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THE COST OF MIXING REINSURANCE

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Abstract

In his 1986 paper, "The Cost of Mixing Reinsurance," Ron Wiser analyzed the consequences of mixing pro rata and excess of loss reinsurance. He concluded that such mixed reinsurance situations were always unfavorable to the ceding company, both in terms of financial cost and loss ratio stability. This paper refutes that conclusion and shows that Wiser proved his argument only for the special case of a particular, inefficient reinsurance structure. Some of his reported "cost of mixing" was actually a result of purchasing redundant reinsurance. The degree of inefficiency in a mixed reinsurance structure can be quantified as the "cost of overlap" between the types of reinsurance coverage. This should be measured separately from the cost of mixing per se. A case of negative cost of mixing is presented, and a relatively simple test of whether the mixing cost will be positive or negative is derived and demonstrated.

1. INTRODUCTION

Ron Wiser's "The Cost of Mixing Reinsurance" [1] is an excellent presentation of many of the major issues involved in the analysis of the effect of mixing proportional and excess of loss reinsurance. It is a well-written paper that introduces the reader to several important concepts. Because I see that paper as so important, I believe it is necessary to comment on some aspects the author has omitted. My primary concern is that the Wiser paper may leave the casual reader with the impression that mixing excess of loss and proportional reinsurance *always* has adverse cost and stability consequences for the ceding company. That is not true, as I will show.

This discussion is best read with a copy of Wiser's paper close at hand, since it frequently refers in detail to the examples presented in that paper. An effort has been made to use the terminology and notation of the original paper, in order to make it easier to read the two together.

2. EFFICIENT MIXING OF REINSURANCE

The key point of this discussion is to make it clear that Wiser proved his Mixing Loss Ratio Rule *only* for the special case in which the ceding company buys excess of loss coverage all the way up to the top of the policy limits it has issued. In such circumstances, the purchase of proportional reinsurance is redundant and is thus inherently inefficient. That it has a cost should come as no surprise. This is understood by most insurers, and they do not usually structure their reinsurance in this way. As a result, Wiser's conclusion that mixed reinsurance situations are always costly to the ceding company is relevant only to reinsurance arrangements that are not often found in practice.

An insurer normally will determine the net retention it desires for a particular risk and use a mix of proportional and excess of loss reinsurance to absorb the exposure between the retention and the policy limit. In property insurance the proportional reinsurance is typically provided under "surplus share" treaties, which essentially provide the insurer with the capacity to write policy limits larger than it could with its excess of loss reinsurance alone. These treaties act like a sponge, soaking up the surplus exposure above its excess reinsurance coverage. Normally, if there is no "surplus" policy limit exposure, the insurer does not cede any exposure to the surplus share treaties. If it does, it is generally according to a line guide agreed to in advance with the surplus and excess reinsurers, and the price and terms of the reinsurance contracts will reflect the expected cessions implied by the line guide. Insurers also use a mixture of excess and proportional reinsurance to extend their capacity for casualty policy limits, though here, the use of surplus share treaties is less common than in property insurance.

Figure 1 is a graphical illustration of this common type of property reinsurance program. It shows: A) a net retention of $500,000; B) a $500,000 excess of $500,000 treaty; and C) a five-line surplus share treaty. This gives the insurer total gross line capacity of $6,000,000. Policies larger than $6,000,000 would require facultative reinsurance (either proportional or excess of loss).

If P denotes the policy limit and F denotes the facultative coverage limit, an insurer with the treaty reinsurance program shown in Figure 1 generally would keep a share, equal to $1,000,000/($P - F$), of any "net and treaty" policy limit, $P - F$, greater than $1,000,000. The excess reinsurer remains exposed for its entire $500,000 limit, though the composition of that exposure is different after proportional reinsurance is introduced.

Figure 2 illustrates the allocation of loss exposure arising from a $5 million policy issued by an insurer that has the reinsurance program summarized in Figure 1.

3. ORDER OF REINSURANCE RECOVERIES

In mixed reinsurance situations, i.e., where a policy is protected by a number of reinsurance contracts, working out the correct loss recoveries can be fairly involved. In theory the interplay between the applicable coverages is negotiable but, in practice, the following principles are normally applied as standard unless otherwise agreed:

1. *More specific coverage responds before less specific coverage.* For example, facultative always responds before treaty. Per risk covers always respond before catastrophe (i.e., per occurrence) covers.
2. *Proportional coverage responds before excess of loss coverage, subject, however, to Principle 1.* For example, a property surplus treaty responds before a per risk excess cover but after a facultative excess cover.

This discussion assumes the application of these recovery principles. The conclusions will not be applicable to those relatively rare instances where a proportional treaty has been negotiated to respond *after* an excess of loss treaty.

