Second Revision – Educational Note

Premium Liabilities

Committee on Property and Casualty Insurance Financial Reporting

July 2016

Document 216076

Ce document est disponible en français

Members should be familiar with educational notes. Educational notes describe but do not recommend practice in illustrative situations. They do not constitute standards of practice and are, therefore, not binding. They are, however, intended to illustrate the application (but not necessarily the only application) of the Standards of Practice, so there should be no conflict between them. They are intended to assist actuaries in applying standards of practice in respect of specific matters. Responsibility for the manner of application of standards of practice in specific circumstances remains that of the members.

MEMORANDUM

To: Members in the property and casualty insurance area

From: Pierre Dionne, Chair
Practice Council

Julie-Linda Laforce, Chair
Committee on Property and Casualty Insurance Financial Reporting

Date: July 12, 2016

Subject: **Second Revision – Educational Note: Premium Liabilities**

This educational note has been prepared by the Committee on Property and Casualty Insurance Financial Reporting in accordance with the Institute's Policy on Due Process for the Approval of Guidance Material other than Standards of Practice and Research Documents, and received final approval for distribution from the Practice Council on July 11, 2016.

As outlined in subsection 1220 of the Standards of Practice, "The actuary should be familiar with relevant Educational Notes and other designated educational material". That subsection explains further that a "practice that the Educational Notes describe for a situation is not necessarily the only accepted practice for that situation and is not necessarily accepted actuarial practice for a different situation". As well, "Educational Notes are intended to illustrate the application (but not necessarily the only application) of the standards, so there should be no conflict between them".

Questions or comments regarding this educational note may be directed to Julie-Linda Laforce at julielindalaforce@axxima.ca.

PD, JLL

1. Introduction

Appointed Actuaries (AA) for property & casualty (P&C) insurers in Canada are required to value insurance contract liabilities which include both claim liabilities and premium liabilities. Specifically, the Standards of Practice (SOP) state that

> 2130.01 The actuary should value the insurance contract liabilities and the reinsurance recoverables for the balance sheet and the changes in them for the income statement.

In actuarial literature, much attention is given to the definition of claim liabilities. Premium liabilities are often defined by exception. The SOP has the following definitions:

> 1110.27.2 Insurance contract liabilities in an insurer's statement of financial position are the liabilities at the date of the statement of financial position on account of the insurer's insurance contracts, including commitments, which are in force at that date or which were in force before that date.
>
> 1110.13 Claim liabilities are the portion of insurance contract liabilities in respect of claims incurred on or before the balance sheet date.
>
> 1110.37 Premium liabilities are the portions of insurance contract liabilities that are not claim liabilities.

The SOP provides additional details on premium liabilities in subsection 2230.

Specifically, premium liabilities include the expected costs in connection with the unexpired portion of the in-force insurance contract (i.e., incurred after the valuation date) and all other liabilities related to premium development adjustments (e.g., retro-rated premium, contingent profit commissions, etc.).

Elements to consider when analyzing premium liabilities generally include the following:

- Unearned premium reserve;
- Deferred policy acquisition expenses;
- Unearned (ceded) commission;
- Premium deficiency;
- Ceded deferred premium tax (applicable for inter-company pooling arrangements);
- Anticipated broker/agent commission;
- Expected adjustments (plus or minus) to swing-rated policies;
- Expected changes to premiums as a result of audits, late reporting, or endorsements; and
- Expected commission adjustments on policies with variable commissions.

Currently, premium liabilities are not shown explicitly in an insurer's financial statements. Some elements of the premium liabilities are calculated by the insurer and carried in the P&C Annual Return, while others must be estimated by the AA.

It is the AA's responsibility to evaluate the gross and net policy liabilities in connection with the unearned premium, including the assessment of the need for a premium deficiency reserve, and the maximum amount of deferrable policy acquisition expense.

The main elements related to premium liabilities are included in the regulatory expression of opinion in the AA report (see appendix D).

