Credibility-Type Smoothing Using Ghost Trend

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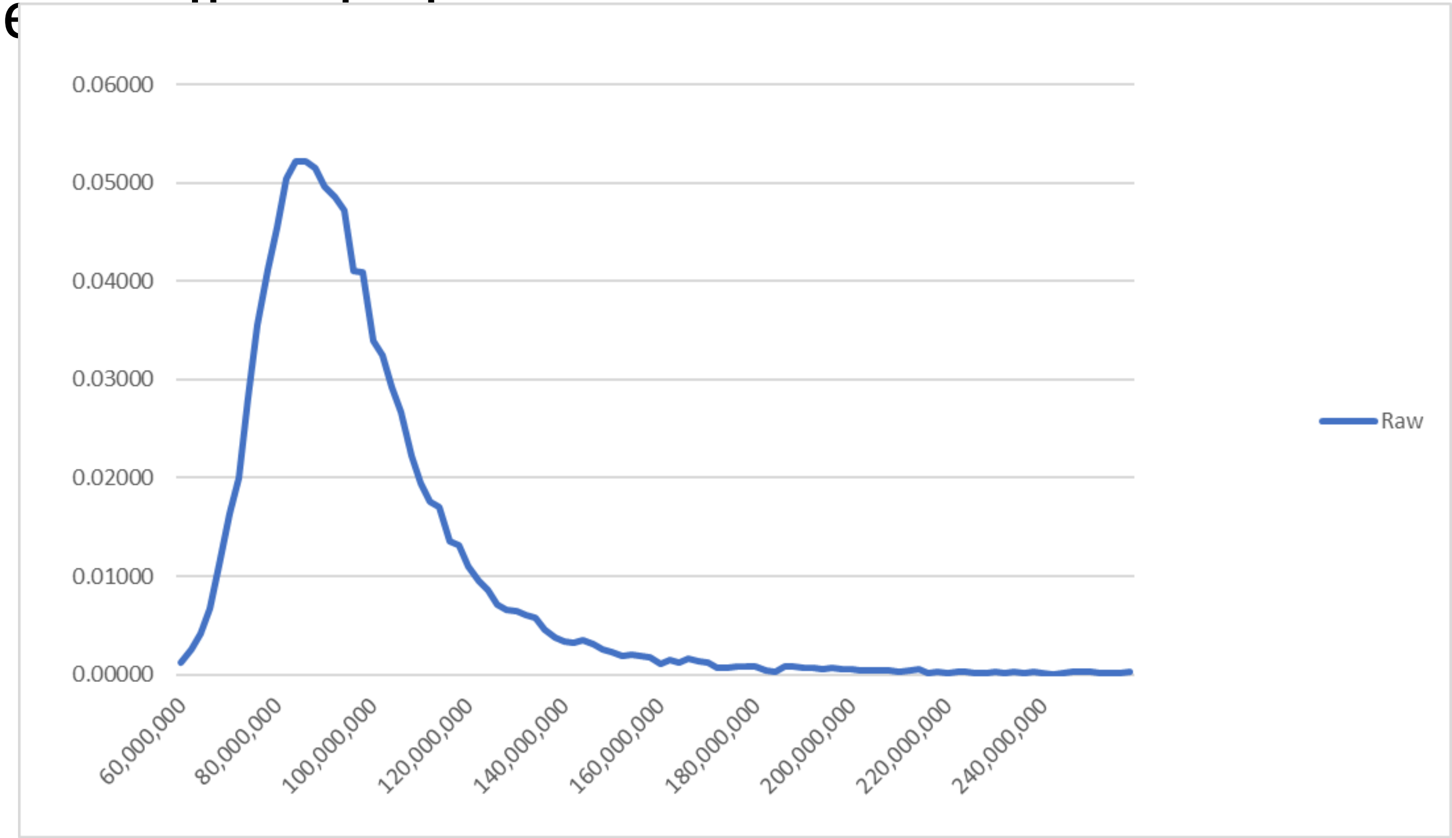
How This is Relevant

- In the paper that goes with the other half of this session, I needed to illustrate (graph) what an aggregate loss distribution representing the claims of a medical malpractice insurer looks like.
- For detail, it involves Poisson(500) claims that come from a truncated and shifted Pareto (α=1.5) distribution with a mean of $100,000.

