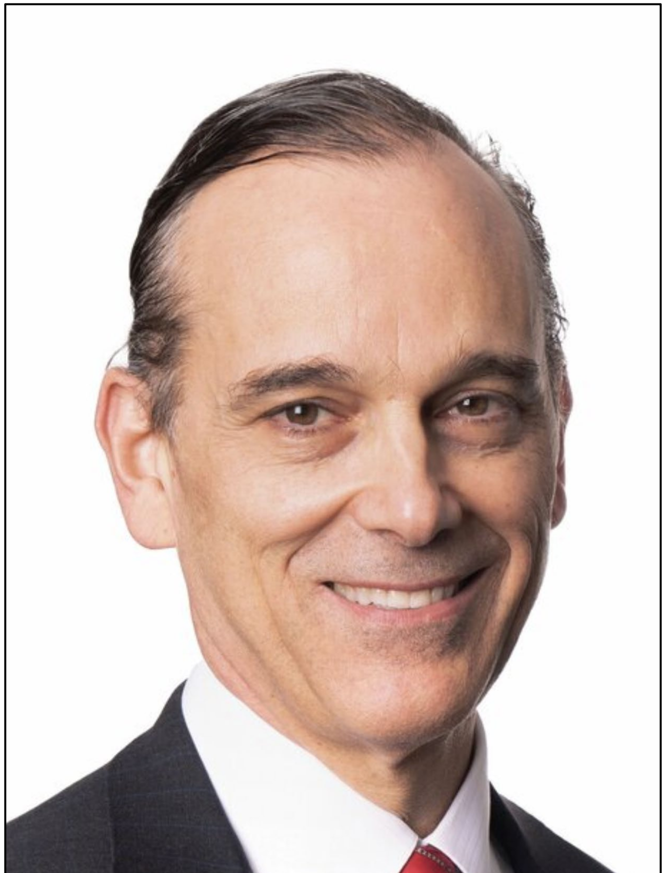
Reinsurance Decision Making: Past, Present and Future

John A. Major, ASA
Stephen J. Mildenhall

CAS Seminar on Reinsurance, June 2022

Donald F. Mango
1963-2022



INSURANCE CAPITAL AS A SHARED ASSET
BY
DONALD MANGO

ASTIN BULLETIN, Vol. 35, No. 2, 2005, pp. 471-486

Capital Tranching: A RAROC Approach to Assessing Reinsurance Cost Effectiveness

by Donald Mango, John Major, Avraham Adler, and Claude Bunick

CASUALTY ACTUARIAL SOCIETY VOLUME 7/ISSUE 1

Reinsurance Decision Making: Past, Present and Future
Part I: Pricing with Spectral Risk Measures

Stephen J. Mildenhall

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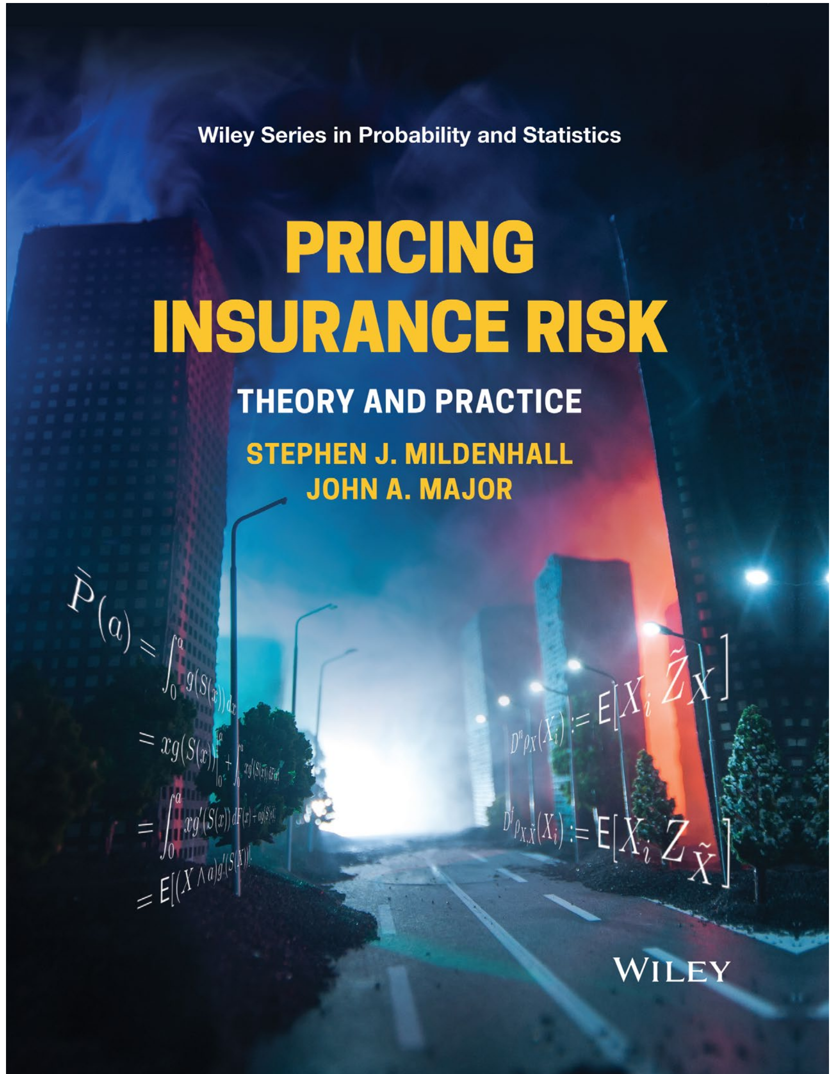
Wiley Series in Probability and Statistics
PRICING
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Portfolio Pricing: Five Simplifying Assumptions and One Objective

1. One-period model	2. No existing business	3. No taxes
4. No investment income	5. No expenses (handled separately)	Load loss cost for risk

Caveat: Everything presented is true most of the time, nothing is true all the time.

Pricing Functional: The Idea

Risk → Premium

$X \rightarrow \rho(X)$

X = random variable of outcomes

- Cat model
- Casualty simulation model
- Parametric distribution

convex risk

Pricing Functional: Desirable Properties

- Consistent with prices in a competitive market

1. Monotone: $X \le Y$ implies $\rho(X) \le \rho(Y)$
2. Respects diversification: $\rho(X + Y) \le \rho(Y) + \rho(X)$
3. But...no credit when no diversification
   – If outcomes X and Y imply same event order, then $\rho(X + Y) = \rho(Y) + \rho(X)$
4. $\rho(X)$ only depends on the distribution of X

- Jargon: 2 = sub-additive, 3 = comonotonic additive, 4 = law invariant (**SCALI**)

SCALI Risk Measure = Spectral Risk Measure (SRM)

- SRMs have **four** different representations of ρ(X)
  - Weighted average of VaRs
  - Weighted average of TVaRs
  - Worst over a set of probability scenarios
  - Distorted expected value

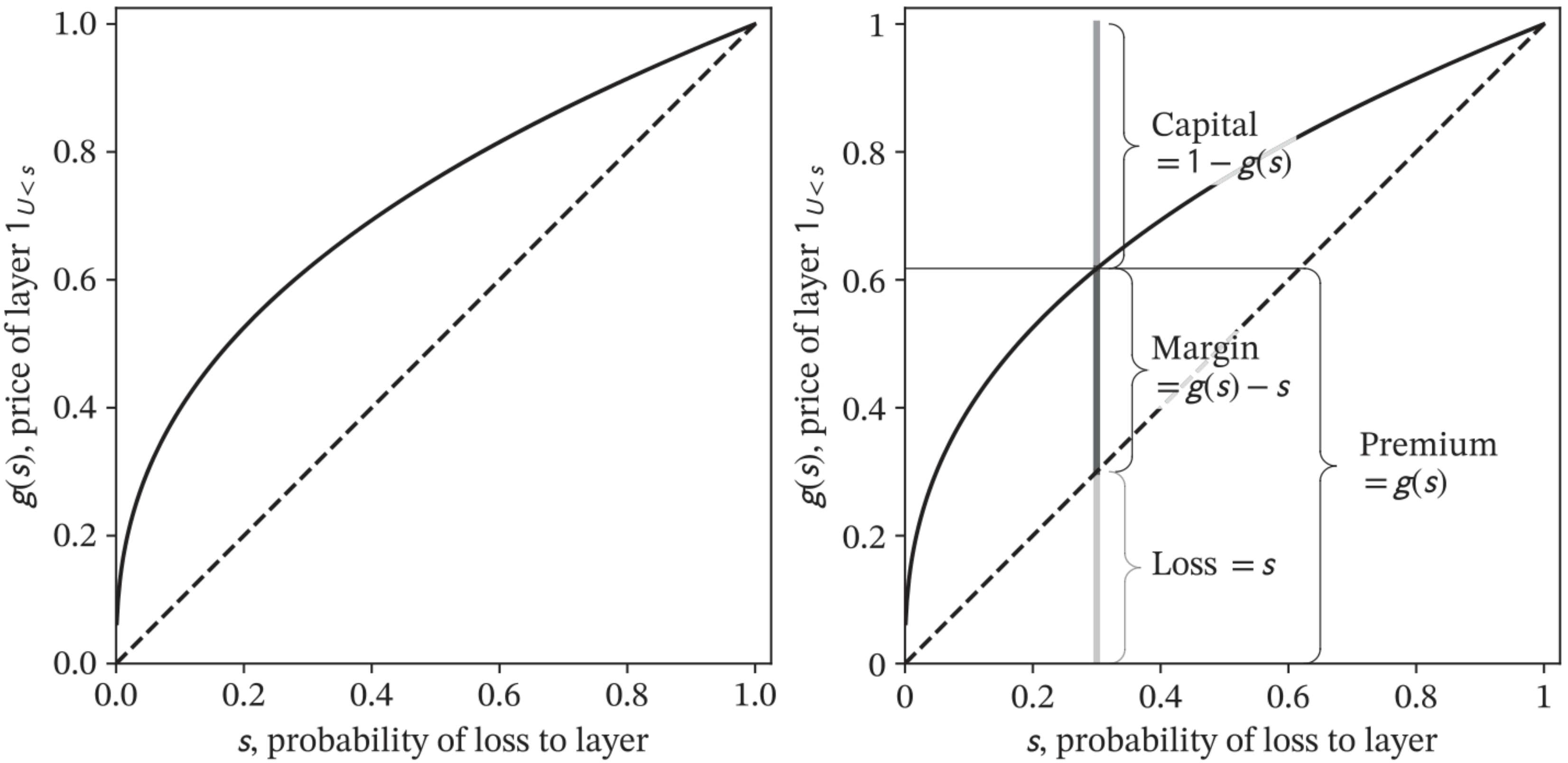
- Distorted expected value: there exists an increasing, concave distortion function g so that

$$\rho_g(X) := \int_0^\infty g(S_X(x))\,dx = \mathsf{E}[Xg'(S(X))]$$

where $S_X(x) = \Pr(X>x)$ is the survival function of X

- Expectation representation shows SRMs have a natural allocation, which also equals the marginal allocation

