Exam S

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam S

Statistics and Probabilistic Models

October 30, 2015 **4 HOURS**

INSTRUCTIONS TO CANDIDATES

1. This 90 point examination consists of 45 multiple choice questions worth 2 points.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2015 and that the exam name is S.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam S" included in your exam packet.

   - Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table.

   - Candidates should employ a non-parametric test unless otherwise specified in the problem, or when there is a standard distribution that is logically or commonly associated with the random variable in question. Examples of problems with a logical or commonly associated distribution would include exponential wait times for Poisson processes and applications of the Central Limit Theorem.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2015.

END OF INSTRUCTIONS

1.

You are given the following information:

- Buses depart from the bus stop at a Poisson rate of 5 per hour.
- Today, Joe arrives at the bus stop just as a bus is leaving and will have to wait for the next bus to depart.
- Yesterday, Joe arrived at the bus stop 5 minutes after the prior bus departed and had to wait for the subsequent bus to depart.

Calculate how much longer Joe's expected wait time is today compared to yesterday.

A. Less than 1 minute
B. At least 1 minute, but less than 2 minutes
C. At least 2 minutes, but less than 3 minutes
D. At least 3 minutes, but less than 4 minutes
E. At least 4 minutes

Exam Continued on Next Page

2.

You are given the following:

- A Poisson process with $\lambda = 1.2$ has its 4^{th} event occur at time 10.
- This process began operating at time 0.

Calculate the expected value of the time at which the 3^{rd} event occurred.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

Exam Continued on Next Page

3.

You are given the following information:

- Lucy finds coins at a Poisson rate of 1 coin per 10 minutes.
- The denominations are randomly distributed as follows:
    - 65% of the coins are worth 1 each;
    - 20% of the coins are worth 5 each;
    - 15% of the coins are worth 10 each.

Calculate the probability that in the first 30 minutes she finds at least 1 coin worth 10 each and in the first hour finds at least 2 coins worth 10 each.

A. Less than 0.165
B. At least 0.165, but less than 0.175
C. At least 0.175, but less than 0.185
D. At least 0.185, but less than 0.195
E. At least 0.195

Exam Continued on Next Page

4.

You are given the following information on a workers' compensation policy:

- Claims occur according to a Poisson process with rate $\lambda = 10$ per week.
- Claims are classified independently into the following claim types:

Claim Type	Probability
1	0.2
2	0.3
3	0.5

- For each claim a random amount X_i, $i = 1, 2, 3$, is added to a fund.
- The fund starts with an amount of zero at time zero.
- The random variables X_i follow the Exponential distribution with the following means:

Claim Type	Mean
1	500
2	300
3	200

Calculate the 95th percentile of the amount of the fund after 13 weeks, using the Normal approximation.

A. Less than 43,000
B. At least 43,000, but less than 45,000
C. At least 45,000, but less than 47,000
D. At least 47,000, but less than 49,000
E. At least 49,000

Exam Continued on Next Page

5.

You are given the following information:

- The amount of damage involved in a home theft loss is an Exponential random variable with mean 2,000.
- The insurance company only pays the amount exceeding the deductible amount of 500.
- The insurance company is considering changing the deductible to 1,000.

Calculate the absolute value of the change in the expected value of the amount the insurance company pays per theft loss by changing the deductible from 500 to 1,000.

A. Less than 330
B. At least 330, but less than 350
C. At least 350, but less than 370
D. At least 370, but less than 390
E. At least 390

Exam Continued on Next Page

6.

You are given the following information:

- An engine system consists of a series of 4 independent pumps, each of which is in operation at time zero.
- The engine system will fail when any one of the 4 pumps stops running.
- The amount of time a pump runs (in hours) is distributed uniformly over (0, 100).

Calculate the expected run time of the engine system in hours.

A. Less than 17.5
B. At least 17.5, but less than 18.5
C. At least 18.5, but less than 19.5
D. At least 19.5, but less than 20.5
E. At least 20.5

Exam Continued on Next Page

7.

You are given the following information about a series system:

- There are three components: Component 1, Component 2, and Component 3.
- The lifetimes of Component 1 and Component 2 are each distributed U(0,6).
- The lifetime of Component 3 is distributed U(0,12).
- The component lifetimes are independent.
- U(a,b) is used to define a uniform distribution between a and b.

