CLASSICAL PARTIAL CREDIBILITY WITH APPLICATION TO TREND

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Abstract

Even with the recent advances in Bayesian credibility theory, there remain situations in which some may prefer the classical approach. Such situations may include data limitations, the failure of Bayesian model assumptions, the desire to incorporate a broader class of auxiliary information, ease of calculation and explanation, or just the force of tradition.

This paper discusses a probabilistic interpretation of the classical square root rule which provides some rationale for its use. The same rationale applied to trend projections leads to a similar rule, which utilizes the relative goodness of fit of the trend line.

While classical credibility for pure premiums is calculated from the volume of data used, the importance of volume is only in determining certain confidence intervals, which in turn determine credibility. In the trend model, the relative goodness of fit determines the confidence intervals. Using these confidence intervals in the same manner as in the pure premium case yields classical credibilities for the trend.

Volume is important here only to the extent that the stability it imparts contributes to the goodness of fit. As there may be other influences affecting the fit, volume alone does not guarantee high credibility in the trend case.

Credibility requirements under the Normal Power approximation also are reviewed. For these a partial credibility method different from the square root formula is indicated.

Partial credibility in the "classical" approach (Longley-Cook [5]) has often been presented in a somewhat ad hoc fashion, not particularly related to the statistical development of the full credibility standard.

An exception is provided by the "limited fluctuation" development (DeVylder [2] and Hossack, Pollard, and Zehnwirth [4]), which shows that the square root rule can be given a reasonable probabilistic interpretation when the full credibility standard is developed from a normal approximation to aggregate losses. The limited fluctuation concept is similar to an interpretation of credibility theory found in the 1932 *PCAS* (Perryman [8]).

The present paper outlines the limited fluctuation interpretation of credibility and uses it to develop a classical credibility approach to trend. As a further illustration, this method is extended to the computation of partial credibilities when the normal power approximation to aggregate losses is used to develop a full credibility standard (Mayerson, Jones, and Bowers [6]). To present this method clearly, a review of the standard credibility procedure is in order.

ELEMENTARY CREDIBILITY FROM AN ADVANCED STANDPOINT

The primary focus in classical credibility is the establishment of a full credibility standard. This is viewed as the expected number of claims needed to meet a predefined standard of stability of the aggregate losses. ("Aggregate losses" refers to the total dollar amount of the claims.) The standard is expressed in terms of confidence intervals. A typical standard would be that there be a 90% probability of observed aggregate losses for a year being within ±5% of the expected aggregate losses.

The limited fluctuation approach to partial credibility proceeds by establishing a confidence interval of the same precision and width as desired for full credibility, but centered at the credibility weighted estimate rather than at the observed mean. Some rationale for this method will be discussed below. This approach turns out to yield the square root rule for partial credibility in the case that aggregate losses are adequately approximated by a normal probability distribution. This distribution may not be a very good approximation in practice, but it is useful to illustrate the development of the theory. The development of the full credibility standard under this assumption proceeds as follows.

Since the normal distribution is symmetric about its mean, a 90% probability of the aggregate losses T being within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ corresponds to a 95% probability of T being below $\mathrm{E}(T)(1 + k)$. In general, a probability p of T being within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ translates to a probability of $.5(1 + p)$ of T being below $\mathrm{E}(T)(1 + k)$. For notational convenience, then, let $d = .5(1 + p)$ and y_d denote the dth quantile of the standard normal distribution, i.e., there is a probability d that a standard normal variate is less than y_d. For example, $y_{.95} = 1.645$.

Thus, to meet the standard of T being within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ with probability p, $k\mathrm{E}(T)$ must equal y_d standard deviations of T, i.e., $k\mathrm{E}(T) = y_d\sqrt{\mathrm{Var}(T)}$. To express this standard in terms of the number of claims requires an expression for the variance of T in terms of the moments of N, the number of claims, and

X, the claim size. This expression, derived in Appendix 2, is

$$\mathrm{Var}(T) = \mathrm{Var}(X)\mathrm{E}(N) + \mathrm{Var}(N)\,\mathrm{E}(X)^2.$$

Thus, the full credibility requirement is

$$k^2\mathrm{E}(X)^2\mathrm{E}(N)^2 = y_d^2\,(\mathrm{Var}(X)\mathrm{E}(N) + \mathrm{Var}(N)\mathrm{E}(X)^2)$$

or

$$\mathrm{E}(N) = (y_d/k)^2((\mathrm{Var}(X)/\mathrm{E}(X)^2) + (\mathrm{Var}(N)/\mathrm{E}(N))).$$

Now, this is supposed to be an equation for E(N), but E(N) also occurs on the right side. However, the ratio Var(N)/E(N) can often be treated as a constant of the frequency distribution. In fact for a Poisson frequency, this constant is 1.0. The negative binomial distribution with parameters x and p has $\mathrm{E}(N) = x(1-p)/p$ and $\mathrm{Var}(N) = x(1-p)/p^2$, so the ratio of variance to mean is $1/p$. As long as p does not change, the expected number of claims can increase or decrease due to the x parameter without influencing the variance to mean ratio.

