Tables for

CAS Exam MAS-II

Version date 06/25/2018

The following tables will be provided to the candidate with the exam. The tables on pages 4 through 13 are reprinted with the permission of the Society of Actuaries; the tables on pages 14 through 18 are copyright material of the Casualty Actuarial Society.

We are furnishing a set of tables for statistical tests as well as a set of distribution functions for Exam MAS-II. We do not have a single authoritative textbook for MAS-II. The nomenclature used to describe the distribution functions may vary from one textbook to the next. To avoid confusion on the part of the candidates we will use the formulae, tables and distribution functions definitions that follow when writing exam questions for Exam MAS-II.

Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)

AIC and BIC are two common measures of relative model fit often used as criteria to select among competing models. These topics are covered in a number of the syllabus materials. Although the conceptual treatment is the same in each, the actual AIC and BIC formulae can vary by source.

Candidates are expected to understand AIC and BIC as presented in all of the texts. However, if an exam question requires a calculation of AIC and/or BIC, unless an alternative formula is explicitly given, the following will be used:

$$AIC = -2l(\widehat{\boldsymbol{\theta}};\boldsymbol{y}) + 2 * p$$

$$BIC = -2l(\widehat{\boldsymbol{\theta}};\boldsymbol{y}) + \ln(n) * p$$

where:

$l(\widehat{\boldsymbol{\theta}};\boldsymbol{y})$ is the log-likelihood of the observed data, $\boldsymbol{y}$, under the model of interest at the (restricted) maximum likelihood estimates of the parameters, $\widehat{\boldsymbol{\theta}}$.

p is the number of parameters in the model (fixed effects + covariance parameters).

n is the number of observations in the modeled dataset.

Mixed Model Notation

To specify models that may include both fixed and random effects, notation consistent with Linear Mixed Models: A Practical Guide Using Statistical Software will be employed.

Specifically, for the i^{th}subject with n_i repeated observations in a model with p fixed effect covariates and q random effects the model will be specified as follows:

$$\boldsymbol{Y_i} = \boldsymbol{X_i}\boldsymbol{\beta} + \mathbf{Z_i}\boldsymbol{u_i} + \boldsymbol{\epsilon_i}$$

$$\boldsymbol{u_i} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{D})$$

$$\boldsymbol{\epsilon_i} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{R_i})$$

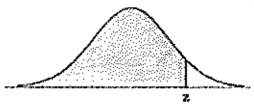
Where

$$\boldsymbol{Y_i} = \begin{pmatrix} Y_{1i} \\ Y_{2i} \\ \vdots \\ Y_{n_i i} \end{pmatrix} \; ; \; \boldsymbol{X_i} = \begin{pmatrix} X_{1i}^{(1)} & X_{1i}^{(2)} & \cdots & X_{1i}^{(p)} \\ X_{2i}^{(1)} & X_{2i}^{(2)} & \cdots & X_{2i}^{(p)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n_i i}^{(1)} & X_{n_i i}^{(2)} & \cdots & X_{n_i i}^{(p)} \end{pmatrix} \; ; \; \boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix}$$

$$\boldsymbol{Z_i} = \begin{pmatrix} Z_{1i}^{(1)} & Z_{1i}^{(2)} & \cdots & Z_{1i}^{(q)} \\ Z_{2i}^{(1)} & Z_{2i}^{(2)} & \cdots & Z_{2i}^{(q)} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n_i i}^{(1)} & X_{n_i i}^{(2)} & \cdots & Z_{n_i i}^{(q)} \end{pmatrix} \; ; \; \boldsymbol{u_i} = \begin{pmatrix} u_{1i} \\ u_{2i} \\ \vdots \\ u_{qi} \end{pmatrix} \; ; \; \boldsymbol{\epsilon_i} = \begin{pmatrix} \epsilon_{1i} \\ \epsilon_{2i} \\ \vdots \\ \epsilon_{n_i i} \end{pmatrix}$$

With the notation above, the fixed effect covariates for the i^{th} subject are contained in the $\boldsymbol{X_i}$ design matrix, the fixed effects are represented by the $\boldsymbol{\beta}$ vector, the random effect covariates for the i^{th} subject are collapsed into the $\boldsymbol{Z_i}$ design matrix, the random effects are stored in the $\boldsymbol{u_i}$ vector which is assumed to be multivariate normal with covariance matrix $\boldsymbol{D}$, and the n_i residuals for the i^{th} subject are represented by $\boldsymbol{\epsilon_i}$ vector which is also assumed multivariate normal with covariance matrix $\boldsymbol{R_i}$.

Tables of the Normal Distribution



Probability Content from -∞ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Values of z for selected values of Pr(Z<z)

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Excerpts from the Appendices to *Loss Models: From Data to Decisions, 2nd edition*

April 21, 2005

Appendix A

An Inventory of Continuous Distributions

A.1 Introduction

The incomplete gamma function is given by

$$\Gamma(\alpha; x) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-t}\, dt, \quad \alpha > 0,\ x > 0$$

$$\text{with } \Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt, \quad \alpha > 0.$$

Also, define

$$G(\alpha; x) = \int_x^\infty t^{\alpha-1} e^{-t}\, dt, \quad x > 0.$$

Integration by parts produces the relationship

$$G(\alpha; x) = -\frac{x^\alpha e^{-x}}{\alpha} + \frac{1}{\alpha} G(\alpha + 1; x)$$

For negative α, this can repeated until the first argument is postive, say at $\alpha + k$. Then the incomplete gamma function can be evaluated from

$$G(\alpha + k; x) = \Gamma(\alpha + k)[1 - \Gamma(\alpha + k; x)].$$

The incomplete beta function is given by

$$\beta(a, b; x) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1 - t)^{b-1}\, dt, \quad a > 0,\ b > 0,\ 0 < x < 1.$$

A.2 Transformed beta family

A.2.3 Three-parameter distributions

A.2.3.1 Generalized Pareto (beta of the second kind)—α, θ, τ

$$
\begin{aligned}
f(x) &= \frac{\Gamma(\alpha+\tau)}{\Gamma(\alpha)\Gamma(\tau)} \frac{\theta^\alpha x^{\tau-1}}{(x+\theta)^{\alpha+\tau}} \qquad F(x) = \beta(\tau, \alpha; u), \quad u = \frac{x}{x+\theta} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(\alpha-k)}{\Gamma(\alpha)\Gamma(\tau)}, \quad -\tau < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k \tau(\tau+1)\cdots(\tau+k-1)}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(\alpha-k)}{\Gamma(\alpha)\Gamma(\tau)} \beta(\tau+k, \alpha-k; u) + x^k[1-F(x)], \quad k > -\tau \\
\text{mode} &= \theta \frac{\tau-1}{\alpha+1}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.2 Burr (Burr Type XII, Singh-Maddala)—α, θ, γ

