Basics of Reinsurance Pricing

Actuarial Study Note

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Introduction

Like primary insurance, reinsurance is a mechanism for spreading risk. A reinsurer takes some portion of the risk assumed by the primary insurer (or other reinsurer) for premium charged. Most of the basic concepts for pricing this assumption of risk are the same as those underlying ratemaking for other types of insurance. This study note will assume a knowledge of basic ratemaking concepts on the part of the reader.

A major difference between reinsurance and primary insurance is that a reinsurance program is generally tailored more closely to the buyer; there is no such thing as the "average" reinsured or the "average" reinsurance price. Each contract must be individually priced to meet the particular needs and risk level of the reinsured. This leads to what might be called the “pricing paradox”:

> If you can precisely price a given contract, the ceding company will not want to buy it.

That is to say, if the historical experience is stable enough to provide data to make a precise expected loss estimate, then the reinsured would be willing to retain that risk. As such, the "basic" pricing tools are usually only a starting point in determining an adequate premium. The actuary proves his or her worth by knowing when the assumptions in these tools are not met and how to supplement the results with additional adjustments and judgment.

For the different types of reinsurance outlined in this study note, the basic pricing tools will be introduced in Section A, and criticisms and advanced topics will be introduced in Section B. Section A will include the methods generally accepted and standard throughout the industry. Section B will include areas which require the actuary's expertise but have not been solved to universal agreement.

This study note will focus on domestic treaty covers. Pricing for facultative covers or international (non-U.S.) treaties will not be addressed explicitly, but may be viewed as variations on the same themes. Differences exist in accounting, loss sensitive features and the amount of judgment needed, but the underlying theory does not change.

Finally, this study note will give numerical examples where needed. The numbers used are meant to illustrate the pricing techniques with realistic amounts, but in no way should be taken as recommendations for actual factors.

1. Proportional Treaties

Section 1A. Basic Tools

A proportional treaty is an agreement between a reinsurer and a ceding company (the reinsured) in which the reinsurer assumes a given percent of losses and premium. The simplest example of a proportional treaty is called "Quota Share". In a quota share treaty, the reinsurer receives a flat percent, say 50%, of the premium for the book of business reinsured. In exchange, the reinsurer pays 50% of losses, including allocated loss adjustment expenses, on the book. The reinsurer also pays the ceding company a ceding commission which is designed to reflect the differences in underwriting expenses incurred.

Another, somewhat more complicated, proportional treaty is known as "Surplus Share"; these are common on property business. A surplus share treaty allows the reinsured to limit its exposure on any one risk to a given amount (the "retained line"). The reinsurer assumes a part of the risk in proportion to the amount that the insured value exceeds the retained line, up to a given limit (expressed as a multiple of the retained line, or "number" of lines). An example should make this clear:

Retained Line:	$100,000
1st Surplus:	4 lines ($400,000)

Risk	Insured Value	Retained Portion	1st Surplus Reinsured Portion	1st Surplus Percent
1	50,000	50,000	0	0%
2	100,000	100,000	0	0%
3	250,000	100,000	150,000	60%
4	500,000	100,000	400,000	80%
5	1,000,000	100,000	400,000	40%
6	10,000,000	100,000	400,000	4%

It is important to remember that this is <u>not</u> excess insurance. The retained line is only being used to establish the percent of the risk reinsured. Once the ceded percent is calculated, the reinsurer is responsible for that percent of any loss on the risk.

Other types of proportional treaties include fixed and variable quota share arrangements on excess business (e.g., commercial umbrella policies). For these contracts, the underlying business is excess of loss, but the reinsurer takes a proportional share of the ceding company's book. Umbrella treaties will be addressed in the section on casualty excess contracts.

The present section will focus primarily on a proportional property treaty. Most of the techniques described follow standard ratemaking procedures.

The following steps should be included in the pricing analysis for proportional treaties:

Step 1: Compile the historical experience on the treaty.

> Assemble the historical premium and incurred losses on the treaty for five or more years. If this is not available, the gross experience (i.e., prior to the reinsurance treaty) should be adjusted "as if" the surplus share terms had been in place, to produce the hypothetical treaty experience. Because a surplus share treaty focuses on large risks, its experience may be different than the gross experience.
>
> The treaty may be on a "losses occurring" basis for which earned premium and accident year losses should be used. Alternatively, the treaty may be on a "risks attaching" basis, which covers losses on policies written during the treaty period. For risks attaching treaties, written premium and the losses covered by those policies are used.

Step 2: Exclude catastrophe and shock losses.

> Catastrophe losses are due to a single event, such as a hurricane or earthquake, which may affect a large number of risks. Shock losses are any other losses, usually affecting a single policy, which may distort the overall results. For property contracts, catastrophes are generally defined on a per-occurrence (multiple risk) basis, whereas shock losses are large losses due to a single risk. For casualty contracts, catastrophes may include certain types of claims impacting many insureds (e.g., environmental liability), whereas shock losses would represent a large settlement on a single policy.

Step 3: Adjust experience to ultimate level and project to future period.

> The historical losses need to be developed to an ultimate basis. If the treaty experience is insufficient to estimate loss development factors, data from other sources may need to be used. Depending on the source of these factors, adjustments for the reporting lag to the reinsurer or the accident year / policy year differences may need to be made.
>
> The next step is to adjust historical premiums to the future level. The starting point is historical changes in rates and average pricing factors (e.g., changes in schedule rating credits). Rate level adjustment factors can be calculated using the parallelogram method for "losses occurring" treaties. The impact of rate

changes anticipated during the treaty period must also be included. This is an area requiring some judgment, as these percents may not actually have been filed or approved at the time the treaty is being evaluated.

If the premium base is insured value (for property), or some other inflation sensitive base, then an exposure inflation factor should also be included in the adjustment of historical premium.

Finally, the losses need to be trended to the future period. Various sources are available for this adjustment, including the amounts used in the ceding company's own rate filings.

Step 4: Select the expected non-catastrophe loss ratio for the treaty.

If the data used in Step 3 is reliable, the expected loss ratio is simply equal to the average of the historical loss ratios adjusted to the future level. It is worthwhile comparing this amount to the ceding company's gross calendar year experience, available in its Annual Statement, and to industry averages.

Step 5: Load the expected non-catastrophe loss ratio for catastrophes.

Typically, there will be insufficient credibility in the historical loss experience to price a loading for catastrophe potential. However, this amount is critical to the evaluation of property treaties.

In the past, reinsurers had priced catastrophe loads based on "spreading" large losses over expected payback periods. A 1-in-20-year event would be included as a loading of 5% of the loss amount. The payback approach may still be used for casualty events but is only referenced as a reasonability check for property.

The most common procedure is now for a company to select a property catastrophe load based on an engineering-based model that incorporates the risk profile of the ceding company. These models will be discussed in Section 5A below.

Step 6: Estimate the combined ratio given ceding commission and other expenses.

After the total expected loss ratio is estimated, the other features of the treaty must be evaluated. These include:

1. Ceding Commission - often on a "sliding scale" basis (see Section 1B)
2. Reinsurer's general expenses and overhead
3. Brokerage fees (where applicable)

If the reinsurer's business is produced through a broker, there is typically a fee paid by the reinsurer as a percent of treaty premium. If the reinsurer markets the business directly to the ceding company, there is no brokerage fee, but the general expense loading may be higher.

Finally, the reinsurer must evaluate whether or not the projected combined ratio on the treaty is acceptable. The evaluation of treaty terms should take into account potential investment income and the risk level of the exposures to determine if they meet the target return of the reinsurer.

The remainder of this section will be devoted to an example of the pricing for a proportional treaty.

The ceding company has requested a property quota share treaty effective 1/1/97, to be written on a "losses occurring" basis. The submission includes six years of historical experience, rate changes, and a loss development triangle.

The first step involves compiling the historical experience, which in this case is six years with a partial period for 1996. The incurred losses shown are on an accident year basis and include case reserves and allocated loss adjustment expenses but do not include IBNR.

Accident Year Experience evaluated 9/30/96:

Accident Year	Earned Premium	Incurred Losses	Loss Ratio to date
1991	1,640,767	925,021	56.4%
1992	1,709,371	2,597,041 *	151.9%
1993	1,854,529	1,141,468	61.6%
1994	1,998,751	1,028,236	51.4%
1995	2,015,522	999,208	49.6%
1996	1,550,393	625,830	40.4%
Total	10,769,333	7,316,804	67.9%

*Includes 1,582,758 due to Hurricane Andrew

The catastrophe loss for Hurricane Andrew is identified in the 1992 period.

The losses, excluding the Andrew loss, are trended at 4% a year and developed to an ultimate basis. The development factor on the 1996 year is selected so as to project losses for the full year.

Accident Year	Incurred Losses (excl. cats)	LDF	Trend Factor at 4%	Trended Ultimate Incurred Losses
1991	925,021	1.000	1.265	1,170,152
1992	1,014,283	1.000	1.217	1,234,382
1993	1,141,468	1.000	1.170	1,335,518
1994	1,028,236	1.000	1.125	1,156,766
1995	999,208	1.075	1.082	1,162,229
1996	625,830	1.600	1.040	1,041,381
Total	5,734,046			7,100,428

In addition, the rate change information shown below is provided. It should be noted that the +10% rate increase to be effective 4/1/97 is an estimate based on the rate filing that the ceding company expects to make in the coming year. The rate level adjustment assumes that this amount will be approved.

Effective Date	Average Rate Change	
1/1/1991	2.00%	
1/1/1993	10.00%	
7/1/1994	-4.00%	
4/1/1997	10.00%	(pending)

The earned premium amounts above are then adjusted to the average 1997 rate level using factors based on a standard parallelogram method. The other adjustments are that the 1996 premium has been adjusted from a 9 month basis to a full year basis, and all premiums are trended based on average property value inflation of 3%.

Accident Year	Unadjusted Earned Premium	On Level Factor	Trend Factor at 3%	Earned Premium at 1997 Level
1991	1,640,767	1.096	1.194	2,147,147
1992	1,709,371	1.086	1.159	2,151,541
1993	1,854,529	1.034	1.126	2,159,198
1994	1,998,751	0.992	1.093	2,167,158
1995	2,015,522	1.023	1.061	2,187,654
1996	2,067,191	1.028	1.030	2,188,825
Total	11,286,131			13,001,523

The non-catastrophe loss ratio is estimated to be 54.6% based on the projections of loss and premium to the 1997 level.

