An Aggregate Trend Renewal Micro Model for Loss Reserves, with Inflation and Discount

Anas Abdallah • CAS Spring Meeting (2022)

joint work with:

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Orlando, FL • May 16-18, 2022

Summary

1. Introduction

Context

- Medical Malpractice Insurance
  - Indemnities
  - Expenses
- Aggregate Trend Renewal Model
  - Trend effects
  - Inflation and Discount Factors
- Loss Reserving
  - Macro-Level Reserving
  - Micro-Level Reserving

Medical Malpractice

- Medical malpractice is generally defined as a professional negligence
- Need for medical professionals to obtain professional liability **insurance** to offset the risk and (high) costs of lawsuits
- Medical malpractice costs and premiums vary by specialties
- Medical malpractice regulations vary by country and even by jurisdiction within countries
- There are essentially two primary types of policies for medical malpractice : "claims-made" and "occurrence"

Medical Malpractice

2. Loss Reserving

The insurance framework and reserving

Inverted production cycle

$\Downarrow$

Insurer receives the price (premium) of the product before knowing its cost (claim)

$\Downarrow$

Estimate this cost and hold enough capital

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Estimate this cost and hold enough capital

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Make sure the company has enough money put aside to pay all the claims that they are on the hook for : Loss **Reserving**

Loss Reserving schools of thoughts

- Aggregated claims : Macro-Level Reserving
  - Loss Triangles
- Individual claims : Micro-Level Reserving
  - Granular Data

From Macro-Level Reserving...

- The current reserving practice consists of using methods based on claims development triangles : Macro-Level Reserving
  - Triangles are organized by accident and dev. periods
- Traditional reserving methods have worked well in several circumstances in the past, however :
  - Limitations to provide robust and realistic estimates in more variable contexts
  - Limited interpretive and predictive power of the accident and development period parameters
  - Huge estimation error for the latest development periods (lack of observed aggregate amounts)
  - Inability to properly capture the pattern of claim development

... to Micro-Level Reserving

- Big data era and increased need for more accurate reserving models
- Taking advantage of the information embedded in individual claims data
- Separates the estimation of the incurred but nor reported (IBNR) and the reported but not settled (RBNS) claims
- Allows for the incorporation of the detailed information about the individual claims

Macro vs Micro Loss Reserving

- Macro-Level Reserving
  - + Comparative ease of use (benchmark)
  - + Availability of triangle data
  - – Heterogeneity of data
  - – Predictive Power
- Micro-Level Reserving
  - + Accuracy
  - + More Flexibility
  - – Availability of data
  - – Expensive computing power

3. ATRP Model

Context

- We use the model that has been proposed and described in Léveillé and Hamel (2017), but in a loss reserving framework
- Contribution to the loss reserving literature :
  - Introduction trend (on occurrence of the events and claims)
  - Inflation factor
  - Discount rate

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We study their impacts on the Macro-level and Micro-level reserving methods

Trend renewal process (TRP)

- The Trend renewal process (TRP) model was first introduced rigorously by Lindqvist et al. (2003) in reliability theory
- Let $\lambda(t)$ a non-negative function defined for $t \geq 0$, satisfying

$$\Lambda(t) = \int_0^t \lambda(u)du \ < \infty, \quad t \geq 0 \ \text{ and } \ \Lambda(\infty) = \infty$$

- The sequence of occurrence times $\{T_k; k \in N^*\}$ generates a TRP$[F, \lambda(.)]$ if the sequence of transformed times $\{\Lambda(T_k); k \in N^*\}$ forms an ordinary renewal process for which the inter-occurrence times have a common (positive) distribution F and a deterministic trend function $\lambda(t)$

Aggregate Trend renewal process (ATRP)

- Now let us introduce the aggregate trend renewal process (ATRP), that will be used as a « data generating process »
- We consider the following process, for $1 \leq i, j \leq t$:

$$Z_{i,j}(t) = \sum_{k=1}^{N(t)} I\left(T_k, \xi_k, \zeta_k; t, i, j\right) \left\{A_1\left(T_k + \xi_k + \zeta_k\right) X_k + A_2\left(T_k + \xi_k + \zeta_k\right) Y_k\right\}$$

