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USING THE HAYNE MLE MODELS: A PRACTITIONER'S GUIDE

Mark R. Shapland, FCAS, FSA, MAAA

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Abstract

Motivation. The Hayne MLE family of models are quite elegant in their application, but, like most models, the modeling framework needs to allow for the flexibility to deal with many different practical issues in order to address the needs of the practicing actuary. While actuaries are accustomed to making practical adjustments to their algorithms, there is motivation to stay as close to the theoretical underpinnings of the models as possible in order to maintain a sound foundation. Whenever the monograph strays a bit from the theory, those departures are noted so practitioners can adequately judge their impact.

Method. This monograph starts by reviewing the Hayne MLE modeling framework using a standard notation. Then it covers a number of practical data issues and addresses the diagnostic testing of the model assumptions. Next it will explore a variety of enhancements to the basic framework to allow the models to address other issues related to reserving and pricing risk. Finally, since no single model is perfect, ways to combine or credibility weight the Hayne MLE model results with various other models are explored in order to arrive at a "best estimate" of the distribution. This is similar to how a deterministic best estimate is generally derived in practice, so ways for the practitioner to correlate models by segment in order to simulate aggregate results are also discussed.

Results. The monograph will illustrate the practical implementation of the Hayne MLE modeling framework as a powerful tool for estimating a distribution of unpaid claims.

Conclusions. The monograph outlines the full versatility of the Hayne MLE models for the practicing actuary.

Availability. In lieu of technical appendices, several companion Excel workbooks are included that illustrate the calculations described in this monograph. The companion materials are summarized in the Supplementary Materials section and are available at https://www.casact.org/sites/default/files/2022-03/Monograph10-ExcelFiles_0.zip.

Keywords. Maximum Likelihood Estimate, Reserve Variability, Reserve Range, Stochastic Reserving, Distribution of Possible Outcomes, Generalized Linear Model, Best Estimate.

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Casualty Actuarial Society
4350 North Fairfax Drive, Suite 250
Arlington, VA 22203
www.casact.org
(703) 276-3100

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By Mark R. Shapland

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Biography of the Author

Mark R. Shapland has a B.S. degree in Integrated Studies (Actuarial Science) from the University of Nebraska-Lincoln. He is a Fellow of the Casualty Actuarial Society, a Fellow of the Society of Actuaries, and a Member of the American Academy of Actuaries. He was the leader of Section 3 of the Reserve Variability Working Party, the Chair of the CAS Committee on Reserves, co-chair of the Tail Factor Working Party, and co-chair of the Loss Simulation Model Working Party. He is also a co-developer and co-presenter of the CAS Reserve Variability Limited Attendance Seminar and has spoken frequently on this subject both within the CAS and internationally. He can be contacted at mrshapland@netzero.com.

1. Introduction

With the introduction of the Hayne [8] MLE family of models, the CAS membership has gained a very powerful and useful new toolset for estimating unpaid claim distributions from a data triangle. The growing need for stochastic models for use as part of enterprise risk management and the changing regulatory landscape makes these new stochastic models all the more important. However, like most papers on stochastic modeling, the Hayne [8] paper focuses primarily on the theory and development of the basic modeling framework, which of course is the critical first step. This monograph is an attempt to build and expand upon the foundation of these models by exploring different aspects of their use on a regular basis so that the practicing actuary has a more complete toolset for solving a wider variety of actuarial problems.

1.1. Objectives

This is the second in a series looking at distributions of loss estimates. The monographs in this series look at the theoretical foundations of stochastic unpaid claims models and practical details of implementing them.

Common objectives of the monographs in this series are:

1. Showing how the models can be used in practice to help the wider adoption of unpaid claims distribution.

   Most of the papers describing stochastic models tend to focus primarily on the theoretical aspects of the model while ignoring the data issues that commonly arise in practice. As a result, the models can be quite elegantly implemented yet suffer from practical limitations such as only being useful for complete triangles or only for positive incremental values. Thus, while keeping as close to the theoretical foundation as possible, this objective is to illustrate how practical adjustments can be made to accommodate common data issues and allow the model to "fit" the data. As a practical matter, it is also possible that the model does not fit the data very well, or fits less well than other models, so the process of diagnosing the reasonability of the assumptions will inform the actuary's judgment when considering adjustments to the parameters or how much weight, if any, to give the model in relation to other models.
2. Showing how stochastic reserving can be similar to deterministic reserving when it comes to analyzing and using the best parts of multiple models.

   Actuaries are still searching for the perfect model to describe "the" distribution of unpaid claims, as if imperfections in a model remove it from all consideration, since it can't be "the one." This notion can also manifest itself when an actuary settles for a model that seems to work the best or is the easiest to use, or believes that each model must be used in its entirety or not at all. Interestingly, this notion was dispelled long ago with respect to deterministic point estimates, as actuaries commonly use many different methods, which range from easy to complex, and judgmentally weight the results to arrive at their best estimate.

Model risk – the risk that the model you have chosen is not the same as the one that generates future losses – is very real. Weighting or combining multiple estimates is a very practical way of addressing model risk. More importantly, the monograph hopes to illustrate the advantage of using a more complete set of risk estimation tools (which can include both stochastic models and deterministic methods) to arrive at an actuarial best estimate of the distribution of possible outcomes, rather than to focus on deterministic methods to select the "mean" and then simply "add on" a simple approximation or use only a favorite model to turn that selected mean into a distribution.

A final objective of this monograph is to review the theoretical foundation of Hayne MLE models to better understand the assumptions and parameters. If model assumptions and parameters do not fit the statistical features found in the data, then the results of a simulation may not be a very good estimate of the distribution of possible outcomes. Thus, the modeling framework must be able to adapt or "fit" the model to the data, so this point will be elaborated on in later sections.

2. Notation

Rather than use the notation in the Hayne [8] paper, the notation from the CAS Working Party on Quantifying Variability in Reserve Estimates Summary Report [4] will be used since it is intended to serve as a "standard notation" for further research.

Many models visualize loss data as a two-dimensional array, (w,d) with accident period or policy period w, and development age d (think w = "when" and d = "delay"). For this discussion, it is assumed that the loss information available is an "upper triangular" subset for rows $w = 1, 2, \ldots, n$ and for development ages $d = 1, 2, \ldots, n - w + 1$. The "diagonal" for which $w + d$ equals the constant, k, represents the loss information for each accident period w as of accounting period k.[1]

For purposes of including tail factors, the development beyond the observed data for periods $d = n + 1, n + 2, \ldots, u$, where u is the ultimate time period for which any claim activity occurs, or the period in which all claims are final and paid in full, must also be considered.

The monograph uses the following notation for certain important loss statistics:

$c(w,d)$: cumulative loss from accident[2] year w as of age d.

$q(w,d)$: incremental loss for accident year w from $d - 1$ to d.

$c(w,n) = U(w)$: total loss from accident year w when claims are at ultimate values at time n.[3]

$R(w)$: future development after age d for accident year w, i.e., $= U(w) - c(w,d)$.

$f(d)$: parameter or factor applied to $c(w,d)$ to estimate $q(w,d + 1)$ or can be used more generally to indicate any parameter or factor relating to age d.

$F(d)$: parameter or factor applied to $c(w,d)$ to estimate $c(w,d + 1)$ or $c(w,n)$ or can be used more generally to indicate any cumulative parameter or factor relating to age d.

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1 For a more complete explanation of this two-dimensional view of the loss information, see the *Foundations of Casualty Actuarial Science* [6], Chapter 5, particularly pages 210–226.

2 The use of accident year is used for ease of discussion. All of the discussion and formulas that follow could also apply to underwriting year, policy year, report year, etc. Similarly, year could also be half-year, quarter or month. Finally, while year is implied in the formulas, in many of the tables that follow the equivalent number of months are shown.

3 This would imply that claims reach their ultimate value without any tail factor. This is generalized by changing $\boldsymbol{n}$ to $\boldsymbol{n} + \boldsymbol{t} = \boldsymbol{u}$, where $\boldsymbol{t}$ is the number of periods in the tail.

$T = T(n)$: ultimate tail factor at end of triangle data, which is applied to the estimated $c(w,n)$ to estimate $c(w,u)$.

$G(w)$: parameter or factor relating to accident year w – capitalized to designate ultimate loss level.

$h(k)$: parameter or factor relating to the diagonal k along which $w + d$ is constant.[4]

$M(w,d)$: matrix factors relating to both accident year w and development year d parameters.

$e(w,d)$: a random fluctuation, or error, which occurs at the w,d cell.

$b(w,d)$: cumulative claim count from accident year w as of age d.

$p(w,d)$: incremental claim count for accident year w from $d - 1$ to d.

$N(w)$: the exposures for accident year w.

$A(w,d)$: the incremental average for accident year w from $d - 1$ to d.

$E[x]$: the expectation of the random variable x.

$Var[x]$: the variance of the random variable x.

κ, ρ: variance parameters.

What are called factors here could also be summands, but if factors and summands are both used, some other notation for the additive terms would be needed. The notation does not distinguish paid vs. incurred, but if this is necessary, capitalized subscripts P and I could be used.

[4] Some authors define $\boldsymbol{d} = 0, 1, \ldots, \boldsymbol{n} - 1$ which intuitively allows $\boldsymbol{k} = \boldsymbol{w}$ along the diagonals, but in this case the triangle size is $\boldsymbol{n} \times \boldsymbol{n} - 1$ which is not intuitive. With $\boldsymbol{d} = 1, 2, \ldots, \boldsymbol{n}$ defined as in this monograph, the triangle size $\boldsymbol{n} \times \boldsymbol{n}$ is intuitive, but then $\boldsymbol{k} = \boldsymbol{w} + \mathbf{1}$ along the diagonals is not as intuitive. A way to think about this which helps tie everything together is to assume the $\boldsymbol{w}$ variables are the beginning of the accident periods and the $\boldsymbol{d}$ variables are at the end of the development periods. Thus, if years are used, then cell $\boldsymbol{c}(\boldsymbol{n},\mathbf{1})$ represents accident year $\boldsymbol{n}$ evaluated at $12/31/n$, or essentially $1/1/n + 1$.

3. The Hayne MLE Models

The Hayne MLE models[5] are based on a triangular array of incremental values:

		d					
		1	2	3	. . .	n – 1	n
w	1	q(1,1)	q(1,2)	q(1,3)	. . .	q(1,n – 1)	q(1,n)
	2	q(2,1)	q(2,2)	q(2,3)	. . .	q(2,n – 1)	
	3	q(3,1)	q(3,2)	q(3,3)	. . .		
	. . .	. . .	. . .				
	n – 1	q(n – 1,1)	q(n – 1,2)				
	n	q(n,1)					

By incorporating an exposure adjustment, the variety of methods available for analysis is widened as the focus shifts to the incremental averages:

$$A(w,d) = \frac{q(w,d)}{N(w)}. \tag{3.1}$$

Hayne [8] notes that the exposure adjustment for average incremental values (3.1) can be based on exposure counts or premium amounts, which would commonly be referred to as an average pure premium or burning cost. In addition, the exposure adjustment can be based on an estimate of ultimate claim counts, which would be commonly referred to as an average claim severity:[6]

$$A(w,d) = \frac{q(w,d)}{b(w,u)}. \tag{3.2}$$

In the case of the average claim severity, the ultimate claim counts are often only estimates and as such could be treated as random variables, which will be addressed in Section 4.

[5] While condensed for ease of exposition, significant portions of Section 3 are based on Hayne [8].

[6] For both premiums and exposures, their magnitude can result in very small average values that may have some disadvantages with respect to estimating the model parameters. A practical solution in either case is to use premiums or exposures in thousands or a similar value.

The Hayne MLE models are then based on a generalized framework that expresses each underlying method as a matrix-valued function of a parameter vector θ:

$$A(w,d) = M(\boldsymbol{\theta}). \tag{3.3}$$

In order to turn this general framework into a stochastic model, two key assumptions are made.[7] First, the variance of each incremental value is assumed to be proportional to a power of the square of the mean. It is quite common to assume the variance is proportional to a power of the expected values, but the square of the mean is used to allow incremental values to be negative. Also, the constant of proportionality is exponential, allowing the parameter to take on any value while assuring positive values for the variance. Second, as the variance of an average of a sample with a finite variance will be inversely proportional to the number of items in the sample, the constant of proportionality is assumed to vary inversely to the number of exposures.

The stochastic model is then expressed as follows:

$$E\left[A(w,d)\right] = \mu \tag{3.4}$$

$$Var\left[A(w,d)\right] = \frac{e^{\kappa}\left(\mu^{2}\right)^{\rho}}{N(w)} = e^{\kappa-\ln[N(w)]}\left(\mu^{2}\right)^{\rho}. \tag{3.5}$$

Hayne [8] notes that this model includes an implicit structural heteroscedasticity and that both the expected values and variances differ by accident and development year. The two variance parameters, κ and ρ, provide a mechanism to approximate the variance structure of the data without over-parameterizing the model. However, the formulae can be modified to allow κ to vary by development period if additional control over the heteroscedasticity is desired.

Hayne [8] eloquently describes additional assumptions and processes for estimating the parameters for the stochastic model expressed in (3.4) and (3.5), including R code in the appendix. As this can't be improved upon here, it is left to the reader to review the Hayne [8] paper for further details, but the focus will turn to the five different implementations of this general framework before moving on to various practical implementation issues. For anyone not familiar with R, the implementation of the process of estimating model parameters in R is replicated in Excel in the companion "Hayne MLE Models.xlsm" file. In Excel, the MLE estimates for the parameter vector θ are found using the Solver function. Note, however, that while the Solver algorithm in Excel should estimate parameters that are very close to those estimated in R, there can be differences and in some cases constraints may need to be added to the Excel Solver algorithm.

[7] It is also important to understand that by appealing to the Central Limit Theorem we note that $A(w,d)$ is approximately Normal provided the number of claims is sufficiently large, since it is an average of independent events from equation (3.1) or (3.2).

While the monograph continues to illustrate the Hayne MLE models with the more advanced "Hayne MLE Models.xlsm" file, also included in the companion Excel files are a set of "Hayne Framework 6 __.xlsm" files that illustrate the calculations for each of these different models using a 6×6 triangle. These simpler files are designed to help the reader gain a deeper understanding of each model.

3.1. Berquist-Sherman Model

Berquist and Sherman [2] developed methods to recognize that incremental severities can have different "levels" by accident year as well as different trends by development year. Hayne [8] simplifies this approach by assuming a uniform trend from one accident year to the next, which replaces different levels with uniform changes in level and indirectly impacts the development for each year.

$$E\left[A(w,d)\right] = f(d) \times e^{wG} \tag{3.6}$$

In the Hayne Berquist-Sherman model, the $f(d)$ parameters represent an average incremental by development period. The G parameter is a constant accident year trend where $w = 1, 2, 3, \ldots, n$. Using the data from Hayne [8], the companion Excel file summarizes the Berquist-Sherman model parameters as in Table 3.1.

In addition to the mean and standard deviation of each parameter, which are nearly identical to those in Hayne [8], the coefficient of variation ("CoV") row is added so that the heteroscedastistic variance by parameter is more apparent. The decay ratios row is simply the mean of the development parameter divided by the mean of the prior

Table 3.1. Summary of Berquist-Sherman Parameters

	Development Period Parameters (Average Incremental)									
	12	24	36	48	60	72	84	96	108	120
Mean	620.95	760.66	708.15	553.57	349.99	181.39	70.96	43.88	11.08	15.21
Std Dev	40.50	46.55	43.00	35.49	26.17	17.66	10.39	8.74	4.22	7.34
Decay Ratios:		122.5%	93.1%	78.2%	63.2%	51.8%	39.1%	61.8%	25.2%	137.3%
CoV:	*6.5%*	*6.1%*	*6.1%*	*6.4%*	*7.5%*	*9.7%*	*14.6%*	*19.9%*	*38.1%*	*48.3%*

	Accident Year						
	Trend	K	p	AIC	BIC		Parameters
Mean	0.045	11.216	0.654	643.4	669.5	Acc Period	0
Std Dev	0.009	1.037	0.085			Dev Period	10
CoV:	*18.9%*	*9.2%*	*12.9%*			Trend	1
							11

Table 3.2. Expected Incremental Mean Values for Berquist-Sherman Model

Predicted Incremental Mean [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	649.69	795.86	740.93	579.17	366.20	189.78	74.25	45.91	11.59	15.92	0.00
2009	679.73	832.66	775.18	605.95	383.13	198.56	77.69	48.03	12.13	16.65	16.65
2010	711.16	871.16	811.03	633.96	400.84	207.74	81.28	50.25	12.69	17.42	30.11
2011	744.04	911.43	848.52	663.28	419.37	217.34	85.04	52.57	13.27	18.23	84.07
2012	778.44	953.57	887.75	693.94	438.76	227.39	88.97	55.00	13.89	19.07	176.93
2013	814.43	997.66	928.80	726.03	459.05	237.90	93.08	57.55	14.53	19.95	423.01
2014	852.08	1,043.79	971.74	759.59	480.27	248.90	97.38	60.21	15.20	20.88	922.84
2015	891.48	1,092.05	1,016.67	794.71	502.48	260.41	101.89	62.99	15.90	21.84	1,760.22
2016	932.70	1,142.54	1,063.67	831.46	525.71	272.45	106.60	65.90	16.64	22.85	2,905.28
2017	975.82	1,195.36	1,112.85	869.90	550.02	285.05	111.53	68.95	17.41	23.91	4,234.97
											10,554.09

development parameter, which will be used in later discussions about model fit and tail extrapolation.

Using formulas (3.6) and (3.5) to calculate the expected mean and standard deviation, the results for each incremental value are shown in Tables 3.2 and 3.3, respectively.

Reviewing Table 3.2 you can see how the expected mean values for each development period relate to the model parameters for $f(d)$ in Table 3.1 by looking at each column. Also, comparing rows allows you to see how the trend parameter G impacts each accident year. Reviewing Table 3.3, you can see how expected standard deviation values follow a similar pattern to the expected mean values by column and row. In addition, the standard deviations are related to the exposures (i.e., ultimate claims in this example), which can produce interesting differences, such as the values for 2017 being smaller than for 2016 due to a significantly larger estimate of ultimate claims. Table 3.4 shows the incremental coefficients of variation so that you can easily see the heteroscedasticity implied in the model.

3.2. Cape Cod Model

Hayne [8] notes that the traditional Bornhuetter-Ferguson [3] method estimates future losses by accident year as a percent of an a priori estimate of the ultimate losses for that year. In contrast, a feature of the Cape Cod method is that it derives the a priori estimates directly from the data. Hayne [8] essentially combines these methods by assuming that the incremental average amounts are the product of an accident year

Table 3.3. Incremental Standard Deviation Values for Berquist-Sherman Model

Predicted Incremental Standard Deviation [Model Fitted] (Paid [÷ Ultimate Claims])

Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	95.13	108.63	103.66	88.24	65.39	42.55	23.04	16.82	6.84	8.42	0.00
2009	98.60	112.59	107.44	91.46	67.77	44.10	23.88	17.43	7.09	8.72	8.72
2010	97.68	111.54	106.45	90.61	67.14	43.69	23.65	17.27	7.02	8.64	11.14
2011	100.06	114.26	109.04	92.82	68.78	44.75	24.23	17.69	7.19	8.85	21.05
2012	104.03	118.79	113.36	96.50	71.51	46.53	25.19	18.39	7.48	9.20	33.37
2013	108.82	124.26	118.59	100.95	74.80	48.67	26.35	19.24	7.82	9.63	59.90
2014	107.65	122.92	117.31	99.86	74.00	48.15	26.07	19.03	7.74	9.52	94.79
2015	112.81	128.81	122.93	104.64	77.54	50.45	27.32	19.95	8.11	9.98	144.29
2016	114.36	130.58	124.62	106.08	78.61	51.15	27.69	20.22	8.22	10.12	192.16
2017	110.40	126.07	120.31	102.41	75.89	49.38	26.73	19.52	7.94	9.77	224.29
											349.83

Table 3.4. Incremental Coefficients of Variation for Berquist-Sherman Model

Predicted Incremental Coefficient of Variation [Model Fitted] (Paid [÷ Ultimate Claims])

Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	14.6%	13.6%	14.0%	15.2%	17.9%	22.4%	31.0%	36.6%	59.0%	52.9%	
2009	14.5%	13.5%	13.9%	15.1%	17.7%	22.2%	30.7%	36.3%	58.5%	52.4%	52.4%
2010	13.7%	12.8%	13.1%	14.3%	16.8%	21.0%	29.1%	34.4%	55.4%	49.6%	37.0%
2011	13.4%	12.5%	12.9%	14.0%	16.4%	20.6%	28.5%	33.7%	54.2%	48.6%	25.0%
2012	13.4%	12.5%	12.8%	13.9%	16.3%	20.5%	28.3%	33.4%	53.9%	48.3%	18.9%
2013	13.4%	12.5%	12.8%	13.9%	16.3%	20.5%	28.3%	33.4%	53.8%	48.2%	14.2%
2014	12.6%	11.8%	12.1%	13.1%	15.4%	19.3%	26.8%	31.6%	50.9%	45.6%	10.3%
2015	12.7%	11.8%	12.1%	13.2%	15.4%	19.4%	26.8%	31.7%	51.0%	45.7%	8.2%
2016	12.3%	11.4%	11.7%	12.8%	15.0%	18.8%	26.0%	30.7%	49.4%	44.3%	6.6%
2017	11.3%	10.5%	10.8%	11.8%	13.8%	17.3%	24.0%	28.3%	45.6%	40.9%	5.3%
											3.3%

factor and lag factor, which are usually taken as ultimate loss for the year and the percentage of losses emerging that year.

$$E\left[A(w,d)\right]=\begin{cases} G(1,1), & w=1, d=1 \\ G(1,1)\times G(w), & w>1, d=1 \\ G(1,1)\times f(d), & w=1, d>1 \\ G(1,1)\times G(w)\times f(d), & w>1, d>1 \end{cases} \tag{3.7}$$

In the Hayne Cape Cod model, the $G(1,1)$ parameter, or scale, is a constant from which all other parameters are based. The $G(w)$ parameters are factors multiplied by the constant, which essentially adjust the base for average exposure changes by accident year. The $f(d)$ parameters are factors multiplied by the constant, or constant adjusted by the $G(w)$ parameters, which essentially adjust the base (by accident year) for average incremental changes by development year. Using the data from Hayne [8], the companion Excel file summarizes the Cape Cod model parameters as in Table 3.5.

Using formulas (3.7) and (3.5) to calculate the expected mean and standard deviation, the results for each incremental value are shown in Tables 3.6 and 3.7, respectively.

Table 3.5. Summary of Cape Cod Parameters

	Accident Period Parameters									
	Scale	2009	2010	2011	2012	2013	2014	2015	2016	2017
Mean	620.067	1.160	1.123	1.322	1.376	1.521	1.533	1.580	1.169	1.164
Std Dev	30.048	0.066	0.064	0.072	0.075	0.082	0.084	0.091	0.082	0.105
CoV	*4.8%*	*5.7%*	*5.7%*	*5.4%*	*5.4%*	*5.4%*	*5.5%*	*5.8%*	*7.1%*	*9.0%*
		Development Period Parameters (Average Incremental)								
		24	36	48	60	72	84	96	108	120
Mean		1.181	1.063	0.838	0.534	0.284	0.111	0.067	0.015	0.024
Std Dev		0.041	0.040	0.036	0.029	0.023	0.016	0.016	0.009	0.017
Decay Ratios			*90.0%*	*78.8%*	*63.7%*	*53.2%*	*39.0%*	*60.7%*	*22.8%*	*158.0%*
CoV		*3.5%*	*3.8%*	*4.3%*	*5.5%*	*8.1%*	*14.8%*	*23.1%*	*61.1%*	*70.4%*
		K	p	AIC	BIC				Parameters	
Mean		13.104	0.435	659.4	701.6		Acc Period		9	
Std Dev		1.010	0.083				Dev Period		9	
CoV		*7.7%*	*19.0%*				Scale		1	
									19	

Table 3.6. Expected Incremental Mean Values for Cape Cod Model

Predicted Incremental Mean [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	620.07	732.01	659.04	519.38	330.98	176.23	68.79	41.73	9.53	15.05	0.00
2009	719.49	849.38	764.70	602.65	384.04	204.48	79.82	48.43	11.05	17.47	17.47
2010	696.47	822.21	740.24	583.37	371.76	197.94	77.27	46.88	10.70	16.91	27.61
2011	819.84	967.86	871.37	686.71	437.61	233.01	90.95	55.18	12.60	19.90	87.68
2012	853.00	1,006.99	906.61	714.48	455.31	242.43	94.63	57.41	13.11	20.71	185.86
2013	943.01	1,113.26	1,002.28	789.88	503.36	268.01	104.62	63.47	14.49	22.89	473.48
2014	950.77	1,122.42	I,010.52	796.38	507.50	270.22	105.48	63.99	14.61	23.08	984.87
2015	979.71	1,156.58	1,041.28	820.62	522.95	278.44	108.69	65.94	15.05	23.78	1,835.47
2016	725.16	856.08	770.74	607.41	387.08	206.10	80.45	48.81	11.14	17.60	2,129.33
2017	721.47	851.72	766.81	604.31	385.10	205.05	80.04	48.56	11.08	17.52	2,970.19
											8,711.96

Table 3.7. Incremental Standard Deviation Values for Cape Cod Model

Predicted Incremental Standard Deviation [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	58.08	62.43	59.64	53.77	44.20	33.60	22.31	17.95	9.44	11.52	0.00
2009	62.35	67.02	64.03	57.73	47.45	36.07	23.96	19.28	10.14	12.37	12.37
2010	59.13	63.56	60.72	54.75	45.00	34.21	22.72	18.28	9.61	11.73	15.17
2011	63.13	67.86	64.83	58.45	48.04	36.52	24.26	19.52	10.26	12.52	25.36
2012	64.83	69.69	66.58	60.02	49.34	37.51	24.91	20.04	10.54	12.86	36.04
2013	68.78	73.94	70.63	63.68	52.35	39.79	26.43	21.26	11.18	13.64	55.18
2014	66.30	71.26	68.08	61.38	50.45	38.35	25.47	20.49	10.78	13.15	73.31
2015	68.34	73.45	70.17	63.27	52.01	39.53	26.26	21.12	11.11	13.56	98.55
2016	59.01	63.43	60.59	54.63	44.90	34.14	22.67	18.24	9.59	11.70	104.47
2017	55.18	59.32	56.67	51.09	42.00	31.92	21.20	17.06	8.97	10.95	114.30
											210.79

Table 3.8. Incremental Coefficients of Variation for Cape Cod Model

Predicted Incremental Coefficient of Variation [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	9.4%	8.5%	9.0%	10.4%	13.4%	19.1%	32.4%	43.0%	99.1%	76.5%	
2009	8.7%	7.9%	8.4%	9.6%	12.4%	17.6%	30.0%	39.8%	91.7%	70.8%	70.8%
2010	8.5%	7.7%	8.2%	9.4%	12.1%	17.3%	29.4%	39.0%	89.8%	69.4%	54.9%
2011	7.7%	7.0%	7.4%	8.5%	11.0%	15.7%	26.7%	35.4%	81.5%	62.9%	28.9%
2012	7.6%	6.9%	7.3%	8.4%	10.8%	15.5%	26.3%	34.9%	80.4%	62.1%	19.4%
2013	7.3%	6.6%	7.0%	8.1%	10.4%	14.8%	25.3%	33.5%	77.2%	59.6%	11.7%
2014	7.0%	6.3%	6.7%	7.7%	9.9%	14.2%	24.1%	32.0%	73.8%	57.0%	7.4%
2015	7.0%	6.4%	6.7%	7.7%	9.9%	14.2%	24.2%	32.0%	73.8%	57.0%	5.4%
2016	8.1%	7.4%	7.9%	9.0%	11.6%	16.6%	28.2%	37.4%	86.1%	66.5%	4.9%
2017	7.6%	7.0%	7.4%	8.5%	10.9%	15.6%	26.5%	35.1%	80.9%	62.5%	3.8%
											2.4%

Reviewing Table 3.6, you can see that the scale, or constant, is the value for 2008 at 12 months of development. The $G(w)$, or accident year, parameters are used to adjust the scale in the 12-month column and then the $f(d)$, or development year, parameters are used to adjust the scale, or scale adjusted by accident year, for each development column. Table 3.8 shows the incremental coefficients of variation so that you can easily see the heteroscedasticity implied in the model.

3.3. Chain Ladder Model

For the traditional chain ladder method, average development factors are multiplied by the cumulative amounts by accident year to estimate the expected future incremental values. Hayne [8] also uses the cumulative amounts by accident year, but instead derives parameters which represent the proportion of the incremental value in each development year. The parameters are constrained so that the incremental values sum to the cumulative values. In addition, $n - 1$ parameters are used with the last development year parameter derived so that the sum of all parameters is 100%.

In the Hayne chain ladder model, the $G(w)$ parameters are the actual cumulative values for each accident year. The $f(d)$ parameters are factors multiplied times the cumulative values to derive the expected incremental values by development year. Only $n - 1$ parameters are derived and the "parameter" for the last development period is one minus the sum of the $n - 1$ parameters. In order to constrain the sum of the expected incremental values to equal the cumulative values, the $f(d)$ parameters are divided by the sum of the parameters for that accident year so that the proportional factors for that accident year up to the diagonal sum to 100%.

$$E\left[A(w,d)\right]=\begin{cases} G(w)\times f(d), & w=1, d<n \\ G(w)\times\left[1-\sum_{d=1}^{d=n-1} f(d)\right], & w=1, d=n \\ \dfrac{G(w)\times f(d)}{\sum_{k=1}^{k=n+1-w} f(k)}, & w>1, d<n \\ \dfrac{G(w)\times\left[1-\sum_{d=1}^{d=n-1} f(d)\right]}{\sum_{k=1}^{k=n+1-w} f(k)}, & w>1, d=n \end{cases} \qquad (3.8)$$

Using the data from Hayne [8], the companion Excel file summarizes the chain ladder model parameters as in Table 3.9.

The parameter for 120 months is greyed since it is derived by subtracting the sum of the other parameters from one. Using formulas (3.8) and (3.5) to calculate the expected mean and standard deviation, the results for each incremental value are shown in Tables 3.10 and 3.11, respectively.

Reviewing Table 3.10, it is not as obvious how the parameters relate to the incremental values compared to the Berquist-Sherman or Cape Cod models. However, if you sum the incremental values up to the diagonal for each accident year, you will discover that they sum to the cumulative value for each accident year. Thus, the $f(d)$ parameters can be seen as representing an average proportion of the incremental values compared to the cumulative values. Table 3.12 shows the incremental coefficients of variation so that you can easily see the heteroscedasticity implied in the model.

Table 3.9. Summary of Chain Ladder Parameters

	Development Period Parameters (Average Incremental)									
	12	24	36	48	60	72	84	96	108	120
Mean	0.195	0.231	0.208	0.164	0.104	0.056	0.022	0.013	0.003	0.005
Std Dev	0.005	0.005	0.005	0.005	0.005	0.004	0.003	0.003	0.002	0.003
Decay Ratios:		118.1%	90.0%	78.8%	63.7%	53.2%	39.0%	60.8%	22.9%	157.7%
CoV:	*2.5%*	*2.3%*	*2.5%*	*3.1%*	*4.5%*	*7.3%*	*14.3%*	*22.6%*	*60.4%*	*69.6%*
		K	p	AIC	BIC				Parameters	
Mean		13.074	0.438	619.4	661.5		Acc Period		10	
Std Dev		1.007	0.082				Dev Period		9	
CoV:		*7.7%*	*18.8%*				Trend		0	
									19	

Table 3.10. Expected Incremental Mean Values for Chain Ladder Model

Predicted Incremental Mean [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	617.57	729.07	656.33	517.19	329.57	175.45	68.44	41.59	9.51	15.00	0.00
2009	715.93	845.18	760.86	599.56	382.05	203.39	79.34	48.21	11.03	17.39	17.39
2010	695.14	820.64	738.76	582.16	370.96	197.49	77.04	46.81	10.71	16.89	27.60
2011	823.53	972.20	875.21	689.67	439.47	233.96	91.26	55.46	12.69	20.01	88.15
2012	854.54	1,008.81	908.16	715.64	456.02	242.77	94.70	57.55	13.16	20.76	186.17
2013	943.04	1,113.29	1,002.22	789.76	503.25	267.91	104.51	63.51	14.53	22.91	473.37
2014	951.15	1,122.87	l,010.84	796.55	507.58	270.22	105.41	64.06	14.65	23.11	985.02
2015	981.03	1,158.13	1,042.59	821.57	523.52	278.70	108.72	66.07	15.11	23.83	1,837.53
2016	726.85	858.06	772.46	608.70	387.88	206.49	80.55	48.95	11.20	17.66	2,133.89
2017	723.30	853.88	768.69	605.74	385.99	205.49	80.16	48.71	11.14	17.57	2,977.37
											8,726.49

Table 3.11. Incremental Standard Deviation Values for Chain Ladder Model

Predicted Incremental Standard Deviation [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	58.10	62.48	59.67	53.76	44.14	33.49	22.18	17.84	9.35	11.41	0.00
2009	62.38	67.08	64.06	57.72	47.38	35.96	23.81	19.15	10.04	12.25	12.25
2010	59.23	63.69	60.83	54.80	44.99	34.14	22.61	18.18	9.53	11.63	15.04
2011	63.44	68.22	65.15	58.70	48.19	36.57	24.22	19.47	10.21	12.46	25.27
2012	65.08	69.98	66.84	60.22	49.44	37.51	24.84	19.98	10.47	12.78	35.91
2013	69.01	74.21	70.87	63.86	52.42	39.78	26.34	21.18	11.11	13.56	55.07
2014	66.53	71.54	68.32	61.56	50.54	38.35	25.40	20.42	10.71	13.07	73.29
2015	68.61	73.78	70.46	63.48	52.12	39.55	26.19	21.06	11.04	13.48	98.71
2016	59.22	63.68	60.82	54.79	44.98	34.14	22.61	18.18	9.53	11.63	104.68
2017	55.39	59.56	56.88	51.25	42.07	31.93	21.14	17.00	8.91	10.88	114.60
											211.05

Table 3.12. Incremental Coefficients of Variation for Chain Ladder Model

Predicted Incremental Coefficient of Variation [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	9.4%	8.6%	9.1%	10.4%	13.4%	19.1%	32.4%	42.9%	98.3%	76.1%	
2009	8.7%	7.9%	8.4%	9.6%	12.4%	17.7%	30.0%	39.7%	91.0%	70.5%	70.5%
2010	8.5%	7.8%	8.2%	9.4%	12.1%	17.3%	29.3%	38.8%	89.0%	68.9%	54.5%
2011	7.7%	7.0%	7.4%	8.5%	11.0%	15.6%	26.5%	35.1%	80.5%	62.3%	28.7%
2012	7.6%	6.9%	7.4%	8.4%	10.8%	15.5%	26.2%	34.7%	79.6%	61.6%	19.3%
2013	7.3%	6.7%	7.1%	8.1%	10.4%	14.8%	25.2%	33.4%	76.4%	59.2%	11.6%
2014	7.0%	6.4%	6.8%	7.7%	10.0%	14.2%	24.1%	31.9%	73.1%	56.6%	7.4%
2015	7.0%	6.4%	6.8%	7.7%	10.0%	14.2%	24.1%	31.9%	73.1%	56.6%	5.4%
2016	8.1%	7.4%	7.9%	9.0%	11.6%	16.5%	28.1%	37.1%	85.1%	65.9%	4.9%
2017	7.7%	7.0%	7.4%	8.5%	10.9%	15.5%	26.4%	34.9%	80.0%	61.9%	3.8%
											2.4%

3.4. Hoerl Curve Model

The Hoerl Curve is a three-parameter exponential model that uses the development lag for all three parameters, i.e., number of periods, number of periods squared and the natural log of the number of periods. Hayne [8] combines these three parameters with a constant level parameter and an accident year trend factor.

$$E\left[A(w,d)\right]=e^{G(1)+d\times f(1)+d^2\times f(2)+\ln(d)\times f(3)+w\times G(2)} \tag{3.9}$$

In the Hayne Hoerl Curve model, the $G(1)$ parameter is the constant level on a log scale. The $G(2)$ parameter is a constant trend that adjusts the level by accident year. The $f(1)$, $f(2)$, and $f(3)$ parameters are factors multiplied times the development lags, i.e., by d, d^2, and $\ln(d)$, respectfully. Using the data from Hayne [8], the companion Excel file summarizes the Hoerl Curve model parameters as in Table 3.13.

Using formulas (3.9) and (3.5) to calculate the expected mean and standard deviation, the results for each incremental value are shown in Tables 3.14 and 3.15, respectively.

Reviewing Table 3.14, the link to the parameters must be viewed on a log scale. Starting with the first development column, the beginning "level" for each accident year on a log scale is the $G(1)$ parameter plus the trend times the number of years, plus one of the $f(1)$ and $f(2)$ parameters. Moving from left to right across the development years, the combination of the three development parameters acts to first increase the incremental values, then to decrease the incremental values in a smooth curve. Table 3.16 shows the incremental coefficients of variation so that you can easily see the heteroscedasticity implied in the model.

Table 3.13. Summary of Hoerl Curve Parameters

	Parameters (Average Incremental)						
	Level	d	d^2	ln(d)	Trend		
Mean	6.496	0.005	(0.065)	0.596	0.043		
Std Dev	0.220	0.240	0.019	0.323	0.008		
CoV:	*3.4%*	*4687.1%*	*–28.4%*	*54.2%*	*19.5%*		
		K	p	AIC	BIC		Parameters
Mean		13.147	0.506	639.7	653.8	Level	1
Std Dev		1.014	0.083			Development	3
CoV:		*7.7%*	*16.3%*			Trend	1
							5

Table 3.14. Expected Incremental Mean Values for Hoerl Curve Model

Predicted Incremental Mean [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	651.30	813.57	751.30	567.59	362.10	197.86	93.30	38.14	13.55	4.20	0.00
2009	679.90	849.29	784.29	592.51	378.00	206.55	97.40	39.81	14.15	4.38	4.38
2010	709.75	886.58	818.73	618.53	394.60	215.62	101.67	41.56	14.77	4.57	19.34
2011	740.92	925.51	854.68	645.68	411.93	225.08	106.14	43.38	15.42	4.77	63.58
2012	773.45	966.15	892.20	674.04	430.01	234.97	110.80	45.29	16.09	4.98	177.16
2013	807.41	1,008.57	931.38	703.63	448.90	245.28	115.66	47.28	16.80	5.20	430.22
2014	842.86	1,052.85	972.28	734.53	468.61	256.05	120.74	49.35	17.54	5.43	917.72
2015	879.87	1,099.08	1,014.97	766.78	489.18	267.30	126.04	51.52	18.31	5.67	1,724.80
2016	918.50	1,147.34	1,059.53	800.45	510.66	279.03	131.58	53.78	19.11	5.92	2,860.07
2017	958.83	1,197.72	1,106.06	835.59	533.08	291.28	137.35	56.14	19.95	6.18	4,183.37
											10,380.64

Table 3.15. Incremental Standard Deviation Values for Hoerl Curve Model

Predicted Incremental Standard Deviation [Model Fitted] (Paid [÷ Ultimate Claims])

Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	95.72	107.11	102.88	89.29	71.14	52.41	35.84	22.80	13.51	7.47	0.00
2009	98.43	110.15	105.81	91.82	73.16	53.89	36.85	23.45	13.90	7.68	7.68
2010	96.76	108.28	104.00	90.26	71.91	52.98	36.23	23.05	13.66	7.55	15.61
2011	98.34	110.05	105.71	91.73	73.09	53.84	36.82	23.42	13.88	7.67	28.29
2012	101.45	113.52	109.04	94.63	75.39	55.54	37.98	24.16	14.32	7.92	47.90
2013	105.29	117.83	113.18	98.22	78.25	57.65	39.42	25.08	14.86	8.22	76.12
2014	103.35	115.65	111.08	96.40	76.81	56.58	38.69	24.61	14.59	8.06	107.15
2015	107.45	120.25	115.50	100.23	79.86	58.83	40.23	25.59	15.17	8.38	149.87
2016	108.08	120.95	116.18	100.82	80.33	59.18	40.47	25.74	15.26	8.43	190.32
2017	103.53	115.85	111.28	96.57	76.94	56.68	38.76	24.66	14.62	8.08	216.00
											354.98

Table 3.16. Incremental Coefficients of Variation for Hoerl Curve Model

Predicted Incremental Coefficient of Variation [Model Fitted] (Paid [÷ Ultimate Claims])

Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	14.7%	13.2%	13.7%	15.7%	19.6%	26.5%	38.4%	59.8%	99.7%	178.0%	
2009	14.5%	13.0%	13.5%	15.5%	19.4%	26.1%	37.8%	58.9%	98.2%	175.4%	175.4%
2010	13.6%	12.2%	12.7%	14.6%	18.2%	24.6%	35.6%	55.5%	92.5%	165.1%	80.7%
2011	13.3%	11.9%	12.4%	14.2%	17.7%	23.9%	34.7%	54.0%	90.0%	160.8%	44.5%
2012	13.1%	11.7%	12.2%	14.0%	17.5%	23.6%	34.3%	53.4%	89.0%	158.9%	27.0%
2013	13.0%	11.7%	12.2%	14.0%	17.4%	23.5%	34.1%	53.0%	88.5%	158.0%	17.7%
2014	12.3%	11.0%	11.4%	13.1%	16.4%	22.1%	32.0%	49.9%	83.2%	148.5%	11.7%
2015	12.2%	10.9%	11.4%	13.1%	16.3%	22.0%	31.9%	49.7%	82.9%	147.9%	8.7%
2016	11.8%	10.5%	11.0%	12.6%	15.7%	21.2%	30.8%	47.9%	79.8%	142.5%	6.7%
2017	10.8%	9.7%	10.1%	11.6%	14.4%	19.5%	28.2%	43.9%	73.2%	130.8%	5.2%
											3.4%

3.5. Wright Model

The Wright model also uses the three-parameter Hoerl curve, but instead of constant level and trend parameters, individual parameters for each accident year "level" are used.

$$E\left[A\left(w,d\right)\right]=e^{G(w)+d\times f(1)+d^2\times f(2)+\ln(d)\times f(3)} \tag{3.10}$$

In the Hayne Wright model, the $G(w)$ parameters are the individual levels for each accident year. Similar to the Hoerl Curve model, the $f(1)$, $f(2)$, and $f(3)$ parameters are factors multiplied times the development lags, i.e., by d, d^2, and $\ln(d)$, respectfully. Using the data from Hayne [8], the companion Excel file summarizes the Wright model parameters as in Table 3.17.

Using formulas (3.10) and (3.5) to calculate the expected mean and standard deviation, the results for each incremental value are shown in Tables 3.18 and 3.19, respectively.

Reviewing Table 3.18, you can see the similarities to Table 3.14. Starting with the first development column, the beginning "level" for each accident year on a log scale is the $G(w)$ parameter plus one of the $f(1)$ and $f(2)$ parameters. Moving from left to right across the development years, the combination of the three development parameters acts to first increase the incremental values, then to decrease the incremental values in a smooth curve. Table 3.20 below shows the incremental coefficients of variation so that you can easily see the heteroscedasticity implied in the model.

Table 3.17. Summary of Wright Parameters

	Accident Period Parameters									
	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Mean	6.312	6.472	6.436	6.587	6.636	6.738	6.742	6.771	6.475	6.468
Std Dev	0.168	0.167	0.167	0.166	0.167	0.167	0.166	0.164	0.166	0.184
CoV	*2.7%*	*2.6%*	*2.6%*	*2.5%*	*2.5%*	*2.5%*	*2.5%*	*2.4%*	*2.6%*	*2.8%*

	Development Period Parameters (Average Incremental)					
	d	d^2	ln(d)			
Mean	0.192	(0.078)	0.290			
Std Dev	0.183	0.015	0.232			
CoV	*95.4%*	*–19.5%*	*80.0%*			
	K	p	AIC	BIC		Parameters
Mean	14.592	0.319	612.3	642.4	Acc Period	10
Std Dev	0.909	0.075			Dev Period	3
CoV	*6.2%*	*23.4%*				13

Table 3.18. Expected Incremental Mean Values for Wright Model

Predicted Incremental Mean [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	617.75	724.24	668.24	509.80	326.60	176.91	81.32	31.79	10.59	3.01	0.00
2009	724.55	849.44	783.76	597.94	383.06	207.49	95.38	37.29	12.42	3.52	3.52
2010	698.92	819.39	756.04	576.79	369.51	200.15	92.00	35.97	11.98	3.40	15.38
2011	813.22	953.39	879.68	671.11	429.93	232.88	107.05	41.85	13.94	3.96	59.74
2012	854.17	l,001.41	923.98	704.91	451.59	244.61	112.44	43.96	14.64	4.16	175.19
2013	945.66	1,108.66	1,022.94	780.41	499.95	270.81	124.48	48.67	16.21	4.60	464.77
2014	949.61	1,113.29	1,027.21	783.67	502.04	271.94	125.00	48.87	16.27	4.62	968.75
2015	977.65	1,146.17	1,057.55	806.81	516.87	279.97	128.70	50.31	16.75	4.76	1,804.17
2016	726.83	852.12	786.23	599.82	384.26	208.14	95.68	37.41	12.46	3.54	2,127.54
2017	721.95	846.40	780.95	595.80	381.68	206.75	95.04	37.16	12.37	3.51	2,959.65
											8,578.71

Table 3.19. Incremental Standard Deviation Values for Wright Model

Predicted Incremental Standard Deviation [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	57.98	61.00	59.45	54.53	47.30	38.89	30.35	22.48	15.83	10.59	0.00
2009	61.39	64.59	62.95	57.74	50.09	41.18	32.13	23.81	16.76	11.21	11.21
2010	58.37	61.41	59.86	54.90	47.63	39.16	30.55	22.64	15.93	10.66	19.17
2011	60.93	64.10	62.48	57.31	49.71	40.87	31.89	23.63	16.63	11.13	30.96
2012	62.47	65.73	64.06	58.76	50.97	41.91	32.70	24.23	17.05	11.41	45.57
2013	65.55	68.96	67.21	61.65	53.48	43.97	34.31	25.42	17.89	11.97	64.96
2014	63.03	66.32	64.64	59.29	51.43	42.28	32.99	24.44	17.21	11.51	80.91
2015	64.73	68.10	66.38	60.88	52.81	43.42	33.88	25.10	17.67	11.82	103.01
2016	57.96	60.98	59.43	54.51	47.28	38.88	30.33	22.47	15.82	10.58	109.72
2017	54.21	57.03	55.58	50.98	44.22	36.36	28.37	21.02	14.80	9.90	117.40
											225.23

Table 3.20. Incremental Coefficients of Variation for Wright Model

Predicted Incremental Coefficient of Variation [Model Fitted] (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	9.4%	8.4%	8.9%	10.7%	14.5%	22.0%	37.3%	70.7%	149.5%	352.3%	
2009	8.5%	7.6%	8.0%	9.7%	13.1%	19.8%	33.7%	63.8%	135.0%	318.0%	318.0%
2010	8.4%	7.5%	7.9%	9.5%	12.9%	19.6%	33.2%	62.9%	133.0%	313.5%	124.7%
2011	7.5%	6.7%	7.1%	8.5%	11.6%	17.6%	29.8%	56.5%	119.3%	281.2%	51.8%
2012	7.3%	6.6%	6.9%	8.3%	11.3%	17.1%	29.1%	55.1%	116.5%	274.5%	26.0%
2013	6.9%	6.2%	6.6%	7.9%	10.7%	16.2%	27.6%	52.2%	110.4%	260.2%	14.0%
2014	6.6%	6.0%	6.3%	7.6%	10.2%	15.5%	26.4%	50.0%	105.7%	249.2%	8.4%
2015	6.6%	5.9%	6.3%	7.5%	10.2%	15.5%	26.3%	49.9%	105.5%	248.5%	5.7%
2016	8.0%	7.2%	7.6%	9.1%	12.3%	18.7%	31.7%	60.1%	127.0%	299.3%	5.2%
2017	7.5%	6.7%	7.1%	8.6%	11.6%	17.6%	29.9%	56.6%	119.6%	281.8%	4.0%
											2.6%

3.6. The Simulation Process

For each of the Hayne MLE models, using the parameters to calculate the expected mean and standard deviation for each incremental cell is only the starting point, since this framework allows us to use simulation to generate a distribution of possible outcomes. Additional outputs for each model are the standard deviations for each parameter (shown in Tables 3.1, 3.5, 3.9, 3.13, and 3.17) and the variance-covariance matrix of all the parameters (not shown). Using the means and variance-covariance matrix, the simulation process starts by sampling a random set of new parameters using the multi-variate normal distribution. For example, a sample iteration for the Berquist-Sherman model could look like Table 3.21.

Table 3.21. Sample of Berquist-Sherman Parameters

Berquist-Sherman: Development Period Parameters (Average Incremental)									
12	24	36	48	60	72	84	96	108	120
668.32	704.13	645.21	559.41	380.69	165.37	84.01	33.80	26.55	15.75
Trend	K	p							
0.047	11.268	0.661							

Using the sample parameters, the next step in the simulation process is to calculate the mean and standard deviation for each cell, as in Tables 3.22 and 3.23. As a check, we can also review the coefficients of variation from the sample incremental parameters in Table 3.24.

Table 3.22. Sampled Incremental Mean Values for Berquist-Sherman

Generate Incremental Mean from Random Parameters (Paid [÷ Ultimate Claims])

Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	700.40	737.93	676.18	586.26	398.96	173.31	88.05	35.42	27.82	16.50	0.00
2009	734.02	773.35	708.64	614.40	418.11	181.63	92.27	37.12	29.16	17.29	17.29
2010	769.26	810.47	742.66	643.89	438.18	190.35	96.70	38.90	30.56	18.12	48.68
2011	806.18	849.37	778.30	674.79	459.21	199.48	101.34	40.77	32.02	18.99	91.78
2012	844.88	890.14	815.66	707.18	481.25	209.06	106.21	42.73	33.56	19.91	202.40
2013	885.43	932.87	854.81	741.13	504.35	219.09	111.31	44.78	35.17	20.86	431.21
2014	927.93	977.65	895.84	776.70	528.56	229.61	116.65	46.93	36.86	21.86	980.47
2015	972.47	1,024.57	938.84	813.98	553.93	240.63	122.25	49.18	38.63	22.91	1,841.51
2016	1,019.15	1,073.75	983.91	853.06	580.52	252.18	128.11	51.54	40.48	24.01	2,913.81
2017	1,068.07	1,125.29	1,031.13	894.00	608.39	264.29	134.26	54.01	42.43	25.16	4,178.97
											10,706.12

Table 3.23. Sampled Incremental Std. Dev. Values for Berquist-Sherman

Generate Incremental Standard Deviation from Random Parameters (Paid [÷ Ultimate Claims])

Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	107.53	111.30	105.06	95.60	74.12	42.71	27.30	14.95	12.74	9.02	0.00
2009	111.61	115.53	109.04	99.23	76.93	44.33	28.33	15.52	13.23	9.37	9.37
2010	110.73	114.62	108.19	98.45	76.33	43.98	28.11	15.40	13.12	9.29	16.08
2011	113.59	117.58	110.98	100.99	78.30	45.12	28.84	15.79	13.46	9.53	22.84
2012	118.27	122.42	115.55	105.15	81.52	46.98	30.02	16.44	14.02	9.92	38.30
2013	123.90	128.25	121.05	110.15	85.40	49.21	31.45	17.23	14.69	10.40	63.50
2014	122.74	127.05	119.92	109.12	84.60	48.75	31.16	17.07	14.55	10.30	105.43
2015	128.81	133.33	125.84	114.51	88.79	51.16	32.70	17.91	15.27	10.81	159.23
2016	130.77	135.36	127.76	116.26	90.14	51.94	33.20	18.18	15.50	10.97	206.04
2017	126.42	130.86	123.52	112.40	87.14	50.22	32.09	17.58	14.98	10.61	238.34
											376.95

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Table 3.24. Sampled Incremental CoVs for Berquist-Sherman

Incremental Coefficient of Variation from Generated Random Parameters (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	15.4%	15.1%	15.5%	16.3%	18.6%	24.6%	31.0%	42.2%	45.8%	54.7%	
2009	15.2%	14.9%	15.4%	16.2%	18.4%	24.4%	30.7%	41.8%	45.4%	54.2%	54.2%
2010	14.4%	14.1%	14.6%	15.3%	17.4%	23.1%	29.1%	39.6%	43.0%	51.3%	33.0%
2011	14.1%	13.8%	14.3%	15.0%	17.1%	22.6%	28.5%	38.7%	42.0%	50.2%	24.9%
2012	14.0%	13.8%	14.2%	14.9%	16.9%	22.5%	28.3%	38.5%	41.8%	49.9%	18.9%
2013	14.0%	13.7%	14.2%	14.9%	16.9%	22.5%	28.3%	38.5%	41.8%	49.8%	14.7%
2014	13.2%	13.0%	13.4%	14.0%	16.0%	21.2%	26.7%	36.4%	39.5%	47.1%	10.8%
2015	13.2%	13.0%	13.4%	14.1%	16.0%	21.3%	26.7%	36.4%	39.5%	47.2%	8.6%
2016	12.8%	12.6%	13.0%	13.6%	15.5%	20.6%	25.9%	35.3%	38.3%	45.7%	7.1%
2017	11.8%	11.6%	12.0%	12.6%	14.3%	19.0%	23.9%	32.5%	35.3%	42.2%	5.7%
											3.5%

Next, using the sampled mean and standard deviation for each incremental cell, process variance is added by randomly generating an observation for each cell using the normal distribution and the sampled mean and standard deviation for that cell. Continuing the example, U(0,1) random values are shown in Table 3.25 and the random observations based on the means and standard deviations by cell in Tables 3.22 and 3.23, respectively, are shown in Table 3.26.

