Exam 3L

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

November 1, 2012

Exam 3L

Life Contingencies and Statistics

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

    - Fill in that it is Fall 2012 and that the exam number is 3L.
    - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
    - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
    - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

    - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 15, 2012.

END OF INSTRUCTIONS

Exam 3L Fall 2012

1.

You are given the following information for an individual who is currently age 25:

- $\mu(x) = \dfrac{1}{110-x}$, for $0 \le x < 110$

Calculate the (complete) expected number of years lived between ages 30 and 70 for that individual.

A. Less than 27
B. At least 27, but less than 28
C. At least 28, but less than 29
D. At least 29, but less than 30
E. At least 30

EXAM CONTINUED ON NEXT PAGE

2.

You are given the following information:

- $F(x) = -\frac{x^2}{10{,}000} + \frac{2x}{100}$, for $0 \le x \le 100$

Calculate $\lambda(20)$, the hazard rate, at $x = 20$.

A. Less than 0.02
B. At least 0.02, but less than 0.03
C. At least 0.03, but less than 0.04
D. At least 0.04, but less than 0.05
E. At least 0.05

EXAM CONTINUED ON NEXT PAGE

3.

You are given the following information:

- The force of mortality is known to be constant with $\mu = 0.2$.
- J is the exact value of ${}_{0.75}q_{30.25}$ calculated using the known force of mortality for all ages including fractional ages.
- K is the estimated value of ${}_{0.75}q_{30.25}$ calculated assuming uniform distribution of deaths for fractional ages.

Calculate $J - K$, the difference between the exact value and the estimated value of ${}_{0.75}q_{30.25}$.

A. Less than -0.005
B. At least -0.005, but less than 0.005
C. At least 0.005, but less than 0.015
D. At least 0.015, but less than 0.025
E. At least 0.025

EXAM CONTINUED ON NEXT PAGE

4.

You are given the following information about two electronic devices, A and B:

- Device A with age x and Device B with age y are independent.
- Device A: $q_{x+n} = 0.07$, for n = 0,1,2,...
- Device B: $q_{y+n} = 0.14$, for n = 0,1,2,...
- Let K_x denote the random variable for the interval of failure for the status x; (first interval, K_x =1; second interval, K_x =2; etc.).

Calculate the probability that the difference in the interval of failure for the last-survivor status and the joint-life status for Device A and Device B is two, $\Pr(K_{\overline{xy}} - K_{xy} = 2)$.

A. Less than 0.08
B. At least 0.08, but less than 0.09
C. At least 0.09, but less than 0.10
D. At least 0.10, but less than 0.11
E. At least 0.11

EXAM CONTINUED ON NEXT PAGE

5.

You are given the following information for a last-survivor insurance of 100,000 on two independent lives that are age 80 and age 90:

- The benefit is payable at the end of the year of death.
- The benefit is paid only if the second death occurs during year 5.
- Mortality follows the Illustrative Life Table.
- $i = 0.06$

Calculate the actuarial present value of this insurance.

A. Less than 6,500
B. At least 6,500, but less than 6,600
C. At least 6,600, but less than 6,700
D. At least 6,700, but less than 6,800
E. At least 6,800

EXAM CONTINUED ON NEXT PAGE

6.

You are given the following information:

- In a triple-decrement table, lives are subject to decrements of death (d), withdrawal (w), and disability (s).
- $\ell_{x+1}^{(\tau)} = 9{,}768$
- $d_x^{(d)} = 150$
- $d_x^{(w)} = 450$
- $q_x^{(s)} = 0.0126$
- $q_{x+1}^{(w)} = 0.046$

Calculate ${}_2q_x^{(w)}$, the probability of decrement by withdrawal within two years for an individual age x.

A. Less than 0.085
B. At least 0.085, but less than 0.090
C. At least 0.090, but less than 0.095
D. At least 0.095, but less than 0.100
E. At least 0.100

EXAM CONTINUED ON NEXT PAGE

7.

