Modeling Inflation Driven Reserving Cycles

2022 CLRS

James Ely, FCAS

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Session Organization

Part 1

Outline how to build a single model that finds the historical development pattern and reserving cycle effects using wave methods.

Part 2

Explain why we might want to use transforms, knowing that this is not our dominant paradigm.

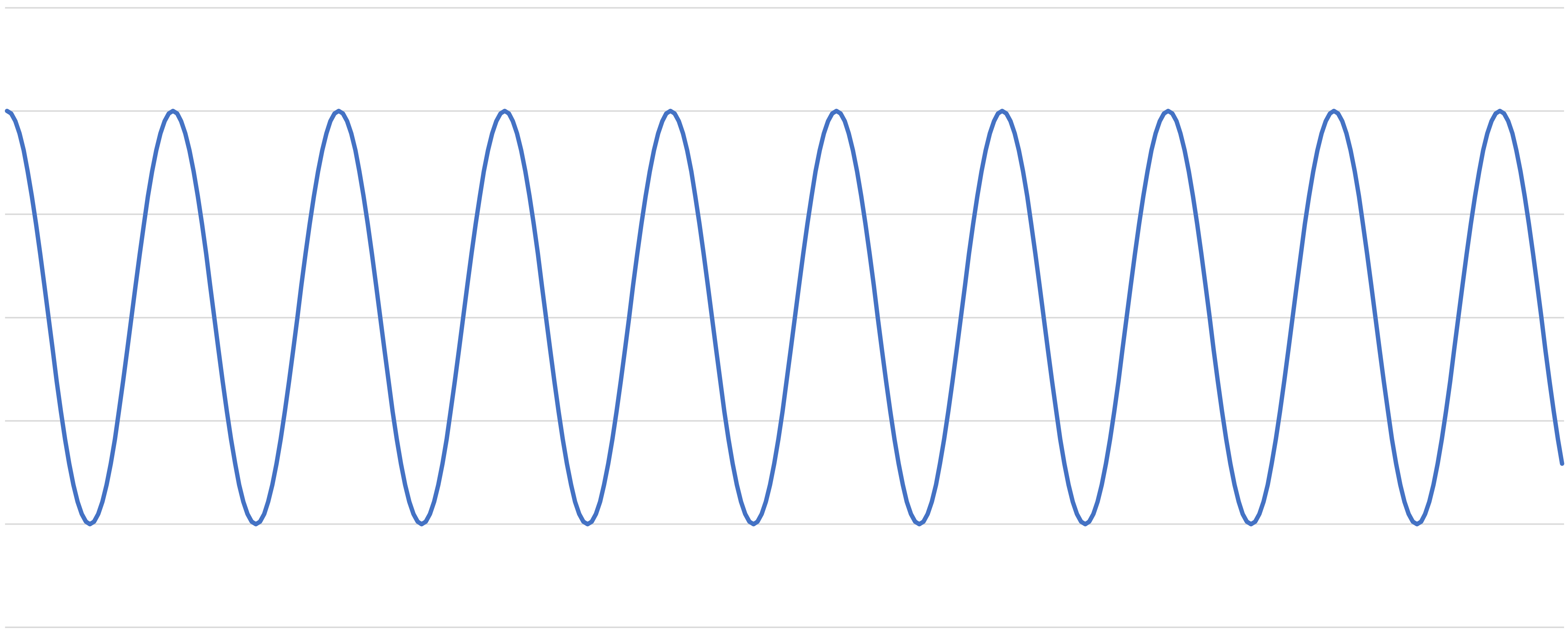
How To Build a Wave Model for Loss Development Cycles