Distortion Function g Prices Bernoulli 0/1 Risk



Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Spectral Risk Measure Portfolio Pricing

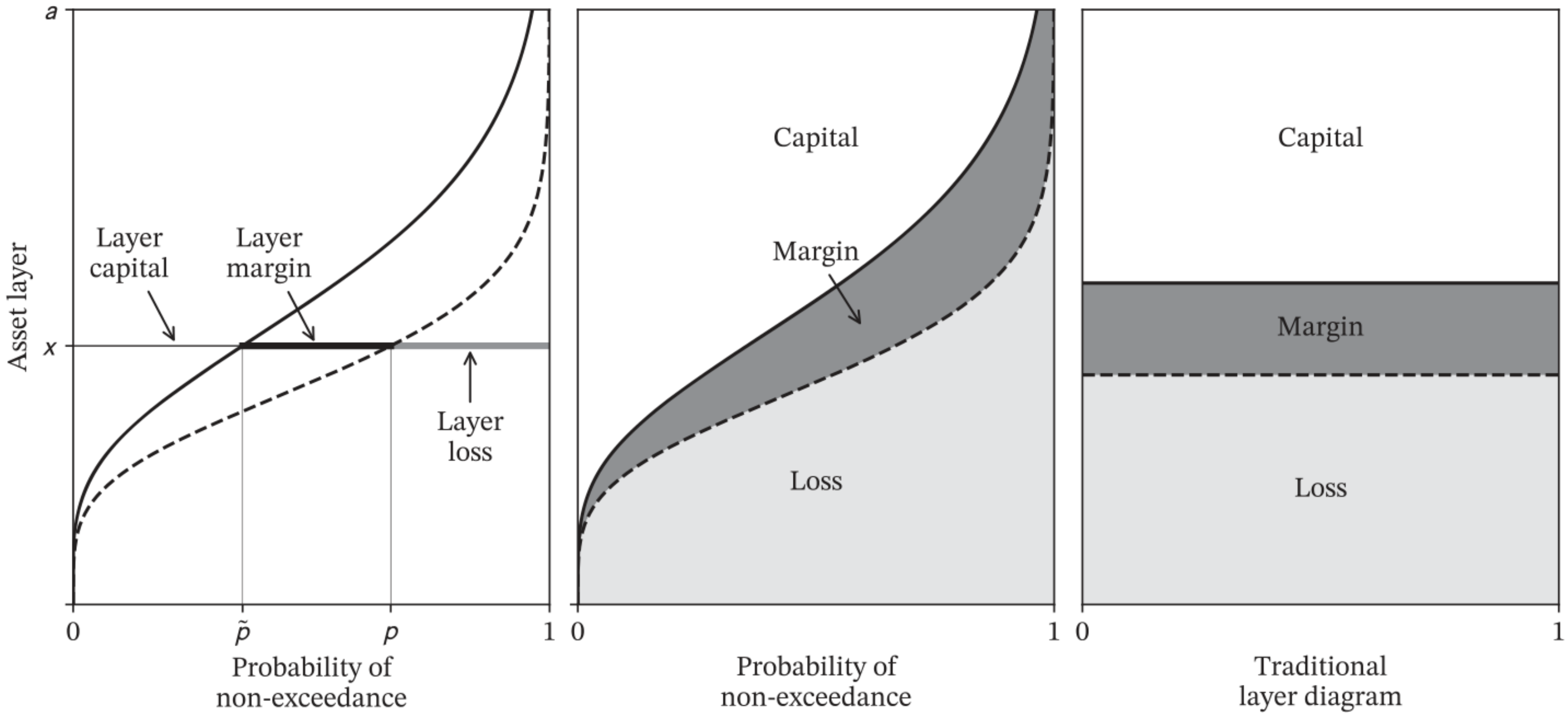


Figure 10.5 Continuum of risk sharing varying by layer of loss (dashed) and premium (solid): by layer (left), in total summed over layers (middle), and traditional (right). The total loss, margin, and capital areas are equal in the middle and right plots. Losses in the left and middle plots are Lee diagrams.

Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Case Study: Financial Model

- InsCo. has only **two sources** of assets
  - **Policyholders** pay **premium** by buying policies at InsCo's asking price
  - **Investors** contribute **capital** by buying residual value at their bid price

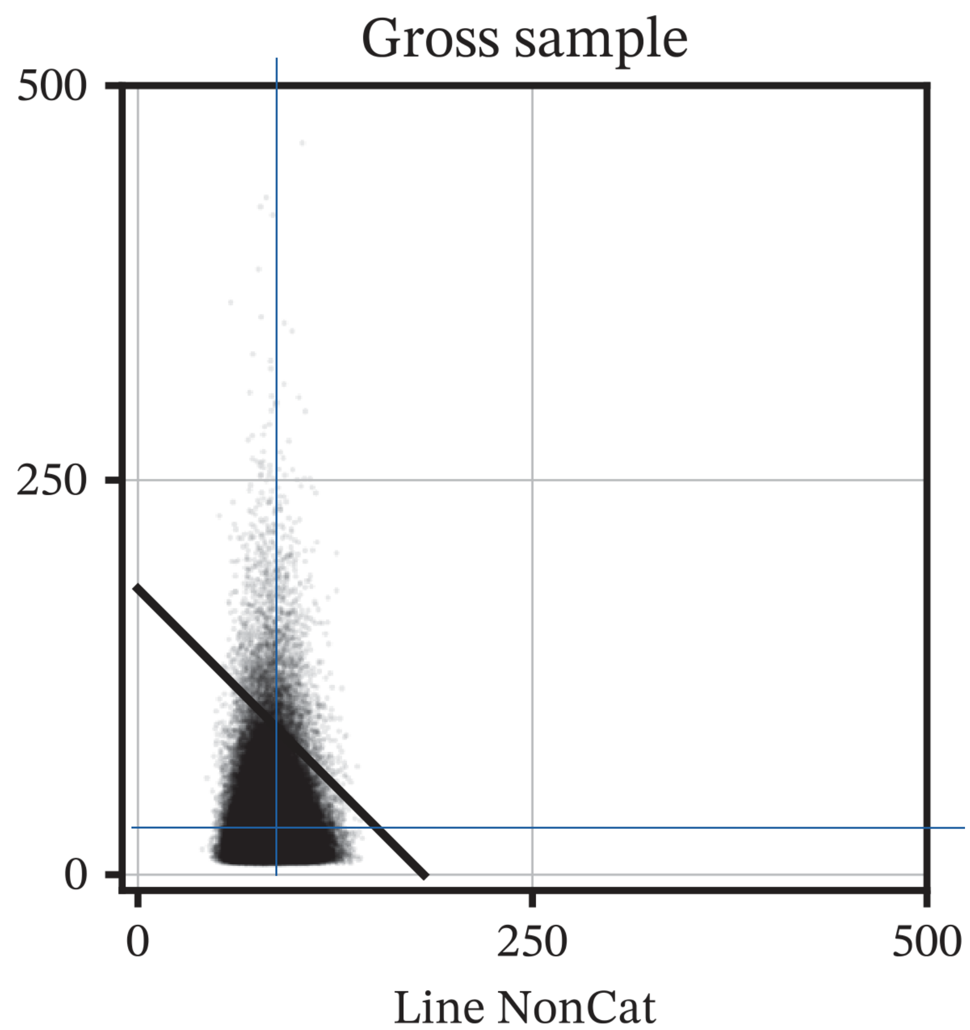
- At time 0
  - Premium P
  - Capital Q
  - Assets a = P + Q
  - Asset amount a is set by regulator/rating agency

- At time 1
  - Claims X revealed
  - Policyholder payments $X \wedge a = \min(X, a)$
  - Investor return $(a - X)^+ = \max(0, a - X)$

Case Study: Cat/NonCat Stochastic Model

- Non-cat: Gamma
  - mean 80, cv 0.15
- Cat: Lognormal
  - mean 20, cv 1.0
- Independent
- Total
  - mean 100, cv 0.233

- Asset requirement
  - VaR 99.9% = 267.2



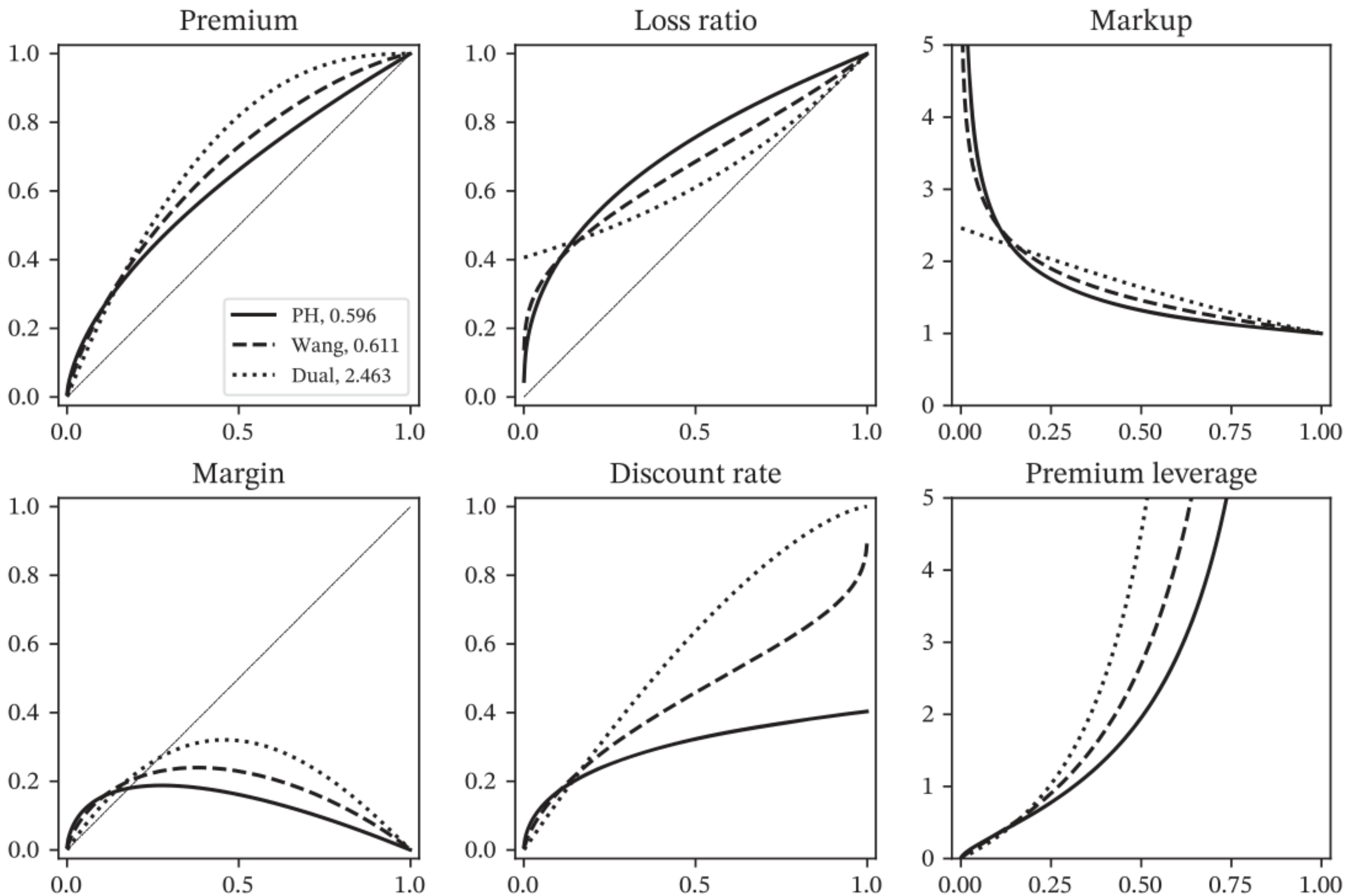
Example g, Shape of g and Properties of ρ

Table 11.5 Parameters for the six SRMs and associated distortions that are applied in Section 11.4.