$$
\begin{aligned}
f(x) &= \frac{\alpha\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^{\alpha+1}} \qquad F(x) = 1 - u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(1+k/\gamma)\Gamma(\alpha-k/\gamma)}{\Gamma(\alpha)}, \quad -\gamma < k < \alpha\gamma \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(1+k/\gamma)\Gamma(\alpha-k/\gamma)}{\Gamma(\alpha)} \beta(1+k/\gamma, \alpha-k/\gamma; 1-u) + x^k u^\alpha, \quad k > -\gamma \\
\text{mode} &= \theta \left(\frac{\gamma-1}{\alpha\gamma+1} \right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Inverse Burr (Dagum)—τ, θ, γ

$$
\begin{aligned}
f(x) &= \frac{\tau\gamma(x/\theta)^{\gamma\tau}}{x[1+(x/\theta)^\gamma]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k/\gamma)\Gamma(1-k/\gamma)}{\Gamma(\tau)}, \quad -\tau\gamma < k < \gamma \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k/\gamma)\Gamma(1-k/\gamma)}{\Gamma(\tau)} \beta(\tau+k/\gamma, 1-k/\gamma; u) + x^k[1-u^\tau], \quad k > -\tau\gamma \\
\text{mode} &= \theta \left(\frac{\tau\gamma-1}{\gamma+1} \right)^{1/\gamma}, \quad \tau\gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.4 Two-parameter distributions

A.2.4.1 Pareto—α, θ

$$\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}\beta[k+1, \alpha-k; x/(x+\theta)] + x^k\left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}$$

A.2.4.2 Inverse Pareto—τ, θ

$$\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k(-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\tau\int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k}dy + x^k\left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}$$

A.2.4.3 Loglogistic (Fisk)—γ, θ

$$\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}$$

A.2.4.4 Paralogistic—α, θ

This is a Burr distribution with $\gamma = \alpha$.

$$\begin{aligned}
f(x) &= \frac{\alpha^2 (x/\theta)^\alpha}{x[1+(x/\theta)^\alpha]^{\alpha+1}} \qquad F(x) = 1 - u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\alpha} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(1+k/\alpha)\Gamma(\alpha - k/\alpha)}{\Gamma(\alpha)}, \quad -\alpha < k < \alpha^2 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(1+k/\alpha)\Gamma(\alpha - k/\alpha)}{\Gamma(\alpha)} \beta(1+k/\alpha, \alpha - k/\alpha; 1-u) + x^k u^\alpha, \quad k > -\alpha \\
\text{mode} &= \theta \left(\frac{\alpha - 1}{\alpha^2 + 1} \right)^{1/\alpha}, \quad \alpha > 1, \text{ else } 0
\end{aligned}$$

A.2.4.5 Inverse paralogistic—τ, θ

This is an inverse Burr distribution with $\gamma = \tau$.

$$\begin{aligned}
f(x) &= \frac{\tau^2 (x/\theta)^{\tau^2}}{x[1+(x/\theta)^\tau]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\tau}{1+(x/\theta)^\tau} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k/\tau)\Gamma(1 - k/\tau)}{\Gamma(\tau)}, \quad -\tau^2 < k < \tau \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k/\tau)\Gamma(1 - k/\tau)}{\Gamma(\tau)} \beta(\tau+k/\tau, 1 - k/\tau; u) + x^k [1-u^\tau], \quad k > -\tau^2 \\
\text{mode} &= \theta \left(\tau - 1 \right)^{1/\tau}, \quad \tau > 1, \text{ else } 0
\end{aligned}$$

A.3 Transformed gamma family

A.3.2 Two-parameter distributions

A.3.2.1 Gamma—α, θ

$$\begin{aligned}
f(x) &= \frac{(x/\theta)^\alpha e^{-x/\theta}}{x\Gamma(\alpha)} \qquad F(x) = \Gamma(\alpha; x/\theta) \\
M(t) &= (1-\theta t)^{-\alpha}, \quad t < 1/\theta \qquad \mathrm{E}[X^k] = \frac{\theta^k \Gamma(\alpha+k)}{\Gamma(\alpha)}, \quad k > -\alpha \\
\mathrm{E}[X^k] &= \theta^k (\alpha+k-1)\cdots\alpha, \quad \text{if } k \text{ is an integer}
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\alpha+k)}{\Gamma(\alpha)} \Gamma(\alpha+k; x/\theta) + x^k [1 - \Gamma(\alpha; x/\theta)], \quad k > -\alpha \\
&= \alpha(\alpha+1)\cdots(\alpha+k-1)\theta^k \Gamma(\alpha+k; x/\theta) + x^k [1 - \Gamma(\alpha; x/\theta)], \quad k \text{ an integer} \\
\text{mode} &= \theta(\alpha - 1), \quad \alpha > 1, \text{ else } 0
\end{aligned}$$

A.3.2.2 Inverse gamma (Vinci)—α, θ

$$\begin{aligned}
f(x) &= \frac{(\theta/x)^\alpha e^{-\theta/x}}{x\Gamma(\alpha)} \qquad F(x) = 1 - \Gamma(\alpha; \theta/x) \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad k < \alpha \qquad \mathrm{E}[X^k] = \frac{\theta^k}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\alpha-k)}{\Gamma(\alpha)}[1 - \Gamma(\alpha-k; \theta/x)] + x^k \Gamma(\alpha; \theta/x) \\
&= \frac{\theta^k \Gamma(\alpha-k)}{\Gamma(\alpha)} G(\alpha-k; \theta/x) + x^k \Gamma(\alpha; \theta/x), \text{ all } k \\
\text{mode} &= \theta/(\alpha+1)
\end{aligned}$$

A.3.2.3 Weibull—θ, τ

$$\begin{aligned}
f(x) &= \frac{\tau(x/\theta)^\tau e^{-(x/\theta)^\tau}}{x} \qquad F(x) = 1 - e^{-(x/\theta)^\tau} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1 + k/\tau), \quad k > -\tau \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(1 + k/\tau) \Gamma[1 + k/\tau; (x/\theta)^\tau] + x^k e^{-(x/\theta)^\tau}, \quad k > -\tau \\
\text{mode} &= \theta \left(\frac{\tau - 1}{\tau} \right)^{1/\tau}, \quad \tau > 1, \text{ else } 0
\end{aligned}$$

A.3.2.4 Inverse Weibull (log Gompertz)—θ, τ

$$\begin{aligned}
f(x) &= \frac{\tau(\theta/x)^\tau e^{-(\theta/x)^\tau}}{x} \qquad F(x) = e^{-(\theta/x)^\tau} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1 - k/\tau), \quad k < \tau \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(1 - k/\tau)\{1 - \Gamma[1 - k/\tau; (\theta/x)^\tau]\} + x^k \left[1 - e^{-(\theta/x)^\tau}\right], \quad \text{all } k \\
&= \theta^k \Gamma(1 - k/\tau) G[1 - k/\tau; (\theta/x)^\tau] + x^k \left[1 - e^{-(\theta/x)^\tau}\right] \\
\text{mode} &= \theta \left(\frac{\tau}{\tau + 1} \right)^{1/\tau}
\end{aligned}$$