1. Organize our data

2. Calculate Model Parameters

3. Apply our Judgement

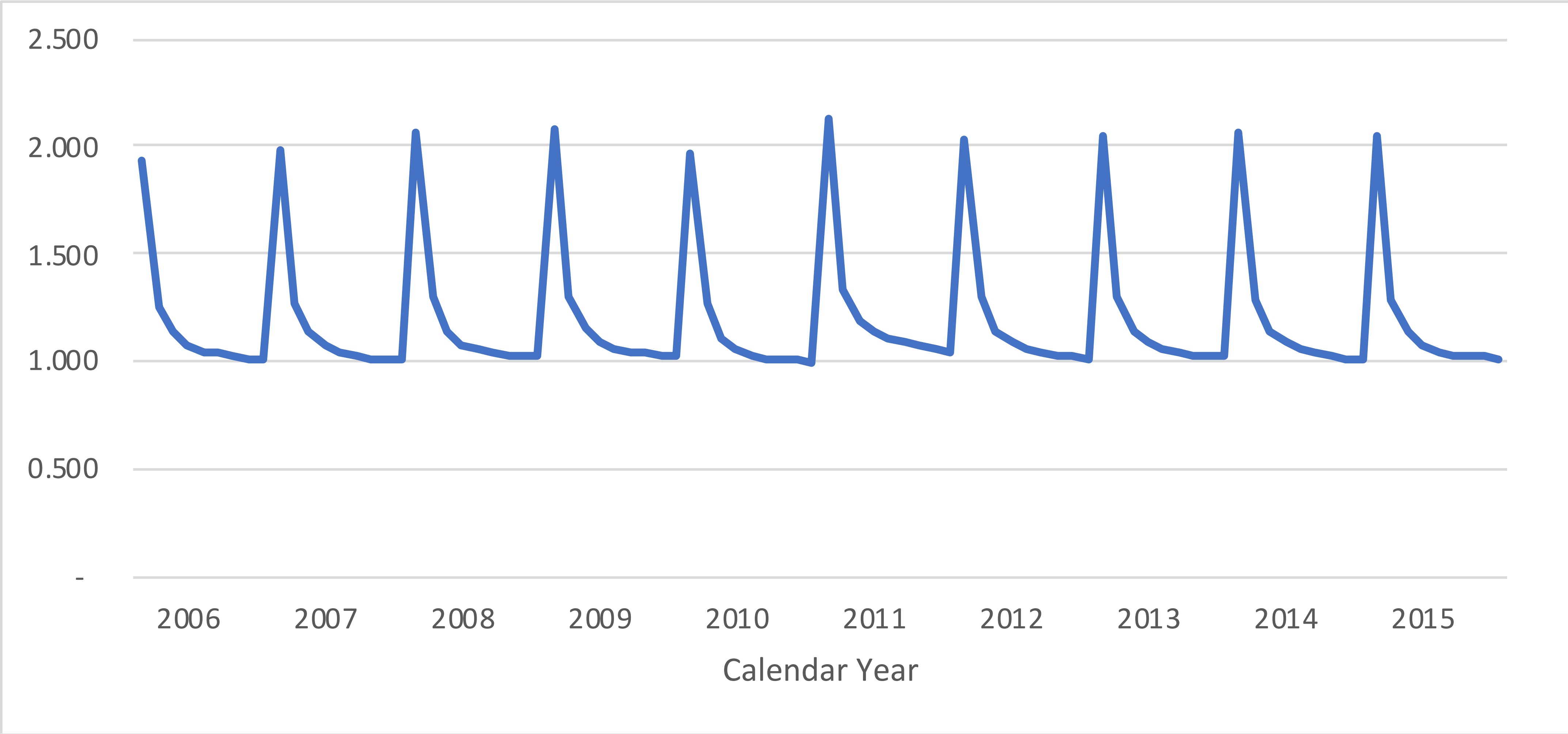
Periodic Function or Wave



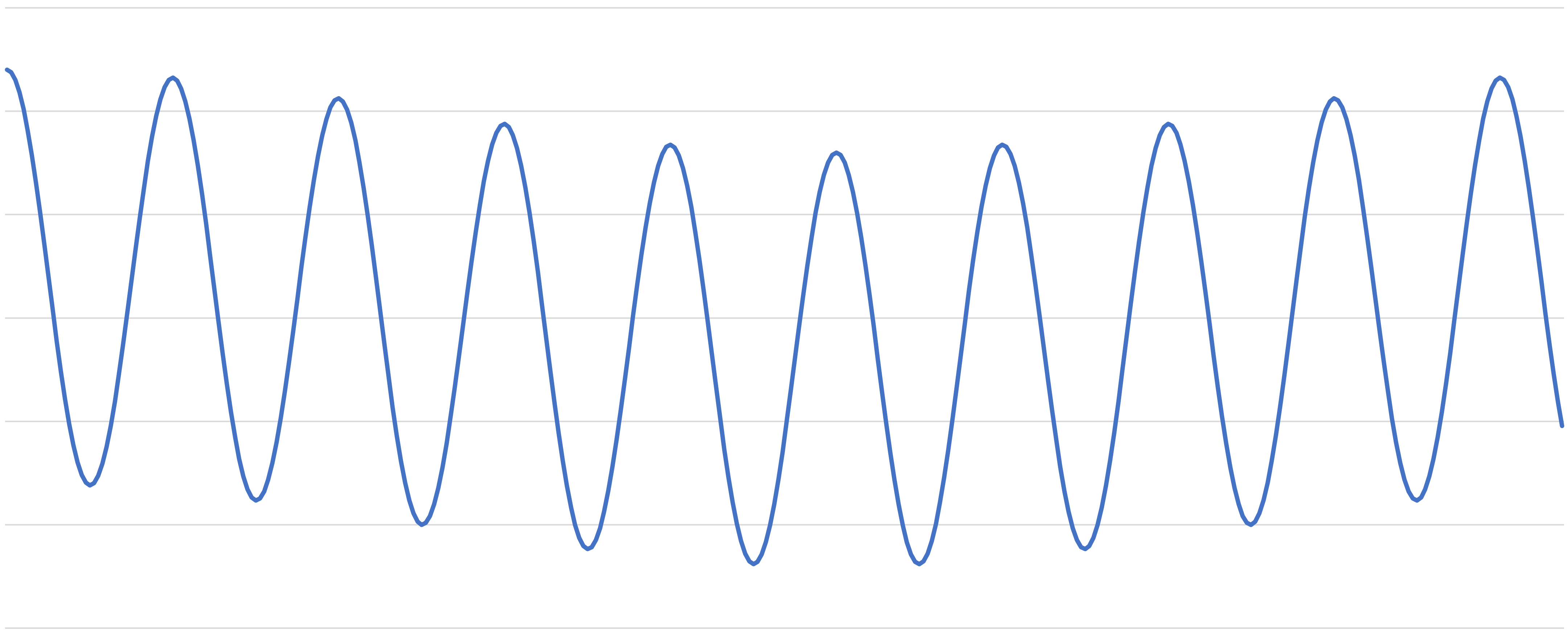
US Workers Compensation
Paid Link Ratios

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
AY									
1997									1.011
1998								1.018	1.010
1999							1.026	1.013	1.022
2000						1.037	1.019	1.021	1.036
2001					1.048	1.025	1.033	1.034	1.002
2002				1.082	1.044	1.043	1.042	1.005	1.052
2003			1.141	1.084	1.058	1.043	1.012	1.061	1.019
2004		1.261	1.138	1.084	1.061	1.014	1.075	1.026	1.022
2005	1.931	1.274	1.136	1.092	1.026	1.087	1.029	1.027	1.016
2006	1.981	1.297	1.161	1.053	1.101	1.038	1.036	1.020	1.017
2007	2.056	1.307	1.113	1.135	1.058	1.044	1.029	1.022	
2008	2.072	1.267	1.198	1.088	1.058	1.040	1.026		
2009	1.971	1.344	1.143	1.089	1.058	1.033			
2010	2.122	1.299	1.142	1.089	1.046				
2011	2.030	1.297	1.148	1.078					
2012	2.049	1.289	1.134						
2013	2.068	1.284							
2014	2.041								