Calculate the expected system lifetime.

A. Less than 2.00
B. At least 2.00, but less than 2.25
C. At least 2.25, but less than 2.50
D. At least 2.50, but less than 2.75
E. At least 2.75

Exam Continued on Next Page

8.

Ben and Allison each decide to wager 1 unit against the other person on flips of a fair coin, until one of them runs out of money. At the start of the contest, Ben has 20 units and Allison has 55 units.

Calculate the variance of Ben's final wealth.

A. Less than 1
B. At least 1, but less than 10
C. At least 10, but less than 100
D. At least 100, but less than 1,000
E. At least 1,000

Exam Continued on Next Page

9.

You are given the following Markov chain transition matrix:

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.4 & 0.4 & 0.2 & 0.0 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.4 & 0.2 & 0.1 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Determine the number of transient states in this Markov chain.

A. 0
B. 1
C. 2
D. 3
E. 4

Exam Continued on Next Page

10.

An insurance company classifies its customers into three categories:

1. Good Risk
2. Acceptable Risk
3. Bad Risk

Customers independently transition between categories at the end of each year according to a Markov process with the following transition matrix:

$$P = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.4 & 0.4 & 0.2 \\ 0.0 & 0.5 & 0.5 \end{bmatrix}$$

Calculate the stationary probability of a customer being classified as a Good Risk.

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

Exam Continued on Next Page

11.

You are given the following information

- For a group of automobile insurance policies, there are three premium levels: P, 1.1P, and 1.2P. Each premium level corresponds to a State 0, 1 or 2 that is determined by claim filed by the policyholder with movement to the next state taking place at the beginning of the policy term.
- The transition of policyholders between these levels is modeled by a Markov Chain as follows:

	Current Annual	Next State if		
Current State	Premium	No Claims	One Claim	Two Claims
0	P	0	1	2
1	1.1P	0	2	2
2	1.2P	1	2	2

- For example, if a policyholder is in State 0, the premium the policyholder would pay is P. If that policyholder has a claim, next year the policyholder would move to State 1 and pay 1.1P for the premium.
- For each policyholder, the probability of number of claims is:

Number of Claims	Probability
No Claim	0.5
One Claim	0.4
Two Claims	0.1

Calculate the long run mean value of the annual premium.

A. Less than 1.10P
B. At least 1.10P, but less than 1.12P
C. At least 1.12P, but less than 1.14P
D. At least 1.14P, but less than1.16P
E. At least 1.16P

Exam Continued on Next Page

12.

You are given the following information for a policyholder age 65:

- A 3-year term insurance policy on (65) provides for a death benefit of 1,000 payable at the end of the year of death.
- This policy is purchased by a single premium, P, at time 0.
- If (65) lives to age 68, the single premium is returned without interest.
- Mortality rates are:

x	q_x
65	0.15
66	0.20
67	0.25

- $i = 0.10$.

Calculate P using the equivalence principle.

A. Less than 640
B. At least 640, but less than 650
C. At least 650, but less than 660
D. At least 660, but less than 670
E. At least 670

Exam Continued on Next Page

13.

You are given the following information:

- A life insurance company issues a special 3-year discrete insurance to a life (x).
- If the policyholder dies, at the end of the year of death, there is a random drawing. With probability 0.2, the death benefit is 50,000. With probability 0.8, the death benefit is 0.
- At the beginning of each year the policy is in effect while (x) is alive, there is a random drawing. With probability 0.8, the premium π is paid. With probability 0.2, no premium is paid.
- The random drawings are independent.
- The probability of surviving an additional k years is
  ${}_kp_x = 0.9^k, \textit{ for } k = 0,1,2,...$
- $i = 0.06$.

Calculate π using the equivalence principle.

A. Less than 1,100
B. At least 1,100, but less than 1,200
C. At least 1,200, but less than 1,300
D. At least 1,300, but less than 1,400
E. At least 1,400

14.