Minimum Capital Test (MCT) Calculation

Since 2012, the AA's estimate of premium liabilities and the duration on these liabilities, net of reinsurance recoverables (the net premium liabilities), is used in the calculation of the Interest Rate Risk Margin in the MCT pages of the P&C Annual Return. With the MCT Guideline effective January 1, 2015, the estimated net premium liabilities is also used to derive the Insurance Risk Margin in the P&C Annual Return, replacing unearned premium as the basis for the calculation of a premium-related margin. The formula requires the Insurance Risk Margin to be calculated by class of insurance. In this context, "class of insurance" is intended to be consistent with definitions prescribed by the regulator for statutory financial reporting.

Appendices B and C provide guidance on the evaluation of the gross and net premium liabilities respectively, by class of insurance, and provide detail on the corresponding entries in the AA's expression of opinion in appendix D.

Definitions

Deferred policy acquisition expenses (DPAE but also referred to as DPAC or DAC): Prepaid acquisition expenses which relate to the unexpired portion of the policy. These expenses are paid up front when the policy is issued but are not expensed on the income statement until the premium is earned. A deferred asset is set up for these prepaid expenses on the balance sheet. These expenses generally include broker/agent commissions and premium taxes but may also include an allocation of operating expenses such as renewal costs, advertising, licenses and fees, associations and dues, etc. The DPAE is an asset which recognizes the prepaid expenses over the policy period provided that such costs are recoverable from the equity in the net unearned premium reserve (UPR), as evaluated by the AA.

Earning pattern: Premiums should be earned on a basis consistent with the occurrence of losses. For most lines of business, losses are assumed to occur at a uniform rate during the year and premiums are earned on a pro rata basis over the term of the policy. However, for some lines of business this assumption is not appropriate. For example, most motorcycle losses occur during the period April to October and the earning pattern would recognize the timing of the loss exposure. Similarly, extended warranty premiums would be earned according to the expected payment of losses: for a three-year warranty there may be no exposure in year one if there is a manufacturer's warranty in force. The exposure would be expected to increase in the second and third years.

Equity in the gross unearned premium: amount by which the gross UPR exceeds the gross policy liabilities in connection with unearned premium.

Equity in the net unearned premium: amount by which the net UPR plus unearned (reinsurance) commissions exceeds the net policy liabilities in connection with unearned premium.

General expenses: these expenses include operational overhead and servicing expenses that are not related to the acquisition of new of renewal business. These expenses generally come from the P&C Annual Return (page 20.30) and exclude loss adjustment and investment expenses.

Maximum allowable DPAE: is the maximum of the equity in the UPR and zero. If the equity in the net UPR drops below zero, then the booked DPAE is reduced to zero and a premium deficiency amount is required to be carried on the balance sheet for the difference. Section 3 discusses the determination of the maximum allowable DPAE.

Policy liabilities in connection with unearned premium (premium liabilities): liabilities for future events consisting of the following items relating to the actuarial present value (APV) on the unexpired portion of the policies in-force at the valuation date:

- Future claims and adjustment expenses;
- Expected reinsurance costs based on anticipated contracts that are not yet underwritten; and
- Maintenance costs: i.e., administrative costs of servicing the in-force policies.

Premium deficiency: a provision that is determined by the AA when the equity in net UPR is negative. It is the amount which, when added to the net UPR and unearned (reinsurance) commissions, makes an appropriate provision for future costs arising from the unexpired portion of in-force policies.

Unearned (ceded) commissions: Unearned (reinsurance) commissions arise from commission revenue on reinsurance ceded premium. Quota-share reinsurance agreements generally provide for reinsurance commissions to be paid to the insurer by the reinsurer on the ceded premium. The reinsurance commission relating to the unexpired portion of a policy (i.e., the reinsurance commission on the unearned ceded premium) is carried as a liability. These unearned commissions generally include a total provision for broker/agent commissions, premium taxes, and other acquisition and servicing expenses.

Unearned premium reserve (UPR): the written premium associated with the exposure remaining on the unexpired portion of the policy under a contract of insurance. The UPR at the valuation date usually is established based on the written premium, the policy term, and an assumed earning pattern.

2. Policy Liabilities in Connection with Unearned Premium

The largest component of the policy liabilities in connection with unearned premium relates to future claims and adjustment expenses. They are estimated by applying a selected expected loss ratio to the UPR. This evaluation is generally undertaken by line of business or using a business segmentation that is consistent with the analysis of claim liabilities.

In order to facilitate the MCT calculations referred to in section 1, the AA may wish to use a business segmentation which produces estimated future costs that can be aggregated to an Annual Return class of insurance level. An example provided in the appendices illustrates the estimation of the net premium liabilities by class of insurance for purposes of deriving the MCT insurance risk margin on page 30.64 of the P&C Annual Return.

Expected Losses

The evaluation of future expected loss ratios in connection with the unexpired portion of in-force policies is a critical aspect of determining the future expected losses. Many evaluation methods may be used depending on the complexity of the business segments and characteristics of the insurer. Projected loss ratios may be based on the AA's valuation of claim liabilities, on the insurer's budget, on the results of a ratemaking analysis or on an ad hoc analysis, as considered appropriate. Generally, future expected losses are based on the insurer's recent experience adjusted to the period during which the unearned premium will be earned. The AA would consider the earning pattern underlying the calculation of the UPR, assess whether it reflects the exposure to risk, and select assumptions accordingly.

Examples of adjustments to the historical experience would include, but are not limited to, the following:

- Loss trends applied to adjust historical cost levels to the average accident date underlying the UPR;
- Expected legislative change impacts (including mandated benefit modifications);
- Recent court decision impacts relating to insurance coverage;
- Mix of business change impacts;
- On-level factors applied to adjust historical experience to the rate level underlying the UPR;
- Catastrophe and large losses loadings;
- Seasonality adjustments to the indicated expected loss ratios may need to be applied if the claims occurrence pattern is not uniform throughout the exposure period of the UPR (e.g., seasonal occurrences of hurricanes). Depending on the line of business, the seasonality adjustment may not be significant. However, for some portfolios (e.g., property catastrophe treaty reinsurance) seasonality may be a meaningful consideration; and
- Policy term assumptions taking into account the term of the policy and the future period covered by the UPR. For example, for policies with a term longer than 12 months (such as warranties or multi-year contracts), assumptions for the expected loss ratio need to take into consideration trends that are expected over the remaining term of these polices.

Various considerations for claim liabilities and premium liabilities are listed in the educational note Valuation of Policy Liabilities – P&C Insurance Considerations Regarding Claim Liabilities and Premium Liabilities.

Loss Adjustment Expenses

AAs may choose to include allocated loss adjustment expenses (ALAE) in their estimation of losses. If ALAE is not included with losses, an estimate of future ALAE would be derived by the

AA by applying an approach similar to the expected loss approach, or on another suitable basis consistent with the AA's valuation of claim liabilities.

Similarly, AAs may choose to include unallocated loss adjustment expenses (ULAE) in their estimation of losses. If ULAE is not included with losses, an estimate of future ULAE would be derived by the AA on a suitable basis consistent with the AA's valuation of claim liabilities. A typical calculation is to apply a ULAE ratio based on historical experience reflecting any expected changes in claims practices to the expected losses.

In order to facilitate the MCT calculations referred to in section 1, the AA may wish to consider the extent to which ALAE ratios and ULAE ratios might vary by Annual Return class of insurance.

Expected Reinsurance Costs

For the net policy liabilities in connection with unearned premium, in addition to the above considerations, the AA would also consider expected reinsurance costs. The manner to properly reflect reinsurance costs will depend on the type of reinsurance treaty and its terms and conditions.

For example, for a line of business covered by a proportional reinsurance treaty, the net unearned premium will be lower than the gross unearned premium and the loss ratio will be the same on a gross and net basis. For a line covered by an excess of loss treaty expiring at the valuation date, the gross and net unearned premium are the same and the ceded unearned premium is $0 at the end of the contract period. However, the cost of reinsurance in relation to the unexpired portion of the policies would be taken into account. The assumptions used would reflect the reinsurance rates and expected recoveries consistent with the reinsurance structure in place over the exposure period of the unearned premium.

Maintenance Expenses

Maintenance expenses also need to be included to reflect the future cost of servicing the policies in force. These expenses include expenses associated with endorsements, mid-term cancellations, changes in reinsurance contracts, etc. Maintenance expenses are generally expressed as a percentage of gross UPR and are evaluated as a portion of general expenses with typical ratios ranging from 25 percent to 50 percent.

In order to facilitate the MCT calculations referred to in section 1, the AA may wish to consider the extent to which general expense ratios vary by line of business based on the following considerations:

- The availability of historical and/or company plan expense information by line of business;
- Distribution model of the insurer;
- Characteristics of the insurer's portfolio (e.g., two-year contracts); and
- Any other considerations.

Maintenance expense assumptions are usually consistent from year-to-year; but may vary in the event of rapid growth or changes in the insurer's operations.

Discounting for Time Value of Money

Premium liabilities should be calculated on an APV basis, consistent with the SOP. The APV includes the time value of money and explicit provisions for adverse deviations (claims development, investment return rates, and recovery from reinsurance ceded) which is discussed below.

With regards to the time value of money, the cash flows other than losses and loss adjustment expenses (i.e., reinsurance costs and maintenance expenses) would also be considered but are not generally material to the calculation of the premium liabilities.

The discount rate for premium liabilities would generally be consistent with the discount rate used for discounting the claim liabilities, and generally would be based on a cash flow modelling of all policy liabilities including any premium payment lags (i.e., installment premiums).

The payment pattern for discounting premium liabilities would normally be consistent with that used for claim liabilities. As illustrated in appendix B, sheet 5, a future accident period payment pattern (for a given line of business) would typically be discounted to the valuation date for claims liabilities purposes. However, an adjustment would be required to reflect differences in the average accident date (AAD) of a future accident period versus the average accident date underlying the UPR (which is a partial accident period).

For example, assuming premium writings occur uniformly in a calendar year and the corresponding losses are also incurred uniformly throughout the year, the mean earning date and the mean accident date of a future accident year occurs at 0.50 years or halfway through the year. However, for the expected losses underlying the UPR, the average accident date is not as straightforward a calculation. This exercise involves calculating a weighted average or mean of the future accident dates using uniformly declining exposures as weights.

The average accident date or earning date of the UPR can be calculated by using integrals in calculating the weighted average.

Let x = future accident date underlying the UPR relating to 12-month policies

Let $f(x)$ = the loss exposure earned on a given future accident date

$= 1 - x$; where $0 \leq x \leq 1$

And x = 0 is the valuation date and x = 1 is one year later (assuming annual policies) i.e., the last date the loss exposure exists.

Let the average accident (or earned) date equal $\int_x f(x)$ integrating over the values 0 through 1, divided by the sum of the probability.

$$= \frac{\int_0^1 x f(x) dx}{\int_0^1 f(x) dx} = \frac{1}{3} \text{ year}$$

Thus, the average accident date or earning date of the UPR can be calculated as one-third of a year or four months (May 1 assuming a December 31 year end).

If the calculation were to be repeated for six-month policies, the results would show an average accident period of two months (or March 1 assuming a December 31 year end).

The median may be a reasonable approximation of the mean accident date. The median accident date is calculated using simple trigonometry assuming premiums and losses are uniformly distributed and the sides of the UPR triangle are set equal to 1.00. The equation to solve for the length of the triangle sides, x, that yields half the area of the triangle or ¼ would be ½ x^2 = ¼ or x = √0.5. Note that the length of the triangle side is defined here as the time from the median accident date to the end of the period or time 1. Therefore, the desired timing from time 0 (or the valuation date) to the median accident date would be the complement or 1 - √0.5 or 0.2929 years.

Once the average accident date underlying the UPR is determined, the present value may be derived directly from the present value of a future accident year, as illustrated in appendix B, sheet 5, by assuming that the present value of an average loss discounted to the accident date is equivalent to the present value of the UPR's expected losses at its average accident date. The final adjustment is to then discount from the average accident date back to the valuation date. A mathematical proof for this approximation is shown in appendix A.

