Exam S

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CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam S

Statistics and Probabilistic Models

May 8, 2017

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 90 point examination consists of 45 multiple choice questions each worth 2 points.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2017 and that the exam name is S.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam S" included in your exam packet.

   - Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table.

   - Candidates should employ a non-parametric test unless otherwise specified in the problem, or when there is a standard distribution that is logically or commonly associated with the random variable in question. Examples of problems with a logical or commonly associated distribution would include exponential wait times for Poisson processes and applications of the Central Limit Theorem.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 22, 2017.

END OF INSTRUCTIONS

1.

You are given the following information about a random variable, X.

- For all $r, s \geq 0$, $\Pr(X > r + s) = \Pr(X > r)\Pr(X > s)$
- $E[X|X > 10] = 30$

Calculate the result for the expression: $E[X|X > 20]$.

A. Less than 30
B. At least 30, but less than 40
C. At least 40, but less than 50
D. At least 50, but less than 60
E. At least 60

Exam Continued on Next Page

2.

You are given the following information about a senior-year college student from an actuarial science program:

- Job offers arrive according to a Poisson process with a rate of three per month.
- A job offer is acceptable if the salary offered is at least 50,000 per year.
- The salaries offered are mutually independent and follow a lognormal distribution with $\mu = 10.5$ and $\sigma = 0.2$.

Calculate the probability that it will take this senior-year college student more than four months to receive an acceptable job offer.

A. Less than 0.505
B. At least 0.505, but less than 0.515
C. At least 0.515, but less than 0.525
D. At least 0.525, but less than 0.535
E. At least 0.535

Exam Continued on Next Page

3.

You are given the following information about emergency room visits at a local hospital:

- The amount of time between emergency room visits for broken bones is exponentially distributed with a mean of two hours.
- The amount of time between emergency room visits for the flu is exponentially distributed with a mean of five hours.
- Emergency room visits for broken bones and emergency room visits for the flu are independent.

Calculate the probability that two patients will come into the hospital with broken bones before three patients come into the hospital with the flu.

A. Less than .875
B. At least .875, but less than .900
C. At least .900, but less than .925
D. At least .925, but less than .950
E. At least .950

Exam Continued on Next Page

4.

Auto physical damage claims from 100 policyholders of an insurance company arise according to a Poisson process with rate 20 per year. You are given the following:

- Claim sizes are independent and follow an exponential distribution with mean 3,000.
- Claim sizes and claim counts are independent.
- For claims less than 10,000, the insurance company pays the policyholder the claim amount.
- For claims greater than or equal to 10,000, the insurance company pays the policyholder 10,000.

Calculate the expected cost to the insurance company per policyholder per year.

A. Less than 560
B. At least 560, but less than 570
C. At least 570, but less than 580
D. At least 580, but less than 590
E. At least 590

Exam Continued on Next Page

5.

You are given:

- R and S are two independent components in a series system.
- X and Y are the time-to-failure random variables of R and S, respectively.
- X and Y both follow the exponential distribution but with different hazard rates.
- The probability that R fails before S is 0.2.
- The mean lifetime of X is 1.
- R and S start operation at the same time.

Calculate the expected time until the system fails.

A. Less than 0.25
B. At least 0.25, but less than 0.50
C. At least 0.50, but less than 0.75
D. At least 0.75, but less than 1.00
E. At least 1.00

Exam Continued on Next Page

6.

X and Y are two independent exponential random variables with hazard rates $\lambda_X = 2$ and $\lambda_Y = 8$, respectively.

Calculate the expected value of X, conditional on $1 < X < Y$.

A. Less than 1.20
B. At least 1.20, but less than 1.40
C. At least 1.40, but less than 1.60
D. At least 1.60, but less than 1.80
E. At least 1.80

Exam Continued on Next Page

7.

