CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

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Exam 7

Estimation of Policy Liabilities, Insurance Company Valuation, and ERM

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4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 66 point examination consists of 22 problem and essay questions.

2. For the problem and essay questions the number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer the questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid.

   - Write your Candidate ID number and the examination number, 7, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER.** Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specific number of items is requested, do not offer more items than the number requested. For example, if three items are requested, only the first three responses will be graded.
   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Anything written in the examination booklet will not be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 17, 2013.

END OF INSTRUCTIONS

1. (3 points)

Given the following information:

Age-to-Age Loss Development Factors

	12-24	24-36	36-48	48-60	60-72
2007	2.45	1.73	1.19	1.05	1.05
2008	5.42	1.26	1.23	1.09	
2009	2.64	1.35	1.16		
2010	2.04	1.55			
2011	6.23				

z-value for 90th percentile of the normal distribution: 1.645

a. (2.5 points)

The null hypothesis is that the triangle does not display calendar year effects. Conduct a test to determine whether the null hypothesis should be accepted or rejected at the 90% confidence level.

b. (0.5 point)

Briefly describe two potential causes of calendar year effects in loss development data.

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2. (2.75 points)

Given the following information:

Incremental Paid Loss ($000)

Accident Year	0-12	12-24	24-36	Premium ($000)
2010	1,400	650	280	3,600
2011	1,000	850		3,700
2012	1,500			3,650
Total	3,900	1,500	280	10,950

Assume that there is no further development beyond 36 months.

a. (2 points)

Calculate the individual loss ratio claims reserve R^{ind} for the total of accident years 2010 through 2012.

b. (0.75 point)

Calculate the collective loss ratio claims reserve R^{coll} for the total of accident years 2010 through 2012.

CONTINUED ON NEXT PAGE

3. (5 points)

Given the following information:

Accident Year	Cumulative Paid Loss ($000) 12	24	36
2010	2,750	4,250	5,100
2011	2,700	4,300	
2012	2,900		

The expected accident year loss emergence pattern (growth function) is approximated by a Weibull function of the form:

$$G(x|\omega,\theta) = 1 - \exp(-(x/\theta)^{\omega})$$

Parameter estimates are: $\omega = 1.5$ and $\theta = 20$

a. (3 points)

Calculate the process standard deviation of the reserve estimate for accident years 2010 through 2012 using the LDF method.

b. (2 points)

Graph the normalized residuals plotted against the increment age of loss emergence. Based on your graphical results, discuss the appropriateness of the Weibull model above.

CONTINUED ON NEXT PAGE

4. (2 points)

Given the following information:

Cumulative Paid Loss ($000)

Accident Year	12	24	36	48
2009	5,751	10,640	11,491	12,181
2010	5,528	9,287	10,680	
2011	4,120	7,004		
2012	5,304			

Calculated Ultimate Loss ($000)

Accident Year	Bornhuetter-Ferguson Ultimate	Benktander Ultimate
2009	12,181	12,181
2010	11,246	11,316
2011	8,428	8,204
2012	10,403	10,609

a. (1.5 points)

Calculate the 24-month-to-ultimate cumulative development factor that would result in the ultimate loss estimates shown above.

b. (0.5 point)

For accident year 2011, suppose that the Bornhuetter-Ferguson method is performed over multiple iterations. Deduce the ultimate loss estimate that will be produced as the number of iterations approaches infinity.

CONTINUED ON NEXT PAGE

5. (2.25 points)

Given the following information as of December 31, 2012:

	Cumulative Reported Loss ($000)			
Accident Year	12	24	36	48
2009	233	522	570	601
2010	204	480	512	
2011	242	541		
2012	255			

- Claim data in the triangle is on an unlimited basis
- Basic limit: $50,000
- Calendar period ground-up unlimited loss trend: 5% per year
- An exponential distribution fit to the claim level data at each maturity produced the following unlimited claim size means at the accident year 2012 level:

	12	24	36	48
Unlimited Claim Size Mean	26,000	48,000	58,000	66,000

- Mean of exponential distribution: θ
- Variance of exponential distribution: θ^2
- Limited mean of exponential distribution at limit *k*: $\theta\left(1 - e^{\frac{k}{\theta}}\right)$

Using the method described by Sahasrabuddhe, calculate the latest diagonal of the cumulative reported loss triangle based on accident year 2012 cost levels for the basic limit of loss.

CONTINUED ON NEXT PAGE

6. (2.75 points)

Given the following information regarding a book of retrospectively rated policies:

	Retrospective Adjustment		
	First	Second	Third
Loss Age	18	30	42
Premium Age	27	39	51
Loss Capping Ratio	80%	50%	70%
LDF to Ultimate	1.800	1.150	1.050

- Tax multiplier: 1.04
- Loss conversion factor: 1.15
- Total basic premium: $12,456,000
- Total standard premium: $54,309,000
- Expected loss ratio to standard premium: 75%

a. (1 point)

Calculate the PDLD ratio as of the first retrospective adjustment.

b. (0.5 point)

Calculate the PDLD ratio as of the second retrospective adjustment.

c. (0.5 point)

Calculate the PDLD ratio as of the third retrospective adjustment.

d. (0.75 point)

Based on the results of parts a., b. and c. above, discuss the reasonableness of the loss capping ratios shown above.

CONTINUED ON NEXT PAGE

7. (2 points)

The following cumulative reported loss triangle is to be used in a bootstrap process, by building a generalized linear model (GLM) based on incremental loss.

	Cumulative Reported Loss		
Accident Year	12	24	36
2010	$200,000	225,000	250,000
2011	300,000	325,000	
2012	450,000		

a. (0.75 point)

Construct a log-link triangle that can be used to solve for the GLM parameters.

b. (0.5 point)

Now assume that the 2010 cumulative reported loss value at 24 months is $170,000 instead of $225,000 and that all other cumulative values stay the same. Briefly explain the obstacles to setting up and solving a GLM bootstrap model for this revised triangle.

c. (0.75 point)

For the revised triangle described in part b. above, construct a log-link triangle that can be used to solve for the parameters of the GLM and briefly describe how to then modify the GLM's fitted values.

CONTINUED ON NEXT PAGE

8. (5.75 points)

Given the following information for an insurance company's outstanding claim liabilities as of December 31, 2012:

		Coefficients of Variation (CoV)		
Line of business	Central Estimate	Independent risk	Internal systemic risk	External systemic risk
Workers compensation	$110,000,000	7.2%	7.5%	3.4%
Auto	95,000,000	5.2%	4.5%	4.3%
Homeowners	45,000,000	5.8%	5.0%	6.7%
Total	250,000,000			4.7%

- Assume that the claim liabilities are normally distributed.
- The z-value for the 75th percentile of the normal distribution is: 0.674
- The selected correlation between each pair of lines of business for internal systemic risk is: 0.25
- Assume that each source of independent risk is uncorrelated with any other source of risk.

a. (0.5 point)

Calculate the independent risk CoV for all lines combined.

b. (0.5 point)

Describe why it is reasonable to assume that there is positive correlation between lines of business for internal systemic risk.

c. (1 point)

Calculate the internal systemic risk CoV for all lines combined.

d. (0.5 point)

Calculate the consolidated CoV for all sources of risk.

QUESTION 8 CONTINUED ON NEXT PAGE

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(8 continued)

e. (0.5 point)

Calculate the risk margin (in dollars) for the overall book of business that provides a 75% probability of reserve adequacy.

f. (0.5 point)

Discuss how the risk margin in e. above would change if the selected pair-wise correlation between each pair of lines of business for internal systemic risk were higher than 0.25.

g. (1.5 points)

Analyze the consistency of the selected CoVs by line of business using internal benchmarking checks. Discuss all three main sources of risk.

h. (0.75 point)

Discuss an external systemic risk for which the assumption of constant correlation between lines of business across their loss distributions does not hold. Suggest a method to incorporate variable correlation into the risk margin.

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9. (3.5 points)

An actuary is considering a Bayesian approach to developing a predictive loss distribution based on the deterministic chain-ladder method using the all-years weighted average link ratios.

a. (0.5 point)

To produce results that closely resemble the deterministic chain-ladder outcome, explain whether the actuary should select high or low variances for the prior distributions of the link ratios.

b. (0.5 point)

The actuary decides to override the link ratios suggested by the data for the 36-48 month maturity interval with a judgmental selection. However, the actuary is less confident of this selection than of the all-years weighted averages used for the other maturity intervals. Describe how to change the prior distributions of the link ratios to adjust for this.

c. (1 point)

Discuss the effect of the change implemented in part b. above on the simulated results, addressing both the mean and the prediction error.

d. (1 point)

Describe one advantage that a Bayesian approach has over a bootstrapping algorithm and one advantage that a Bayesian approach has over the Mack method.

e. (0.5 point)

Discuss a modification to the Bayesian framework for the chain-ladder method so that it applies to the Bornhuetter-Ferguson method.

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10. (4 points)

Given the following information ($000) as of December 31, 2012:

Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag
2010	9,200	9,200	3,600	70%
2011	8,500	7,400	2,500	50%
2012	10,000	10,000	4,200	30%

a. (1.5 points)

Use the Stanard-Bühlmann method to estimate the IBNR for accident years 2010 through 2012 combined.

b. (2 points)

Use a credibility-weighted combination of the Stanard-Bühlmann and chain ladder estimates to calculate a total IBNR estimate for accident years 2010 through 2012 combined. Assume credibility is a linear function of the report lag with a credibility factor of 0.35.

c. (0.5 point)

Briefly describe one advantage and one disadvantage of the Stanard-Bühlmann method as compared to the chain-ladder method.

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11. (3.5 points)

Given the following information for an insurance company ($000,000):

	Calendar Year		
	2013	2014	2015
Beginning US GAAP equity	100		
Net income	15	16	17
Minimum capital needed at year-end to maintain current rating	110	120	130

- Risk-free rate: 2.0%
- Expected market return for peer companies: 10.0%
- Insurance company's equity beta: 1.25

- CAPM is an appropriate model for determining the company's risk-adjusted discount rate.
- The company plans to maintain its current rating by meeting the minimum capital requirements.

a. (3 points)

Determine the value of the insurance company as of January 1, 2013 based on the free cash flow to equity method.

b. (0.5 point)

Explain why the free cash flow to equity method is preferable to the free cash flow to the firm method for valuing a property and casualty insurer.

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12. (2.5 points)

Beginning in 2013, an insurance company entered a niche market that other companies had not yet entered. The company plans to grow aggressively in this market. Given the following information for this insurance company:

	Calendar Year Projections ($000,000)		
	2013	2014	2015
Beginning US GAAP equity	1,000	1,105	1,221
Net income	150	166	183
Dividends to be paid at year-end	45	50	55

- Risk-free rate: 2.0%
- Expected equity market risk premium: 6.0%
- Insurance company equity beta: 1.5

- CAPM is an appropriate model for determining the company's risk-adjusted discount rate.
- The company's plan is to continue its current dividend payout ratio.

a. (2 points)

Determine the value of this company as of January 1, 2013 based on the abnormal earnings valuation method. Assume that after the forecast horizon abnormal earnings will decrease to zero, linearly, by 2020.

b. (0.5 point)

Explain why an investor might prefer the abnormal earnings valuation method over the dividend discount model valuation for this particular company.

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13. (1.75 points)

An insurance company has a portfolio of workers compensation, medical malpractice and commercial property business. The portfolio has been in existence for over ten years and currently the volume of business written in each line is stable.

a. (1 point)

The company believes that the unpaid claim reserves for its property business have recently become more uncertain than its reserves for workers compensation and medical malpractice. Discuss two possible reasons for this conclusion.

b. (0.75 point)

Identify three sources of risk the company should consider in order to avoid underestimating the degree of uncertainty in its unpaid claim reserves for workers compensation.

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14. (2.25 points)

Given the following bond portfolio for an insurance company:

	Years to Maturity	Bond Rating	Credit Risk Solvency Capital Requirement ($000)	Bond Issuer's Current Annual Default Probability
Bond 1	1	BBB	45	0.2%
Bond 2	3	BBB	157	0.2%

Assume that the best diversification can be approximated by correlation coefficients of 0.5.

a. (1.5 points)

Identify and briefly describe three sources of credit risk that may be associated with this bond portfolio.

b. (0.75 point)

Calculate the capital requirement due to credit risk for this bond portfolio assuming that both bonds are from the same issuer.