FIGURE 1

COMMON PROPERTY REINSURANCE STRUCTURE

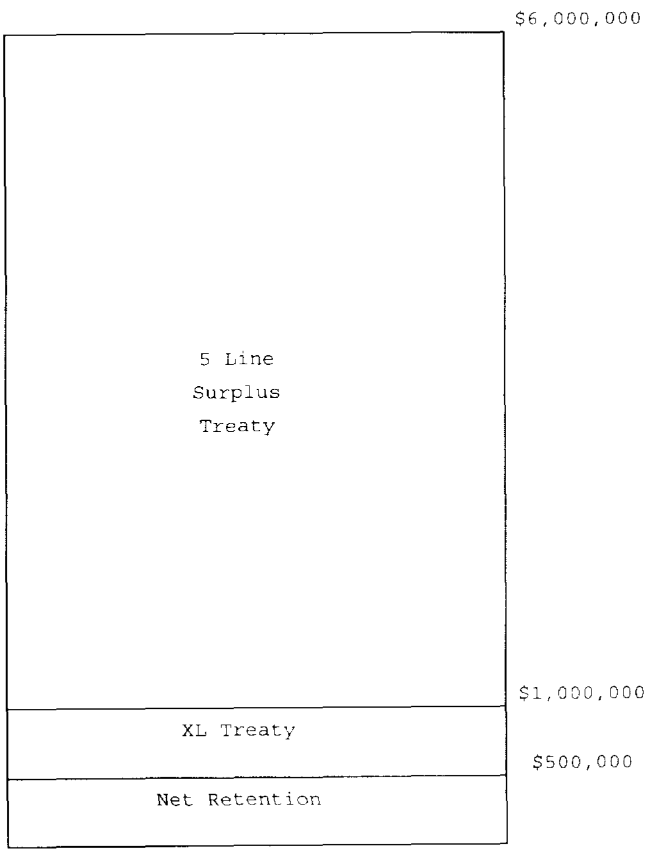
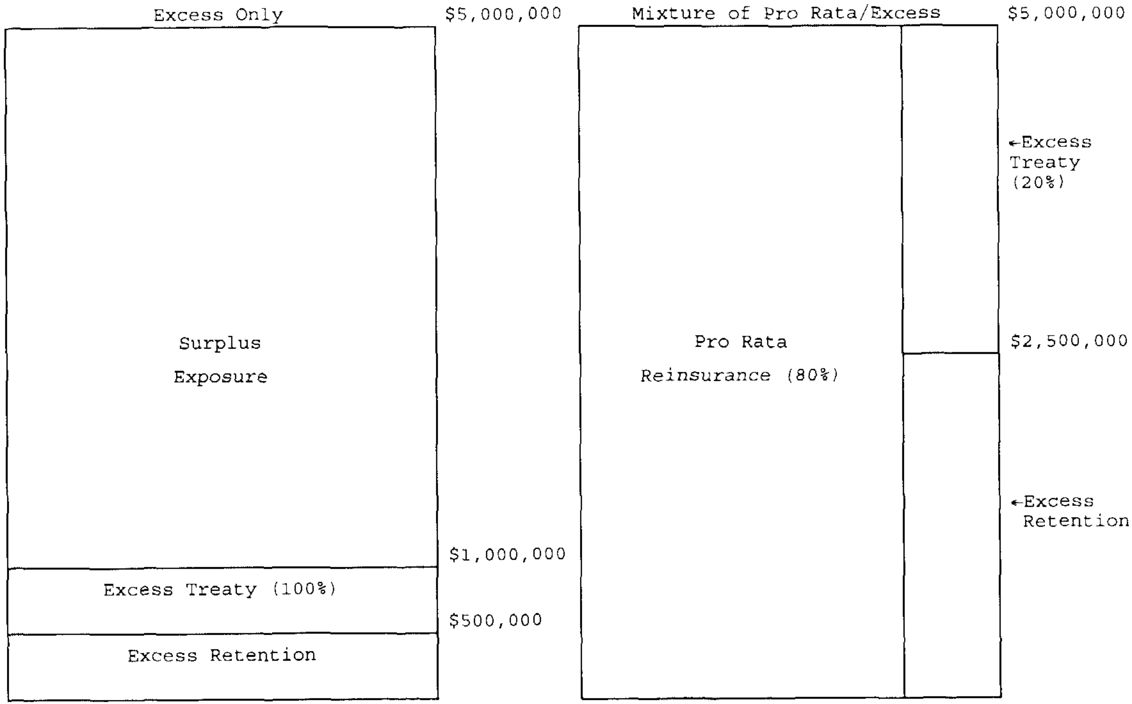


FIGURE 2
ALLOCATION OF LOSS EXPOSURE
COMMON PROPERTY REINSURANCE STRUCTURE / $5 MILLION POLICY



4. EXCESS LIMIT EXPOSURE

At least a portion of the "cost of mixing" demonstrated by Wiser is a result of the inefficient reinsurance structure he assumed, for both his examples and his general case. Consider his examples.

In the casualty case presented in Wiser's paper [1, pp. 175-187], the excess coverage is \$2 million excess of \$250,000. For some reason, the insurer buys 50% pro rata facultative coverage on a \$1 million excess of \$100,000 policy. Without the facultative placement, the excess reinsurer is exposed for \$750,000 excess of \$250,000. But with it, the excess exposure is only \$250,000 excess of \$250,000. We should not find it surprising that the excess reinsurer has a lower expected pure premium and the insurer a higher expected cost of reinsurance! The allocation of loss exposure is shown graphically in Figure 3, where it is evident that the proportional coverage has crowded out the excess of loss protection. The bar on the left shows the allocation of exposure in the pure excess case. The bar on the right illustrates the mixed case. The much smaller area corresponding to excess treaty exposure in the mixed case is indicative of the reduced exposure compared to the pure excess case.

The same is true of Wiser's property example [1, pp. 191-201], though the inefficiency of the reinsurance is less extreme than in the first case. The excess coverage is \$2 million excess of \$250,000. The insurer issues a \$20 million policy and cedes 90% of the risk (\$18 million) on a pro rata basis. The pro rata placement reduces the excess reinsurer's maximum exposure from \$2 million excess of \$250,000 to \$1.75 million excess of \$250,000. Again, it should be obvious that the excess reinsurer will benefit and the insurer's expected net cost of reinsurance will be increased. The allocation of exposure in the pure excess and mixed scenarios is shown graphically in Figure 4.

In both of these examples the insurer has bought *overlapping* reinsurance coverage. While there might be valid reasons for doing this under certain circumstances (e.g., "protecting the excess treaty"), it will always be more expensive than not doing so. In the casualty case, it is not clear why the insurer bought any proportional cover at all, since it had adequate coverage under its excess of loss treaty alone. In the property case, the