For any frequency distribution, increasing E(N) by adding independent identically distributed exposure units does not change this ratio, because Var(N) will increase proportionally. (For independent risks, $\mathrm{E}(N + M) = \mathrm{E}(M) + \mathrm{E}(N)$ and $\mathrm{Var}(N+M) = \mathrm{Var}(N)+\mathrm{Var}(M)$. From this it follows that if $\mathrm{Var}(N)/\mathrm{E}(N) = r = \mathrm{Var}(M)/\mathrm{E}(M)$, then also $\mathrm{Var}(N + M)/\mathrm{E}(N + M) = r$.) In more sophisticated models, large risks or portfolios are not assumed to behave as aggregations of independently distributed exposure units, and then this ratio is not a constant (Meyers and Schenker [7]). However, this constancy will be assumed here. Thus, the full credibility standard can be written as

$$\mathrm{E}(N) = c\,(y_d/k)^2,$$

where $c = (\mathrm{Var}(X)/\mathrm{E}(X)^2) + (\mathrm{Var}(N)/\mathrm{E}(N))$ is a constant of the distribution. The first term of c can be denoted as CV^2 with CV the severity coefficient of variation. For example, with a Poisson frequency, $c = 1 + CV^2$.

A standard example (Longley-Cook [5]) is given by a Poisson frequency and a severity distribution with $CV = 0$ (constant severity) and thus $c = 1$. Taking $y_d = 1.645$ and $k = .05$ then yields $\mathrm{E}(N) = 1082.4$. This might be a reasonable standard for claim frequency, or for aggregate losses with constant severity. To achieve the same confidence intervals, still with a Poisson frequency, this standard would have to be multiplied by $1 + CV^2$ to account for severity variation. In Longley-Cook [5], this factor is referred to as $1 + S_c^2/M_c^2$ and in Hossack [4] as $1 + (\sigma/m)^2$.

It is of interest to note that c is also invariant under scale changes in severity, since $\text{Var}(X)/\text{E}(X)^2$ has this invariance. A scale change is a transformation that affects every claim by a uniform factor, such as simple monetary inflation (Venter [11]). Real world inflation may affect different claim sizes differently, however (Rosenberg and Halpert [9]). $\text{Var}(X)/\text{E}(X)^2$ is invariant in this sense because numerator and denominator both change by r^2 under a scale change of r. Thus, for given constants p and k, the credibility standard will not change due to growth of the business (i.e., addition of independent identically distributed exposure units) or uniform inflation.

In practical applications, $\text{E}(N)$ is often estimated by the number of claims arising. Thus, for example, if 1,082 expected claims is the full credibility standard, a body of experience with 1,082 claims may be deemed fully credible. The model, however, specifies a standard in terms of the exact expected number of claims. Using an estimate of this expected number changes the confidence intervals. Expected claims of 1,000 or of 1,164, for example, could occasionally produce 1,082 claims. Using $k = y_a\sqrt{c}/\text{E}(N)$ yields k's of .052 and .048 for these two expected values. Thus, the confidence interval widths arising in practice may be slightly different than those contemplated by the theory. This problem seems to be minor, given the degree of judgment used to select k originally.

PARTIAL CREDIBILITY

When $\text{E}(N)$ is less than the full credibility standard, a weighting scheme is used to estimate $\text{E}(T)$. The estimate, u, is a weighted average of the observed aggregate claims T with v, a previous estimate of $\text{E}(T)$. The previous estimate v can be regarded as the best available estimate of $\text{E}(T)$ without the observation T. Thus

$$u = zT + (1 - z)v.$$

Under the limited fluctuation partial credibility approach, the weight z is calculated so that there will be a probability p of u being within $k\text{E}(T)$ of $z\text{E}(T) + (1 - z)v$, where p and k are the defining constants of the full credibility standard. Thus the credibility estimate u is, with probability p, within the originally desired distance $k\text{E}(T)$ of a weighted average of $\text{E}(T)$ and the previous estimate v.

For u to meet this criterion, zT must be within $k\text{E}(T)$ of $z\text{E}(T)$ with probability p, as can be seen from the definition of u. This is equivalent to requiring T to be within $(k/z)E(T)$ of $E(T)$ with probability p. But this is just the full credibility requirement with k replaced by k/z. Thus, under the above assumptions, the expected number of claims needed for credibility z is

$$E(N) = c(y_d/(k/z))^2.$$

Comparing the resulting expected number of claims N_z needed for a credibility of z to the full credibility standard N_f yields that

$$N_z = z^2 N_f$$

or

$$z = \sqrt{N_z/N_f}.$$

That is, the credibility factor z for an expected number of claims N_z is just the square root of the ratio of N_z to the full credibility standard N_f, with a maximum of $z = 1$.

Also, since $(k/z)E(T)$ is the width of the p confidence interval when $E(N) = N_z$, then z is just the ratio of the target p confidence interval $kE(T)$ to the wider p confidence interval around $E(T)$, $(k/z)\,E(T)$, that arises for $E(N) = N_z$. As a result, the p confidence interval around $zE(T)$ is of the targeted width $kE(T)$, and thus there is a probability p of the credibility estimate u being within this target width of $zE(T) + (1 - z)v$.

This gives a reasonable probabilistic interpretation to the square root rule for partial credibilities. It does not, however, rule out other possible partial credibility rules which also may be reasonable. The classical approach is essentially pragmatic, and does not claim optimality.

For an example, again assume Poisson frequency and constant severity, so $c = 1$. Suppose 683 claims are observed, and this is taken as the estimate of $E(N)$. Using $k = y_d\sqrt{c/E(N)} = .063$, a 90% confidence interval of 683 $(1 \pm .063) = 683 \pm 43$ is computed. However, suppose an interval half width of $(.05)\,(683) = 34$ is desired, which is smaller than the actual by the ratio of $.050/.063 = .79$. The 90% confidence interval around $.79N = (.79)\,(683)$ is of the desired half width $34 = (.79)\,(43)$. Adding the constant $(1 - .79)\,v$ does not change this half width. Thus, taking $z = .79$ meets the limited fluctuation criterion, and this z can be simply calculated as the square root of the ratio 683/1082.