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15. (1.5 points)

Identify and briefly discuss three tools that can be used in an internal model to quantify the effect of extreme events on an insurer's solvency.

CONTINUED ON NEXT PAGE

16. (1 point)

An insurer with a large, stable book of personal auto insurance is building an internal model for solvency. The actuary building the model assumes that the loss ratios are positively correlated from one year to the next.

a. (0.5 point)

Provide two reasons why this assumption is reasonable.

b. (0.5 point)

Describe how the model should incorporate this assumption.

CONTINUED ON NEXT PAGE

17. (2.75 points)

Given the following data:

	Policy Year Ultimate Loss ($000)				
Line of Business	2008	2009	2010	2011	2012
Personal Auto	900	1,150	6,000	850	1,100
Homeowners	340	200	1,060	150	250

a. (1 point)

Calculate the Pearson product-moment correlation between the personal auto loss data and the homeowners loss data.

b. (1 point)

Calculate the Spearman rank correlation between the personal auto loss data and the homeowners loss data.

c. (0.75 point)

Explain the difference between these two measures of correlation as applied to this data set.

CONTINUED ON NEXT PAGE

18. (1.25 points)

A reinsurance company is pricing a casualty excess-of-loss reinsurance treaty that covers loss and allocated loss adjustment expense (ALAE).

a. (0.75 point)

Discuss the relationship between loss and ALAE amounts for a casualty line of business, specifically, how the relationship typically differs between large and small claims.

b. (0.5 point)

Describe how the relationship between loss and ALAE can be modeled in the pricing of a casualty excess-of-loss reinsurance treaty.

CONTINUED ON NEXT PAGE

19. (6.25 points)

Given the following information for an insurance company:

Loss distribution for earthquake (EQ)

Losses (x, in $000,000)	f(x)
0	0.91
200	0.07
400	0.02

Loss distribution for coastal property (CP)

Losses (y, in $000,000)	g(y)
0	0.80
250	0.20

- Losses for these two perils are independent.
- The Wang transform has the form $F^*(x) = \Phi\{\Phi^{-1}[F(x)] - \Phi^{-1}(\alpha)\}$, where Φ represents the cumulative standard normal distribution.
- The Wang transform risk measure for CP is $197.5 million.
- The Wang transform risk measure for CP and EQ combined is $323 million.

z	$\Phi(z)$
0.304	0.62
0.409	0.66
0.804	0.79
0.842	0.80
1.341	0.91
1.645	0.95
2.054	0.98

a. (1.75 points)

For EQ, calculate:

i. Tail value-at-risk (TVaR) at the 95% level.
ii. Wang transform at the 95% level.
iii. Expected policyholder deficit (EPD) at the 5% level.

b. (0.5 point)

Evaluate whether using TVaR as a risk measure for EQ would satisfy the company's requirement to have a pricing charge for any risk taken.

QUESTION 19 CONTINUED ON NEXT PAGE

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(19 continued)

c. (0.5 point)

Evaluate whether using the Wang transform as a risk measure for EQ would satisfy the company's requirement to have a pricing charge for any risk taken.

d. (0.5 point)

Evaluate whether using TVaR as a risk measure for EQ would reflect the company's increasing risk aversion to larger losses.

e. (0.5 point)

Evaluate whether using the Wang transform as a risk measure for EQ would reflect the company's increasing risk aversion to larger losses.

f. (2.5 points)

Calculate the TVaR and Wang transform risk measures at the 95% level for the combined CP and EQ book of business. Evaluate whether these measures exhibit the diversification benefits of writing both coverages.

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20. (5 points)

Given the following 2013 information for an insurance company:

Gross premiums	\$250 million
Gross loss ratio	60%
Gross expense ratio	32%
Risk-free interest rate	1.5%
Investment yield	3.5%
Beginning surplus	\$1 billion
Probability of distress without reinsurance	2%

The insurance company is considering a reinsurance treaty with the following characteristics:

Proposed reinsurance	20% quota share
Ceding commission	10% of reinsurance premiums
Probability of distress with reinsurance	0.4%

- All premiums, expenses and commissions are collected or paid at the beginning of the year.
- All claims are paid at the end of the year.
- The insurance portfolio composition is not expected to grow or change in future years.

a. (4 points)

Using the risk-adjusted present value of future earnings, quantify the effect on the firm's value of purchasing the quota share reinsurance.

b. (1 point)

Identify and briefly describe two alternative approaches to quantifying the value of risk transfer to the insurance company.

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21. (1.25 points)

a. (0.75 point)

Within the context of Basel II, describe how a firm's mix of business affects the basic indicator approach and the standardized approach to calculating the required capital for operational risk.

b. (0.5 point)

Describe a feature of the standardized approach that could make a firm prefer it to the basic indicator approach.

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22. (4 points)

Consider two companies:

- Company 1 is a basic limits private passenger automobile insurer that invests in duration-matched US government securities.
- Company 2 is a large multi-line commercial insurance company that invests in a variety of fixed-income securities and in equity markets.

For each of the following risk categories, discuss the issues faced by each company.

a. (1 point)

Pricing risk

b. (1 point)

Claim variability

c. (1 point)

Market risk

d. (1 point)

Correlation

END OF EXAMINATION

Exam 7
Estimation of Policy Liabilities, Insurance Company Valuation, and ERM

QUESTION	POINT VALUE OF QUESTIONS	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
1	3	2.5	0.5						
2	2.75	2	0.75						
3	5	3	2						
4	2	1.5	0.5						
5	2.25	2.25							
6	2.75	1	0.5	0.5	0.75				
7	2	0.75	0.5	0.75					
8	5.75	0.5	0.5	1	0.5	0.5	0.5	1.5	0.75
9	3.5	0.5	0.5	1	1	0.5			
10	4	1.5	2	0.5					
11	3.5	3	0.5						
12	2.5	2	0.5						
13	1.75	1	0.75						
14	2.25	1.5	0.75						
15	1.5	1.5							
16	1	0.5	0.5						
17	2.75	1	1	0.75					
18	1.25	0.75	0.5						
19	6.25	1.75	0.5	0.5	0.5	0.5	2.5		
20	5	4	1						
21	1.25	0.75	0.5						
22	4	1	1	1	1				
TOTAL	66								

Question 1 Sample Answer

Solution 1

a)

	12-24	24-36	36-48	48-60	60-72
2007	S	L	*	S	*
2008	L	S	L	L	
2009	*	S	S		
2010	S	L			
2011	L				

S= Less than median   L=Greater than median*= median

Diagonal	#S	#L	Z	n	m	c_n	$E[Z_n]$	$VAR[Z_n]$
2	0	2	0	2	0	.5	.5	.25
4	3	1	1	4	1	.75	1.25	.4375
5	1	3	1	5	1	.75	1.25	.4375
Total			2				3	1.125

Z= min(#S, #L)   n= # of elements   m=floor$\left[\frac{n-1}{2}\right]$ $c_n = \binom{n-1}{m}\frac{n}{2^3}$

$E[Z_n] = \frac{n}{2} - c_n$ $var[Z_n] = \frac{n(n-1)}{4} - (n-1)c_n + E[Z_n] - (E[Z_n])^2$

Test statistic =2

90% CI = 3 $\pm 1.645(\sqrt{1.125})$= (1.26, 4.74)

Since z is within range, accept H_0 and conclude there is not a calendar year effect

b)

* A Change on claim handling could cause this. For instance, the claims department might decide to strengthen outstanding case reserves for all claims in a given calendar year

* Legislative Changes: A law change might impact all claims settled after that date

Solution 2

a)

Development Period	12-24	24-36	36-48	48-60	60-72
Median Age to Age Factor	2.64	1.45	1.19	1.07	1.05

	12-24	24-36	36-48	48-60	60-72
2007	L	H	-	L	-
2008	H	L	H	H	
2009	-	L	L		
2010	L	H			
2011	H				

L= Less than median H=above the median*= median

CY Diag	#H	#L	Z	n	m	c_n	$E[Z_n]$	$VAR[Z_n]$
2007	0	1						
2008	2	0	0	2	0	.50	0.50	.250
2009	0	1						
2010	1	3	1	4	1	.75	1.25	.4375
2011	3	1	1	4	1	.75	1.25	.4375
Total			2				3	1.125

N= # of L's and H's

Z= min(#L, #L) n= # of elements m=floor$\left[\frac{n-1}{2}\right]$ $c_n = \binom{n-1}{m}\frac{n}{2^3}$

$E[Z_n] = \frac{n}{2} - c_n$ $var[Z_n] = \frac{n(n-1)}{4} - (n-1)c_n + E[Z_n] - (E[Z_n])^2$

CI= $E(z_n) \pm z_{90\%}\sqrt{Var(z)}$

CI= $3 \pm 1.645\sqrt{1.125)} = (1.2552, 4.74487)$

Our z=2 is within the 90% confidence interval so we accept the null hypothesis that the triangle does not display calendar year effects

b)

* A change in the legal environment could affect all the claims in a CY

* If the claims department strengthened reserves for all open claims that would increase the ldf's for that CY.

Examiner Comment

Part a) Most candidates were able to correctly create the S/L/* table, but failed to mention the median or incorrectly used the mean. Some candidates used the rankings based on the calendar years and not accident years.

Candidates were expected to calculate the z, E(z), and Var(z) as well as provide formulas for calendar years with at least 2 elements. Common mistakes included using the wrong diagonal, not calculating the mean and variance correctly, and not providing formulas. Credit was granted as long as the numbers were consistent with the S/L/* table.

Most candidates were able to provide the confidence interval and provide accept/reject decision correctly. Common errors included missing the square root, not using the correct test statistic, and incorrect decision. Credit was granted if answers were consistent with previous steps.

Credit was given whether the CI was calculated based on the z-value of 1.645 provided in the question or 1.96.

We provided limited credit for candidates that described how to calculate the CI and the accept/reject criterion, but did not provide numbers.

b) We accepted standard comments on changes in inflation/trends, claim reserving practices, and legislative/judicial changes that would have a calendar year impact.

We also accepted answers with adequate explanation on how they would impact calendar year results.

The common mistakes were the same type of item twice and also including items that would cause accident year effects and not calendar year.