FIGURE 3

ALLOCATION OF LOSS EXPOSURE

FIRST CASUALTY EXAMPLE $1 MILLION EXCESS OF $100,000 POLICY

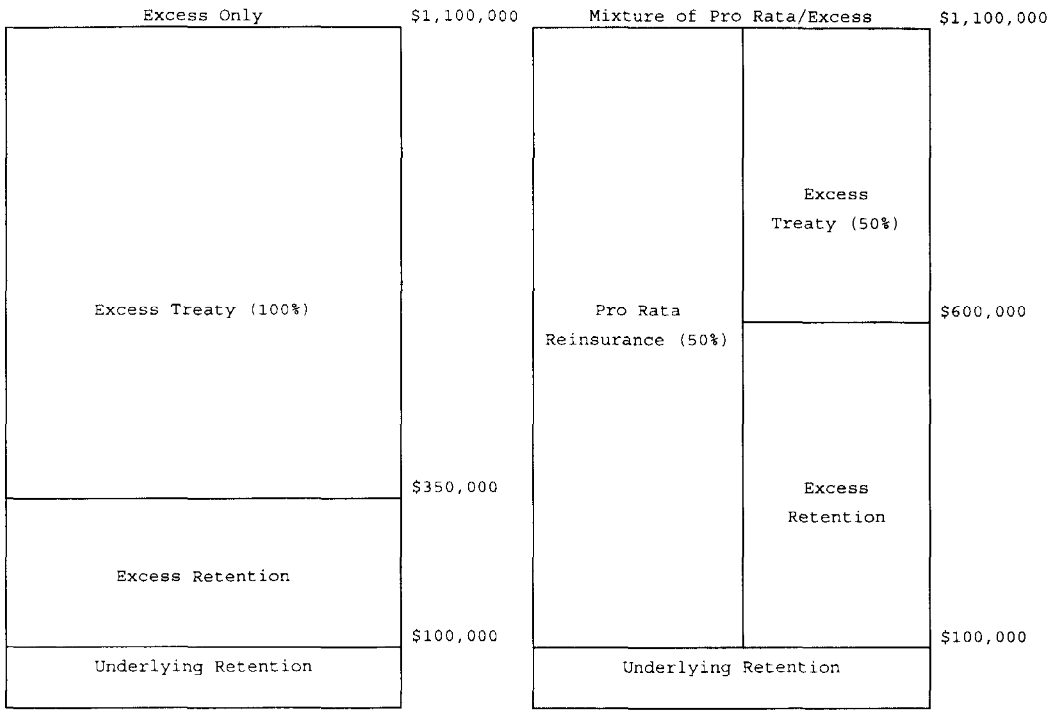
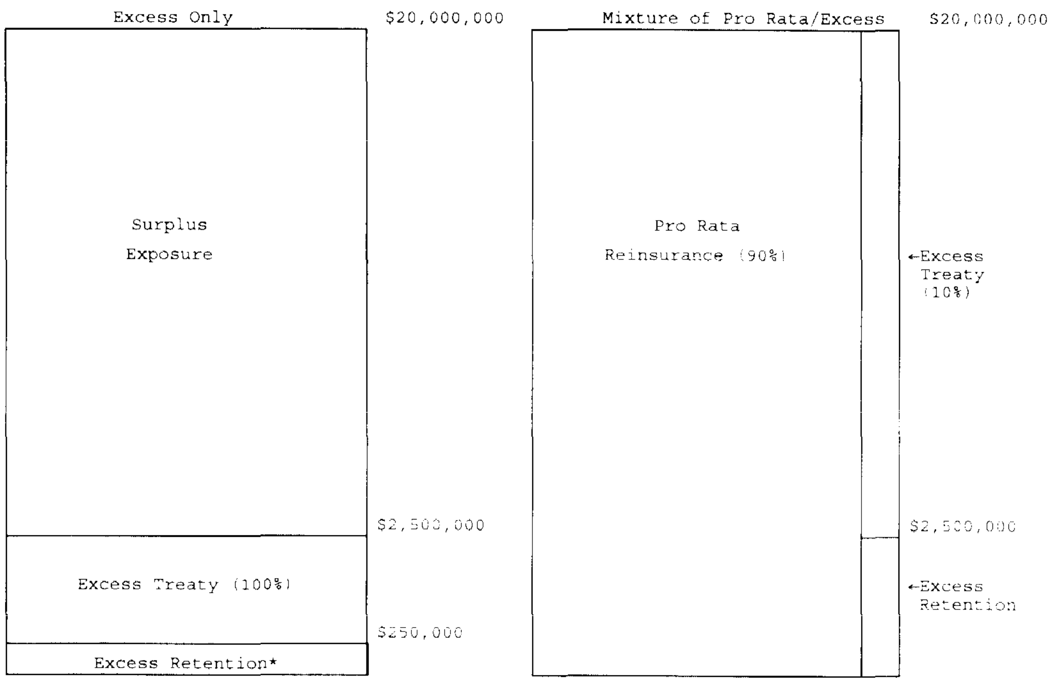


FIGURE 4
ALLOCATION OF LOSS EXPOSURE
PROPERTY EXAMPLE / $20 MILLION POLICY



* Area not to scale

correct pro rata purchase to avoid overlap would have been $17.75 million, or 88.75%, instead of $18 million (90%).

The point is that the "costs of mixing" shown on Wiser's Exhibits 2 and 3 and implied by his Mixing Loss Ratio Rule include not only the effect of mixing but also the cost of buying down the excess reinsurer's limits exposure. Indeed, we cannot be certain that the effect of mixing per se is unfavorable without further analysis.

5. COST OF OVERLAP AND COST OF MIXING PER SE

In the casualty example presented by Wiser, the reinsurer's total policy limit exposure in the pure excess case is $750,000 excess of $250,000, which implies expected losses of $85,144, or 35.48% of the policy losses subject to the excess coverage. See Wiser's Exhibit 1.1 [1, p. 178]. This layer can be subdivided into two layers of $250,000 excess of $250,000 and $500,000 excess of $500,000. The expected losses in these two layers are 19.71% and 15.77% of subject losses, respectively. These ratios are easily computed using values from the limited mean function of the size of loss distribution. (See Exhibit 6, or Wiser's Exhibit 1.2, for a partial tabulation of this function.)

In the mixed reinsurance case, the excess reinsurer's total policy limit exposure is only $250,000 excess of $250,000 with respect to subject loss exposure of $500,000. (This is equivalent to 50% of $500,000 excess of $500,000 with respect to original policy exposure of $1 million.) The $500,000 excess of $500,000 layer now has no policy limit exposure at all. The expected excess losses in these two layers are 15.77% and 0% of subject losses, respectively.

