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Applying Stochastic Loss Reserving to quantify variability in IBNR
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1. Abstract

This report provides intuitive and in-depth description of actuarial stochastic reserving methods in insurance. Firstly, the context around reserving is established by reviewing underwriting, pricing, data and inherent uncertainty. Then various methods utilized in General Insurance for stochastic reserving for claim reserves is elaborated. The aim is to drill down into the dynamics and to comment on interpretation of the models and their results. The R ‘ChainLadder’ package has been used in arriving at these results.
2. Introduction

Situating the Context

‘In reserving, are we swapping specific risk for systematic risk? ’¹

This question was asked previously in the previous CAS working paper “Developments on the Reserving Uncertainty Frontier” (Syed Danish Ali; Posted 28 June 2016). This question is highlighted again so as to progress from point estimates in IBNR to quantifying the variability within these point estimates. This brings the uncertainty onto the surface instead of hiding it beneath deterministic ‘rugs’. In the previous paper, survey was undertaken to describe various methods for quantifying variability within loss reserves; here we will work in detail upon one of these methods, i.e., stochastic reserving.

The key questions postulates the hypothesis that in normal market conditions, reserving results are kept at consistent levels and volatility of their results is reduced. The traditional approach requires precise figures (point estimates) and so leads to understatement of uncertainty.

This keeps a comfort level for us but the hidden risk of uncertainty in our reserve estimates is hardly given the attention it merits. The uncertainty crops up from the rug it was shrugged under in stressed market conditions and reserves that are then systematically proven to be insufficient.

In other words, are we causing the fat tail problem² by our practices? What can be done to reduce the fatness of such tails and bring the hidden uncertainties onto the surface explicitly?

A fat tail exhibits large skew and kurtosis and so there is a higher probability for large losses compared to other distributions like normal distributions. This higher loss tendency remains hidden under normal market conditions only to resurface in times of higher volatility.

Some material on stochastic reserving is taken from the previous working paper for the sake of completeness and so that reader is able to get a brief overview of stochastic reserving before jumping into R software and its results.

Stochastic Reserving

Loss reserving covers Incurred but Not Reported (IBNR) claims. These reserves are usually estimated based on historical claims paid based on accident/reporting patterns. In the past a point estimator for the reserves was sufficient. New regulatory requirements (such as Solvency II and other risk management regimes) foster stochastic methods for loss reserving. Stochastic Reserving has been applied on hypothetical data which is realistic using statistical software R.

A standard approach in quantifying uncertainty revolving around the reserving point estimate is stochastic reserving. Stochastic methods for reserving are used in capital modeling exercises but deterministic methods like BF and Chain Ladder dominate the reserving landscape.³ The bootstrap method breaks development factors into two important areas; random noise and the underlying historical pattern. While underlying historical pattern is constant, random noise is shuffled across the triangles for a number of simulations to create probability distribution of IBNR results. In this process, random noise is assumed to lie uniformly at every point in the triangle.

Mack Method facilities Claims data in telling the story as it is distribution-free for claim amounts. Instead of tagging any distribution to the claim amounts, normal distribution is applied onto the mean and lognormal onto the standard deviation of the claims so as to generate a full distribution of ultimate claims.

¹ Idea adapted from 'The Economist; In Plato’s Cave; January 2009'.
² http://lexicon.ft.com/Term?term=fat-tails
³ Actuarial Post: Making uncertainty explicit-stochastic modeling
Stochastic methods are important in preventing and exposing fat tails because randomness and simulations allow us a greater access than what we do from learning from history and extrapolating it to the future.

Instead of placing all of our hedges on history being a reliable indicator of the future, stochastic measures uncover greater possibilities than history has shown us to shed greater light on fat tails and how they can manifest.

It is important to understand Coefficient of variation (CoV) when applying stochastic reserving too. CoV is quite simply the ratio of the standard deviation $\sigma$ to the mean $\mu$.