ID	Distortion	$g(s)$		Tame	Cat/Non-Cat	Hu/SCS
CCoC	CCoC	$(s+\iota)/(1+\iota)$	$\iota =$	0.10	0.10	0.10
PH	Proportional hazard	s^α	$\alpha =$	0.683	0.596	0.449
Wang	Wang	$\Phi(\Phi^{-1}(s)+\lambda)$	$\lambda =$	0.375	0.611	1.190
Dual	Dual moment	$1-(1-s)^m, m \ge 1$	$m =$	1.576	2.463	12.029
TVaR	TVaR	$s/(1-p) \wedge 1$	$p =$	0.227	0.482	0.899
Blend	PWL					

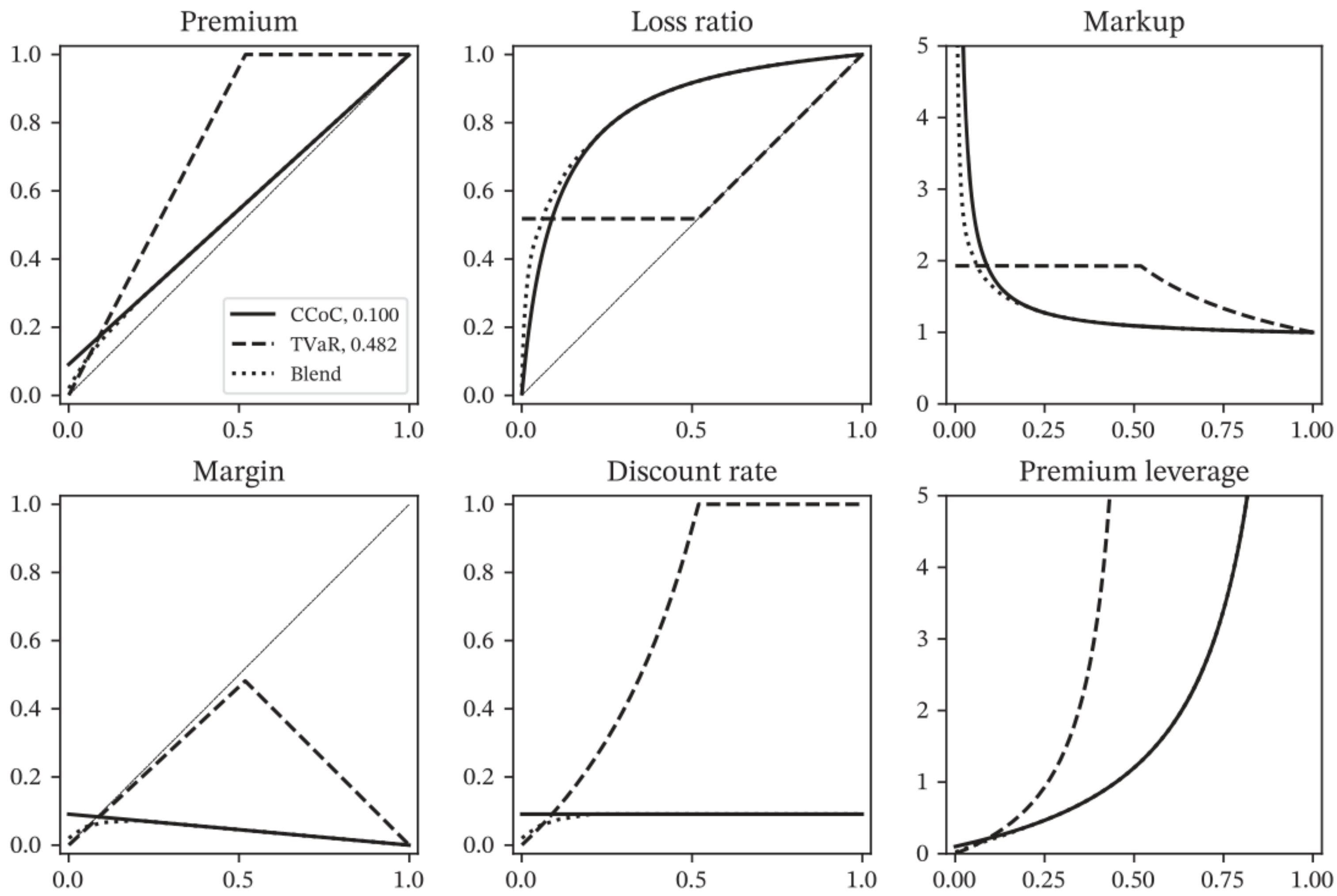
Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Shape of g and Properties of ρ



Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Shape of g and Properties of ρ

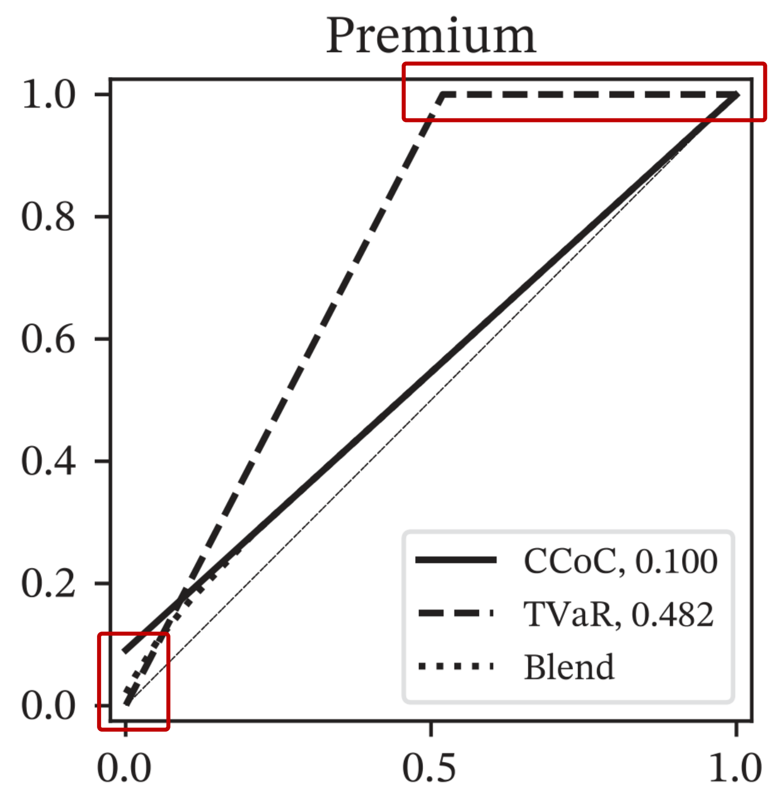


Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

convex risk

Shape of g and Properties of ρ

1. If g is steep near s=0 it has expensive tail-risk capital
   - CCoC > PH > Wang > Dual > TVaR
2. If g is flat near s=1 it has expensive body-risk capital
   - Opposite order
3. CCoC vertical at 0: has the most expensive tail-risk and cheapest body-risk capital
4. TVaR flat at 1: has the most expensive body-risk and cheapest tail-risk capital



See "Similar Risks have Similar Premiums", IME 2022 for more, https://authors.elsevier.com/a/1e%7EbLc7vgdMA6

Stand-Alone Pricing: Cat low 30s, NonCat upper 80s

		Gross				Net		
Statistic	**Distortion**	**Cat**	**Non-Cat**	**SoP**	**Total**	**Cat**	**SoP**	**Total**
Loss	Blend	19.95	80.00	99.94	99.95	17.73	97.73	97.73
Margin	CCoC	15.03	3.84	18.87	15.21	7.99	11.83	8.17
	PH	13.81	6.37	20.17	15.21	7.38	13.75	9.82
	Wang	12.88	7.51	20.38	15.21	7.95	15.45	11.06
	Dual	11.88	8.6	20.48	15.21	8.83	17.43	12.40
	TVaR	11.21	9.17	20.38	15.21	9.15	18.32	13.15
	Blend	6.49	2.39	8.88	6.81	3.44	5.83	4.09
Premium	CCoC	34.98	83.84	118.8	115.2	25.72	109.6	105.9
	PH	33.75	86.36	120.1	115.2	25.11	111.5	107.6
	Wang	32.82	87.50	120.3	115.2	25.68	113.2	108.8
	Dual	31.83	88.59	120.4	115.2	26.57	115.2	110.1
	TVaR	31.15	89.17	120.3	115.2	26.88	116.0	110.9
	Blend	26.43	82.39	108.8	106.8	21.17	103.6	101.8
Loss ratio	CCoC	0.57	0.954	0.841	0.868	0.689	0.892	0.923
	PH	0.591	0.926	0.832	0.868	0.706	0.877	0.909
	Wang	0.608	0.914	0.831	0.868	0.691	0.863	0.898
	Dual	0.627	0.903	0.83	0.868	0.667	0.849	0.887
	TVaR	0.64	0.897	0.831	0.868	0.66	0.842	0.881
	Blend	0.755	0.971	0.918	0.936	0.838	0.944	0.96

Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Allocated Pricing using the Natural Allocation

		Gross			Net			Ceded
Statistic	Method	Cat	Non-Cat	Total	Cat	Non-Cat	Total	Diff
Loss	Expected loss	19.96	79.99	99.95	17.75	79.98	97.73	2.21
Margin	Expected loss	3.04	12.17	15.21	1.78	8.04	9.82	5.38
	Dist ROE	22.02	−6.82	15.21	13.94	−5.77	8.17	7.04
	Dist PH	12.94	2.27	15.21	6.32	3.51	9.82	5.38
	Dist Wang	11.31	3.9	15.21	6.15	4.91	11.06	4.15
	Dist dual	9.72	5.49	15.21	6.47	5.94	12.40	2.8
	Dist TVaR	8.74	6.47	15.21	6.65	6.5	13.15	2.05
	Dist blend	7.32	−0.507	6.81	4.31	−0.217	4.09	2.73
Premium	Expected loss	22.99	92.16	115.2	19.53	88.02	107.6	7.6
	Dist ROE	41.98	73.17	115.2	31.68	74.22	105.9	9.25
	Dist PH	32.89	82.26	115.2	24.06	83.49	107.6	7.6
	Dist Wang	31.26	83.89	115.2	23.89	84.89	108.8	6.36
	Dist dual	29.68	85.48	115.2	24.21	85.92	110.1	5.02
	Dist TVaR	28.69	86.46	115.2	24.39	86.49	110.9	4.27
	Dist blend	27.28	79.48	106.8	22.05	79.77	101.8	4.94
Loss ratio	Expected loss	0.868	0.868	0.868	0.909	0.909	0.909	0.291
	Dist ROE	0.475	1.09	0.868	0.560	1.08	0.923	0.239
	Dist PH	0.607	0.972	0.868	0.738	0.958	0.909	0.291
	Dist Wang	0.638	0.954	0.868	0.743	0.942	0.898	0.348
	Dist dual	0.673	0.936	0.868	0.733	0.931	0.887	0.441
	Dist TVaR	0.696	0.925	0.868	0.728	0.925	0.881	0.519
	Dist blend	0.732	1.01	0.936	0.805	1	0.96	0.448

