CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

October 30, 2013

Exam 8

Advanced Ratemaking

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 57.5 point examination consists of 25 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

- Verify that you have received the reference materials:

  a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2013 Study Kit*).
  b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan.
  c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2013 Study Kit*).

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 18, 2013.

END OF INSTRUCTIONS

1. (2.25 points)

A company is considering introducing a risk accumulation surcharge into its homeowners rating plan. The surcharge is a territory factor beyond expected pure premium differences and will be applied to territories where the total number of homeowner's risks in the territory is beyond a selected threshold.

Construct an argument for or against the introduction of the risk accumulation surcharge and whether it accomplishes the three primary purposes of risk classification listed in the American Academy of Actuaries "Risk Classification Statement of Principles."

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2. (3.5 points)

An actuary at a private passenger auto insurance company wishes to use a generalized linear model to create an auto frequency model using the data below.

Number of Claims

Gender	Territory A	Territory B
Male	700	600
Female	400	420

Number of Exposures

Gender	Territory A	Territory B
Male	1,400	1,000
Female	1,000	1,200

The model will include three parameters: β_1, β_2, and β_3, where β_1 is the average frequency for males, β_2 is the average frequency for Territory A, and β_3 is an intercept.

a. (0.5 point)

Define the design matrix [X].

b. (0.25 point)

Define the vector of responses [Y].

c. (2.25 points)

Assuming $\beta_3 = 0.35$, solve a generalized linear model with a normal error structure and identity link function for β_1.

d. (0.5 point)

The actuary determines that the analysis results would be improved by assuming a Poisson error structure with a log link function. Identify two reasons this structure may better suit this data.

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3. (1.75 points)

An actuary is creating a workers compensation classification rating plan. The actuary has access to frequency data from years 2002-2012 for fatal claims, which he believes may have low credibility.

After developing and testing a multivariate credibility procedure, the actuary finds the following results:

	Sum of Squared Prediction Errors – Fatal Claims		
Hazard Group	Prediction based on Hazard Group	Prediction based on raw data less the holdout sample	Prediction based on credibility procedure
A	100	90	80
B	110	70	105
C	90	115	75

a. (0.25 point)

Briefly describe the purpose of a holdout sample.

b. (0.5 point)

Justify an appropriate holdout sample from the available frequency data for the actuary to use in the classification analysis.

c. (0.5 point)

Discuss what the actuary's predicted results imply about his credibility procedure.

d. (0.5 point)

Suppose the actuary suspects that there may be an intrinsic downward trend in frequencies of fatal claims between 2002 and 2012 due to improved safety in his clients' workplaces. Propose a way for the actuary to test this theory.

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4. (3 points)

An actuary is helping design a new internet liability product that would use industry as a rating factor. Different business types such as restaurants, auto manufacturers, and dairy farms would fall into different industry groups. The actuary wants to create several industry factors from a combination of insurance and demographic data, and use this to classify business types into industry groups.

a. (1 point)

Describe two reasons that a generalized linear model might not be appropriate for developing industry factors.

b. (0.25 point)

Describe a benefit that a principal component method would have over a generalized linear model for determining the industry factors.

c. (1 point)

Briefly describe the major steps in using a cluster analysis to group the industry factors.

d. (0.75 points)

Describe two test statistics that could be used to determine the optimal number of groups from the cluster analysis. Identify which statistic would be preferred when variables are correlated.

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5. (2.25 points)

The following increased limits factors (ILFs) are used to price a general liability policy:

Aggregate Limit (000)	Occurrence Limit (000)				
	$25	$50	$100	$250	$500
$25	1.00				
$50	1.50	1.70			
$100	1.80	2.05	2.50		
$250	2.00	X	2.80	3.15	
$500	2.17	2.47	3.05	3.45	3.60

Calculate the range of possible values for the $50,000 occurrence / $250,000 aggregate ILF such that all the factors in the table pass the two-dimensional consistency test.

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6. (2 points)

Given the following information:

Amount of Loss	Probability of Loss
$0	80%
$100,000	15%
$500,000	5%

- The basic limit is $200,000.
- The actuary selects 20% of the standard deviation as the risk load.
- Assume there are no expenses.

Calculate the risk-loaded increased limit factor for a policy limit of $400,000.

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7. (2.5 points)

An actuary is modeling the impact of dispersion on loss development and excess ratios. The actuary has assumed that undeveloped losses are uniformly distributed between \$0 and \$120,000.

a. (1 point)

Calculate the excess ratio at \$75,000.

b. (1 point)

Assume a simple dispersion model such that each loss has an equal likelihood of developing by a multiplicative factor of 0.75, 1.00 or 1.25.

Given the following, calculate the excess ratio at \$75,000 with simple dispersion.

Loss	Excess Ratio
\$50,000	0.3403
\$56,250	0.2822
\$60,000	0.2500
\$93,750	0.0479
\$100,000	0.0278
\$110,000	0.0069

c. (0.5 point)

Briefly explain two impacts that simple dispersion has on excess ratios.

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8. (2.75 points)

Given the following information for a general liability policy, determine the value of X that yields an experience modification of +4.5%.

- Effective period of the policy: January 1 to December 31, 2014.
- Expected loss ratio: 65.6%.
- Type of policy being rated: Claims-made.

Loss Experience		
Policy period	Type of Policy	Includable Losses in Experience Period (Limited by MSL)
January 1, 2012, to December 31, 2012	Claims-made	X
January 1, 2011, to December 31, 2011	Occurrence	$72,234
January 1, 2010, to December 31, 2010	Occurrence	$30,484
Total		$102,718 + X

Policy Period	Coverage	Company Subject Loss Cost
Latest Policy Year	Prem/Ops	$32,160
	Products	$6,679
Prior Policy Year	Prem/Ops	$42,832
	Products	$14,137
Next Prior Policy Year	Prem/Ops	$38,695
	Products	$13,327
Total		$147,830

Subject Loss Cost	Credibility	Expected Experience Ratio	Maximum Single Loss
$138,763 - $145,183	0.33	0.866	$109,200
$145,184 - $151,800	0.34	0.870	$111,400
$151,801 - $158,621	0.35	0.874	$113,700
$158,622 - $165,658	0.36	0.878	$116,050
$165,659 - $172,920	0.37	0.882	$118,450

Loss Development Factors			
Subline	Latest Policy Year	Prior Policy Year	Next Prior Policy Year
Prem/Ops	0.519	0.338	0.198
Products	0.766	0.637	0.528

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9. (4.5 points)

Suppose that workers compensation risks are subject to a no-split experience rating plan under which credibility, as a function of expected loss, is calculated as follows:

$$Z = \frac{E}{E + 50{,}000}$$

During the experience rating period, a group of homogeneous risks had the following experience:

Risk	Actual Loss	Expected Loss
1	$130,000	$125,000
2	$60,000	$85,000
3	$160,000	$150,000
4	$200,000	$130,000
5	$100,000	$150,000
6	$250,000	$175,000

After the experience modifications were applied, the same group had the following experience:

Risk	Manual Premium	Actual Loss
1	$50,000	$35,000
2	$50,000	$25,000
3	$70,000	$60,000
4	$75,000	$50,000
5	$65,000	$40,000
6	$65,000	$100,000

a. (3.75 points)

Assess how effectively the experience rating plan corrected for the differences it identified for these particular workers compensation risks. Group the risks as appropriate.

b. (0.75 point)

Propose and justify a change to the plan that would improve its ability to correct the differences it identifies.

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10. (1.5 points)

An actuarial analyst has experience rated five groups of policies under the current rating plan and two alternatives, Plan A and Plan B. The results are as follows:

		Manual Loss Ratio			Standard Loss Ratio		
Risks with	Manual Premium ($000)	Current Plan	Proposed Plan A	Proposed Plan B	Current Plan	Proposed Plan A	Proposed Plan B
Lowest mod	$50,000	0.70	0.70	0.70	1.03	0.97	0.89
Next lowest	$70,500	0.85	0.80	0.90	1.02	1.02	0.92
Middle	$98,000	1.05	1.05	1.05	0.98	0.96	0.93
Next highest	$150,000	1.20	1.15	1.15	0.97	1.02	1.06
Highest mod	$10,000	1.45	1.55	1.35	0.96	1.06	1.11

a. (0.75 point)

The off-balance factor for the current plan is 1.05 and for proposed Plan A is 0.99. The analyst says that proposed Plan A performs better because the off-balance is less than 1. Critique this statement.

b. (0.75 point)

The actuarial analyst recommends staying with the current plan because it has made the higher mod groups more attractive to write and it has the least standard loss ratio spread. Critique this reasoning.

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11. (2 points)

Assume that workers compensation rate adequacy in a particular state has improved for several successive years.

a. (0.25 point)

Briefly describe the impact this improvement will have on the statewide off-balance.

b. (0.5 point)

Discuss the effect that the off-balance impact will have on the state's indicated rate level.

c. (0.5 point)

Discuss the effect on the off-balance factor and the state's premium adequacy over time if inadequate rates are approved.

d. (0.75 point)

Fully explain why the experience rating off-balance is frequently a credit.

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12. (2.5 points)

Given the following loss ratios for a set of five identical risks:

Risk	Loss Ratio
1	40%
2	40%
3	80%
4	100%
5	140%

Assume that the sample loss ratio of 80% equals the expected loss ratio.

Construct a Table M showing the insurance charges for entry ratios from 0 to 2.0 in increments of 0.50.

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13. (1.75 points)

A cohort of policies has a loss elimination ratio of k = 0.1.

a. (1 point)

   Draw a graph with three curves showing the relationship between the Table L charge (y-axis) and the entry ratio (x-axis) for policies with premiums of $20,000; $500,000; and $1,000,000.

b. (0.75 point)

   Briefly explain three main features of the curves that describe their proper relationship to the axes and to each other.

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14. (1.5 points)

A policy subject to a balanced retrospective rating plan is written with the following values:

- Expected loss ratio = 70% of standard premium
- Tax multiplier = 1.10
- Expense ratio = 20%
- Loss conversion factor = 1.125
- Maximum retrospective premium = 125% of standard premium
- Minimum retrospective premium = 75% of standard premium

The appropriate values from Table M for the current year are as follows:

- Insurance charge at entry ratio associated with maximum = 0.653
- Insurance savings at entry ratio associated with minimum = 0.031

Calculate the expected retrospective premium as a percentage of standard premium for the policy.

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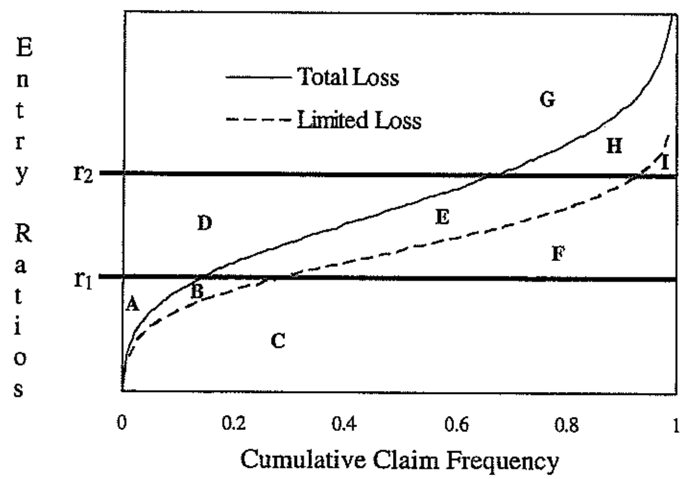
15. (3.25 points)

The following diagram depicts a book of business for a retrospectively rated workers compensation plan:



The descriptions of the labels on the diagram are as follows:

- r_1 = Aggregate minimum.
- r_2 = Aggregate maximum.
- Total loss - Total aggregate losses with no per-accident limit.
- Limited loss - Total aggregate losses after application of a per-accident limit.

a. (1.5 points)

Using the letter labels above to represent portions of the graph, describe the following quantities:

1.) ϕ - The Table M insurance charge at r_2.
2.) ψ – The Table M savings at r_1.
3.) ϕ^* - The Table L insurance charge at r_2.
4.) ψ^* - The Table L savings at r_1.
5.) I - The amount expected to be paid by the insured with an aggregate limit but no per-accident limit.
6.) I* - The amount expected to be paid by the insured in the presence of both an aggregate and a per-accident limit.

<<QUESTION 15 CONTINUED ON NEXT PAGE>>

b. (0.75 point)

A change in relevant workers compensation law goes into effect that causes a significant increase in the most severe losses. Briefly explain what effect this is likely to have on the areas of E and H in the above diagram.

c. (1 point)

Assume loss frequency and severity are independent, all individual losses come from the same distribution, and the only difference between large and small accounts is the number of expected claim counts. Determine whether the above diagram is accurate for both large and small accounts. Justify your answer.

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16. (1.25 points)

An actuary calculates the insurance charges on an aggregate deductible for a general liability policy for house painters. All the losses in the historical data used in the analysis resulted from inadequate and/or sloppy paint jobs, which were relatively inexpensive to fix. Later, it is discovered that some paint contained a toxic substance and those painters are liable for very expensive remediation of the painted properties.

The new claims are 10% as common as the historical claims. For every 10 claims that would have been expected before, there are now 11, one of which is cleaning up toxic paint.

Had this been known, the expected cost of a policy would have been twice the cost the actuary used.

a. (0.75 point)

At an entry ratio of 2.00, with no per-occurrence loss limit, explain whether the insurance charge would increase, decrease, or stay the same.

b. (0.5 point)

Explain how a per-occurrence limit would affect the change in the insurance charge for the aggregate deductible.