You are given:

- A factory machine is processing parts sequentially in an assembly line.
- After a part is processed, the machine moves to the next part in line.
- There is a chance that while waiting to be processed, a part will fall to the floor and drop out of the processing queue. The probability a part drops on the floor in a given time is distributed exponentially with rate θ for each part. Once a part falls on the floor, it drops out of the processing line permanently.
- The time to complete processing, once the machine has begun work on a part, is exponentially distributed with mean 5 and is independent for each part.
- A part is now 5^{th} in line for processing and has a probability of being processed of 0.10.
- P_7 is the probability that the part that is seventh in line for processing will eventually be processed.

Calculate P_7.

A. Less than 0.060
B. At least 0.060, but less than 0.065
C. At least 0.065, but less than 0.070
D. At least 0.070, but less than 0.075
E. At least 0.075

Exam Continued on Next Page

8.

You are given the following information:

- Engineers are comparing two systems.
- System U is a 4-out-of-5 system consisting of 5 identical components.
- For each component of System U, its lifetime follows the exponential distribution with a hazard rate of 0.5.
- System V is a series system consisting of 10 identical components.
- For each component of System V, its lifetime follows the exponential distribution with a hazard rate of 1.0.
- T_U and T_V are the lifetimes of Systems U and V, respectively.

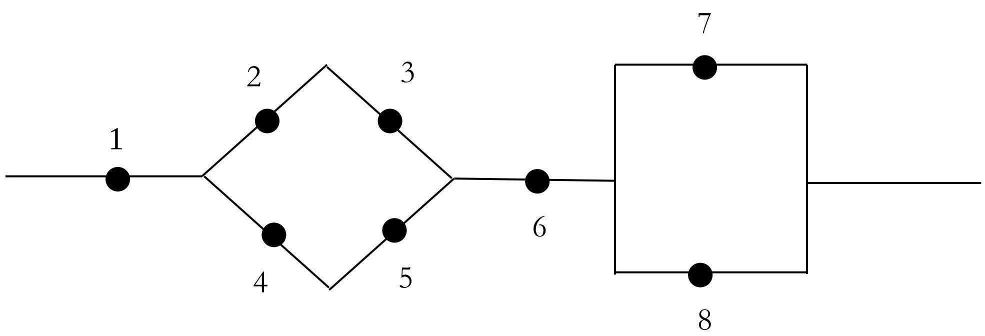
Calculate the results for the expression: $\mathrm{Min}[E[T_U], E[T_V]]$.

A. Less than 0.05
B. At least 0.05, but less than 0.15
C. At least 0.15, but less than 0.25
D. At least 0.25, but less than 0.35
E. At least 0.35

Exam Continued on Next Page

9.

You are given a system whose structure is represented in the diagram below:



The structure has identical components, with probabilities of working all equal to 0.6.

u is the upper bound reliability of the system by using the first two inclusion-exclusion bounds, defining the events in terms of minimal path sets.

Calculate *u*.

A. Less than 0.150
B. At least 0.150, but less than 0.250
C. At least 0.250, but less than 0.350
D. At least 0.350, but less than 0.450
E. At least 0.450

Exam Continued on Next Page

10.

You are given:

- A basic series system has 3 components.
- The series system can be simplified to 2 components.
- All components are independent and function for an amount of time that is uniformly distributed over (0, 1).

Calculate the increase in expected system life by simplifying from 3 components to 2 components.

A. Less than 0.03
B. At least 0.03, but less than 0.05
C. At least 0.05, but less than 0.07
D. At least 0.07, but less than 0.09
E. At least 0.09

Exam Continued on Next Page

11.

You are given:

- Loan borrowers move among three states at the end of each month:
    - State 1: Current
    - State 2: Delinquent
    - State 3: Default
- 75% of Current borrowers remain Current; 25% move to Delinquent.
- 60% of Delinquent borrowers remain Delinquent; 20% move to Current; 20% move to Default.
- Once in Default, the loan is written off so that the borrower remains in Default forever.

