CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

October 31, 2013

Exam 3L

Life Contingencies and Statistics

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2013 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 18, 2013.

END OF INSTRUCTIONS

1.

A life table has been constructed using the following information:

- $S_0(x) = \dfrac{r-2x}{r+2x}$, $x \geq 0$
- $\ell_0 = 5{,}000$
- $\ell_{20} = 3{,}000$
- ω is defined as the smallest number for which $S_0(\omega) = 0$

Calculate ω.

A. Less than 77.5
B. At least 77.5, but less than 78.5
C. At least 78.5, but less than 79.5
D. At least 79.5, but less than 80.5
E. At least 80.5

EXAMINATION CONTINUED ON NEXT PAGE

2.

You are given the following information:

- $S(x) = (1 - \frac{x}{\omega})^2$,for $0 \leq x < \omega$
- $\overset{o}{e}_{40} = 20$

Calculate ${}_{2|3}q_{50}$.

A. Less than 0.10
B. At least 0.10, but less than 0.12
C. At least 0.12, but less than 0.14
D. At least 0.14, but less than 0.16
E. At least 0.16

3.

You are given the following information:

- Deaths are uniformly distributed between integer ages.
- $l_x = 11{,}236$
- $\mu_{x+0.2} = 0.11247$
- $d_{x+1} = 1{,}200$

Calculate ${}_{0.5|1.25}q_x$.

A. Less than 0.130
B. At least 0.130, but less than 0.134
C. At least 0.134, but less than 0.138
D. At least 0.138, but less than 0.142
E. At least 0.142

EXAMINATION CONTINUED ON NEXT PAGE

4.

You are given the following information:

- The lifetime of a die-casting machine is assumed to follow a uniform distribution with $\omega = 15$.
- A manufacturer operates two die-casting machines at its production facility.
- Currently, one machine is age 5 and the other is age 9.
- The lifetimes of the two machines are independent of each other.

Calculate the probability that the second failure will occur during the third year.

A. Less than 0.075
B. At least 0.075, but less than 0.100
C. At least 0.100, but less than 0.125
D. At least 0.125, but less than 0.150
E. At least 0.150

EXAMINATION CONTINUED ON NEXT PAGE

5.

You are given the following information:

- The future lifetimes of an individual, aged (75), and an individual, aged (80), are independent.
- Mortality follows the Illustrative Life Table, except that q_{82} is twice that given by the Illustrative Life Table.

Calculate the three-year temporary curtate expectation of the joint life status $e_{75:80:\overline{3}|}$

A. Less than 2.25
B. At least 2.25, but less than 2.35
C. At least 2.35, but less than 2.45
D. At least 2.45, but less than 2.55
E. At least 2.55

EXAMINATION CONTINUED ON NEXT PAGE

6.

Students entering a 3-year program are subject to two decrements – Academic Failure and Withdrawal.

Year	Number of Academic Failures	Number of Withdrawals	Number who Survive to End of Year
1			68
2		16	
3			28

- Three times as many students fail in year 2 than withdraw in year 3.
- Five times as many students survive year 2 as fail in year 3.

Calculate the probability that a student who survives 2 years does not survive to graduation.

A. Less than 27%
B. At least 27%, but less than 29%
C. At least 29%, but less than 31%
D. At least 31%, but less than 33%
E. At least 33%

EXAMINATION CONTINUED ON NEXT PAGE

7.

For a continuous triple-decrement model, you are given the following information:

- $\mu_{x+t}^{(1)} = \mu$
- $\mu_{x+t}^{(2)} = 2\mu$
- $\mu_{x+t}^{(3)} = 0.06$
- ${}_t p_x^{(\tau)} = e^{-0.21t}$

Calculate ${}_5 q_x^{(1)}$.

A. Less than 0.150
B. At least 0.150, but less than 0.152
C. At least 0.152, but less than 0.154
D. At least 0.154, but less than 0.156
E. At least 0.156

EXAMINATION CONTINUED ON NEXT PAGE

8.

Assume workers transition through the labor force independently with the transitions following a homogenous Markov Chain process for three states:

1. Employed full-time
2. Employed part-time
3. Retired

The transition matrix is:

$$Q = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.0 & 0.6 & 0.4 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

- Worker A is currently employed full-time
- Worker B is currently employed part-time

Calculate the probability that at least one of the workers will be employed part-time after two transitions.

A. Less than 0.42
B. At least 0.42, but less than 0.46
C. At least 0.46, but less than 0.50
D. At least 0.50, but less than 0.54
E. At least 0.54

EXAMINATION CONTINUED ON NEXT PAGE

9.

You are given that claim counts follow a non-homogeneous Poisson Process with $\lambda(t) = 30t^2 + t^3$.

Calculate the probability of at least two claims between time 0.2 and 0.3.

A. Less than 1%
B. At least 1%, but less than 2%
C. At least 2%, but less than 3%
D. At least 3%, but less than 4%
E. At least 4%

EXAMINATION CONTINUED ON NEXT PAGE

10.

