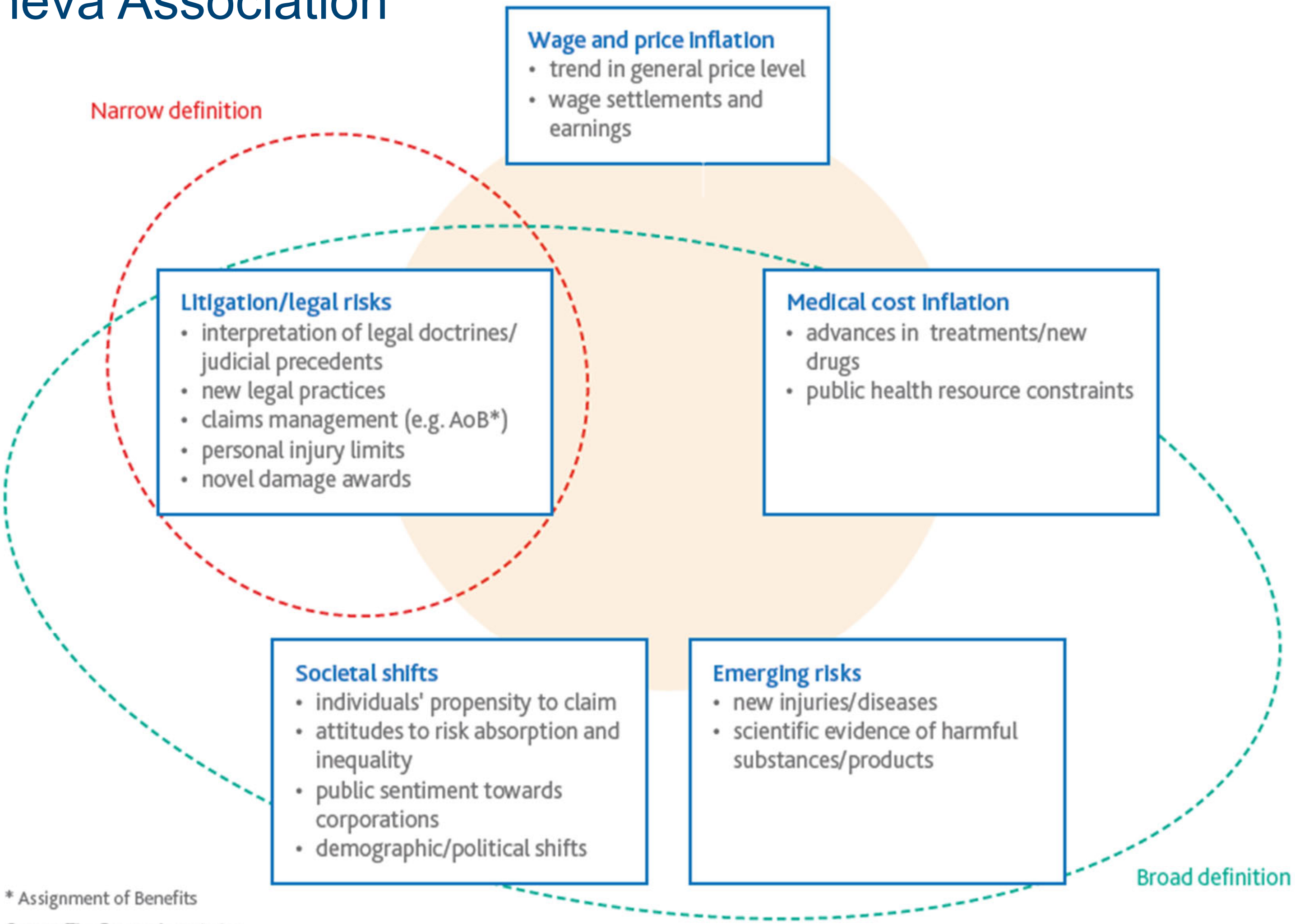
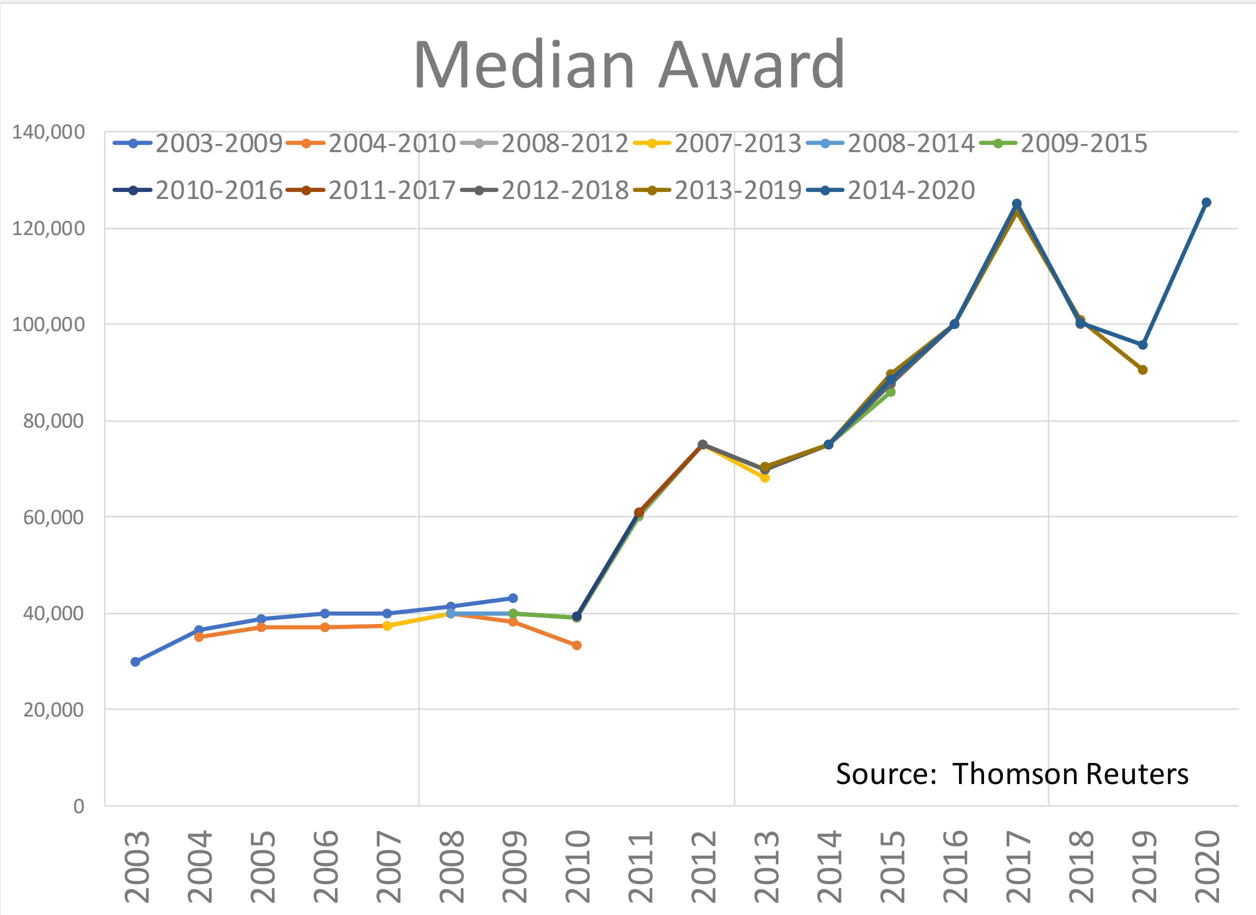


* Assignment of Benefits

Source: The Geneva Association

Thomson Reuters – Personal Injury Awards



The median award numbers show a relatively "flat" period from 2003 to 2010, with an upward trend in subsequent years.

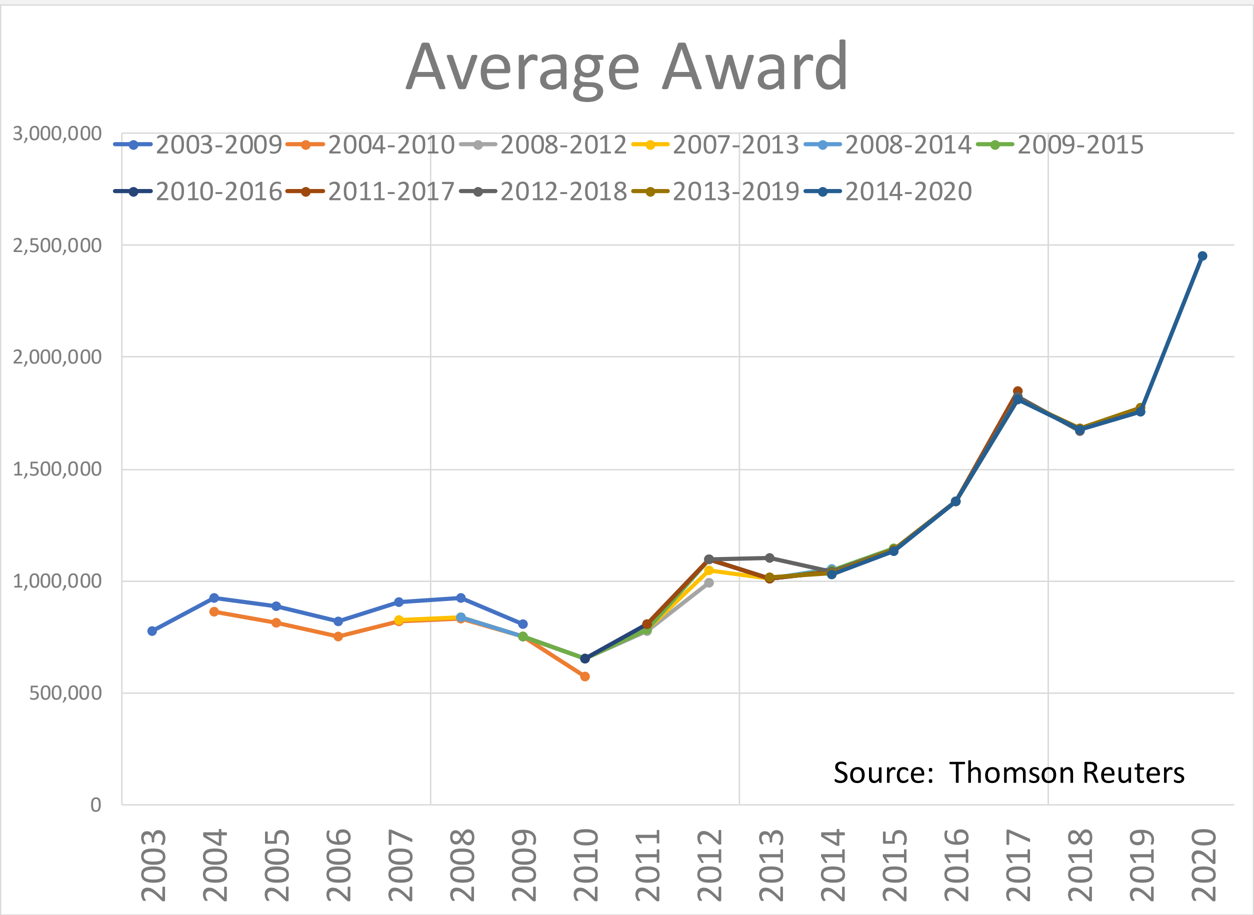
We do not have a good way to evaluate if this is due to change in cases that settle versus those that go to verdict.