Since the model is typically based on average severities, the final step is to multiply the random observations times the ultimate claim counts[8] by year to convert the sample to total claim values, as in Table 3.27.

Repeating these steps a large number of times, the results for all iterations can be saved and summarized by accident year as shown in Figure 3.1.

In addition to results by accident year, results by calendar year and other possibilities can also be created from the same simulations. The output will be discussed in more detail in Sections 5 and 6.

[8] This step depends on the original exposure basis used to parameterize the model. For example, if the model is based on pure premiums then the last step is to multiply times exposures by year.

Table 3.25. Random Values

Simulated Random Values [Correlated] (Paid)										
Year	12	24	36	48	60	72	84	96	108	120
2008	0.4009	0.4189	0.9459	0.3101	0.3192	0.1740	0.4005	0.0364	0.1201	0.0822
2009	0.3078	0.7144	0.5731	0.1989	0.4034	0.4817	0.3595	0.8254	0.8173	0.6103
2010	0.3334	0.8134	0.5619	0.9379	0.3830	0.0163	0.1479	0.8463	0.9088	0.9352
2011	0.9491	0.2084	0.7126	0.2911	0.4702	0.6269	0.7621	0.4779	0.1540	0.0921
2012	0.7837	0.4402	0.1229	0.8062	0.4995	0.3770	0.3096	0.5040	0.8820	0.0521
2013	0.1960	0.2693	0.0002	0.3931	0.1450	0.0349	0.1155	0.0600	0.3554	0.0203
2014	0.7020	0.0977	0.2878	0.7736	0.5855	0.0297	0.9950	0.3926	0.7570	0.6794
2015	0.5225	0.0925	0.9975	0.3746	0.1550	0.5164	0.0112	0.7273	0.1654	0.5295
2016	0.4272	0.7301	0.3417	0.6337	0.3146	0.7889	0.2524	0.8902	0.8295	0.6409
2017	0.0630	0.4542	0.8377	0.4535	0.9946	0.1432	0.5699	0.1098	0.7175	0.1494

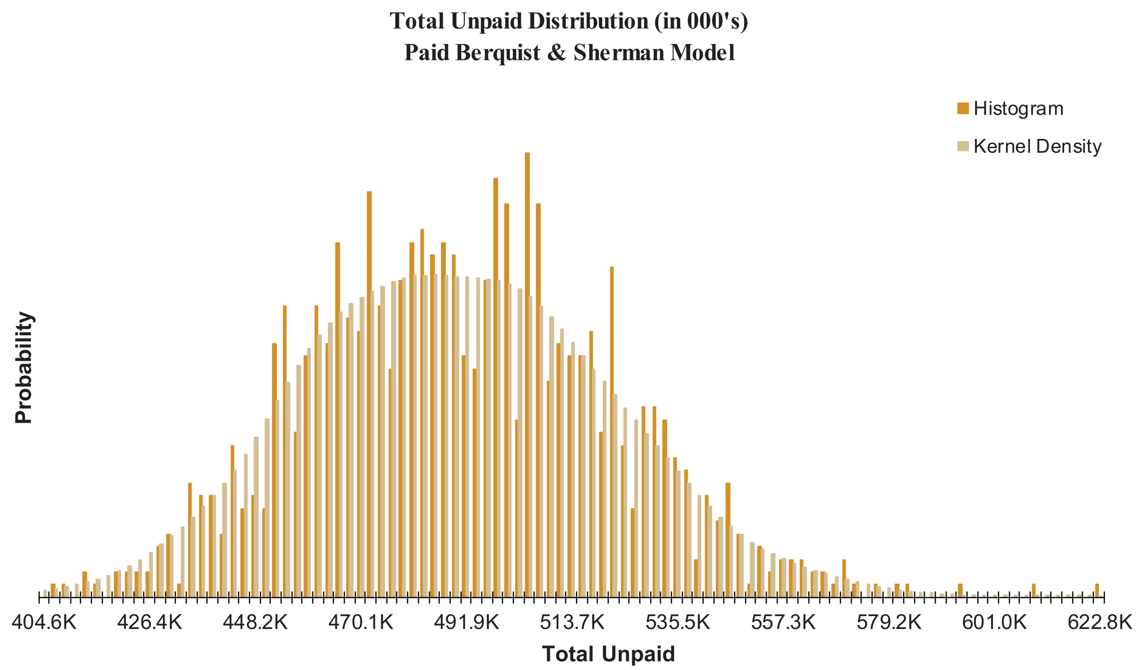
Table 3.26. Sample Observations for Berquist-Sherman

Generate Random Observation from Sampled Incremental Mean & Variance (Paid [÷ Ultimate Claims])											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	667.37	709.01	844.47	532.84	359.51	130.00	79.63	7.10	11.80	3.16	0.00
2009	670.92	834.93	723.93	523.28	394.97	177.35	80.40	51.29	40.82	19.53	19.53
2010	714.78	909.75	754.66	794.60	411.07	91.53	65.10	54.30	47.90	32.15	80.05
2011	991.65	745.44	836.84	612.69	449.33	212.28	121.06	39.09	17.25	5.51	61.86
2012	934.48	865.17	672.08	795.40	477.14	191.62	89.39	42.09	49.95	2.84	184.27
2013	770.31	845.46	403.96	704.98	407.24	124.96	71.03	16.39	28.85	(1.54)	239.69
2014	988.80	802.30	820.93	855.55	543.17	132.74	197.56	41.30	46.56	26.29	987.63
2015	973.59	836.35	1,296.12	770.69	456.90	240.28	43.88	59.43	22.61	23.20	1,617.00
2016	988.06	1,152.40	924.07	888.17	531.34	292.49	103.72	73.59	54.89	27.54	2,895.81
2017	862.99	1,103.37	1,150.12	874.98	832.28	206.77	138.49	30.96	50.55	13.31	4,400.84
											10,486.67

Table 3.27. Conversion to Total Value for Berquist-Sherman

Convert Incremental Severity (Paid [÷ Ultimate Claims]) to Total Incremental Value (in 000's)											
Year	12	24	36	48	60	72	84	96	108	120	Future Totals
2008	26,135	27,766	33,070	20,866	14,079	5,091	3,118	278	462	124	0
2009	25,946	32,289	27,996	20,236	15,275	6,859	3,109	1,983	1,578	755	755
2010	29,878	38,029	31,546	33,215	17,183	3,826	2,721	2,270	2,002	1,344	3,346
2011	41,910	31,505	35,368	25,894	18,990	8,972	5,116	1,652	729	233	2,614
2012	38,763	35,888	27,879	32,994	19,792	7,948	3,708	1,746	2,072	118	7,643
2013	30,978	34,000	16,245	28,350	16,377	5,025	2,856	659	1,160	(62)	9,639
2014	43,110	34,979	35,791	37,301	23,682	5,787	8,613	1,801	2,030	1,146	43,059
2015	41,006	35,226	54,590	32,460	19,244	10,120	1,848	2,503	952	977	68,105
2016	42,960	50,106	40,178	38,617	23,103	12,717	4,510	3,200	2,387	1,197	125,908
2017	42,712	54,608	56,922	43,305	41,192	10,234	6,854	1,532	2,502	659	217,808
											478,879

Figure 3.1. Distribution for Berquist-Sherman



4. Practical Issues

Now that the basic Hayne MLE framework has been described, a variety of practical issues needed for addressing many common problems can be addressed. In order to distinguish whether the underlying model has parameters associated with individual development period, the underlying models can be categorized into two families. The first family has parameters tied to individual development age—Berquist Sherman, Cape Cod, and chain ladder models fall into this family. The other family has no parameters specific to individual development periods and the parameters are more comparable to coefficient of regression on development age (operational time)—Hoerl Curve and Wright models belong to this family.

4.1. Negative Incremental Values

In general for the Hayne MLE framework, no special care is required in modeling triangles with a few negative entries. When the total incremental values for a given development period are significantly lower than zero, models from the first family have no problem dealing with this type of triangle. Calibrated development period parameters, most likely, will turn out to be negative to reflect negative expected incremental values for the period. For models from the second family, incremental means are exponential and hence are always positive, so negative incremental values in the triangle are difficult to model, which typically implies inappropriateness of the model and resulting in a bad fit to the data. However, negative numbers can still be simulated due to the process variance during simulation, so a close fit may still work.

4.2. Standardized Residuals

As the Hayne MLE framework is based on an assumed distribution, i.e., the normal distribution for incremental values, this implies that the standardized residuals should be normally distributed with mean of zero and a standard deviation of one. The goodness of fit to a standard normal distribution of standardized residuals, to some degree, implies the appropriateness of the chosen model. Unlike the ODP bootstrap model, however, the standardized residuals are not used during the simulation process.

While the residuals are not sampled, the mean and standard deviation of the residuals can be used to adjust the process variance simulations. For the mean, an average of the residuals greater than zero implies that the mean of the predicted values are "low" compared to means that would result in an average of zero. Thus, the adjustment for the mean is to increase the mean for each cell by the standard deviation for that cell times the average of the residuals. Similarly, a standard deviation of the residuals greater than one implies "less" variability than would be "normal," so the standard deviation for each cell can be increased by multiplying it times the standard deviation of the residuals.

Another way of thinking about this adjustment is to remember that the process variance in the simulations is based on N(0, 1), so if the residuals exhibit a mean and standard deviation which differ from zero and one, respectively, then this adjustment allows the process variance to more closely match the residuals. In the "Hayne MLE Models.xlsm" file, the "Include Residual Adjustment" option on the Inputs sheet allows the user to use this adjustment, or not, as this will move away from the calculated Hayne MLE parameters, but it could be a way of fitting the model to the data.

4.3. Using an *L*-Year Average

It is quite common for actuaries to use averages that are less than all years in their chain-ladder and related methods. Similarly, the Hayne MLE models can be adjusted to only consider the data in the most recent diagonals. For the Hayne MLE framework, only the most recent $L + 1$ diagonals (since an L-year average uses $L + 1$ diagonals) could be used to parameterize the model. The shape of the data to be modeled essentially becomes a trapezoid instead of a triangle and the excluded diagonals are given zero weight in the models. When running the simulations the entire triangle can still be used, since the parameterization of the model has already been constrained by the number of diagonals.

The companion "Hayne MLE Models.xlsm" file has not been specifically designed to select an L-year model, but that can easily be accomplished by using the outlier table to give zero weight to the prior diagonals. Another possibility for an L-year model includes an additional payment year term (e.g., a function set to 0 for $k < n - L$ and 1 for $k \geq n - L$) that would produce a similar effect (i.e., using the last $L + 1$ diagonals) while allowing all of the data to be included in the model fit, perhaps enabling better diagnostic testing and parameter estimation.

4.4. Missing Values

Sometimes the loss triangle will have missing values. For example, values may be missing from the middle of the triangle, or a triangle may be missing the oldest diagonals, if loss data was not kept in the early years of the book of business.

If values are missing, then the following calculations will be affected:

- Fitted parameters
- Variance-Covariance Matrix
- Fitted triangle

- Residuals
- Degrees of freedom

There are several solutions. The missing value may be estimated using the surrounding values. Or the parameterization of the model can exclude the missing values as long as the missing value is not compromising the surrounding incremental values, or for the chain ladder model the cumulative values. In any case, zero weights are applied to corresponding entries in maximizing log-likelihood functions. The mean and standard deviation of the incremental corresponding to the missing value can be derived from simulated parameters.

If the missing value lies on the most recent diagonal, parameters can be calibrated without any issue except for the chain ladder model, which relies on paid-to-date losses to estimate average incremental values. A solution is to use the value in the second most recent diagonal to fit the triangle and the average incremental formula should be adjusted to be divided by the sum of the first $n - w$ parameters rather than $n - w + 1$ parameters. Of course for other MLE models, simply using the outliers to assign zero weight to the corresponding cell will allow the model to be parameterized without disturbing the overall framework.

4.5. Outliers

There may be a few extreme or incorrect values in the original triangle dataset that could be considered outliers. These may not be representative of the variability of the dataset in the future and, if so, the modeler may want to remove their impact from the model. These values could be removed and dealt with in the same manner as missing values by applying zero weight to the corresponding incremental.

If there are a significant number of outliers, then this could be an indication that the model is not a good fit to the data. For example, since the Hayne MLE models include an underlying assumption of normality, too many outliers could imply that the underlying claims distribution is too severe for the central limit theorem to apply—i.e., the model may be fine but the normality assumption is not. Outliers should always be removed only after careful consideration of the underlying data to make sure it is truly an unusual event.

4.6. Heteroscedasticity

As noted earlier, the Hayne MLE models include variance parameters that adjust the variance for each cell instead of assuming a constant variance throughout. In essence, the modeling framework assumes heteroscedasticity. However, since the variance for the incremental value is only specified using two parameters, it is still possible that the modeled heteroscedasticity does not match up well with the variances in the data. In this case, additional variance parameters can be specified as described in Hayne [8], but that is outside the scope of this monograph.

4.7. Heteroecthesious Data

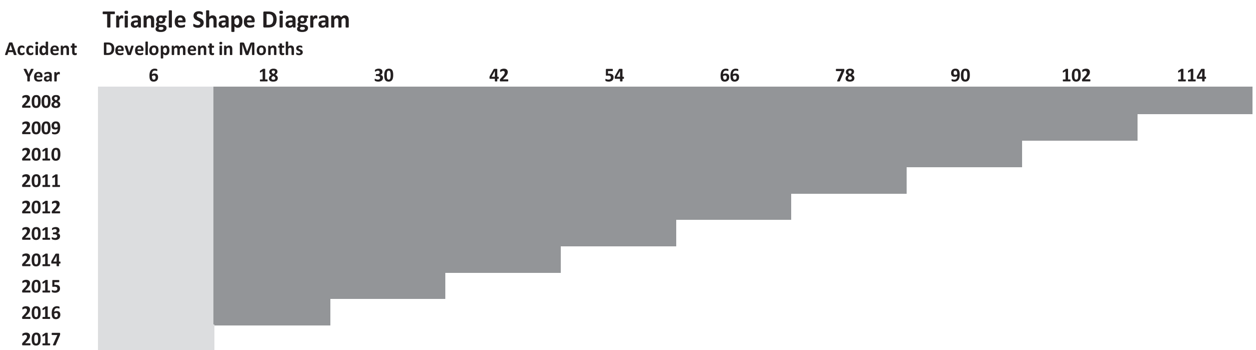
The basic Hayne MLE framework assumes both a symmetrical shape (e.g., annual by annual, quarterly by quarterly, etc., triangles) and homoecthesious data (i.e., similar exposures).[9] Other non-symmetrical shapes (e.g., annual x quarterly data) can also be modeled with the Hayne MLE framework as assumptions are independent from triangle shapes.

Most often, the actuary will encounter heteroecthesious data (i.e., incomplete or uneven exposures) at interim evaluation dates, with the two most common data triangles being either a partial first development period or a partial last calendar period. For example, with annual data evaluated as of June 30, partial first development period data would have all development periods ending at 6, 18, 30, etc., months, while partial last calendar period data would have development periods as of 12, 24, 36, etc., months for all of the data in the triangle except the last diagonal, which would have development periods as of 6, 18, 30, etc., months. In either case, not all of the data in the triangle has full annual exposures—i.e., it is heteroecthesious data.

4.7.1. Partial First Development Period Data

For partial first development period data, the first development column has a different exposure period than the rest of the columns (e.g., in the earlier example the first column has six months of development exposure, while the rest have 12), as illustrated in Figure 4.1. In models such as Berquist Sherman, Cape Cod and chain ladder, where a parameter is specified for each development period, it is not an issue in the parameterization process. Likewise, for the Hoerl Curve or Wright models, development age or operational time is embedded in the model so the development age component should reflect this partial first development period and no further adjustment is required when fitting the model.

Figure 4.1. Triangle Shape for Partial First Development Period



[9] The terms *homoecthesious* and *heteroecthesious* are a combination of the Greek *homos* (or Όμός) meaning the same or *hetero* (or έτερο) meaning different and the Greek *ekthesē* (or έκθεση) meaning exposure. They were introduced in Shapland [16].

After simulation, an additional adjustment for this type of heteroecthesious data is applied in the projection of future incremental values. In a deterministic analysis, the most recent accident year needs to be adjusted to remove exposures beyond the evaluation date. For example, continuing the previous example, the development periods at 18 months and later are all for an entire year of exposure, whereas the six-month column is only for six months of exposure. Thus, the 18-month incremental values will effectively extrapolate the first six months of exposure in the latest accident year to a full accident year's exposure. Accordingly, it is common practice to reduce the projected future payments by half to remove the exposure from June 30 to December 31.[10]

The simulation process for Hayne MLE models can be adjusted similarly to the way a deterministic analysis would be adjusted. After simulated parameters are used to project the future incremental values, the last accident year's values can be reduced (in the previous example by 50%) to remove the future exposure and then process variance can be simulated as before. Alternatively, the future incremental values can be reduced after the process variance step. For example, Table 4.1 can be compared to Table 3.27 to see the reduction in the future exposures for the last accident year.

Table 4.1. Total Values Adjusted to Remove Future Exposures

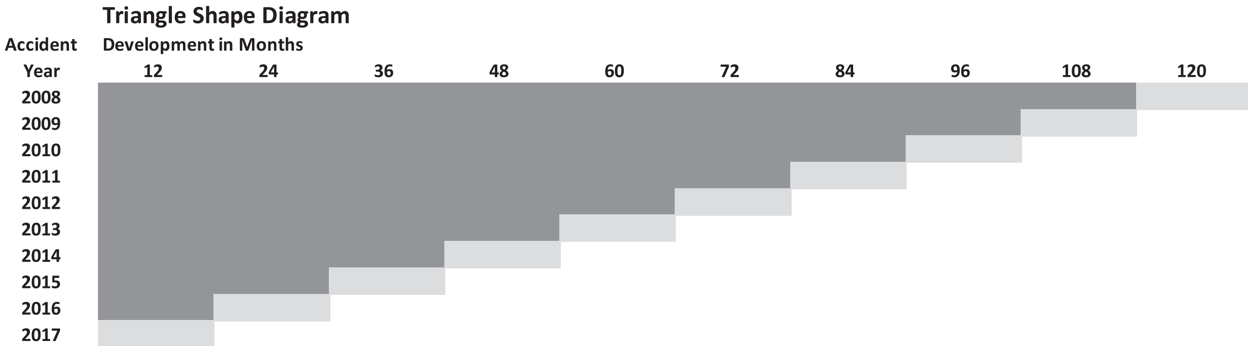
Adjust Total Incremental Value to Remove Future Exposures (Paid) (in 000's)											
Year	6	18	30	42	54	66	78	90	102	114	Acc Yr Unpaid
2008	26,135	27,766	33,070	20,866	14,079	5,091	3,118	278	462	124	0
2009	25,946	32,289	27,996	20,236	15,275	6,859	3,109	1,983	1,578	755	755
2010	29,878	38,029	31,546	33,215	17,183	3,826	2,721	2,270	2,002	1,344	3,346
2011	41,910	31,505	35,368	25,894	18,990	8,972	5,116	1,652	729	233	2,614
2012	38,763	35,888	27,879	32,994	19,792	7,948	3,708	1,746	2,072	118	7,643
2013	30,978	34,000	16,245	28,350	16,377	5,025	2,856	659	1,160	(62)	9,639
2014	43,110	34,979	35,791	37,301	23,682	5,787	8,613	1,801	2,030	1,146	43,059
2015	41,006	35,226	54,590	32,460	19,244	10,120	1,848	2,503	952	977	68,105
2016	42,960	50,106	40,178	38,617	23,103	12,717	4,510	3,200	2,387	1,197	125,908
2017	42,712	*27,304*	*28,461*	*21,653*	*20,596*	*5,117*	*3,427*	*766*	*1,251*	*329*	108,904
											369,975

[10] Reduction by half is actually an approximation since it would also make sense to account for the differences in development between the first and second half years.

4.7.2. Partial Last Calendar Period Data

For a partial last calendar period data, most of the data in the triangle has annual exposures and annual development periods, except for the last diagonal that, continuing the example, only has a six-month development period as illustrated in Figure 4.2. A simple approach is to adjust the raw data incremental values along the diagonal to a full development period to make them consistent with the rest of the triangle. The parameterization process can then be done with the adjusted incremental values.

Figure 4.2. Triangle Shape for Partial Last Calendar Period



During the Hayne MLE simulation process, incremental means and standard deviations can be calculated from the fully annualized sample parameters and used to simulate incremental values. Then, the last diagonal from the sample triangle can be adjusted to de-annualize the incremental values in the last diagonal—i.e., reversing the annualization of the original last diagonal—as illustrated in Table 4.2. Note that

Table 4.2. Total Values Adjusted to De-annualize Incremental Values

Adjust Total Incremental Value for Exposures (Paid) (in 000's)												
Year	12	24	36	48	60	72	84	96	108	120	132	Future Totals
2008	26,135	27,766	33,070	20,866	14,079	5,091	3,118	278	462	*62*	*62*	62
2009	25,946	32,289	27,996	20,236	15,275	6,859	3,109	1,983	*789*	*1,167*	*378*	1,544
2010	29,878	38,029	31,546	33,215	17,183	3,826	2,721	*1,135*	*2,136*	*1,673*	*672*	4,481
2011	41,910	31,505	35,368	25,894	18,990	8,972	*2,558*	*3,384*	*1,191*	*481*	*116*	5,172
2012	38,763	35,888	27,879	32,994	19,792	*3,974*	*5,828*	*2,727*	*1,909*	*1,095*	*59*	11,618
2013	30,978	34,000	16,245	28,350	*8,188*	*10,701*	*3,941*	*1,758*	*910*	*549*	*(31)*	17,827
2014	43,110	34,979	35,791	*18,650*	*30,491*	*14,734*	*7,200*	*5,207*	*1,915*	*1,588*	*573*	61,710
2015	41,006	35,226	*27,295*	*43,525*	*25,852*	*14,682*	*5,984*	*2,176*	*1,728*	*965*	*489*	95,401
2016	42,960	*25,053*	*45,142*	*39,397*	*30,860*	*17,910*	*8,613*	*3,855*	*2,793*	*1,792*	*599*	150,961
2017	*10,678*	*59,338*	*55,765*	*50,114*	*42,248*	*25,713*	*8,544*	*4,193*	*2,017*	*1,580*	*329*	249,842
												598,618

Table 4.3. Total Values Adjusted to Remove Future Exposures

Adjust Total Incremental Value to Remove Future Exposures (Paid) (in 000's)												
Year	12	24	36	48	60	72	84	96	108	120	132	Acc Yr Unpaid
2008	26,135	27,766	33,070	20,866	14,079	5,091	3,118	278	462	62	62	62
2009	25,946	32,289	27,996	20,236	15,275	6,859	3,109	1,983	789	1,167	378	1,544
2010	29,878	38,029	31,546	33,215	17,183	3,826	2,721	1,135	2,136	1,673	672	4,481
2011	41,910	31,505	35,368	25,894	18,990	8,972	2,558	3,384	1,191	481	116	5,172
2012	38,763	35,888	27,879	32,994	19,792	3,974	5,828	2,727	1,909	l,095	59	11,618
2013	30,978	34,000	16,245	28,350	8,188	10,701	3,941	1,758	910	549	(31)	17,827
2014	43,110	34,979	35,791	18,650	30,491	14,734	7,200	5,207	1,915	1,588	573	61,710
2015	41,006	35,226	27,295	43,525	25,852	14,682	5,984	2,176	1,728	965	489	95,401
2016	42,960	25,053	45,142	39,397	30,860	17,910	8,613	3,855	2,793	1,792	599	150,961
2017	10,678	*29,669*	*27,883*	*25,057*	*21,124*	*12,856*	*4,272*	*2,097*	*1,009*	*790*	*165*	124,921
												473,697

since the model parameters are annual, the "de-annualization" process includes a partial shifting of the future incremental values to the next future period. Finally, the future incremental values for the last accident year must be reduced (in the previous example by 50%) to remove the future exposure, as illustrated in Table 4.3.[11]

4.8. Parameter Adjustments

The Hayne MLE framework will find the optimal parameters for the specified model. Like all models, this also means that there will be times that the noise in the data will lead to "distortions" in the parameters. This is akin to the need to select age-to-age factors to smooth the development pattern. The ability to judgmentally adjust some of the parameters is also possible with the Hayne MLE models. For example, consider the plot of the decay ratios for the Berquist-Sherman model in Figure 4.3.

In Figure 4.3, notice the "outlier" for the 120 month development period. This is an indication that the fitted or modeled parameter for 108 months may be lower than would have been expected. Reviewing the development year parameters, the choice for the modeler boils down to deciding whether to accept the parameters as reasonable or adjusting them to smooth out some of the noise in the data. For this Berquist-Sherman model example, the manual adjustment in Table 4.4 can be compared to the parameters in Table 3.1.[12]

[11] These heteroecthesious data issues can be addressed in the "Hayne MLE Models.xlsm" file by using the Exposure Factors sheet. While not included in the Excel file, it should be noted that these adjustments can add more uncertainty and the estimates can be highly sensitive to the adjustments. To address this, extra uncertainty could be added to the adjustment factors in the simulation process.

[12] Similar manual adjustments for each of the models are illustrated in Appendix A.

Figure 4.3. Decay Ratios for Berquist-Sherman Model

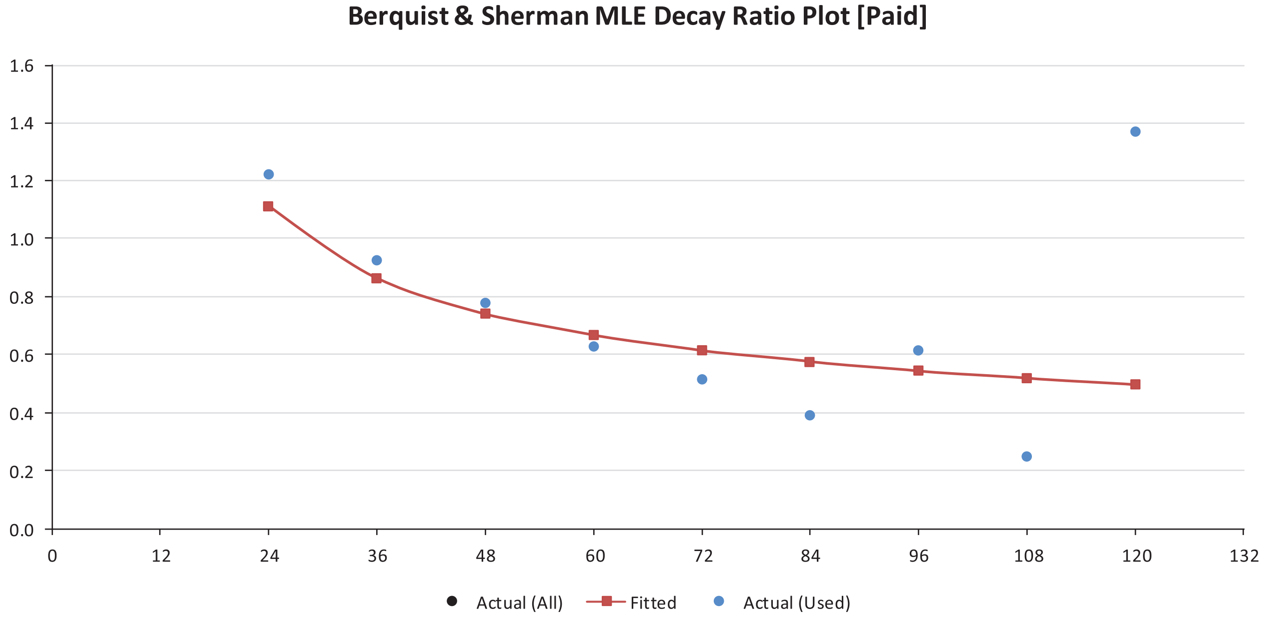


Table 4.4. User Selected Parameters for Berquist-Sherman

	User Selected Parameters:									
	12	24	36	48	60	72	84	96	108	120
Mean	620.96	760.67	708.16	553.57	350.00	181.39	70.97	43.88	26.00	15.21
Std Dev	40.50	46.55	43.00	35.49	26.17	17.66	10.40	8.75	7.60	7.36
Decay Ratios:		122.5%	93.1%	78.2%	63.2%	51.8%	39.1%	61.8%	59.3%	58.5%
CoV:	*6.5%*	*6.1%*	*6.1%*	*6.4%*	*7.5%*	*9.7%*	*14.7%*	*19.9%*	*29.2%*	*48.4%*

	Accident Year				
	Trend	K	p	AIC	BIC
Mean	0.045	11.216	0.654	647.9	674.0
Std Dev	0.009	1.094	0.089		
CoV:	*18.9%*	*9.3%*	*13.6%*		

To adjust the mean for 108 months, the decay ratios were reviewed and the original mean of 11.08 was seen to be low compared to the surrounding parameters due to the low decay ratio for 108 months and high decay ratio for 120 months. The parameter of 26.00 was selected by smoothing the decay ratios for the last three development periods.[13] Notice that only the mean parameters need to be adjusted since the MLE framework allows the variance-covariance parameters to be recalculated based on the

[13] Compared to the original parameter of 11.08, the parameter of 26.00 is more than 3 standard deviations larger, which is quite large, but it would be consistent with a strong a priori belief that the decay should be smooth.

selected parameter. In essence, we are assuming the expected incremental losses are derived from selected parameters, or the "true" parameters for the data.

Rather than judgmentally selecting new parameters, another option would be to change the parametric setup of the model such that a group of parameters (e.g., the last 3 development periods for the Berquist-Sherman model) are not independent. The Excel companion files do not include this type of adjustment to the model setup.

As you balance the competing goals of goodness of fit and reasonability of assumption, the diagnostics will give an indication of the significance of the changes to the model parameters. Finally, while user-selected parameters will tend to move the statistics away from optimal, the goal is to reasonably replicate the statistical features of the data and other adjustments, like the residual adjustment discussed in section 4.2, can also be made if the impact on the residuals is significant.

4.9. Tail Extrapolation

One of the most common data issues is that claim development is not complete within the loss triangle and tail factors are commonly used to extrapolate beyond the end of the data triangle. There are many common methods for calculating tail factors and a useful reference in this regard is the CAS Tail Factor Working Party Report [5]. However, for the Hayne MLE models a different approach is required in order to extrapolate the parameters so that a multi-variant normal distribution can continue to be used. Once extrapolation is used to extend the parameters, incremental values can all be extended to include development periods beyond the end of the triangle—i.e., the tail periods.

For the first family of models (i.e., Berquist-Sherman, Cape Cod, and chain ladder) the decay ratios shown in Tables 3.1, 3.5, and 3.9 can be used as a way of extrapolating the development parameters for each model similarly to how a tail factor might be calculated for a deterministic method. In the "Hayne MLE Models.xlsm" file, five different regression models (i.e., average, linear, logarithmic, power, and polynomial) can be used to extrapolate decay ratios for up to 5 years from either the modeled or user selected parameters.[14] For example, Table 4.5 illustrates the extrapolation for the Berquist-Sherman model, which is based on the user-selected parameters in Table 4.4, so the graph in Table 4.5 can be compared to Figure 4.3.

From these regression models, the implied tail decay mean is the fitted decay ratio from the regression and the decay standard deviation is the average deviation for the actual decay ratios from the regression curve. The length of the tail period can then be determined by reviewing the means of the incremental periods beyond the triangle and then including enough periods such that the means in the final development column are close to zero. Using the decay ratio statistics and selected number of periods in the tail, the Hayne MLE framework will also extend the variance-covariance

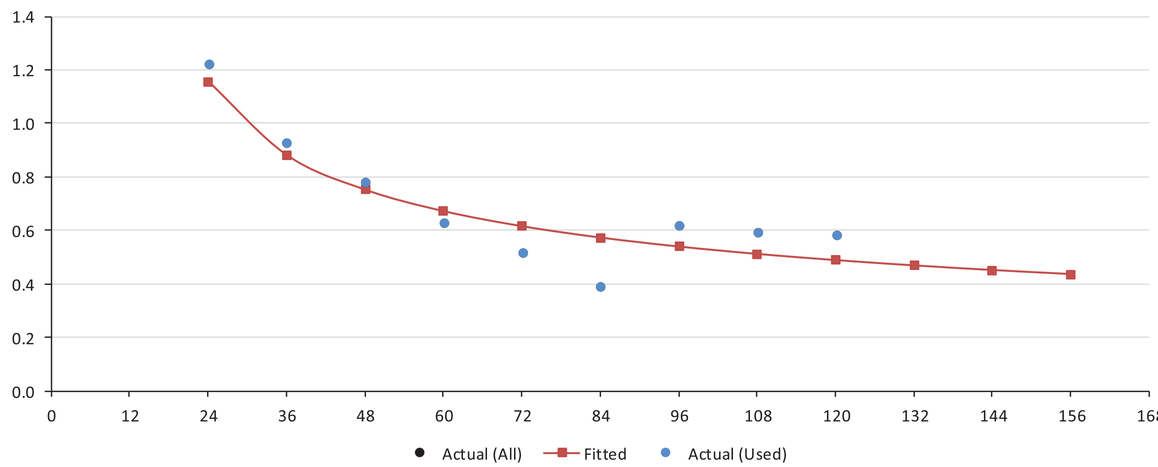
[14] The limit of 5 years in the "Hayne MLE Models.xlsm" file is only based on a practical need to limit the size of the file.

Table 4.5. Berquist-Sherman Model Tail Extrapolation

Decay Ratio Analysis:					
Parameters: User	Curve Type: Power	3	Least Squares Regression Coefficients:		Goodness of Fit Statistics:
			x^a	–0.3916	R^2 Statistic 0.710
			coefficient	1.1559	Regression Deviation 13.5%
					Suggested Decay Parameters:
					Mean 45.3%
					Standard Deviation 13.7%

Periods	Decay Ratio	Outliers	Selected Age	Decay Ratio	Fitted Factors
12–24	1.225	0	1	1.225	1.156
24–36	0.931	0	2	0.931	0.881
36–48	0.782	0	3	0.782	0.752
48–60	0.632	0	4	0.632	0.672
60–72	0.518	0	5	0.518	0.615
72–84	0.391	0	6	0.391	0.573
84–96	0.618	0	7	0.618	0.539
96–108	0.593	0	8	0.593	0.512
108–120	0.585	0	9	0.585	0.489
120–132					0.469
132–144					0.452
144–156					0.437
156–168					0.423
168–180					0.411

Berquist & Sherman MLE Decay Ratio Plot [Paid]



matrix to include the tail periods.[15] Continuing the Berquist-Sherman example, the extended parameters for 3 years are illustrated in Table 4.6, which can be compared to Table 4.4.[16]

Table 4.6. Extended Parameters for Berquist-Sherman Model

	User Selected Parameters:												
	12	24	36	48	60	72	84	96	108	120	132	144	156
Mean	620.96	760.67	708.16	553.57	350.00	181.39	70.97	43.88	26.00	15.21	*6.89*	*3.12*	*1.41*
Std Dev	40.50	46.55	43.00	35.49	26.17	17.66	10.40	8.75	7.60	7.36	*4.05*	*2.13*	*1.09*
Decay Ratios:		122.5%	93.1%	78.2%	63.2%	51.8%	39.1%	61.8%	59.3%	58.5%			
CoV:	*6.5%*	*6.1%*	*6.1%*	*6.4%*	*7.5%*	*9.7%*	*14.7%*	*19.9%*	*29.2%*	*48.4%*	*58.8%*	*68.4%*	*77.6%*

	Accident Year					Tail Extrapolation			Implied Tail Factor		
	Trend	K	p	AIC	BIC	Decay Ratio	Periods	Distribution	Adjusted	Actual	Tail Sampling Option
Mean	0.045	11.216	0.654	647.9	674.0	45.3%	3	Gamma	1.0034	1.0034	Conditional Variance
Std Dev	0.009	1.094	0.089			13.7%					
CoV:	*18.9%*	*9.3%*	*13.6%*								

One of the interesting features of this extrapolation process is that coefficients of variation in the tail parameters are increasing, which is a statistical feature you would expect to find. The implied tail factor is also shown in the table in order to better compare with other models and traditional methods.[17] Finally, two different "Tail Sampling Options" are included for use in the simulation process. For the "Conditional Variance" option, the parameters in the tail are sampled using the multivariate normal along with all the other parameters. For the "Sampling" option, a decay ratio is sampled using the mean and standard deviation from the regression and the selected distribution (i.e., Gamma, Normal, or Lognormal can be selected).

For the second family of models (i.e., Hoerl Curve and Wright), there are no parameters tied specifically to development age, so it is a simple matter to extend the "development" ages. The length of the tail period can be determined by reviewing the means of the incremental periods beyond the triangle and then including enough periods such that the means in the final development column are close to zero.

A key ingredient for all of these considerations is to verify that the simulations in the tail are reasonable. For example, the tail period represents the extension of development parameters and using just a single period may not produce appropriate incremental results.

15 The extension of the variance-covariance matrix is shown below the simulated values in the "Hayne MLE" sheets in the "Hayne MLE Models.xlsm" file.

16 The modeled parameters are also extended in the companion file, but they are not illustrated in the monograph.

17 The "adjusted" tail factor would be for annualized data if there were exposure issues as discussed in Section 4.7, whereas the "actual" tail factor would be for the data as is.

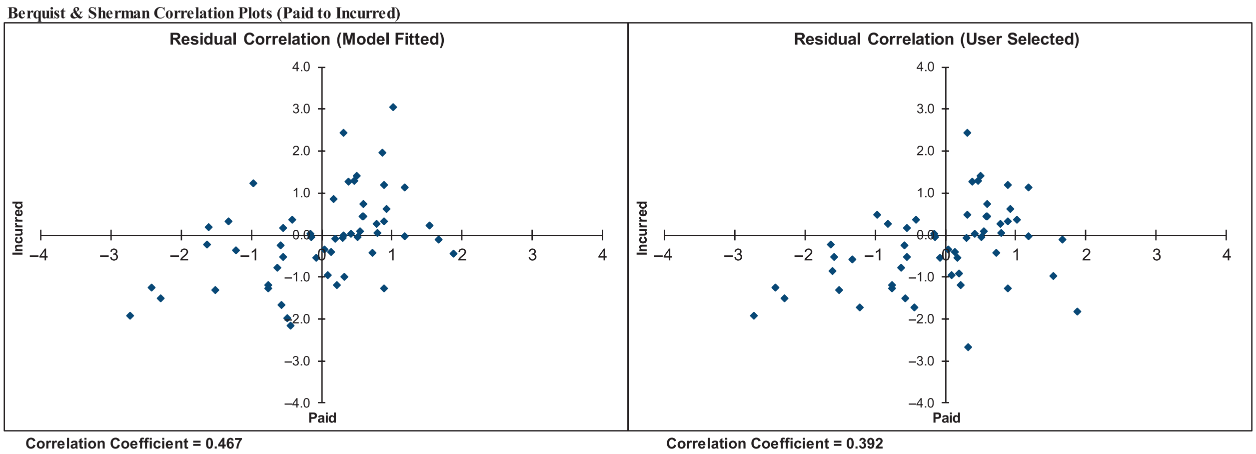
4.10. Incurred Data

The Hayne MLE models can be used to model both paid and incurred loss data. Using incurred data incorporates case reserves, thus perhaps improving the ultimate estimates. However, the resulting distribution from using incurred data will be possible outcomes of the IBNR, not a distribution of the unpaid. There are two possible approaches for modeling an unpaid loss distribution using incurred loss data: modeling incurred data and converting the ultimate values to a payment pattern, or modeling paid and case reserves separately.

Using the first approach, a convenient way of converting the results of an incurred data model to a payment stream is to run the paid data model in parallel with the incurred data model and use the random payment pattern from each iteration from the paid data model to convert the ultimate values from each corresponding iteration from the incurred data to a payment pattern for each iteration (for each accident year individually). The "Hayne MLE Models.xlsm" file illustrates this concept. It is worth noting, however, that this process allows the "added value" of using the case reserves to help predict the ultimate results to work its way into the calculations, thus perhaps improving the ultimate estimates, while still focusing on the payment stream for measuring risk. In effect, it allows a distribution of IBNR to become a distribution of IBNR and case reserves.

This process can also be made more sophisticated by correlating the multi-variate normal simulation of the paid and incurred models (e.g., the model parameters and/or process variance). In order to specify a correlation coefficient between the paid and incurred models, the correlation of the standardized residuals can be measured as, for example, in Figure 4.4 for the Berquist-Sherman model.

Figure 4.4. Correlation of Paid and Incurred Standardized Residuals



From Figure 4.4 observe that there is a positive correlation between the paid and incurred standardized residuals for the Berquist-Sherman model. This is not surprising, as incurred data includes paid data, but using this to correlate the paid and incurred simulations is a way of including this statistical feature of the data in the model. In the

"Hayne MLE Models.xlsm" file the correlation assumption is specified in the Inputs sheet and it will only be used to correlate the process variance portion of the paid and incurred data models.

The second approach could be accomplished by applying the Hayne MLE models to the case reserve triangle and then "combining" the case reserve and paid claim simulations. This has the advantage over the first approach of not modeling the paid losses twice, but it would also require specifying the correlation of the paid and outstanding losses. For both approaches it may be possible to extend the Hayne MLE models to include both paid and incurred data in a combined modeling framework, but that is beyond the scope of this monograph.

4.11. Claim Count Data

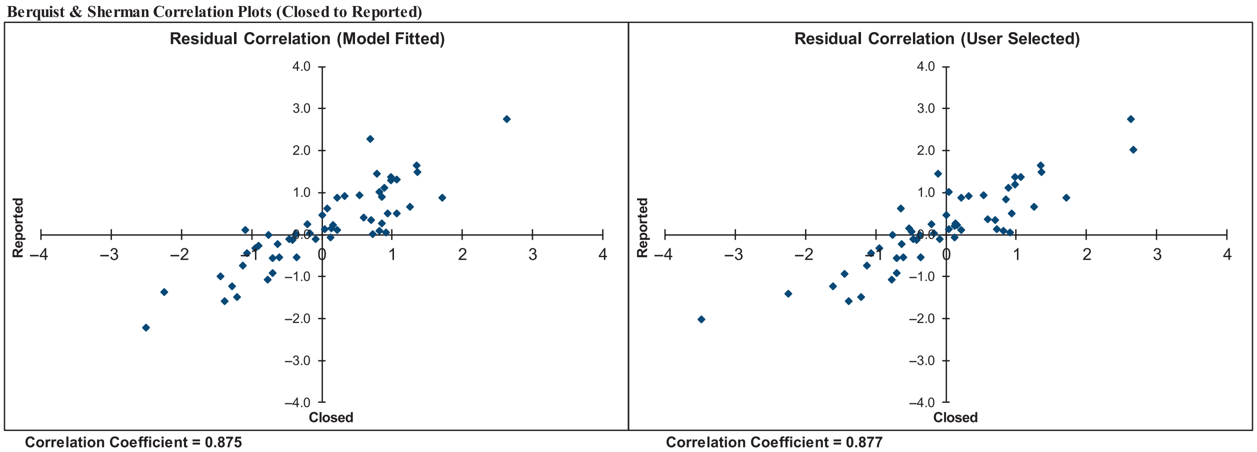
For a sufficient volume of claims, the count distribution can be approximated with a normal distribution and it follows that the Hayne MLE models can also be used to model claim count data. Indeed, as the models are typically based on average claim severity, this assumes that the ultimate claim count has been estimated. As a first step to estimating unpaid amounts, the Hayne MLE models can all be used to estimate unclosed and ultimate claim counts using either closed or reported claim count triangles.[18] Similarly to loss amount models, models based on reported counts would result in a distribution of possible outcomes of the IBNR claim count, not a distribution of the unclosed count. The two possible approaches for modeling an unpaid loss distribution using incurred loss data also apply to modeling an unclosed count distribution using reported count data: modeling reported count data and converting the ultimate values to a closed pattern, or, modeling closed and open counts separately.

Using the first approach, the "Hayne MLE Models.xlsm" file illustrates a convenient way of converting the results of a reported count data model to a closed stream by running the closed count data model in parallel with the reported count data model and using the random closed pattern from each iteration from the closed count data model to convert the ultimate values from each corresponding iteration from the reported data to a closed pattern for each iteration (for each accident year individually). It is worth noting that this process allows the "added value" of using the open count to help predict the ultimate results to work its way into the calculations, thus perhaps improving the ultimate count estimates, while still focusing on the closed stream for measuring risk and combining model results. In effect, it allows a distribution of IBNR counts to become a distribution of IBNR and open counts.

This process can also be made more sophisticated by correlating the multi-variate normal simulation of the closed and reported models (e.g., the model parameters and/or process variance). In order to specify a correlation coefficient between the closed and reported models, the correlation of the standardized residuals can be measured as, for example, in Figure 4.5 for the Berquist-Sherman model.

[18] In the "Hayne MLE Models.xlsm" file, claim count data and model selections can be made on the Inputs sheet. All of the sheets with "(Cnt)" in the sheet name show model details for the claim count models.

Figure 4.5. Correlation of Closed and Reported Standardized Residuals



From Figure 4.5 observe that there is a strong positive correlation between the closed and reported standardized residuals for the Berquist-Sherman model. This is not surprising, as reported count data includes closed count data, but using this to correlate the closed and reported simulations is a way of including this statistical feature of the data in the model. In the "Hayne MLE Models.xlsm" file, the correlation assumption is specified in the Inputs sheet and it will only be used to correlate the process variance portion of the closed and reported data models.

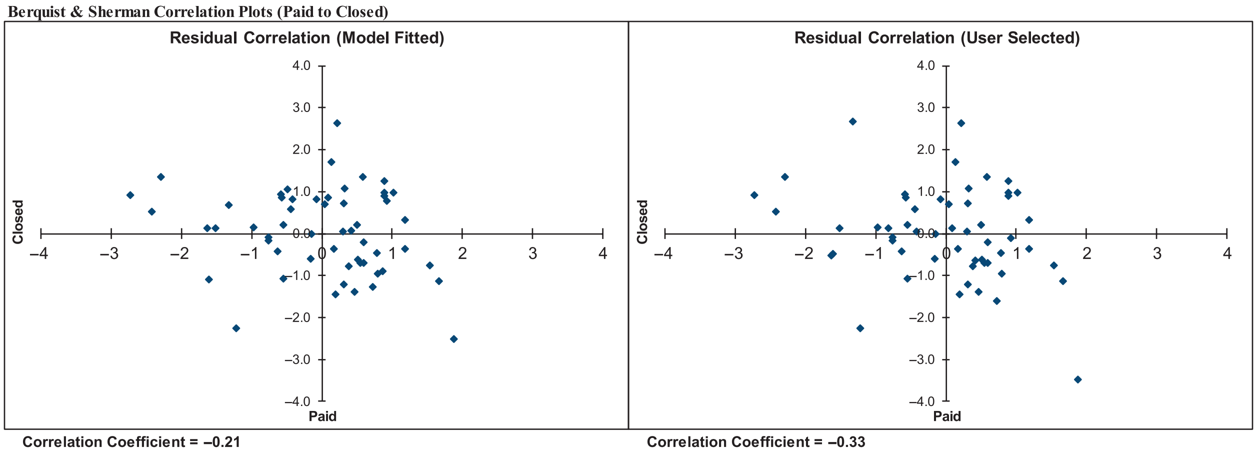
The second approach could be accomplished by applying the Hayne MLE models to the open count triangle and then "combining" the open count and closed count simulations. This has the advantage over the first approach of not modeling the closed counts twice, but it would also require specifying the correlation of the closed and open counts. For both approaches it may be possible to extend the Hayne MLE models to include both closed and reported data in a combined modeling framework, but that is beyond the scope of this monograph.

Finally, as noted at the beginning of this section, the normality assumption relies on a sufficient volume of claims and, without this, the Hayne MLE models may not fit the statistical features of the data. From a practical standpoint, it stands to reason that count distributions may be less likely to be diagnostically normal, thus requiring extra care, especially for an open count model. It also follows that if the claim count distribution is not normal, then it also has implications for the quality of the average severity models.

4.12. Frequency and Severity Modeling

In addition to modeling only the claim counts, including both the count and value data allows the modeler to use all of the data in the simulations. This process can also be made more sophisticated by correlating the multi-variate normal simulation of the paid claims and closed count models (e.g., the model parameters and/or process variance). In order to specify a correlation coefficient between the paid and closed models, the correlation of the standardized residuals can be measured as, for example, in Figure 4.6 for the Berquist-Sherman model.

Figure 4.6. Correlation of Paid and Closed Standardized Residuals



From Figure 4.6 observe that there is a negative correlation between the paid and closed standardized residuals for the Berquist-Sherman model.[19] This is not surprising, as average paid would tend to be lower than average when the closed count is higher than average, but using this to correlate the paid and closed simulations is a way of including this statistical feature of the data in the model. In the "Hayne MLE Models.xlsm" file the correlation assumption is specified in the Inputs sheet and it will only be used to correlate the process variance portion of the paid and closed data models when they are run jointly.

In theory it may be possible to extend the Hayne MLE models to include all of the paid, incurred, closed, and reported data in a combined modeling framework, but that is beyond the scope of this monograph. While it seems this theory would need to combine all of the data into a framework for one model at a time, without a single cohesive model it is possible to mix and match models between the value data (i.e., paid and incurred) and the count data (i.e., closed and reported). This allows for a larger number of combinations of models and allows the user to combine the best model for each data type.

[19] In practice, a more sophisticated approach could be used, for example, to see if there is more correlation in the later development periods.

5. Diagnostics

The quality of any model depends on the quality of the underlying assumptions. When a model fails to "fit" the data, it is unlikely to produce a good estimate of the distribution of possible outcomes.[20] However, a balance must be considered between parsimony of parameters and the goodness-of-fit. Over-parameterization may cause the model to be less predictive of future losses. On the other hand, no model will perfectly "fit" the data, so the best you can hope for with any model is that it reasonably represents the data and your understanding of the processes that impact the data. Therefore, diagnostically evaluating the assumptions underlying a model is important for evaluating whether it will produce reasonable results or not and whether it should stay in your selected group of reasonable models.

The CAS Working Party [4], in the third section of their report on quantifying variability in reserve estimates, identified 20 criteria or diagnostic tools for gauging the quality of a stochastic model. The Working Party also noted that, in trying to determine the optimal fit of a model, or indeed an optimal model, no single diagnostic tool or group of tools can be considered definitive. Depending on the statistical features found in the data, a variety of diagnostic tools are necessary to best judge the quality of the model assumptions and to adjust the parameters of the model. This monograph will discuss some of these tools in detail as they relate to the Hayne MLE models.

The key diagnostic tests are designed for three purposes: to test various assumptions in the model, to gauge the quality of the model fit to the data, and to help guide the adjustment of model parameters, if needed. Some tests are relative in nature, enabling results from one set of model parameters to be compared to those of another, for a specific model, allowing a modeler to improve the fit of the model. For the most part, however, the tests can't be used to compare different models. The objective, consistent with the goals of a deterministic analysis, is ***not*** to find the one best model, but rather a set of reasonable models.

Some diagnostic measures include statistical tests, providing a pass/fail determination for some aspects of the model assumptions. This can be useful even though a "fail" does not necessarily invalidate an entire model; it only points to areas where improvements can be made to the model or its parameterization. The goal is to find the

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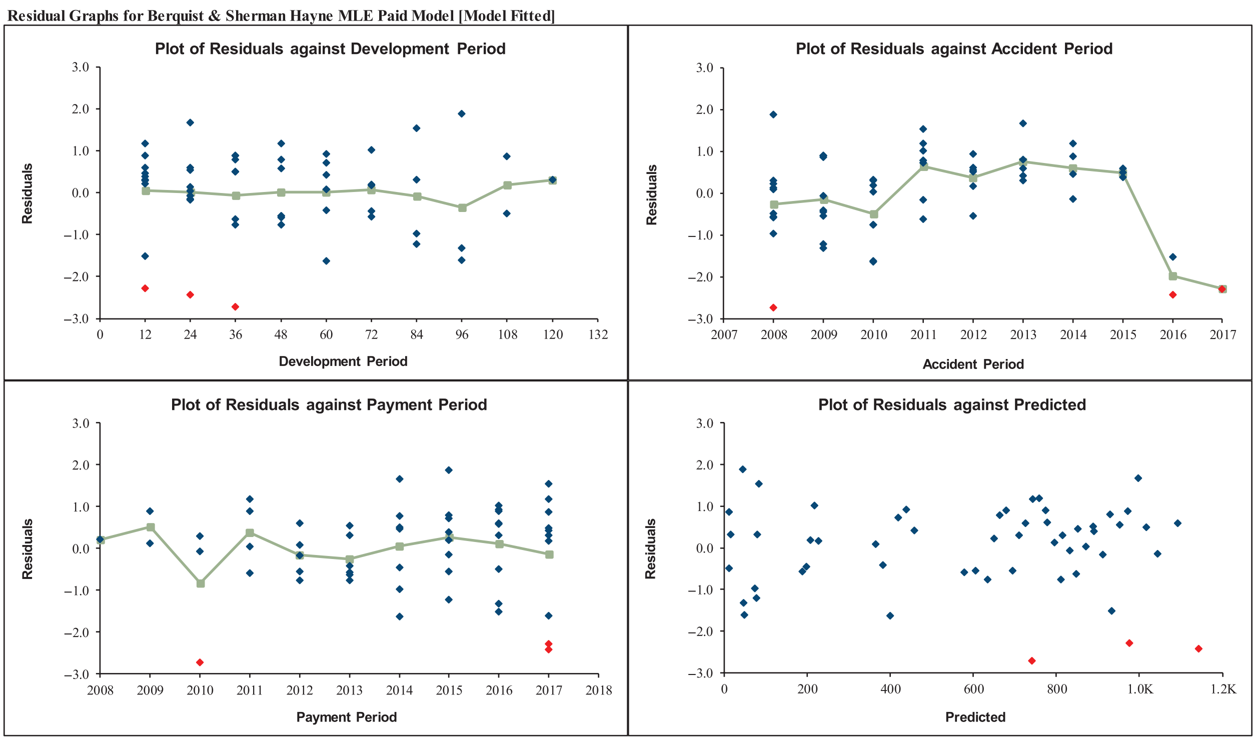
[20] While the examples are different, significant portions of sections 5 and 6 are based on IAA [10] and Milliman [13]. Many of the diagnostic graphs from Hayne [8] have also been reproduced in this monograph.

sets of models and parameters that will yield the most realistic, most consistent simulations, based on statistical features found in the data.[21]

5.1. Residual Graphs

As noted earlier, the Hayne MLE models rely on the normal distribution assumption for incremental values and the standardized residuals are independent and identically distributed about the standard normal distribution conditional on parameters. Graphing residuals is a good way to check this. Consider the residual graphs for the Berquist-Sherman model in Figure 5.1 for the modeled parameters.