The difference in $250,000 excess of $250,000 layer expected losses between the pure excess and mixed reinsurance cases is approximately 3.93% (19.71% - 15.77%) of subject losses, or about $4,720. This is the true cost of *mixing* excess and proportional reinsurance in this example.

See Exhibit 1 of this discussion for a summary of the key limits, limited means, and relative exposure for the first layer. This exhibit measures the cost of *mixing per se*.

Since the ceding company has the same net retention of $250,000 after buying the pro rata coverage as in the pure excess case, all it has achieved is to buy down the excess reinsurer's limit exposure. This is manifested by the exposure in the $500,000 excess of $500,000 layer going to zero in the mixed case. The cost of the limit buy-down is about $18,927, which is 15.77% of subject losses. This is the measure of the inefficiency of this particular mixed reinsurance structure, the "cost of *overlapping* reinsurance." See Exhibit 2, which is analogous to Exhibit 1, but summarizes the cost of *overlap* calculations.

Wiser reports the cost of mixing in this example to be $23,653. In fact, the cost of mixing per se is only $4,720, or 20%, of this total. The remaining 80% is due to buying unnecessary reinsurance.

In the property example, the excess coverage can be layered as $1.75 million excess of $250,000 and $250,000 excess of $2 million. In the pure excess case the expected losses in these two layers are 32.88% and 1.41% of subject policy losses, respectively. (This can be verified using values from Exhibit 7.)

Matching up these layers with their counterparts in the mixed case, the reinsurer's exposure in the first layer is 13.88% of subject losses and the second layer is not exposed at all. The cost of mixing is given by the difference in first layer expected losses, which is approximately 19% of $30,000, or $5,699. The cost of overlap is the difference in second layer expected losses. This is equal to 1.41% of $30,000, or $422. Thus, in this example, 93% of the cost of mixing reported by Wiser is due to mixing itself and 7% is due to the purchase of redundant coverage. The cost of overlap is much lower than in the casualty example because, here, the degree of overlap between excess and proportional reinsurance is much less.

Exhibits 3 and 4 summarize the cost of mixing and cost of overlap calculations, respectively, for this example.

6. NEGATIVE COST OF MIXING

To see that mixing pro rata and excess coverage is sometimes actually favorable to an insurer, suppose an insured having the same loss severity characteristics as in Wiser's casualty example buys total insurance coverage of $10 million.

Assume the $5 million excess of $5 million layer is written by an insurer that has excess of loss reinsurance of $1.75 million excess of $250,000. With no proportional reinsurance, the insurer has a total net retention of $3.25 million ($250,000 at the bottom and $3 million at the top). The left portion of Figure 5 illustrates this graphically.

Exhibit 5 shows the calculation for the cost of mixing per se for this example. With excess reinsurance only, the expected excess reinsurance recovery is 49.77% of subject losses. If expected subject losses are $50,000, the expected recovery amount is $24,884.

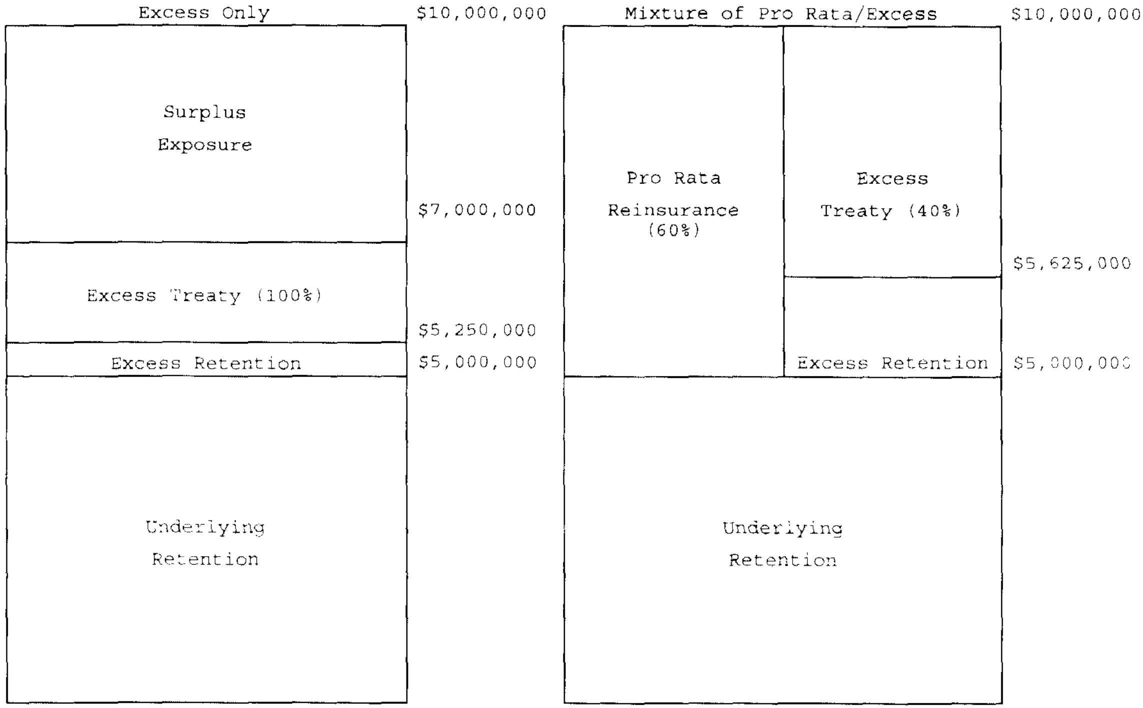
With proportional reinsurance of 60%, the insurer's net retention is reduced to $250,000. This eliminates all net exposure above the excess of loss protection, but leaves the excess reinsurer's maximum loss exposure unchanged at $1.75 million. This is illustrated by the right portion of Figure 5. The expected excess recovery is 40% of $37,695 (the expected losses in the layer $4.375 million excess of $5.625 million) or 75.35% of subject losses (40% of $50,000).

The cost of mixing is (49.77% – 75.35%) of $20,000, or –$5,116. The cost consequences of mixing are adverse to the reinsurer. The insurer sees a cost benefit. It should be clear on the face of it that the insurer also benefits in terms of stabilization: without mixing, it has a much higher net retention, most of it in the form of the unreinsured layer $3 million excess of $2 million.