In actuarial terms it implies standard deviation of the estimated reserve as a percentage of estimated reserve which is the mean reserve calculated by the actuary.\(^4\) Panning takes CoV as a yardstick for measuring uncertainty in reserves. Linear regression is applied onto a dependent variable $Y$ (such as paid losses) with other independent variable(s) $X$. Panning charters a way out of key limitations of linear regression like homoskedasticity, correlated disturbances, bias and so on through a number of important modifications in the application of the linear regression.

The linear regression is then fitted and forecasted paid losses are generated. Standard deviation of these forecasts is calculated and both are put together in the CoV ratio.

Model diagnostics such as analysis of standardized residuals is followed at the same time to ensure accuracy of applying the CoV method. Apart from being an accurate measure of loss reserve uncertainty it is also simple to understand and implement using spreadsheets which actuaries use very often. Most actuaries work in companies 81% and struggle to bring new analytics because of fractured IT systems and risk averse culture especially in emerging developing markets. CoV can be used for its ease and simplicity when it is not possible due to organizational hurdles to implement more ambitious analytics.

Moreover, the CoV is comparable across different lines of business as well. It is recommended that CoV be utilized as a first and credible step towards systematically recognizing uncertainty in loss reserving by actuaries.

\(^4\) William Panning: Measuring loss reserve uncertainty
3. Underwriting and Pricing Adequacy

It is clear that underwriting, pricing and reserving are all interlinked and share the same fortunes. Moreover, the business objectives & planning, insurance cycles and risk management situates these three interlinked functions in their proper context.

Regarding underwriting, there are usually tiers like preferred risk, normal risk, adverse risk and declined risks. The business strategy defined how much proportion of business should be targeted for each of these tiers. The business strategy might be that it is a new company and so has plenty of capital available; it will aim to establish market share and so will usually try to except adverse risks and quote at lower premiums than other competitors. This is directly linked to claims and reserves; for instance, if we are underwriting undue proportion of old age members for health insurance than it will be no surprise to see high loss ratios.

The insurance cycle also plays its role; in soft markets there will be pressure to sustain market share and profitability will be reduced. In hard markets, the insurer can be more ambitious in terms of new products, focusing on profitability and so on. Lastly, a firm on the brink of non-compliance with minimum solvency standards set by the regulator cannot afford to be more ambitious than a company which has ample unappropriated profits and capital.

Likewise, reserving for Premium Deficiency Reserve (PDR) and pricing are interlinked. The pricing exercise will be given a clear signal to increase prices if the reserving shows that PDR is required because premiums are not adequate enough to cover the risks undertaken. Similarly, if we compare actual claim experience to expected reserves held to cover these, and there arises a negative shortfall, than pricing needs to see whether the premium assumptions on loss ratios, expenses etc. were appropriate or not.

The underwriting results are obviously a function of reserves (and especially claims reserves). These should be separately reviewed in actuarial reserving report and satisfaction obtained that the reserves set out are adequate.
Having low prices for capturing market share alone is not an appropriate justification because it underprices the risk that is actually being underwritten. The pricing report can determine a set of net rates (i.e., net of loadings for commissions, expenses and profits) are low relative to the rates observed in the market and still be justified. This can be due to a pricing strategy of risk avoidance, where the underwriters utilize policy terms which included sizeable deductibles, increasing deductibles in cases of risk which experienced multiple claims within a calendar year, and other surcharges to discourage risks that are not attractive in the eyes of the underwriter.

In aggregate, current net book rates for normal group business can appear adequate when compared to the underlying claims experience but significant cross-subsidies can exist between the book rates of the various products within the current rating structure.

With this holistic framework, a company has to decide on the appropriate groupings to undertake; for instance, long tailed and short tailed, commercial general insurance and personal general insurance (mostly motor and medical) and so on.
4. Reserving Methodology for Property & Casualty Insurance

Overview

This valuation process can be succinctly shown as follows:

The range of reserves calculated for Short Term General Insurance business can be illustrated below:

In judging adequacy of reserves, risk management focuses on procedural efficiency and distributive efficiency. Procedural efficiency is analysis of processes and methodology adopted in reaching amounts for
particular reserves. Distributive efficiency is concerned with the result as to how prudent are the reserve figures arrived at.