But the pattern is consistent with what we have heard on "social inflation" increasing more recently.

Available from Insurance Information Institute:

https://www.iii.org/table-archive/22445

Thomson Reuters – Personal Injury Awards



Amounts for verdict awards are highly skewed, so the average is generally much higher than the median. In fact, about 7 out of 8 cases are below the average award in any year.

But both show similar pattern: flat amounts for 2003-2010 and higher inflation in last decade.

Median = “midpoint” where half the verdicts are above this amount and half below.

Average = sum all verdicts and divide by the total number

Smoothing Splines

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$$\text{minimize} \quad \sum_{i=1}^{n} \left(y_i - g(x_i)\right)^2 + \lambda \cdot \int g''(t)^2 \, dt$$

$\lambda = 0$ is the data itself $g(x_i) = y_i$

$\lambda \to \infty$ is linear regression (no curvature)

$$\hat{g}_\lambda = S_\lambda \cdot Y$$

Smoothing Splines remove the need for the user to select change points.

They are derived as a solution to the curve that minimizes squared error subject to a "penalty" for nonlinearity.

This turns out to be a cubic spline.

The Smoother Matrix $\boldsymbol{S_\lambda}$ plays the same role as the Hat Matrix in regression, allowing us to approximate:

- Effective number of parameters
- Confidence intervals around the smoothed curve

Linear Regression Splines

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$$\hat{Y} = X \cdot \beta$$

$$\hat{\beta} = (X^T \cdot X)^{-1} \cdot X^T \cdot Y$$

$$\hat{Y} = X \cdot \underbrace{(X^T \cdot X)^{-1} \cdot X^T}_{\text{Hat Matrix}} \cdot Y = H \cdot Y$$

In linear regression, the design matrix ***X*** is used to create a "hat matrix" that is used to transform the original data ***Y*** to the fitted vector ***Ŷ***.

The "hat matrix" ***H*** is useful:

- Trace of ***H*** (sum of diagonal) is equal to number of parameters
- Can be used to estimate prediction error around the fitted regression line

Technical Details

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The mathematical details are not critical for making use of smoothing splines.

Intuitively, we may consider the smoothed result ***g*** as a weighted average of the data itself ***y*** and the least-squares regression fit.

The “weight” used in the average is a matrix ***Z***, rather than a single number.

$$\hat{g}_\lambda = Z_\lambda \cdot y + (I - Z_\lambda) \cdot \hat{y}_{LS}$$

Smoothed | Data | Regression Fit

Smoothing Splines

How do we estimate the smoothing parameter?

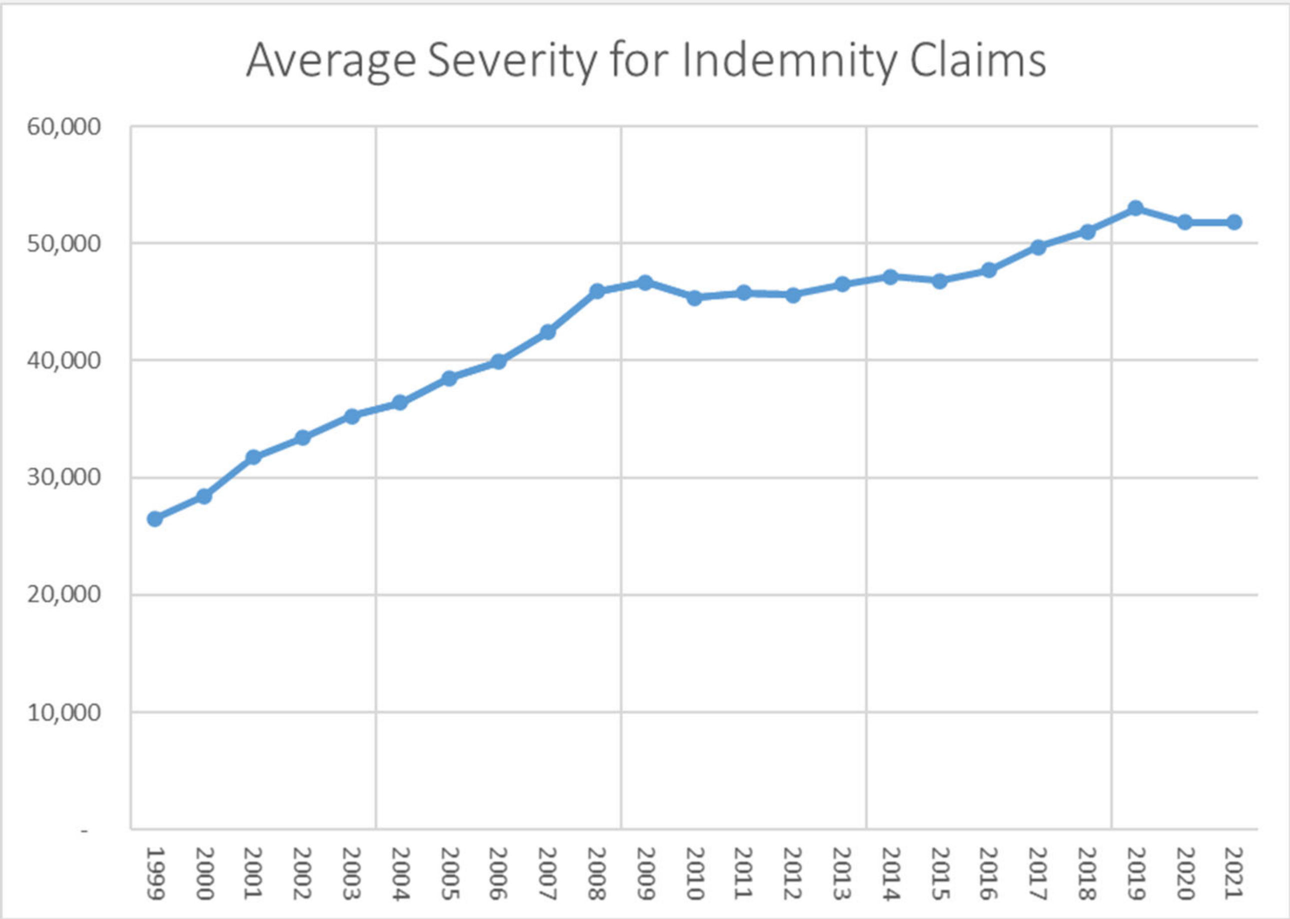
- Smooth “by eye” = often sufficient in exploratory analysis
- Consider the effective number of parameters relative to the number of data points
- Cross Validation
  - Leave One Out (LOO)
  - Generalized Cross Validation (GCV) is an easier short-cut

$$GCV_\lambda = \frac{n}{\left(n - Tr(S_\lambda)\right)^2} \cdot \sum_{i=1}^{n} \left(y_i - g(x_i)\right)^2$$

“*Our experience and that of others has indicated that GCV tends to undersmooth… particularly in small datasets*.”

- Hastie & Tibshirani; Generalized Additive Models

Example #1: Workers' Compensation Severity



The average severity for Workers' Compensation "lost time" claims has historically outpaced inflation.

The trend is not constant over time, with a moderating pattern during 2009-2015.

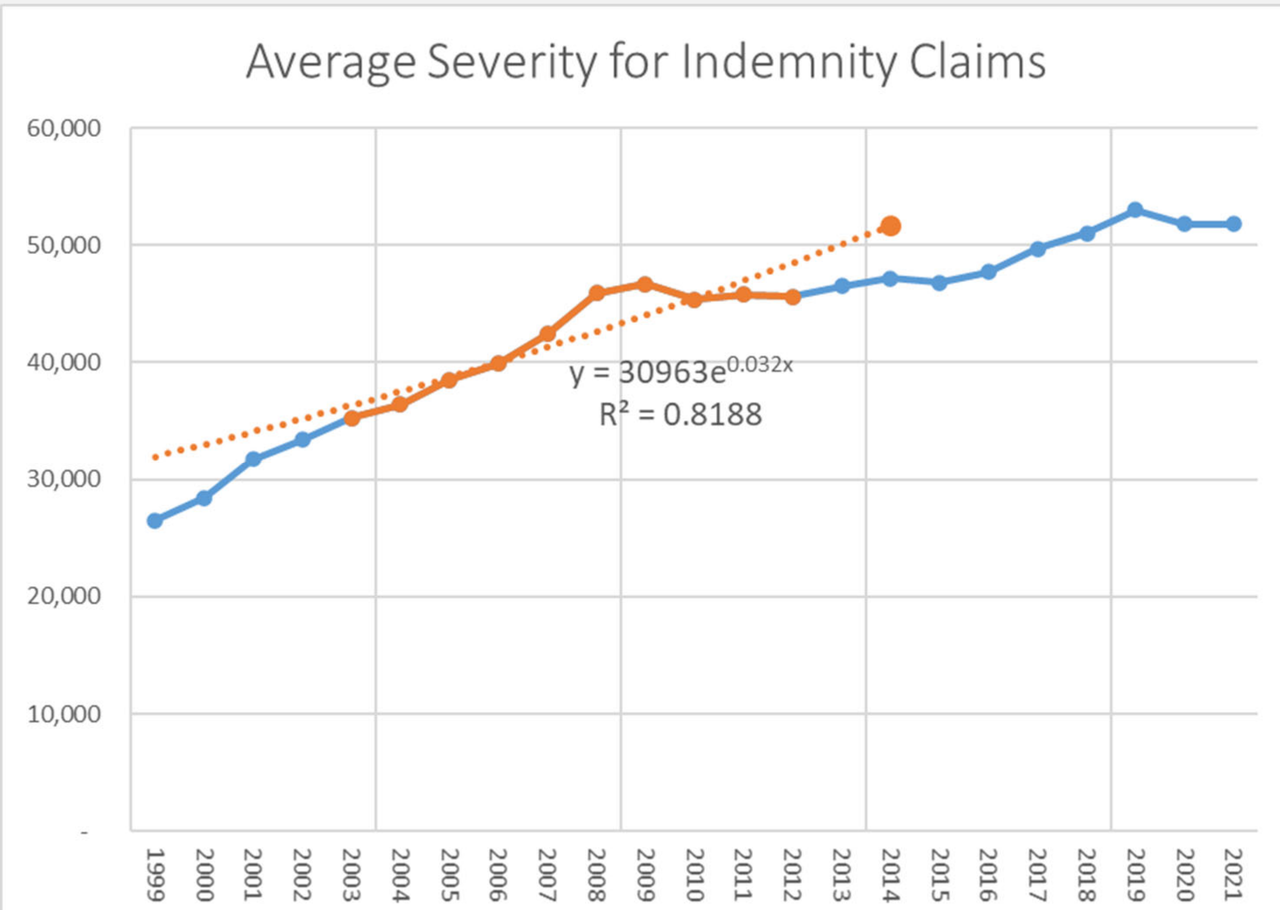
Many factors may contribute to this change:

- ACA/Obamacare
- Opioid crisis
- Economic factors moving from manufacturing to service industries

Source:

https://www.ncci.com/SecureDocuments/SOLGuide2022.html#WC_Loss_Drivers

Example #1: Workers' Compensation Severity



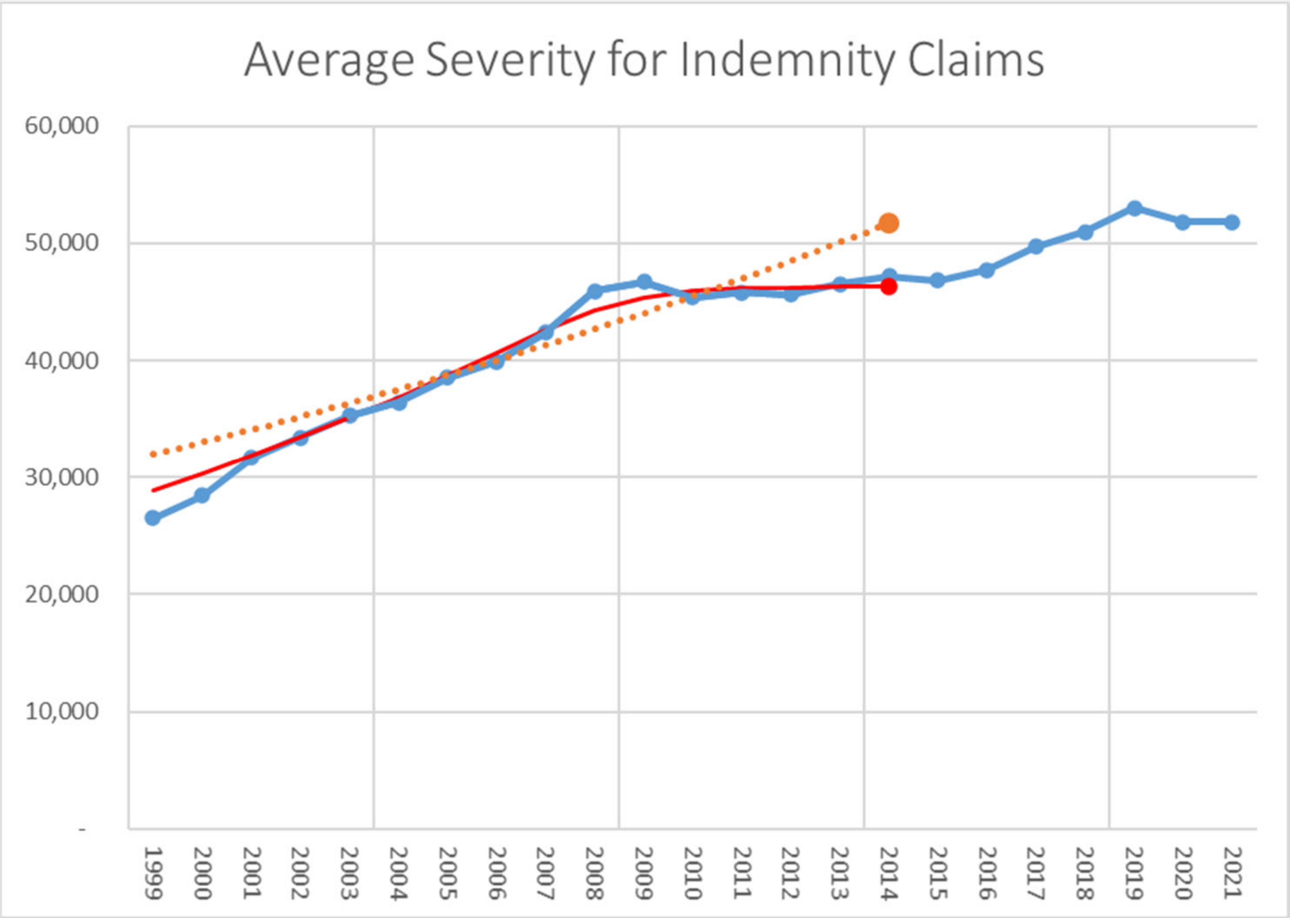
A naïve projection for 2014 might have used a log-linear regression on prior ten complete years 2003-2012.

Simple extrapolation would lead to about a 10% miss in the severity.

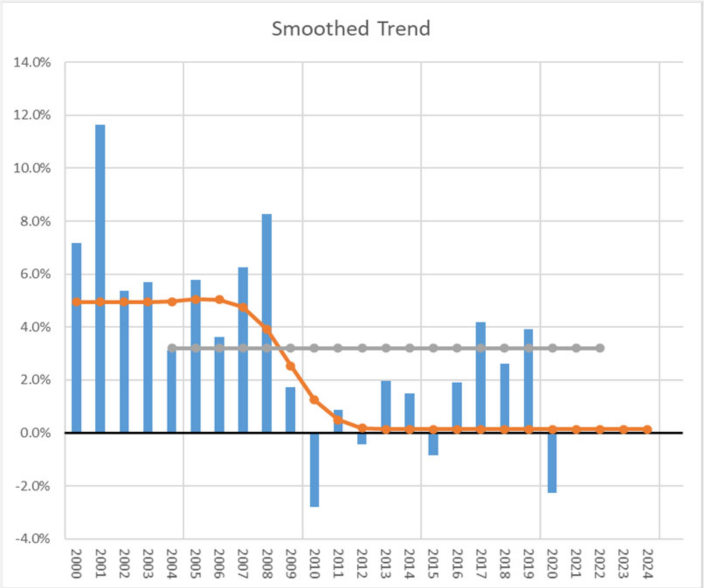
Source:

https://www.ncci.com/SecureDocuments/SOLGuide2022.html#WC_Loss_Drivers

Example #1: Workers' Compensation Severity



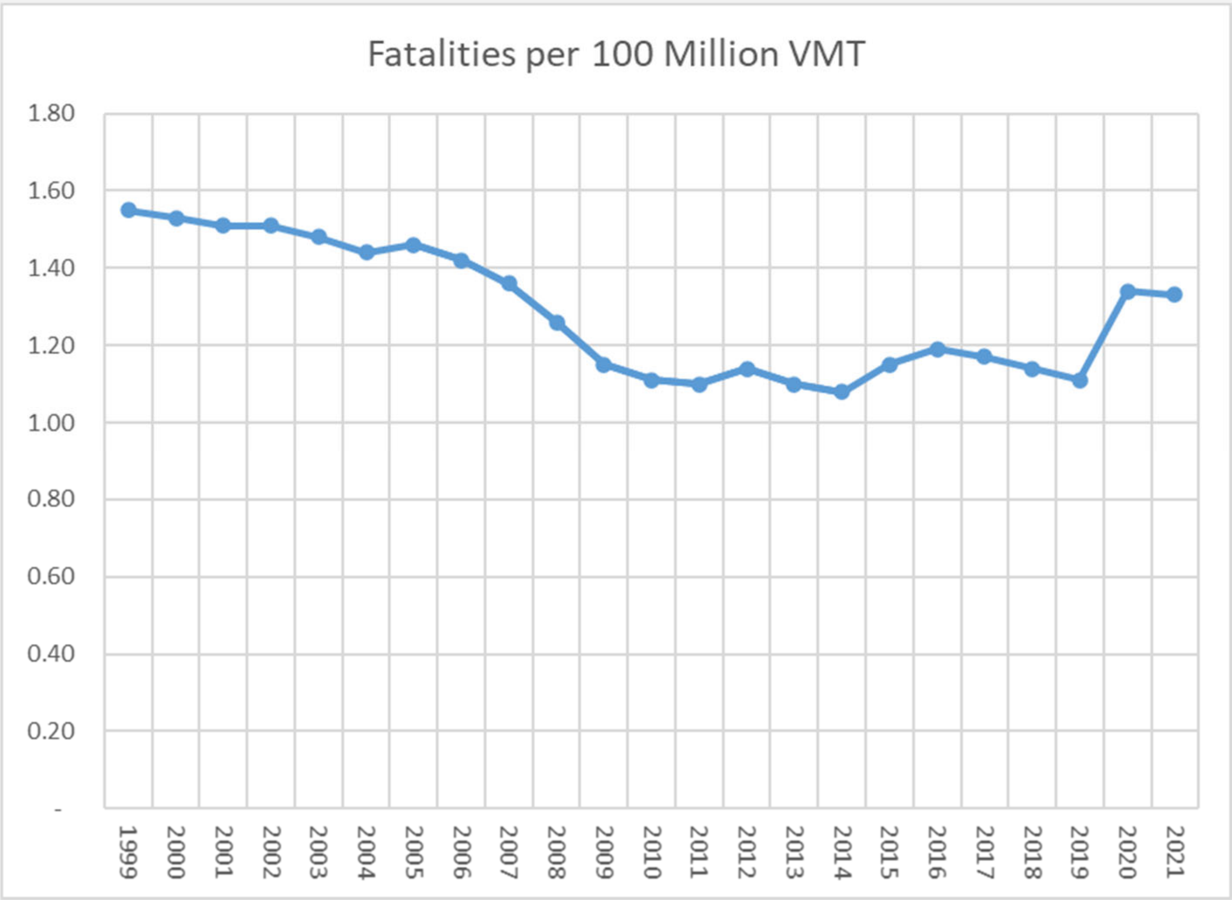
A smoothing spline with smoother of 5.0 is much more representative of the data than the all-year fit



Source:

https://www.ncci.com/SecureDocuments/SOLGuide2022.html#WC_Loss_Drivers

Example #2: Auto Fatalities



Data from the National Highway Traffic Safety Administration (NHTSA) shows generally improving frequency in fatalities.

The rate of improvement is not constant and may be influenced by economic conditions and technology changes.