A.3.3 One-parameter distributions

A.3.3.1 Exponential—θ

$$\begin{aligned}
f(x) &= \frac{e^{-x/\theta}}{\theta} \qquad F(x) = 1 - e^{-x/\theta} \\
M(t) &= (1 - \theta t)^{-1} \qquad \mathrm{E}[X^k] = \theta^k \Gamma(k+1), \quad k > -1 \\
\mathrm{E}[X^k] &= \theta^k k!, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[X \wedge x] &= \theta(1 - e^{-x/\theta}) \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(k+1) \Gamma(k+1; x/\theta) + x^k e^{-x/\theta}, \quad k > -1 \\
&= \theta^k k! \Gamma(k+1; x/\theta) + x^k e^{-x/\theta}, \quad k \text{ an integer} \\
\text{mode} &= 0
\end{aligned}$$

A.3.3.2 Inverse exponential—θ

$$\begin{aligned} f(x) &= \frac{\theta e^{-\theta/x}}{x^2} \qquad F(x) = e^{-\theta/x} \\ \mathrm{E}[X^k] &= \theta^k \Gamma(1-k), \quad k<1 \\ \mathrm{E}[(X\wedge x)^k] &= \theta^k G(1-k;\theta/x) + x^k(1-e^{-\theta/x}), \quad \text{all } k \\ \text{mode} &= \theta/2 \end{aligned}$$

A.4 Other distributions

A.4.1.1 Lognormal—μ,σ (μ can be negative)

$$\begin{aligned} f(x) &= \frac{1}{x\sigma\sqrt{2\pi}}\exp(-z^2/2) = \phi(z)/(\sigma x), \quad z = \frac{\ln x - \mu}{\sigma} \qquad F(x) = \Phi(z) \\ \mathrm{E}[X^k] &= \exp(k\mu + k^2\sigma^2/2) \\ \mathrm{E}[(X\wedge x)^k] &= \exp(k\mu + k^2\sigma^2/2)\Phi\left(\frac{\ln x - \mu - k\sigma^2}{\sigma}\right) + x^k[1-F(x)] \\ \text{mode} &= \exp(\mu - \sigma^2) \end{aligned}$$

A.4.1.2 Inverse Gaussian—μ,θ

$$\begin{aligned} f(x) &= \left(\frac{\theta}{2\pi x^3}\right)^{1/2}\exp\left(-\frac{\theta z^2}{2x}\right), \quad z = \frac{x-\mu}{\mu} \\ F(x) &= \Phi\left[z\left(\frac{\theta}{x}\right)^{1/2}\right] + \exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{1/2}\right], \quad y = \frac{x+\mu}{\mu} \\ M(t) &= \exp\left[\frac{\theta}{\mu}\left(1-\sqrt{1-\frac{2t\mu^2}{\theta}}\right)\right], \quad t < \frac{\theta}{2\mu^2}, \qquad \mathrm{E}[X] = \mu, \quad \mathrm{Var}[X] = \mu^3/\theta \\ \mathrm{E}[X\wedge x] &= x - \mu z\Phi\left[z\left(\frac{\theta}{x}\right)^{1/2}\right] - \mu y\exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{1/2}\right] \end{aligned}$$

A.4.1.3 log-t—r,μ,σ (μ *can be negative*)

Let Y have a t distribution with r degrees of freedom. Then $X = \exp(\sigma Y + \mu)$ has the log-t distribution. Positive moments do not exist for this distribution. Just as the t distribution has a heavier tail than the normal distribution, this distribution has a heavier tail than the lognormal distribution.

$$\begin{aligned} f(x) &= \frac{\Gamma\left(\frac{r+1}{2}\right)}{x\sigma\sqrt{\pi r}\,\Gamma\left(\frac{r}{2}\right)\left[1+\frac{1}{r}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]^{(r+1)/2}}, \\ F(x) &= F_r\left(\frac{\ln x - \mu}{\sigma}\right) \text{ with } F_r(t) \text{ the cdf of a } t \text{ distribution with } r \text{ d.f.,} \end{aligned}$$

$$F(x) = \begin{cases} \dfrac{1}{2}\beta\left[\dfrac{r}{2},\dfrac{1}{2};\dfrac{r}{r+\left(\dfrac{\ln x-\mu}{\sigma}\right)^2}\right], & 0 < x \le e^{\mu}, \\ 1-\dfrac{1}{2}\beta\left[\dfrac{r}{2},\dfrac{1}{2};\dfrac{r}{r+\left(\dfrac{\ln x-\mu}{\sigma}\right)^2}\right], & x \ge e^{\mu}. \end{cases}$$

A.4.1.4 Single-parameter Pareto—α, θ

$$f(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, \quad x > \theta \qquad F(x) = 1-(\theta/x)^\alpha, \quad x > \theta$$

$$\mathrm{E}[X^k] = \frac{\alpha\theta^k}{\alpha-k}, \quad k < \alpha \qquad \mathrm{E}[(X\wedge x)^k] = \frac{\alpha\theta^k}{\alpha-k} - \frac{k\theta^\alpha}{(\alpha-k)x^{\alpha-k}}$$

$$\text{mode} = \theta$$

Note: Although there appears to be two parameters, only α is a true parameter. The value of θ must be set in advance.

A.5 Distributions with finite support

For these two distributions, the scale parameter θ is assumed known.

A.5.1.1 Generalized beta—a, b, θ, τ

$$\begin{aligned} f(x) &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}u^a(1-u)^{b-1}\frac{\tau}{x}, \quad 0 < x < \theta, \quad u = (x/\theta)^\tau \\ F(x) &= \beta(a,b;u) \\ \mathrm{E}[X^k] &= \frac{\theta^k\Gamma(a+b)\Gamma(a+k/\tau)}{\Gamma(a)\Gamma(a+b+k/\tau)}, \quad k > -a\tau \\ \mathrm{E}[(X\wedge x)^k] &= \frac{\theta^k\Gamma(a+b)\Gamma(a+k/\tau)}{\Gamma(a)\Gamma(a+b+k/\tau)}\beta(a+k/\tau,b;u) + x^k[1-\beta(a,b;u)] \end{aligned}$$

A.5.1.2 beta—a, b, θ

$$\begin{aligned} f(x) &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}u^a(1-u)^{b-1}\frac{1}{x}, \quad 0 < x < \theta, \quad u = x/\theta \\ F(x) &= \beta(a,b;u) \\ \mathrm{E}[X^k] &= \frac{\theta^k\Gamma(a+b)\Gamma(a+k)}{\Gamma(a)\Gamma(a+b+k)}, \quad k > -a \\ \mathrm{E}[X^k] &= \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}, \quad \text{if } k \text{ is an integer} \\ \mathrm{E}[(X\wedge x)^k] &= \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}\beta(a+k,b;u) \\ &\quad +x^k[1-\beta(a,b;u)] \end{aligned}$$

Appendix B

An Inventory of Discrete Distributions

B.2 The $(a, b, 0)$ class

B.2.1.1 Poisson—λ

$$\begin{aligned} p_0 &= e^{-\lambda}, \quad a = 0, \quad b = \lambda \qquad\qquad p_k = \frac{e^{-\lambda}\lambda^k}{k!} \\ \mathrm{E}[N] &= \lambda, \quad \mathrm{Var}[N] = \lambda \qquad\qquad P(z) = e^{\lambda(z-1)} \end{aligned}$$

B.2.1.2 Geometric—β

$$\begin{aligned} p_0 &= \frac{1}{1+\beta}, \quad a = \frac{\beta}{1+\beta}, \quad b = 0 \qquad\qquad p_k = \frac{\beta^k}{(1+\beta)^{k+1}} \\ \mathrm{E}[N] &= \beta, \quad \mathrm{Var}[N] = \beta(1+\beta) \qquad\qquad P(z) = [1-\beta(z-1)]^{-1}. \end{aligned}$$

This is a special case of the negative binomial with $r = 1$.