You are given the following information:

- You draw a random sample of five observations from a distribution with density function:

$$f(y|\alpha,\theta) = \left(\frac{1}{\Gamma(\alpha)\theta^{\alpha}}\right) y^{\alpha-1} e^{-\frac{y}{\theta}}, \qquad y > 0$$

- The sampled values are 0.5, 2.0, 10.0, 1.5, 7.0
- $\alpha = 2$.

Calculate the maximum likelihood estimate of θ.

A. Less than 2.00
B. At least 2.00, but less than 2.25
C. At least 2.25, but less than 2.50
D. At least 2.50, but less than 2.75
E. At least 2.75

Exam Continued on Next Page

15.

You are given a random sample of five observations from a Poisson distribution with an unknown parameter, λ:

$$0, 0, 1, 4, 5$$

Calculate the maximum likelihood estimate of $\frac{1}{\lambda}$.

A. Less than 0.4
B. At least 0.4, but less than 0.8
C. At least 0.8, but less than 1.2
D. At least 1.2, but less than 1.6
E. At least 1.6

Exam Continued on Next Page

16.

A sample of independent and identically distributed random variables, $(X_1, X_2, \ldots, X_n)$, is drawn from a distribution with the following probability density:

$$\Pr(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad \lambda > 0, \qquad x \in \{0,1,2,\ldots\}$$

Determine which of the following is a sufficient statistic for λ.

a. X_1
b. $\min(X_1, X_2, \ldots, X_n)$
c. $\sum_{i=1}^{n} X_i$
d. $\sum_{i=1}^{n} X_i^2$
e. $\prod_{i=1}^{n}(X_i!)$

Exam Continued on Next Page

17.

Let $(X_1, X_2, \dots, X_n)$ be a sample of independent and identically distributed random variables from an inverse exponential distribution with parameter θ.

Calculate the Rao-Cramer lower bound on the variance of the estimator of the mode.

A. $\dfrac{\theta^4}{8n}$

B. $\dfrac{\theta^4}{4n}$

C. $\dfrac{\theta^2}{8n}$

D. $\dfrac{\theta^2}{4n}$

E. $\dfrac{\theta^2}{n}$

Exam Continued on Next Page

18.

You are given the following information:

- The time to failure of a machine is uniformly distributed on $[0, \omega]$.
- The time to failure of each machine is independent.
- An inspection of 20 randomly selected machines revealed the following:
  - The average time to failure of the sample was 10.5 years.
  - The minimum time to failure of the sample was 2.3 years.
  - The maximum time to failure of the sample was 15.6 years.
  - The median time to failure of the sample was 12 years.

Calculate the maximum likelihood estimate for ω based on this sample.

A. Less than 14 years
B. At least 14 years, but less than 16 years
C. At least 16 years, but less than 18 years
D. At least 18 years, but less than 20 years
E. At least 20 years

Exam Continued on Next Page

19.

You are testing whether actuarial students average more than 6 hours of sleep per night. You assume that they get at least 6 hours of sleep ($H_o: \mu \geq 6$) and take a random sample to test if they actually get less than six hours ($H_a: \mu < 6$).

- The chosen significance level (α) is 0.10.
- The p-value of this test is 0.07 using the random sample.
- After the test, you are given that the distribution underlying the number of hours that actuarial students sleep and find out the true underlying average is average 6.2 hours of sleep per night.
- In this case, you were asked to accept or reject the null hypothesis before the true underlying distribution is known using the results of the random sample and the chosen significance level.

Determine which of the following statements is true once you know the true distribution:

A. You have correctly rejected the null hypothesis.
B. You have correctly failed to reject the null hypothesis.
C. You have correctly accepted the alternative hypothesis.
D. You have committed a Type I error.
E. You committed a Type II error.

Exam Continued on Next Page

20.

X is a single observation from the probability density function:

$f(x) = 2\theta x + 1 - \theta$, for $0 < x < 1$.

You are testing the hypothesis:

$H_0: \theta = 0$ vs $H_1: \theta = 1$.

For the most powerful test of significance of size α, determine the critical region for which you would reject the null hypothesis.

A. $x < \alpha$
B. $x > \alpha$
C. $x < 1 - \alpha$
D. $x > 1 - \alpha$
E. $x < \alpha/2$

Exam Continued on Next Page

21.

