CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

May 3, 2013

Exam 3L

Life Contingencies and Statistics

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2013 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 17, 2013.

END OF INSTRUCTIONS

1.

You are given the following information:

- $s(x) = \begin{cases} 1 & , \quad x = 0 \\ 0.8 & , \quad x = 1 \\ r^{x-1} & , \quad x \geq 2, 3, \ldots \end{cases}$

- ${}_{2|2}q_1 = 0.288$

- $s(x+1) < s(x)$ for all x

Calculate s(2).

A. Less than 0.56
B. At least 0.56, but less than 0.62
C. At least 0.62, but less than 0.68
D. At least 0.68, but less than 0.74
E. At least 0.74

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2.

You are given the following information:

- ${}_2p_0 = 0.180$
- $\mu_x = A + e^{.02x}$ for $x \geq 0$

Calculate ${}_{1|1}q_0$.

A. Less than 0.275
B. At least 0.275, but less than 0.285
C. At least 0.285, but less than 0.295
D. At least 0.295, but less than 0.305
E. At least 0.305

CONTINUED ON NEXT PAGE

3.

You are given the following information:

- Deaths are uniformly distributed between integer ages.
- $q_{70} = 0.17$

Calculate the probability that a person age 70.5 will die within 4 months.

A. Less than 0.04
B. At least 0.04, but less than 0.06
C. At least 0.06, but less than 0.08
D. At least 0.08, but less than 0.10
E. At least 0.10

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4.

You are given the following information for two independent lives aged 45 and 65:

- Mortality for the life age 45 follows the Illustrative Life Table.
- Mortality for the life age 65 follows: $f(y) = 0.2e^{-0.2y}$ for $y > 0$

Calculate ${}_{10}p_{\overline{45:65}}$.

A. Less than 0.925
B. At least 0.925, but less than 0.935
C. At least 0.935, but less than 0.945
D. At least 0.945, but less than 0.955
E. At least 0.955

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5.

You are given the following information:

- A life table that defines the mortality for an individual (x) age 25 and an individual (y) age 27.

Age	l_x	l_y
25	920	
26	915	
27	905	880
28		850
29		810

- Assume the future lifetimes of an individual (x) age 25 and an individual (y) age 27 are independent.

Calculate the two-year temporary curtate expectation of the last-survivor status, $e_{\overline{25:27}:\overline{2}|}$

A. Less than 1.80
B. At least 1.80, but less than 1.85
C. At least 1.85, but less than 1.90
D. At least 1.90, but less than 1.95
E. At least 1.95

CONTINUED ON NEXT PAGE

6.

You are given the following double-decrement table:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(\tau)}$	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
48			0.075			260
49	0.020	0.060		3,700		
50					132	

Calculate ${}_3q_{48}^{(1)}$.

A. Less than 0.0625
B. At least 0.0625, but less than 0.0650
C. At least 0.0650, but less than 0.0675
D. At least 0.0675, but less than 0.0700
E. At least 0.0700

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7.

Assume that Gross Domestic Product (GDP) compound growth rates follow a homogeneous Markov Chain process, with the following states:

State 1: 4%
State 2: 2%
State 3: -1%

The transition matrix is:

$$Q = \begin{bmatrix} 0.05 & 0.65 & 0.30 \\ 0.30 & 0.50 & 0.20 \\ 0.10 & 0.80 & 0.10 \end{bmatrix}$$

- GDP is currently 100.
- The starting point is State 3.

Calculate the probability that GDP will be at least 103 after two transitions.

A. Less than 0.68
B. At least 0.68, but less than 0.70
C. At least 0.70, but less than 0.72
D. At least 0.72, but less than 0.74
E. At least 0.74

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8.

The monthly status of a commercial real estate loan is modeled by a homogeneous Markov Chain, with the following states:

State 1: Current
State 2: 30 Days Delinquent
State 3: In Default
State 4: In Foreclosure

The transition probability matrices are:

Strong Economy:

$$Q = \begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.65 & 0.25 & 0.10 & 0 \\ 0 & 0 & 0.50 & 0.50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Weak Economy:

$$Q = \begin{bmatrix} 0.83 & 0.17 & 0 & 0 \\ 0.40 & 0.35 & 0.25 & 0 \\ 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The borrower is in the 30 Days Delinquent state at time n.

The probability that the borrower is in the In Foreclosure state at time $n+2$ is k_1 when the economy follows the Strong Economy transition probability matrix over the entire two-year period.

The probability that the borrower is in the In Foreclosure state at time $n+2$ is k_2 when the economy follows the Weak Economy transition probability matrix over the entire two-year period.

Calculate $k_2 - k_1$.

A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15, but less than 0.20
E. At least 0.20

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9.

You are given the following:

- An actuary takes a vacation where he will not have access to email for eight days.
- While he is away, emails arrive in the actuary's inbox following a non-homogeneous Poisson process where
  $\lambda(t) = 8t - t^2$ for $0 \leq t \leq 8$.(t is in days)

Calculate the variance of the number of emails received by the actuary during this trip.