It is sometimes claimed in casual conversation that the classical credibility criterion is biased against downward estimates. There are two lines of reasoning used for this. The first notes that a portfolio with a smaller expected number of claims has a smaller confidence interval radius than one with more expected claims, and thus asserts that it is unfair to give it lower credibility.

This argument in effect questions the use of a target confidence interval expressed as a percentage of the expected losses, and favors an absolute confidence interval. There are good reasons for using a relative confidence interval, however. For instance, the resources to absorb adverse fluctuations are usually available in approximate proportion to expected losses. These resources may include surplus, investment income, and a profit/contingency provision in the rates. It should also be noted that a criterion based on absolute confidence intervals would give the greatest weight to the smallest volumes of data, which is just the opposite of what is intended by credibility.

The other argument for bias applies when the actual rather than the expected number of claims is used for credibility. The model assumes random fluctuations occur equally on either side of the expected value. However, downward fluctuations get lower credibility than upward ones, giving the whole procedure a slight upward bias.

To illustrate this, consider a case where the full credibility standard is $1089 = 33^2$ claims, and $E(N) = 1000$. Assume also that the previous estimate $v = 1000$. The credibility should be .958 based on $E(N) = 1000$. However, if credibility is based on the actual number of claims it will usually differ somewhat from this value. The credibility z and credibility estimate u are shown below for several n's that could arise.

n	z	u	n	z	u
1023	.969	1022	977	.947	978
1063	.988	1062	937	.928	942
1088	1.000	1088	912	.915	919
		1047.6			954.8

As can be seen, the fluctuations above the expected value do produce slightly larger indicated changes than do those below the mean. In fact the average estimate produced is 1001, so there is a 0.1% expected upward bias in this case. The weights used for each row to compute this average are .4679, .3607, and .1713. These and the n's selected derive from the 6 point Gaussian quadrature integration procedure for the interval (905, 1095), which under the normal approximation contains about 99¾% of the values of n that could arise.

The 0.1% expected bias in this case comes about because the practice departs from the theory, i.e., the credibility is calculated based on the latest observed rather than the expected number of claims. This problem need not occur in other applications of the limited fluctuation theory which use some other estimate for expected claims.

To summarize classical normal approximation credibility, then, a full credibility standard is first established, based on a specified high probability of the data being within a specified narrow band around the expected value being estimated. Partial credibility standards are then derived by requiring that the credibility weighted estimate be within just as narrow a confidence band, but this confidence band is now centered at the credibility estimate. The partial credibility z then turns out to be the ratio of the width of the target full credibility confidence interval to the corresponding confidence interval produced by the actual data.

Does classical credibility theory make sense in this form, and if so, under what circumstances?

Assumptions for aggregate losses (e.g., approximately normally distributed) that lead to the confidence interval properties of the credibility estimator have been given, but the relationship between the observed aggregate losses, those being estimated, and the previous estimate need to be clarified in order to evaluate the methodology.

Without formulating a specific model, the credibility estimate seems useful when a situation like the following is involved.

Things (i.e, the underlying processes) tend to be fairly stable over time, but occasionally they change, and these changes are of varying degrees and directions. Observations fluctuate randomly around the underlying processes, and the degree of this latter fluctuation is fairly well known. Rates should respond to fundamental changes but not to fluctuations.

Under such a scenario, it seems reasonable to set up a target confidence criterion with respect to the random fluctuations so that the latest year's indication will be used at face value if the confidence interval this experience produces is tight enough, in reference to selected constants p and k. This would delimit the degree of random fluctuation that would be deemed acceptable.

At the other extreme, if no observation can be made, the previous estimate will continue to be used. Between these extremes, a weighted average of the observation and the previous estimate seems like a reasonable and appropriate choice. What should be the weights? One possibility is to attribute just enough weight to the last observation so that that observation gives the resulting weighted average only the degree of random fluctuation that has already been deemed acceptable. As the above analysis has demonstrated, this is the result the classical credibility procedure produces, given the assumptions involved.

Thus, although no claims about optimality are advanced, the classical procedure can at least be seen to have a reasonable probabilistic interpretation. It may be particularly useful when the premises of Bayesian credibility, such as homogenity over time, cannot be assumed to hold, when the data is not available to do a full Bayesian credibility analysis, and when the auxiliary data to be incorporated comes from a different source, such as broader economic indices.

In the next section, the above procedure will be used to develop a classical credibility standard for trend projections. In Appendix 1, it is used to produce partial credibility when the normal power approximation to aggregate losses is employed.

CREDIBILITY FOR A TIME TREND

To apply classical credibility to a trend projection, a full credibility standard relative to p and k must first be determined. In the classical spirit, this can be specified as follows: a projected point will be deemed fully credible relative to p and k if there is a probability of at least p that the actual value being projected will fall within $1 \pm k$ of the projected point.

Note that this standard is more restrictive than in the aggregate loss credibility framework in that it requires the realization of the random variable, not just its expected value, to be in the interval. Accordingly, a larger value of k may be deemed appropriate for a given p in this situation than for aggregate losses. There may be other ways to specify a reasonable full credibility standard, but the above definition will be used herein. As in the classical approach, the target confidence interval is expressed as a percentage of the estimate, which seems appropriate for most of the reasons advanced above.

There are standard statistical formulas, in texts covering regression, for calculating confidence intervals around a trended point. In general, these utilize the number of points in the experience period, the number of points forward the projection is carried, and the goodness of fit of the least squares line. Let us suppose, then, that the line is based on n equally spaced observed points and the projection of interest is m points beyond the midpoint of the observations. Goodness of fit will be measured by s, where $(n - 2)s^2 = SSR$, the sum of the squares of the residuals, i.e., the sum of the squared differences between the observed and fitted points. The $n - 2$ is an adjustment for degrees of freedom, because 2 parameters are required for fitting a line.