For Jones's political poll the question is what is the probability, p, that a voter would vote for Jones:

- 15 voters were sampled to test:
  $H_0: p = 0.5$ vs. $H_1: p < 0.5$.
- The test statistic Y is the number of sampled voters favoring Jones.
- $RR = \{y \leq 3\}$ is selected as the rejection region.
- Assume a binomial distribution for the probability that a voter will favor Jones.

Calculate the probability of a Type I error, α.

A. Less than 0.010
B. At least 0.010, but less than 0.015
C. At least 0.015, but less than 0.020
D. At least 0.020, but less than 0.025
E. At least 0.025

Exam Continued on Next Page

22.

A company is evaluating its customer satisfaction through an anonymous survey of 100 randomly selected customers.

Given the following information:

- Surveyed customers score the company continuously between 1 (least satisfied) to 5 (most satisfied).
- The company is considered "best in class" if average customer satisfaction is 4.3 or higher.
- The null hypothesis is that the company is not "best in class".
- The standard deviation of the sample observations is 1.3.
- The selected significance level is 5%.

Use the normal approximation to determine the lowest sample mean, μ, such that the null hypothesis is rejected.

A. Less than 4.4
B. At least 4.4, but less than 4.5
C. At least 4.5, but less than 4.6
D. At least 4.6, but less than 4.7
E. At least 4.7

Exam Continued on Next Page

23.

We are testing the hypothesis that policyholders have no preference concerning the choice of one of 3 big insurers. The table below shows a random sample of the number of policyholders for the 3 insurers, for a total of $n = 120$ insureds:

	Insurer 1	Insurer 2	Insurer 3
Observed Insureds	46	36	38

Calculate the χ^2 test statistic.

A. Less than 1.00
B. At least 1.00, but less than 1.50
C. At least 1.50, but less than 2.00
D. At least 2.00, but less than 2.50
E. At least 2.50

Exam Continued on Next Page

24.

The following data represent a tutor's ranking of 10 psychology students as to their suitability for their career and their knowledge of psychology:

Student	Career	Psychology
1	4	5
2	10	8
3	3	6
4	1	2
5	9	10
6	2	3
7	6	9
8	7	4
9	8	7
10	5	1

Calculate the Kendall's Tau correlation statistic for Career and Psychology.

A. Less than 0.35
B. At least 0.35, but less than 0.55
C. At least 0.55, but less than 0.75
D. At least 0.75, but less than 0.95
E. At least 0.95

Exam Continued on Next Page

25.

A statistician wishes to know whether longevity is related to heavy smoking. The table below shows the number of cigarettes per day in the past 5 years paired with the number of years lived for 9 subjects:

Subject	Cigarettes	Years Lived
1	25	63
2	35	68
3	10	72
4	40	62
5	85	65
6	75	46
7	60	51
8	45	60
9	50	55

Calculate the Spearman rho correlation statistic for Cigarettes and Years Lived.

A. Less than -0.70
B. At least -0.70, but less than -0.65
C. At least -0.65, but less than -0.60
D. At least -0.60, but less than -0.55
E. At least -0.55

Exam Continued on Next Page

26.

Losses are separated into two types of claims: Type W and Type X.

- The values of 18 randomly selected claims are provided in the table below:

Type W		Type X	
Loss Amount	Rank	Loss Amount	Rank
4,500	9	3,450	1
4,600	10	3,475	2
4,700	11	3,625	3
4,800	12	3,650	4
4,900	13	3,675	5
7,000	14	4,100	6
8,000	15	4,200	7
10,000	16	4,300	8
		11,000	17
		15,000	18

- H_0: $Median_{Type\,W} = Median_{Type\,X}$
- H_A: $Median_{Type\,W} \neq Median_{Type\,X}$

Calculate the minimum significance level, α, for which H_0 can be rejected using the Mann-Whitney Wilcoxon Test.

A. Less than 0.010
B. At least 0.010, but less than 0.025
C. At least 0.025, but less than 0.050
D. At least 0.050, but less than 0.100
E. At least 0.100

Exam Continued on Next Page

27.

