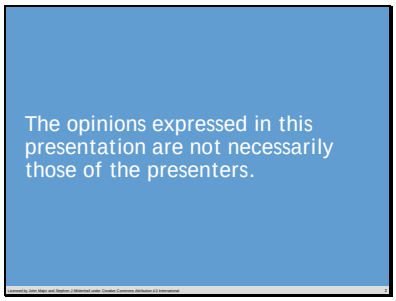
Slide 1

Slide 2



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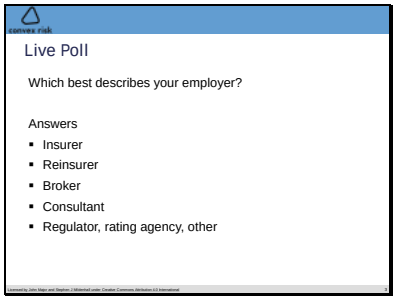

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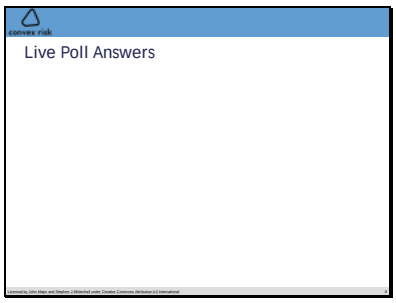

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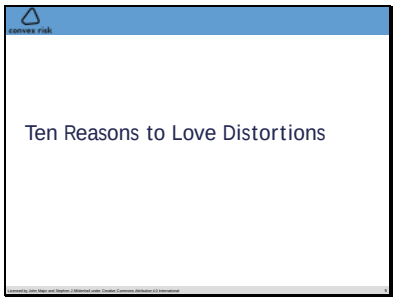

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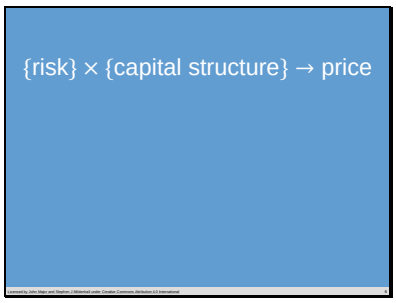

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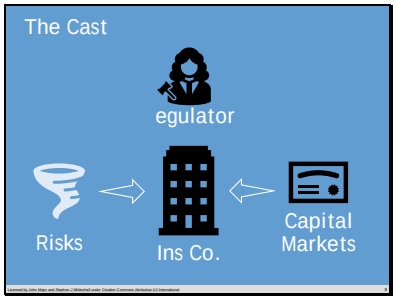

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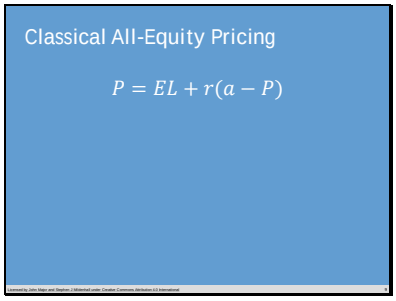

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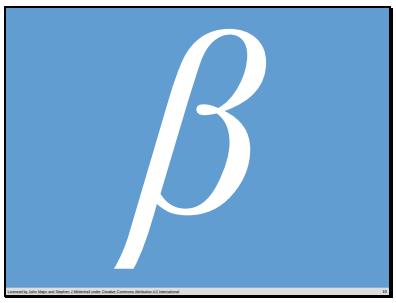

Slide 11

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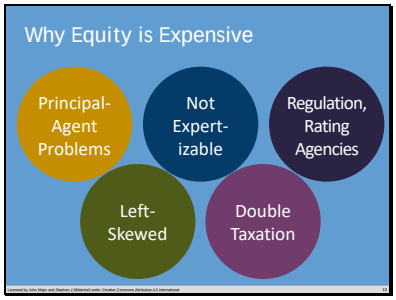

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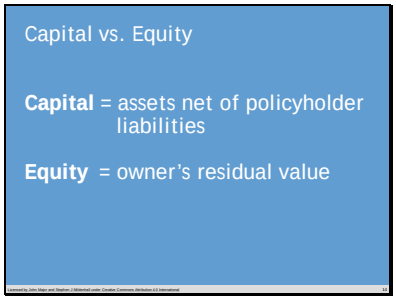

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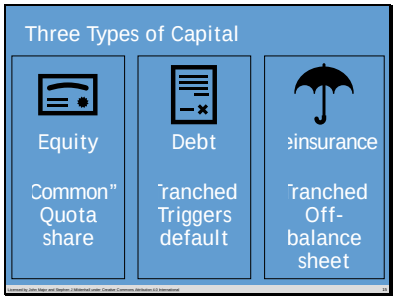