Question 2 Sample Answer

Solution 1

a) R^{ind} for total (2010-2012) = $\sum q_i / p_i \times C_i$

$m_k = \frac{sum(loss\ @\ dev\ age\ i)}{sum(Premium)}$

$m_{0\text{-}12} = \frac{1400+1000+1500}{3600+3700+3650} = 0.356$

$m_{12\text{-}24} = \frac{650+850}{3600+3700} = 0.205$

$m_{24\text{-}36} = \frac{280}{3600} = 0.078$

$p_i = \frac{\sum mi\ to\ date}{\sum mi}$

$p_{2010} = \frac{0.356+0.205+0.078}{0.356+0.205+0.078} = 1.000$

$p_{2011} = \frac{0.356+0.205}{0.356+0.205+0.078} = 0.878$

$p_{2012} = \frac{0.356}{0.356+0.205+0.078} = 0.557$

$q_i = 1 - p_i$

$q_{2010} = 1 - 1 = 0$

$q_{2011} = 1 - 0.878 = 0.122$

$q_{2012} = 1 - 0.557 = 0.443$

$C_i = \sum$paid losses to date

$C_{2010} = 1{,}400 + 650 + 280 = 2{,}330$

$C_{2011} = 1{,}000 + 850 = 1{,}850$

$C_{2012} = 1{,}500$

$R^{ind}_{2010} = 0/1 \times 2{,}330 = 0$

$R^{ind}_{2011} = 0.122/0.878 \times 1{,}850 = 256$

$R^{ind}_{2012} = 0.443/0.557 \times 1{,}500 = 1{,}193$

$R^{ind}_{Total} = 0 + 256 + 1{,}193 = 1{,}449$

b) Process variance = $\sigma^2 \sum_i R_i = 10000(2032979)$

Process SD = $\sqrt{}$(P Variance) = 142,582.56

R^{coll} for total = $q_i \times U^{BC}_i = q_i \times m_{total} \times V_i$

$R^{coll}_1 \quad = (0 \times .639) \times 3{,}600 = 0$

$R^{coll}_2 = (.122 \times .639) \times 3{,}700 = 288$

$R^{coll}_3 = (.443 \times .639) \times 3{,}650 = 1{,}034$

$R^{coll}_{Total} = 0 + 288 + 1034 = 1{,}322$

Solution 2

a) R^{ind} for total (2010-2012) = $\sum q_i / p_i \times C_i$

$m_k = \frac{sum(loss\ @\ dev\ age\ i)}{sum(Premium)}$

$m_{0\text{-}12} = \frac{1400+1000+1500}{3600+3700+3650} = 0.356$

$m_{12\text{-}24} = \frac{650+850}{3600+3700} = 0.205$

$m_{24\text{-}36} = \frac{280}{3600} = 0.078$

$p_i = \frac{\sum mi\ to\ date}{\sum mi}$

$p_{2010} = \frac{0.356+0.205+0.078}{0.356+0.205+0.078} = 1.000$

$p_{2011} = \frac{0.356+0.205}{0.356+0.205+0.078} = 0.878$

$p_{2012} = \frac{0.356}{0.356+0.205+0.078} = 0.557$

$q_i = 1 - p_i$

$q_{2010} = 1 - 1 = 0$

$q_{2011} = 1 - 0.878 = 0.122$

$q_{2012} = 1 - 0.557 = 0.443$

$C_i = \sum$paid losses to date

$C_{2010} = 1{,}400 + 650 + 280 = 2{,}330$

$C_{2011} = 1{,}000 + 850 = 1{,}850$

$C_{2012} = 1{,}500$

$R^{ind}_{2010} = 2{,}330 / 1.00 - 2{,}330 = 0$

$R^{ind}_{2011} = 1{,}850 / .878 - 1{,}850 = 256$

$R^{ind}_{2012} = 1{,}500/.557 - 1{,}500 = 1{,}193$

$R^{ind}_{Total} = 0 + 256 + 1{,}193 = 1{,}449$

b) U^{BC}_i = Premium × ELR = $V_i \times \sum m_i$

$U^{BC}_{2010} = 3{,}600 \times (0.356+0.205+0.078) = 2{,}302$

$U^{BC}_{2011} = 3{,}700 \times (0.356+0.205+0.078) = 2{,}366$

$U^{BC}_{2012} = 3{,}650 \times (0.356+0.205+0.078) = 2{,}334$

R^{coll}_1 $= 0 \times 2{,}302 = 0$

R^{coll}_2 $= .122 \times 2{,}366 = 288$

R^{coll}_3 $= .443 \times 2{,}334 = 1{,}034$

R^{coll}_{Total} $= 0 + 288 + 1{,}034 = 1{,}322$

Solution 3

b)

R^{coll}_1	0	$= (63.9\% - 35.6\% - 20.5\% - 7.8\%) \times 3{,}600$
R^{coll}_2	288	$= (63.9\% - 35.6\% - 20.5\%) \times 3{,}700$
R^{coll}_3	1,034	$= (63.9\% - 35.6\%) \times 3{,}650$
R^{coll}_{Total}	1,322	

Examiner Comment

Several candidates used a standard paid loss development method instead of calculating the individual loss ratio claims reserve R^{ind} as requested. Another common mistake was miscalculating the m_k's using the total earned premium for all accident years. Another common mistake was candidates misidentifying R^{ind} as R^{coll}, and vice versa. Candidates making these mistakes did not receive full credit.

Question 3 Sample Answer

Solution 1

a)

Process st dev = $\sqrt{R\sigma^2}$ $\sigma^2 = 1/(n-p)\sum(c-\mu)^2/\mu$ n=6 p=2+3=5

↑ G,W ↑ Ult 2010, Ult 2011, Ult 2012

$G(6)=1-e^{(-(6/20)^{\wedge}1.5)} = .1515$

G(18) = .5742

G(30) = .8407

* Check if truncation is needed:

- Extend Δ out 3 years G(66) = .9975
- G(66) is reasonably close to 1 and so the function does not need to be truncated

AY	G(x)	LDF = 1/G(x)	Ult = Cum Pd×LDF	Reserve
2010	0.8407	1.1895	6,066.4	966.4
2011	0.5742	1.7416	7,488.7	3,188.7
2012	0.1515	6.6007	19,142	16,242
				20,397.1

avg age:	(6)	(18)	(30)
expected	12	24	36
10	919	2564.3	1616.7
11	1134.5	3165.5	1995.7
12	2900	8091.3	5101.3

μ = [G(y) – G(x)] Ult AY

1995.7 = (.8407-.5742)(7488.7)

actual

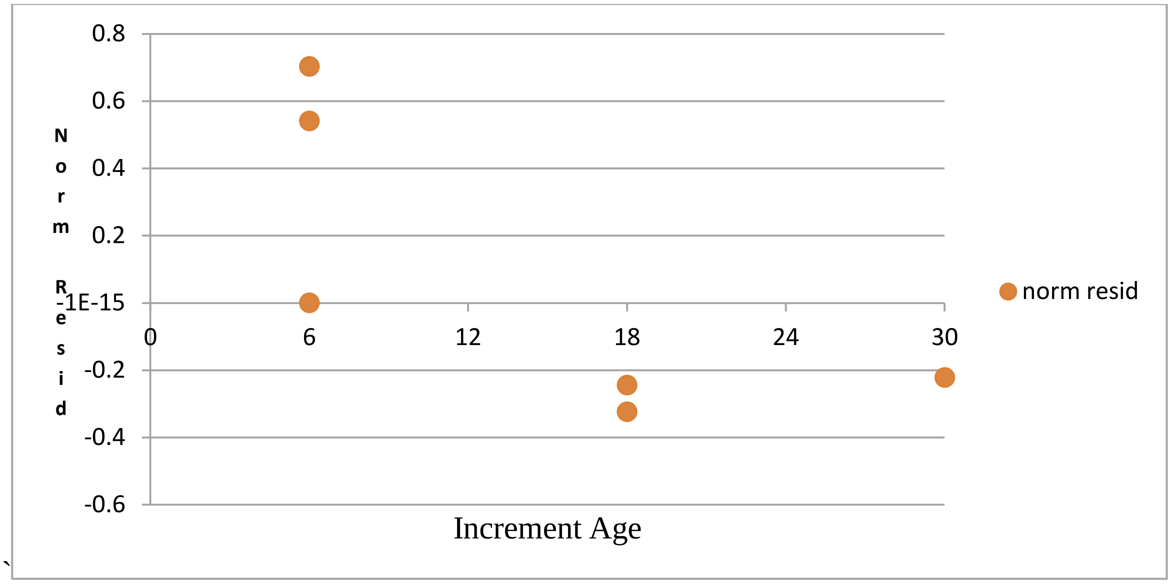
10	2750	1500	850
11	2700	1600	
12	2900		↖4300-2700

$\sigma^2=1/(6-5)\times\left[\frac{(2750-919)^2}{919}+\frac{(2700-1134.5)^2}{1134.5}+\ldots\right] = 7387.84$

process standard dev = $\sqrt{R\sigma^2} = \sqrt{7387.84 * 20397.1}$ = 12,275.607

b) $\varepsilon = (c-\mu)/\sqrt{\mu\sigma^{\wedge}2}$

6	18	30
0.7027	-0.245	-0.222
0.5407	-0.324	↑
0		(850-1616.7)/(7387.4×1616.7)^0.5



*expect points randomly scattered around zero, but it appears that the residuals at age 6 are all positive (μ is underestimating) then the residuals are negative at later ages (μ is over estimating). Thus Weibull model used may not be appropriate for this loss experience.

<u>Solution 2</u>

a) Avg Age

<u>Avg Age</u>	<u>G(Age)</u>
6	1-e^-(6/20)^1.5=.152
18	=.574
30	=.841
12(6)-6=66	=.998 Truncate at twice the triangle or 3(2)=6 years

AY	Paid to Date	% paid	LDF-Ultimate	LDF Truncated
12	2,900	0.152	1/.152=6.58	6.58(.998)=6.57
11	4,300	0.574	1.74	1.736
10	5,100	0.841	1.19	1.188

AY	Ult Loss	Reserve
12	6.58×2900-19053	19053-2900=16153
11	1.736×4300=7465	3165
10	1.188×5100=6059	959
		Total=20277 <-- Total Reserves

Incremental Expected Triangle = Ult (% paid at age)

AY	12	24	36
10	921	2557	1618
11	1135	3150	
12	2896		

6059×(.152)=921

Actual Incremental Triangle

AY	12	24	36
10	2750	1500	850
11	2700	1600	
12	2900		

(actual - expected)^2 / expected

AY	12	24	36
10	3632	437	365
11	2158	763	
12	0.006		

Sum of triangle = 7358

Total Reserves = 20,277

Process Variance = 20277(σ^2)

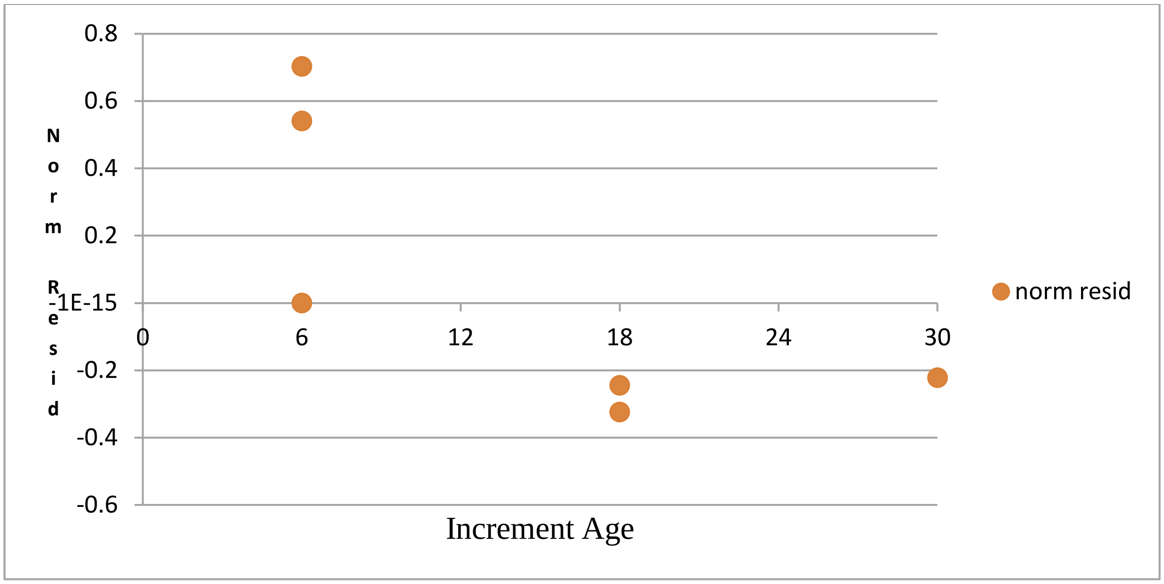
σ^2=1/(n-p) $\sum(A-E)^2/E$ = 1/(6-5)(7335) = 7335

Process standard deviation = $\sqrt{20277(7335)}$ = 12196

b) normalized residual = (actual-expected)/(σ(expected)$^{0.5}$) $\sigma=\sqrt{7335}$ = 85.6

Residual Triangle

AY	12	24	36
10	0.704	-0.244	-0.223
11	0.543	-0.323	
12	0		



If the Weibull curve were appropriate, we would see residual randomly scattered around 0 at each age. In our graph, there is a clear trend from positive to negative as age increases. This means this Weibull model is not appropriate.

<u>Examiner Comment</u>

a) The candidate was asked to determine the process standard deviation given a specific model. To get there, several basic calculations were needed such as determining ultimate losses given specific data and a specific curve, resulting reserves, and the chi squared formula – including corresponding degrees of freedom. Common errors included using cumulative instead of incremental actual and fitted data triangles, not recognizing the use of the average age in the application of the Weibull model, not correctly determining the degrees of freedom to use, and general math errors. Candidates often did not calculate the process standard deviation correctly - either incorrectly taking the square root of σ^2 or calculating the correct process variance but forgetting that the standard deviation was requested.

   Truncation was not specifically mentioned in the question and could be subject to candidate interpretation of model results. Both truncated and untruncated approaches were accepted for full credit.

b) The candidate is asked to calculate and graph normalized residual using the results from part a). A very common error in the determination of the normalized residuals was to fail to divide by the σ^2 resulting in a set of residuals incorrectly scaled. Another common mistake was to take the square root of the chi-squared results from part a) which failed to recognize the possibility of negative residuals. Candidates generally graphed their calculated residuals correctly, but a common mistake observed was to graph the residual against incremental loss size which was not what the question specified. In the evaluation of the graph, most candidates recognized that the Weibull was not appropriate given the data and resulting residuals; however, explanations as to why were oftentimes incomplete or non-existent.

Question 4 Sample Answer

Solution 1

a) $R_{GB} = q_k \times U_{BF}$
$(8204 - 7004) = (1 - 1/LDF_{24\text{-}ULT}) \times 8428$
$1200/8428 = 1 - 1/LDF_{24\text{-}ULT}$
$LDF = 1.1660$

b) As you iterate on the BF method to infinity, the result will approach the chain ladder estimate.

Solution 2

a) AY 2011
$U_{BF} = 8428 = 7004 + q_i \times U_0$
$U_{BT} = 8204 = 7004 + q_i \times 8428 \rightarrow q_i = .1424$
p_i = % Reported = $1 - q_i = .8576$
$LDF_{24\text{-}ULT} = 1/.8576 = \boxed{1.166}$

b) $U_{BF} = U_{CL} \times (1- q_i) + U_0 \times q_i \rightarrow 1^{st}$ iteration
$U_{BT} = U_{CL} \times (1- q_i^2) + U_0 \times q_i^2 \rightarrow 2^{nd}$ iteration
3^{rd} iteration: $U_{CL} \times (1- q_i) + q_i\, [U_{CL} \times (1- q_i^2) + U_0 \times q_i^2]$
$U_{CL} \times (1- q_i) + q_i \times U_{CL} \times (1- q_i^2) + U_0 \times q_i^3$
$U_{CL} \times (1- q_i)\, [1+ q_i \times (1- q_i)] + U_0 \times q_i^3$
$U_{CL} \times (1- q_i)\, [1+ q_i - q_i^2] + U_0 \times q_i^3$
…
$U_{CL} \times (1- q_i^3) + U_0 \times q_i^3$

As the number of iterations approaches infinity
$\rightarrow U_{CL} (1 - q_i^{\infty}) + U_0 \times q_i^{\infty}$

where $\lim_{t \to \infty} q_i^{\,t} = 0$

thus the reserve estimate becomes $U_{CL} (1 - 0) + U_0 \times 0 = U_{CL}$

$U_{CL} = 7004 \times LDF_{24\text{-}ULT} = 7004 \times 1.166 = \boxed{8167}$

Examiner Comment

This question was generally answered well by candidates.

Part a)

The majority of candidates received full credit for part a). Errors, if any, were generally due to calculation/arithmetic errors. The critical component of the solution was the relationship between the Benktander and Bornhuetter Ferguson method.

Part b)

The question in part b) did not explicitly state that candidates needed to estimate the value, so full credit was given to candidates that mentioned chain-ladder ultimate without explicitly calculating the value of the chain-ladder ultimate.

A few candidates used a cumulative development factor inconsistent with their calculated cumulative development factor in part a). Partial credit was given to candidates with this response.

Some candidates derived the estimate showing iterations to infinity. These candidates received full credit if their derivation was correct.

Partial credit was given to candidates that calculated the value of the chain-ladder estimate, without explicitly stating that it is the chain-ladder estimate, or showing a derivation.

Question 5 Sample Answer

Solution 1

Triangle of Trend

AY	12	24	36	48
09	1	1.05	1.103	1.158
10	1.05	1.103	1.158	1.216
11	1.103	1.158	1.216	1.276
12	1.158	1.216	1.276	1.34

Triangle of Trended Θ's

AY	12	24	36	48
09	22,453	41,447	50,136	57,036
10	23,575	43,539	52,636	
11	24,765	45,711		
12	26,000	48,000	58,000	66,000

AY 09 at 48 months = 66,000×(1.158/1.34)

LEV's for basic limit – last row only

12	24	36	48
22,200	31,062	33,507	35,059

^

$26{,}000(1-e^{-50{,}000/26{,}000})$

AY	Age	cum loss @ B cost level
2009	48	601 × 35,059 / 57,036 = 369.42
2010	36	512 × 33,507 / 52,636 = 325.93
2011	24	541 × 31,026 / 45,711 = 367.63
2012	12	255 × 22,200 / 26,000 = 217.73

Solution 2

Mean clm at BC at 2012 lvl

12	24	36	48
22,200	31,062	33,507	35,059

$22{,}200 = 26{,}000 \times (1-e^{-50{,}000/26{,}000})$

For latest CY
2012 Adjusted = 255 × 22,200 / 26,000 = 218
2011 Adjusted = 541 × 1.05 × 31,062 / 48,000 = 368
2010 Adjusted = $512 \times 1.05^2 \times 33{,}507 / 58{,}000 = 326$
2009 Adjusted = $601 \times 1.05^3 \times 35{,}509 / 66{,}000 = 370$

Examiner Comment

The question was fairly straightforward, with a number of acceptable approaches to the same answer. There was a typo in the given exponential limited mean formula, but this was recognized and corrected by almost all of the candidates. The most common error (other than the most basic of trending errors) was calculating and using the LEV's on the latest diagonal for each AY, as opposed to AY 2012 at the various evaluations. Following this line, some candidates knew how to calculate the individual pieces, but just couldn't wrap it all together.

Question 6 Sample Answer

Solution 1

a) PDLD$_{18}$ = $\frac{P_{18}}{L_{18}} = \frac{(BP + CL_{18} xLCF)xTM}{L_{18}}$

$$= \frac{12,456,000 \times 1.04 + 18,103,000 \times 1.15 \times 1.04}{22,628,750} = 1.53$$

E(L) = 75%×54,309,000 = 40,731,750

Loss$_{18}$ = 40,731,750 × 1/1.8 = 22,628,750

CL$_{18}$ = 22,628,750 × 80% = 18,103,000

b) PDLD$_2$ = $\left(\frac{CL_{30} - CL_{18}}{L_{30} - L_{18}}\right)$×LCF×TM

$$= \left(\frac{17,709,457 - 18,103,000}{35,418,913 - 22,628,750}\right) \times 1.15 \times 1.04 = -0.0368$$

CL$_{30}$ = E(L)/1.15 × 50% = 17,709,457

L$_{30}$ = E(L)/1.15 = 35,418,913

c) CL$_{42}$ = E(L)/1.05×0.7 = 27,154,500

L$_{42}$ = E(L)/1.05 = 38,792,143

PDLD = $\left(\frac{CL_{42} - CL_{30}}{L_{42} - L_{30}}\right)$×LCF×TM

$$= \frac{27,154,500 - 17,709,457}{38,792,143 - 35,418,913} \times 1.15 \times 1.04 = 3.35$$

d) Loss capping ratio should be decreasing ideally, however, in the question, 70% is higher than 50%. So, it's not reasonable.

Because as losses matures, more policies will hit the max premium and per occurrence limit and more losses will develop outside the caps. So capping ratios should decrease as losses mature.

<u>Solution 2</u>

a) $PDLD_1 = \dfrac{BP}{SPxELRx\%L_1} \times TM + \dfrac{CL_1}{L_1} \times LCF \times TM$

$= \dfrac{12,456,000}{54,309,000 \times 75\% \times (1/1.8)} \times 1.04 + 0.8 \times 1.15 \times 1.04$

$= 1.529268 \approx 1.53$

b) $PDLD_2 = \left(\dfrac{CL_2 - CL_1}{L_2 - L_1}\right) \times LCF \times TM$ (Assuming loss capping ratio given is incremental loss capping ratio)

$= 0.5 \times 1.15 \times 1.04$

$= 0.598$

c) $PDLD_3 = \left(\dfrac{CL_3 - CL_2}{L_3 - L_2}\right) \times LCF \times TM$ (Assuming loss capping ratio given is incremental loss capping ratio)

$= 0.7 \times 1.15 \times 1.04$

$= 0.8372$

d) Typically we expect premium responsiveness to decline with increasing maturity, since as more time passes and more losses emerge, we expect more losses to be capped by the per-occurrence limit or the maximum premium, and only capped losses contribute to additional premium. Based on a-c above, we see that premium responsiveness at the 3rd adjustment is higher than the 2nd, which is contrary to the explanation above. This is because the loss capping ratio is higher at the 3rd adjustment, suggesting fewer losses are being capped, which is not a reasonable assumption.

Examiner Comment

Part A, this is a relatively straightforward application of the formula for PDLD ratio as provided within Teng and Perkins which calculates the Initial Premium as of the first adjustment as compared to the expected losses to have emerged as of that time. Common errors include arithmetic errors and errors in memorizing formulas.

Part B, the graders were expecting the candidates to understand the difference between cumulative and incremental activity. The true retrospective premium adjustments are based on the incremental activity from the prior adjustment to the subsequent adjustment. The loss capping ratios are cumulative unless they are labeled as incremental loss capping ratio. And candidates are expected to know how to use cumulative loss capping ratios to calculate the incremental ones. Common errors include confusing cumulative loss capping ratios to be incremental ones, forgetting TM and LCF, and arithmetic errors. For candidates who are using loss capping ratios as incremental ones, they must state their assumptions clearly; otherwise they will need to treat loss capping ratios as cumulative.

Part C, please see part B.

Part D, the candidates are expected to understand that the loss capping ratios given in the question were not reasonable in normal circumstance and why. The candidates are also expected to answer the question by using the data given in the question. Common errors will be candidates just giving generic comments/conclusions without tying them back to the situation that was given in the question.

Question 7 Sample Answer

Solution 1

a) Incremental losses

AY	12	24	36
10	200,000	25,000	25,000
11	300,000	25,000	
12	450,000		

Log link triangle = ln(incr. loss)

AY	12	24	36
10	12.206	10.127	10.127
11	12.612	10.127	
12	13.017		

b) Incremental losses

AY	12	24	36
10	200,000	-30,000	80,000
11	300,000	25,000	
12	450,000		

The GLM bootstrap model assumes all incremental losses are non-negative. The log link function fails for negative inputs. Furthermore, the sum of the 12-24 columns is negative, so the first method of fixing the triangle (-ln(abs(incr. loss))) fails.

c) Subtract the largest negative number from every value in triangle, then take log.

revised incr

AY	12	24	36
10	230K	0	110K
11	330K	55K	
12	480K		

log link

AY	12	24	36
10	12.346	0	11.608
11	12.707	10.915	
12	13.082		

modify the fitted values by adding -30K to each value (i.e. subtract 30,000 from all fitted values)

Solution 2

a) $\ln m = \eta_{w,d}$

convert triangle to incremental triangle and take ln of each incremental.

AY	12	24	36
2010	ln (200,000) = 12.2061	10.1266	ln(25,000) = 10.1266
2011	12.6115	10.1266	
2012	13.0170		

b) There will now be a negative incremental value at 24 months for 2010.

ie 170,000 - 200,000 = -30,000

Cannot use negative numbers in the model: ln (negative #) does not work.

c) With b) change, incremental loss triangle loss becomes:

AY	12	24	36
'10	200,000	-30,000	80,000
'11	300,000	25,000	
'12	450,000		

Since sum of 24 month column is negative, we will take the largest negative value (ie -30,000) and subtract from all values in the table, then take the ln of those values to create the triangle. Set ln 0 = 0.

AY	12	24	36
'10	ln(230,000) = 12.3458	0	ln(80,000-(-30,000)) = 11.6082
'11	12.7068	10.9151	
'12	13.0815		

Solution 3

c) Now, incremental value is 170K - 200K = -30K. Sum of column of incremental values is -30K + 25K = -5K

We now adjust all incremental values by adding Ψ, where Ψ = -(-5K) = 5,000.

Our incremental value at AY 2010, 24 months is still negative so we adjust it by using the log link function as –ln(-(-25,000)) since -30K + 5K = -25K.

Once we run GLM, we must remove Ψ from all of the fitted values so we subtract all fitted values by 5,000.

Log-link triangle:

AY	12	24	36
2010	12.231	-10.127 = -ln(-(-25,000))	ln(85,000) = 11.350

2011	12.628	10.309 = ln (30,000)
2012	13.028 = ln (455,000)	

Examiner Comment

Most candidates knew how to construct the log link triangle for part a. Common errors for this part included taking the ln of the cumulative amounts rather than incremental or taking the incremental of the ln of the cumulative amount. For part b, most candidates knew that you can't take the ln of a negative number but very few mentioned the secondary issue of the negative sum of the column. Candidates who only indicated that the incremental value was negative without explaining why that was a problem were given no credit. In part c, the candidate had to recognize that the substitution of the –ln(-(-30,000)) wouldn't work due to the negative sum of the column of incremental values. The most common errors included adjusting a subset of the incremental triangle rather than all values in the triangle, using an incorrect value for adjusting, and not correcting the 36 month incremental value for the adjusted 24 month cumulative value.

Question 8 Sample Solution

Solution 1

a) The Φ of independent risk = $(\Sigma (\omega_i \times \Phi_i) \wedge 2) \wedge 0.5$;

$= [(.44 \times 0.72)\wedge2 + (.38 \times .052)\wedge2 + (.18 \times .058)\wedge2]\wedge0.5$;

$= .039$

b) In the case of the same individual performing actuarial analyses of the central estimates for all lines of business, the "same actuary effect" can be present, resulting in estimates that are subject to the same biases and therefore positively correlated.

c) $\Phi_{Internal} = [\Sigma \ \rho_{ij} \times (\omega_i \times \Phi_i)(\omega_j \times \Phi_j)] \wedge 0.5$ where $\rho_{ij} = 0.25$ for all i,j

$= [.44 \times 0.75 \times (1 \times .44 \times .075 + .25 \times .38 \times .045 + .25 \times .18 \times .05)$ <--- WC

$+ .38 \times .045 \times (.25 \times .44 \times .075 + 1 \times .38 \times .045 + .25 \times .18 \times .05)$ <--- Auto

$+ .18 \times .05 \times (.25 \times .44 \times .075 + .25 \times .38 \times .045 + 1 \times .18 \times .05)] \wedge 0.5$ <--- HO

$= .044$

d) $\Phi_{External} = .047$ (given in problem)

$\Phi_{Total} = [.039 \wedge2 + .044\wedge2 + .047\wedge2] \wedge 0.5$

$= .075$

e) Risk margin = $Z_\alpha \times \Phi_{Total} = .674 \times .075 = .05055 \times \$250m = \$12,637,500$

f) A high cross correlation would raise the internal risk CoV, since adverse results of one class would mean adverse results in another class with a stronger dependency. The overall CoV would then increase as well as the risk margin.

g) 1-Independent Risk: This includes both process risk (randomness) and parameter risk, the risk that parameters selected do not fit the line of business. The CoV's rank from least complex line (auto) to most complex (work comp), and it makes sense that a more complex line such as work comp, which involves additional parameters (inflation, court decisions, disability schedules) would have a higher CoV for independent risk.

2-Internal Systemic Risk: This includes specification error, parameter selection error, and data error. Among the risks here are the same actuary effect, lack of knowledge about a line of business, and the idea that no model will accurately capture the entire risk. The resulting CoVs follow a similar pattern as independent risk, that is a less complex line (auto) has a lower CoV than work comp. The follows the logic discussed above.

3-External Systemic Risk: This includes economic and social risk, legislative, political and claims inflation risk, claims management process risk, event risk, expense risk, latent claim risk, and recovery risk. HO has the highest CoV here, likely due to the influence of event risk on this book of business. Also, since HO is a smaller liability than work comp or auto, its CoV will be higher here. It is surprising that auto's CoV is higher than work comp, though this may also be due to event risk and due to a smaller liability size than WC.

h) Event risk – There will be increased correlation in the tail between auto and homeowners in the event of a catastrophe (and work comp as well if it is an earthquake with damage to a building while employees are in it). To incorporate this variable correlation, a copula could be used to join the distributions of different lines of business. This would be more rigorous computationally, but would result in a more accurate view of increased tail dependencies in the case of a catastrophic event.

<u>*Solution 2*</u>

a) Total cov^2 = $\Sigma\ (\ \omega_i \times \Phi_i\ )\ \hat{}\ 2$

$= \{\ [\ (110/250) \times .075\ ]\ \hat{}2 + [\ (95/250) \times .052\ ]\ \hat{}2 + [\ (45/250) \times .058\ ]\hat{}2\ \}\ \hat{}0.5$

$= .0388$

b) Internal systemic risk is risk internal to the insurance liability valuation modeling process. Possible risks include data errors, parameter mis-specification, or even the "same actuary" effect, all of which will impact multiple lines of business in the same way thus resulting in positive correlation.

c) $[\ \Sigma\ \ \Sigma\ \ \rho_{ij} \times (\ \omega_i \times \Phi_i\ )(\ \omega_j \times \Phi_j\ )\ ]\ \hat{}\ 0.5$

$= [\ \ (110/250 \times .075\ )\hat{}2\ \ + (95/250 \times .045)\hat{}2 + (45/250 \times .05)\hat{}2$

$+\ 2 \times (.25)\times(110/250 \times .075)(95/250 \times .045)$ <-- WC + Auto

$+\ 2 \times (.25)\times(110/250 \times .075)(45/250 \times .05)$ <-- WC + HO

$+\ 2 \times (.25)\times(95/250 \times .045)(45/250 \times .05)\ ]\ \hat{}0.5$ <-- Auto + HO

$= [\ .00146 + .00028 + .000149 + .000077\ ]\ \hat{}0.5$

$= .0444$

d) $= [\ .0388\ \hat{}2 + .0444\ \hat{}2 + .047\ \hat{}2\ ]\ \hat{}0.5 = .0754$

e) $= \mu \times (1 + Z \times \text{CoV}\ ) - \mu$ = Risk Margin

$250 \times (1 + .674 \times .0754) - 250 = 12,705,615$

f) If the correlations were higher, this would increase the overall CoV for internal systemic risk, which would increase the total CoV and thus increase the risk margin.

g) For independent risk, we should consider the portfolio size and length of claim runoff when selecting CoV's. As we might expect, WC has a higher CoV because it has a longer claim runoff. This should be tempered though by the fact that it is the largest in size of reserves – law of large numbers should drive this CoV down.

For internal systemic risk, the low CoV's for HO and auto suggest we are more confident in our models and parameters compared to WC. Perhaps we have fewer models for WC or haven't identified the best predictors of claim cost outcomes.

For external systemic risk, we might expect the HO CoV to be lower since it is for O/S claim liability only (the prem liability CoV would be higher due to event risk). The WC CoV seems low given the risk of claim inflation and the potential for legislative changes for this LOB.

h) Event risk for auto and homeowners. Event risk is the risk of a single event causing a large number of claims. Auto and HO results are likely to be uncorrelated except in the tails of the distribution, for instance when there is a large catastrophe. We could use copulas to model this correlation structure so that the results only show dependency in tails.

Examiner Comment

Part a) of the problem depended upon an understanding that the weighted CoV values by line should be squared and then added, prior to taking the square root for the overall independent risk. Common errors included not using the line of business proportional weights in the formula, not squaring the weighted CoV values prior to taking their sum, or squaring the individual line CoV values but not squaring the individual line weights.

Part b) of the problem could be answered in several ways for full credit, however simply mentioning "the same actuary effect" without any elaboration did not qualify for full credit. A number of candidates referred to internal system issues that would affect multiple lines of business (e.g., policy issuance problems) but had no mention of uncertainty arising from actuarial valuation models and therefore that type of response did not receive any credit.

Part c) of the problem depended upon the proper application of the formula using weighted CoV values by lines of business and incorporating the effect of correlations between lines. Candidate errors on this part included the omission of line of business weights in the formula, failure to include a factor of 2.0 in the correlation component of the formula for each line of business pair, and failure to square the line of business weights in the stand-alone CoV components for each line of business, all leading to minor point reductions. Part d) of the problem required the use of the CoV values calculated in parts a) and c) of the problem along with the total external systemic risk CoV value stated in the problem. If the candidate had miscalculated either or both of the CoV values in parts a) and c) of the problem but used the errant figures correctly in the setup to part d), there was no additional penalty applied in this part of the problem. Also, due to the complexity of the formula in this part, candidates were more likely to make a computational error in evaluating the formula value, but as long as the formula set-up was clearly shown only a minor penalty was applied for that computational error.

Part e) of the problem required the candidate to use the consolidated CoV value determined in part d) along with the Z-value given in the problem and calculate a risk margin in dollars. If a candidate had miscalculated the consolidated CoV in part d) of the problem but used the errant figure correctly in the setup to part e), there was no additional penalty applied in this part. Some candidates also failed to calculate the risk margin in dollars as explicitly required by the problem or else provided the low/high range of reserve values instead of a reserve margin, both of which resulted in only partial credit. Part f) of the problem required some explanation of how the risk margin in part e) would change; therefore candidate solutions that stated solely that the margin would increase received only partial credit.

Part g) of the problem required some discussion of the relative CoV values within each risk category (independent, internal systemic, and external systemic) for full credit, along with some recognition that the external systemic risk CoV for the homeowners line appears abnormally high relative to the other two lines of business. Part h) required the candidate to explain why the assumption of constant correlation between lines of business across their loss distributions does not hold. Some candidates mentioned event risk and noted that correlations would be higher in the tail but did not explain why or give any example which resulted in a minor point reduction.

Question 9 Sample Answer

Solution 1

a) High Variances – If we are not confident in our prior estimates, a wide variance will cause the model to output parameters based on the actual data rather than our prior opinion.

b) Let the distribution for the $LDF_{36\text{-}48}$ have the actuary's opinion as the expected value, but leave a wide variance around the estimate. This will let the model consider the actuary's selection to some degree, but will still use the historical data to determine the parameter.

c) It will pull the $LDF_{36\text{-}48}$ closer to the actuary's estimate. Because the LDF's distribution specified a large variance, prediction error will similar to the chain ladder, though probably larger as the variance selection is large.

d) Over bootstrapping: Can insert your opinion into the parameter selection without much difficulty. Over Mack: We will get a full distribution of the loss estimate, not just the first two moments.

e) We could insert row parameters, one for each accident year, and specify relatively tight variance around them. This will make the model use the prior estimate more, as in BF where the reserve estimate is based on our prior expected LR.

Solution 2

a) Use large variance for the prior distribution to put more weight on the chain ladder outcome. The larger variance reflects we are not as confident in our prior distribution.

b) Put a distribution with a mean equal to his selection, but with large variance. For example, for the 36-48 interval LDF,
$\gamma' = 1.5\ , variance(\gamma') = w; such\ that\ w\ with\ large\ variance$

c) The simulated results will incorporate the expert opinion for the 36-48 LDF, and the prediction error will be higher since we are less confident in our selection for this LDF than the weighted average.

d) Compared to bootstrapping, the Bayesian approach can incorporate expert knowledge into the selection of the ratios. Compared to Mack, the full predictive distribution can be easily calculated and we can calculate the prediction error as the square root of the MSEP of the distribution.

e) If we use really strong priors (i.e. low variance) for the row parameters, this allows us to set row parameters equal to the BF estimate of ultimate for each year. This will replicate the BF method.

<u>Examiner Comment</u>

a) Responses with low variance were not given credit.

b) In order to receive full credit candidate responses had to include the use of high variance for the 36-48 LDF interval due to the actuary's high uncertainty with the selected LDF.

c) In order to receive full credit candidate responses had to discuss the effect of the change in b. on mean and predication error of the simulated results. A candidate's response may receive part c. credit even if they did not receive full credit for part b.

d) Candidates were given full credit for response which describing the same advantage of Bayesian over both. For example full credit could be given for response which identified and described the accommodation of actuary's expert opinion in the predictive distribution for reserves afforded by Bayesian approach is an advantage over both Mack and Bootstrapping.

e) Overall this question part appeared to be the most difficult for candidates. Candidate's responses frequently discussed the BF modification to the chain ladder method but did not address predictive or stochastic features. Credit was not given to candidates who only discussed the deterministic, non-stochastic BF model modification to the chain ladder.

Question 10 Sample Answer

Solution 1

a)

AY	Used premium = (Adjusted prem) × (Report Lag)
2010	9200(0.7) = 6440
2011	3700
2012	3000
	13140

$$ELR = \frac{\Sigma \text{ Reported losses}}{\Sigma \text{ Used Premium}} = \frac{3600 + 2500 + 4200}{13140} = 0.784$$

AY	IBNR = (% unreported) × (Adjusted prem) × ELR
2010	(1 - 0.7)(9200)(0.784) = 2164
2011	(1 - 0.5)(7400)(0.784) = 2901
2012	5488
	10,553

b)

AY	LDF	IBNR = Reported loss × (LDF - 1)	Credibility	Cred. wtd IBNR
2010	1/0.7 = 1.429	3600(1.429 - 1) = 1543	0.35×0.7= 0.245	2,012
2011	1/0.5	2500	0.35×0.5= 0.175	2,831
2012	1/0.3	9800	0.105	5,941
				10,784

Cred. Weighted IBNR = Z × [Chain ladder IBNR] + (1 - Z)[Stannard-Buh…IBNR]
Where Z = 0.35 × Reporting lag

c) Advantage: more stable at less mature years

Disadvantage: requires many adjustments to earned premium that could be time consuming or difficult

Solution 2

a)

	Used Up Premium	Unpaid = ELR × Adj Prem × (1 – Loss Lag)
2010	6440	2164
2011	3700	2901
2012	3000	5488
		10,533

$$ELR = \frac{\Sigma Rep\ Loss}{\Sigma Used\ Up\ Premium} = 0.784$$

b)

	Unpaid CL = $\frac{Loss}{Loss\ Lag} \times (1 - Loss\ Lag)$	Cred Factor = $\rho_i \times 0.35$	Cred Weighted = CL × (z) + SB × (1-z)
2010	1543	.245	2012
2011	2500	.175	2831
2012	9800	.105	5941
			10,781

c) The SB method uses a expected loss ratio, resulting in a more stable estimate of IBNR. The CL method is more responsive to emerged loss experience than the S-B method.

Examiner Comment

This problem was fairly straightforward though calculation intense. Calculation errors were very common. For parts a) and b), several candidates forgot to provide the total IBNR as instructed in the question, but just provided it by year. For part c, a common error was providing an advantage of the Stanard-Bühlmann method relative to a method other than the chain ladder method.

Question 11 Sample Answer

Solution 1

a) $k = r_f + \beta (E(r_m) - r_f) = 2\% + 1.25 \times (10\% - 2\%) = 12\%$

$FCF_1 = 100 + 15 - 110 = 5$
$FCF_2 = 110 + 16 - 120 = 6$
$FCF_3 = 120 + 17 - 130 = 7$

Assume $g = Capital_{2015}/Capital_{2014} - 1 = 130/120 - 1 = 8.3\%$
Firm Value $= \Sigma\ FCF / (1+k)^i = 5/(1+12\%) + 6/(1+12\%)^2 + 7/(1+12\%)^3$
$+ [7(1+8.3\%)/(12\% - 8.3\%)]/(1+12\%)^3$

$= 160.07$ (\$000,000)

b) 1. Debt vs Policyholder Liability: FCFF method firstly calculates total firm value and then subtract debt. There is no economic rationale for different treatments of two kinds of liability.
2. WACC and unlevered return k: FCFF use either WACC or unlevered return for discount rate k. Due to policyholder liability, it is hard to precisely define and estimate either one.

Solution 2

a) FCFE = NI + Non-cash charges (0) + net borrowing (0) – net working cap (0) - Δ capital req.

	2013	2014	2015
FCFE	15 – (110 – 100) = 5	6	7

$k = r_f + \beta (E(r_m) - r_f) = .02 + 1.25 \times (0.01 - 0.02) = 12\%$
need g = ROE x Reinvestment Rate

	2013	2014	2015	selected
ROE = $\frac{NI}{Beg\ Eq.}$	0.15	0.14545	0.14167	0.14167
Reinv Rate = $\frac{NI-FCFE}{NI}$	=0.667	0.625	0.58823	0.58823

Since ROE & Reinv rate are decreasing over years, I will select latest year ROE & Rein. Rate to use to calculate g.

➔ $g = 0.14167 \times 0.58823 = 0.0833 = 8.33\%$

$$V_0 = \frac{5}{(1.12)} + \frac{6}{(1.12)^2} + \frac{7}{(1.12)^3} + \frac{1}{(1.12)^3} \times \frac{[7(1.0833)]}{.12-.0833}$$

$$= 14.2299 + \frac{1}{(1.12)^3}\,TV$$

$$= 14.2299 + 147.07 = 161.3$$

b) Because policyholder liability is considered debt, & the FCFF method must incorporate debt into formula. However, incorporating policyholder liability both explicitly & in the WACC is not easy task, and therefore prefer FCFE method.

Solution 3

a) (1) Determine K with CAPM

$K = 0.02 + 1.25 \times (0.10 - 0.02) = 0.12$

(2)

Year	2013	2014	2015
Net Income	15	16	17
Req. Cap.	110	120	130
Increase in Req Cap.	10	10	10
FCFE	5	6	7
Reinvested Cap.	10	10	10

➔ FCFE = NI – Increase in Req. Capital

For g, we use $ROE_{2015} \times \text{Reinvestment Rate}_{2015}$

$$= \left(\frac{17}{120}\right) \times \left(\frac{10}{17}\right) = 0.083$$

(3) Terminal Value at 12/31/2015 $= \dfrac{7\times(1.083)}{0.12-0.083} = 204.89$

(4) $V = \dfrac{5}{1.12} + \dfrac{6}{1.12^2} + \dfrac{7}{1.12^3} + \dfrac{204.89}{1.12^3} = 160.07$

b) Since policyholder's liabilities account for most of an insurer's debt, it is hard to precisely define the WACC or the discount rate for the APV.

Solution 4

a) $k = r_f + \beta\ (E(r_m) - r_f)$
$= 2\% + 1.25 \times (10\% - 2\%) = 12\%$

	2013	2014	2015	
Δ Equity	110 – 100 = 10	120-110 = 10	10	
NI - Δ Equity	15 – 10 = 5	6	7	
				Selected
ROE = $\frac{NI}{Beg\ Cap}$	$\frac{15}{100}$ =.15	0.145	0.142	0.146
ρ	$\frac{10}{15}$	$\frac{10}{16}$	$\frac{10}{17}$	.627

$$\frac{5}{(1.12)} + \frac{6}{(1.12)^2} + \frac{7}{(1.12)^3} = 14.23$$

growth = (.146 × .627) = .0915

$$\frac{7\ (1.0915)}{1.12^3\ (.12 - .0915)} = 190.82$$

$$190.82 + 14.23 = 205.05$$

b) FCFE calculates the value of Firm then subtracts the value of debt to the equity value. However, the distinction between policyholder liabilities + debt is arbitrary + there is no economic rationale for treating them differently. The FCFE met alleviates this problem and calculates the equity directly

Solution 5

a) $k = .02 + 1.25\ (.1 - .02)$

$= .12$

$= NI - \Delta Cap$
$= 15 - (110 - 100)$

	Beginning Equity	FCFE	ROE	ΔFCFE
2013	100	5	.15	
2014	110	6	.145	20%
2015	120	7	.1416	16.667%
Selected			$\boxed{.14}$	$\boxed{.10}$

$= \frac{NI}{Beg\ eq.}$

Select a growth rate of 10%

This seems a little bit high even w/ the low selection

PV Time Horizon

$$= \frac{5}{1.12} + \frac{6}{1.12^2} + \frac{7}{1.12^3} = 14.23\ M$$

PV Extended Horizon = $\frac{7\ (1.10)}{.12 - .10} = \frac{FCFE\ (1+g)}{k-g\ (1.12)^3}$

$$= \frac{385}{1.12^3} = 274$$

$$\boxed{= 288.26\ M}$$

Examiner Comment

Part a) was generally answered well by candidates.

We accepted several valid methods for calculating the growth rate.

- g = Reinvestment Rate × ROE
- $g = FCFE_t / FCFE_{t-1} - 1$
- $g = Capital_t / Capital_{t-1} - 1 = \Delta Capital / Capital_{t-1}$

Full credit was not given if candidates merely assumed a growth rate without calculating and reviewing one of the above metrics.

Some candidates neglected to discount the terminal value, or made other calculation/arithmetic errors.

Some candidates included the beginning capital value in the calculation of the total value of the company. Full credit was not given in these instances.

Many candidates supplied partial answers to part b). Common responses that received partial credit include:

- Mention of taking into account policyholder liabilities for the FCFE method <u>without mentioning that the distinction between debt and policyholder liabilities is arbitrary.</u>
- Mention that the interest rate is hard to define, <u>without explanation (i.e., due to presence of policyholder liabilities)</u>

Candidates that supplied 2 incomplete answers for part b) only received partial credit.

Question 12 Sample Answer

Solution 1

a) Required Return = $k = r_f + \beta(E[r_m] - r_f) = 2.0\% + 1.50\ (6.0\%) = 11.0\%$
Abnormal Earnings $(AE)_{2013} = NI_{2013} - k \times BV_{1/1/2013} = 150.0 - 11.0\% \times 1{,}000 = 40.0$

$AE_{2014} = NI_{2014} - k \times BV_{1/1/2014} = 166.0 - 11.0\% \times 1{,}105 = 44.5$

$AE_{2015} = NI_{2015} - k \times BV_{1/1/2015} = 183.0 - 11.0\% \times 1{,}221 = 48.7$

Total Equity $Value_{1/1/2013} = BV_{1/1/2013} + \Sigma PV(AE's)$

Year	Abnormal Earnings: Forecast Horizon	Abnormal Earnings: Beyond Forecast Horizon		PV Factor = $1 / (1 + k)^{(Year - 2012)}$		PV of AE's
2013	40.0			90.1%	$= 1 / 1.11$	36.0
2014	44.5			81.2%	$= 1 / 1.11^2$	36.1
2015	48.7			73.1%	$= 1 / 1.11^3$	35.6
2016		39.0	$= 48.7 \times 4 / 5$	65.9%	$= 1 / 1.11^4$	25.7
2017		29.2	$= 48.7 \times 3 / 5$	59.3%	$= 1 / 1.11^5$	17.3
2018		19.5	$= 48.7 \times 2 / 5$	53.5%	$= 1 / 1.11^6$	10.4
2019		9.7	$= 48.7 \times 1 / 5$	48.2%	$= 1 / 1.11^7$	4.7
					$BV_{1/1/2013}$	1,000.0
					Total Value	**1,165.8**

Solution 2

a) Required Return = $k = r_f + \beta(E[r_m] - r_f) = 2.0\% + 1.50\ (6.0\%) = 11.0\%$

Abnormal Earnings $(AE)_{2013} = NI_{2013} - k \times BV_{1/1/2013} = 150.0 - 11.0\% \times 1{,}000 = 40.0$

$AE_{2014} = NI_{2014} - k \times BV_{1/1/2014} = 166.0 - 11.0\% \times 1{,}105 = 44.5$

$AE_{2015} = NI_{2015} - k \times BV_{1/1/2015} = 183.0 - 11.0\% \times 1{,}221 = 48.7$

Total Equity Value$_{1/1/2013}$ = BV$_{1/1/2013}$ + ΣPV(AE's)

Year	Abnormal Earnings: Forecast Horizon	Abnormal Earnings: Beyond Forecast Horizon		PV Factor = $1 / (1 + k)^{(Year - 2012)}$		PV of AE's
2013	40.0			90.1%	= 1 / 1.11	36.0
2014	44.5			81.2%	= 1 / 1.11^2	36.1
2015	48.7			73.1%	= 1 / 1.11^3	35.6
2016		40.6	= 48.7 × 5 / 6	65.9%	= 1 / 1.11^4	26.7
2017		32.5	= 48.7 × 4 / 6	59.3%	= 1 / 1.11^5	19.3
2018		24.3	= 48.7 × 3 / 6	53.5%	= 1 / 1.11^6	13.0
2019		16.2	= 48.7 × 2 / 6	48.2%	= 1 / 1.11^7	7.8
2020		8.1	= 48.7 × 1 / 6	43.4%	= 1 / 1.11^8	3.5
					BV$_{1/1/2013}$	1,000.0
					Total Value	**1,178.1**

Solution 1

b) Starting in 2013, this company entered into a new market other companies had not yet entered. The DDM method assumes dividend growth in perpetuity which may be unsustainable as competition enters the market. Whereas the AE method assumes abnormal earnings will decrease to zero over some forecasted time horizon which may be more realistic as competition enters the market.

Other acceptable answers may include but are not limited to:

The DDM method is highly leveraged to the terminal value and the assumptions that underlie the terminal value whereas the AE method is less leveraged to the terminal value.

The AE method focuses on the source of value creation – the company's ability to earn a return on equity in excess of the investors' required return. The DDM focuses only on the effect of this value creation – the company's ability to pay cash flows to its owners.

The actual ROE is considerably higher than the required return, k, and these types of earning are unsustainable as competition enters the market.

Examiner Comment

For part a), the question specified that the abnormal earning would linearly decrease to zero by 2020, i.e., abnormal earning would be zero in the year 2020. However, a lot of candidates interpreted this as abnormal earnings decreasing to zero linearly by the year 2021. Both solutions, if calculated correctly, were accepted.

A common mistake was that candidates incorrectly identified the "expected equity market risk premium" as the "expected market return" therefore incorrectly calculating the required return, k. Another common mistake is that candidates did not include the company's beginning book value in the calculated value of this company.

For part b), a mistake several candidates made was saying that the DDM would understate the value of the company. In this particular scenario, the DDM is actually higher than the AE valuation method. Additionally, some candidates lost at least partial credit for not fully assessing and explaining the reasoning behind their answer.

Question 13 Sample Answer

Solution 1

a) One possibility is an increase in significant catastrophe activity in its property book over the past few years (such as from Irene and Sandy) that is impacting their cat models. Another possibility is a significant change in fuel prices causing the supply chain of building materials to become more costly. This will impact replacement costs in an unpredictable fashion.

   This could be due to a recent catastrophe affecting the property business. This could cause evaluation of unpaid claims to be volatile, especially if the event happened near the balance sheet date. Also, the insurer could have purchased excess of loss reinsurance on the casualty lines, mitigating the concern on these lines; alternatively, they may have dropped their treaties on property and now retain all the losses.

b)

- Legal changes to the benefit levels;
- Inflation, due to the long-tailed nature of this line;
- Latent exposures such as asbestos.

- Legislative changes that could affect the long-tailed lines;
- Inflationary pressures causing upward development on claims;
- Late reported claims in the pipeline.

Examiner Comment

Most candidates did reasonably well on part a) and very well on part b). Most of the full credit responses for part a) focused on the catastrophe exposure on the property line.

Question 14 Sample Answer

Solution 1

a) Direct Default: Both bonds have non-zero possibility to go into default and not paying the coupon/principal

   Downgrade risk: role that bonds will downgrade and lose value in market and possibly increase the probability of default

   Concentration risk: Only invest in BBB rated bond and no higher grade bonds. Also all bonds seem to be of lower rating type (no mention of industry/geography. But may still have concentration risk)

b) Because they are from the same issuer I would assume the credit risk correlation is 1.00
   In this case total capital required is= Cap Req+ Cap Req=45+157=202

Solution 2

a) Default Risk: Risk that the bond issuer will default on the interest and/or principal payment of the bond

   Downgrade: Risk that the bond will get a rating downgrade which will lower the value of the bond

   Settlement Risk: risk related to the lag between the end data of the bond and the payment of capital

b) 45 +157=202= Total Capital Requirement. The two values are added because they are 100% correlated

Examiner Comment

a) The candidate was expected to identify and describe 3 credit risks.

Many candidates did not sufficiently describe the risk (Default risk is the risk of default) or described a different risk than identified. Some did not identify the risk. The question specifically asked for credit risk and financial risks were not allowed (interest rate risk, asset-liability mismatch, reinvestment risk, etc.)

Credit was given if the list included Key driver of credit risk (Credit quality, Maturity, Size of expected loss)

b) Bonds from the same issue should have an $\alpha = 0$ and the standard formula will simplify to the sum of the individual risks. And the answer should include 45+157=202

Some credit was given for the formula, but candidates must show the final answer and how it was derived.

Question 15 Sample Answer

Solution 1

(1) CAT models – Used to measure multiple CAT risk, as well as assist in purchase of reinsurance. Help quantify magnitude of extreme event under various scenarios
(2) Expert Opinion – Draw on knowledge of industry experts to gain insight as to company's exposure to extreme events and what they can do to migrate. This can be subject to problems of inherent use of judgment or be biased
(3) Economic Scenario Generators – Used to generate different scenarios of inflation rates and other economic factors that can quantify impact extreme event have

Solution 2

(1) Can explicitly use Cat Models to model the effect (loss distribution) of natural and man-made disaster on the insurance company's losses
(2) Can use economic scenario generates to simulate different economic scenarios (inflation, risk of credit default, equity /property risks) and how they effect solvency in different scenarios
(3) Can use Copulas to combine the loss distribution of multiple lines of business (auto, home) and specify how correlated are in different parts of the distribution (tail, for example)

Examiner Comment

Many candidates listed three of the six items from Brooks, et. al. Other answers were accepted as long as consideration was given to "fattening the tail" of the underlying severity distribution. Copulas were also accepted as long as they properly referenced the correlation of lines of business during extreme events.

Often candidates did not provide an explanation of why the tool was relevant to extreme events.

Question 16 Sample Answer

Solution 1

a) Any two of the following common responses. Others were accepted if they were explained well and made sense.
   i. It is a stable book, so you will have the same policyholders year after year, so the book should perform similarly over time.
   ii. It is a large book, so it has high credibility and you would not expect it to have large swings in loss ratio from year to year.
   iii. Personal auto is highly regulated, so it is difficult to get significant rate changes and so rate levels will not fluctuate wildly.
   iv. Low catastrophe exposure, which would lead to more consistent results and positively correlated loss ratios.
   v. The underwriting cycle causes rate levels to be low for a period of time, and then high for a period of time. If the years are all in the same period of the cycle, the rate level would be consistent and this would make it more likely that the loss ratios would be positively correlated across years.

b) To address this assumption:
   i. Input for expected rating environment
   ii. Volatility around the mean selected
   iii. Volatility should increase over new projection period
   iv. Appropriate correlation between premium rating movements for different projection years and between different classes

Examiner Comment

We accepted other answers for partial or full credit if they were well explained and made sense. We received a wide variety of answers on this question, as it was very open ended. A common way to lose points was to not give enough explanation, such as simply stating that you would build correlation into the model, but not explaining how. This was seen as restating the question, and no credit was given unless there was more detail.

Question 17 Sample Answer

Solution 1

a) Pearson $\rho = \dfrac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum (X_i - \overline{X})^2}\sqrt{\sum (Y_i - \overline{Y})^2}}$

$= \dfrac{3298500}{\sqrt{20065000 \times 564200}}$

= 0.98035

	PA		HOME	$X_i - \overline{X}$	$Y_i - \overline{Y}$
	900		340	-1100	-60
	1150		200	-850	-200
	6000		1060	4000	660
	850		150	-1150	-250
	1100		250	-900	-150
$\overline{X}$	2000	$\overline{Y}$	400		

$\sum (X_i - \overline{X})(Y_i - \overline{Y})$ = 3298500

$\sum (X_i - \overline{X})^2$ = 20065000

$\sum (Y_i - \overline{Y})^2$ = 564200

Solution 2

a) $\rho = \dfrac{E(XY) - E(X)E(Y)}{\sigma_X \sigma_Y}$

$= \dfrac{1459700 - 2000x400}{(2003.25)(335.92)}$

= 0.98

E(X) = 2000

E(Y) = 400

$E(XY) = 1{,}459{,}700$

$\sigma_X = 2003.25$

$\sigma_Y = 335.92$

Solution 1

b) $\rho = 1 - \frac{6 \times \sum d_j^2}{N(N^2 - 1)}$ = 1 - $\frac{8}{5(5^2-1)/6}$ = 0.60

X	Y	Rank X	Rank Y	Diff^2
900	340	2	4	4
1150	200	4	2	4
6000	1060	5	5	0
850	150	1	1	0
1100	250	3	3	0
				8

Solution 1

c) Pearson’s correlation is calculated based on values, while Spearman’s correlation is based on ranks. Thus, Pearson’s correlation can be impacted by large outliner amounts. This impact can be seen here, where the 2010 year is largely influencing the correlation factor in the Pearson calculation. The use of ranks in Spearman’s calculation is not impacted by this, so while it shows some positive correlation with 0.6, it is not nearly as high as the Pearson result.

Solution 2

c) Pearson is a cardinal measure & is driven by the magnitude of the values. It will be highly leveraged by points far away from the mean. This occurs in 2010 which both Auto & HO have losses above the mean. This point drives the high correlation factor of 0.98. Spearman is an ordinal measure, weighted equally among all 5 years.

Examiner Comment

Part a), this is an application of the formulas for Pearson's correlation. The syllabus material provides two approaches to calculate the correlation using the provided data. Common errors include arithmetic errors and mixing two formulas together.

Part b), this is an application of the formulas for Spearman's rank correlation. The syllabus material provides a formula and approach for estimating the correlation using the provided data. Common errors include arithmetic errors and memorizing the incorrect formula.

Part c), common errors include only give generic statements of difference without tying back to the data given in the question.

Question 18 Sample Answer

Solution 1

a) Usually, the correlation between loss and ALAE is not the same for all claims amounts. For large claims, ALAE are usually large too because the insurer will try to pay less, which means higher attorney fees, higher expenses to investigate the claims, etc. For small claims, the correlation is harder to establish because the insurer will investigate all claims that could be frauds, and usually not spend much time and money on the other small claims. So, for small claims, you could have a great range of ALAE, from almost zero to many thousands.

b) One method to do so is to use a copula. A copula describes the correlation between two sets of values, loss and ALAE for this example. For example, if the copula selected is the Heavy Right Tail, it will mean that for high losses the two distributions are correlated heavily. There are a lot of copula types, and the actuary has to test the data to find which one best describes the correlation.

Solution 2

a) Loss & ALAE amounts can be correlated together especially when looking at the magnitude of claims. Large claims will likely require more ALAE as insurers are likely to fight these claims in court to drive down the overall size of the payment or not to pay at all. Small claims will likely incur little to no ALAE because an insurer will be more likely to pay the claim, but could also have claims where a lot of ALAE is spent and no loss has to be paid. Because of this, there will be less correlation between the two payments for small claims.

b) Based on the nature of the relationship between loss & ALAE, I would include a copula in my model that reflects the high correlation in the tails. (Remember, large losses will have large ALAE.) I would further suggest using a Gumbel copula or Heavy Right Tail copula because both will have high correlation in tail.

Examiner Comment

a) Most candidates were able to identify the positive correlation between loss & ALAE. Not so many were able to discuss the change in the correlation through the two distributions. Some candidates mentioned that you could have high ALAE on a zero loss through a successful claim defense, but were unable to tie that to a more general statement about the nature of the correlation. Many other candidates misunderstood the question to be about reinsurance treaty terms, discussing treating ALAE as part of loss or pro-rata with loss, and these responses earned no credit.

b) Most candidates correctly identified the need for a copula, but many of these candidates failed to explain the features of a copula that make it suited for the purpose in question.