You are asked to test the hypothesis that the median claim severity is different in year 1 than it is in year 2. Claim severities in years 1 and 2 for 5 randomly selected policyholders are in the table below:

Policyholder	Year 1 Severity	Year 2 Severity
1	250	150
2	350	300
3	1,000	100
4	100	85
5	35	90

Calculate the p-value for this hypothesis using the Sign Test.

A. Less than 0.20
B. At least 0.20, but less than 0.25
C. At least 0.25, but less than 0.30
D. At least 0.30, but less than 0.35
E. At least 0.35

Exam Continued on Next Page

28.

A manufacturer of gunpowder has developed a new powder, which was tested in 8 shells.

- Let $y_1, \ldots, y_8$ denote the resulting muzzle velocities.
- $\sum y = 23{,}672$.
- Muzzle velocities are normally distributed with unknown mean μ and variance of 225.
- μ follows a normal distribution with mean of 2,800 and variance of 2,500.

Calculate the weight assigned to the Maximum Likelihood Estimate, $\bar{Y}$, in the Bayes estimator.

A. Less than 0.20
B. At least 0.20, but less than 0.40
C. At least 0.40, but less than 0.60
D. At least 0.60, but less than 0.80
E. At least 0.80

Exam Continued on Next Page

29.

You are given the following information for an insurance policy:

- Daily claim frequencies follow a Poisson process with parameter λ.
- The prior distribution of λ follows a gamma distribution with parameters α=4 and β=5.
- This policy had 3 claims per day in the first n days.
- The posterior mean of daily claims for this policy after five days is $\frac{10}{3}$.

Calculate n.

A. Less than 5.5
B. At least 5.5, but less than 7.5
C. At least 7.5, but less than 9.5
D. At least 9.5, but less than 11.5
E. At least 11.5

Exam Continued on Next Page

30.

You are given the following information about the random variable Y:

- Y is binomial with parameters m and $q = \theta$.
- The prior distribution of θ is beta with parameters $a = 9$ and $b = 5$.
- A sample of size 40 is drawn.
- The number of successes in the sample was 15.

Calculate the mean of the posterior distribution of θ.

A. Less than 0.3
B. At least 0.3, but less than 0.4
C. At least 0.4, but less than 0.5
D. At least 0.5, but less than 0.6
E. At least 0.6

Exam Continued on Next Page

31.

Given the following information:

- Y is a random variable in the exponential family

$$f(y) = c(y, \phi) * \exp\left[\frac{y\theta - a(\theta)}{\phi}\right]$$

- $a(\theta) = -\sqrt{-2\theta}$
- $\theta = -0.3$
- $\phi = 1.6$

Calculate $E(Y)$.

A. Less than -1
B. At least -1, but less than 0
C. At least 0, but less than 1
D. At least 1, but less than 2
E. At least 2

Exam Continued on Next Page

32.

A GLM is used to model claim size. You are given the following information about the model:

- Claim size follows a Gamma distribution.
- Log is the selected link function.
- Scale parameter is estimated to be 2.
- Model Output:

Variable	$\hat{\beta}$
(Intercept)	2.32
Location - Urban	0.00
Location - Rural	-0.64
Gender - Female	0.00
Gender - Male	0.76

Calculate the variance of the predicted claim size for a rural male.

A. Less than 25
B. At least 25, but less than 100
C. At least 100, but less than 175
D. At least 175, but less than 250
E. At least 250

Exam Continued on Next Page

33.

You are given the following output from a GLM to estimate the probability of a claim:

- Distribution selected is Binomial.
- Link selected is Logit.

Parameter	β
Intercept	-1.485
Vehicle Body	
Coupe	-0.881
Roadster	-1.047
Sedan	-1.175
Station wagon	-1.083
Truck	-1.118
Utility	-1.330
Driver's Gender	
Male	-0.025
Area	
B	0.094
C	0.037
D	-0.101

Calculate the estimated probability of a claim for:
- Driver Gender: Female
- Vehicle Body: Sedan
- Area: D

A. Less than 0.045
B. At least 0.045, but less than 0.050
C. At least 0.050, but less than 0.055
D. At least 0.055, but less than 0.060
E. At least 0.060

Exam Continued on Next Page

34.