Under normal least squares assumptions, to be discussed further below, the usual formulas yield that the standard deviation of the projected point is

$$s\sqrt{1 + (1/n) + 12m^2/(n^3 - n)}$$

and the p confidence interval measures

$$t(d,n - 2)s\sqrt{1 + (1/n) + 12m^2/(n^3 - n)} \tag{1}$$

on each side of the projected point, where $t(d,n - 2)$ is the $100d$th percentile of the t distribution with $n - 2$ degrees of freedom, and, as before, $d = .5(1 + p)$. Formulas that reduce to these for a time trend can be found in many regression texts. The confidence interval incorporates both the variance of the subsequent point from its expected value on the line and the uncertainty as to where the line really is, since its parameters are estimated.

To use this confidence interval for credibility, it is first necessary to select p and k. For example, a 90% confidence interval of ± 10% of the projected value might be chosen as the full credibility standard. Then the actual p confidence interval is measured for the data at hand. Suppose, for example, the 90% confidence interval around the projected point is found in fact to be ± 12.5% of the projected value. Then, following the principles of classical credibility, the partial credibility for the particular case at hand would be the ratio of the full credibility interval to the actual interval. In this case the ratio is .10/.125 = .80, and thus the trend projection receives 80% credibility.

Applying a credibility factor in this manner limits the possible random deviation of the credibility weighted estimate to the targeted amount, i.e., to $\pm k$ of the projected point. However, the resulting confidence band, while of the desired width, is not centered on the value being estimated, but rather on the weighted average of this value with a previous estimate v. This is precisely what the classical procedure does in the aggregate loss case as well.

In other words, the credibility estimate is z times the projected point plus $1 - z$ times the prior expectation. The p confidence band around this estimate has been shrunk by a factor of z, which is chosen to give the resulting confidence band a width of k times the projected point. Then p expresses the probability that the credibility weighted average of the actual value being projected and the prior expectation will be within the given interval around the credibility estimate.

The theory does not specify what the prior estimate v should be, but it seems reasonable to stipulate that v is the best estimate available prior to the current projection. Possibilities may include a previous projection; a projection based on a wider population, e.g., countrywide data; or a projection based on a broader economic perspective, e.g., pure inflationary considerations.

An example of this method is given in Appendix 3, for a loss ratio trend. A loss ratio of .647 at current rate level is projected, with a 90% confidence interval of $\pm .159$. If the full credibility standard is taken to be a 90% confidence interval of $\pm .0647$, a credibility of $z = .0647/.159 = .41$ results. Thus $1 - z = .59$ will apply to the prior estimate. Suppose the prior estimate is $v = .620$. Then the credibility estimate is $u = (.59)(.620) + (.41)(.647) = .631$.

The probabilistic interpretation of this procedure is then as follows. There is a 90% probability that the expected loss ratio E being estimated is within .159 of .647. Thus, there is also a 90% probability that $.41E$ is within $(.41)(.159) = .065$ of $(.41)(.647) = .265$. Adding $.59v = .366$ to this shows that there is then a 90% probability that the credibility estimate $u = .265 + .59v$ is within .065 of $.41E + .59v$.

PROS AND CONS OF THE METHOD

The confidence interval approach to credibility for trend has several advantages and some disadvantages, as enumerated below. Some features of the method have positive and negative aspects, and thus are listed under both.

Advantages

1. The method is derived explicitly from a statistical model. Thus, it is possible to describe the estimate in probabilistic terms. It is not based on analogy or ad hoc reasoning.
2. Credibility bears a direct relationship to the goodness of fit of the trend line.
3. Since the model is simple, the concepts are relatively easy to explain and the estimation is not difficult to carry out.

4. The method leaves room for the informed judgment of trained experts, both in the selection of the full credibility standard and in the choice of the prior estimate. This makes the method responsive to the needs of different constituencies, which may have different evaluations of the applicability of the various sources of prior data, such as countrywide data or broader economic trends.

Disadvantages

1. The method does not optimize anything. This is in contrast to the modern least squares credibility approach, which does optimize a specific error function.
2. Subjective judgment is required. This is again in contrast to the least squares approach, in which all estimates are produced strictly from the data with no input from subjective probabilities called for. While informed judgment can truly be an advantage over purely data driven methods, judgment can be inconsistent over time and circumstances, and poor judgment can be a disadvantage.
3. The model requirements, while simple, are restrictive. The usual regression assumptions, for example, include normality of the residuals. This assumption can be tested, however, as is discussed further in Appendix 4. If normality is not found, it still may be possible to estimate confidence intervals by other means.

In summary, classical credibility, which can be thought of as a ratio of confidence intervals, can be extended directly to apply to trend. This has several advantages, including flexibility and ease of application and exposition. It is a pragmatic approach with a probabilistic interpretation, but is not derived as a statistical optimization. This leaves open the possibility that, under further assumptions about the statistical relationship between the data and a specific prior estimate, a different credibility procedure can be derived that optimizes a specified error measure.

LEAST SQUARES ASSUMPTIONS

The normal least squares assumptions provide that the various years' observations T_i are normally distributed random variables, each with the same variance, and with the expected value for each given as a linear function of time, i.e., $E(T_i) = a + bi$.

In application, these assumptions may only hold as approximations. In some cases, for instance, the expected values may move as a non-linear function of time. Also, the data is often adjusted to remove systematic influences, e.g., rate changes and benefit changes, before the linear model is fit.