Stand-Alone

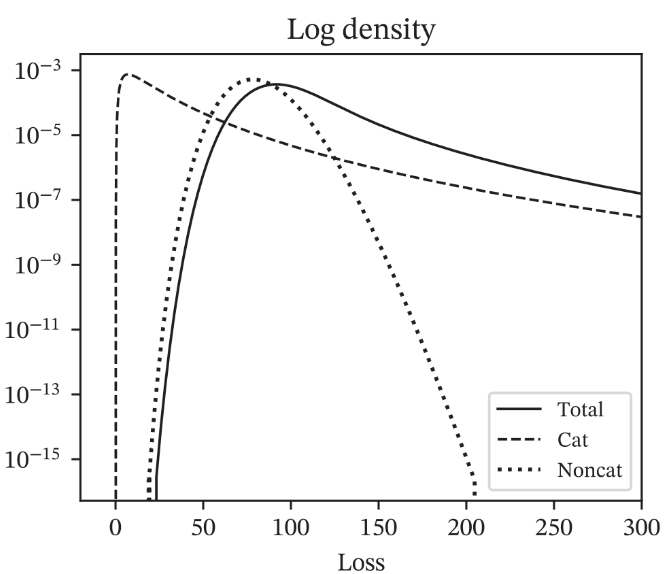
Premium	CCoC	34.98	83.84
	PH	33.75	86.36
	Wang	32.82	87.50
	Dual	31.83	88.59
	TVaR	31.15	89.17
	Blend	26.43	82.39

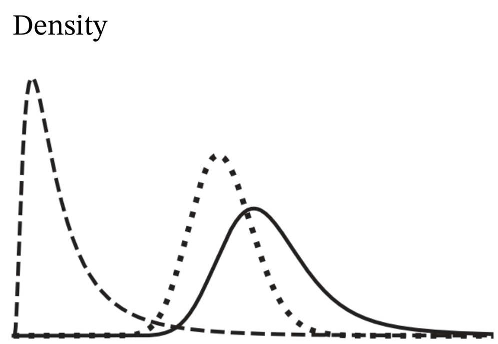
Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley [**CORRECTED**]

Why Do the Allocations Make Sense?

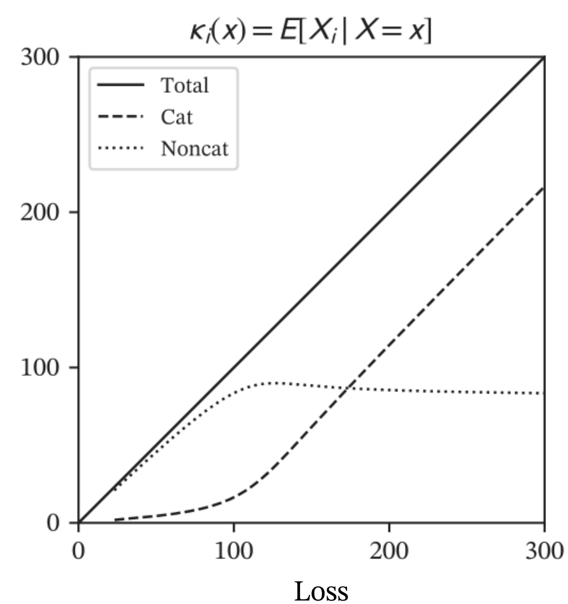
	Cat	Total	NonCat
	PH, Tail-Centric		
Stand-alone	**33.75**	**120.11**	**86.36**
Allocation	**32.89**	**115.15**	**82.26**
Change	0.86 ←	4.96	→ 4.1
Pct	17%		83%

	Cat	Total	NonCat
	TVaR, Body-Centric		
Stand-alone	**31.15**	**120.32**	**89.17**
Allocation	**28.69**	**115.15**	**86.46**
Change	2.46 ←	5.17	→ 2.71
Pct	48%		52%





- Cat: thick tail, narrow body
- NonCat: thin tail, broad body



Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Reinsurance Decision Making: Past, Present and Future

Part 2: Reinsurance Applications

John A. Major, ASA
MajorAnalytics@yahoo.com

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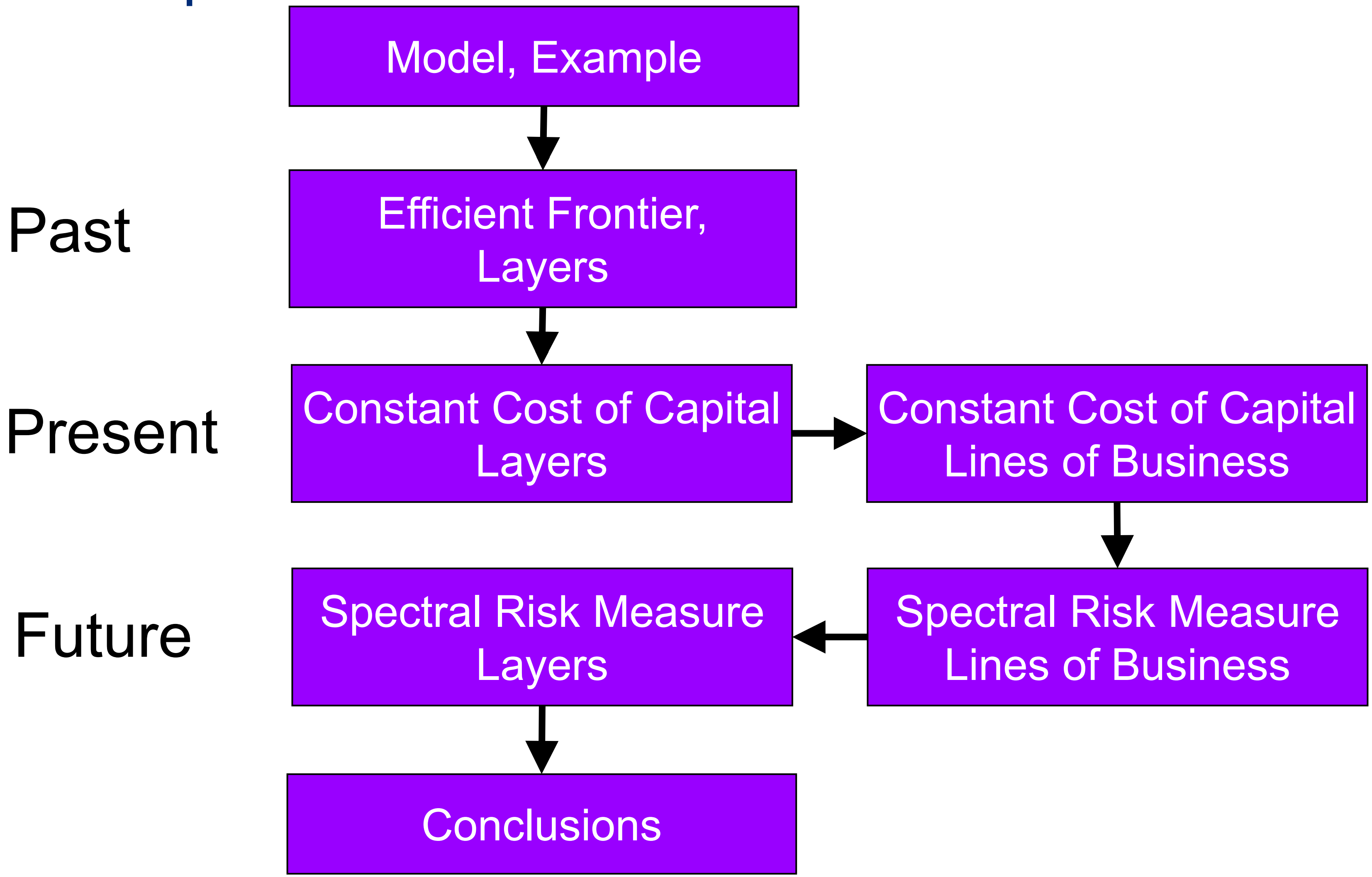
Roadmap



Model, Example
Efficient Frontier, Layers
Past
Constant Cost of Capital Layers
Constant Cost of Capital Lines of Business
Present
Spectral Risk Measure Lines of Business
Spectral Risk Measure Layers
Future
Conclusions

Adding Reinsurance to the Financial Model

- **Three** sources of assets
  - Policyholders P_n
  - Investors Q_n
  - Reinsurance X_c (ceded losses)

