Exam ST

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

May 9, 2016

Exam ST

Models for Stochastic Processes and Statistics

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2016 and that the exam name is ST.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

- Verify that you have a copy of "Tables for CAS Exam ST" included in your exam packet.

- Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table.

- Candidates should employ a non-parametric test unless otherwise specified in the problem, or when there is a standard distribution that is logically or commonly associated with the random variable in question. Examples of problems with a logical or commonly associated distribution would include exponential wait times for Poisson processes and applications of the Central Limit Theorem.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 23, 2016.

END OF INSTRUCTIONS

1.

You are given that $N(t)$ follows the Poisson process with rate $\lambda = 2$.

Calculate $\Pr[N(2) = 3 \mid N(5) = 7]$.

A. Less than 0.25

B. At least 0.25, but less than 0.35

C. At least 0.35, but less than 0.45

D. At least 0.45, but less than 0.55

E. At least 0.55

Examination Continued on Next Page

2.

For a health insurance policy the annual number of claims follows the Poisson process with mean $\lambda = 5$.

Calculate the probability that the time interval between the second and third claims exceeds 0.6 years.

A. Less than 0.01
B. At least 0.01, but less than 0.04
C. At least 0.04, but less than 0.07
D. At least 0.07, but less than 0.10
E. At least 0.10

Examination Continued on Next Page

3.

For a block of one-year insurance policies you are given:

- Aggregate losses follow a compound Poisson process.
- Claim frequency follows a non-homogenous Poisson process with the following monthly rates:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Rate	5	5	5	2	2	2	2	2	2	3	3	3

- Claim severities follow an exponential distribution with $\theta = 500$.

Calculate the standard deviation of the annual aggregate losses.

A. Less than 4,500
B. At least 4,500 but less than 5,000
C. At least 5,000 but less than 5,500
D. At least 5,500 but less than 6,000
E. More than 6,000

4.

You are given:

- Loss amounts follow the probability density function

$$f(x) = \frac{\alpha(3)^{\alpha}}{(x)^{\alpha+1}}, \quad x > 3$$

- Observed loss amounts of 20, 30, 60, and 90.

Calculate the maximum likelihood estimate of α.

A. Less than 0.25

B. At least 0.25, but less than 0.35

C. At least 0.35, but less than 0.45

D. At least 0.45, but less than 0.55

E. At least 0.55

5.

You are given:

- A random variable X has an exponential distribution with mean θ.
- An estimator of θ has a mean of 2 and a variance of 0.5.

Calculate the mean square error of this estimator for $\theta = 2.1$.

A. Less than 0.3

B. At least 0.3, but less than 0.4

C. At least 0.4, but less than 0.5

D. At least 0.5, but less than 0.6

E. At least 0.6

Examination Continued on Next Page

6

A random variable, Y, with parameter θ is a member of the exponential family of distributions if its density function has the form:

$$f(y{:}\,\theta) = \exp\{a(y)b(\theta) + c(\theta) + d(y)\}.$$

It is known that, for the exponential family of distributions,

$$E[a(Y)] = -\frac{c'(\theta)}{b'(\theta)}.$$

For one member of the exponential family, you are given, for y = 0, 1, 2, ..., n

- $a(y) = y$
- $b(\theta) = log\theta$
- $c(\theta) = -\theta$
- $d(y) = -\log y!$
- $\hat{\theta}$ is an unbiased estimator of θ

Find the smallest possible value of the variance of $\hat{\theta}$.

A. θ

B. θ/n

C. $\theta/\sqrt{n}$

D. θ^2/n

E. 0

Examination Continued on Next Page

7.

You are given the following information:

- Loss severity follows an exponential distribution with mean θ.
- Small claims are handled directly by the insured, and no information about losses less than \$5,000 is available to you.
- You observe three losses with the following claim payments:

\$6,000 \$7,500 \$10,000

Calculate the maximum likelihood estimate of θ.