A. Less than 60
B. At least 60, but less than 70
C. At least 70, but less than 80
D. At least 80, but less than 90
E. At least 90

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10.

You are given the following information:

- The number of gold nuggets found per day on a property follows a Poisson process.
- The expected number of nuggets found in a 30 day time period is 15.

Calculate the probability that the time between finding the 8^{th} and 9^{th} nuggets will be greater than 3 days.

A. Less than 20%
B. At least 20%, but less than 25%
C. At least 25%, but less than 30%
D. At least 30%, but less than 35%
E. At least 35%.

CONTINUED ON NEXT PAGE

11.

You are given the following information:

- Assume all claims are either auto or homeowners claims.
- Auto claims occur under a Poisson process and λ=0.3.
- Homeowners claims occur under a Poisson process and λ=0.4.
- The number of auto and homeowners claims are independent.
- The distributions of the number of people involved in a given claim are as follows:

Number of People Auto Claim	Probability
1	0.6
2	0.4

Number of People Homeowners Claim	Probability
1	0.5
2	0.3
3	0.2

Determine the variance of the total number of people that are involved in claims by time 1.

A. Less than 0.5
B. At least 0.5, but less than 1.0
C. At least 1.0, but less than 1.5
D. At least 1.5, but less than 2.0
E. At least 2.0

CONTINUED ON NEXT PAGE

12.

For a special annuity product on a life aged 50 with a single benefit premium of $50,000 paid immediately, you are given the following information:

- An annual benefit of K at the beginning of each year is guaranteed for the first five years.
- After five years, an annual benefit of K at the beginning of each year will be given until death.
- $i = 0.06$
- $a_{50} = 12.267$
- $A^{\;1}_{50:\overline{5|}} = 0.029$
- $A_{55} = 0.305$

Calculate the annual benefit K.

A. Less than $3,620
B. At least $3,620, but less than $3,660
C. At least $3,660, but less than $3,700
D. At least $3,700, but less than $3,740
E. At least $3,740

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13.

You are given the following information about an annuity contract:

- A life aged 40 will put \$10,000 in a fund at the beginning of each year for 20 years.
- At age 60, the individual will begin to collect the amount x, for 3 years.
- All transactions take place at the beginning of the time interval.
- Mortality follows the Illustrative Life Table.
- $i = 6\%$

Calculate x using the equivalence principle.

A. Less than \$152,000
B. At least \$152,000, but less than \$153,000
C. At least \$153,000 but less than \$154,000
D. At least \$154,000, but less than \$155,000
E. At least \$155,000

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14.

You are given the following information:

- An individual aged 40 purchases a 20-year term life insurance policy with a $1,000 death benefit and a 40-year deferred whole life insurance policy also with a $1,000 death benefit.
- Mortality follows the Illustrative Life Table.
- Premiums are payable at the beginning of each year as long as the insured is alive.
- $i = .06$

Calculate the level annual benefit premium using the equivalence principle.

A. Less than $5
B. At least $5, but less than $6
C. At least $6, but less than $7
D. At least $7, but less than $8
E. At least $8

CONTINUED ON NEXT PAGE

15.

You are given the following information:

- An individual life aged 40 purchases a fully discrete whole life insurance policy with a death benefit of $10,000.
- $i = .05$
- $p_{40} = .98$
- $\ddot{a}_{40} = 12$
- Premiums have been calculated according to the equivalence principle.

Calculate the terminal benefit reserve at time t = 1.

A. Less than $165
B. At least $165, but less than $175
C. At least $175, but less than $185
D. At least $185, but less than $195
E. At least $195

CONTINUED ON NEXT PAGE

16.

You are given the following information:

- Mortality for an individual can be described using a non-homogenous Markov Chain process with two states:

  State 1: Alive
  State 2: Deceased

- You are given the following transition probability matrices for this individual:

$$Q_0 = \begin{bmatrix} 0.9 & 0.1 \\ 0 & 1 \end{bmatrix} \quad Q_1 = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 1 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 0.7 & 0.3 \\ 0 & 1 \end{bmatrix} \quad Q_3 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

- An insurance policy is issued to this individual at time 0.
- The insured is in state 1 at the time the policy is issued.
- A benefit of $100,000 is paid out upon transition of the insured from state 1 to state 2.
- Transitions occur at the end of each time period.
- The insurance company receives a premium of $25,000 at the beginning of each time period, if the insured is in state 1 at that time.
- $i = 5\%$

Calculate the benefit reserve for this policy at time 1, assuming the insured is in state 1 and the premium for this time period has not yet been paid.

A. Less than $0
B. At least $0, but less than $10,000
C. At least $10,000, but less than $20,000
D. At least $20,000, but less than $30,000
E. At least $30,000

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17.