Further, nothing in the model assumptions requires the variance to be due to frequency and severity distributions alone. Price levels, the level of economic activity, and reserving changes could all contribute to the variance of the individual results from their expected values on the line. Thus the volume of experience underlying each point is not the sole determinant of the variance, and in fact may be overshadowed by other factors.

DEVELOPING A WORKING FORMULA

A projected point is fully credible p,k if the p confidence interval around the projected point has radius no more than k times that point. By (1) and the definition of s, this criterion will be fulfilled if

$$k\ PRO = t(d,n-2)\sqrt{(1+(1/n)+12m^2/(n^3/n))SSR/(n-2)}. \tag{2}$$

Here PRO denotes the projected point. Also, for credibility z, a confidence interval of $(k/z)\ PRO$ is required, by the limited fluctuation principle. This can be expressed by substitution (k/z) for k in (2).

Rearranging terms then leads to

$$SSR_z/PRO^2 = k^2(n-2)/[z^2 t(d,n-2)^2(1+(1/n)+12m^2/(n^3-n))] \tag{3}$$

where SSR_z is associated with credibility z. Thus

$$SSR_1/PRO^2 = k^2(n-2)/[t(d,n-2)^2(1+(1/n)+12m^2/(n^3-n))] \tag{4}$$

gives the full credibility standard relative sum of squared residuals in terms of p and k, the selected criteria; n, the number of points used to fit the line; and m, the number of points projected beyond the midpoint of the n original points.

Full credibility is expressed by a relative SSR, not an absolute SSR, because the target confidence interval is specified as a percentage of the projected value. As with credibility for aggregate losses, a smaller absolute confidence interval can lead to lower credibility if that interval is wider relative to the value being estimated, and again this appears to be entirely appropriate.

From (4), the full credibility relative *SSR*'s are given for various increasingly specific assumptions below. First, take $p = .90$, so $d = .95$, and assume 5 points are used to fit the line so $n = 5$. Then $t(d, n - 2) = 2.353$ and:

$$SSR_1/PRO^2 = 5.418\, k^2/(12 + m^2).$$

If $k = .06$, this becomes

$$SSR_1/PRO^2 = .0195/(12 + m^2).$$

A typical projection may be to point 7.5, so $m = 4.5$ points beyond the midpoint of the data. This yields

$$SSR_1/PRO^2 = .0006.$$

This full credibility standard for the relative *SSR* only coincidentally is $.01k$. For $k = .05$, the standard is .0004, and for $k = .07$ it is .0008.

For a given *PRO*, (4) can be divided by (3) to yield

$$z = \sqrt{SSR_1/SSR_z},$$

which is the square root rule for partial credibility for trend. Here SSR_z is the actual *SSR* for the fitted line, and SSR_1 is the target relative *SSR* multiplied by PRO^2.

MAKING THE JUDGMENTS

Given the above working formulas, choosing p and k can be replaced by selecting a target full credibility relative *SSR*. This is perhaps a more reasonable judgment to make. Instead of picking p's and k's in advance, experienced actuaries, having a feel for the ratemaking process as a whole, and also for their corporate goals, may prefer to review a collection of fitted lines and select those which can be regarded as fully credible for ratemaking use. However, the resulting p's and k's may be a useful part of this review.

Such a process is also advantageous in that it is less tied to the normal distribution assumption of the model. The selection of the full credibility relative *SSR* can be made with recognition that the residuals may not be normally distributed, and that the confidence intervals involved might actually be wider than the model would predict for that relative *SSR*.

A judgment could also be made that a wider confidence interval may be acceptable when a longer projection is necessary, in recognition of the inherently greater uncertainty involved with a longer projection. One way to reflect this is to keep the target relative *SSR* constant under various projection periods.

Under these circumstances, the Actuarial Committee of the National Council on Compensation Insurance adopted a target relative *SSR* of .0006 for a five-year fitted trend line. As noted above, this results in $k = .060$, that is, a 90% confidence interval radius of 6.0% of the projected value when $m = 4.5$, which corresponds to a 2.5-year projection. For this relative *SSR* and $m = 5.5$, a 3.5-year projection, $k = .068$, that is, there is a 90% confidence interval radius of 6.8% of the projected value. Normally, workers' compensation ratemaking uses a projection period of 2.5 to 3.5 years.

The more general target relative *SSR* of $.0195/(12 + m^2)$ can be used for other projection periods. This maintains the target relative 90% confidence interval radius at 6.0% regardless of the length of the projection.

The Committee also noted that the above formula for the confidence interval around a projected point allows for random fluctuation of the projected loss ratio as well as for uncertainty about the parameters of the regression line. If only the latter were to be considered, the resulting confidence interval would actually be tighter than the formulas indicate.

This indication of a tighter interval may in part be counterbalanced by the possibility that residuals are not normally distributed. Although that distribution was not rejected by standard tests, the tests are not definitive in this context. To the extent that the residuals are from a skewed distribution, the target confidence interval may be wider than the formulas suggest.

Practical considerations such as these support the approach of selecting a target relative *SSR* based on informed judgment which considers, but is not strictly limited by, the implications of the statistical model.

The complement of credibility in this framework should apply to the best estimate of the trended point available prior to the projection that is being weighted. Logical candidates for this are projections based on the countrywide trend, the previous trend in the state, or broader economic indices. The assumption of no trend would not be appropriate unless there is an a priori reason to believe the trend is in fact flat. There may be, for example, good reason to believe this for the ratio of workers' compensation indemnity losses to payroll. However, as medical costs have been increasing faster than payroll in the economy at large, the ratio of medical losses to payroll could not be expected a priori to show no trend.

As medical benefits are quite similar across states, and are subject to similar inflationary influences, the latest available countrywide trend factor was selected as the prior estimate to be used with the complement of credibility for the medical pure premium trend. For indemnity trend, this was felt to be inappropriate, due to widely differing benefit laws. Zero trend was chosen as the prior estimate because of its a priori reasonableness.

APPENDIX 1
CREDIBILITY WITH THE NP APPROXIMATION

Mayerson, Jones, and Bowers [6] note that the normal approximation is inappropriate for casualty insurance aggregate claims distributions because these are almost always positively skewed. They suggest using the NP (normal power) approximation instead, although they never use that term. The NP adjusts the normal approximation for skewness. If t_d is the dth quantile of T, i.e., $\Pr(T \leqq t_d) = d$, and y_d is the dth quantile of the standard normal distribution, then the NP approximation is

$$t_d = \mathrm{E}(T)(1 + c_T(y_d + s_T(y_d^2 - 1)/6)), \tag{5}$$

where for any random variable X, c_X is the coefficient of variation (ratio of standard deviation to mean) and s_X is the coefficient of skewness (ratio of third central moment to the cube of the standard deviation). In this notation, the normal approximation is

$$t_d = \mathrm{E}(T)(1 + c_T y_d),$$

which is the NP with zero skewness.

The NP approximation arises from the first few terms of the Cornish-Fisher expansion, an infinite series expansion which expresses the percentiles of a distribution in terms of its moments. This is an alternating series expansion and is not necessarily convergent. Thus, adding more terms may or may not significantly improve the accuracy of this approximation. See Beard, Pentikainen, and Pesonen [1] for further discussion of this approximation.

Full Credibility with the NP

As with the normal approximation, the starting point for credibility is to find a full credibility standard such that T is within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ with probability p. The NP does not in general provide a symmetric distribution of T around $\mathrm{E}(T)$; however, requiring T to be below $(1 + k)\mathrm{E}(T)$ with probability $d = (1 + p)/2$ is generally assumed to be sufficient for T to be within $\mathrm{E}(T) \pm k\mathrm{E}(T)$ with probability p for positively skewed aggregate claim distributions. This will be assumed for now, but it is discussed further below. With this assumption, the full credibility requirement gives the equation $\mathrm{E}(T)(1 + k) = t_d$, which must be solved for $\mathrm{E}(N)$ to get the full credibility standard. $\mathrm{E}(N)$ does not appear in this equation, but it is an element of both sides.

Employing the NP approximation (5) at this point and solving for k yields

$$k = c_T(y_d + s_T\,(y_d^2 - 1)/6).$$

To solve for E(N), c_T and s_T must be expressed in terms of N and X, i.e., frequency and severity. The methods of Appendix 2 provide the following formulas for the coefficients of variation and skewness of aggregate losses

$$c_T^2 = (c_x^2 + n_2)/\mathrm{E}(N), \text{ and}$$
$$s_T = (s_x c_x^3 + 3n_2 c_x^2 + n_3)/c_T^3\mathrm{E}(N)^2,$$

where n_i is defined by $\mathrm{E}(N)n_i = \mathrm{E}(N - \mathrm{E}(N))^i$. For example, n_2 is the frequency ratio of variance to mean. For the Poisson, $n_2 = n_3 = 1$.

Introducing further notation for the numerators of the aggregate moments will simplify these expressions. Let

$$M_2 = c_x^2 + n_2, \text{ and}$$
$$M_3 = s_x c_x^3 + 3n_2 c_x^2 + n_3.$$

M_2 and M_3 are shape descriptors for the aggregate loss distribution. For example, $M_2/\mathrm{E}(N)$ is the square of the coefficient of variation of aggregate claims, and, in fact, M_2 is the adjustment factor c referred to above in the discussion of the normal approximation credibility formulas. M_3 is a third moment measure for aggregate losses, and comes up frequently in calculations. With this notation the formula for k becomes

$$k = y_d\sqrt{M_2/\mathrm{E}(N)} + (M_3/M_2)(y_d^2 - 1)/6\mathrm{E}(N).$$

This equation can be solved in general for the full credibility standard. Considering it to be a quadratic equation in $\sqrt{\mathrm{E}(N)}$ gives

$$2k\sqrt{\mathrm{E}(N)} = y_d\sqrt{M_2} + \sqrt{y_d^2 M_2 + 2k\,(y_d^2 - 1)M_3/3M_2}.$$

The resulting value of E(N) is the full credibility standard based on the NP approximation. Setting the last term under the square root to zero gives the formula for the normal approximation full credibility standard. Thus, that term is the end result of the NP adjustment.

Partial Credibilities

The limited fluctuation method can be used to calculate partial credibilities under the NP approximation. Following the development in the text, for an E(N) less than N_f, the partial credibility z represents a scaling factor that scales the p confidence interval that would arise for that number of claims down to the target p confidence interval of $k\mathrm{E}(N)$. The number of claims that generates a credibility of z, i.e. N_z, can be developed from the above full credibility formula by replacing k with k/z. This allows for the calculation of credibility tables, but no simple relationship, such as the square root rule of the normal approximation, is evident.

An example would probably be useful at this point. Consider a case with a Poisson frequency distribution and a lognormal severity with a coefficient of variation of 7.0. For the Poisson, $n_2 = n_3 = 1$. Thus $M_2 = 50$. For the lognormal generally, $s_x = c_x^3 + 3c_x$, so in this case $s_x = 364$. Thus $M_3 = 125{,}000$. Take a 90% confidence interval, so $y_d = 1.645$. Then $2k\sqrt{\mathrm{E}(N)} = 11.632 + \sqrt{135.3 + 2843k}$. If $k = .05$, $\mathrm{E}(N) = 80{,}026$ is then the full credibility standard. Replacing k by k/z gives the following standards N_z for partial credibility z

z:	.25	.50	.75
N_z:	9,103	25,786	49,468
z^2N_f:	5,001	20,007	45,015

The square root rule partial credibility criteria z^2N_f for this N_f are consistently lower.

The high credibility requirements in this example derive in part from the large severity CV assumed. For high limits of insurance or unlimited coverage, CV's of this magnitude have been reported by actuaries involved in various lines of commercial property and liability insurance (LeRoy Simon in his review [10] of the Mayerson, Jones, and Bowers paper).

Instead of the Poisson, a negative binomial frequency can be assumed. The negative binomial can be described by means of two parameters, x and p, so that $\Pr(N = n) = \binom{x+n-1}{n} p^x(1-p)^n$, with moments $\mathrm{E}(N) = x(1-p)/p$, $n_2 = 1/p$, $n_3 = (2-p)/p^2$. This illustrates that n_2 and n_3 can be considered fundamental measures of the shape of the frequency distribution, in that they are functions of p only, while the mean can be changed by moving x. Dropkin [3] found that $n_2 = 1.184$ in an automobile insurance study. This implies $p = .8446$ and so $n_3 = 1.620$. In the above example, this increases M_2 to 50.184 and M_3 to 125,027.668. Thus, $2k\sqrt{\mathrm{E}(N)} = 11.653 + \sqrt{135.8 + 2834k}$. For $k = .05$ this yields $\mathrm{E}(N) = 80{,}153$ expected claims for full credibility. Thus, in this case, the full credibility standard is not significantly changed by going to the negative binomial assumption.

Another example of a negative binomial frequency distribution is provided by Meyers and Schenker [7]. They discuss a workers' compensation setting in which the compound process of picking a risk at random from a class and observing its number of claims can be described by a negative binomial distribution. In their case each risk has a Poisson claim count distribution, and the risks' Poisson parameters are gamma distributed across the class. The resulting negative binomial distribution with mean E(N) is estimated to have $n_2 = 1 + .037\mathrm{E}(N)$. Since n_2 is increasing with risk size, very large values can arise for large risks. In this model, a portion of the uncertainty about a risk's claim count comes from the distribution of risks in the class, and this portion is not reduced by increasing the risk size. Essentially n_2 is no longer a fundamental frequency constant, but depends on E(N).

Ignoring the context, however, suppose a negative binomial distribution is given with a constant but large n_2, say $n_2 = 51$. Then $p = 1/51$, and $n_3 = 5151$. In the above example these values give $M_2 = 100$ and $M_3 = 137{,}500$. Thus $2k\sqrt{\mathrm{E}(N)} = 16.45 + \sqrt{270.6 + 1564k}$, and $k = .05$ gives $\mathrm{E}(N) = 123{,}385$. Thus the negative binomial model does make a considerable difference when n_2 is large.

The lognormal assumption increases the skewness in these examples over what some other distributions would provide. A Weibull distribution with a CV of 7 has a shape parameter of .2678046 and thus skewness of 44.44. In the Poisson case above this reduces M_3 to 15,391. Thus $2k\sqrt{\mathrm{E}(N)} = 11.632 + \sqrt{135.3 + 350.1k}$, or, for $k = .05$, $\mathrm{E}(N) = 57{,}568$.

One-and Two-sided Intervals for Skewed Distributions

Previously it was stated that the NP approach to credibility usually assumes that if T has a probability of $d = (1 + p)/2$ of being below $(1 + k)\mathrm{E}(T)$, then T will be within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ with probability at least p. That this is not necessarily true for positively skewed distributions is shown in the following example.

Assume T is Pareto distributed with distribution function $F(t) = 1 - (1 + t/2.5)^{-3.5}$. Then $\mathrm{E}(T) = 1.0$ and $F(1.0) = .6920$. Take $p = .5$, so a 50% symmetric confidence interval around 1.0 is sought. This interval is $1.0 \pm .6898$ as can be verified using $F(t)$. Since $p = .5$, $d = .75$, and since $F(1.215) = .75$, $t_1 = 1.215$. However, the probability of T being in the interval $1.0 \pm .215$ is less than 50%; in fact it is only 13.45%.

This arises because E(T) is well above the median of the distribution, so going only a small distance above E(T) reaches the 100dth percentile for this d. For this distribution, this situation holds up to a p = 72.14% confidence interval around the mean, i.e. up to 1.0 ± .8905. For this interval, the probability d of T being less than E(T)(1 + k) is given by (p + 1)/2 = 86.07% exactly. For higher p, the desired relationship does hold, i.e., being below the (1 + p)/2 quantile is enough to guarantee that the corresponding symmetric interval contains at least p in probability.

Since it is higher confidence intervals that are of interest in credibility, the assumed relationship would be fulfilled in this case. However, a more highly skewed distribution would place the mean at an even higher percentile, which would aggravate this problem. However, most loss distributions for which this credibility procedure is intended are not so highly skewed that this would be likely to occur. In fact the NP itself is of questionable accuracy for highly skewed distributions.