For a triple-decrement model, you are given the following information:

- $\mu^{(1)}(x) = \dfrac{1}{100-x}$, for $0 \le x < 100$
- $\mu^{(2)}(x) = \dfrac{1}{120-x}$, for $0 \le x < 120$
- $\mu^{(3)}(x) = 0.08$

Calculate $p_{50}^{(\tau)}$.

A. Less than 0.87
B. At least 0.87, but less than 0.88
C. At least 0.88, but less than 0.89
D. At least 0.89, but less than 0.90
E. At least 0.90

EXAM CONTINUED ON NEXT PAGE

8

You are given the following information about a homogeneous Markov Chain process:

- There are three states: I, II and III.
- $Q = \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.40 & 0.50 & 0.10 \\ 0.20 & x & y \end{bmatrix}$
- The probability that an entity in state III at t = 0 is in state III at t = 2 is 0.16.

Calculate y.

A. Less than 0.275
B. At least 0.275, but less than 0.300
C. At least 0.300, but less than 0.325
D. At least 0.325, but less than 0.350
E. At least 0.350

EXAM CONTINUED ON NEXT PAGE

9 .

You are given the following information:

- Claims are given by a Poisson Process with claims intensity $\lambda = 8$.
- Frequency and severity of claims are independent.
- Claim severity follows a discrete distribution that is given in the table below

Claim Amount Interval	**Probability**
Less than \$7,000	.40
At least \$7,000 but Less than \$20,000	.50
At least \$20,000	.10

What is the probability that by time 0.6 there will be at least two claims with severity less than \$7,000?

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

EXAM CONTINUED ON NEXT PAGE

10.

You are given the following information about processing time for wildfire claims:

- The time to process a group of claims follows the Gamma distribution.
- The expected time to process 100 claims received at the same time is 50 days.

Calculate the probability that an individual claim will not be processed on the first day it is received.

A. Less than 8%
B. At least 8%, but less than 10%
C. At least 10%, but less than 12%
D. At least 12%, but less than 14%
E. At least 14%

EXAM CONTINUED ON NEXT PAGE

11.

You are given the following information for a Workers Compensation policy:

- Accidents occur under a Poisson process at a rate of 10 per month during the first half of the year, and 15 per month during the second half of the year.
- The payment for each accident follows a Pareto distribution with $\alpha = 5$ and $\theta = 2{,}000$.

Calculate the standard deviation of the total claim payment within a year.

A. Less than 9,985
B. At least 9,985, but less than 9,995
C. At least 9,995, but less than 10,005
D. At least 10,005, but less than 10,015
E. At least 10,015

EXAM CONTINUED ON NEXT PAGE

12.

You are given the following information:

- An insurance company has agreed to make payments to a worker who is age x and was injured at work.
- The payments are 120,000 per year, paid annually, starting immediately and continuing for the remainder of the worker's life.
- After the first 500,000 is paid by the insurance company, the remainder will be paid by a reinsurance company.
- $_t p_x = \begin{cases} 0.6^t, & 0 \le t \le 6.5 \\ 0, & 6.5 < t \end{cases}$
- $i = 0.05$

Calculate the actuarial present value of the payments to be made by the reinsurer.

A. Less than 17,500
B. At least 17,500, but less than 19,000
C. At least 19,000, but less than 20,500
D. At least 20,500, but less than 22,000
E. At least 22,000

EXAM CONTINUED ON NEXT PAGE

13.

You are given the following information about two policyholders who are age x and age y respectively:

- The future lifetimes of (x) and (y) are independent.
- The force of mortality is constant, with $\mu_x = 0.03$ and $\mu_y = 0.08$
- $\delta = 0.05$
- A fully continuous last survivor insurance on (x) and (y) pays a benefit of 100,000.

Calculate the actuarial present value of this insurance benefit.