Accident Year	Earned Premium at 1997 Level	Trended Ultimate Incurred Losses	Projected Loss Ratio
1991	2,147,147	1,170,152	54.5%
1992	2,151,541	1,234,382	57.4%
1993	2,159,198	1,335,518	61.9%
1994	2,167,158	1,156,766	53.4%
1995	2,187,654	1,162,229	53.1%
1996	2,188,825	1,041,381	47.6%
Total	13,001,523	7,100,428	54.6%

The loading for catastrophe losses now needs to be made. For the historical period, the catastrophe loss associated with Hurricane Andrew would have added about 15% to the loss ratio if it had not been excluded. A loading from a catastrophe model might add in a smaller amount. For our example, we will assume that we have selected a 10% loading for catastrophe losses, making our final expected loss ratio approximately 65%.

The final step in the evaluation is the determination of the reinsurer’s combined ratio. A ceding commission of 30% has been suggested by the reinsured. The other expenses are listed below:

Expected Loss Ratio	65.0%
Ceding Commission	30.0%
Brokerage fees	5.0%
Administrative expenses	1.0%
Unallocated expenses	1.0%
Indicated Combined Ratio	102.0%

The reinsurance actuary must then evaluate the profitability of these proposed terms. A 102% combined ratio is unlikely to produce an acceptable return for the reinsurer so a reduction in the ceding commission may be the actuary's recommendation. Other provisions, such as a loss occurrence limit or adjustable features (discussed in the next section) may also be considered.

Section 1B. Special Features of Proportional Treaties

After the expected loss ratio is estimated for a proportional treaty, the actuary's work is not yet done. There will often remain disagreement between the ceding company and reinsurer about the loss ratio and the appropriate ceding commission. In theory, a reinsurer should "follow the fortunes" of the ceding company, but in practice their results may be quite different. Reinsuring a profitable insurer is no guarantee of profits for the reinsurer. In the negotiations to resolve these differences, adjustable features are often built into the treaty.

a) Sliding Scale Commission

A common adjustable feature is the "sliding scale" commission. A sliding scale commission is a percent of premium paid by the reinsurer to the ceding company which "slides" with the actual loss experience, subject to set minimum and maximum amounts.

For example:

Given the following commission terms:

Provisional Commission:	30%
Minimum Commission:	25% at a 65% loss ratio
Sliding 1:1 to	35% at a 55% loss ratio
Sliding .5:1 to a Maximum	45% at a 35% loss ratio

Then the results may follow, for different loss scenarios,

Actual Loss Ratio	Adjusted Commission
30% or below	45.0%
35%	45.0%
40%	42.5%
45%	40.0%
50%	37.5%
55%	35.0%
60%	30.0%
65% or above	25.0%

In a "balanced" plan, it is fair to simply calculate the ultimate commission for the expected loss ratio. However, this may not be appropriate if the expected loss ratio is towards one end of the slide. For example, if the expected loss ratio is 65%, the commission from a simple calculation is 25%, producing a 90% technical ratio (i.e., the sum of the loss and commission ratios). If the actual loss ratio is worse than 65%, the reinsurer suffers the full amount, but if the actual loss ratio is better than 65%, the reinsurer must pay additional commission.

It is more correct to view the loss ratio as a random variable and the expected loss ratio as the probability-weighted average of all possible outcomes. The expected ultimate commission ratio is then the average of all possible outcomes based on the loss ratio.

The simplest approach is to estimate the expected commission based on the historical loss ratios, adjusted to future level as above but including the catastrophe and shock losses. This is a good calculation to make as a reasonability check but may be distorted by historical catastrophes or years with low premium volume. It also leaves out many possible outcomes.

A better approach is the use of an aggregate loss distribution model. Several models are available and these are described in Section 4. The results of any of these models may be put into the following format:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of being in Range	Sliding Scale Commission
0% - 35%	31.5%	0.025	45.0%
35% - 55%	46.9%	0.311	39.0%
55% - 65%	59.9%	0.222	30.1%
65% or above	82.2%	0.442	25.0%
0% or above	65.0%	1.000	31.0%

Note that in this example, the expected technical ratio is 96% (=65%+31%) rather than the 90% (=65%+25%) naively estimated above.

A further complication is the introduction of a carryforward provision in the commission. A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, then the excess loss amount can be included with the current year's loss in the estimate of the current year's commission. In the long run, this should help smooth the results.

Two approaches may be taken to pricing the impact of carryforward provisions. The first is to include any carryforward from past years and estimate the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward. For example, if the carryforward from prior years amounts to a 5% addition to the loss ratio, the terms above would become:

Minimum Commission:	25% at a 60% current year loss ratio
Sliding 1:1 to	35% at a 50% current year loss ratio
Sliding .5:1 to a Maximum	45% at a 30% current year loss ratio

The analysis above would then be restated as follows:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of being in Range	Sliding Scale Commission
0% - 30%	27.4%	0.006	45.0%
30% - 50%	43.0%	0.221	38.5%
50% - 60%	55.1%	0.222	29.9%
60% or above	78.3%	0.551	25.0%
0% or above	65.0%	1.000	29.2%

The problem with this approach is that it ignores the potential for carryforward beyond the current year. For example, in the first year of the program we would calculate the expected commission for the current year as though the program would be cancelled at the end of the year. The same price would result with or without the carryforward provision - which does not seem right because the benefit of the carryforward is ignored.

A second approach is to look at the "long run" of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less than the variance of the loss ratio for a single year (roughly equal to dividing by the number of years in the block). As an example:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of being in Range	Sliding Scale Commission
0% - 35%	34.1%	0.000	45.0%
35% - 55%	51.6%	0.118	36.7%
55% - 65%	60.4%	0.408	29.6%
65% or above	72.3%	0.474	25.0%
0% or above	65.0%	1.000	28.3%

This example reduces the aggregate variance, putting greater probability in the ranges closer to the expected loss ratio of 65%. The first problem with this approach is that the method for reducing the variance is not obvious; the example above reduces the standard deviation of the aggregate distribution by the square root of 5, assuming that the commission applies to a five-year block. A second problem is that it ignores the fact that the contract may not renew the following year, potentially leaving the reinsured with no carryforward benefit.

This issue is further complicated when a commission deficit can be carried forward but not a credit. There is no standard method for handling these questions so far as this author is aware.

b) Profit Commission

A profit commission subtracts the actual loss ratio, ceding commission and a "margin" for expenses from the treaty premium and returns a percent of this as additional commission. For example:

Actual Loss Ratio	55%	
Ceding Commission	25%	
Margin	10%	
Reinsurer's Profit	10%	(100%-55%-25%-10%)
Percent Returned	50%	(as a percent of Reinsurer's Profit)
Profit Commission	5%	(10% profit times 50%)

Like the sliding scale commission, this should be evaluated using an aggregate distribution on the loss ratio. Also like the sliding scale commission, there is some ambiguity concerning the handling of carryforward provisions.

c) Loss Corridors

A loss corridor provides that the ceding company will reassume a portion of the reinsurer's liability if the loss ratio exceeds a certain amount. For example, the corridor may be 75% of the layer from an 80% to a 90% loss ratio. If the reinsurer's loss ratio is 100% before the application of the loss corridor, then it will have a net ratio of 92.5% after its application, calculated as:

	Before Corridor	After Corridor	
Below corridor	80.0%	80.0%	100% capped at 80%
Within corridor	10.0%	2.5%	10% minus 75% of 90%-80%
Above corridor	10.0%	10.0%	100% minus 90%
Total Loss Ratio	100.0%	92.5%	

As above, the proper estimate of the impact of the loss corridor should be made using an aggregate distribution. The probability and expected values for the ranges below, within and above the corridor can be evaluated.

Range of Loss Ratios	Average Loss Ratio in Range	Probability of being in Range	Loss Ratio Net of Loss Corridor
0% - 80%	64.1%	0.650	64.1%
80% - 90%	84.7%	0.156	81.2%
90% or above	103.9%	0.194	96.4%
0% or above	75.0%	1.000	73.0%

For this example, the expected loss ratio is 75.0% before the application of the loss corridor. Even though this is less than the 80% attachment point for the corridor, the corridor still has the effect of lowering the reinsurer's expected loss ratio.

Many variations on these features can be used with a proportional treaty. Bear and Nemlick [1] provide further background on handling loss sensitive features. This should serve to illustrate that the actuary's job is not finished after the expected loss ratio is calculated.

2. Property Per Risk Excess Treaties

Section 2A. Experience and Exposure Rating Models

Property per risk excess treaties provide a limit of coverage in excess of the ceding company’s retention. The layer applies on a "per risk" basis, which typically refers to a single property location. This is narrower than a "per occurrence" property excess treaty which applies to multiple risks to provide catastrophe protection.

The treaty premium is set as a percent of a subject premium base. The subject premium goes by the oxymoronic title "gross net earned premium income” (GNEPI) for losses occurring policies or "gross net written premium income” (GNWPI) for risks attaching policies. This premium is net of any other reinsurance inuring to the benefit of the per risk treaty, such as a surplus share treaty, but gross of the per risk treaty being priced.

The main tools available for pricing per risk treaties are experience and exposure rating.

a) Experience Rating

Experience rating is sometimes referred to as a "burn cost" model though that phrase more commonly denotes just the unadjusted experience and not the projected cost. The basic idea of experience rating is that the historical experience, adjusted properly, is the best predictor of future expectations. The analysis proceeds as follows:

Step 1:

> Gather the subject premium and historical losses for as many recent years as possible. Ten years should be sufficient, though the number of years relied upon in the final analysis should be a balance between credibility and responsiveness.

The historical losses should include all losses that would pierce the layer being priced after the application of trend factors.

Step 2:

Adjust the subject premium to the future level using rate, price and exposure inflation factors as outlined in the section on proportional treaties.

Step 3:

Apply loss inflation factors to the historical large losses and determine the amount included in the layer being analyzed. Sum up the amounts which fall in the layer for each historical period. If allocated expense (ALAE) applies pro-rata with losses, it should be added in individually for each loss.

Step 4:

Apply excess development factors to the summed losses for each period. As in any experience rating model, the loss development factors should be derived from the same ceding company data if possible. Along with the LDF, frequency trend, if determined to be needed, should be applied at this step.

Step 5:

Dividing the trended and developed layer losses by the adjusted subject premium produces loss costs by year. These may be averaged to project the expected loss cost.

The projected loss costs from this analysis should be randomly distributed about the average. If the loss costs are increasing or decreasing from the earliest to latest years in the experience period, then the assumptions of the model may need to be reexamined. The trend or development factors may be too high or low. Alternatively, there may have been shifts in the types of business or sizes of risks written by the ceding company.

As an example of experience rating for a property excess of loss treaty, assume the following terms are requested:

Effective Date:	1/1/97
Treaty Limit:	$400,000
Attachment Point:	$100,000

The losses shown below have been recorded for the treaty. For each loss, a 4% annual trend rate is applied to project the loss from its accident date to the average date in the prospective period. For each trended loss, we then calculate the portion that penetrates into the treaty layer being priced.