In terms of procedural efficiency, it is inefficient if there is no proper or improper reserving methodology for calculation of IBNR reserves for general insurance. On the front of distributive efficiency, if the reserves appear inadequate through run-off analysis of comparing reserves held (expectations) to actual claims experience, reserves are inefficient on this front as well. An insurer should strive to improve on both fronts simultaneously.

Differences between our projections and actual amounts are contingent to the extent to which future experience conforms to the assumptions made for this analysis. It is quite certain that actual experience will not conform precisely to the assumptions used in this analysis. Actual amounts in all reasonability will deviate from projected amounts to the extent that actual experience differs from expected experience.

Variability is relatively higher for the more recent accident years where relatively few payments have been made and where sufficient information might not be available to establish accurate development of reserves.

Key variables that can lead to different experience than forecasted include the selections of loss development factors, expected loss ratios, in addition to trend and tail factors. The overall estimates are potentially sensitive to any of these and reasonable alternative selections can change the estimates in either direction.

Estimation of claims liabilities involves assessment of the following elements;
   a) Claims reported but unpaid (Outstanding)
   b) Claims incurred but not enough reported (IBNER), i.e. possible increase in the eventual cost of claims which are reported.
   c) Claims incurred but not report (Pure IBNR)

Since claim payments also includes payments for claim adjustment expenses, legal fees and other claims related expense, each of the related expense requires proper analysis, along with the actual claim payment cost.
The main deterministic methods described to determine and evaluate IBNR reserves are:
1. Chain Ladder Method
2. Bornhuetter-Ferguson (BF) method
3. Cape Cod method
4. Hybrid method

**Chain Ladder Method**

The Chain Ladder method tracks the development of paid claims (usually on an accident year basis) until all claims related to the accident year is paid off. This can be adjusted so that shorter periods of reference are used (e.g. a quarter or a month). However, this assumes availability of adequate data so that the patterns are statistically credible.

In order to get a clearer idea of the volatility dynamics of the development at a detailed level, we can adopt quarterly or monthly triangles. Sometimes this is not possible due to lack of data or because the paid claims development factors are not overall credible or reliable due to lack of sufficient experience as well as lack of sufficient development of claims.

Incurred is usually more stable than paid claim triangle as it takes change in outstanding as well. However, paid claims triangle based on accident date to paid date estimates the IBNER instead of only the Pure IBNR. If paid claims is leading to significantly distorted IBNR as compared to Incurred, a hybrid solution can be adopted. IBNER as a proportion of claim reserves can be arrived at using paid claim and this percentage can then be applied as loading on the incurred claims based on accident date to reporting date.

The following diagram elaborates on the methodology behind triangulation:
This claim development triangle can be visualized as follows:

![Claims Development Chart; Origin to development periods](image)

Regarding the Development Pattern of Claims-to-Ultimate, the following (illustrative only) table shows paid claims of each year relative to its ultimate claims paid. This ratio can be used to analyze the development period of claims paid. As can be seen, hypothetical claims paid develop almost completely over the 3 years. This is shown in table and visualized as follows:

<table>
<thead>
<tr>
<th>Paid Claims Development to Ultimate</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the end of accident year</td>
<td>37%</td>
<td>57%</td>
<td>64%</td>
<td>67%</td>
<td>65%</td>
<td>62%</td>
<td>61%</td>
</tr>
<tr>
<td>One year later</td>
<td>79%</td>
<td>92%</td>
<td>94%</td>
<td>95%</td>
<td>89%</td>
<td>91%</td>
<td></td>
</tr>
<tr>
<td>Two years later</td>
<td>96%</td>
<td>96%</td>
<td>98%</td>
<td>98%</td>
<td>97%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three years later</td>
<td>100%</td>
<td>100%</td>
<td>99%</td>
<td>99%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four years later</td>
<td>100%</td>
<td>100%</td>
<td>99%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five years later</td>
<td>100%</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Six years later</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Bornhuetter-Ferguson (BF) method**

The basic Chain Ladder method tends to rely on the assumption that there is a statistically reliable body of claims data. Even if the payment development pattern is slow and lumpy, a reasonable estimate of the liabilities can be derived using the chain ladder method applied to the incurred claim development, provided it is reasonably stable.