A. Less than 35,000
B. At least 35,000, but less than 55,000
C. At least 55,000, but less than 75,000
D. At least 75,000, but less than 95,000
E. At least 95,000

EXAM CONTINUED ON NEXT PAGE

14.

You are given the following information for a fully continuous whole life insurance policy sold to someone age x at policy issuance:

- $\overline{P} = 0.13$
- $\overline{P}$ is **not** calculated according to the equivalence principle.
- $\overline{A}_{x:\overline{10|}} = 0.86$
- $\overline{a}_{x:\overline{10|}} = 7.19$
- ${}_{10}E_x = 0.50$

Calculate ${}_{10}\overline{V}_x$, the net benefit reserve at ten years on this fully continuous whole life insurance policy sold to someone age x at policy issuance, using the values given above.

A. Less than 0.25
B. At least 0.25, but less than 0.50
C. At least 0.50, but less than 0.75
D. At least 0.75, but less than 1.00
E. At least 1.00

EXAM CONTINUED ON NEXT PAGE

15.

For a fully discrete two-year term insurance of $1,000 sold to a policyholder age x at issue, you are given the following information:

- $i = 0.05$
- $1000\, P^{1}_{x:\overline{2}|} = 185.825$
- $1000\, {}_1V^{1}_{x:\overline{2}|} = 41.450$
- The benefit premium is calculated according to the equivalence principle.

Calculate the variance of the loss random variable at issue, ${}_0L$.

A. Less than 235,000
B. At least 235,000, but less than 240,000
C. At least 240,000, but less than 245,000
D. At least 245,000, but less than 250,000
E. At least 250,000

EXAM CONTINUED ON NEXT PAGE

16.

You are given the following information about a homogenous Markov Chain with two states:

- The transition probability matrix is $\begin{bmatrix} 0.7 & 0.3 \\ 0.8 & 0.2 \end{bmatrix}$
- Transitions occur at the end of each time period.
- The subject is in state 1 at the beginning of the time period 0.
- There is a cash flow of 10 at the end of the period for each transition from state 1 to state 2 during the next two time periods.
- The interest rate is 5%.

Calculate the variance of the present value of the cash flow over the next two time periods.

A. Less than 23
B. At least 23, but less than 29
C. At least 29, but less than 35
D. At least 35, but less than 41
E. At least 41

17.

A colleague produces a maximum likelihood estimate of $\hat{p} = 0.40$ for a binomial distribution using the sample data below.

Observation	1	2	3
Sample Size	100	50	y
Successes	35	18	41

Determine the value of y.

A. Less than 80
B. At least 80, but less than 82
C. At least 82, but less than 84
D. At least 84, but less than 86
E. At least 86

EXAM CONTINUED ON NEXT PAGE

18.

You are given the following information:

- $E[X^2] = 10$
- $f(x) = \alpha x$ where $0 \le x \le \theta$

Estimate α using the method of moments.

A. Less than 0.15
B. At least 0.15, but less than 0.30
C. At least 0.30, but less than 0.45
D. At least 0.45, but less than 0.60
E. At least 0.60

EXAM CONTINUED ON NEXT PAGE

19.

Two different estimators, μ and γ, are available for estimating the parameter, θ, of a given loss distribution. To test their performance, 100 simulation trials have been conducted for each estimator using $\theta = 1.4$, with the following results:

$$\sum_{i=1}^{100} \mu_i = 150 \qquad \sum_{i=1}^{100} \mu_i^2 = 250 \qquad \sum_{i=1}^{100} \gamma_i = 125 \qquad \sum_{i=1}^{100} \gamma_i^2 = 325$$

Calculate the absolute value of the difference of the mean squared error of estimator μ and the mean squared error of estimator γ.

A. Less than 1.2
B. At least 1.2, but less than 1.4
C. At least 1.4, but less than 1.6
D. At least 1.6, but less than 1.8
E. At least 1.8

EXAM CONTINUED ON NEXT PAGE

20.