Source:

https://www-fars.nhtsa.dot.gov/Trends/TrendsGeneral.aspx

Example #2: Auto Fatalities

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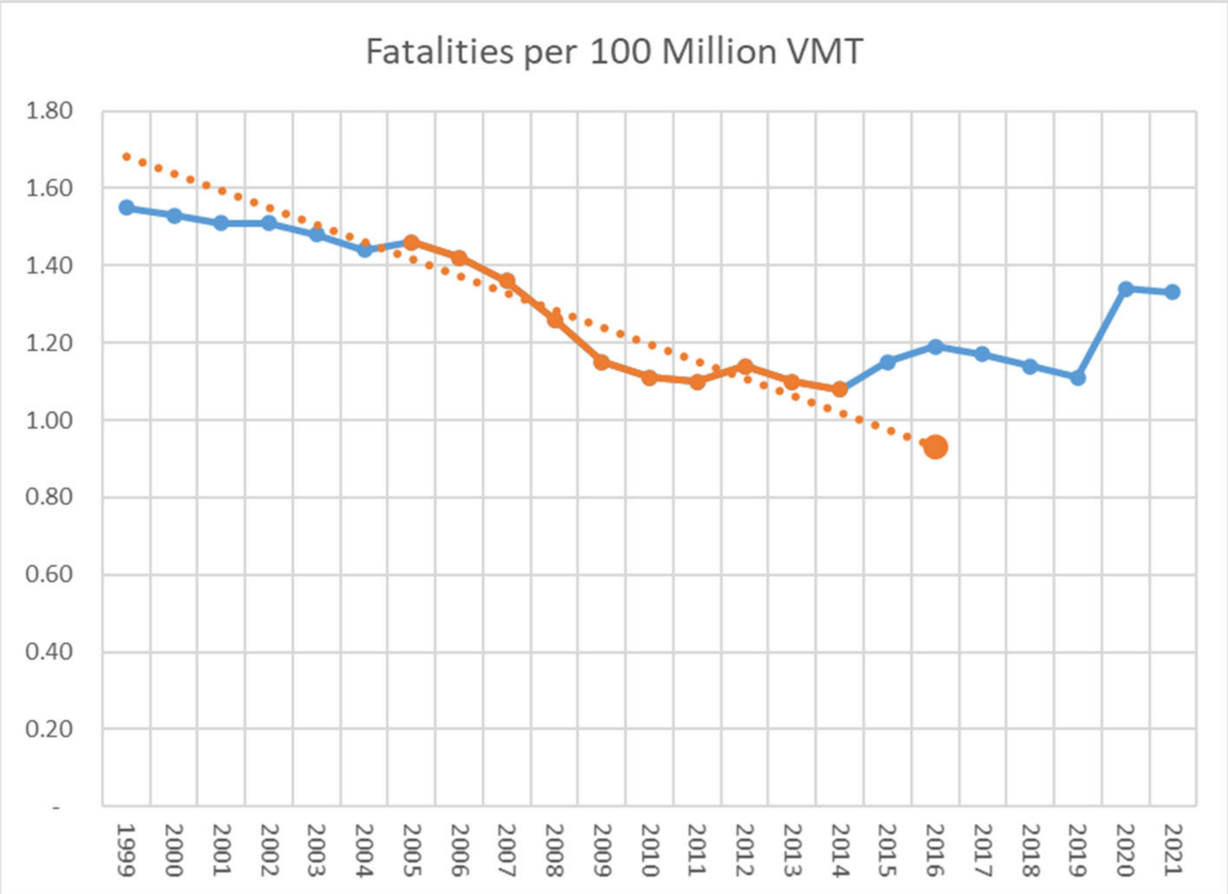

A naïve projection for 2016 might have used a log-linear regression on prior ten complete years 2005-2014.

Simple extrapolation would lead to about a 20% miss in the frequency.

Source:

https://www-fars.nhtsa.dot.gov/Trends/TrendsGeneral.aspx

Example #2: Auto Fatalities smoothing spline on 2005-2014

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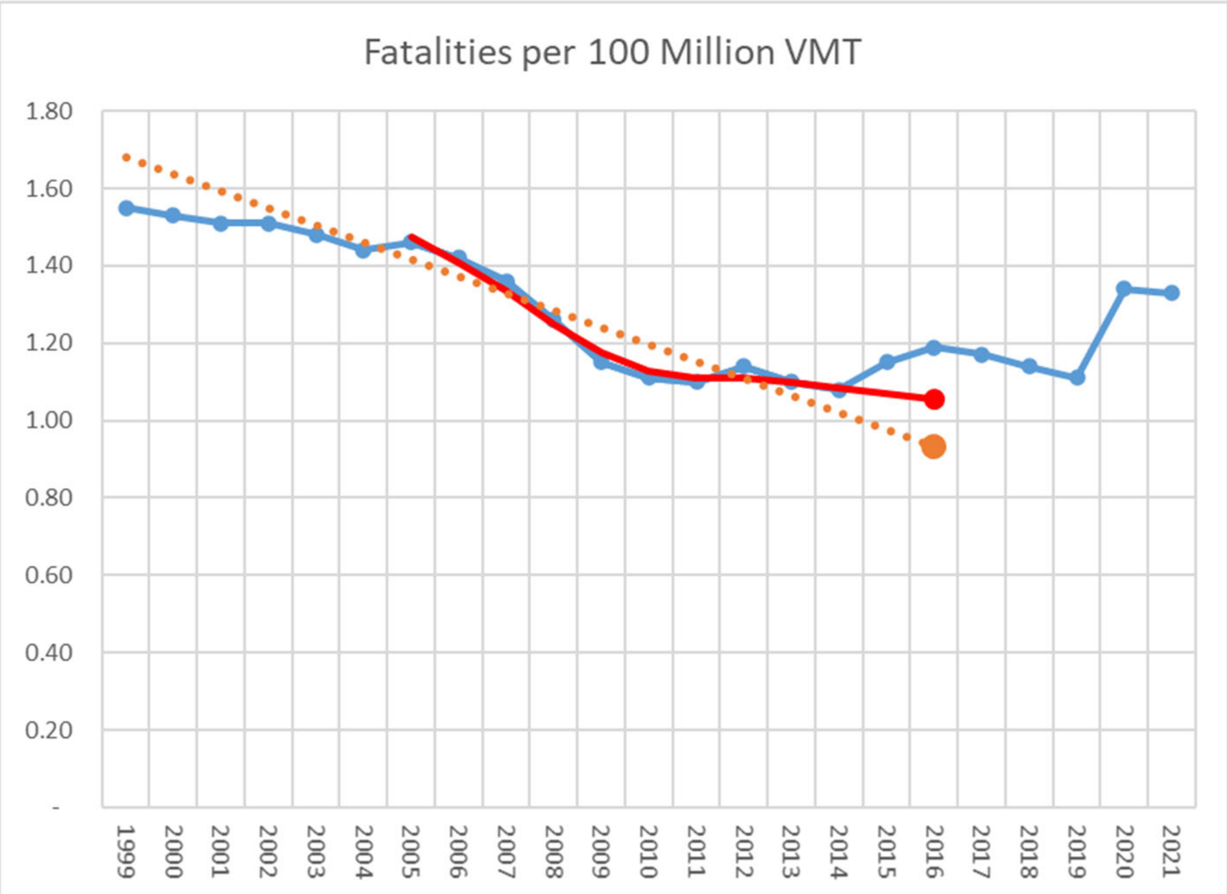
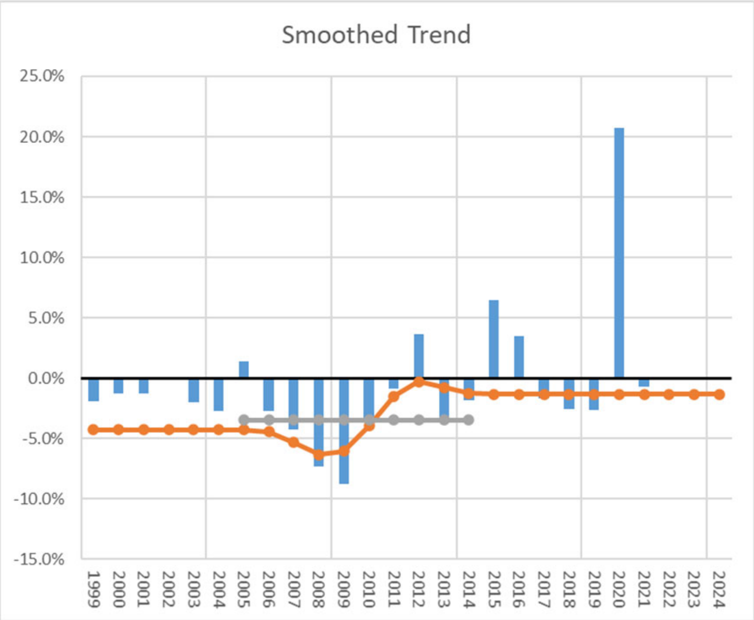


A smoothing spline with smoother of 1.0 is much more representative of the data than the 10-year fit



Source:

https://www-fars.nhtsa.dot.gov/Trends/TrendsGeneral.aspx

Example #2: Auto Fatalities smoothing spline on 2005-2019

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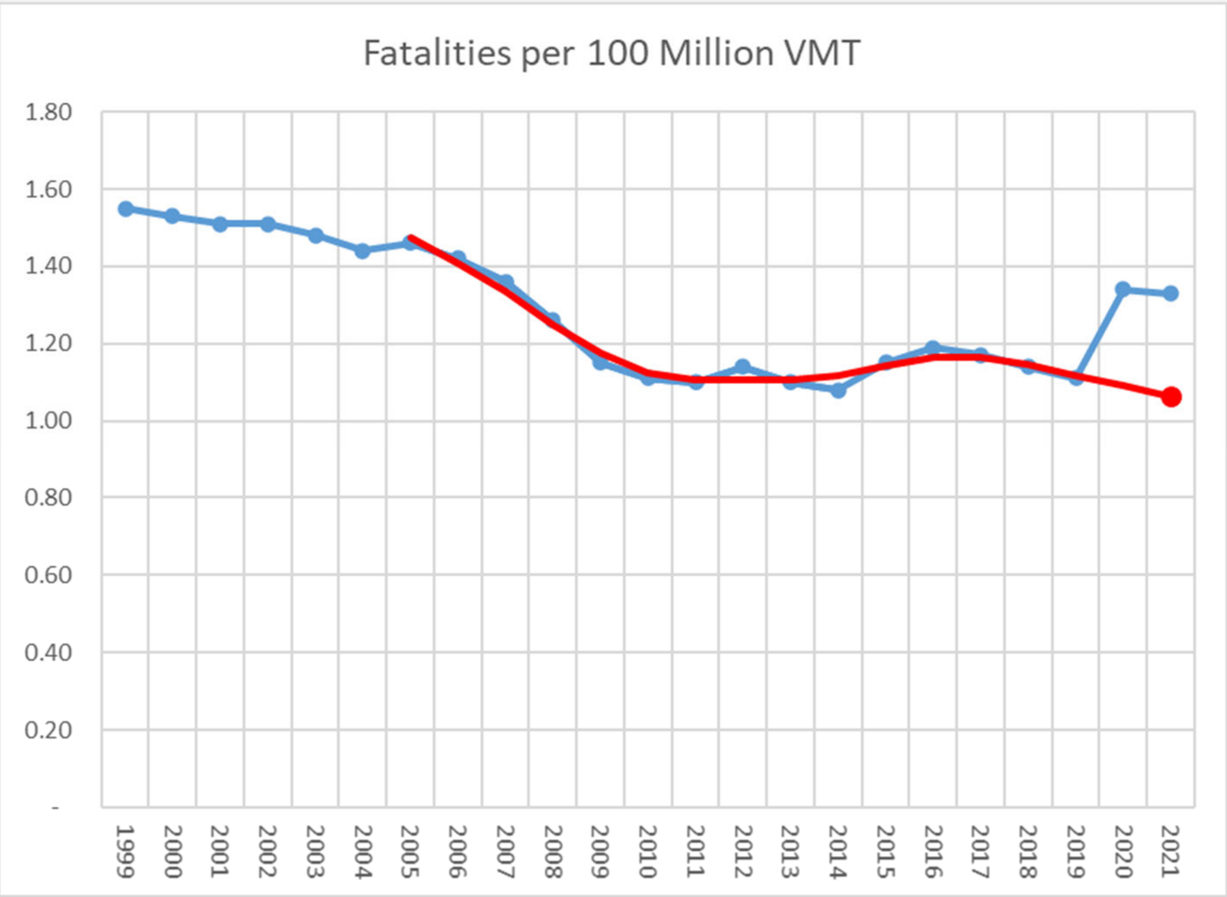
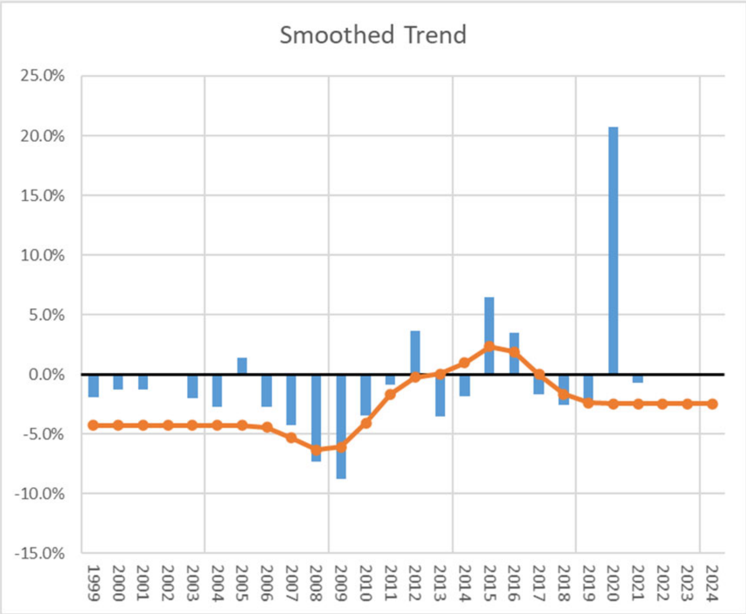


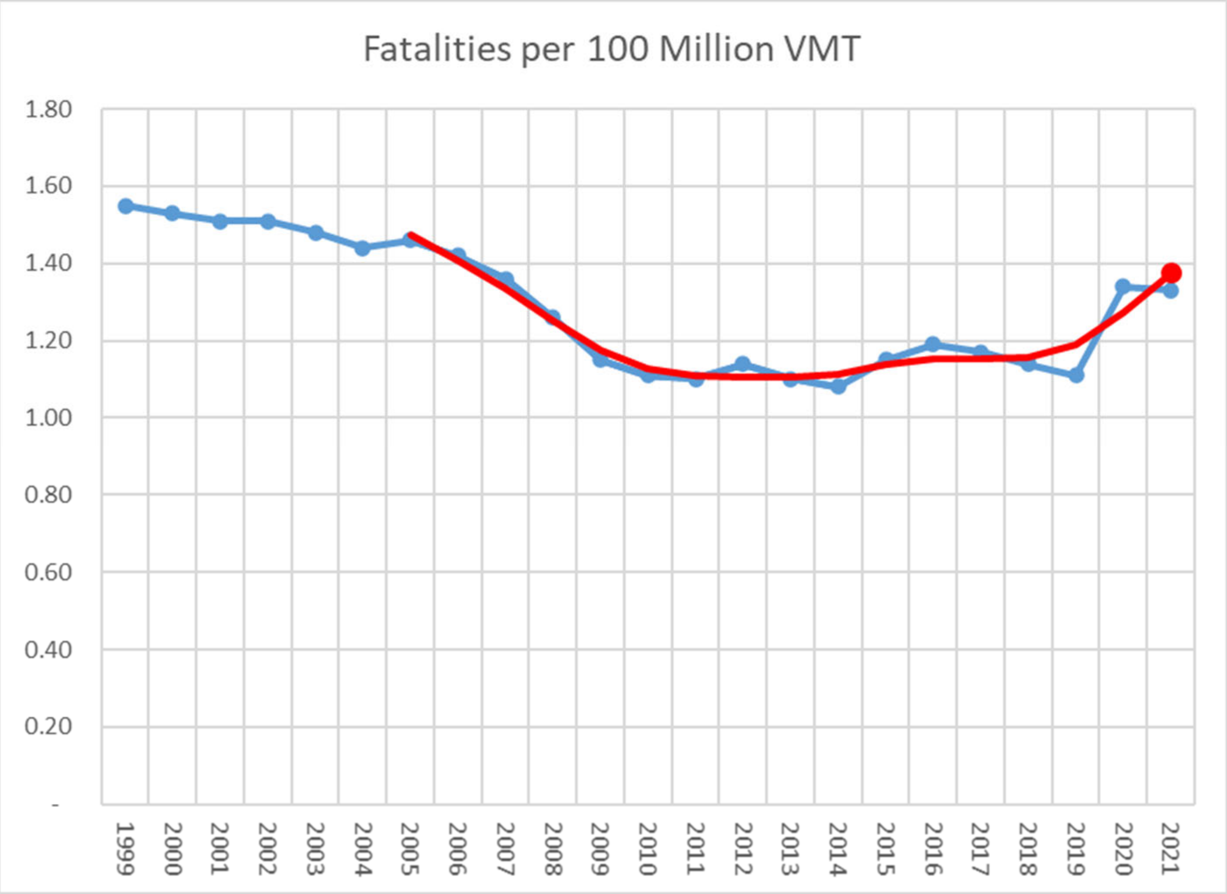
Including data through 2019, the forecast would miss the change caused by the pandemic.



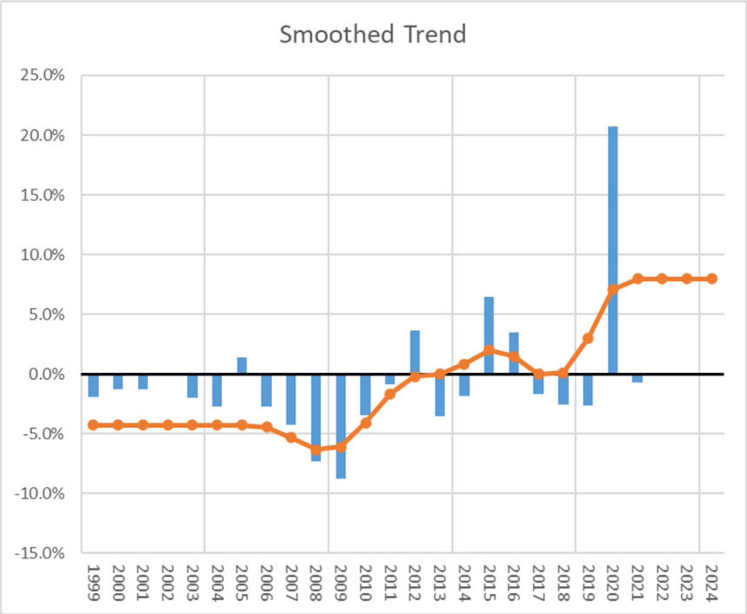
Source:

https://www-fars.nhtsa.dot.gov/Trends/TrendsGeneral.aspx

Example #2: Auto Fatalities smoothing spline on 2005-2020



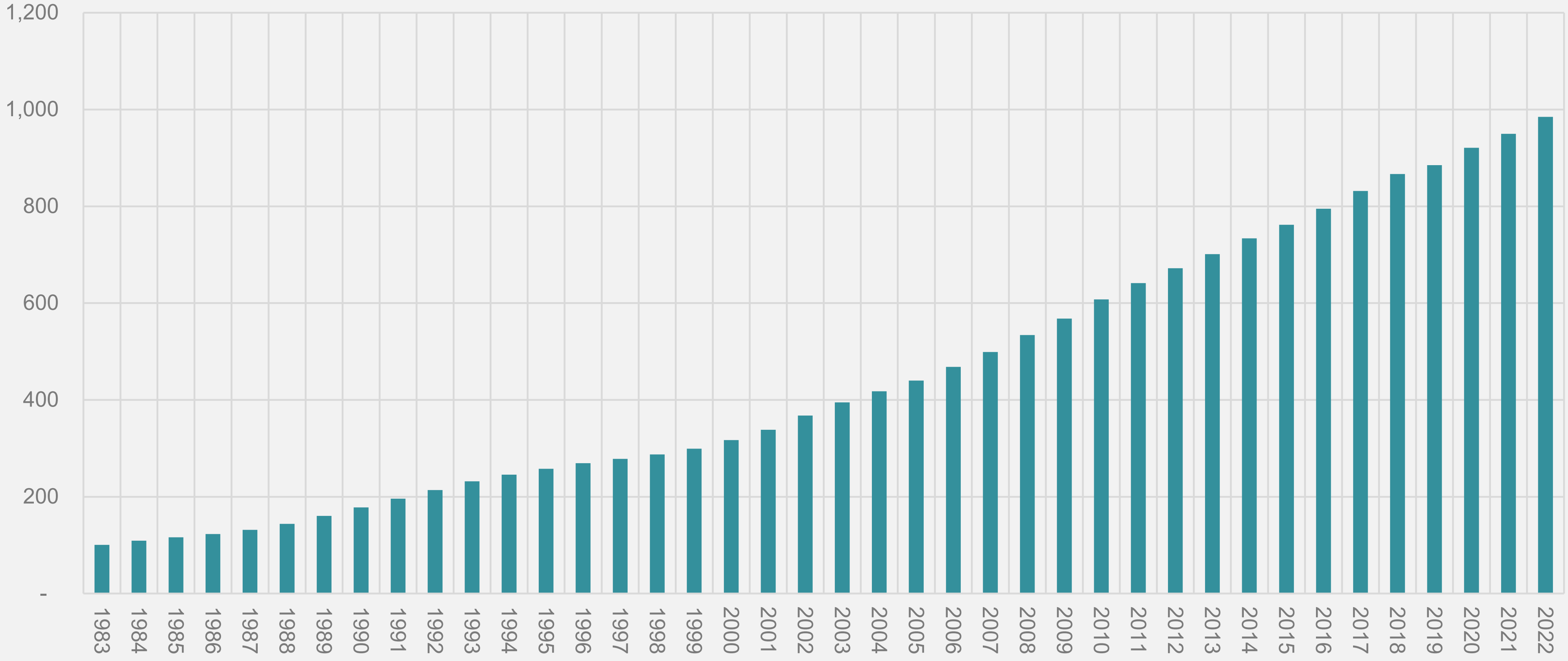
The 2020 data point is extremely leveraged