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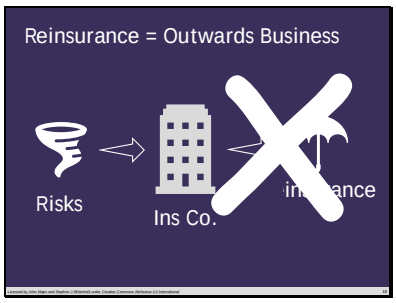

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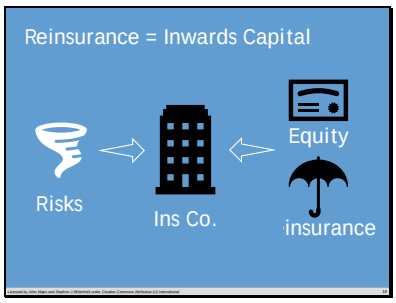

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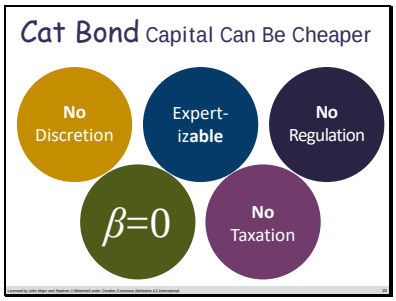

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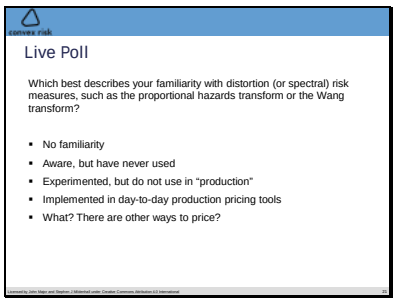

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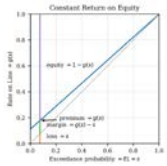
convex risk

Live Poll Answers

Slide 23

Fund Risk-Bearing Assets by Layer



$$P = vEL + da$$
$$P(x) = vS(x) + d$$

Slide 24

How to Calibrate Distortion g

{risk} × {capital structure} → price

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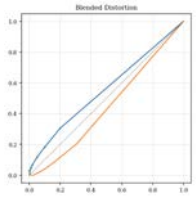
Equity + Reinsurance Distortion

x	ROE	$g(s)$
1e-10	0.03	0.0291
0.01	0.04	0.0481
0.02	0.05	0.0667
0.05	0.075	0.116
0.1	0.1	0.182
0.2	0.15	0.304



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Portfolio Pricing

- $EL = E[X] = \int S(x)\,dx = \int x f(x)\,dx$
- $P = E[Xg'S(X)] = \int gS(x)\,dx$
  $= \int xg'S(x)f(x)\,dx$
- $g'S(X)$ is a risk adjustment, co-measure
- No default!

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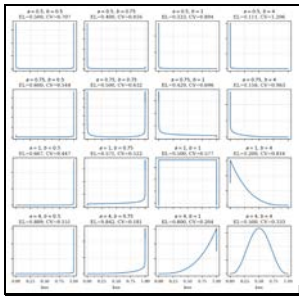

Beta Loss Densities

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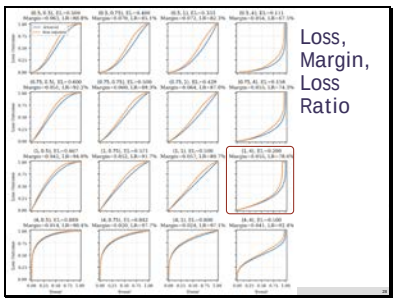

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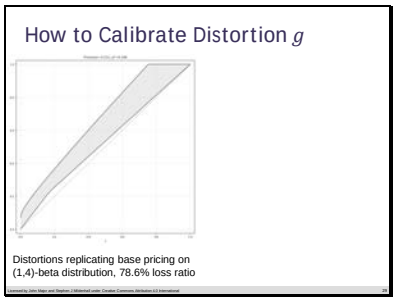

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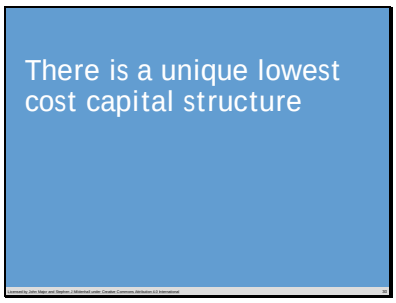

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Allocation When $X = \sum X_i$

- $EL_i = E[X_i] = \int x_i f(x)\, dx$
- $P_i = E[X_i g'S(X)] = \int x_i g'S(x) f(x)\, dx$
- Call P_i the **natural allocation**
- Terms and conditions apply

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Allocation in Practice

Spreadsheet or SQL or R or Python or ...