A. Less than 2,500

B. At least 2,500, but less than 3,000

C. At least 3,000, but less than 3,500

D. At least 3,500, but less than 4,000

E. At least 4,000

Examination Continued on Next Page

8.

You are given the following random sample:

0.15 0.25 0.55 0.60 1.10

The probability density function given below is selected to be fit to the random sample:

$$f(x) = \begin{cases} \frac{1}{2} & for\ 0 \leq x \leq a \\ 1 & for\ a < x \leq 1 + \frac{a}{2} \end{cases}$$

where $0 \leq a \leq 2$.

Select the range of the maximum likelihood estimate of a.

A. Less than 0.15

B. At least 0.15, but less than 0.20

C. At least 0.20, but less than 0.25

D. At least 0.25, but less than 0.30

E. At least 0.30

Examination Continued on Next Page

9.

You are given the following information:

- A random variable X is uniformly distributed on the interval $(0, \theta)$.
- θ is unknown.
- For a random sample of size n, an estimate of θ is $Y_n = \max\{X_1, X_2, \ldots, X_n\}$.
- $E(Y_n) = [n/(n+1)]*\theta$

Determine which one of the following is an unbiased estimator of θ.

A. Y_n

B. $Y_n/(n-1)$

C. $(n/2) * Y_n$

D. $(n+1)/n* Y_n$

E. $1/(n+1)* Y_n$

Examination Continued on Next Page

10.

Some people are right-hand dominant; others are left-hand dominant. The same is true for eyes. You are given the following table of observed values:

	Dominant Hand		
Dominant Eye	Right Hand	Left Hand	**Total**
Right Eye	8402	207	**8609**
Left Eye	1308	104	**1412**
Total	**9710**	**311**	**10021**

- H_0: Eye and hand dominance are independent
- H_A: Eye and hand dominance are not independent

Calculate the chi-squared statistic.

A. Less than 60
B. At least 60, but less than 70
C. At least 70, but less than 80
D. At least 80, but less than 90
E. At least 90

Examination Continued on Next Page

11.

You are given the following information:

- A random variable follows the normal distribution with mean μ and standard deviation 3.
- H_0: $\mu = 8$.
- H_1: $\mu = 7$.
- It is required that errors of Type I and II have probabilities of 0.05 and 0.01, respectively.

Calculate that necessary sample size based on the Neyman-Pearson Lemma.

A. Less than 50

B. At least 50, but less than 100

C. At least 100, but less than 150

D. At least 150, but less than 200

E. At least 200

Examination Continued on Next Page

12.

You are given the following information:

- Your company has developed a new training program for claims adjusters to increase their productivity.
- Assume that the number of claims closed follows a normal distribution, with equal variance before and after the training program.
- H_0: the number of claims closed is the same before and after training.
- H_1: the number of claims closed is not the same before and after training.
- The table below shows the paired results (before and after training) by adjuster.

Adjuster ID	# Claims Closed Before Training	# Claims Closed After Training
1	7	5
2	6	3
3	7	7
4	8	12
5	4	10
6	7	13
7	5	10
8	7	14
9	8	6
10	2	6
TOTAL	61	86

Calculate the minimum significance level at which you reject H_0.

A. Less than 0.1%

B. At least 0.1%, but less than 1.0%

C. At least 1.0%, but less than 2.0%

D. At least 2.0%, but less than 5.0%

E. At least 5.0%

Examination Continued on Next Page

13.

You are given the following information about a process that follows the normal distribution:

- The variance is known and $\sigma^2 = 25$.
- H_0: $\mu = 35$.
- H_1: $\mu = 30$.
- The sample size, n, is equal to 16.

Determine the minimum possible Type I error such that the probability of Type II error is no more than 2.5%.

A. Less than 2%
B. At least 2%, but less than 3%
C. At least 3%, but less than 4%
D. At least 4%, but less than 5%
E. At least 5%

Examination Continued on Next Page

14.