The revised educational note Discounting and Cash Flow Considerations for P&C Insurers provides additional guidance on discounting associated with premium liabilities.

Margins for Adverse Deviations (MfAD)

The SOP states that (emphasis added)

> 2250.02 *The selected margin for adverse deviations should vary*
>
> ***between premium liabilities and claim liabilities,***
>
> *among lines of business, and*
>
> *among accident years, policy years, or underwriting years, as the case may be,*
>
> *according to how those considerations so vary.*

The AA would consider different MfADs if premium liabilities and claim liabilities exhibit different levels of uncertainty. Generally, there would be more uncertainty for claims that have yet to occur such as those underlying the unearned premiums.

The educational note Margins for Adverse Deviations for Property and Casualty Insurance provides additional guidance on the selection of MfADs.

Examples

Illustrative examples of the evaluation of premium liabilities, gross and net, are presented in appendices B and C.

The key concepts discussed above are illustrated in these exhibits, including the derivation of losses and ALAE by line of business, the addition of ULAE, the estimated cash flows on a discounted basis, and the addition of provisions for adverse deviations. In the net calculation of the amount of the expected loss and ALAE, the cost of reinsurance coverage in relation to the unexpired portion of the policies in force is explicitly subtracted from the UPR before applying the expected loss ratio.

Further details regarding the calculation of the expected gross loss ratios, maintenance expenses, and discount factors are presented in appendix B, sheets 4, 5, and 6. A similar exercise would be followed for the net assumptions presented in appendix C.

These examples are for illustrative purposes only. Other approaches to estimate the various components of the policy liabilities in connection with unearned premium may be appropriate. For example, premium liabilities for a block of business that is renewing on a single date would have average payment dates different than in the examples presented.

3. Premium Deficiencies and Maximum Deferrable Policy Acquisition Expenses

The current presentation of the financial statements allows for the reporting of the premium deficiency on a net basis only. In some situations, the calculation of the equity in the gross UPR may provide valuable information that the AA may wish to discuss with management. The remainder of this section will refer to the calculation and presentation of the premium deficiency derived by assessing the equity in the net UPR.

A premium deficiency exists when the net policy liabilities in connection with unearned premium exceed the sum of net UPR and unearned (reinsurance) commissions. In this situation, a premium deficiency liability is established in the amount by which the estimated premium liabilities exceed the sum of the net UPR and unearned (reinsurance) commissions.

The maximum deferrable policy acquisition expenses are commonly referred to as the equity in the UPR. The AA is required to test the adequacy of premium liabilities in the insurer's financial statements, including all future costs arising from the unexpired portion of in-force policies. The AA may do so by considering whether the carried DPAE is less than or equal to the equity in the UPR. In the event of a carried DPAE greater than the maximum estimated, the DPAE would be reduced to the maximum estimated amount. In the event of negative equity in the UPR, the DPAE would be reduced to zero and a premium deficiency would be required.

The previous section demonstrates how the AA calculates the policy liability in connection with unearned premium. The maximum deferrable policy acquisition expenses (net) is defined as follows:

Net UPR

\+ Premium deficiency

\+ Unearned (reinsurance) commissions

– Net policy liabilities in connection with unearned premium

UPR, unearned (reinsurance) commissions and initial DPAE are usually provided by the insurer's accounting department.

The equity in unearned premium is usually calculated on an all lines combined basis. On an all lines combined basis, deficiencies in some lines are offset by redundancies in other lines. This approach is appropriate on an ongoing concern basis to the extent that a company's mix of business does not change significantly from year to year. It is appropriate since it is unlikely that

a company would stop writing its more profitable lines. However, the calculation can be done by line of business when segregation of different books of business is desired. In that case, it is possible to record a premium deficiency on one line of business with a partially offsetting DPAE on the other books of business.

Once the maximum deferrable policy acquisition expense is calculated it will be compared with the initial DPAE amount. If the initial DPAE is higher than the maximum deferrable, management is informed that the DPAE should be reduced to the maximum deferrable amount. In addition, any amount by which the net policy liabilities in connection with unearned premium exceed the sum of the net UPR and unearned (reinsurance) commissions would be recorded as a premium deficiency.