7. DETERMINING COST OR BENEFIT OF MIXING REINSURANCE

Apart from a minor refinement or two, the following analysis uses Wiser's notation.

FIGURE 5
ALLOCATION OF LOSS EXPOSURE
SECOND CASUALTY EXAMPLE / $5 MILLION EXCESS OF $5 MILLION POLICY



Let *RateXS*(a, M, L) denote the excess pure premium rate for excess treaty retention, M, excess treaty limit, L, and pro rata treaty retention percentage, a. Wiser states that the most general characterization of the excess pure premium rate where there is no proportional reinsurance is:

$$RateXS(1, M, L) = \frac{\int_{M}^{M+L} (x - M) \cdot f(x)dx + L \cdot \int_{M+L}^{\infty} f(x)dx}{\text{Subject Premium}}, \qquad (7.1)$$

where $f(x)$ is the p.d.f. describing the distribution of policy losses by size.[1] (Note that the variable name on the left side of the equation has been modified slightly from Wiser's notation to record the reinsurance limit.)

The numerator of formula 7.1 can be expressed in terms of a difference of limited means of $f(x)$:

$$RateXS(1, M, L) = \frac{\mathrm{E}_{M+L}(x) - \mathrm{E}_{M}(x)}{\text{Subject Premium}}. \qquad (7.2)$$

The discerning reader will have noticed that, as the definition of the *pure premium* rate, formula 7.2 is incorrect—it reflects only claim severity. The subject claim count, E(N), has been left out of the numerator. Correcting for this omission, formula 7.2 becomes:

$$RateXS(1, M, L) = \frac{\mathrm{E}(N) \cdot \left[\mathrm{E}_{M+L}(x) - \mathrm{E}_{M}(x)\right]}{\text{Subject Premium}}. \qquad (7.3)$$

Subsequent reference to the formulae in the Wiser paper will assume this correction.

In his analysis Wiser assumed that the insurer buys sufficient excess reinsurance to cover the largest policies issued. In such a case, policy limits always truncate excess exposure at or before the point where the

[1] The formula appearing in the paper shows the factor before the integral in the second term as ($L + M$), but it clearly should be L as shown here.

reinsurance limit does. That makes the second term of formula 7.1 unnecessary, and Wiser proceeds with the following simplification, which does not depend on L:

$$RateXS(1, M) = \frac{E(N) \cdot \left[\int_M^\infty (x - M) \cdot f(x) dx \right]}{\text{Subject Premium}} \tag{7.4}$$

$$= \frac{E(N) \cdot [E(x) - E_M(x)]}{\text{Subject Premium}}.$$

However, because insurers normally do not structure their reinsurance in the way he assumes, the second term cannot be dropped for the analysis of a more realistic scenario, much less for the general case. Therefore, the following discussion rests on formula 7.3, which applies in the general case.

Let us investigate the relationship between expected excess losses in mixed and pure excess cases. In the mixed reinsurance case, the Mixed Pricing Rule tells us to divide the excess retention and limit by the pro rata retention ratio, a, to determine the effective excess layer in terms of the policy loss function, $f(x)$. The limited mean claim size for a net limit, k, reflecting a pro rata retention, a, is given by the following:

$$a \cdot E_k(x, a) = a \cdot \left[\int_0^{k/a} x \cdot f(x) dx + (k/a) \cdot \int_{k/a}^\infty f(x) dx \right] = a \cdot E_{k/a}(x). \tag{7.5}$$

The excess pure premium rate in the mixed case is given by:

$$RateXS(a, M, L) = \frac{a \cdot E(N) \cdot \left[E_{M+L}(x, a) - E_M(x, a) \right]}{a \cdot (\text{Subject Premium})}$$

$$= \frac{E(N) \cdot \left[E_{M+L}(x, a) - E_M(x, a) \right]}{\text{Subject Premium}}. \tag{7.6}$$

If R is a relativity describing the relationship between $RateXS(a, M, L)$ and $RateXS(1, M, L)$, then:

$$R = \frac{RateXS(a, M, L)}{RateXS(1, M, L)} = \frac{\mathrm{E}_{M+L}(x, a) - \mathrm{E}_M(x, a)}{\mathrm{E}_{M+L}(x) - \mathrm{E}_M(x)} . \qquad (7.7)$$

If $R < 1$, the ceding insurer's expected loss recoveries from the excess reinsurer will be lower in the mixed reinsurance case than in the pure excess case—the cost of mixing is positive. On the other hand, if $R > 1$, the insurer will recover more in the mixed case and the cost of mixing is negative.

Let m and m_a denote the slope of the limited mean function between 1) $\mathrm{E}_M(x)$ and $\mathrm{E}_{M+L}(x)$, and 2) $\mathrm{E}_M(x, a)$ and $\mathrm{E}_{M+L}(x, a)$, respectively. Then formula 7.7 can be restated as:

$$R = \frac{m_a \cdot L/a}{m \cdot L} = \frac{m_a}{a \cdot m} . \qquad (7.8)$$

From formula 7.8, we can see that whether the cost of mixing is positive ($R < 1$) or negative ($R > 1$) depends on the shape of the limited mean function between the points that define the excess reinsurance coverage. If the slope on the portion of the curve that defines mixed coverage is less than $100 \cdot a\%$ of the slope in the pure excess case, then $R < 1$ and the cost of mixing is positive. But if the mixed coverage slope exceeds $100 \cdot a\%$ of the pure excess slope, then $R > 1$ and mixing has a negative cost.

A result proved by Miccolis [2] can be used to define a simple test of whether mixing has a positive or negative cost to the ceding insurer. Though he used a different notation invented for his discussion of increased limits factors, Miccolis showed that

$$d\,[\,E_k(x)/C\,]/dk = [1 - f\#(k)]/C , \qquad (7.9)$$

where C is a constant and $f\#(k)$ is the c.d.f. of policy losses by size (Wiser's notation). Since $1 - f\#(k) = \mathrm{Prob}\,(x > k)$, this tells us that the slope of the limited mean function at k is the same as the probability, given a claim, that the claim exceeds k.