Source:

https://www-fars.nhtsa.dot.gov/Trends/TrendsGeneral.aspx

Example #3: CPI for All Urban Consumers
Hospital and Related Services in US City Average as of July 2022



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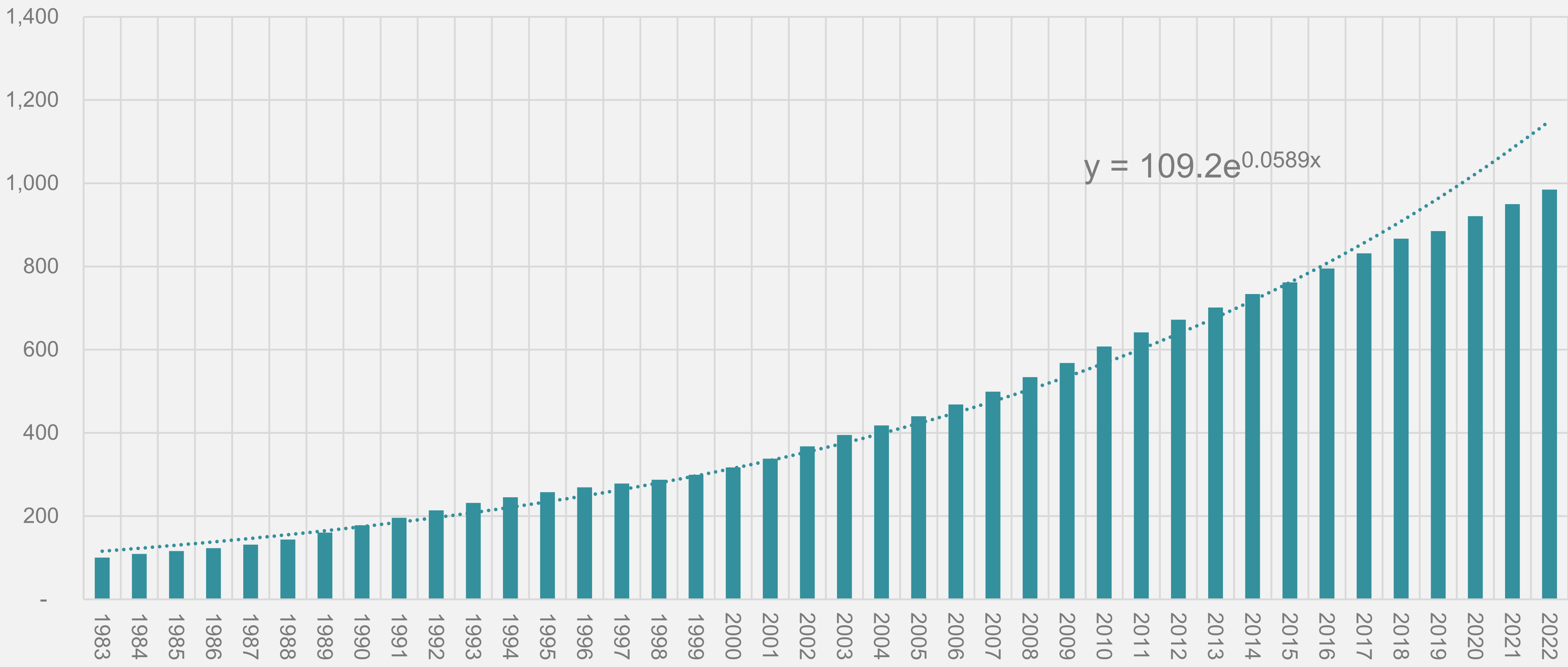

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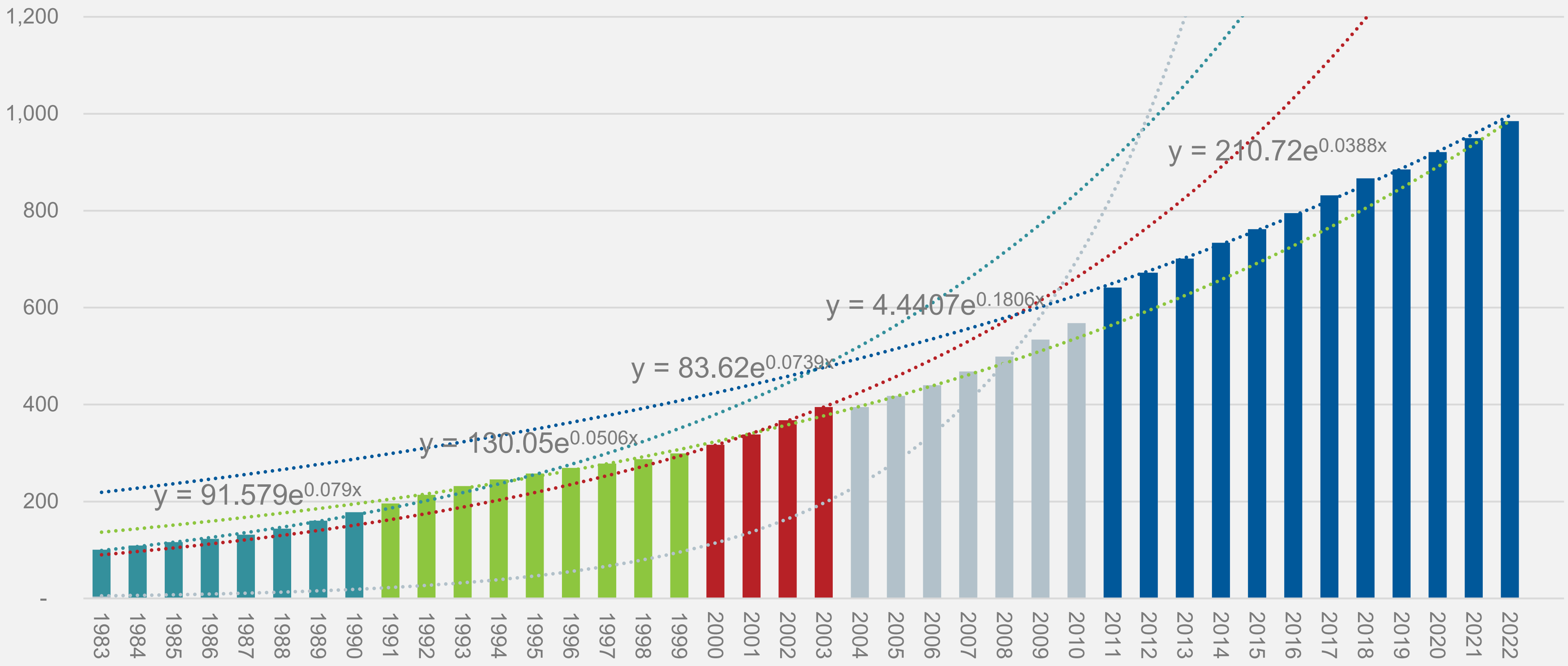
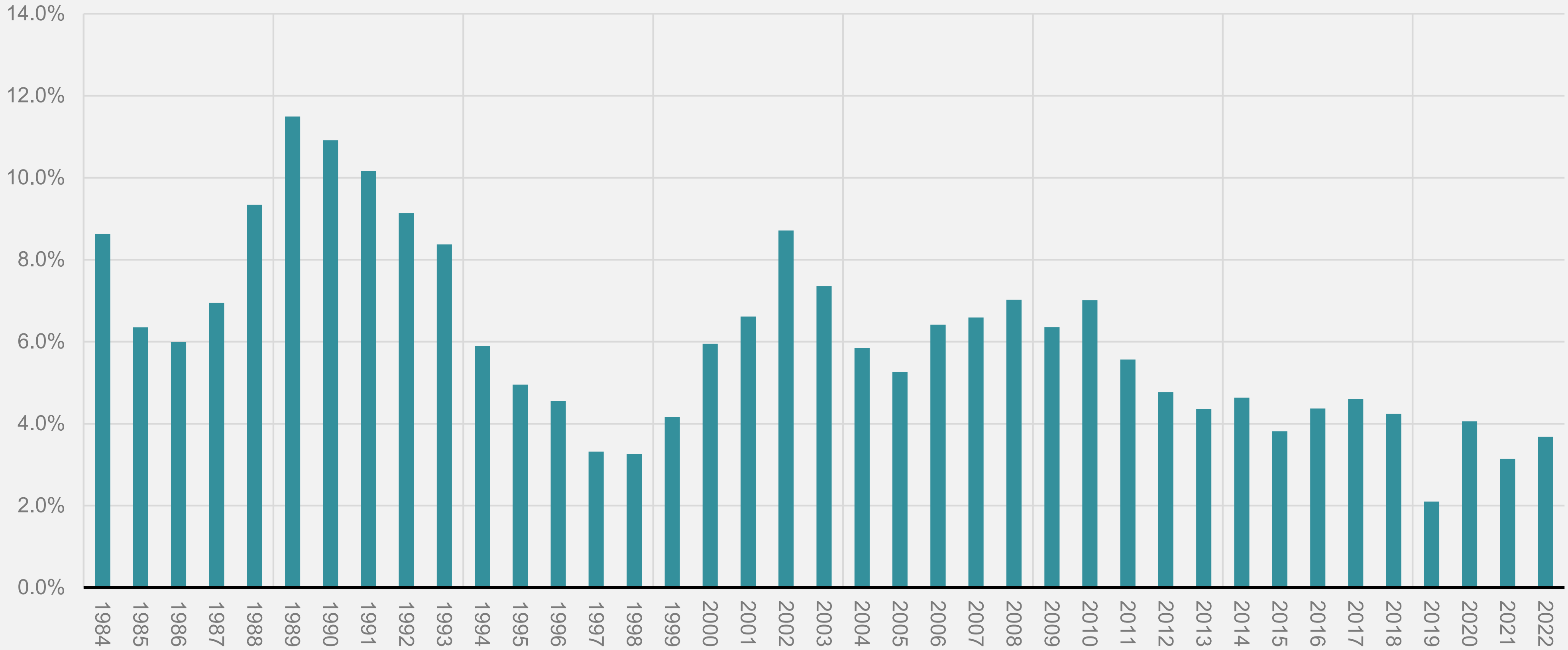
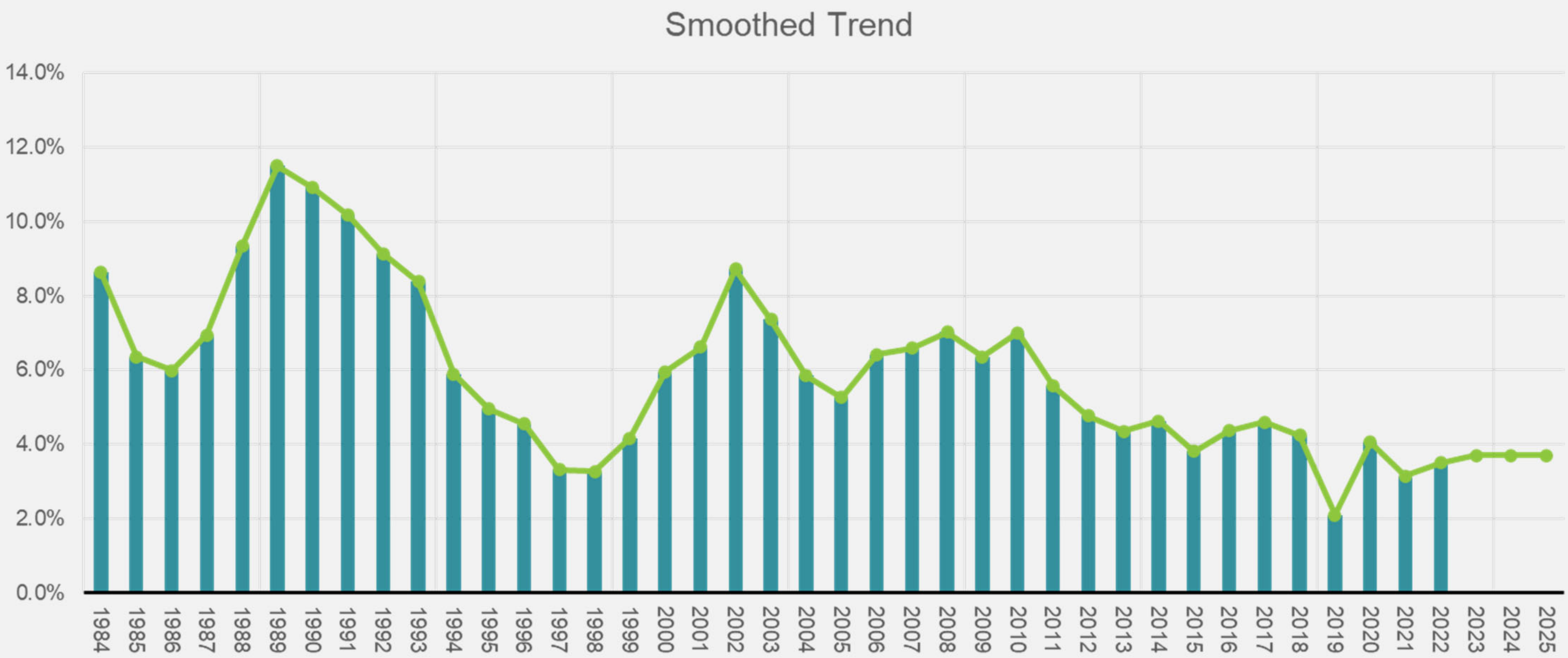