Example

The example in the appendices demonstrates the general calculation of the maximum policy acquisition expenses deferrable and determination of the premium deficiency on both a gross and net of reinsurance basis for a company that is required to book a premium deficiency on its balance sheet:

- Appendix B, sheets 1-3 illustrate the gross premium liabilities calculations. Although a gross premium deficiency is indicated, it is not reported on the balance sheet, however, these exhibits may still be useful for discussion with management.
- Appendix B, sheets 4-6 illustrate the supporting loss ratios, discount factor and maintenance expense ratio calculation for a given class of insurance (personal property) on a gross of reinsurance basis.
- Appendix C illustrates the net premium liabilities calculations. As seen on sheet 3, the initial DPAE would be reduced to zero in column (33) and a premium deficiency reserve, shown in column (34), would be posted equal to the amount by which the net premium liabilities exceed the sum of the net UPR and unearned (ceded) commissions.
- Appendix D shows the premium liabilities portion of the AA's Expression of Opinion based on the figures from the example.

4. Other Net Liabilities and Unearned (Reinsurance) Commissions

"Other Net Liabilities" can be grouped into two major categories: those which relate to commission adjustments, and those which relate to premium adjustments, each of which is booked as a separate accrued liability on the balance sheet.

Contingent commissions (profit sharing commissions) are commissions that insurers pay their agents or brokers based on the profitability and the volume of business of individual producers (agents/brokers). These agreements vary by company and are often established over one- to three-year periods. Some commissions may be incurred as of the statement date and may need to be accrued.

Swing-rated contracts generate premium adjustments between insurers and reinsurers which are payable based on a pre-determined target loss ratio and the actual loss ratio of the book of

business reinsured. An example of a swing-rated contract giving rise to other net liabilities is provided below:

> A reinsurance contract for a long-tail line of business incepting on October 31, 2012 has a 3 percent swing rate adjustment payable by the insurer to the reinsurer if the loss ratio exceeds 73 percent and a 3 percent swing rate adjustment payable by the reinsurer to the insurer if the loss ratio is below 67 percent. Payment is due three years following contract inception.
>
> Assuming the reinsurance premium subject to the 3 percent adjustment is $1,000,000, the maximum swing rate adjustment would be in the amount of $30,000.
>
> As at December 31, 2015 for example, the AA would evaluate the other net liabilities related to this contract. If the AA evaluates that the ultimate loss ratio subject to the reinsurance contract is 75 percent, then the other net liabilities would equal the future swing rate adjustment payable to the reinsurer of $30,000 and would be presented in the AA's opinion.

A provision for retro-rated policy liabilities is booked when insurers issue policies for which the premium is adjusted yearly based on the actual experience on the policy. The final premium is not known until all losses are reported and settled. The provision to be accrued is equal to the difference (either positive or negative) between the estimated final premium and the cumulative paid premium at the valuation date.

Other examples of premium development to be evaluated as part of the premium liabilities are the following:

- Audit premiums where the final premium is not known until the coverage expires;
- Premium development on reinsurance assumed; and
- Premium development on retro-rated reinsurance ceded.

Communication with the accounting department is needed to ensure that the AA's adjustments and the insurer's accounting policy are consistent.

The sources of development on reinsurance assumed or ceded to be considered include, but are not limited to, the following:

- Changes in subject matter premium which is usually unknown until the end of the contract period;
- Swing-rated excess of loss treaties which call for a rate adjustment based on the loss experience during the coverage period; and
- Reinstatement premium for catastrophic or other layer (additional premium to be paid when the limit of coverage provided by the layer has been exhausted unless netted from expected claims recovery).

Some insurers with large quota share treaties may have significant unearned (reinsurance) commissions on their ceded unearned premiums. The unearned (reinsurance) commissions are booked as a liability and are earned pro rata over the terms of the policies.