So for the infinitesimal layer Δx excess of k, the following specifies a precise test for the cost to the ceding insurer of mixing per se:

$$\begin{aligned} &\text{Prob}\,(x > k/a) < \text{Prob}\,(x > k) \cdot a \iff \text{Positive Cost} \\ &\text{Prob}\,(x > k/a) = \text{Prob}\,(x > k) \cdot a \iff \text{No Cost} \\ &\text{Prob}\,(x > k/a) > \text{Prob}\,(x > k) \cdot a \iff \text{Negative Cost} \end{aligned} \tag{7.10}$$

For excess layers of practical importance, Formula 7.10 is not a precise test, but it suggests a way of screening. There are numerous such screens of varying complexity that could be employed, but here is a simple one that is easy to apply: test the layer endpoints, denoted by k_1 and k_2.

$$\begin{aligned} &\text{Prob}\,(x > k_1/a) < \text{Prob}\,(x > k_1) \cdot a \\ &\qquad\text{and} \qquad\qquad\qquad\qquad\qquad \iff \text{Positive Cost} \\ &\text{Prob}\,(x > k_2/a) < \text{Prob}\,(x > k_2) \cdot a \\[1em] &\text{Prob}\,(x > k_1/a) > \text{Prob}\,(x > k_1) \cdot a \\ &\qquad\text{and} \qquad\qquad\qquad\qquad\qquad \iff \text{Negative Cost} \\ &\text{Prob}\,(x > k_2/a) > \text{Prob}\,(x > k_2) \cdot a \\[1em] &\qquad\text{Other Combinations} \qquad\qquad \iff \text{Test Inconclusive} \end{aligned} \tag{7.11}$$

Refinements to this test would be possible, but at the cost of introducing additional complexity.

8. APPLYING THE TEST

To confirm that formula 7.11 correctly identifies the mixing cost characteristics of the examples discussed earlier, let us apply the test to those cases.

The first example involved a \$1 million casualty policy that attached in excess of a \$100,000 self-insured retention (SIR). Recall that despite

\$750,000 of loss exposure to the excess reinsurer before proportional reinsurance, after the 50% pro rata cession, the excess limit exposure is only \$250,000. The cost of mixing calculation was done in respect of this \$250,000 that is present in both the pure excess and mixed cases. Exhibit 1 shows that the excess reinsurer's exposure is higher in the pure excess case, demonstrating a positive cost of mixing.

Exhibit 6 tabulates various information about the loss distribution "from the ground up" (FGU), i.e., including the SIR, so in the pure excess case, the reinsurance retention of \$250,000 equates to \$350,000 on the FGU loss table. From Exhibit 6, Prob ($x >$ \$350,000) = 1.18%. Fifty percent of that (reflecting the pro rata retention $a = 50\%$) is 0.59%. This compares to a mixed case retention in FGU terms of \$600,000 (\$100,000 + \$250,000/.50). Since Prob ($x >$ \$600,000) = 0.52% < 0.59%, this is an indication that the cost of mixing may be positive.

Now test the upper end of common excess coverage. In the pure case, this is \$600,000 FGU (\$100,000 + \$250,000 + \$250,000). Prob ($x >$ \$600,000) = 0.52%. Fifty percent of this is 0.26%. For the mixed case, the upper end of coverage on an FGU basis is \$1,100,000 (\$100,000 + \$250,000/.50 + \$250,000/.50). Prob ($x >$ \$1,100,000) = 0.188% < 0.26%, which confirms the indication of a positive cost of mixing.

The second example involved a \$20 million property policy with no SIR. Available excess reinsurance coverage is \$2 million excess of \$250,000, but after a 90% pro rata cession, only \$1.75 million is used. The cost of mixing calculation done on Exhibit 3 in respect of this \$1.75 million shows a positive cost of mixing.

Exhibit 7 tabulates information about the loss distribution for this example. There is no SIR in this case, so it will not be necessary to make any special adjustments. The excess retention is \$250,000 in the pure excess case; according to Exhibit 7, Prob ($x >$ \$250,000) = 4.613%. Ten percent of this (reflecting the pro rata retention, $a = 10\%$) is 0.461%. The effective mixed case retention in FGU terms is \$2,500,000 (\$250,000/.10). Prob ($x >$ \$2,500,000) = 0.293%, which is less than 0.461%. This indicates a possible positive cost of mixing.

Testing the upper end of common excess coverage, which in the pure excess case is \$2 million (\$250,000 + \$1.75 million), yields Prob ($x >$ \$2 million) = 0.401%. Ten percent of this is 0.04%. The effective upper bound of coverage in the mixed case is \$20 million FGU. Prob ($x >$ \$20 million) = 0.01% < 0.04%, which confirms a positive cost of mixing.

The third example involved a \$5 million excess casualty policy attaching excess of \$5 million. There was excess coverage of \$1.75 million excess of \$250,000 and a proportional cession of 60%. The cost of mixing calculation for this example is summarized on Exhibit 5. It shows a negative cost of mixing.

Exhibit 6 is the source of loss information. Since the attachment point of the policy is \$5 million, in the pure excess case the reinsurance retention of \$250,000 equates to \$5.25 million on the FGU loss table. From Exhibit 6, Prob ($x >$ \$5.25 million) = 0.008%. Reflecting the pro rata retention, 40% of this is 0.003%. This compares to a mixed case retention in FGU terms of \$5.625 million (\$5 million + \$250,000/0.40). Since Prob ($x >$ \$5.625 million) = 0.007% > 0.003%, this is an indication that the cost of mixing may be negative.

Now examine the upper end of excess coverage. In the pure excess case this is \$7 million FGU (\$5 million + \$250,000 + \$1.75 million). Prob ($x >$ \$7 million) = 0.004%. Forty percent of this is 0.0016%. This compares to an upper end of \$10 million FGU in the mixed case (\$5 million + \$250,000/0.40 + \$1.75 million/0.40). Confirming that the cost of mixing is negative, Prob ($x >$ \$10 million) = 0.002% > 0.0016%.