Change in Hospital CPI YOY



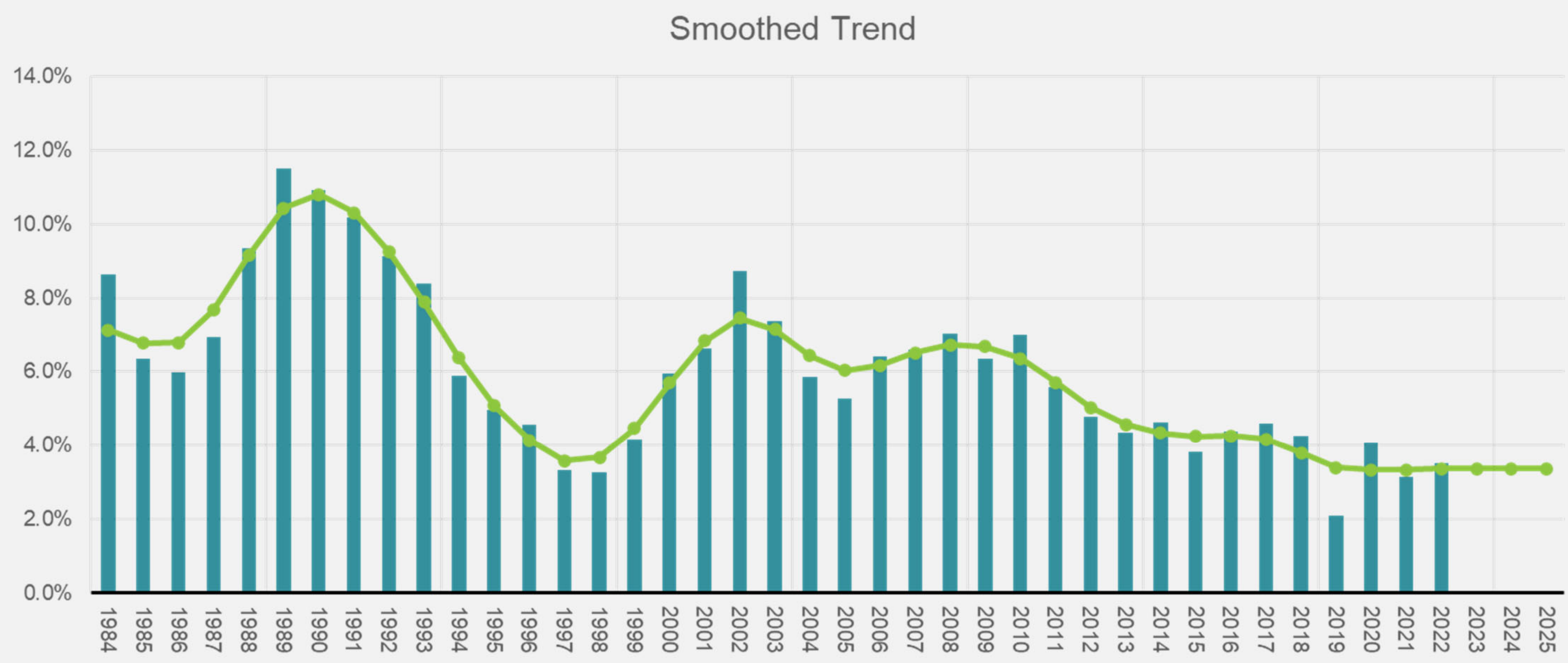
Change in Hospital CPI YOY

Smoothing Parameter = 0, # Parameters = 40, Std Error = 0



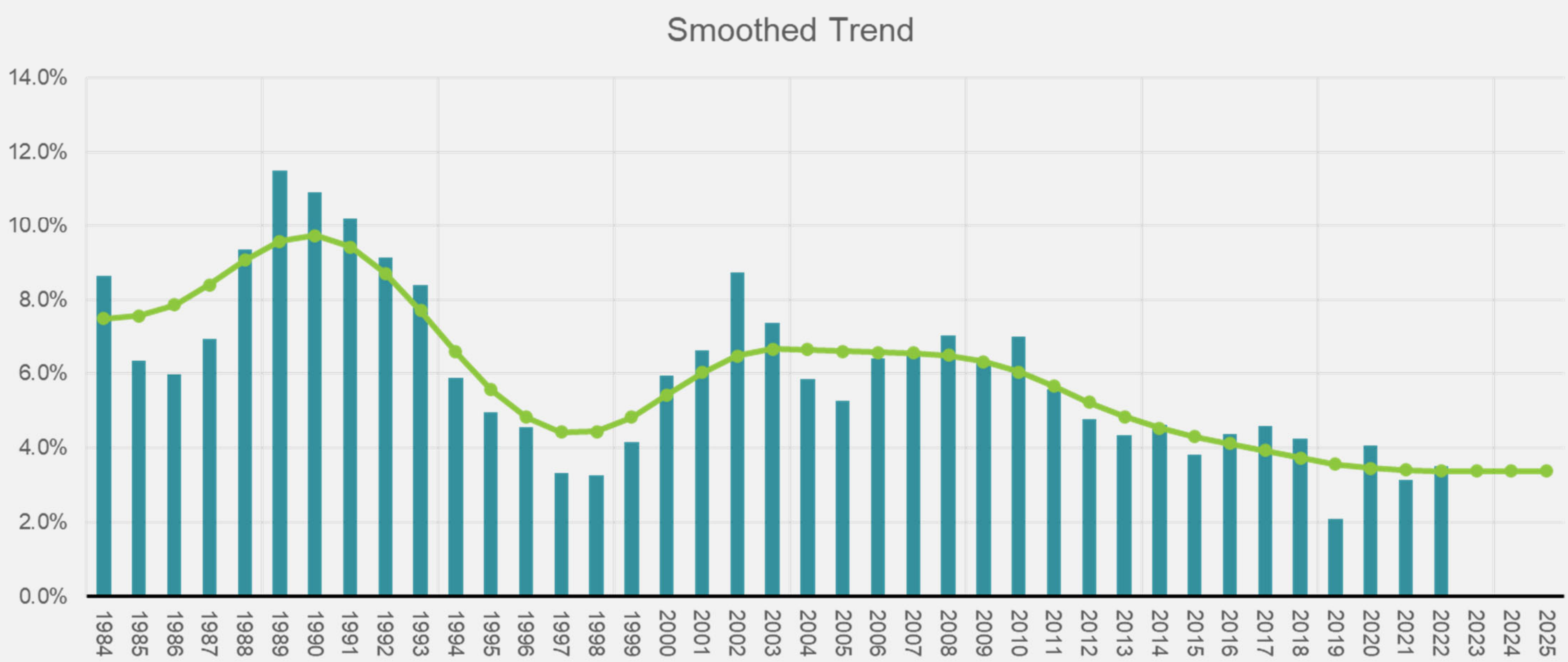
Change in Hospital CPI YOY

Smoothing Parameter = 1, # Parameters = 15.02, Std Error = 2.29



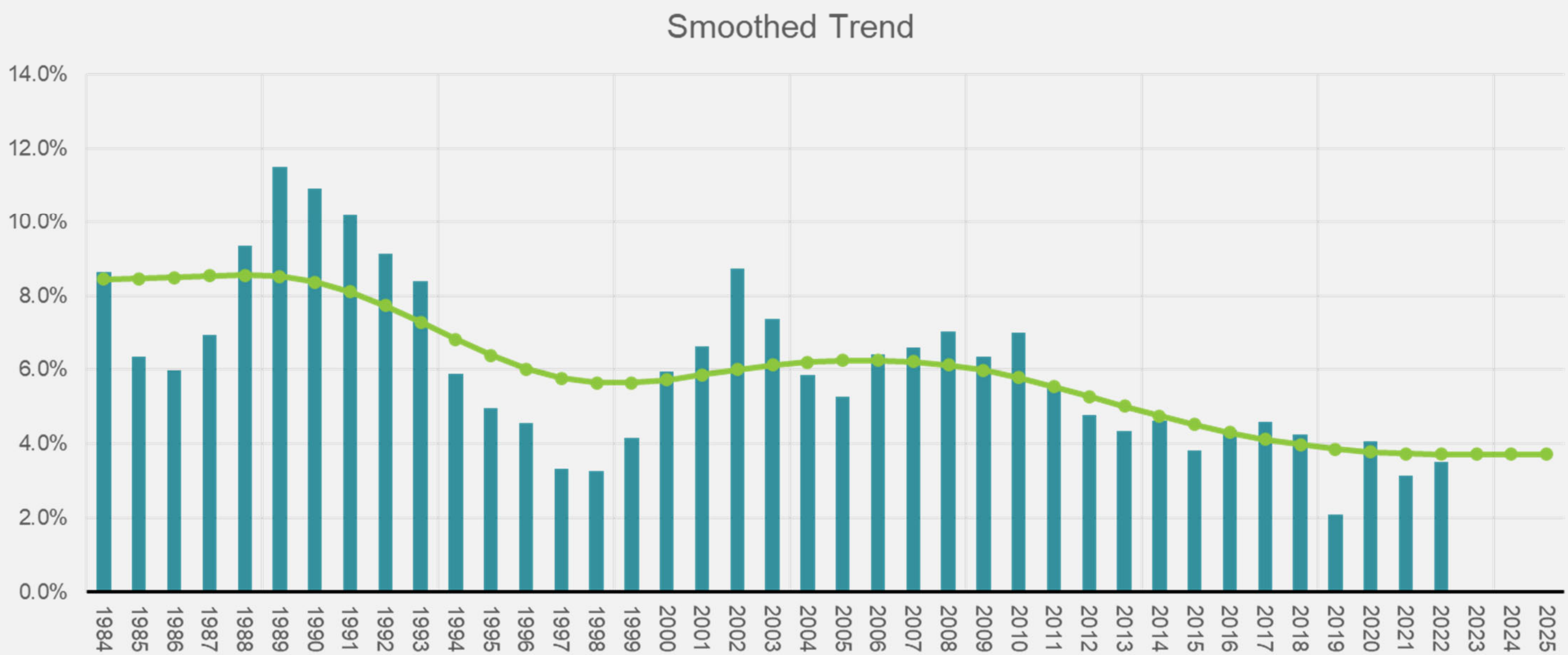
Change in Hospital CPI YOY

Smoothing Parameter = 10, # Parameters = 8.92, Std Error = 3.93



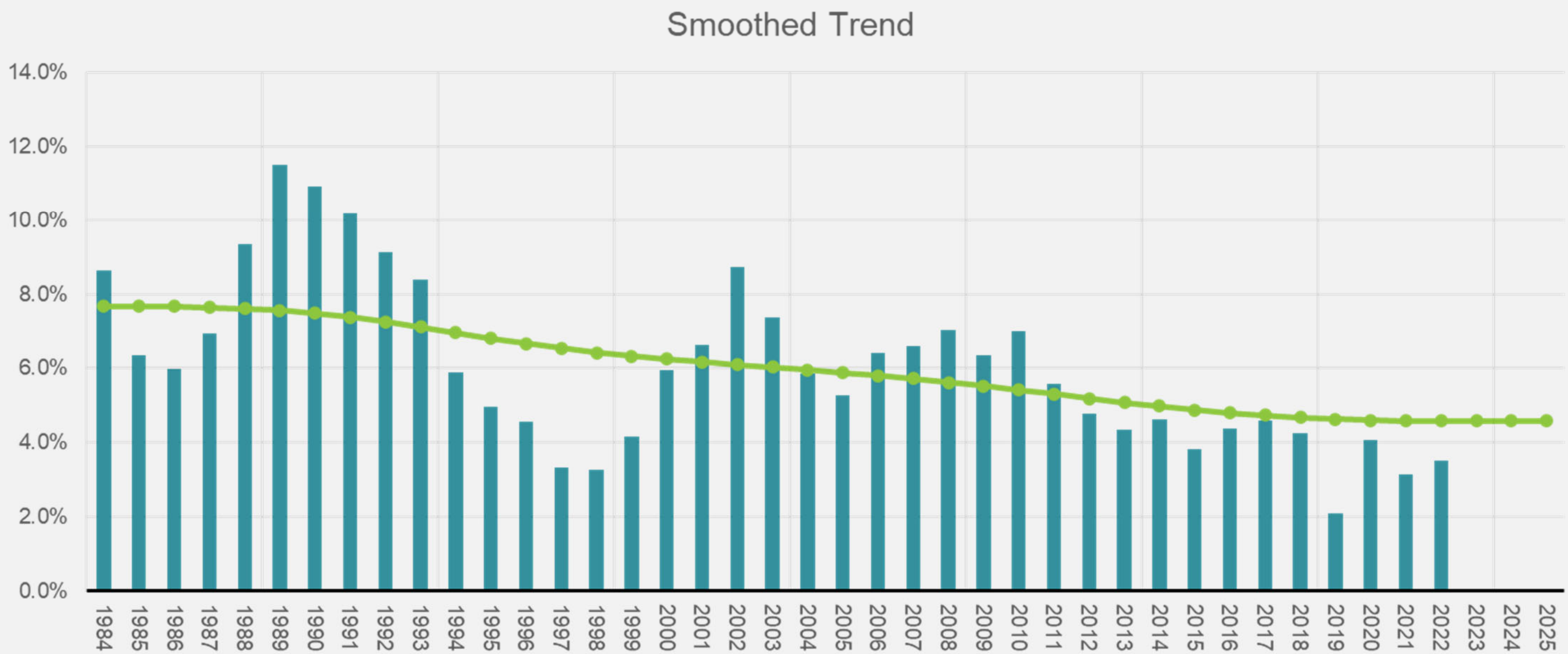
Change in Hospital CPI YOY

Smoothing Parameter = 100, # Parameters = 5.46, Std Error = 8.03



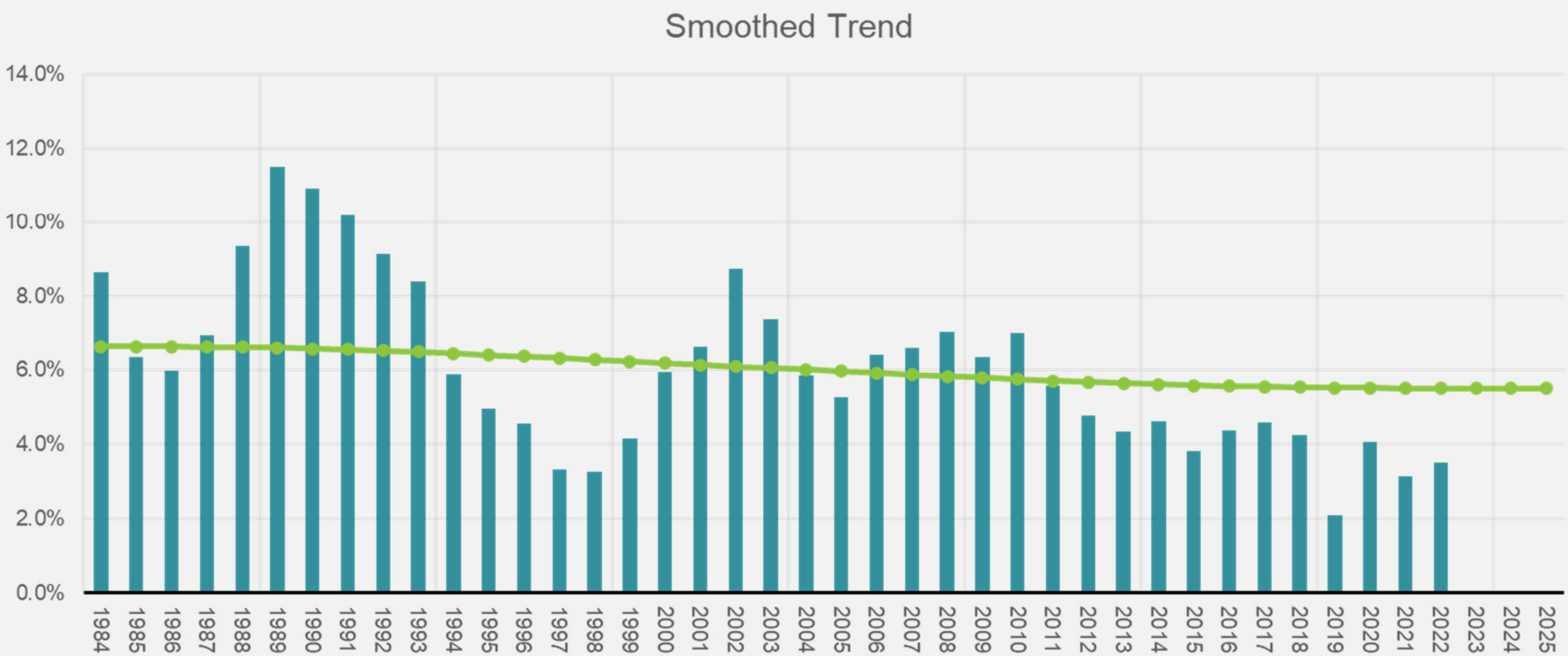
Change in Hospital CPI YOY

Smoothing Parameter = 1000, # Parameters = 3.51, Std Error = 16.93



Change in Hospital CPI YOY

Smoothing Parameter = 10000, # Parameters = 2.43, Std Error = 33.34



Limitations of Statistical Methods

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Smoothing Splines can help us explore our data and identify where trends may be changing.

- They do not tell us <u>why</u> the trends are changing.
  - If there are multiple causes at play, it is not easy to say which is the main driver of change.

- Forecasts beyond the latest point can be difficult.
    - For example, COVID may have significantly impacted the latest year but it does not necessarily represent a trend that will continue into the future.
    - Hyndman paper in the Reference slide is helpful for examining forecasts.

References and Resources

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Tibshirani lecture notes on splines [great 4-page introduction]

http://www.stat.cmu.edu/~ryantibs/advmethods/notes/smoothspline.pdf

Wu, Tongtong, “Introduction to Smoothing Splines”, online lecture slides (2004) available at:

https://www.scribd.com/presentation/421924201/smsp-ppt

Hyndman, Rob, et al, ”Local Linear Forecasts Using Cubic Smoothing Splines” (2005)
https://www.researchgate.net/publication/5179833_Local_Linear_Forecasts_Using_Cubic_Smoothing_Splines

[Good explanation of connection of smoothing splines to ARIMA models, and **confidence intervals on forecasts**]

Dave’s article “Smoothing Splines for Trend”, Actuarial Review Jan/Feb 2021

https://ar.casact.org/smoothing-splines-for-trend/

Recommended R packages: pspline

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