B.2.1.3 Binomial—$q, m, (0 < q < 1,\ m$ an integer$)$

$$\begin{aligned} p_0 &= (1-q)^m, \quad a = -\frac{q}{1-q}, \quad b = \frac{(m+1)q}{1-q} \\ p_k &= \binom{m}{k} q^k (1-q)^{m-k}, \quad k = 0, 1, \ldots, m \\ \mathrm{E}[N] &= mq, \quad \mathrm{Var}[N] = mq(1-q) \qquad\qquad P(z) = [1+q(z-1)]^m. \end{aligned}$$

B.2.1.4 Negative binomial—β, r

$$\begin{aligned} p_0 &= (1+\beta)^{-r}, \quad a = \frac{\beta}{1+\beta}, \quad b = \frac{(r-1)\beta}{1+\beta} \\ p_k &= \frac{r(r+1)\cdots(r+k-1)\beta^k}{k!(1+\beta)^{r+k}} \\ \mathrm{E}[N] &= r\beta, \quad \mathrm{Var}[N] = r\beta(1+\beta) \qquad\qquad P(z) = [1-\beta(z-1)]^{-r}. \end{aligned}$$

Tail areas (two-sided) for t-distributions

df	0.20	0.10	0.05	0.02	0.01
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
400	1.284	1.649	1.966	2.336	2.588
Inf	1.282	1.645	1.960	2.326	2.576