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17. (1.5 points)

In "Workers Compensation Excess Ratios: An Alternative Method of Estimation," Mahler discusses a method of estimating excess ratios using empirical data up to a certain truncation point and a fitted curve to handle larger loss sizes.

a. (0.5 point)

Briefly discuss how to choose a truncation point for this method.

b. (1 point)

Part of Mahler's method to handle larger loss sizes involves fitting a mixed Exponential-Pareto curve for estimating excess ratios. Evaluate the following alternative distributions in estimating excess ratios:

- Pareto curve only
- Mixed exponential – lognormal

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18. (4 points)

An actuary is pricing a Large Dollar Deductible (LDD) workers compensation policy. To price the excess loss portion, an actuary uses a blend of empirical data and a fitted curve to estimate excess loss pure premium factors. The cut-off for empirical data is $250,000. Using the data below, calculate an LDD premium for a policy with a $500,000 deductible and no aggregate limit.

Historical adjusted loss and ALAE for similarly sized risks:

Loss and ALAE	Observed Percentage Of Claim Counts
8,000	80%
100,000	11%
250,000	6%
500,000	2%
1,000,000	0.7%
2,000,000	0.3%

Total expected losses per risk = 55,400

Excess ratios based on fitted mixed Exponential-Pareto distribution on losses Truncated and shifted at $250,000:

Entry Ratio	Excess Ratio
0.1	0.92
0.2	0.84
0.3	0.73
0.4	0.69
0.5	0.65
0.6	0.60
0.7	0.55
0.8	0.51
0.9	0.47
1.0	0.44

Standard premium	$100,000
Loss based assessment (% of loss and ALAE)	4%
ULAE (% of loss and ALAE)	8%
General expenses (% of standard premium)	5%
Credit risk (% of standard premium)	5%
Acquisition expense (% of net premium)	8%
Tax (% of net premium)	3%
Profit Load (% of net premium)	5%

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19. (3.25 points)

The following information is available for a LDD policy:

Standard premium	$1,500,000
Expected ultimate loss ratio	0.75
State hazard group relativity	1.1
Deductible	$200,000
Excess loss factor	0.21
Aggregate limit on deductible	$1,000,000

The following Table M information is applicable to this policy:

Expected Loss Group	Range Rounded Values
30	$600,001 – $750,000
29	$750,001 – $925,000
28	$925,001 – $1,100,000
27	$1,100,001 – $1,300,000
26	$1,300,001 – $1,600,000
25	$1,600,001 – $1,950,000
24	$1,950,001 – $2,200,000

	Expected Loss Group						
Entry Ratio	30	29	28	27	26	25	24
0.75	0.4069	0.3989	0.3911	0.3833	0.3755	0.3677	0.3599
0.81	0.3777	0.3690	0.3605	0.3521	0.3436	0.3352	0.3267
1.07	0.2764	0.2661	0.2557	0.2453	0.2349	0.2245	0.2141
1.15	0.2522	0.2417	0.2310	0.2203	0.2096	0.1989	0.1882
1.23	0.2347	0.2241	0.2134	0.2027	0.1920	0.1813	0.1706
1.53	0.1690	0.1583	0.1476	0.1369	0.1261	0.1154	0.1047

a. (2.25 points)

Calculate the expected loss costs for the policy using the Insurance Charge Reflecting Loss Limitation (ICRLL) procedure.

b. (1 point)

Explain why the ICRLL procedure produces reasonably accurate insurance charges.

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20. (2 points)

The aggregate loss experience of an insurer's book of business is described by the following distribution function:

$$F(x) = x^{0.25} \text{ where } 0 \leq x \leq 1$$

a. (1 point)

Derive an exposure curve from the above cumulative distribution function.

b. (1 point)

Given that the maximum possible loss is \$2,000,000, use the derived exposure curve in part a. above to determine the ratio of pure risk premium in the layer \$1,000,000 excess of \$500,000.

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21. (1.5 points)

A property catastrophe treaty covers the layer $50,000,000 excess of $50,000,000 with an annual premium of $3,000,000 and a reinstatement provision that is 120% pro-rata as to amount with no limit on the number of reinstatements. The treaty is issued for a one-year term effective January 1, 2013.

a. (0.75 point)

Given the three following ground-up catastrophe losses during 2013:

- A loss of $65,000,000 on June 1.
- A loss of $85,000,000 on September 1.
- A loss of $115,000,000 on November 1.

Calculate how much the ceding company pays in reinstatement premiums during 2013.

b. (0.5 point)

Calculate the annual total reinstatement premium with the same three losses as above if the reinstatement provision was pro-rata as to amount and pro-rata to time.

c. (0.25 point)

Briefly explain why relatively few contracts include reinstatements pro-rata as to time.

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22. (1.5 points)

An actuary for a reinsurer uses the following exposure curve to price a non-proportional treaty with the assumption that b = 0.1:

$$G(x) = \frac{(1 - b^{x})}{(1 - b)}$$

The maximum possible loss for the reinsurer is $50 million and the ratio of pure risk premium retained by the cedant is 65%.

Calculate the cedants maximum retention under the treaty.

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23. (2 points)

A primary insurance company's actuary is evaluating the following three types of reinsurance contracts:

- 60% ceded quota share.
- Five-line surplus share treaty with retained line = $100,000.
- $400,000 xs $100,000 per-risk excess of loss.

In the most recent accident year, the company has experienced the following losses on its policies:

Risk	Insured Value	Loss
A	$250,000	$120,000
B	$1,000,000	$245,000
C	$85,000	$85,000
D	$1,250,000	$490,000
E	$400,000	$180,000
Total	$2,985,000	$1,120,000

Determine which reinsurance contract would result in the lowest retained losses for the insurance company.

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24. (2.25 points)

The following Occurrence Exceedance Probability curve is available for an insurance company's portfolio:

Return Period	Occurrence Exceedance Probability	Loss
10,000	0.0001	$200,000,000
500	0.0020	$50,000,000
200	0.0050	$20,000,000
100	0.0100	$12,000,000
50	0.0200	$7,000,000
33	0.0300	$3,500,000
25	0.0400	$1,500,000
20	0.0500	$500,000

a. (1 point)

The insurer specifies that its acceptable risk level is 1-in-250 year PML. Define PML and calculate the 1-in-250 year PML.

b. (1.25 points)

The insurer decides to buy property catastrophe reinsurance protection up to the 1-in-500 year PML in the following treaties:

Quota share, where 30% is ceded up to a $40 million loss limit, which inures to the benefit of the following:

- 100% placed 1st layer property catastrophe excess of loss treaty $6 million xs $4 million
- 90% placed 2nd layer property catastrophe excess of loss treaty $10 million xs $10 million
- 75% placed 3rd layer property catastrophe excess of Loss treaty $30 million xs $20 million

During the treaty year, the insurer suffers a $45 million earthquake loss.

Calculate the amount of loss ceded to each of the reinsurance treaties and the net retained loss by the primary insurer.

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25. (1.25 points)

An insurance company is considering a sliding-scale commission structure for its niche casualty excess of loss program, comprised of independent retailers. Using the program's loss history, two actuaries are tasked with calculating an aggregate loss distribution.

Given the following information:

- All policies have a $2,000,000 per occurrence insured retention.
- All policies have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention.
- All policies have a $2,000,000 aggregate limit.
- The only claims that are reported to the insurer are those that exceed the insured's retention.

Policy Year	Aggregate Loss in Excess of $2,000,000	Reported Claim Count
2000	-	0
2001	$1,506,002	2
2002	$1,070,358	1
2003	-	0
2004	$977,602	1
2005	$2,490,714	2
2006	-	0
2007	$512,933	1
2008	-	0
2009	-	0

Actuary A wants to use the lognormal distribution to determine the aggregate loss distribution. Actuary B prefers Panjer's recursive formula, using a Poisson frequency distribution.

Evaluate each actuary's selection and propose the more appropriate method.

Exam 8
Fall 2013

September 9, 2013

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	2.25							
2	3.50	0.50	0.25	2.25	0.50			
3	1.75	0.25	0.50	0.50	0.50			
4	3.00	1.00	0.25	1.00	0.75			
5	2.25							
6	2.00							
7	2.50	1.00	1.00	0.50				
8	2.75							
9	4.50	3.75	0.75					
10	1.50	0.75	0.75					
11	2.00	0.25	0.50	0.50	0.75			
12	2.50							
13	1.75	1.00	0.75					
14	1.50							
15	3.25	1.50	0.75	1.00				
16	1.25	0.75	0.50					
17	1.50	0.50	1.00					
18	4.00							
19	3.25	2.25	1.00					
20	2.00	1.00	1.00					
21	1.50	0.75	0.50	0.25				
22	1.50							
23	2.00							
24	2.25	1.00	1.25					
25	1.25							
TOTAL	57.50							

Exam 8

October 2013

Examiners' Report with Sample Solutions

The following table summarizes the statistics for this exam. The pass mark was set at 40.75 points, which was equivalent to 70.9% of the total points.

Exam Statistics

Number of questions:	25
Available points:	57.50 points
Pass mark:	40.75 points
Total candidates:	592
Passing candidates:	283
Effective % passing:	49.3%

Question 1

Model Solution 1

The three purposes of risk classification are:
1. Protect the financial soundness of the insurance system
2. Enhance fairness
3. Encourage availability of coverage by offering economic incentives to operate

Charging an accumulation surcharge accomplishes all three.
1. If companies were unable to charge a factor beyond expected pure premium it is likely that rates would be inadequate, not on a pure premium basis but on a risk-adjusted basis. Business in these territories is more likely to drain capital and cause insolvency if a catastrophe were to occur.
2. For the same reason above, risks in territories are riskier than risks within the threshold. To charge both types of risks the same (likely to be subsidies) would be unfair.
3. If companies are unable to charge a surcharge in highly concentrated territories, they may choose to stop offering coverage in these territories. That will cause coverage availability issues. Thus the surcharge improves availability.

Model Solution 2

Against the surcharge

A rating plan should be based on factors that reflect expected cost differences. This would make it more acceptable to the public. Otherwise, as it stands, this factor unfairly discriminates against certain risks. This factor does not reflect expected cost differences; rather it is just a business strategy for insurance since it will overcharge a risk if the insurer itself insures too many risks in the territory.

Protect the insurance system's financial soundness: it seems that if the insurer has too many risks in the territory, it will overcharge certain risks in that territory. This may lead those risks to seek out lower premiums from competitors. This results in adverse selected which harms financial soundness.

Enhance fairness: This rating surcharge is not controllable by the insured since it depends on the entire book of the insurer's business. It also doesn't show direct causality to loss, therefore the public acceptability would be very low. While these considerations are not crucial to a risk classification system, they are preferable.

Ensure widespread availability of coverage and allow economic incentives to operate: With a higher premium, good risks may leave the company and go to other companies with lower premium, and only the bad risks will remain. Eventually it would lead to bankruptcy of the company, which would decrease availability of coverage in the market.

Examiner's Comments

****

Many different answers were accepted for full or partial credit for this question.

The majority of candidates were able to identify the three principles of rate classification. Most were also able to offer some argument for or against the surcharge for each principle, but many of the arguments only received partial credit. To receive full credit, the argument needed to be well thought out and articulated, as well as sound. Some common "weak" arguments included:

1. Arguing that the charge isn't fair because it's beyond pure premium differences. A rate provides for all costs associated with the transfer of risk, including reinsurance and risk-adjusted cost of capital, both of which would be higher in concentrated areas. To argue that pure premium should be the sole determinant of premium differences only warranted partial credit.
2. Arguing that insurance would become less available because the surcharge makes this insurer more expensive was a weak argument. If, however, the candidate argued that if the surcharge became widespread and caused affordability issues throughout the market, that was a strong argument.
3. Many candidates mentioned "equal profit potential" or some variation thereof in their reasoning for providing economic incentives to provide coverage. In general this was considered weak reasoning. The AAA paper never specifically mentions "profit" in this regard, and unless the candidate made a strong connection between this and the insurer's willingness to provide coverage in the territory, the candidate would only receive partial credit.
4. Just stating that the surcharge would or wouldn't lead to adverse selection wasn't enough for full credit – the candidate needed to demonstrate how the adverse selection would take place.

A candidate did not need to have 3 arguments for, or 3 against to receive full credit. A fair amount of candidates argued 2 reasons for and 1 against or vice versa and received full credit.

****

Question 2:

Model Solution 1

a) $[x] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) $[y] = \begin{pmatrix} 700/1400 \\ 600/1000 \\ 400/1000 \\ 420/1200 \end{pmatrix}$

$= \begin{bmatrix} 0.5 \\ 0.6 \\ 0.4 \\ 0.35 \end{bmatrix}$

c) Classical linear model

$0.5 = \beta_1 + \beta_2 + 0.35 + \varepsilon_1 \qquad \varepsilon_1 = 0.15 - \beta_1 - \beta_2$
$0.6 = \beta_1 + 0.35 + \varepsilon_2 \qquad \varepsilon_2 = 0.25 - \beta_1$
$0.4 = \beta_2 + 0.35 + \varepsilon_3 \qquad \varepsilon_3 = 0.05 - \beta_2$
$0.35 = .35 + \varepsilon_4 \qquad \varepsilon_4 = 0$

$SSE = (0.15 - \beta_1 - \beta_2)^2 + (0.25 - \beta_1)^2 + (0.05 - \beta_2)^2$

$dSSE/d\,\beta_1 = -2(0.15 - \beta_1 - \beta_2) - 2(0.25 - \beta_1) = 0$
$.8 = 4\beta_1 + 2\beta_2$
$\beta_1 = 0.2 - 0.5\beta_2$
$\beta_1 = 0.2 - 0.5(.1 - .5\beta_1)$
$\beta_1 = 0.2$

$dSSE/d\,\beta_2 = -2(0.15 - \beta_1 - \beta_2) - 2(0.05 - \beta_2) = 0$
$.4 = 2\beta_1 + 4\beta_2$
$B_2 = .1 - .5\beta_1$

d) Frequency is non-negative, so normal distribution is a poor fit.
A multiplicative relationship fits frequency better than additive relationship.

Model Solution 2

Average Frequencies = # claims/ # Exposures

Gender	Territory A (β_2)	Territory B
Male (β_1)	0.5	0.6
Female	0.4	0.35

Intercept (β_3)	Male (β_1)	Territory A (β_2)	Y
1	1	1	0.5
1	1	0	0.6
1	0	1	0.4
1	0	0	0.35

a) $[x] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) $y = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.4 \\ 0.35 \end{bmatrix}$

$$\begin{bmatrix} 0.5 \\ 0.6 \\ 0.4 \\ 0.35 \end{bmatrix}$$

c) $\beta_3 = 0.35$

Normal error structure, identity link -> $g(x) = x$; $g^{-1}(x) = x$

$$\mu = E[Y] = \begin{pmatrix} \beta_1 + \beta_2 + \beta_3 \\ \beta_1 + \beta_3 \\ \beta_2 + \beta_3 \\ \beta_3 \end{pmatrix}$$

Identify likelihood function:

$$L(y;\mu,\sigma^2) = \prod_{i=1}^{n} \exp\{-\frac{(y_i - \mu_i)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)\}$$

Take the logarithm to convert the product of many terms into a sum:

$$l(y;\mu,\sigma^2) = \sum_{i=1}^{n} - \frac{(y_i - \sum_{j=1}^{p} X_{ij}.\beta_j)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)$$

$\mu = \Sigma x_{ij}\ \beta_j$

*Ignoring constant term $1/2\ln(2\pi\sigma^2)$

$$\to l(y, \mu, \sigma^2) = -\frac{(0.5 - (\beta_1 + \beta_2 + \beta_3))^2}{2\ \sigma^2} - \frac{(0.6 - (\beta_1 + \beta_3))^2}{2\ \sigma^2} - \frac{(0.4 - (\beta_2 + \beta_3))^2}{2\ \sigma^2} - \frac{(0.35 - (\beta_3))^2}{2\ \sigma^2}$$

Maximize the logarithm of the likelihood function by taking partial derivatives with respect to each covariate setting equation to 0 and solving system of equations. Question only asks to solve for β_1.

$$\frac{dl(y, \mu, \sigma^2)}{d\ \beta_1} = \frac{-2(-1)(0.5 - \beta_1 - \beta_2 - \beta_3)}{2\ \sigma^2} - \frac{2(-1)(0.6 - \beta_1 - \beta_3)}{2\ \sigma^2} = 0$$

➔ $0.5 - \beta_1 - \beta_2 - \beta_3 + 0.6 - \beta_1 - \beta_3 = 0 \to 0.4 = 2\ \beta_1 + \beta_2$

$$\frac{dl(y, \mu, \sigma^2)}{d\ \beta_2} = \frac{-2(-1)(0.5 - \beta_1 - \beta_2 - \beta_3)}{2\ \sigma^2} - \frac{2(-1)(0.4 - \beta_2 - \beta_3)}{2\ \sigma^2} = 0$$

➔ $0.5 - \beta_1 - \beta_2 - \beta_3 + 0.4 - \beta_2 - \beta_3 = 0 \to 0.2 = \beta_1 + 2\beta_2$

$0.4 = 2\ \beta_1 + \beta_2$
$0.2 = \beta_1 + 2\beta_2$

$\to 0.8 = 4\beta_1 + 2\beta_2 \ \to 2\ \beta_2 = 0.8 - 4\ \beta_1$
$\to 0.2 = \beta_1 + 0.8 - 4\ \beta_1$
$\to 0.6 = 3\ \beta_1$
$\to 0.2 = \beta_1$

d) The Poisson structure is more appropriate for this data use for a frequency model because:

a. Values for freq. are restricted to positive values; normality violates this assumptions
b. Normality assumes a fixed variance while Poisson structure allows the variance to increase with mean (more weight to observations at left of distribution)

Examiner's Comments
****

Part a
For full credit, the candidate must correctly populate a design matrix **X** based on the information given in the problem. There were multiple correct solutions to this problem, dependent on the order of the βs the candidate chose.

The most common error was omitting or incorrectly representing the intercept term in the matrix.

Part b
For full credit, the candidate must correctly populate the vector of responses **Y** based on the information given in the problem. The answer to part b is dependent on the order of βs the candidate chose in part a.

The most common error was providing the exposures instead of the frequencies as the values for **Y.**

Part c
For full credit, the candidate must solve the GLM by recognizing that the normal error structure and identity link function is identical to a classical linear model. The correct approach involved setting up linear equations, minimizing the sum of squared errors by taking the partial derivatives of each β, and solving the system of equations to arrive at $\beta_1 = 0.2$. Full credit was also given to candidates who did not identify that a normal error structure and identity link function was equivalent to a CLM, but solved by using the appropriate likelihood function, recognizing that solving the log-likelihood was equivalent.

A common error was to differentiate for β_3, which was a given in the problem, and attempting to solve the system of 3 equations.

Part d
For full credit, the candidate must identify two unique reasons that the Poisson error structure and/or a log link error function are a better fit for the data.
The most common incorrect or no credit response was "Poisson is a common distribution for frequencies", as this response did not show an understanding of *why* this was the case.
****

<u>Question 3:</u>

Model Solution 1

a. The holdout sample allows a model to be tested on real data which it hasn't been fitted to. This gives the best estimate of its predictive power.
b. Even years as holdout (odd years as model fit). Using even years (as opposed to say using the last 6) will correct for changes in risk profiles over time.
c. SSE(HG): 300 (= 100 + 110 + 90)
   SSE(Raw): 275
   SSE(Cred): 260
   The credibility procedure has the best SSE for hazard groups A and C (i.e. lowest). The raw data has the best for B.
d. Mahler provides a correlations test for this by comparing the correlation between years and lag. Less correlation as lag increases would imply shifting risk parameters. Looking at the raw data should be sufficient to confirm the direction of the shift.

Model Solution 2

a. The purpose of the holdout sample is to get an estimate of the unobserved population mean, to compare model results to.
b. Use odd years as the holdout sample and even years for estimation. This way any emerging trends in the data will likely be present in both even and odd years.
c. The credibility procedure produces an improvement for hazard group A and C but not for hazard group B. The SSE for hazard group B is greater for the credibility procedure than for the hazard group mean.
d. Calculate the Chi-squared test statistic = sum of (O – E)^2/E and compare it to a selected critical value threshold. If the test statistic > the critical value then the frequency is shifting over time.
   H(0): Parameters do not shift
   H(A): Parameters shift

Examiner's Comments:
****

Part a
For full credit, the candidate must display understanding that the holdout sample is not part of the data used to develop the model and its parameters or is representative of an independent mean. The candidate must state its purpose is to test or validate model results.

Common errors include failing adequately display such understanding.

Part b

For full credit, the candidate must identify a holdout sample and provide adequate justification for it.

The most common error is providing inadequate justification, such as holdout and training samples are similar in size.

Part c

For full credit, the candidate must observe that the method with the lowest sum of squared errors provided the best estimate of the three shown. Comparisons provided in total as well as separately by hazard group are acceptable.

Part d

For full credit, the candidate must identify a method to test the theory and how to apply its accept or reject criterion. Candidates did not have to provide a way to identify the direction of the trend.

The most common error was failing to provide an accept or reject criterion.

****

<u>Question 4:</u>

Part a

Model Solution 1

- There would likely be Intrinsic Aliasing, since you are using qualitative variables.
- It is not apparent what type of error structure or link function should be used for Industry type.

Model Solution 2

- GLM assumes that each obs is independent of each other which is probably not the case here as restaurants are clearly affected by dairy farms.
- The observations may not come from exponential family distribution.

Examiner's Comments

****

Candidates generally performed poorly on this subpart. A common response was that GLMs could not be used for grouping business types into industries. However, this question was not focused on that grouping task but rather on the deficiencies in the GLM assumptions as they relate to using a complex data set (industry and demographic data) when modeling response variables that differ from actuarial norms (frequency, severity, claims).

Examples of other acceptable responses include:

- GLMs are vulnerable to aliasing. If aliasing occurs, convergence of model results may be difficult. Aliasing can be intrinsic (category design), extrinsic (nature of data) or near. In this case, actuary is combining demographic data that is likely to be incomplete across risks and may lead to some form of aliasing.
- Need to select error and link functions. In general, actuaries have an idea of error functions for certain things (severities, claims) but would not have a good place to start with this data. Also, there might not be any functions that this kind of data well.
- Hard to identify the error structure and link function---we don't have a feel if the exponential family is a good model for these factors.
- There could be issues in the data (missing an industry code, etc.) that could lead to aliasing or near aliasing, where all the missing data values are correlated.

****

Part b

Model Solution1

The principal component (analysis) will identify the representative variable and determine the most significant factors---maximizing the proportion of total variance explained.

Model Solution 2

Principal component analysis identifies variables that are most predictive of [the] outcome, allowing one to eliminate other correlated variables from the model making the model simpler without much loss of function.

Examiner's Comments

****

Candidates generally performed poorly on this subpart. We were looking for answers relating to reducing dimensionality of the explanatory variables or accounting for correlation between variables/finding the most predictive variables.

****

Part c

Model Solution 1

1) Select the number of groups for k-means clustering using the methods in part (d)
2) Randomly assign factors to groups
3) Compute average factor for each group, called the centroid
4) Calculate distance of each factor to centroids, re-assigning to the closest centroid
5) If any factors changed groups, go back and repeat at step 3 with new groups until the algorithm stabilizes

Model Solution 2

1) Choose the number of clusters
2) Randomly assign risks to clusters
3) Compute centroid of each cluster and assign each risk to the closest centroid based on euclidean mean criteria.
4) If any of the risks move to a different cluster, repeat step 3.

Examiner's Comments

****

Candidates generally performed well on this subpart.

****

Part d

Model Solution 1

Calinski-Harbasz statistic: Measures the between variance of the clusters divided by the within variance.

Cubic Clustering Criterion (CCC): Measures variance explained by the clusters compared to clusters formed at random according to a multi-dimension uniform distribution.

If correlation is present, CCC performs worse, so use Calinski-Harbasz Statistic.

Model Solution 2

Calinski and Harbasz test statistic

$$\frac{\text{Trace (B)} / (k-1)}{\text{Trace (W)}/(n-k)}$$

Preferred when variables are correlated

Cubic Clustering Criterion
Compares variance explained by clusters to that explained by randomly assigned clusters.

Examiner's Comments

****

Candidates generally performed well on this subpart. The most common mistakes when describing the CCC statistic were failing to mention variance or comparing cluster variance with "total" variance instead of variance of a "random group". Some candidates either failed to mention which statistic was preferred or indicated that CCC was "less" preferred.

****

Question 5:

Model Solution 1

First, hold the agg limit constant:

$$\frac{X-2}{25} \geq \frac{2.8-X}{50} \geq \frac{3.15-2.8}{150}$$

$$\frac{X-2}{25} \geq \frac{2.8-X}{50} \geq .0023$$

$$2X-4 \geq 2.8-X \qquad\qquad 2.68 \geq X$$

$$3X \geq 6.8$$

$$X \geq 2.27$$

Next hold the occ limit constant:

$$\frac{2.05-1.7}{50} \geq \frac{X-2.05}{150} \geq \frac{2.47-X}{250}$$

$$.007 \geq \frac{X-2.05}{150} \qquad\qquad 250X-512.5 \geq 370.5-150X$$

$$3.1 \geq X \qquad\qquad 400X \geq 883$$

$$X \geq 2.208$$

so $2.27 \geq X \geq 2.68$

Model Solution 2

Same occc limits, vary agg limit

\$50K/25K occ limits:	2.05 – 1.8 < x – 2.0 < 2.47 - 2.17
	2.25 < x < 2.30
\$100K/\$50K occ limits:	2.5 – 2.05 < 2.80 – x < 2.05 – 2.47
	2.35 < x < 2.22

Same agg limits, vary occ limit

\$100K/250K agg	2.0 – 1.8 < x – 2.05 < 2.8 – 2.5
	2.25 < x < 2.35
\$250K/500K agg	2.17 – 2.0 < 2.47 – x < 3.05 – 2.8
	2.3 > x > 2.22

$\boxed{2.25 < x < 2.3}$

Examiner's Comments:

****

The majority of candidates received full credit on this question by performing a two dimensional consistency test on the table of ILF's provided using either:

- Miccolis' marginal consistency test as illustrated on page 60 of Rosenberg's discussion
- Factor comparison illustrated on page 61 of Rosenberg's discussion

To receive full credit, the candidate should have:

- Calculated either consistency test along both dimensions:
    1. varying occurrence limit while holding aggregate limit constant
    2. varying aggregate limit while holding occurrence limit constant
- Utilized other limits shown – above/below and left/right of missing ILF.

Credit for candidates doing part of the marginal consistency test and part of the factor comparison (subtraction) was given based on how fully the candidate satisfied the above criteria for full credit. For example, varying the occurrence limit under both tests received less credit than varying the occurrence limit under one test and varying the aggregate limit under the other test.

It was acceptable but not required to cap the upper limit at 2.47 based on the factor for the \$500K Aggregate/\$50K Occurrence limit provided in the table.
Both strict and non-strict inequalities were equally acceptable.