- Gross loss $X = X_n + X_c$
- Cost of reinsurance (ceded premium) π

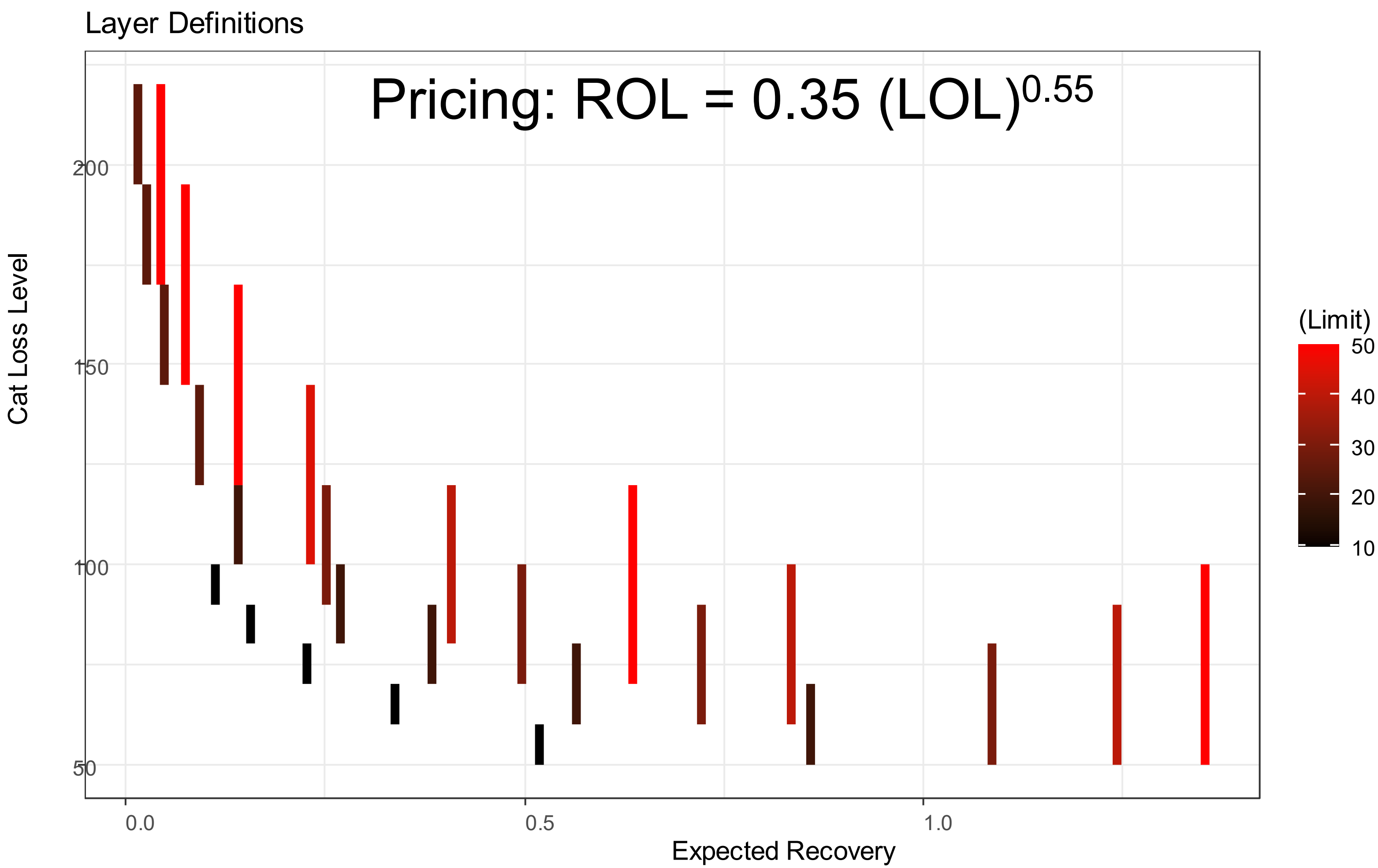
- Capital requirement
  - a_n = net asset requirement $a_n = P_n + Q_n$
  - $\Delta a = a - a_n$ = capital benefit from reinsurance

Example Cat/NonCat Portfolio

- Non-cat: Gamma mean 80, cv 0.15
- Cat: Lognormal mean 20, cv 1.0
- Independent
- Total mean 100, cv 0.233

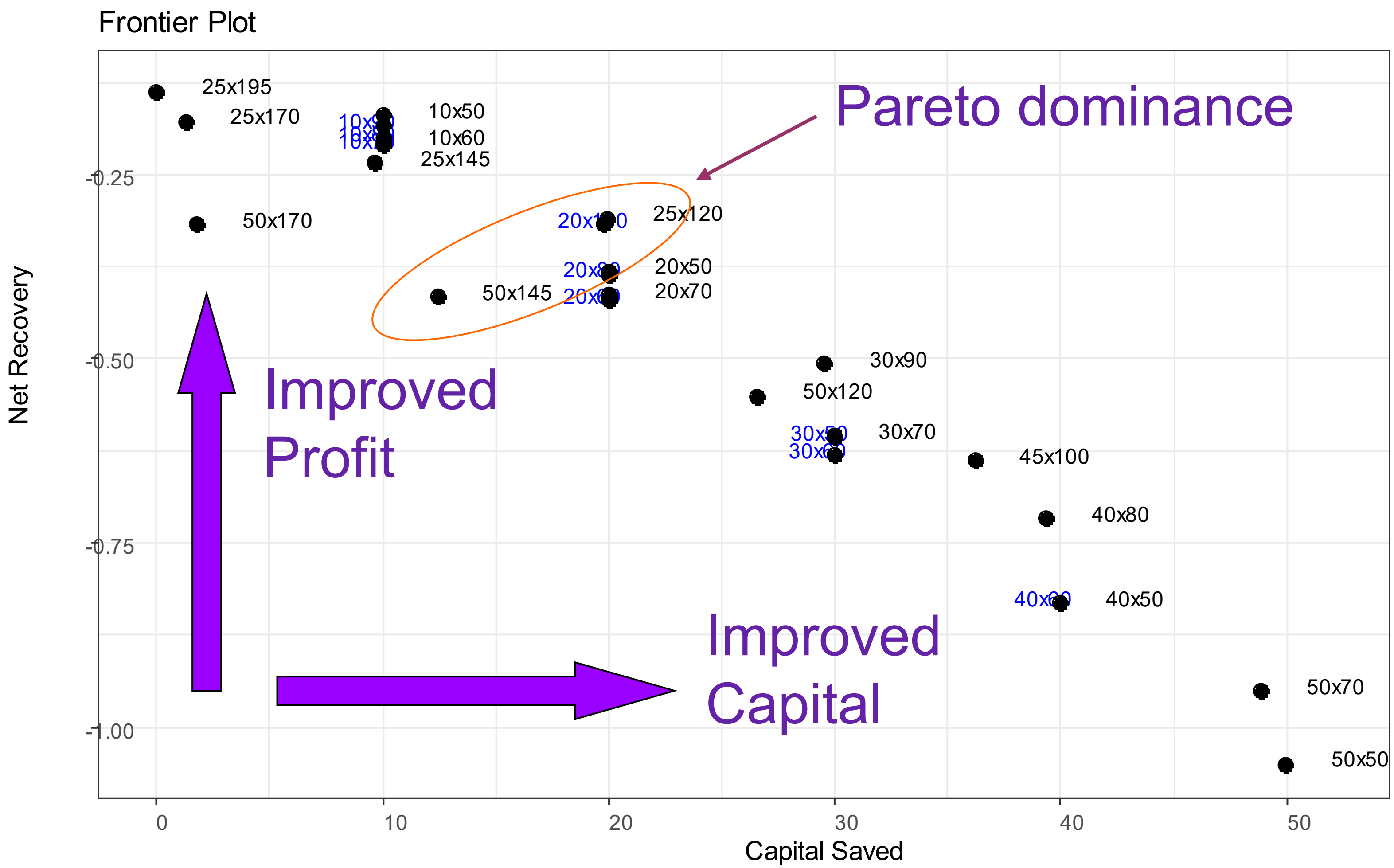
- **Asset requirement**
  - **VaR 99.8% = 237.5**
- **Target return 6%**

Reinsurance Cat Agg XOL Layers

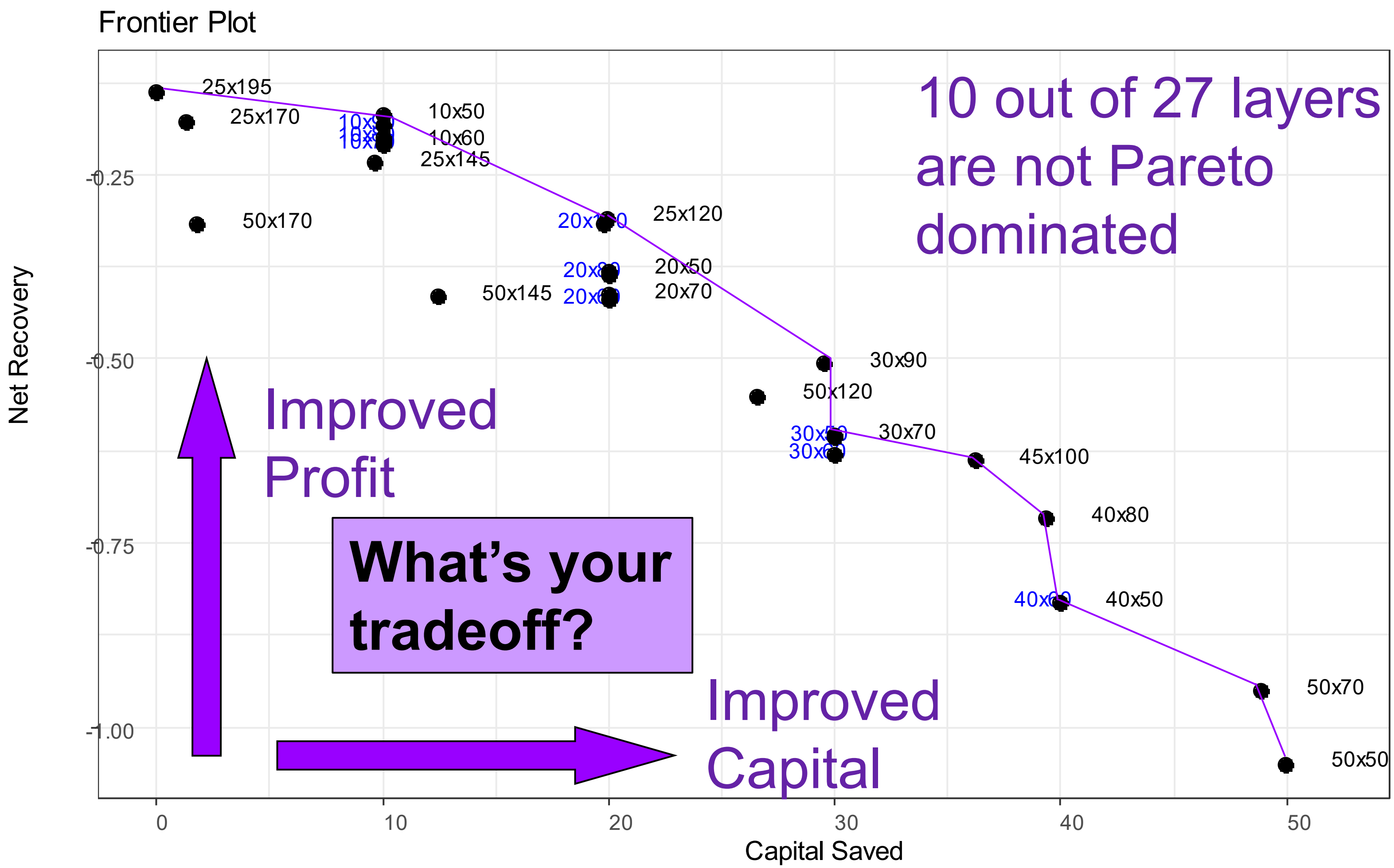


Decisions Past: Multiple Criteria

- Goals:
  - Maximize net recovery $E[X_c] - \pi$
  - Maximize capital savings Δa