- Rows = simulations
- Columns = loss by line and in total
- Group-by & sort total loss and average
- Compute $S(x)$ and $gS(x)$, difference
- Sum-product

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Ambiguity

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Ambiguity is not Bayes-able

- Tossing an uncertain coin
- Bayes solution ➔ Law of Large Numbers
- Adverse selection ➔ Wrong until the end of time

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Ambiguity and Distortions

- Pr(H) > 0.5 and Pr(T) > 0.5
- Non-additive probabilities: $g(\Pr(\mathrm{A}))$
- Example: for $g(s) = x^{1/2}$
  $g(\Pr(H)) = 0.71 = g(\Pr(T))$
  $g(\Pr(H,T)) = 1$

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Underwriters and Distortions

- Distortion pricing = worst of a set of scenario outcomes
- The distortion controls how much event probabilities can increase

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convex risk

Live Poll

A legitimate pricing rule that is consistent with finance theory must have which of the following properties?

1. It only charges for non-diversifiable risk
2. It is linear: the price of a bundle is the sum of the prices of the parts
3. It does not allow arbitrage opportunities

Answers

- 1 only
- 2 only
- 3 only
- 1, 2, and 3
- Don't understand the question

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convex risk

Live Poll Answers

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Finance Theory and Distortions

- Bid-ask spread = market imperfection
- Decreases the effectiveness of no-arbitrage
- Distortion pricing is consistent with financial theory

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Optimization and Distortions

- Optimization = marginal improvements
- Natural allocation has a marginal interpretation

$$\lim_{t \downarrow 0} \frac{\rho(X + tX_i) - \rho(X)}{t} = E[X_i g'S(X)]$$

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Top Ten Reasons To Love Distortions

1. Calibrate to capital structure
2. Can calibrate to market pricing
3. Practical spreadsheet implementation
4. Easy to work with catastrophe model output
5. Sensitive to shape of risk
6. Marginal risk interpretation
7. Weighted average of TVaR interpretation
8. Worst-over-scenarios interpretation
9. Consistent with underwriting
10. Consistent with financial theory

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Major Analytics

Part II: Reinsurance Applications

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Major Analytics

SRM Use Cases

- Policy pricing / technical premium
- LOB assessment
- LOB capital cost allocation
- Reinsurance decisions

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Major Analytics

SRM Principal Modes

- "A/B": Evaluate $\rho(X_G)$ and $\rho(X_N)$; compare
- Allocate: $NA(X_C; X_G)$ where $X_C = X_G - X_N$

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Major Analytics

Applying the Modes

Use Case	A/B	Alloc
Policy pricing / technical premium	No	Yes
LOB assessment	??	Yes
LOB capital cost allocation	No	Yes
Reinsurance decisions	Yes	Yes

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Major Analytics

Case Study

- Use reinsurance to minimize total cost of risk
- VERY simple portfolio: 2 lines
- VERY simple XOL contract: on line 1
- Illustrate the two modes

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Major Analytics

Reinsurance Example

Gross				Net 2 xs 2 (X1), premium = 0.38			
X1	X2	X_G	p	$X1_C$	$X1_N$	X2	X_N
1	1	2	0.7	0	1	1	2
2	3	5	0.1	0	2	3	5
4	2	6	0.1	2	2	2	4
3	4	7	0.1	1	2	4	6

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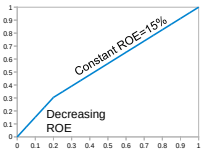
Major Analytics

Our Distortion Function



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Major Analytics

First Mode: A/B

- A/B the gross and net positions
- Which one has lower total cost?

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Major Analytics

Gross Valuation

X_G	X_N	p	S	g(S)	dg_G
2	2	0.7	0.3	0.391	0.609
5	5	0.1	0.2	0.304	0.087
6	4	0.1	0.1	0.152	0.152
7	6	0.1	0	0	0.152

$\Sigma\, x_{Gi}\, dg_i = 3.630$
= gross premium

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Major Analytics

Net Valuation

	X_G	X_N	p	S	g(S)	dg_N
	2	2	0.7	0.3	0.391	0.609
Re-order!	6	4	0.1	0.2	0.304	0.087
	5	5	0.1	0.1	0.152	0.152
	7	6	0.1	0	0	0.152

$\Sigma\, x_{Ni}\, dg_i = 3.239$
= net premium

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Major Analytics

A/B Gross and Net

3.239	Net premium
+0.380	Cost of reinsurance
=3.619	Total cost, net position
vs	
3.630	Gross premium + 0 r/i cost
0.011	Difference

Conclusion: r/i a marginally good deal.