Calculate the probability that a Current borrower at the beginning of a calendar quarter will not be in Default by the end of the same quarter.

A. Less than 0.200
B. At least 0.200, but less than 0.400
C. At least 0.400, but less than 0.600
D. At least 0.600, but less than 0.800
E. At least 0.800

Exam Continued on Next Page

12.

You are given the following information about a homogeneous Markov chain:

- There are three daily wildfire risk states: Green (state 0), Yellow (state 1) and Red (state 2).
- Transition between states occurs at the end of each day.
- The daily transition matrix, P = $\begin{bmatrix} .82 & m & n \\ .61 & .28 & .11 \\ .40 & .31 & .29 \end{bmatrix}$
- The wildfire risk was Yellow on Wednesday.
- Today is Thursday and the wildfire risk is Green.
- The probability that the wildfire risk will be Red on Saturday is 0.07.

Calculate the absolute difference between *m* and *n*.

A. Less than 0.065
B. At least 0.065, but less than 0.075
C. At least 0.075, but less than 0.085
D. At least 0.085, but less than 0.095
E. At least 0.095

Exam Continued on Next Page

13.

You are given the following information about a Markov chain with a transition probability matrix:

- $P = \begin{bmatrix} 0.60 & 0.20 & 0.20 \\ 0.40 & 0.40 & 0.20 \\ 0.10 & 0 & 0.90 \end{bmatrix}$
- The three states are 0, 1, and 2.

Calculate the long run proportion of time in state 2.

A. Less than 0.680
B. At least 0.680, but less than 0.700
C. At least 0.700, but less than 0.720
D. At least 0.720, but less than 0.740
E. At least 0.740

Exam Continued on Next Page

14.

An insurance company is considering either charging a cancellation fee for customers who cancel their policies in the first three years, or charging an additional fixed premium of 10 in each of the first three years for customers that renew to cover the cost of writing a new policy.

- Customers only cancel their policy at the end of a year.
- The cancellation fee is paid to the company upon cancellation.
- The remaining customers pay the additional fixed premium of 10 at the end of each year for the first three years their policy is in effect.
- The survival distribution of a policy is uniform between 0 and 5 years.
- Annual interest rate: $i = 5\%$.

Calculate the cancellation fee required so that the expected present value is equivalent to the expected present value of the additional fixed premium of 10 for new customers that renew for at least three years.

A. Less than 20
B. At least 20, but less than 25
C. At least 25, but less than 30
D. At least 30, but less than 35
E. At least 35

Exam Continued on Next Page

15.

You are given the following information:

- There are two independent lives (40) and (60).
- Mortality is assumed to follow the Illustrative Life Table.

Calculate the probability that life (40) lives to age (55) and life (60) dies before age (70).

A. Less than 0.170
B. At least 0.170, but less than 0.190
C. At least 0.190, but less than 0.210
D. At least 0.210, but less than 0.230
E. At least 0.230

Exam Continued on Next Page

16.

Buses arrive at a bus stop according to a Poisson process with parameter λ. Starting from time $t = 0$, you observe bus inter arrival times of $t = 7, 3, 5, 3, 2$.

Calculate the maximum likelihood estimate of λ.

A. Less than 0.1
B. At least 0.1, but less than 0.2
C. At least 0.2, but less than 0.3
D. At least 0.3, but less than 0.4
E. At least 0.4

Exam Continued on Next Page

17.

Let $y_1, y_2, \dots, y_{10}$ be a random sample from a population with mean, μ, and variance , σ^2. Consider the following two unbiased estimators for μ:

- $\hat{\mu}_1 = \frac{1}{2}(y_1 + y_2)$
- $\hat{\mu}_2 = \frac{1}{3}y_1 + \frac{1}{24}(y_2 + \cdots + y_9) + \frac{1}{3}y_{10}$

Calculate the efficiency of $\hat{\mu}_1$ relative to $\hat{\mu}_2$.