Figure 5.1. Berquist-Sherman Residual Graphs [modeled parameters]



For each model, going clockwise, and starting from the lower-left-hand corner, the graphs in Figure 5.1 show the residuals (blue and red dots[22]) by calendar period, development period, and accident period and against the fitted incremental value (in the lower-right-hand corner). In addition, the graphs include a trend line (in green) that highlights the averages for each period.

Most residuals from the Berquist-Sherman model appear reasonably random and the averages do not deviate significantly from zero by development periods and

[21] Using the data from Hayne [8], diagnostic graphs and tests for all five of the Hayne MLE models are included in Appendix A.

[22] In the graphs that follow, the red dots are outliers as identified in Figure 5.3.

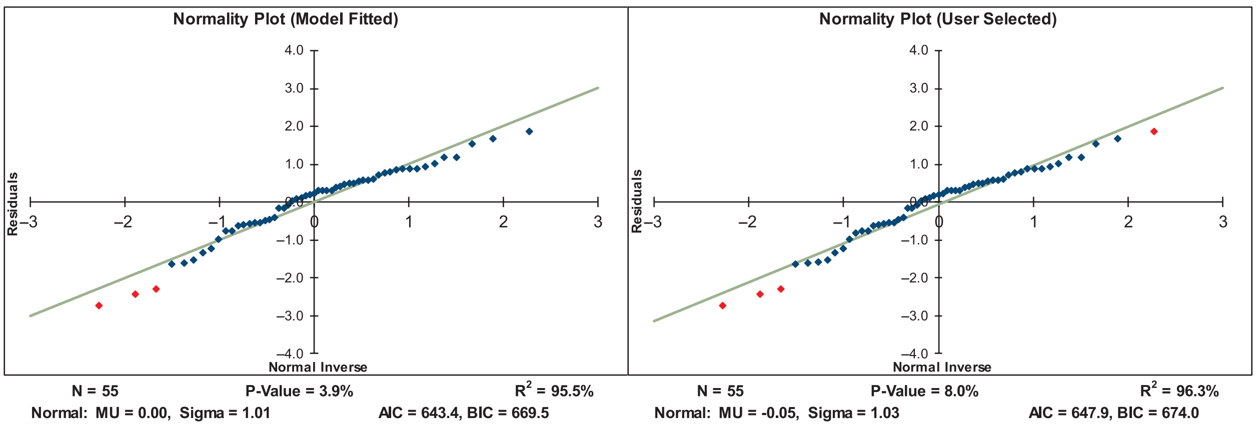
payment periods. The averages by development period are not surprising, since there is a parameter for each development period, but the lack of a trend by payment year is more useful since without a calendar year trend parameter this would be problematic for the Berquist-Sherman model. The averages by accident period appear significantly different from zero, which may indicate that a single trend component is not enough to model the level of incremental values by exposure periods.

Also of interest are the three large negative residuals in early development periods that are indicated in red as outliers. This could indicate the need to adjust those development period parameters, although adjustments to remove outliers is typically a last resort compared to other options.

5.2. Normality Test

To see whether the standardized residuals are normally distributed, tests comparing the residuals against a normal distribution are useful. This also enables a comparison of the modeled parameters to the user-selected parameter sets and gauging the skewness of the residuals in order to further validate the suitability of the chosen model. For example, Figure 5.2 shows the normality tests for the Berquist-Sherman model comparing the modeled and user selected parameters.

Figure 5.2. Normality Plots for Berquist-Sherman



The residual plots appear close to normally distributed, with the data points tightly distributed around the diagonal line. While there is an additional outlier for the user-selected parameters, the *p*-value, a statistical pass-fail test for normality, improved from 3.9% to 8.0%, and the R^2 improved from 95.5% to 96.3%. The *p*-value is generally considered a "passing" score of the normality test when it is greater than 5.0%.[23] The graphs in Figure 5.2 also show N (the number of data points).

[23] Note that this doesn't indicate whether the Hayne MLE model itself passes or fails, it only tests whether the residuals can be judged to be normally distributed.

While the p-value and R^2 tests assess the goodness of fit of the model to the data, they do not penalize for added parameters. Adding more parameters will almost always improve the fit of the model to the data, but the goal is to have a good fit with as few parameters as possible. Two other tests, the Akaike information criteria (AIC) and the Bayesian information criteria (BIC), address this limitation, using the difference between each residual and its normal counterpart from the normality plot to calculate the residual sum squared (RSS) and include a penalty for additional parameters, as shown in (5.1) and (5.2), respectively.[24]

$$AIC = 2 \times p + n \times \left[\ln\left(\frac{2 \times \pi \times RSS}{n} \right) + 1 \right] \tag{5.1}$$

$$BIC = n \times \ln\left(\frac{RSS}{n} \right) + p \times \ln(n) \tag{5.2}$$

A smaller value for the AIC and BIC tests indicate an improvement, especially with respect to overcoming the penalty of adding a parameter. For the Berquist-Sherman model test in Figure 5.2, there were no parameters added but the values increased a little, which is expected since the user selected parameters are not the optimal parameters. It is important to remember that the AIC and BIC tests tend to be model specific in the sense that they are less well suited for comparing different models and better suited for different parameterizations of the same model.[25]

5.3. Outliers

Identifying outliers in the data provides another useful test in determining model fit. Outliers can be represented graphically in a box-whisker plot, which shows the inter-quartile range (the 25th to 75th percentiles) and the median (50th percentile) of the residuals—the so-called box. The whiskers then extend to the largest values within three times this inter-quartile range.[26] Values beyond the whiskers may generally be considered outliers and are identified individually with a point. For example, the box-whisker plots in Figure 5.3 compare the modeled and user selected parameters for the Berquist-Sherman model.

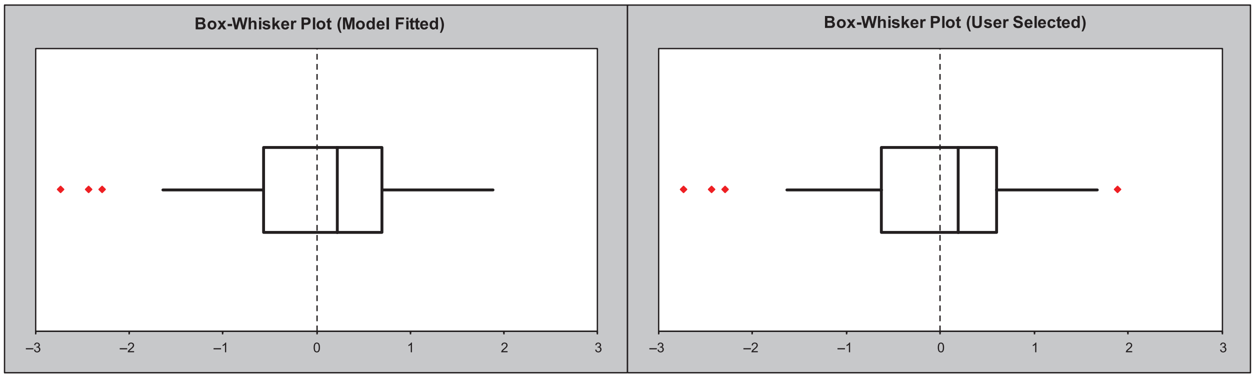
If the data in those outlier cells genuinely represent events that cannot be expected to happen again, the outlier(s) may be removed from the model (by giving it/them zero

[24] There are different versions of the AIC and BIC formula from various authors and sources, but the general idea of each version is consistent. Other similar formulas could also be used.

[25] To be clear, using diagnostic tests to compare different data sets is generally not a good idea. For the same data set and different models, using AIC and BIC (and other diagnostics) can help with model comparison, but even for similar diagnostic results the models could still produce wildly different estimates.

[26] Various authors and textbooks use widths for the whiskers that tend to span from 1.5 to 3 times the inter-quartile range. Changing the multiplier will therefore make the box-whisker plot more or less sensitive to outliers. It is also possible to illustrate "mild" outliers with a multiplier of 1.5 and the more "extreme" outliers with a multiplier of 3 using different colors and/or symbols in the graphs. Of course, the actual multipliers can be adjusted based on personal preference.

Figure 5.3. Berquist-Sherman Box-Whisker Plots



Interquartile Range = [–0.56, 0.70] Median = 0.22
Outliers = 3

Interquartile Range = [–0.62, 0.60] Median = 0.19
Outliers = 4

weight). But extreme caution should be taken even when the removal of outliers seems warranted. The possibility always remains that apparent outliers may actually represent realistic extreme values, which, of course, are critically important to include as part of any sound analysis.

Additionally, when residuals are not normally distributed, a significant number of outliers tend to result—i.e., the distributional shape of the residuals may be skewed or otherwise not normal.[27] In this case, it is impossible for the Hayne MLE simulation to capture this shape, as it relies on the normality assumption, although adjusting the parameters may help "restore" normality. Finally, a significant number of residuals can also mean the underlying model is not a good fit to the data, so other models should be used or this model given less weight (see Section 6).

While the three diagnostic tests shown above demonstrate techniques commonly used with most types of models, they are not the only tests available.[28] Next, we'll take a look at the flexibility of the Hayne MLE framework and some of the diagnostic elements of the simulation results. For a more extensive list of other tests available, see the report "CAS Working Party on Quantifying Variability in Reserve Estimates" [4].

5.4. Model Results

Once the parameter diagnostics have been reviewed, simulations should be run for each model.[29] These simulation results provide an additional diagnostic tool to aid in evaluation of the model, as described in section 3 of CAS Working Party [4]. As an example, the results for the Berquist-Sherman Hayne MLE model will be reviewed. The estimated-unpaid results shown in Table 5.1 were simulated using 10,000 iterations with the parameters from Table 4.6.

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[27] To help assess normality, the interquartile range could be compared to a normally distributed range of ±0.67 standard deviations.

[28] For an example, see Venter [19].

[29] Throughout the monograph, all simulations include both parameter uncertainty and process uncertainty as illustrated in Tables 3.21 through 3.27.

Table 5.1. Estimated Unpaid Results for Berquist-Sherman

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	441	573	129.9%	(1,475)	2,372	391	823	1,420	1,881
2009	140,983	1,083	825	76.2%	(1,675)	4,401	1,048	1,611	2,466	3,113
2010	147,516	2,459	1,168	47.5%	(1,527)	6,082	2,417	3,252	4,462	5,274
2011	174,349	4,793	1,595	33.3%	(172)	11,597	4,758	5,809	7,391	8,954
2012	173,637	8,629	1,992	23.1%	1,588	16,582	8,542	9,810	11,955	13,951
2013	174,996	18,214	3,136	17.2%	7,989	30,302	18,135	20,292	23,509	25,381
2014	169,224	41,402	5,008	12.1%	25,322	59,952	41,302	44,862	49,756	53,216
2015	134,010	75,281	7,480	9.9%	53,427	105,936	74,961	80,194	87,930	93,542
2016	68,911	127,141	11,108	8.7%	93,649	164,080	127,078	134,809	144,791	152,998
2017	35,798	210,599	16,205	7.7%	159,908	275,851	210,505	221,397	236,756	253,297
Totals	1,343,162	490,041	31,334	6.4%	405,127	622,322	488,329	510,471	542,250	566,151

5.4.1. Estimated-Unpaid Results

It's recommended to start a diagnostic review of the estimated unpaid results with the standard error (standard deviation) and coefficient of variation (standard error divided by the mean), shown in Table 5.1. Keep in mind that for books of business with relatively stable volume the standard error should increase when moving from the oldest years to the most recent years, as the standard errors (value scale) should follow the magnitude of the mean of unpaid estimates. In Table 5.1, the standard errors conform to this pattern. At the same time, the standard error for the total of all years should be larger than any individual year.

Also, the coefficients of variation should generally decrease when moving from the oldest year to the more recent years and the coefficient of variation for all years combined should be less than for any individual year.

The main reason for the decrease in the coefficient of variation has to do with the independence in the incremental claim-payment stream. Because the oldest accident year typically has only a few incremental payments remaining, or even just one, the variability is nearly all reflected in the coefficient. For more current accident years, random variations in the future incremental payment stream may tend to offset one another, thereby reducing the variability of the total unpaid loss.[30]

While the coefficients of variation should go down, they could also start to rise again in the most recent years. Such reversals are from a couple of issues:

- With an increasing number of parameters used in a model, the parameter uncertainty tends to increase when moving from the oldest years to the more recent years, particularly for models with accident year parameters, where uncertainty could increase in more recent accident years.

[30] To visualize this reducing coefficient of variation, recall that the standard deviation for the total of several independent variables is equal to the square root of the sum of the squares.

- In the most recent years, parameter uncertainty can grow to overpower process uncertainty, which may cause the coefficient of variation to start rising again. At a minimum, increasing parameter uncertainty will slow the rate of decrease in the coefficient of variation.

The model may be overestimating the uncertainty in recent accident years if the increase is significant. In that case, another model may need to be used. Keep in mind that the standard error or coefficient of variation for the total of all accident years will be less than the sum of the standard errors or coefficients of variation for the individual years. This is because the model assumes that the random process generating the process uncertainty in each accident year is independent.

It is important to note that this diagnostic review of the model output should consider direction and consistency separately from magnitude of the variability. In other words, the standard error patterns are about direction and consistency (i.e., is the pattern consistent with expectations), but the standard error values are about whether the model includes enough uncertainty or not (e.g., does the magnitude indicate enough uncertainty has been incorporated).

Minimum and maximum results are the next diagnostic element in the analysis of the estimated unpaid claims in Table 5.1, representing the smallest and largest values from all iterations of the simulation. These values will need to be reviewed in order to determine their veracity. If any of them seem implausible, the model assumptions would need to be reviewed. Their effects could materially alter the mean indication.

5.4.2. Mean, Standard Deviation and CoV of Incremental Values

The mean, standard deviation and coefficients of variation for every incremental value from the simulation process can also provide useful diagnostic results, enabling a deeper review into potential coefficient of variation issues that may be found in the estimated unpaid results. Consider, for example, the mean, standard deviation and coefficient of variation results shown in Tables 5.2, 5.3, and 5.4, respectively.

Table 5.2. Mean of Incremental Values for Berquist-Sherman

Sample Insurance Company
Hayne Paper Data
Accident Year Incremental Values by Development Period
Paid Berquist & Sherman Model

Accident	Mean Values (in 000's)												
Year	12	24	36	48	60	72	84	96	108	120	132	144	156
2008	25,064	31,145	28,656	22,440	14,281	7,309	2,843	1,814	1,079	613	269	116	56
2009	25,835	32,119	29,703	23,223	14,691	7,552	2,961	1,878	1,113	617	278	134	54
2010	29,579	36,189	33,544	26,237	16,817	8,639	3,384	2,100	1,250	695	309	138	66
2011	31,088	38,234	35,446	27,737	17,569	9,087	3,546	2,182	1,318	747	329	139	78
2012	31,976	39,197	36,545	28,680	18,113	9,362	3,640	2,270	1,354	789	336	162	80
2013	32,175	39,680	36,868	29,088	18,350	9,495	3,691	2,294	1,384	767	343	162	78
2014	36,809	45,089	42,259	32,820	20,700	10,715	4,251	2,642	1,571	883	374	184	82
2015	36,915	45,693	42,709	33,487	20,936	10,886	4,241	2,615	1,582	860	396	192	85
2016	40,158	49,481	45,856	36,060	22,785	11,583	4,600	2,882	1,699	972	425	189	88
2017	47,924	58,862	54,790	43,026	27,063	13,895	5,533	3,402	2,037	1,139	498	234	118

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Table 5.3. Standard Deviation of Incremental Values for Berquist-Sherman

Sample Insurance Company
Hayne Paper Data
Accident Year Incremental Values by Development Period
Paid Berquist & Sherman Model

Accident	Standard Error Values (in 000's)												
Year	12	24	36	48	60	72	84	96	108	120	132	144	156
2008	4,010	4,911	4,418	3,910	2,782	1,895	1,037	776	619	498	365	238	162
2009	4,203	5,015	4,679	3,993	2,819	1,920	1,079	791	626	465	365	254	171
2010	4,524	5,085	4,684	4,094	3,163	1,992	1,185	906	688	533	407	285	191
2011	4,337	5,232	5,277	4,218	3,313	2,126	1,228	929	743	544	439	286	190
2012	4,665	5,270	5,114	4,576	3,282	2,213	1,243	911	708	563	420	301	199
2013	4,639	5,546	5,234	4,562	3,410	2,243	1,240	955	759	589	441	303	196
2014	5,184	6,314	5,718	4,887	3,589	2,420	1,337	996	800	655	473	324	220
2015	5,169	6,197	6,028	5,178	3,800	2,491	1,415	1,022	788	625	479	337	226
2016	5,652	6,619	6,140	5,421	4,013	2,649	1,467	1,142	881	676	548	353	239
2017	6,057	7,346	7,284	6,284	4,601	3,062	I,707	1,244	982	789	605	416	288

Table 5.4. Coefficient of Variation of Incremental Values for Berquist-Sherman

Sample Insurance Company
Hayne Paper Data
Accident Year Incremental Values by Development Period
Paid Berquist & Sherman Model

Accident	Coefficient of Variation Values												
Year	12	24	36	48	60	72	84	96	108	120	132	144	156
2008	16.0%	15.8%	15.4%	17.4%	19.5%	25.9%	36.5%	42.8%	57.3%	81.3%	135.7%	204.6%	290.1%
2009	16.3%	15.6%	15.8%	17.2%	19.2%	25.4%	36.4%	42.1%	56.2%	75.4%	131.3%	189.1%	319.2%
2010	15.3%	14.1%	14.0%	15.6%	18.8%	23.1%	35.0%	43.2%	55.1%	76.6%	131.6%	206.4%	291.3%
2011	14.0%	13.7%	14.9%	15.2%	18.9%	23.4%	34.6%	42.6%	56.4%	72.9%	133.3%	206.4%	241.8%
2012	14.6%	13.4%	14.0%	16.0%	18.1%	23.6%	34.1%	40.2%	52.3%	71.4%	125.1%	186.2%	248.6%
2013	14.4%	14.0%	14.2%	15.7%	18.6%	23.6%	33.6%	41.7%	54.8%	76.8%	128.4%	187.1%	249.9%
2014	14.1%	14.0%	13.5%	14.9%	17.3%	22.6%	31.4%	37.7%	50.9%	74.2%	126.3%	175.8%	267.6%
2015	14.0%	13.6%	14.1%	15.5%	18.1%	22.9%	33.4%	39.1%	49.8%	72.7%	121.0%	175.4%	265.3%
2016	14.1%	13.4%	13.4%	15.0%	17.6%	22.9%	31.9%	39.6%	51.9%	69.6%	128.8%	187.2%	271.3%
2017	12.6%	12.5%	13.3%	14.6%	17.0%	22.0%	30.8%	36.6%	48.2%	69.3%	121.4%	177.3%	243.1%

The mean values in Table 5.2 appear consistent throughout and support the increases in estimated unpaid by accident year that are shown in Table 5.1. In fact, the future mean values, which lie beyond the stepped diagonal line in Table 5.2, sum to the results in Table 5.1. The standard deviation values in Table 5.3 also appear consistent, but the standard deviations can't be added because the standard deviations in Table 5.1 represent those for aggregated incremental values by accident year, which are less than perfectly correlated. The coefficient of variation values in Table 5.4 help the user efficiently review both the incremental mean and standard deviation values in Tables 5.2 and 5.3, as inconsistencies in a column will highlight issues with either the means or standard deviations or both. The coefficients by column in Table 5.4 all appear consistent, so the other main use of this table is to review the progression of CoVs by development period, which should increase over time as they do in Table 5.4, indicating that the final incremental payments in the tail tend to be the most uncertain.

For comparison with the Berquist-Sherman model, Appendix A contains the model parameters, diagnostics and estimated unpaid claims for each of the Hayne MLE models using the paid data. While not included in Appendix A, it will generally be useful to include the models for incurred data in any comparison. Another possible comparison is with the combined frequency and severity models, as described at the end of Section 4. Between the various paid and incurred models, the relative variability of each model doesn't necessarily conform to any rules, as the variability tends to depend on the relative fit of each model. In contrast, the combined frequency and severity models should increase the variability of the results, all else being equal, just based on the interactions of the simulated ultimate claim counts and simulated severities. An example of this is shown in Table 5.5, which can be compared to Table 5.1.

Table 5.5. Estimated Unpaid Results for Berquist-Sherman (frequency and severity)

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Berquist & Sherman Model

Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	308	402	130.6%	(974)	2,496	256	528	980	1,449
2009	140,983	948	790	83.4%	(2,252)	5,003	861	1,387	2,313	3,205
2010	147,516	2,483	1,266	51.0%	(844)	7,005	2,399	3,275	4,687	6,123
2011	174,349	5,451	2,029	37.2%	(392)	12,879	5,345	6,674	9,030	10,807
2012	173,637	10,316	3,051	29.6%	2,623	21,675	10,119	12,224	15,679	17,875
2013	174,996	23,281	5,344	23.0%	10,049	40,607	22,718	26,710	32,776	36,838
2014	169,224	49,206	9,718	19.7%	20,354	82,036	48,819	55,358	65,983	73,214
2015	134,010	83,591	16,492	19.7%	29,154	151,014	83,194	93,445	111,712	125,440
2016	68,911	121,920	25,036	20.5%	42,859	221,483	120,165	136,883	163,665	192,108
2017	35,798	161,577	33,743	20.9%	44,343	288,934	162,200	182,677	216,533	242,895
Totals	1,343,162	459,081	57,062	12.4%	268,331	687,553	457,132	496,597	554,955	603,727

6. Using Multiple Models

So far the focus has only been on one model. In practice, multiple stochastic models should be used in the same way that multiple methods should be used in a deterministic analysis. First the results for each model must be reviewed and finalized, after an iterative process of diagnostic testing and reviewing model output to make sure the model "fits" the data, has reasonable assumptions, and produces reasonable results. Then these results can be combined by assigning a weight to the results of each model.

Two primary methods exist for combining the results for multiple models:

- **Run models with the same random variables.** For this algorithm, every model uses the exact same random variables. In the "Hayne MLE Models.xlsm" file, the random values are simulated before they are used to simulate results, which means that this algorithm may be accomplished by reusing the same set of random variables for each model. At the end, the incremental values for each model, for each iteration by accident year (that have a partial weight), can be weighted together.
- **Run models with independent random variables.** For this algorithm, every model is run with its own random variables. In the "Hayne MLE Models.xlsm" file, the random values are simulated before they are used to simulate results, which means that this algorithm may be accomplished by simulating a new set of random variables for each model.[31] At the end, the weights are used to randomly select a model for each iteration by accident year so that the result is a weighted "mixture" of models.

Both algorithms are similar to the process of weighting the results of different deterministic methods to arrive at an actuarial best estimate. The process of weighting the results of different stochastic models produces an actuarial best estimate of a distribution. In practice it is also common to further "adjust" or "shift" the weighted results by year after considering case reserves and the calculated IBNR. For example, in an older year the weighted value could result in a negative IBNR which offsets case reserves and a reasonable adjustment could be to accept the case reserves by "shifting" the IBNR to zero. This "shifting" can also be done for weighted distributions, either

[31] In general, in order to simulate new random values, a new seed value must be selected (or a seed value of zero can be used), otherwise the same random values will be simulated. In the "Hayne MLE Models.xlsm" file, the seed value is incremented for each model and data type so that different seed values are being used as long as new random numbers are generated for each model and data type.

additively to maintain the exact shape and width of the distribution by year or multiplicatively to maintain the exact shape of the distribution but adjusting the width of the distribution.

By comparing the results for all ten models (or fewer, depending on how many are used),[32] a qualitative assessment of the relative merits of each model may be determined. Bayesian methods can be used to determine weighting based on the quality of each model's forecasts.[33] The weights can be determined separately for each year. The values in Table 6.1 show an example of weights for the Hayne MLE data.[34] The weighted results are displayed in the "Best Estimate" column of Table 6.2.

Table 6.1. Model Weights by Accident Year

Accident	Model Weights by Accident Year										
Year	Paid BS	Incd BS	Paid CC	Incd CC	Paid CL	Incd CL	Paid HC	Incd HC	Paid WR	Incd WR	TOTAL
2008	25.0%	25.0%	25.0%	25.0%							100.0%
2009	25.0%	25.0%	25.0%	25.0%							100.0%
2010	25.0%	25.0%	25.0%	25.0%							100.0%
2011	25.0%	25.0%	25.0%	25.0%							100.0%
2012	25.0%	25.0%	25.0%	25.0%							100.0%
2013	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%					100.0%
2014	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%			100.0%
2015	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%			100.0%
2016	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%			100.0%
2017	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%	12.5%			100.0%

Table 6.2. Summary of Mean Results by Model

Sample Insurance Company
Hayne Paper Data
Summary of Results by Model (in 000's)

	Mean Estimated Unpaid										
Accident	Berquist & Sherman		Cape Cod		Chain Ladder		Hoerl Curve		Wright		Best Est.
Year	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred	(Weighted)
2008	441	528	485	488	168	177	86	91	64	65	471
2009	1,083	1,164	1,201	1,228	477	507	269	281	218	218	1,148
2010	2,459	2,494	2,355	2,427	1,281	1,389	919	937	694	718	2,453
2011	4,793	4,812	5,172	5,182	3,975	4,278	2,872	2,861	2,715	2,769	4,945
2012	8,629	8,400	9,239	8,940	8,073	8,721	7,681	7,516	7,597	7,429	8,642
2013	18,214	17,179	20,571	20,421	19,370	20,588	17,664	16,874	19,119	19,046	19,280
2014	41,402	38,115	44,568	42,079	43,332	44,793	40,416	37,923	42,804	40,657	41,487
2015	75,281	66,959	78,842	74,018	77,959	80,697	73,354	67,037	76,810	72,994	74,398
2016	127,141	110,465	93,698	93,653	93,147	101,410	125,089	112,174	93,415	94,782	107,115
2017	210,599	178,646	147,763	150,595	147,782	162,612	207,924	182,932	147,450	151,814	173,575
Totals	490,041	428,763	403,895	399,031	395,563	425,172	476,274	428,627	390,884	390,491	433,516

[32] Other models in addition to the Hayne MLE models could also be included in the weighting process as long as the simulated results are in the form of random incremental payment streams.

[33] Quality of the forecast could be defined in a number of ways, but the essential idea is to measure the relative predictive power of competing models.

[34] For simplicity, the weights are only illustrative and not derived using Bayesian methods.

As a parallel to a deterministic analysis, the means from the eight models given some weight could be used to derive a reasonable range from the modeled results (i.e., from $395,563 to $490,041), as shown in Table 6.3. Alternatively, if only results by accident year which are given some weight when deriving the best estimate are considered, then the "weighted range" may be a more representative view of the uncertainty of the actuarial central estimate.[35] In a sense, the range of mean estimates reflects some of the uncertainty as it relates to the central estimate and then the weighted distribution represents a more complete view of the entire uncertainty.[36]

Table 6.3. Summary of Ranges by Accident Year

Sample Insurance Company Sample
Hayne Paper Data
Summary of Results by Model (in 000's)

		Ranges			
		Weighted		Modeled	
Accident Year	Best Est. (Weighted)	Mininum	Maximum	Mininum	Maximum
2008	471	441	528	86	528
2009	1,148	1,083	1,228	269	1,228
2010	2,453	2,355	2,494	919	2,494
2011	4,945	4,793	5,182	2,861	5,182
2012	8,642	8,400	9,239	7,516	9,239
2013	19,280	17,179	20,588	16,874	20,588
2014	41,487	37,923	44,793	37,923	44,793
2015	74,398	66,959	80,697	66,959	80,697
2016	107,115	93,147	127,141	93,147	127,141
2017	173,575	147,763	210,599	147,763	210,599
Totals	433,516	380,045	502,488	395,563	490,041

When selecting weights for stochastic models, the standard deviations should also be considered in addition to the means by model since the weighted best estimate should reflect the actuary's judgments about the entire distribution, not just a central estimate. Thus, coefficients of variation by model can be used for this purpose, as illustrated in Table 6.4. In addition to the diagnostic considerations discussed in section 5.4, judgments about the magnitude of the uncertainty are also important to the weighting process as the goal is to estimate the "correct" uncertainty and not to minimize the uncertainty.[37]

[35] The "modeled range" in Figure 6.3 is derived using each model that is given at least some weight for any accident year—i.e., if the model is used. Note also that the Totals are based on the models where at least some weight is used and not the sum of the values in the respective columns. In contrast, the "weighted range" is derived using only the models given weight for each accident year, which are highlighted in grey in Tables 6.2 and 6.4.

[36] For a more complete discussion of ranges and distributions in the reserving context, see Shapland [15]. For a more complete discussion of ranges and distributions in an Enterprise Risk Management context, see Shapland and Courchene [18].

[37] Note that the selected weights in Table 6.1 are purely illustrative and are not intended to reflect a complete analysis of the means in Table 6.2 or the standard deviations in Table 6.4.

Table 6.4. Summary of CoV Results by Model

Sample Insurance Company
Hayne Paper Data
Summary of Results by Model (in 000's)

	Coefficient of Variation									
	Berquist & Sherman		Cape Cod		Chain Ladder		Hoerl Curve		Wright	
Accident Year	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred
2008	129.9%	118.0%	131.4%	131.3%	239.3%	254.9%	279.2%	281.7%	632.5%	639.9%
2009	76.2%	78.5%	97.7%	98.2%	156.0%	163.8%	166.4%	168.8%	303.7%	311.0%
2010	47.5%	48.6%	64.5%	64.5%	78.5%	82.7%	92.4%	93.5%	146.0%	147.4%
2011	33.3%	33.7%	38.6%	38.3%	39.7%	45.7%	51.1%	51.4%	58.0%	58.2%
2012	23.1%	25.1%	27.2%	27.1%	25.0%	32.5%	32.2%	32.7%	31.1%	30.8%
2013	17.2%	17.0%	15.6%	15.0%	14.1%	24.0%	20.8%	20.6%	17.1%	16.3%
2014	12.1%	13.5%	10.0%	9.8%	9.3%	22.4%	13.4%	13.9%	10.7%	10.1%
2015	9.9%	10.6%	7.7%	7.0%	6.4%	20.8%	10.2%	10.5%	7.7%	6.7%
2016	8.7%	9.4%	8.5%	7.0%	5.9%	24.0%	8.5%	9.0%	8.2%	6.5%
2017	7.7%	8.4%	9.4%	5.8%	5.2%	22.0%	7.2%	7.8%	9.4%	5.4%
Totals	6.4%	6.1%	5.9%	4.6%	4.1%	11.8%	6.0%	5.7%	5.5%	3.9%

With a focus on the entire distribution, the weights by year are used to randomly sample the specified percentage of iterations from each model. A more complete set of the results for the "weighted" iterations can be created similar to the tables shown in section 5. The companion "Best Estimate.xlsm" file can be used to weight ten different models together in order to calculate a weighted best estimate. An example is shown in Table 6.5 for the Hayne [8] data.

Table 6.5. Estimated Unpaid Model Results (Weighted)

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	471	644	136.7%	(2,545)	4,909	405	829	1,576	2,333
2009	140,983	1,148	1,049	91.4%	(3,520)	6,314	1,092	l,780	2,957	3,957
2010	147,516	2,453	1,357	55.3%	(3,302)	10,083	2,408	3,290	4,714	5,954
2011	174,349	4,945	1,789	36.2%	(4,448)	12,718	4,898	6,102	7,933	9,502
2012	173,637	8,642	2,208	25.5%	(1,331)	19,227	8,604	10,106	12,319	14,029
2013	174,996	19,280	3,656	19.0%	4,625	39,886	19,143	21,530	25,382	28,830
2014	169,224	41,487	6,136	14.8%	16,382	75,478	41,413	45,225	51,128	58,189
2015	134,010	74,398	9,887	13.3%	25,947	157,876	74,300	79,822	90,176	104,245
2016	68,911	107,115	17,580	16.4%	28,733	187,403	104,724	120,254	137,020	148,299
2017	35,798	173,575	30,419	17.5%	9,842	285,509	170,237	197,558	224,280	240,117
Totals	1,343,162	433,516	38,243	8.8%	254,901	599,252	432,354	460,201	497,529	524,069
Normal Dist.		433,516	38,243	8.8%			433,516	459,310	496,420	522,483
logNormal Dist.		433,522	38,456	8.9%			431,826	458,398	499,520	530,586
Gamma Dist.		433,516	38,243	8.8%			432,392	458,661	498,279	527,410
TVaR							464,489	483,287	513,512	536,643
Normal TVaR							464,029	482,127	512,401	535,442
logNormal TVaR							464,105	483,728	518,630	546,956
Gamma TVaR							463,996	483,043	516,174	542,419

As one final check of the weighted results, it would be common to review the implied IBNR to make sure there are no issues, as shown in Table 6.6. By reviewing this reconciliation, and perhaps also comparing it to deterministic results, additional adjustments could be made to various assumptions. For example, from year 2008 in Table 6.6 it may be more realistic to revisit the tail factor assumptions or the weights by model so that the unpaid estimate is more consistent with the case reserves. Finally, after the interactive process of reviewing results and adjusting assumptions is complete, it may still be prudent to make adjustments to the best estimate of the unpaid by shifting the results as noted earlier in this section.

Table 6.6. Reconciliation of Total Results (Weighted)

Sample Insurance Company
Hayne Paper Data
Reconciliation of Total Results (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Incurred To Date	Case Reserves	IBNR	Estimate of Ultimate	Estimate of Unpaid
2008	123,738	124,486	748	(277)	124,209	471
2009	140,983	141,488	505	643	142,131	1,148
2010	147,516	150,057	2,541	(88)	149,969	2,453
2011	174,349	180,737	6,388	(1,443)	179,294	4,945
2012	173,637	182,952	9,315	(673)	182,279	8,642
2013	174,996	193,196	18,200	1,080	194,276	19,280
2014	169,224	199,879	30,655	10,832	210,711	41,487
2015	134,010	189,518	55,508	18,890	208,408	74,398
2016	68,911	132,561	63,650	43,465	176,026	107,115
2017	35,798	110,269	74,471	99,104	209,373	173,575
Totals	1,343,162	l,605,143	261,981	171,535	1,776,678	433,516

7. Additional Output and Results

This discussion of stochastic modeling is not complete without considering some of the additional output and results that can be included as part of the modeling process. Much of the additional output and results can be derived simply by reorganizing the model output to capture results in a different way. This has important uses, such as cash flows for discounting and unpaid claim runoff for calculating risk margins using the cost of capital method, that are often obtained using very little additional effort. Finally, while the examples in this section are all based on the weighted result, they all apply equally well to any individual model.

7.1. Additional Output

Three rows of percentile numbers for the normal, lognormal, and gamma distributions, which have been fitted to the total unpaid-claim distribution, may be seen at the bottom of Table 6.5.[38] The fitted mean, standard deviation, and selected percentiles are in their respective columns. The smoothed results can be used to assess the quality of fit,[39] parameterize a dynamic financial analysis ("DFA") model, or smooth the estimate of extreme values,[40] among other applications.

Four rows of numbers indicating the tail value at risk (TVaR), defined as the average of all of the simulated values greater than or equal to the percentile value, may also be seen at the bottom of Table 6.5. For example, in this table, the 99th percentile value for the total unpaid claims for all accident years combined is 524,069, while the

[38] The fitted distribution values are calculated by matching the selected distribution parameters to the mean and standard deviation of the total unpaid claim distribution.

[39] Since the mean and standard deviations for each distribution are generally very close to the same measures for the simulated distribution, it makes more sense to base quality of fit considerations on the more extreme percentiles. Assuming the fitted distributions are stable, meaning new simulations with different random numbers result in similar fitted distributions, the extreme percentiles are typically of greater interest for uses such as capital requirements. For example, from Table 6.5 it appears that either the normal or gamma distributions are a better fit than the lognormal. The closeness of the normal fit could be evidence that the distribution is close to symmetrical or gamma could be the better choice to reflect some skewness.

[40] A random instance of an extreme percentile can be quite erratic compared to the same percentile of a distribution fitted to the simulated distribution. This random noise for extreme percentiles could be cause for increasing the number of iterations, but if the same percentiles for the fitted distributions are stable, perhaps they can be used in lieu of more iterations. Of course, the use of the extreme values assumes that the models are reliable.

average of all simulated values that are greater than or equal to the 99th percentile is 536,643. The normal TVaR, lognormal TVaR, and gamma TVaR rows are calculated similarly, except that they use the respective fitted distributions in the calculations rather than actual simulated values from the model.

An analysis of the TVaR values is likely to help clarify a critical issue: if the actual outcome exceeds the X percentile value, by how much will it exceed that value on average? This type of assessment can have important implications related to risk-based capital calculations and other technical aspects of enterprise risk management. But it is worth noting that the purpose of the normal, lognormal, and gamma TVaR numbers is to provide "smoothed" values—that is, that some of the random statistical noise is essentially prevented from distorting the calculations.

7.2. Estimated Cash Flow Results

A model's output may also be reviewed by calendar year (or by future diagonal), as shown in Table 7.1. A comparison of the values in Tables 6.5 and 7.1 indicates that the total rows are identical, because summing the future payments horizontally or diagonally will produce the same total. Similar diagnostic issues (as discussed in Section 5) may be reviewed in Table 7.1, with the exception of the relative values of the standard errors and coefficients of variation moving in opposite directions for calendar years compared to accident years. This phenomenon makes sense on an intuitive level when one considers that "final" payments, projected to the furthest point in the future, should actually be the smallest, yet relatively most uncertain.

Table 7.1. Estimated Cash Flow (Weighted)

Sample Insurance Company
Hayne Paper Data
Calendar Year Unpaid (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2018	160,184	14,166	8.8%	109,684	222,966	159,583	169,553	184,716	195,263
2019	116,073	12,102	10.4%	72,833	166,146	115,235	124,202	136,915	145,439
2020	75,084	8,938	11.9%	34,373	111,295	74,509	80,772	90,836	97,566
2021	42,212	6,021	14.3%	16,605	71,311	41,859	46,173	52,711	57,524
2022	21,143	3,889	18.4%	8,545	37,308	20,894	23,666	27,935	30,994
2023	9,680	2,613	27.0%	(212)	20,773	9,541	11,348	14,156	16,596
2024	4,960	1,802	36.3%	(2,713)	13,036	4,900	6,101	8,021	9,492
2025	2,371	1,338	56.4%	(3,187)	8,932	2,299	3,229	4,684	5,783
2026	1,102	992	90.0%	(2,827)	6,547	1,003	1,691	2,847	3,857
2027	462	632	136.8%	(3,435)	4,443	376	790	1,644	2,350
2028	182	383	210.8%	(2,728)	2,866	122	357	865	1,365
2029	61	221	363.4%	(1,545)	1,829	24	130	460	799
Totals	433,516	38,243	8.8%	254,901	599,252	432,354	460,201	497,529	524,069

7.3. Estimated Ultimate Loss Ratio Results

Another output, Table 7.2, shows the estimated ultimate loss ratios by accident year. Similar to the estimated unpaid and estimated cash-flow tables, the values in this table are calculated using all simulated values, not just the values beyond the end of the historical triangle. Because the simulated sample triangles represent additional possibilities of what could have happened in the past, even as the "squaring of the triangle" and process variance represent what could happen as those same past values are played out into the future, there is sufficient information to enable estimation of the variability in the loss ratio from day one until all claims are completely paid and settled for each accident year.[41]

Table 7.2. Estimated Time Zero to Ultimate Loss Ratio (Weighted)

Sample Insurance Company
Hayne Paper Data
Accident Year Ultimate Loss Ratios (Premiums in 000's)
Best Estimate (Weighted)

Accident Year	Earned Premium	Mean Loss Ratio	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	184,450	71.9%	7.2%	10.0%	48.2%	105.4%	70.9%	76.1%	85.2%	91.9%
2009	237,093	60.5%	4.5%	7.5%	38.0%	84.3%	60.4%	63.3%	67.9%	71.8%
2010	297,807	52.3%	3.9%	7.5%	37.0%	71.5%	52.1%	54.7%	59.0%	62.6%
2011	349,324	49.3%	3.6%	7.3%	28.3%	61.3%	49.6%	51.8%	54.8%	56.9%
2012	361,198	48.1%	3.4%	7.1%	32.3%	61.8%	48.3%	50.5%	53.3%	55.2%
2013	374,921	50.0%	6.7%	13.4%	14.0%	100.2%	50.3%	52.8%	60.8%	73.3%
2014	370,904	54.2%	6.3%	11.6%	20.5%	102.4%	54.3%	57.3%	62.8%	77.2%
2015	345,267	58.3%	7.0%	12.0%	21.2%	125.7%	58.3%	61.6%	68.9%	82.2%
2016	301,114	61.4%	9.5%	15.5%	16.3%	104.4%	59.9%	68.8%	76.9%	82.8%
2017	277,987	77.0%	13.1%	17.0%	4.4%	129.2%	75.3%	87.5%	98.8%	105.2%
Totals	3,100,065	57.0%	2.2%	3.9%	48.8%	66.6%	56.9%	58.4%	60.7%	62.5%

Reviewing the simulated values indicates that the standard errors in Table 7.2 should be proportionate to the means, while the coefficients of variation should be relatively constant by accident year. In terms of diagnostics, any increases in standard error and coefficient of variation for the most recent years would be consistent with the reasons previously cited in Section 5.4 for the estimated unpaid tables.[42] Risk management-wise, the loss ratio distributions have important implications for projecting pricing risk—the mean loss ratios can be used to view any underwriting cycles and help inform the projected mean for the next few years, while the coefficients of variation can be used to select a standard deviation for the next few years.[43]

[41] If one is only interested in the "remaining" volatility in the loss ratio, then the values in the estimated unpaid (Table 6.5) can be added to the cumulative paid values by year and divided by the premiums.

[42] The theoretical consistency of the coefficients of variation by accident year is based on all years having the same number of independent incremental payment periods. In practice, the increasing coefficients in Table 7.2 could be due to an increasing impact of parameter uncertainty as discussed in section 5.4.

[43] The coefficients of variation measure the variability of the loss ratios, given the movements by year. Without this information, it is common to base the future standard deviation on the standard deviation of the historical mean loss ratios, but this is not ideal, since the variability of the mean loss ratios is not the same as the possible variation in the actual outcomes given movements in the means.

7.4. Estimated Unpaid Claim Runoff Results

Table 7.3 shows the runoff of the total unpaid claim distribution by future calendar year. Like the estimated unpaid and estimated cash-flow tables, the values in this table are calculated using only future simulated values, except that future diagonal results are sequentially removed so that only the unpaid claims at the end of each future calendar period are remaining. These results are quite useful for calculating the runoff of the unpaid claim distribution when calculating risk margins using the cost of capital method.

Table 7.3. Estimated Unpaid Claim Runoff (Weighted)

Sample Insurance Company
Hayne Paper Data
Calendar Year Unpaid Claim Runoff (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2017	433,516	38,243	8.8%	254,901	599,252	432,354	460,201	497,529	524,069
2018	273,331	27,072	9.9%	142,347	383,736	272,131	292,254	319,167	337,111
2019	157,258	17,542	11.2%	67,252	220,814	156,499	169,104	187,514	200,341
2020	82,174	10,939	13.3%	32,880	123,782	81,702	89,376	100,832	108,855
2021	39,962	6,966	17.4%	14,345	69,981	39,632	44,447	52,029	57,298
2022	18,819	4,746	25.2%	1,463	41,958	18,626	21,805	27,058	30,892
2023	9,139	3,442	37.7%	(4,763)	26,381	8,926	11,285	15,161	18,408
2024	4,178	2,466	59.0%	(5,361)	15,768	3,933	5,672	8,598	11,114
2025	1,807	1,647	91.2%	(7,328)	10,335	1,565	2,713	4,837	6,709
2026	704	938	133.2%	(4,654)	6,189	539	1,172	2,474	3,628
2027	243	491	202.5%	(3,442)	3,710	152	455	1,135	1,876
2028	61	221	363.4%	(1,545)	1,829	24	130	460	799

7.5. Distribution Graphs

A final model output to consider is a histogram of the estimated unpaid amounts for the total of all accident years combined, as shown in the graph in Figure 7.1. The histogram is created by counting the number of outcomes within each of 100 "buckets" of equal size spread between the minimum and maximum outcome. To smooth the histogram, a kernel density function[44] is often used, which is represented by the green bars in Figure 7.1.

Another useful strategy for graphing the total unpaid distribution may be accomplished by creating a summary of the ten model distributions used to determine the weighted "best estimate" and distribution. An example of this graph using the kernel density functions is shown in Figure 7.2 and dots for the mean estimates, which would represent a traditional range,[45] are also included.

[44] A kernel density function uses weighed values of the surrounding values, with decreasing weight the further from the value in question, in order to smooth the values. There are many choices for kernel density functions, with whole books describing different functions.

[45] A traditional range would use deterministic point estimates instead of means of the distributions, but the intent is consistent. While the points would technically have an infinitesimal probability and should therefore sit on the x-axis, they are elevated above the zero probability level purely for illustration purposes.

Figure 7.1. Total Unpaid Claims Distribution

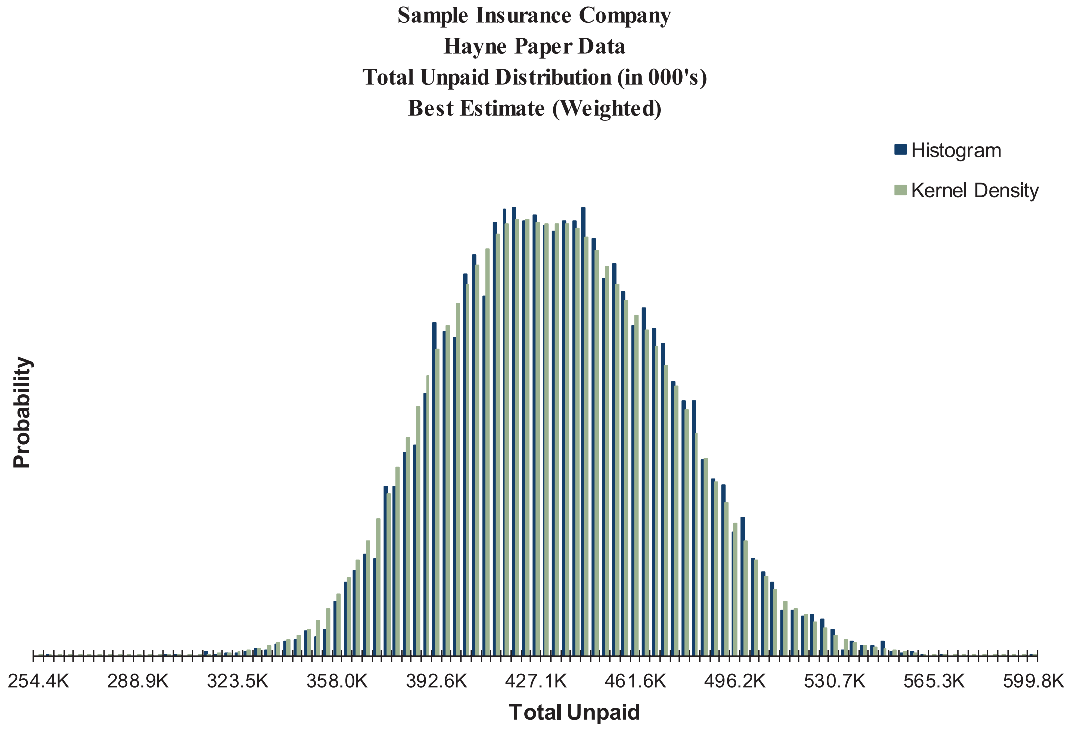
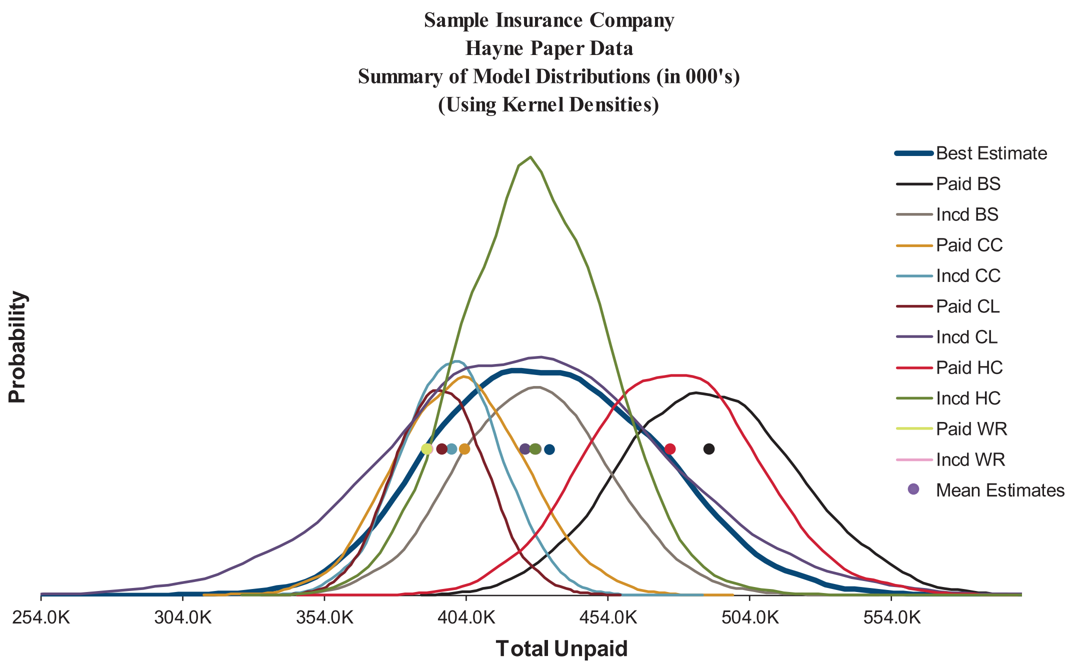


Figure 7.2. Summary of Model Distributions



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8. Correlation and Aggregation

Results for an entire business unit can be estimated, after each business segment has been analyzed and weighted into best estimates, using aggregation. This represents another area where caution is warranted. The procedure is not a simple matter of adding up the distributions for each segment. In order to estimate the distribution of possible outcomes for a company as a whole, a correlation of results among segments must be used. To illustrate aggregation, data from the "Industry Data.xls" file for Parts A, B, and C are used. The various model tables and graphs for the Part A, Part B, and Part C results are shown in Appendices B, C, and D, respectively.

Simulating correlated variables is commonly accomplished with a multi-variate distribution whose parameters and correlations have been previously specified. This type of simulation is most easily applied when distributions are uniformly identical and known in advance (for example, all derived from a multi-variate normal distribution). Unlike the ODP bootstrap framework, in which the characteristics of the overall distribution are unknown in advance, the multi-variate normal distribution assumption in the Hayne MLE framework could allow model correlation for multiple business segments. However, the correlation among parameters from each segment has to be defined before consolidating the variance-covariance matrices to simulate parameters for all segments. Thus, a fair amount of parameters are needed for correlation and it is difficult to visualize the gigantic aggregated variance-covariance matrix, so it is beyond the scope of this monograph.

Alternatively, two useful correlation processes for the Hayne MLE model are synchronized parameter simulation and re-sorting.[46]

With synchronized parameter simulation, in each iteration, independent normal random values are simulated for each parameter and each segment, then correlation is applied to adjust the simulated random numbers for the second segment and beyond, and modified random numbers are used for multi-variate normal distribution sampling of the parameters used by each segment for each iteration. For each iteration, once the correlated parameters are sampled for each segment, the independent random [0,1] values used for process risk could also be correlated between segments.

[46] For a useful reference see Kirschner, et al. [11]. The Kirschner paper is about correlation for the ODP bootstrap model, but the two processes can be used with other models.

The synchronized simulation process can be implemented in Excel once a correlation matrix has been estimated. There are, however, three potential drawbacks to this process. First, since multiple LOB/segments are being simulated simultaneously, either the size of the workbook needs to increase to accommodate all of the segments or the random number streams need to be correlated in a separate process. Second, it makes sense to correlate the parameters and process risk for the same model for all segments, but different triangle sizes will create "gaps" wherein some segments may have more parameters than other segments. Third, when the multiple models are weighted to get a "best estimate" for each segment the coordination of multiple models and segments is even more complex.