You are given the following random sample:

0.15 0.25 0.55 0.60 1.10

The probability density function given below is selected to be fit to the random sample:

$$f(x) = \begin{cases} \frac{1}{2} & \text{for } 0 \le x \le \alpha \\ 1 & \text{for } \alpha < x \le 1 + \frac{\alpha}{2} \end{cases}$$

where $0 \le \alpha \le 2$.

Select the range of the maximum likelihood estimate of α.

A. Less than 0.15
B. At least 0.15, but less than 0.20
C. At least 0.20, but less than 0.25
D. At least 0.25, but less than 0.30
E. At least 0.30

EXAM CONTINUED ON NEXT PAGE

21.

You are given the following information:

- The weight of chips in a "20-ounce bag" is assumed to be $N(\mu, \sigma^2)$.
- H_0: $\mu = 20.2$
- H_1: $\mu > 20.2$
- For a random sample of size $n = 24$,
    - Sample mean is 20.5.
    - Sample standard deviation is 0.8.

Determine the p-value for the test of the null hypothesis.

A. Less than 0.025
B. At least 0.025, but less than 0.050
C. At least 0.050, but less than 0.075
D. At least 0.075, but less than 0.100
E. At least 0.100

EXAM CONTINUED ON NEXT PAGE

22.

You are given the following:

- A random variable X follows a normal distribution with a known variance of 20.
- Sample size = 100.
- H_0: $\mu = 4$
- H_1: $\mu = 5$
- We reject the null hypothesis when $\overline{X} \geq 4.75$.

Calculate the absolute value of the difference in the probabilities of Type I and Type II error.

A. Less than 5%
B. At least 5%, but less than 10%
C. At least 10%, but less than 15%
D. At least 15%, but less than 20%
E. At least 20%

EXAM CONTINUED ON NEXT PAGE

23.

A six-sided die is rolled 120 times with the following distribution of outcomes:

Outcome	Frequency
1	15
2	13
3	28
4	25
5	12
6	27

The following hypothesis test has been set up:

H_0: The die is fair (outcomes are equally likely).
H_1: The die is not fair.

Determine the significance level at which one would reject the null hypothesis given the outcomes in the table above.

A. Less than 0.5%
B. At least 0.5%, but less than 1.0%
C. At least 1.0%, but less than 2.5%
D. At least 2.5%, but less than 5.0%
E. At least 5.0%

EXAM CONTINUED ON NEXT PAGE

24.

Let $Y_1 < Y_2 < \ldots < Y_5$ be the order statistics associated with five independent observations $X_1, X_2, \ldots, X_5$ from a Pareto distribution with parameter $\alpha = 2$ and $\theta = 1$.

Calculate the probability that $Y_4 < 0.4$.

A. Less than 0.15
B. At least 0.15, but less than 0.16
C. At least 0.16, but less than 0.17
D. At least 0.17, but less than 0.18
E. At least 0.18

EXAM CONTINUED ON NEXT PAGE

25.

You are given the following information:

- A random variable X is uniformly distributed on the interval (0,1).
- Five values of X are observed, X_1, X_2, X_3, X_4, X_5, with order statistics
  $Y_1<Y_2<Y_3<Y_4<Y_5$
- The probability density function of the order statistic Y_k, with a sample size n, is
  $\frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k} f(y)$.

Calculate the expected value of Y_4.

A. Less than 0.60
B. At least 0.60, but less than 0.65
C. At least 0.65, but less than 0.70
D. At least 0.70, but less than 0.75
E. At least 0.75

END OF EXAMINATION

Fall 2012 Exam 3L Answer Key

Question	Answer
1	C
2	B
3	B
4	A
5	B
6	B
7	D
8	C
9	C
10	D
11	C
12	E
13	A
14	E
15	B
16	A
17	D
18	A
19	C
20	C
21	B
22	E
23	C
24	D
25	C