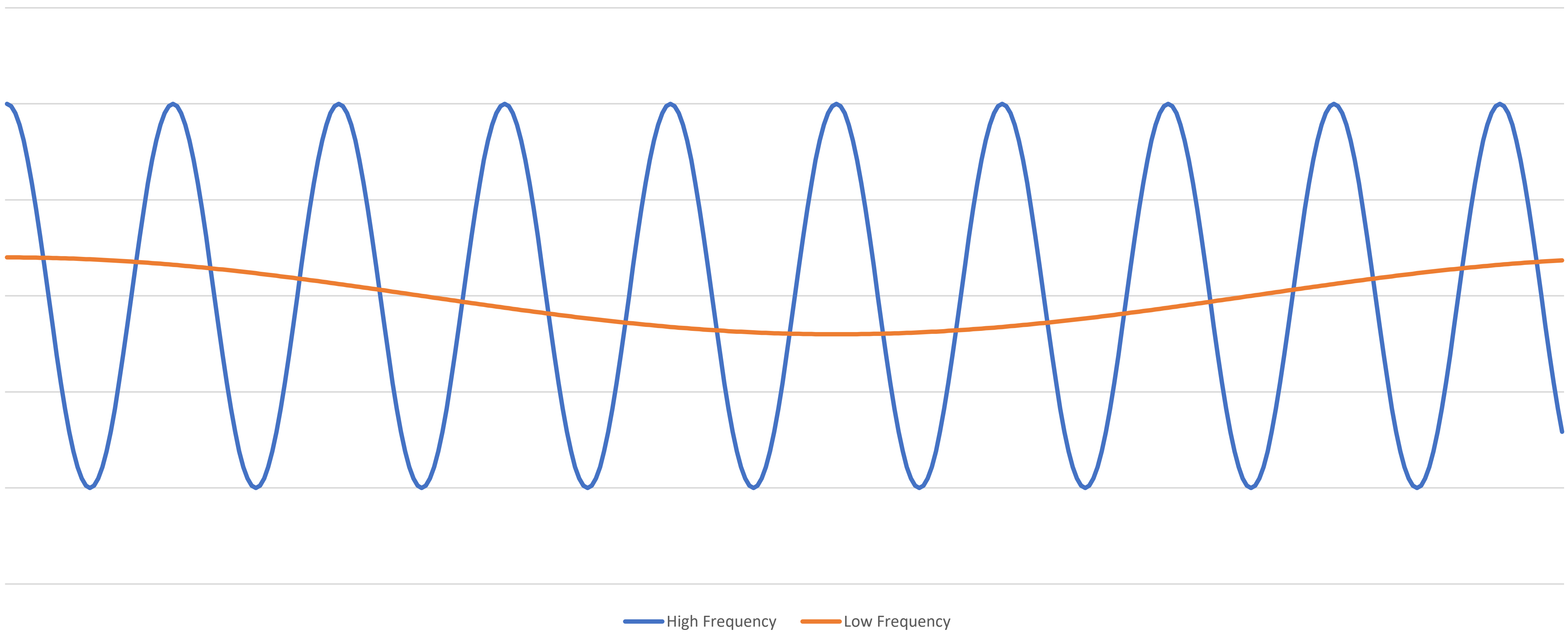
US Workers Compensation
Paid Link Ratios as a Periodic Sequence



A High Frequency Pattern with a Low Frequency Cycle



Our Job is to Separate the Two Components



Steps in Fourier Analysis

1. Calculate Fourier Coefficients

   The coefficients give the amplitude of component waves

2. Separate Pattern, Cycle and Noise

Modeling Periodic Data

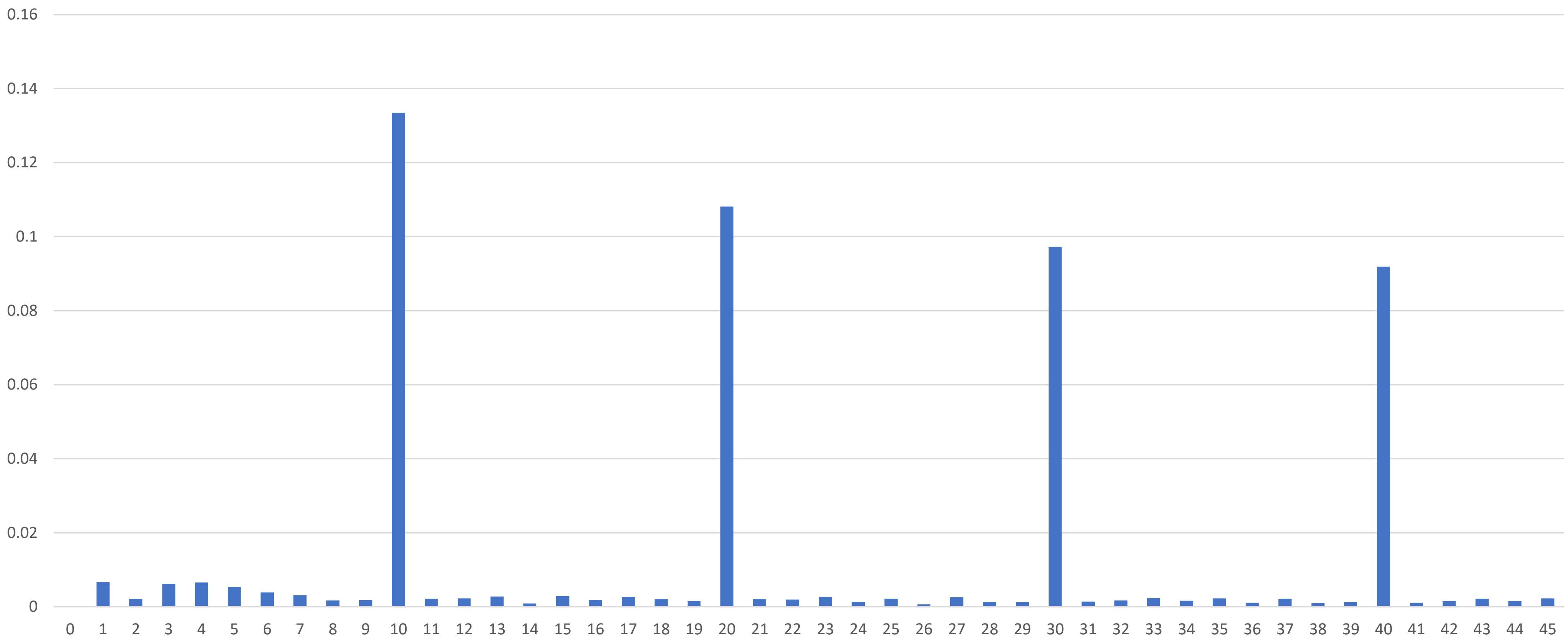
We can find the component waves by interpolating our sequential data with wave functions.