You are given the following information for an auto liability book of business:

- Auto liability claim severity follows an exponential distribution.
- The sum of all claim amounts for the book is \$2.5 million.
- All policies have a \$50,000 per claim limit.
- The total number of claims is 100, of which 15 are capped by the \$50,000 limit.

Estimate the average uncapped claim severity for this book using the maximum likelihood method.

A. Less than \$26,000
B. At least \$26,000 but less than \$27,000
C. At least \$27,000 but less than \$28,000
D. At least \$28,000 but less than \$29,000
E. At least \$29,000

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18.

You are given a sample from a random process:

5 10 7 4

The underlying distribution is assumed to be an Inverse Weibull distribution, where $\tau = 2$ and $\theta > 0$ is unknown.

Determine the maximum likelihood estimate of θ for this distribution using this data.

A. Less than 3.0
B. At least 3.0, but less than 4.0
C. At least 4.0, but less than 5.0
D. At least 5.0, but less than 6.0
E. At least 6.0

CONTINUED ON NEXT PAGE

19.

You are given the following information:

- A random variable X is uniformly distributed on the interval $(0, \theta)$.
- θ is unknown.
- For a random sample of size n, an estimate of θ is $Y_n = \max\{X_1, X_2, \ldots, X_n\}$.
- $E(Y_n) = [n/(n+1)]*\theta$

Determine which of the following is an unbiased estimator of θ.

A. Y_n
B. $Y_n/(n-1)$
C. $(n/2) * Y_n$
D. $((n+1)/n)* Y_n$
E. $(1/(n+1))* Y_n$

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20.

You are given the following:

- Accidents happen during a work day at a probability of p when a machine is operated.
- The null hypothesis H_0 is that the probability of an accident is 0.05; the alternative hypothesis H_1 is that the probability is less than 0.05.
- If less than 20 accidents are observed in 365 work days, then reject the null hypotheses.

Using the Normal approximation, calculate the probability of Type II error using the value 0.03 as the true probability of an accident occurring.

A. Less than 0.0028
B. At least 0.0028, but less than 0.0029
C. At least 0.0029, but less than 0.0030
D. At least 0.0030, but less than 0.0031
E. At least 0.0031

CONTINUED ON NEXT PAGE

21.
You are given the following:

- A hypothesis test is set up for data sampled from a Normal distribution.
- H_0: $\mu = k$
- H_1: $\mu \neq k$
- k is a constant

Determine the percentage increase in sample size required to reduce the critical region from 10% to 5%.

A. Less than 35%
B. At least 35%, but less than 40%
C. At least 40%, but less than 45%
D. At least 45%, but less than 50%
E. At least 50%

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22.

You are given the following:

- A sample of size 25 from a Normal distribution has a sample mean of 5.
- The unbiased sample variance is 31.
- H_0: $\mu = 3$
- H_1: $\mu > 3$

Determine which of the following is true.

A. Reject H_0 at 0.5%
B. Do not reject H_0 at 0.5%. Reject H_0 at 1.0%
C. Do not reject H_0 at 1.0%. Reject H_0 at 2.5%
D. Do not reject H_0 at 2.5%. Reject H_0 at 5.0%
E. Do not reject H_0 at 5.0%

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23.

The probability density of Y_k, the k^{th} order statistic for a sample of size n is:

$$g_k(y_k) = \frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y)$$

Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of five independent observations from a continuous distribution where 30 is at the with 80^{th} percentile for the cumulative distribution function.

Find Pr $(Y_3 < 30 < Y_5)$.

A. Less than 0.25
B. At least 0.25, but less than 0.35
C. At least 0.35, but less than 0.45
D. At least 0.45, but less than 0.55
E. At least 0.55

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24..

You are given the following information:

- Wait times between calls at a claims office follow the Exponential distribution with a mean of 10 minutes.

- Five wait times will be recorded.

- The probability density function of the order statistic Y_k with a sample size n is
$$\frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y).$$

Calculate the expected value of the shortest wait time.

A. Less than 1 minute
B. At least 1 minute, but less than 4 minutes
C. At least 4 minutes, but less than 7 minutes
D. At least 7 minutes, but less than 10 minutes
E. At least 10 minutes

CONTINUED ON NEXT PAGE

25.

You are given the following data set with two variables, X and Y:

X	Y
10	22
13	20
20	6
15	18
5	10

Using the least squares method for a linear regression with Y as the dependent variable, calculate the absolute value of the residual where the x and y values are 12 and 18 respectively.

A. Less than 1
B. At least 1, but less than 2
C. At least 2, but less than 3
D. At least 3, but less than 4
E. At least 4

END OF EXAMINATION

Spring 2013 Exam 3L Answer Key

Question	Answer
1	B
2	A
3	C
4	D
5	E
6	A
7	C
8	A
9	D
10	B
11	E
12	E
13	C
14	B
15	C
16	D, E
17	E
18	D
19	D
20	A, E
21	C
22	D
23	E
24	B
25	C