Issue with Graphing the Distribution

- I ran 30,000 samples (using NTRAND) and got the following graph from the

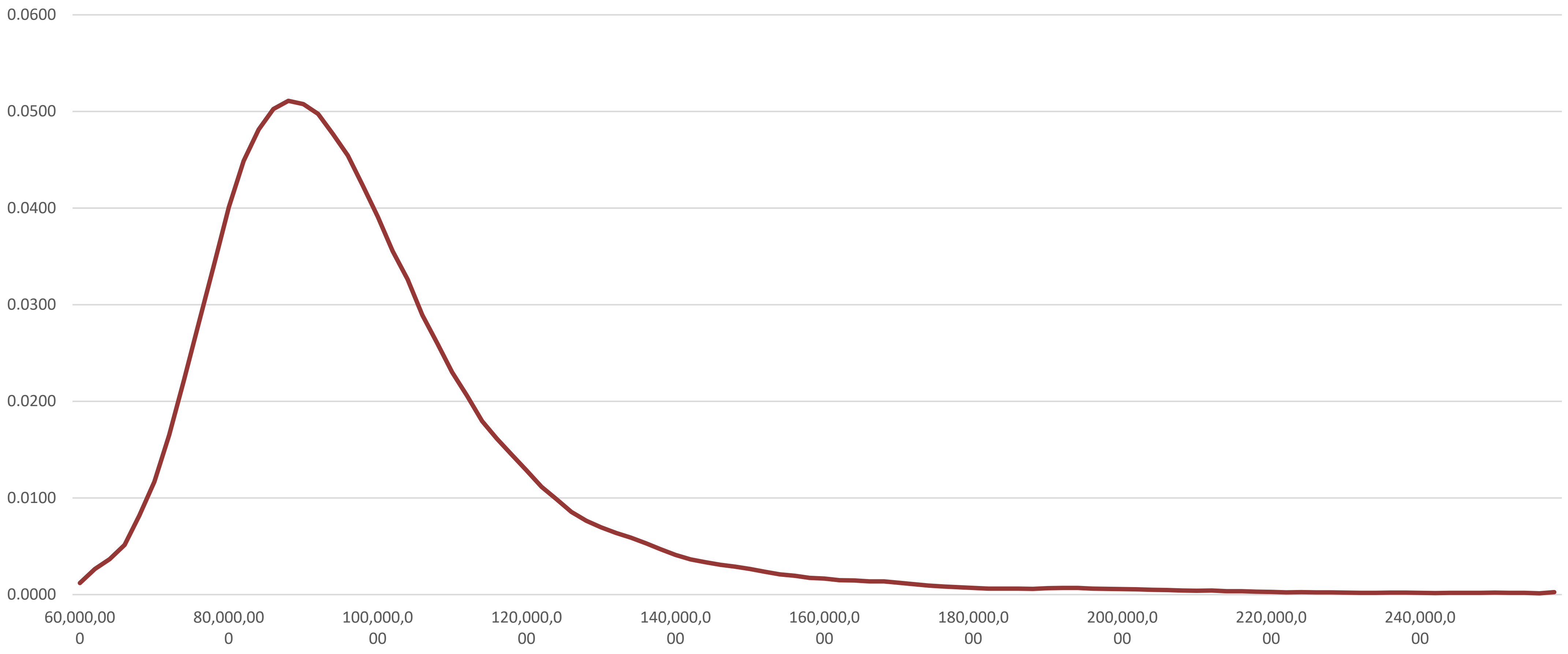


Removing the Bumps

- Certainly enough samples would remove the bumpiness, but my sample size was very, very, high already
- I chose to put the ghost trend approach I had to work.