The second correlation process, re-sorting, can be accomplished with algorithms such as Iman-Conover[47] or copulas, among others. The primary advantages of re-sorting include:

- The correlation is a combination of parameter uncertainty and process variance,
- Different correlation assumptions may be employed without re-running all of the simulations, and
- Different correlation algorithms may also have other beneficial impacts on the aggregate distribution.

For example, using a *t*-distribution copula with low degrees of freedom rather than a normal-distribution copula will effectively "strengthen" the focus of the correlation in the tail of the distribution, all else being equal. This type of consideration is important for risk-based capital and other risk modeling issues.

To induce correlation among different segments in the "Aggregation.xlsm" file, a correlation matrix can be calculated using Spearman's Rank Order for each data/model type combination in order to select a correlation assumption. Using the selected correlation, re-sorting based on the ranks of the total unpaid claims for all accident years combined can be done. The calculated correlations for Parts A, B, and C based on the paid residuals for Berquist-Sherman may be seen in the first part of Table 8.1. A second part of Table 8.1 is the *p*-values for each correlation coefficient, which are an indication of whether a correlation coefficient is significantly different than zero as the *p*-value gets close to zero.[48]

By reviewing the correlation coefficients for each "pair" of segments, along with the *p*-values, from different sets of correlations matrices (e.g., from paid or incurred data for each model) judgment can be used to select a correlation matrix assumption. As noted above, caution is warranted as these calculated correlation matrices are limited to the data used in the calculation and the impact of other systemic issues, such as contagion, may also need to be considered.

[47] For a useful reference see Iman and Conover [9] or Mildenhall [12]. In the "Aggregate Estimate.xlsm" file the Iman-Conover algorithm is used to "Generate Rank Values" on the Inputs sheet.

[48] While judgment is clearly appropriate, the typical threshold is a *p*-value of 5%—i.e., a *p*-value of 5% or less indicates the correlation is significantly different than zero, while a *p*-value greater than 5% indicates the correlation is not significantly different than zero.

Table 8.1. Estimated Correlation and P-values

Rank Correlation of Residuals Paid BS Model—[Modeled]			
LOB	HO	PPA	CA
HO	1.00	0.26	0.22
PPA	0.26	1.00	0.15
CA	0.22	0.15	1.00
P-Value of Rank Correlation of Residuals Paid BS Model—[Modeled]			
LOB	HO	PPA	CA
HO	0.00	0.06	0.11
PPA	0.06	0.00	0.29
CA	0.11	0.29	0.00

Using these correlation coefficients, the "Aggregate Estimate.xlsm" file, and the simulation data for Parts A, B, and C, the aggregate results for the three lines of business were calculated and summarized in Table 8.2. A more complete set of tables for the aggregate results is shown in Appendix E.

Note that using residuals to correlate the lines of business (or other segments), as in the synchronized simulation method, and measuring the correlation between residuals, as in the re-sorting method, both tend to create correlations that are close to zero.

Table 8.2. Aggregate Estimated Unpaid

Sample Insurance Company Aggregate Three Lines of Business Accident Year Unpaid (in 000's)										
Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2006	18,613	146	1,002	688.1%	(2,013)	74,778	37	55	421	2,422
2007	20,618	198	993	500.3%	(1,523)	37,034	70	94	503	3,069
2008	22,866	246	927	377.4%	(5,763)	54,447	128	162	542	3,227
2009	22,842	367	1,286	350.7%	(2,918)	90,399	230	268	695	3,778
2010	22,351	535	1,359	254.3%	(1,875)	69,139	406	452	860	3,458
2011	22,422	869	1,266	145.7%	(3,632)	68,690	760	826	1,253	4,003
2012	24,350	1,589	939	59.1%	(4,107)	27,387	1,518	1,633	2,198	4,927
2013	19,973	2,814	1,424	50.6%	(8,046)	80,667	2,785	2,963	3,667	6,153
2014	18,919	5,418	4,384	80.9%	(8,120)	407,319	5,420	5,768	6,863	9,408
2015	15,961	13,369	3,352	25.1%	(11,431)	98,644	13,319	14,627	17,722	21,777
Totals	208,915	25,550	9,304	36.4%	(815)	476,278	24,635	26,612	32,642	55,933
Normal Dist.		25,550	9,304	36.4%			25,550	31,826	40,854	47,195
logNormal Dist.		25,528	6,217	24.4%			24,803	29,163	36,812	43,354
Gamma Dist.		25,550	9,304	36.4%			24,430	31,065	42,526	52,000
TVaR							28,995	32,475	48,429	89,074
Normal TVaR							32,974	37,377	44,742	50,348
logNormal TVaR							30,371	33,900	40,865	47,165
Gamma TVaR							32,838	38,140	48,373	57,295

For reserve risk, the correlation that is desired is between the total unpaid amounts for two segments. The correlation that is being measured is the correlation between each incremental future loss amount, given the underlying model describing the overall trends in the data. This may or may not be a reasonable approximation.

While not the direct measure being sought, keep in mind that some level of implied correlation between lines of business will naturally occur due to correlations between the model parameters—e.g., similarities in development parameters—so correlation based on the correlation between the remaining random movements in the incremental values given the model parameters (i.e., residuals) may be reasonable. However, an example of an issue not particularly well suited to measurement via residual correlation is contagion between lines of business—i.e., single events that result in claims in multiple lines of business. To account for this, and to add a bit of conservatism, the correlation assumption can be easily changed based on actuarial judgment.

Correlation is often thought of as being much stronger than "close to zero," but in this case the correlation being considered is typically the loss ratio movements by line of business. For pricing risk, the correlation that is desired is between the loss ratio movements by accident year between two segments. This correlation is not as likely to be close to zero, so correlation of loss ratios (e.g., for the data in Table 7.2) is often done with a different correlation assumptions compared to reserving risk.

9. Future Research

While common use of the Hayne MLE models may be in its infancy, the hope is that this monograph will spur more widespread use of the models. Nevertheless, there are many areas where further research can add value, but only a few key areas are offered up here.

- **Use of Other Distributions** – The key assumption which allows the framework for the Hayne MLE is the normal distribution that is appropriate whenever the central limit theorem is reasonable. Other distribution assumptions, while more complex mathematically, may provide useful alternatives when the central limit theorem does not apply, such as small portfolios or skewed distributions;
- **Combined Models** – Instead of simulating paid and incurred (or closed and reported) in parallel and then converting the incurred (reported) estimate to a random payment (closed) stream, it is possible that the Hayne MLE could be expanded to include both types of data in one combined framework with all parameters being correlated. A more ambitious undertaking would be to combine all four data types into a single modeling framework;
- **A Flexible Model** – Similar to the GLM bootstrap or incremental log models, it may be possible to develop a model using the Hayne MLE framework where the user can specify the place for parameters and include a diagonal parameter;
- **Time Horizon Models** – As other models have been adapted for calculation of the one-year time horizon for Solvency II purposes, the Hayne MLE models could also be so adapted; and
- **Pricing Models** – In order to expand the usefulness of the models, they could be extrapolated into future underwriting periods.

10. Conclusions

While this monograph endeavored to show how the Hayne MLE models can be used in a variety of practical ways, and to illustrate the diagnostic tools the actuary needs to assess whether the model is working well, it should not be assumed that a given Hayne MLE model is well suited for every data set. However, it is hoped that the Hayne MLE "toolsets" can become an integral part of the actuary's regular estimation of unpaid claim liabilities, rather than just a "black box" to be used only if necessary or after the deterministic methods have been used to select a point estimate. Finally, the modeling framework allows the actuary to "adjust" the model parameters to smooth anomalies in the data instead of simply accepting the model as is and essentially forcing the data to "fit" the model.

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Supplementary Material

There are several companion files designed to give the reader a deeper understanding of the concepts discussed in the monograph. The files are all in the "Hayne MLE Practitioners Guide.zip" file. The files are:

Model Instructions.pdf – this file contains a written description of how to use the primary Hayne MLE modeling files.

Primary modeling files:

Industry Data.xls – this file contains Schedule P data by line of business for the entire U.S. industry and five of the top 50 companies, for each LOB that has 10 years of data.

Hayne MLE Models.xlsm – this file contains the detailed model steps described in this monograph as well as various modeling options and diagnostic tests. Data can be entered and simulations run and saved for use in calculating a weighted best estimate.

Best Estimate.xlsm – this file can be used to weight the results from ten different models to get a "best estimate" of the distribution of possible outcomes.

Aggregate Estimate.xlsm – this file can be used to correlate the best estimate results from 3 LOBs/segments.

Correlation Ranks.xlsm – this file contains examples of ranks used to correlate results by LOB/segment.

Simple example calculation files:

Hayne Framework 6 BS.xlsm – this file illustrates the calculations for the Hayne MLE framework using the Berquist & Sherman model for a simple 6×6 triangle.

Hayne Framework 6 CC.xlsm – this file illustrates the calculations for the Hayne MLE framework using the Cape Cod model for a simple 6×6 triangle.

Hayne Framework 6 CL.xlsm – this file illustrates the calculations for the Hayne MLE framework using the Chain Ladder model for a simple 6×6 triangle.

Hayne Framework 6 HC.xlsm – this file illustrates the calculations for the Hayne MLE framework using the Hoerl Curve model for a simple 6×6 triangle.

Hayne Framework 6 WR.xlsm – this file illustrates the calculations for the Hayne MLE framework using the Wright model for a simple 6×6 triangle.

Appendix A—User Selected Parameters and Diagnostics

In this appendix, the selected parameters, diagnostics, and simulated unpaid claims are shown for paid data for each model.

Figure A.1. User-selected Parameters for Berquist-Sherman

User Selected Parameters:

	12	24	36	48	60	72	84	96	108	120	132	144	156
Mean	620.96	760.67	708.16	553.57	350.00	181.39	70.97	43.88	**26.00**	15.21	*7.05*	*3.27*	*1.51*
Std Dev	40.50	46.55	43.00	35.49	26.17	17.66	10.40	8.75	7.60	7.36	*6.47*	*4.32*	*2.67*
Decay Ratios:		122.5%	93.1%	78.2%	63.2%	51.8%	39.1%	61.8%	59.3%	58.5%			
CoV:	*6.5%*	*6.1%*	*6.1%*	*6.4%*	*7.5%*	*9.7%*	*14.7%*	*19.9%*	*29.2%*	*48.4%*	*91.8%*	*132.3%*	*176.2%*

	Accident Year						Tail Extrapolation		Implied Tail Factor		
	Trend	**K**	**p**	**AIC**	**BIC**	**Decay Ratio**	**Periods**	**Distribution**	**Adjusted**	**Actual**	**Tail Sampling Option**
Mean	0.045	11.216	0.654	647.9	674.0	46.3%	3	Gamma	1.0036	1.0036	Conditional Variance
Std Dev	0.009	1.094	0.089			32.5%					
CoV:	*18.9%*	*9.8%*	*13.6%*								

Decay Ratio Analysis:

Parameters: Model **Curve Type:** Power 3

Least Squares Regression Coefficients:

x^a	−0.3673
coefficient	1.1166

Goodness of Fit Statistics:

R^2 Statistic	0.240
Regression Deviation	32.5%
Suggested Decay Parameters:	
Mean	46.3%
Standard Deviation	32.5%

Periods	Decay Ratio	Outliers	Selected Age	Selected Decay Ratio	Incremental Fitted Factors
12–24	1.225	0	1	1.225	1.117
24–36	0.931	0	2	0.931	0.866
36–48	0.782	0	3	0.782	0.746
48–60	0.632	0	4	0.632	0.671
60–72	0.518	0	5	0.518	0.618
72–84	0.391	0	6	0.391	0.578
84–96	0.618	0	7	0.618	0.546
96- 108	0.252	0	8	0.252	0.520
108–120	1.373	0	9	1.373	0.498
120–132					0.479
132–144					0.463
144–156					0.448
156–168					0.435
168–180					0.424

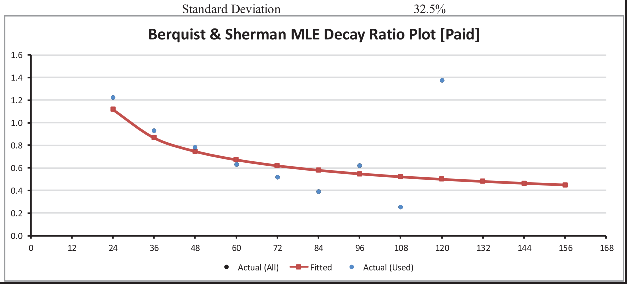


Figure A.2. Residual Graphs for Berquist-Sherman [Modeled Parameters]

Residual Graphs for Berquist & Sherman Hayne MLE Paid Model [Model Fitted]

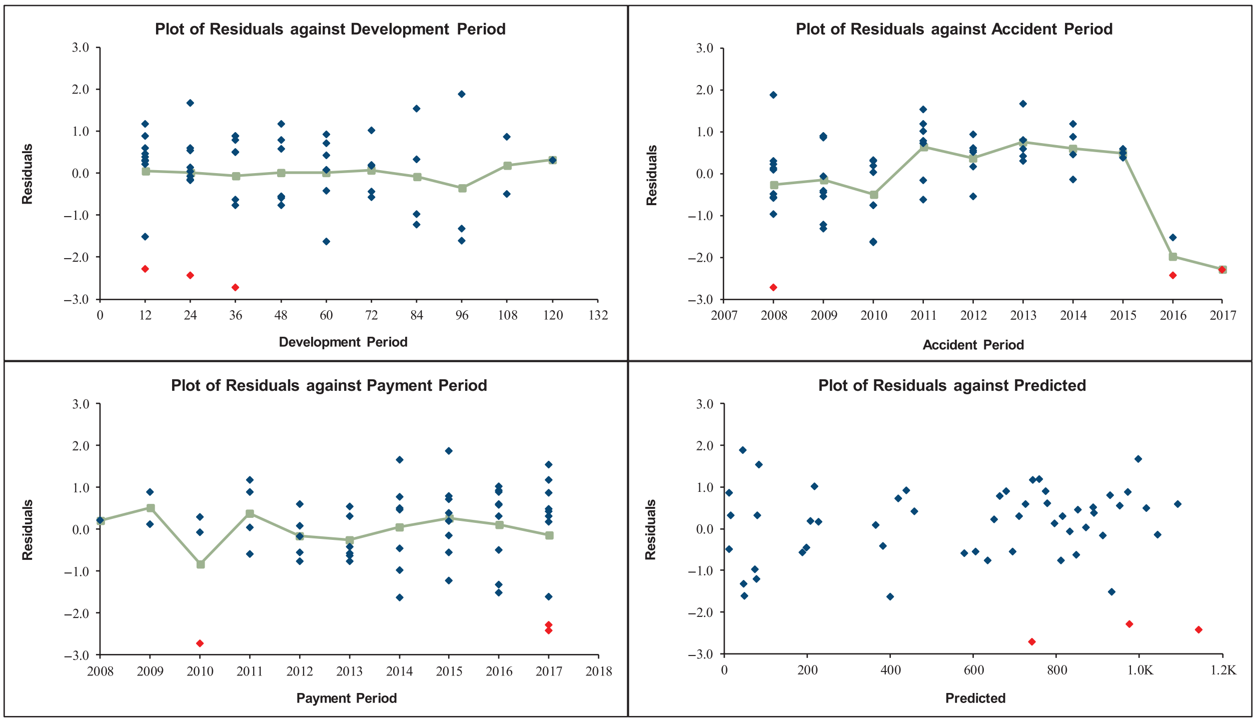


Figure A.3. Residual Graphs for Berquist-Sherman [Selected Parameters]

Residual Graphs for Berquist & Sherman Hayne MLE Paid Model [User Selected]

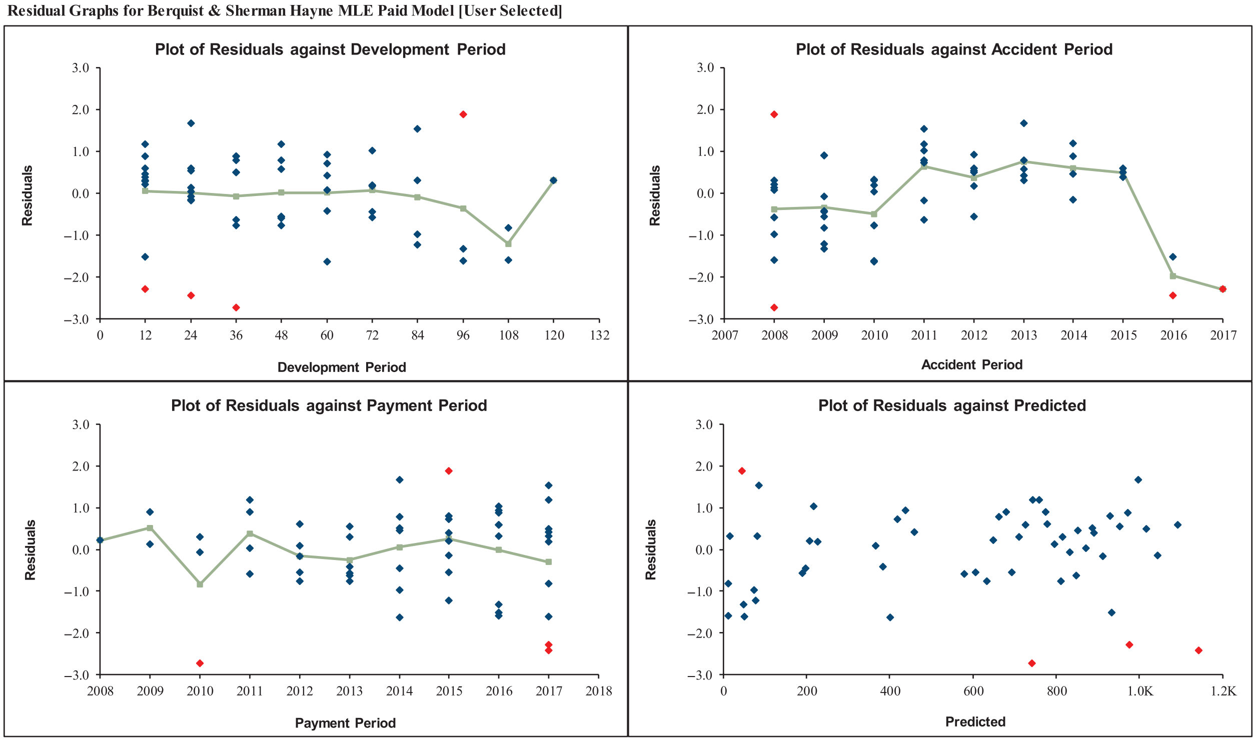


Figure A.4. Normality Plots for Berquist-Sherman

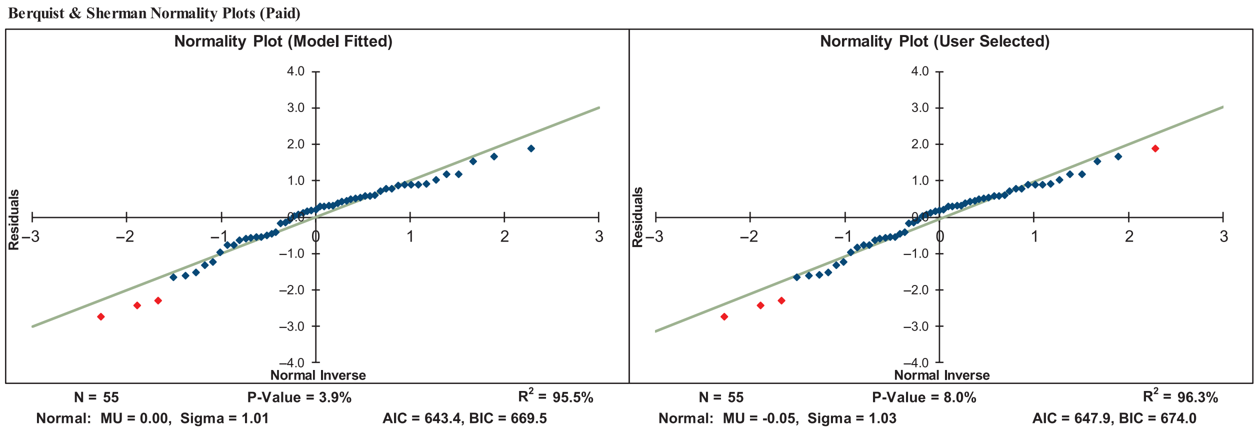


Figure A.5. Box-whisker Plots for Berquist-Sherman

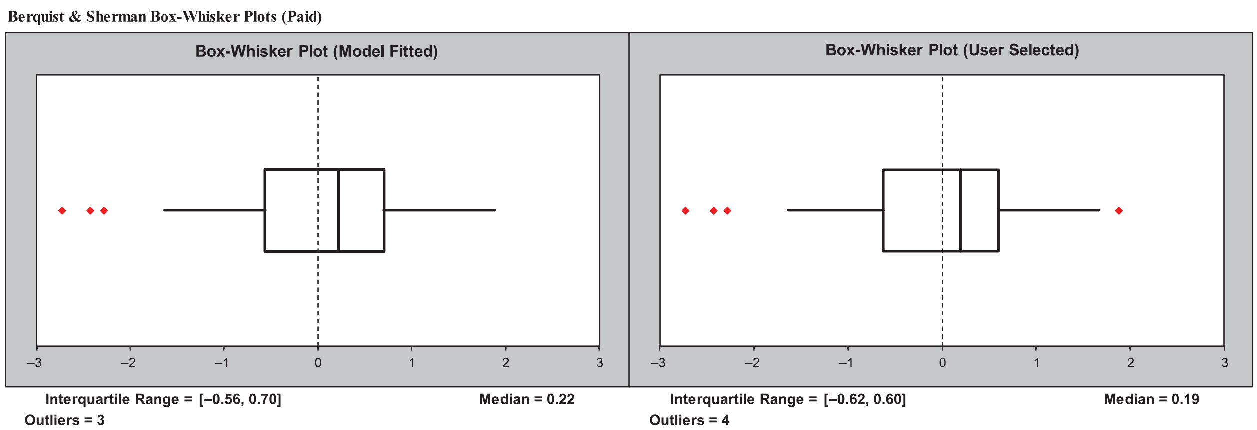


Figure A.6. Model Structure Graphs for Berquist-Sherman

Berquist & Sherman Model Structure [User Selected] (Paid)

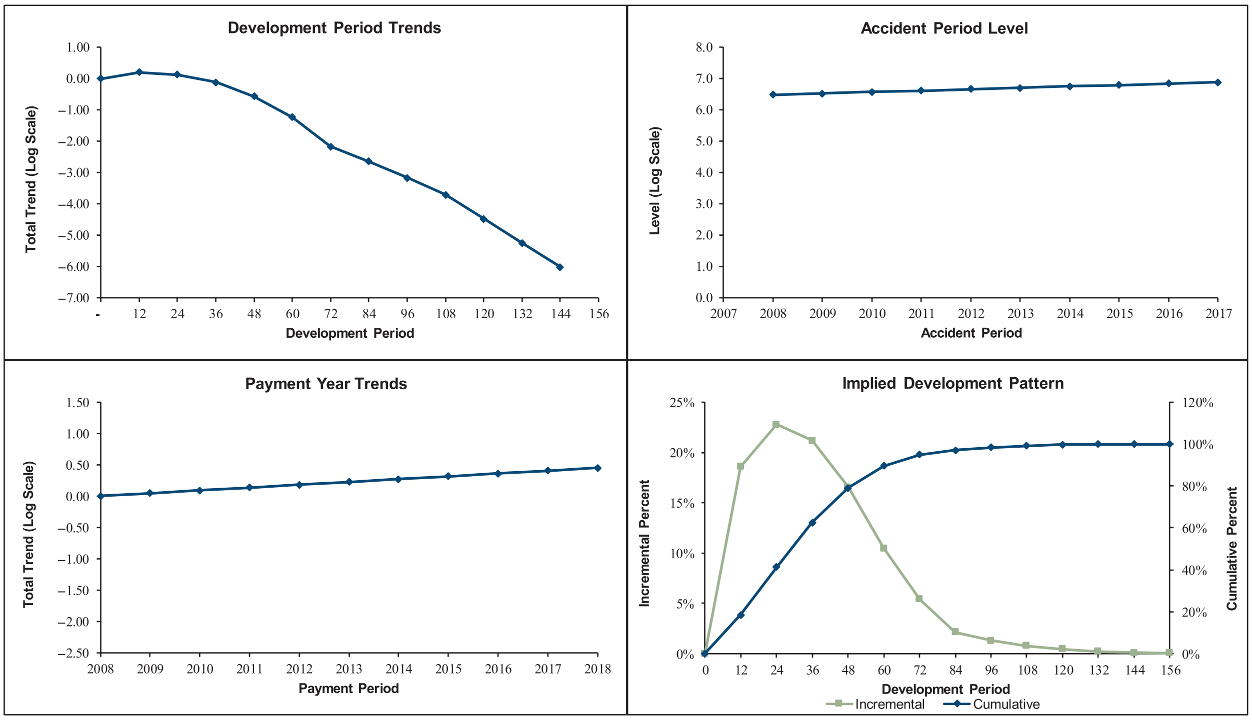


Figure A.7. Estimated Unpaid Results for Berquist-Sherman

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	441	573	129.9%	(1,475)	2,372	391	823	1,420	1,881
2009	140,983	1,083	825	76.2%	(1,675)	4,401	1,048	1,611	2,466	3,113
2010	147,516	2,459	1,168	47.5%	(1,527)	6,082	2,417	3,252	4,462	5,274
2011	174,349	4,793	1,595	33.3%	(172)	11,597	4,758	5,809	7,391	8,954
2012	173,637	8,629	1,992	23.1%	1,588	16,582	8,542	9,810	11,955	13,951
2013	174,996	18,214	3,136	17.2%	7,989	30,302	18,135	20,292	23,509	25,381
2014	169,224	41,402	5,008	12.1%	25,322	59,952	41,302	44,862	49,756	53,216
2015	134,010	75,281	7,480	9.9%	53,427	105,936	74,961	80,194	87,930	93,542
2016	68,911	127,141	11,108	8.7%	93,649	164,080	127,078	134,809	144,791	152,998
2017	35,798	210,599	16,205	7.7%	159,908	275,851	210,505	221,397	236,756	253,297
Totals	1,343,162	490,041	31,334	6.4%	405,127	622,322	488,329	510,471	542,250	566,151

Figure A.8. User-selected Parameters for Cape Cod

	Scale	2009	2010	2011	2012	2013	2014	2015	2016	2017
Mean	620.067	1.160	1.123	1.322	1.376	1.521	1.533	1.580	1.169	1.164
Std Dev	30.027	0.066	0.064	0.072	0.075	0.082	0.084	0.091	0.082	0.105
CoV	*4.8%*	*5.7%*	*5.7%*	*5.4%*	*5.4%*	*5.4%*	*5.5%*	*5.8%*	*7.0%*	*9.0%*

Development Period Parameters (Average Incremental)

	24	36	48	60	72	84	96	108	120	132	144	156
Mean	1.181	1.063	0.838	0.534	0.284	0.111	0.067	0.040	0.024	*0.011*	*0.005*	*0.002*
Std Dev	0.041	0.040	0.036	0.029	0.023	0.016	0.016	0.015	0.017	0.009	0.004	0.002
Decay Ratios		*90.0%*	*78.8%*	*63.7%*	*53.2%*	*39.0%*	*60.7%*	*59.4%*	*60.7%*			
CoV	*3.5%*	*3.8%*	*4.3%*	*5.5%*	*8.1%*	*14.9%*	*23.1%*	*37.8%*	*70.6%*	*77.1%*	*83.4%*	*89.7%*

					Tail Extrapolation			Implied Tail Factor		
	K	p	AIC	BIC	Decay Ratio	Periods	Distribution	Adjusted	Actual	Tail Sampling Option
Mean	13.104	0.435	622.3	664.4	46.4%	3	Gamma	1.0037	1.0037	Sampling
Std Dev	1.061	0.087			11.8%					
CoV	*8.1%*	*19.9%*								

Decay Ratio Analysis:

Parameters: User **Curve Type:** Power 3

Least Squares Regression Coefficients:

x^a	−0.3195
coefficient	1.0261

Goodness of Fit Statistics:

R^2 Statistic	0.435
Regression Deviation	11.6%
Suggested Decay Parameters:	
Mean	46.4%
Standard Deviation	11.8%

Periods	Decay Ratio	Outliers	Selected Age	Selected Decay Ratio	Incremental Fitted Factors
24–36	0.900	0	2	0.900	0.822
36–48	0.788	0	3	0.788	0.722
48–60	0.637	0	4	0.637	0.659
60–72	0.532	0	5	0.532	0.614
72–84	0.390	0	6	0.390	0.579
84–96	0.607	0	7	0.607	0.551
96-108	0.594	0	8	0.594	0.528
108–120	0.607	0	9	0.607	0.509
120–132					0.492
132–144					0.477
144–156					0.464
156–168					0.452
168–180					0.442

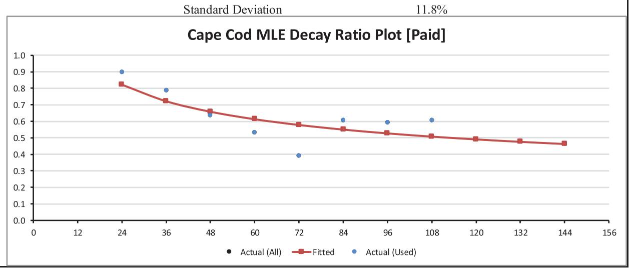


Figure A.9. Residual Graphs for Cape Cod [Modeled Parameters]

Residual Graphs for Cape Cod Hayne MLE Paid Model [Model Fitted]

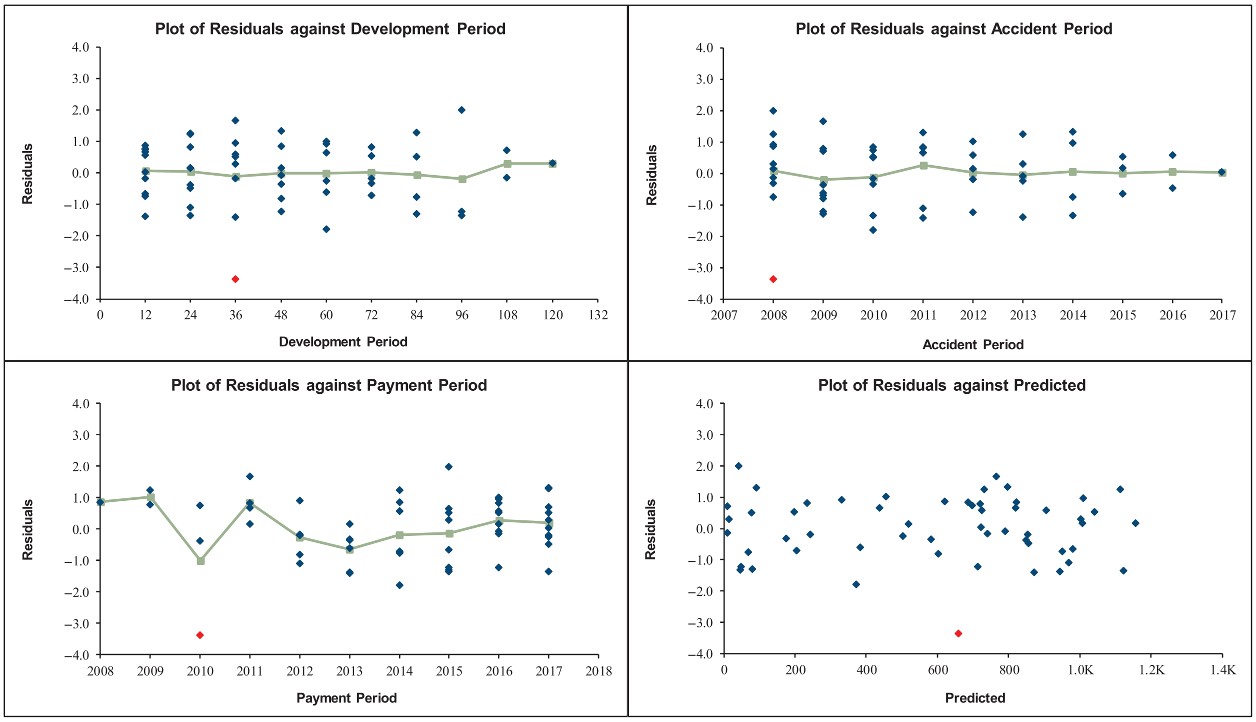


Figure A.10. Residual Graphs for Cape Cod [Selected Parameters]

Residual Graphs for Cape Cod Hayne MLE Paid Model [User Selected]

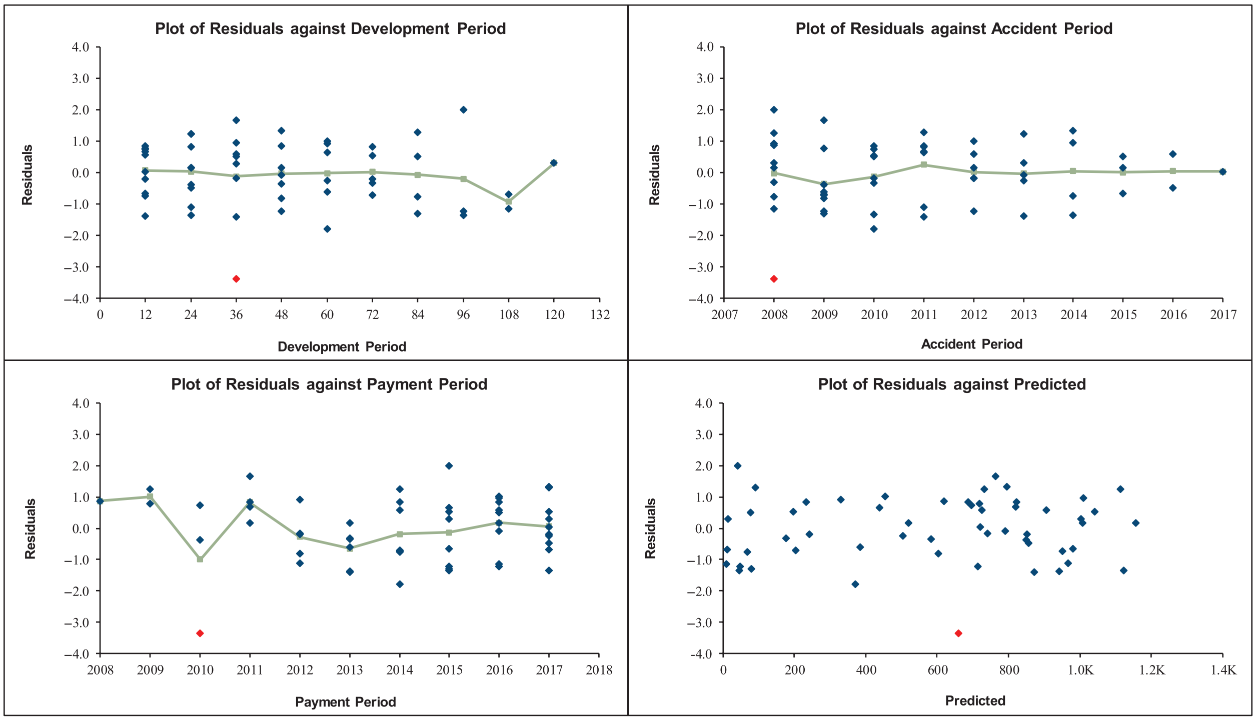
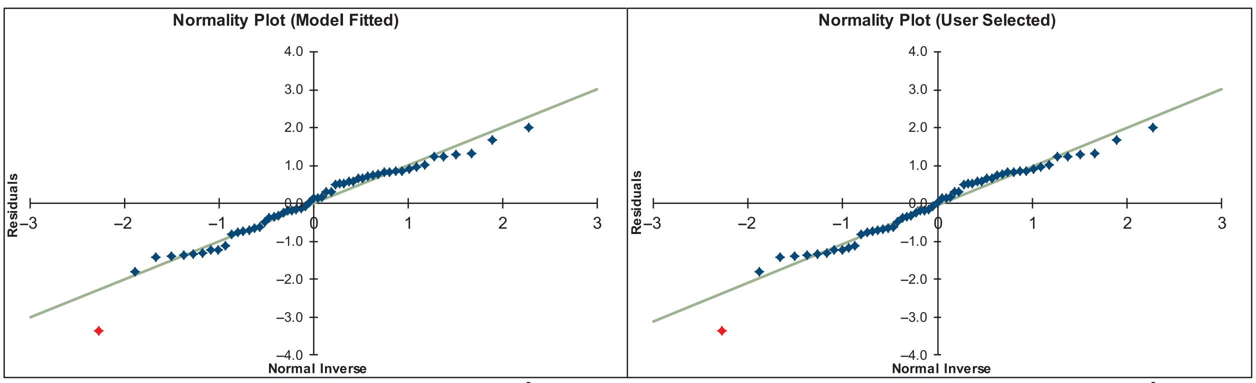


Figure A.11. Normality Plots for Cape Cod

Cape Cod Normality Plots (Paid)



N = 55 P-Value = 5.7% R^2 = 95.9%
Normal: MU = 0.00, Sigma = 1.01 AIC = 581.3, BIC = 585.3

N = 55 P-Value = 8.7% R^2 = 96.4%
Normal: MU = -0.04, Sigma = 1.02 AIC = 584.3, BIC = 588.3

Figure A.12. Box-whisker Plots for Cape Cod

Cape Cod Box-Whisker Plots (Paid)

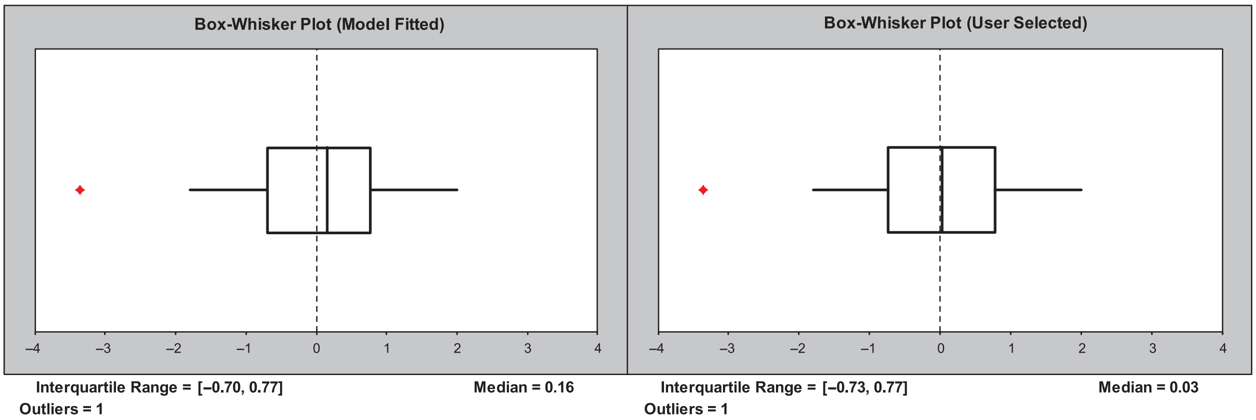


Figure A.13. Model Structure Graphs for Cape Cod

Cape Cod Model Structure [User Selected] (Paid)

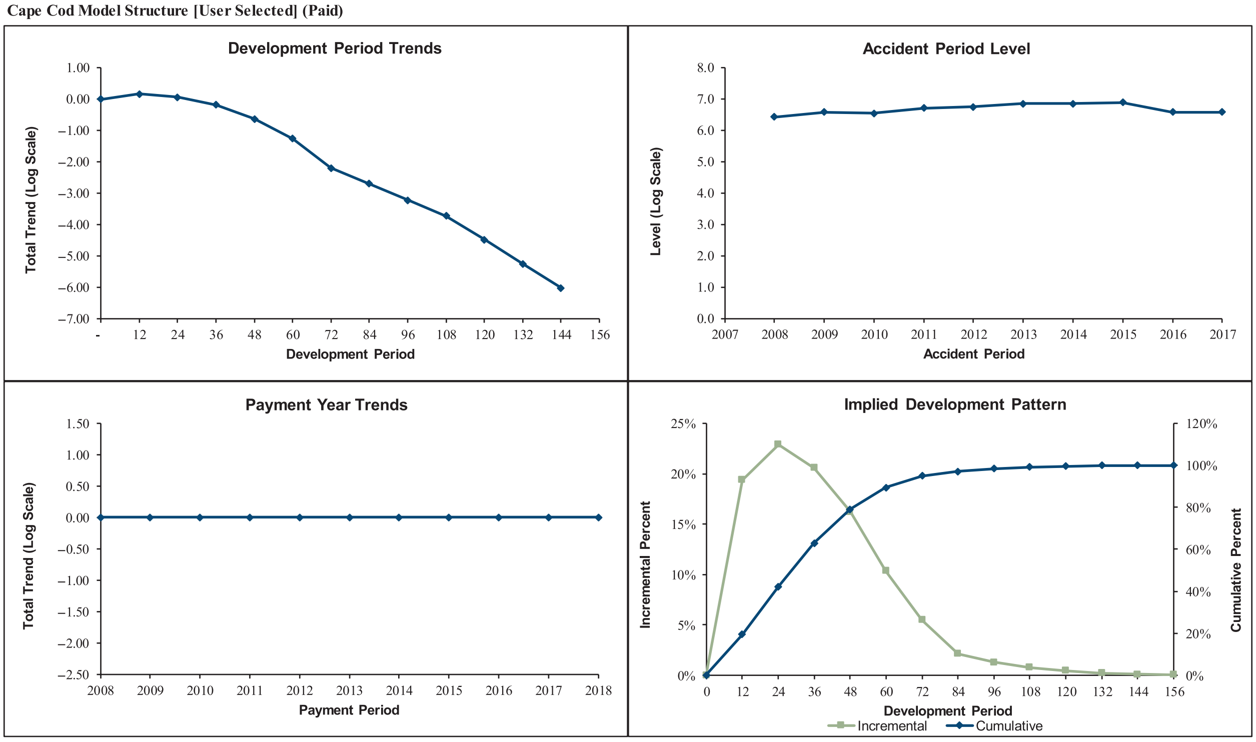


Figure A.14. Estimated Unpaid Results for Cape Cod

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	485	637	131.4%	(2,193)	3,075	421	785	1,636	2,456
2009	140,983	1,201	1,173	97.7%	(2,990)	6,332	1,160	1,900	3,229	4,144
2010	147,516	2,355	1,519	64.5%	(3,142)	9,157	2,276	3,334	4,886	6,102
2011	174,349	5,172	1,995	38.6%	(1,900)	13,402	5,129	6,491	8,478	9,747
2012	173,637	9,239	2,512	27.2%	292	17,346	9,182	10,836	13,327	15,490
2013	174,996	20,571	3,205	15.6%	10,947	31,846	20,594	22,724	25,753	27,992
2014	169,224	44,568	4,470	10.0%	27,639	57,928	44,413	47,663	51,822	55,511
2015	134,010	78,842	6,091	7.7%	56,709	98,780	78,954	82,736	88,985	94,935
2016	68,911	93,698	7,922	8.5%	71,173	119,141	93,937	99,118	106,485	112,562
2017	35,798	147,763	13,927	9.4%	102,107	193,192	147,105	157,329	171,694	182,349
Totals	1,343,162	403,895	23,704	5.9%	324,609	495,709	404,213	418,598	443,687	457,476

Figure A.15. User-selected Parameters for Chain Ladder

User Selected Parameters:

	12	24	36	48	60	72	84	96	108	120	132	144	156
Mean	0.195	0.231	0.208	0.164	0.104	0.056	0.022	0.013	0.006	0.002	*0.001*	*0.000*	*0.000*
Std Dev	0.005	0.005	0.005	0.005	0.005	0.004	0.003	0.003	0.002	0.002	*0.001*	*0.000*	*0.000*
Decay Ratios:		118.1%	90.0%	78.8%	63.7%	53.2%	39.0%	60.8%	41.8%	41.1%			
CoV:	*2.5%*	*2.3%*	*2.5%*	*3.1%*	*4.5%*	*7.3%*	*14.3%*	*22.6%*	*45.1%*	*96.4%*	*103.0%*	*109.6%*	*116.3%*

	K	p	AIC	BIC	Tail Extrapolation Decay Ratio	Tail Extrapolation Periods	Tail Extrapolation Distribution	Implied Tail Factor Adjusted	Implied Tail Factor Actual	Tail Sampling Option
Mean	13.074	0.438	621.6	663.8	37.6%	3	Gamma	1.0013	1.0013	Sampling
Std Dev	1.018	0.083			9.8%					
CoV:	*7.8%*	*19.0%*								

Decay Ratio Analysis:
Parameters: User **Curve Type:** Power 3

Least Squares Regression Coefficients:
x^a –0.4965
coefficient 1.2345

Goodness of Fit Statistics:
R2 Statistic 0.870
Regression Deviation 9.5%
Suggested Decay Parameters:
Mean 37.6%
Standard Deviation 9.8%

Periods	Decay Ratio	Outliers	Selected Age	Selected Decay Ratio	Incremental Fitted Factors
12–24	1.181	0	1	1.181	1.235
24–36	0.900	0	2	0.900	0.875
36–48	0.788	0	3	0.788	0.715
48–60	0.637	0	4	0.637	0.620
60–72	0.532	0	5	0.532	0.555
72–84	0.390	0	6	0.390	0.507
84–96	0.608	0	7	0.608	0.470
96- 108	0.418	0	8	0.418	0.440
108–120	0.411	0	9	0.411	0.415
120–132					0.394
132–144					0.375
144–156					0.359
156–168					0.345

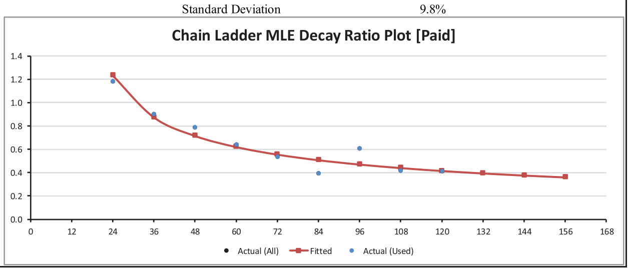


Figure A.16. Residual Graphs for Chain Ladder [Modeled Parameters]

Residual Graphs for Chain Ladder Hayne MLE Paid Model [Model Fitted]

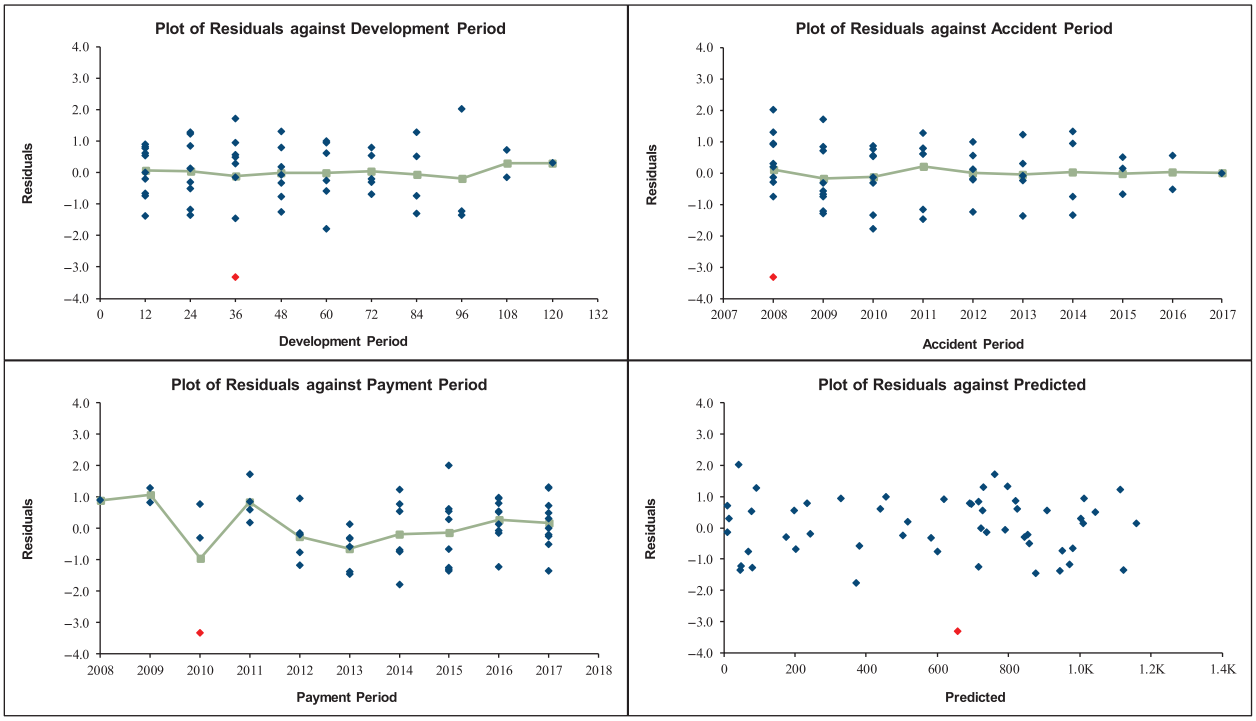


Figure A.17. Residual Graphs for Chain Ladder [Selected Parameters]

Residual Graphs for Chain Ladder Hayne MLE Paid Model [User Selected]