For use on the CAS exams

Selected Upper-tail areas for F-distributions

Numerator df

Denominator df	Upper-tail	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	Inf		
1	0.20	9.472	12	13.064	13.644	14.008	14.258	14.439	14.577	14.685	14.772	14.844	14.904	14.998	15.07	15.171	15.238	15.306	15.374	15.415	15.47	15.497	15.539	15.563	15.58	0.20	1
1	0.10	39.86	49.5	53.59	55.83	57.24	58.2	58.91	59.44	59.86	60.19	60.47	60.71	61.07	61.35	61.74	62	62.26	62.53	62.69	62.9	63.01	63.17	63.26	63.33	0.10	1
1	0.05	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	243	243.9	245.4	246.5	248	249.1	250.1	251.1	251.8	252.6	253	253.7	254.1	254.3	0.05	1
1	0.02	1013	1249	1351	1406	1441	1464	1482	1495	1505	1514	1521	1526	1535	1542	1552	1558	1565	1571	1575	1581	1583	1587	1590	1591	0.02	1
1	0.01	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6083	6106	6143	6170	6209	6235	6261	6287	6303	6324	6334	6350	6360	6366	0.01	1
2	0.20	3.556	4	4.156	4.236	4.284	4.317	4.34	4.358	4.371	4.382	4.391	4.399	4.41	4.419	4.432	4.44	4.448	4.456	4.461	4.468	4.471	4.476	4.479	4.481	0.20	2
2	0.10	8.526	9	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392	9.401	9.408	9.42	9.429	9.441	9.45	9.458	9.466	9.471	9.478	9.481	9.486	9.489	9.491	0.10	2
2	0.05	18.51	19	19.16	19.25	19.3	19.33	19.35	19.37	19.38	19.4	19.4	19.41	19.42	19.43	19.45	19.45	19.46	19.47	19.48	19.48	19.49	19.49	19.49	19.5	0.05	2
2	0.02	48.51	49	49.17	49.25	49.3	49.33	49.36	49.37	49.39	49.4	49.41	49.42	49.43	49.44	49.45	49.46	49.46	49.47	49.48	49.48	49.49	49.49	49.5	49.5	0.02	2
2	0.01	98.5	99	99.17	99.25	99.3	99.33	99.36	99.37	99.39	99.4	99.41	99.42	99.43	99.44	99.45	99.46	99.47	99.47	99.48	99.49	99.49	99.49	99.5	99.5	0.01	2
3	0.20	2.682	2.886	2.936	2.956	2.965	2.971	2.974	2.976	2.978	2.979	2.98	2.981	2.982	2.982	2.983	2.983	2.984	2.984	2.984	2.984	2.984	2.985	2.985	2.985	0.20	3
3	0.10	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.24	5.23	5.222	5.216	5.205	5.196	5.184	5.176	5.168	5.16	5.155	5.148	5.144	5.139	5.136	5.134	0.10	3
3	0.05	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.763	8.745	8.715	8.692	8.66	8.639	8.617	8.594	8.581	8.563	8.554	8.54	8.532	8.526	0.05	3
3	0.02	20.62	18.86	18.11	17.69	17.43	17.25	17.11	17.01	16.93	16.86	16.81	16.76	16.69	16.63	16.55	16.5	16.45	16.39	16.36	16.32	16.3	16.26	16.24	16.23	0.02	3
3	0.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.13	27.05	26.92	26.83	26.69	26.6	26.5	26.41	26.35	26.28	26.24	26.18	26.15	26.13	0.01	3
4	0.20	2.351	2.472	2.485	2.483	2.478	2.473	2.469	2.465	2.462	2.46	2.457	2.455	2.452	2.449	2.445	2.442	2.439	2.436	2.434	2.432	2.43	2.428	2.427	2.426	0.20	4
4	0.10	4.545	4.325	4.191	4.107	4.051	4.01	3.979	3.955	3.936	3.92	3.907	3.896	3.878	3.864	3.844	3.831	3.817	3.804	3.795	3.784	3.778	3.769	3.764	3.761	0.10	4
4	0.05	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.936	5.912	5.873	5.844	5.803	5.774	5.746	5.717	5.699	5.676	5.664	5.646	5.635	5.628	0.05	4
4	0.02	14.04	12.142	11.344	10.899	10.616	10.419	10.274	10.162	10.074	10.003	9.944	9.894	9.815	9.755	9.67	9.612	9.554	9.495	9.46	9.412	9.388	9.352	9.33	9.315	0.02	4
4	0.01	21.2	18	16.69	15.98	15.52	15.21	14.98	14.8	14.66	14.55	14.45	14.37	14.25	14.15	14.02	13.93	13.84	13.75	13.69	13.61	13.58	13.52	13.49	13.46	0.01	4
5	0.20	2.178	2.259	2.253	2.24	2.228	2.217	2.209	2.202	2.196	2.191	2.187	2.184	2.178	2.173	2.166	2.161	2.156	2.151	2.148	2.144	2.141	2.138	2.136	2.134	0.20	5
5	0.10	4.06	3.78	3.619	3.52	3.453	3.405	3.368	3.339	3.316	3.297	3.282	3.268	3.247	3.23	3.207	3.191	3.174	3.157	3.147	3.133	3.126	3.116	3.109	3.105	0.10	5
5	0.05	6.608	5.786	5.409	5.192	5.05	4.95	4.876	4.818	4.772	4.735	4.704	4.678	4.636	4.604	4.558	4.527	4.496	4.464	4.444	4.418	4.405	4.385	4.373	4.365	0.05	5
5	0.02	11.323	9.454	8.67	8.233	7.953	7.758	7.614	7.503	7.415	7.344	7.285	7.235	7.156	7.095	7.009	6.951	6.893	6.833	6.797	6.749	6.724	6.687	6.665	6.65	0.02	5
5	0.01	16.258	13.274	12.06	11.392	10.967	10.672	10.456	10.289	10.158	10.051	9.963	9.888	9.77	9.68	9.553	9.466	9.379	9.291	9.238	9.166	9.13	9.075	9.042	9.02	0.01	5
6	0.20	2.073	2.13	2.113	2.092	2.076	2.062	2.051	2.042	2.034	2.028	2.022	2.018	2.01	2.004	1.995	1.989	1.982	1.976	1.972	1.966	1.963	1.959	1.956	1.954	0.20	6
6	0.10	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937	2.92	2.905	2.881	2.863	2.836	2.818	2.8	2.781	2.77	2.754	2.746	2.734	2.727	2.722	0.10	6
6	0.05	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.06	4.027	4	3.956	3.922	3.874	3.841	3.808	3.774	3.754	3.726	3.712	3.69	3.678	3.669	0.05	6
6	0.02	9.876	8.052	7.287	6.859	6.585	6.393	6.251	6.141	6.055	5.984	5.925	5.876	5.797	5.737	5.651	5.593	5.534	5.474	5.438	5.389	5.364	5.327	5.304	5.289	0.02	6
6	0.01	13.745	10.925	9.78	9.148	8.746	8.466	8.26	8.102	7.976	7.874	7.79	7.718	7.605	7.519	7.396	7.313	7.229	7.143	7.091	7.022	6.987	6.934	6.902	6.88	0.01	6
7	0.20	2.002	2.043	2.019	1.994	1.974	1.957	1.945	1.934	1.925	1.918	1.911	1.906	1.897	1.89	1.879	1.872	1.865	1.857	1.852	1.845	1.842	1.837	1.833	1.831	0.20	7
7	0.10	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703	2.684	2.668	2.643	2.623	2.595	2.575	2.555	2.535	2.523	2.506	2.497	2.484	2.476	2.471	0.10	7
7	0.05	5.591	4.737	4.347	4.12	3.972	3.866	3.787	3.726	3.677	3.637	3.603	3.575	3.529	3.494	3.445	3.41	3.376	3.34	3.319	3.29	3.275	3.252	3.239	3.23	0.05	7
7	0.02	8.988	7.203	6.454	6.035	5.765	5.576	5.435	5.327	5.241	5.171	5.113	5.064	4.985	4.925	4.839	4.781	4.722	4.662	4.625	4.576	4.551	4.513	4.49	4.475	0.02	7
7	0.01	12.246	9.547	8.451	7.847	7.46	7.191	6.993	6.84	6.719	6.62	6.538	6.469	6.359	6.275	6.155	6.074	5.992	5.908	5.858	5.789	5.755	5.702	5.671	5.65	0.01	7
8	0.20	1.951	1.981	1.951	1.923	1.9	1.883	1.868	1.856	1.847	1.838	1.831	1.825	1.815	1.807	1.796	1.787	1.779	1.77	1.765	1.757	1.753	1.748	1.744	1.742	0.20	8
8	0.10	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538	2.519	2.502	2.475	2.455	2.425	2.404	2.383	2.361	2.348	2.33	2.321	2.307	2.298	2.293	0.10	8
8	0.05	5.318	4.459	4.066	3.838	3.687	3.581	3.5	3.438	3.388	3.347	3.313	3.284	3.237	3.202	3.15	3.115	3.079	3.043	3.02	2.99	2.975	2.951	2.937	2.928	0.05	8
8	0.02	8.389	6.637	5.901	5.489	5.223	5.036	4.897	4.79	4.705	4.635	4.577	4.528	4.449	4.389	4.304	4.245	4.186	4.125	4.088	4.038	4.013	3.975	3.952	3.936	0.02	8
8	0.01	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.734	5.667	5.559	5.477	5.359	5.279	5.198	5.116	5.065	4.998	4.963	4.911	4.88	4.859	0.01	8
9	0.20	1.913	1.935	1.901	1.87	1.846	1.826	1.811	1.798	1.787	1.778	1.771	1.764	1.753	1.745	1.732	1.723	1.714	1.704	1.698	1.69	1.686	1.68	1.676	1.673	0.20	9
9	0.10	3.36	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.44	2.416	2.396	2.379	2.351	2.329	2.298	2.277	2.255	2.232	2.218	2.199	2.189	2.174	2.165	2.159	0.10	9
9	0.05	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.23	3.179	3.137	3.102	3.073	3.025	2.989	2.936	2.9	2.864	2.826	2.803	2.771	2.756	2.731	2.717	2.707	0.05	9
9	0.02	7.961	6.234	5.51	5.103	4.84	4.654	4.517	4.41	4.325	4.256	4.198	4.149	4.071	4.011	3.925	3.866	3.806	3.745	3.708	3.658	3.632	3.593	3.57	3.554	0.02	9
9	0.01	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.178	5.111	5.005	4.924	4.808	4.729	4.649	4.567	4.517	4.449	4.415	4.363	4.332	4.311	0.01	9