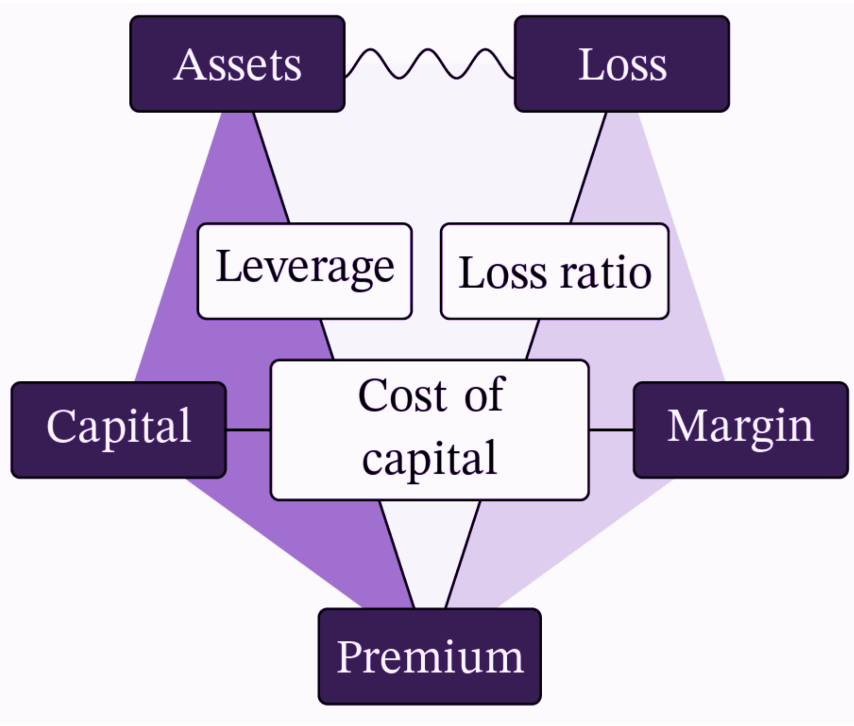

Frontier Plot
Pareto dominance
Improved Profit
Improved Capital
Net Recovery
Capital Saved
-0.25
-0.50
-0.75
-1.00
0
10
20
30
40
50
25x195
25x170
10x50
10x60
25x145
50x170
25x120
20x50
20x70
50x145
30x90
50x120
30x70
45x100
40x80
40x50
50x70
50x50

So where do you want to be?



Decisions Present: The Portfolio Cost of Capital

- Basic logic:
  - Policyholder premiums P = EL + M
  - Investor capital Q = a – P
  - Expected return ι = M / Q
- Conclusion
  - P = EL + ι (a – P)
  - = (EL + ι a) / (1 + ι)
  - = v EL + d a ←
- Note
  - v = 1 / (1 + ι) is the risk discount factor
  - d = ι / (1 + ι) = ιv is the rate of risk discount
  - v + d = 1



Impact of Reinsurance on Premium

- Funded with capital only

$$P = v\,E[X \wedge a] + d\,a$$

- Funded with capital and reinsurance

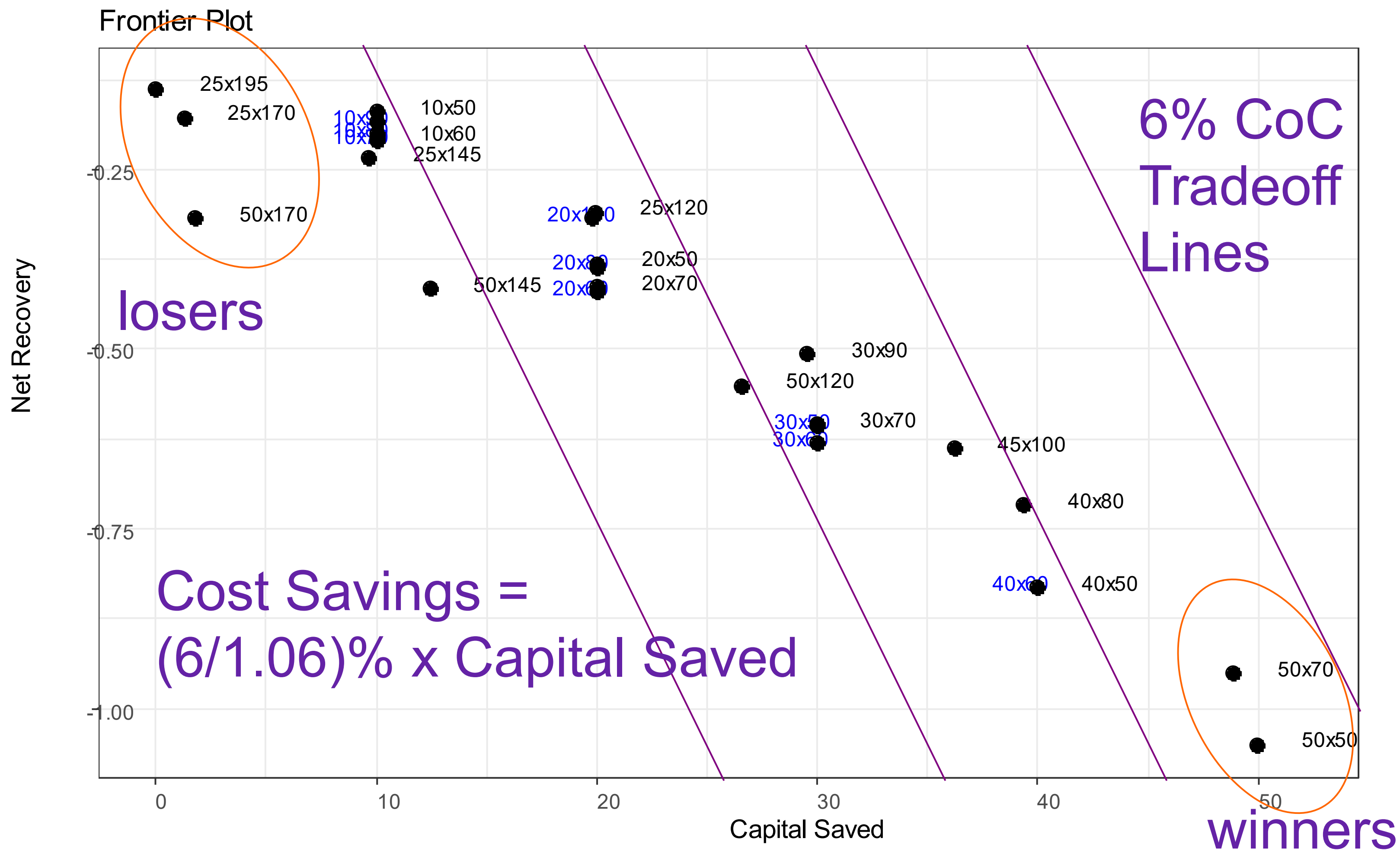
$$P = v\,E[X_n \wedge a_n] + d\,a_n + \pi$$

- Difference

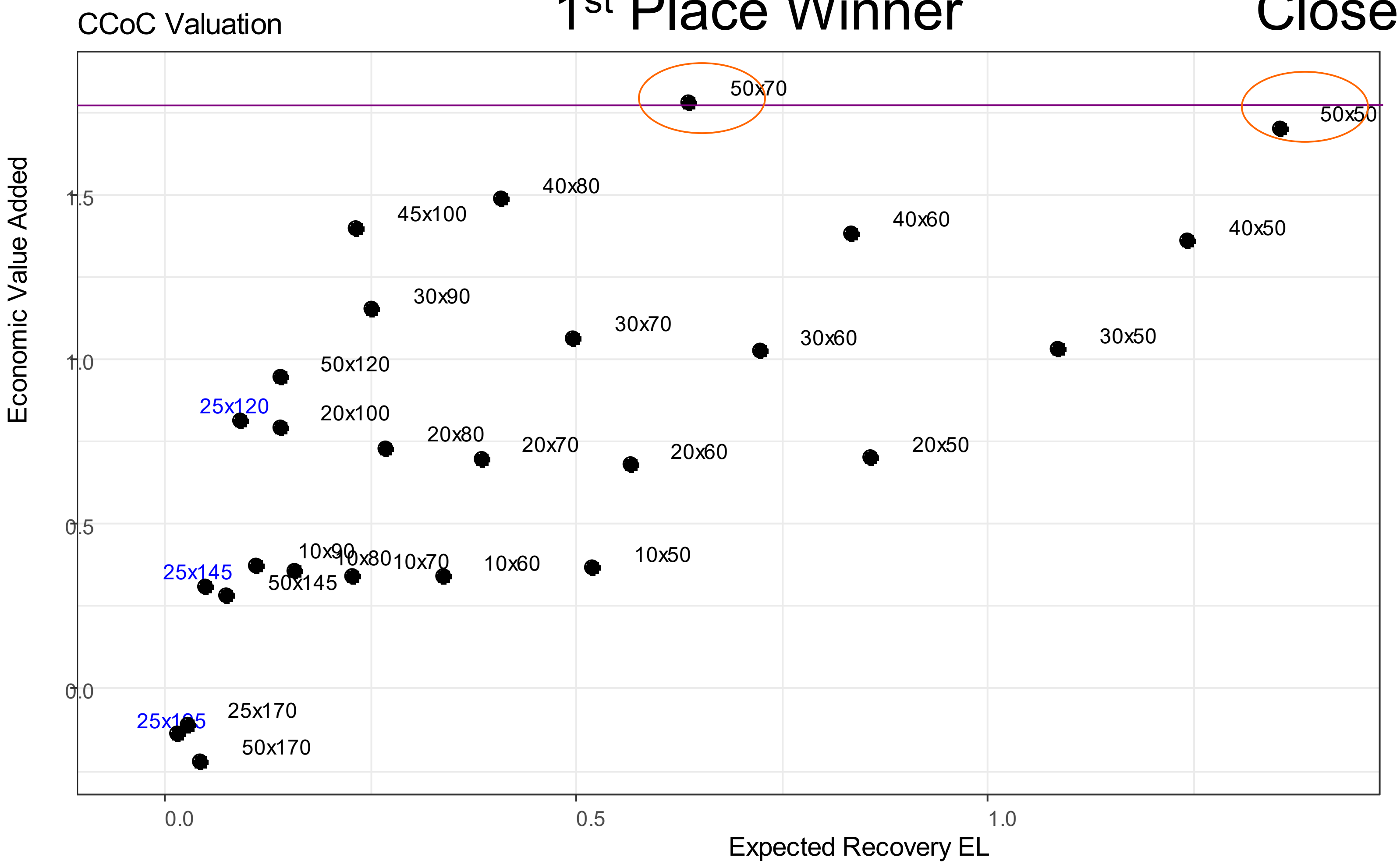
$$v(E[X \wedge a] - E[X_n \wedge a_n]) + d\,\Delta a - \pi$$