A. Less than 0.15
B. At least 0.15, but less than 0.35
C. At least 0.35, but less than 0.55
D. At least 0.55, but less than 0.75
E. At least 0.75

Exam Continued on Next Page

18.

Let $y_1, y_2, \ldots, y_n$ be a random sample from an exponential distribution with density:

$$f(y) = \lambda e^{-\lambda y}, \quad \text{for } y > 0$$

Determine the Rao-Cramer lower bound for the variance of any unbiased estimator of λ.

A. λ/n
B. λ/n^2
C. λ^2/n
D. $(\lambda/n)^2$
E. $n\lambda^2$

Exam Continued on Next Page

19.

Let 150, 204, 310, 480, 500 be a random sample from the density function given by:

$$f(y|\alpha,\theta) = \begin{cases} \left(\frac{1}{\Gamma(\alpha)\theta^{\alpha}}\right) y^{\alpha-1} e^{\frac{-y}{\theta}}, & \text{for } y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

- $\alpha = 20$
- The coefficient of variation (CV) is the standard deviation divided by the mean
- The mean of θ is estimated using maximum likelihood

Calculate the CV of the maximum likelihood estimate.

A. Less than 0.05
B. At least 0.05, but less than 0.15
C. At least 0.15, but less than 0.25
D. At least 0.25, but less than 0.35
E. At least 0.35

Exam Continued on Next Page

20.

You are given:

- The current manufacturing process for bullets has a mean exit velocity of 2900 MPH and a standard deviation of 200 MPH.
- A new process for manufacturing bullets was developed.
- 100 bullets using the new process were tested and had a mean exit velocity of 2950 MPH.
- H_0: Mean exit velocity = 2900 MPH
- H_1: Mean exit velocity > 2900 MPH
- The standard deviation is the same as the existing manufacturing process.
- Exit velocities are assumed to follow the normal distribution.

Calculate the minimum significance level at which you would reject H_0.

A. Less than 0.010
B. At least 0.010, but less than 0.025
C. At least 0.025, but less than 0.050
D. At least 0.050, but less than 0.100
E. At least 0.100

Exam Continued on Next Page

21.

You are given:

- $x_1, x_2, \dots, x_{10}$ is a random sample from a Bernoulli distribution with parameter $p = Pr(X = 1)$
- $\mathrm{H}_0: p = 0.5$
- $\mathrm{H}_1: p > 0.5$

The critical region, $C = \left\{\sum_{j=1}^{10} x_j \geq 7\right\}$.

Calculate the probability of a Type I error.

A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15, but less than 0.20
E. At least 0.20

Exam Continued on Next Page

22.

Two independent random samples of size 12 and 8 are drawn from two normal populations, X and Y. Let S_X^2 and S_Y^2 be the unbiased sample variances.

Using the test statistic $W = S_X^2/S_Y^2$ you wish to test:

- H_0: the variance of population Y is one-third the variance of population X
- H_1: the variance of population Y is less than one-third the variance of population X

Calculate the critical value for a one-sided test of significance 0.02:

A. Less than 1.20
B. At least 1.20, but less than 1.40
C. At least 1.40, but less than 1.60
D. At least 1.60, but less than 1.80
E. At least 1.80

Exam Continued on Next Page

23.

You are given the mid-term and final exam scores of 10 students, and you would like to test whether the mid-term and final scores have the same distribution using the Wilcoxon Signed-Rank Test for matched pairs.

Student	**Mid-Term Score**	**Final Score**
1	86	93
2	90	85
3	63	76
4	97	90
5	33	0
6	82	85
7	88	87
8	81	90
9	74	88
10	90	85

Calculate the test statistic.

A. Less than 10
B. At least 10, but less than 15
C. At least 15, but less than 20
D. At least 20, but less than 25
E. At least 25

Exam Continued on Next Page

24.