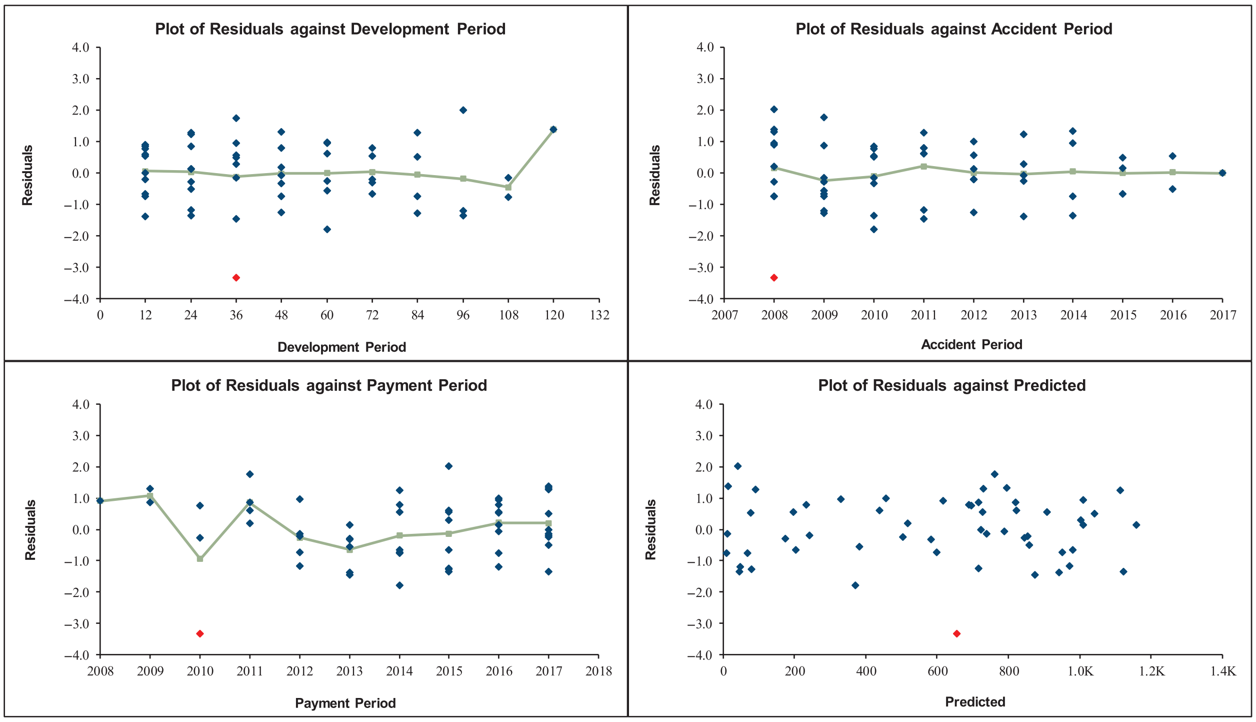


Figure A.18. Normality Plots for Chain Ladder

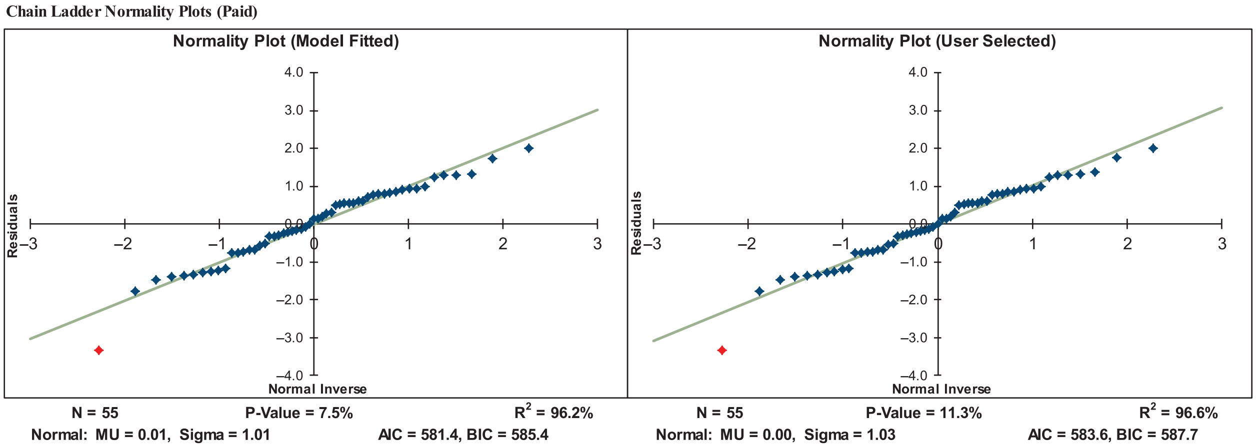


Figure A.19. Box-whisker Plots for Chain Ladder

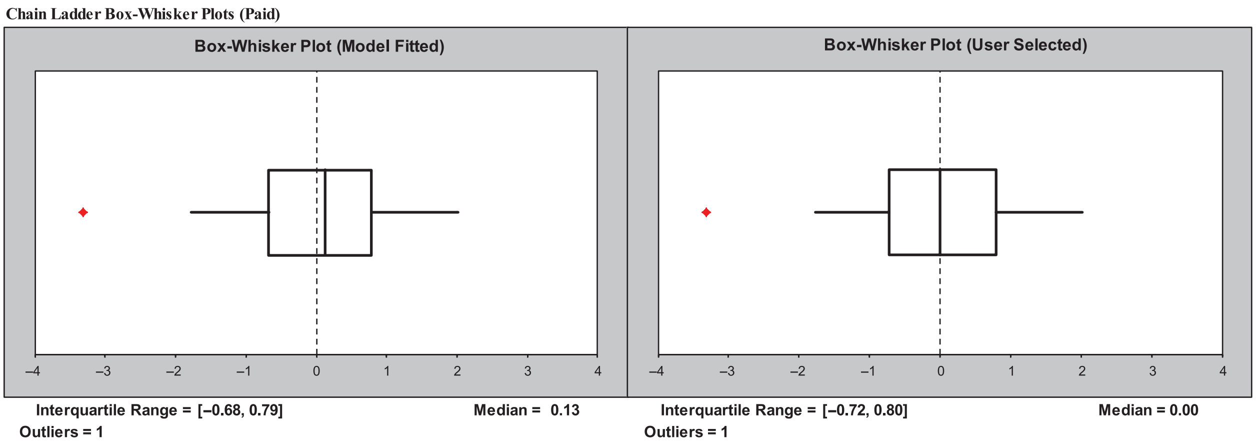


Figure A.20. Model Structure Graphs for Chain Ladder

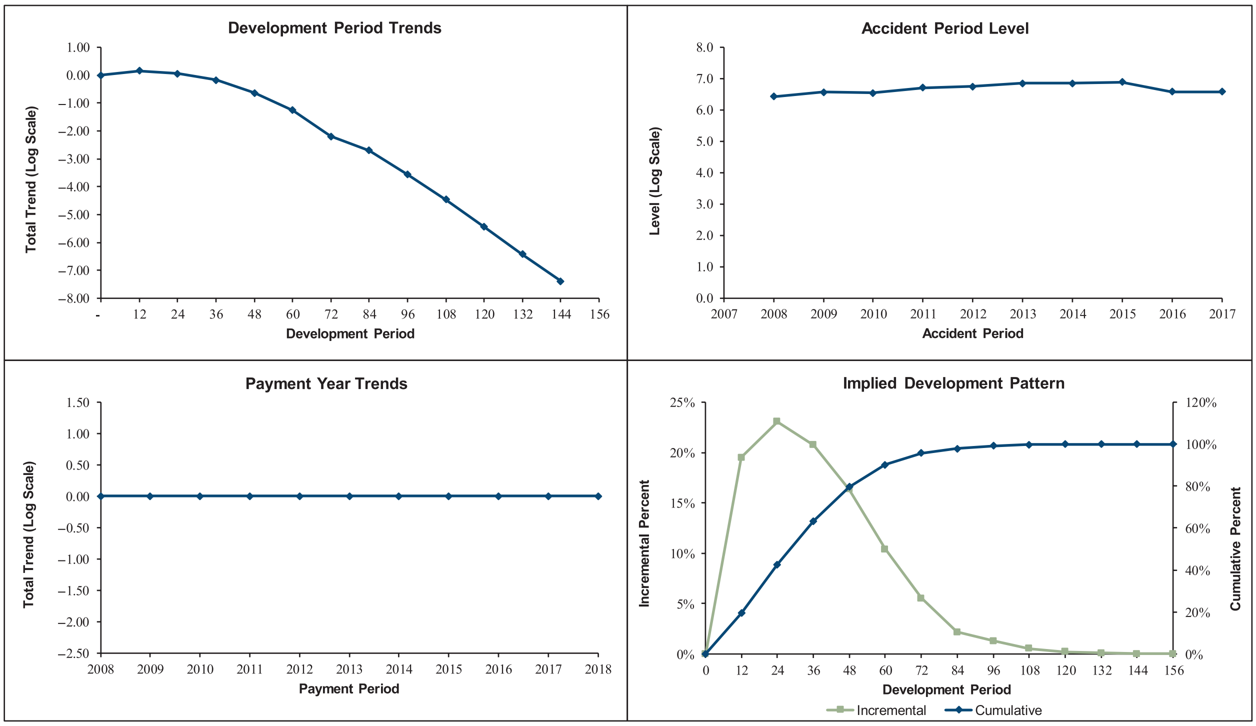


Figure A.21. Estimated Unpaid Results for Chain Ladder

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	168	402	239.3%	(1,683)	2,108	130	353	870	1,376
2009	140,983	477	744	156.0%	(2,553)	3,939	366	917	1,801	2,334
2010	147,516	1,281	1,006	78.5%	(2,716)	4,601	1,265	1,862	2,952	3,881
2011	174,349	3,975	1,577	39.7%	(1,884)	9,378	3,961	4,936	6,674	8,256
2012	173,637	8,073	2,018	25.0%	1,479	14,671	8,054	9,463	11,359	12,842
2013	174,996	19,370	2,737	14.1%	9,886	27,967	19,378	21,243	23,668	26,080
2014	169,224	43,332	4,046	9.3%	29,894	55,614	43,274	46,198	49,868	52,771
2015	134,010	77,959	5,022	6.4%	58,459	97,798	77,888	81,248	85,985	90,065
2016	68,911	93,147	5,499	5.9%	78,946	109,522	93,001	96,830	102,453	106,589
2017	35,798	147,782	7,657	5.2%	126,331	174,935	147,669	152,763	160,526	166,069
Totals	1,343,162	395,563	16,256	4.1%	332,424	451,082	395,463	405,138	423,244	436,972

Figure A.22. User-Selected Parameters for Hoerl Curve

User Selected Parameters:

	Level	d	d^2	ln(d)	Trend
Mean	6.496	0.005	(0.065)	0.596	0.043
Std Dev	0.220	0.240	0.019	0.323	0.008
CoV:	*3.4%*	*4687.1%*	*–28.4%*	*54.2%*	*19.5%*

	K	p	AIC	BIC	Tail Extrapolation Periods	Implied Tail Factor Adjusted	Implied Tail Factor Actual
Mean	13.147	0.506	635.9	649.9	3	1.0004	1.0004
Std Dev	1.014	0.083					
CoV:	*7.7%*	*16.3%*					

Figure A.23. Residual Graphs for Hoerl Curve [Modeled Parameters]

Residual Graphs for Hoerl Curve Hayne MLE Paid Model [Model Fitted]

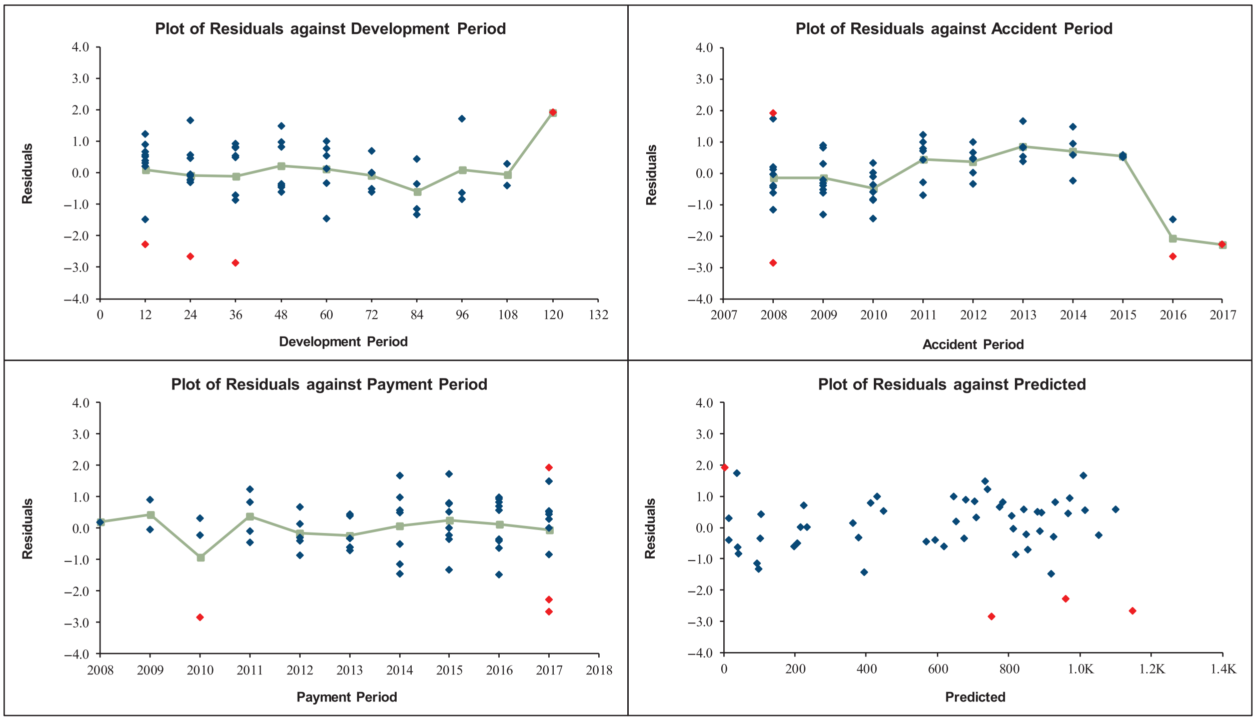


Figure A.24. Residual Graphs for Hoerl Curve [Selected Parameters]

Residual Graphs for Hoerl Curve Hayne MLE Paid Model [User Selected]

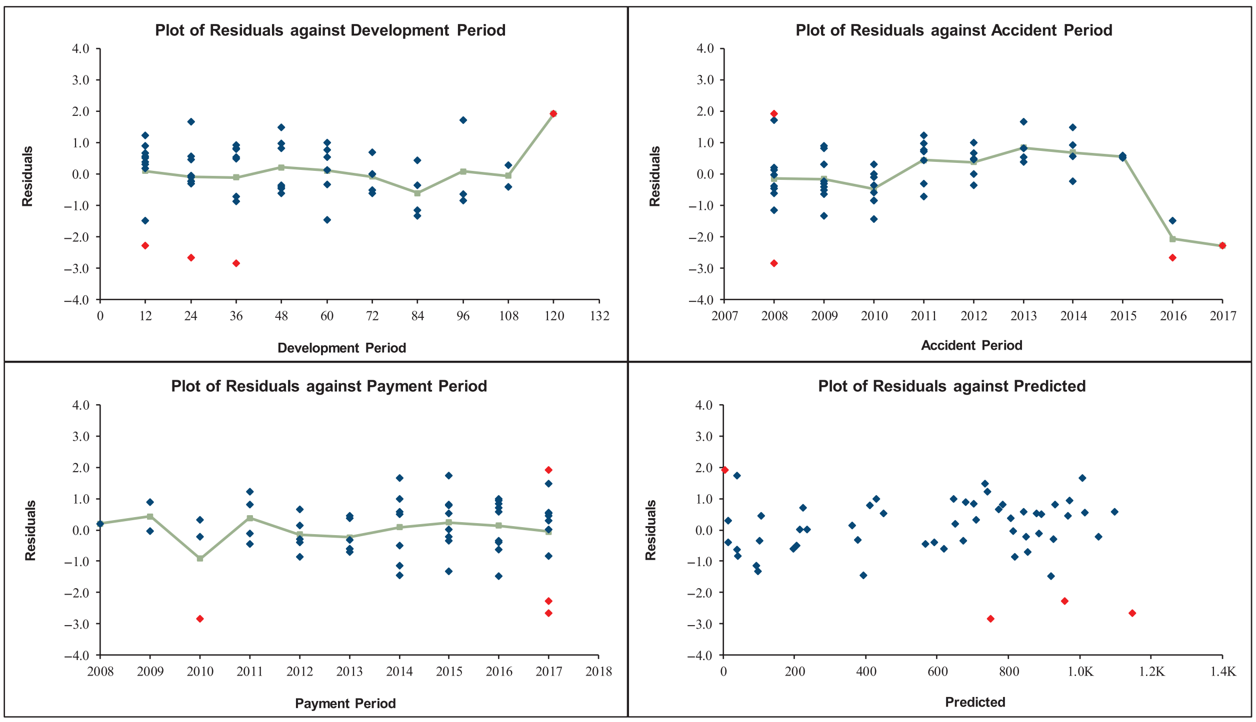


Figure A.25. Normality Plots for Hoerl Curve

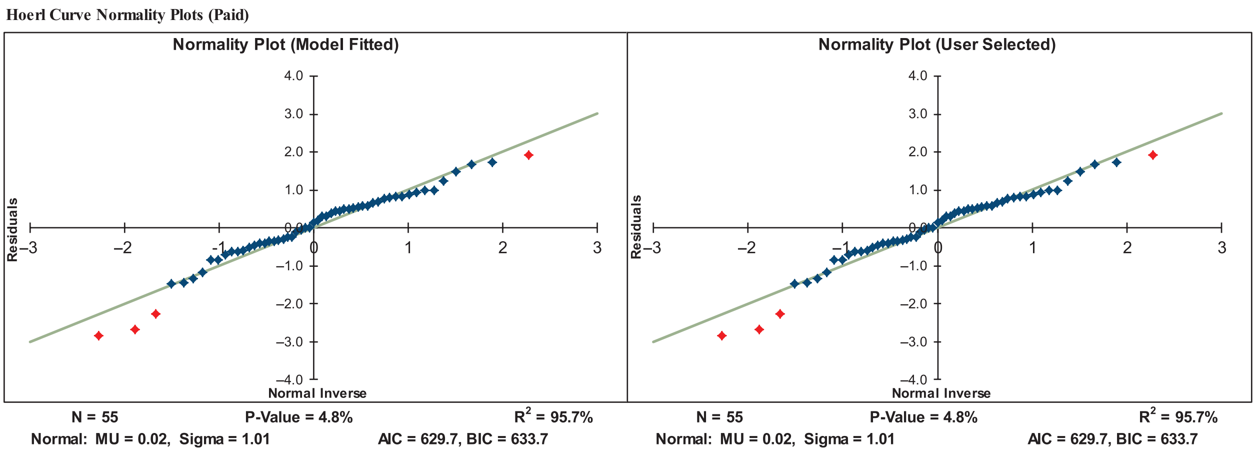


Figure A.26. Box-Whisker Plots for Hoerl Curve

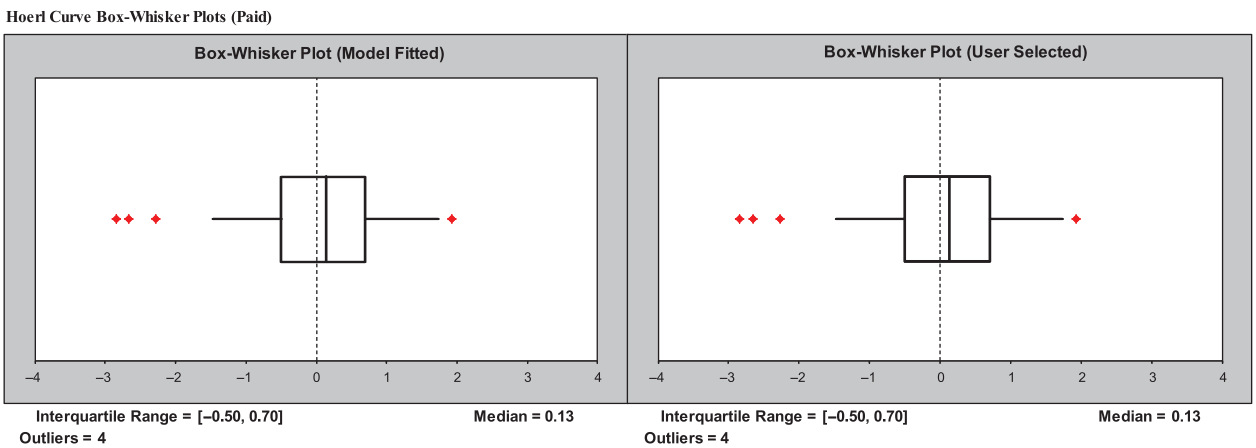


Figure A.27. Model Structure Graphs for Hoerl Curve

Hoerl Curve Model Structure [User Selected] (Paid)

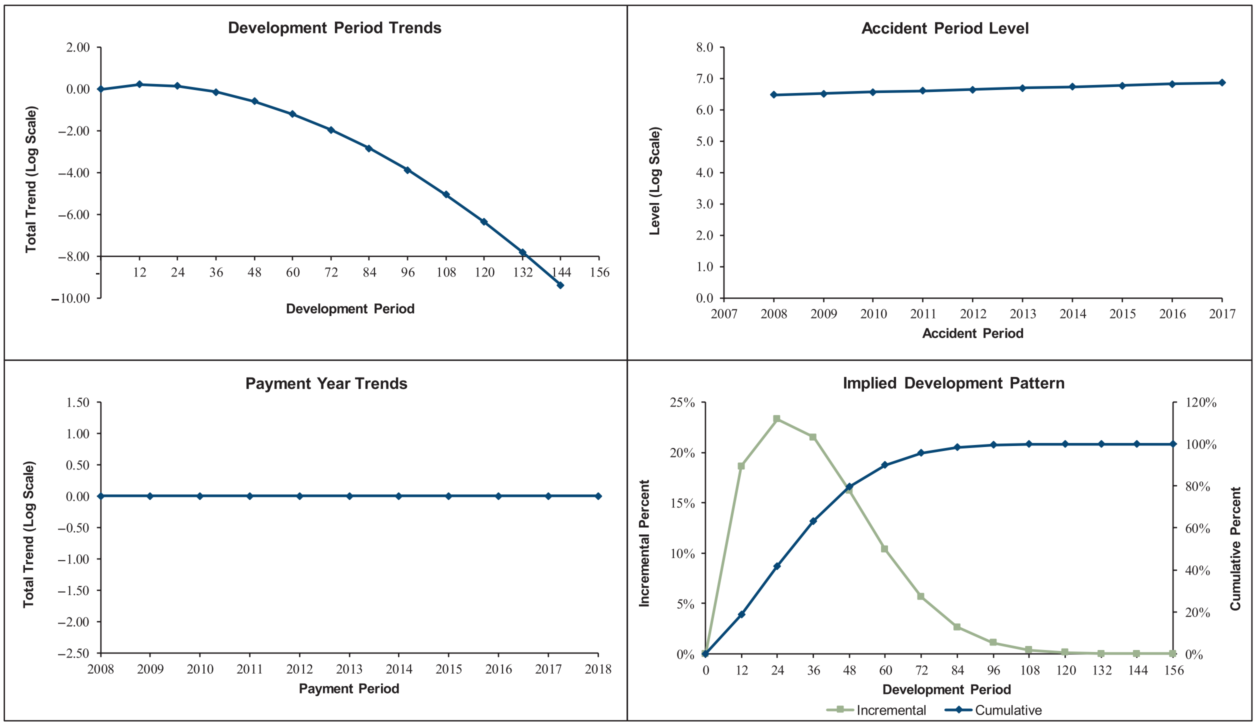


Figure A.28. Estimated Unpaid Results for Hoerl Curve

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	86	241	279.2%	(1,438)	1,992	79	206	453	764
2009	140,983	269	447	166.4%	(1,769)	2,015	269	516	1,001	1,509
2010	147,516	919	849	92.4%	(2,819)	4,501	868	1,406	2,391	3,153
2011	174,349	2,872	1,469	51.1%	(1,462)	7,843	2,871	3,766	5,234	6,712
2012	173,637	7,681	2,471	32.2%	(2,851)	15,565	7,663	9,285	11,843	13,506
2013	174,996	17,664	3,672	20.8%	6,019	31,220	17,487	20,159	24,031	26,361
2014	169,224	40,416	5,400	13.4%	21,019	58,254	40,552	44,130	48,818	53,580
2015	134,010	73,354	7,454	10.2%	45,679	94,722	73,494	78,399	85,417	91,003
2016	68,911	125,089	10,640	8.5%	94,695	157,632	125,071	131,940	142,229	151,840
2017	35,798	207,924	15,017	7.2%	153,768	252,433	208,208	217,912	232,223	241,497
Totals	1,343,162	476,274	28,803	6.0%	387,439	571,460	476,387	494,362	522,640	546,673

Figure A.29. User-Selected Parameters for Wright

	User Selected Parameters:									
	2008	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**
Mean	6.312	6.472	6.436	6.587	6.636	6.738	6.742	6.771	6.475	6.468
Std Dev	0.168	0.167	0.167	0.166	0.167	0.167	0.166	0.164	0.166	0.184
CoV	*2.7%*	*2.6%*	*2.6%*	*2.5%*	*2.5%*	*2.5%*	*2.5%*	*2.4%*	*2.6%*	*2.8%*
	Development Period Parameters (Average Incremental)									
		d	**d^2**	**ln(d)**						
Mean		0.192	(0.078)	0.290						
Std Dev		0.183	0.015	0.232						
CoV		*95.4%*	*-19.5%*	*80.0%*						
							Tail Extrapolation		**Implied Tail Factor**	
		K	**p**	**AIC**	**BIC**		**Periods**		**Adjusted**	**Actual**
Mean		14.592	0.319	612.3	642.4		3		1.0003	1.0003
Std Dev		0.909	0.075							
CoV		*6.2%*	*23.4%*							

Figure A.30. Residual Graphs for Wright [Modeled Parameters]

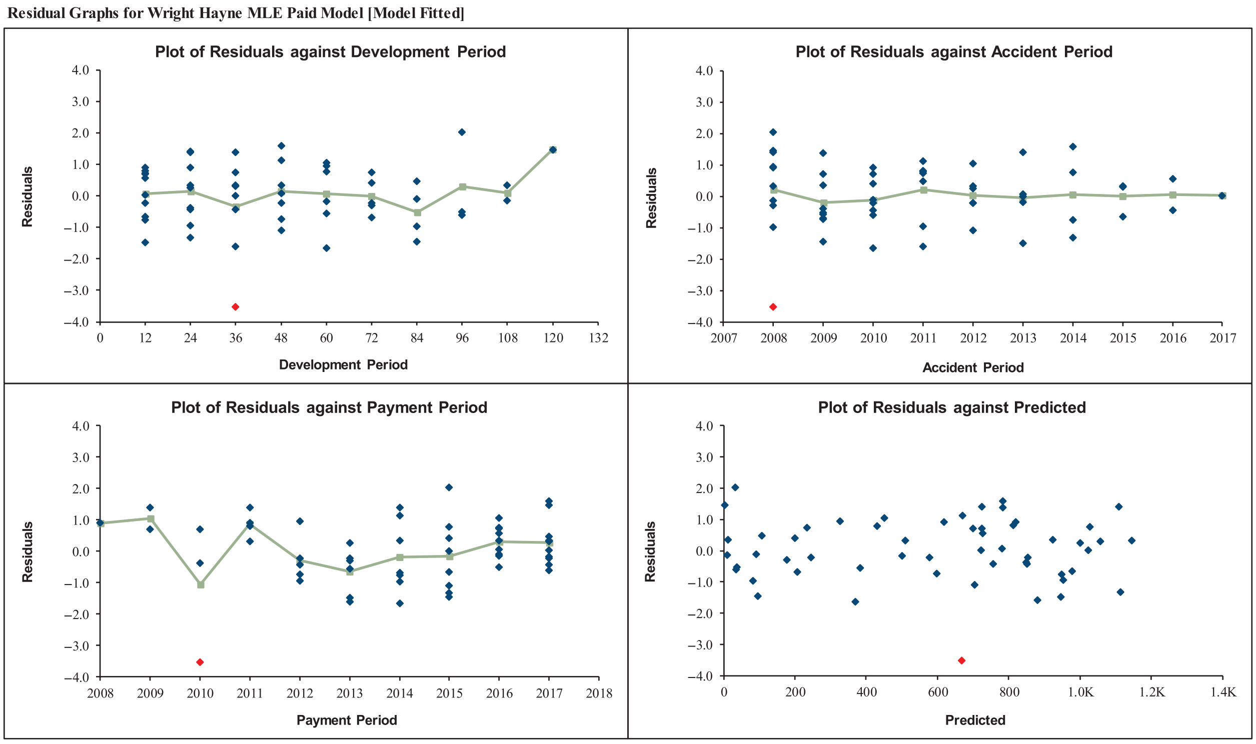


Figure A.31. Residual Graphs for Wright [Selected Parameters]

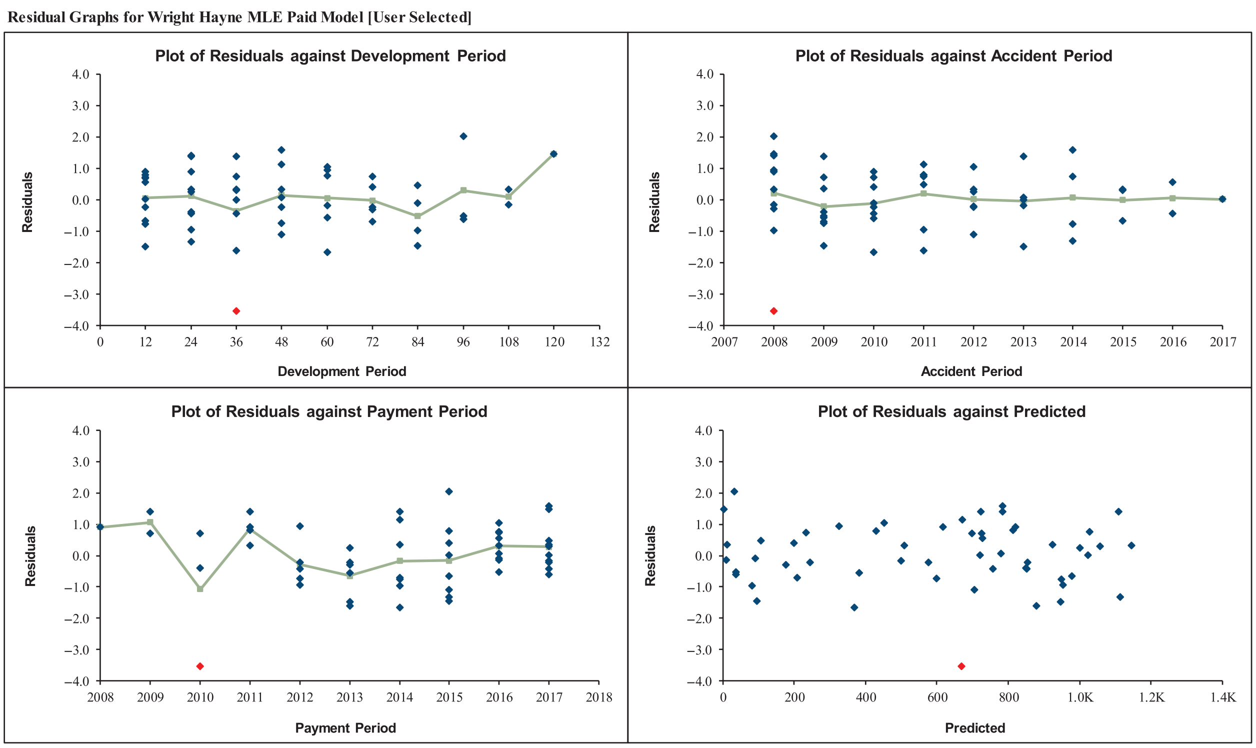


Figure A.32. Normality Plots for Wright

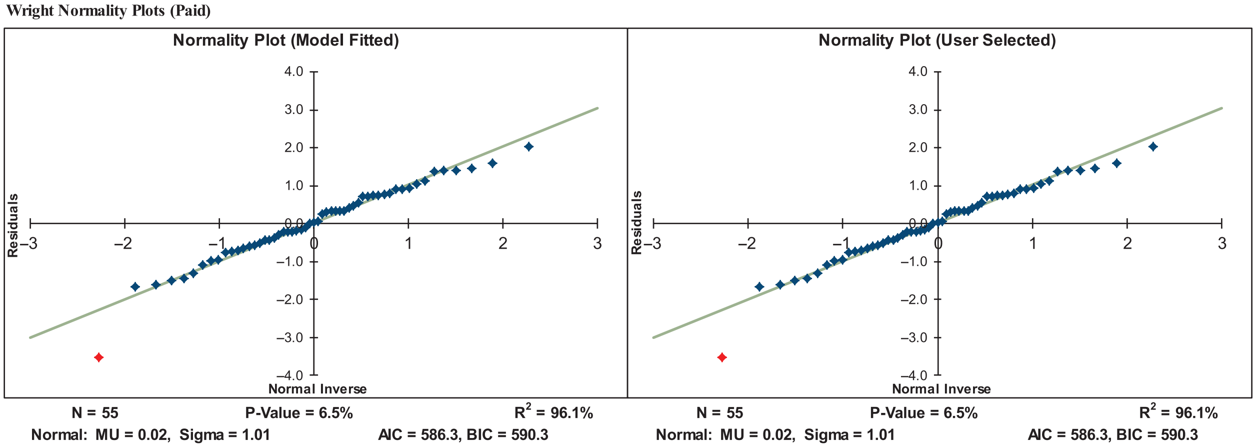


Figure A.33. Box-Whisker Plots for Wright

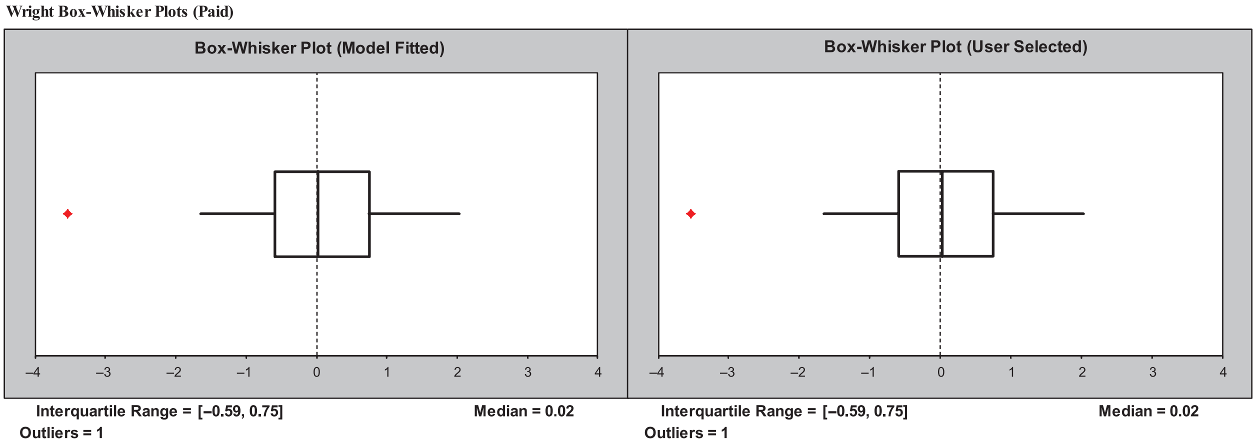


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Figure A.34. Model Structure Graphs for Wright

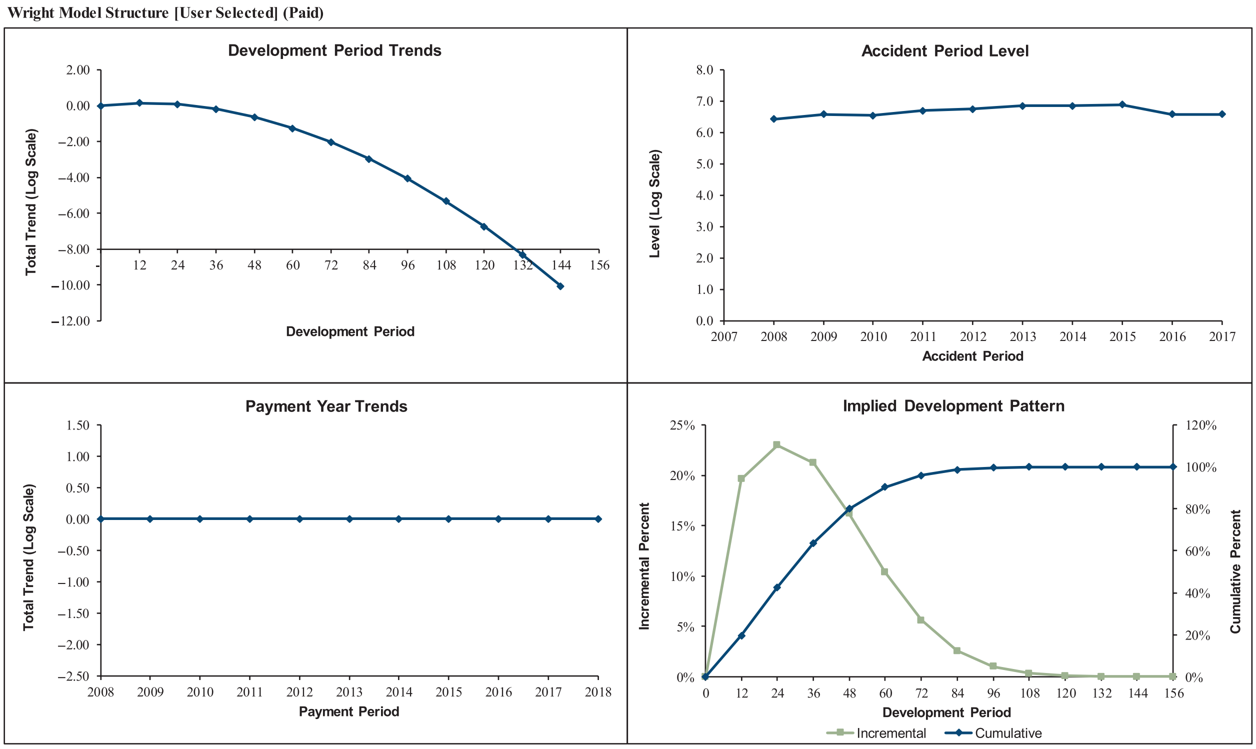

Figure A.35. Estimated Unpaid Results for Wright

Sample Insurance Company
Hayne Paper Data
Accident Year Unpaid (in 000's)
Paid Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	123,738	64	406	632.5%	(2,161)	2,001	65	287	682	1,112
2009	140,983	218	661	303.7%	(3,077)	3,353	227	578	1,290	1,812
2010	147,516	694	1,013	146.0%	(3,572)	5,289	703	1,263	2,316	3,461
2011	174,349	2,715	1,574	58.0%	(3,983)	8,066	2,693	3,714	5,356	6,613
2012	173,637	7,597	2,363	31.1%	(686)	15,029	7,634	9,211	11,331	12,805
2013	174,996	19,119	3,264	17.1%	8,387	29,546	19,046	21,478	24,484	26,398
2014	169,224	42,804	4,595	10.7%	24,538	58,172	42,809	45,807	50,456	53,909
2015	134,010	76,810	5,904	7.7%	57,127	98,936	76,908	80,434	87,177	91,415
2016	68,911	93,415	7,652	8.2%	70,266	119,305	93,165	98,465	106,517	112,357
2017	35,798	147,450	13,815	9.4%	108,429	194,441	146,745	156,152	172,807	182,299
Totals	1,343,162	390,884	21,640	5.5%	321,950	490,197	389,943	405,111	427,283	442,582

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Appendix B—Schedule P, Part A Results

In this appendix the results for Schedule P, Part A (Homeowners/Farmowners) are shown.

Figure B.1. Estimated Unpaid Model Results (Paid Berquist-Sherman)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Paid Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	1	2	180.3%	(13)	12	1	3	6	9
2009	6,470	3	5	145.9%	(12)	25	3	6	11	19
2010	7,848	9	11	119.8%	(26)	45	8	15	27	36
2011	7,020	18	19	106.5%	(47)	103	18	30	50	65
2012	7,291	38	33	88.7%	(84)	218	38	59	94	118
2013	8,134	80	60	75.4%	(120)	263	79	120	177	219
2014	10,800	181	113	62.5%	(211)	575	181	253	362	478
2015	7,522	342	207	60.6%	(274)	1,106	343	470	707	810
2016	7,968	789	427	54.2%	(727)	2,126	800	1,062	1,461	1,789
2017	9,309	4,880	1,850	37.9%	(2,872)	11,865	4,846	6,061	7,993	9,246
Totals	77,596	6,340	1,916	30.2%	(896)	13,657	6,355	7,623	9,484	10,650
Normal Dist.		6,340	1,916	30.2%			6,340	7,632	9,491	10,797
logNormal Dist.		6,791	3,793	55.9%			5,929	8,426	13,971	19,927
Gamma Dist.		6,340	1,916	30.2%			6,149	7,507	9,785	11,624

Figure B.2. Total Unpaid Claims Distribution (Paid Berquist-Sherman)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Total Unpaid Distribution (in 000's)
Paid Berquist & Sherman Model

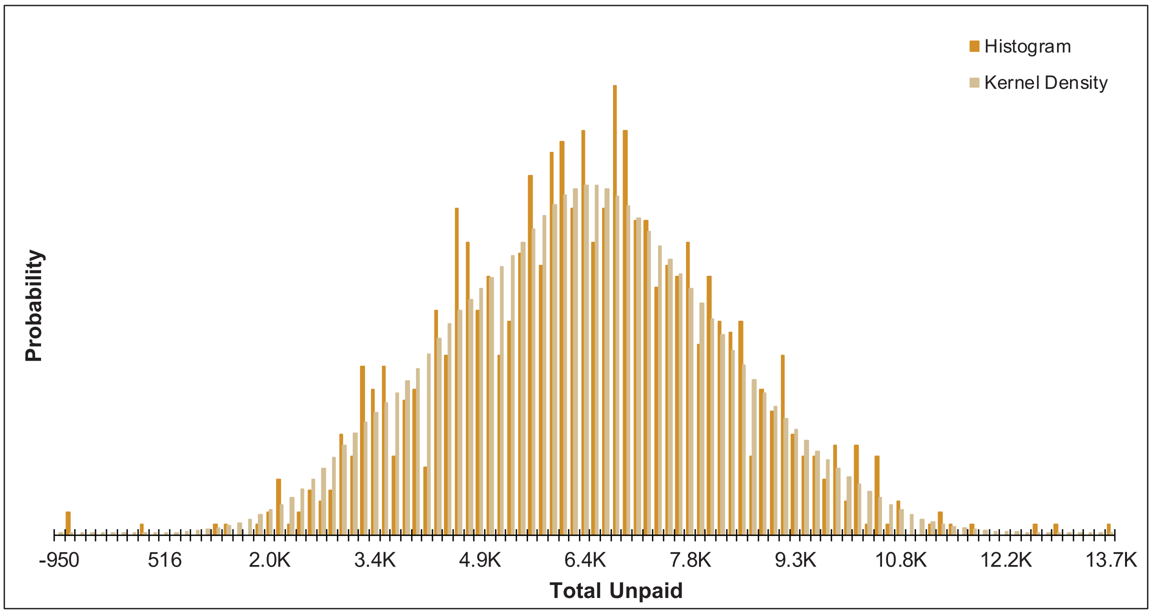

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Figure B.3. Estimated Unpaid Model Results (Incurred Berquist-Sherman)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Incurred Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	1	4	296.4%	(54)	50	1	3	7	13
2009	6,470	3	9	267.7%	(172)	109	3	6	15	23
2010	7,848	10	35	354.9%	(735)	675	8	16	31	50
2011	7,020	21	41	189.4%	(106)	1,032	18	31	60	96
2012	7,291	44	138	311.7%	(155)	3,281	32	56	107	251
2013	8,134	82	105	129.2%	(1,215)	1,430	70	114	218	400
2014	10,800	181	289	159.6%	(5,037)	5,874	159	252	419	713
2015	7,522	339	684	201.7%	(12,497)	9,046	282	453	902	1,762
2016	7,968	794	2,795	351.9%	(63,725)	50,307	656	965	1,816	3,496
2017	9,309	4,260	2,334	54.8%	(695)	46,021	4,081	5,048	7,206	11,774
Totals	77,596	5,736	3,744	65.3%	(56,400)	54,796	5,441	6,633	9,385	14,683
Normal Dist.		5,736	3,744	65.3%			5,736	8,262	11,895	14,446
logNormal Dist.		6,881	6,211	90.3%			5,108	8,597	18,185	30,775
Gamma Dist.		5,736	3,744	65.3%			4,945	7,637	12,945	17,771

Figure B.4. Total Unpaid Claims Distribution (Incurred Berquist-Sherman)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Total Unpaid Distribution (in 000's)
Incurred Berquist & Sherman Model

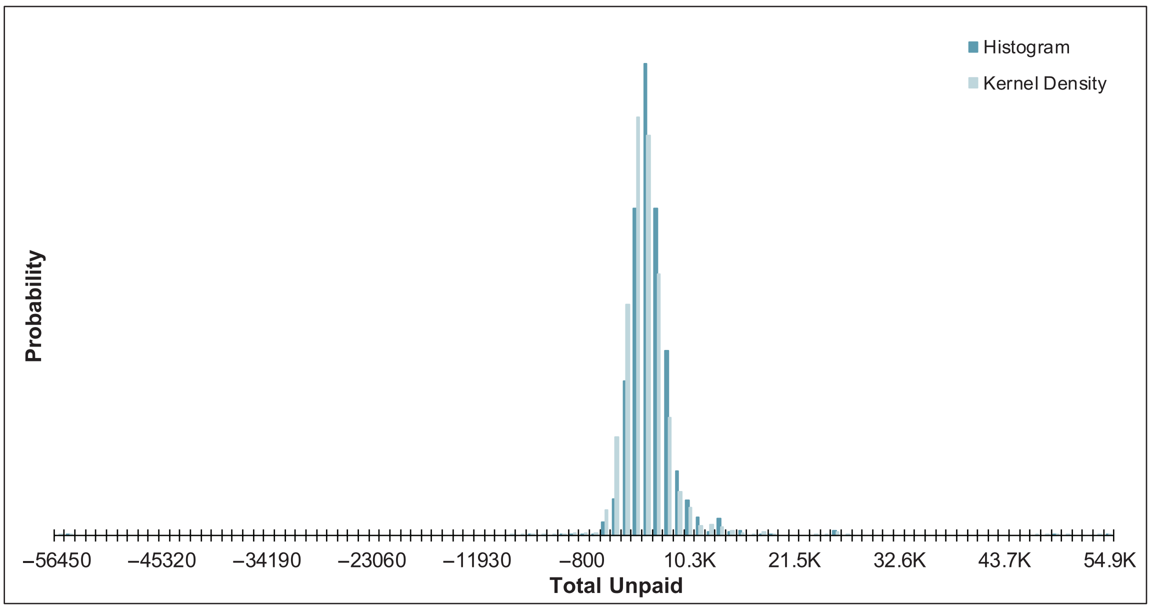

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Figure B.5. Estimated Unpaid Model Results (Paid Cape Cod)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Paid Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	2	2	145.7%	(7)	13	1	3	6	9
2009	6,470	4	4	112.8%	(8)	29	3	5	10	17
2010	7,848	10	9	88.7%	(19)	40	10	16	26	34
2011	7,020	21	16	76.6%	(32)	76	22	32	47	62
2012	7,291	40	28	70.0%	(50)	130	40	60	84	106
2013	8,134	81	48	59.7%	(88)	286	81	112	156	199
2014	10,800	240	119	49.6%	(124)	659	240	323	441	501
2015	7,522	298	157	52.7%	(398)	969	301	395	553	677
2016	7,968	717	336	46.8%	(322)	1,894	711	933	1,276	1,577
2017	9,309	3,937	1,416	36.0%	(6)	8,153	3,904	4,835	6,319	7,312
Totals	77,596	5,350	1,478	27.6%	1,256	10,155	5,392	6,272	7,856	8,680
Normal Dist.		5,350	1,478	27.6%			5,350	6,347	7,781	8,788
logNormal Dist.		5,374	1,707	31.8%			5,121	6,313	8,529	10,535
Gamma Dist.		5,350	1,478	27.6%			5,214	6,259	7,990	9,374

Figure B.6. Total Unpaid Claims Distribution (Paid Cape Cod)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Total Unpaid Distribution (in 000's)
Paid Cape Cod Model

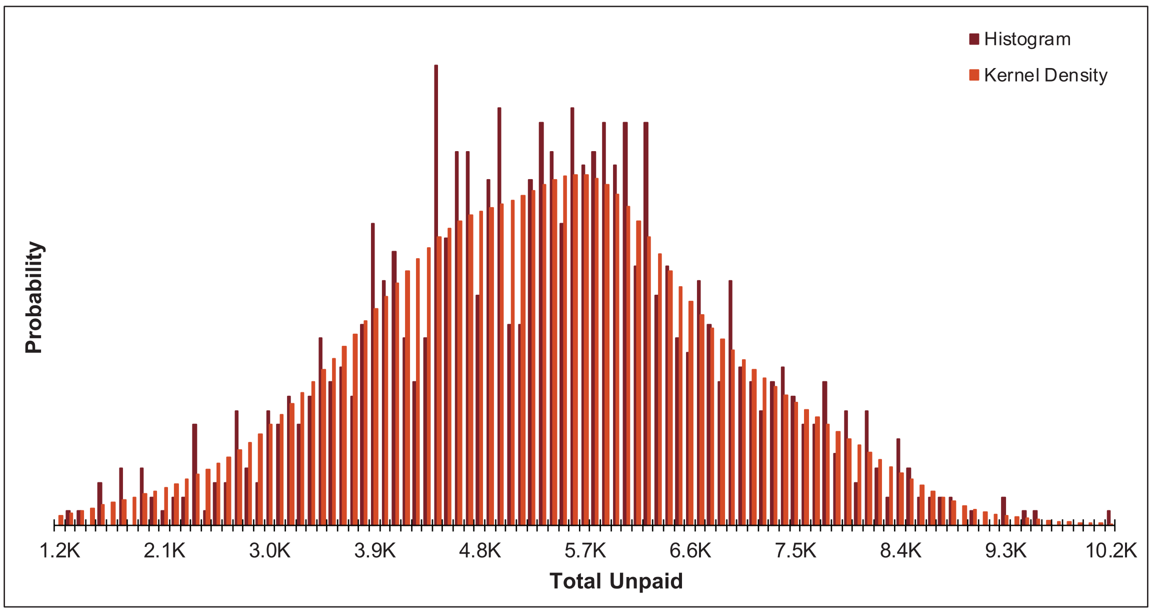

Figure B.7. Estimated Unpaid Model Results (Incurred Cape Cod)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Incurred Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	1	3	185.0%	(28)	20	1	2	6	10
2009	6,470	3	10	283.2%	(235)	59	3	6	11	25
2010	7,848	11	13	120.2%	(160)	154	10	17	30	49
2011	7,020	26	28	110.4%	(136)	428	23	36	65	111
2012	7,291	50	114	226.4%	(72)	2,555	40	63	113	204
2013	8,134	92	99	107.8%	(1,066)	1,254	79	122	214	385
2014	10,800	211	164	78.0%	(72)	3,242	189	270	452	668
2015	7,522	297	698	234.9%	(16,494)	5,425	272	404	768	1,308
2016	7,968	1,315	15,993	1216.4%	(9,454)	498,887	652	944	1,711	2,637
2017	9,309	3,884	1,745	44.9%	(5)	21,243	3,736	4,672	6,505	9,280
Totals	77,596	5,890	16,156	274.3%	(12,718)	504,979	5,150	6,196	8,438	11,426
Normal Dist.		5,890	16,156	274.3%			5,890	16,788	32,465	43,476
logNormal Dist.		5,943	3,903	65.7%			4,967	7,439	13,302	20,006
Gamma Dist.		5,890	16,156	274.3%			150	3,384	33,123	80,833

Figure B.8. Total Unpaid Claims Distribution (Incurred Cape Cod)

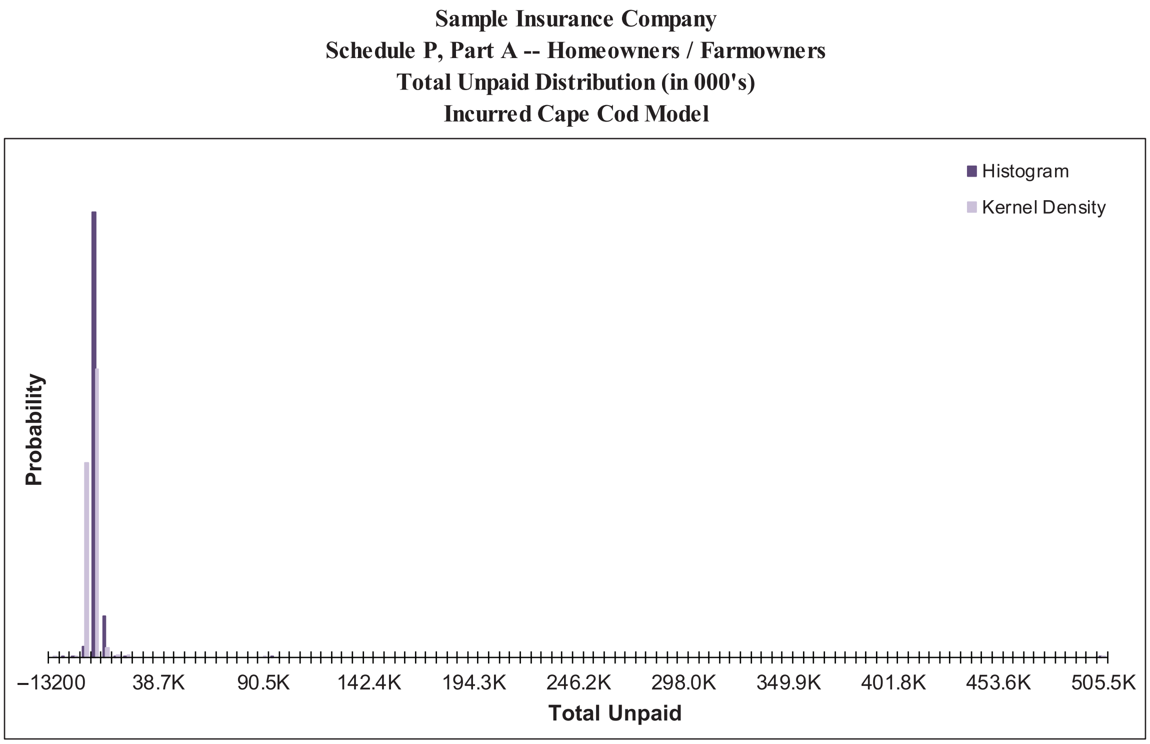

Figure B.9. Estimated Unpaid Model Results (Paid Chain Ladder)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Paid Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	12	11	93.8%	(16)	66	10	17	32	45
2009	6,470	23	17	73.8%	(21)	98	22	33	53	71
2010	7,848	44	23	52.8%	(18)	131	42	59	86	104
2011	7,020	53	25	47.1%	(13)	165	51	70	95	116
2012	7,291	75	29	38.3%	(4)	188	74	95	125	146
2013	8,134	125	36	28.9%	(6)	259	125	149	183	215
2014	10,800	244	57	23.3%	60	413	245	282	339	372
2015	7,522	311	68	21.7%	55	506	311	358	417	451
2016	7,968	698	113	16.1%	355	1,036	694	771	881	969
2017	9,309	3,841	364	9.5%	2,667	4,806	3,832	4,091	4,437	4,673
Totals	77,596	5,425	443	8.2%	3,925	6,815	5,422	5,731	6,159	6,411
Normal Dist.		5,425	443	8.2%			5,425	5,724	6,154	6,456
logNormal Dist.		5,425	449	8.3%			5,407	5,717	6,194	6,552
Gamma Dist.		5,425	443	8.2%			5,413	5,717	6,174	6,509

Figure B.10. Total Unpaid Claims Distribution (Paid Chain Ladder)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Total Unpaid Distribution (in 000's)
Paid Chain Ladder Model