- And I got ……

Curve After Ghost Trend Adjustment (and 5 Point Averaging)



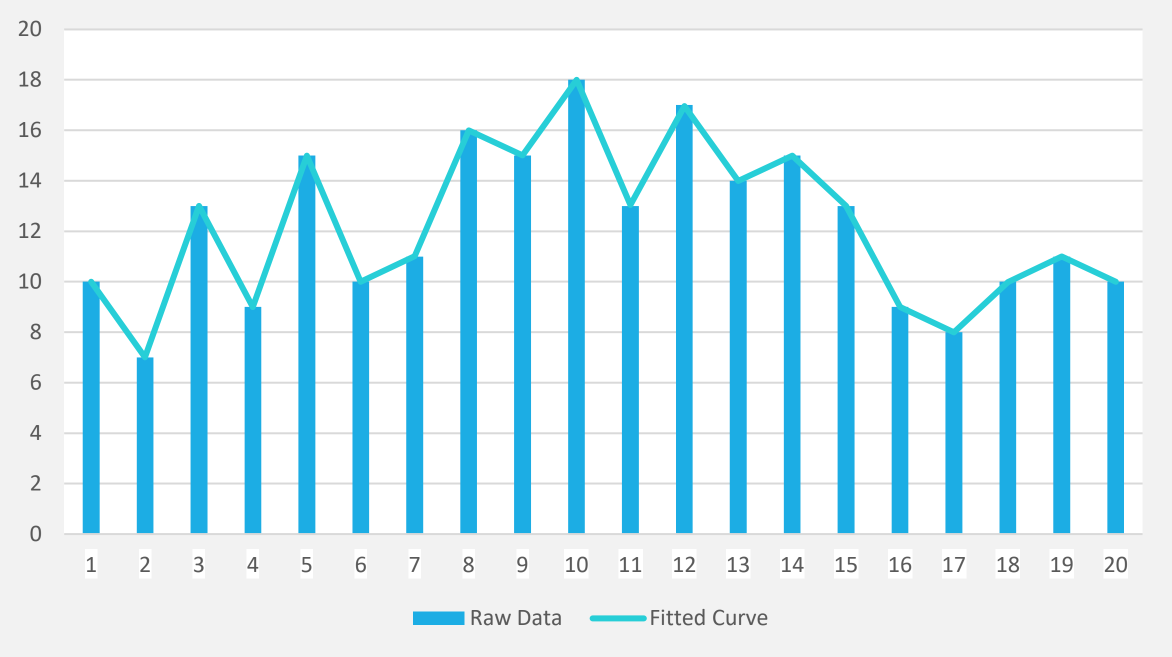
How the Process Works

Competing Concerns When Smoothing

- Want the curve to match the data points as closely as possible
- But also want the changes from point-to-point to be smooth and consistent
  - Even if the data is wildly bumpy and volatile
  - Need a smoothing mechanism that addresses both as well as possible…a smooth curve that is close to the points

Flow Thru the Steps that Produce the Method

- Start by solely requiring that curve match the points as closely as possible-straight match but very “bumpy”