****

Question 6:

Model Solution 1

ILF will be equal to $\frac{E[g(x;k)]+.2(stddev\ g(x;k))}{E[g(x;b)]+.2(stdev\ g(x;b))}$

$E[g(x;400k)] = .15(100k) + .05(400k) = 35{,}000$

$E[g(x;400k)^2] = .15(100k^2) + .05(400k^2) = 9.5B$

$std\ dev = \sqrt{9.5B - (35000)^2} = 90{,}967.027$

$risk\ load = .2(90967.027) = 18193.4054$

Similarly:

$E[g(x;200k)] = .15(100k) + .05(200k) = 25000$

$E[g(x;200k)^2] = .15(100k^2) + .05(200k^2) = 3.5B$

$std\ dev = \sqrt{3.5B - (25000)^2} = 53619.026$

$risk\ load = .2(53619.026) = 10723.81$

$$ILF = \frac{35000 + 18193.405}{25000 + 10723.81} = 1.489$$

Model Solution 2

<u>Basic Limit</u>

$$E[L] = (.80)(0) + .15(100{,}000) + .05(200{,}000)$$
$$= 25{,}000$$
$$E[L^2] = (.80)(0) + .15(100{,}000^2) + .05(200{,}000^2)$$
$$= 35\ x\ 10^8$$
$$\sigma_{200} = \sqrt{E(L^2) - E(L)^2} = 53{,}619$$

<u>400K Limit</u>

$$E[L] = (.80)(0) + .15(100{,}000) + .05(400{,}000)$$
$$= 35{,}000$$
$$E[L^2] = (.80)(0) + .15(100{,}000^2) + .05(400{,}000^2)$$
$$= 95\ x\ 10^8$$
$$\sigma_{400} = \sqrt{E(L^2) - E(L)^2} = 90{,}967$$
$$ILF = \frac{35{,}000 + (.2)(90{,}967)}{25{,}000 + (.2)(53{,}619)} = 1.489$$

Examiner's Comments:

****

For full credit, a candidate needed to select the correct limits for evaluation, as well as correctly apply all the formulas, especially recognizing that the Risk Load was 20% of the Standard Deviation for each limit. Common mistakes that resulted in minimal credit included using the Variance or Second Moment for calculating the risk load at each limit. Given the magnitude of the Variance and Second Moment, the use of these produce unrealistic risk adjusted premiums at each limit. Another

common mistake resulting in minimal credit was using the Standard Distribution from the total distribution. Risk loads should vary by limit to appropriately reflect the difference in risk.
****

Question 7:

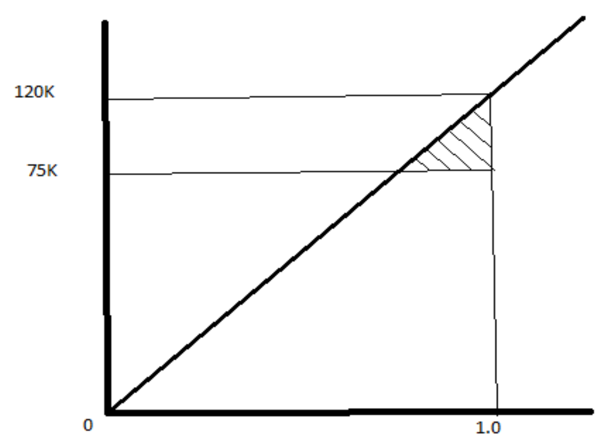
Part a
Model Solution 1

$$E(X) = \frac{1}{120}\int_0^{120} x\,dx = \frac{1}{120} * \left(\frac{120^2}{2}\right) = 60$$

$$R(75) = \frac{\left[\frac{1}{120} * \int_{75}^{120}(x-75)\,dx\right]}{E(x)}$$

$$R(75) = \frac{8.4375}{60} = 0.1406$$

Model Solution 2



Excess ratio at 75,000 = {(120-75)/120}^2 = 0.1406

Examiner's Comments:
****
Part (a) was very straightforward and most candidates did very well.

****

Part b
Model Solution

$$R^\wedge(75) = \frac{1}{3}\left[0.75 * R\left(\frac{75}{0.75}\right) + 1.00 * R\left(\frac{75}{1.00}\right) + 1.25 * R\left(\frac{75}{1.25}\right)\right]$$

$$R^\wedge(75) = \frac{1}{3}[0.75 * R(100) + 1.00 * R(75) + 1.25 * R(60)]$$

R(75) = 0.1406 (from part a)

R(100) = 0.0278 (from excess ratio table)

R(60) = 0.25 (from excess ratio table)

$$R^\wedge(75) = \frac{1}{3}[0.75 * 0.0278 + 0.1406 + 1.25 * 0.25] = 0.1580$$

Examiner's Comments:
****
On part (b), most candidates did well and utilized the developed loss excess ratio formula correctly. The most common mistakes were:

- Confusing multipliers with divisors
- Simply averaging R(60,), R(75) and R(100) instead of weighting them by the multiplier

If the candidate interpolated the value of R(75) from the table using linear interpolation, instead of using the answer from part (a), we have given full credit for this approach. The interpolated value of R(75) = 0.1602

There were a handful of candidates that calculated the values of R(100) and R(60), instead of picking the values directly from the table. If the calculation was done correctly, the answer should be the same as the value in the table and they were not penalized for this.

****

Part c
Model Solution

Impacts of dispersion on excess ratios:

- Dispersion produces more excess losses without affecting total expected losses.
- Dispersion raises the excess ratios for higher limits and alters those for lower limits.

Examiner's Comments:
****
On part (c), points were given to other valid answers that were not listed in the model.

<u>Other possible solutions:</u>

- The higher the variation (or C.V) of dispersion, the higher the excess ratio
- Dispersion increases the variation (CV /uncertainty/variability) of excess ratio
- Simple dispersion will result excess ratio > 0, when excess ratio = 0 when no simple dispersion
- The impact of simple dispersion on excess ratios is less than the impact of the Gamma distribution.
- The impact of uniform dispersion on excess ratios is less than the impact of simple dispersion

****

Question 8:

Model Solution 1

CSLC = 147,830 → z = .034 EER = 0.870 MSL = 111,400

PY	Covg	CSLC	EER	LDF	ARULL
2012	Prem	32,160	0.870	0	0
2012	Prod	6,679	0.870	0	0
2011	Prem	42,832	0.870	0.338	12,595
2011	Prod	14,137	0.870	0.637	7,835
2010	Prem	38,695	0.870	0.198	6,666
2010	Prod	13,327	0.870	0.528	6,122
Total		147,830			33,218

ARULL = CSLC x EER x LDF
Mod = z*(AER – EER)/EER
0.045 = 0.34*(AER – 0.87)/0.87
AER = 0.985147059 = (102,718 + X + 33,218)/147,830
X = 9,698

Examiner's Comments:
****
Common errors that resulted in partial credit:

- LDFs were used for the Claims Made (2012) Policy
- Did not multiply by EER in calculation of the ARULL
- In the formula for the Mod, some candidates used 1+Mod instead of just the mod itself on the left side of the equation
- Candidates did not do the entire first step to calculate the ARULL
- There were some candidates who did not explicitly calculate the AER, however if they arrived at the correct answer, they were given full credit. If the candidate did not explicitly calculate the AER, but got the wrong answer, additional deductions were taken since the steps were not clearly documented.
- Incorrect formulas
- Did not calculate includable losses correctly
- Some candidates wrote down the correct credibility to use somewhere on the page, but actually used something different in formulas and calculations

****

Question 9:

Model Solution 1

a.

Calculate mod as: $\left(\frac{A-E}{E}\right)Z+1$

Act Loss = A

Expected Loss = E

Risk	Z	Mod
1	.714	1.029
2	.63	.815
3	.75	1.05
4	.722	1.389
5	.75	.75
6	.778	1.333

$\frac{175}{175+50}$

$=\left(\frac{250-175}{175}\right).778+1$

$=\frac{40+25}{48.75+40.75}$

Rank f/ low mod to High Mod

Risk	Mod	MP	SP = Mod * MP	Loss	Stand. LR	Man LR
5	.75	65	48.75	40	.726	.565
2	.815	50	40.75	25		
1	1.029	80	51.45	35	.7603	.792
3	1.05	70	73.5	60		
6	1.333	65	86.645	100	.786	1.071
4	1.389	75	104.175	50		

Group into 3 buckets

mod < 1

Mods close to 1

Mods significantly > 1

To see how well the plan corrects risk differences, look at standard loss ratio. Expect to see little variability in standard loss ratio by bucket. We see the Std LR increase as the mod increases → The plan is giving too little credibility to actual experience and thus does a poor job of correcting for risk differences. The plan does do a good job of identifying risk differences though, as shown in the increasing manual LRs as mod increases.

b.

One change that could be made is to use a split plan, breaking losses into primary and excess components. Since WC losses are highly skewed, there is a substantial difference between the optimal estimate of credibility and the best linear estimate. This plan uses a linear estimate. By splitting losses into primary and excess, the distributions will be less skewed and predictive accuracy will be enhanced.

Model Solution 2

a.

$Z = \dfrac{E}{E + 50{,}000}$ K = 50,000

Risk	Actual loss (000)	Exptd (000)	M = $\dfrac{A+K}{E+K}$
1	130	125	1.029
2	60	85	0.815 $\longrightarrow = \dfrac{60+50}{85+50}$
3	160	150	1.05
4	200	130	1.389
5	100	150	0.75
6	250	175	1.333

Use 3 groupings to test the plan. (by mod)

b.

Since K is constant, it implies that the variance of loss ratios will decrease as the size of risk increases. We know that is not necessarily the case, due to changing conditions and diversifying operations as risk size increases. Therefore, allowing K to increase as risk size increases would improve the predictive accuracy of the plan, since it will avoid problem of large risks essentially being self-rated and will assign credibility < 1.

Examiner's Comments:
****

Part a

Many candidates did not pay attention to the instructions that they needed to group the risks in order to appropriately assess the experience rating plan. The optimal grouping was to group the risks by mod (low, mid, high) as explained in the Venter (and Gillam) paper. Some candidates grouped credit vs. debit risks, which was acceptable. Those who did not appropriately group the risks could only receive partial credit.

Some candidates performed a quintiles test, but given that there were 6 risks, it would not make sense to create 5 groups. Candidates needed to recognize that the mods were clearly divided into three groups of 2. Without grouping in this fashion, one could not assess the plan in a valid way due to volatility in individual risk experience over 1 year.

Many candidates who did not group the risks chose to perform the efficiency test, which is not an appropriate method to apply here because there is no other experience rating plan with which to compare those test statistics.

Two common conceptual errors were:

- Re-calculating the mod after combining risks; and
- Taking straight averages of the standard loss ratios instead of properly combining losses and standard premiums for each mod group

Candidates did not need to calculate manual loss ratios to receive full credit on this question.

<u>Part b</u>

This question asked candidates to identify a change to the experience rating plan itself that would improve its ability to correct for the differences it identifies. Thus, this part was not directly related to the specific numerical example given in part a.

In general, candidates did well, but overall they needed to improve their justification of the identified change. Explaining what the change is in more detail does not serve as justification for the same.

Candidates who misunderstood the question and simply said that K should be lowered (to increase credibility) received partial credit, since it is true that if K is lowered, the standard loss ratios calculated in part a. become nearly flat across mod group.
****

Question 10:

Part a
Model Solution 1

An Off balance factor is a measure of the weighted modification factor. An off balance less than 1 does not necessarily mean that the plan is better. To decide which plan is best, we can find the plan that can best identify the differenced in the groups, and also best corrects the differences. We can do this by looking at the manual loss ratios for whether the plan identifies the differences. The more spread the manual loss ratios, the better the plan identifies. We then look at standard loss ratio for correcting the differences. The flatter the loss ratio, the better.
By examining the loss ratios of the 3 plans, all of them do well for identifying differences because the manual loss ratio increases as risk potential increases. Plan A does well for correcting the difference because the standard loss ratios are flat and no trend.

Examiner's Comments:
****
Many candidates ignored the crux of the question as it related to the off balance. To receive full credit, the candidate must state that the off balance is not an indicator of plan performance, what the off balance is or represents (weighted average mod, or standard premium/manual premium) and finally what metrics can be used to evaluate the performance of the experience rating plan. Many candidates stated why A is the better plan, which is relevant to part (b) of the question, but did not comment on the actuary's reasoning that it is better because of the Off Balance, and therefore did not receive full credit.
****

Part b
Model Solution 1

The current plan has a better loss ratio for higher mod groups. In fact, the current plan has a decreasing trend on standard loss ratios. This means that too much credibility is assigned to the actual data in current plan. Our goal of the plan should be resulting a standard loss ratio close to 1 and with no clear trend. The current plan violates this rule. Plan B has an increasing trend on the standard loss ratio, meaning too little credibility is assigned. Plan A is the best among the three because the standard loss ratio is flat and no trend.

Model Solution 2

Having a low standard loss ratio variance is good, but you also need to consider any pattern in the LRs. Dorweiler's necessary condition is that all standard Loss ratios across risks must be equal, or at least similar enough so insurers do not prefer one over the other. The current plan <u>did</u> make higher mod groups more attractive but it also made lower mod groups less attractive to write. The current plan gives too much credibility to the risks' experience, which is causing the downward trend. Plan B on the other hand, doesn't give enough credibility. Based on Dorweiler's test, Plan A would be the better option.

Model Solution 3
Unfortunately, competitive pressure may lead to adverse selection under this plan. Essentially, Mods are too high for some of the worse than expected risks because too much credibility is assigned to their experience. This is what is driving the favorable standard loss ratios under the current plan. Plan A may be more equitable because of seemingly random nature of its standard loss ratios.

Examiner's Comments:
****
Most of the candidates answered satisfactorily by pointing out the trends in the SLRs or MLRs and stating what these trends indicated (e.g. oversensitivity, balance, identifying differences among risk groups).
****

<u>Question 11:</u>

Model Solution 1

a. Lower the off-balance

b. Off-balance reduces the degree of indicated rate level change

c. Off-balance will go up and reduce the degree of premium inadequacy

d. Larger risks that go into the experience rating usually perform better than small risks that are not eligible for experience rating. Off-balance factor is assumed to be 1 for these small risks when calculating experience rating off-balance resulting in a credit.

Model Solution 2

a. Off-balance should have move down closer to 1.0 since it would be greater than 1.00 with inadequate rates

b. Off-balance tends to correct for rate inadequacy so it is greater than 1.0. If adequate rates approved, off-balance move down to 1.0 but rates still could be inadequate since off-balance masks true rate need.

c. Off-balance tends to partially correct for inadequate rates. If inadequate rates approved, off-balance move greater than 1.0

d. Large risks which receive greater weight tend to have better experience and receive credit which is not offset by smaller risks with worse experience. Since smaller risks are not as credible and given less weight.

Model Solution 3

a. Decrease in off-balance

b. Off-balance decrease may cause rates to still be inadequate and cause indicated rate increases (because rate changes are based off standard premium, aka modified premium, and the rate changes assume that the off-balance will not change.

c. Off-balance will increase and partially correct for the inadequacy in the rates. However, rates will still be inadequate even after considering and taking into account the off-balance increases.

d. Smaller risks that don't qualify for experience rating tend to have worse experience than expected. Larger risks tend to have better experience than expected and receive credit. So overall off-balance is less than 1.

Examiner's Comments:

****

a. Candidates needed to state that the off-balance would decrease. Stating the off-balance would be 1.0 or move to 1.0 was not given credit.

b. Candidates needed to demonstrate that they understood that if rate adequacy was improving indicated rate levels would be decreasing, but the off-balance would make the decrease less than expected.

   Partial credit was given for stating that the off-balance masks the indicated rate change.

   Common answers that did not receive credit included:

   True statements that do not answer the question

   - Off-balance near 1 implies rates are adequate.
   - Rate level is based on manual premium which the off-balance does not impact
   - There is no change in the off-balance between the experience and prospective period

   Incorrect statement

   - As the off-balance decreases so does standard premium therefore the indicate rate level increases

c. Most candidates correctly answered that the off-balance factor would increase. To receive full credit candidates also needed to clearly state that the increasing off-balance reduces premium inadequacy. Common answers that did not receive full credit were: Off-balance offsets inadequate rates and premium adequacy is not impacted.

d. To earn full credit candidates needed to clearly state three key pieces of information:

   1 – There are only certain risks that qualify or receive more credibility
   2 – Risks that qualify and/or receive more credibility are large
   3 – These risks have better experience or perform better

Stating that risks have a credit mod or better risk instead of better experience would not have earned credit.

****

Question 12:

Model Solution 1
("The Upward Sum Method"):

E = (.4 + .4 + .8 + 1.0 + 1.4) / 5 = 80%

L/R
0%
40%
80%
120%
160%
200%

Using the double sum method, need more entry ratios to create rectangle areas
r = entry ratios = L / E

L/R	**r**	**# risks at r**	**# risks above r**	**double sum**	**Ø(r)**
0%	0	0	5	20	1.0
20%	.25	0	5	15	0.75
40%	.50	2	3	10	0.50
60%	.75	0	3	7	0.35
80%	1.0	1	2	4	0.20
100%	1.25	1	1	2	0.10
120%	1.50	0	1	1	0.05
140%	1.75	1	0	0	0
160%	2.0	0	0	0	0

Entry Ratio	**Ø(r) = insurance charge**
0	1.0
.5	0.5
1.0	0.2
1.5	0.05
2.0	0

Model Solution 2
(“The % Method”)

Risk	**entry ratio (r_i) = Loss Ratio / ELR**
1	0.5
2	0.5
3	1.0
4	1.25
5	1.75

Entry R	**# Risks**	**# Risk >**	**% Risk >**	**Ø(r_i)**
0	0	5	100%	1.0
0.5	2	3	60%	0.5
1.0	1	2	40%	0.2
1.25	1	1	20%	0.1
1.5	0	1	20%	0.05
1.75	1	0	0	0
2.0	0	0	0	0

$Ø(r_i) = Ø(r_{i+1}) + (r_{i+1} - r_i)$ x % Risks >
So

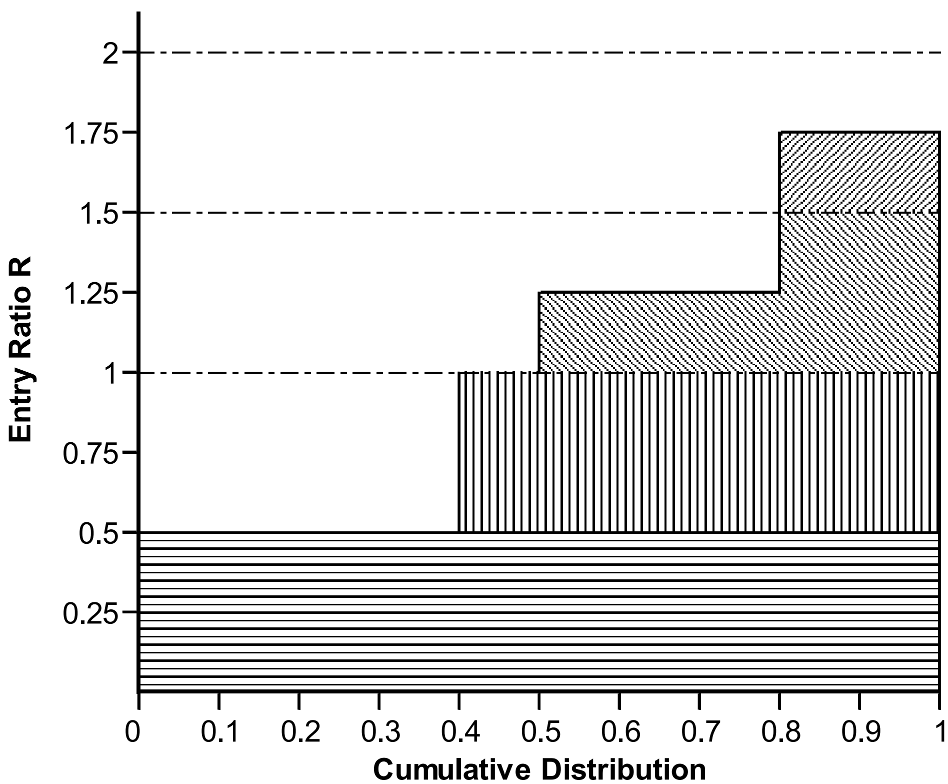
Entry r_i	**Ø(r_i)**
0	1.0
.5	0.5
1.0	0.2
1.5	0.05
2.0	0

Model Solution 3
("The Direct Method")

Entry Ratio	Portion Above Entry Ratio	Charge
0.0	(0.5 + 1 + 1.25 + 1.75 + .5) / 5 = 1	1.0
0.5	(0.5 + .75 + 1.25) / 5 = 0.5	0.5
1.0	(.25 + .75) / 5 = 0.2	0.2
1.5	(.25) / 5 = 0.05	0.05
2.0	0 / 5 = 0	0.0

Model Solution 4
("The Graphical Method")

Entry Ratio	# of Risks at	# of Risks Above	Percent Above	Charge
0.0	0	5	1.0	0.5 + 0.5 = 1
0.5	2	3	0.6	0.2 + (1-0.5)(1-0.4) = .5
1.0	1	2	0.4	.05 + (1.5-1.25)(1-.8) + (1.25-1)(1-.6) = 0.2
1.5	0	1	0.2	(1-.8)(1.75-1.5) = 0.05
2.0	0	0	0.0	0



Examiner's Comments:
****

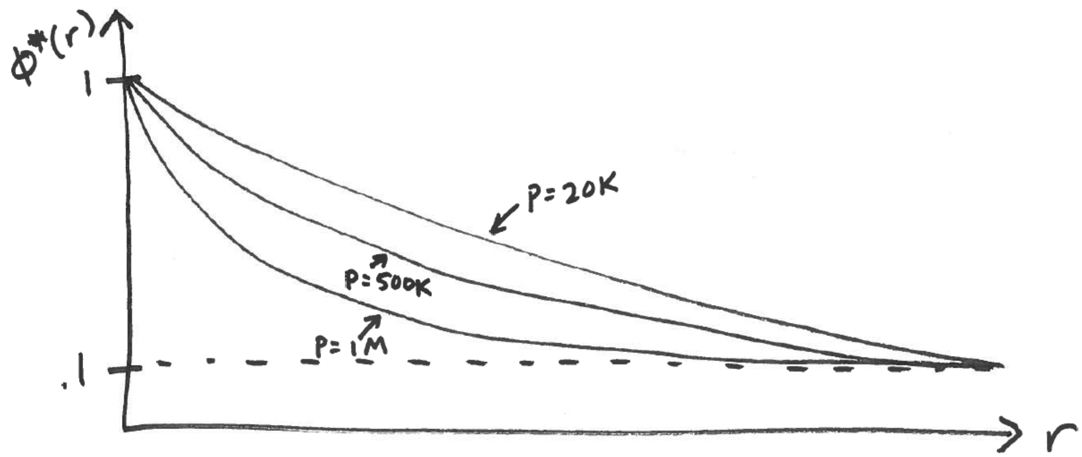
Most candidates were successful in calculating the correct insurance charges using one of the four model solutions. Candidates were not required to show the rows for entry ratios of 1.25 and 1.75, but no credit was deducted if those rows were in the final answer. The stated assumption "Assume that the sample loss ratio of 80% equals the expected loss ratio" should cue a well-prepared candidate that there is no need to normalize per the Brosius paper. Several candidates also included the calculation of the savings, which was not requested. This did not affect scores, but would have been an inefficient use of time.

By far the most common errors involved the table intervals. Using the upward sum method, intervals need to be evenly-spaced and have rows for each observed value to work correctly. For the % method, the intervals needn't be evenly-spaced, but table entries are required for observed values in order to calculate the required values. Some candidates did not include rows for entry ratios of 1.25 and 1.75, which produced erroneous final answers. Other, less common, errors include: improper conversion of loss ratios to entry ratios and arithmetic errors.

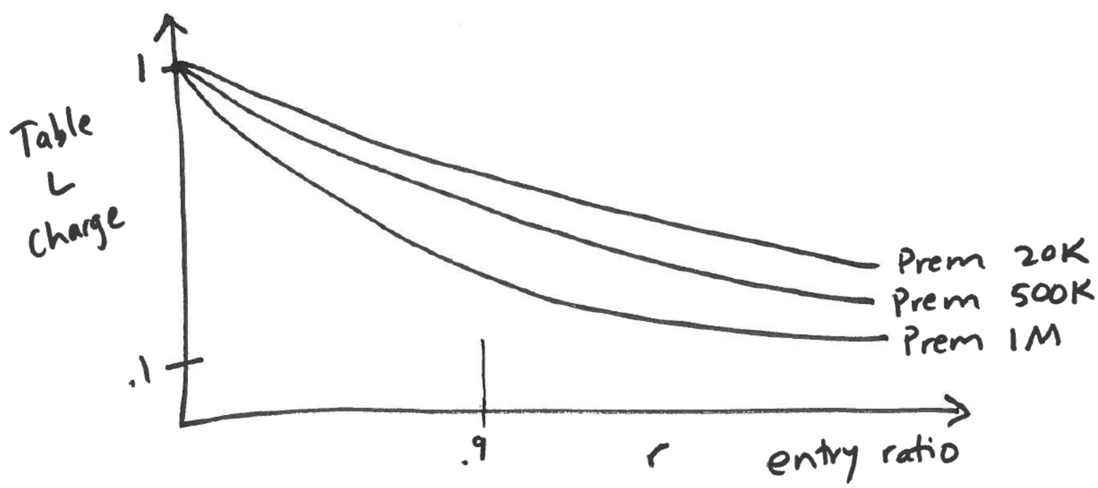
****

__Question 13:__

Part a
Model Solution 1



Model Solution 2



Part b
Model Solution 1

1. Lower premium amounts have a higher charge for a given entry ratio.
2. No charge goes below LER of 0.1.
3. $\Phi^*(0) = 1$ for all premium levels, and charge gradually approaches LER as entry ratios increase.

Model Solution 2

1. $\Phi^*(0) = 1$
2. $\Phi^*(\infty) = k$
3. $\Phi^{*\prime}(r) < 0$

Examiner's Comments:

****

This item asked candidates to draw a graph showing the Table L charges for a number of policies at different entry ratios and then describe the features of that graph. Since both parts are related, candidates' scores for parts a and b were often correlated. The result is that many candidates either did very well or very poorly on this item.

For both parts of this item, the most common error was reversing the order of the curves. Many candidates answered that, for a given entry ratio, the Table L charge for a larger policy was larger than that of a smaller policy. Many candidates made this mistake both in drawing the graph for part a and describing the features of the graph in part b.

Other common errors included:

- not starting all curves at an entry ratio of zero and a Table L charge of one;
- not reflecting that the curves asymptotically approach the loss elimination ratio;
- and not reflecting that the curves should be concave up (or convex).

For Part b, candidates were asked to supply three features. Many correct solutions included some variation of the items in the following list:

- For a fixed premium size, the charge is a decreasing function of entry ratio.
- For a fixed premium size, the charge approaches k as the entry ratio increases.
- For a fixed premium size, the charge approaches 1 as the entry ratio approaches 0.
- For a fixed entry ratio, the charge approaches 1 as the premium size approaches 0.
- For a fixed entry ratio, the charge decreases as premium increases.
- For an entry ratio of 0, the charge equals 1.
- As premium increases, the Table L charge approaches k if $r \geq 1-k$, and $1-r$ if $r < 1-k$.

****

<u>Question 14:</u>

Model Solution 1

Step 1: Calculate the basic premium factor:

b = [expense in basic] + [converted insurance charge]
b = [e - (c - 1) * E] + [cI]
b = [0.2 - (1.125 - 1) * 0.7] + [1.125 * (0.653 - 0.031) * 0.7]
b = [0.1125] + [0.4898]
b = 0.6023

Step 2: Calculate expected ratable losses

E[L] = E * (1 - charge + savings)
E[L] = 0.7 * (1 - 0.653 + 0.031)
E[L] = 0.2646

Step 3: Calculate the expected retrospective premium:

E[RP] = T * (b + c * E[L])
E[RP] = 1.1 * (0.6023 + 1.125 * 0.2646)
E[RP] = 0.9900

Model Solution 2

The following solution can be used IF AND ONLY IF the candidate stated that it's a balanced plan.

E[RP] = Tax Multiplier * (E[Expense Ratio] + E[Loss Ratio])
E[RP] **=** 1.1 * (0.20 + 0.70)
E[RP] = 0.99

Examiner's Comments:
****
The majority of candidates used model solution one and received full credit for this question. The most common error was made in calculating the expected ratable losses. Of the candidates that made this error, a significant number incorrectly used the expected loss ratio (E) as the expected ratable losses (E[L]).

The other common error was made when calculating the converted insurance charge. The candidates who made this error did not multiply the [charge – savings] by the expected loss ratio (E).
****

Question 15:

Part a
Model Solution 1

1. H+I
2. A
3. I + B + E + H
4. A + B
5. A + B + C +E + F
6. A + B + C + F

Model Solution 2

1. H+I
2. A
3. I + B + E + H
4. A + B
5. H + I - A
6. H + I + E - A

Model Solution 3

1. H+I
2. A
3. I + B + E + H
4. A + B
5. Assuming plan in balance, should pay for expected losses
H + I + E + F + B + C
6. H + I + E + F + B + C, same reasoning as 5

Examiner's Comments:
****
Most candidates answered all parts of this question correctly. On part 5 and 6, several answers were accepted: some candidates interpreted the question to mean the expected amount to be paid in total, others only gave the net insurance charge, and others gave the total expected losses minus the net insurance charge; all of these answers were accepted for full credit.
****

Part b
Model Solution 1

E&H section will get much larger. Since the law change affects the most severe losses, these are likely already over the occurrence limit, so the limited loss curve will not change much. However the total loss curve will increase, widening the gap between the two curves. This will affect the right region (higher aggregate loss region) of the graph more. So E&H will increase in comparison to other regions. (After everything is renormalized to 1.)

Model Solution 2

If there is an increase in most severe losses, then by limiting losses, the LER becomes higher (a higher portion of losses is eliminated by the limit).
LER = B + E + H, therefore E & H will increase in area.

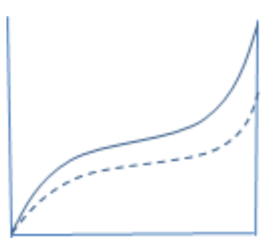
Examiner's Comments:
****
Most candidates received full credit on part b. The most common errors were candidates who either indicated that the Lee diagram was a severity distribution rather than an aggregate distribution, and candidates who did not explicitly identify that the change in the areas of E and H occurs because of the different impact on limited and unlimited losses.
****

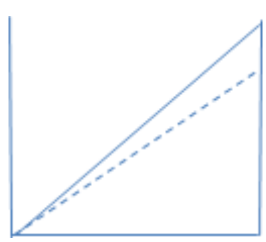
Part c
Model Solution 1

No. Large account are more stable than small account due to law of large numbers, for small accounts, the curve should be steeper, or the insurance charge should be higher to reflect this. Therefore the same curve is not appropriate for large and small accounts. Different size accounts should have different curves, like different ELGs in NCCI manual have different charge values.

Model Solution 2

No, the diagram will look different for large and small accounts. Even if the individual losses come from the same distribution, the aggregate distribution will look different (something approaching but not quite the curve predicted by the law of large numbers)


Large


Small

Examiner's Comments:

****

Candidates received credit for noting that there would be a significant difference in the shape of the large and small accounts aggregate loss curves due to relative variance between the two (either discussing account stability, relative variance, law of large numbers, or differences between ELGs in the NCCI plan.) Many candidates incorrectly focused on the normalization of entry ratios by loss ratio as a reason why the curves would be appropriate for both. Other candidates wrongly implied that large accounts would have a higher chance of breaching the respective aggregate limit, disregarding that the aggregate shown is an entry ratio limit rather than a fixed dollar limit.

****

Question 16:

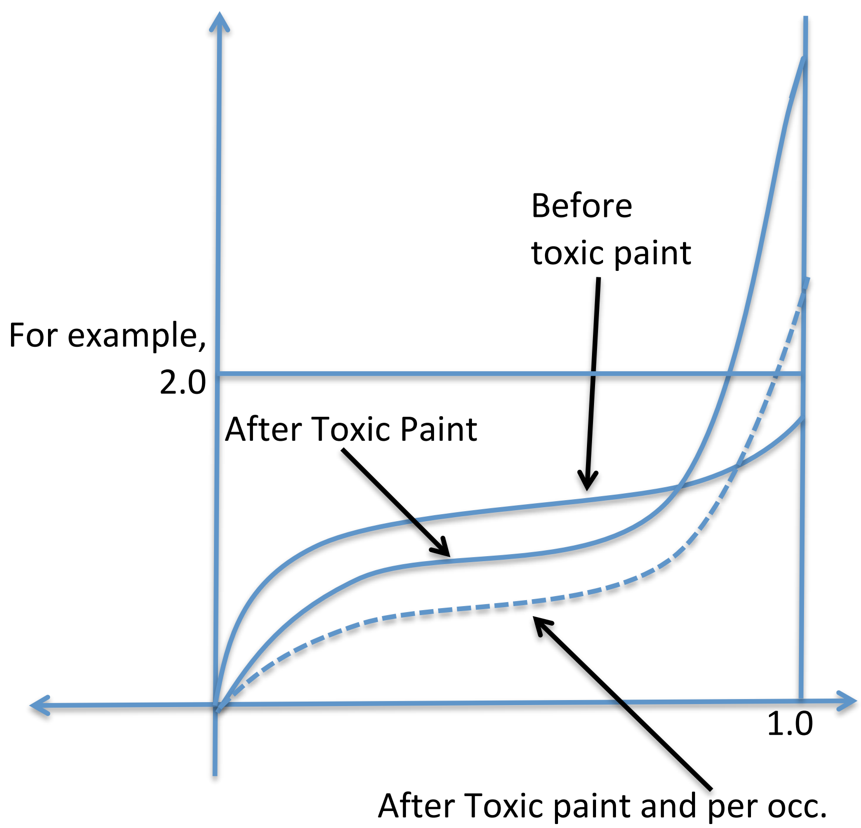
Model Solution 1

a) Since one out of every eleven claims is extremely expensive, the shape of the tail of the distribution changes dramatically.
This will result in increase to the insurance charge at an entry ratio of 2.00.
The actuary used an understated estimate of the expected losses. All else equal, when expected losses are understated the insurance charge is understated.

b) It would mitigate the change in the insurance charge. Since the toxic paint claims would be very large, most of the losses would now become excess losses and not aggregate losses therefore reducing the increase in the insurance charge.

Model Solution 2

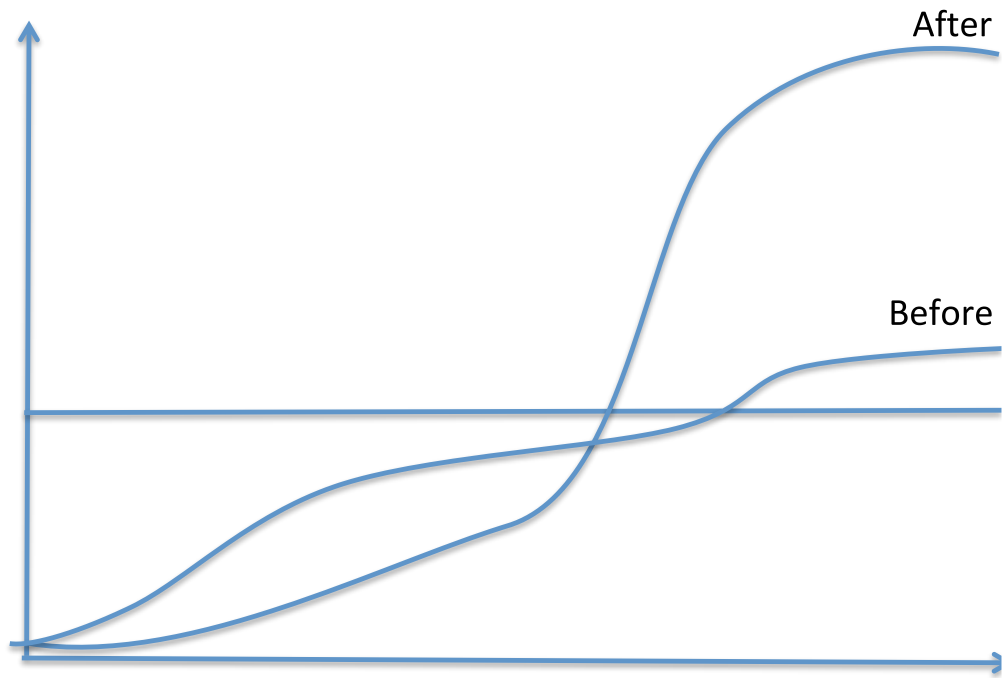
a) The insurance charge will increase because these new toxic paint claims have made the agg loss distribution more volatile, mostly because of the increase in volatility of severity losses.



b) a per occurrence limit would make the change in the insurance charge less. The aggregate loss distribution would be less volatile. See dotted line above.

Model Solution 3

a) I am assuming question meant for next year, i.e. ER 2 of new loss rate. Charge will increase b/c volatility has increased. A larger volatility will increase charges. This is similar to the concept in Venter, that if small and large losses are combined results are more volatile and you can have a loss multiples of the mean which will cause the charge to go up. The Lee diagram below would illustrate how losses shift higher.



b) W per occurrence limit, change in AD would go down b/c losses exceeding the limit would not be counted towards blowing the agg, but would be charged for separately using an ELF.

Model Solution 4

a) The insurance charge would increase because expected losses and the CV of the losses has increased.

b) A per-occurrence limit would reduce the increase in the insurance charge because the new large claims would affect the aggregate deductible less.

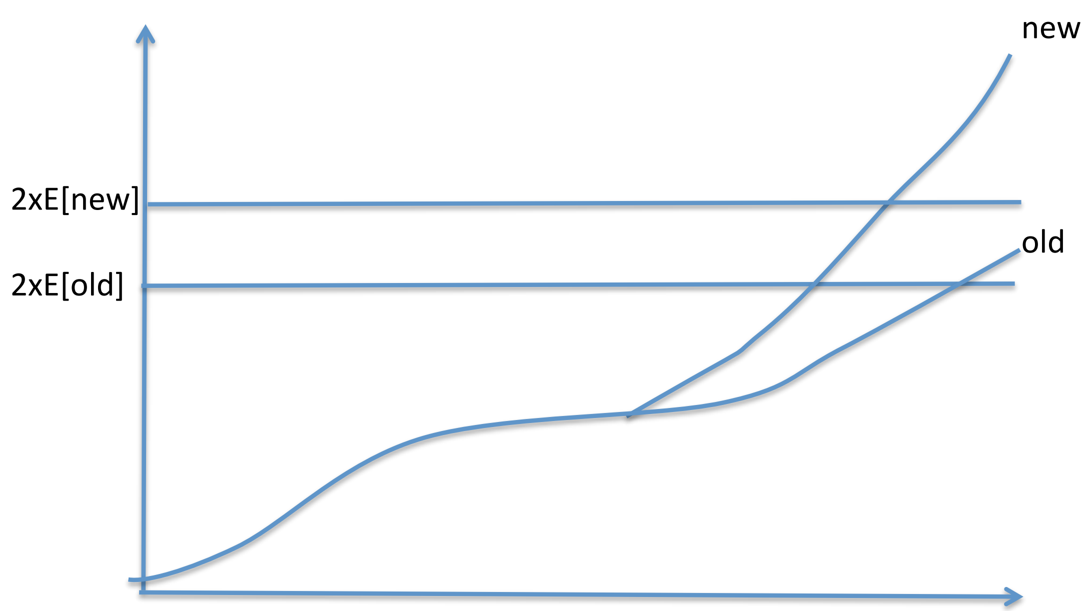
Model Solution 5

a) X_1 = old severity distribution

X_2 = toxic severity distribution

$X_{new} = 10/11\ X_1 + 1/11\ X_2$.

Because X_2 affects the tail distribution, for an entry ratio = 2, the insurance charge would increase.



b) Since the per occurrence limit would be very much affected by new higher severity claims and since at high entry ratios a larger portion of the total charge is from the per occurrence limit, I would say the charge would be even higher than in a.

Examiner's Comments:

****

Part a

A candidate could interpret the phrase "At an entry ratio of 2.00" as being either the entry ratio under the original assumptions, or under the new assumptions. That is, the actuary might be pricing the next year knowing that the distribution is different and looking at the charge under a new entry ratio of 2.0, or, the actuary might be reserving the previous year and observe that the expected entry ratio shouldn't have been 2.0 but rather should have been 1.0. In both cases, the insurance charge increased, and candidates could get credit for answering under either assumption. However, a key feature of the problem is that the actuary now knows there are individual claims much larger than any historical claims, and candidates could not get full credit without noting that these larger claims increased the variance/dispersion of the loss distribution, which is the major driver of insurance charge at any given entry ratio. While this situation is, of course, extreme, the NCCI insurance charge contemplates this sort of effect with the state/hazard group adjustment factor.

Some candidates wrote vague verbal answers but included clear and explicit Lee diagrams. In general, if the Lee diagram made the point explicitly enough, candidates were given credit. Many other candidates wrote correct verbal answers but drew Lee diagrams that didn't support that answer. Because the question did not require a Lee diagram, those diagrams were generally not considered.

Many candidates appeared to confuse aggregate charges with excess charges, and gave answers that implied the charge for losses excess of a particular limit would increase, rather than that aggregate losses would increase. While the phrase "insurance charge" is sometimes used to describe the sum of the aggregate and excess charges (for example, in the Skurnick paper) it does not refer to an excess charge alone, especially if there is an aggregate on the policy. So this was an error.

Many candidates explicitly cited inflation. While inflation does have a disproportional effect on an excess charge at a fixed dollar limit, it will generally have no impact on the aggregate charge as a percent of expected loss at a fixed entry ratio. However, candidates could get some credit for just for stating that the charge increased.

Some candidates argued that the charge should decrease, noting that more losses would push the risk into a higher expected loss group and thus reduce the aggregate charge. This is incorrect. It is true that we expect larger policies to have lower aggregate charges when all else is the same, because the law of large numbers suggests the variance of results will be lower when there are more total claims drawn from the same distribution. However, in the situation described all else is very much NOT the same, and the true distribution clearly has more variance than the initially expected distribution.

A few candidates made a lot of observations but never actually addressed the question being asked. In general, these answers scored poorly.

Part b

Part b asked the candidate to explain how a per occurrence limit would affect the change in the insurance charge, described in part a. Depending on whether the candidate wrote about a Table L insurance charge or a Table M insurance charge, a deductible could either increase or decrease the change in the charge. Candidates could get full credit for either answer, so long as their explanation supported the change they described. They could also get credit for clearly stating both answers, although this was not required.

However, candidates did not earn full credit unless they picked a change and supported it. Partial credit was awarded to candidates who correctly described the impact of a per occurrence limit on the insurance charge but did not evaluate the change in the insurance charge after knowing about the toxic claims. Similarly, the candidate was required to give an explanation. No credit was given for an unsupported claim that the charge increased or decreased.

It was possible to earn full credit on part b with an incorrect or partially correct answer in part a. However, many candidates lost credit on this problem for giving extremely generic answers about what a deductible would do to an insurance charge in any situation. The question asked the candidate to explain what would happen to the change described in part a, and to earn full credit the answer had to address the details of the situation – in particular, it had to address how the deductible would differentially affect the newly discovered large claims.
****

<u>Question 17:</u>

Model Solution 1

a. The actuary should maximize the reliance on the reported data while leaving enough data above the truncation point to permit reasonable curve fitting. Also desirable that truncation point be a nice round number prior to the data thinning out.

b. A Pareto only curve has a higher mean residual like than the mixed distribution – has a heavier tail. This would result in higher excess ratios for limits above the truncation point.
The lognormal curve is less heavy tailed than the Pareto (but heavier than exponential). The excess ratios at very high limits would be modeled more on the lognormal curve, and since this distribution is less heavy tailed than Pareto, the excess ratios would be lower.

Model Solution 2

a. The truncation point should be selected to leave enough data below such that there is enough credibility to use actual data to calculate excess ratios. Also, there should be enough data above to fit a curve.

b. The smaller losses wouldn't fit well with the heavier tailed Pareto. The excess ratio for lower limits would likely be overstated.
The lognormal doesn't have as heavy a tail as the Pareto, so the larger limits' excess ratios would likely be understated.

Examiner's Comments:
****

Part a

Mahler listed several components of an ideal truncation point, two of which were deemed more meaningful than the others (maximum reliance on reported; enough data above truncation point to allow for reasonable fit) with most candidates receiving full credit for a response that was nearly verbatim from the paper on these two points.

Part B

Full credit responses discussed both the tail of the distributions and the impact on the excess ratios. The Pareto curve only was more straightforward as both it and the Exponential were discussed in the paper, while the Lognormal was never mentioned in the paper. The majority of candidates commented on the tails of the distributions, but most did not tie that back to the excess ratios.
****

Question 18:

Model Solution 1

$$R^{\wedge}(500) = R(250) x R_Y(500-250)$$
$$R(250) = 1 - \frac{8(.80)+100(.11)+250(.09)}{55.4} = 0.2798$$
$$R_Y(500-250): r = \left(\frac{500-250}{AvgTruncated\ \&\ Shifted\ @\ 250K}\right)$$
$$= \frac{250}{\frac{(500-250)x.02+(1000-250)x.007+(2000-250)x.003}{.02+.007+.003}} = 0.48$$
$R_Y(0.48) = 0.658$ (interpolate from table)
$$R^{\wedge}(500) = 0.2798 x 0.658 = 0.1841$$
$$Ee = 0.1841 x 55{,}400 = 10{,}199$$

$$LDD\,Prem = \frac{Ee + E(LBA+ULAE) + SP(GO+CR)}{1-A-T-p} = \frac{10{,}199 + 55{,}400(4\%+8\%)+100{,}000(5\%+5\%)}{1-8\%-3\%-5\%}$$
LDDPrem = 31,961

Model Solution 2

R(250) = Losses above 250,00 / Total Losses
Losses above 250K =
2%*(500K – 250K) + 0.7%*(1000K – 250K) + 0.3%*(2000K – 250K) = 15.5K
Total Losses = 55.4K
R(250) = 15.5K / 55.4K = 0.28
Above truncation 250K, mean = 15.5K / (.02+.007+.003) = 516.67K
➔r = 250K / 516.67K = 0.4839

Interpolation: $\left(\frac{0.4839-0.4}{0.1}\right)x0.65 + \left(1-\frac{0.4839-0.4}{0.1}\right)x0.69 = 0.656$

➔XL= 0.656 x 0.28 = 0.184

LDDPrem=

$$\frac{ELx(ULAE+LBA+XL)+SPx(GO+CR)}{1-A-T-p} = \frac{55{,}400x(.08+.04+0.184)+100{,}000x(.05+.05)}{1-.08-.03-.05}$$
=31,941.52

Model Solution 3

$$\text{LDDPrem}=\frac{ELx(ULAE+LBA+XL)+SPx(GO+CR)}{1-A-T-p}$$

E(loss) = 55,400

$$R(250{,}000)=1-\frac{8000(.80)+100000(.11)+250000(.06+.02+.007+.003)}{55{,}400}=0.2798$$

e(250,000)=

$$\frac{0.02(500{,}000-250{,}000)+0.007(1{,}000{,}000-250{,}000)+0.003(2{,}000{,}000-250{,}000)}{0.03}=\frac{15{,}500}{0.03}=516{,}667$$

$$r=\frac{500{,}000-250{,}000}{516{,}667}=0.4839$$ → round to 0.5 and use xs ratio at 0.

$$\therefore R(500{,}000)=R(250{,}000)xR(500{,}000-T)=0.2798x0.65=0.18187$$

$$\therefore LDD\,\text{Pr}\,em=\frac{55{,}400(0.18187+0.08+0.04)+100{,}000(0.05+0.05)}{1-0.08-0.03-0.05}=31{,}814$$

Examiner's Comments:

****

Generally the candidates did well on this question. Most understood that they needed to calculate an empirical piece, and a model piece, and combine them. Probably the hardest part is getting the formulas right, and if they did, the calculation was pretty straightforward. We did see some papers that showed answers with no discernible work.

There were 5 main calculations:

- R(250) – the empirical excess ratio at $250,000
- Average Truncated and Shifted losses at $250,000
- Entry Ratio – to use in the table of excess ratios for the $500,000 level
- R(500) = XL – the total excess loss ratio
- LDD Prem - being able to remember the formula correctly, and substituting in the figures given for XL, expenses & profit, and calculating the final answer.

Most candidates received full credit for the R(250) calculation. Common errors here were for calculating the losses limited to 250, rather than the excess.

Average Truncated and Shifted (AT&S) was the most problematic of the calculations. Some skipped it completely, not realizing that this was the denominator in the entry ratio calculation. Others calculated the limited instead of the excess. There was actually a short cut here, as the numerator of the R(250) and the AT&S are the same.

The Entry Ratio calculation was straighforward if the AT&S was calculated correctly. If the AT&S was not correctly calculated, most candidates gave the correct numerator of the entry ratio. Then, being able to select the appropriate excess ratio from the table was straightforward. Rounding the entry ratio to 0.50 to select a value in the table was given full credit.

Nearly all candidates knew that the final XL was the product of two figures, and most knew that it was the product of R(250) and the figure they looked up in the entry ratio table.

The LDD Premium formula was a challenge to quite a few candidates; nearly all got the expense portions correct, but were stymied by the portion that involved the XL. While most gave the correct formula for an LDD premium, several seemed to confuse it with an Excess premium.

Depending how and where the candidate rounded their calculations, the final answer could vary from roughly 31,700 to roughly 32,100. If work was shown and the answer fell in this range, the candidate received full credit.
****

Question 19:

Part a

Model Solution 1

Expected Unlimited Loss = 1.5M * 0.75 = 1.125M
Loss Elimination Ratio = 0.21 / 0.75 = 0.28
F = (1 + 0.8*LER) / (1 – LER) = 1.7
Adj Loss = 1.125M * 1.7 * 1.1 = 2.10375M -> ELG 24
Expected Limited Loss = 1.125 * (1-LER) = 810,000
Expected Excess Loss = 1.125M * LER = 315,000
Entry Ratio = 1M / 810,000 = 1.234
Charge = 0.1706 (from table)
Expected Loss Cost = Charge * Exp Lim Loss + Expected Excess Loss
= 0.1706 * 810,000 +315,000 = 453,186

Model Solution 2

Expected Unlimited Loss = 1.5M * 0.75 = 1.125M
Loss Elimination Ratio = 0.21 / 0.75 = 0.28
F = (1 + 0.8*LER) / (1 – LER) = 1.7
Adj Loss = 1.125M * 1.7 * 1.1 = 2.10375M -> ELG 24
Expected Limited Loss = 1.125 * (1-LER) = 810,000
Expected Excess Loss = 1.125M * LER = 315,000
Entry Ratio = 1M / 810,000 = 1.234
Charge = 0.1706 (from table)
Expected Loss Cost = Charge * Exp Lim Loss + Expected Excess Loss
= 0.1706 * 810,000 +315,000 = 453,186
Converting to Loss Cost as a % of Standard Premium: 453,186 / 1.5M = 0.3021

Examiner's Comments:
****
Most candidates received full credit or a majority of partial credit on this problem. The most common errors included using unlimited expected losses to calculate the entry ratio instead of the limited expected losses or incorrectly calculating the loss elimination ratio. Some candidates lost points for using the excess loss factor as a percent of loss, without explicitly stating that as an assumption. Those candidates that did state that as an assumption did not lose partial credit.
****

Part b

Model Solution 1

Because the LUGS adjustment shifts the aggregate loss distribution used to estimate the insurance charge to a loss distribution associated with a larger (less skewed) loss distribution. This less skewed distribution approximates the limited loss distribution that is really acting on the risk being rated due to the presence of the per loss limit. This will lead to insurance charges that are smaller than would be indicated in the absence of a loss limit reflecting the removal of the overlap between the “loss limitation” and the aggregate loss limit.

Model Solution 2

The use of the per-occurrence limit (or a deductible in this case) reduces the losses that count toward the aggregate limit. If one pulled an insurance charge without adjusting losses, there would be overlap between the losses eliminated by the deductible and implied losses eliminated by the aggregate limit. To reduce overlap, an expected loss group with more stable losses (resulting in a lower charge) is selected by finding an equivalent ELG based on the losses eliminated and the state hazard group relativity. Because the lower ELG produces lower charges for the aggregate limit, this helps to eliminate overlap and provide accurate charges

Examiner’s Comments:
****
Most candidates received partial credit. Responses that did not receive partial credit include those that described the process involved in part a rather than explain why the procedure produces accurate insurance charges. While most candidates were able to identify at least one reason as to why the procedure produces reasonable results, few were able to give a complete answer.
****

Question 20:

Part a

Model Solution 1

$$G(d) = \frac{\int_0^d 1 - F(x)\,dx}{\int_0^1 1 - F(x)\,dx} = \frac{\int_0^d 1 - x^{0.25}dx}{\int_0^1 1 - x^{0.25}dx} = \frac{x - 0.8x^{1.25}\Big|_0^d}{x - 0.8x^{1.25}\Big|_0^1} = \frac{d - 0.8\cdot d^{1.25}}{1 - 0.8\cdot 1^{1.25}}$$

$$= 5d - 4d^{1.25}$$

Model Solution 2

$$F(x) = 1 - \frac{G'(x)}{G'(0)} = x^{0.25}$$

$$G'(0)\left(1 - x^{0.25}\right) = G'(x)$$

$$\frac{1}{G'(0)} = 0.2 = E(X) \qquad \Rightarrow G'(0) = 5$$

$$\Rightarrow G(x) = \int_0^d G'(x) = \int_0^d G'(0)\left(1 - x^{0.25}\right) dx = 5\int_0^d \left(1 - x^{0.25}\right) dx = 5d - 4d^{1.25}$$

Examiner's Comments:

****

Many candidates attempted to validate whether the given CDF was an exposure curve (ex. defining the properties required to be an exposure curve) rather deriving an exposure curve from the given CDF which was what the item asked for.

Some candidates set up most of the integration correctly with either an error in the limits of integration, or did not complete solving it.

The majority of candidates received at least some credit for this item.

****

Part b

Model Solution

$$G\left(\frac{1.5M}{2M}\right) - G\left(\frac{0.5M}{2M}\right) = G(0.75) - G(0.25)$$

$$= \left(5\cdot 0.75 - 4\cdot 0.75^{1.25}\right) - \left(5\cdot 0.25 - 4\cdot 0.25^{1.25}\right)$$

$$= 0.4153$$

Examiner's Comments:

****

The vast majority of candidates identified the layers correctly. Regardless of the exposure curve defined in part a, work was followed through to determine partial or full credit. Candidates whose chosen exposure curve produced a final answer between 0 and 1, and followed through all steps correctly, received full credit.

The majority of candidates received full credit.

****

Question 21:

Model Solution 1

a.

Date	Layer Loss	Reinstatement Premium	Total
6/1/2013	15,000,000	$3M * 1.2 * (15M / 50M)	1,080,000
9/1/2013	35,000,000	$3M * 1.2 * (35M / 50M)	2,520,000
11/1/2013	50,000,000	$3M * 1.2 * (50M / 50M)	3,600,000
			7,200,000

In order to reinstate the full layer coverage after each loss, the total reinstatement premium would be $7.2M.

b.

Date	Layer Loss	Reinstatement Premium	Total
6/1/2013	15,000,000	$3M * 1.2 * (7/12) * (15M / 50M)	630,000
9/1/2013	35,000,000	$3M * 1.2 * (4/12) * (35M / 50M)	840,000
11/1/2013	50,000,000	$3M * 1.2 * (2/12) * (50M / 50M)	600,000
			2,070,000

c.

Property catastrophes are seasonal in nature, so you cannot expect the same loss experience uniformly across the year, which this method assumes.

Model Solution 2

a.

Reinstatement 1 = (65M-50M)/50M*3M*1.2 = 1.08M
Reinstatement 2 = (85M-50M)/50M*3M*1.2 = 2.52M
Reinstatement 3 = (100M-50M)/50M*3M*1.2 = 3.60M
Total reinstatement premium = 1.08M+2.52M+3.60M = 7.20M

b.

Reins. 1 = 1.08M*7/12 = 0.63M
Reins. 2 = 2.52M*4/12 = 0.84M
Reins. 3 = 3.60M*2/12 = 0.60M
Total = 0.63M+0.84M+0.60M = 2.07M

c.

There is seasonality effect (e.g. hurricances do not usually occur in winter). So assumption that losses are uniform throughout year is a poor assumption.

Examiner's Comments:
****
Part a.

Most candidates received full credit on this part.

The most common error was that the candidate did not reinstate after each loss, but instead reinstated after the 2^{nd} or 3^{rd} loss.

Part b.

This part was more difficult for candidates.
Common errors included:

- Not including pro-rata as to amount
- Not including the 1.2 pro-rata factor in the calculation
- Not correctly determining the pro-rata as to time factors
- Not reinstating after each loss, but instead reinstating after the 2^{nd} or 3^{rd} loss
- Calculation error

Part c.

Most candidates received full credits on this part. Answers that addressed the seasonality of events or the lack of uniformity of exposure throughout the years received full credit.
****

Question 22:

Model Solution 1

Denote the max retention as a, thus $G(a / (50 + a)) = 0.65$

- ⇨ $(1 – 0.1 ^\wedge (a / (50 + a)) / (1 – 0.1) = 0.65$
- ⇨ $a / (50 + a) = 0.382$
- ⇨ $a = 30.9$

Then the cedant's maximum retention is 30.9 million

Model Solution 2

$G(x) = (1 - .1 ^\wedge x) / .9 = .65 \Rightarrow .1 ^\wedge x = .415 \Rightarrow x = .382$ of the total possible loss
Reins max loss = 50 = M (1 - .382) => M = 80.9 million
Then cedants max retention under policy = M x G(d) = 80.9 (.382) = 30.9 M

Model Solution 3

$G(x) = (1 – 0.1^\wedge x) / 0.9$
G(attachment point) + 1 – G(attachment point + reinsurance policy limit) = 0.65
0.35 = G(AP + 50) / IV – G(AP) / IV where AP = attachment point, IV = insured value
0.35 = (1 – 0.1 ^((AP + 50) / IV) - 1 + 0.1 ^ (MP / IV)) / 0.9
0.315 = 0.1 ^ (AP / IV) – 0.1 ^ ((AP + 50) / IV)) I'll assume that AP + 50 = IV
0.315 = 0.1 ^ (AP / IV) – 0.1
AP / IV = 0.38195
Assume (AP + 50) / IV = 1
50 / IV = 1 – 0.38195
IV = 80.9
Therefore cedant's retention is 80.9M x 0.38195 = 30.9 million

Examiner's Comments:

****

The most common solution was solving for x accurately (.382) and then solving .382 * 50 = 19.1 and not recognizing that 50 was the reinsurer maximum loss and not the maximum total loss.

Some variations of this solution was solving for x (.382) and recognizing that $50M was the reinsurer maximum loss but deriving the maximum loss inaccurately as follows:

Max loss = 50M / .35 max loss = 143M and then x = .382 * 143M x = 55M. Significant partial credit was given to these solutions.

Other common errors that resulted in partial point reductions were:

- Solving correctly for the max loss of 80.9M but not subtracting the reinsurer loss and leaving 80.9 as the final answer.
- Setting up the original equation to be equal to .35 instead of .65.
- Any sort of calculation error in the process of otherwise solving the problem correctly.

No credit was given for solutions which solved for G(.65) by setting x =.65 in the original formula (instead of solving G(x) = .65) and solving for G(x) = .8624 and then solving .8624 * 50 = 43M as the initial set up is incorrect.

****

Question 23:

Model Solution 1

- 60 % ceded quota share
  - ⇨ Retain 40 % of total losses
  - ⇨ 0.4 x total losses = 0.4 x 1,120,000 = $448,000
- Five-line surplus share with retained line = $100,000

(1)	(2)	(3) = Min {(2) , 100,000}	(4) = Min {(2) – (3) , 500,000}	(5) = (4) / (2)	(6) = (1 – (5)) x loss
Risk	**Insured value**	**Retained**	**Excess**	**% ceded**	**$ retained**
A	250,000	100,000	150,000	0.6	48,000
B	1,000,000	100,000	500,000	0.5	122,500
C	85,000	85,000	0	0	85,000
D	1,250,000	100,000	500,000	0.4	294,000
E	400,000	100,000	300,000	0.75	45,000
Total					**$594,500**

- $400,000 xs $100,000 per-risk excess of loss

(1)	(2)	(3) = Max { Min {(2) – 100,000 , 400,000} , 0 }	(4) = (2) – (3)
Risk	**Loss**	**In layer**	**$ retained**
A	120,000	20,000	100,000
B	245,000	145,000	100,000
C	85,000	0	85,000
D	490,000	390,000	100,000
E	180,000	80,000	100,000
Total			**$485,000**

⇨ <u>60 % ceded quota share would result in the lowest retained losses.</u>

Model Solution 2

- Five-line surplus share with retained line = $100,000

(1)	(2)	(3) = Min {(2) , 100,000}	(4) = Min {(2) – (3) , 500,000}	(5) = (4) / (2)	(6)	(7) = (5) x (6)
Risk	**Insured value**	**Retained line**	**Reinsured line**	**Reinsured line as a % of insured value**	**Loss**	**$ ceded**
A	250,000	100,000	150,000	0.6	120,000	72,000
B	1,000,000	100,000	500,000	0.5	245,000	122,500
C	85,000	85,000	0	0	85,000	0
D	1,250,000	100,000	500,000	0.4	490,000	196,000
E	400,000	100,000	300,000	0.75	180,000	135,000
Total						**$525,500**

- 60 % ceded quota share
  - ⇨ Ceded losses = 0.6 x total losses = 0.6 x 1,120,000 = $672,000
- $400,000 xs $100,000 per-risk excess of loss

(1)	(2) = Max { Min {Loss – 100,000 , 400,000} , 0 }
Risk	**Loss ceded**
A	20,000
B	145,000
C	0
D	390,000
E	80,000
Total	**$635,000**

⇨ <u>60 % ceded quota share results in the lowest retained losses since it has the highest ceded losses.</u>

Examiner's Comments:
****

The question asked candidates to calculate retained losses for three types of reinsurance contracts and compare them in order to determine which contract would result in the lowest retained losses for the insurance company.

The above calculations were pretty straightforward, especially for ceded quota share and per-risk excess of loss contracts. Most candidates were able to complete the appropriate calculations shown in the model solution.

The most common mistakes made by candidates were:

- Calculating the ceded losses instead of the retained ones without mentioning it anywhere in the answer, especially for the per-risk excess of loss contract. Note that candidates who calculated ceded losses throughout the question and interpreted the lowest retained losses' contract as the one with the highest ceded losses were awarded full credit.
- Using a 4-line surplus instead of a 5-line in the surplus share contract
- Forgetting to apply the $500,000 cap by reinsured losses in the surplus share contract
- Making a calculation error in the process of computing the retained losses

****

<u>Question 24:</u>

Part a
Model Solution 1

PML = Probable Maximum Loss – largest loss likely to occur for the insurer

1/250 = 0.004

Must interpolate between 0.002 and 0.005

$\frac{(0.004-0.002)}{(0.005-0.002)}$*(20 – 50) = -20

50,000,000 – 20,000,000 = 30,000,000 is 1/250 PML

Model Solution 2

PML is probable maximum loss and it is the largest likely loss that the insurer uses to make sure that they can withstand (remain solvent) that loss if it occurs.

Here, 1 in 250 yrs corresponds to an OEP of 1/250=0.004

Thus the 1 in 250 yrs PML = [1 – $\frac{0.005-0.004}{0.005-0.002}$]*20M + 50M * $\frac{0.005-0.004}{0.005-0.002}$ = 30M

Linear Interpolation Here

Model Solution 3

PML is the Probable maximum loss. The 1 in 250 year threshold means the probable maximum loss that occurs with probability 1/250=0.004, 0.4%. It means the loss amount that will likely be exceeded 0.4% of the time in a given year.

Linearly interpolate between 0.002 & 0.005 OEPs

20M + $\frac{(50-20)}{0.003}$* 0.001 = 30M

Examiner's Comments:
****
Full credit was given if the candidate can define PML as probable maximum loss, explain the PML as in the above solutions and calculate the 1 in 250 yrs PML correctly using linear interpolation. Most candidates could define PML and knew to

use linear interpolation to calculate the 1/250 PML. Quite a few candidates were not able to provide an appropriate explanation of PML.
****

Part b
Model Solution 1

Loss = 45m Loss Limit = 40m for quota share of 30%
Layers: 6m x 4m, 10m x 10m, 30m x 20m
0.3*(40m) = 12m ceded for quota share
45m – 12m = 33m remains
6M x 4M layer: 6m (1) = 6m ceded
10M x 10M layer: 10m * (0.9) = 9m ceded
30M x 20M layer: (33m – 20m)*(0.75) = 9.75m ceded
Total = 24.75m
Net retained loss = 45m – 12m – 24.75m = 8.25m

Model Solution 2

Ceded to Quota Share = 0.3 * 40m = 12m
Retained from QS = 45 -12 = 33m
Ceded to first layer = 100% * 6m = 6m (retained 4m)
Ceded to 2nd layer = 90% * 10m = 9m (retained 1m)
Ceded to 3rd layer = 75% * 13m = 9.75m (retained 3.25m)

Net retained = 4 + 1 + 3.25 = 8.25m

Examiner's Comments:
****
Full credit was given if the candidate could correctly calculate the ceded amount to quota share, amount retained from QS, XOL calculation for all layers and lastly the net retained loss. In general, the candidates could calculate the ceded amount to quota share, but quite a few candidates didn't understand the inuring reinsurance concept and didn't know how to calculate the amount ceded to each XOL layer.
****

Question 25:

Model Solution 1

There are a few practical issues that arise with actuary A's method. First, using a single distribution to model aggregate loss it is difficult to reflect the impact of the different occurrence limits. Second, with a lognormal distribution there is no allowance for the loss free scenario. As can be seen in the loss history there were several loss free years.

The main issue with actuary B's method is that only a single severity distribution can be used with the recursive formula. However, this method does work well for low frequency scenarios. Based on the volume of reported claim counts over the historical period the method seems appropriate.
Actuary B's method is preferred because there are too many limitations to actuary A's method.

Model Solution 2

Lognormal Distribution – It doesn't permit loss free scenario since it is not defined at 0. This situation occurred in the loss history. It is also difficult to consider different occurrence limits. This is not a good choice.
Recursive Formula – The occurrences do appear to be independent since they come from retailers. For this method, frequency and severity can be analyzed separately. It also works well for low frequency scenarios, which is the case here as few claims were reported in the layer.
Choose actuary B's method.

Examiner's Comments:

A sizable portion of candidates left this question blank. Of those who answered, candidates responded reasonably well considering the novelty of the question. Many candidates were able to discuss the zero point mass and differing occurrence limits as weaknesses of the lognormal, as well as the appropriateness of the recursive formula in low frequency situations.

Candidates commonly lost partial credit for not providing enough valid responses and for proposing the lognormal as the more appropriate method. Candidates did not receive credit for merely choosing the recursive formula if they did not provide any actual arguments either in favor of the method or in opposition to the lognormal.

Examples of other responses which were acceptable and received partial credit include:

- Stating that Poisson was a common industry standard for modeling frequency.
- Stating that the short layer of losses would make it feasible to construct a severity distribution for the recursive formula.
- Performing calculations to show that E(N) was approximately equal to V(N), thus making the Poisson an appropriate choice.
- Stating that the lognormal has a straightforward formula for incorporating inflation.
- Mentioning the "black box" nature of collective risk models.
- Stating that the recursive formula would be difficult to use without individual severities. While the intent of the question was that individual severities would be available, since this information is not explicitly stated within the problem, this was considered an acceptable response.

****