Interpolation Options

1. Fourier Series – real valued waves
2. Transforms – complex valued waves

Amplitude of Component Waves

for WC Link Ratios



Separating Signal from Noise
(by Analogy to Color Vision)

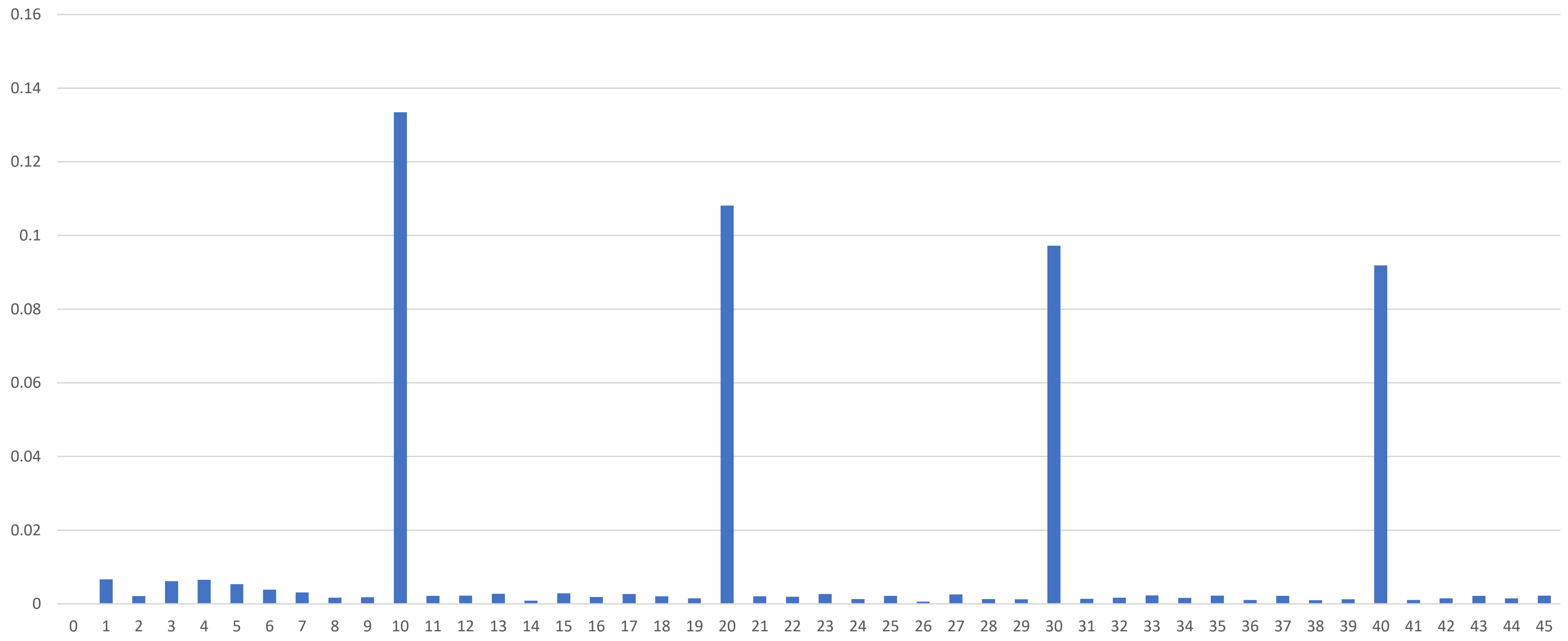
- Look for a particular frequency or a narrow band of frequencies

  e.g. Find something green

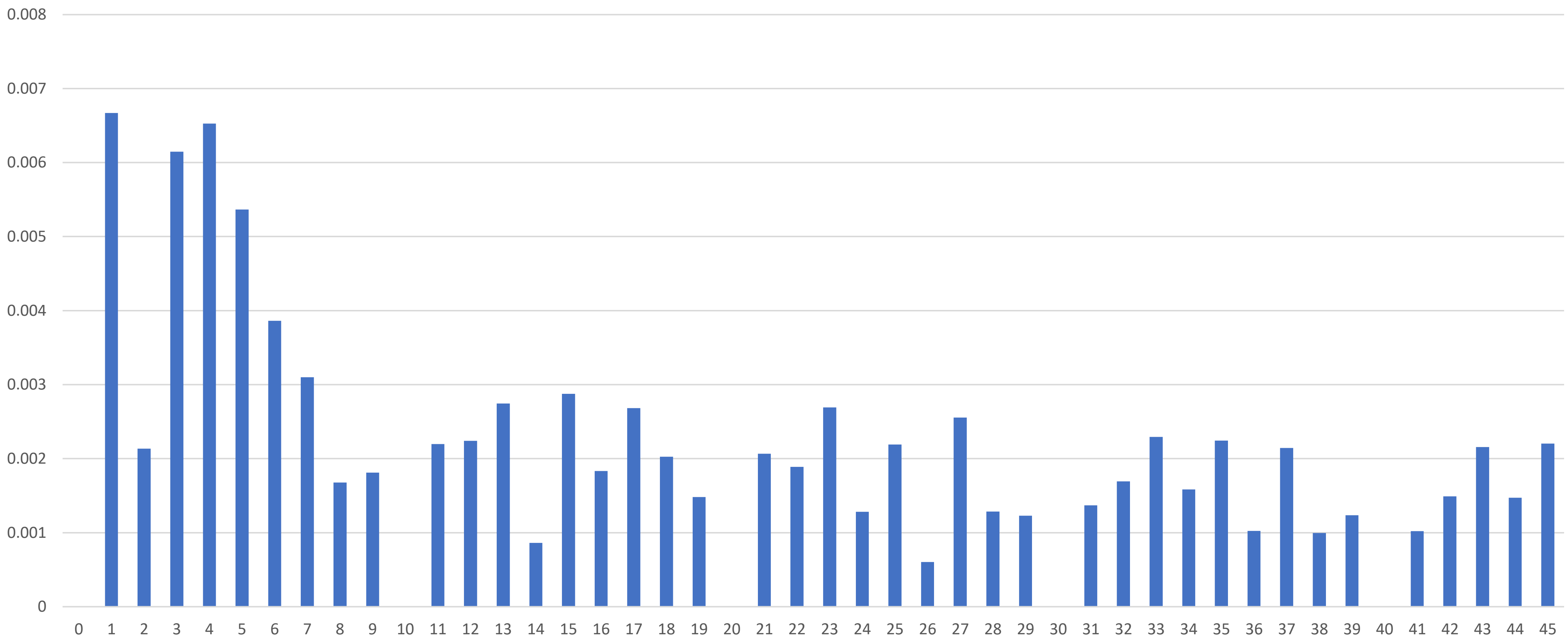
- Find the frequencies that are most significant

  e.g. What color is it?