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Major Analytics

Second Mode: Allocation

- Allocate premium from Gross to Net and Ceded components
- Allocate premium from Net to Gross and (-)Ceded components
- Is Ceded worth the r/i premium?

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Major Analytics

Allocating Ceded from Gross Position

$X_G = X_N + X1_C$

X1	X2	X_G	dg_G	$X1_C$
1	1	2	0.609	0
2	3	5	0.087	0
4	2	6	0.152	2
3	4	7	0.152	1

$\Sigma\ X1c_i\, dg_i = 0.456 > 0.38$ by 0.076

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Major Analytics

Allocating Ceded from Net Position

$X_N = X_G - X1_C$

	$X1_N$	X2	X_N	dg_N	$X1_C$
	1	1	2	0.609	0
Re-order!	2	2	4	0.087	2
	2	3	5	0.152	0
	2	4	6	0.152	1

$\Sigma\ X1c_i\, dg_i = 0.326 < 0.38$ by 0.054

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Major Analytics

Conclusions about the r/i deal so far

- Direct A/B:
  - Eh, okay. +0.011
- Allocate starting at gross
  - Great! +0.076
- Allocate starting at net
  - Bad! -0.054
- How do I explain this to my boss?
  - What does it all really mean?

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Major Analytics

A More Thorough A/B

- Fractional participation t
- $X_N(t) = X_G - t \cdot X_C$
- Minimize total cost = $\rho(X_N(t)) + t \cdot$ (r/i prem)

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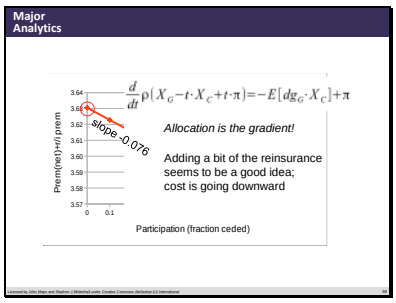

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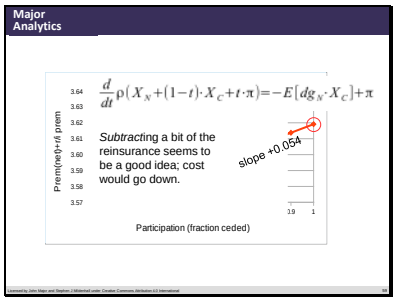

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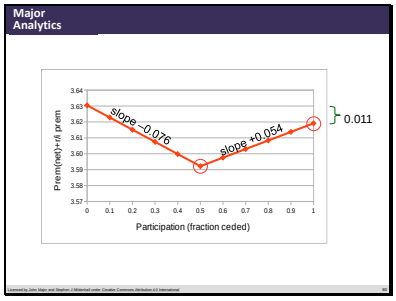

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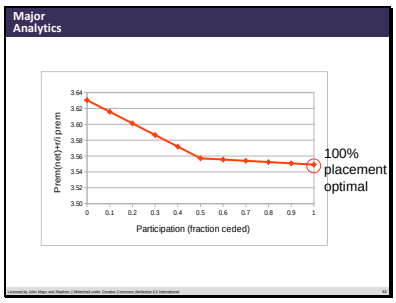

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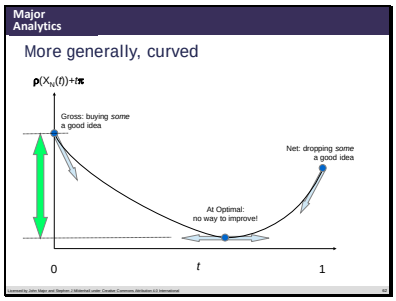

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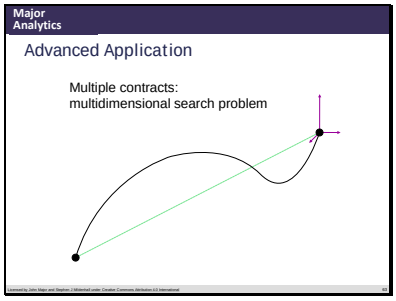

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Major Analytics

Summary: Allocation for Decisions

- Allocation = gradient
- Faster than A/B
- Like using Taylor's Theorem
  - OK for small changes
  - Iffy for big

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Major Analytics

Applying the Modes, Redux

Use Case	A/B	Alloc
Policy pricing / technical premium	No	Yes
LOB assessment	??	Yes
LOB capital cost allocation	No	Yes
Reinsurance decisions	Yes	Yes

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Questions?