	Numerator df	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	Inf		
Denominator df	Upper-tail																										
10	0.20	1.883	1.899	1.861	1.829	1.803	1.782	1.766	1.752	1.741	1.732	1.723	1.716	1.705	1.696	1.682	1.673	1.663	1.653	1.646	1.637	1.633	1.626	1.621	1.618	0.20	10
10	0.10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323	2.302	2.284	2.255	2.233	2.201	2.178	2.155	2.132	2.117	2.097	2.087	2.071	2.062	2.055	0.10	10
10	0.05	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.02	2.978	2.943	2.913	2.865	2.828	2.774	2.737	2.7	2.661	2.637	2.605	2.588	2.563	2.548	2.538	0.05	10
10	0.02	7.638	5.934	5.218	4.816	4.555	4.371	4.235	4.129	4.044	3.975	3.917	3.868	3.79	3.73	3.644	3.585	3.525	3.463	3.425	3.374	3.348	3.309	3.285	3.269	0.02	10
10	0.01	10.044	7.559	6.552	5.994	5.636	5.386	5.2	5.057	4.942	4.849	4.772	4.706	4.601	4.52	4.405	4.327	4.247	4.165	4.115	4.048	4.014	3.962	3.93	3.909	0.01	10
11	0.20	1.859	1.87	1.83	1.796	1.768	1.747	1.73	1.716	1.704	1.694	1.685	1.678	1.666	1.656	1.642	1.632	1.622	1.611	1.604	1.594	1.589	1.582	1.577	1.574	0.20	11
11	0.10	3.225	2.86	2.66	2.536	2.451	2.389	2.342	2.304	2.274	2.248	2.227	2.209	2.179	2.156	2.123	2.1	2.076	2.052	2.036	2.016	2.005	1.989	1.979	1.972	0.10	11
11	0.05	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.818	2.788	2.739	2.701	2.646	2.609	2.57	2.531	2.507	2.473	2.457	2.431	2.415	2.404	0.05	11
11	0.02	7.388	5.701	4.993	4.594	4.336	4.153	4.017	3.912	3.828	3.758	3.701	3.652	3.573	3.513	3.427	3.367	3.307	3.245	3.207	3.155	3.129	3.089	3.065	3.048	0.02	11
11	0.01	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.462	4.397	4.293	4.213	4.099	4.021	3.941	3.86	3.81	3.742	3.708	3.656	3.624	3.602	0.01	11
12	0.20	1.839	1.846	1.804	1.768	1.74	1.718	1.7	1.686	1.673	1.663	1.654	1.646	1.634	1.624	1.609	1.598	1.587	1.576	1.568	1.558	1.553	1.545	1.54	1.537	0.20	12
12	0.10	3.177	2.807	2.606	2.48	2.394	2.331	2.283	2.245	2.214	2.188	2.166	2.147	2.117	2.094	2.06	2.036	2.011	1.986	1.97	1.949	1.938	1.921	1.911	1.904	0.10	12
12	0.05	4.747	3.885	3.49	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.717	2.687	2.637	2.599	2.544	2.505	2.466	2.426	2.401	2.367	2.35	2.323	2.307	2.296	0.05	12
12	0.02	7.188	5.516	4.814	4.419	4.162	3.98	3.845	3.74	3.656	3.587	3.529	3.48	3.402	3.341	3.254	3.195	3.134	3.071	3.033	2.981	2.954	2.913	2.889	2.872	0.02	12
12	0.01	9.33	6.927	5.953	5.412	5.064	4.821	4.64	4.499	4.388	4.296	4.22	4.155	4.052	3.972	3.858	3.78	3.701	3.619	3.569	3.501	3.467	3.414	3.382	3.361	0.01	12
13	0.20	1.823	1.826	1.783	1.746	1.717	1.694	1.676	1.661	1.648	1.637	1.628	1.62	1.607	1.596	1.581	1.57	1.558	1.546	1.539	1.528	1.523	1.514	1.509	1.506	0.20	13
13	0.10	3.136	2.763	2.56	2.434	2.347	2.283	2.234	2.195	2.164	2.138	2.116	2.097	2.066	2.042	2.007	1.983	1.958	1.931	1.915	1.893	1.882	1.864	1.853	1.846	0.10	13
13	0.05	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.635	2.604	2.554	2.515	2.459	2.42	2.38	2.339	2.314	2.279	2.261	2.234	2.218	2.206	0.05	13
13	0.02	7.024	5.366	4.669	4.276	4.02	3.84	3.705	3.6	3.516	3.447	3.39	3.341	3.262	3.201	3.114	3.054	2.993	2.93	2.891	2.838	2.811	2.77	2.745	2.728	0.02	13
13	0.01	9.074	6.701	5.739	5.205	4.862	4.62	4.441	4.302	4.191	4.1	4.025	3.96	3.857	3.778	3.665	3.587	3.507	3.425	3.375	3.307	3.272	3.219	3.187	3.165	0.01	13
14	0.20	1.809	1.809	1.765	1.727	1.697	1.674	1.655	1.639	1.626	1.615	1.606	1.598	1.584	1.573	1.557	1.546	1.534	1.521	1.513	1.502	1.497	1.488	1.482	1.479	0.20	14
14	0.10	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.122	2.095	2.073	2.054	2.022	1.998	1.962	1.938	1.912	1.885	1.869	1.846	1.834	1.816	1.805	1.797	0.10	14
14	0.05	4.6	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.565	2.534	2.484	2.445	2.388	2.349	2.308	2.266	2.241	2.205	2.187	2.159	2.142	2.131	0.05	14
14	0.02	6.888	5.241	4.549	4.158	3.904	3.724	3.589	3.485	3.401	3.332	3.274	3.225	3.146	3.086	2.998	2.938	2.876	2.812	2.773	2.72	2.692	2.651	2.625	2.608	0.02	14
14	0.01	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.14	4.03	3.939	3.864	3.8	3.698	3.619	3.505	3.427	3.348	3.266	3.215	3.147	3.112	3.059	3.026	3.004	0.01	14
15	0.20	1.797	1.795	1.749	1.71	1.68	1.656	1.637	1.621	1.608	1.596	1.587	1.578	1.564	1.553	1.537	1.525	1.513	1.5	1.491	1.48	1.474	1.465	1.459	1.455	0.20	15
15	0.10	3.073	2.695	2.49	2.361	2.273	2.208	2.158	2.119	2.086	2.059	2.037	2.017	1.985	1.961	1.924	1.899	1.873	1.845	1.828	1.805	1.793	1.774	1.763	1.755	0.10	15
15	0.05	4.543	3.682	3.287	3.056	2.901	2.79	2.707	2.641	2.588	2.544	2.507	2.475	2.424	2.385	2.328	2.288	2.247	2.204	2.178	2.142	2.123	2.095	2.078	2.066	0.05	15
15	0.02	6.773	5.135	4.447	4.058	3.805	3.626	3.492	3.387	3.303	3.235	3.177	3.128	3.049	2.988	2.9	2.84	2.777	2.713	2.674	2.62	2.592	2.55	2.524	2.506	0.02	15
15	0.01	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.73	3.666	3.564	3.485	3.372	3.294	3.214	3.132	3.081	3.012	2.977	2.923	2.891	2.868	0.01	15
16	0.20	1.787	1.783	1.736	1.696	1.665	1.641	1.621	1.605	1.591	1.58	1.57	1.561	1.547	1.536	1.519	1.507	1.494	1.481	1.472	1.46	1.454	1.445	1.439	1.435	0.20	16
16	0.10	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.055	2.028	2.005	1.985	1.953	1.928	1.891	1.866	1.839	1.811	1.793	1.769	1.757	1.738	1.726	1.718	0.10	16
16	0.05	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.456	2.425	2.373	2.333	2.276	2.235	2.194	2.151	2.124	2.087	2.068	2.039	2.022	2.01	0.05	16
16	0.02	6.674	5.046	4.361	3.974	3.721	3.543	3.409	3.304	3.221	3.152	3.094	3.045	2.966	2.905	2.817	2.756	2.693	2.628	2.589	2.534	2.506	2.463	2.437	2.419	0.02	16
16	0.01	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.89	3.78	3.691	3.616	3.553	3.451	3.372	3.259	3.181	3.101	3.018	2.967	2.898	2.863	2.808	2.775	2.753	0.01	16
17	0.20	1.778	1.772	1.724	1.684	1.652	1.628	1.608	1.591	1.577	1.566	1.555	1.547	1.532	1.52	1.503	1.491	1.478	1.464	1.455	1.443	1.437	1.427	1.421	1.416	0.20	17
17	0.10	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.028	2.001	1.978	1.958	1.925	1.9	1.862	1.836	1.809	1.781	1.763	1.738	1.726	1.706	1.694	1.686	0.10	17
17	0.05	4.451	3.592	3.197	2.965	2.81	2.699	2.614	2.548	2.494	2.45	2.413	2.381	2.329	2.289	2.23	2.19	2.148	2.104	2.077	2.04	2.02	1.991	1.973	1.96	0.05	17
17	0.02	6.589	4.968	4.286	3.901	3.649	3.471	3.337	3.233	3.149	3.08	3.023	2.973	2.894	2.833	2.745	2.683	2.62	2.555	2.515	2.46	2.431	2.388	2.361	2.343	0.02	17
17	0.01	8.4	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593	3.519	3.455	3.353	3.275	3.162	3.084	3.003	2.92	2.869	2.8	2.764	2.709	2.676	2.653	0.01	17
18	0.20	1.77	1.762	1.713	1.673	1.641	1.616	1.596	1.579	1.565	1.553	1.543	1.534	1.519	1.507	1.489	1.477	1.463	1.449	1.44	1.428	1.421	1.411	1.404	1.4	0.20	18
18	0.10	3.007	2.624	2.416	2.286	2.196	2.13	2.079	2.038	2.005	1.977	1.954	1.933	1.9	1.875	1.837	1.81	1.783	1.754	1.736	1.711	1.698	1.678	1.665	1.657	0.10	18
18	0.05	4.414	3.555	3.16	2.928	2.773	2.661	2.577	2.51	2.456	2.412	2.374	2.342	2.29	2.25	2.191	2.15	2.107	2.063	2.035	1.998	1.978	1.948	1.929	1.917	0.05	18
18	0.02	6.515	4.9	4.221	3.837	3.586	3.408	3.275	3.171	3.087	3.018	2.96	2.911	2.832	2.77	2.682	2.62	2.557	2.491	2.45	2.395	2.366	2.322	2.295	2.277	0.02	18
18	0.01	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.434	3.371	3.269	3.19	3.077	2.999	2.919	2.835	2.784	2.714	2.678	2.623	2.589	2.566	0.01	18
19	0.20	1.763	1.754	1.704	1.663	1.631	1.605	1.585	1.568	1.554	1.542	1.531	1.522	1.507	1.495	1.477	1.464	1.45	1.436	1.427	1.414	1.407	1.396	1.39	1.385	0.20	19
19	0.10	2.99	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.984	1.956	1.932	1.912	1.878	1.852	1.814	1.787	1.759	1.73	1.711	1.686	1.673	1.652	1.639	1.631	0.10	19
19	0.05	4.381	3.522	3.127	2.895	2.74	2.628	2.544	2.477	2.423	2.378	2.34	2.308	2.256	2.215	2.155	2.114	2.071	2.026	1.999	1.96	1.94	1.91	1.891	1.878	0.05	19
19	0.02	6.449	4.84	4.164	3.781	3.531	3.353	3.22	3.116	3.032	2.963	2.906	2.856	2.777	2.715	2.626	2.564	2.501	2.434	2.394	2.337	2.308	2.264	2.236	2.218	0.02	19
19	0.01	8.185	5.926	5.01	4.5	4.171	3.939	3.765	3.631	3.523	3.434	3.36	3.297	3.195	3.116	3.003	2.925	2.844	2.761	2.709	2.639	2.602	2.547	2.512	2.489	0.01	19