You are given the following information for a model of vehicle claim counts by policy:

- The response distribution is Poisson and the model has a log link function.
- The model uses two categorical explanatory variables: Number of Youthful Drivers and Number of Adult Drivers.
- The parameters of the model are given:

Parameter	**Degrees of Freedom**	$\hat{\beta}$
Intercept	1	-2.663
Number of Youthful Drivers		
0		
1	1	0.132
Number of Adult Drivers		
1		
2	1	-0.031

Calculate the predicted claim count for a policy with one adult driver and one youthful driver.

A. Less than 0.072
B. At least 0.072, but less than 0.074
C. At least 0.074, but less than 0.076
D. At least 0.076, but less than 0.078
E. At least 0.078

Exam Continued on Next Page

35.

You are given a GLM of liability claim size with the following potential explanatory variables only:

- Vehicle price, which is a continuous variable modeled with a third order polynomial
- Average driver age, which is a continuous variable modeled with a first order polynomial
- Number of drivers, which is a categorical variable with four levels
- Gender, which is a categorical variable with two levels
- There is only one interaction in the model, which is between gender and average driver age.

Determine the maximum number of parameters in this model.

A. Less than 9
B. 9
C. 10
D. 11
E. At least 12

Exam Continued on Next Page

36.

You are given the following information for two potential logistic models used to predict the occurrence of a claim:

- Model 1: (AIC = 262.68)

Parameter	$\hat{\beta}$
(Intercept)	-3.264
Vehicle Value ($000s)	0.212
Gender-Female	0.000
Gender-Male	0.727

- Model 2: (AIC = 263.39)

Parameter	$\hat{\beta}$
(Intercept)	-2.894
Gender-Female	0.000
Gender-Male	0.727

- AIC is used to select the most appropriate model.

Calculate the probability of a claim for a male policyholder with a vehicle valued $12,000 by using the selected model.

A. Less than 0.15
B. At least 0.15, but less than 0.30
C. At least 0.30, but less than 0.45
D. At least 0.45, but less than 0.60
E. At least 0.60

Exam Continued on Next Page

37.

You are given the following table for model selection:

Model	Deviance $\Delta\ (= -2\ell)$	Number of Parameters (p)	AIC
Intercept + Age	A	5	435
Intercept + Vehicle Body	392	11	414
Intercept + Age + Vehicle Value	392	X	446
Intercept + Age + Vehicle Body + Vehicle Value	B	Y	501

Calculate Y.

A. Less than 36
B. 36
C. 37
D. 38
E. At least 39

Exam Continued on Next Page

38.

You are testing the addition of a new categorical variable into an existing GLM. You are given the following information:

- The change in model deviance after adding the new variable is -53.
- The change in AIC after adding the new variable is -47.
- The change in BIC after adding the new variable is -32.
- Prior to adding the new variable, the model had 15 parameters.

Calculate the number of observations in the model.

A. Less than 1,000
B. At least 1,000, but less than 1,100
C. At least 1,100, but less than 1,200
D. At least 1,200, but less than 1,300
E. At least 1,300

Exam Continued on Next Page

39.

You are given the following model output from five candidate models:

Model #	Included Parameters	R^2	AIC	BIC
1	Gender	0.0162	7532	7899
2	Age	0.0230	7524	7865
3	Age + Gender	0.0390	7508	7885
4	Age + Gender + Income	0.0755	7469	7645
5	Age + Gender + Income + Uninsured	0.0762	7471	7659

Determine which statement is correct based on the selection criteria above.

A. Model 1 is the best according to R^2.
B. Model 1 is the best according to AIC.
C. Model 4 is the best according to R^2.
D. Model 4 is the best according to AIC.
E. Model 5 is the best according to BIC.

Exam Continued on Next Page

40.

You are given the following information related to the following linear model:

$$y \sim x_1 + x_2 + x_3.$$

- $x_1 \sim x_2 + x_3$

Source	Degrees of Freedom	Sum of Squares
Regression	2	3.73
Error	503	3.04

- $x_2 \sim x_1 + x_3$

Source	Degrees of Freedom	Sum of Squares
Regression	2	24.74
Error	503	226.34

- $x_3 \sim x_1 + x_2$

Source	Degrees of Freedom	Sum of Squares
Regression	2	214,258
Error	503	85,883

Calculate the variable inflation factor for the variable which exhibits the greatest collinearity in the original model.