9. CONCLUSION

I hope this discussion will be seen as building on the foundation of Wiser's original paper. Its purpose has been to aid the reader in understanding more clearly the effect of mixing proportional and excess of loss reinsurance, and the importance of distinguishing between the cost of overlap and the cost of mixing per se.

REFERENCES

[1] Wiser, Ronald F., "The Cost of Mixing Reinsurance," *PCAS* LXXIII, 1986, p. 168.

[2] Miccolis, Robert S., "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, 1977, p. 27.

EXHIBIT 1

ANALYSIS OF REINSURED LOSS EXPOSURE VS. INSURED LOSS EXPOSURE*
CASUALTY EXAMPLE
REINSURED LAYER: $250,000 EXCESS OF $250,000**
CALCULATION OF THE COST OF MIXING PER SE

		Critical Limits of Insured and Reinsured Loss Layers and the Corresponding Limited Means										
		Subject Policies				Reinsured Layer			Relative Exposure			
		A	B	C	D	E	F	G	H	I	J	K
Reinsurance Structure		Under-lying Retention	Policy Limit	Policy + Underlying Retention	Subject Share *a*	Effective Retention	Effective Rein-surance Limit	Effective Reten-tion + Limit	Reinsured Exposure as % of Subject Loss	Subject Loss	Reinsured Layer Loss	Adjusted Rein-sured Layer Loss #
Pure XS	Limits	100,000	1,000,000	1,100,000	100.00%	350,000	250,000	600,000				
	Lim Means	18,034	9,916	27,950		24,432	1,954	26,386	19.71%	240,000	47,293	23,647
Mixed	Limits	100,000	1,000,000	1,100,000	50.00%	600,000	500,000	1,100,000				
	Lim Means	18,034	9,916	27,950		26,386	1,564	27,950	15.77%	120,000	18,927	18,927
							Pure - Mixed		3.93%			4,720

* Loss model is lognormal: $\mu = 8.6799043$; $\sigma = 1.8050198$. Calculations use limited means to precision displayed. See Exhibit 6 for corresponding loss distribution table.

** Both pure excess and mixed reinsurance structures fully expose the excess reinsurer in this layer.

Based on subject premiums from mixed case.

B = C - A
F = G - E
H = F / B
J = I * H

EXHIBIT 2

ANALYSIS OF REINSURED LOSS EXPOSURE VS. INSURED LOSS EXPOSURE*
CASUALTY EXAMPLE
REINSURED LAYER: $1,750,000 EXCESS OF $500,000**
CALCULATION OF THE COST OF OVERLAP

		Critical Limits of Insured and Reinsured Loss Layers and the Corresponding Limited Means										
		Subject Policies				Reinsured Layer			Relative Exposure			
		A	B	C	D	E	F	G	H	I	J	K
Reinsurance Structure		Under-lying Retention	Policy Limit	Policy + Underlying Retention	Subject Share *a*	Effective Retention	Effective Rein-surance Limit	Effective Retention + Limit	Reinsured Exposure as % of Subject Loss	Subject Loss	Reinsured Layer Loss	Adjusted Rein-sured Layer Loss #
Pure XS	Limits	100,000	1,000,000	1,100,000	100.00%	600,000	500,000	1,100,000				
	Lim Means	18,034	9,916	27,950		26,386	1,564	27,950	15.77%	240,000	37,854	18,927
Mixed	Limits	100,000	1,000,000	1,100,000	50.00%	1,100,000	0	1,100,000				
	Lim Means	18,034	9,916	27,950		27,950	0	27,950	0.00%	120,000	0	0
								Pure - Mixed	15.77%			18,927

* Loss model is lognormal: $\mu = 8.6799043$; $\sigma = 1.8050198$. Calculations use limited means to precision displayed. See Exhibit 6 for corresponding loss distribution table.

** Only pure excess structure exposes the excess reinsurer in this layer.

Based on subject premiums from mixed case.

B = C - A
F = G - E
H = F / B
J = I * H

EXHIBIT 3
ANALYSIS OF REINSURED LOSS EXPOSURE VS. INSURED LOSS EXPOSURE*
PROPERTY EXAMPLE
REINSURED LAYER: $1,750,000 EXCESS OF $250,000**
CALCULATION OF THE COST OF MIXING PER SE

		Critical Limits of Insured and Reinsured Loss Layers and the Corresponding Limited Means										
		Subject Policies				Reinsured Layer			Relative Exposure			
		A	B	C	D	E	F	G	H	I	J	K
Reinsurance Structure		Underlying Retention	Policy Limit	Policy + Underlying Retention	Subject Share *a*	Effective Retention	Effective Reinsurance Limit	Effective Retention + Limit	Reinsured Exposure as % of Subject Loss	Subject Loss	Reinsured Layer Loss	Adjusted Reinsured Layer Loss #
Pure XS	Limits	0	20,000,000	20,000,000	100.00%	250,000	1,750,000	2,000,000				
	Lim Means	0	65,577	65,577		33,205	21,559	54,764	32.88%	300,000	98,628	9,863
Mixed	Limits	0	20,000,000	20,000,000	10.00%	2,500,000	17,500,000	20,000,000				
	Lim Means	0	65,577	65,577		56,475	9,102	65,577	13.88%	30,000	4,164	4,164
								Pure - Mixed	19.00%			5,699

* Loss model is lognormal: $\mu = 8.8123226$; $\sigma = 2.1482831$. Calculations use limited means to precision displayed. See Exhibit 7 for corresponding loss distribution table.

** Both pure excess and mixed reinsurance structures fully expose the excess reinsurer in this layer.

Based on subject premiums from mixed case.