You are given the following information:

- Wilcoxon's rank test and Mann-Whitney U test are both used for comparing two independent random samples from two populations: Population I and Population II.
- Both random samples have a sample size of 10.
- For the sample from Population I, the Wilcoxon's rank-sum statistic equals 85.5.

Calculate the Mann-Whitney U statistic for the sample from Population I.

A. Less than 60
B. At least 60, but less than 90
C. At least 90, but less than 120
D. At least 120, but less than 150
E. At least 150

Exam Continued on Next Page

25.

You are given the following information about two samples and the test parameters:

- Sample X is of size 5
- Sample Y is of size 9
- The rank sum of Sample Y is 84
- H_0: The two samples are from the same distribution
- You are to do a two-sided test of H_0
- The Mann-Whitney-U procedure without a normal approximation is used

Calculate the *p*-value for this test.

A. Less than 0.005
B. At least 0.005, but less than 0.010
C. At least 0.010, but less than 0.015
D. At least 0.015, but less than 0.020
E. At least 0.020

Exam Continued on Next Page

26.

You are given the following information:

- Y has a binomial distribution with parameters m and $q = \lambda$.
- The prior distribution of λ is beta with parameters $a = 5, b = 5$ and $\theta = 1$.
- A sample of size 26 is drawn.
- The number of successes in the sample was 19.

Calculate the posterior mean of λ.

A. Less than 0.50
B. At least 0.50, but less than 0.57
C. At least 0.57, but less than 0.64
D. At least 0.64, but less than 0.71
E. At least 0.71

Exam Continued on Next Page

27.

For a portfolio of homeowner's insurance contracts, λ represents the proportion of policies for which there were claims made within one year. λ is a random variable with a beta prior distribution with parameters a, b and $\theta = 1$.

Prior to reviewing the results from a random sample of policies described below, it has been estimated that the mean and variance of λ are $\frac{2}{3}$ and $\frac{1}{72}$, respectively.

In a given year, for 150 randomly selected policies from the portfolio, there were 70 claims.

Calculate the posterior mean of λ.

A. Less than 0.45
B. At least 0.45, but less than 0.50
C. At least 0.50, but less than 0.55
D. At least 0.55, but less than 0.60
E. At least 0.60

Exam Continued on Next Page

28.

You are given:

- A random sample is drawn from a Poisson distribution with parameter λ, where the prior density of λ is proportional to the following function:

$$g(\lambda) = \lambda^2 e^{-2\lambda}, \qquad \text{for } \lambda > 0$$

- The sample size is $n = 98$.
- The sum of the sampled values equals 100.

Calculate the mean of the posterior distribution of λ.

A. Less than 0.98
B. At least 0.98, but less than 1.00
C. At least 1.00, but less than 1.02
D. At least 1.02, but less than 1.04
E. At least 1.04

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29

You are given the following output from a GLM to estimate an insured's pure premium:

Response variable	pure premium	
Response distribution	gamma	
Link	log	
Dispersion parameter (ϕ)	1	
Parameter	Df	$\hat{\beta}$
Intercept	1	3.25
Risk Group	2	
Group 1	0	0.00
Group 2	1	0.30
Group 3	1	0.40
Vehicle Symbol Group	1	
Group 1	0	0.00
Group 2	1	0.45

Calculate the variance of the pure premium for an insured in Risk Group 3 with Vehicle Symbol Group 2.

A. Less than 3,000
B. At least 3,000, but less than 4,000
C. At least 4,000, but less than 5,000
D. At least 5,000, but less than 6,000
E. At least 6,000

Exam Continued on Next Page

30.

You are given the following information for a fitted GLM:

Response variable	Y	
Response distribution	Poisson	
Link	log	
AIC	221.254	

Parameter	$\hat{\beta}$	$\mathrm{s.e.}(\hat{\beta})$
Intercept	5.421	0.228
Gender		
Male	0.000	0.000
Female	-0.557	0.217
Age	0.107	0.002

Calculate the predicted value of Y for a Female with an Age value of 22.