A. Less than 1.0
B. At least 1.0, but less than 2.0
C. At least 2.0, but less than 3.0
D. At least 3.0, but less than 4.0
E. At least 4.0

Exam Continued on Next Page

41.

You are building a model of claim counts. To evaluate the risk of overdispersion, you find the following:

- The log-likelihood of the Poisson regression is $l_P = 350$.
- The log-likelihood of the Negative Binomial regression is l_{NB}.
- At a significance level of 0.5%, you reject the hypothesis that a Poisson response is more appropriate than a Negative Binomial response.

Calculate the smallest possible value of l_{NB}.

A. Less than 350
B. At least 350, but less than 352
C. At least 352, but less than 354
D. At least 354, but less than 356
E. At least 356

Exam Continued on Next Page

42.

An AR(1) model was fit to the following time series data through time t=7.

- The mean was subtracted from the data before the parameter was estimated.
- The estimated parameter for the model is -0.79.

time(t)	x_t
1	8.70
2	7.00
3	8.60
4	7.40
5	8.30
6	7.60
7	8.50
mean	8.01

Calculate the forecast for the observation at t=9, $\hat{x}_9$.

A. Less than 8.25
B. At least 8.25, but less than 8.35
C. At least 8.35, but less than 8.45
D. At least 8.45, but less than 8.55
E. At least 8.55

Exam Continued on Next Page

43.
The three models AR(2), MA(1) and ARMA(2,1) are fitted to the following time series:

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2010	112	104	100	96
2011	101	101	105	94
2012	106	106	108	110

The results using R are as follows:

AR(2)

	ar1	ar2	intercept
	0.2618	0.0868	104.227
Std Err	0.3289	0.3326	2.3972

$\sigma^2 = 25.02$ log likelihood = -36.4

MA(1)

	ma1	intercept
	0.1899	103.798
Std Err	0.2574	1.7373

$\sigma^2 = 25.66$ log likelihood = -36.51

ARMA(2,1)

	ar1	ar2	ma1	intercept
	-0.5547	0.4437	0.9779	104.4542
Std Err	0.3329	0.3327	0.235	2.4584

$\sigma^2 = 21.31$ log likelihood = -36.02

The models are ranked using the Akaike Information Criterion (AIC).

Determine the order from best to worst model.

A. AR(2), MA(1), ARMA(2,1)

B. AR(2), ARMA(2,1), MA(1)

C. MA(1), AR(2), ARMA(2,1)

D. MA(1), ARMA(2,1), AR(2)

E. ARMA(2,1), AR(2), MA(1)

Exam Continued on Next Page

44.

The two time series $\{x_t\}$ and $\{y_t\}$ are stationary in the mean and the variance. The following 8 sample values are available.

t	x_t	y_t
1	10	8
2	12	9
3	11	11
4	16	12
5	19	15
6	22	16
7	25	20
8	29	21

Calculate the sample cross covariance function for lag 4.

A. Less than -9
B. At least -9, but less than -8
C. At least -8, but less than -7
D. At least -7, but less than -6
E. At least -6

Exam Continued on Next Page

45.

You are given the following AR processes:

Model I: $x_t = \frac{1}{2} \cdot x_{t-1} + w_t$

Model II: $x_t = x_{t-1} + 2 \cdot x_{t-2} + w_t$

Model III: $x_t = \frac{1}{2} \cdot x_{t-1} + \frac{1}{2} \cdot x_{t-2} + w_t$

Determine which processes are stationary.

A. Model I only
B. Model II only
C. Model III only
D. Model I and II only
E. Model I and III only

End of Examination

Fall 2015 Exam S Answer Key

Question_Num	Solution
1	A
2	D
3	C
4	C
5	B
6	D
7	A
8	E
9	B
10	B
11	B
12	C
13	B
14	B
15	B
16	C
17	D
18	B
19	D
20	D
21	C
22	C
23	B
24	B
25	C
26	C
27	E
28	E
29	Invalid Question
30	C
31	D
32	E
33	D
34	E
35	C
36	D
37	C
38	B
39	D
40	D
41	C
42	B

43	C
44	B
45	A