You are given the following information about two loss severity distributions fit to a sample of 275 closed claims:

- For the Exponential distribution, the natural logarithm of the likelihood function evaluated at the maximum likelihood estimate is −828.37.
- For the Weibull distribution, the natural logarithm of the likelihood function evaluated at the maximum likelihood estimate is −826.23.
- The Exponential distribution is a subset of the Weibull distribution.
- The null hypothesis is that the exponential distribution provides a better fit than the Weibull distribution.

Calculate the significance level at which one would reject the null hypothesis.

A. Less than 0.5%

B. At least 0.5%, but less than 1.0%

C. At least 1.0%, but less than 2.5%

D. At least 2.5%, but less than 5.0%

E. At least 5.0%

Examination Continued on Next Page

15.

You are given the following information:

- $f(x;\theta)$ is the pdf of X, where θ represents one or more unknown parameters θ
- Ω is the set of all possible parameters for θ.
- $H_0: \theta \in \omega$ where ω is a subset of Ω
- $H_1: \theta \in \Omega$.
- $\lambda = \dfrac{L(\hat{\omega})}{L(\hat{\Omega})}$, where the numerator is the maximum likelihood function with respect to θ under the null hypothesis .

Determine which of the following are true.

I. $\lambda \leq 1$.

II. λ can be less than zero.

III. As λ increases the likelihood of rejecting the null hypothesis increases.

A. I only
B. II only
C. I and II only
D. I and III only
E. II and III only

Examination Continued on Next Page

16.

Ten runners were ranked by judges on their speed (1 = fastest) prior to a race in which they then recorded the following times:

Runner	Rank	Time (min:sec)
1	4	4:45
2	5	4:50
3	1	4:31
4	6	4:49
5	9	5:05
6	8	4:55
7	7	5:30
8	3	4:20
9	2	4:25
10	10	5:55

Calculate Spearman's Rank statistic, r_S, for this data.

A. Less than -0.75

B. At least -0.75, but less than -0.25

C. At least -0.25, but less than 0.25

D. At least 0.25, but less than 0.75

E. At least 0.75

Examination Continued on Next Page

17.

For a general liability policy, loss amounts, Y, follow the exponential distribution with probability density function:

$$f(y) = \frac{1}{\theta}\, e^{-y/\theta} \qquad \theta = 1000, \quad 0 < y.$$

For reinsurance purposes we are interested in the distribution of the median loss amount in a random sample of size 3, which is denoted by $Y_{(2)}$.

Calculate the probability that $Y_{(2)}$ is less than 2000.

A. At least 0.73, but less than 0.78

B. At least 0.78, but less than 0.83

C. At least 0.83, but less than 0.88

D. At least 0.88, but less than 0.93

E. At least 0.93

Examination Continued on Next Page

18.

You are given:

- A tutor claims that students will get more than 89.5 out of 100 questions right on a particular exam if the students use the tutor's service for 15 minutes a day.
- A random sample of 200 previous students was taken.
- No student scored exactly 89.5.
- H_0: Students using the service have a mean score of 89.5 on the exam.
- H_1: Students using the service have a mean score greater than 89.5 on the exam.
- H_0 was rejected at the 5% significance level using the sign test.
- The normal approximation with continuity adjustment is used.

Calculate the minimum number of students that scored more than 89.5 in the sample.

A. 111 or fewer

B. 112

C. 113

D. 114

E. 115 or more

Examination Continued on Next Page

19.

A gun is fired using two different brands of bullets.

- The results of the tests are provided

Bullet Brand A		Bullet Brand B	
Muzzle Velocity	Rank	Muzzle Velocity	Rank
1011	1	1014	4
1012	2	1015	5
1013	3	1016	6
1020	10	1017	7
1021	11	1018	8
1029	12	1019	9
1030	13	1050	19
1031	14	1051	20
1032	15	1052	21
1035	16	1055	22
1040	17		
1045	18		

- H_0: $Median_{Brand\,A} = Median_{Brand\,B}$.
- H_1: $Median_{Brand\,A} \neq Median_{Brand\,B}$.
- The normal approximation is used.