Accident Date	Untrended Total Loss	Trend Factor at 4%	Trended Total Loss	Loss in Treaty Layer
9/20/1988	240,946	1.411	339,975	239,975
10/11/1988	821,499	1.408	1,156,671	400,000
3/15/1989	158,129	1.385	219,009	119,009
6/21/1990	114,051	1.317	150,205	50,205
10/24/1990	78,043	1.300	101,456	1,456
1/10/1991	162,533	1.289	209,505	109,505
2/23/1992	324,298	1.234	400,184	300,184
4/30/1992	100,549	1.225	123,173	23,173
9/22/1992	75,476	1.206	91,024	0
1/1/1993	171,885	1.193	205,059	105,059
5/18/1993	94,218	1.175	110,706	10,706
8/1/1993	170,297	1.166	198,566	98,566
8/15/1994	87,133	1.119	97,502	0
7/12/1995	771,249	1.080	832,949	400,000

The losses that trend into the proposed layer are then summed for each historical accident year. The subject premium for each year is listed after adjustment for rate level changes and inflation trend of the insured values. The application of the loss development factor projects the ultimate trended loss cost for the treaty.

Accident Year	On Level Subject Premium	Trended Losses in Layer	LDF	Trended Ultimate in Layer	Loss Cost
1988	1,422,554	639,975	1.000	639,975	45.0%
1989	1,823,103	119,009	1.000	119,009	6.5%
1990	2,054,034	51,661	1.000	51,661	2.5%
1991	2,147,147	109,505	1.000	109,505	5.1%
1992	2,151,541	323,357	1.010	326,591	15.2%
1993	2,159,198	214,331	1.050	225,048	10.4%
1994	2,167,158	0	1.150	0	0.0%
1995	2,187,654	400,000	1.300	520,000	23.8%
Total	16,112,389	1,857,838		1,991,789	12.4%

b) Exposure Rating

The second pricing tool for property per risk treaties is exposure rating. The advantage of this approach over experience rating is that the current risk profile is modeled, not what was written years earlier. The exposure rating model is fairly simple, but may at first appear strange since nothing similar is found on the primary insurance side.

The approach was first developed by Ruth Salzmann in 1963 for Homeowners business and eventually adapted for commercial property as well. The method centers on an exposure curve (P). This represents the amount of loss capped at a given percent (*p*) of the insured value (*IV*) relative to the total value of the loss. This may be represented mathematically as:

$$P(p) = \frac{\int_0^{p \cdot IV} x \cdot f(x)\,dx + \int_{p \cdot IV}^{\infty} p \cdot IV \cdot f(x)\,dx}{\int_0^{\infty} x \cdot f(x)\,dx} = \frac{\int_0^{p \cdot IV} [1 - F(x)]\,dx}{E[X]}$$

where *f(x)* = distribution of individual loss dollar amount

For a property of a given insured value, we calculate the retention and limit as percents of that insured value. The portion of the expected loss on the risk which falls in the treaty layer is then given by:

P((Retention+Limit)/Insured Value) - P(Retention/Insured Value)

As an example, suppose the proposed treaty is intended to cover a per-risk layer of $400,000 excess of $100,000. For a single risk with an insured value of $500,000, we would calculate the difference between the exposure factors for 20% (from $100,000 / $500,000) and 100% (from $400,000+$100,000 / $500,000). From the table below, this results in an exposure factor of 44% (= 93%-49%).

Percent of I.V.	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

The exposure curve provided above is for illustration purposes only. The curve does allow for exposure above the insured value; this is due to the fact that often the limits profile provided does not include business interruption coverage for commercial policies or living expenses for homeowners policies.

For a portfolio of risks, this same calculation is performed on a distribution of premium by different ranges of insured values, known as the "limits profile". The limits profile must also be questioned to verify that the size of risk ranges are on a per location basis. If it is assembled using total values for policies covering multiple locations, distortions will result.

For the example below, it is assumed that all locations within the range are exactly equal to the midpoint of the range.

Treaty Limit:	$400,000
Treaty Retention:	$100,000

Range of Insured Values ($000s)	Midpoint	Retention as % of I.V.	Ret+Limit % of I.V.	Exposure Factor
20 - 100	60	167%	833%	0%
100 - 250	175	57%	286%	26%
250 - 1,000	625	16%	80%	41%
1,000 - 2,000	1,500	7%	33%	33%

Range of Insured Values ($000s)	Subject Premium	Expected Loss Ratio	Expected Losses	Reinsurer's Losses
20 - 100	682,000	65%	443,300	0
100 - 250	161,000	65%	104,650	27,209
250 - 1,000	285,000	65%	185,250	75,953
1,000 - 2,000	1,156,000	65%	751,400	247,962
Grand Total	2,284,000	65%	1,484,600	351,124

The reinsurer's loss cost is 15.37% (Reinsurer's Losses 351,124 over Subject Premium 2,284,000). This loss cost is then loaded for expenses and profit.

The expected loss ratio is of critical importance as the final rate will move proportionally with this amount. A rigorous projection of the expected loss ratio, following the procedures for proportional treaties, should be made.

An implicit assumption in the exposure rating approach outlined above is that the same exposure curve applies regardless of the size of the insured value. For example, the likelihood of a $10,000 loss on a $100,000 risk is equal to the likelihood of a $100,000 loss on a $1,000,000 risk. This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks. The Lloyds scales, previously an industry standard, did not recognize this shortcoming.

Ludwig [4] gives an excellent, more detailed description of this topic.

Section 2B. Other Issues on Property Per Risk Treaties

After loss costs are estimated using the experience and exposure rating models, the actuary's task is to reconcile the results and select a final expected loss cost.

a) Free Cover

One difficulty in this reconciliation is the issue of "free cover". This refers to an experience rating in which no losses trend into the highest portion of the layer being priced. For example, if you are comparing prices for a layer $750,000 excess of $250,000 with a layer $250,000 excess of $250,000, and your largest trended loss is $500,000 from ground up, then you will produce the same loss cost for either option. The top $500,000 excess of $500,000 layer would be implicitly a "free cover". One approach to this problem is to use the experience rating as a basis for the lowest portion of the layer and then use the relativities in the exposure rating to project the higher layer.

The table below gives an example of this approach:

Layer to be Priced	Experience Rating Loss Cost	Exposure Rating Loss Cost	Selected Loss Cost
$250k xs $250k	16%	20%	16%
$500k xs $500k	0%	10%	8% *
$750k xs $250k	16%	30%	24%

* 8% = 16% x (10%/20%)

b) Credibility

A first measure of credibility is the number of claims expected during the historical period. Note that this is not the same as the actual number observed during the period. If credibility is set based solely on the historical number, then more credibility will be assigned to experience rating projections that are fortuitously worse than average.

Because the expected number of claims may not be easily calculable, the dollars of expected loss, based on the exposure rating, may be used. For example, if the exposure rating indicates that $2,000,000 in losses was expected during the historical period, but only $1,000,000 was actually observed, then the credibility given to the experience rating should still be based on the $2,000,000 expected.

As a second measure of credibility, it is appropriate to look at the year-to-year variation in the projected loss cost from each of the historical periods. Stability in this rate should add credibility even if the number of claims is relatively small.

Assigning credibility is, in part, a subjective exercise. Often significant credibility is given to experience rating simply because there are too many limitations to the exposure rating alternative. Discussions on reconciling the experience and exposure rating results are given in Mashitz and Patrik [5] and Clark [9].

c) Inuring Reinsurance

An additional problem which may be encountered in both methods is that the excess treaty may apply to the ceding company's retention after a surplus share treaty is applied. The $750k xs $250k layer may apply to a $1,000,000 loss which is actually a 10% share of a $10,000,000 loss. For experience rating, the only accurate way to reflect this underlying reinsurance is to restate the historical loss experience on a basis net of the inuring reinsurance.

The exposure rating can be applied directly to a risk profile adjusted to reflect the terms of the inuring surplus share treaty. However, if the actuary has exposure curves varying by size of insured value, the curve should be selected based on the insured value before the surplus share is applied, but the exposure factor should apply to the subject premium after the surplus share is applied.

For example, suppose the ceding company from Section 2A decides to purchase a surplus share treaty in which it retains a maximum of $200,000 on any one risk. On its net retention, it then wishes to purchase a per-risk excess cover of $100,000 excess of $100,000. Its risk profile and the single exposure rating curve are the same as used in the earlier example.

Range of Insured Values ($000s)	Midpoint	Ins. Value after S/S	Gross Premium	GNEPI
20 - 100	60	60	682,000	682,000
100 - 250	175	175	161,000	161,000
250 - 1,000	625	200	285,000	91,200
1,000 - 2,000	1,500	200	1,156,000	154,133
Grand Total			2,284,000	1,088,333

Range of Insured Values ($000s)	Net Ins. Value	Retention % of I.V.	Ret+Limit % of I.V.	Exposure Factor
20 - 100	60	167%	333%	0%
100 - 250	175	57%	114%	24%
250 - 1,000	200	50%	100%	23%
1,000 - 2,000	200	50%	100%	23%

Range of Insured Values ($000s)	Subject Premium	Expected Loss Ratio	Expected Losses	Reinsurer's Losses
20 - 100	682,000	65%	443,300	0
100 - 250	161,000	65%	104,650	25,116
250 - 1,000	91,200	65%	59,280	13,634
1,000 - 2,000	154,133	65%	100,186	23,043
Grand Total	1,088,333	65%	707,416	61,793

The loss cost for the $100,000 excess of $100,000 layer is 5.68% (Reinsurer's Losses 61,793 over Subject Premium 1,088,333) for the per-risk excess treaty net of the surplus share. The exposure factors for the two highest ranges are the same because a single exposure curve is used.

3. Casualty Per Occurrence Excess Treaties

Section 3A. Experience and Exposure Rating Models

Like property excess, casualty lines use experience and exposure rating models. This discussion of casualty will refer to general liability (including products), auto liability and workers compensation. The same techniques described can be adapted for other casualty lines, such as professional liability, with some modifications.

Casualty per occurrence excess treaties are often separated into three categories:

Working Layer:

> Low layer attachment which is expected to be penetrated, often multiple times in each annual period.

Exposed Excess:

> Excess layer which attaches below some of the policy limits on the underlying business - that is, there are policies for which a full limit loss would cause a loss to the treaty. Typically, these losses will be less frequent and there will be some years in which the treaty layer is not penetrated.

Clash Covers:

> High layer attachment excess - typically a loss on a single policy will not penetrate the treaty layer. A clash cover will be penetrated due to multiple policies involved in a single occurrence, or when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits

(XPL) are determined in a settlement. The method for including allocated loss adjustment expenses in the treaty may also expose the clash layer.

