1.

An actuary is using the inversion method to simulate a gamma random variable with mean and variance both equal to 2. Two random draws from the uniform distribution (0,1) are independently made, and their values are 0.2 and 0.8.

Calculate the value of the simulated gamma random variable.

A. Less than 0.5
B. At least 0.5, but less than 1.0
C. At least 1.0, but less than 1.5
D. At least 1.5, but less than 2.0
E. At least 2.0

1. Solution
Key: D

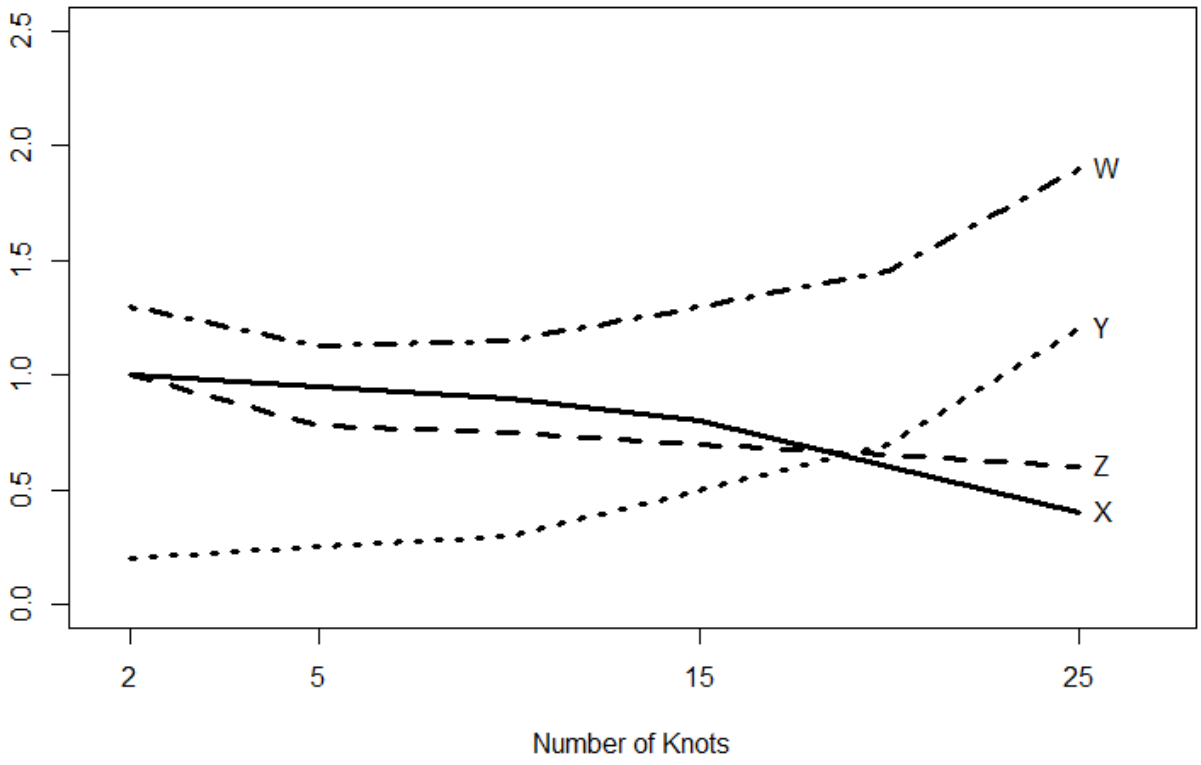
Solution: By hypothesis, we know that the actuary is simulating a gamma random variable with $(\alpha, \theta) = (2,1)$. Thus, it can be thought of the sum of two independent exponential random variables each with rate 1. Using the inversion method, we have:

$$-\ln(0.2) - \ln(0.8) = 1.833.$$

Classification: A.8.c – Simulation – Inversion Method
Text Reference: *Ross*, 11.2.1 (example 11.3) & 11.3.2
Item Notes: The author states that when simulating exponential random variables using either U or (1-U) as values is valid (example 11.3, p 650). However in this question the simulated values are chosen such that the answer is the same either way.

2.

You want to fit a cubic spline to a large dataset and need to determine the number of knots to use. Below is a chart of four statistics from this model valued for various numbers of knots:



Determine which set of statistics below best describes each line.

A.	W is Test MSE	X is Variance	Y is Squared Bias	Z is Train MSE
B.	W is Variance	X is Squared Bias	Y is Test MSE	Z is Train MSE
C.	W is Train MSE	X is Test MSE	Y is Variance	Z is Squared Bias
D.	W is Test MSE	X is Train MSE	Y is Variance	Z is Squared Bias
E.	W is Variance	X is Train MSE	Y is Test MSE	Z is Squared Bias

2. Solution
Key: D

Solution:
All else equal, a spline with more knots will be more flexible, and allow the model to better fit the training data.

Variance increases with flexibility → Line Y
Bias squared and Train MSE both decrease with flexibility → Lines X&Z
Test MSE generally exhibits a 'U' shaped behavior → Line W

→ Only answer D meets these conditions

Also Test MSE is the sum of Variance + Bias Squared + Irreducible Error, and only Line W can be the sum of two of the other values

Classification: C.2.q – Test vs Train Error
Text Reference: *James et al,* p 29-37
Item Notes:

3.

You are given the following statements about different resampling methods:

I. Leave-one-out cross-validation (LOOCV) is a special case of k-fold cross-validation
II. k-fold cross-validation has higher variance than LOOCV when $k < n$
III. LOOCV tends to overestimate the test error rate in comparison to validation set approach

Determine which of the above statements are correct.

A. I only
B. II only
C. III only
D. I, II, and III
E. The correct answer isn't given by (A), (B), (C), or (D)

3. Solution
Key: A

Solution:

I. TRUE: LOOCV is just k-fold cross-validation where k = n
II. FALSE: LOOCV has higher variance than k-fold validation, for k < n, because of the high degree of correlation in the training data sets.
III. FALSE: Validation set approach tends to overestimate the test error rate more than LOOCV, which is one of its drawbacks.

Classification: C.2.p – Cross Validation
Text Reference: *James et al,* Chapter 5.1
Item Notes:

4.

You are given the following models which contain regression splines:

Model	Numbers of Spline Knots	Degree of Regression Spline
A	6	4
B	5	5
C	8	2
D	10	3

Calculate the total number of the regression coefficients in the four models.

A. Less than 5
B. At least 15, but less than 25
C. At least 25, but less than 35
D. At least 35, but less than 45
E. At least 45

4. Solution
Key: E

Solution:

# of coefficients = Degree of Regression Spline + Numbers of Spline Knots + Intercept

Number of coefficients sum for Model A: $6 + 4 + 1 = 11$
Number of coefficients sum for Model B: $5 + 5 + 1 = 11$
Number of coefficients sum for Model C: $8 + 2 + 1 = 11$
Number of coefficients sum for Model D: $10 + 3 + 1 = 14$

Total number of coefficients: $11 + 11 + 11 + 14 = 47$

Classification: C.4.i – Piecewise Linear and Smoothing Splines
Text Reference: *James et al,* p 273
Item Notes: