

# CASUALTY ACTUARIAL SOCIETY AND THE CANADIAN INSTITUTE OF ACTUARIES



Steven D. Armstrong Vice President-Admissions

William Wilder Chairperson Examination Committee

Jason Russ Assistant Chairperson Examination Committee

May 4, 2016

# Exam 9

Financial Risk and Rate of Return

Examination Committee
General Officers
Aadil Ahmad
Derek Jones
Sharon Mott
James Sandor
Thomas Struppeck
Christopher Styrsky
Rhonda Walker

4 HOURS

#### INSTRUCTIONS TO CANDIDATES

- 1. This 62.5 point examination consists of 29 problem and essay questions.
- 2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use <u>dark</u> pencil or ink. Do not use multiple colors or correction fluid/tape.
  - Write your Candidate ID number and the examination number, 9, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do <u>not</u> include leading zeroes. Your name, or any other identifying mark, must not appear.
  - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper DO NOT WRITE ON THE BACK OF THE PAPER. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
  - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
  - <u>In order to receive full credit</u> or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, <u>showing calculations</u> where necessary. Also, you must clearly <u>specify any additional assumptions</u> you have made to answer the question.
- 3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

#### CONTINUE TO NEXT PAGE OF INSTRUCTIONS

©2016 Casualty Actuarial Society

- 4. Prior to the start of the exam, you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
  - Verify that the table of the Normal Distribution is attached to the examination after the last question.
- 5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. <u>Do not remove this label</u>. Keep a record of your Candidate ID number for future inquiries regarding this exam.
- 6. <u>Candidates must remain in the examination center until two hours after the start of the examination.</u> The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, <u>candidates may not leave the exam room during the last fifteen minutes of the examination.</u>
- 7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
- 8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. <u>Do not put the self-addressed stamped envelope inside the Examination Envelope.</u> Interoffice mail is not acceptable.
  - If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. <u>Do not put scrap paper in the Examination Envelope</u>. The supervisor will collect your scrap paper.
  - Candidates may obtain a copy of the examination from the CAS Web Site.
  - All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.
- 9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
- 10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 23, 2016.

#### Exam 9, Spring 2016

#### 1. (1.25 points)

An investor has a risk aversion factor of A = 2. The investor has \$100,000 to invest in a combination of an S&P 500 index fund and risk-free assets. Given the following information:

- The investor's utility is given by  $U = E(r_c) 0.5A\sigma_c^2$ .
- The investor's risk-free lending rate is 4%.
- The investor's risk-free borrowing rate is 6%.
- The S&P 500 index offers an expected return of 11.5% with a standard deviation of 20%.

#### a. (0.5 point)

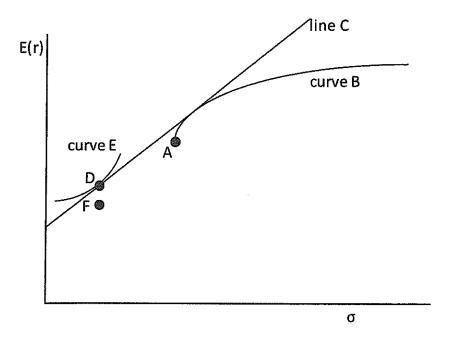
The investor would like to achieve the greatest possible return, subject to a maximum standard deviation of 25%. Calculate the amount the investor should allocate to risk-free assets.

#### b. (0.75 point)

Calculate the expected return and standard deviation of the portfolio in part a. above.

# 2. (1.75 points)

The following graph represents the Markowitz portfolio selection model.



A rational investor decides to invest in portfolio D.

#### a. (0.5 point)

Identify what each of the following represents:

- point A
- curve E

## b. (0.25 point)

Briefly explain where the optimal risky portfolio would be on the graph above.

#### c. (0.5 point)

Explain why another rational investor may choose a portfolio other than portfolio D, with reference to the graph above.

# d. (0.5 point)

Identify and describe one reason from behavioral finance that may lead an investor to choose portfolio F instead of portfolio D.

#### 3. (4.5 points)

The table below describes the assets available to the investor. All assets are uncorrelated.

Asset	Expected Return	Standard Deviation
A	10%	25%
В	10%	25%
С	10%	25%
D	2%	0%

The table below describes two different portfolio investment options available to the investor:

Asset	Portfolio 1 Weights	Portfolio 2 Weights
Α	30%	30%
В	0%	30%
С	0%	30%
D	70%	10%

#### a. (1 point)

Define risk pooling and risk sharing. Provide an example of how each is achieved in the insurance industry.

## b. (1.5 points)

Calculate each portfolio's expected return, variance and Sharpe ratio.

#### c. (0.5 point)

An investor currently holds Portfolio 1 and is considering moving all investments from Portfolio 1 into Portfolio 2. Explain how this will affect the investor's overall risk.

#### d. (1.5 points)

Construct a portfolio that would give the investor the same expected return as Portfolio 1 and the same Sharpe ratio as Portfolio 2. Briefly explain how this will affect the investor's overall risk.

#### Exam 9, Spring 2016

# 4. (1 point)

Given the following information under the single index model:

- Portfolio A's  $\alpha = 2\%$ .
- Portfolio A's  $\beta = 1.25$ .
- a. (0.5 point)

Construct a tracking portfolio that matches the systematic component of portfolio A's return.

# b. (0.5 point)

Demonstrate how to use the tracking portfolio above to earn a positive excess return with no exposure to systematic risk.

#### 5. (2.75 points)

Given the following information:

- Risk free rate is 5%.
- Expected market return is 15%.
- The market price of risk is 40%.
- The covariance of stock A with the market is 20%.
- The current price of stock A is \$90.

#### a. (0.75 point)

Calculate the expected return for stock A using the Capital Asset Pricing Model.

## b. (0.5 point)

Explain how the current price of stock A would be expected to change if investors forecast the price of stock A to be \$99 in one year.

#### c. (0.5 point)

The current price of stock A remains at \$90 despite the forecasted price of \$99 in one year. Briefly describe an arbitrage opportunity that exists.

#### d. (1 point)

Describe two limitations to being able to take advantage of an arbitrage opportunity.

# 6. (2 points)

Given the following information:

	Total Market		Average Annual
Portfolio	Value	β	Return
Α	\$ 1 million	0.9	7.1%
В	\$ 58 million	1.5	7.8%
С	\$120 million	1.1	6.7%

- The risk-free rate is 2%.
- The expected market return is 6%.

#### a. (1 point)

Calculate the alpha of each portfolio.

#### b. (0.5 point)

Describe a market anomaly associated with these three portfolios that appears to contradict the semi-strong form of the efficient market hypothesis.

# c. (0.5 point)

Describe an extension of CAPM that addresses the cause of the anomaly identified in part b. above.

# Exam 9, Spring 2016

# 7. (1.75 points)

a. (0.75 point)

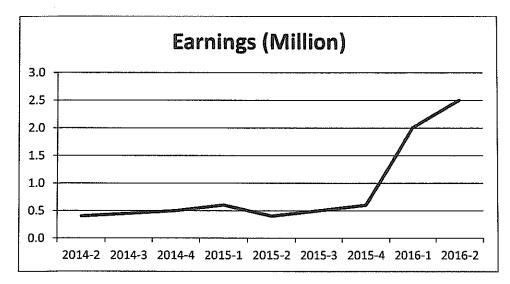
Briefly describe the three different versions of the efficient market hypothesis.

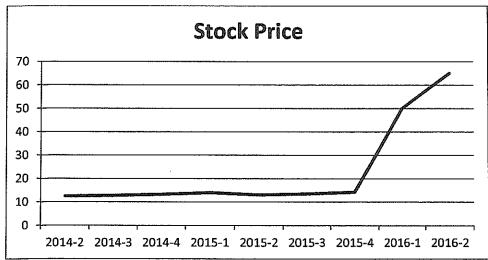
b. (1 point)

Describe two reasons why it is difficult to determine conclusively that the market is efficient.

#### 8. (1.5 points)

Given the following quarterly historical information for a firm:





- The market forecasted earnings for 2016-3 are \$3.5 million.
- The market forecasted stock price for 2016-3 is \$90.

#### a. (1 point)

Identify and briefly describe two potential information processing errors in the scenario above.

#### b. (0.5 point)

An individual investor believes the forecast contains information processing errors as described in part a. above. Briefly explain how the investor expects the stock price to change and identify an investment strategy this investor should pursue.

# 9. (2 points)

An actuary is testing the expected return-beta relationship implied by the Capital Asset Pricing Model (CAPM) using a two-step regression procedure. To run the regression, the actuary collected monthly rates of return on various stocks, the S&P 500 index, and 1-month (risk-free) T-bills over a five-year sample period.

The first-pass estimates were used as inputs for the following second-pass regression:

$$\overline{r_i - r_f} = \gamma_0 + \gamma_1 b_i + \gamma_2 \sigma^2(e_i)$$

The second-pass regression produced the following coefficients:

Coefficient	Statistically Significant?
$\gamma_0 = 0.003$	No
$\gamma_1 = 0.175$	Yes
$\gamma_2 = 0.300$	Yes

The market risk premium is 9%.

#### a. (1.5 points)

Briefly describe what each coefficient represents and, using the information above, assess whether or not each coefficient is consistent with CAPM.

#### b. (0.5 point)

Briefly describe two criticisms with this regression approach for testing CAPM.

# 10. (3 points)

Given the following information:

Bond	Par Value	Time To Maturity (years)	Annual Coupon Rate	Current Bond Price
1	\$100	1	3%	\$98.10
2	\$100	2	5%	\$99.10
3	\$100	3	5%	X
4	\$100	4	3%	\$86.65

- The 1-year forward rate for year 4 is 10%.
- The liquidity premium in year 3 is 1%.
- Coupons are paid annually.
- Assume annual compounding.
- a. (1.5 points)

Calculate X.

b. (0.75 point)

Calculate the price of bond 3 at time 2 under the expectations hypothesis theory. Assume the coupon has already been paid.

c. (0.75 point)

Calculate the price of bond 3 at time 2 under the liquidity preference theory. Assume the coupon has already been paid.

## 11. (2 points)

Company X entered into a forward rate agreement in which it will lend \$100,000 at 4% to company Y from January 1, 2017 to January 1, 2018. The value of this agreement at January 1, 2016 is \$193.12.

#### Assume:

- The one-year spot rate at January 1, 2016 is 3.5%.
- The one-year forward rate from January 1, 2018 to January 1, 2019 is 4.1%.
- Annual compounding.

# a. (1.25 points)

Draw the three-year, implied yield curve on January 1, 2016. Label all axes and data points.

#### b. (0.75 point)

Briefly describe the implied yield curve using the three main theories for the term structure of interest rates.

# 12. (2 points)

Given the following securities:

- Fixed maturity bond
- Callable bond
- Mortgage-backed security
- a. (0.75 point)

Draw the price-yield curve for each security. Where appropriate label axes, call price, and principal balance.

b. (0.75 point)

Briefly describe the convexity of each security.

c. (0.5 point)

Discuss the difference between the features that cap the maximum price of a callable bond and the features that cap the maximum price of a mortgage-backed security.

# Exam 9, Spring 2016

# 13. (1.75 points)

a. (1 point)

Briefly explain four potential problems a company may face with a volatile Market Value Surplus.

b. (0.75 point)

Explain what a positive duration gap of surplus means for a company in a rising interest rate environment.

#### 14. (2.5 points)

Given the following information for an insurer's property-casualty line of business for accident year 2015:

- The risk-free rate is 2%.
- The new money yield is 6%.
- The insurer has the following projected loss reserve payout pattern:

Development Year 1 2 3
Percentage Paid 27.3% 33.7% 19.0%

The remaining liabilities are paid out evenly over the following two years.

• Losses are paid at the end of each year.

#### a. (1 point)

Calculate the Macaulay duration for the accident year 2015 loss reserves.

#### b. (0.5 point)

The company is considering funding this liability with a bond portfolio having a duration equal to the duration of loss reserves. Explain if this is an appropriate choice if the inflation rate is currently 3%.

#### c. (1 point)

Explain why having mismatched duration of assets and liabilities is not necessarily a major problem for P&C insurers, and identify a situation when it becomes a problem.

## 15. (2.75 points)

Given the following annual information regarding an insurance company:

- Surplus is \$400,000.
- Written Premium is \$398,352.
- Expenses are \$100,000.
- Expected Loss + ALAE is \$300,000.
- Risk-free rate is 4%.
- Total Market Value is \$657,152.
- Premiums are collected and expenses are paid at the beginning of each year.
- Losses are paid at the end of each year.

#### a. (1.25 points)

Assume the company uses a pricing strategy that involves a fixed target return on surplus.

Calculate the assumed client retention rate implied by the market value.

#### b. (0.5 point)

Assume the company uses a pricing strategy that varies the target return on surplus (k) according to this formula: k = a + b \* y

where:

a = 8%

b = 1.5

Calculate the assumed client retention rate implied by the market value.

#### c. (0.5 point)

Describe which pricing strategy the company is more likely to be using.

#### d. (0.5 point)

Calculate the duration of franchise value using the pricing strategy assumptions in part b. above.

# 16. (2 points)

A bank is considering acquiring a large portfolio of similar loans. The two portfolio options the bank is considering have the following characteristics:

<u> </u>	Portfolio A	Portfolio B
Exposure	100 million	100 million
Average probability of default in 3 years	1.5%	3.0%
Estimated correlation between loans	0.20	0.05
Average Recovery Rate	55%	50%

- Assume equal default rates for each loan within both portfolios.
- Assume equal correlation between each loan within both portfolios.
- Assume the Value at Risk (VaR) follows a Normal distribution.

#### a. (1 point)

Using the 3-year 99% credit VaR as a decision framework, evaluate which portfolio the bank should acquire.

#### b. (1 point)

The CRO wants to have a more robust analysis of the credit risks of the portfolios before making a decision. Discuss an alternative approach that would provide a more accurate assessment of the 3-year 99% credit VaR.

## 17. (2.25 points)

Below is a table summarizing derivatives with three counterparties. The value columns are from the perspective of XYZ Corporation.

Counterparty	Credit Rating	Number of Transactions	Value of in-the- money derivatives (Million)	Value of out-of- the-money derivatives (Million)
Α	AAA	20	\$1,200	\$ 0
В	В	1	\$1,200	\$ 0
C	С	38	\$ 480	\$380

The CFO of XYZ Corporation has the option to add credit mitigation clauses to outstanding derivatives with the three counterparties.

Assume all contracts expire on the same date.

For each counterparty, describe a strategy to mitigate credit risk and justify your choice.

#### 18. (2 points)

Given the following for a group of mortgages:

- Principal amounts are \$1 million each.
- Probability of default is 5% for each mortgage.
- Defaults are uncorrelated.
- There are no recoveries on defaults.

Assume all CDOs and CDO2s contain two underlying securities, and all securities have two tranches.

#### a. (1.5 points)

Construct a synthetic security from the junior tranche of a CDO and the junior tranche of a CDO<sup>2</sup> with tranches of size \$1 million and calculate the default probability of the senior tranche of the security.

# b. (0.5 point)

Explain how an increase in the correlation between the underlying mortgages should affect the rating agency's ratings for the senior tranche in part a. above.

#### 19. (3.75 points)

ABC Reinsurance Company is aiming to mitigate its catastrophe risk through a purchase of an Industry Loss Warranty (ILW).

#### Assume that:

- ABC provides \$10 million excess of \$5 million reinsurance to Company XYZ on XYZ's catastrophe exposures.
- ABC pays a premium rate on line (ROL) of 10% to purchase \$10 million coverage from a binary triggered ILW with a \$1 million retention trigger and a \$25 billion warranty trigger.
- 1,000 simulations of annual catastrophe loss events with equal probabilities were conducted.
- Only 10 out of 1,000 simulated scenarios generated non-zero modeled losses to either XYZ or to the Industry.
- ABC's management sets its risk capital to equal to the 99.5% Conditional Tail Expectation (CTE) minus the expected loss of the simulated loss distribution.
- ABC's internal guidelines are such that for the ILW to serve as an effective risk
  mitigation tool, it has to provide a risk capital reduction in excess of the premium
  paid.

Ignore any acquisition expenses.

o:	Industry Loss	Ground Up Loss
Simulation	(Million)	for XYZ (Million)
1	50,000	5,500
2	40,000	25,000
3	35,000	7,000
4	30,000	10,000
5	20,000	5,500
6	10,000	10,000
7	10,000	6,500
8	5,000	10,000
9	5,000	7,000
10	5,000	2,000

#### a. (3.25 points)

Using ABC's internal guidelines, evaluate whether the above ILW is an effective risk mitigation tool for ABC. Show all calculations and assumptions.

#### b. (0.5 point)

Discuss what basis risk means in the context of using the ILW to mitigate catastrophe risk.

#### 20. (2.5 points)

An insurance company writes one line of business with the following probability distribution of incurred losses:

Incurred Loss	Probability
\$10,000	0.50
\$20,000	0.30
\$30,000	0.20

The company wants to reduce their capital requirement by writing two independent lines of business with the same loss distribution above and participating in a 50% quota share reinsurance agreement.

- The company targets an EPD (expected policyholder deficit) ratio of 5%.
- Assume assets are invested at a risk-free rate of 0%.

Calculate the reduction in capital required by adding the second line of business and entering into the quota share reinsurance agreement.

## Exam 9, Spring 2016

# 21. (1.5 points)

For each situation below, discuss the firm's need to engage in managing the volatility of its financial assets.

a. (0.5 point)

A firm with a low debt-to-equity ratio and an AAA credit rating.

b. (0.5 point)

A firm with a medium debt-to-equity ratio and a Baa credit rating.

c. (0.5 point)

A firm with a high debt-to-equity ratio and is in distress.

#### 22. (1.75 points)

A newly formed insurance company is considering whether to write one or all three of the lines shown below. The insurer will base its decision on the expected risk-adjusted economic performance of each of the choices available. Given the following:

Line of Business	Premium	Discounted Loss Ratio	Expense Ratio	Stand Alone Capital
Α	1,493,000	65.0%	27.5%	800,000
В	1,750,000	75.0%	15.0%	1,250,000
С	2,520,000	57.5%	30.0%	2,250,000
Total	5,763,000			4,300,000

_	Lines of Business	Line of Business Added	Merton-Perold Marginal Capital	Myers-Read Marginal Capital
	A&B	С	700,000	800,000
	A&C	В	900,000	1,200,000
	B&C	Α	1,250,000	1,350,000
_				
	Total		2,850,000	3,350,000

- Cost of capital is 15%.
- Expenses are paid at the beginning of the year.
- · Assume no investment income on capital.

#### a. (0.5 point)

Based on the information provided, recommend a total amount of capital needed to write all three lines.

# b. (0.25 point)

Explain why Merton-Perold's total allocated capital is lower than the sum of the standalone capital.

#### c. (1 point)

Discuss whether the company should write a standalone line of business or all three lines together.

# 23. (1.25 points)

Management is choosing between two new lines of business with the following characteristics:

	Line A	Line B
Expected Premium (million)	6	4
Required Capital (million)	4	2
Expense Ratio	20%	30%
Discounted Loss Ratio	75%	65%

#### For both lines:

- Investment return is 5%.
- Expenses are paid immediately.
- Losses are paid at the end of the year.
- · Capital is released at the end of the year.

Using a risk-adjusted evaluation, determine which line should be pursued by management.

#### 24. (3.25 points)

Assume the following for a property and casualty insurance company writing Workers Compensation insurance:

- All policies are annual policies written on January 1<sup>st</sup>.
- Initial premium is collected at inception in the amount of \$1,000.
- Expenses are paid at inception in the amount of \$200.
- Expected loss ratio is 60% relative to initial premium collected at inception.
- 35% of losses are paid at the end of the first year, and the remainder are paid at the end of the third year.
- The company allocates surplus to maintain a 2:1 undiscounted loss reserve-tosurplus ratio.
- Cost of Capital is 5%.
- Investment yield is 7.5%.
- Investment income is paid at the end of each year.
- Ignore taxes.
- An exposure audit is performed three months after all policies expire.
- An audit premium or credit is expected as the result of the exposure audit.

Calculate the amount of audit premium that will result in an IRR of 10%.

# 25. (1.5 points)

Given the following information about insurance companies A and B:

# Company A

Investment gain or loss (after appropriate tax charge)	5%
Premium-to-Surplus ratio	0.5
Underwriting profit or loss as a percentage of premium	2.4%
Reserves-to-Surplus ratio	0.2

# Company B

Investment gain or loss (after appropriate tax charge)		5%			
Underwriting profit or loss (after appropriate tax charge)	\$	6			
Capital, surplus, and equity in unearned premium reserves					
Reserves and other liabilities (excluding equity in unearned premium reserves)	\$2	00			

# a. (1 point)

Calculate the return on owners' equity for each company.

# b. (0.5 point)

Briefly describe the "interest" that an insurer pays for the use of reserve capital and contrast this with the interest a financial institution pays for debt capital.

#### 26. (2.25 points)

Consider the following formula for the Present Value of Income over Present Value of Equity Model:

$$PVI/PVE = \frac{\sum_{j=0}^{j} INC_{j} \cdot v_{(m)}^{j}}{\sum_{j=1}^{j} EQB_{j} \cdot v_{(m)}^{j}}$$

# a. (0.5 point)

Describe one issue that would arise when switching from an annual frequency model to a quarterly frequency model for computing the underwriting profit provision using the formula above.

#### b. (0.75 point)

Identify three rationales for using a pre-tax, risk-free rate for discounting in the numerator.

#### c. (1 point)

Suppose there has been a decrease to the discount rate while the positive target return remains constant. Discuss the leverage impact and the resulting change to the underwriting profit provision in the annualized PVI/PVE model.

# Exam 9, Spring 2016

# 27. (1 point)

a. (0.5 point)

Briefly describe two sources of risks contributing to surplus need.

b. (0.5 point)

Briefly describe two reasons why allocation of surplus by line of business using premium to surplus ratios may be problematic for a multi-line national insurer.

# 28. (2.5 points)

Given the following information:

- Assume a reinsurance contract covering two years with a single loss payment at the end of the two years.
- Assume no expenses.

Mean of loss distribution	200,000
Standard deviation of loss distribution	300,000
Dollar safety level associated with loss distribution	2,000,000
Annual expected yield rate of target investment	7%
Standard deviation of investment yield over one year	20%
Standard deviation of investment yield over two years	30%
Annual risk free rate	3%

# a. (1.5 points)

Use the swap technique to calculate the time-0 asset that needs to be allocated by the reinsurer for the reinsurance contract.

#### b. (1 point)

Calculate the reinsurance premium.

#### 29. (2.5 points)

A property reinsurer currently writes the following two renewal accounts with modeled losses as shown below:

	Event	Losses For Account (in \$000's)				
Event	Probability	X	Y	X + Y		
1	0.5%	\$100	\$85	\$185		
2	1.0%	\$ 75	\$60	\$135		
3	2.5%	\$ 35	\$25	\$ 60		
4	5.0%	\$ 20	\$15	\$ 35		
Coefficient of Variation		3.97	4.11	4.03		

- The target return on marginal surplus is 12.5%.
- Using the marginal surplus method, the reinsurer calculated the sum of the risk loads for account X and Y to be \$4,071.

# a. (2 points)

Determine the probability that the actual result will require more surplus than what was allocated.

# b. (0.5 point)

Identify an alternative approach to using the marginal surplus method for calculating risk loads and briefly describe an advantage to that approach.

Exam 9
Financial Risk and Rate of Return

# POINT VALUE OF QUESTIONS

OHECTION	VALUE	SUB-PART OF QUESTION						
QUESTION	OF QUESTON	(a)	(b)	(c)	(d)	(e)	<b>(f)</b>	(g)
1	1.25	0.50	0.75					
2	1.75	0.50	0.25	0.50	0.50			
3	4.50	1.00	1.50	0.50	1.50			
4	1.00	0.50	0.50					
5	2.75	0.75	0.50	0.50	1.00			
6	2.00	1.00	0.50	0.50				
7	1.75	0.75	1.00					
8	1.50	1.00	0.50					
9	2.00	1.50	0.50					
10	3.00	1.50	0.75	0.75				
11	2.00	1.25	0.75					
12	2.00	0.75	0.75	0.50				
13	1.75	1.00	0.75			·		
14	2.50	1.00	0.50	1.00				
15	2.75	1.25	0.50	0.50	0.50			
16	2.00	1.00	1.00					
17	2.25	2.25						
18	2.00	1.50	0.50					
19	3.75	3.25	0.50					
20	2.50	2.50						
21	1.50	0.50	0.50	0.50				
22	1.75	0.50	0.25	1.00				
23	1.25	1.25						
24	3.25	3.25						
25	1.50	1.00	0.50					
26	2.25	0.50	0.75	1.00				
27	1.00	0.50	0.50					
28	2.50	1.50	1.00					
29	2.50	2.00	0.50					
30	0.00							
31	0.00			_				
32	0.00							
33	0.00							
34	0.00							
35	0.00							
36	0.00		,.					
37	0.00							
38	0.00							
39	0.00							
40	0.00							
41	0.00							
42	0.00							
43	0.00							
44	0.00							
45	0.00							

TOTAL 62.50

# Tables of the Normal Distribution

Probability Content from -∞ to Z								
Z   0.00	0.01 0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0   0.5000	0.5040 0.508	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1   0.5398								
0.2   0.5793 (								
0.3   0.6179 (								
0.4   0.6554	0.6591 0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
	0.6950 0.698							
0.6   0.7257 (								
0.7   0.7580 (								
0.8   0.7881 (								
0.9   0.8159 (								
1.0   0.8413 (								
	0.8665 0.8686							
1.2   0.8849								
1.3   0.9032 0								
1.4   0.9192 0								
1.5   0.9332 0								
1.6   0.9452 0								
1.7   0.9554 0								
1.8   0.9841 0								
2.0   0.9772 0								
2.1   0.9821 0								
2.2   0.9821 0								
2.3   0.9893 0								
	0.9920 0.9922							
	0.9940 0.9941							
2.6   0.9953 0								
2.7   0.9965 0								
2.8   0.9974 0								
2.9   0.9981 0								
3.0   0.9987 0	0.9987 0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.3300

# Values of z for selected values of Pr(Z<z)

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z <z)< th=""><th>0.800</th><th>0.850</th><th>0.900</th><th>0.950</th><th>0.975</th><th>0.990</th><th>0.995</th></z)<>	0.800	0.850	0.900	0.950	0.975	0.990	0.995

#### SAMPLE ANSWERS AND EXAMINER'S REPORT

#### **GENERAL COMMENTS:**

- Candidates should note that the instructions to the exam explicitly say to show all work; graders
  expect to see enough support on the candidate's answer sheet to follow the calculations
  performed. While the graders made every attempt to follow calculations that were not welldocumented, lack of documentation may result in the deduction of points where the
  calculations cannot be followed or are not sufficiently supported.
- Candidates should justify their arguments/positions. It is generally not sufficient to choose an
  option. An explanation should be provided for the reasoning behind the choice. Candidates
  should note that a restatement of a numerical selection in words is not a justification. See the
  specific question comments for more examples.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- Candidates should try to be cognizant of the way an exam question is worded. They must look
  for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future
  Fellows article from December 2009 entitled "The Importance of Adverbs" for additional
  information on this topic.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).

#### **EXAM STATISTICS:**

Number of Candidates: 521Available Points: 62.50

Passing Score: 44.00

Number of Passing Candidates: 254

Raw Pass Ratio: 48.8%Effective Pass Ratio: 50.4%

#### SAMPLE ANSWERS AND EXAMINER'S REPORT

#### **QUESTION 1**

TOTAL POINT VALUE: 1.25 LEARNING OBJECTIVE: A1

#### **SAMPLE ANSWERS**

Part a: 0.5 point(s)

Since the expected return of the S&P 500 index exceeds the risk-free borrowing rate, the investor should allocate as much funds as possible to achieve the greatest expected return.

$$.25 = y^*.2 \rightarrow y^* = 1.25 .25 = y^* (.2) \rightarrow y^* = 1.25$$

$$(1-1.25)(\$100,000) = -\$25,000$$

The investor should borrow \$25,000 at the risk-free rate

#### **Part b:** 0.75 point(s)

$$E(r_c) = y^* E(r_p) + (1 - y^*) r_f = (1.25)(.115) + (-.25)(.06) = 12.88\%$$
  
$$\sigma_c = y^* \sigma_p = (1.25)(.2) = .25$$

#### **EXAMINER'S REPORT**

Candidates were expected to know calculations needed to identify optimal portfolio investments given different assumptions so as to calculate investment mix and expected returns and standard deviations.

Performance was generally good on this question.

Common error was in calculation of amount to invest in risky portfolio along with assuming incorrect risk-free rate in part b.

#### Part a

Candidates were expected to know calculations needed to determine amount to invest in risk-free rate with given assumptions.

Common mistake included incorrect calculations for y\*, such as using this formula:

$$y^* = (E(rp) - rf)/AOp2$$

#### Part b

Candidates were expected to know how to calculate portfolio expected return and standard deviation given response to part a.

Common mistake involved incorrect assumption around which risk-free rate to use. Depending on the answer to Part a (positive or negative y\*), candidates should use 4% risk-free rate for lending or 6% risk-free rate for borrowing).

QUESTION 2	
TOTAL POINT VALUE: 1.75	EARNING OBJECTIVE: A3, A10

#### **SAMPLE ANSWERS**

# Part a: 0.5 point(s)

- Global Minimum Variance Portfolio
- Minimum Variance Portfolio
- The asset on the on the efficient frontier with minimum variance
- A point on the efficient frontier
- Minimum variance point of efficient frontier
- Indifference Curve
- Graphs the risk + return relationships an investor is indifferent about given their risk aversion level
- Curve of equal utility for investor

## Part b: .25 point(s)

- Where curve c tangent with curve b is the optimal risky portfolio
- Optimal risky portfolio is point a which curve b and line c meet
- Where the efficient frontier(b) intersects the CAL(c)
- The point at the intersection of line c and curve b

## Part c: 0.5 point(s)

- Different level of risk aversion will result in varying allocation of risk free asset and optimal risky portfolio along line c.
- Depends on their risk aversion (A) -> would dictate the %invested in the r<sub>f</sub> part.
- May have different risk preference, so rational can invest along line c (CAL) and allocated btw risky and r<sub>f</sub> asset.
- Based on separation property we already know where optimal risky portfolio is so only thing that matter now is how to allocate between r<sub>f</sub> and optimal risky portfolio. Based on Utility function of investor. Rational investors will select a point on line c that maximizes the function.
- Investor have different Utility functions so while the ORP are the same, the OCP with risk free asset differ. Other investors will have different indifference curves with different tangency points.

## Part d: 0.5 point(s)

- Forecasting error F may have had recent good experience and investor gives too much weight to it, believes high return will continue in the future.
- Conservatism The Investor could be slow to react to new info that would push their portfolio to a new CAL where portfolio D lies.
- Overconfidence An investor may exhibit overconfidence in her ability to beat the market by active investing, resulting a suboptimal allocation at F, rather than adopting a passive approach and selecting D.
- Sample Size Neglect and Representativeness An investor might identify a pattern too quickly extrapolate trends too far out into the future.
- Framing An investor may act differently where a decision is framed in different ways. They could be risk seeking when it comes to losses but risk averse when it comes to gains.

- Mental Accounting Investors tend to make decisions in isolated buckets rather than for the portfolio as a whole, this may lead to suboptimal securities within the portfolio.
- Regret Avoidance If portfolio F has more conventional assets while D is less conventional, then investors with regret avoidance may feel worse if he loses money in portfolio D than portfolio F.
- Prospect Theory alters the traditional indifference curves. For example, an investor may be more concerned with changes in wealth than levels of wealth.
- Affect F has negative alpha suggesting below market returns but if F contains
  investments that serve a social purpose, investors may be willing to accept lower returns
  to "feel good".
- Loss Avoidance An investor is hesitant or unwilling to realize losses on a bad investment, so he holds on to an inefficient portfolio.
- Information Processing Error- like forecasting errors may misestimate probabilities and/or parameters which may lead to a different capital allocation line.

#### **EXAMINER'S REPORT**

- The candidate was expected to identify the components of the Markowitz Portfolio Selection Process and explain the process in detail. Candidates also were expected to demonstrate their understanding of behavioral finance and how it influences sub-optimal (inefficient) decisions by individual investors.
- Candidates generally performed well on this question. The common mistakes were the
  misclassification of the indifference curve, the switching of optimal risky portfolio for
  optimal complete portfolio and the misclassification/poor explanation of behavioral
  finance topics.

#### Part a

- The candidate was expected to identify the components of the Markowitz Portfolio selection
- For full credit point A was expected to be labeled as the point on the efficient frontier with the lowest variance, or the global minimum variance portfolio. Curve E was the indifference curve that plotted the Expected return & standard deviation of all portfolios that yielded the same utility given the investor's risk aversion level.
- Most candidates scored well on the first item. The most common mistake on the 2<sup>nd</sup> item was calling the indifference curve the utility curve. In the discussion of the Markowitz Portfolio Selection Process, utility was only used in determining the indifference curve which is what was plotted on a graph with E(r) and σ on the axes.
- Because only the efficient frontier of the minimum variance frontier was shown, it was
  expected that the candidate would recognize the global minimum variance, however credit
  was awarded if the candidate recognized the minimum variance frontier and/or the
  efficient frontier without a direct reference to the global minimum. If candidates
  explained the indifference curve as having the same utility value without labeling it as
  "indifference curve" they were awarded full credit.

#### Part b

- The candidate was expected to know where the optimal risky portfolio was located on the graph depicting the MPS.
- Majority of candidates scored full credit. A few incorrectly inferred that point D was the optimal risky portfolio, when it was actually the optimal complete.

## Part c

- Candidate was expected to know how investor preferences played into the selection of the optimal complete portfolio (OCP)
- For full credit, candidates must explain how the individual preference for risk or utility led to different allocations of the risk free and optimal risk portfolio. The explanation could have been conceptual or graphical, the point being that each investor will still have a portfolio on the CAL to the left or right of point D, with more or less of the risky portfolio depending on risk aversion/utility levels.
- A common omission was not explaining how being more or less risk averse related to the nature of the optimal complete portfolio. Some candidates gave explanations from irrelevant sections of the syllabus, such as behavioral finance. Some candidates attempted to explain using reasons relating to maximizing expected return or minimizing variance.

#### Part d

- The candidate was expected to know how behavioral finance explained investors making
  decisions contrary to the efficient market hypothesis. Although many different topics were
  discussed, candidates were only expected to describe one topic in detail
- Candidates received partial credit if they were able to identify but not able to describe how the behavior affected the decision making of a rational investor.

#### **QUESTION 3**

TOTAL POINT VALUE: 4.50 LEARNING OBJECTIVE: A4

#### **SAMPLE ANSWERS**

#### Part a: 1 point(s)

- Risk Pooling: increase investment by adding uncorrelated securities to a portfolio
- Risk Sharing: keep total investment fixed and allocate amongst additional securities
- Risk Pooling Example: selling additional policies to uncorrelated insureds
- Risk Sharing Example (1): selling more insurance but cede some business to reinsurers
- Risk Sharing Example (2): selling shares of the company to additional investors

#### Part b: 1.5 point(s)

## Portfolio 1

Exp Return = (.3)(.1) + (.7)(.02) = .044Variance =  $(.3^2)(.25^2) + (.7^2)(0^2) = .005625$ Sharpe =  $(.044 - .02)/(.005625^{1/2}) = .32$ 

## Portfolio 2

Exp Return = (.3)(.1)(3) + (.1)(.02) = .092Variance =  $(.3^2)(.25^2)(3) + (.1^2)(0^2) = .016875$ Sharpe =  $(.092 - .02)/(.016875^{1/2}) = .5543$ 

#### Part c: 0.5 point(s)

This will increase risk as portfolio 2 has a greater variance than portfolio 1

# Part d: 1.5 point(s)

# Sample 1

Invest in 10% in each of the asset A,B,C and invest the remaining 70% in risk free.

 $E(r_p) = (0.1)(3)(0.1) + (0.7)(0.02) = 0.044$ 

 $Var_p = (3)(0.1)^2(0.25)^2 = 0.00188$ 

Sharpe<sub>p</sub> =  $(.044 - .02)/(.00188)^{1/2} = 0.5543$ 

The overall risk is reduced. Since the risk measure standard deviation reduced from 0.075 to 0.043.

# Sample 2

$$\begin{split} &E(r_c) = .044 = w(r_p) + (1\text{-}w)(.02) \\ &Sharpe_c = (r_p - .02)/sd_p = .5543 \\ &sd_c = .0433 = (.044 - .02)/.5543 \\ &sd_c = w*sd_p = w(.1299) = .0433 \\ &w = .3333 \end{split}$$

Invest 33.33% in Portfolio 2 and 66.67% in D

 $sd_c = (.3333)(.1299) = .0433 < .075$  so this will reduce overall risk.

#### **EXAMINER'S REPORT**

#### Part a

In this part the candidate was expected to define and provide insurance relevant examples for both risk pooling and risk sharing.

In order to receive full credit the candidates needed to fully define risk pooling and risk sharing including that additional risks were uncorrelated (or similar adjective) and that for risk sharing the portfolio size remained fix.

Common mistakes included not fully defining risk pooling or risk sharing and not providing applicable insurance related examples.

## Part b

In this part the candidate was expected to calculate the expected return, variance, and Sharpe ratio for two different portfolios.

In order to receive full credit the candidates needed to correctly calculate each of the six required elements.

Common mistakes included calculating standard deviation, rather than variance, and using incorrect formulas for variance and/or Sharpe ratio.

## Part c

In this part the candidate was expected to identify the impact to the total risk by shifting an investment entirely in Portfolio 1 to Portfolio 2.

In order to receive full credit the candidate needed to correctly identify the impact to overall risk and identify a reason for the impact. Simply stating, "risk increases" was not enough for full credit.

Common mistakes included not providing a reason for the change in risk.

# Part d

In this part the candidate was asked to construct a portfolio with certain characteristics of Portfolio 1 and Portfolio 2.

In order to receive full credit the candidate needed to correctly calculate the percentage allocation to each of assets A, B, C, and D as well as illustrate that the resulting expected return matched that of Portfolio 1 and the resulting Sharpe ratio matched that of Portfolio 2. The candidate also needed to state the impact to overall risk of the new portfolio compared to Portfolio 1 or 2.

Common mistakes included incorrectly calculating the percentages to each asset, and not identifying the impact to overall risk.

QUESTION 4	
TOTAL POINT VALUE: 1	LEARNING OBJECTIVE: A5

#### **SAMPLE ANSWERS:**

#### Part a: 0.5 point(s)

## Sample 1

Tracking portfolio would consist of 125% market portfolio and borrowing 25% at the risk free rate. This will result in a portfolio with  $\beta$ = 1.25 and  $\alpha$ = 0. Thus the systematic component is matched.

## Sample 2

Invest 125% of funds in market, and borrow at risk free rate

# Part b: 0.5 point(s)

#### Sample 1

To use this tracking portfolio, purchase portfolio A, and sell short the tracking portfolio. The resulting combination of holdings will have  $\alpha$ = 2% and  $\beta$ =0 as the tracking portfolio as the tracking, portfolio will cancel out  $\beta$ A. Thus, no exposure to systematic risk, but a positive excess return

#### Sample 2

```
Short tracking portfolio + use proceeds to invest in A. \alpha= 1 x .02 – 1.25 x 0 + .25 x 0 = .02, \beta = 1 x 1.25 - 1.25 x 1 +.25 x 0 = 0
```

#### **EXAMINER'S REPORT**

Candidates performed well on this question with the majority receiving half or more points. Few candidates skipped or received no points.

## Part a

- This section required the construction of the tracking portfolio, not just a listing of its properties.
- Credit was awarded to candidates who correctly identified the long position of the portfolio as being the market (or index) portfolio. Factor portfolio with an associated beta=1 was accepted for full credit as well.
- Candidates who stated that 25% of the portfolio should be borrowed but did not state that this should happen at the risk-free rate were not awarded full credit.
- Candidates who inverted the long/short transaction order were not awarded full credit.

## Part b

- More candidates received full credit for part B than part A.
- This sub-part required candidates to demonstrate (not simply state) that there would be no exposure to systemic risk. They were expected to show how the betas/systematic risk would be offset in the transaction.
- Candidates who incorrectly designed the tracking portfolio in part A were not penalized if they went on to correctly show how combining the two portfolios would eliminate systematic risk. Many candidates received such partial credit.
- Candidates were required to identify that the overall return was the alpha of portfolio A, or 2%.

#### **QUESTION 5**

TOTAL POINT VALUE: 2.75 LEARNING OBJECTIVE: A6, A8, A10

# **SAMPLE ANSWERS**

# Part a: 0.75 point(s)

1) Market price of risk =  $0.4 = \frac{E(R_M)}{\sigma_M^2}$ 

$$\sigma_{\rm M}^2 = \frac{{\rm E}({\rm R}_{\rm M})}{0.4} = \frac{(0.15 - 0.05)}{0.4} = 0.25$$

$$\beta_{\rm A} = \frac{{\rm Cov}({\rm R}_{\rm A}, {\rm R}_{\rm M})}{\sigma_{\rm M}^2} = \frac{0.2}{0.25} = 0.8$$

$$E(r_A) = r_f + \beta_A [E(r_M) - r_f] = 0.05 + 0.8[.15 - .05] = 13\%$$

2) Market price of risk =  $0.4 = \frac{E(R_M)}{\sigma_M}$ 

$$\sigma_{M} = \frac{E(R_{M})}{0.4} = \frac{(0.15 - 0.05)}{0.4} = 0.25$$

$$\beta_A = \frac{\text{Cov}(R_A, R_M)}{\sigma_M^2} = \frac{0.2}{0.0625} = 3.2$$

$$E(r_A) = r_f + \beta_A [E(r_M) - r_f] = 0.05 + 3.2[.15 - .05] = 37\%$$

# Part b: 0.50 point(s)

## Sample 1

Investors forecasted E (RA) =  $\frac{99}{90}$  – 1= < .13 price of A will drop.

## Sample 2

Since 90 (1 + .13) = 101.7 > 99, the forecast projects lesser returns, so the price should be depressed (assume no dividend payments.)

#### Sample 3

This implies expected return of  $\frac{99}{90}$  – 1= 10%. If CAPM is accurate, current price should be X (1.13) =99 => X = 87.612

#### Sample 4

The stock price would drop. Investors expect to earn a return of 37% which is greater than  $\frac{99}{90} - 1 = 10$ %. So, the price will fall until investors can buy the stock at a price they expect to earn 37%.

# Part c: 0.50 point(s)

## Sample 1

Construct a portfolio with .8 portion in market index portfolio. .2 portion in risk-free asset. We can short 1 portion stock A, and buy 1 portion of this portfolio.

$$E[R] = .13 - .1 = .03$$

R=0

## Sample 2

An arbitrage opportunity exists BE/E stock A is overpriced and has a -X to take advantage, we must construct a portfolio of 3.2 units of exposure to the market. And borrow 2.2 units of risk-free, for an inv. Of \$1. Then we sell (Short) on Stock A for \$1 making our net investment 0, no exposure to suggest risk, and a riskless - • (-2) =2.

#### Sample 3

Create a tracking portfolio with the same Beta as Stock A. Purchase this and short sell Stock A.

#### Sample 4

Short sell Stock A for 90. Invest \$.8 (90) = 72 in market portfolio. And 0.2 in risk-free asset. Expected return =  $72 \times 1-15 + 18(1.05) = 101.7... 101.7 - 99 = 2.7$ . This creates a risk-free return of 2.7 per share without upfront investment.

#### Sample 5

Investors could create a tracking portfolio comprised of the market index and risk free asset that matches the systematic risk of stock. Investors could purchase the tacking portfolio and short sell the stock. This would produce an expected return of .03.

## **Part d:** 1.00 point(s)

#### Sample 1

Model risk:- Maybe the model is not right

Fundamental risks- the market may persist with overpriced securities- The overpricing can get worse.

#### Sample 2

Model risk- The current price of the stock is indeed valid I see that the arbitrageur's model has some issues which led it to falsely indicate mispricing.

#### Sample 3

Fundamental risk- risk that the market doesn't go as expected and arbitrage doesn't pay off. Transaction costs- Actual cost of implementation may make arbitrage a wash or even cost more money than earned.

#### **EXAMINER'S REPORT**

#### Part a

Candidates were expected to show that they knew the formulae for the market price of risk,  $\beta_A$ , and the expected return of an asset. In general, candidates performed very well on this part. Because of the confusion in the literature, between the syllabus readings and other sources on the same topic, as to whether the market price of risk was divisible by the market standard deviation or market variance, both were accepted. As a result, the expected return on the stock price was very different. It should be noted that the syllabus reading is clear on which is correct, and is more up-to-date than other sources on this topic. Moreover, candidates are encouraged, as always, to use the readings cited in the syllabus as their primary source of study. Therefore, candidates should not assume that both options will continue to be accepted in future sittings.

The most common error on this part of the question was using the market price of risk as the market standard deviation.

# Part b

Candidates were expected to explain that investors were expecting a lower return on the stock than CAPM, thus leading to a drop in the price due to its inferior return relative to its risk.

If the candidate came up with an expected return of less than 10% in part a, then full credit was given for a response of the price increasing due to increased demand.

The most common mistake was to assume the price would change from 90 to 99 as a result of investors' expectations.

#### Part c

Candidates were able to recognize that there was an arbitrage opportunity in shorting the stock. However, most candidates had trouble determining how to take advantage of the arbitrage opportunity. As with part b, if the candidate came up with an expected return of less than 10% in part a, full credit was given for a response of needing to short a tracking portfolio with the same beta as the stock and buy stock A.

#### Part d

Most candidates were able to at least list two limitations to arbitrage based on BKM chapter 12 and explain them. The biggest errors here were that candidates could either list the limitation but not necessarily explain them or vice versa. As an example, someone may have said that forecasting price isn't correct but not explicitly state model risk as a limitation.

#### **QUESTION 6**

TOTAL POINT VALUE: 2.0 LEARNING OBJECTIVE: A6, A9, A7

#### **SAMPLE ANSWERS**

#### Part a: 1.0 point

## Sample 1

$$\begin{split} E(r_A) &= \alpha_A + r_A + \beta_A \left[ E(r_M) - r_f \right] \\ .071 &= \alpha_A + .02 + .9 \left( .06 - .02 \right) \ => \ \alpha_A = \ .015 \\ .078 &= \alpha_B + .02 + 1.5 \left( .06 - .02 \right) \ => \ \alpha_B = \ -.002 \\ .067 &= \alpha_C + .02 + 1.1 \left( .06 - .02 \right) \ => \ \alpha_C = \ .003 \end{split}$$

# Sample 2

$$\begin{split} & E(r_A) = 0.02 + 0.9 \ (0.06 - 0.02) \\ & = 0.056 \\ & \alpha_A = 0.071 - 0.056 = 1.5\% \\ & E(r_B) = 0.02 + 1.5 \ (0.04) \\ & = 0.08 \\ & \alpha_B = 0.078 - 0.08 = -0.2\% \\ & E(r_C) = 0.02 + 1.1 \ (0.04) \\ & = 0.064 \\ & \alpha_C = 0.067 - 0.064 = 0.3\% \end{split}$$

# Parts b and c: 0.5 points for each part

## Small/Neglected Firm Effect

# Part b:

## Sample 1

Small firm effect – the lowest capitalization firm has earned an excess return ( $\alpha$ ) of 1.5% which is higher than all other firms/portfolios. Small firms tend to be researched less/have lower investor confidence. This idea of "lesser known" may cause these firms to be less liquid, resulting in an excess return that is actually demanded due to illiquid nature

# Sample 2

Small/Neglected Firm -> smaller/neglected firms have less information published about them so investors require higher abnormal returns to bear that additional risk

# Sample 3

Anomaly is that smaller market value portfolio has highest alpha. Publically available info such as market size should not allow for excess returns ( $\alpha$ ) if markets are efficient

#### Part c:

#### Sample 1

Liquidity Adjusted CAPM – CAPM adjusted to account for illiquidity of firm due to lack of analysis/trading described in (b). Thus, liquidity-adj. CAPM will account for  $\uparrow$  expected return not predicted in normal CAPM, reducing part A's  $\alpha$ 

## Sample 2

Fama French 3 factor model includes a small minus big factor in the equation to adjust for the excess returns that small firms experience over larger firms

## Alpha/Beta Relationship

#### Part b:

Low Beta portfolios earn a higher risk adj return than portfolios w/ high Beta. This contradicts S-S EMH b/c all public information should be reflected in the stock price, and  $\beta$ 's are public.

# Part c:

The Zero Beta CAPM reflects that many investors cannot actually borrow at the risk free rate and that assumption of CAPM is relaxed. As such, we see a higher y intercept and flatter SML than traditional CAPM would predict which we see in this data.

#### **EXAMINER'S REPORT**

Overall, candidates performed well on this question. Candidates were expected to calculate alphas of the 3 portfolios, describe a market anomaly indicated by the alphas, and describe an extension of CAPM that addresses the anomaly. Many candidates interpreted the portfolios to be individual firms; this was considered acceptable and full credit could be earned.

#### Part a

For this part, candidates were expected to calculate the alphas for all three portfolios. Most candidates received full credit on this part by correctly calculating the portfolio alphas. A common mistake was leaving the risk-free rate out of the formula, resulting in all positive alphas.

#### Part b

Most candidates noted that portfolio/firm "A" had the smallest market value and an alpha significantly greater than 0, and concluded that the data indicated the Small/Neglected Firm anomaly. To receive full credit, candidates needed to identify this anomaly and describe either why it can happen, or why it contradicts the Semi-Strong Form of the Efficient Market Hypothesis.

Alternatively, some candidates noted a relationship between the alphas and betas of the portfolios. To receive full credit, candidates were expected to note this relationship and describe how it contradicts the Semi-Strong Form of the Efficient Market Hypothesis.

The most common mistake was identifying but not describing an anomaly. Another common mistake involved using the portfolios' annual returns to justify an anomaly instead of looking at excess returns/alphas.

## Part c

Acceptable answers for this part depended on which anomaly the candidate identified in part (b). To receive full credit, candidates needed to identify and describe an extension of CAPM that addresses the anomaly identified in part (b).

<u>Small/Neglected Firm Effect:</u> The most common response was the Fama-French 3 Factor Model; this model was considered for full credit even though it is not necessarily an "extension" of CAPM. Another acceptable response is CAPM with liquidity adjustments.

<u>Alpha-beta relationship:</u> The most common answers in this scenario were Zero-Beta CAPM and Labor/Human Capital extension, which were both considered for full credit.

ICAPM and Consumption CAPM were also considered for partial or full credit depending on the description and the response to part (b). Common mistakes were inadequate descriptions and/or not tying answers to part (b).

$\cap$ I	JEST	$\Gamma$	NI 7
u	JEJ	IIU	IN /

TOTAL POINT VALUE: 1.75 LEARNING OBJECTIVE: A9

#### **SAMPLE ANSWERS**

Part a: 0.75 point(s)

Weak Form: stock prices reflect all information regarding past trading/market data

Semi-Strong Form: stock prices reflect all publically available fundamental information about the firm's prospects

Strong From: stock prices reflect all available information about a firm, including insider information

## Part b: 1.0 point(s)

Any two of the below reasons to earn full credit:

Selection Bias: investors don't share strategies than can beat the market. If they became publically known, they would no longer work since all investors would use it, so we only know about failed strategies.

Luck Event Issue: Given the large number of investors, it is likely to see some of them consistently outperform the market. However, it is not easy to determine if this is due to luck or skill.

Magnitude Issue: Due to the large volatility of market returns, it is difficult to identify excessive returns since they lie within the standard deviation of market returns.

Unobservable Market: The true market portfolio is unobservable due to non-traded assets and other factors. Therefore, even if we test a market proxy, we can only conclusively determine if the proxy is efficient; it is not truly indicative of the market.

Actual vs. Expected Returns: The efficient market hypothesis is built on expected returns; however we can only observe actual returns. It is therefore difficult to directly test the EMH.

Behavioral Bias: Some investors are not always rational and these irrational behaviors introduce inefficiencies into the market. Therefore, we cannot determine it is completely efficient.

Anomalies: There are a number of known anomalies (such as small firm in January effect) that have been identified. The existence of these anomalies makes it difficult to determine that the market is completely efficient.

Insider Trading: It is difficult to know whether some investors are using insider information. If they are and other investors are not, the prices wouldn't fully reflect this information and hence the market can't be strong form efficient.

#### **EXAMINER'S REPORT**

The candidate was expected to understand the three forms of the EMH and their implications as well as tests of the EMH.

Candidates generally scored well. Most candidates earned full credit for part (a), but may not have offered complete descriptions as required for full credit in part (b).

#### Part a

Where candidates lost credit for (a), a common mistake was to exclude reference to "market/trading" information specifically for the Weak Form EMH.

A few candidates misinterpreted what the exam was asking in part (a) and wrote descriptions for Expectations Hypothesis, Liquidity Preference Theory, and the Segmentation Theory instead of describing EMH forms.

## Part b

Where candidates lost credit for (b), a common mistake was to not offer complete descriptions. For example, if the candidate referred to the "selection bias", they may not have explained why the investor would not want to share his secret and how it would then be difficult to conclude efficiency.

QUESTION 8	
TOTAL POINT VALUE: 1.5	LEARNING OBJECTIVE: A

#### **SAMPLE ANSWERS**

Part a: 1.0 point(s)

#### Sample 1

Forecasting error – the investors may be forecasting the recently good performance too far into the future

Overconfidence – investors are often overconfident in their decision making. If this stock has started performing better, maybe they think this is due to something they associate with higher returns and over-trust their instincts

#### Sample 2

Forecasting error – too much weight is often put on recent experience when forecasting expected earnings/stock prices. Both were flat for a while and recently have had steep increases. Expectations are forecasted that these steep increases will continue instead of flatten out.

Overconfidence – people sometimes place too much confidence in the precision of their estimates. The steep increasing continuing is not a very conservative assumption, so predictors must be pretty confident in their skills to predict continued drastic increase.

#### Sample 3

Forecasting error -> investors tend to give too much weight to recent experience and project it forward.

Here recent good performance could be leading to over optimistic future predictions and therefore investors are paying too much for the security

Sample size neglect & representativeness -> investors overestimate the credibility of small sample size giving this info too much weight in there model.

w/only 2 Qtrs of strong earnings growth investors may be overreaching and overprojecting future earnings growth.

Possibly leading them to pay to much for the security

#### Sample 4

**Forecasting Errors** 

The recent (short period) of high stock growth may be projected into the future resulting in an extreme forecast.

Sample Size Neglect and representativeness

• The investors may be identifying a growth pattern from limited data i.e. growth from 2015-4 to 2016-2. Such a short period is not adequate to credibly identify a pattern

#### Sample 5

Forecast errors: investors may give too much weight to recent performance and make extreme forecasts. In this case they see prices increasing possibly due to recent success of new product and so expect that to continue

Sample size neglect/Representativeness: deriving patterns from limited sample size. In this case they saw an increase in price from 15-4 to 16-1 followed by another increase. They may infer a pattern based on just two samples

# Sample 6

Sample size neglect. We only have 9 qtrs of data but the stock price for 2016Q3 is expected to increase like earnings because each of the past 9 qtrs have followed earnings

Forecasting error – investors are putting too much weight on recent qtr data because price of last 2 qtrs increased w/earnings they expect next qtr to as well

## Sample 7

Forecasting error – too much weight given to recent experience when forecasting

-> May explain P/E effect when earnings forecasts are too high based on recent performance

Overconfidence – overestimate precision of beliefs and forecasting abilities

-> May explain higher trading activity

#### Sample 8

Forecast errors – people tend to extrapolate recent earnings growth too far into the future & tend to forecast the stock prices going up a lot as a result b/c people bid it up

Sample size neglect & representativeness – looks like the company has only been around for 2 yrs & the earnings have only been good for the last 2 quarters, so the investors are relying too much on such little data & time span.

#### **Part b:** 0.5 point(s)

#### Sample 1

Stock should come down so short at current prices.

#### Sample 2

The stock price may fall after the investors in the market realize the overreactions and correct the mistake. The investor who identify this in advance can short sell this stock now and profit.

## Sample 3

The investor recognizes the information process error so he expects the stock price to drop in the future.

He should either exclude this stock in his portfolio or short-sell on it.

## Sample 4

Investor believes price will decrease in the next few quarters as actual experience is less positive than forecast and investors correct for their overreaction. The investor could short-sell or buy puts on the stock if they want to make a speculative profit, but this is risky (they may have overconfidence). If more risk adverse, avoid buying stock until future price correction.

## Sample 5

The investor will think either price will go down, or, that it wont go up to \$90. Investor could buy put on stock. Since market expects stock to go up a lot, a put at high price will be relatively cheap. When price doesn't actually reach that level he can exercise put and sell at higher price.

#### Sample 6

Would expect stock price to be lower than expected.

Can buy a put option with strike equal to the forecasted price and earn a profit

# Sample 7

The investor expects the stock price to increase but not by as much as the forecast. Should buy the stock though since he still believes it will increase.

#### Sample 8

Given trends, investor may still expect price to rise, just not by as much as forecast. Investor could reduce exposure to this stock or diversify (assuming they hold just this one) to protect from the potential error in prediction.

#### Sample 9

Current prices is \$65, and investor believes \$90 is to high next quarter. Investor could sell a call option at the \$90 strike, when price fails to reach \$90 investor profits! Investor expects price to be below \$90

## Sample 10

Investor expects stock price to decline in the future as markets realize the irrational excitement built into price

An investment strategy could be to short the security. This is speculative so to reduce the risk could also long a portfolio w/same  $\beta$ .

## Sample 11

The underlying growth rate would be lower than what the latest 2 quarters indicate. The next quarter is likely to converge back to the company's underlying growth rate The stock price would likely to be lower than the forecasted 90, or even drop back to the historical level of \$10-\$20 range

I would suggest to not invest in this firm since it's not clear if the recent growth is real or temporary.

## Sample 12

Based on the forecasting error mentioned above the investor believes that the market forecasted stock price is too high. However, it still may increase from the current price of 60-70. Therefore the investor would still purchase the stock (but maybe not as much of the stock as he would if he believed the forecasted stock price of \$90).

#### Sample 13

- The forecasted earning is not going to be as high as forecasted and price goes down
- We over-react due to the trend we see in the small sample and the price falls back down as a result later

If the investor believes that the market is efficient, he should just use passive portfolio management and hold a well diversified portfolio that mirrors market portfolio.

#### Sample 14

He must think the price will fall, in which case he should sell the stock

#### Sample 15

Since stock price increases tremendously due to information processing errors above, the stock price will reverse downwards in the future. Investor should pursue contrarian strategy and short the stock.

#### **EXAMINER'S REPORT**

- The candidates were expected to recognize the information processing errors in a specific
  example of earnings and stock price forecast presented in the question. Additionally, the
  candidates were expected to describe how the stock price will be affected as a result of
  those errors being recognized and what an appropriate investment strategy would be in
  this case.
- Candidates generally scored well on the question as a whole
- The most common errors resulted from the candidates relating the textbook material to the specific stock price forecast example presented in the question
  - In majority of such cases, the candidates included the correct description of "conservatism" error in Part A and a correct investment strategy associated with that error in Part B, while failing to recognize that the conservatism error is not applicable to the specific stock forecast presented

#### Part a

- The candidates were expected to identify and describe two information processing errors
- Common errors made by candidates:
  - Listing "Conservatism"; while it is a possible information processing error as described in the exam material, it is not applicable to the specific stock forecast presented in the question. The forecast of a further jump in both earnings and price is indicative of an overreaction by analysts to the recent dramatic growth in earnings and stock price for this firm, whereas conservatism would lead investors to react more slowly to new information. A number of candidates suggested that the fact that the price chart didn't perfectly follow the earnings chart in some

- earlier quarters prior to 2016 was indicative of the presence of conservatism, but this was more representative of some minor random fluctuations rather than a delayed reaction to a trend.
- The use of the statement "forecasting too far into the future" without further explanation or without further relating it to the specific stock in the questions was not a sufficient response for describing either the forecasting error or the sample size neglect error
- Describing the sample size neglect error as "using data form a single firm" or discussing the small firm size instead of the small data sample size
- Overconfidence error was frequently confused with investors optimism about the stock price outlook
- Post earnings announcement drift was listed as an information processing error

#### Part b

- The candidates were expected to describe how the stock price will be affected as a result
  of information processing errors in Part A being recognized and what an appropriate
  investment strategy would be in that case.
- Common errors made by candidates:
  - Suggestions of trying to "time" the market (for example, wait until stock reaches its peak price to sell)
  - o "Contrarian strategy" or "buying recent losers" without explanation of what that strategy means in the context of the specific stock in question
  - Strategy of short selling the stock without expectations of the stock price going down
  - Candidates took the earnings/growth forecasts as fact (assumed it is correct as is), and based their discussion of the accuracy of the stock price forecast on how that relates to the earnings/growth forecast
- Candidates that provided a correct strategy that aligns with a conservatism error still
  received partial credit for Part B as long as their response in Part A also stated and
  correctly described conservatism.

QUESTION 9	
TOTAL POINT VALUE: 2.0	LEARNING OBJECTIVE: A11
SAMPLE ANSWERS	
Part a: 1.5 point(s)	

# Sample 1

YO is the non-market return based on investors' unique analysis (alpha). According to CAPM, this should be 0. Regression shows the coefficient is close to zero R is not statistically significant, so this is generally consistent W/ CAPM.

Y1 is the market risk premium. According to CAPM, should be equal to  $r_m$ - $r_f$ , which is 9%. The coefficient in the regression was found to be 17.5%, which is much greater than 9%, and it was statistically significant, so this is inconsistent with CAPM.

Y2 is the risk premium for firm specific risk. According to CAPM, this should be 0 b/c investors are only compensated for systematic risk. In regression, coefficient was found to be 0.3,, which is greater than 0, and statistically significant, so found that return is generated from firm specific risk. This is inconsistent W/ CAPM.

## Sample 2

Y0 = The alpha or excess return

Y1= Market risk premium

Y2 = Non-systematic risk

We expect Y0 = Y2 = 0 and Y1 = .09

Y0 is.003 but not significant – this is consistent

Y2 = .175 > .09 => Inconsistent

 $Y3 = .3 > 0 \Rightarrow$  Inconsistent

#### Part b: 0.5 point(s)

- stock returns are highly volatile which reduces accuracy of the test
- not using true market portfolio
- investors can't borrow at risk-free rate
- betas are measured with significant error in 1<sup>st</sup> pass regression.

#### **EXAMINER'S REPORT**

Candidates were expected to understand the 2 step regression test of CAPM, interpret the results of the test, and know the limitations of the test. The majority of candidates scored very well.

#### Part a

Most candidates did quite well on this part. The candidate was expected to know that the first coefficient represented either the intercept of the regression line, or "alpha". Any response that indicated an understanding that it represented an excess return, which would contradict CAPM, was awarded credit.

Some candidates struggled with the second coefficient, as they confused the market risk premium term with beta, and thus thought that the coefficient should equal 1. Any response that indicated the coefficient represented the market risk premium, or systemic/market risk, received credit.

Another common error was stating that the coefficient should be greater than 0, but not stating that it should be equal to the market risk premium.

For the third coefficient, any response indicating firm specific risk, variance, residual or error term received credit.

The statistical significance aspect of the entire part seemed to confuse some candidates. For all three coefficients, credit was given for any reasonable assumption. For example, some candidates thought that  $y_0$  being .003 and not statistically significant meant that no conclusions could be drawn. They received credit as long as they indicated that they knew it should be 0 according to CAPM and made a reasonable assertion about whether the test was consistent with CAPM or not.

#### Part b

This part was straightforward, and there were many possible answers, all falling into one of 4 broad categories- investors cannot realistically borrow at the risk free rate, volatility of stock returns, S&P doesn't represent the entire market, and the sampling error of beta from the first pass regression impacts the second pass regression. The most common error was only coming up with one answer, or two answers that were essentially identical, but most candidates were able to list two.

QUESTION 10	
TOTAL POINT VALUE: 3	LEARNING OBJECTIVE: B1, B2
SAMPLE ANSWERS	LEANNING OBJECTIVE. B1, B2
Part a: 1.5 point(s)	
Sample 1	
Bond 1: $\frac{103}{9810}$ = 1.05 => R1=0.05	
Bond 2: $\frac{\frac{98.10}{5}}{1.05} + \frac{1.05}{(1+R2)(1.05)} = 99.10 => R2$ =	= 0.06
Bond 4: 86.65 = $\frac{5}{1.05} + \frac{5}{(1.05)(1.06)} + \frac{1}{(1.05)(1.06)}$	$\frac{3}{1.05)(1.06)(1+(R3))} + \frac{1.03}{(1.05)(1.06)(1+(R3))(1.1)} =$
86.65	
$81.097 = \frac{2.6954}{1.097} + \frac{84.1297}{1.097} = \frac{86,825}{1.097} = 86,825$	3 = 0.07
$81.097 = \frac{2.6954}{1+R3} + \frac{84.1297}{1+R3} = \frac{86,825}{1+R3} =  R3$ $X = \frac{5}{1.05} + \frac{5}{(1.05)(1.06)} + \frac{1.05}{(1.05)(1.06)(1.06)}$	
$X = \frac{5}{1.05} + \frac{5}{(1.05)(1.06)} + \frac{1.05}{(1.05)(1.06)(1.06)}$	<del></del>
Sample 2	
$98.10 = \frac{103}{1+S1} \implies S1 = 5\%$	
1+31 5 105	
$99.10 = \frac{5}{1.05} + \frac{105}{(1+S2)^2} = 52 = 5.5\%$	
$86.65 \frac{3}{1.05} + \frac{3}{1.055^2} + \frac{3}{(1+S3)^3} + \frac{10}{(1+S3)^3}$	03
$81.0975 = 3x + \frac{103}{110} \chi$	↑ 1 year forward rate for year 4
Let $x = (1 + S3)^{-3}$	
$X = .8392 = (1+S3)^{-3}$ S3 = 6.02%	
33- 0.02%	
Pand 3 Price (v) 5 5 10	05 = 07.26
Bond 3 Price (x) = $\frac{5}{1.05} + \frac{5}{1.055^2} + \frac{10}{1.06}$	$\frac{1}{10000000000000000000000000000000000$
Sample 3	
Year 1 98.1= 103X	
0.9524 = X	
Year 2	
99.1 = 5(0.9524) + 105y	
0.8985 =y	
Year 4	
86.65 = 3 (0.9524) + 3 (0.8985) + 3(0.8985	$(z) + 103 (1.1)^{-1} (z) (0.8985)$
81.097 = 86.8235z	
0.93405= z	
Year 3	
TCut 5	

Price (X) - Assume there is no liquidity premium X= 5(0.9524) + 5(0.8985) + 105(0.93405) (0.8985) = 97.371

# Part b: .75 point(s)

#### Sample 1

Under expectations hypothesis, the forward rate at time 1 is the actual expected interest rate for that time period. Thus, we use 0.07 from a.)

$$\frac{105}{1.07}$$
 = 98.13

#### Sample 2

Only coupon & principle left to be paid in 1 year.

1 year forward rate for year 3 =  $\frac{1.0602^3}{1.055^2}$  - 1 = 7.07%

Under expectations hypothesis forward rate = expected future spot rate. So, 1 year spot rate at time 2 is 7.07%

$$Price = \frac{1.05}{1.0707} = 98.07$$

#### Sample 3

**Expectations Hypothesis** 

Forward rate= expected future short rate

At time=0, Price = 93.74

$$1.055^2$$
 (1+F3) =  $1.0603^3$ 

$$\frac{105}{1.071} = 98.04$$

#### Sample 4

$$(97.37(1.05) - 5)1.06 - 5 = 98.075$$

$$E(R1) = 1.05$$

$$F^2 = \frac{1.055^2}{1.05} = 1.06$$

## Part c: .75 point(s)

#### Sample 1

Under liquidity preference, the premium for year 3 is 1%. Now this premium is no longer applicable as the maturity is now 1. Thus we use 0.07 - 0.01 = 0.06

$$\frac{105}{1.06}$$
 = 99.06

## Sample 2

Under liquidity preference theory, forward rates include a liquidity premium over the expected spot rate. 1 year spot rate at time 2 = 7.07% - liquid premium (1%) = 6.07%

Price = 
$$\frac{1.05}{1.0607}$$
 = 98.99

#### Sample 3

$$F2 = 7.06 \% = \frac{1}{0.93405}$$

Because of liquidity E [R2] = 7.06% - 1% = 6.06%Price=  $105 (1.0606)^{-1} = 98.9998$ 

#### **EXAMINER'S REPORT**

Candidates were expected to demonstrate knowledge of the term structures theories and bootstrapping from the yield curve. Candidates generally scored well, with the majority of candidates receiving full credit on parts a and b.

#### Part a

Candidates were expected to be able to bootstrap spot and forward rates from the given bonds. Generally candidates received full credit by calculating the 4 year spot rate and then either the 3 year spot or forward rate. Common errors included:

- Candidates mistakenly added or removed the liquidity premium in their bootstrapping calculations, but this should not be included because prices are available at time 0
- Candidates confused yield to maturity with the spot rate

#### Part b

Candidates were expected to demonstrate or describe knowledge of the expectations hypothesis. The majority of candidates performed well, using or calculating the forward rate from time 2 to time 3. Common errors included:

- Incorrectly removed the liquidity premium from the forward rate
- Calculated the forward rate for the wrong time period
- Discounted to the wrong time
- Used the spot rate instead of the forward rate to discount

#### Part c

Candidates were expected to demonstrate or describe knowledge of the liquidity preference theory. Candidates should have recognized that the expected future short rate was equal to the forward rate minus the liquidity premium, since the liquidity premium will disappear at time 2, when the liquidity risk no longer exists according to the theory. Candidates did not perform as well on this part of the question. Common errors included:

- Incorrectly added the liquidity premium to the forward rate
- Removed the liquidity premium from the spot rate

## **QUESTION 11**

**TOTAL POINT VALUE: 2.0 LEARNING OBJECTIVE:** B2

#### **SAMPLE ANSWERS**

# **Part a: 1.25** point(s)

#1

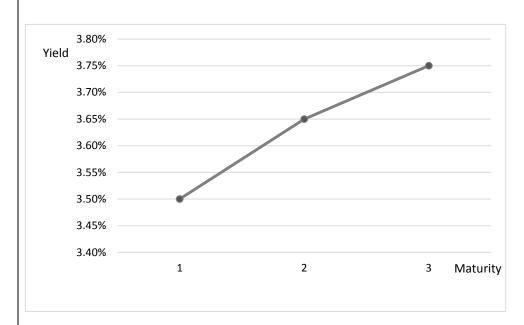
$$193.12 = \frac{100,000 \times (0.04 - f2)}{(1.035) \times (1 + f2)}$$

$$199.8792 \times (1 + f2) = 4,000 - 100,000 \times f2$$

$$199.8792 + 199.8792 \times f2 = 4,000 - 100,000 \times f2$$
  $f2 = 0.0379$ 

$$s2 = (1.035 \times 1.0379) ^{(1/2)} - 1.0 = 3.65\%$$

$$s3 = (1.035 \times 1.0379 \times 1.041) ^{(1/3)} - 1.0 = 3.80\%$$



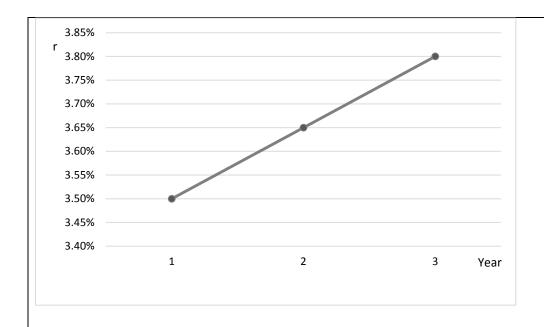
$$193.12 = \frac{100,000 \times (0.04 - f2)}{(1 + r2)^2}$$

$$f2 = \frac{(1+r2)^2}{(1+r1)} - 1.0 = \frac{(1+r2)^2}{1.035} - 1.0$$

 $(1 + r2)^2$ 

$$\frac{(1+r3)^3}{(1+r2)^2} = 1.041$$
  $\frac{(1+r3)^3}{1.0365^2} = 1.041$   $r3 = 0.038$ 

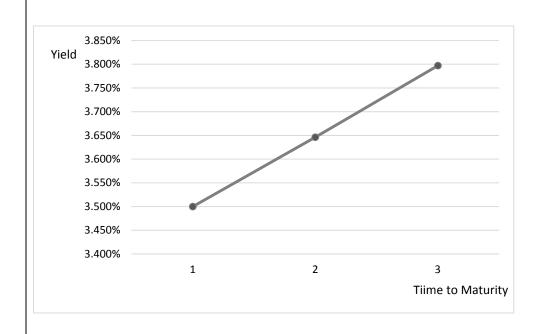
$$r3 = 0.038$$



#3
$$193.12 = -\frac{100,000}{(1.035)} + \frac{104,000}{(1+y2)^2}$$
y2 = 3.646%

f 3 = 4.1%

$$(1 + y3)^2 = (1 + y2)^2 \times (1 + f3)$$
  
 $(1 + y3)^2 = (1.03646)^2 \times (1.041)$   $y3 = 3.797\%$ 



# **Part b: 0.75** point(s)

#### #1

Expectation hypothesis: investors expect yield curve to increase.

Liquidation preference theory: short term investors dominate the market, require premium to invest in long-term bonds.

Segmentation hypothesis: short term bonds have a higher demand than long-term bonds, thus lower yield for short term bonds.

#### #2

Expectation hypothesis: All investors care about is expected return. Currently they expect yields to rise thus yield curve is upward sloping.

Liquidity Preference Theory: Investors prefer bonds that mature when they need the money. In this case investors prefer short-term bonds so they are asking for a premium to buy long term bonds. Hence the curve is upward sloping.

Segmentation Theory: Each investor has a preference for either short term, median term or long term, and do not switch easily from a range to another. The yield in each range is determined by supply & demand. So demand is higher relative to supply for shorter term bonds in this case. So the curve is upward sloping.

## **EXAMINER'S REPORT**

#### Part a

Candidates should be able to determine the U.S. Treasury zero rates at different maturities. In order to receive full credit, candidates needed to use the forward rate agreement to calculate the forward rate and then use that rate to calculate the two-year and three-year spot rate.

Candidates could also use the forward rate agreement to calculate the two-year spot rate directly. Candidates also needed to correctly graph the first, second, and third year spot rates as well as correctly label the graph.

Candidates generally performed well on this question. The most common mistake was not discounting the forward rate agreement for the forward rate. Other common mistakes included:

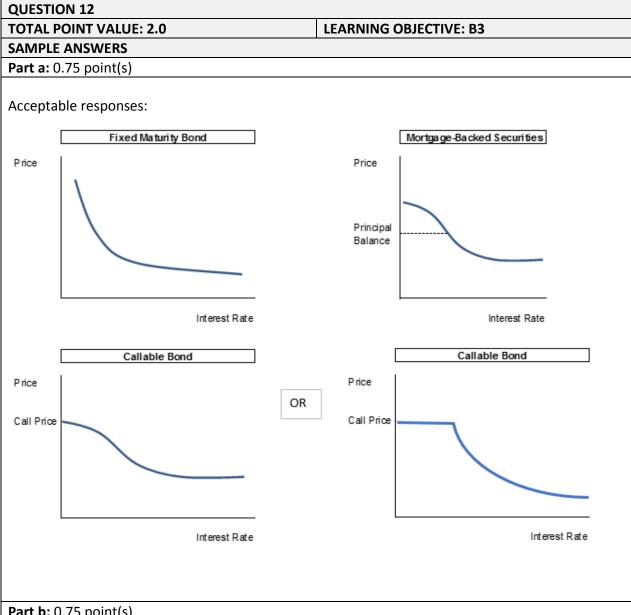
- Calculation errors in the forward rate and the spot rates
- Graphing the forward rates instead of the spot rates

#### Part b

Candidates should know the different term structure theories and explain the implied yield curve using the theories to obtain full credit.

Generally candidates responded well to this question. Common mistakes:

- Describing the theories without applying them to the implied curve.
- Another common mistake is how demand & supply relationship impacts the yield in Market Segmentation Theory. A number of candidates incorrectly thought that a higher demand would correspond to a higher yield.



# Part b: 0.75 point(s)

Fixed maturity bond acceptable responses:

- Positive convexity
- Concave up
- Convex
- Some convexity
- Normal/regular convexity
- Convex down
- Convex throughout (the range of interest rates)
- As yield increases, price decreases at a decreasing rate
- Slope becomes less negative at higher interest rates
- Price increase from an interest rate decrease is greater than the price decrease from a

corresponding interest rate increase

- Price increases at a faster rate as yield decreases
- For non-callable bonds the duration approximation understates the value of the bond (underestimates the increase in bond price as yield falls and overestimates the decline in price when yield rises) as it does not consider the convexity of the bond

# Callable bond acceptable responses:

- Displays negative convexity near the call price
- Displays negative convexity as interest rates decrease
- Has positive convexity at higher yields and negative convexity at lower yields
- Behaves like fixed bond for higher yields but at lower yields it switches convexity (credit was given if the fixed maturity bond response was correct)
- For callable bond when rates drop, the bond can be called and price cannot rise anymore the convexity is negative there
- Callable bond has negative convexity because it's callable
- Concave up until it appears it will possibly get called, then it switches
- Callable bond is the same as fixed maturity bond when yields increase but will be called when yields fall so there is a price cap and a flat part of the curve (credit was given if the fixed maturity bond response was correct)
- Negative convexity when bond price hits the cap; below the price the convexity is positive
- Callable bond has an area of negative convexity because of the price cap
- Concave near the call price
- Concave down near the call price
- Positive convexity at higher interest rates but price reaches a maximum at call price, so there is no convexity there
- Price cannot go above call price, so the size of a price drop (due to increase in yield) has
  potential to be more than the price increase (due to same size decrease in yield)

#### Mortgage-backed securities (MBS) acceptable responses:

- Exhibits negative convexity because of embedded prepayment options
- Displays negative convexity around the principal balance
- Displays negative convexity as interest rates decrease
- Has positive convexity at higher yields and negative convexity at lower yields
- Behaves like fixed bond for higher yields but at lower yields it switches convexity (credit was given if the fixed maturity bond response was correct)
- Similar to callable bond (credit was given if the callable bond response was correct)
- MBS similar to callable bond as borrower can refinance when yield is low (credit was given if the callable bond response was correct)
- Concave above principal balance because mortgage holders will refinance
- Max price is limited, so the size of a price drop (due to increase in yield) has potential to be more than the price increase (due to same size decrease in yield)

## **Part c:** 0.50 point(s)

#### Acceptable responses:

- The call price caps the callable bond. The MBS principal amount won't necessarily cap the value of the MBS because of transaction cost of refinancing or selling your mortgage and mortgage holders' unwillingness or ignorance in their ability to do so.
- Callable bonds max price is the call price. MBS max price is a little higher than the principal balance because not all borrowers will refinance.
- The callable bond will be called when the bond hits the call price but not all homeowners will refinance when interest rates drop.
- A single party (the bond issuer) can recall the callable bond. However it takes many parties (mortgage holders) to recall their mortgages.

#### **EXAMINER'S REPORT**

#### Part a

Candidates were expected to:

- Show the fixed maturity bond graph with positive convexity throughout the range, and label the axes correctly. We accepted either "interest rate" or "yield" for all graphs.
- Show the callable bond graph with positive convexity at higher yields and negative convexity at lower yields (near the call price). We also accepted responses showing positive convexity at high yields with an abrupt transition to a flat curve at the call price. The call price needed to be correctly labeled as the maximum price. Axes needed to be labeled correctly.
- Show the MBS graph with positive convexity at higher yields and negative convexity near the principal balance. The principal balance needed to be depicted slightly below the maximum price of the MBS graph. Axes needed to be labeled correctly.

Candidates generally did well on the fixed maturity and callable bond graphs. Fewer candidates were correctly able to depict the MBS graph. Common errors included:

- Didn't label principal balance (MBS) or call price (callable bond)
- Depicting curves with no convexity (straight lines)
- For MBS and callable bond, showing curves that ONLY exhibited negative convexity (missed that the graphs exhibit positive convexity at higher yields)
- Depicting prices that increased as interest rates increased
- Incorrectly depicting the call price as a floor of the callable bond price
- Incorrectly depicting the principle balance as a floor of the MBS price
- Incorrectly showing the call price below the max price for the callable bond
- Displaying the principal balance as the max price to the MBS graph
- Displaying curves where prices went below zero
- Labeling errors (confusing price and yield with "change in price" and "change in yield")

Most candidates put "Price" on the y-axis and "Yield" on the x-axis. However, candidates were not penalized for choosing the opposite convention. Many candidates chose to display all three curves on the same graph, which was also acceptable.

#### Part b

Candidates were expected to:

- Fixed maturity bond know it exhibits positive convexity
- Callable bond know it exhibits negative convexity (or alternatively, a plateau on price)
   AND:
  - o demonstrate knowledge of 'where' the negative convexity existed (e.g. at low interest rates) OR
  - o demonstrate knowledge of 'why' negative convexity existed (e.g. due to callable nature)
- MBS know it exhibits negative convexity (or alternatively, a plateau on price) AND:
  - o demonstrate knowledge of 'where' the negative convexity existed (e.g. at low interest rates) OR
  - demonstrate knowledge of 'why' negative convexity existed (e.g. due to prepayment risk)

Candidates generally did well on this part. For callable and MBS, we accepted a wide variety of responses. Common errors included:

- Responses that only discussed the general relationship between price and yield (e.g. when rates rise, bond prices decrease).
  - o Such responses were insufficient because they did not describe *convexity*.
- For callable bond and MBS simply mentioning "negative convexity" with no additional response. This answer was insufficient because these callable bond and MBS curves have regions of both negative and positive convexity.
  - We gave credit to responses which demonstrated knowledge of 'where' the negative convexity existed (e.g. at lower rates). Such responses implied that positive convexity existed elsewhere (e.g. at higher rates).
  - We also gave credit to responses explaining 'why' negative convexity existed (e.g. due to callable nature or prepayment risk). Such responses demonstrated knowledge of why callable/MBS behave differently from the fixed maturity case (which is always positively convex).
- For callable bond and MBS responses that only discussed positive convexity at higher interest rates
- Attempts to rank the securities from "most convex" to "least convex" (e.g. low/medium/high)
  - Such responses did not receive credit, because the question did not ask for a comparison, and not enough information was given to compute or quantify convexity to make comparisons.
- Responses that failed to discuss convexity
- Responses stating that there was no convexity

Credit was given to candidates who (instead of writing a full response to part b) chose to label regions of "positive convexity" and "negative convexity" on the graphs in part a). Additionally, many candidates who made convexity errors in the graphs in part a) were able to obtain full credit in part b) by providing correct verbal descriptions of the convexity of each security.

#### Part c

## Candidates were expected to:

- Identify the callable bond's maximum price is limited to the call price
- Identify the maximum price of the MBS is influenced by prepayments/refinancing
- Identify the difference in the features:
  - The issuer of a callable bond will always call the bond if interest rates decrease enough to make the bond callable (making the call price a fixed cap on the price of the callable bond), whereas
  - Homeowners may not choose to prepay/refinance when interest rates drop (due to closing costs, moving soon, time left on loan, etc) so the principle balance is not a hard cap on the price of the MBS.

#### Common errors included:

- Failure to identify the features which create ceilings on the maximum price of callable and/or MBS
- *Identifying* the features, but not explaining the difference

Sometimes candidates' responses to a) and b) contained elements of the answer to part c). If an attempt was made to answer part c), and the response was incomplete, responses for parts a) and b) were considered in the grading of part c).

QUESTION 13	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE: B4
SAMPLE ANSWERS	

Part a: 1 point

Possible sample responses include:

- A volatile MVS would make it difficult to estimate the future book value of the firm, since book value converges to market value as assets and liabilities mature.
- Unable to properly serve the goals of the company's owners (e.g. higher future stock prices for stockholders or higher future dividends for policyholders).
- Increased regulatory scrutiny Regulators may become concerned with the company's capital adequacy
- A volatile surplus account could be damaging to insurers since regulators typically limit premium volume to a small multiple of reported surplus.
- Increase taxes due to surplus volatility due to the convexity of the tax structure
- Cash flow volatility issues, e.g. high payment to stakeholders (e.g. employees, management) to compensate for risk
- Rating agencies may lower rating because of increased risk due to volatility of MVS.
- If MVS falls low, increased potential for insolvency and bankruptcy costs related to it
- Exposure to interest rate risk as assets and liabilities do not match / need to rebalance durations of assets and liabilities so they are immunized.
- Company may need to hold more surplus, and would have less capital to invest >
  company would lose out on other potential investment opportunities
- If a catastrophe or unexpected large loss occurs, claims may be greater than the reserves available. If MVS is low, may not have enough funds to pay for losses.
- High borrowing costs creditors may charge higher rates for borrowing
- Lose investors, as investors demand a higher cost of capital due to volatility

Note: Additional answers were accepted as valid responses for identifying potential problems.

## Part b: 0.75 points

A positive duration gap of surplus indicates that assets have longer duration than liabilities. This means that any rise in interest rates would lower the absolute value of MVS, since it would cause asset market values to decline relatively more than liability market values. A large positive DGs implies a potentially volatile surplus account, which could lead to the issues listed in (a).

#### **EXAMINER'S REPORT**

#### Part a

Candidates were expected to identify potential problems that may occur if MVS is not managed and becomes volatile. In order to receive full credit, the candidate had to list 4 separate issues or challenges related to a volatile MVS.

Candidates generally performed well on this part of the question. A common error was that candidates did not list responses that were inherently different from each other. For example, listing that there would be increased exposure to interest rate risk and assets and liabilities would not be duration matched would only receive credit for one problem not two.

#### Part b

Candidates were expected to be able to:

- Define that a positive duration gap of surplus indicates that assets have longer duration than liabilities; i.e. assets longer than liabilities
- Come to the conclusion that MVS would fall in a rising interest rate environment
- Mention that this implies increased volatility or interest rate risk or that this implies a potentially volatile surplus account (which could lead to the issues in part a).

Candidates generally received partial credit for this part of the question. Common errors included not including a definition of a positive duration gap of surplus, and not mentioning that this could lead to increased volatility or a volatile MVS.

#### **QUESTION 14**

TOTAL POINT VALUE: 2.5 LEARNING OBJECTIVE: B4, B5

#### **SAMPLE ANSWERS**

#### Part a: 1 point(s)

100% - (27.3%+33.7%+19%) =20%, or 10% each year over years 4&5. (0.25 pts)

Using the new money yield of 6% and keeping in mind that losses are paid at year end (0.25 pts):

#### Duration =

$$=\frac{27.3*1*1.06^{-1}+33.7*2*1.06^{-2}+19.0*3*1.06^{-3}+10.0*4*1.06^{-4}+10.0*5*1.06^{-5}}{27.3*1.06^{-1}+33.7*1.06^{-2}+19.0*1.06^{-3}+10.0*1.06^{-4}+10.0*1.06^{-5}}\\=2.327~\text{years}.$$

## Part b: 0.5 point(s)

If inflation is anticipated, it is already reflected in the price of the bond and would not be an issue.

However, if it is unexpected, the value of liabilities will grow, while the bond will still mature for the same amount.

## Part c: 1 point(s)

- 1. P&C insurers, unlike life insurers, are not subject to the risk of disintermediation, so it is not as important that assets are sufficient to cover the obligations at any given point in time.
- 2. P&C insurer can rely on the cash flows from current premium and investment to pay claims, since they expect a steady stream of premium inflow regardless of interest rate changes.
- 3. P&C insurers do not segment funds, so investment returns must be sufficient for the company as a whole, not for any given block of policies.
- 4. However, having mismatched duration of assets and liabilities will become a problem if current income is insufficient to support the losses.

## **EXAMINER'S REPORT**

- Candidates are expected to be able to:
- Calculate the Macaulay duration of loss reserves and the Macaulay duration of surplus of P&C insurers (LO B4);
- Understand the importance of Asset/Liability Matching (ALM) for P&C insurers (LO B5).
- Majority of candidates demonstrated solid understanding in this question. Many candidates did not receive full credits for Part b or c because they did not provide a robust explanation.

#### Part a

- Part a is a calculation of Macaulay duration of loss reserve (LO B4). It is very similar to the example in Feldblum's paper.
- To obtain full credit, candidates need to express the correct payout pattern for years 1 through
   use correct interest rate, use end of year discounting, list correct formula for MaCaulay
   Duration and calculate correctly. Most candidates scored very well in this part.
- Some candidates did not receive full credit for reasons such as they did not use new money yield, or did not calculate the %loss paid in 4<sup>th</sup> and 5<sup>th</sup> year correctly, or did not treat loss as paid at the end of each year as question specified.

#### Part b

- Part b tests inflation sensitivity, which was a key point in Feldblum's paper and is highly important in ALM.
- A majority of candidates received full credit on this part. In order to get full credit, candidates are expected to make an assessment whether this is appropriate or not and further support their assessment with robust explanation.
- Some candidates lost partial points because of missing an explanation part.

#### Part c

- Part c tests if candidates understand the importance of ALM for P&C insurers.
- This question requires candidates to provide robust explanations for why mismatched duration is not necessarily a major problem for P&C insurers and also identify one situation where it becomes a problem.
- To receive full credit, 3 explanations and 1 problem are required. No more than 0.75 points can be awarded for the explanations, and no more than 0.25 points can be awarded to identify the problem of duration mismatch.
- Credits in the increment of 0.25 points for each valid bullet point were awarded as long as candidates provided a relevant and reasonable explanation.
- Credit is not given if the provided statement is not applicable to the underlying question, though the statement itself may be a correct statement.
- Candidates in general were able to correctly answer 1 or 2 explanations along with listing 1 problem

QUESTION 15			
TOTAL POINT VALUE: 2.75	LEARNING OBJECTIVE: B6		
SAMPLE ANSWERS			
Part a: 1.25 point(s)			
Current Economic Value =	S + P - E - L/(1+y) 400+398.352-100-300/(1+.04) = 409.890		
Franchise Value =	Total Economic Value - Current Economic Value 657.152 – 409.890 247.262		
Franchise Value =	{ P - E - L /(1+y) }*d/(1-d) 247.262 = {398.352-100-300/(1+.04)}*d/(1-d)		
Solve for d	d = .96154		
Solve for cr	d = cr/(1+y) cr = d*(1+y) cr = .96154 * (1+.04) cr = 1.000		
Part b: 0.5 point(s)			
Sample 1			
Franchise value is same as above	ve = 247.262		
Franchise value	= $\{cr * S * [a + y*(b-1)]\} / (1+y) / (1+y-cr)$		
	247.262 = {cr * 400 * [.08 + .04 * (1.5-1)]} / (1+.04) / (1+.04-cr)		
Solve for cr	cr = .90		
Sample 2			
Franchise value is same as abo	ve = 247.262		
Franchise value	= {cr * S * [a + y*(b-1)]} / (d / (1-d) )		
	247.262 = {cr * 400 * [.08 + .04 * (1.5-1)]} / (d / (1-d) )		
Solve for d	=0.86538		
Solve for cr	= d * (1+y) = 0.86538 * 1.04 =0.90		
Sample 3 k	= .08 + 1.5 * .04 = .14		

Written Premium	= { (S * [k-y])+ L} / (1+y) + E ={ (400 * [0.1404])+ 300} / (1+.04) + 100 = 426.923
Current Economic Value	= S + P - E - L/(1+y) = 400 + 426.923 - 100 - 300/(1+.04) = 438.461
Franchise Value =	Total Economic Value - Current Economic Value = 657.152 - 438.461 = 218.691
Franchise Value =	{ P - E - L /(1+y) }*d/(1-d) 218.691 = {426.923-100-300/(1+.04)}*d/(1-d)
Solve for d	d = .0.8504
Solve for cr	d = cr/(1+y) cr = d*(1+y) cr = .8504 * (1+.04) cr = .884

## Part c: 0.5 point(s)

## Sample 1

It is more likely to be using the variable pricing strategy in part b. It is unlikely that a company has 100% retention or that a company would assume 100% retention. A retention of 90% is more reasonable.

### Sample 2

Since the duration of the franchise value under the variable pricing strategy is lower, the company is more likely using this strategy so that the duration of franchise value is less sensitive to changes in interest rates.

#### Sample 3

Use the variable pricing strategy as this allows you to manage the duration of franchise value (and thus the duration of market value) in a way that is invisible to the regulators.

Part d: 0.5 point(s)	
Df	= (a-b+1)/(1+y)/(a+b*y-y) + 1/(1+y-cr)
Df	= (.08-1.5+1)/(1+.04)/(.08+1.5*.0404) + 1/(1+.0490)
Df	= 3.104

### **EXAMINER'S REPORT (BY PART, AS APPLICABLE)**

#### Part a

Candidates were expected to know the formulas for Total Economic Value, Current Economic Value and Franchise Value. They were also expected to know the relationship between d and cr. Candidates generally did well in this part. Due to multiple steps involved, it was quite common to see a calculation error.

#### Common errors included:

- 1. Assuming Total Economic Value = Franchise Value + Surplus
- 2. When converting from d to cr (last step): dividing by 1.04 instead of multiplying by 1.04

#### Part b

Candidates were expected to know the formula for Franchise Value under the variable pricing strategy. Again, candidates generally did well in this part. There were two primary approaches to this part where the candidate could assume the same Franchise Value as part a (ie. Sample 1 and Sample 2) or assume the Premium and Franchise Value needed to both be re-calculated (ie. Sample 3).

#### Common errors included:

- 1. Assuming the premium is unaffected and thus assuming the same retention as Part a
- 2. For Sample 1, forgetting (1+y) in the denominator
- 3. For Sample 3, not recalculating the Franchise value after having recalculated Premium

#### Part c

Candidates were expected to know at least one advantage of the variable pricing strategy or be able to recognize that a company generally would not assume 100% retention. Again, candidates generally did well in this part and frequently commented on the retention being unreasonable. Many candidates recognized that companies would use a variable pricing strategy to minimize the sensitivity of the Franchise Value to interest rate changes.

#### Common errors included:

• Not clearly stating the pricing strategy that the company will more likely use

#### Part d

Candidates were expected to know the formula for the duration of Franchise Value. Candidates didn't do as well in this part as the other parts. Candidates either had trouble recalling the formula for the duration or they had calculation errors.

### Common errors included:

- Switching the +/- signs in the numerator of the duration formula (eg. a + b 1)
- Mistaking the portion of the denominator of the duration formula as (a+b\*y-1)

#### **QUESTION 16**

TOTAL POINT VALUE: 2.0 points LEARNING OBJECTIVE: C1

#### **SAMPLE ANSWERS**

Part a: 1.0 point(s)

$$\text{V(X,T)} = N \Big[ \frac{N^{-1} \Big( Q(T) \Big) + \sqrt{\rho} \ N^{-1} \ (X)}{\sqrt{1-\rho}} \Big]$$

Credit VaR = L \* (1 - R) \* V(X,T)

#### Portfolio A:

$$\begin{split} &\text{V(0.99,3)} = N \Big[ \frac{N^{-1}(0.015) + \sqrt{0.2} \ N^{-1} \ (0.99)}{\sqrt{1 - 0.2}} \Big] \\ &N^{-1}(0.015) = \text{-}2.17 \\ &N^{-1}(0.99) = 2.326 \\ &= N \Big[ \frac{-2.17 + \sqrt{0.2} * 2.326}{\sqrt{1 - 0.2}} \Big] \\ &= N \Big[ -1.26 \Big] \\ &= 1 - 0.8962 \\ &= 0.1038 \\ &\text{VaR} = 100M * (1 - 0.55) * 0.1038 \\ &\text{VaR} = 4.671M \end{split}$$

### Portfolio B:

$$\begin{split} &\text{V(0.99,3)} = N \Big[ \frac{\text{N}^{-1}(0.03) + \sqrt{0.05} \, \text{N}^{-1} \, (0.99)}{\sqrt{1 - 0.05}} \Big] \\ &\text{N}^{-1}(0.03) = \text{-}1.81 \\ &\text{N}^{-1}(0.99) = 2.326 \\ &= N \Big[ \frac{-1.88 + \sqrt{0.05} * 2.326}{\sqrt{1 - 0.05}} \Big] \\ &= N \Big[ -1.395 \Big] \\ &= 1 - 0.9177 \\ &= 0.0823 \\ &\text{VaR} = 100M * (1 - 0.50) * 0.0823 \\ &\text{VaR} = 4.115M \end{split}$$

#### Pick B as it has a lower VaR

#### **Part b: 1.0** point(s)

- Use a cash flow simulation because:
  - o It allows you to see the path not just the end result
  - o It is not dependent on the normal distribution which may not have a heavy enough tail
  - Can account for severity of losses not just the fact that they occur
- VaR only looks at the dollar loss associated with an arbitrary probability. A better

- approach would be to use shortfall risk which sets a level below which the company's assets cannot fall & produces estimates of risk accordingly. Shortfall risk is better because it calculates the probability associated with a given loss (as a real concern) as opposed to VaR which calculates an arbitrary probability.
- Use an EPD framework. This will consider severity of the downside events. Run a simulation of 1000's of events using normal distribution. Then look at the events where losses exceed capital. Pick the worst 0.5% of all events. Calculate the average shortage in assets to cover said events. Multiply by prob. of each event. This is the EPD. This considers severity of bad events, not just an exceedence probability.
- Use Credit Metrics:
  - Simulates rating change of each bond over the period and revalues the portfolio based on new credit rating
  - o Calc. Credit VaR based on losses implied from this
  - Advantage -> Considers downgrades (not just defaults), can consider any credit risk mitigation clauses
  - Disadvantage -> Time/computationally intensive
- Recommend a more robust analysis involving incorporation of the severity of default during instances of default. One method to account for this would be a conditional tail expectation which looks at the severity in the 1/100 year loss events.

#### **EXAMINER'S REPORT**

This question required knowledge of the Credit VaR calculation, as well as its weaknesses. Candidates generally did not do well with few receiving full credit.

Of the candidates who attempted part a, most knew the Credit VaR formula. However, many lost credit due to a lack of understanding of what the variables represent and/or calculation errors.

For part b, there was a wide range of acceptable answers. Because the question states "discuss", the candidate was expected to not only provide an alternative but also sufficiently explain how that alternative is more robust and/or accurate for the CRO. Generally candidates did not sufficiently discuss their alternative and did not receive full credit.

#### Part a

Candidates generally knew the Credit VaR formula. If the candidate made any errors calculating the Credit VaR for portfolios A or B, they still received partial credit if they picked the portfolio with the lower VaR and explained their reasoning. Some candidates did not remember the Credit VaR formula but explained why portfolio B was the superior choice; these candidates received partial credit.

Some candidates did not pick a superior portfolio despite calculating the Credit VaR's correctly; these candidates only received partial credit.

Common mistakes included:

- Incorrect Credit VaR formula
- Looking up numbers from the normal table incorrectly
- Assuming the default rate was annual instead of three years, as stated in the question
- Forgetting to multiply the normal calculation by L \* (1 R)
- Not picking a superior portfolio

#### Part b

Candidates came up with a wide range of responses for this part. In order to have sufficiently discussed the alternative and received full credit, we expected the candidate to state both a "what" and a "why". The "what" being an explanation of an alternative, and the "why" being an explanation of how the alternative is more robust or accurate over Credit VaR.

Most candidates lost credit on this part for three reasons:

- Not stating a "what"
- Not stating a "why"
- Answering the question in a vague way that did not demonstrate knowledge of Credit Risk, e.g.:
  - Explaining a generic simulation without tying it to Credit Risk
  - Simply stating other metrics (CTE, EPD, Shortfall, etc.) without tying it to Credit
  - Explaining a generic predictive model without tying it to Credit Risk

QUESTION 17	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE: C1, C2
SAMPLE ANSWERS	

# 2.25 point(s)

### Counterparty A:

- Select a collateralization clause. A is highly rated company so they shouldn't run into difficulties if additional collateral needs to be posted. Netting will be ineffective as all contractors are in the money. A downgrade trigger isn't ideal as A has a number of outstanding transactions
- Collateral given its rating, the company should be able to obtain and post collateral if
  its derivatives values change
  Definition: when the value of a counterparty's derivatives exceed a given threshold the
  counterparty must post collateral in excess of the threshold. This only works if
  counterparty will be responsive and able to post collateral

### Counterparty B:

- Downgrade trigger:
  - when rating of the counterparty is downgraded, the company can withdraw/cancel the contract.
  - since there is only one contract with the counterparty, no need to worry about the need to cancel many contracts when the counterparty's rating is downgraded.
- Choose collateralization clause because it has no contract to net against and its credit
  rating is already poor so he may not be protected by credit downgrade if he has a large
  drop. Collateralization is that when the total value of contracts exceed a predetermined
  threshold, the counterparty will post collateral to cover the excess. The amount under the
  threshold will not be protected and counterparty may stop responding if its financial
  condition deteriorates.

### Counterparty C:

For company C, I would include a netting clause which states that if they default on one
contract, they must default on all contracts. This will be effective as XYZ has both in the
money and out of the money derivatives with company C. These will offset each other
under a netting clause.

#### **EXAMINER'S REPORT**

Candidates are expected to demonstrate knowledge on the three risk mitigation strategies through describing their mechanics and applying them in different situations.

To gain full credit from this question, candidates are expected to describe the strategy chosen and justify their selections. Points were taken off for simply listing the strategies: collateralization, downgrade trigger and netting. To justify the chosen strategies, candidates can either negate the other two strategies or provide a good argument that support the selection.

Description of Strategies:

Collateral: Credit only given to candidates who recognize collateral will be required on the difference between market value and predetermined threshold. Some candidates lost points for simply saying requiring a CP to provide a collateral.

Downgrade Trigger: candidates generally did well. Some candidates lost points from not recognizing the trigger of the clause is a credit downgrade.

Netting: candidates did very well in this section. Some candidates lost points from not demonstrating knowledge that if CP default on one contract, they have to default on all.

#### **Justifications**

CP A: A lot of candidates chose downgrade trigger for CP A. This answer is not accepted because downgrade trigger does not provide for protection in case of large drop in credit rating. An AAA company that gets downgraded to AA is still highly rated. Collateral works best because there A is highly rated and there is little concern that A does not respond to collateral call.

CP B: Both collateral and downgrade triggers are acceptable answers. To justify collateral, candidates should recognize that netting does not work with only 1 transaction and downgrade trigger does not work because its rating is already low. The justification of downgrade is the fact that there is only 1 transaction and downgrade trigger does not work when too many contracts have this clause as the company will be unlikely to be able to afford closing out many contracts.

CP C: Candidates generally did well in identifying netting as the strategy for counterparty (CP) C because of the benefits from significant offsetting between in the money and out of money transactions.

QUESTION 18	
TOTAL POINT VALUE: 2.0	LEARNING OBJECTIVE: C3
SAMPLE ANSWERS	
Part a: 1.5 noint(s)	

# <u>Sample 1</u>: CDO<sup>2</sup> constructed using two junior CDOs

	Default Probability	Calculation	Description
Mortgage	0.05		P(Default)
CDO Junior	0.0975	=2*0.05*0.95 + 0.05 <sup>2</sup>	P(1 Default) + P(2 Defaults)
CDO <sup>2</sup> Junior	0.18549375	=2*0.0975*(1-0.0975) + 0.0975 <sup>2</sup>	P(1 Default) + P(2 Defaults)
Synthetic Senior	0.018085641	=0.0975*0.18549375	P (2 Defaults)

# <u>Sample 2</u>: CDO^2 constructed using two Senior CDOs

	Default Probability	Calculation	Description
Mortgage	0.05		P(Default)
CDO Junior	0.0975	=2*0.05*0.95 + 0.05 <sup>2</sup>	P(1 Default) + P(2 Defaults)
CDO Senior	0.0025	0.05^2	P(2 defaults)
CDO <sup>2</sup> Junior	0.00499375	=2*0.0025*(1-0.0025)+ 0.0025^2	P(1 Default) + P(2 Defaults)
Synthetic Senior	0.000486891	=0.0975*0.00499375	P (2 Defaults)

Sample 3: CDO^2 constructed using one Junior CDO and one Senior CDO

	Default Probability	Calculation	Description
Mortgage	0.05		P(Default)
CDO Junior	0.0975	=2*0.05*0.95 + 0.05 <sup>2</sup>	P(1 Default) + P(2 Defaults)
CDO Senior	0.0025	0.05^2	P(2 defaults)
CDO <sup>2</sup> Junior	0.09975625	=0.0975*(1- 0.0025)+0.0025*(1- 0.0975)+0.0975*0.0025	P(1 Default) + P(2 Defaults)
Synthetic Senior	0.009726234	=0.0975*0.09975625	P (2 Defaults)

### Part b: 0.5 point(s)

An increase in the default correlations would increase the default probability of the Senior tranche of the synthetic security. This should decrease the rating agency's rating of the security

### **EXAMINER'S REPORT**

Candidates did well on this question. Candidates were expected to know how to calculate the expected probability of default for CDO's CDO^2 and synthetic securities. The candidate was also expected to understand the impact from a change in correlation of the mortgages.

#### Part a

The candidates did well on this part. The candidate was expected to know how to construct a securitized product and calculate expected probability of defaults for CDO's CDO^2 and synthetic securities.

The model solution proposes alternative ways of constructing Junior tranche CDO<sup>2</sup> given that the definition of CDO<sup>2</sup> in the source material is broad enough that any combination of tranches from the underlying CDOs is acceptable. However, from the standpoint of the issuer, it wouldn't make much sense to make a CDO<sup>2</sup> from senior tranches, but the intention of the question was to test the understanding of the securitization mechanism. Although Solution 2 and 3 were accepted for full credit, the best way to solve this problem would be to construct Junior CDO<sup>2</sup> from two Junior CDOs (Solution 1).

#### Part b

The candidates did well on this part. They were expected to understand the impact of a change in correlation of the mortgages on the default probability of the Senior tranche of a securitized product. Candidates were also expected to explain the impact on the credit rating.

**QUESTION 19** 

TOTAL POINT VALUE: 3.75 LEARNING OBJECTIVE: C5, C7, D8

#### **SAMPLE ANSWERS (BY PART, AS APPLICABLE)**

#### Part a: 3.25 points

Graders considered the typographical error as they graded and accept answers derived either from accepting the headers as correct (despite the contradictions that this generated) or from assuming that the headers were in error and proceeding accordingly. Additionally, we adjusted the MQC standard downward for this item to reflect the work that could be done despite the error while making an allowance for confusion that may have been caused by the error.

There were two variations of solutions that were accepted for full credit. For candidates that recognized that the industry losses were mislabeled as millions (instead of billions), there was one set of values that was acceptable for full credit. For candidates that attempted the problem as it was actually stated in the problem, a different set of values were deemed acceptable for full credit.

### Sample 1:

### Step 1

	Industry					
	Loss	XYZ Ground	XYZ Net	ABC Incurred	ILW Payout	Net Loss to
	(Billions)	Up Loss	Losses	Losses on (2)	to ABC*	ABC
Scenarios	(1)	(2)	(3)	(4) = (2) - (3)	(5)	(6) = (4)-(5)
1	50	5,500,000	5,000,000	500,000	0	500,000
2	40	25,000,000	15,000,000	10,000,000	10,000,000	0
3	35	7,000,000	5,000,000	2,000,000	10,000,000	(8,000,000)
4	30	10,000,000	5,000,000	5,000,000	10,000,000	(5,000,000)
5	20	5,500,000	5,000,000	500,000	0	500,000
6	10	10,000,000	5,000,000	5,000,000	0	5,000,000
7	10	6,500,000	5,000,000	1,500,000	0	1,500,000
8	5	10,000,000	5,000,000	5,000,000	0	5,000,000
9	5	7,000,000	5,000,000	2,000,000	0	2,000,000
10	5	2,000,000	2,000,000	0	0	0

<sup>\*</sup>Payout is always either 10,000,000 or 0 with a binary trigger.

### Step 2

Gross Expected Loss to ABC = 31,500 = Sum of Column (4) / 1,000 Expected ILW Payout to ABC = 30,000 = Sum of Column (5) / 1,000 Net Loss to ABC = 1,500 = 31,500 – 30,000

## Step 3

Calculate Gross CTE for ABC = Sum of 5 largest losses from Column (4) / 5 5,400,000 = (10+5+5+5+2)\*1,000,000/5 Calculated Net CTE for ABC = Sum of 5 largest losses from Column (6) / 5 2,800,000 = (5+5+2+1.5+0.5)\*1,000,000/5

### Step 4

Gross Risk Capital for ABC = 5,368,500 = 5,400,000 - 31,500Net Risk Capital for ABC = 2,798,500 = 2,800,000 - 1,500Reduction in Risk Capital = 2,570,000 = 5,368,500 - 2,798,500

#### Step 5

Premium paid by ABC for ILW = 10% Rate on Line times coverage limit of \$10 million 1,000,000 = .1\*10,000,000

#### Step 6

Conclusion: 2,570,000 >1,000,000, therefore ILW is effective risk mitigation tool per ABC guidelines.

#### Sample 2:

#### Step 1

	Industry Loss (Billions)	XYZ Ground Up Loss (Billions)	XYZ Net Losses	ABC Incurred Losses on (2)	ILW Payout to ABC	Net Loss to ABC
Scenarios	(1)	(2)	(3)	(4) = (2) - (3)	(5)	(6) = (4)-(5)
1	50	5.5	5,490,000,000	10,000,000	10,000,000	0
2	40	25	24,990,000,000	10,000,000	10,000,000	0
3	35	7	6,990,000,000	10,000,000	10,000,000	0
4	30	10	9,990,000,000	10,000,000	10,000,000	0
5	20	5.5	5,490,000,000	10,000,000	0	10,000,000
6	10	10	9,990,000,000	10,000,000	0	10,000,000
7	10	6.5	6,490,000,000	10,000,000	0	10,000,000
8	5	10	9,990,000,000	10,000,000	0	10,000,000
9	5	7	6,990,000,000	10,000,000	0	10,000,000
10	5	2	1,990,000,000	10,000,000	0	10,000,000

#### Step 2

Gross Expected Loss to ABC = 100,000 = Sum of Column (4) / 1,000 Expected ILW Payout to ABC = 40,000 = Sum of Column (5) / 1,000 Net Loss to ABC = 60,000 = 100,000 – 40,000

#### Step 3

Calculate Gross CTE for ABC = Sum of 5 largest losses from Column (4) / 5 10,000,000 = (10+10+10+10+10)\*1,000,000/5 Calculated Net CTE for ABC = Sum of 5 largest losses from Column (6) / 5 10,000,000 = (10+10+10+10+10)\*1,000,000/5

#### Step 4

Gross Risk Capital for ABC = 9,900,000 = 10,000,000 - 100,000Net Risk Capital for ABC = 9,940,000 = 10,000,000 - 60,000Reduction in Risk Capital = -40,000 = 9,900,000 - 9,940,000

### Step 5

Premium paid by ABC for ILW = 10% Rate on Line times coverage limit of \$10 million 1,000,000 = .1\*10,000,000

### Step 6

Conclusion: -40,000<1,000,000, therefore ILW is NOT effective risk mitigation tool per ABC guidelines. Risk capital actually increases.

#### Part b: 0.5 points

Basis risk can be defined as the risk that the payout of the ILW will be insufficiently correlated with ABC's actual losses. For example, ABC suffers a loss that is below the \$1 Million retention but the industry suffers large losses above the industry trigger. ABC may also suffer a significant loss but the ILW Industry trigger of \$25 Billion may not be met. Lastly, ABC may suffer a loss greater than the \$1 Million retention but below the \$10 Million ILW coverage limit while the industry loss is above \$25 Billion. This last scenario actually results in ABC receiving a larger payout (\$10 Million) from the ILW than actual losses incurred.

#### **EXAMINER'S REPORT**

#### Part a

To complete this problem, candidates were expected to know how an ILW functions, in particular one with a dual trigger. In addition, candidates needed to understand how to calculate net and gross losses under the ILW and how to apply concepts of a CTE to those losses. Lastly, the candidate was expected to calculate the premium for the ILW and compare it to the change in capital per the internal guidelines of ABC

A good portion of the candidates were able to calculate a Gross CTE. However, fewer candidates were able to apply the Gross or Net CTE's to the correct column of losses. Many candidates calculated CTE's for the industry or company XYZ rather than ABC. Once the CTE was calculated, many candidates did not subtract off the expected loss to calculate the Risk Capital.

Most candidates were able to calculate the premium paid by ABC for the ILW, and most candidates who calculated both a gross and net Risk Capital were able to compare the difference to the premium to come up with an appropriate conclusion.

Many candidates were also able to identify the correct years that the ILW would pay out, but few candidates realized that the payout would always be \$10,000,000. Many candidates assumed the ILW payout would not be greater than the losses sustained by ABC in any scenario.

#### Part b

For part b, candidates were expected to understand the definition of basis risk and how that definition can result in various scenarios for this ILW. Candidates who received full credit for this problem were able to determine that this particular ILW could result in scenarios where ABC is not covered for losses due to the industry trigger as well as scenarios where ABC is reimbursed from ILW payouts that are actually greater than losses incurred by ABC.

Most candidates were able to either define basis risk accurately or describe a scenario where ABC would not be covered for losses due to the industry trigger not being met. To receive full credit a candidate must have discussed more than one scenario, or defined basis risk along with providing commentary on how the industry trigger introduces basis risk while the indemnity trigger minimizes basis risk.

#### **QUESTION 20**

TOTAL POINT VALUE: 2.5 LEARNING OBJECTIVE: C6

#### **SAMPLE ANSWERS**

Expected Loss = \$10K\*0.5+\$20K\*0.3+\$30K\*0.2 = \$17K

Expected Deficit = \$17K \* 5% = 850

 $850 = 0.2 * (\$30,000 - A_1)$ 

 $A_1 = $25,750$  (this works because  $A_1$  is greater than \$20K)

 $C_1 = A_1 - E(L) = $25,750 - $17,000 = $8,750$ 

Writing 2 Independent Lines with 50% Quota Share:

	Assets	Loss	Probability
Scenario 1	A <sub>2</sub>	\$10,000	.5*.5=0.25
Scenario 2	A <sub>2</sub>	\$15,000	2*.5*.3=0.30
Scenario 3	A <sub>2</sub>	\$20,000	.3*.3+2*.5*.2=0.29
Scenario 4	A <sub>2</sub>	\$25,000	2*.3*.2= 0.12
Scenario 5	A <sub>2</sub>	\$30,000	.2*.2=0.04

Expected Loss =  $10K^*.25+15K^*.3+20K^*.29+25K^*.12+30K^*.04 = $17K$ 

We'll test the situation where assets are between \$25K and \$30K first.

EPD Ratio =  $5\% = [0.04 * (\$30,000 - A_2)] / \$17,000$ 

 $A_2 = \$8,750$  (this does not work because not between \$25K and \$30K as assumed)

We'll test the situation where assets are between \$20K and \$25K.

EPD Ratio =  $5\% = [0.04 * (\$30,000 - A_2) + 0.12 * (\$25,000 - A_2)] / \$17,000$ 

 $A_2 = $20,938$  (this works because it is between \$20K and \$25K as assumed)

$$C_2 = A_2 - E(L) = $20,938 - $17,000 = $3,938$$

Acceptable answers receiving full credit:

- Reduction in Capital =  $A_1 A_2 = $25,750 $20,938 = $4,813$  Reduction
- Reduction in Capital =  $C_1 C_2 = \$8,750 \$3,938 = \$4,813$  Reduction
- Reduction in Capital = \$3,938 / \$8,750 1 = 55% Reduction
- Reduction in Capital = \$20,938 / \$25,750 1 = 18.6% Reduction

#### **EXAMINER'S REPORT**

Candidates were expected to know how to calculate the expected loss, the joint probability distribution, deficit in each scenario, the EPD, and remember its relationship to the EPD ratio.

In the current single line, no quota share situation, the candidate was expected to figure out that only the \$30K scenario loss will produce a deficit and correctly use the 5% target EPD ratio to get to a required assets / capital.

With the additional line and quota share, the candidate was expected to figure out that only the \$25K and \$30K loss scenarios will produce a deficit. Keeping the 5% target EPD, the candidate would arrive at a new required assets / capital.

After that, the candidate was expected to calculate the reduction in assets / capital resulting from adding the 2<sup>nd</sup> line and entering the quota share.

Candidates in general scored well, with a majority of candidates receiving full credit.

After calculating assets, some candidates did not go back and verify that a deficit was present under the loss scenarios they selected, thereby disproving their assumption.

Some candidates incorrectly calculated the 2 line probability distribution. Some candidates incorrectly defined the deficit in the EPD formula.

Some candidates made calculation mistakes.

QUESTION 21	
TOTAL POINT VALUE: 1.5	LEARNING OBJECTIVE: C7, C8
SAMPLE ANSWERS	
Part a: 0.5 point(s)	

- Firm AAA is a strong performer and has no probability of entering distress in a given year does not need to hedge. Should actively take bets if it has specialized knowledge / good insight / competitive advantage.
- No need to hedge, in fact it can seek speculative bets if it has specialized knowledge.
- This firm should not waste resources on risk management since investors can better/more cheaply manage this risk. Should take bets if they have specialized knowledge.
- Firm could benefit from risk management, as it would allow firm to increase use of debt to benefit from tax advantages or optimize capital structure.

### Part b: 0.5 point(s)

- High need to hedge as company is close to possible distress. They should hedge assets and not take any bets.
- Use risk management to reduce the probability of distress.
- Firm should engage in risk management, as it will decrease the probability of financial distress. Distress would have significant costs (e.g. legal, bankruptcy)
- With a relatively low rating and medium debt, a downward surprise in financial assets could result in bankruptcy. To avoid the costs associated with this, the firm should manage volatility.
- Should hedge against risk to avoid going into distress. Should not let management bias affect hedging ratio.

### Part c: 0.5 point(s)

- This firm is already in distress. Has no need to hedge and should take bets to try in increase positive tail outcomes.
- Company is already in financial distress. Not only should they take bets, they should actively seek new ones.
- This firm is already in distress and equity is close to worthless. Investors would want the company to increase risk. Do not hedge. Increase risk where possible.

#### **EXAMINER'S REPORT**

#### Part a

The candidate was expected to know that a highly rated firm with low debt does not need to engage in risk management as shareholders can do the same job more cost-effectively. However, full credit was also awarded if a candidate responded that this firm could actively manage volatility in order to increase leverage. Debt leverage has tax advantages over equity financing and also has the potential to strengthen management incentives.

Most candidates received full credit on this response.

#### Part b

The candidate was expected to know that a firm with a lower than average credit rating faces real costs of distress (such as bankruptcy) and should engage in risk management since reducing the costs of distress provides real benefit to shareholders. Additionally, the firm should not take bets or let management influence the hedge ratio.

Most candidates received full credit on this response.

#### Part c

The candidate was expected to know that a firm in distress should not use risk management. Reducing risk once the firm is in distress is not in the shareholder's best interest. Management should accept bets that present themselves and also seek new ones as the possibility arises.

Most candidates received full credit on this response.

QUESTION 22	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE: C8
SAMDLE ANSWERS	

# Part a: 0.5 point(s)

To receive full credit, candidates were expected to identify \$3,350,000 or the Myers-Read method as the total amount of capital required and to explain that:

- This is because 100% of required capital was allocated using this method
- Contrast with Merton-Perold which does not allocate 100% of the capital
- Explain that the Merton-Perold method has \$500,000 of non-allocated capital (or corporate capital)

### **Part b:** 0.25 point(s)

For full credit, candidates needed to explain that allocated capital was lower than standalone due to diversification benefits or a variant such as:

- Imperfect correlation leading to lower capital when adding lines of business
- Clearly explaining that the sum of square roots of standard deviations was lower than the sum of the individual parts.

#### Part c: 0.5 point(s)

For full credit, candidates were expected to provide and/or calculate the following:

- The correct formula for RAROC, EVA, or the UW Profit.
- The RAROC or EVA using standalone capital
- The RAROC or EVA using the M-R method (or the method answered in part (a))
- Correctly concluding that the company should all three lines together due to diversification benefits or that the return was higher combined than standalone.

#### **EXAMINER'S REPORT**

#### Part a

The candidate was expected to understand that the Myers-Read method always adds up to the total required capital or that the Merton-Perold does not fully allocate to this required capital amount

Common mistakes made were:

- Only identifying the required amount or best method without explaining why.
- Identifying the Merton-Perold total capital as the total amount needed.
- Simply calculating required capital as profit divided by CoC (does not account for diversification benefits)
- Only describing that the M-R method was additive without explaining what that meant.

#### Part b

The candidate was expected to demonstrate an understanding that the main reason why Merton-Perold total capital is lower than standalone is that there are diversification benefits accounted for by M-P.

The most common mistake on this question was explaining that there was unallocated capital using M-P or that method was sub-additive.

#### Part c

Candidates were expected to know how to calculate RAROCs for standalone lines as well as combined using a marginal allocation method.

Candidates were expected to prove numerically that the total RAROC under Myers-Read was higher than the cost of capital (and the RAROC using standalone capital).

#### Common mistakes were:

- -Not showing any calculations and stating that there were diversification benefits to writing all three lines together. This may be true, but this did not necessarily mean that writing all 3 lines together would provide a return higher than the cost of capital.
- -Using a marginal capital method to calculate the RAROCs for the standalone lines. This would lead to a conclusion to write Line C on a standalone basis.
- -Reallocating the \$4.3M of standalone capital between lines A, B, and C using the Myers-Read capital.
- -Not enough calculation detail (only showing totals without the methodology to get to them)
- -Calculating the total RAROC using Myers-Read without comparing to the standalone RAROC.

**QUESTION 23** 

TOTAL POINT VALUE: 1.25 LEARNING OBJECTIVE: C9

#### **SAMPLE ANSWERS**

**Part a:** 1.25 point(s)

#### Sample 1

Net Income

Line A: 6,000,000 (1-.20) (1.05) --6,000,000(.75) = 540,000 B: 4,000,000 (1-.30) (1.05) - 4,000,000(.65) = 340,000

Line A: RAROC: 540,000/4,000,000 = 13.75%

B: RAROC: 340,000/2,000,000 = 17% 17%> 13.5%

Therefore B is preferable and should be pursued if the cost of capital is less than 17%.

#### Sample 2

Line B has higher risk adjusted return on capital and should be pursued.

### **EXAMINER'S REPORT**

Candidates were expected to be able to assess the comparable value of investments in two lines of business based on risk based measures.

In order to obtain full credit, the candidates were expected to calculate income based on underwriting income and investment income (excluding the investment income on capital, which belongs to the stockholders in the RAROC methodology). The income is divided by capital to get the estimates of RAROC for each line, and then the answers needed to be compared. Some candidates assumed loss ratios were discounted to t=0 instead of discounted to t=1, this was also accepted as long as RAROC was calculated at t=1.

Common errors were around the calculation of the investment income, including:

- 1. omission of the investment income calculation entirely
- 2. including capital in the base for investment income
- 3. failing to reduce the premium by the expenses.

QUESTION 24	
<b>TOTAL POINT VALUE:</b> 3.25	LEARNING OBJECTIVE: D1
SAMPLE ANSWERS	

Part a: 3.25 points

## Sample 1

Year	0	1	1.25	2	3
Premium	1000	0	х	0	0
Expense	200	0	0	0	0
Losses Paid	0	210	0	0	390
Loss Reserves	600	390	390	390	0
Surplus	300	195	195	195	0
U/W Income	200	0	х	0	0
Inv. Income	0	67.5	0	43.875	43.875
Net Income	200	67.5	х	43.875	43.875
All. Surplus	-300	105	0	0	195
Cash Flow	-100	172.5	x	43.875	238.875

$$0 = -100 + \frac{172.5}{1.1} + \frac{x}{1.1^{1.25}} + \frac{43.875}{1.1^2} + \frac{238.875}{1.1^3}$$

$$x = -307.033$$

Sample 2
Assuming audit premium is paid at time 2

Year	0	1	2	3
Premium	1000	0	Х	0
Expense	200	0	0	0
Losses Paid	0	210	0	390
Loss Reserves	600	390	390	0
Surplus	300	195	195	0
U/W Income	200	0	х	0
Inv. Income	0	67.5	43.875	43.875
Net Income	200	67.5	43.875 + x	43.875
All. Surplus	-300	105	0	195
Cash Flow	-100	172.5	43.875 + x	238.875

$$0 = -100 + \frac{172.5}{1.1} + \frac{43.875 + x}{1.1^2} + \frac{238.875}{1.1^3}$$

$$x = -329.78$$

#### **EXAMINER'S REPORT**

- Candidates were expected to be able to evaluate the internal rate of return framework.
   They were expected to correctly calculate the loss payments and the loss reserves in order to calculate the required surplus. Candidates should have then calculated the investment income and the resulting equity flows. They should have used these equity flows in the IRR equation to solve for the audit premium.
- Candidates generally scored well on this question. Most candidates that lost credit either
  had a calculation error or did not understand the treatment of the audit premium. They
  either missed the timing of the audit premium or did not recognize that the audit
  premium was separate from the original premium. Some candidates missed the timing or
  amounts of the loss payments or missed the investment income at time 2.

#### **QUESTION 25**

**TOTAL POINT VALUE: 1.50 LEARNING OBJECTIVE: D2** 

#### **SAMPLE ANSWERS**

Part a: 1 point(s)

### **Company A**

Sample 1

$$\frac{T}{S} = \frac{I}{A} \left( 1 + \frac{R}{S} \right) + \frac{U}{P} \frac{P}{S}$$

$$\rightarrow \frac{T}{S} = 5\% (1 + 0.2) + (2.4\%)(0.5) = 7.2\%$$

Sample 2

$$\frac{T}{S} = \frac{I}{A} + \left(\frac{U}{R} + \frac{I}{A}\right)\frac{R}{S} = \frac{I}{A} + \left(\frac{U}{P} * \frac{P}{S} * \frac{S}{R} + \frac{I}{A}\right)\frac{R}{S}$$

$$\rightarrow \frac{\text{T}}{\text{S}} = 5\% + (2.4\% * \frac{0.5}{0.2} + 5\%)*0.2 = 7.2\%$$

### **Company B**

Sample 1

$$\frac{T}{S} = \frac{I}{A} + \left(\frac{U}{R} + \frac{I}{A}\right) \frac{R}{S}$$

$$\rightarrow \frac{T}{S} = 5\% + \left(\frac{6}{200} + 5\%\right) \frac{200}{500} = 8.2\%$$

Sample 2

$$\frac{T}{S} = \frac{I}{A} \left( 1 + \frac{R}{S} \right) + \frac{U}{P} \frac{P}{S}$$

$$\rightarrow \frac{T}{S} = 5\% * \left( 1 + \frac{200}{500} \right) + \left( \frac{6}{500} \right) = 8.2\%$$

Sample 3

$$\frac{T}{S} = {\frac{I}{A}(R+S) + U}/(S)$$

$$\rightarrow \frac{T}{S} = {5\% * (200 + 500) + 6}/(500) = 8.2\%$$

### Part b: 0.50 point(s)

#### Interest in reserve capital

Accepted answer 1: The "interest" that an insurer pays for the use of reserve capital is the underwriting return/ losses (as a percent of reserves and other liabilities).

Accepted answer 2: The "interest" that an insurer pays for the use of capital is the U/R term in Ferrari's 2nd equation.

## Difference of Interest between reserve capital and debt capital

Accepted answer 1: The interest paid for the use of debt capital is fixed (predetermined /set/ specific/ not variable) whereas the underwriting losses are variable (not fixed/volatile/ not know / expected value with variance).

Accepted answer 2: An increase in the relative amount of debt capital generally entails demands by the creditors for a progressively higher interest rate to reflect the increased risk of larger fixed commitments, but the relative profitability of expanding an insurance portfolio is not as predictable. The ability to reduce the relative variance of underwriting results by sheer volume and logical diversification may offset the costs of taking additional and possibly poorer risks.

#### **EXAMINER'S REPORT**

#### Part a

- The candidates were expected to know the components of return
- Candidates scored well with most receiving full credit
- Common errors were in identifying surplus in company
  - o Using the Reserves (200)
  - Using the sum of Reserves and Surplus (200+500)

#### Part b

- The candidates were expected to understand what portion of the calculation in part (a) represent "interest" and how this interest varies from typical financial institutions
- Candidates generally scored well especially in the first half of this part
- Common errors included:
  - o Interest stated to P or S instead of R
  - o Stating that insurers can have negative interest but financial institutions cannot
  - Stating that insurers will always decrease volatility by taking on more risks

QUESTION 26	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE: D5
SAMPLE ANSWERS	

#### Part a: 0.5 point(s)

Candidates were expected to identify an issue and to provide a brief description of the issue. For the latter part, either describing the mechanics of the issue (what causes it) or providing a workaround to fix this issue were accepted.

Candidates had to identify the issue raised by Robbin when switching to the quarterly frequency model. The following answers were accepted:

- Switching to a quarterly frequency model will result in an undesired increase in the Present Value of Equity (PVE)
- Switching to a quarterly frequency model will result in a decrease in the PVI/PVE ratio without any change to the equity.

Variants of the above were also accepted.

As the question asked to "describe," a complete answer had to include a brief description. The following elements were accepted for the remaining points:

- The equity will be counted almost four times, once for each quarter.
- Equity is a balance item and not a flow, so counting it multiple time makes no sense.
- The equity is artificially inflated, which distorts the measured PVI/PVE ratio.
- Switching to a quarterly model creates the need to annualize the equity.
- Switching to a quarterly model creates the need to adjust the equity base by dividing it by the sum of the present value factors.

Variants of the above were also accepted.

#### **Part b:** 0.75 point(s)

The three elements expected (Robbin discussion):

- A pre-tax rate is used because taxes are already accounted for/removed/excluded in the income calculation.
- A risk-free rate is used because it eliminates the need to measure/quantify the default risk;
- A risk-free rate is used because the underwriting profit provision will not depend on the investment strategy of the insurer.

Several other answers were also accepted, mainly with respect to the rationales for using a risk-free rate. Some of the other answers accepted were:

- Creates a level playing field between insurers from a regulatory perspective;
- Easier to justify to a regulator;
- The risk-free rate is easier to obtain and/or verify;
- The policyholder should not be penalized by the investors' gains above the risk-free rate.
- Using a risk-free rate creates a breakeven U/W profit from the investor's perspective.

#### Part c: 1.00 point(s)

The following were deemed to be complete answers:

### Sample 1

- o The PVE will increase as a result of the decrease in the discount rate.
- This leverage will make PVI/PVE less positive.
- o The target return must remain constant so the decrease has to be compensated.
- This will result in an increase in the indicated Underwriting Profit Provision.
   Most candidates did not reference the "leverage" explicitly, but if the candidate mentioned the effect on the PVI/PVE ratio full credit was given.

### <u>Sample 2</u> same as answer 1 but with an alternate first point:

 Both the PVI and the PVE will increase, however the PVE will increase more than PVI.

Full credit were also given to candidates who mentioned the effect of the PVI increase "winning" over the PVE increase, assuming sufficient assumptions were provided as this goes against Robbin's logical flow.

### Sample 3

- PVE remains constant provided the candidate specifies that this is because the equity is held in block and/or the PVE used is "annualized"
- o PVI increase with a decrease in the discount rate
- Keeping PVI/PVE constant, this places downward pressure on the Underwriting Profit Provision

Answers that captured the elements above, whether explicitly or implicitly, received full credit.

#### **EXAMINER'S REPORT**

This was a challenging question. Many candidates answered the question by referencing texts other than Robbin that did not apply well with this method. Many candidates did not realize that the method was based on the investor's perspective (being a measure of the profitability of equity) and answered from the policyholder's view.

#### Part a

Candidates performed well on this sub-part compared to the other parts of the question. It was expected that candidates identify and briefly describe the issue described by Robbin when switching to a quarterly frequency model.

A common mistake was to be too vague in the answer ("the ratio will change", "the equity will be impacted") without providing a direction. Many candidates also only mentioned the "fix" (annualizing the equity) without describing the issue itself (which was what the question was asking). Candidates who correctly identified the issue of PVE being too high and who either provided the correct fix to this issue, or a description of the mechanic of it, were awarded full credit.

#### Part b

Candidates did not perform well on this part. A high proportion of candidates were able to provide one or two elements from either remembering Robbin's discussion or from logical deduction. Many candidates correctly identified the rationale for using a pre-tax rate, but fewer identified rationales for using a risk-free rate. A very common mistake was to mention how the policyholders should not benefit from the gains made above the risk-free rate because they do not bear the investment risk. In the context of this question, this could not be an acceptable answer because a discount rate higher than the risk-free rate for discounting the numerator would result in a temporary decrease in PVI, which would have to be compensated by an increase in the Underwriting Profit Provision, thereby penalizing the policyholders via higher premiums. The policyholder would thus not benefit from a higher discount rate, but be penalized by it. Those candidates answered as if the question was about the investment yield that goes into calculating the investment income (which is part of the overall income), however the question was about the discount rate used to discount the overall income which is a discount rate that is to be interpreted from the investor's perspective, not the policyholder's.

### Part c

Candidates generally did not perform well on this subpart. Many candidates did not include Robbin's discussion on the leverage effect in their review. Many candidates correctly identified the effect on PVE increasing and some candidates also identified the effect on the PVI/PVE ratio. Very few candidates successfully identified the effect up to the Underwriting Profit Provision for full credit.

Robbin references "leverage" as the impact of a change in PVE on the PVI/PVE ratio making a positive one more positive (and a negative one more negative) in the context of an increasing discount rate.

In general, candidates that were able to correctly justify the movements in PVE, PVI, and PVI/PVE along with the resulting impact on the UPP were awarded full credit.

Some candidates stated that the denominator (PVE) was discounted at the target return, which did not change, and as a result PVE remained constant. This rationale and resulting conclusions were not accepted as they contradict Robbin's discussion on "leverage effect" where the change in PVE and its impact on the UPP is discussed in most detail.

QUESTION 27	
TOTAL POINT VALUE: 1.0	LEARNING OBJECTIVE: D6
SAMPLE ANSWERS	·

#### Part a: 0.5 points

- Catastrophe risk is the risk of large catastrophe striking your business
- Credit risk is risk of not being able to collect premiums
- Pricing Premium collected will be insufficient to cover actual losses and expense from policy written
- Reserve Reserves established will be insufficient to cover PHS liabilities.
- Risk that value of invested assets will decrease
- Risk that interest rate changes will impact assets and liabilities differently because their durations are mismatched.

### Part b: 0.5 points

- All surplus is available for every risk, so it is an artificial allocation
- Some risk sources are not proportional to premium, like catastrophe risk.
- All of the surplus is able to support only one line of business in an extreme event;
- May understate risk for some policies (i.e. excess policies are riskier and may need additional surplus.)
- It is difficult to allocate to line because it is difficult to account for diversification benefit between lines.
- Allocation by premium/surplus does not appropriately account for differences in risk between lines with different payout patterns.
- There is no general leverage ratio to apply to all insurers. Every insurer has unique surplus needs that are specific to the risks it is exposed to an industry ratio is not necessarily applicable to an individual insurer.
- Allocation by current year premium/surplus does not account well for the past growth or decline in the line, which can be important for long-tailed lines.

#### **EXAMINER'S REPORT**

#### Part a

In this part, candidates were expected to be able to describe risks that insurers face that may cause them to need to draw on their surplus. Most candidates did well on this part, receiving full credit. The answers listed above are just a few examples of the multitude of valid descriptions of risks facing insurers.

The question asked the candidates to describe the risk, but not to label them. Some candidates did the opposite – naming risks but not describing them. These responses did not receive full credit.

Full credit was given for any reasonable description of a risk that would draw on surplus. Some candidates gave a bad definition of a given label, but the description was a valid risk that would draw on surplus.

Some candidates gave two descriptions of essentially the same risk. For example, "Asset Risk – the risk that invested assets will decline; Interest Rate risk – the risk that interest rates will increase causing bond investments to decline." Only partial credit was given for these duplicative answers.

#### Part b

In this part, candidates were expected to be able to describe issues large insurers face when they try to allocate capital using premium-to-surplus ratios. Most candidates scored well on this part. The answers listed above are just a few examples of the multitude of valid ways that these challenges can be expressed.

The Roth paper makes the case that because all surplus backs every risk, any allocation is artificial. Some candidates tried to use this thought in both answers in part b: i.e., " – All surplus is available for every risk; - allocation is artificial." Only partial credit was given for these answers unless the candidate supported the second part with further reasons why allocation is artificial.

Many candidate responses centered around the valid argument that not all risks covered by surplus are proportional to premium. However, some candidates stated this as being that some risks aren't included in the consideration of the premium level. Some candidates successfully argued that current year premium may not include consideration for all risks that the insurer is currently exposed to because of some reason (like changes in coverage writings.)

#### **QUESTION 28**

TOTAL POINT VALUE: 2.5 LEARNING OBJECTIVE: D8

#### **SAMPLE ANSWERS**

Part a: 1.5 points

#### Sample 1

A Variance = 
$$\frac{300,000}{.3}$$
 = 1,000,000  
A safety =  $\frac{2,000,000-200,000}{1.07^2}$  = 1,572 190< Use higher 1,157, 190 need asset

#### Sample 2

LSC= 
$$(1+\sqrt{F})^2 F_2^7 S$$
  
Need A $(1+y)^2 = F(1+RF)^2 = M^2$   
A $(1+Y)^2 = (P+A)(1+RT)^2 = M^2$   
A $(1+1.07)^2 = (P+A)(1.03)^2 - 200,000$ 

IVC is investment Variance Constraint LSC is loss satisfactory constraint

LSC=> 
$$1.03^2$$
(A+P)  $\geq 2M$   
A $\geq 1.885192$ M- P

IVC=> A 
$$\geq \frac{300,000}{0.3}$$
 = 1,000,000  
LSC Floor=> P= 1.885192M-A  
A(1.07)<sup>2</sup>= (1.885192M- A+A) (1.03)<sup>2</sup>- 200,000  
A= 1,572, 190, Larger than IV from Investment Variance Constraint Dominant A= \$1,572,190

#### Sample 3

For candidates assuming the given loss distribution mean and standard deviation were the one year statistics:

Safety Constraint

$$R = \frac{0.1449 - 0.0609}{1.0609} \bullet \frac{2,000,000 - 200,000}{1.1449} \bullet 2$$
= 110651.5

Variance Constraint 
$$A = \frac{VL}{VY}$$
  
 $A = \sqrt{2} \cdot 300,000 = \frac{424,269}{0.3} = 1414213.5$   
 $R = \frac{0.1449 - 0.0609}{1.0609}$  (1414213.5) = 111975  
> 1110851.5

Since risk load is greater under the variance constraint, it's dominant.

R= 11975 A= 1414213.5

## Part b: 1 point

## Sample 1

mple 1

R= 1, 572, 190 
$$\frac{1.07^2 - 1.03^2}{1.03^2}$$
 = 124, 483

= A  $\frac{Y - RF}{Y + RM}$ 

P= R+ UL/1.03  $^2$  = 313, 002

### Sample 2

A(1 + 
$$Y$$
)<sup>2</sup> = (A+P) (1 +  $r$ )<sup>2</sup> - $L$ <sup>2</sup>  
1.8M= 1.03<sup>2</sup> • P + 1.468M  
P=.313M

# Sample 3

(P+A) 
$$(1 + RF)^2$$
= S  
(P+ 1,572,189.71)(1.03)  $^2$ = 200,000  
P = 313,002.11

### Sample 4

For candidates assuming the given loss distribution mean and standard deviation were the one year statistics:

$$P = R + \frac{UL}{1 + RF} = 111,975 + 400,000 \div 1.0609$$

$$P = $489,013$$

#### **EXAMINER'S REPORT**

Candidates were expected to know how to use Kreps' swap technique to calculate the allocated assets and reinsurance premium for the contract.

Candidates performed well on this question. Common mistakes included failing to properly adjust for the two-year contract.

#### Part a

Candidates were expected to calculate the assets allocated under both the safety and variance constraints and select the higher of the two as the final allocated asset. Candidates also needed to account for the two-year contract by calculating the investment yield and risk free rate over a two-year period and using the standard deviation of investment yield over two years.

#### Common errors:

- Not adjusting or incorrectly adjusting the investment yield for the two-year contract in the safety constraint formula
- Using the standard deviation of investment yield over one year in the variance constraint formula
- Using the risk free rate instead of the investment yield in the safety constraint formula

#### Part b

Candidates were expected to calculate the reinsurance premium for the contract given the allocated asset calculated in part a. There were several equivalent formulas that candidates used to arrive at the correct reinsurance premium (see sample answers). As in part a, candidates needed to account for the two-year contract by calculating the investment yield and risk free rate over a two-year period.

### Common errors:

- Not adjusting or incorrectly adjusting the investment yield and risk free rate for the twoyear contract
- Calculating the risk load but not the reinsurance premium
- Calculation errors

## **QUESTION 29**

TOTAL POINT VALUE: 2.5 LEARNING OBJECTIVE: D9

#### **SAMPLE ANSWERS**

### Part a: 2.0 points

#### Sample 1

Account X + Y:

$$\mu_{x+y}$$
 = 0.005 \* 185,000 + 0.01 \* 135,000 + 0.025 \* 60,000 + 0.05 \* 35,000 = 5,525   
  $\sigma_{x+y}$  = 4.03 \*  $\mu_{x+y}$  = 4.03 \* 5,525 = 22,266

Account X:

$$\mu_x$$
 = 0.005 \* 100,000 + 0.01 \* 75,000 + 0.025 \* 35,000 + 0.05 \* 20,000 = 3,125  $\sigma_x$  = 3.97 \*  $\mu_x$  = 3.97 \* 3,125 = 12,406

Account Y:

$$\mu_{y}$$
 = 0.005 \* 85,000 + 0.01 \* 60,000 + 0.025 \* 25,000 + 0.05 \* 15,000 = 2,400  $\sigma_{y}$  = 4.11 \*  $\mu_{y}$  = 4.11 \* 2,400 = 9,864

Risk Loads:

$$R_x = \lambda * (\sigma_{x+y} - \sigma_y) = \lambda * (22,266 - 9,864) = 12,402 \lambda$$
  
 $R_y = \lambda * (\sigma_{x+y} - \sigma_x) = \lambda * (22,266 - 12,406) = 9,860\lambda$   
 $R_{x+y} = 12,402 \lambda + 9,860 \lambda = 4,071$ 

$$\lambda = 0.183$$

$$\lambda = y * Z / (1+y)$$

$$0.183 = 0.125 * Z / (1+0.125)$$

$$Z = 1.645$$

Look up Z in standard normal distribution table to get 95%.

Therefore, probability is 5% (100% - 95%)

#### Sample 2

Use Mango Binomial Approximation:

$$\begin{split} &\sigma^2 = \sum_i p_i * (1-p_i) * L_i^2 \\ &\sigma_x^2 = .005^*.995^*100,000^2 + .01^*.99^*75,000^2 + .025^*.975^*35,000^2 + .05^*.95^*20,000^2 \\ &\sigma_x^2 = 154,296,875 \\ &\sigma_x = 12,422 \\ &\sigma_y^2 = .005^*.995^*85,000^2 + .01^*.99^*60,000^2 + .025^*.975^*25,000^2 + .05^*.95^*15,000^2 \\ &\sigma_y^2 = 97,506,250 \\ &\sigma_y = 9,875 \\ &\sigma_{x+y}^2 = .005^*.995^*185,000^2 + .01^*.99^*135,000^2 + .025^*.975^*60,000^2 + .05^*.95^*35,000^2 \\ &\sigma_{x+y}^2 = 496,634,375 \\ &\sigma_{x+y} = 22,285 \\ &4,071 = \frac{.125^*z}{1.125} * (22,285 - 9.875) + \frac{.125^*z}{1.125} * (22,285 - 12,422) \\ &z = 1.645 \\ &1 - N(z) = .05 \end{split}$$

### Part b: 0.5 points

- Use the Covariance Share method--The advantage is that it is renewal additive.
- Covariance Share Method. It spreads risk loads more equitably by using expected loss to allocate the covariance.
- The Shapley method allocates covariance equally to each account. Advantage is that this method is renewal additive.
- Use Myers-Read marginal surplus method--advantage is that it is renewal additive.

#### **EXAMINER'S REPORT**

Candidate performance was mixed on this question with most receiving partial credit. Candidates were expected to know the marginal surplus equation for part A) and why another method might be more appropriate for part B).

#### Part a

In part a, candidates were expected to derive the key components of the marginal surplus renewal risk load equation for accounts X and Y as described in the Mango paper. Candidates were expected to use these items to compute the last remaining input--the probability that the result would require more surplus than what was allocated.

As opposed to using the correct procedure applied above,  $R_{x+y} = \lambda * ((\sigma_{x+y} - \sigma_y) + (\sigma_{x+y} - \sigma_x))$ , many candidates used incorrect equations for the risk load.

The most common variations were:

$$\begin{split} R_{x+y} &= \lambda * (\sigma_{x+y}) \\ R_{x+y} &= \lambda * (\sigma_x + \sigma_y) \\ R_{x+y} &= \lambda * ((\sigma_{x+y}) + (\sigma_{x+y} - \sigma_y)) \end{split}$$

Due to the high correlation of losses for accounts X and Y, as well as rounding, many candidates who applied one of the incorrect procedures above ended up with a similar probability to the correct procedure, resulting in partial credit.

#### **Other Common Errors:**

- Using an incorrect formula for the coefficient of variation
- Assuming additional surplus was equal to the risk load plus expected loss
- Calculating the probability that the surplus would be sufficient rather than the probability that more surplus would be required (i.e. forgetting to subtract the probability from 1)
- Assuming the Risk Load provided was in \$1000's
- Calculation Errors

#### Part b

Most candidates picked up on a major deficiency of the marginal surplus method from the paper and were able to show a method that would be superior. The vast majority of candidates received full credit. Some candidates provided multiple advantages: only the first one was considered.

### Some Common Fully or Partially Incorrect Responses Were:

- Marginal Variance because it is more conservative
- Build Up Method
- Describing an alternate approach but not explicitly naming an advantage
- Naming an incorrect method