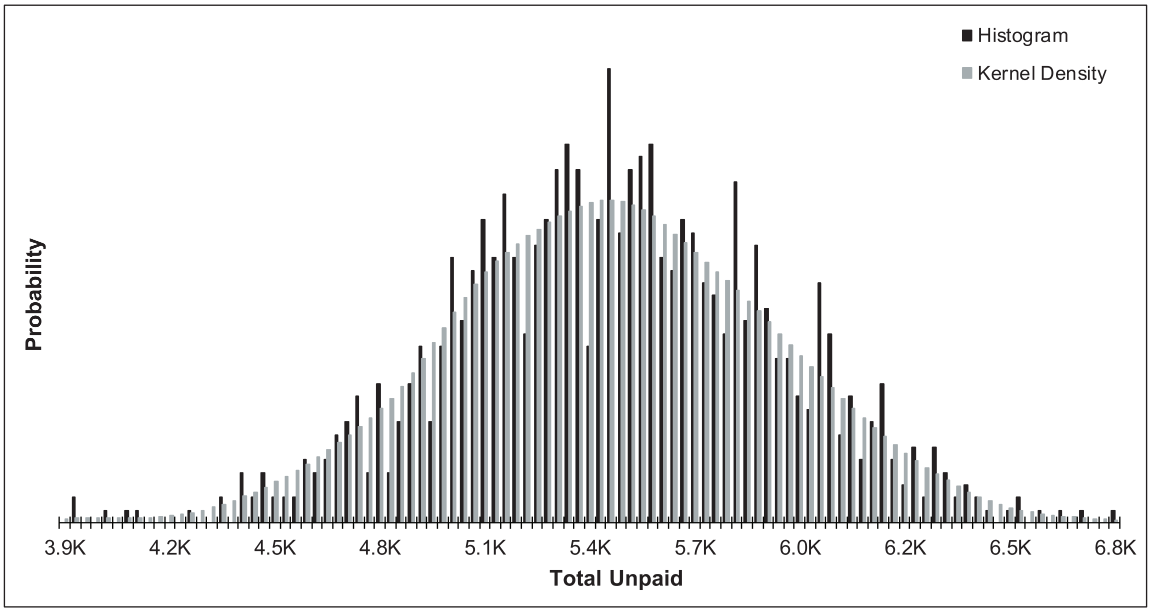

Figure B.11. Estimated Unpaid Model Results (Incurred Chain Ladder)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Incurred Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	12	11	97.4%	(17)	86	10	17	31	45
2009	6,470	23	18	77.9%	(14)	126	21	33	56	71
2010	7,848	43	24	55.5%	(17)	129	41	59	87	109
2011	7,020	52	28	53.3%	(16)	178	49	68	99	135
2012	7,291	74	32	43.2%	(3)	240	71	94	131	161
2013	8,134	124	42	34.4%	(6)	259	122	150	198	237
2014	10,800	243	68	28.0%	45	470	242	287	362	402
2015	7,522	304	87	28.5%	42	633	300	357	459	528
2016	7,968	704	157	22.4%	240	1,476	697	793	969	1,130
2017	9,309	3,701	596	16.1%	1,605	5,935	3,684	4,060	4,695	5,169
Totals	77,596	5,279	651	12.3%	3,057	7,701	5,258	5,697	6,332	6,838
Normal Dist.		5,279	651	12.3%			5,279	5,718	6,349	6,793
logNormal Dist.		5,279	664	12.6%			5,238	5,700	6,437	7,010
Gamma Dist.		5,279	651	12.3%			5,252	5,702	6,393	6,910

Figure B.12. Total Unpaid Claims Distribution (Incurred Chain Ladder)

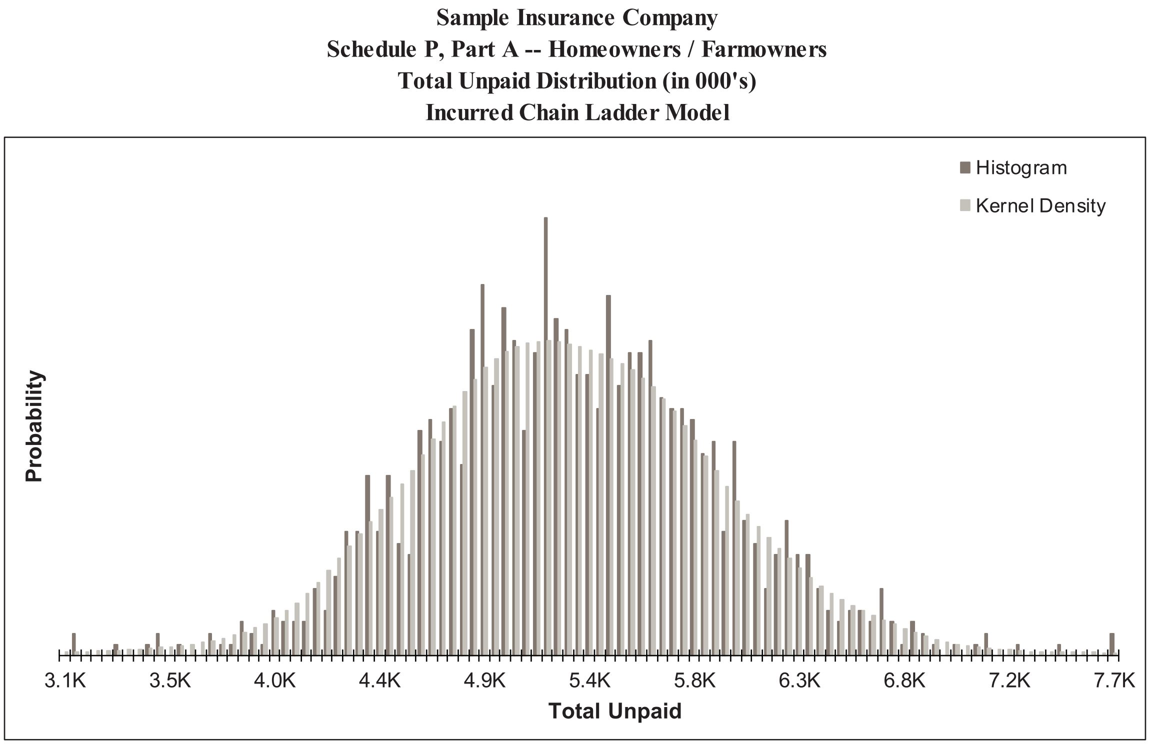

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Figure B.13. Estimated Unpaid Model Results (Paid Hoerl Curve)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Paid Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	-	-		-	-	-	-	-	-
2009	6,470	47	29	63.3%	(37)	170	44	63	100	129
2010	7,848	79	42	52.3%	(47)	262	75	102	154	208
2011	7,020	97	45	46.3%	(42)	291	93	124	173	224
2012	7,291	111	46	41.7%	(34)	329	106	140	193	232
2013	8,134	148	56	38.0%	2	396	142	180	250	317
2014	10,800	236	71	30.0%	35	523	233	277	361	422
2015	7,522	320	78	24.5%	21	613	318	368	452	502
2016	7,968	798	137	17.2%	345	1,259	796	888	1,028	1,127
2017	9,309	4,428	451	10.2%	3,062	5,849	4,422	4,724	5,179	5,546
Totals	77,596	6,264	616	9.8%	4,469	8,520	6,225	6,665	7,308	7,868
Normal Dist.		6,264	616	9.8%			6,264	6,680	7,278	7,698
logNormal Dist.		6,264	618	9.9%			6,234	6,662	7,329	7,837
Gamma Dist.		6,264	616	9.8%			6,244	6,668	7,311	7,786

Figure B.14. Total Unpaid Claims Distribution (Paid Hoerl Curve)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Total Unpaid Distribution (in 000's)
Paid Hoerl Curve Model

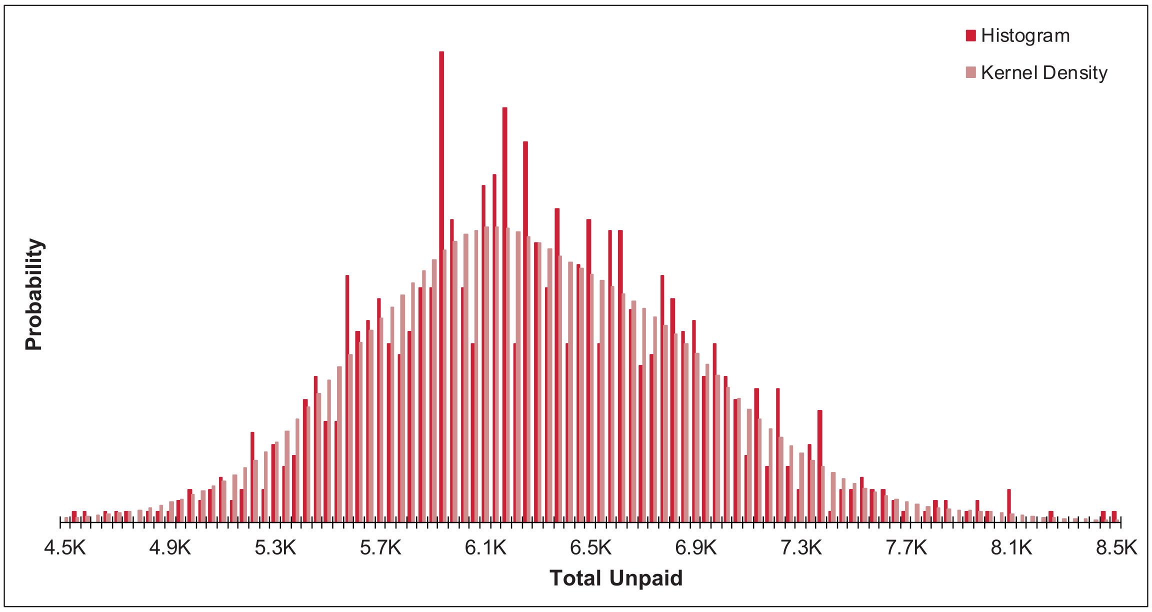

Figure B.15. Estimated Unpaid Model Results (Incurred Hoerl Curve)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Incurred Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	-	-		-	-	-	-	-	-
2009	6,470	47	31	64.7%	(45)	177	44	64	101	137
2010	7,848	80	42	52.7%	(50)	278	76	103	155	205
2011	7,020	96	45	47.2%	(37)	317	91	124	175	220
2012	7,291	110	47	42.5%	(31)	340	105	136	191	237
2013	8,134	145	56	38.6%	2	423	140	177	243	307
2014	10,800	229	69	30.3%	36	532	225	269	348	405
2015	7,522	305	78	25.6%	22	574	302	353	441	497
2016	7,968	759	137	18.1%	349	1,500	756	844	991	1,102
2017	9,309	4,140	424	10.2%	3,012	5,953	4,134	4,401	4,861	5,250
Totals	77,596	5,911	554	9.4%	4,278	8,496	5,876	6,251	6,842	7,399
Normal Dist.		5,911	554	9.4%			5,911	6,285	6,822	7,199
logNormal Dist.		5,911	553	9.4%			5,885	6,268	6,862	7,313
Gamma Dist.		5,911	554	9.4%			5,894	6,275	6,850	7,275

Figure B.16. Total Unpaid Claims Distribution (Incurred Hoerl Curve)

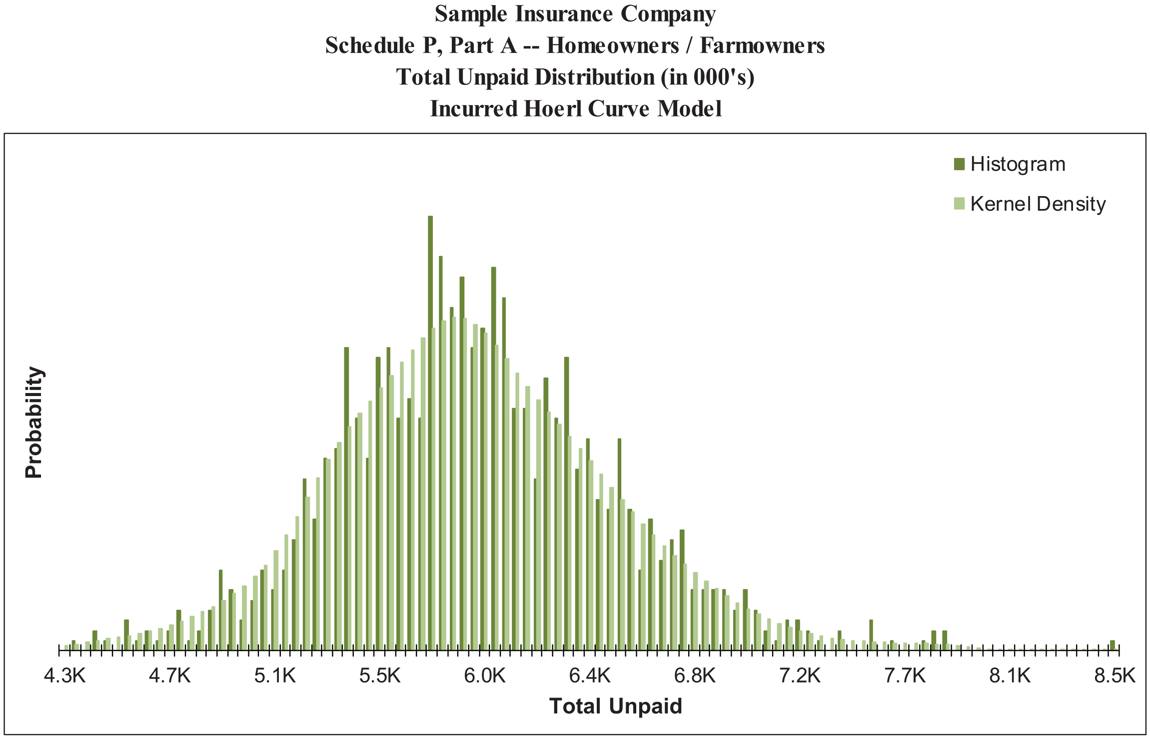

Figure B.17. Estimated Unpaid Model Results (Paid Wright)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Paid Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	-	-		-	-	-	-	-	-
2009	6,470	47	31	65.1%	(31)	194	44	65	105	138
2010	7,848	83	44	52.6%	(51)	281	81	107	161	210
2011	7,020	93	45	48.5%	(30)	262	86	118	176	212
2012	7,291	111	49	43.9%	(5)	346	106	143	195	236
2013	8,134	150	58	38.9%	(17)	399	147	185	252	304
2014	10,800	265	81	30.6%	56	615	257	312	411	480
2015	7,522	304	75	24.8%	41	603	300	353	430	487
2016	7,968	791	124	15.7%	373	1,197	788	868	993	1,077
2017	9,309	3,905	343	8.8%	2,992	4,995	3,902	4,135	4,465	4,703
Totals	77,596	5,750	514	8.9%	4,193	7,586	5,711	6,056	6,678	7,075
Normal Dist.		5,750	514	8.9%			5,750	6,096	6,595	6,946
logNormal Dist.		5,750	514	8.9%			5,727	6,082	6,632	7,048
Gamma Dist.		5,750	514	8.9%			5,734	6,088	6,621	7,013

Figure B.18. Total Unpaid Claims Distribution (Paid Wright)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Total Unpaid Distribution (in 000's)
Paid Wright Model

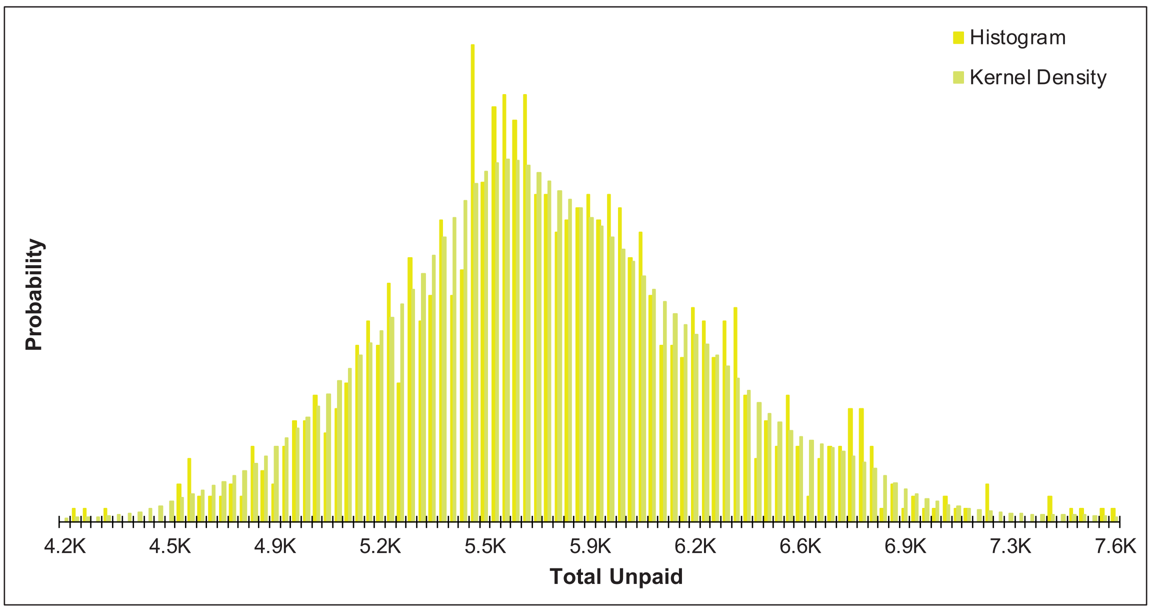

Figure B.19. Estimated Unpaid Model Results (Incurred Wright)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Incurred Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	-	-		-	-	-	-	-	-
2009	6,470	44	28	63.3%	(39)	192	41	61	93	122
2010	7,848	81	43	52.5%	(39)	283	77	104	160	205
2011	7,020	94	47	50.6%	(46)	358	88	118	180	236
2012	7,291	115	51	44.0%	(14)	392	111	146	204	256
2013	8,134	154	58	37.5%	(8)	497	150	185	253	322
2014	10,800	251	72	28.7%	55	530	248	296	383	442
2015	7,522	297	73	24.4%	86	540	295	346	416	477
2016	7,968	777	114	14.7%	412	1,187	775	854	971	1,053
2017	9,309	3,812	266	7.0%	3,013	4,733	3,808	3,988	4,264	4,448
Totals	77,596	5,625	440	7.8%	4,333	7,210	5,605	5,887	6,392	6,829
Normal Dist.		5,625	440	7.8%			5,625	5,923	6,350	6,650
logNormal Dist.		5,625	438	7.8%			5,608	5,911	6,374	6,721
Gamma Dist.		5,625	440	7.8%			5,614	5,916	6,369	6,701

Figure B.20. Total Unpaid Claims Distribution (Incurred Wright)

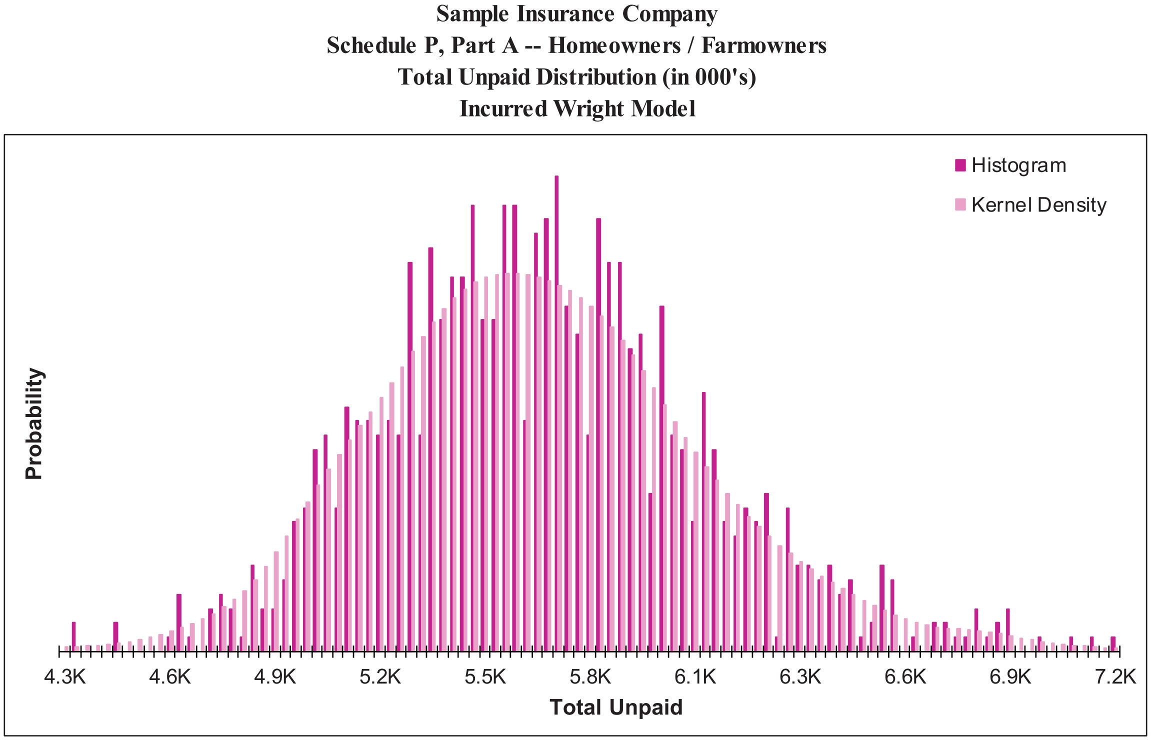

Figure B.21. Model Weights by Accident Year

Accident Year	Model Weights by Accident Year										
	Paid BS	Incd BS	Paid CC	Incd CC	Paid CL	Incd CL	Paid HC	Incd HC	Paid WR	Incd WR	TOTAL
2008	40.0%		30.0%		30.0%						100.0%
2009	40.0%		30.0%		30.0%						100.0%
2010	40.0%		30.0%		30.0%						100.0%
2011	40.0%		30.0%		30.0%						100.0%
2012	40.0%		30.0%		30.0%						100.0%
2013	40.0%		30.0%		30.0%						100.0%
2014	40.0%		30.0%		30.0%						100.0%
2015	40.0%		30.0%		30.0%						100.0%
2016	40.0%		30.0%		30.0%						100.0%
2017	40.0%		30.0%		30.0%						100.0%

Figure B.22. Estimated Mean Unpaid by Model

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Summary of Results by Model (in 000's)

Accident Year	Mean Estimated Unpaid										
	Berquist & Sherman		Cape Cod		Chain Ladder		Hoerl Curve		Wright		Best Est.
	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred	(Weighted)
2008	1	1	2	1	12	12	-	-	-	-	5
2009	3	3	4	3	23	23	47	47	47	44	9
2010	9	10	10	11	44	43	79	80	83	81	20
2011	18	21	21	26	53	52	97	96	93	94	29
2012	38	44	40	50	75	74	111	110	111	115	49
2013	80	82	81	92	125	124	148	145	150	154	94
2014	181	181	240	211	244	243	236	229	265	251	217
2015	342	339	298	297	311	304	320	305	304	297	318
2016	789	794	717	1,315	698	704	798	759	791	777	739
2017	4,880	4,260	3,937	3,884	3,841	3,701	4,428	4,140	3,905	3,812	4,312
Totals	6,340	5,736	5,350	5,890	5,425	5,279	6,264	5,911	5,750	5,625	5,792

Figure B.23. Estimated Ranges

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Summary of Results by Model (in 000's)

Accident Year	Best Est. (Weighted)	Ranges			
		Weighted		Modeled	
		Minimum	Maximum	Mininum	Maximum
2008	5	1	12	1	12
2009	9	3	23	3	23
2010	20	9	44	9	44
2011	29	18	53	18	53
2012	49	38	75	38	75
2013	94	80	125	80	125
2014	217	181	244	181	244
2015	318	298	342	298	342
2016	739	698	789	698	789
2017	4,312	3,841	4,880	3,841	4,880
Totals	5,792	5,166	6,587	5,350	6,340

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Figure B.24. Reconciliation of Total Results (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Reconciliation of Total Results (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Incurred To Date	Case Reserves	IBNR	Estimate of Ultimate	Estimate of Unpaid
2008	5,234	5,237	3	2	5,239	5
2009	6,470	6,479	9	1	6,480	9
2010	7,848	7,867	19	1	7,868	20
2011	7,020	7,046	26	3	7,050	29
2012	7,291	7,341	50	(1)	7,340	49
2013	8,134	8,225	91	3	8,228	94
2014	10,800	11,085	285	(68)	11,017	217
2015	7,522	7,810	288	30	7,840	318
2016	7,968	8,703	735	4	8,707	739
2017	9,309	12,788	3,478	834	13,621	4,312
Totals	77,596	82,580	4,984	808	83,388	5,792

Figure B.25. Estimated Unpaid Model Results (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Unpaid (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	5,234	5	8	169.9%	(37)	63	2	6	21	35
2009	6,470	9	14	148.1%	(30)	103	4	11	40	59
2010	7,848	20	22	110.9%	(38)	156	13	28	65	92
2011	7,020	29	25	85.5%	(71)	227	26	43	76	99
2012	7,291	49	35	70.7%	(90)	210	49	72	107	133
2013	8,134	94	55	58.3%	(132)	318	96	130	180	219
2014	10,800	217	106	49.0%	(281)	659	222	284	385	478
2015	7,522	318	162	51.0%	(438)	1,177	314	400	600	759
2016	7,968	739	335	45.3%	(1,016)	2,588	719	903	1,341	1,678
2017	9,309	4,312	1,512	35.1%	(2,872)	12,591	4,060	5,087	7,090	8,710
Totals	77,596	5,792	1,571	27.1%	(800)	14,273	5,568	6,609	8,652	10,410
Normal Dist.		5,792	1,571	27.1%			5,792	6,851	8,375	9,446
logNormal Dist.		5,846	1,890	32.3%			5,562	6,880	9,343	11,583
Gamma Dist.		5,792	1,571	27.1%			5,651	6,760	8,594	10,056

Figure B.26. Estimated Cash Flow (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Calendar Year Unpaid (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2018	3,871	1,443	37.3%	(3,836)	11,833	3,690	4,610	6,468	7,923
2019	959	413	43.0%	(878)	3,461	923	1,183	1,709	2,114
2020	446	221	49.6%	(643)	1,488	429	563	848	1,073
2021	214	113	52.7%	(371)	721	208	279	408	514
2022	124	69	55.9%	(183)	529	122	165	240	304
2023	72	44	61.1%	(103)	286	71	99	146	185
2024	44	29	66.8%	(83)	180	43	62	94	120
2025	28	22	78.5%	(51)	167	26	40	66	88
2026	16	16	104.0%	(38)	132	12	23	47	68
2027	10	12	124.9%	(26)	125	7	14	33	51
2028	6	9	155.5%	(34)	87	3	9	24	39
2029	3	7	220.4%	(23)	100	1	3	16	29
Totals	5,792	1,571	27.1%	(800)	14,273	5,568	6,609	8,652	10,410

Figure B.27. Estimated Loss Ratio (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Ultimate Loss Ratios (in 000's)
Best Estimate (Weighted)

Accident Year	Earned Premium	Mean Loss Ratio	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	7,878	69.8%	21.6%	31.0%	-26.9%	168.0%	67.7%	80.4%	108.4%	129.3%
2009	8,257	79.7%	22.8%	28.6%	-29.4%	192.7%	78.8%	90.5%	120.1%	141.0%
2010	8,812	89.6%	24.5%	27.4%	-11.8%	254.9%	89.0%	100.9%	132.4%	155.1%
2011	9,823	75.4%	22.6%	29.9%	-57.7%	189.6%	73.1%	86.1%	116.6%	138.5%
2012	11,499	66.1%	20.0%	30.3%	-44.6%	173.3%	64.5%	75.4%	101.7%	122.0%
2013	12,965	65.2%	19.1%	29.4%	-37.5%	169.8%	64.0%	74.1%	99.1%	117.4%
2014	13,875	84.3%	25.1%	29.8%	-32.7%	231.7%	80.7%	96.3%	130.5%	157.9%
2015	14,493	57.6%	18.6%	32.3%	-36.6%	160.7%	55.3%	66.5%	91.7%	109.1%
2016	15,202	60.4%	19.3%	32.0%	-17.0%	175.8%	58.1%	70.0%	95.2%	114.0%
2017	15,148	96.2%	26.7%	27.7%	-16.9%	235.7%	90.5%	110.0%	146.5%	172.7%
Totals	117,952	74.0%	7.2%	9.7%	47.5%	109.0%	73.9%	78.6%	85.9%	91.8%

Figure B.28. Estimated Unpaid Claim Runoff (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Calendar Year Unpaid Claim Runoff (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2017	5,792	1,571	27.1%	(800)	14,273	5,568	6,609	8,652	10,410
2018	1,920	505	26.3%	(102)	4,329	1,876	2,206	2,823	3,308
2019	961	275	28.6%	(206)	2,428	950	1,116	1,441	1,713
2020	515	159	30.8%	(97)	1,203	515	616	779	913
2021	301	110	36.5%	(101)	828	299	371	485	574
2022	178	80	45.3%	(135)	674	171	225	321	402
2023	106	62	58.8%	(132)	515	98	139	221	297
2024	62	48	77.6%	(74)	417	51	84	153	217
2025	34	35	102.2%	(96)	305	24	47	103	156
2026	18	23	123.2%	(58)	243	11	26	63	100
2027	9	14	154.2%	(38)	162	4	12	37	61
2028	3	7	220.4%	(23)	100	1	3	16	29

Figure B.29. Mean of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Mean Values (in 000's)												
Year	12	24	36	48	60	72	84	96	108	120	132	144	156
2008	3,865	1,191	233	103	43	25	14	8	6	3	2	1	1
2009	4,622	1,425	285	123	53	30	17	10	7	3	2	2	1
2010	5,563	1,705	335	148	61	35	20	12	9	4	3	2	2
2011	5,203	1,608	317	138	59	33	18	11	8	4	3	2	2
2012	5,342	1,647	323	144	61	34	19	11	8	4	3	2	2
2013	5,969	1,800	359	159	67	38	22	13	9	4	3	2	2
2014	8,260	2,509	495	217	91	51	29	18	12	6	4	3	2
2015	5,857	1,818	356	160	67	37	21	13	9	4	3	2	2
2016	6,467	1,975	393	172	73	41	24	14	10	5	3	2	2
2017	10,266	3,145	620	276	114	64	37	22	15	8	5	4	3

Figure B.30. Standard Deviation of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Standard Error Values (in 000's)												
Year	12	24	36	48	60	72	84	96	108	120	132	144	156
2008	1,557	610	166	88	43	27	17	11	8	5	4	3	3
2009	1,742	676	187	99	49	30	19	12	9	6	5	4	4
2010	2,010	765	209	111	55	34	22	14	11	7	6	5	4
2011	2,042	785	212	111	55	34	21	14	10	7	5	4	4
2012	2,106	807	223	116	59	35	22	14	11	7	5	4	4
2013	2,300	869	240	127	63	38	24	15	12	8	6	5	4
2014	3,134	1,159	309	158	80	48	30	20	14	9	8	6	5
2015	2,476	953	263	139	68	41	26	16	12	8	6	5	4
2016	2,723	1,050	284	148	74	44	28	18	13	8	6	5	5
2017	3,609	1,403	374	200	97	59	36	23	17	12	9	8	7

Figure B.31. Coefficient of Variation of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part A -- Homeowners / Farmowners
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Coefficients of Variation												
Year	12	24	36	48	60	72	84	96	108	120	132	144	156
2008	40.3%	51.2%	71.1%	85.1%	100.7%	107.7%	120.5%	128.4%	136.7%	174.4%	200.4%	226.6%	249.8%
2009	37.7%	47.5%	65.4%	80.9%	92.4%	99.3%	112.8%	119.4%	129.6%	172.6%	195.5%	218.5%	244.5%
2010	36.1%	44.9%	62.3%	75.2%	89.6%	95.6%	107.4%	114.5%	123.2%	165.8%	185.0%	206.7%	235.0%
2011	39.2%	48.8%	66.8%	80.7%	93.0%	101.2%	114.9%	121.9%	127.6%	172.6%	189.1%	214.2%	239.5%
2012	39.4%	49.0%	68.8%	80.7%	97.1%	104.3%	115.3%	123.1%	130.5%	172.1%	192.4%	210.9%	237.6%
2013	38.5%	48.3%	66.8%	79.8%	94.1%	99.3%	112.5%	120.7%	128.3%	168.0%	185.7%	209.3%	236.7%
2014	37.9%	46.2%	62.3%	72.7%	88.2%	94.8%	102.4%	109.4%	117.7%	159.1%	176.0%	198.6%	228.9%
2015	42.3%	52.4%	73.9%	86.7%	101.8%	110.0%	119.8%	129.4%	136.3%	170.6%	190.1%	213.5%	244.3%
2016	42.1%	53.2%	72.3%	85.8%	101.0%	107.9%	117.3%	125.6%	133.1%	169.4%	191.4%	212.9%	232.3%
2017	35.2%	44.6%	60.4%	72.2%	85.1%	90.8%	99.1%	104.9%	115.8%	153.8%	171.3%	193.9%	220.4%

Figure B.32. Total Unpaid Claims Distribution (Weighted)

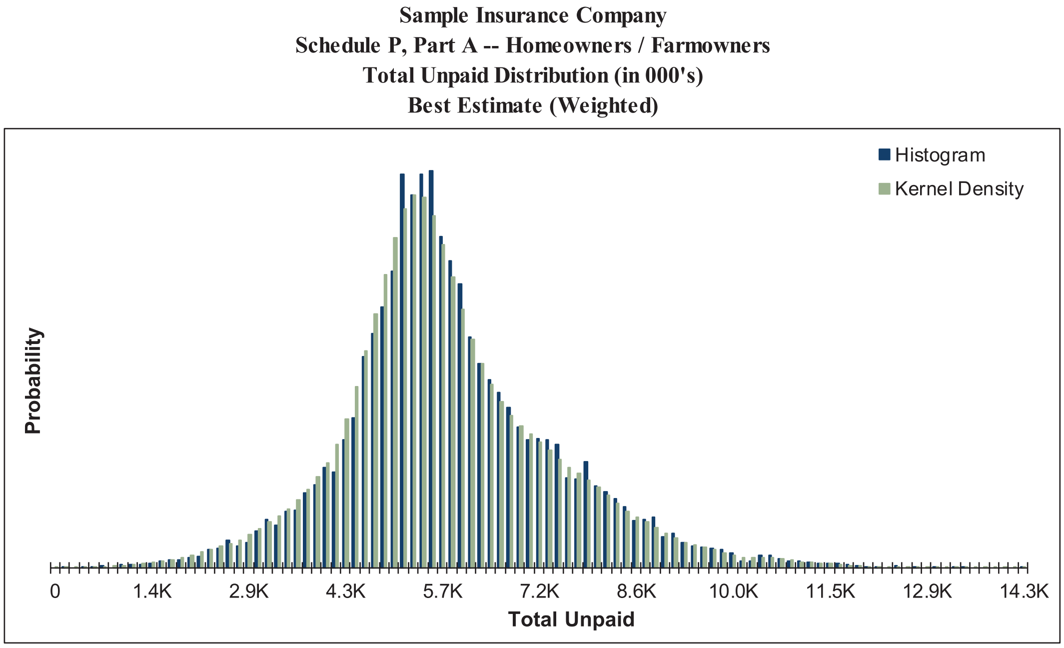
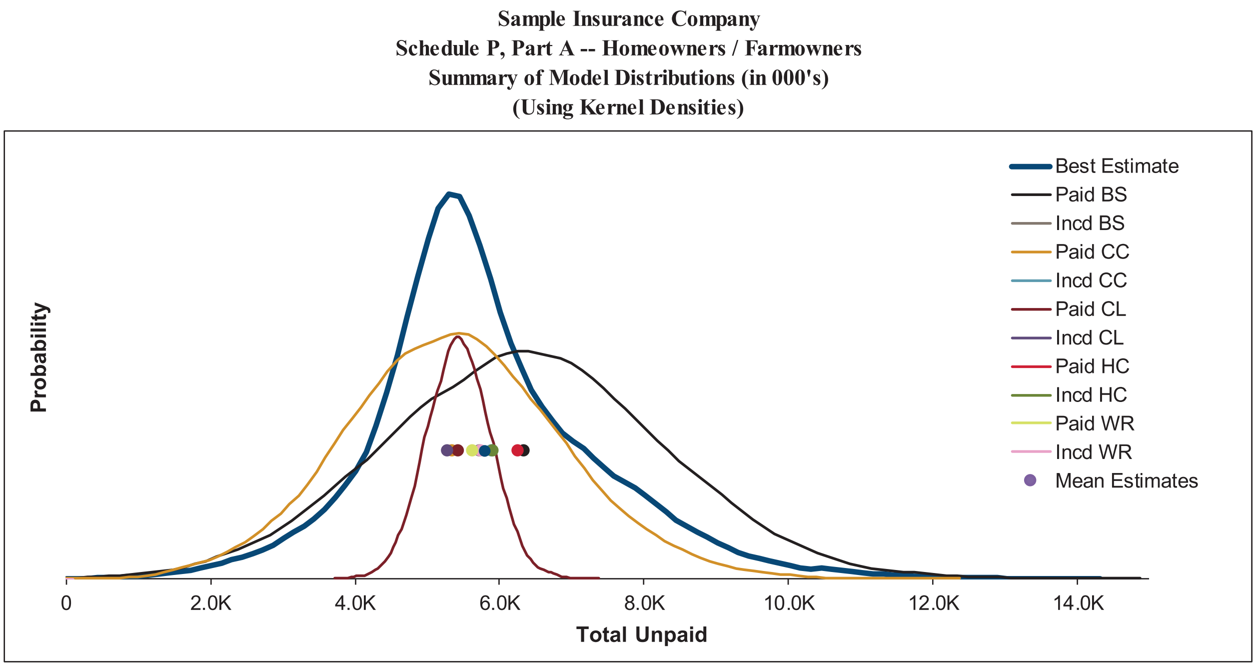


Figure B.33. Summary of Model Distributions



Appendix C—Schedule P, Part B Results

In this appendix the results for Schedule P, Part B (Private Passenger Auto Liability) are shown.

Figure C.1. Estimated Unpaid Model Results (Paid Berquist-Sherman)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Paid Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	39	8	20.8%	13	72	39	45	53	59
2009	12,679	68	9	13.6%	36	103	68	74	83	89
2010	13,631	108	10	9.4%	75	144	108	114	124	132
2011	14,472	184	11	6.2%	151	224	185	192	204	212
2012	13,717	311	14	4.4%	263	369	312	320	333	343
2013	13,090	571	18	3.2%	510	627	572	583	602	618
2014	12,490	1,107	29	2.6%	1,025	1,215	1,108	1,128	1,154	1,171
2015	11,598	2,110	48	2.3%	1,964	2,276	2,112	2,140	2,192	2,223
2016	10,306	3,964	87	2.2%	3,680	4,247	3,962	4,021	4,109	4,167
2017	6,357	8,078	173	2.1%	7,523	8,628	8,074	8,192	8,369	8,484
Totals	120,157	16,541	271	1.6%	15,759	17,433	16,553	16,724	16,991	17,159
Normal Dist.		16,541	271	1.6%			16,541	16,724	16,987	17,172
logNormal Dist.		16,541	271	1.6%			16,538	16,722	16,991	17,182
Gamma Dist.		16,541	271	1.6%			16,539	16,723	16,989	17,178

Figure C.2. Total Unpaid Claims Distribution (Paid Berquist-Sherman)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Paid Berquist & Sherman Model

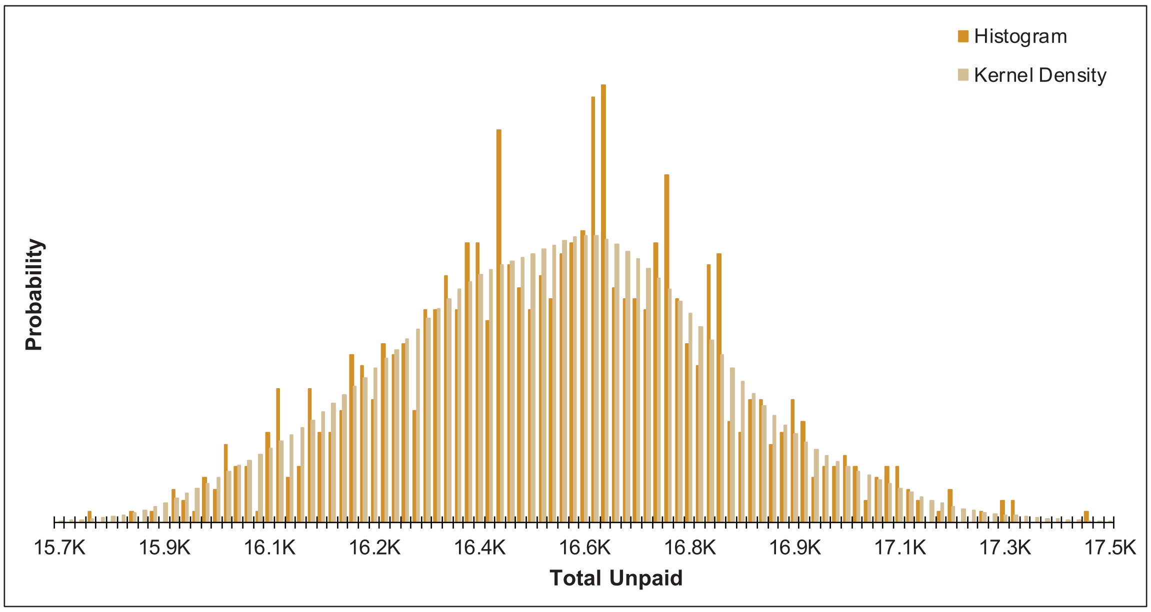


Figure C.3. Estimated Unpaid Model Results (Incurred Berquist-Sherman)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Incurred Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	41	9	21.3%	13	77	41	46	55	62
2009	12,679	69	10	14.6%	39	107	69	76	87	93
2010	13,631	110	11	10.5%	74	156	109	117	129	137
2011	14,472	187	14	7.5%	144	240	187	196	211	222
2012	13,717	315	19	6.0%	261	391	315	328	347	363
2013	13,090	576	30	5.3%	468	707	576	596	623	646
2014	12,490	1,113	54	4.8%	845	1,264	1,116	1,147	1,199	1,243
2015	11,598	2,109	96	4.6%	1,787	2,498	2,111	2,177	2,266	2,330
2016	10,306	3,950	178	4.5%	3,393	4,637	3,952	4,066	4,239	4,355
2017	6,357	8,041	366	4.6%	6,334	9,228	8,026	8,288	8,650	8,948
Totals	120,157	16,511	492	3.0%	14,729	18,489	16,495	16,835	17,297	17,794
Normal Dist.		16,511	492	3.0%			16,511	16,843	17,320	17,655
logNormal Dist.		16,511	492	3.0%			16,504	16,838	17,332	17,687
Gamma Dist.		16,511	492	3.0%			16,506	16,840	17,328	17,676

Figure C.4. Total Unpaid Claims Distribution (Incurred Berquist-Sherman)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Berquist & Sherman Model

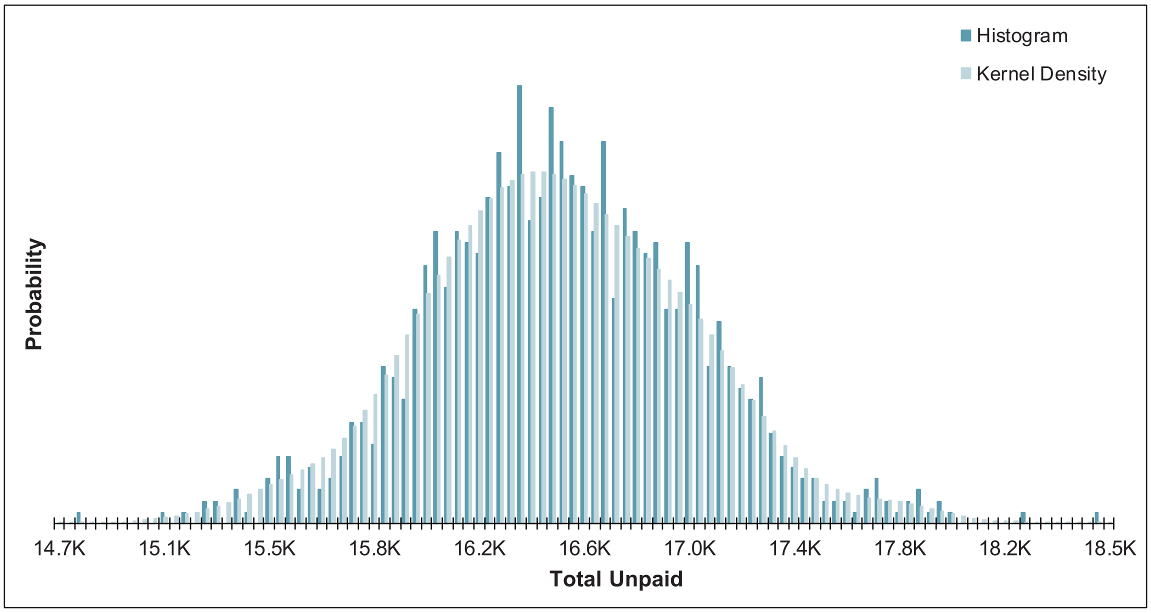


Figure C.5. Estimated Unpaid Model Results (Paid Cape Cod)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Paid Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	305	1,958	641.5%	0	36,642	16	69	956	4,630
2009	12,679	351	2,087	595.3%	22	39,584	43	100	1,033	4,939
2010	13,631	413	2,214	535.9%	59	41,095	84	146	1,180	5,464
2011	14,472	511	2,371	463.5%	130	44,495	161	226	1,273	5,749
2012	13,717	633	2,322	366.8%	249	45,048	294	355	1,426	5,889
2013	13,090	884	2,272	256.9%	484	43,956	555	611	1,665	5,851
2014	12,490	1,401	2,230	159.2%	976	44,387	1,082	1,142	2,089	6,281
2015	11,598	2,374	2,201	92.7%	1,858	43,272	2,069	2,130	3,085	7,659
2016	10,306	4,212	2,322	55.1%	3,532	48,773	3,897	3,997	4,965	9,352
2017	6,357	8,351	2,347	28.1%	7,248	52,155	8,056	8,265	9,150	13,969
Totals	120,157	19,435	22,304	114.8%	15,233	439,407	16,251	16,796	26,583	69,103
Normal Dist.		19,435	22,304	114.8%			19,435	34,479	56,121	71,321
logNormal Dist.		18,404	5,673	30.8%			17,587	21,550	28,867	35,446
Gamma Dist.		19,435	22,304	114.8%			11,848	26,812	64,241	103,101

Figure C.6. Total Unpaid Claims Distribution (Paid Cape Cod)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Paid Cape Cod Model

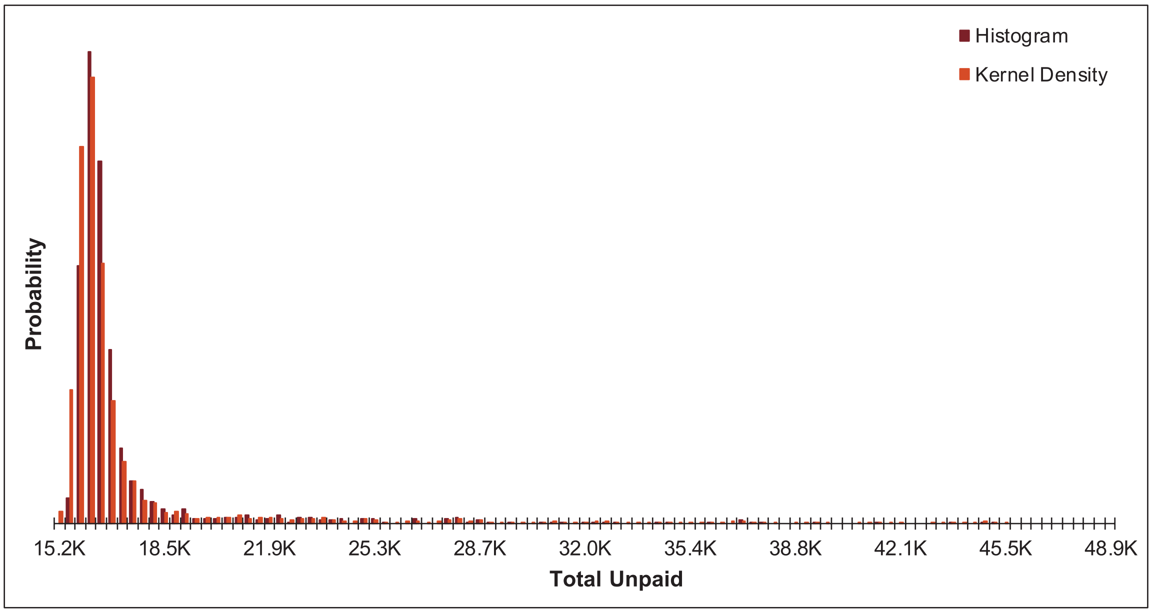


Figure C.7. Estimated Unpaid Model Results (Incurred Cape Cod)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Incurred Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	190	746	391.9%	0	9,272	16	69	877	3,473
2009	12,679	223	768	344.3%	22	9,932	43	100	942	3,562
2010	13,631	286	852	297.8%	55	10,795	86	146	1,087	3,948
2011	14,472	384	914	238.4%	128	11,267	169	234	1,251	4,318
2012	13,717	508	878	172.8%	253	11,161	304	365	1,316	4,388
2013	13,090	742	826	111.3%	469	10,301	557	612	1,488	4,300
2014	12,490	1,255	778	62.0%	885	10,899	1,091	1,149	1,966	4,634
2015	11,598	2,195	726	33.1%	1,762	10,855	2,059	2,139	2,836	5,297
2016	10,306	4,034	675	16.7%	3,364	11,716	3,923	4,062	4,582	6,785
2017	6,357	8,415	515	6.1%	7,434	14,114	8,371	8,612	9,068	10,103
Totals	120,157	18,232	7,536	41.3%	15,207	109,195	16,630	17,144	25,086	50,824
Normal Dist.		18,232	7,536	41.3%			18,232	23,315	30,627	35,763
logNormal Dist.		18,025	3,936	21.8%			17,610	20,370	25,116	29,095
Gamma Dist.		18,232	7,536	41.3%			17,205	22,599	32,131	40,152

Figure C.8. Total Unpaid Claims Distribution (Incurred Cape Cod)

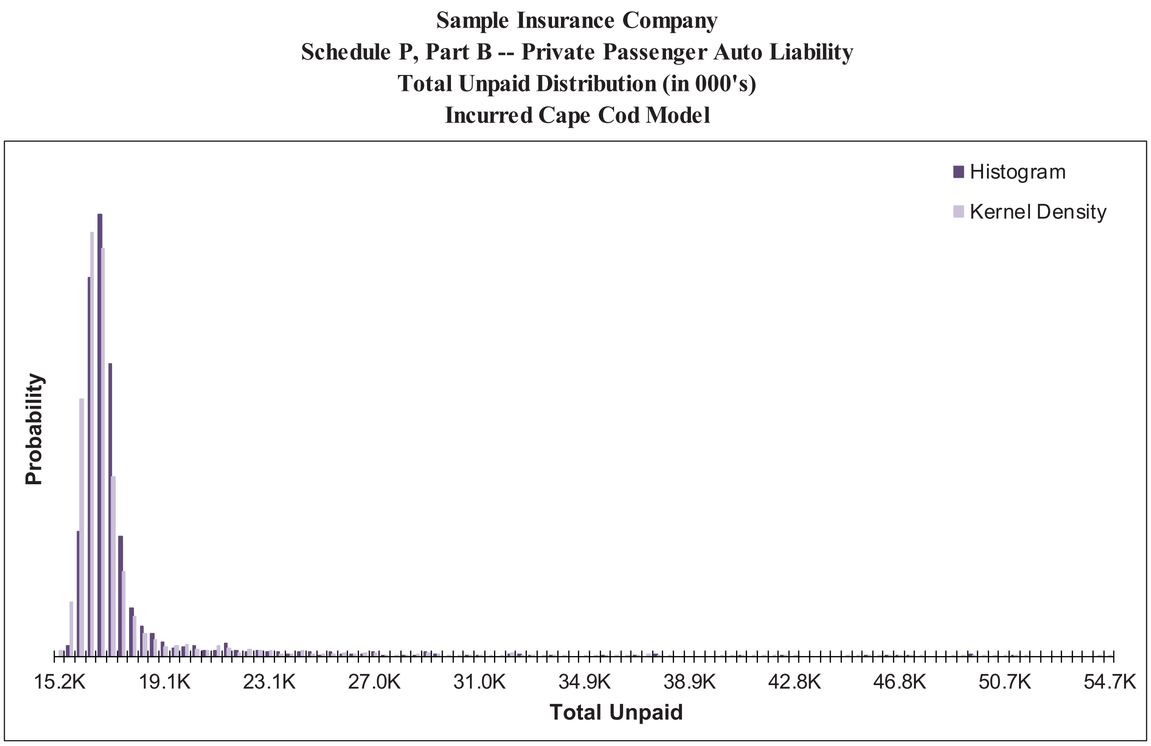


Figure C.9. Estimated Unpaid Model Results (Paid Chain Ladder)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Paid Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	536	3,745	698.6%	0	70,078	19	90	1,532	8,689
2009	12,679	602	4,030	669.2%	20	75,408	48	126	1,704	9,201
2010	13,631	681	4,276	628.2%	57	79,197	88	170	1,892	10,040
2011	14,472	798	4,564	572.0%	129	85,847	165	259	2,008	10,661
2012	13,717	901	4,410	489.2%	243	84,539	294	379	2,129	10,702
2013	13,090	1,135	4,285	377.7%	474	82,537	547	628	2,315	10,327
2014	12,490	1,649	4,245	257.4%	953	81,846	1,072	1,149	2,810	10,613
2015	11,598	2,636	4,296	163.0%	1,896	82,205	2,061	2,142	3,817	12,255
2016	10,306	4,493	4,528	100.8%	3,577	89,297	3,897	3,996	5,690	14,021
2017	6,357	8,629	4,501	52.2%	7,540	92,918	8,051	8,195	9,809	18,554
Totals	120,157	22,060	42,872	194.3%	15,256	823,872	16,189	16,962	34,096	115,071
Normal Dist.		22,060	42,872	194.3%			22,060	50,977	92,579	121,796
logNormal Dist.		19,588	7,896	40.3%			18,168	23,603	34,394	44,805
Gamma Dist.		22,060	42,872	194.3%			4,314	23,741	104,947	208,038

Figure C.10. Total Unpaid Claims Distribution (Paid Chain Ladder)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Paid Chain Ladder Model