A. Less than 1,000
B. At least 1,000, but less than 1,500
C. At least 1,500, but less than 2,000
D. At least 2,000, but less than 2,500
E. At least 2,500.

Exam Continued on Next Page

31.

If Y has a distribution from the exponential family, and $f(y)$ is in canonical form $f(y) = \exp[a(y)b(\theta) + c(\theta) + d(y)]$, then:

I. $\mathrm{E}[a(y)] = -\frac{c'(\theta)}{b'(\theta)}$

II. $\mathrm{Var}[a(y)] = \frac{b''(\theta)c'(\theta) - c''(\theta)b'(\theta)}{[b'(\theta)]^3}$

III. $\mathrm{E}[(y^2)] = -\frac{c''(\theta)}{b''(\theta)}$

Determine which of the above equations are correct.

A. None are correct
B. I and II
C. I and III
D. II and III
E. The answer is not given by (A), (B), (C), or (D)

Exam Continued on Next Page

32.

You are given:

- A generalized linear model includes three explanatory variables: X_1, X_2, and X_3
- $R^2_{(j)}$ is the coefficient of determination obtained from regressing the j^th explanatory variable against all other explanatory variables.
- $R^2_{(1)} = 0.05$, $R^2_{(2)} = 0.83$, and $R^2_{(3)} = 0.47$
- The threshold of the Variance Inflation Factor for variable *j* (VIF_j) for determining excessive collinearity is $VIF_j > 5$

Determine which of the 3 variables under consideration will exceed the threshold established above.

A. X_1 only
B. X_2 only
C. X_3 only
D. X_1, X_2 and X_3
E. The answer is not given by (A), (B), (C), or (D)

Exam Continued on Next Page

33.

You are given the following table of eight individual adults' heights in inches, y_{ij}, categorized by their season of birth, and selected summary statistics:

Observation	Summer/Fall	Winter/Spring
1	70	0
2	67	0
3	64	0
4	66	0
5	0	60
6	0	62
7	0	68
8	0	63

- $\bar{y} = 65$
- Season of birth is either Summer/Fall (i=1) or Winter/Spring (i=2).
- Observation is indexed by j.
- $\sum_{i=1}^{2} \sum_{j=1}^{8} y_{ij}^2 = 33{,}878$
- $\left(\sum_{i=1}^{2} \sum_{j=1}^{8} y_{ij}\right)^2 = 270{,}400$
- $\sum_{j=1}^{8} (y_{1,j} - \bar{y}_1)^2 = 18.75$
- $\sum_{j=1}^{8} (y_{2,j} - \bar{y}_2)^2 = 34.75$

Calculate the mean square for error of the observations.

A. Less than 7.5
B. At least 7.5, but less than 8.5
C. At least 8.5, but less than 9.5
D. At least 9.5, but less than 10.5
E. At least 10.5

Exam Continued on Next Page

34.

You are given the following information for a one-way ANOVA:

- The ANOVA F-statistic for a sample is 3.75.
- The mean square for treatments is 606.69 with 2 degrees of freedom.
- The total sum of squares is 2670.25.
- *n* is the number of observations in the sample.

Calculate *n*.

A. Less than 14
B. At least 14, but less than 17
C. At least 17, but less than 20
D. At least 20, but less than 23
E. At least 23

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35.

You have fit a generalized linear model using a Poisson distribution. One observation of the response variable is 59. The corresponding GLM fitted value is 71. The corresponding leverage of this point is 22%.

Calculate the standardized deviance residual corresponding to this observation.

A. Less than -1.8
B. At least -1.8, but less than -1.7
C. At least -1.7, but less than -1.6
D. At least -1.6, but less than -1.5
E. At least -1.5

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36.