$$\approx\ v\,E[X_c] + d\,\Delta a - \pi$$

Evaluating Reinsurance with CCoC



1st Place Winner   Close 2nd

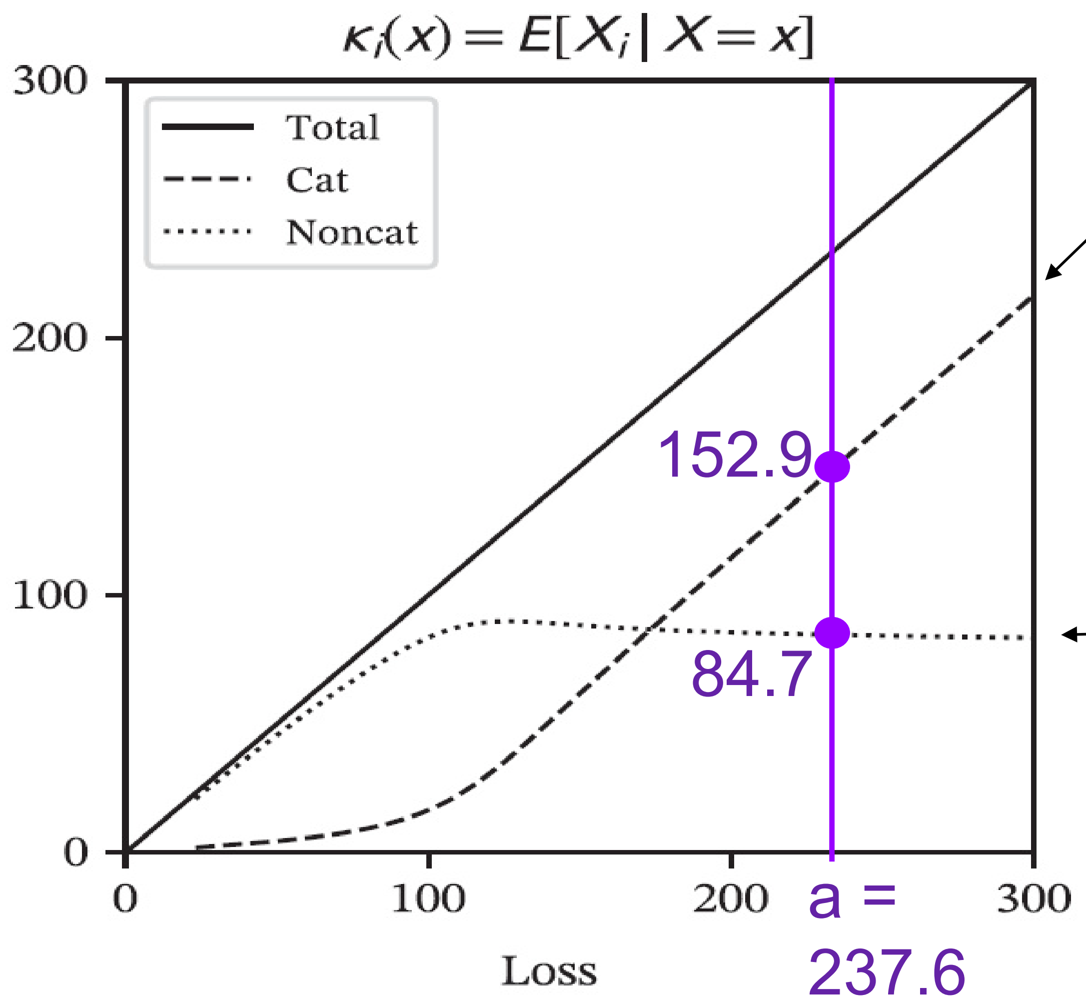


Evaluating Lines of Business

- $X = w_{cat} X_{cat} + w_{nc} X_{nc}$ *yeah, yeah, not really*

  $w_{cat} = w_{nc} = 1$
- $a = VaR_{0.998} (X)$
- $\partial a/\partial w_{cat} = E[X_{cat} \mid X = a]$ *marginal asset = co-VaR*
- $P = v\ EL + d\ a$
- $M = P - EL$
- $\partial M/\partial w_{cat} = d\ (E[X_{cat} \mid X = a] - E[X_{cat}])$

Co-VaR, a.k.a Kappa Function



Cat
loss in tail much greater than expected: 20

Noncat
loss in tail approx. equal expected: 80

Source: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Let’s apply to our LOBs!

LOB	Con-ditional EL	Uncon-ditional EL	Shared Liability	Allocation by CCoC Method	
				Profit Margin	Share
Non-Cat	84.7	80	4.7	0.26	3%
Cat	152.9	20	132.9	7.53	97%
Total	237.6	100	137.6	7.79	100%

Is this reasonable? Many people don't think so.

LOB	EL	CCoC Margin	90^{th} %ile	σ^2
Non-Cat	80	0.26 *3%*	+16 *62%*	144 *26%*
Cat	20	7.53 *97%*	+21 *81%*	400 *74%*
Total	100	7.79 *100%*	+26 *100%*	544 *100%*

What's Going On Here

- P = v EL + d a

- Agent is
  - Risk neutral *v* of the time: P=EL
  - Doom and gloom *d* of the time: P=a

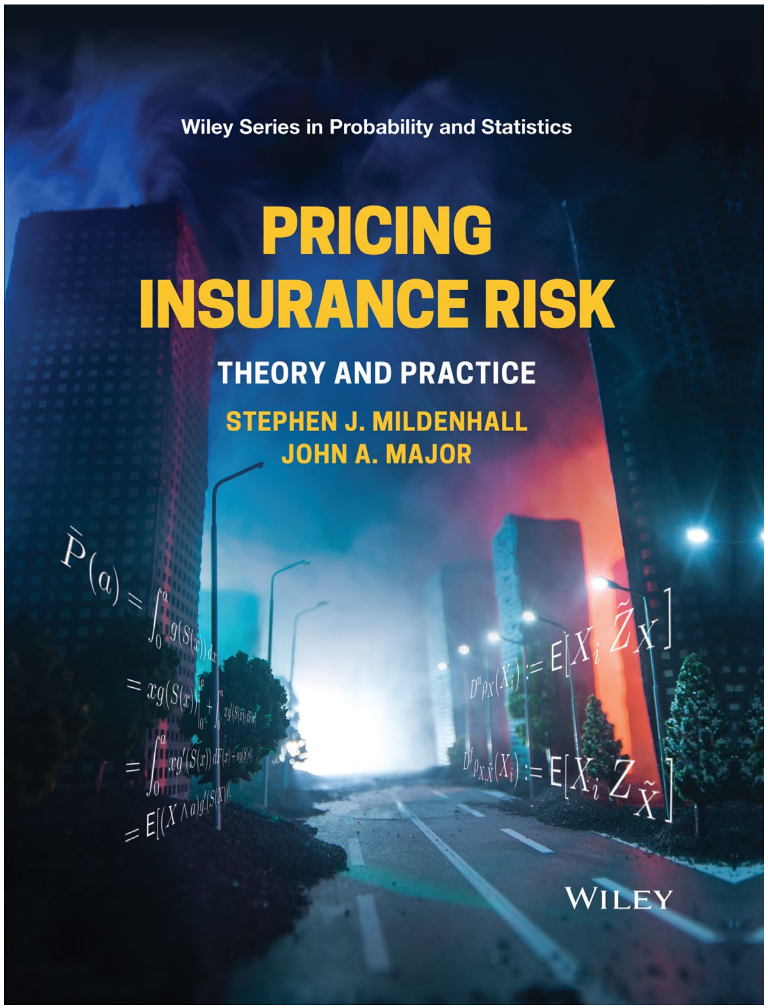
- CCoC only sensitive to mean + extreme

A Deeper Critique of CCoC

- Capital has a **range of costs**
  - Bonds: credit yield curve
  - Cat bonds at different attachments

- "One return to rule them all" ???
  - Same ROE
  - All LOBs
  - gross & net

Decisions Future? Spectral Risk Measures



15

Modern Price Allocation Practice

In this chapter, we show how to apply the natural allocations and unit funding analysis developed in Chapter 14 to discrete data. We apply the formulas to an adjusted version of Simple Discrete Example introduced in Section 2.4.1. We illustrate how the auxiliary functions κ, α, and β can be used to diagnose risk characteristics of the three Case Studies. We compute the lifted natural allocations for each Case Study. As always, the reader is encouraged to replicate all the calculations.

15.1 Applying the Natural Allocations to Discrete Random Variables

In this section, we show how to compute the various natural allocations of $\rho(X)$ to X_i as part of X by extending the algorithm in Chapter 11 that computes $\rho(X)$. We work with a multivariate discrete distribution as produced by a simulation or catastrophe model.

15.1.1 Algorithm to Compute the Linear Natural Allocation for Discrete Random Variables

Algorithm Inputs:

i. The outcome values $(X_{1,j},\dots,X_{m,j})$, $j = 1,\dots,n$, of a discrete m-dimensional multivariate loss random variable. Outcome j occurs with probability p_j. $X_j = \sum_i X_{i,j}$ denotes the total loss for outcome j.
ii. A spectral risk measure ρ associated with the distortion function g.

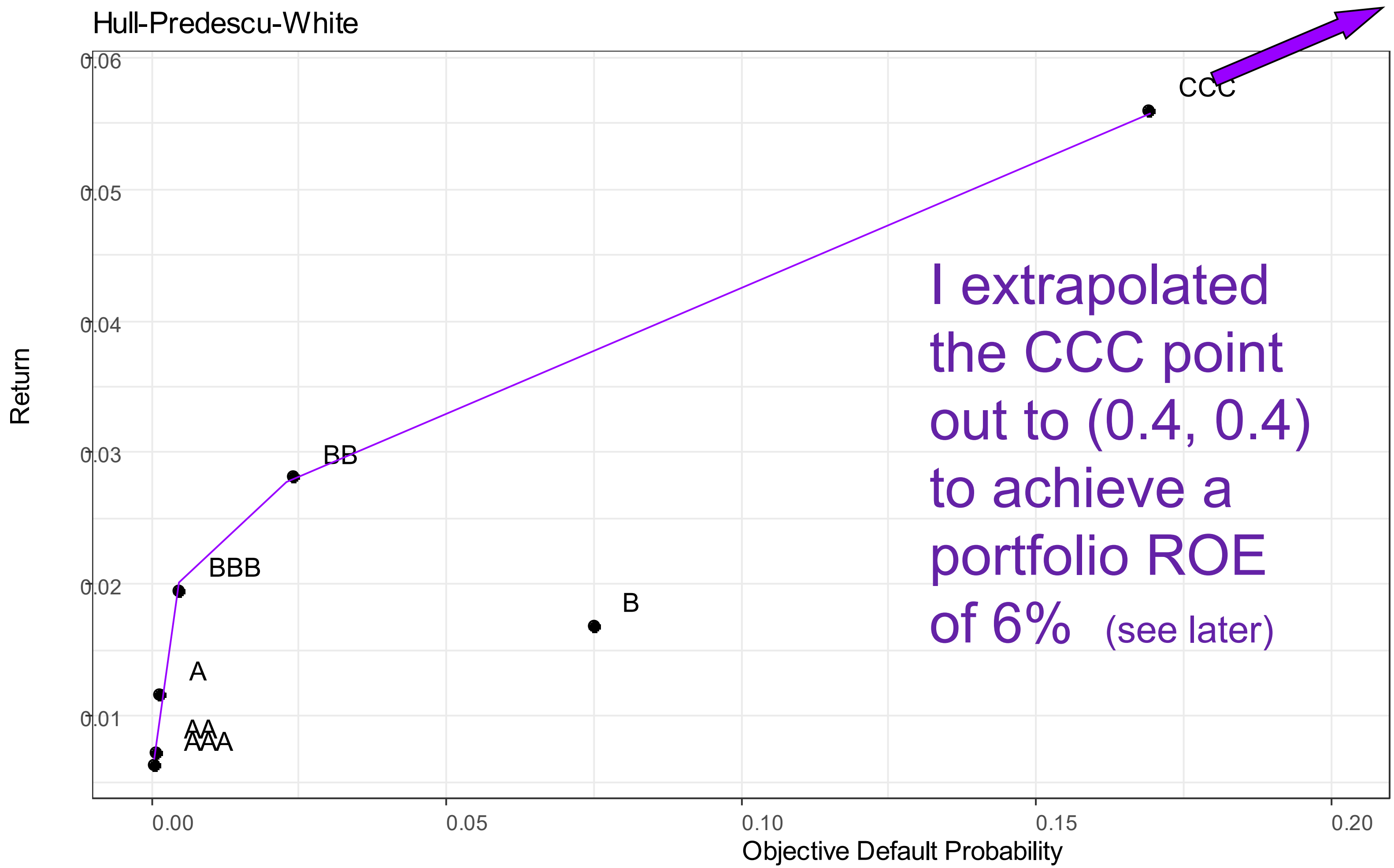
Follow these steps to determine $D^n\rho_X(X_{i,\cdot})$, the natural allocation of $\rho(X)$ to unit i.