- $N(t)$ is a $\mathbf{TRP}[F, \lambda(.)]$ process generated by a sequence of continuous positive random variables $\{T_k; k \in N^*\}$
- T_k represents the **occurrence** time of the k^{th} event
- ξ_k is the **delay** taken by the insured from T_k to **report** the claim to the insurer
- ζ_k is the **delay** taken by the insurer from $T_k + \xi_k$ to **pay** the indemnity and the expenses

Aggregate Trend renewal process (ATRP)

- X_k represents the deflated amount of the k^{th} **indemnity** paid
- Y_k represents the deflated amount of the k^{th} **expenses** paid
- $A_k = \exp\left\{-\int_0^x \beta_k(x)dx + \int_0^t \alpha_k(x)dx\right\}$, $k = 1, 2$, are the net **discount** factors at time t corresponding to the payments of the indemnities :

  α_i and β_i are the corresponding forces of **inflation** and **interest**

Conditional ATRP

- Here, we will consider the incremental payments of the indemnities and expenses
- Let $W_{i,j}(t)$ represent future incremental payments of the indemnities and expenses for the accident year i and development period j :

$$W_{i,j}(t) = Z_{i,j}(t) | B_i$$

- B_i is related to the (strict) **lower** triangular matrix

4. Illustrations

Data and hypotheses

- A real closed claims database from the state of Florida in USA is used for numerical calculations and calibration
- The database contains information on individual claims related to malpractice insurance
- We will focus on 11 years of data (2005-2016)
- We use the same conclusions for parameters estimation obtained from this data in Léveillé and Hamel (2017)
- These hypotheses include parameters calibration for : density functions of the **delays** of reporting and payment, the **expenses** and **indemnities** distributions, **inflation** and **interest** and **trend** function

Methods

- Macro-Model : GLM
- Micro-Model : ATRP

Model Selection

- Akaike Information criteria (AIC)

$$AIC = logL\left(\hat{\theta}; x\right) - p, \tag{1}$$

where $logL\left(\hat{\theta}; x\right)$ is the log-likelihood evaluated at the MLE $\hat{\theta}$, and p is the number of estimated parameters in the model. Based on this approach the model with the largest AIC is selected.

- Bayesian Information criteria (BIC)

$$BIC = logL\left(\hat{\theta}; x\right) - \frac{p}{2} log(n), \tag{2}$$

and the largest statistic is again favored.

Macro-Model : GLM (ODP)

- Model Calibration on data (upper triangle) generated by the ATRP model
- The fit statistics suggest the Over-Dispersed Poisson (ODP)

Model	AIC	BIC
Poisson Log-Link, Not overdispersed	11,800,261	11,800,307
Poisson Log-Link, Overdispersed, Saddle Point Approximation (ODP)	2,010.078	2,056.06
Negative Binomial Log-Link	2,071.292	2,119.464
Gamma Log-Link	2,071.319	2,119.492
Gamma Inverse-Link	2,083.098	2,131.27

Macro-Model : GLM (ODP)

- We define the incremental claims as

$$Y_{i,j} = C_{i,j} - C_{i,j-1},\ j \in \{1, ..., I\}\ ,\ \ ,\forall i.$$

- The predicted values of the runoff triangle will be obtained as followed :

$$\begin{aligned} E[Y_{i,j}] &= \mu_{i,j} = exp\left(\gamma + \omega_i + \psi_j\right), \\ Var[Y_{i,j}] &= \varphi\ \mu_{i,j} = \varphi\ exp\left(\gamma + \omega_i + \psi_j\right) \end{aligned}$$

Micro-Model : ATRP - conditional moments

- According to the hypotheses and definitions of section 3, a closed-form expression is obtained for the **first moment** $E[W_{i,j}]$.
- **Theorem** : According to the hypotheses and definitions of the section, the **second moment** $E[W_{i,j}^2]$ is also obtained (and then variance).

