

Wavelet Based Equity VaR Estimation

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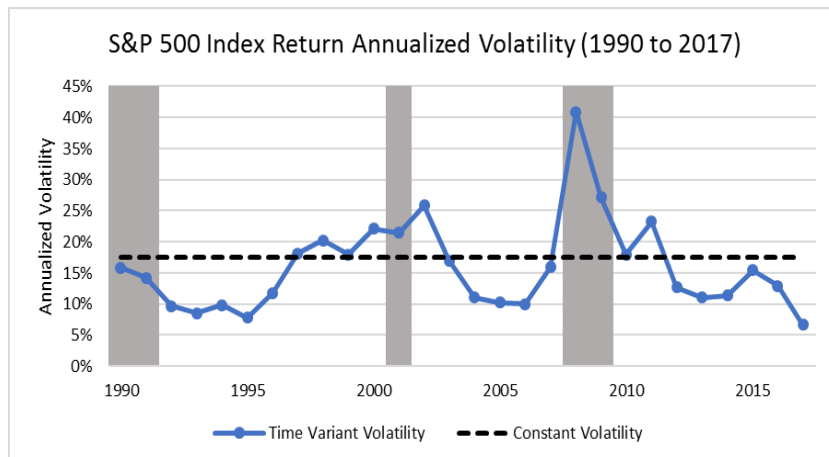
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Agenda

- **Problem: Time vs. Frequency**
- **Time Series Analysis**
- **Wavelet Analysis**
- **Implementation and Extension**

Time vs. Frequency



Time vs. Frequency

S&P 500 Index Return Volatility and VaR by Frequency

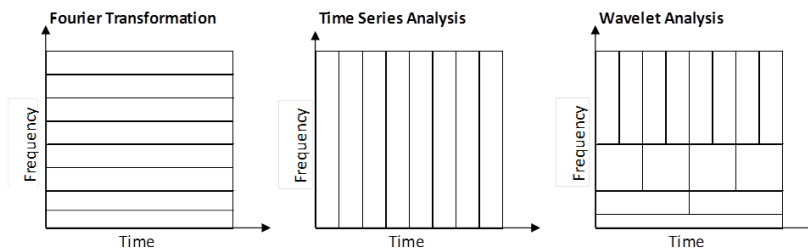
Frequency	Time-Invariant Volatility	Annualized Volatility	99.5% Empirical VaR	Annualized Empirical VaR
Daily	1.1%	17.5%	3.9%	69.3%
Monthly	4.2%	14.5%	19.3%	75.3%
Quarterly	7.9%	15.5%	26.9%	64.2%
Yearly	17.7%	17.5%	43.5%	43.5%

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Time vs. Frequency



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Time Series Analysis

- Equity return: **ARMA**

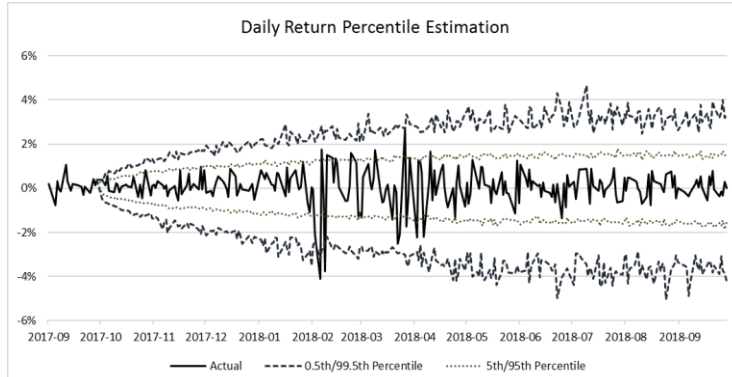
$$ARMA(p, q) \sim r_t = c + \varepsilon_t + \sum_{i=1}^p \phi_i r_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j}$$

- Conditional Vol: **GARCH**

$$GARCH(p, q) \sim \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

- Skewed generalized error distribution

$$f_{SGED}(x; \mu, \sigma, \lambda, p) = \frac{pe^{-\left(\frac{|x-\mu+m|}{\lambda\sigma(1+\lambda \text{sign}(x-\mu+m))}\right)^p}}{2v\sigma\Gamma(1/p)}$$



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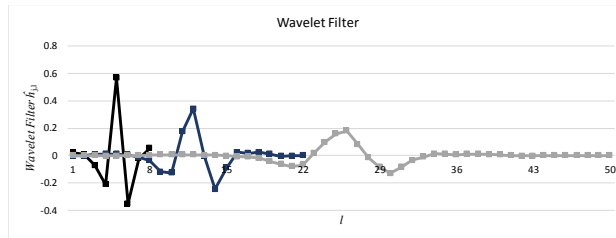
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Wavelet Analysis