The BF method is a technique that combines actual past claims experience and any prior information or expectations that might be available concerning claims, for example expected ultimate loss ratios. The method progressively blends actual incurred claims development with an IBNR (including IBNER) based on the expected development.

We have to calculate earned premiums for the purposes of reserving (premiums are a proxy for the level of exposure a company is subject to and useful in Bornhuetter-Ferguson (BF) reserving) and for considering unexpired risk reserves as well as changes in writing and earnings patterns.

The Bornhuetter-Ferguson method is most useful as an alternative to other models for immature accident years. For these immature years, the amounts reported or paid may be small and unstable and therefore not predictive of future development. Therefore, future development is assumed to follow an expected pattern that is supported by more stable historical data or by emerging trends. This method is also useful when changing reporting patterns or payment patterns distort the historical development of losses.

**Cape Cod Method**

The Cape Cod method may be thought of as the Bornhuetter Ferguson with the initial expected (loss) ratios derived from smoothed development factor based ultimates. At one extreme, all the initial expected ratios may be set to the same value derived from an overall weighted average ratio. This is the simple Cape Cod method. At the other extreme, if no smoothing is applied, the initial expected ratios are the development factor based ultimates and the reserves are identical to those derived from the development factor method. The Generalised Cape Cod method allows for initial expected ratios to lie anywhere between these two extremes.

The amount of smoothing is determined by the applied "decay factor". A factor of 0.0 means no smoothing is applied (development factor results) and a factor of 1.0 gives a constant initial ratio (Simple Cape Cod). A factor between these two extremes is used in the generalised method.

**Hybrid’ Methods**

Results from the runoff analysis should be taken into consideration while setting reserves for IBNR claims. Actuarial techniques using Chain-Ladder (CL) method and Bornheutter- Ferguson (BF) method should be used to analyze claims. An alternative approach to determine the total IBNR is by dividing it into ‘normal’ and ‘resubmission’ IBNR separately.

It is also very much possible that the IBNR claim provision be based on the Chain Ladder method (which uses the pattern of claim payments to project the ultimate liability for each exposure period) but Run off analyses to reveal later that the reserves determined using chain ladder method were not sufficient.

This can be due to many reasons all indicating higher volatility and uncertainty than expected. For e.g., surge in resubmission of initially rejected claims as a result of which payment patterns of recent time periods became distorted. ‘hybrid” method should be used in such cases where the development of loss ratios for
each exposure quarter be closely monitored and the IBNR claim reserve determined by using target loss ratios (using BF and/or Cape Cod factors) for recent time periods and the Chain Ladder Method for earlier periods.

It is important to take time series context of structural fluctuations. For e.g., highest motor claims tend to be on weekends. A lower medical loss ratio for the 3rd quarter has been a consistent feature in the last few years largely as a result of seasonality with lower utilization during summer months and on long holidays. Heat flashes or sudden storms can also lead to spikes in the claims data. Time series decomposition in software R can be used to show such trends easily.

It is important also whether the triangles be ‘monthly’, ‘quarterly’ or ‘yearly’. Short tailed lines with massive amounts of data like motor can lend itself to monthly triangles. Quarterly triangles should be followed instead of yearly triangles when we have adequate data to capture volatility dynamics better than on yearly triangles. However, commercial general insurance can have inadequate data because of low-frequency and high-severity dynamics and because these high severities take a lot of time to develop and/or are long tailed.