You are given the following information for a model fitted using ordinary least squares (OLS):

Response variable	Rating
Response distribution	normal
Link	identity
MSE	6.993

Parameter	df	$\hat{\beta}$	standard error $(\hat{\beta})$
Intercept	1	14.37632	6.61999
Complaints	1	0.75461	0.09753

Summary Statistic	Rating	Complaints
N	30	30
Min	40	37
Max	85	90
Sum	1939	1998
Median	65.5	65.0
Sample Std. Dev.	12.1725619	13.3147572

Calculate the upper bound of the 95% prediction interval for Rating, for an observation with a Complaints value of 50.

A. Less than 50
B. At least 50, but less than 60
C. At least 60, but less than 70
D. At least 70, but less than 80
E. At least 80

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37.

Let $Y_1, \dots, Y_n$ be independent Poisson random variables, each with respective mean μ_i for $i = 1, 2, \dots, n$, where:

$$\ln(\mu_i) = \begin{cases} \alpha, & \text{for } i = 1,2,\dots,m \\ \beta, & \text{for } i = m+1, m+2, \dots, n \end{cases}$$

The claims experience for a portfolio of insurance policies with $m = 50$ and $n = 100$ is:

$$\sum_{i=1}^{50} y_i = 563$$

$$\sum_{i=51}^{100} y_i = 1{,}261$$

Denote by $\hat{\alpha}$ and $\hat{\beta}$ the maximum likelihood estimates of α and β, respectively.

Calculate the ratio $\dfrac{\hat{\alpha}}{\hat{\beta}}$.

A. Less than 0.40
B. At least 0.40, but less than 0.60
C. At least 0.60, but less than 0.80
D. At least 0.80, but less than 1.00
E. At least 1.00

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38.

You are given a Poisson regression model of the form:

$$\mathrm{E}(Y_i) = \beta_1 + \beta_2 x_i$$

Maximum likelihood estimates of the beta coefficients are obtained using iterative weighted least squares. W is the weight matrix. X is the design matrix. Z has the beta values from the prior iteration applied to the explanatory variables as well as the correction term.

Using the estimates of the initial iteration, the following matrices are calculated:

$$\mathrm{b}^{(1)} = \begin{bmatrix} 3.60 \\ 0.12 \end{bmatrix}$$

$$\mathrm{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$(\mathrm{X}^T\mathrm{WX})^{(1)} = \begin{bmatrix} 2.336 & 13.249 \\ 13.249 & 102.538 \end{bmatrix}$$

$$\left[(\mathrm{X}^T\mathrm{WX})^{(1)}\right]^{-1} = \begin{bmatrix} 1.602 & -0.207 \\ -0.207 & 0.0365 \end{bmatrix}$$

$$(\mathrm{X}^T\mathrm{Wz})^{(1)} = \begin{bmatrix} 9.158 \\ 54.671 \end{bmatrix}$$

Calculate the estimate for the first diagonal element of the weight matrix, w_{11}, on the second iteration.

A. Less than 0.22
B. At least 0.22, but less than 0.24
C. At least 0.24, but less than 0.26
D. At least 0.26, but less than 0.28
E. At least 0.28

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39.

The random variable, Y, is assumed to follow a Poisson distribution:

$$P(Y) = \frac{\lambda^Y e^{-\lambda}}{Y!}$$

The observed values of Y are 0, 1, 2, 1, 4, 0, 3, 1, 2.

U in the table below is the score function.

The method of scoring is used to estimate λ, from which you are given the following partial table:

Iteration	1	2	3
λ	1.300	1.514	
U			
U'	-8.284		
U/U'		X	

Calculate X.

A. Less than -0.10
B. At least -0.10, but less than -0.05
C. At least -0.05, but less than 0.00
D. At least 0.00, but less than 0.05
E. At least 0.05

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40.

I. For a binary response variable with a continuous explanatory variable, logistic regression is an inappropriate method of statistical analysis.
II. Ordinal variables are a type of continuous explanatory variable.
III. ANOVA is a useful approach for analyzing the means of groups of continuous response variables, where the groups are categorical.