The distinctions between these categories are generally soft in the pricing process. A perfect working layer would produce stable enough results to be retained by the ceding company. Experience rating techniques are still used even when the experience approaches the "exposed excess" category. On the other hand, for large ceding carriers, "clash" losses may be common enough that the experience rating procedure provides guidance for the price.

a) Experience Rating

The steps in the experience rating procedure follow those of property experience rating, but some additional complications arise.

Step 1:

Gather the subject premium and historical losses for as many years as possible. Along with the historical losses, it is very important that allocated loss adjustment expenses (ALAE) be captured separately from losses. For general liability and auto liability losses, the underlying policy limit should also be listed. For auto losses on a split limits rather than a combined single limit (CSL) basis, other modifications may be needed in order to separately cap losses for bodily injury and property damage.

Workers compensation (WC) losses will not have an explicit limit associated with them. However, because large workers compensation losses are often shown on a discounted case reserve basis, a request should be made for these losses on a full undiscounted basis. Further discussion of handling WC losses will be given in the next section.

Step 2:

Adjust the subject premium to the future level using rate, price and exposure inflation factors. These adjustment factors will vary for each line of business included.

Step 3:

Apply loss inflation factors to the individual historical losses. Inflation factors should also vary by line of business.

The selection for a source of loss inflation is difficult. The Insurance Services Office (ISO) estimates basic and total limits trend factors for general and auto liability for use in ratemaking. Theoretically, what should be used is an unlimited trend factor derived from large losses only. Using losses capped at the underlying policy limit as a source may understate the final results. There is also an implicit assumption that the same trend factor applies to all losses regardless of amount. In the final analysis, the actuary must make a selection of loss inflation rates by year.

The trended losses must then be capped at applicable policy limits. This represents another problem for which there is no generally accepted solution. Theoretically, we want to cap losses at the limit applicable if the same policy were written in the future treaty period. One possible approach is to apply the historical policy limit to each trended loss; this leaves out the fact that the insured will generally increase its policy limits over time. A second approach is to apply the trend factor to the historical loss without applying a policy limit cap; this assumes that policy limits "drift" upwards to precisely match inflation. If this second approach is used, then the subject premium must also be adjusted to the level that would have been charged had the higher limits been in effect; otherwise an overstatement of the expected loss cost will result.

The discussion by Mata and Verheyen [10] gives some more advanced concepts on making use of exposure rating techniques to adjust for changes in the policy limit profile.

After the loss and ALAE amounts are trended, the portion of each in the treaty layer is calculated. Allocated expenses are usually included in one of two ways:

Pro-rata with loss:
ALAE in the layer allocated in proportion to losses.

ALAE as Part-of-Loss (aka "on top" or "add-on"):
ALAE added to loss and the treaty limit applies to the sum.

Example 1:

Trended Loss:	$640,000
Trended ALAE:	$320,000
Treaty Attachment:	$400,000
Treaty Limit:	$600,000

	Pro-Rata Treatment of ALAE			ALAE as Part-of-Loss
	Loss	ALAE	Loss+ALAE	Loss+ALAE
Retained	400,000	200,000	600,000	400,000
In Treaty	240,000	120,000	360,000	560,000
Above Treaty	0	0	0	0
Total	640,000	320,000	960,000	960,000

Example 2:

Trended Loss:	$920,000
Trended ALAE:	$460,000
Treaty Attachment:	$400,000
Treaty Limit:	$600,000

	Pro-Rata Treatment of ALAE			ALAE as Part-of-Loss
	Loss	ALAE	Loss+ALAE	Loss+ALAE
Retained	400,000	200,000	600,000	400,000
In Treaty	520,000	260,000	780,000	600,000
Above Treaty	0	0	0	380,000
Total	920,000	460,000	1,380,000	1,380,000

These two examples should serve to illustrate the two methods of including ALAE in a treaty. It should also be noted that the amount in the treaty layer is not necessarily higher or lower for either method, but depends on the actual experience.

Step 4:

Apply excess development factors to the summed losses for each period. For casualty lines, this step is critical due to the very large factors needed to reflect future development. If possible, historical patterns should be derived for the excess layer using ceding company data. Where this is not available, other benchmarks are needed.

The Reinsurance Association of America (RAA) publishes a loss development study on a biennial basis, which is considered an industry benchmark. The historical data in that study includes more than thirty years of development, broken out by line of business. Its statistics show a significant lag between reported losses for a primary company and a reinsurer. The graphs included in the 2012 edition of the RAA Study, attached as a supplement to this study note, illustrate this lag.

The use of compiled industry data gives a level of stability to the estimate of excess development patterns that is often superior to that for individual ceding companies.

While the RAA statistics may be considered a benchmark, the user should remember that the data is simply what is reported by its members. Some cautions:

1. The reporting lag from the occurrence of an event to the establishment of a reinsurer's case reserve may vary by company. Included in the data is retrocessional business which may include several levels of reporting lag.

2. The mix of attachment points and limits is not cleanly broken out. In recent studies, the RAA has begun publishing statistics by attachment point ranges, but this data is considerably less stable than the total triangle. Loss development varies significantly for different attachment points so every effort should be made to adjust the selected factors to the layer of the treaty being priced.

3. The RAA requests data exclusive of Asbestos and Environmental claims which could distort the patterns. It cannot be known if all member companies have done this consistently. Other long term exposure claims, such as medical products, mold, or tobacco, are not excluded.

4. For workers compensation, the members may not handle the tabular discount on large claims in a consistent manner. If a ceding company reports a loss on a discounted basis, and the reinsurer establishes a case reserve as the amount of the discounted value that falls into the reinsured layer, a very high development factor may result due to the unwinding of the discount.

As a practical matter, having a very slow development pattern will often produce results showing either zero or very high projected ultimate layer losses by year. The actuary will often need to use smoothing techniques, such as a Bornhuetter-Ferguson approach or Cape Cod (aka Stanard-Bühlmann), to produce a final experience rate.

Step 5:

Dividing the trended and developed layer losses by the adjusted subject premium produces loss costs by year. These amounts are averaged and a final expected loss cost selected. The loss cost may be adjusted for the time value of money, expenses and risk load; these adjustments are dealt with in the last section of this study note.

b) Exposure Rating

The second pricing method is exposure rating. As was the case for property, this method estimates a loss cost based on the premium and limits expected to be exposed during the treaty period. The exposure rating approach uses a severity distribution, based on industry statistics, to estimate layer losses. The severity distribution is used to calculate increased limits factors (ILF) for general liability and auto liability, and excess loss factors (ELF) for workers compensation. The theory is the same for these different lines, but the practical calculation is different.

For all of these approaches, we begin with a Cumulative Distribution Function (CDF) representing the probability that a loss is a given size or smaller.

x =	random variable for size of loss
F(x) =	probability a loss is x or smaller, the CDF
f(x) =	density function, first derivative of F(x)
E[x] =	expected value or average unlimited loss
E[x;L] =	expected value of losses capped at L

The severity distribution is used to calculate expected losses in any given layer.

We define:

$$E[x;L] = E[min(x,L)] = \int_0^L x\, f(x)\, dx + \int_L^\infty L\, f(x)\, dx$$

$$ILF_{L,U} = \frac{E[x;U]}{E[x;L]} \qquad\qquad ELF_L = \frac{E[x] - E[x;L]}{E[x]}$$

For general liability and auto liability, one option is to use the truncated Pareto distribution for loss severity. The form of E[x;L] is given by

$$E[x;L] = P \cdot S + \left(\frac{1-P}{Q-1}\right) \cdot \left[(B + Q \cdot T) - (B + L) \cdot \left(\frac{B+T}{B+L}\right)^Q\right]$$

for $Q<>1, L>T.$

The five parameters for this distribution follow some intuitive meanings:

T = Truncation point, "small" losses are below this point, "large" losses follow a Pareto distribution
P = probability of a "small" loss
S = average small loss severity
B = scale parameter for Pareto distribution
Q = shape parameter for Pareto distribution

The scale of the distribution is easily adjusted for inflation by multiplying the parameters T, S and B by the same amount. Two limitations of this formula should be noted:

1. The formula shown above only applies for losses above the truncation point T. As a practical matter, this is not a problem as that parameter is set at an amount well below any treaty attachment point.
2. The excess factors for higher layers become very dependent on the Q parameter. This parameter must be watched very carefully when the curves are updated.

A curious note on the truncated Pareto distribution is that when B=0 and Q=1, the distribution becomes a log-logistic distribution of the form below.

$$E[x;L] = P \cdot S + (1-P) \cdot T \cdot \left[1 - ln\left(\frac{T}{L}\right)\right]$$

This has the property that expected losses in layers are equal if the limit and attachment point are in the same ratio.

$$E[x;U] - E[x;L] = E[x;kU] - E[x;kL] \quad \text{for any constant k}$$

This property may be approximated when the B parameter is small and the Q parameter is close to 1. It should be remembered, however, that this relationship holds for severities for individual claims, but not necessarily for treaty loss costs, which will decrease for higher layers due to fewer policies being exposed.

The “BQPST” form of the Pareto is not the only choice available. There is great flexibility possible with discrete mixture models. A discrete mixture is a weighted average of relatively simple curve forms that approximates a more complex but realistic shape.

A popular example of a mixture model is the Mixed Exponential, which is a weighted average of several exponential distributions.

$$E[x;L] = \sum_{j=1}^{N} w_j \cdot \mu_j \cdot \left(1 - exp\left(-\frac{L}{\mu_j}\right)\right)$$

$$\text{where} \qquad 1 = \sum_{j=1}^{N} w_j$$

Once a severity distribution is selected, an exposure factor can be calculated. This factor is analogous to the factor used for excess property and should likewise be applied to ground-up expected losses to estimate the loss cost to the treaty layer.

$$\text{Exposure Factor} = \frac{E[x; min(PL, AP + Lim)] - E[x; min(PL, AP)]}{E[x; PL]}$$

Where
PL = Ceding Company Policy Limit
AP = Treaty Attachment Point
Lim = Treaty Limit

If the treaty includes ALAE in proportion to losses, this exposure factor can be applied to subject premium times an expected loss and ALAE ratio. If the ALAE is included with losses, the following exposure factor formula can be used:

$$\text{Exposure Factor} = \frac{E\left[x; min\left(PL, \frac{AP + Lim}{(1+e)}\right)\right] - E\left[x; min\left(PL, \frac{AP}{(1+e)}\right)\right]}{E[x; PL]}$$

Where:
PL = underlying Policy Limit applying to loss only
AP = Treaty Attachment Point applying to ALAE plus loss capped at PL
Lim = Treaty Limit applying to ALAE plus loss capped at PL
e = ALAE as a percent of loss capped at PL

The key assumption in both cases is that ALAE varies directly with capped indemnity loss. This is not an accurate model in that ALAE is not a constant percent of any given loss. For example, losses which close without an indemnity payment may still incur a large expense. In general, as the size of a loss increases, the ALAE as a percent of the loss will tend to decrease. The assumption that loss and ALAE are perfectly correlated will tend to result in an overstatement of expected amounts in the higher layers.