5. Subsequent Event Application in the Case of Premium Liabilities

A subsequent event is defined in the SOP as "an event of which an actuary first becomes aware after a calculation date but before the corresponding report date." The treatment of a subsequent event in the AA's work is described as follows:

> 1520.02 For work with respect to an entity, the actuary should take a subsequent event into account (other than in a pro forma calculation) if the subsequent event
>
> provides information about the entity as it was at the calculation date,
>
> retroactively makes the entity different at the calculation date, or
>
> makes the entity different after the calculation date and a purpose of the work is to report on the entity as it will be as a result of the event.

Subsequent events need to be analyzed in the context of claim liabilities and premium liabilities. Some subsequent events will be taken into consideration in the same manner in both claim and premium liabilities, while some events may only need to be taken into consideration in premium liabilities. For example, the educational note Subsequent Events discusses the case of the 1998 ice storm. The event occurred on January 5, 1998; it did not make the entity different at the valuation date of December 31, 1997 and therefore did not need to be recognized in claim or premium liabilities. However, it was recognized that the actual premium liabilities could be materially different from the expected premium liabilities and that this should be disclosed as a note to the financial statements.

Appendix A: Premium Liabilities Present Value Factor

Assume the following:

- Let i = discount rate used in the valuation of claims liabilities based on a cash flow model that includes consideration of any premium payment lags (e.g., monthly installment plans);
- PV_{AY} represents the present value of the future accident year losses to January 1 divided by the ultimate losses;
- PV_{UPR} represents the present value of the losses underlying the UPR to January 1 divided by the ultimate losses;
- PV represents the present value of an average loss discounted to the accident date divided by the amount of the ultimate loss;
- 0<PV≤1; and
- Assume losses are uniformly distributed and premiums are annual and evenly distributed.

$$PV_{AY} = \int_0^1 PV\,(1+i)^{-t}dt$$

where $\int_0^1 (1+i)^{-t}\,dt = \int_0^1 (1 - ti + (t)(t+1)i^2 + \cdots)\,dt$

$\approx \int_0^1 (1-ti)\,dt = 1 - i/2$

$PV_{AY} \approx PV\,[\,1 - i/2\,]$

$PV_{UPR} = \frac{\int_0^1 PV(1-t)(1+i)^{-t}dt}{\int_0^1 (1-t)dt}$

$= 2\,PV\,[\int_0^1 (1+i)^{-t}dt - \int_0^1 t\,(1+i)^{-t}dt\,]$

where $\int_0^1 t\,(1+i)^{-t}dt \approx \int_0^1 t\,(1-ti)\;\;dt$

$$= \int_0^1 (t - t^2 i)\,dt = \left[\frac{t^2}{2} - \frac{t^3 i}{3}\right]_0^1 = \frac{1}{2} - \frac{i}{3}$$

$PV_{UPR} \approx 2\,PV\,[1 - \frac{i}{2} - \frac{1}{2} + \frac{i}{3}] \approx PV\,[1 - \frac{i}{3}]$

- Substituting the value of PV from above we can solve for the PV_{UPR} in terms of PV_{AY}

$$PV_{UPR} \approx PV_{AY} \frac{\left[1-\frac{i}{3}\right]}{\left[1-\frac{i}{2}\right]} \approx PV_{AY}[1 + i/6\ ] \approx PV_{AY}(1+i)^{x}$$

where x = difference between the *mean* average accident date of an accident year and that of the UPR of 1/6 (i.e., 1/2 less 1/3).

If, however, the cash flow model used to derive the discount rate underlying the claim liabilities does not incorporate premium payment lags, then the following adjustment is required to the discount rate to reflect the decrease in anticipated investment income:

- Let k = the portion of the unearned premium supported by invested assets (or % of portfolio paid in full at inception) as of the valuation date;
- Let j = k x i , 0% ≤ k ≤ 100%;
- k = 100% if all policies are paid in full at inception then j = i; and
- k = 0% if all policies are on an installment payment plan then j = 0 and no additional discount is needed to the valuation date.

$$PV_{UPR} \approx PV_{AY}(1+i)^{.5}\ (1+j)^{-.33}$$