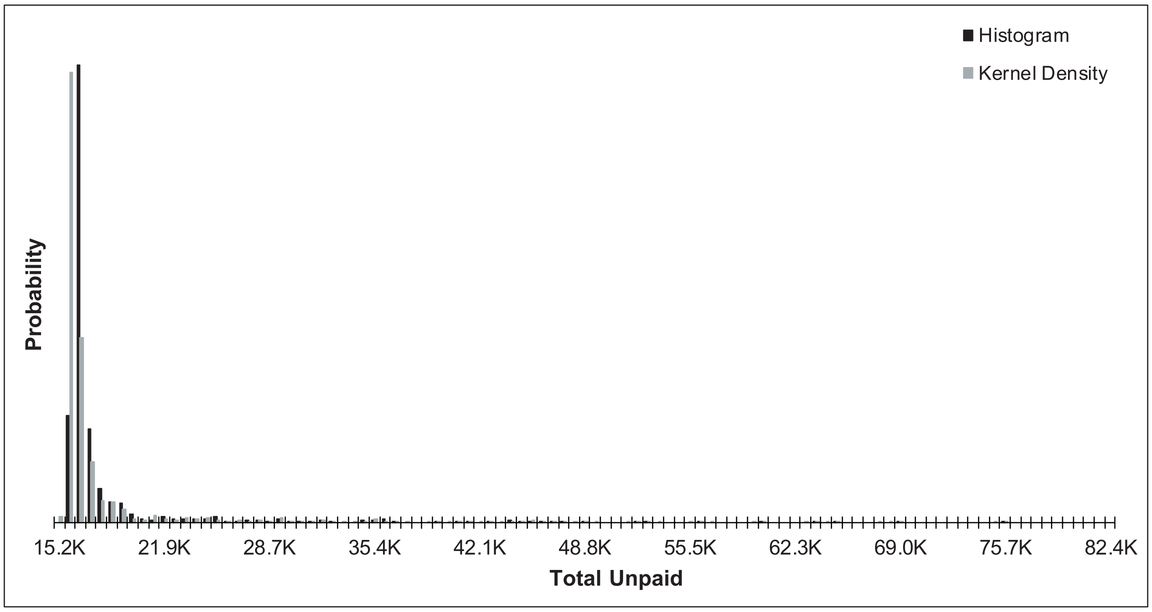


Figure C.11. Estimated Unpaid Model Results (Incurred Chain Ladder)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Incurred Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	212	994	469.1%	(1,028)	16,639	10	55	904	4,948
2009	12,679	226	1,009	446.6%	(5,368)	16,438	38	85	1,047	3,770
2010	13,631	243	867	357.1%	(4,064)	10,852	70	135	1,029	4,617
2011	14,472	375	1,083	288.8%	(601)	14,903	151	227	1,236	5,709
2012	13,717	445	1,049	235.9%	(3,824)	15,213	255	399	1,511	4,644
2013	13,090	598	1,083	181.0%	(2,561)	13,647	441	674	1,523	5,014
2014	12,490	990	1,092	110.3%	(2,239)	15,426	897	1,315	2,329	4,854
2015	11,598	1,704	1,577	92.6%	(3,079)	13,891	1,641	2,364	3,908	7,614
2016	10,306	3,106	2,388	76.9%	(5,810)	20,138	3,137	4,479	6,789	8,088
2017	6,357	6,652	4,635	69.7%	(14,979)	22,158	6,703	9,505	14,259	16,821
Totals	120,157	14,551	9,189	63.1%	(13,211)	115,434	13,628	17,226	25,849	49,616
Normal Dist.		14,551	9,189	63.1%			14,551	20,749	29,666	35,928
logNormal Dist.		25,561	52,784	206.5%			11,140	26,572	92,799	223,349
Gamma Dist.		14,551	9,189	63.1%			12,669	19,278	32,188	43,849

Figure C.12. Total Unpaid Claims Distribution (Incurred Chain Ladder)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Chain Ladder Model

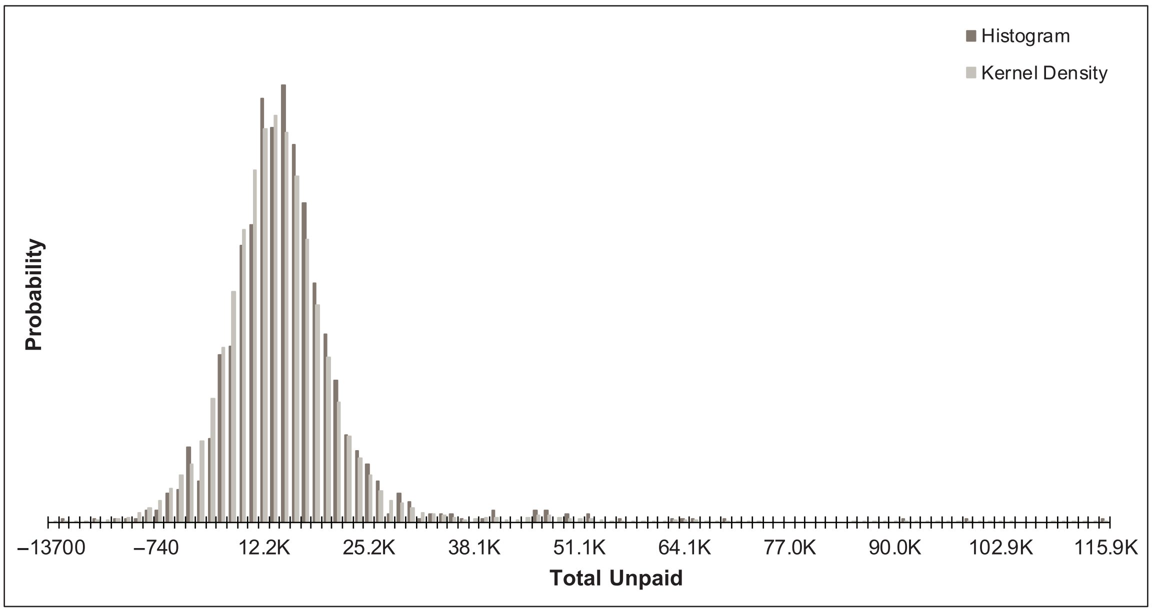


Figure C.13. Estimated Unpaid Model Results (Paid Hoerl Curve)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Paid Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	31	6	19.3%	15	55	31	35	42	47
2009	12,679	55	8	15.0%	35	90	54	60	69	78
2010	13,631	98	11	11.3%	66	137	97	105	118	127
2011	14,472	180	15	8.6%	140	234	179	190	206	220
2012	13,717	309	22	7.0%	246	384	309	323	345	356
2013	13,090	561	34	6.1%	459	688	560	583	618	641
2014	12,490	1,057	57	5.4%	880	1,275	1,058	1,093	1,154	1,188
2015	11,598	2,052	101	4.9%	1,716	2,381	2,052	2,114	2,222	2,283
2016	10,306	4,145	201	4.9%	3,441	4,783	4,154	4,273	4,458	4,653
2017	6,357	8,030	386	4.8%	6,852	9,105	8,032	8,286	8,689	8,951
Totals	120,157	16,517	562	3.4%	14,682	18,267	16,519	16,894	17,410	17,867
Normal Dist.		16,517	562	3.4%			16,517	16,896	17,442	17,825
logNormal Dist.		16,517	563	3.4%			16,507	16,891	17,459	17,869
Gamma Dist.		16,517	562	3.4%			16,511	16,893	17,453	17,853

Figure C.14. Total Unpaid Claims Distribution (Paid Hoerl Curve)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Paid Hoerl Curve Model

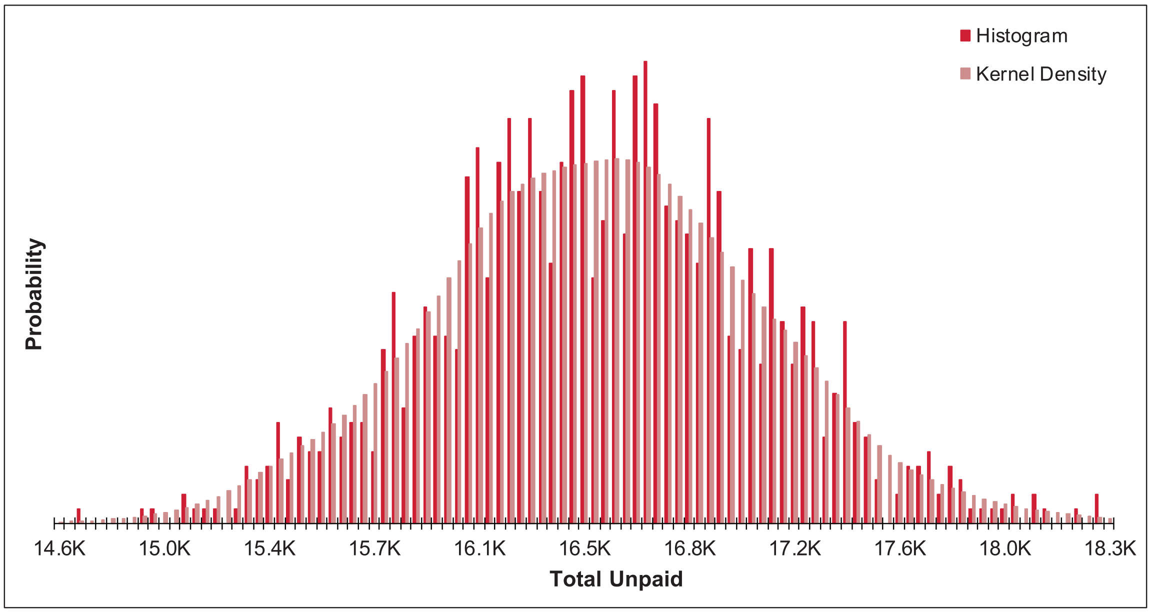


Figure C.15. Estimated Unpaid Model Results (Incurred Hoerl Curve)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Incurred Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	32	6	20.1%	15	55	31	36	43	50
2009	12,679	56	9	15.9%	34	97	55	61	72	79
2010	13,631	100	12	12.4%	66	143	99	107	120	131
2011	14,472	183	17	9.5%	132	238	182	194	214	227
2012	13,717	314	26	8.2%	212	400	314	332	357	375
2013	13,090	570	43	7.5%	426	716	569	597	645	679
2014	12,490	1,074	73	6.8%	835	1,329	1,074	1,122	1,196	1,251
2015	11,598	2,082	132	6.3%	1,641	2,530	2,084	2,169	2,300	2,401
2016	10,306	4,203	241	5.7%	3,414	5,065	4,196	4,363	4,600	4,783
2017	6,357	8,167	443	5.4%	6,639	10,105	8,175	8,444	8,904	9,166
Totals	120,157	16,781	549	3.3%	14,964	19,012	16,788	17,130	17,678	18,120
Normal Dist.		16,781	549	3.3%			16,781	17,151	17,684	18,059
logNormal Dist.		16,781	549	3.3%			16,772	17,146	17,698	18,097
Gamma Dist.		16,781	549	3.3%			16,775	17,148	17,695	18,085

Figure C.16. Total Unpaid Claims Distribution (Incurred Hoerl Curve)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Hoerl Curve Model

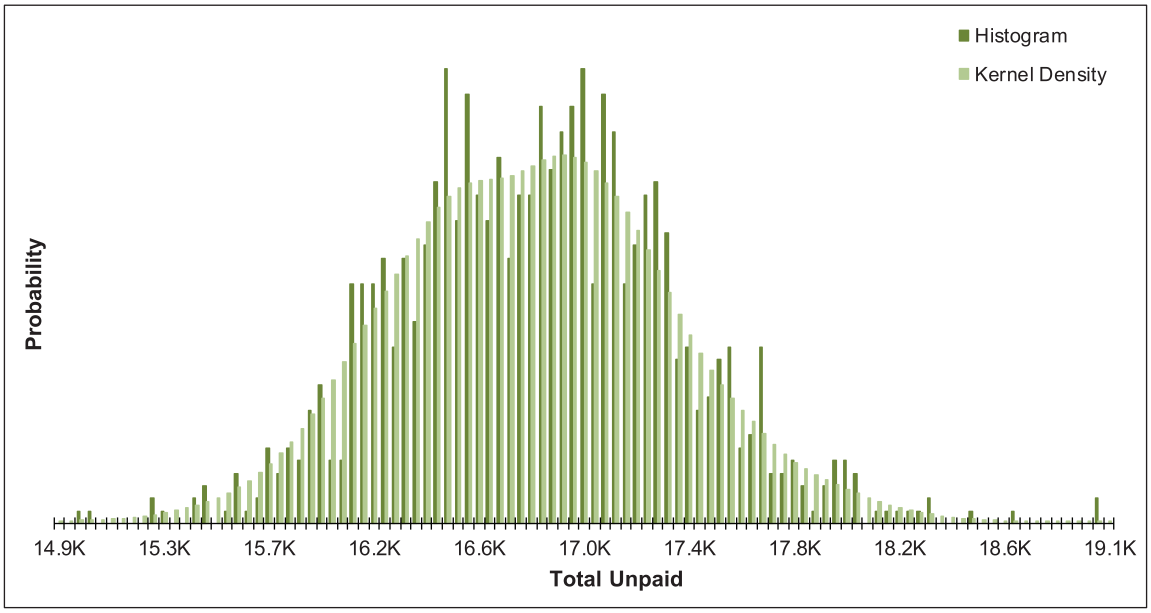


Figure C.17. Estimated Unpaid Model Results (Paid Wright)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Paid Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	33	6	18.0%	19	52	32	36	44	49
2009	12,679	57	8	14.2%	35	87	56	62	72	77
2010	13,631	103	12	11.2%	69	149	102	110	124	131
2011	14,472	188	16	8.5%	132	238	188	198	215	227
2012	13,717	322	23	7.2%	252	392	321	337	361	375
2013	13,090	572	36	6.4%	471	719	572	597	635	656
2014	12,490	1,039	63	6.0%	832	1,219	1,042	1,082	1,140	1,185
2015	11,598	1,982	116	5.8%	1,603	2,345	1,983	2,057	2,174	2,245
2016	10,306	4,172	259	6.2%	3,263	5,107	4,178	4,339	4,619	4,755
2017	6,357	7,932	596	7.5%	6,151	10,392	7,894	8,315	8,943	9,467
Totals	120,157	16,399	712	4.3%	14,387	18,935	16,364	16,858	17,619	18,179
Normal Dist.		16,399	712	4.3%			16,399	16,879	17,570	18,055
logNormal Dist.		16,399	710	4.3%			16,383	16,869	17,593	18,119
Gamma Dist.		16,399	712	4.3%			16,389	16,873	17,587	18,100

Figure C.18. Total Unpaid Claims Distribution (Paid Wright)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Paid Wright Model

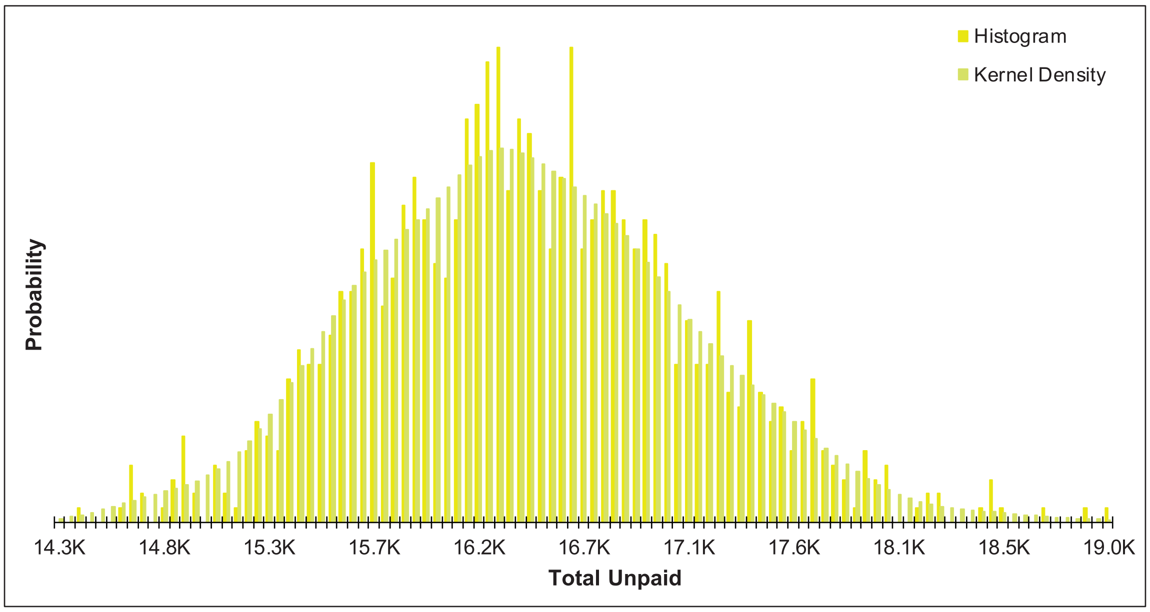


Figure C.19. Estimated Unpaid Model Results (Incurred Wright)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Incurred Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	3,966,020	89,251,382	2250.4%	11	2,562,613,734	32	37	47	182
2009	12,679	6,836,660	151,785,937	2220.2%	35	4,285,467,202	57	63	75	397
2010	13,631	11,394,086	249,064,047	2185.9%	70	6,902,830,083	105	114	130	631
2011	14,472	22,062,894	473,328,887	2145.4%	(19)	12,454,255,734	197	209	231	1,248
2012	13,717	33,137,459	697,879,848	2106.0%	175	17,782,233,386	332	349	382	2,198
2013	13,090	57,164,640	1,225,268,490	2143.4%	307	33,399,406,687	577	606	652	3,679
2014	12,490	112,525,697	2,407,249,389	2139.3%	479	64,465,798,848	1,066	1,105	1,196	6,437
2015	11,598	217,196,589	4,589,378,234	2113.0%	876	115,499,405,106	2,020	2,103	2,279	12,504
2016	10,306	395,484,469	8,302,943,031	2099.4%	(546)	208,307,514,180	4,137	4,285	4,552	24,508
2017	6,357	854,159,749	18,202,471,420	2131.0%	2,861	478,840,892,606	8,366	8,599	9,075	50,131
Totals	120,157	1,713,928,261	36,360,742,828	2121.5%	6,088	944,500,417,566	16,866	17,188	17,873	101,915
Normal Dist.		1,713,928,261	36,360,742,828	2121.5%			1,713,928,261	26,238,876,608	61,522,027,980	86,301,665,037
logNormal Dist.		42,038	82,461	196.2%			19,093	44,556	150,791	355,000
Gamma Dist.		1,713,928,261	36,360,742,828	2121.5%			0	0	41	4,737,757,328

Figure C.20. Total Unpaid Claims Distribution (Incurred Wright)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Wright Model

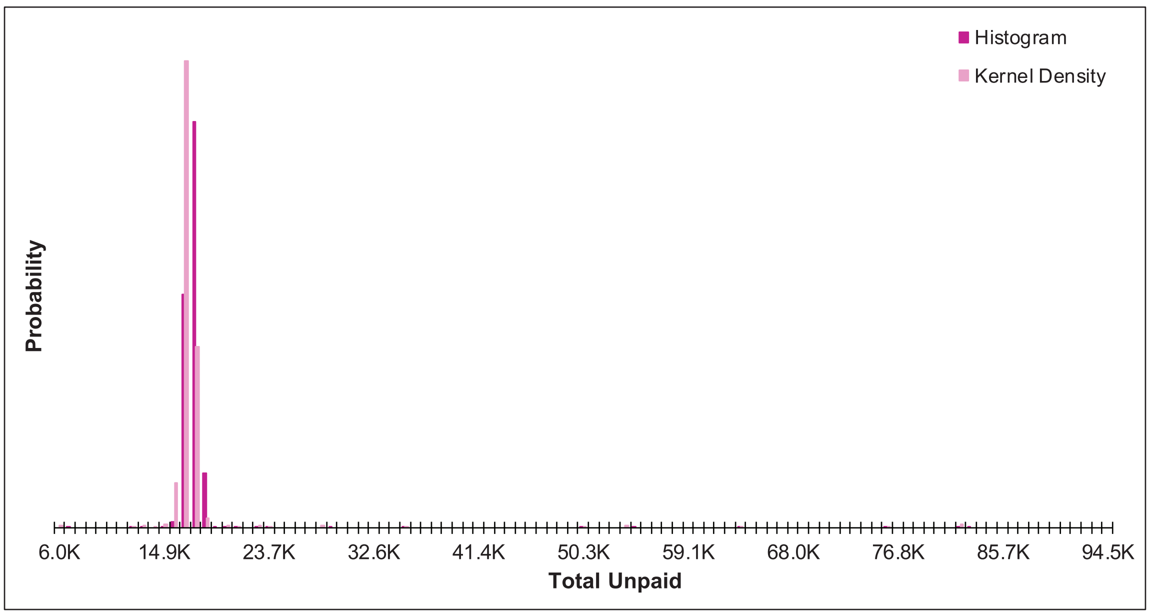


Figure C.21. Model Weights by Accident Year

Accident Year	Model Weights by Accident Year										
	Paid BS	Incd BS	Paid CC	Incd CC	Paid CL	Incd CL	Paid HC	Paid WR	[redacted]		TOTAL
2008	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%	[redacted]		100.0%
2009	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2010	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2011	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2012	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2013	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2014	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2015	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2016	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%
2017	10.0%	20.0%	10.0%	20.0%	10.0%	20.0%	5.0%	5.0%			100.0%

Figure C.22. Estimated Mean Unpaid by Model

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Summary of Results by Model (in 000's)

Accident	Mean Estimated Unpaid								
	Berquist & Sherman		Cape Cod		Chain Ladder		Hoerl Curve		Best Est.
Year	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	Incurred	(Weighted)
2008	39	41	305	190	536	212	31	33	140
2009	68	69	351	223	602	226	55	57	186
2010	108	110	413	286	681	243	98	103	215
2011	184	187	511	384	798	375	180	188	314
2012	311	315	633	508	901	445	309	322	439
2013	571	576	884	742	1,135	598	561	572	677
2014	1,107	1,113	1,401	1,255	1,649	990	1,057	1,039	1,165
2015	2,110	2,109	2,374	2,195	2,636	1,704	2,052	1,982	2,093
2016	3,964	3,950	4,212	4,034	4,493	3,106	4,145	4,172	3,923
2017	8,078	8,041	8,351	8,415	8,629	6,652	8,030	7,932	7,928
Totals	16,541	16,511	19,435	18,232	22,060	14,551	16,517	16,399	17,079

Figure C.23. Estimated Ranges

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Summary of Results by Model (in 000's)

Accident		Ranges			
	Best Est.	Weighted		Modeled	
Year	(Weighted)	Minimum	Maximum	Mininum	Maximum
2008	140	31	536	31	536
2009	186	55	602	55	602
2010	215	98	681	98	681
2011	314	180	798	180	798
2012	439	309	901	309	901
2013	677	561	1,135	561	1,135
2014	1,165	990	1,649	990	1,649
2015	2,093	1,704	2,636	1,704	2,636
2016	3,923	3,106	4,493	3,106	4,493
2017	7,928	6,652	8,629	6,652	8,629
Totals	17,079	13,687	22,060	14,551	22,060

Figure C.24. Reconciliation of Total Results (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Reconciliation of Total Results (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Incurred To Date	Case Reserves	IBNR	Estimate of Ultimate	Estimate of Unpaid
2008	11,816	11,863	47	92	11,956	140
2009	12,679	12,752	72	113	12,865	186
2010	13,631	13,743	112	103	13,846	215
2011	14,472	14,687	216	99	14,786	314
2012	13,717	14,079	362	77	14,156	439
2013	13,090	13,691	600	76	13,767	677
2014	12,490	13,683	1,193	(28)	13,655	1,165
2015	11,598	13,912	2,313	(221)	13,691	2,093
2016	10,306	14,625	4,319	(396)	14,229	3,923
2017	6,357	15,188	8,830	(902)	14,285	7,928
Totals	120,157	138,223	18,066	(987)	137,236	17,079

Figure C.25. Estimated Unpaid Model Results (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Unpaid (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	11,816	140	1,002	717.3%	(2,012)	74,732	33	46	409	2,417
2009	12,679	186	992	534.2%	(1,534)	37,021	59	75	490	3,024
2010	13,631	215	926	431.1%	(5,790)	54,408	101	118	513	3,196
2011	14,472	314	1,285	408.8%	(2,963)	90,358	179	200	646	3,686
2012	13,717	439	1,359	309.7%	(1,945)	69,048	308	333	765	3,336
2013	13,090	677	1,264	186.9%	(3,824)	68,442	562	598	1,051	3,798
2014	12,490	1,165	928	79.7%	(4,552)	27,150	1,088	1,144	1,762	4,562
2015	11,598	2,093	1,405	67.1%	(8,529)	79,999	2,066	2,153	2,880	5,341
2016	10,306	3,923	4,359	111.1%	(9,679)	405,947	3,935	4,095	5,126	7,619
2017	6,357	7,928	2,727	34.4%	(16,198)	92,918	8,087	8,384	10,346	14,962
Totals	120,157	17,079	8,888	52.0%	(8,740)	467,516	16,313	17,135	22,900	45,682
Normal Dist.		17,079	8,888	52.0%			17,079	23,074	31,698	37,755
logNormal Dist.		17,362	6,693	38.5%			16,200	20,823	29,882	38,509
Gamma Dist.		17,079	8,888	52.0%			15,565	21,956	33,823	44,176

Figure C.26. Estimated Cash Flow (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Calendar Year Unpaid (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2018	7,556	1,252	16.6%	(4,926)	16,377	7,784	8,077	8,947	10,799
2019	3,729	587	15.7%	(1,542)	7,781	3,832	3,979	4,440	5,204
2020	2,056	329	16.0%	(1,017)	4,299	2,111	2,196	2,449	2,901
2021	1,120	275	24.5%	(599)	12,854	1,116	1,175	1,441	1,965
2022	672	853	126.9%	(349)	60,793	576	625	1,032	2,954
2023	426	833	195.4%	(435)	33,332	306	346	791	2,860
2024	293	815	277.8%	(956)	44,837	170	205	692	2,742
2025	244	1,086	445.1%	(1,312)	70,895	101	132	643	3,052
2026	209	1,139	544.5%	(744)	60,995	63	93	574	3,034
2027	180	1,049	584.4%	(1,512)	53,125	34	64	578	2,993
2028	159	825	519.5%	(1,570)	23,121	19	43	550	3,195
2029	156	1,260	805.5%	(3,523)	74,981	11	25	478	3,172
2030	169	3,600	2134.8%	(7,667)	342,488	6	12	412	2,742
2031	110	1,288	1168.6%	(1,140)	66,389	2	4	196	2,239
Totals	17,079	8,888	52.0%	(8,740)	467,516	16,313	17,135	22,900	45,682

Figure C.27. Estimated Loss Ratio (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Ultimate Loss Ratios (in 000's)
Best Estimate (Weighted)

Accident Year	Earned Premium	Mean Loss Ratio	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	15,679	72.5%	22.1%	30.5%	-127.2%	554.0%	76.2%	78.6%	87.9%	125.3%
2009	15,510	77.8%	22.5%	28.9%	-120.8%	321.8%	81.5%	83.9%	94.7%	134.4%
2010	16,428	79.2%	22.7%	28.7%	-80.0%	418.0%	83.0%	85.4%	94.4%	135.3%
2011	18,432	76.0%	20.6%	27.1%	-121.9%	568.8%	78.9%	81.6%	90.2%	128.0%
2012	20,376	66.2%	18.8%	28.5%	-71.4%	405.5%	68.9%	71.2%	79.5%	112.5%
2013	20,821	63.2%	18.2%	28.8%	-133.5%	391.5%	65.9%	67.8%	76.8%	108.6%
2014	20,445	64.2%	18.6%	29.0%	-112.3%	193.2%	66.9%	68.8%	78.9%	114.7%
2015	20,724	63.2%	19.0%	30.1%	-110.9%	441.9%	66.1%	67.9%	76.3%	110.3%
2016	20,414	67.4%	27.9%	41.3%	-151.2%	2038.6%	69.9%	72.0%	81.6%	118.0%
2017	20,467	68.8%	20.5%	29.8%	-139.5%	485.5%	70.7%	73.2%	87.2%	128.7%
Totals	189,295	69.3%	7.0%	10.1%	30.3%	277.8%	70.1%	73.0%	78.1%	84.1%

Figure C.28. Estimated Unpaid Claim Runoff (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Calendar Year Unpaid Claim Runoff (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2017	17,079	8,888	52.0%	(8,740)	467,516	16,313	17,135	22,900	45,682
2018	9,523	8,743	91.8%	(3,814)	460,913	8,426	9,009	14,012	40,239
2019	5,794	8,725	150.6%	(2,272)	457,482	4,538	4,972	10,156	37,168
2020	3,738	8,694	232.6%	(1,255)	455,334	2,400	2,757	8,053	35,056
2021	2,619	8,557	326.8%	(656)	452,390	1,281	1,572	6,762	33,341
2022	1,946	8,054	413.8%	(307)	438,625	705	938	5,628	29,716
2023	1,520	7,587	499.1%	(156)	432,875	400	597	4,727	25,537
2024	1,227	7,152	582.9%	(101)	427,826	231	388	3,904	21,800
2025	983	6,647	676.3%	(9,318)	427,376	133	257	3,207	18,459
2026	774	6,071	784.7%	(9,365)	424,965	72	160	2,498	14,866
2027	594	5,472	921.1%	(9,345)	411,264	39	93	1,874	11,321
2028	435	4,936	1134.2%	(9,102)	393,463	19	46	1,261	7,898
2029	279	3,988	1430.4%	(7,664)	342,491	8	17	690	5,233
2030	110	1,288	1168.6%	(1,140)	66,389	2	4	196	2,239

Figure C.29. Mean of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Mean Values (in 000's)														
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
2008	4,987	3,176	1,405	801	432	214	108	56	31	23	14	12	16	29	68
2009	5,286	3,367	1,491	850	458	226	114	59	32	25	15	14	18	34	80
2010	5,706	3,639	1,611	918	495	244	123	63	35	26	16	14	18	32	73
2011	6,134	3,912	1,729	987	532	263	133	68	38	28	17	15	20	38	90
2012	5,909	3,764	1,667	950	512	253	128	66	36	27	16	15	20	38	92
2013	5,763	3,671	1,625	927	499	247	125	64	35	27	16	15	20	37	91
2014	5,748	3,660	1,619	924	498	246	124	64	35	27	16	15	20	36	84
2015	5,728	3,651	1,617	921	497	245	124	64	35	27	16	15	20	38	92
2016	6,012	3,829	1,695	967	521	258	130	67	37	28	17	15	21	43	125
2017	6,161	3,925	1,737	991	534	264	133	69	38	29	17	16	22	43	110

Figure C.30. Standard Deviation of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Standard Error Values (in 000's)														
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
2008	1,504	955	425	241	131	65	33	17	9	7	11	22	58	187	752
2009	1,503	957	426	241	131	65	33	17	9	7	12	25	68	210	696
2010	1,629	1,033	460	261	142	70	35	18	10	8	12	25	65	192	656
2011	1,622	1,031	458	260	141	70	35	18	10	8	13	28	76	249	951
2012	1,621	1,027	459	260	141	70	35	18	10	8	13	28	79	269	994
2013	1,612	1,022	458	258	140	69	35	18	10	8	13	28	77	250	917
2014	1,673	1,061	472	268	145	72	36	19	10	8	13	27	67	192	618
2015	1,670	1,063	473	268	145	72	36	19	10	8	13	28	78	254	950
2016	1,709	1,083	485	275	149	74	37	19	11	8	13	31	108	572	3,565
2017	1,730	1,096	486	277	150	74	38	20	11	9	14	31	92	328	1,288

Figure C.31. Coefficient of Variation of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part B -- Private Passenger Auto Liability
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Coefficients of Variation														
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
2008	30.1%	30.1%	30.2%	30.1%	30.3%	30.4%	30.3%	30.6%	30.9%	31.7%	77.8%	175.4%	359.0%	643.8%	1099.6%
2009	28.4%	28.4%	28.5%	28.4%	28.6%	28.6%	28.7%	28.9%	29.2%	30.0%	78.7%	186.6%	373.9%	612.0%	865.2%
2010	28.5%	28.4%	28.6%	28.4%	28.7%	28.6%	28.7%	28.9%	29.3%	30.2%	78.3%	178.7%	355.4%	591.5%	898.4%
2011	26.4%	26.4%	26.5%	26.4%	26.6%	26.6%	26.6%	26.8%	27.3%	28.2%	77.7%	181.3%	372.8%	661.7%	1057.6%
2012	27.4%	27.3%	27.5%	27.4%	27.5%	27.6%	27.6%	27.9%	28.3%	29.0%	78.5%	188.1%	394.4%	708.1%	1079.9%
2013	28.0%	27.8%	28.1%	27.8%	28.1%	28.0%	28.2%	28.5%	28.9%	29.6%	79.8%	189.0%	390.0%	668.2%	1007.6%
2014	29.1%	29.0%	29.1%	29.0%	29.2%	29.1%	29.1%	29.4%	29.8%	30.8%	79.4%	180.6%	340.5%	529.0%	733.9%
2015	29.2%	29.1%	29.3%	29.1%	29.3%	29.3%	29.4%	29.5%	29.8%	30.6%	80.1%	190.3%	391.1%	673.6%	1037.3%
2016	28.4%	28.3%	28.6%	28.4%	28.6%	28.5%	28.7%	28.8%	29.3%	29.9%	78.3%	202.5%	513.3%	1333.7%	2845.1%
2017	28.1%	27.9%	28.0%	27.9%	28.2%	28.1%	28.2%	28.6%	28.8%	29.9%	80.0%	196.5%	420.2%	757.1%	1168.6%

Figure C.32. Total Unpaid Claims Distribution (Weighted)

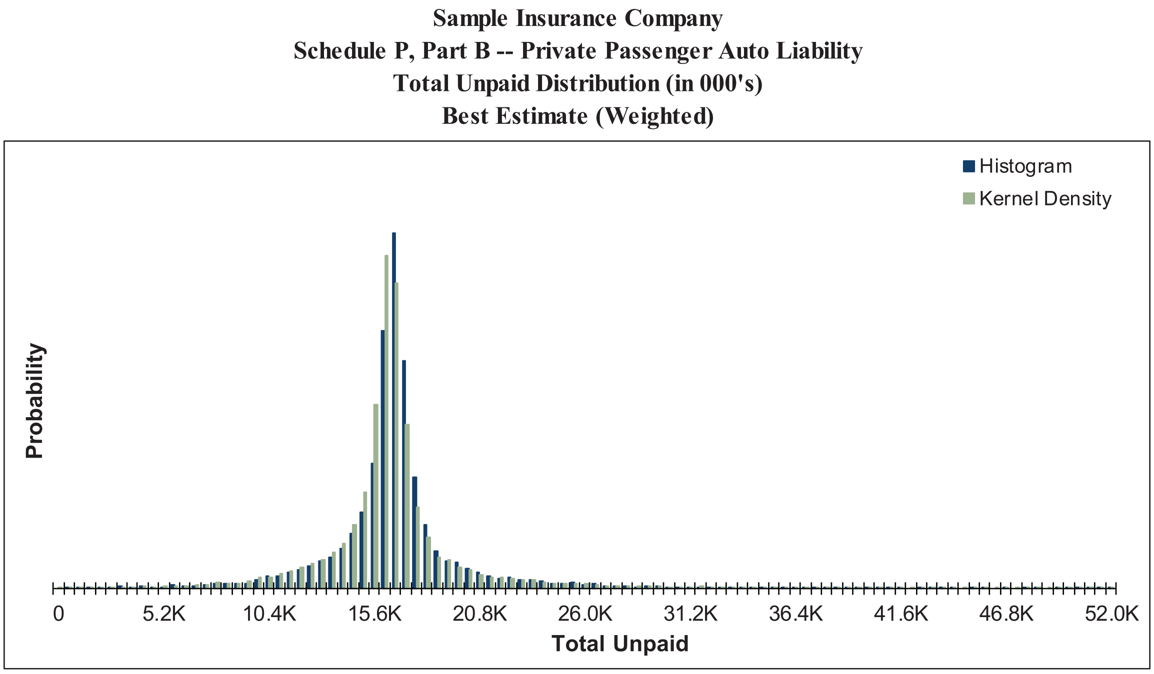
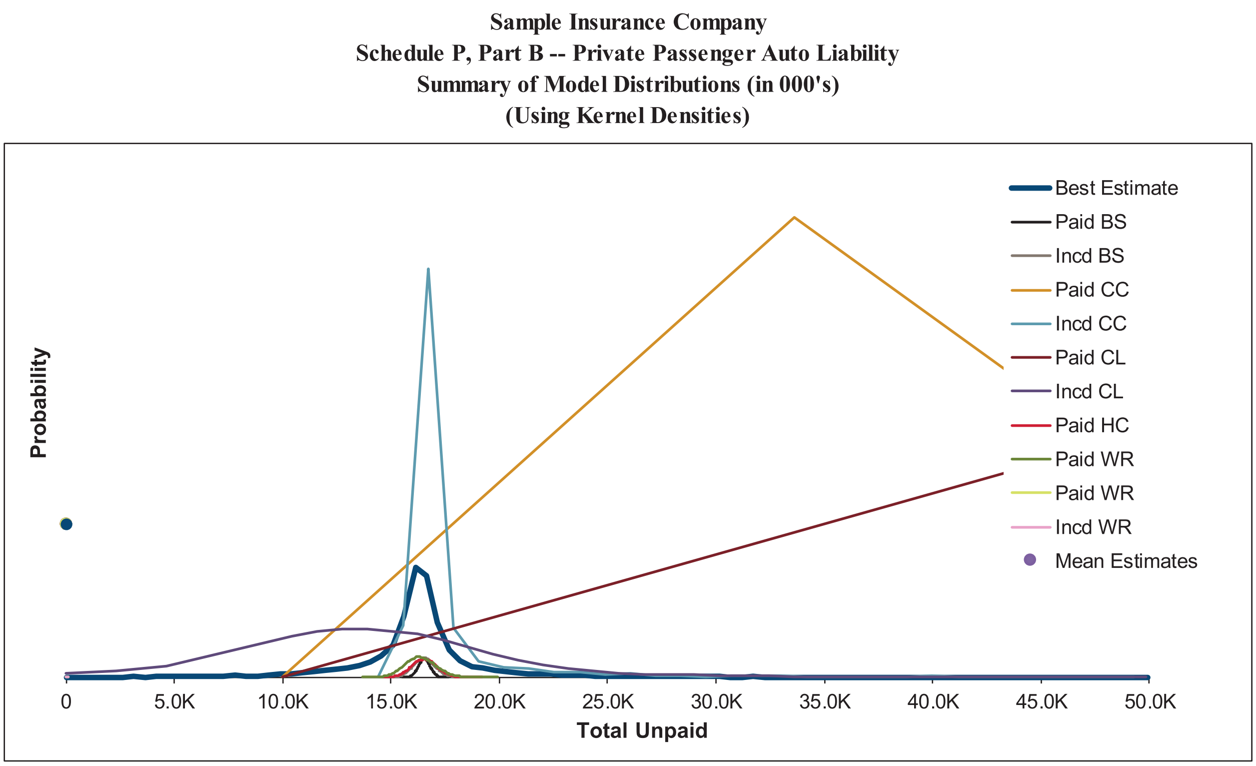


Figure C.33. Summary of Model Distributions



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Appendix D—Schedule P, Part C Results

In this appendix the results for Schedule P, Part C (Commercial Auto Liability) are shown.

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Figure D.1. Estimated Unpaid Model Results (Paid Berquist-Sherman)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Paid Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	3	251.9%	(20)	14	1	3	7	10
2009	1,469	4	5	149.6%	(23)	22	3	7	13	18
2010	1,387	14	8	54.9%	(21)	43	14	19	28	34
2011	1,350	28	10	35.4%	(9)	62	27	34	44	51
2012	1,342	50	13	25.4%	5	91	50	58	71	81
2013	1,198	103	16	15.6%	55	157	103	114	128	140
2014	1,061	209	22	10.3%	110	303	210	224	243	256
2015	853	402	28	6.9%	308	490	401	421	448	467
2016	645	742	40	5.4%	619	888	741	768	809	842
2017	294	1,176	51	4.4%	1,026	1,341	1,173	1,212	1,260	1,299
Totals	11,162	2,729	111	4.1%	2,361	3,140	2,725	2,798	2,918	2,983
Normal Dist.		2,729	111	4.1%			2,729	2,804	2,911	2,986
logNormal Dist.		2,729	111	4.1%			2,727	2,802	2,915	2,996
Gamma Dist.		2,729	111	4.1%			2,727	2,803	2,913	2,993

Figure D.2. Total Unpaid Claims Distribution (Paid Berquist-Sherman)

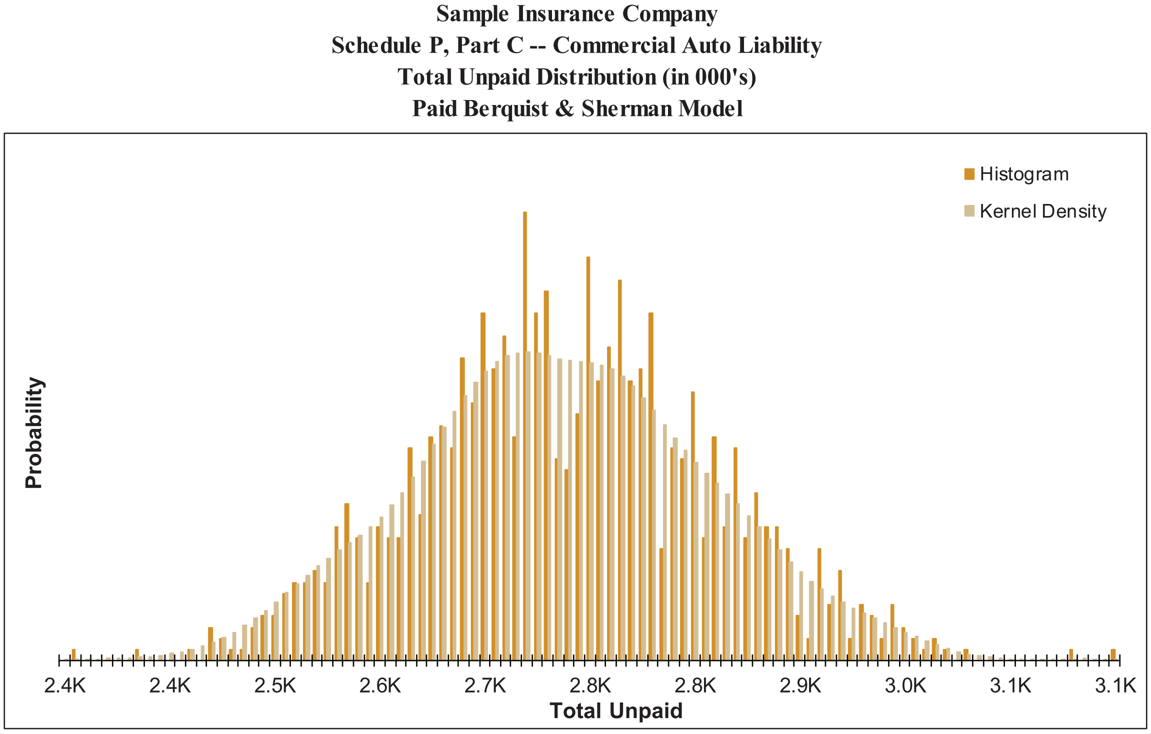

Figure D.3. Estimated Unpaid Model Results (Incurred Berquist-Sherman)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Incurred Berquist & Sherman Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	3	258.7%	(25)	13	1	3	6	9
2009	1,469	3	5	152.9%	(27)	24	3	6	12	17
2010	1,387	14	8	56.8%	(23)	45	13	18	27	33
2011	1,350	27	10	38.6%	(7)	76	26	33	45	53
2012	1,342	49	15	30.0%	4	107	49	59	75	88
2013	1,198	103	23	22.1%	46	186	102	117	142	162
2014	1,061	213	39	18.1%	107	334	212	239	275	304
2015	853	418	67	16.0%	160	655	418	463	531	577
2016	645	786	122	15.6%	363	1,249	783	864	981	1,074
2017	294	1,271	182	14.3%	655	1,879	1,274	1,387	1,570	1,699
Totals	11,162	2,885	276	9.6%	1,805	3,901	2,887	3,072	3,340	3,553
Normal Dist.		2,885	276	9.6%			2,885	3,072	3,340	3,528
logNormal Dist.		2,885	281	9.8%			2,872	3,066	3,370	3,601
Gamma Dist.		2,885	276	9.6%			2,876	3,066	3,354	3,567

Figure D.4. Total Unpaid Claims Distribution (Incurred Berquist-Sherman)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Berquist & Sherman Model

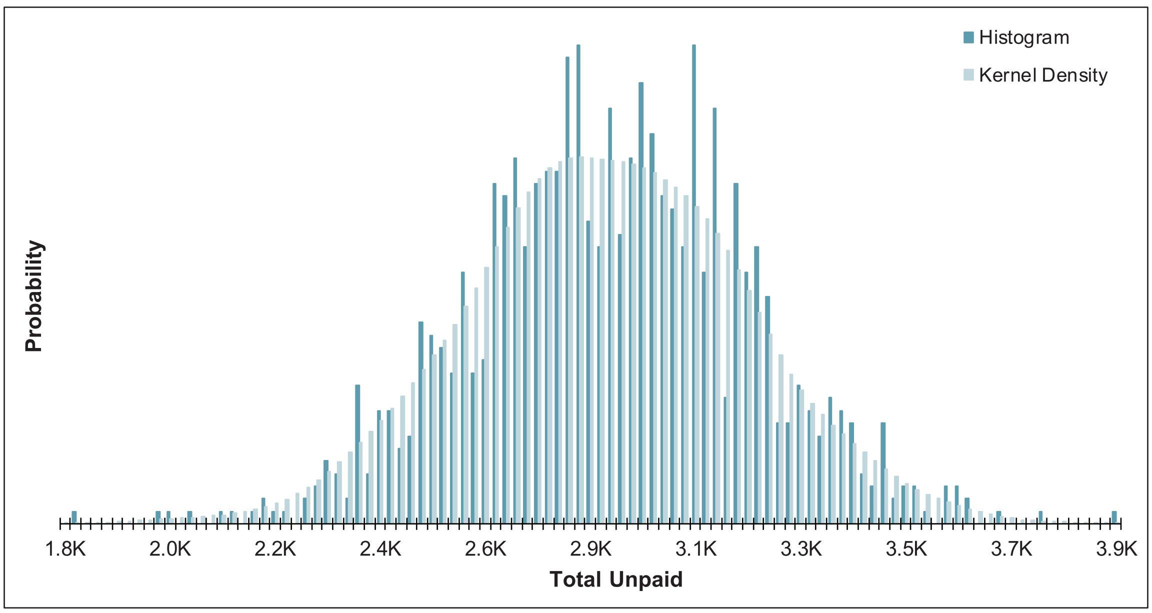

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Figure D.5. Estimated Unpaid Model Results (Paid Cape Cod)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Paid Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	2	7	352.9%	(28)	39	2	5	13	21
2009	1,469	5	10	202.5%	(35)	45	4	11	22	33
2010	1,387	15	12	80.5%	(24)	71	14	22	34	45
2011	1,350	29	14	49.5%	(25)	97	28	38	52	63
2012	1,342	53	17	32.0%	2	103	53	65	82	92
2013	1,198	101	19	18.9%	29	165	101	114	133	149
2014	1,061	212	24	11.3%	131	303	211	228	250	266
2015	853	407	30	7.4%	296	509	406	426	455	477
2016	645	767	46	6.0%	619	932	766	796	845	882
2017	294	1,093	73	6.7%	826	1,319	1,093	1,142	1,216	1,265
Totals	11,162	2,684	130	4.8%	2,267	3,103	2,680	2,772	2,895	2,998
Normal Dist.		2,684	130	4.8%			2,684	2,772	2,898	2,987
logNormal Dist.		2,684	131	4.9%			2,681	2,770	2,904	3,002
Gamma Dist.		2,684	130	4.8%			2,682	2,770	2,901	2,996

Figure D.6. Total Unpaid Claims Distribution (Paid Cape Cod)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Paid Cape Cod Model

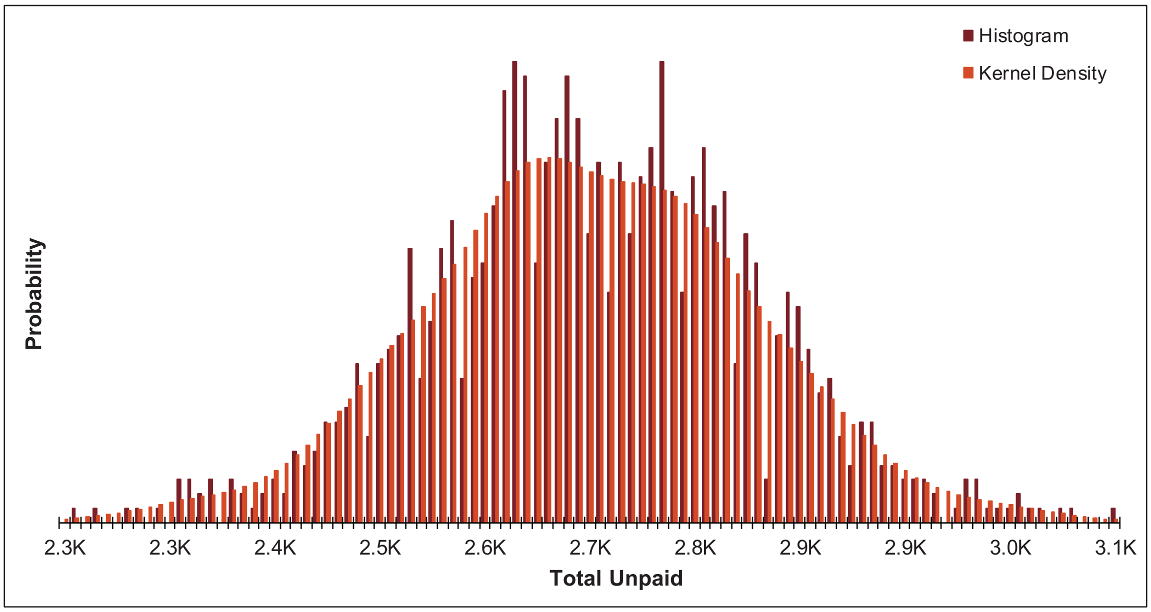

Figure D.7. Estimated Unpaid Model Results (Incurred Cape Cod)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Incurred Cape Cod Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	(34)	1,036	-3051.6%	(32,754)	310	0	3	10	25
2009	1,469	(35)	1,169	-3319.0%	(36,931)	820	2	7	20	56
2010	1,387	42	790	1886.5%	(2,137)	18,514	10	19	40	110
2011	1,350	11	643	5723.4%	(18,141)	8,214	25	37	62	107
2012	1,342	(19)	1,741	-9140.2%	(53,588)	3,106	53	70	102	194
2013	1,198	112	749	666.5%	(13,123)	18,673	113	133	182	506
2014	1,061	459	8,951	1951.6%	(36,649)	279,556	223	255	353	979
2015	853	533	4,998	937.7%	(46,566)	125,917	417	464	613	2,008
2016	645	1,277	16,899	1323.6%	(21,260)	524,102	732	793	1,051	3,347
2017	294	402	10,559	2627.1%	(306,693)	24,226	1,047	1,143	1,540	3,321
Totals	11,162	2,747	23,026	838.1%	(130,300)	711,166	2,611	2,788	3,131	4,659
Normal Dist.		2,747	23,026	838.1%			2,747	18,278	40,621	56,313
logNormal Dist.		7,743	34,578	446.6%			1,692	5,487	29,805	97,831
Gamma Dist.		2,747	23,026	838.1%			0	0	3,034	#NUM!