Reserves Predictions

- Conditional ATRP model without Markov chain Monte Carlo (MCMC)
- Conditional ATRP model with Markov chain Monte Carlo (MCMC)
- GLM-ODP without Bootstrap
- GLM-ODP with Bootstrap

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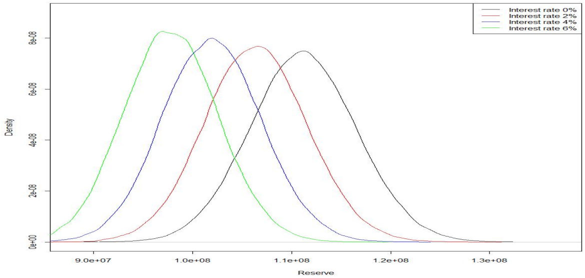
- Discounting impact
- Inflation impact

Reserves Predictions - Discounting Impact (GLM-ODP)

- The Figure below presents the estimated reserve densities for discount rates of 0%, 2%, 4% and 6%, for the **GLM-ODP**
- The relatively weak asymmetry and the maximum of the density functions decrease with the decreasing of interest rate



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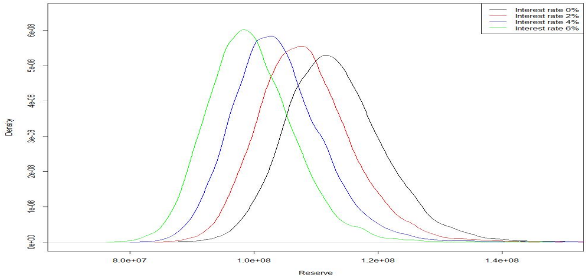

Reserves Predictions - Inflation Impact

- Assessing the inflation impact (without discounting) and comparing the **GLM-ODP** and the **ATRP** model
- Knowing the full claims developments from our generating process makes the **true reserve** available
- We observe a higher inflation impact for the ATRP model, compared to the classical methods
- A smaller coefficient of variation for the ATRP model

Reserves Predictions - Inflation Impact

	Inflation			No-Inflation			Inflation impact		
Method	Reserve	St. dev.	CV	Reserve	St. dev.	CV	Reserve	St. Dev.	CV
True Res.	113,983,485	-	-	63,596,509	-	-	79.2%	-	-
Mack	111,187,449	18,476,949	16.6%	62,814,370	11,238,832	17.9%	77%	64.4%	-7.3%
ODP Boot.	112,819,697	19,034,902	16.9%	63,821,991	11,424,602	17.9%	76.8%	66.6%	-5.6%
ATRP	113,229,168	8,016,520	7.1%	63,347,117	4,426,208	7.0%	78.7%	81.1%	1.4%

Risk Capital Implications

- The predictive reserves distribution is very informative for actuaries, but also from a risk capital standpoint
- Comply with regulatory standards (buffer for adverse scenarios)
- Insurance companies have to hold risk capital relating to (unexpected) losses that are incurred but not yet paid
- Risk Capital is the difference between the risk measure and the reserve : Risk Capital $= TVaR_{95\%}(R) - TVaR_{60\%-80\%}(R)$

$$\Downarrow$$

We study the combined impact of discounting and inflation on Risk Capital

Risk Capital Implications

- Impact on Risk Capital - **GLM-ODP** Model

ODP Bootstrap	With Inflation			Without Inflation			...
Discount rate	TVaR60	TVaR95	Risk Capital	TVaR60	TVaR95	Risk Capital	Inflation impact
0%	132,171,851	158,601,247	26,429,396	75,295,015	90,982,406	15,687,391	68,5%
2%	126,383,920	151,576,492	25,192,573	72,178,338	87,165,105	14,986,767	68,1%
4%	120,944,030	144,882,365	23,938,335	69,277,584	83,586,120	14,308,536	67,3%
6%	115,752,524	138,362,809	22,610,285	66,534,695	80,183,528	13,648,834	65,7%