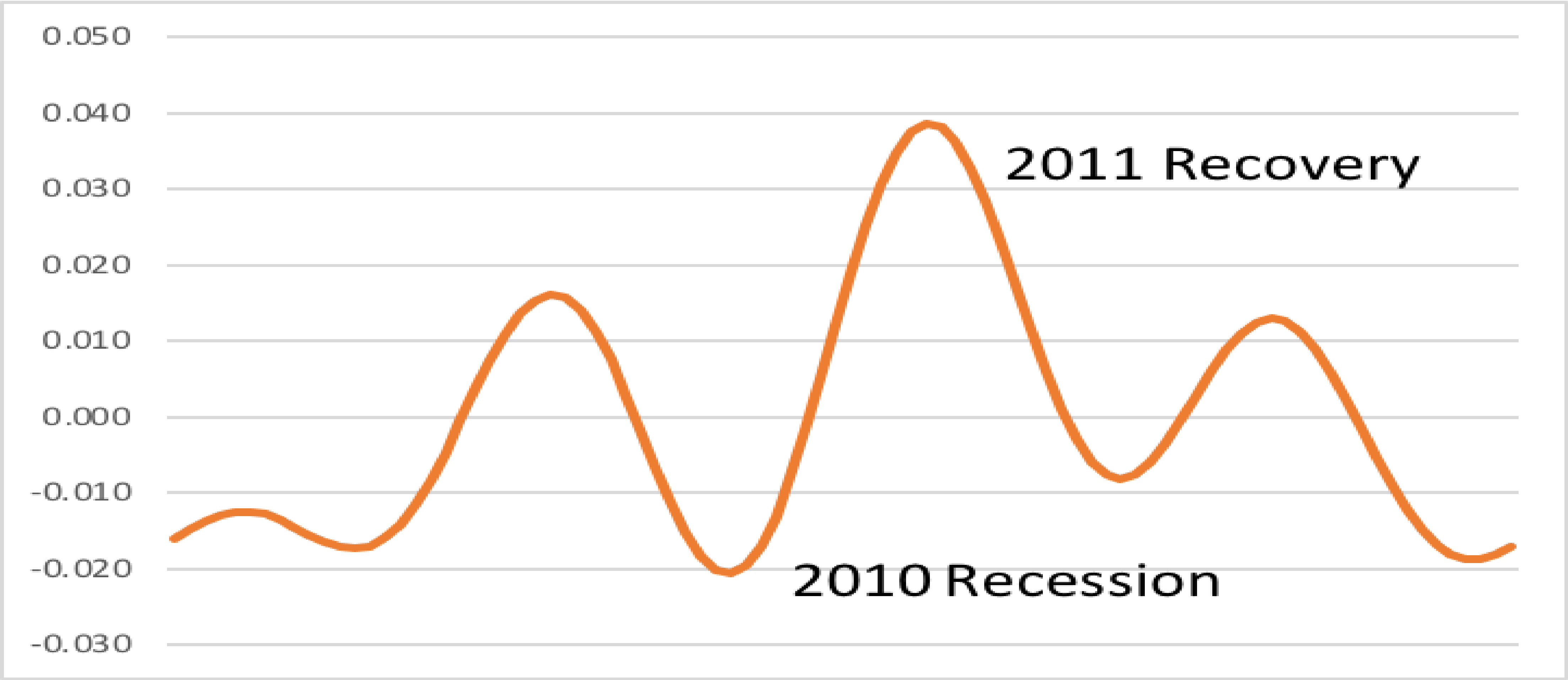
Amplitude of Component Waves
for WC Link Ratios