Algorithm Steps

1. **Pad** the input by adding a zero outcome $X_{1,0} = \cdots = X_{m,0} = X_0 = 0$ with probability $p_0 = 0$.
2. **Sort** events by total outcome X_j into ascending order.

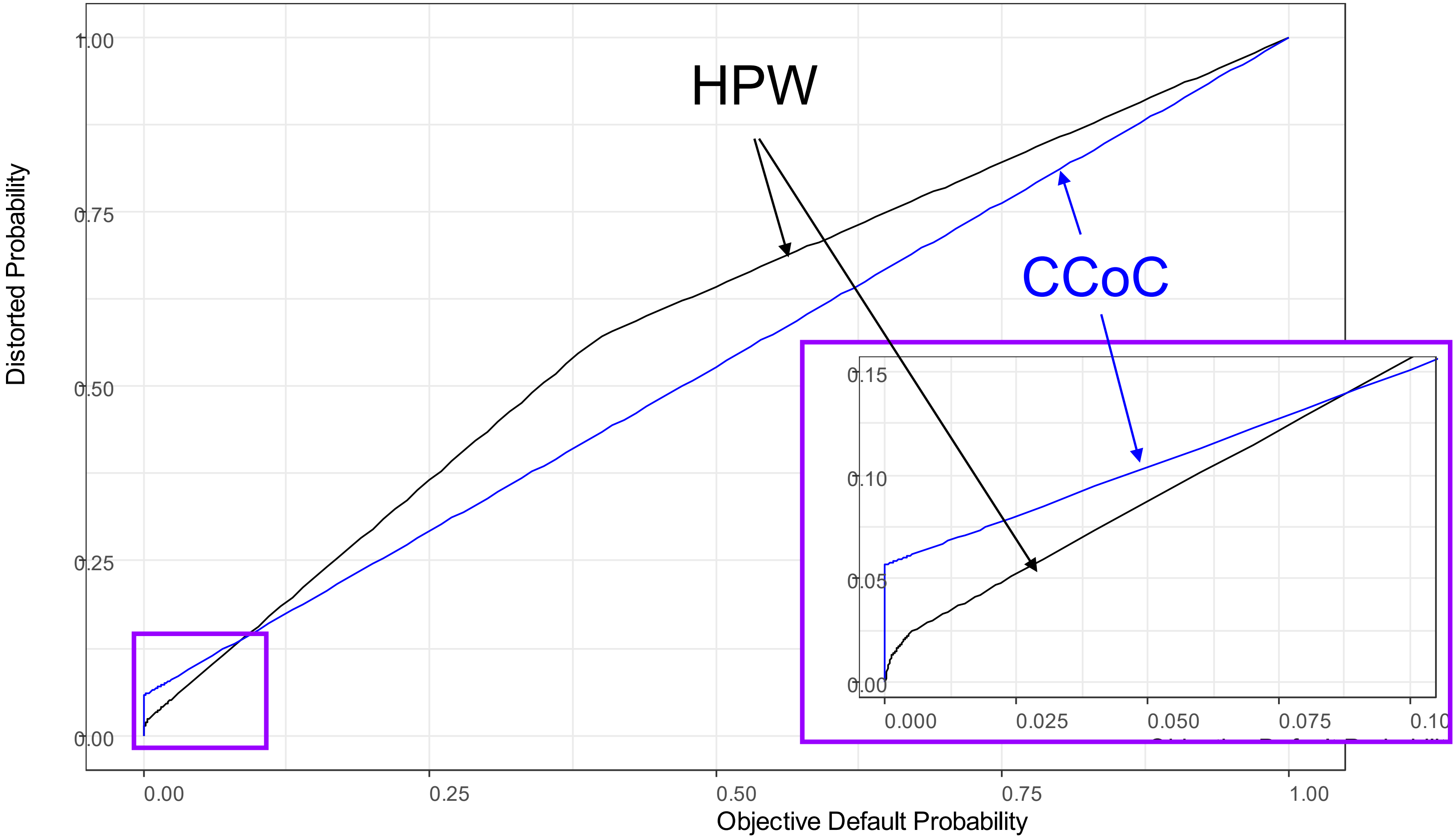
Pricing Insurance Risk: Theory and Practice, First Edition. Stephen J. Mildenhall and John A. Major.

Our Return Model



I extrapolated the CCC point out to (0.4, 0.4) to achieve a portfolio ROE of 6% (see later)

Our Model Distortion Function vs CCoC



Piecewise Linear g(s)

Rating	s	g(s)
0	0.0	0.0
AAA	0.000400	0.006700
A	0.001300	0.012800
BBB	0.004700	0.023800
BB	0.024000	0.050700
CCC	0.396580	0.569995
1	1.0	1.0

So you can reproduce this example on your own.

Applying SRM to ***LOB*s**

- Simulated scenarios sorted by portfolio loss
- Every scenario *j* has
  - Probability p_j
  - Exceedance probability s_j
  - Distorted EP $g(s_j)$
  - Distorted probability $\Delta g(s_j)$
- Expected loss for LOB *i* is $EL_i = \sum_j X_{i,j} \, p_j$
- Technical Premium $\rho_i = \sum_j X_{i,j} \, \Delta g(s_j)$
- Margin $= \rho_i - EL_i$

Videos!
go.guycarp.com/
cas2018

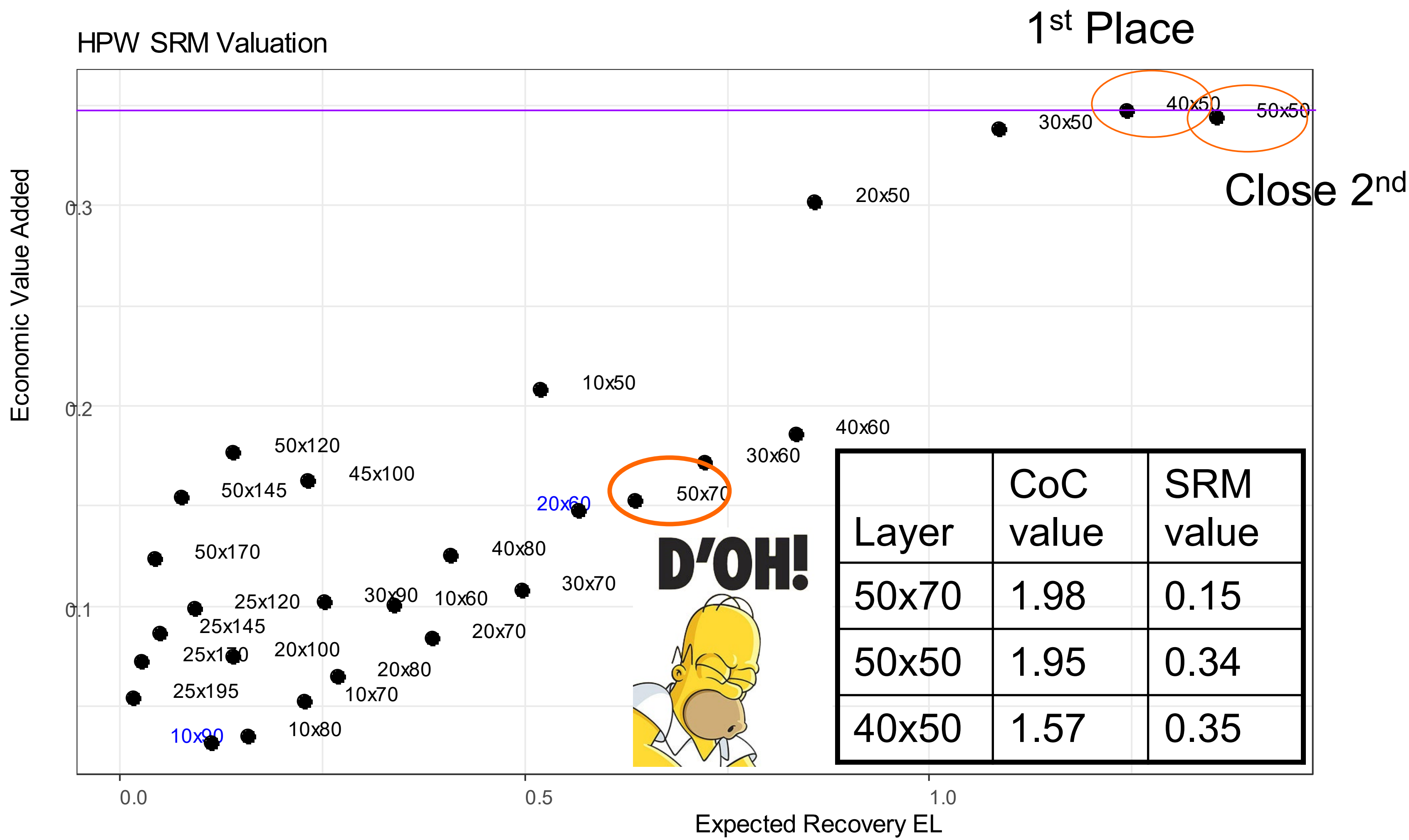
SRM Conclusions about LOBs

LOB	EL	CCoC Method		SRM Method	
		Margin	Share	Margin	Share
Non-Cat	80	0.26	3%	2.00	26%
Cat	20	7.53	97%	5.80	74%
Total	100	7.79	100%	7.80	100%

Applying SRM to ***Reinsurance***

- Economic Value Added
  - $\rho(X_{gross}) - \rho(X_{net})$
  - "A/B method"

- Approximation
  - $\sum_j X_{ceded,\,j}\, \Delta g(s_j)$
  - "Allocate gross"
    - Technically, "linear allocation" $D^n\rho_X(X_c)$



Layer	CoC value	SRM value
50x70	1.98	0.15
50x50	1.95	0.34
40x50	1.57	0.35

Takeaways

- Past: Efficient frontiers … meh
- Present: CCoC … extreme
- Future: SRMs easy[1]
  - a. Variable capital cost
  - b. Risk-adjusted probabilities

[1] Terms and conditions apply