You are given the following information:

- Cars arrive according to a Poisson Process at a rate of 20 cars per hour.
- 75% of cars are red and 25% of cars are blue.
- 28 red cars and 32 blue cars have arrived after three hours have passed.

Calculate the total expected number of red cars that will have arrived eight hours have passed.

A. Less than 80
B. At least 80, but less than 100
C. At least 100, but less than 120
D. At least 120, but less than 140
E. At least 140

EXAMINATION CONTINUED ON NEXT PAGE

11.

You are given the following information:

- The number of claims received by an insurance company follows a Poisson Process.
- Claims arrive at a rate of 20 claims per day on Mondays.
- Claims arrive at a rate of 10 claims per day on the other 6 days of the week.
- The size of each claim follows a Gamma distribution with $\alpha = 5$ and $\theta = 200$.
- Claim frequency and severity are independent.

Calculate the standard deviation of the total loss amount for a seven day period.

A. Less than 10,000
B. At least 10,000, but less than 11,000
C. At least 11,000, but less than 12,000
D. At least 12,000, but less than 13,000
E. At least 13,000

EXAMINATION CONTINUED ON NEXT PAGE

12.

For a whole life annuity-due of $1,000 on a life aged ($x$), payable annually, you are given the following information:

- $q_x = 0.012$
- $q_{x+1} = 0.015$
- Interest rate $i = 0.06$
- $\ddot{a}_{x+1} = 10.904$

If p_{x+1} is decreased by 0.01, calculate the absolute change in the actuarial present value of this annuity-due.

A. Less than $90
B. At least $90, but less than $95
C. At least $95, but less than $100
D. At least $100, but less than $105
E. At least $105

EXAMINATION CONTINUED ON NEXT PAGE

13.

You are given the following information:

- $\overline{A}_x = 0.375$
- $\overline{A}^{1}_{x:\overline{10}|} = 0.299$
- The force of mortality and force of interest are constant.

Calculate $\overline{a}_x$.

A. Less than 6.5
B. At least 6.5, but less than 7.5
C. At least 7.5, but less than 8.5
D. At least 8.5, but less than 9.5
E. At least 9.5

EXAMINATION CONTINUED ON NEXT PAGE

14.

For a 30-payment whole life insurance of $10,000 on a life aged x, you are given the following information:

- Interest rate $i = 0.06$
- The level annual benefit premium is $150.46 paid at the beginning of the year
- The benefit reserve at the end of year 29 is $5612.04
- $p_{x+29} = 0.9527$

Calculate $10{,}000P_{x+30}$, the level annual benefit premium for a whole life insurance of $10,000 on $(x+30)$.

A. Less than $750
B. At least $750, but less than $800
C. At least $800, but less than $850
D. At least $850, but less than $900
E. At least $900

EXAMINATION CONTINUED ON NEXT PAGE

15.

You are given the following information:

- $\ddot{a}_x = 14.817$
- $a_{x+15} = 11.275$
- $l_x = 1000$
- ${}_{15}d_x = 72$
- $i = 0.06$

Calculate $1000\ {}_{14}V_{x:\overline{15}|}$.

A. Less than 899
B. At least 899, but less than 902
C. At least 902, but less than 905
D. At least 905, but less than 908
E. At least 908

EXAMINATION CONTINUED ON NEXT PAGE

16.

An insurance company uses a homogenous Markov Chain process to model a group of claims.

You are given the following information:

- The four states in the transition matrix are, in order, : Pre-Trial, Litigation, Settlement, and Judgment;
- Transitions occur on January 1 of each year;
- The transition matrix is $\begin{bmatrix} 0.4 & 0.1 & 0.5 & 0 \\ 0 & 0.6 & 0.3 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$;
- Today is December 31, 2013;
- 1,000 claims are in Pre-Trial and 500 are in Litigation;
- $i = 10\%$
- The company will encourage its attorneys to settle claims more quickly by paying the following bonuses:
  - 5,000 upon transition from Pre-Trial to Settlement;
  - 2,500 upon transition from Litigation to Settlement.

Calculate the actuarial present value of the bonuses paid by January 2, 2015 (i.e., after two transitions).

A. Less than 3,000,000
B. At least 3,000,000 but less than 3,500,000
C. At least 3,500,000 but less than 4,000,000
D. At least 4,000,000 but less than 4,500,000
E. At least 4,500,000

17.

You are given the following:

I. $\frac{\sum X_i}{n}$ as an estimator of the underlying mean of $(x_1,\ldots,x_n)$

II. $\sum\frac{(X_i-\overline{X})^2}{n}$ as an estimator of the variance of the Normal Distribution

III. $\frac{\sum X_i}{n}+\frac{1}{n}$ as an estimator of the underlying mean $(x_1,\ldots,x_n)$

IV. x_1 as an estimator of the underlying mean $(x_1,\ldots,x_n)$

Determine which of the above estimators is NOT consistent.

A. I
B. II
C. III
D. IV
E. All of the estimators are consistent

EXAMINATION CONTINUED ON NEXT PAGE

18.