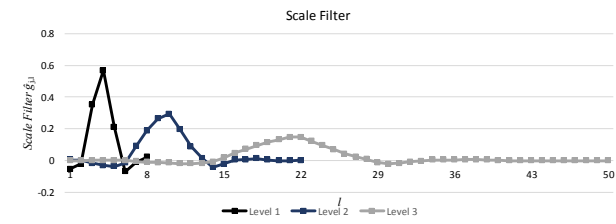
Wavelet coefficient

$$\tilde{W}_{j,t} = \sum_{i=0}^{L_j-1} \tilde{h}_{j,i} X_{t-l \text{ MOD } N}$$



Scale coefficient

$$\tilde{V}_{j,t} = \sum_{i=0}^{L_j-1} \tilde{g}_{j,i} X_{t-l \text{ MOD } N}$$

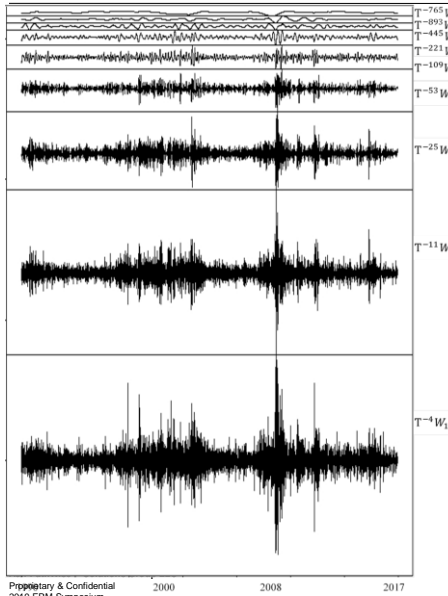


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Wavelet Analysis



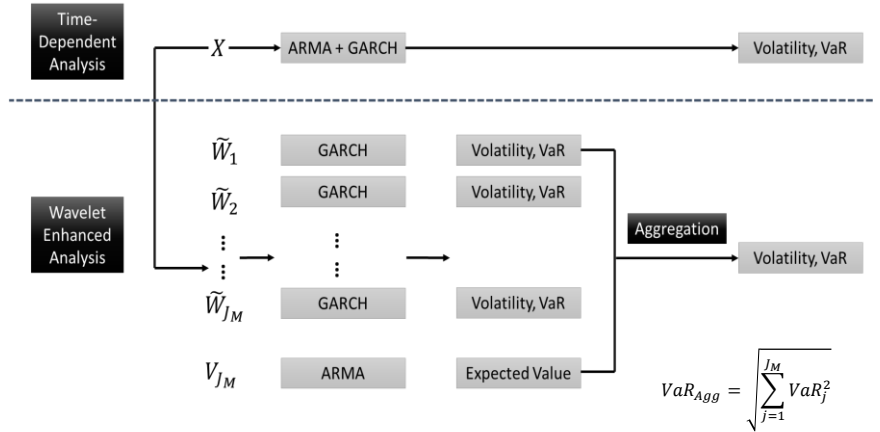
Level (j)	Frequency	Scale (1/Frequency)
1	[1/4, 1/2]	2 – 4 days
2	[1/8, 1/4]	4 – 8 days
3	[1/16, 1/8]	8 – 16 days
4	[1/32, 1/16]	16 – 32 days
5	[1/64, 1/32]	32 – 64 days
6	[1/128, 1/64]	64 – 128 days
7	[1/256, 1/128]	128 – 256 days
8	[1/512, 1/256]	256 – 512 days

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Wavelet Analysis

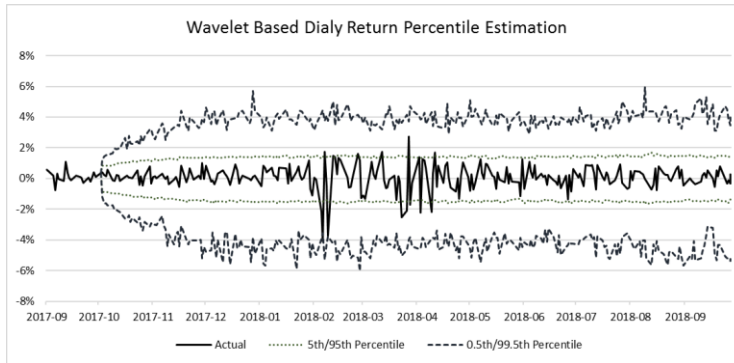


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Wavelet Analysis



Back-testing in the paper

- Medium Volatility (2007 Year End: 1.1%)
- Low Volatility (Sept. 2017: 0.5%)
- High Volatility (Sept. 2008: 2.6%)

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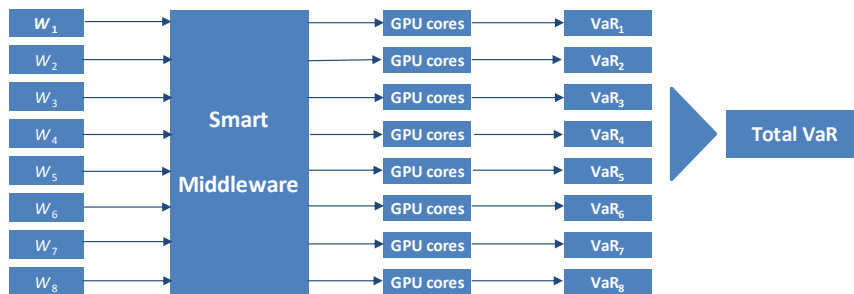


Implementation and Extension

Challenge

- Separate calibration and calculation at 8 or more levels
- Simulation based VaR estimation at each level: 10k simulation for each

Parallel Computing

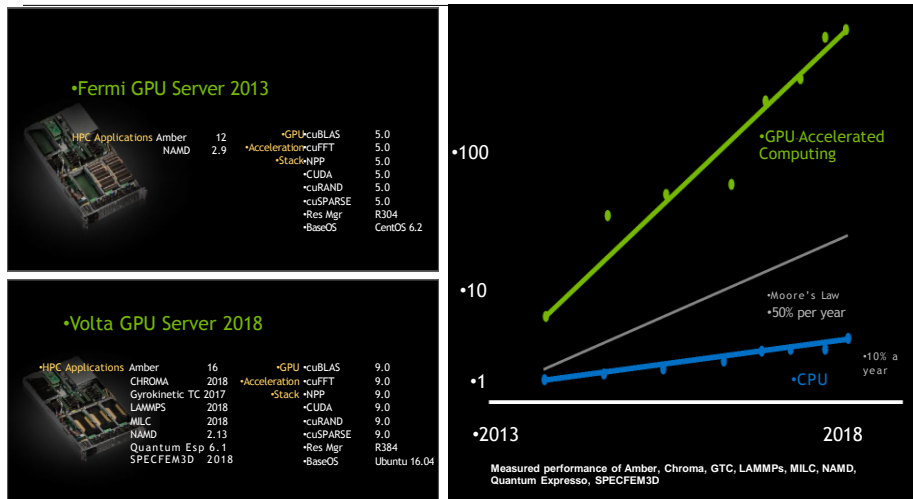


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The Age of Accelerated Computing Continued...



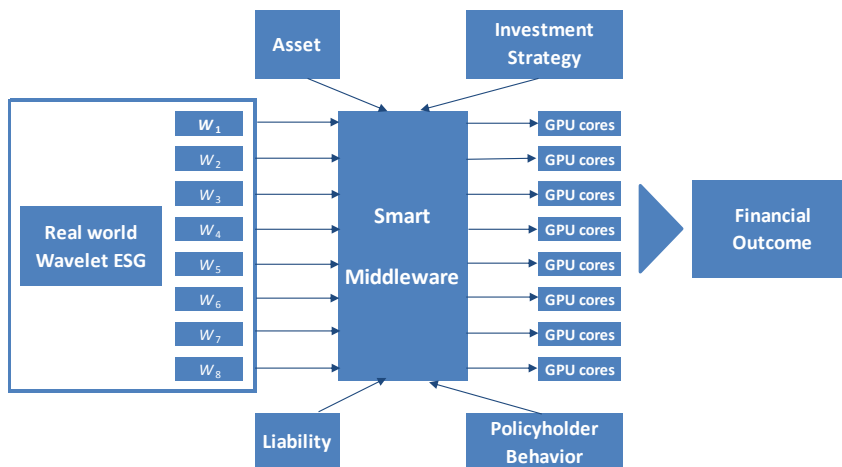
Advances in Technology allows us to perform complicated calculations in a **fast and cost-effective** way. In 5 years, memory size went from **6 GB to 516 GB**, and memory speed went from **192 GB/s to 900 GB/s**, and from throughput from **1.5 Tflops to 16 Tflops** in single precision ---eye watering figures

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Implementation and Extension

Use Wavelet based ESG for ALM, SAA, and Capital Analysis



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Thank you!

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