B = C - A
F = G - E
H = F / B
J = I * H

EXHIBIT 4

ANALYSIS OF REINSURED LOSS EXPOSURE VS. INSURED LOSS EXPOSURE*
PROPERTY EXAMPLE
REINSURED LAYER: $250,000 EXCESS OF $2,000,000**
CALCULATION OF THE COST OF OVERLAP

		Critical Limits of Insured and Reinsured Loss Layers and the Corresponding Limited Means										
		Subject Policies				Reinsured Layer			Relative Exposure			
		A	B	C	D	E	F	G	H	I	J	K
Reinsurance Structure		Under-lying Reten-tion	Policy Limit	Policy + Underlying Retention	Subject Share *a*	Effective Retention	Effective Rein-surance Limit	Effective Retention + Limit	Reinsured Exposure as % of Subject Loss	Subject Loss	Reinsured Layer Loss	Adjusted Rein-sured Layer Loss #
Pure XS	Limits	0	20,000,000	20,000,000	100.00%	2,000,000	250,000	2,250,000				
	Lim Means	0	65,577	65,577		54,764	922	55,686	1.41%	300,000	4,218	422
Mixed	Limits	0	20,000,000	20,000,000	10.00%	20,000,000	0	20,000,000				
	Lim Means	0	65,577	65,577		65,577	0	65,577	0.00%	30,000	0	0
								Pure - Mixed	1.41%			422

* Loss model is lognormal: $\mu = 8.8123226$; $\sigma = 2.1482831$. Calculations use limited means to precision displayed. See Exhibit 7 for corresponding loss distribution table.

** Only the pure excess structure exposes the excess reinsurer in this layer.

Based on subject premiums from mixed case.

B = C - A
F = G - E
H = F / B
J = I * H

EXHIBIT 5

ANALYSIS OF REINSURED LOSS EXPOSURE VS. INSURED LOSS EXPOSURE*
REVISED CASUALTY EXAMPLE
REINSURED LAYER: $1,750,000 EXCESS OF $250,000**
CALCULATION OF THE COST OF MIXING PER SE

		Critical Limits of Insured and Reinsured Loss Layers and the Corresponding Limited Means										
		Subject Policies				Reinsured Layer			Relative Exposure			
		A	B	C	D	E	F	G	H	I	J	K
Reinsurance Structure		Underlying Retention	Policy Limit	Policy + Underlying Retention	Subject Share a	Effective Retention	Effective Reinsurance Limit	Effective Retention + Limit	Reinsured Exposure as % of Subject Loss	Subject Loss	Reinsured Layer Loss	Adjusted Reinsured Layer Loss #
Pure XS	Limits	5,000,000	5,000,000	10,000,000	100.00%	5,250,000	1,750,000	7,000,000				
	Lim Means	29,665	215	29,880		29,687	107	29,794	49.77%	50,000	24,884	9,953
Mixed	Limits	5,000,000	5,000,000	10,000,000	40.00%	5,625,000	4,375,000	10,000,000				
	Lim Means	29,665	215	29,880		29,718	162	29,880	75.35%	20,000	15,070	15,070
								Pure - Mixed	-25.58%			(5,116)

* Loss model is lognormal: $\mu = 8.6799043$; $\sigma = 1.8050198$. Calculations use limited means to precision displayed. See Exhibit 6 for corresponding loss distribution table.

** Both pure excess and mixed reinsurance structures fully expose the excess reinsurer in this layer.

Based on subject premiums from mixed case.

B = C - A F = G - E

H = F / B J = I * H

EXHIBIT 6

Loss Distribution Table—Casualty Example

Log Normal Model*

Limit k	Limited Mean $LM(k)$	Limit Factor $LF(k)$	$f\#(k)$	$f\$(k)$	Prob $(x > k)$
100,000	18,034	0.60112	0.94174	0.40691	0.05826
350,000	24,432	0.81439	0.98820	0.67672	0.01180
600,000	26,386	0.87954	0.99480	0.77552	0.00520
1,000,000	27,745	0.92485	0.99778	0.85087	0.00222
1,100,000	27,950	0.93165	0.99812	0.86280	0.00188
1,250,000	28,201	0.94003	0.99851	0.87774	0.00149
1,500,000	28,517	0.95058	0.99893	0.89703	0.00107
2,000,000	28,926	0.96419	0.99938	0.92280	0.00062
2,100,000	28,985	0.96616	0.99944	0.92664	0.00056
2,350,000	29,111	0.97037	0.99955	0.93493	0.00045
3,000,000	29,341	0.97805	0.99972	0.95041	0.00028
4,600,000	29,624	0.98748	0.99989	0.97033	0.00011
5,000,000	29,665	0.98884	0.99991	0.97331	0.00009
5,250,000	29,687	0.98958	0.99992	0.97493	0.00008
5,625,000	29,718	0.99061	0.99993	0.97722	0.00007
7,000,000	29,794	0.99312	0.99996	0.98292	0.00004
10,000,000	29,880	0.99601	0.99998	0.98972	0.00002
100,000,000	29,998	0.99995	1.00000	0.99984	0.00000

* $\mu = 8.6799043$ $\sigma = 1.8050198$

EXHIBIT 7

Loss Distribution Table—Property Example

Log Normal Model*

Limit k	Limited Mean $LM(k)$	Limit Factor $LF(k)$	$f\#(k)$	$f\$(k)$	Prob $(x > k)$
250,000	33,205	0.49193	0.95387	0.32108	0.04613
500,000	41,145	0.60956	0.97759	0.44353	0.02241
1,000,000	48,520	0.71881	0.99007	0.57168	0.00993
1,500,000	52,336	0.77534	0.99409	0.64408	0.00591
2,000,000	54,764	0.81132	0.99599	0.69262	0.00401
2,222,000	55,590	0.82356	0.99654	0.70963	0.00346
2,250,000	55,686	0.82498	0.99660	0.71162	0.00340
2,500,000	56,475	0.83667	0.99707	0.72813	0.00293
3,000,000	57,760	0.85570	0.99775	0.75554	0.00225
4,000,000	59,577	0.88262	0.99853	0.79558	0.00147
5,000,000	60,813	0.90093	0.99896	0.82377	0.00104
8,889,000	63,347	0.93848	0.99959	0.88447	0.00041
10,000,000	63,760	0.94459	0.99966	0.89480	0.00034
20,000,000	65,577	0.97151	0.99990	0.94239	0.00010
100,000,000	67,208	0.99567	1.00000	0.98995	0.00000

*$\mu = 8.8123226 \quad \sigma = 2.1482831$