Risk Capital Implications

- Impact on Risk Capital - **ATRP** Model

ATRP model	With Inflation			Without Inflation			...
Discount rate	TVaR60	TVaR95	Risk Capital	TVaR60	TVaR95	Risk Capital	Inflation impact
0%	120,921,634	132,655,648	11,734,013	67,696,362	74,134,396	6,438,034	82,3%
2%	115,701,240	126,805,051	11,103,811	64,932,597	71,075,991	6,143,393	80,7%
4%	110,839,934	121,413,899	10,573,964	62,394,648	68,296,073	5,901,425	79,2%
6%	106,366,460	116,332,116	9,965,657	60,009,379	65,689,797	5,680,418	75,4%

Risk Capital Implications - Inflation and Discount

- Impact of the inflation on the risk capital is very high for both models
- This finding confirms the importance of incorporating the inflation analysis in loss reserves and risk capital analyses
- The inflation impact seems to be slightly correlated, negatively, with the discount factor
- Also, independently from the inflation, the risk capital decreases when we increase the discount factor

Risk Capital Implications - Models Comparison

- Inflation impact seems to be constantly higher for the ATRP model with MCMC, compared to the GLM-ODP model with Bootstrap
- GLM-ODP model underestimates the impact of the inflation on the Risk Capital (this is consistent with the reserves findings)

Risk Capital Implications - Models Comparison

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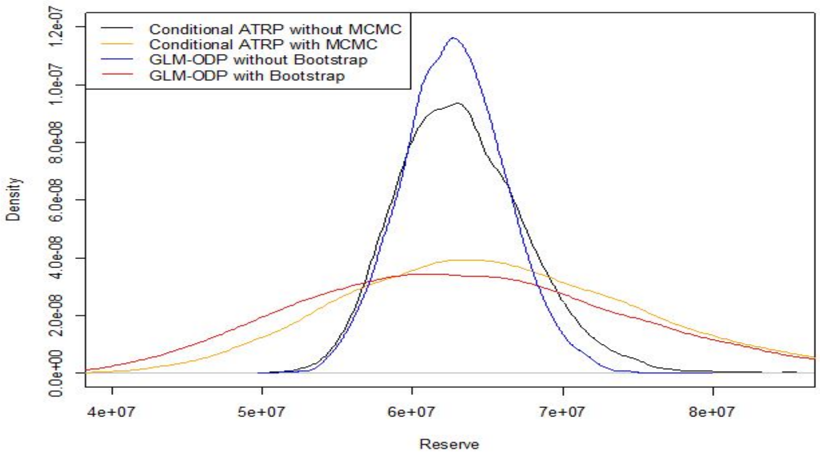
- GLM-ODP model underestimates the impact of the inflation on the Risk Capital (this is consistent with the reserves findings)
- Inflation is explicitly incorporated in the ATRP model
- The ODP model only incorporates the inflation impact through the estimated parameters
- Discount factor is not applied at the same time for both models (due to their different forms)

Risk Capital Implications - Parameter uncertainties

- The two models have relatively the same behavior at the tail of the distribution (more conservative)
- This difference is also explained by the parameter uncertainties
- We observe that the incorporation of parameters uncertainty (parameter risk) is over-estimating the risk capital
- We observe a lighter tailed distribution (a much less conservative scenario) without parameter uncertainties

Risk Capital Implications - Parameter uncertainties



5. Concluding Remarks

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With the conditional ATRP model, we have incorporated many useful features :

- Separating the expenses and indemnities
- Delays before reporting the claims and before payments
- Inflation and Discount
- A Trend renewal model for the (accidents) counting process

Concluding Remarks

With the conditional ATRP model, we have incorporated many useful features :

- Separating the expenses and indemnities
- Delays before reporting the claims and before payments
- Inflation and Discount
- A Trend renewal model for the (accidents) counting process
- Flexibility : assumptions can be relaxed if the economic environment or legislative and regulatory measures are changing

Limitations and extensions

- Work in progress
    - Evaluating the model assumptions impact, under different scenarios (sensitivity analysis)
    - Incorporating stochastic inflation and discount (more realistic conditions of investments and credits)

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  - Evaluating the model assumptions impact, under different scenarios (sensitivity analysis)
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- Extensions
  - P&C : Auto Coverages (example : Accident Benefits with Disability Income) with real-world data
  - Dependence between coverages (Multivariate Trend Renewal Model)

Selected References

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End of the presentation

Thank you for your attention

Questions ?

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