Question 19 Sample Answer

Solution 1

a) $TVaR_{0.95}$ = conditional mean of losses greater than the 95th percentile
$TVaR_{0.95} = (0.03 \times 200 + 0.02 \times 400) / (0.03 + 0.02) =$ **280M**

Wang transform

(1)	(2)	(3)	(4)	(5) = (3) - (4)	(6) = Φ[(5)]	
Loss	F(x)	$\Phi^{-1}[F(x)]$	$\Phi^{-1}(\alpha)$		F*(x)	f*(x)
0	0.91	1.341	1.645	-0.304	0.38	0.38
200	0.98	2.054	1.645	0.409	0.66	0.28
400	1.00				1.00	0.34

WT(0.95) = $(0 \times 0.38) + (200 \times 0.28) + (400 \times 0.34) =$ **192M**

$EPD_\alpha = (1-\alpha) \times (TVaR_\alpha - VaR_\alpha)$
$= (1-.95) \times (280 - 200)$
= **4M**

Alternate solution: Some used .05 for α, resulting in **76M.** This was accepted as full credit.

b) $TVaR_{0.95}$ only includes tail risk and therefore does not meet the criteria to charge for any risk taken.

c) The Wang transform uses the entire loss distribution to calculate the risk measure and therefore does meet the criteria to charge for all risks taken (not just tail risk).

d) No. By taking the average of losses above the threshold, TVaR is linear in the tail and fails to demonstrate that the company is more risk averse as the loss size increases.

Alternate solution: Yes. Since it uses tail losses, which would reflect the largest losses of the distribution, it does reflect an increasing risk aversion to larger losses.

e) Yes. The Wang transform modifies the probability distribution to put more weight on large losses. That is, f*(x) > f(x) for large losses. This reflects an aversion to larger loss sizes.

f) Assuming independence,

EQ+CP	f(EQ+CP)
0	0.728
200	0.056
250	0.182
400	0.016
450	0.014
650	0.004

$TVaR_{0.95}$ (EQ+CP) = (0.016×250) + (0.016×400) + (0.014×450) + (0.004×650)

= **386M**

$TVaR_{0.95}$ (EQ+CP) < $TVaR_{0.95}$ (EQ) + $TVaR_{0.95}$ (CP)

386M < 280M + 250M = 530M

$TVaR_{0.95}$ is sub-additive and reflects the diversification benefits.

Wang transform

WT(EQ) = 192M

WT(CP) = 197.5M

WT(EQ+CP) = **323M**

WT(EQ+CP) < WT(EQ) + WT(CP)

323M < 192M + 197.5M = 389.5M

Wang transform is sub-additive and reflects the diversification benefits.

For calculation of the Wang Transform, the value was given in the problem. Full credit was awarded to candidates who calculated the number, or for those who pointed out that it was given as 323M.

Examiner Comment

Candidates tended to do well on the TVaR portion of parts a) and f), but not as well on the Wang Transform. They performed best on parts b)-e), even for the parts that focus on the Wang Transform. A common place to lose points was to answer parts b) and c) with a solution that was more applicable to the question asked in parts d) and e).

Question 20 Sample Answer

Solution 1

a) Without Reinsurance:

$$D = \frac{1-d}{1+r} = \frac{1-0.02}{1+0.015} = 0.9655$$
$$Value = Earnings \times \frac{D}{1-D}$$
UW Earnings = 250M – (0.6+0.32) × 250M = 20M
Investment = (1B +250M – 0.32 × 250M) × 0.035 = 40.95M
Value = (20M +40.95M) × $\frac{0.966}{1-0.966}$ = 1705.7M

With Reinsurance:
$$D = \frac{1-d}{1+r} = \frac{1-0.004}{1+0.015} = 0.9813$$

UW Earnings = 250M×0.8 – 250M×0.8×0.6 – 250M×0.32 + 250M×0.1×0.2=5M
Investment = (1B +250M × 0.8 – 0.32 × 250M + 250 × 0.1 × 0.2) × 0.035 = 39.375M
Value = (5M+39.375M)× $\frac{00.9813}{1-0.9813}$ = 2328.6M

The value increases by 2328.6M–1705.7M = 622.9M

b)
(1) Efficient Frontier Graph – which plot U/W profit against different risk measure such as, VaR, or TvaR. The reinsurance option in the upper left corner is preferred since it has higher U/W profit with low risk
(2) Cost Allocation Method – holding capital is not free, which required a return of capital. We compare the total cost of holding capital and reinsurance net cost, to see which option has lower cost

Solution 2

a) $E \times M = E \times \frac{D}{1-D}$ $where\ D = \frac{1-d}{1+r}$

Without Reinsurance:

E=UW income + Investment Income

E=250 – 0.92×250 + (1000+250×(1–0.32))×0.035 = 20 + 1170 × 0.035 = 60.95

D = $\frac{1-2\%}{1+1.5\%} = 0.965517$

Risk adjusted PV (Earnings) = 60.95 × $\frac{0.965517}{1-0.965517}$=1706.6

With Reinsurance:

UW income = 0.8×250-0.8×0.6×250-0.32×250+-.1×0.2×250 = 5

Investment Income = (1000 + 0.8×250-0.32×250+0.1×0.2×250)×3.5% = 1125×0.035=39.375

D = $\frac{1-0.4\%}{1+1.5\%} = 0.98128$

Risk adjusted PV (Earnings) = (5+39.375)× $\frac{0.98128}{1-0.98128} = 2326.18421$

Difference: Impact of RI – 2326.18421-1706.6=619.58421 M

b)

(1) Simple Factor: Value = prob of distress × Equity
Difference with or without reinsurance quantifies the risk transfer
(2) Efficient Frontier Graph: the expected U/W profit against a risk measure. More U/W profit expected should be associated with a higher risk measures. The closest to the efficient frontier of the program is the more efficient we are

Examiner Comment

a) Many candidates did not properly calculate underwriting income and/or investment income for either the gross case or the reinsured case. Most successful candidates properly calculated the firm value using the Panning model of V = E × M where M = D/(1-D).

Often candidates didn't acknowledge expenses are paid at front, and claims are paid at year end.

Candidates rarely provided written explanation of what their calculations represented. Often, candidates just provided large quantities of numbers and left it to the graders to figure out what they were trying to accomplish.

To make sure a question with many calculations being answered more properly and cleanly, it might be better to split it into small pieces.

b) Candidates often did not provide an adequate description of their alternative. At other times, candidates correctly stated an alternative but incorrectly described the alternative.

Question 21 Sample Answer

Solution 1

a) The basic indicator applies a single factor to the entire business without giving consideration to the mix of business. Specially, apply a factor of $\alpha = .15$ to the entire entity (3 year average gross income). The standardized approach, on the other hand, applies a different factor, β, to different lines of business, specifically for retail banking, β=.12, for commercial banking, β=.15, and for everything else, β=.18. So the standardized approach does account for the firm's mix of business by applying different factors to different business segments while the Basic Indicator does not.

b) The standardized approach effectively gives credit for years with negative (or $0) gross income, while the basic indicator does not. For the basic indicator, multiply α times 3 years gross income average, but years with negative or $0 gross income are excluded from average (so if one year is negative, will only average over two other years). But for the standardized approach, add up all years (use $0 for negative years), and divide by three (even if $0 is used for some years). This can lower the capital requirement if the firm had a year with negative gross income.

Solution 2

a) Basic Approach = [1 / Z_n × ∑ from i=1 to i=3 of max (Gross Income, 0)] × α

Where Z_n = # years where gross income > 0 for the last 3 years. The mix of business doesn't have an effect on the basic approach as the factor α is constant (15%) for all lines.

Standardized approach = 1/3 × ∑ max (∑ $\beta_j GI_j$, 0). Here the mix of business affects the capital as each line of business has a specific β (usually in the range of 12-18%) that is applied to the Gross Income for that line (e.g., Retail Banking: β = 12%, Commercial Banking: β = 15%, Other: β = 18%). The capital requirement therefore depends on the mix.

b) The standardized approach allows negative gross income in one area to be offset by positive gross income in another line by year. Also, the standardized approach always averages the last 3 years, but the Basic only divides by # of years where Gross Income > 0. So, this may result in a lower capital need using the Standardized Approach.

Examiner Comments

Part a) of the problem required understanding of a key difference in how capital requirements are done under two methods described in the syllabus reading and how mix of business affected the capital calculations. Candidates were allowed to demonstrate that understanding either through formula presentation or narrative descriptions, although often both forms of response were entered in the candidate solutions. In those cases where candidates entered a solution but did not receive full credit, it was generally because the response was vague or incomplete in some respect, the candidate failed to indicate that capital charges varied by type of business in the standardized approach, or else the candidate did not explicitly answer the question as to whether mix of business impacted the capital calculation under each method.

Part b) of the problem could be answered in several ways, any one of which could receive full credit. Common reasons for not assigning full credit to a response in this part included insufficient explanation as to why positive and negative income offsets (or "netting") across lines of business within a given year could generate different indicated capital amounts under the standardized method, or failure to describe what circumstances were necessary to provide a benefit under standardized approach's unique capital formula (e.g., an overall loss in one year or a higher capital charge on a line of business with a loss).

Question 22 Sample Answer

Solution 1

a) Company 1: Investment income will likely be steady, albeit not very large for any given year; will need to rely heavily on u/w profit, so must price risks accordingly.
Company 2: More aggressive investment strategy which will lead to volatile investment income; when pricing, should take a long-term view when determining if rates are adequate.

For Company 1, government securities typically have high rating but lower yield, may not generate a lot of investment income. Company 1 needs to price high enough to compensate for lower investment yield.
For Company 2, the securities held can have higher yield but very volatile. The value could drop significantly; the company needs to price to include a risk margin for that.

b) Company 1: Because auto is relatively short tailed, there is less variability in claims unless changes in claim handling caused changes in reserve adequacy or speed of payment which would cause basic methods to reserve incorrectly.
Company 2: There is much more risk of claim variability in long tailed lines that are difficult to estimate, especially with inflation, economic, legal and other environmental changes.

Company 1 could face claim variability from a number of sources including catastrophes, especially winter storms that drive up PPA claims. Company 1 could also see claim variability come from new laws involving claims and social inflation which could cause insureds to increase frequency of claim filings.
Company 2 could observe claim variability from natural catastrophes such as hurricanes and earthquakes in the lines they insure. There is also attritional risk for both companies. Attritional risk is the risk of increased frequency of small claims (low severity) claims. Both companies also face risk that large claims could be more severe than expected.

c) Company 1: Market risks that this company needs to consider include interest rate risk and concentration risk.
Company 2: This company should consider asset liability mismatch risk and risk associated with return on equity or reinvestment risk. There is more variability in their investments and values depend heavily on the market.

Company 1 won't be too much exposed to market risk since it is invested in secure government securities. The duration matched approach also limits the amount of liquidity risk company 1 would be exposed to.
Company 2 is heavily exposed to default risk on their investments. The liquidity risk is also present if assets are not matched with liabilities. Although it can be rewarding as far

as returns, if bonds and stocks don't perform well our asset base will deplete with liabilities still coming in.

d) Company 1: Generally PPA claims are independent. There is some regional correlation to weather and countrywide correlation to medical costs.
Correlation could be significant:
* Equity portfolio & CAT property during event; terrorism
* Liability lines; inflation, latent claims, and regulatory changes
* Limited historical data to model means use expert judgment.

Company 1: Should be concerned about correlation of Loss & ALAE, especially in the tail. Also, auto is subject to catastrophes, which could impact policyholders in many states at once.
Company 2: Should be concerned with the correlation among all lines of business. They can be correlated in the tail. Inflation will cause liabilities to increase and the market value of fixed income securities to decrease, so that is a concern. Deterioration of economy could harm assets and liabilities.

Examiner Comment

This is an ERM question and many candidates failed to identify the nature of the categories. Also a number of candidates did not answer or address the issue stated in the question. For example, for pricing risk, some candidates gave a definition of what pricing risk is rather than the specific issues faced in each organization mentioned in the question. Alternatively, several candidates discussed correlation matters under the pricing risk part of the question.