Calculate the smallest value for significance level for which H_0 can be rejected using the Mann-Whitney Test.

A. Less than 1%

B. At least 1%, but less than 5%

C. At least 5%, but less than 10%

D. At least 10%, but less than 15%

E. At least 15%

Examination Continued on Next Page

20.

You are given:

- A multiple linear regression model was fit to 20 observations.
- There are five explanatory variables with fitted values for β_1 through β_5.
- The following summarizes the fitted coefficients excluding the intercept:

	Point Estimate	Standard Error	t-statistic
β_1	0.020000	0.012000	1.661
β_2	-0.004950	0.008750	-0.565
β_3	0.216000	0.043200	5.000
β_4	-0.034600	0.115000	-0.301
β_5	-0.000294	0.000141	-2.090

Determine the number of coefficients in the table above for the five explanatory variables that are not statistically different from zero at a significance level of $\alpha = 10\%$, based on a two-tailed test.

A. 1

B. 2

C. 3

D. 4

E. 5

Examination Continued on Next Page

21.

You are given the following linear regression model which is fitted to 11 observations:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

The coefficient of determination is $R^2 = 0.25$.

Calculate the F-statistic used to test for a linear relationship.

A. Less than 1.5

B. At least 1.5, but less than 2.5

C. At least 2.5, but less than 3.5

D. At least 3.5, but less than 4.5

E. At least 4.5

Examination Continued on Next Page

22.

The following two models were fit to 18 observations:

- Model 1: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$
- Model 2: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_1^2 + \beta_5 X_2^2 + \varepsilon$

The results of the regression are:

Model Number	Error Sum of Squares	Regression Sum of Squares
1	102	23
2	78	39

Calculate the value of the F-statistic used to test the hypothesis that $\beta_3 = \beta_4 = \beta_5 = 0$.

A. Less than 1.30
B. At least 1.30, but less than 1.40
C. At least 1.40, but less than 1.50
D. At least 1.50, but less than 1.60
E. At least 1.60

Examination Continued on Next Page

23.

You are given the following information:

- The number of claims per year for an individual follows a Poisson distribution with parameter λ.
- The parameter λ follows a gamma distribution with parameters $\alpha = 1$ and $\theta = 5$.
- A randomly selected insured had 0 claims last year.

Calculate the upper bound of the 95% equal-tailed Bayesian credible interval for the posterior distribution of λ.

A. Less than 2.2

B. At least 2.2, but less than 2.9

C. At least 2.9, but less than 3.6

D. At least 3.6, but less than 4.3

E. At least 4.3

Examination Continued on Next Page

24.

The annual number of claims for a policy follows the Poisson distribution with mean λ, which varies by policy. The prior probability density function of λ is

$$h(\lambda) = \lambda e^{-\lambda}, \qquad \lambda > 0$$

In Year 1, three claims are reported for a policy.

Calculate the coefficient of variation (the standard deviation divided by the mean) of the posterior distribution of λ for this policy.

A. Less than 0.5
B. At least 0.5, but less than 1.0
C. At least 1.0, but less than 1.5
D. At least 1.5, but less than 2.0
E. More than 2.0

Examination Continued on Next Page

25.

You are given:

- The number of claims per year for a policyholder follows a Poisson distribution with mean λ.
- The mean λ follows a gamma distribution with parameters $\alpha = 3$ and $\theta = 10$.
- A policyholder had 1 claim last year.

Calculate the mean of the posterior distribution of λ.

A. Less than 2.0

B. At least 2.0, but less than 4.0

C. At least 4.0, but less than 6.0

D. At least 6.0, but less than 8.0

E. At least 8.0

End of Examination

Spring 2016 Exam ST Answer Key

Question#	Answer
1	B
2	C
3	A
4	C
5	D
6	B
7	B
8	C
9	D
10	E
11	C
12	E
13	B
14	D
15	A
16	E
17	E
18	C
19	E
20	C
21	C
22	A
23	C
24	A
25	B