Determine which of the above statements are true.

A. I only
B. II only
C. III only
D. I, II and III
E. The answer is not given by (A), (B), (C), or (D)

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41.

You are given the following partial ANOVA table from a two-way analysis of variance with no interaction effect:

Source	Degrees of freedom	Sum of Squares	Mean Square
Factor 1		103280	
Factor 2	6		8835
Residual	54		3095
Total	69		

Calculate the value of the F-statistic used to test the effect of Factor 1.

A. Less than 1.5
B. At least 1.5, but less than 2.0
C. At least 2.0, but less than 2.5
D. At least 2.5, but less than 3.0
E. At least 3.0

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42.

A study was commissioned on the effect of type of fertilizer and type of seed on crop yield. There are four types of fertilizers, and five types of seed included in the study.

Two separate Poisson GLMs with log link functions were fit to the dataset.

1) Using type of fertilizer and type of seed without an interaction term, the log-likelihood of this GLM is -283.
2) Using type of fertilizer and type of seed, and all interaction terms between those two main effect variables, the log-likelihood of this GLM is -272.

Let:

- H_0: The effect of type of fertilizer is independent of type of seed on crop yield.
- H_1: The effect of type of fertilizer is not independent of type of seed on crop yield.

Calculate the smallest significance level at which you reject H_0.

A. Less than 0.5%
B. At least 0.5%, but less than 1.0%
C. At least 1.0%, but less than 2.5%
D. At least 2.5%, but less than 5.0%
E. At least 5.0%

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43.

You are given the following statements about a time series modeled as an AR(3) process:

I. Partial autocorrelation for lag 3 is always equal to zero.
II. Partial autocorrelation for lag 4 is always equal to zero.
III. Partial autocorrelation for lag 4 is always greater than zero.

Determine which of the above statements are true.

A. I only
B. II only
C. III only
D. I and II
E. The answer is not given by (A), (B), (C), or (D)

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44.

You are given the following ARMA(p, q) models:

I. $x_t = \frac{1}{2}x_{t-1} - \frac{1}{2}w_{t-1} + w_t$

II. $x_t = \frac{1}{2}x_{t-1} - \frac{1}{9}w_{t-2} + w_t$

III. $x_t = -\frac{5}{6}x_{t-1} - \frac{1}{6}x_{t-2} + \frac{8}{12}w_{t-1} + \frac{1}{12}w_{t-2} + w_t$

Determine which of the models are parameter redundant.

A. I only
B. II only
C. III only
D. I, II and III
E. The answer is not given by (A), (B), (C), or (D)

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45.

You are given:

- A time series, $\{x_t\}$, is fitted by the model $\hat{x}_t = e^{2+0.025t}$
- The residual series, $\{w_t\}$, of this fitted log-regression model is Gaussian white noise with mean zero and standard deviation $\sigma = 2$.
- $\hat{x}'_{10}$ is the adjusted forecast for time $t = 10$.

Calculate $\hat{x}'_{10}$.

A. Less than 20
B. At least 20, but less than 40
C. At least 40, but less than 60
D. At least 60, but less than 80
E. At least 80

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Spring 2017 Exam S Final Answer Key

Number	Solution	
1	C	
2	C	
3	D	
4	C	
5	A	
6	A	
7	D	
8	B	
9	C	
10	D	
11	E	
12	C	
13	A	
14		Invalid Question
15	B	
16	C	
17	C	
18	C	
19	B	C
20	A	
21	D	
22	E	
23	D	
24	B	A
25	E	
26	D	
27	B	
28	D	
29	B	
30	B	
31	B	
32	B	
33	C	E
34	A	
35	C	
36	B	
37	C	
38	E	
39	C	
40	C	
41	E	
42	D	
43	B	

44 E
45 D