Amplitude of Noise Frequencies
for WC Link Ratios

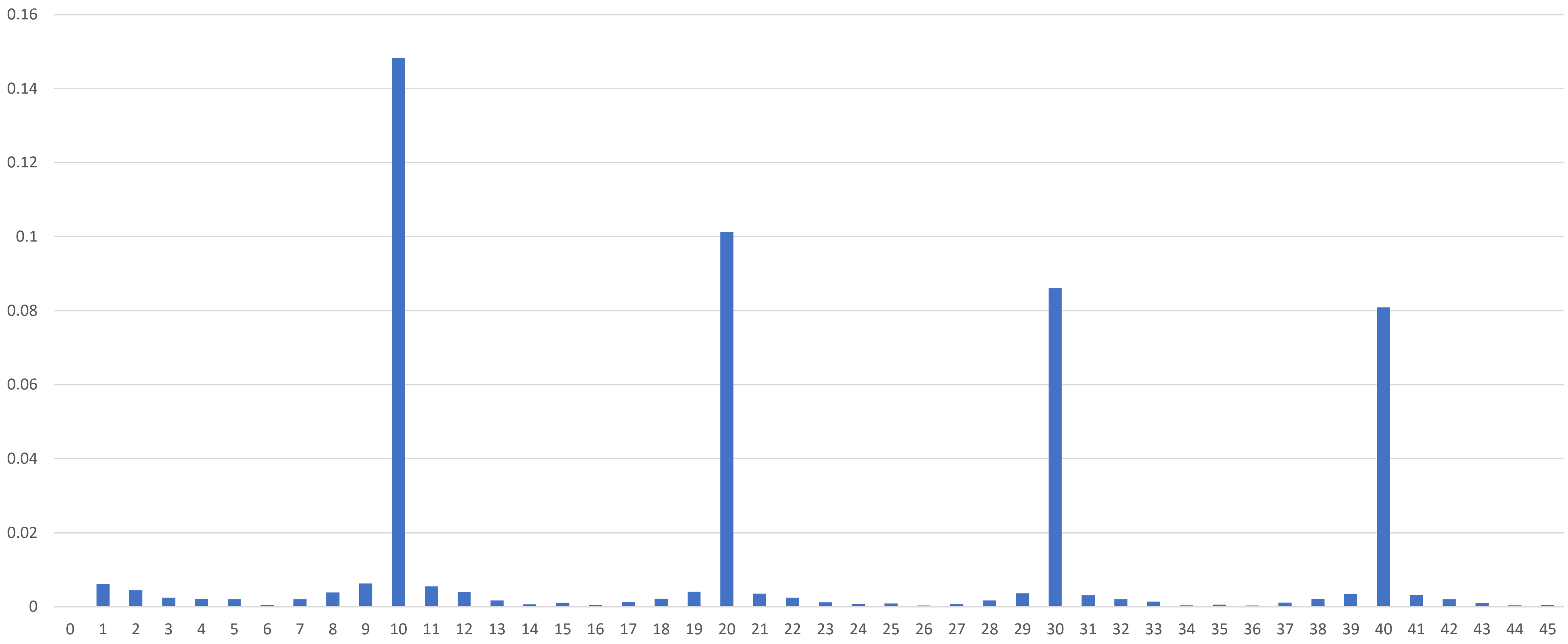


Cyclical Component of WC LDFs



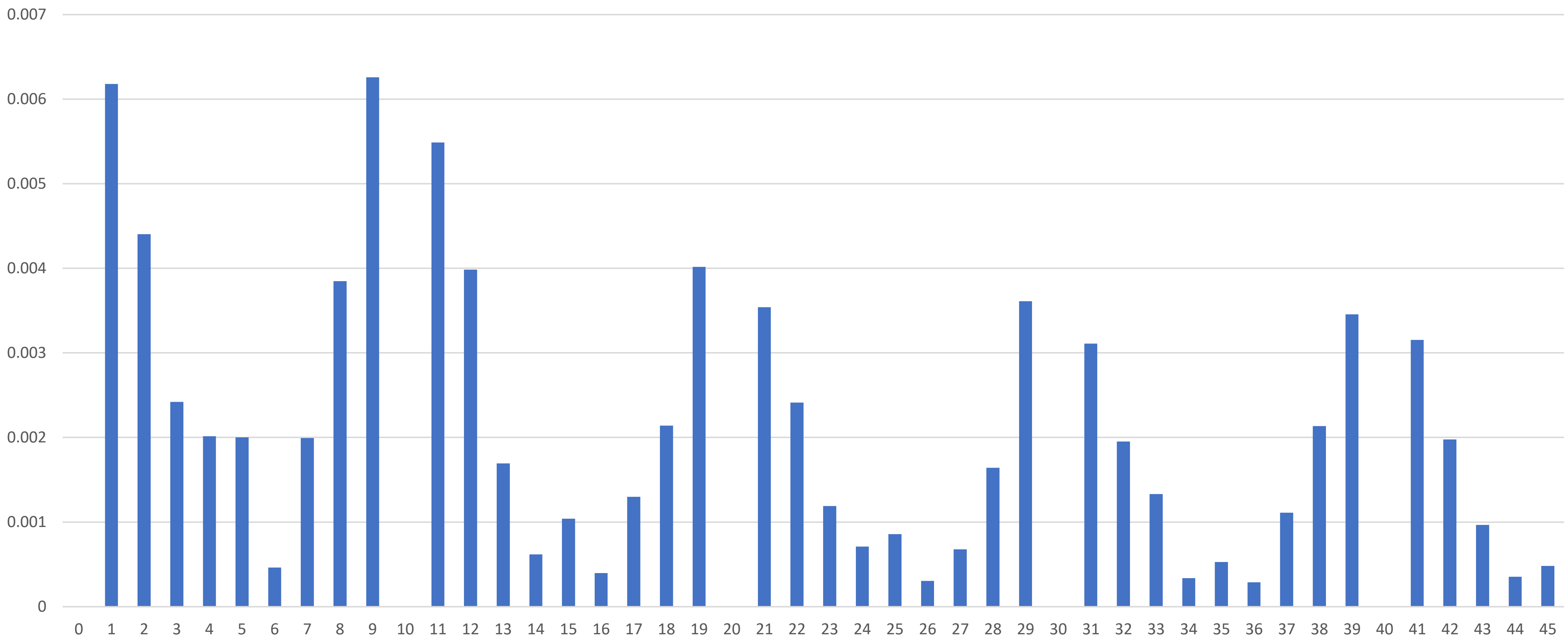
Amplitude of Component Waves

for Comm AL Link Ratios

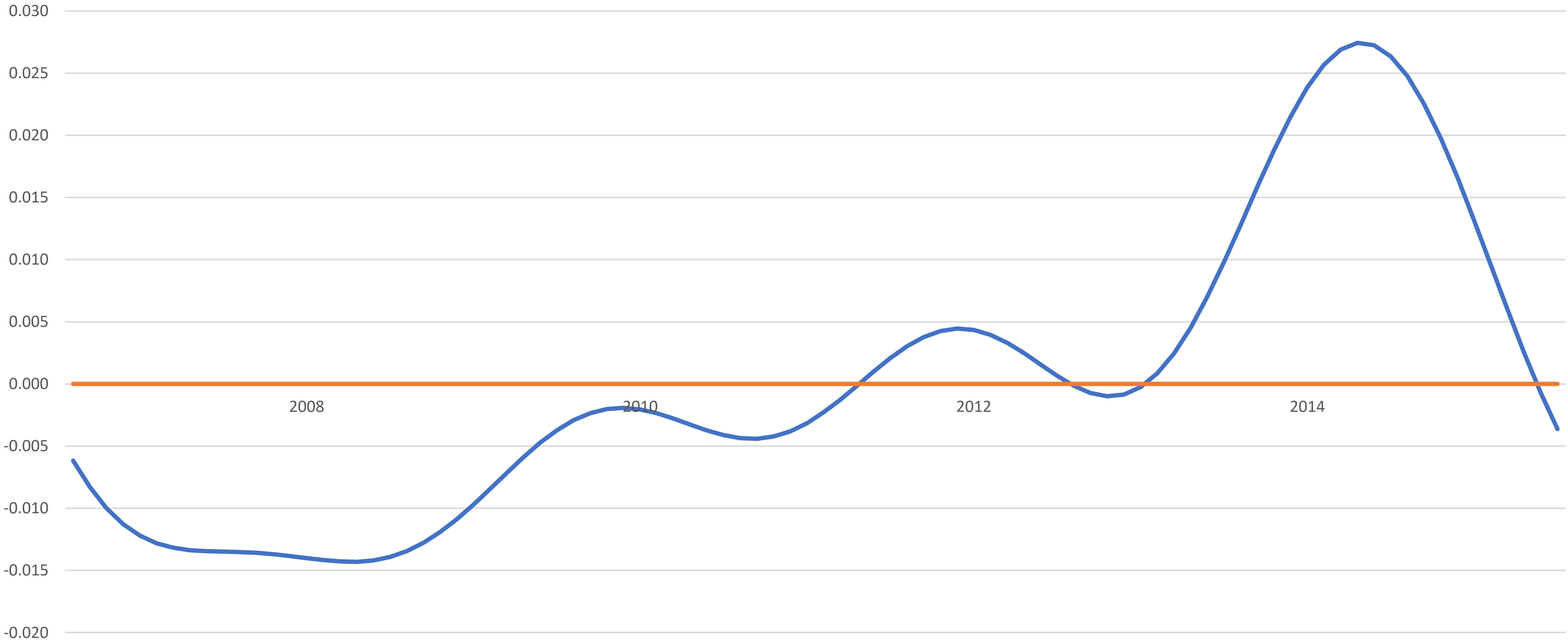


Amplitude of Noise Frequencies

for Comm AL Link Ratios



Cyclical Component of CAL Link Ratios



Questions?

1. How do we organize our data?

2. How do we calculate model parameters?

3. How do we separate pattern, cycle and noise?