Another limitation of the formula for the latter case is that an exposure factor of zero will be applied to high layers which are indeed exposed. For example, if the underlying policy limit is $1,000,000 and the ALAE loading is 1.500, then a treaty attaching at $1,500,000 will not be considered exposed by this formula.

A more refined analysis of the effect of ALAE would require modeling of how ALAE varies with loss size.

Another use for the severity curves is for proportional treaties on excess business. These proportional treaties may be on a quota share basis, where the reinsurer takes a set percent of each contract the ceding company writes, or on a "cessions" basis for which the percent depends on the attachment point and limit written on each policy. A cessions basis treaty will typically require the ceding company to use increased limits factors to price the portion of its policies exposing the treaty. The exposure factors calculated above can be compared with the factors used by the ceding company in pricing its business.

For workers compensation, the severity distributions used most commonly come from the National Council on Compensation Insurance (NCCI), which publishes excess factors for retrospective rating plans in many states. Its curves vary by state and hazard group. The underlying data incorporates different injury types as well. It is not always possible to calculate the underlying severity distribution directly.

The NCCI curves, or other excess factors, can easily be approximated by an inverse power curve of the form:

$$ELF_L = \frac{E[x] - E[x;L]}{E[x]} = a \cdot L^{-b}$$

The parameters "*a*" and "*b*" are estimable from selected excess factors. The fitted factors behave in the higher layers much like the Pareto distribution described above.

Workers compensation does not have policy limits corresponding to those on liability policies. The WC limits refer instead to limitations on annual benefits specific to individual states. The exposure factor is therefore calculated using only the treaty attachment point (AP) and limit.

$$\text{Exposure Factor} = \text{ELF}_{\text{AP}} - \text{ELF}_{\text{AP+Limit}}$$

The exposure factor is estimated separately for each state and hazard group for which the ceding company projects premium for the treaty period. Expected loss ratios are also needed for each of these divisions.

An example of exposure rating would look as follows:

Treaty Limit:	750,000
Treaty Attachment Point:	250,000

State	H.G.	Standard Premium	ELR	ELF at 250,000	ELF at 1,000,000	Exposure Factor	Treaty Losses
AL	B	100,000	70%	0.030	0.006	0.024	1,680
AL	C	100,000	70%	0.040	0.008	0.032	2,240
NJ	B	100,000	85%	0.070	0.020	0.050	4,250
NJ	D	100,000	85%	0.100	0.035	0.065	5,525
		400,000					13,695

The loss cost for the treaty will be 3.42% (Treaty Losses 13,695 over Standard Premium 400,000).

Section 3B. Special Problems on Casualty Excess Treaties

This section will deal with a number of problems which commonly arise with casualty excess treaties. Issues about credibility or "free cover" have been addressed in the section on property per risk excess treaties, but should equally be considered for casualty treaties. The methods described are the author's suggestions and should not be viewed as the consensus opinion. However these issues are addressed, they cannot be ignored in the pricing process.

a) Including Umbrella Policies

The ceding company may include umbrella policies in the business subject to the treaty. These policies are excess of an underlying retention and "drop down" if an underlying aggregate is exhausted.

If the umbrella policies are above primary policies written by the ceding company, then it is best to consider the combination of the primary and excess as a single policy with a higher limit. For experience rating the primary and excess pieces are simply added together. When the umbrella policies are above primary policies from other carriers, the procedures are more difficult.

For experience rating, the main difficulty is in selecting the appropriate trend factor. The limit on the underlying policy should be added to losses on the umbrella policy before the application of trend, then subtracted after it:

Trended Loss = (Loss + Underlying Limit)·(Trend Factor) - Underlying Limit

This procedure will still leave out losses from the underlying policy which historically did not exhaust the underlying limit, but which would have after the application of a trend factor.

For exposure rating, the exposure factor on an excess policy is calculated as:

$$\text{Exposure Factor} = \frac{E[x; min(UL + PL, UL + AP + Lim)] - E[x; min(UL + PL, UL + AP)]}{E[x; UL + PL] - E[x; UL]}$$

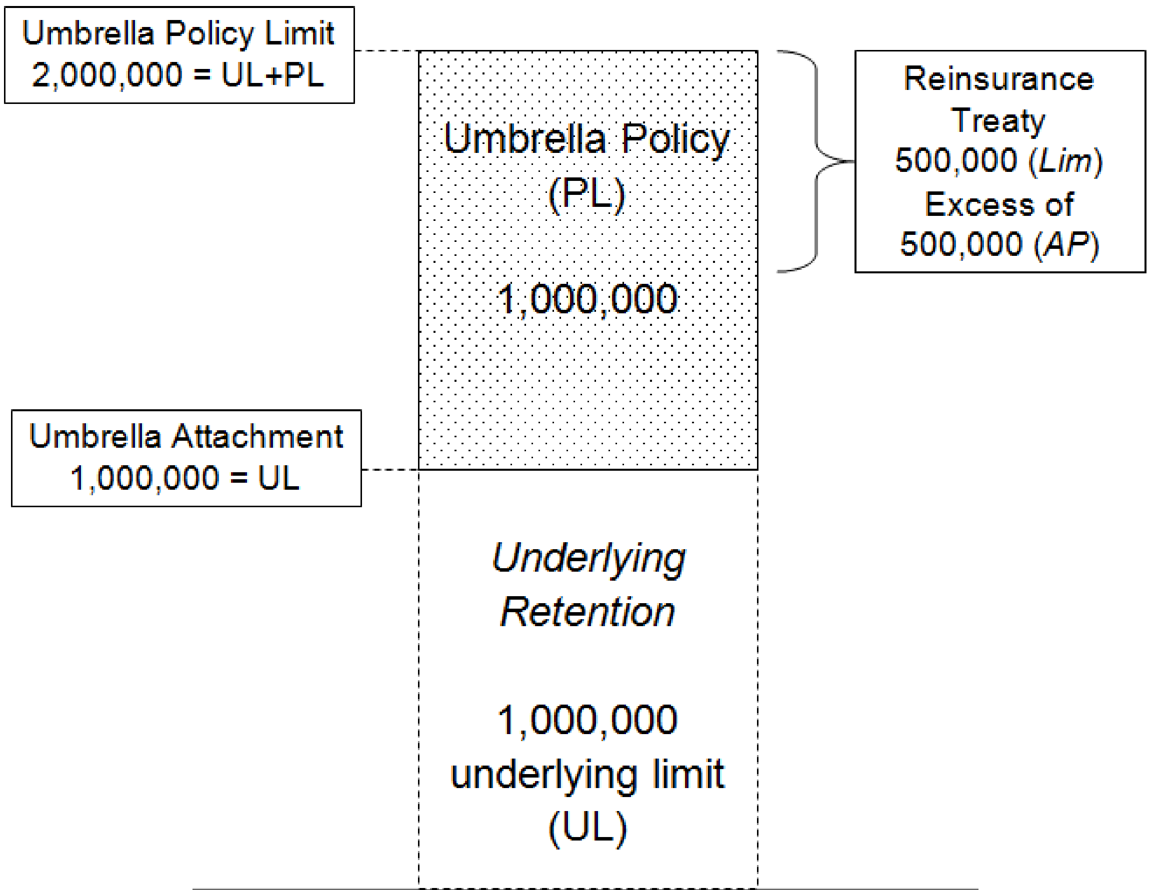
Where:

UL = Limit of Underlying Policies (attachment point of the umbrella)
PL = Policy Limit on Umbrella
AP = Treaty Attachment Point
Lim = Treaty Limit

For example, if the ceding company sells an umbrella policy for $1,000,000 excess of $1,000,000 and the treaty covers losses for the layer $500,000 excess of $500,000, then the exposure factor would be:

$$\text{Exposure Factor} = \frac{E[x; 2] - E[x; 1.5]}{E[x; 2] - E[x; 1]}$$

The graphic below illustrates how this treaty would apply.



This formula leaves out the possibility of the "drop down" feature of the umbrella policy. An approximation to include this additional exposure would be:

$$\text{Exposure Factor} = \frac{\{E[x;2]-E[x;1.5]\}\cdot(1-\phi) + \{E[x;1]-E[x;0.5]\}\cdot\phi}{\{E[x;2]-E[x;1]\}\cdot(1-\phi) + \{E[x;1]-0\}\cdot\phi}$$

The ϕ in the formula represents the aggregate excess factor on the underlying policy. This is analogous to a Table M charge factor, and will be given a more explicit definition in the section on Aggregate Distributions.

b) Loss Sensitive Features

For working layer excess, the ceding company is often willing to retain more of the losses. In these cases, an annual aggregate deductible (AAD) may be used. The AAD allows the ceding company to retain the first losses in the layer, but maintain protection in case there are more losses than anticipated. The treaty then becomes an excess of aggregate cover, where the aggregate losses are per occurrence excess losses in the layer.

The savings due to aggregate deductibles can be estimated directly from the experience rating if they are set at a sufficiently low level (say, half of the expected value). A better approach is the use of an aggregate distribution model. An excess charge factor for a given AAD is defined as:

$$\phi_{AAD} = \frac{\int_{AAD}^{\infty}(y - AAD)\, g(y)\, dy}{E[y]}$$

where *g(y)* is the distribution of aggregate losses in the layer

The form of this expression may be recognized as that underlying Table M; it is also analogous to the ELF calculation used for per occurrence excess. This charge may be estimated from a number of different methods. These methods are outlined in a separate section on aggregate distributions.

The charge factor ϕ_{AAD} is multiplied by the loss cost for the layer gross of the AAD to estimate the net loss cost.

A second type of loss sensitive program is the "swing plan" which is a type of retrospective rating program. Actual losses to the layer are loaded for expenses and the result is charged back to the ceding company, subject to maximum and minimum constraints.

Swing plans likewise require aggregate distribution models to be evaluated correctly. A swing plan formula may work as follows:

Retro Premium = (Actual Layer Losses) x 100/80
Provisional Rate = 15%
Maximum Premium = 30% x Subject Premium
Minimum Premium = 10% x Subject Premium

For example, if actual losses in the layer are $100,000, then the ultimate premium paid to the reinsurer will be $125,000, subject to the maximum and minimum.