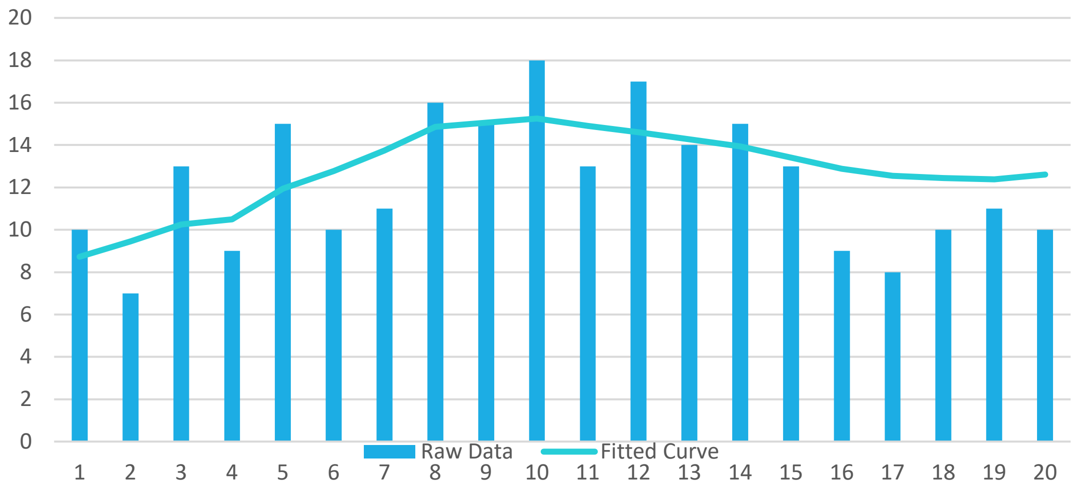


First Step in the Trade-Off
Accuracy vs. Smoothness

- Values on previous slide simply match the data
  - For the trade-off, use the sum of squared differences between the curve and the data points
- For smoothness use a constant “trend” rate, or linear, non-exponential increase from point-to-point.
  - In this case the value to manage is the sum of squared differences between in turn the differences between values at adjacent points
- The tradeoff is set by selecting weights for the two SOS quantities, then minimizing the weighted sum of squares.

First “Ghostlike” Trend Process

- Results are much smoother
- Process is credibility-like if data points are treated as raw data and the fixed trend values are viewed as benchmarks.



The “Hump” in the Last Slide Makes the Fit Challenging

- The data shows a positive trend going up the hump, but negative trend when going down the other side of the hump.
- Solution: Don’t require that the underlying “expected” or “benchmark” trend be constant. Just put a penalty on large changes from point.

Penalty for Large Changes in the Trend Benchmark

- Set actual "trend" between two adjacent points to be the difference between the value at the second point in the two minus the value at the point before it.
- There is a penalty for the squared differences between the actual trend values and the "ghost trend" values.
- The ghost trend is not constant, but the squared differences between the ghost trend in adjacent intervals are added up and get a "weight" multiplier

Add Up Three Penalties, Each for a Different Aspect of the Fit

- Weight 1 times sum of squared differences between the curve and the datapoints.
- Weight 2 times sum of squared differences between the actual point-to-point trends and the corresponding ghost trends
- Weight three times the sum of squared differences between the ghost end values at adjacent intervals.

What Do You Pick to Minimize the Total Weighted Sum?

- Curve values ad ghost trend values
  - Then I generally run “solver” to get the optimum curve
- The choice of weights is, to my knowledge, completely arbitrary- select what works
  - More weight on difference from data – more accuracy, less smoothness
  - More weight on differences from ghost trend- more stiffness, more smoothness.
  - More/less weight on ghost trend, more/less long term stiffness or flexibility

Why Consider Prudence of Purchasing a Reinsurance Contract

- CV approach does (speaker's opinion) a great job of assessing whether a contract makes the business less risky
- Historically, risk transfer was used to test whether contract in some way exploited a company by transfring more funds than necessary to a sister company, etc.)
- The CV approach alone does not address this, but requiring that the contract be prudent purchase does this…

Full Approach

- Minimizing the weighted set of sums to compute the curve can lead to a very substantial reduction in the “bumpiness”
- If you’re working with a large amount of data points and very variable values, using , say, 5 point averaging may be a useful final touch.

Summary

- Ghost trend process, minimizing weighted sum of sums of squares, can create a very practical smoothed version of volatile data values.
- Allows actuary to exercise a great deal of judgment in choosing weights for stiffness vs. accuracy, etc.
- Since it is an unknown (although estimated) benchmark to influence but not govern a trend that governs the curve, I feel that “ghost trend” is a fitting name

Ghost Trend

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