Denominator df	Upper-tail	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	Inf		
20	0.20	1.757	1.746	1.696	1.654	1.622	1.596	1.575	1.558	1.544	1.531	1.521	1.512	1.496	1.484	1.466	1.452	1.439	1.424	1.414	1.401	1.394	1.383	1.377	1.372	0.20	20
20	0.10	2.975	2.589	2.38	2.249	2.158	2.091	2.04	1.999	1.965	1.937	1.913	1.892	1.859	1.833	1.794	1.767	1.738	1.708	1.69	1.664	1.65	1.629	1.616	1.607	0.10	20
20	0.05	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.31	2.278	2.225	2.184	2.124	2.082	2.039	1.994	1.966	1.927	1.907	1.875	1.856	1.843	0.05	20
20	0.02	6.391	4.788	4.113	3.731	3.482	3.304	3.171	3.067	2.984	2.915	2.857	2.808	2.728	2.666	2.577	2.515	2.451	2.384	2.343	2.286	2.257	2.212	2.184	2.165	0.02	20
20	0.01	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.294	3.231	3.13	3.051	2.938	2.859	2.778	2.695	2.643	2.572	2.535	2.479	2.445	2.421	0.01	20
21	0.20	1.751	1.739	1.688	1.646	1.614	1.588	1.567	1.549	1.535	1.522	1.511	1.502	1.487	1.474	1.455	1.442	1.428	1.413	1.403	1.39	1.383	1.371	1.364	1.36	0.20	21
21	0.10	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.948	1.92	1.896	1.875	1.841	1.815	1.776	1.748	1.719	1.689	1.67	1.644	1.63	1.608	1.595	1.586	0.10	21
21	0.05	4.325	3.467	3.072	2.84	2.685	2.573	2.488	2.42	2.366	2.321	2.283	2.25	2.197	2.156	2.096	2.054	2.01	1.965	1.936	1.897	1.876	1.845	1.825	1.812	0.05	21
21	0.02	6.339	4.74	4.068	3.687	3.438	3.261	3.128	3.024	2.94	2.872	2.814	2.764	2.685	2.623	2.533	2.471	2.406	2.339	2.298	2.24	2.211	2.165	2.137	2.118	0.02	21
21	0.01	8.017	5.78	4.874	4.369	4.042	3.812	3.64	3.506	3.398	3.31	3.236	3.173	3.072	2.993	2.88	2.801	2.72	2.636	2.584	2.512	2.475	2.419	2.384	2.36	0.01	21
22	0.20	1.746	1.733	1.682	1.639	1.606	1.58	1.559	1.541	1.526	1.514	1.503	1.494	1.478	1.465	1.446	1.433	1.418	1.403	1.393	1.379	1.372	1.361	1.353	1.349	0.20	22
22	0.10	2.949	2.561	2.351	2.219	2.128	2.06	2.008	1.967	1.933	1.904	1.88	1.859	1.825	1.798	1.759	1.731	1.702	1.671	1.652	1.625	1.611	1.59	1.576	1.567	0.10	22
22	0.05	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.259	2.226	2.173	2.131	2.071	2.028	1.984	1.938	1.909	1.869	1.849	1.817	1.797	1.783	0.05	22
22	0.02	6.292	4.698	4.028	3.647	3.399	3.222	3.089	2.985	2.902	2.833	2.775	2.725	2.646	2.584	2.494	2.431	2.366	2.299	2.257	2.199	2.169	2.123	2.095	2.075	0.02	22
22	0.01	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.184	3.121	3.019	2.941	2.827	2.749	2.667	2.583	2.531	2.459	2.422	2.365	2.329	2.305	0.01	22
23	0.20	1.741	1.728	1.676	1.633	1.599	1.573	1.552	1.534	1.519	1.506	1.495	1.486	1.47	1.457	1.438	1.424	1.41	1.394	1.384	1.37	1.362	1.351	1.343	1.338	0.20	23
23	0.10	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.919	1.89	1.866	1.845	1.811	1.784	1.744	1.716	1.686	1.655	1.636	1.609	1.594	1.572	1.558	1.549	0.10	23
23	0.05	4.279	3.422	3.028	2.796	2.64	2.528	2.442	2.375	2.32	2.275	2.236	2.204	2.15	2.109	2.048	2.005	1.961	1.914	1.885	1.844	1.823	1.791	1.771	1.757	0.05	23
23	0.02	6.249	4.66	3.991	3.611	3.363	3.187	3.054	2.95	2.867	2.798	2.74	2.69	2.61	2.548	2.458	2.395	2.33	2.262	2.22	2.162	2.132	2.085	2.056	2.037	0.02	23
23	0.01	7.881	5.664	4.765	4.264	3.939	3.71	3.539	3.406	3.299	3.211	3.137	3.074	2.973	2.894	2.781	2.702	2.62	2.535	2.483	2.411	2.373	2.316	2.28	2.256	0.01	23
24	0.20	1.737	1.722	1.67	1.627	1.593	1.567	1.545	1.527	1.512	1.499	1.488	1.479	1.463	1.45	1.43	1.416	1.401	1.385	1.375	1.361	1.353	1.341	1.334	1.329	0.20	24
24	0.10	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.906	1.877	1.853	1.832	1.797	1.77	1.73	1.702	1.672	1.641	1.621	1.593	1.579	1.556	1.542	1.533	0.10	24
24	0.05	4.26	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.3	2.255	2.216	2.183	2.13	2.088	2.027	1.984	1.939	1.892	1.863	1.822	1.8	1.768	1.747	1.733	0.05	24
24	0.02	6.211	4.625	3.958	3.579	3.331	3.155	3.022	2.919	2.835	2.766	2.708	2.658	2.578	2.516	2.426	2.363	2.297	2.229	2.187	2.128	2.097	2.05	2.021	2.001	0.02	24
24	0.01	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.094	3.032	2.93	2.852	2.738	2.659	2.577	2.492	2.44	2.367	2.329	2.271	2.235	2.211	0.01	24
25	0.20	1.733	1.718	1.665	1.622	1.588	1.561	1.539	1.521	1.506	1.493	1.482	1.472	1.456	1.443	1.423	1.409	1.394	1.378	1.367	1.353	1.345	1.333	1.325	1.32	0.20	25
25	0.10	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.895	1.866	1.841	1.82	1.785	1.758	1.718	1.689	1.659	1.627	1.607	1.579	1.565	1.542	1.527	1.518	0.10	25
25	0.05	4.242	3.385	2.991	2.759	2.603	2.49	2.405	2.337	2.282	2.236	2.198	2.165	2.111	2.069	2.007	1.964	1.919	1.872	1.842	1.801	1.779	1.746	1.725	1.711	0.05	25
25	0.02	6.176	4.593	3.928	3.549	3.302	3.126	2.993	2.89	2.806	2.737	2.679	2.629	2.549	2.487	2.396	2.333	2.267	2.199	2.156	2.097	2.066	2.018	1.989	1.969	0.02	25
25	0.01	7.77	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	3.056	2.993	2.892	2.813	2.699	2.62	2.538	2.453	2.4	2.327	2.289	2.23	2.194	2.169	0.01	25
26	0.20	1.729	1.713	1.66	1.617	1.583	1.556	1.534	1.516	1.5	1.487	1.476	1.466	1.45	1.437	1.417	1.402	1.387	1.371	1.36	1.345	1.337	1.325	1.317	1.312	0.20	26
26	0.10	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.884	1.855	1.83	1.809	1.774	1.747	1.706	1.677	1.647	1.615	1.594	1.566	1.551	1.528	1.514	1.504	0.10	26
26	0.05	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.22	2.181	2.148	2.094	2.052	1.99	1.946	1.901	1.853	1.823	1.782	1.76	1.726	1.705	1.691	0.05	26
26	0.02	6.144	4.564	3.9	3.522	3.275	3.099	2.967	2.863	2.78	2.711	2.652	2.603	2.523	2.46	2.369	2.306	2.24	2.171	2.128	2.068	2.037	1.989	1.959	1.939	0.02	26
26	0.01	7.721	5.526	4.637	4.14	3.818	3.591	3.421	3.288	3.182	3.094	3.021	2.958	2.857	2.778	2.664	2.585	2.503	2.417	2.364	2.29	2.252	2.193	2.156	2.131	0.01	26