When setting loss ratios for BF or Cape Cod assumptions it is important to calculate ratios on both underwriting and accident year. Any significant divergence between these two can mean a material change in policy and claim nature over the time period considered.
5. Stochastic Loss Reserving

The following delivers reasons for why we should adopt stochastic reserving:

• Computer power and statistical methodology make it possible
• Provides measures of variability as well as location (changes emphasis on best estimate)
• Can provide a predictive distribution
• Allows diagnostic checks (residual plots etc.)
• Useful in Dynamic Financial Analysis (DFA) and regulatory regimes like Solvency 2.
• Useful in satisfying Own Risk Solvency Assessment (ORSA) under different risk management regimes

Stochastic reserving does not just give us the best estimate, it also provides us with the variability in claims reserve. The variability of a forecast is given as prediction error or standard error, where prediction error is

\[ \text{Prediction Error} = \left( \text{Process Variance} + \text{Estimation Variance} \right)^{1/2} \]

The following stochastic loss reserving methods are commonly utilized.5

• Mack Model
• Bootstrap Model
• Generalized Linear Model (GLM) utilizing Over Dispersed Poisson

Munich Chain Ladder deterministic model was also applied in the end.

Kindly note that Mack produces results that are very close to Chain Ladder and GLM using Over Dispersed Poisson distribution produces same results as Mack.

**Mack Model**

Thomas Mack published in 1993 a method which estimates the standard errors of the chain-ladder forecast without assuming any particular distribution.

The Mack-chain-ladder model gives an unbiased estimator for IBNR (Incurred but Not Reported) claims. The Mack-chain-ladder model can be regarded as a weighted log-linear regression through the origin for each development period. Prediction Error is the total variability in the projection of future losses by the Mack Method.

The residual plots show the standardized residuals against fitted values, origin period, calendar period and development period. All residual plots should show no pattern or direction other than broadly converging towards zero. Slight deviation is expected and is not a cause for concern. Variance in reality is also not constant as assumed here; so conditional heteroskedasticity will always be present in insurance data.

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5 Claims Reserving with R: Chain Ladder-0.2.4 package Vignette-Alessandro Carrato, Fabio Concina, Markus Gesmann, Dan Murphy, Mario Wuthrich and Wayne Zhang
The CAS is not responsible for statements or opinions expressed in this working paper. This paper has not been peer reviewed by any CAS Committee.
The advantage of stochastic modeling is that for almost all models, we get a distribution of IBNR instead of only a point estimate. This can be illustrated for instance, as follows:

<table>
<thead>
<tr>
<th>IBNR Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%: 190,000</td>
</tr>
<tr>
<td>2.5%: 200,000</td>
</tr>
<tr>
<td>10%: 225,000</td>
</tr>
<tr>
<td>20%: 240,000</td>
</tr>
<tr>
<td>30%: 248,000</td>
</tr>
<tr>
<td>40%: 255,000</td>
</tr>
<tr>
<td>50%: 265,000</td>
</tr>
<tr>
<td>60%: 269,000</td>
</tr>
</tbody>
</table>

What we usually mean by mean IBNR deterministic reserve is here as 50% percentile figure for IBNR. Risk Margin for adverse deviation can be captured as the difference between reserve held at 75% and that held at 50%. This gives an appropriate way to quantify the risk margin loading.

**Bootstrap Model**

The Bootstrap model uses a two-stage bootstrapping/simulation approach following the paper by England and Verrall [2002]. In the first stage an ordinary chain-ladder method is applied to the cumulative claims triangle. From this we calculate the scaled Pearson residuals which we bootstrap R times to forecast future incremental claims payments via the standard chain-ladder method. In the second stage, we simulate the process error with the bootstrap value as the mean and using the process distribution assumed. The set of reserves obtained in this way forms the predictive distribution, from which summary statistics such as mean, prediction error or quantiles can be derived.

Bootstrap uses distributions like Gamma Distribution in arriving at its estimates of IBNR reserves. The results of simulating bootstrap replicates to generate estimates for IBNR can be depicted as a histogram of frequency as below:
This shows the histogram of the 1000 simulations. Each simulation resulted in one point estimate figure for IBNR.

The mean ultimate claims that have been simulated can be made evident as follows:

![Simulated ultimate claims cost](image)

This illustration shows each ultimate claim estimate in the 1,000 simulation as a dot with black outline. When many dots get concentrated, the result is black concentration of dots over the simulations. First, the interquartile range (demarcating the upper quartile and the lower quartile) is set just like in a box and whisker plot. The mean from that interquartile range is then arrived at and is shown as a red dot here.

Another bootstrap run on 10 thousand simulations (instead of 1,000 simulations) on a different illustrative data shows the following plots:
Here, we can clearly see that majority of the reserve in the histogram comes between 300,000 and 400,000. The cumulative development factor is also shown. In the last two diagrams, we can see each black dots as point estimate of IBNR from 1 simulation. The black dots concentrate to form black think lines and then the box and whisker plot is implemented upon these dots. The interquartile range is arrived and then the red dot of mean ultimate claim and latest actual claim is shown. It is clearly shown that the latest origin period has far more uncertainty and variance then previous origin periods.
Generalized Linear Model

Although GLM is mainly used in pricing of general insurance premiums, GLMS have found their way into reserving as well by adopting Over-dispersed Poisson Distribution. The GLM takes an insurance loss triangle, converts it to incremental losses internally if necessary, and fits the resulting loss data with a Generalised Linear Model where the mean structure includes both the accident year and the development lag effects. The GLM also provides analytical methods to compute the associated prediction errors, based on which the uncertainty measures such as predictive intervals can be computed. This means that zero or negative increment losses will result in error and the model will not work. In “bootstrap beyond the basics”, Mark Shapland overcomes this difficulty in negative or zero increment losses. This should point towards improvement in the future when updating the R package “ChainLadder”.

Only the Tweedie family of distributions are allowed, that is, the exponential family that admits a power variance function \( V(u) = u^p \). For the purposes of stochastic loss reserving, the distribution of Over-Dispersed Poisson can be utilized, especially as this choice of distribution will lead to same mean figure for IBNR as provided in Mack. The P parameter can be put on manually, or it can be taken as Over-Dispersed Poisson, Compound Poisson, or Gamma.

The typical outputs from GLM reserving can be illustrated as follows:

<table>
<thead>
<tr>
<th>Latest</th>
<th>Dev.To.Date</th>
<th>Ultimate</th>
<th>IBNR</th>
<th>S.E</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>174,928</td>
<td>0.800</td>
<td>174,930</td>
<td>2</td>
<td>65</td>
</tr>
<tr>
<td>3</td>
<td>182,286</td>
<td>0.800</td>
<td>182,289</td>
<td>3</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>182,286</td>
<td>0.769</td>
<td>189,662</td>
<td>7,377</td>
<td>3,693</td>
</tr>
<tr>
<td>5</td>
<td>180,978</td>
<td>0.769</td>
<td>188,327</td>
<td>7,350</td>
<td>3,682</td>
</tr>
<tr>
<td>6</td>
<td>209,374</td>
<td>0.769</td>
<td>217,895</td>
<td>8,522</td>
<td>4,045</td>
</tr>
<tr>
<td>7</td>
<td>236,466</td>
<td>0.769</td>
<td>246,325</td>
<td>9,859</td>
<td>4,426</td>
</tr>
<tr>
<td>8</td>
<td>240,051</td>
<td>0.767</td>
<td>250,232</td>
<td>10,181</td>
<td>4,503</td>
</tr>
<tr>
<td>9</td>
<td>256,634</td>
<td>0.764</td>
<td>268,754</td>
<td>12,120</td>
<td>4,926</td>
</tr>
<tr>
<td>10</td>
<td>259,502</td>
<td>0.753</td>
<td>275,739</td>
<td>16,237</td>
<td>5,607</td>
</tr>
<tr>
<td>11</td>
<td>214,286</td>
<td>0.723</td>
<td>237,128</td>
<td>22,842</td>
<td>6,414</td>
</tr>
<tr>
<td>12</td>
<td>104,823</td>
<td>0.304</td>
<td>275,831</td>
<td>171,008</td>
<td>25,757</td>
</tr>
<tr>
<td>total</td>
<td>2,241,614</td>
<td>0.715</td>
<td>2,507,113</td>
<td>265,499</td>
<td>35,606</td>
</tr>
</tbody>
</table>
Call:
\[ \text{glm(formula = value} \sim \text{factor(origin)} + \text{factor(dev)}, \text{family = fam}) \]
\[ \text{data = ldaFit, offset = offset}) \]

Deviance Residuals:

<table>
<thead>
<tr>
<th>M</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.346</td>
<td>-12.774</td>
<td>-3.311</td>
<td>11.136</td>
<td>151.373</td>
</tr>
</tbody>
</table>

Coefficients:

| Effect                          | Estimate | Std. Error | t value | Pr(>|t|) |
|--------------------------------|----------|------------|---------|---------|
| (Intercept)                     | 10.329282| 0.091920   | 123.252 | < 2e-16 *** |
| factor(origin)2                 | -0.002513| 0.122328   | -0.012  | 0.990179 |
| factor(origin)3                 | 0.049697 | 0.121087   | 0.328   | 0.744280 |
| factor(origin)4                 | 0.089350 | 0.121497   | 0.653   | 0.516412 |
| factor(origin)5                 | 0.082283 | 0.121712   | 0.594   | 0.555021 |
| factor(origin)6                 | 0.228119 | 0.117611   | 1.855   | 0.069020 .|
| factor(origin)7                 | 0.350754 | 0.114528   | 3.122   | 0.002858 ** |
| factor(origin)8                 | 0.366492 | 0.114169   | 3.799   | 0.000365 *** |
| factor(origin)9                 | 0.487902 | 0.112632   | 4.034   | 0.00171 *** |
| factor(origin)10                | 0.463560 | 0.112429   | 4.001   | 0.00171 *** |
| factor(origin)11                | 0.312705 | 0.117183   | 2.583   | 0.012478 * |
| factor(origin)12                | 0.463892 | 0.144723   | 3.136   | 0.002747 ** |
| factor(dev)2                   | 0.340575 | 0.049716   | 6.448   | 3.01e-08 *** |
| factor(dev)3                   | -2.337448| 0.132755   | -17.457 | < 2e-16 *** |
| factor(dev)4                   | -3.356485| 0.227510   | -14.577 | < 2e-16 *** |
| factor(dev)5                   | -4.465922| 0.429445   | -10.376 | 1.46e-14 *** |
| factor(dev)6                   | -6.344051| 1.201304   | -5.289  | 2.19e-06 *** |
| factor(dev)7                   | -6.059453| 1.127515   | -5.348  | 1.77e-06 *** |
| factor(dev)8                   | -8.442561| 4.108714   | -2.045  | 0.045646 * |
| factor(dev)9                   | -7.973341| 3.741202   | -2.134  | 0.037326 * |
| factor(dev)10                  | -2.269850| 0.258290   | -8.827  | 4.01e-12 *** |
| factor(dev)11                  | -10.645378| 23.31765  | -0.526  | 0.601226 |
| factor(dev)12                  | -10.646134| 28.612020 | -0.372  | 0.711517 |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for Tweedie family taken to be 1637.278)

Null deviance: 6369245 on 70 degrees of freedom
Residual deviance: 91608 on 50 degrees of freedom
AIC: NA

Number of Fisher Scoring iterations: 6
The default for arriving at prediction errors is set at formulaic approach but we can also carry out bootstrap simulations for computing the prediction errors. The default number of simulations is 999 but we can alter that too. This bootstrapping allows us to arrive at percentiles and instead of one point estimate for IBNR.

The predictive distribution in full can also be visualized for the simulated reserves. For instance,

The distribution here is not accurate or reliable because only 10 simulations have been run. It was quite time consuming on the specific PC configuration to run full 999 simulations. The R Package “H20” drastically reduces the time required for running of models and for handling big data and so should be integrated with R package “ChainLadder” for faster simulations in GLMs.

**Munich Chain Ladder**
Munich chain-ladder is a reserving method that reduces the gap between IBNR projections based on paid losses and IBNR projections based on incurred losses. The Munich chain-ladder method uses correlations between paid and incurred losses of the historical data into the projection for the future.

Plotting the results from Munich chain ladder shows the following:

![Munich Chain Ladder Results](image1)

The residual plots should be random and show no pattern or direction. Otherwise, a better model might be needed and this matter should be investigated more. As can be seen, incurred is more reliable to be used for determining IBNR as the residuals and the line of fit is closer to zero than that of paid residuals.