Figure D.8. Total Unpaid Claims Distribution (Incurred Cape Cod)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Cape Cod Model

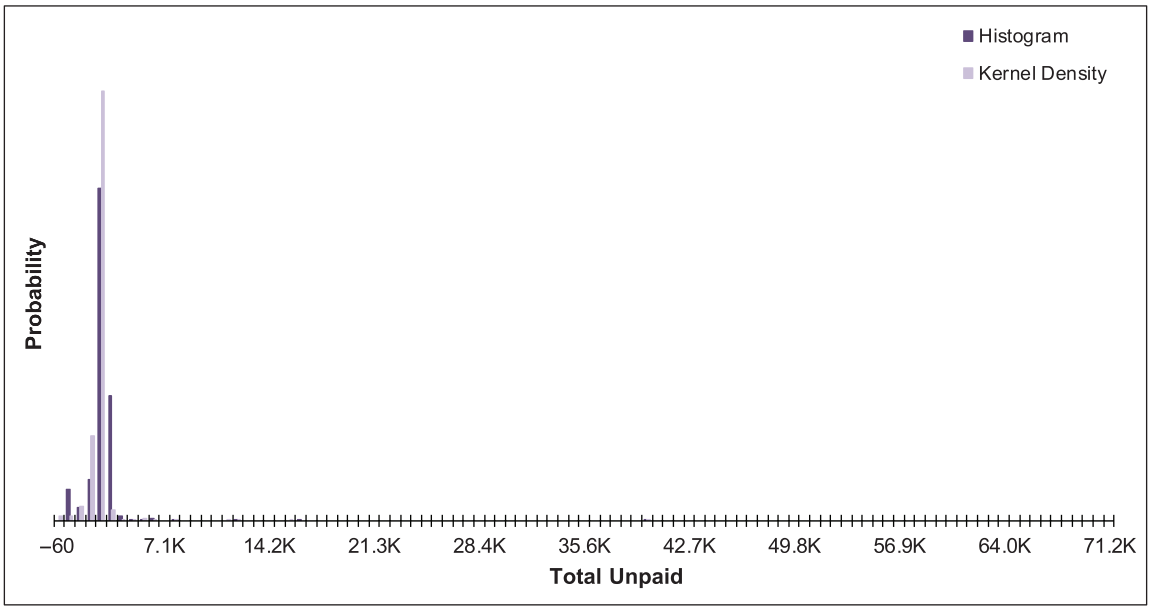

Figure D.9. Estimated Unpaid Model Results (Paid Chain Ladder)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Paid Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	5	494.6%	(26)	28	1	3	9	15
2009	1,469	3	7	245.1%	(38)	30	2	7	15	22
2010	1,387	13	10	75.7%	(27)	46	12	19	28	37
2011	1,350	26	12	46.8%	(29)	71	26	34	46	54
2012	1,342	51	15	29.9%	(13)	103	51	62	76	85
2013	1,198	101	17	17.2%	29	159	101	113	128	142
2014	1,061	211	21	10.2%	121	313	211	225	245	256
2015	853	406	26	6.5%	330	493	405	423	449	470
2016	645	766	34	4.4%	669	882	766	790	822	848
2017	294	1,096	43	3.9%	974	1,249	1,097	1,123	1,168	1,204
Totals	11,162	2,675	95	3.5%	2,374	3,000	2,675	2,739	2,829	2,912
Normal Dist.		2,675	95	3.5%			2,675	2,739	2,831	2,896
logNormal Dist.		2,675	95	3.6%			2,673	2,738	2,834	2,903
Gamma Dist.		2,675	95	3.5%			2,674	2,738	2,833	2,901

Figure D.10. Total Unpaid Claims Distribution (Paid Chain Ladder)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Paid Chain Ladder Model

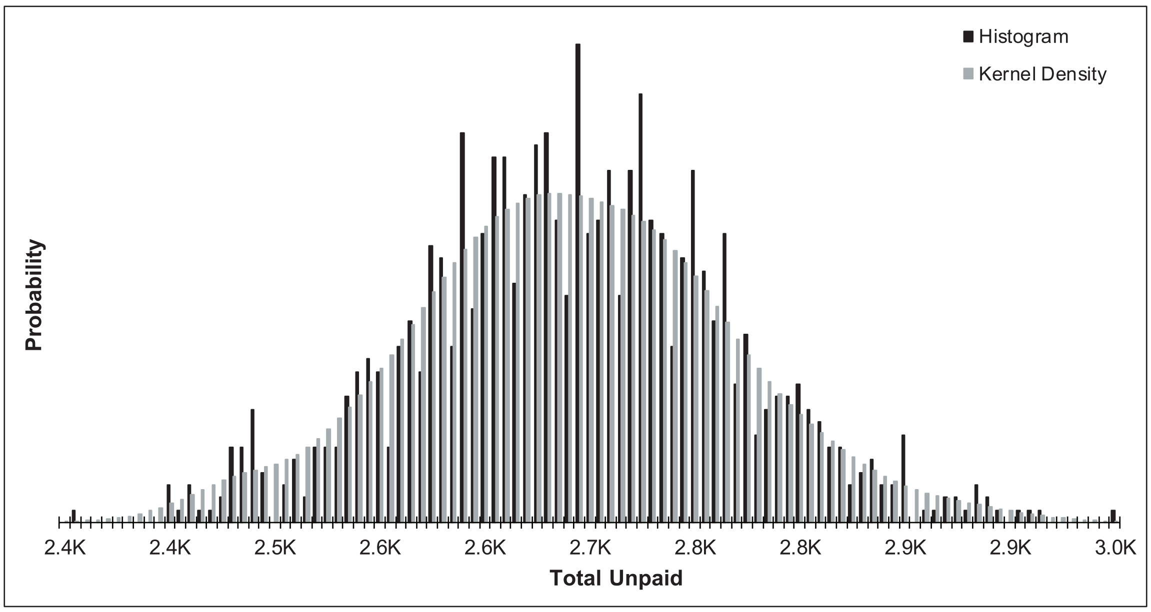

Figure D.11. Estimated Unpaid Model Results (Incurred Chain Ladder)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Incurred Chain Ladder Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	6	693.6%	(33)	36	0	3	9	19
2009	1,469	3	9	357.0%	(89)	58	1	6	18	30
2010	1,387	12	16	127.8%	(44)	85	10	20	40	60
2011	1,350	25	26	100.8%	(54)	148	22	38	70	102
2012	1,342	47	47	100.1%	(147)	214	44	74	131	180
2013	1,198	97	89	92.1%	(191)	493	95	149	252	343
2014	1,061	200	196	98.1%	(548)	955	200	317	527	709
2015	853	384	366	95.2%	(1,275)	1,789	381	616	961	1,279
2016	645	718	638	88.8%	(1,751)	3,360	724	1,122	1,759	2,244
2017	294	1,071	907	84.7%	(3,060)	4,244	1,136	1,645	2,559	3,298
Totals	11,162	2,557	1,221	47.7%	(4,786)	7,504	2,541	3,329	4,603	5,312
Normal Dist.		2,557	1,221	47.7%			2,557	3,381	4,566	5,398
logNormal Dist.		4,521	9,406	208.0%			1,959	4,686	16,441	39,696
Gamma Dist.		2,557	1,221	47.7%			2,366	3,243	4,839	6,213

Figure D.12. Total Unpaid Claims Distribution (Incurred Chain Ladder)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Chain Ladder Model

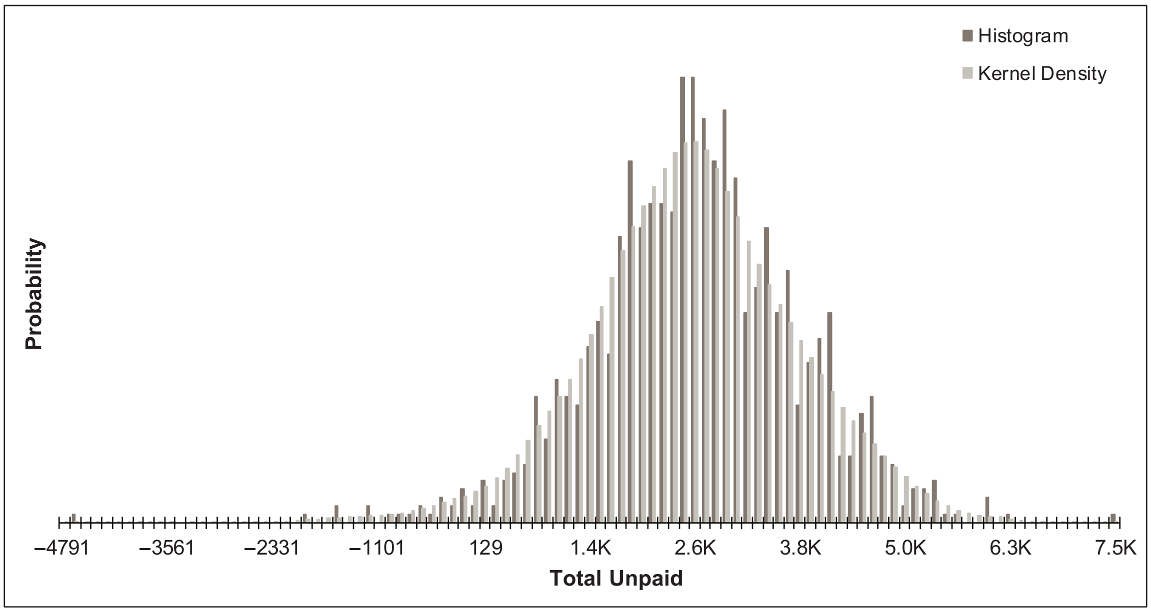

Figure D.13. Estimated Unpaid Model Results (Paid Hoerl Curve)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Paid Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	8	738.6%	(34)	39	1	5	13	25
2009	1,469	2	10	562.6%	(43)	55	2	7	16	27
2010	1,387	6	12	210.0%	(38)	56	5	12	25	39
2011	1,350	14	15	104.5%	(46)	70	13	23	39	50
2012	1,342	36	19	51.6%	(20)	99	36	48	68	89
2013	1,198	91	23	25.0%	(13)	167	91	106	129	143
2014	1,061	203	27	13.4%	116	294	203	220	247	267
2015	853	395	31	7.9%	306	505	395	415	448	467
2016	645	730	40	5.5%	616	867	729	757	798	824
2017	294	1,164	50	4.3%	1,013	1,342	1,165	1,196	1,247	1,284
Totals	11,162	2,641	110	4.2%	2,328	2,997	2,641	2,719	2,822	2,896
Normal Dist.		2,641	110	4.2%			2,641	2,715	2,822	2,896
logNormal Dist.		2,641	110	4.2%			2,639	2,714	2,826	2,907
Gamma Dist.		2,641	110	4.2%			2,640	2,714	2,824	2,903

Figure D.14. Total Unpaid Claims Distribution (Paid Hoerl Curve)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Paid Hoerl Curve Model

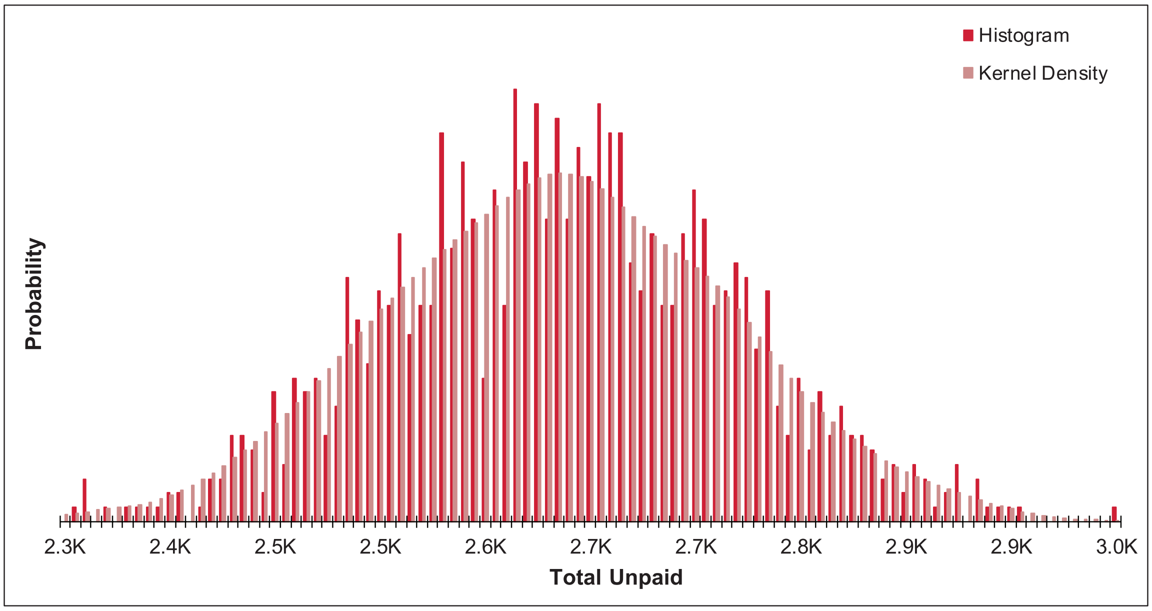

Figure D.15. Estimated Unpaid Model Results (Incurred Hoerl Curve)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Incurred Hoerl Curve Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	###########	###########	-3162.3%	###########	###########	1	4	17	12,332,021
2009	1,469	###########	###########	-3162.3%	###########	###########	2	7	27	37,702,476
2010	1,387	###########	###########	3162.3%	###########	###########	5	12	35	###########
2011	1,350	###########	###########	3162.3%	###########	###########	14	24	182	###########
2012	1,342	###########	###########	3162.3%	(51,418,906)	###########	36	51	4,042	###########
2013	1,198	###########	###########	3162.3%	(487,667)	###########	93	113	17,391	###########
2014	1,061	###########	###########	3162.3%	(5,299,244)	###########	210	244	36,869	###########
2015	853	###########	###########	3162.3%	(2,019,237)	###########	420	475	66,618	###########
2016	645	###########	###########	3162.3%	(54,398,452)	###########	791	877	124,983	###########
2017	294	###########	###########	3162.3%	(157,144)	###########	1,290	1,410	211,790	###########
Totals	11,162	###########	###########	3162.3%	(27,682,830)	###########	2,840	3,023	469,045	###########
Normal Dist.		###########	###########	3162.3%			###########	###########	###########	###########
logNormal Dist.		###########	###########	240623181.5%			7,145	276,650	53,267,753	###########
Gamma Dist.		###########	###########	3162.3%			0	0	###########	###########

Figure D.16. Total Unpaid Claims Distribution (Incurred Hoerl Curve)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Hoerl Curve Model

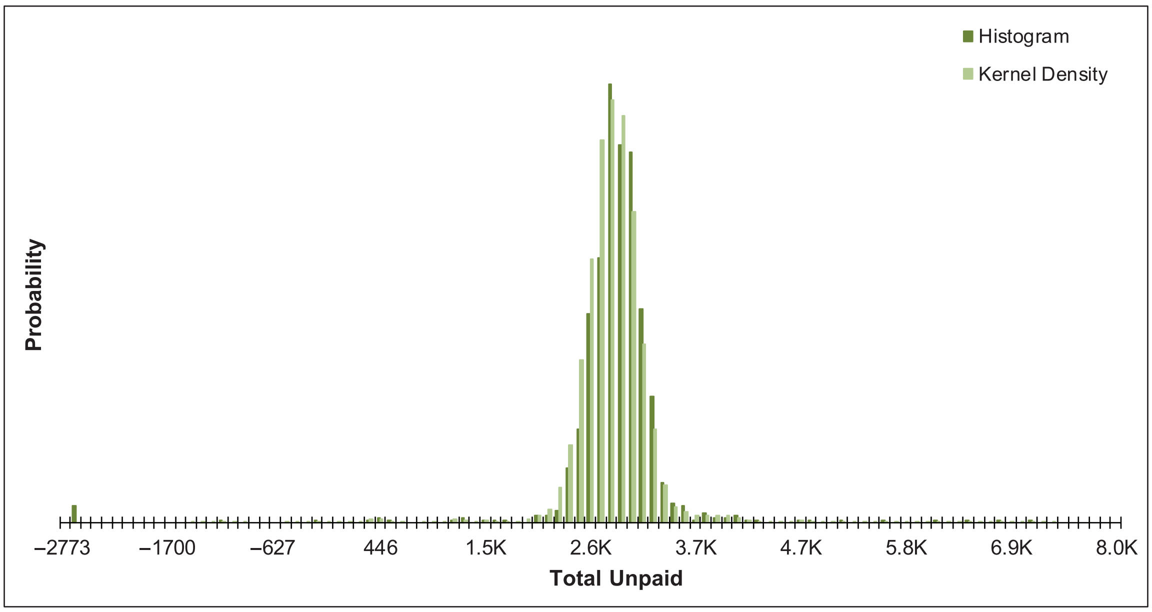

Figure D.17. Estimated Unpaid Model Results (Paid Wright)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Paid Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	14	1071.2%	(101)	93	1	8	21	36
2009	1,469	2	16	642.8%	(69)	100	3	10	27	44
2010	1,387	6	17	301.3%	(61)	106	5	14	34	56
2011	1,350	14	20	145.3%	(73)	145	12	25	45	67
2012	1,342	35	23	67.2%	(68)	141	35	49	73	96
2013	1,198	87	25	29.2%	7	205	86	102	128	157
2014	1,061	202	29	14.6%	88	323	202	220	249	270
2015	853	398	34	8.4%	263	509	399	420	454	476
2016	645	754	45	5.9%	628	942	754	783	824	866
2017	294	1,088	69	6.3%	853	1,350	1,086	1,131	1,204	1,255
Totals	11,162	2,587	121	4.7%	2,151	3,124	2,585	2,664	2,786	2,907
Normal Dist.		2,587	121	4.7%			2,587	2,668	2,786	2,868
logNormal Dist.		2,587	121	4.7%			2,584	2,667	2,790	2,881
Gamma Dist.		2,587	121	4.7%			2,585	2,667	2,789	2,876

Figure D.18. Total Unpaid Claims Distribution (Paid Wright)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Paid Wright Model

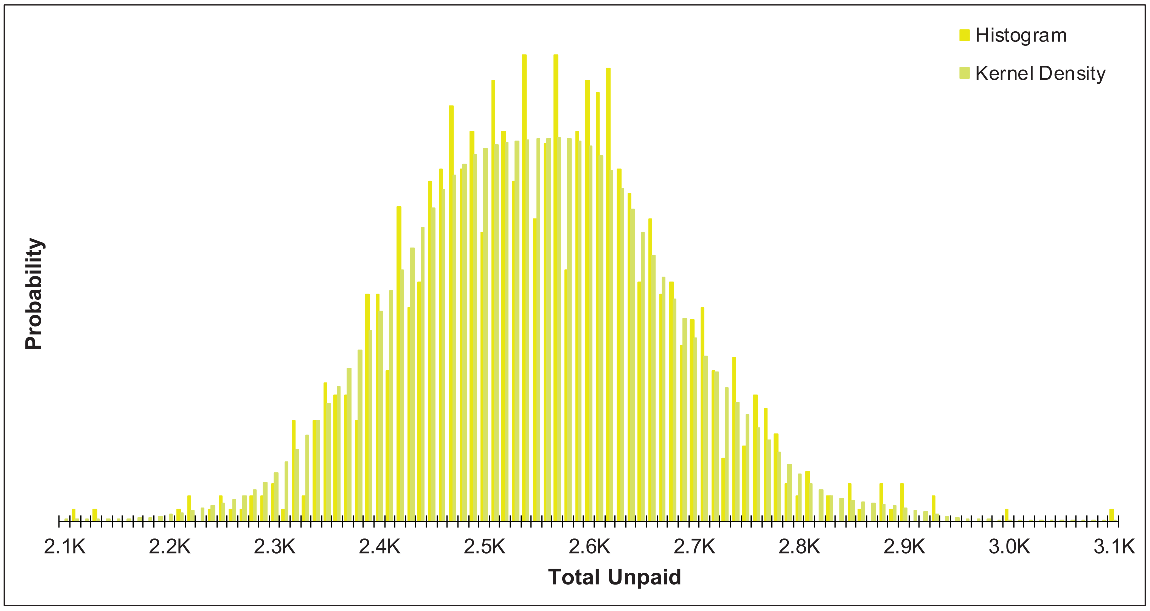

Figure D.19. Estimated Unpaid Model Results (Incurred Wright)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Incurred Wright Model

Accident Year	To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	2	20	1351.2%	(182)	177	1	7	30	62
2009	1,469	6	30	535.4%	(256)	278	2	12	50	94
2010	1,387	11	38	333.5%	(207)	254	4	21	82	134
2011	1,350	38	76	199.8%	(380)	895	18	59	173	329
2012	1,342	105	123	116.8%	(246)	1,244	75	147	339	499
2013	1,198	326	322	98.8%	(243)	2,469	248	443	945	1,616
2014	1,061	841	828	98.4%	(1,401)	11,042	624	1,129	2,378	3,601
2015	853	2,152	2,550	118.5%	(2,155)	49,324	1,576	2,787	5,899	8,999
2016	645	5,361	5,333	99.5%	(2,081)	56,677	3,763	7,162	15,009	24,079
2017	294	14,213	19,620	138.0%	(9,209)	307,488	8,685	17,580	41,810	81,568
Totals	11,162	23,055	22,052	95.6%	(4,260)	324,893	17,086	28,124	57,861	101,310
Normal Dist.		23,055	22,052	95.6%			23,055	37,929	59,327	74,355
logNormal Dist.		24,843	26,123	105.2%			17,120	30,640	70,786	127,452
Gamma Dist.		23,055	22,052	95.6%			16,527	31,928	66,938	101,555

Figure D.20. Total Unpaid Claims Distribution (Incurred Wright)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Total Unpaid Distribution (in 000's)
Incurred Wright Model

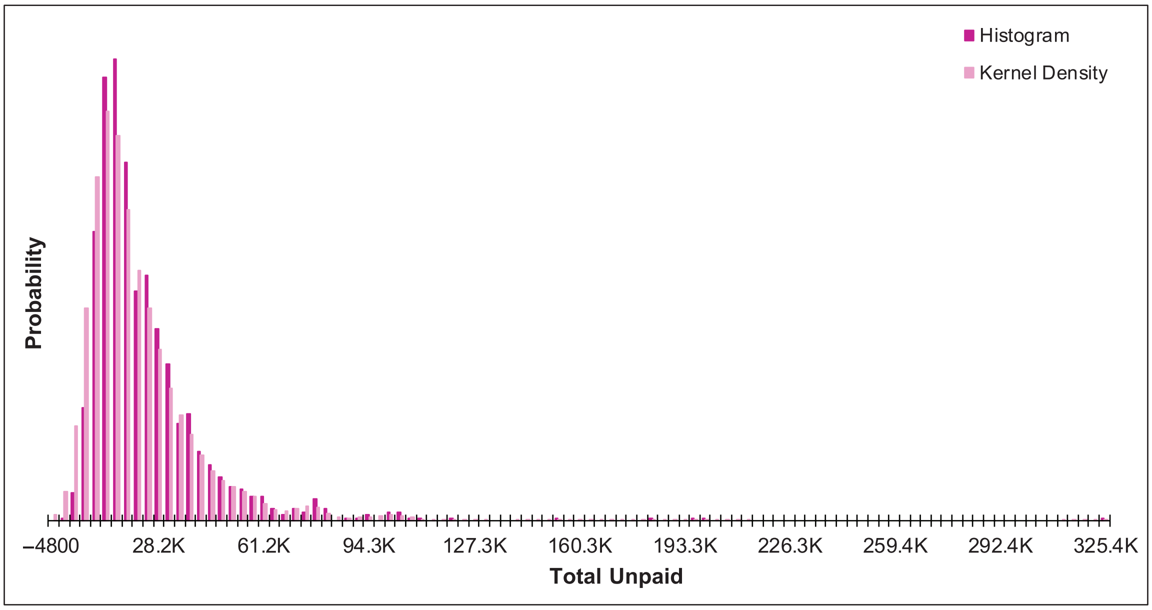

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Figure D.21. Model Weights by Accident Year

Accident	Model Weights by Accident Year								
Year	Paid BS	Incd BS	Paid CC	Paid CL	Incd CL	Paid HC	Paid WR	[redacted]	TOTAL
2008	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2009	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2010	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2011	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2012	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2013	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2014	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2015	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2016	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%
2017	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	[redacted]	100.0%

Figure D.22. Estimated Mean Unpaid by Model

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Summary of Results by Model (in 000's)

	Mean Estimated Unpaid							
Accident	Berquist & Sherman		Cape Cod		Chain Ladder		Hoerl Curve	Best Est.
Year	Paid	Incurred	Paid	Incurred	Paid	Incurred	Paid	(Weighted)
2008	1	1	2	1	1	1	1	1
2009	4	3	5	3	3	2	2	3
2010	14	14	15	13	12	6	6	11
2011	28	27	29	26	25	14	14	23
2012	50	49	53	51	47	36	35	47
2013	103	103	101	101	97	91	87	99
2014	209	213	212	211	200	203	202	207
2015	402	418	407	406	384	395	398	403
2016	742	786	767	766	718	730	754	756
2017	1,176	1,271	1,093	1,096	1,071	1,164	1,088	1,130
Totals	2,729	2,885	2,684	2,675	2,557	2,641	2,587	2,679

Figure D.23. Estimated Ranges

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Summary of Results by Model (in 000's)

		Ranges			
Accident	Best Est.	Weighted		Modeled	
Year	(Weighted)	Minimum	Maximum	Mininum	Maximum
2008	1	1	2	1	2
2009	3	2	5	2	5
2010	11	6	15	6	15
2011	23	14	29	14	29
2012	47	35	53	35	53
2013	99	87	103	87	103
2014	207	200	213	200	213
2015	403	384	418	384	418
2016	756	718	786	718	786
2017	1,130	1,071	1,271	1,071	1,271
Totals	2,679	2,517	2,894	2,557	2,885

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Figure D.24. Reconciliation of Total Results (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Reconciliation of Total Results (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Incurred To Date	Case Reserves	IBNR	Estimate of Ultimate	Estimate of Unpaid
2008	1,563	1,577	14	(12)	1,565	1
2009	1,469	1,505	36	(33)	1,472	3
2010	1,387	1,436	49	(38)	1,398	11
2011	1,350	1,417	67	(44)	1,373	23
2012	1,342	1,445	102	(56)	1,389	47
2013	1,198	1,345	147	(48)	1,297	99
2014	1,061	1,339	278	(71)	1,267	207
2015	853	1,327	474	(71)	1,256	403
2016	645	1,442	797	(41)	1,401	756
2017	294	1,422	1,128	1	1,424	1,130
Totals	11,162	14,255	3,093	(413)	13,841	2,679

Figure D.25. Estimated Unpaid Model Results (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Unpaid (in 000's)
Best Estimate (Weighted)

Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,563	1	7	526.2%	(101)	95	1	4	12	23
2009	1,469	3	9	274.3%	(89)	100	3	8	18	30
2010	1,387	11	13	119.2%	(71)	115	11	18	31	46
2011	1,350	23	17	75.1%	(80)	216	24	33	50	68
2012	1,342	47	24	52.3%	(111)	272	47	59	82	118
2013	1,198	99	39	39.1%	(235)	571	100	115	147	224
2014	1,061	207	80	38.8%	(880)	891	208	228	290	490
2015	853	403	139	34.6%	(974)	1,841	401	428	551	920
2016	645	756	244	32.3%	(1,740)	3,765	755	793	1,014	1,635
2017	294	1,130	370	32.8%	(2,262)	4,783	1,133	1,199	1,533	2,406
Totals	11,162	2,679	474	17.7%	(500)	6,591	2,683	2,837	3,362	4,119
Normal Dist.		2,679	474	17.7%			2,679	2,999	3,458	3,781
logNormal Dist.		2,749	897	32.6%			2,614	3,239	4,411	5,478
Gamma Dist.		2,679	474	17.7%			2,651	2,982	3,503	3,903

Figure D.26. Estimated Cash Flow (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Calendar Year Unpaid (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2018	1,069	176	16.4%	38	2,391	1,072	1,135	1,337	1,599
2019	744	136	18.3%	(166)	1,830	745	794	945	1,154
2020	443	90	20.2%	(178)	1,298	444	478	568	714
2021	229	52	22.7%	(149)	661	229	252	301	385
2022	106	29	27.5%	(90)	313	106	122	150	182
2023	48	19	40.3%	(55)	176	47	60	80	99
2024	23	15	63.7%	(37)	139	23	32	47	61
2025	11	12	104.9%	(49)	94	11	18	30	42
2026	3	8	248.7%	(60)	55	3	8	16	25
2027	1	6	454.7%	(86)	61	1	4	11	18
2028	1	4	777.7%	(46)	59	0	2	7	14
2029	0	3	1416.3%	(27)	40	0	1	4	9
Totals	2,679	474	17.7%	(500)	6,591	2,683	2,837	3,362	4,119

Figure D.27. Estimated Loss Ratio (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Ultimate Loss Ratios (in 000's)
Best Estimate (Weighted)

Accident Year	Earned Premium	Mean Loss Ratio	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	1,748	86.6%	27.2%	31.4%	-264.5%	340.5%	88.5%	91.5%	108.2%	180.7%
2009	1,810	80.6%	24.5%	30.5%	-193.1%	335.3%	81.8%	84.5%	101.8%	166.6%
2010	1,915	73.5%	22.2%	30.3%	-139.9%	287.7%	74.4%	77.5%	93.9%	149.7%
2011	2,275	60.3%	19.4%	32.2%	-126.9%	245.5%	60.6%	62.7%	78.5%	128.9%
2012	2,524	53.4%	18.0%	33.6%	-107.9%	232.9%	54.0%	56.1%	70.8%	116.8%
2013	2,445	53.0%	17.4%	32.9%	-132.3%	248.4%	53.2%	55.0%	69.9%	116.6%
2014	2,543	49.5%	18.2%	36.9%	-190.8%	224.8%	49.7%	51.4%	67.2%	117.4%
2015	2,461	51.1%	17.5%	34.2%	-117.8%	240.1%	50.9%	52.7%	69.3%	116.5%
2016	2,485	56.2%	18.1%	32.2%	-132.1%	278.3%	56.1%	58.3%	75.8%	123.2%
2017	2,383	60.2%	19.8%	32.9%	-118.2%	257.3%	60.3%	63.7%	81.5%	130.0%
Totals	22,588	60.8%	6.2%	10.2%	20.4%	93.6%	61.1%	63.8%	70.9%	77.6%

Figure D.28. Estimated Unpaid Claim Runoff (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Calendar Year Unpaid Claim Runoff (in 000's)
Best Estimate (Weighted)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2017	2,679	474	17.7%	(500)	6,591	2,683	2,837	3,362	4,119
2018	1,610	308	19.1%	(590)	4,201	1,612	1,714	2,049	2,557
2019	865	179	20.7%	(445)	2,522	866	933	1,114	1,419
2020	422	97	23.0%	(268)	1,247	422	465	561	715
2021	193	54	28.1%	(119)	634	193	222	277	342
2022	88	35	39.9%	(86)	339	87	108	143	179
2023	40	25	62.0%	(65)	204	39	54	80	105
2024	16	17	105.8%	(80)	113	16	26	45	64
2025	5	12	222.4%	(77)	93	5	11	25	38
2026	2	8	387.7%	(83)	81	2	6	15	28
2027	1	5	694.5%	(49)	62	0	3	9	18
2028	0	3	1416.3%	(27)	40	0	1	4	9

Figure D.29. Mean of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Mean Values (in 000's)												
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**	**156**
2008	321	373	334	236	133	63	27	13	8	2	1	0	0
2009	309	359	322	227	128	61	26	13	8	2	1	0	0
2010	299	347	311	219	124	59	25	13	8	2	1	0	0
2011	291	338	303	213	121	57	25	12	8	2	1	0	0
2012	286	332	298	209	119	56	24	12	7	2	1	0	0
2013	275	320	286	202	114	54	23	11	7	2	1	0	0
2014	267	310	278	196	110	53	23	11	7	2	1	0	0
2015	267	310	278	196	111	53	23	11	7	2	1	0	0
2016	297	345	309	218	123	58	25	12	8	2	1	0	0
2017	305	353	317	223	126	60	26	12	8	2	1	0	0

Figure D.30. Standard Deviation of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Standard Error Values (in 000's)												
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**	**156**
2008	104	117	106	75	44	23	14	10	9	6	5	4	3
2009	98	110	99	71	41	22	13	10	9	6	4	4	3
2010	94	105	96	67	40	21	13	10	9	6	4	4	3
2011	98	109	99	70	41	22	12	10	8	6	4	4	3
2012	100	112	101	72	42	22	12	10	8	6	4	3	3
2013	94	105	95	68	39	21	12	9	8	5	4	3	3
2014	101	114	103	73	43	22	12	9	8	5	4	3	3
2015	95	106	96	69	39	20	12	9	8	5	4	3	3
2016	99	111	100	71	41	22	12	10	8	5	4	3	3
2017	103	117	106	75	43	23	13	10	8	5	4	3	3

Figure D.31. Coefficient of Variation of Incremental Values (Weighted)

Sample Insurance Company
Schedule P, Part C -- Commercial Auto Liability
Accident Year Incremental Values by Development Period
Best Estimate (Weighted)

Accident	Coefficients of Variation												
Year	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**	**156**
2008	32.4%	31.4%	31.7%	32.0%	33.1%	36.5%	49.8%	75.4%	108.1%	300.4%	599.5%	982.0%	1807.7%
2009	31.6%	30.6%	30.7%	31.2%	31.9%	36.4%	49.4%	75.6%	108.5%	278.9%	601.2%	1156.2%	1193.4%
2010	31.5%	30.4%	30.7%	30.7%	32.1%	35.9%	50.0%	76.0%	110.9%	297.0%	666.9%	1051.0%	1366.6%
2011	33.5%	32.3%	32.5%	32.9%	33.9%	37.9%	49.9%	77.7%	108.6%	292.6%	586.7%	1061.1%	1535.5%
2012	34.8%	33.7%	34.0%	34.3%	35.3%	38.5%	51.0%	80.1%	109.0%	300.1%	572.5%	934.5%	2346.4%
2013	34.0%	32.9%	33.2%	33.6%	34.5%	38.4%	51.9%	78.3%	114.1%	297.8%	581.3%	1007.1%	1454.7%
2014	37.9%	36.9%	37.1%	37.5%	38.5%	42.1%	54.5%	81.5%	117.4%	298.5%	536.2%	1189.7%	1266.2%
2015	35.4%	34.2%	34.5%	35.0%	35.4%	38.9%	51.9%	79.6%	113.7%	294.5%	575.8%	963.2%	1305.7%
2016	33.4%	32.2%	32.5%	32.8%	33.6%	37.7%	48.8%	77.3%	106.5%	278.9%	578.7%	999.3%	1598.3%
2017	33.9%	33.0%	33.3%	33.6%	34.0%	37.5%	49.6%	76.4%	106.0%	271.3%	531.0%	860.7%	1416.3%

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Figure D.32. Total Unpaid Claims Distribution (Weighted)

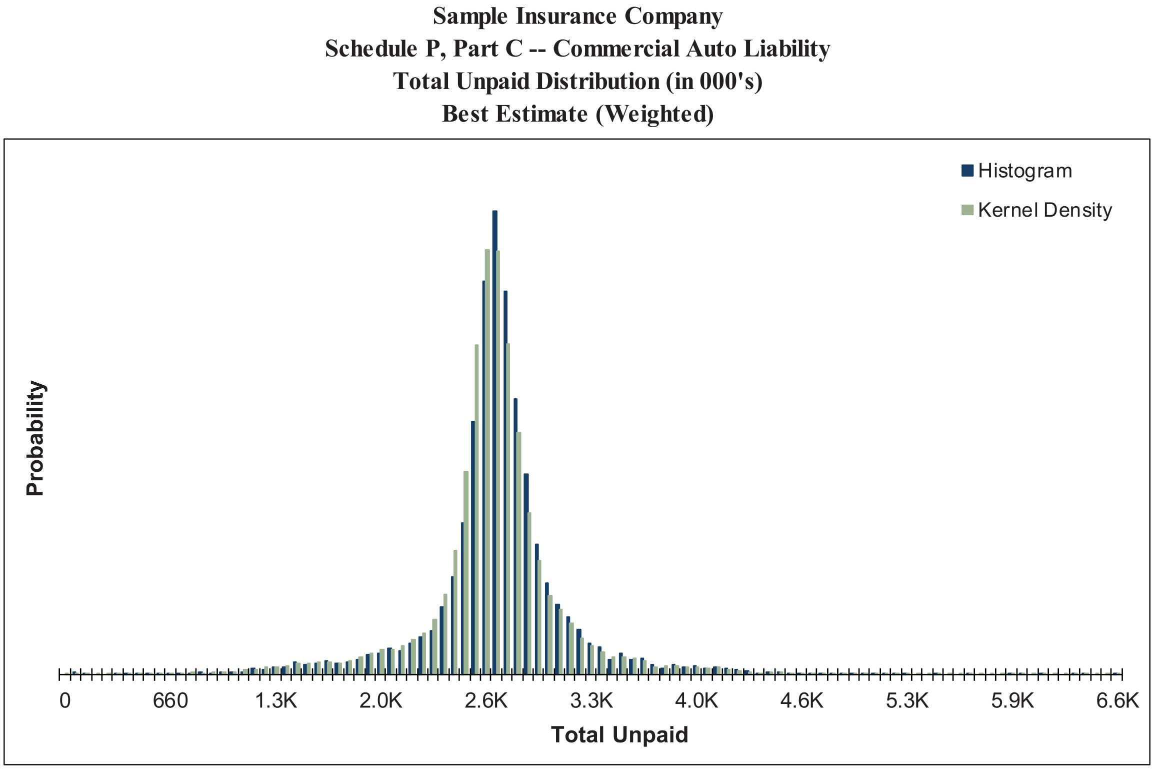
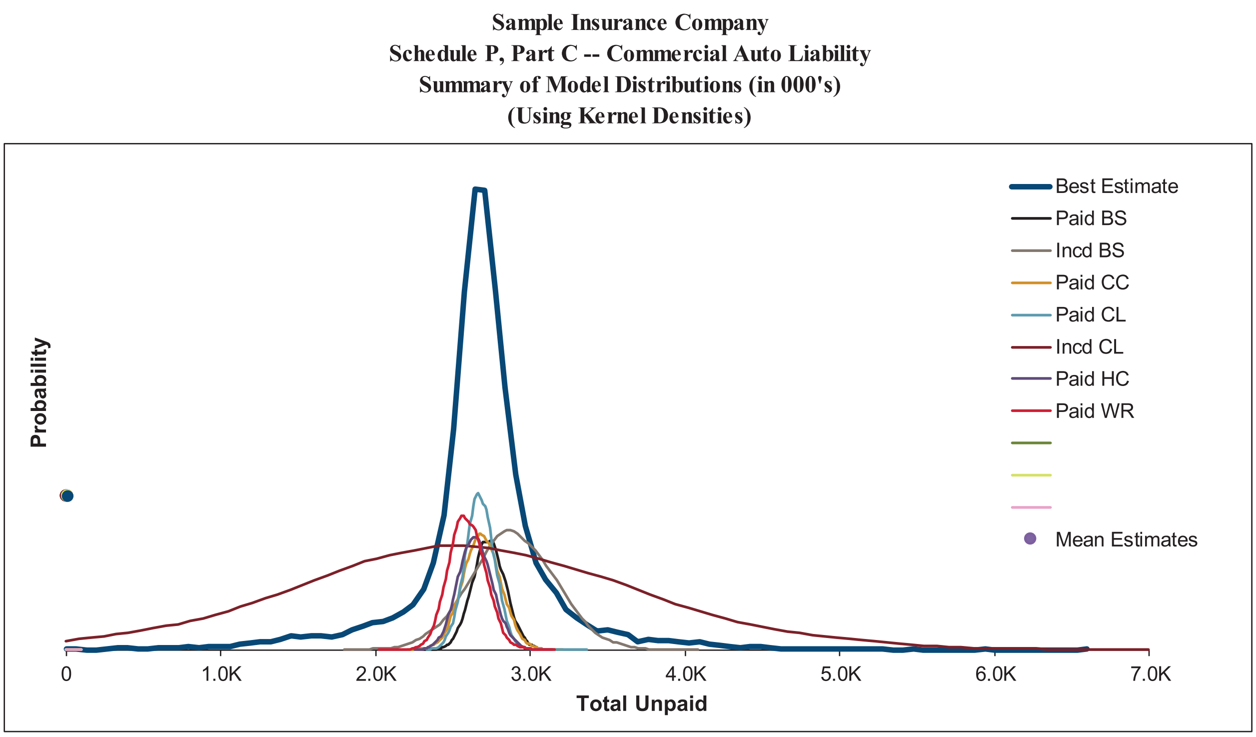


Figure D.33. Summary of Model Distributions



Appendix E—Aggregate Results

In this appendix the results for the correlated aggregate of the three Schedule P lines of business (Parts A, B, and C) are shown, using the correlation calculated from the paid data for the Berquist-Sherman model.

Figure E.1. Estimated Unpaid Model Results

Sample Insurance Company
Aggregate Three Lines of Business
Accident Year Unpaid (in 000's)

Accident Year	Paid To Date	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	18,613	146	1,002	688.1%	(2,013)	74,778	37	55	421	2,422
2009	20,618	198	993	500.3%	(1,523)	37,034	70	94	503	3,069
2010	22,866	246	927	377.4%	(5,763)	54,447	128	162	542	3,227
2011	22,842	367	1,286	350.7%	(2,918)	90,399	230	268	695	3,778
2012	22,351	535	1,359	254.3%	(1,875)	69,139	406	452	860	3,458
2013	22,422	869	1,266	145.7%	(3,632)	68,690	760	826	1,253	4,003
2014	24,350	1,589	939	59.1%	(4,107)	27,387	1,518	1,633	2,198	4,927
2015	19,973	2,814	1,424	50.6%	(8,046)	80,667	2,785	2,963	3,667	6,153
2016	18,919	5,418	4,384	80.9%	(8,120)	407,319	5,420	5,768	6,863	9,408
2017	15,961	13,369	3,352	25.1%	(11,431)	98,644	13,319	14,627	17,722	21,777
Totals	208,915	25,550	9,304	36.4%	(815)	476,278	24,635	26,612	32,642	55,933
Normal Dist.		25,550	9,304	36.4%			25,550	31,826	40,854	47,195
logNormal Dist.		25,528	6,217	24.4%			24,803	29,163	36,812	43,354
Gamma Dist.		25,550	9,304	36.4%			24,430	31,065	42,526	52,000
TVaR							28,995	32,475	48,429	89,074
Normal TVaR							32,974	37,377	44,742	50,348
logNormal TVaR							30,371	33,900	40,865	47,165
Gamma TVaR							32,838	38,140	48,373	57,295

Figure E.2. Estimated Cash Flow

Sample Insurance Company
Aggregate Three Lines of Business
Calendar Year Unpaid (in 000's)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2018	12,497	2,095	16.8%	(797)	23,980	12,494	13,633	15,903	17,708
2019	5,432	760	14.0%	175	10,046	5,475	5,859	6,587	7,241
2020	2,945	423	14.4%	(100)	5,390	2,965	3,176	3,586	3,989
2021	1,562	310	19.8%	(95)	13,391	1,553	1,674	1,959	2,463
2022	902	857	95.0%	(102)	60,941	810	893	1,281	3,189
2023	546	835	153.0%	(320)	33,504	431	490	928	3,022
2024	361	817	226.4%	(880)	44,982	242	289	756	2,813
2025	283	1,087	384.4%	(1,221)	70,925	144	183	681	3,090
2026	228	1,139	499.5%	(714)	61,008	84	120	590	3,055
2027	190	1,049	551.1%	(1,481)	53,144	46	79	587	3,006
2028	165	825	499.4%	(1,571)	23,126	27	54	554	3,206
2029	160	1,260	789.6%	(3,531)	74,987	14	33	480	3,172
2030	169	3,600	2134.8%	(7,667)	342,488	6	12	412	2,742
2031	110	1,288	1168.6%	(1,140)	66,389	2	4	196	2,239
Totals	25,550	9,304	36.4%	(815)	476,278	24,635	26,612	32,642	55,933

Figure E.3. Estimated Loss Ratio

Sample Insurance Company
Aggregate Three Lines of Business
Accident Year Ultimate Loss Ratios (in 000's)

Accident Year	Earned Premium	Mean Loss Ratio	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2008	25,305	72.6%	15.3%	21.1%	-52.5%	368.8%	74.2%	79.3%	91.2%	108.2%
2009	25,577	78.6%	15.6%	19.8%	-34.7%	224.3%	80.3%	85.5%	97.6%	114.8%
2010	27,155	82.2%	16.0%	19.5%	-18.6%	276.9%	84.0%	89.3%	102.3%	119.9%
2011	30,529	74.6%	14.5%	19.4%	-59.9%	373.8%	75.7%	80.8%	93.3%	109.0%
2012	34,399	65.2%	13.0%	19.9%	-24.3%	269.8%	66.3%	70.9%	81.9%	94.6%
2013	36,231	63.2%	12.5%	19.8%	-47.2%	251.4%	64.2%	68.8%	79.9%	92.2%
2014	36,863	70.7%	14.1%	20.0%	-37.8%	146.6%	70.7%	77.5%	92.4%	107.1%
2015	37,678	60.2%	12.8%	21.3%	-34.6%	271.2%	60.8%	65.9%	77.7%	89.1%
2016	38,101	63.9%	16.8%	26.2%	-54.2%	1115.7%	64.2%	69.8%	82.1%	95.7%
2017	37,997	79.2%	15.9%	20.1%	-36.7%	313.1%	78.0%	86.8%	104.4%	121.2%
Totals	329,835	70.4%	4.9%	6.9%	47.9%	195.6%	70.5%	73.2%	77.3%	81.3%

Figure E.4. Estimated Unpaid Claim Runoff

Sample Insurance Company
Aggregate Three Lines of Business
Calendar Year Unpaid Claim Runoff (in 000's)

Calendar Year	Mean Unpaid	Standard Error	Coefficient of Variation	Minimum	Maximum	50.0% Percentile	75.0% Percentile	95.0% Percentile	99.0% Percentile
2017	25,550	9,304	36.4%	(815)	476,278	24,635	26,612	32,642	55,933
2018	13,054	8,804	67.4%	(18)	464,252	11,965	12,832	17,870	43,814
2019	7,621	8,748	114.8%	(193)	459,695	6,389	6,960	12,059	39,214
2020	4,676	8,706	186.2%	(93)	456,649	3,366	3,757	8,953	36,003
2021	3,113	8,561	275.0%	2	452,976	1,799	2,096	7,305	33,834
2022	2,212	8,057	364.3%	67	439,029	986	1,225	5,912	30,057
2023	1,665	7,588	455.6%	21	433,153	557	758	4,879	25,743
2024	1,305	7,152	548.1%	14	427,965	318	480	3,998	21,821
2025	1,022	6,647	650.3%	(9,274)	427,465	177	304	3,245	18,524
2026	794	6,071	764.6%	(9,339)	425,019	94	187	2,507	14,898
2027	604	5,472	906.6%	(9,336)	411,276	49	108	1,873	11,341
2028	438	4,936	1126.0%	(9,105)	393,463	23	52	1,269	7,908
2029	279	3,988	1430.4%	(7,664)	342,491	8	17	690	5,233
2030	110	1,288	1168.6%	(1,140)	66,389	2	4	196	2,239

Figure E.5. Mean of Incremental Values

Sample Insurance Company
Aggregate Three Lines of Business
Accident Year Incremental Values by Development Period

Accident Year	Mean Values (in 000's)														
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
2008	9,173	4,740	1,972	1,141	608	302	149	77	45	28	16	14	17	29	68
2009	10,218	5,151	2,099	1,201	639	318	157	82	47	30	18	16	20	34	80
2010	11,568	5,691	2,256	1,285	680	339	169	88	51	33	19	17	20	32	73
2011	11,629	5,858	2,348	1,338	711	353	176	92	53	34	20	18	22	38	90
2012	11,538	5,742	2,289	1,303	691	343	171	89	52	33	20	18	22	38	92
2013	12,007	5,790	2,270	1,288	680	340	170	88	52	33	20	17	22	37	91
2014	14,275	6,479	2,392	1,337	699	350	176	93	54	34	21	18	22	36	84
2015	11,853	5,778	2,251	1,277	674	335	168	87	51	33	20	17	22	38	92
2016	12,776	6,149	2,397	1,357	716	357	178	94	54	35	21	18	23	43	125
2017	16,732	7,423	2,675	1,491	773	388	195	103	60	38	23	20	25	43	110

Figure E.6. Standard Deviation of Incremental Values

Sample Insurance Company
Aggregate Three Lines of Business
Accident Year Incremental Values by Development Period

Accident Year	Standard Deviation Values (in 000's)														
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
2008	2,168	1,131	467	268	145	74	39	22	15	11	12	22	58	187	752
2009	2,302	1,179	475	272	146	75	40	23	16	11	13	26	68	210	696
2010	2,598	1,299	517	293	156	80	43	25	17	12	14	26	65	192	656
2011	2,610	1,303	513	293	158	81	43	25	16	12	15	29	76	249	951
2012	2,637	1,313	516	292	158	81	44	25	17	12	15	29	79	269	994
2013	2,801	1,342	525	295	158	82	44	26	17	12	15	28	77	250	917
2014	3,566	1,581	577	320	173	90	48	29	20	13	15	28	67	192	618
2015	3,004	1,428	550	312	166	85	47	26	18	12	15	29	78	254	950
2016	3,196	1,509	572	323	171	90	48	28	19	13	15	32	108	572	3,565
2017	4,007	1,903	637	359	187	98	54	32	22	15	17	32	92	328	1,288

Figure E.7. Coefficient of Variation of Incremental Values

Sample Insurance Company
Aggregate Three Lines of Business
Accident Year Incremental Values by Development Period

Accident Year	Coefficients of Variation														
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
2008	23.6%	23.9%	23.7%	23.5%	23.9%	24.6%	26.1%	28.6%	33.7%	38.3%	74.9%	156.3%	332.5%	643.8%	1099.6%
2009	22.5%	22.9%	22.7%	22.7%	22.8%	23.6%	25.6%	27.7%	33.0%	36.7%	74.4%	164.6%	342.8%	612.0%	865.2%
2010	22.5%	22.8%	22.9%	22.8%	23.0%	23.7%	25.7%	28.1%	33.5%	36.9%	73.2%	155.0%	321.5%	591.5%	898.4%
2011	22.4%	22.2%	21.8%	21.9%	22.2%	22.9%	24.4%	26.9%	31.1%	34.5%	72.3%	160.6%	344.1%	661.7%	1057.6%
2012	22.9%	22.9%	22.6%	22.4%	22.9%	23.5%	25.6%	27.8%	32.6%	35.7%	73.4%	165.2%	363.3%	708.1%	1079.9%
2013	23.3%	23.2%	23.1%	22.9%	23.3%	24.0%	26.0%	28.9%	33.9%	37.1%	73.8%	163.4%	354.9%	668.2%	1007.6%
2014	25.0%	24.4%	24.1%	24.0%	24.7%	25.6%	27.5%	31.1%	36.0%	39.1%	73.3%	151.8%	301.9%	529.0%	733.9%
2015	25.3%	24.7%	24.4%	24.5%	24.7%	25.4%	27.8%	30.1%	34.9%	37.6%	74.0%	164.7%	356.7%	673.6%	1037.3%
2016	25.0%	24.5%	23.9%	23.8%	23.8%	25.1%	26.8%	29.9%	34.4%	37.3%	73.1%	175.0%	466.6%	1333.7%	2845.1%
2017	23.9%	25.6%	23.8%	24.0%	24.1%	25.3%	27.7%	31.1%	36.3%	40.0%	72.2%	160.0%	368.1%	757.1%	1168.6%

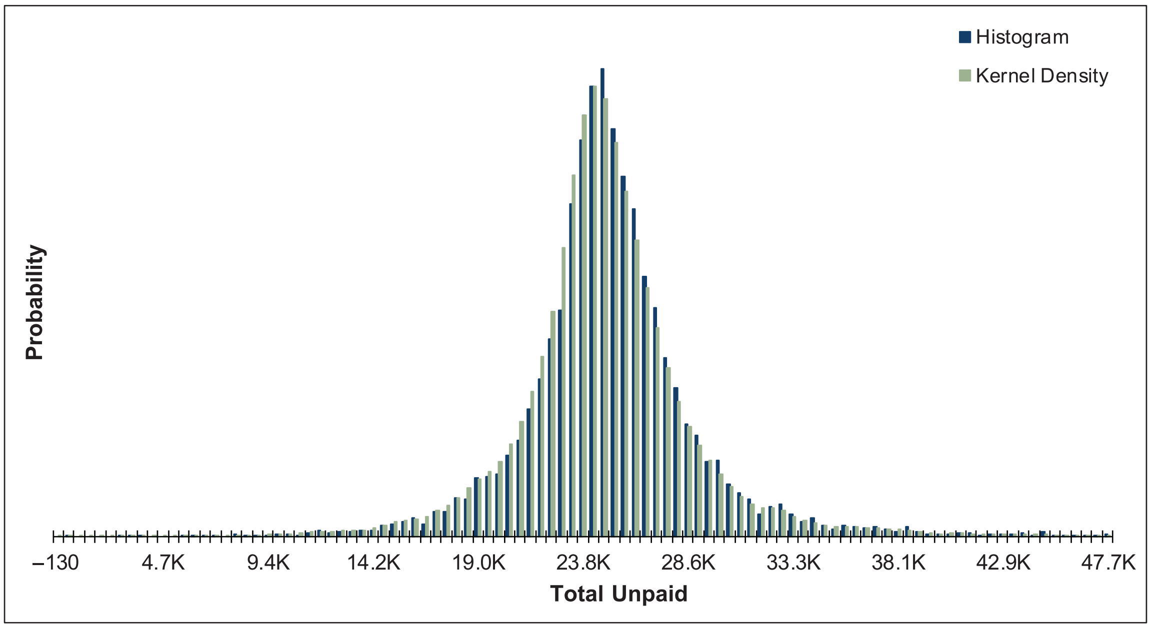
Figure E.8. Calculation of Risk Based Capital

Sample Insurance Company
Aggregate Three Lines of Business
Indicated Unpaid Claim Risk Portion of Required Capital (in 000's)

LOB / Segment	Earned Premium	Mean Unpaid	99.0% Unpaid	Value at Risk Capital	Allocated Capital	Unpaid Ratio	Premium Ratio
Homeowners / Farmowners	15,148	5,792	10,410	4,618	4,048	69.9%	26.7%
Private Passenger Auto Liabilit	20,467	17,079	45,682	28,602	25,072	146.8%	122.5%
Commercial Auto Liability	2,383	2,679	4,119	1,439	1,262	47.1%	52.9%
Total	37,997	25,550	60,210	34,660			
Aggregate	37,997	25,550	55,933	30,382	30,382	118.9%	80.0%

Figure E.9. Total Unpaid Claims Distribution

Sample Insurance Company
Aggregate Three Lines of Business
Total Unpaid Distribution



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Abbreviations and Notations

Collect here in alphabetical order all abbreviations and notations used in the monograph

- AIC, akaiki information criteria
- BIC, bayesian information criteria
- BS, berquist-sherman
- WR, wright
- TVaR, tail value at risk
- CoV, coefficient of variation
- HC, hoerl curve
- CC, cape cod
- CL, chain ladder
- VaR, value at risk

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