Why Should We Use a Transform?

1. Transforms allow us to attach meanings to the quantities in our model

2. Transforms also allow us to solve otherwise difficult problems like convolutions and differential equations

Imaginary Exponents are Frequencies

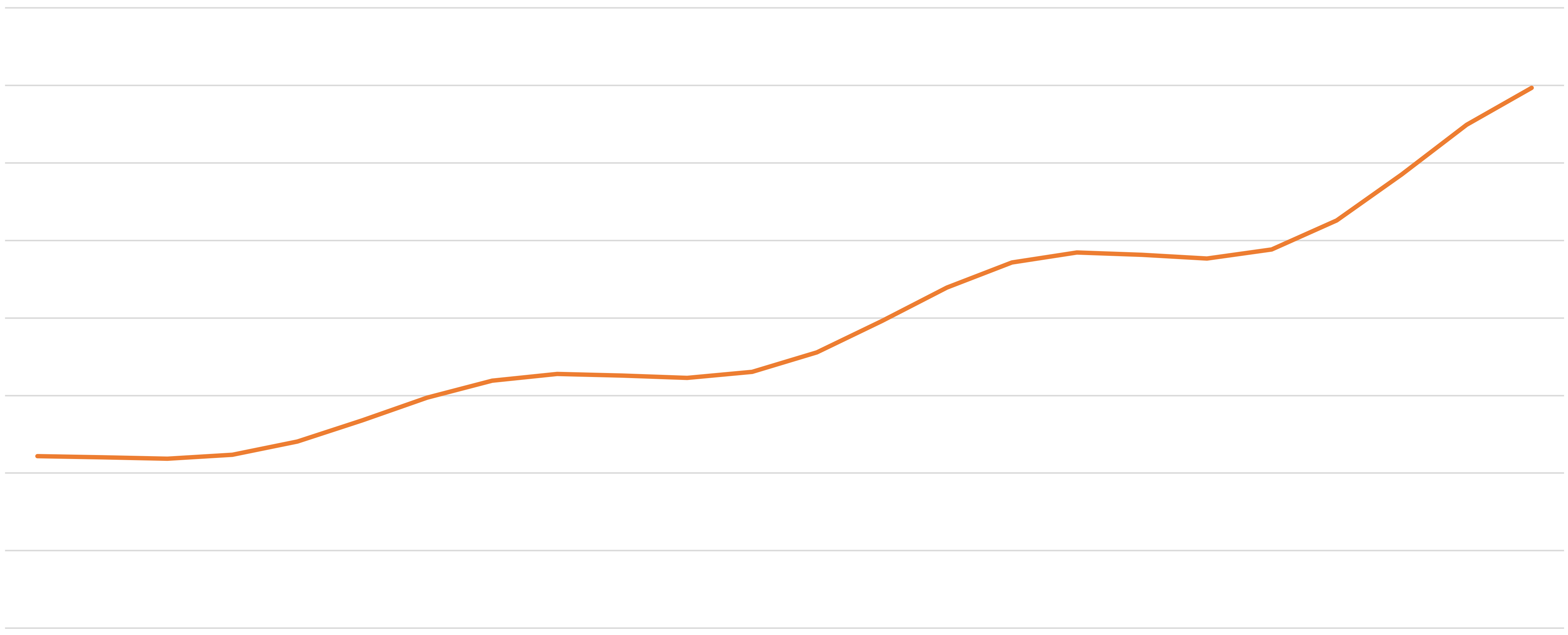
$\cos(\omega t)$ describes a periodic function with frequency ω

$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ describes a periodic function with frequency ω

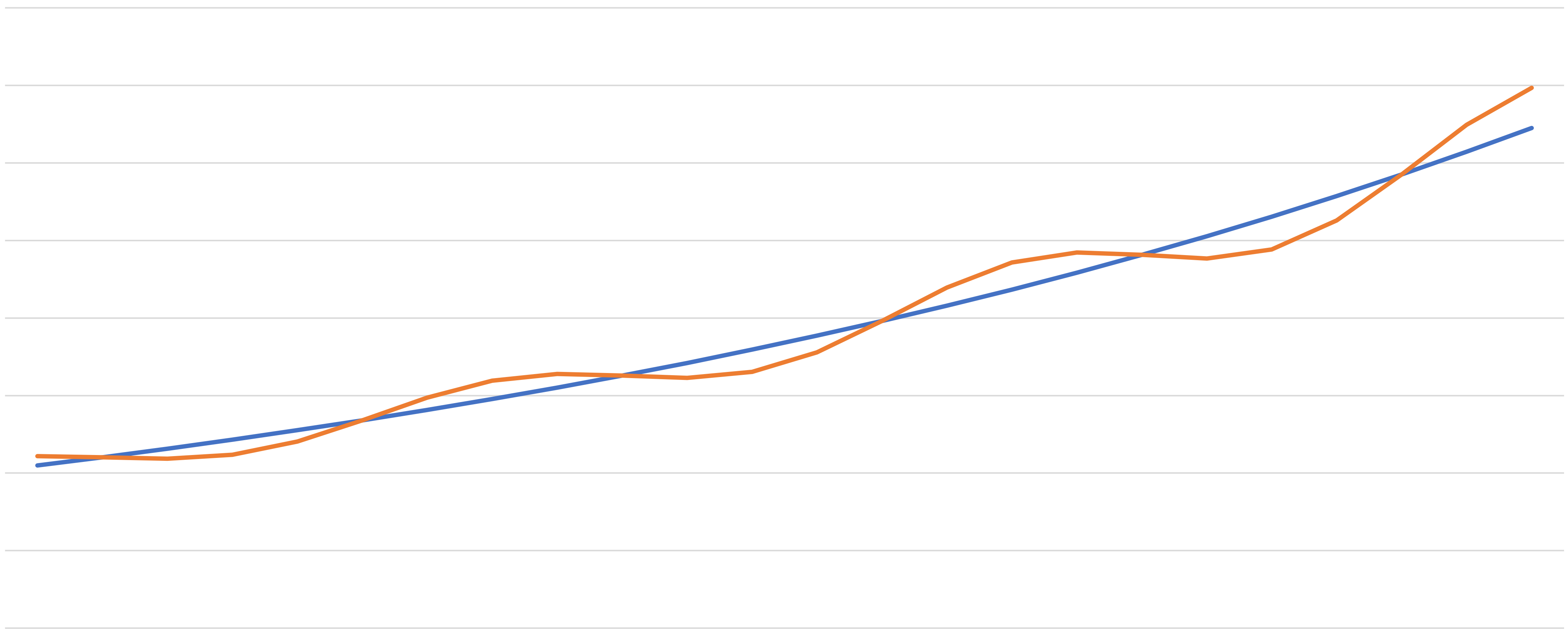
The presence of the imaginary unit i in an exponent changes the meaning of the exponent from a rate to a frequency

Our Typical Inflation Model

Real Life Inflation Exhibits Variability



We Can Model Variable Inflation as A Wave Around an Exponential



Mathematical Motivation

$e^{\alpha t}$ describes an exponential

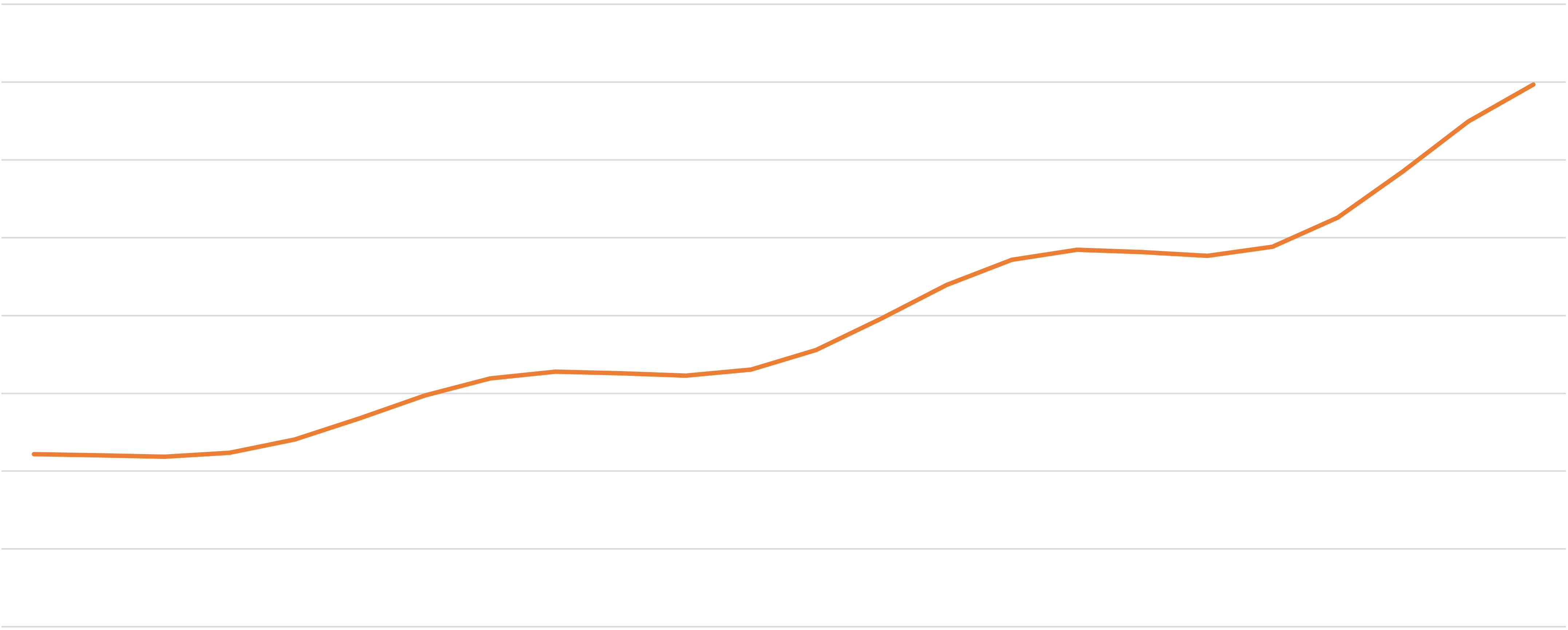
$e^{i\omega t} = \cos(\omega t) + i\, sin(\omega t)$ describes a wave

$e^{(\alpha + i\omega)t}$ describes a wave around an exponential

$$e^{\alpha t}$$

$$Re[e^{i\omega t}]$$

$$Re[e^{(\alpha+i\omega)t}]$$



Transform Formulas

$$T[x_t] = \sum_{t=0}^{N-1} x_t e^{-rt}$$

r is real	T is Present Value
$r = i\omega$	T is the Fourier transform
$r = \alpha + i\omega$	T is the LaPlace transform

Stochastic Present Value

The LaPlace transform of a sequence of cash flows can be interpreted as the stochastic present value (a distribution).

If the exponent represents inflation rather than interest, then the LaPlace transform represents the distribution of total losses at constant cost.