Lower-tail areas for Chi-square distributions

df	0.005	0.010	0.025	0.050	0.950	0.975	0.990	0.995
1	0.00	0.00	0.00	0.00	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
31	14.46	15.66	17.54	19.28	44.99	48.23	52.19	55.00
32	15.13	16.36	18.29	20.07	46.19	49.48	53.49	56.33
33	15.82	17.07	19.05	20.87	47.40	50.73	54.78	57.65
34	16.50	17.79	19.81	21.66	48.60	51.97	56.06	58.96
35	17.19	18.51	20.57	22.47	49.80	53.20	57.34	60.27
36	17.89	19.23	21.34	23.27	51.00	54.44	58.62	61.58
37	18.59	19.96	22.11	24.07	52.19	55.67	59.89	62.88
38	19.29	20.69	22.88	24.88	53.38	56.90	61.16	64.18
39	20.00	21.43	23.65	25.70	54.57	58.12	62.43	65.48
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
41	21.42	22.91	25.21	27.33	56.94	60.56	64.95	68.05
42	22.14	23.65	26.00	28.14	58.12	61.78	66.21	69.34
43	22.86	24.40	26.79	28.96	59.30	62.99	67.46	70.62
44	23.58	25.15	27.57	29.79	60.48	64.20	68.71	71.89
45	24.31	25.90	28.37	30.61	61.66	65.41	69.96	73.17
46	25.04	26.66	29.16	31.44	62.83	66.62	71.20	74.44
47	25.77	27.42	29.96	32.27	64.00	67.82	72.44	75.70
48	26.51	28.18	30.75	33.10	65.17	69.02	73.68	76.97
49	27.25	28.94	31.55	33.93	66.34	70.22	74.92	78.23
50	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49

For use on the CAS exams