You are given the following information:

- The number of trials before success follows a geometric distribution.
- A random sample of size 10 from that process is: 1, 0, 5, 6, 4, 8, 2, 3, 7, 4.

Calculate the maximum likelihood estimate of the variance for the underlying geometric distribution.

A. Less than 10
B. At least 10, but less than 12
C. At least 12, but less than 14
D. At least 14, but less than 16
E. At least 16

EXAMINATION CONTINUED ON NEXT PAGE

19.

$X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables with the following distribution:

$$f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad , 0 < \mathrm{x} < \infty$$

Find the maximum likelihood estimator of $\Pr(X \leq 2)$.

A. $e^{-\frac{\theta}{2}}$

B. $1 - e^{-\frac{2}{\theta}}$

C. $1 - e^{\frac{\overline{X}}{2}}$

D. $e^{-\frac{\overline{X}}{2}}$

E. $1 - e^{-\frac{2}{\overline{X}}}$

EXAMINATION CONTINUED ON NEXT PAGE

20.

You are given the following:

- A random sample of size n = 8 is drawn from the uniform p.d.f.

$$f_X(x;\theta) = \frac{1}{\theta} \quad \text{, where } 0 \le x \le \theta$$

- H_0: $\theta = 2.0$ and H_1: $\theta < 2.0$ at the level of significance $\alpha = 0.10$.
- The statistic used to determine the critical region is Y_8, the largest order statistic.
- The probability density of Y_k, the k^{th} order statistic for a sample of size n is:

$$g_k(y_k) = \frac{n!}{(k-1)!\,(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y)$$

Determine the probability of committing a Type II error when $\theta = 1.6$.

A. Less than 0.15
B. At least 0.15, but less than 0.30
C. At least 0.30, but less than 0.45
D. At least 0.45, but less than 0.60
E. At least 0.60

EXAMINATION CONTINUED ON NEXT PAGE

21.

You are given the following information:

- A random sample consisting of 10 observations is taken from a Normal distribution with unknown variance σ_x^2.
- An independent random sample consisting of 8 observations is taken from another Normal distribution with unknown variance σ_y^2.
- The sample variance of the first random sample is S_x^2.
- The sample variance of the second random sample is 40.

The following hypothesis test is performed:

- H_0: $\sigma_x^2 = \sigma_y^2$
- H_1: $\sigma_x^2 > \sigma_y^2$

The p-value of this test is 1.0%.

Determine the value of S_x^2.

A. Less than 60
B. At least 60, but less than 120
C. At least 120, but less than 180
D. At least 180, but less than 240
E. At least 240

EXAMINATION CONTINUED ON NEXT PAGE

22.

You are given the following:

- $X_1, \ldots, X_{100}$ is a random sample of size 100 from an Normal distribution, with known variance equal to 25.
- $H_0 : u = k$
- $H_1 : u = 1.2 \times k$ (1.2 times k)
- Given the critical region selected, the probability of a Type I error is 0.0013
- Given the critical region selected, the probability of a Type II error is 0.9772

Calculate k.

A. Less than 1.50
B. At least 1.50, but less than 1.75
C. At least 1.75, but less than 2.00
D. At least 2.00, but less than 2.25
E. At least 2.25

EXAMINATION CONTINUED ON NEXT PAGE

23.

You are given the following information:

- Accidents follow a Poisson Process with a rate of 0.05 accidents per day.
- Two people have accidents independently of one another.

Calculate the expected number of days until both people have had at least one accident.

A. Less than 29.5
B. At least 29.5, but less than 30.5
C. At least 30.5, but less than 31.5
D. At least 31.5, but less than 32.5
E. At least 32.5

EXAMINATION CONTINUED ON NEXT PAGE

24.

You are given the following information:

- X is random variable that has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.
- Sample size = 10

Calculate Variance (X_{Min}), the variance of the first order statistic.

A. Less than 20
B. At least 20, but less than 25
C. At least 25, but less than 30
D. At least 30, but less than 35
E. At least 35

EXAMINATION CONTINUED ON NEXT PAGE

25.

You are given the following information:

- A random variable, X, follows the normal distribution with an unknown mean, μ, and an unknown variance, σ^2.
- A random sample of size 17 is drawn and the unbiased sample variance is 2.75
- $H_0 : \sigma_0^2 = 8$
- $H_1 : \sigma^2 < \sigma_0^2$

Calculate the smallest observed significance level at which one would reject the null hypothesis.

A. Less than 0.5%
B. At least 0.5%, but less than 1.0%
C. At least 1.0%, but less than 2.5%
D. At least 2.5%, but less than 5.0%
E. At least 5.0%

END OF EXAMINATION

Fall 2013 Exam 3L Answer Key

Question	Answer
1	D
2	B
3	C
4	B
5	A
6	C
7	D
8	D
9	B
10	C or D
11	A
12	B
13	A
14	C
15	B
16	D
17	D
18	E
19	E
20	C
21	E
22	E
23	B
24	D
25	B