![Munich Chain Ladder vs. Standard Chain Ladder](image2)

![Paid residual plot](image3)

![Inurred residual plot](image4)
6. Criticism of stochastic loss reserving

Stochastic loss reserving is just one of the method to tease out the underlying variability within IBNR reserves. There are many modeling alternatives, including those increasingly being available due to predictive modeling, machine learning and text mining approaches. Here, we discuss only the critique presented by triangle free reserving.

**Triangle free reserving**

The main problem with triangulation as per Parodi is that of information compression. By compacting information into a small triangle (only claims paid/claims incurred amounts by accident date/reported date/paid date), a huge amount of information regarding claim statistics is lost and cannot be taken into account sufficiently, no matter how creative we get in engineering and tweaking the triangle itself by stochastic modeling. Triangle can be satisfactory for reaching a point estimate, but it gets very difficult to figure out the uncertainty revolving around this estimate when determining distribution of the reserves due to significant information lost in making of compact triangles. Thus we need modeling that takes in higher dimensions of more factors that are available in the data instead of utilizing only two-dimensional triangles.

The paper goes on to argue that an alternative triangle free reserving is possible. Date of loss and date of reporting are very important for this triangle free reserving framework. A frequency model of the IBNR claim count based upon lag (between reporting and loss date) distribution is developed on weighted basis so as to avoid biases towards any particular lags. This lag distribution is also used to produce kernel severity model for individual losses. These frequency and severity models can be combined through Monte Carlo simulation to produce an aggregate model for reserves.

All the while we still retain high dimensions and each IBNR figure can be seen on a transactional level.

Parodi argues that this triangle-free approach has a number of advantages over triangulation. These are:

- Higher accuracy and predictive power than triangle based approaches

Any further information on risk can be easily taken into account by this framework. For instance, a different model for losses above a given threshold such as Value at Risk threshold can be made on market statistics as company data might be too sparse.

- Calculation of tail factor can be done more systematically and rigorously than the heuristic fashion adopted in triangle based approaches.

- The results and approach does not become un-credible as easily as triangle based approaches when data is small.

However, it is important not to go overboard with this without doing proper feature engineering. Too many variables in data can mean colinearity and multilinearity can be present so Principal Component Analysis and other methods should be applied on variable selection to get parsimonious variables.

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6 Pietro Parodi: Triangle free reserving; a nontraditional framework for estimating reserves and reserve uncertainty: 4 Feb 2013
Triangle free reserving is important and worth consideration because it does not compressed information and utilizes a number of other parameters included in the data which can be analyzed for their impact upon IBNR on a transactional basis to reduce uncertainty in the reserving process.
7. Conclusion

Applying triangulation method of chain ladder does not just improve deterministic reserving but also enables stochastic reserving to be introduced through comparison of the results of Chain Ladder.

One of the most contentious debates is over the level of complexity to be adopted in reserving between the technical and business sides of the operations. Actuaries advocate higher sophistication (especially in stochastic reserving) whereas managements usually prefer modeling that is understandable to them and where they can make their expert-judgment impact as well 21 Both sides have their own merits. Each side is expressing a different perspective of a difficult problem. Triangle-based deterministic methods introduce a powerful simplicity in the calculations of reserving which renders it easier to narrow the communication gap between the management and the technical specialists.

However, stochastic reserving, data science applications and triangle-free reserving can better exposure underlying variability in reserve estimates.

The Risk culture is foremost for any reserving exercise because financial and insurance sector is not solely run by quantitative numbers, but by the underlying human psychology as well. It is up to the risk culture to not antagonize in binary opposites like complex/simple, good/bad etc., but to reach the middle ground to converge communication and mentalities between different stakeholders.

In conclusion, by measuring and exposing areas of uncertainty, we can reduce our chances of swapping specific risk by systematic risk in our reserving procedures and lessen fatness of the tails.

It is hoped that this report is able to generate further discussions and research into how to measure reserving uncertainty.