This formula may apply to a block of years instead of to a single year. Following the example of sliding scale commissions, the calculation of expected premium is as follows:

Range of Loss Cost	Probability	Average in Range	Loaded Loss Cost	Capped Premium
0% < LC < 8%	0.120	6.0%	7.5%	10.0%
8% < LC < 24%	0.630	18.0%	22.5%	22.5%
24% < LC	0.250	40.0%	50.0%	30.0%
Total	1.000	22.1%	27.6%	22.9%

In this example, the expected loss ratio is 96.5% (= 22.1%/22.9%), not 80% (from the 100/80 loading) because the maximum and minimum amounts are not in "balance". The loading, maximum or minimum rates can be adjusted to produce an acceptable loss ratio. A second issue on the swing plan is that the provisional rate of 15% is well below the expected ultimate swing plan premium rate of 22.9%; this difference is an added cash flow advantage for the ceding company which must be included in the final pricing evaluation.

c) Workers Compensation Experience Rating

As described above, experience rating for workers compensation may be distorted depending on how tabular discounts are taken into account. A way to avoid this distortion is to collect sufficient information for individual claimants to project their expected costs into the treaty layer. The information needed is:

1. Claimant's current age
2. Claimant's sex (M/F)
3. Estimate of annual indemnity cost including escalation, if any
4. Estimate of annual medical cost
5. Amounts paid to date

For claims with the potential for penetrating the layer, all future payments (both indemnity and medical costs adjusted for escalation) should be determined. For those potential payments which fall within the excess layer, an appropriate mortality factor should be applied to determine the expected amount in the treaty layer. It is important to note that some claims, for which the incurred amount reported by the ceding company falls below the treaty retention, will show an expected amount in the layer. A smaller development factor would then be needed to include only "true IBNR" claims.

4. Aggregate Distribution Models

Throughout the pricing discussions above, aggregate distribution models have been used for pricing a variety of treaty features. This section will outline a number of tools which can be used for these calculations. As a general rule, aggregate models produce results which are very sensitive to the input assumptions; wherever possible, sensitivity analysis on the parameters, or even several approaches, should be used.

All of the approaches in this section may be considered "advanced", but this is not to say that they are optional. Improper evaluation of features which vary with loss experience could lead to significant under- or over-pricing.

a) Empirical Distribution

For most of the adjustable features outlined in this paper, the historical experience can be used to estimate the impact of the adjustable feature. For example, if the actuary has five or more years of loss ratios on a surplus share treaty, then a sliding scale commission can be priced by calculating the commission as if the current terms had been in effect over the historical period (adjusted to current rate level).

The empirical approach is generally very easy to calculate and should be examined at least as a check on other methods. However, some caveats should be recognized:

1) The experience does not take into account all possible outcomes, and may miss the possibility of events outside of what has been observed.
2) If the volume or mix of business has been changing, then the volatility of the future period may be very different than the historical period.
3) If loss development has been performed using a Bornhuetter-Ferguson or Cape Cod method, then the historical periods may present an artificially smooth sequence of loss ratios that does not reflect future volatility.

b) Single Distribution Model

The single distribution approach assumes that the aggregate of all losses to the treaty follows a known CDF form. This is in contrast to a "collective risk" model for which there is explicit modeling of frequency and severity distributions.

A commonly used model is the lognormal distribution. The lognormal has been shown to be a reasonable approximation to empirical distributions, and most spreadsheet software applications allow it to be programmed directly.

The lognormal cumulative distribution function (CDF) has the form:

$$CDF = G(y) = \Phi\left(\frac{ln(y) - \mu}{\sigma}\right) = \int_0^y \frac{exp\left(-\frac{(ln(t) - \mu)^2}{2 \cdot \sigma^2}\right)}{t \cdot \sigma \cdot \sqrt{2\,\pi}}\, dt$$

The parameters can be easily set based on a method of moments, given an expected value and coefficient of variation (CV = standard deviation over the mean).

$$\sigma^2 = ln(CV^2 + 1) \qquad \mu = ln(mean) - \frac{\sigma^2}{2}$$

The limited expected loss function is given by:

$$E[y; L] = exp\left(\mu + \frac{\sigma^2}{2}\right) \cdot \Phi\left(\frac{ln(L) - \mu - \sigma^2}{\sigma}\right) + L \cdot \left[1 - \Phi\left(\frac{ln(L) - \mu}{\sigma}\right)\right]$$

Related to that is the excess charge function:

$$\text{Excess Charge Function} = \phi_L = \frac{E[y] - E[y; L]}{E[y]}$$

Finally, the expression for the conditional expected value within a given range is given by the formula below. The first term in the numerator and denominator is replaced by 1 if U is equal to infinity. The second term in the numerator and denominator is replaced by 0 if L=0.

$$E[y \mid L < y < U] = exp\left(\mu + \frac{\sigma^2}{2}\right) \cdot \frac{\Phi\left(\frac{ln(U) - \mu - \sigma^2}{\sigma}\right) - \Phi\left(\frac{ln(L) - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{ln(U) - \mu}{\sigma}\right) - \Phi\left(\frac{ln(L) - \mu}{\sigma}\right)}$$

Where: *L* = lower end of range
U = upper end of range

The formula for the expected value in a given range is very useful because most adjustable features can be broken down into piecewise linear functions, and only the expected value is needed within each linear range.

For example, in the swing plan program illustrated above, the ultimate premium is at the minimum when the loss cost is below 8%, and it increases at a rate of 100/80 with loss until hitting the maximum of 30% at a 24% loss cost. The premium needs only to be estimated for the expected value in each of the three ranges. The ultimate premium estimates are then weighted together using the probabilities of the loss being in each range.

The procedure may be generalized as follows:

Aggregate Loss Range	Expected In Range	Probability In Range
0 to P1	E[y | 0<y<P1]	G(P1)
P1 to P2	E[y | P1<y<P2]	G(P2)-G(P1)
P2 to P3	E[y | P2<y<P3]	G(P3)-G(P2)
…	…	…
Pn & Above	E[y | Pn<y]	1-G(Pn)

This table can be set up for sliding scale commissions, profit commissions, swing plans, loss corridors, or many other common features. The formulae above make use of the lognormal distribution, which is often used in the actuarial literature and can be included in a spreadsheet program. Other curve forms, such as transformed gamma (see Venter [8]) or inverse Gaussian, have been recommended as also providing good fits to aggregate loss data.

The single distribution model has the advantage of being relatively simple to use, even when the source data is limited. A reasonable fit is provided even when frequency and severity distributions are not known. There are two main disadvantages: First, there is no allowance for the loss free scenario; in fact the lognormal is not defined for y=0. Second, there is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses. Bear and Nemlick [1] offer several useful suggestions for modifying the single distribution model to overcome these disadvantages.

c) Recursive Calculation of Aggregate Distribution

The recursive formula, introduced into the actuarial literature by Panjer (see Panjer and Willmot [6]), is a very convenient tool for calculating an aggregate distribution for low frequency scenarios. The frequency distribution is assumed to be Poisson, negative binomial or binomial, and the severity distribution is defined in discrete steps.

For an example, assume that the frequency distribution is Poisson with a mean of λ.

This has the well-known form:

$$Pr(n) = \frac{\lambda^n \cdot e^{-\lambda}}{n\,!}$$

This can also be given the recursive form:

$$Pr(0) = e^{-\lambda} \qquad Pr(n) = \left(\frac{\lambda}{n}\right) \cdot Pr(n-1)$$

Next, a severity distribution must be defined. For the recursive formula, each possible severity must be equally spaced from the preceding amount. The largest severity may be set equal to the per occurrence limit on an excess treaty, or to the limit times a loading for ALAE. In this example, we will define:

Notation	Severity	Probability
S_1	250	.400
S_2	500	.150
S_3	750	.100
S_4	1,000	.350

This example uses four points, but the formula can be expanded to handle any finite number. The severity distribution must sum to one ($1=S_1+S_2+S_3+S_4$).

For the aggregate distribution, the probability of zero losses is simply equal to the Poisson probability of zero, Pr(0)=.050 for E[n]=λ=3. The probability of the aggregate losses totaling 250 is the probability of one loss, Pr(1)=.150, times the probability that that one loss is equal to 250, S_1=.400. This may be restated in terms of A_0:

A_0	$= Pr(0)$	.050
A_1	$= Pr(1) \cdot .400 = (\lambda/1) \cdot S_1 \cdot A_0$	.060

The probability that the aggregate distribution is 500 is the addition of two pieces: the probability of one 500 loss, plus the probability of two 250 losses. Again this can be restated recursively:

$$A_2 = Pr(1) \bullet .150 + Pr(2) \bullet .400 \bullet .400$$
$$= (\lambda/2) \bullet (S_1 \bullet A_1 + 2 \bullet S_2 \bullet A_0) \qquad .059$$

Likewise, the probabilities for higher amounts are easily calculable:

$$A_3 = (\lambda/3) \bullet (1 \bullet S_1 \bullet A_2 + 2 \bullet S_2 \bullet A_1 + 3 \bullet S_3 \bullet A_0) \qquad .057$$

$$A_4 = (\lambda/4) \bullet (1 \bullet S_1 \bullet A_3 + 2 \bullet S_2 \bullet A_2 + 3 \bullet S_3 \bullet A_1 + 4 \bullet S_4 \bullet A_0) \qquad .096$$

$$A_5 = (\lambda/5) \bullet (1 \bullet S_1 \bullet A_4 + 2 \bullet S_2 \bullet A_3 + 3 \bullet S_3 \bullet A_2 + 4 \bullet S_4 \bullet A_1) \qquad .094$$

Notice that for aggregate amounts above the largest possible individual severity, the number of terms does not increase. A simple table can be set up to illustrate this calculation:

Amount	Sev. Prob.	Agg. Prob.	Calculation
0	.000	.050	
250	.400	.060	(3/1)(.400 •.050)
500	.150	.059	(3/2)(.400 •.060+2 •.150 •.050)
750	.100	.057	(3/3)(.400 •.059+2 •.150 •.060+3 •.100 •.050)
1,000	.350	.096	(3/4)(.400 •.057+2 •.150 •.059+3 •.100 •.060+4 •.350 •.050)
1,250	.000	.094	(3/5)(.400 •.096+2 •.150 •.057+3 •.100 •.059+4 •.350 •.060)
1,500	.000	.083	(3/6)(.400 •.094+2 •.150 •.096+3 •.100 •.057+4 •.350 •.059)

This calculation continues indefinitely using the following formula:

$$A_k = \sum_{i=1}^{k} \frac{\lambda}{k} \cdot i \cdot S_i \cdot A_{k-i}$$

When the Poisson frequency distribution is used, the mean and variance of the aggregate distribution are easily estimated as:

$$\text{Mean} = \lambda\,(250 \bullet S_1 + 500 \bullet S_2 + 750 \bullet S_3 + 1000 \bullet S_4)$$

$$\text{Variance} = \lambda\,(250^2 \bullet S_1 + 500^2 \bullet S_2 + 750^2 \bullet S_3 + 1000^2 \bullet S_4)$$

The recursive formula can be generalized for frequency distributions other than Poisson:

$$A_k \ = \ \sum_{i=1}^{k} \left(a + \frac{b}{k} \cdot i \right) \cdot S_i \cdot A_{k-i}$$

The "a" and "b" parameters are defined as follows:

Poisson:

$$a = 0 \qquad\qquad b = \lambda \qquad\qquad Pr(n) \ = \ \frac{\lambda^n \cdot e^{-\lambda}}{n\,!}$$

Negative Binomial:

$$a = (1-p) \qquad b = (\alpha - 1) \cdot (1-p) \qquad Pr(n) = \binom{\alpha + n - 1}{n} \cdot p^{\alpha} \cdot (1-p)^n$$

Binomial:

$$a = \frac{p}{p-1} \qquad b = \frac{(M+1) \cdot p}{1-p} \qquad Pr(n) = \binom{M}{n} \cdot p^n \cdot (1-p)^{M-n}$$

The use of a negative binomial or binomial frequency distribution allows for greater flexibility in the aggregate distribution.

The recursive formula has the major advantage of being simple to work with and providing an accurate handling of low frequency scenarios. The number of points evaluated on the severity distribution can be expanded to closely approximate continuous curves. The disadvantages are: 1) For higher expected frequencies, the calculation is inconvenient because all the probabilities up to the desired level must be calculated and 2) only a single severity distribution can be used in the analysis.

d) Other Collective Risk Models

In general, collective risk models are distributions for which frequency and severity are explicitly recognized. The recursive method outlined above is a straightforward example of a collective risk model. For handling continuous functions and higher expected frequencies, more advanced techniques may be needed.

A collective risk model assumes the severity of loss, represented by the random variable "x", has a given distribution. The aggregate loss is the sum of "n" of these severities, where "n" is also a random variable. Most aggregate loss models allow for more than a single severity distribution to be used.

The aggregate distribution may be evaluated using simulation or numerical methods. Numerical methods have been developed which can provide very close approximations to the theoretical distribution, with efficient computer time. The underlying calculations are well beyond the scope of this paper (see Heckman and Meyers [3], Robertson [7] or Wang [12] for detailed, readable accounts).

The inputs needed are the severity distribution(s) and parameters for the frequency distribution. Most models then will produce the cumulative distribution function G(y) and excess charge factor ϕ(y) at requested points.

The expected aggregate loss in a given range can be estimated as:

$$E[y \mid L < y < U] = \frac{E[y] \cdot \{\phi_L - \phi_U\} + L \cdot \{1 - G(L)\} - U \cdot \{1 - G(U)\}}{G(U) - G(L)}$$

The results of the aggregate model are very useful in pricing the adjustable features described in this study note. They become even more important on "pure" excess of aggregate covers such as Stop Loss treaties, which cover losses in excess of a set loss amount or loss ratio. The collective risk model is generally the best way to price these treaties but some words of caution are in order:

1. The complexity of the calculations can lead to a "black box" mentality - assuming the numbers must be right because of the accuracy of the computer. Whenever possible, more than one set of results should be produced, as a check on the sensitivity of the answer to the starting assumptions. Some basic statistics, such as the coefficient of variation (standard deviation over mean) and percentiles, should be compared to the empirical data for reasonability.
2. Most models assume that each occurrence is independent of the others and that the frequency and severity distributions are independent of each other. This may be a reasonable assumption in many cases, but could be false in others.
3. Some collective risk models use numerical methods with a large error term for low frequency scenarios. Check the output of the model; the expected error term should be given.
4. The aggregate distribution reflects the process variance of losses but does not reflect the full parameter variance. "Process variance" refers to the random fluctuation of actual results about the expected value. "Parameter variance" in the narrow sense refers to uncertainty about the parameters and may be calculable from outside sources. Some models allow for a prior distribution to apply to the selected parameters. "Parameter variance" in the broader sense of not being sure if you are even using the right model is harder to estimate and is

best reflected by repeated sensitivity analysis. This broader sense could perhaps be called "model risk".

5. Property Catastrophe Covers

Section 5A. Traditional Products and Methods

A property catastrophe cover provides protection for a catastrophic event, such as a hurricane or earthquake. The occurrence may often affect multiple risks and multiple policies. Typically, the catastrophe cover applies to the ceding company's retained exposure net of surplus share, per risk excess treaties and facultative certificates. That is, other reinsurance inures to the benefit of the catastrophe cover.

The limit is defined in excess of a total loss amount. A cover may be $10,000,000 in excess of $30,000,000 per occurrence. Because the limit is often a substantial dollar amount, the contract provides a limited number of reinstatements. Without reinstatements, the catastrophe cover would provide $10,000,000 of limit, but after the full layer is exhausted, there is no more protection. Additional reinstatements are available “pro-rata as to amount” and less often "pro-rata as to time".

Pro-rata as to amount means that if half the limit is exhausted, it can be reinstated for premium proportional to the amount reinstated:

Occurrence Limit:	$10,000,000	
Annual Premium:	$2,000,000	
Reinstatement Provision:	110%	pro-rata as to amount
Actual Loss Amount:	$4,500,000	
Reinstatement Premium:	$990,000	(= $2,000,000 X 1.10 X 4.5/10)

The treaty effectively has an aggregate limit equal to one plus the number of reinstatements, times the occurrence limit. For a cover with one reinstatement, the same results will be produced for four losses halfway through the layer as for two full limit losses.

Less frequently, the reinstatement premium is pro-rata as to time, meaning that the premium would be further reduced to reflect only the amount of time left in the policy period. Given the seasonal nature of some types of catastrophes (e.g., hurricanes), relatively few contracts include reinstatements pro-rata as to time.

Before the widespread development of catastrophe models there had been few tools available to systematically price catastrophe covers. The most common method was known as the payback approach, in which premium was set so the offered limit was paid back over a given period of time. For the example above, the payback period is five years, meaning that the $2,000,000 of annual premium would cover a single total loss of $10,000,000 every five years.

Catastrophe models are now the generally accepted approach for pricing of natural and some man-made events. There are four main components of typical catastrophe models:

- Event sets that simulate the covered hazards (e.g., hurricanes, earthquakes, terrorist events). These events cover the full range of possible sizes of a hazard at all relevant locations and are simulated based on estimates of frequency and intensity at specific locations.
- Calculation of local event intensity for each property within a portfolio.
- Estimation of damage for each property within a portfolio impacted by a given event.
- Insured loss estimates based on policies written by the ceding company.

The event sets are generally created and stored within the model prior to pricing of a catastrophe cover. The damage and insured loss estimates are specific to the portfolio written by the ceding company and therefore require additional information.

A catastrophe model will require several types of information:

1. Measure of exposure:
   This should be insured values, construction types, occupancies, along with attachment points for excess contracts.

2. Geographical information:
   Property address information is converted into latitude and longitude coordinates by a geocoding engine that is provided with the model. In some cases insured value information may be less precisely aggregated by zip code or state, resulting in less precise model results.

3. Terms of the insurance policies:
   Include deductible and coinsurance provisions of the original policies.

4. Details of inuring reinsurance:
   If a surplus share treaty inures to the benefit of the catastrophe treaty, any features such as occurrence caps or loss corridors will affect the catastrophe exposure.

The output of a catastrophe model is a distribution of possible losses on the subject business. The expected amount in the treaty layer, usually referred to as the average annual loss (AAL), can be calculated, along with its standard deviation. This can be used as a starting point for a loss cost on the cover.

The model output is also used for management of accumulations, so typically an Occurrence Exceedance Probability (OEP) curve is calculated. The OEP represents the probability that at least one event during the year will exceed a given loss amount. There may also be an Annual Exceedance Probability (AEP) given, which represents the probability that the total of all modeled events in a single year exceeds a given loss amount.

Catastrophe models are a major advance in the ability of insurers and reinsurers to assess their risks. There are additional items that may or may not be included in the results. If not explicitly modeled, these may need to be included more subjectively:

1. Workers compensation losses may be included within the cover. If there is an earthquake during standard working hours, this exposure could be substantial.

2. The inuring reinsurance terms may not be calculable by the model.

3. Even if earthquake coverage is not sold by the ceding company, there may still be exposure due to a "fire following" the earthquake.

4. Other coverage terms, such as the portion of policyholders purchasing replacement cost coverage instead of actual cash value, may be critical. After a major catastrophe event, there may be increased demand for materials and labor which raises the total cost borne by the insurer.

One last complication that should be addressed is due to the basis of coverage for the catastrophe cover: whether it is "losses occurring" during the period or "losses occurring on risks attaching" during the period. As before, "risks attaching" contracts cover losses on policies written during the treaty period. For risks attaching contracts, there is the potential for the reinsurer to pay twice on the same loss event.

Consider a treaty renewing on 1/1/95 for a layer of $10,000,000. A loss event takes place on 3/15/95. The ceding company has policies that are affected, some effective 7/1/94 and some effective 1/1/95. The catastrophe reinsurance treaty effective 1/1/94 covers the losses on the 7/1/94 policies and the treaty effective 1/1/95 covers the losses on the 1/1/95 policies. The reinsurer may end up paying $20,000,000 for the single event. To address this difficulty, many treaties include an “interlocking clause”, designed to equitably apportion losses that may be covered under more than one contract.

Section 5B. Alternative Risk Products

A great variety of products are grouped under the titles "financial reinsurance" or "finite reinsurance". For this study note, the term "finite risk" will refer to property catastrophe covers for which the maximum loss amount is reduced relative to traditional covers. This distinction is very soft because traditional covers are already "finite" in the sense that there is a definite limit that can be paid. Further, the relationship between the ceding company and the reinsurer on traditional covers is often viewed as a partnership; there is an unspoken understanding that the ceding company is expected to pay its own losses over the long term.

Two characteristics are common to most finite risk covers:

1. Multiple year features.
2. Loss sensitive features such as profit commissions and additional premium formulas.

For example, there may be a provision that the contract applies to a three-year period and is cancelable after the first or second year only if premium to date exceeds the loss payments. On the other side, there may be a profit commission which returns, say, 75% of premium if the contract is loss free for three years. In exchange for the profit commission, a relatively high annual premium is charged up front.

These types of features may greatly reduce the downside risk on the contract but it is rarely eliminated. The ceding company can only consider this insurance if two conditions are met:

1. The reinsurer assumes significant insurance risk under the reinsured portions of the underlying insurance agreements.
2. It is reasonably possible that the reinsurer may realize a significant loss from the transaction.

The reinsurance actuary is likely to be called upon to help quantify the risk on the contract to verify that these criteria are met. Timing risk as well as underwriting risk should be evaluated. The actuary charged with this task should refer to the American Academy of Actuaries' "Risk Transfer Testing Practice Note" [11] for guidance.

It is also important to remember that features like profit commissions may substantially reduce the "upside" of the contract from the reinsurer's perspective. It is all well and good to limit the loss to $500,000 in the event of a hurricane, but if this is in exchange for a maximum profit of $10,000 on a loss free year, then more attention needs to be given. In a limited sense, there is an equivalent traditional risk cover corresponding to the possible results from a finite risk cover.

Assume that the following terms are provided on a finite basis:

Annual Premium:	$2,500,000 (25% nominal rate on line)
Occurrence Limit:	$10,000,000
Profit commission:	80% after 10% margin on Annual Premium
Additional Premium:	50% of (Loss + Margin - Annual Premium)

	Loss Free Scenario	One Full Loss Scenario
Premium	$2,500,000	$2,500,000
Loss	$0	$10,000,000
Profit Commission	$1,800,000	$0
Add'l Premium	$0	$3,875,000
U/W Result*	$700,000	($3,625,000)

Now consider the following terms on a traditional basis:

	Loss Free Scenario	One Full Loss Scenario
Premium	$700,000	$700,000
Loss	$0	$4,325,000
U/W Result*	$700,000	($3,625,000)

* U/W result here excludes expenses

The rate on line for the traditional risk program is 16%, and produces an underwriting result equivalent to that of the more complex finite risk program. In this case, the question becomes: would the reinsurer be willing to offer this cover on a traditional basis at a 16% rate on line? If not, the pricing for the alternative cover is also inadequate.

This type of analysis becomes more complicated when reinstatement provisions, expenses, and carryforward provisions from earlier years are taken into account, but those features can be reflected in an expanded analysis. More difficult are provisions in which the additional premium and profit commission percents change each year of the program or depend on whether the ceding company or the reinsurer cancels the cover.

The best approach to these programs is to estimate the different possible outcomes for a one-year time horizon. Using a simplifying assumption that any penetrations into the layer will exhaust the full limit, probabilities can be assigned to each scenario using a Poisson or other distribution.

Using a frequency distribution is convenient because the mean of the distribution is related to the "payback period" for traditional risk covers. The payback which produces an acceptable expected result can be compared to the results of catastrophe models or other pricing analysis.

A final consideration on finite reinsurance relates to the credit risk of the ceding company. In the example above, the reinsurer depends upon the contingent "additional premium" to minimize the downside risk on the contract. However, there is a new risk introduced that the ceding company will be financially unable to make the payment, especially after experiencing the loss that makes it necessary. A careful review of the ceding company's annual statement needs to be made.

6. Calculating the Final Price

Up to this point, this study note has focused on estimating the reinsurer's expected losses. The final program must be structured to cover this amount but also to cover the reinsurer's expenses and the risk that is borne by the stockholder. The timing of the payment of these amounts is also considered because investment income will contribute to profitability.

Turning first to expenses, it should be noted that the reinsurer's expenses are not the same as those of the ceding company. For instance, reinsurers are not subject to premium tax. The reinsurer's expenses can be broken into three types:

1. Expenses varying with premium
    - ceding commission paid to the reinsured
    - brokerage fees (where applicable)
    - federal excise tax (where applicable)

2. Fixed expenses
    - general overhead costs (salaries, real estate)
    - underwriting and claims audit expenses

    While these expenses may vary somewhat with the size of the account, it is clear that they would not increase simply by taking a larger share of a given treaty. These company expenses should be set independently of variable expenses such as ceding commissions.

    Similarly, an excess of loss reinsurance treaty may be quoted with and without an Annual Aggregate Deductible (AAD). The expected loss to the reinsurer net of the AAD is less than for the treaty without an AAD, but the expenses incurred may not be different. The reinsurer needs an expense structure that covers its costs regardless of whether an AAD is selected.

3. Expenses varying with losses
    - reinsurer's unallocated loss adjustment expenses

    This percent should also vary with the type of reinsurance contract. A working layer excess treaty may require extensive work; a quota share contract may require a review of a loss bordereau, but less claim file review. These amounts can be estimated after discussion with the claims department.

If it is desired simply to load the losses for these expense categories, the final premium could be estimated as:

$$\text{Premium} = \frac{\text{Loss Cost} \cdot (1 + \text{ULAE}) + \text{Fixed Expense}}{(1 - \text{Variable Expense \%})}$$

The "traditional" loading of 100/80, often applied on excess treaties, is an example of this formula. In that case, all expenses are considered variable with premium and assumed to total 20%.

However, consideration must also be given to the timing and risk elements of the contract. The cash flows on the treaty need to be estimated for the treaty, including premium and loss payments and any adjustable features (e.g., swing plan premium).

The considerations for profit or risk load are more complex, and beyond the scope of this study note. Feldblum [2] has given a description of various approaches to accounting for risk.

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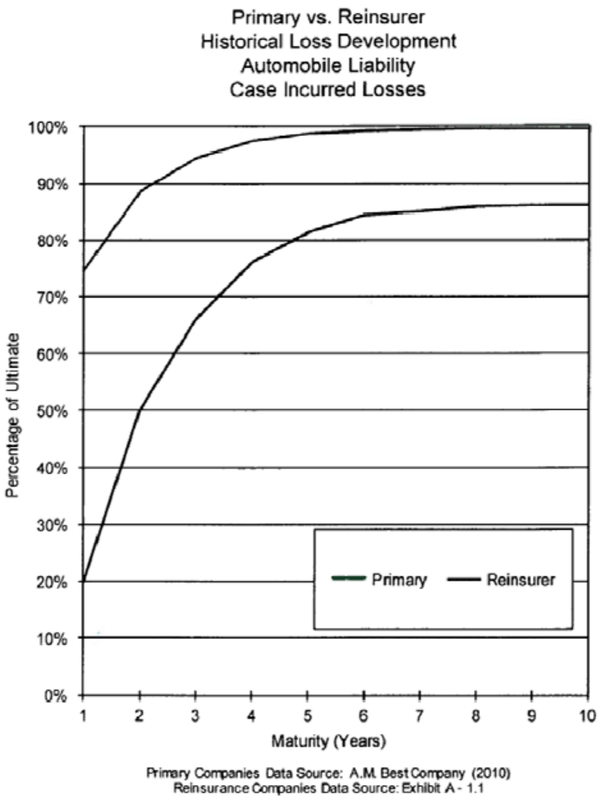
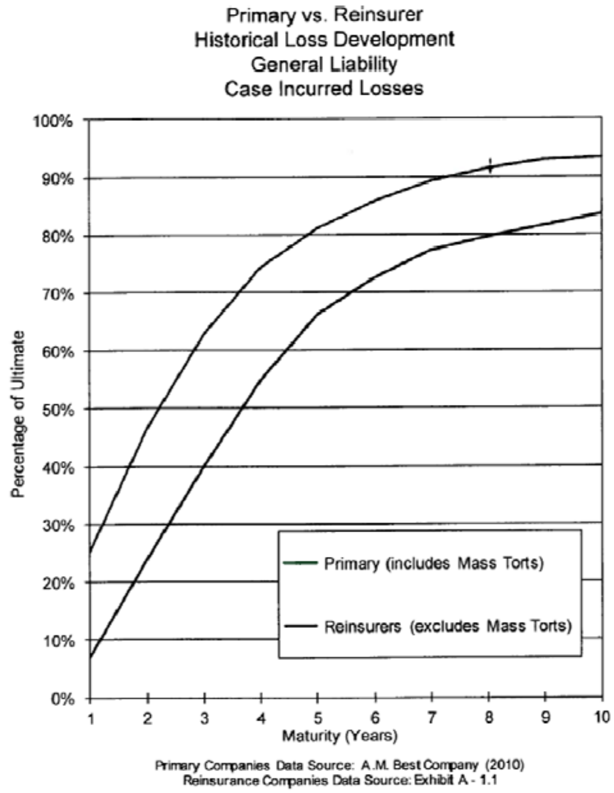
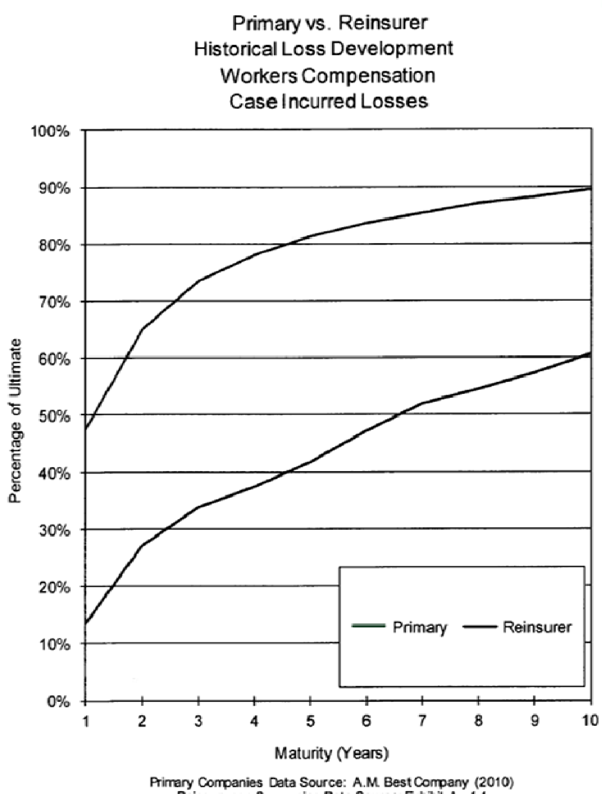
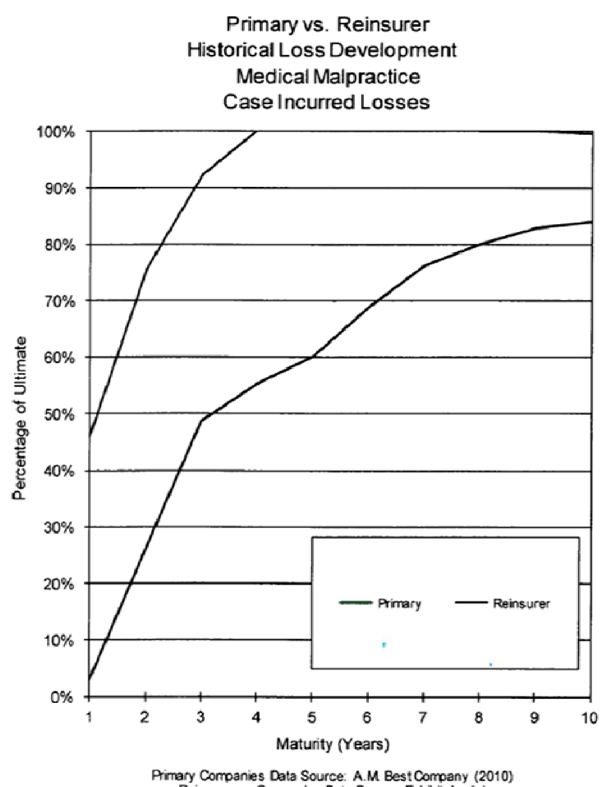
Supplement
Comparison of Primary and Reinsurance Development









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