Applicability of the NP to Skewed Distributions

To investigate this, the percentiles of the several distributions are calculated directly and by the NP approximation. The Pareto distribution $F(t) = 1 - (1 + t/b)^{-s}$ has moments defined by

$$E(T^n) = \prod_{i=1}^{n} ib/(s - i).$$

Using these with the above values (b = 2.5, s = 3.5) yields, after some algebra, $c_T^2 = 7/3$ and $c_T s_T = 18$. Thus the NP approximation becomes

$$t_d = 3y_d^2 + 1.5275y_d - 2.$$

This is compared to the actual values of t_d for this distribution below.

d:	.10	.25	.50	.75	.90	.95
t_d NP:	.9698	−1.665	−2.000	.3952	4.885	8.630
t_d Actual:	.0764	.2142	.5475	1.215	2.327	3.384

The NP approximation is clearly not appropriate for this distribution. From the table, the NP might be reasonably accurate for a small range of values somewhere in between the 75th and 90th percentiles. For the right hand tail, it clearly overstates the percentiles. The problem here apparently is the high skewness.

Distributions of this great a skewness are not likely for large portfolios of risks, for which the NP was originally developed. The aggregate claim distribution for a small portfolio or a single risk could easily be this highly skewed, however, and use of the NP could lead to large errors in such a case.

For less skewed distributions (e.g., skewness below 1.0) the NP can be fairly accurate. Two distributions, the gamma and the Weibull, are compared below to their NP estimates. Both of these distributions are assumed to have mean 1 and standard deviation 1/3, which fixes their parameters. The gamma then has skewness of 2/3, while that for the Weibull is approximately .077. The percentiles are shown below.

d:	.01	.05	.25	.50	.75	.95	.99
t_d Gamma:	.390	.522	.760	.963	1.200	1.604	1.934
t_d NP:	.388	.515	.755	.963	1.205	1.611	1.939
t_d Weibull:	.277	.454	.765	.998	1.231	1.554	1.770
t_d NP:	.243	.459	.773	.996	1.223	1.556	1.794

The NP approximation is reasonably close for both distributions, although at the extremes it is better for the gamma than for the Weibull.

APPENDIX 2
FORMULA FOR VAR(T)

T is the sum of the individual claims X_i, where i runs from 1 to N, the number of claims. Since N is a random variable, both frequency and severity contribute to the variance of T. It is generally assumed that all claims have the same distribution, and that individual claim sizes are independent of each other and of N.

To compute the variance of T under these assumptions, begin by calculating $E(T^2/N = n)$, i.e., fix the number of claims at n and find $E((X_1 + \ldots + X_n)^2)$.

Expanding the square yields n^2 terms of the form X_iX_j. When $i = j$ the expected value of the term is $E(X^2)$. Otherwise, it is $E(X)^2$, since then X_i and X_j are independent. Thus

$$\begin{aligned} E((X_i + \ldots + X_n)^2) &= nE(X^2) + (n^2 - n)E(X)^2 \\ &= n\text{Var}(X) + n^2E(X)^2. \end{aligned}$$

Now, by general considerations of conditional expectations, $E(T^2) = E(E(T^2/N = n))$. Thus, taking the expected value of the above equation with respect to N gives

$$\begin{aligned} E(T^2) &= E(N)\text{Var}(X) + E(N^2)E(X)^2 \\ &= E(N)\text{Var}(X) + \text{Var}(N)E(X)^2 + E(N)^2E(X)^2. \end{aligned}$$

The last term is just $E(T)^2$. Subtracting it from both sides then yields

$$\text{Var}(T) = E(N)\text{Var}(X) + \text{Var}(N)E(X)^2.$$

APPENDIX 3

LOSS RATIO TREND EXAMPLE

The points labeled "Line" below were computed from the formula Line = .9684 − .0428 Year, and represent the least squares fit to "Data".

Year	Data	Line
1	.909	.926
2	.929	.883
3	.819	.840
4	.767	.797
5	.776	.754

The point for year 7.5 is projected to be .647, and the 90% confidence interval around this point is sought. The sum of the squared residuals is .00423, so $s = .03755$, since $3s^2 = .00423$. For year 7.5, $m = 7.5 - 3 = 4.5$, so $1 + 1/n + 12m^2/(n^3 - n) = 3.275 = 1.796^2$. Also, $t(.95,3) = 2.353$. Thus, the 90% confidence interval is $.647 \pm (2.353)(.03755)(1.796) = .647 \pm .159$.

APPENDIX 4
TESTING RESIDUALS FOR NORMALITY

As mentioned in the text, the confidence interval calculation relies on the assumption of normally distributed residuals. To some extent this assumption is testable, but for a trend based on a small number of data points, the tests are not particularly conclusive.

The SAS package provided a test of normality for small samples, namely the Shapiro-Wilk W statistic. W is the ratio of two estimates of the variance of the residuals, one (the numerator) based on order statistics, and the other the usual sample variance approach. This ratio is between 0 and 1, and small values lead to the rejection of normality.

For example, in Appendix 3, $W = .881$ was calculated by SAS. From the critical values provided by Shapiro and Wilk, the probability of a lower value of W arising from a sample of 5 from a truly normal population is 35%. This is not a low enough value to reject normality.

Since tests like this are not conclusive for small samples, one may want to appeal to general principles. In the case at hand, loss ratios are usually believed to have positively skewed distributions, so it may seem inappropriate to assume a normal distribution.

Three comments are in order, however:

1. In some cases the skewness may be small enough that the normal approximation is reasonable.

2. In some cases the deviations of the expected loss ratios for each year from the trend line may follow a normal distribution, and the deviation of the actual loss ratio from the expected for the year a positively skewed distribution. If the deviations of the expected from the line have a greater magnitude than the deviations of the actual from the expected, the normal approximation may not be too bad overall.

3. Confidence intervals using a skewness correction could possibly be developed in cases where a positive skewness is significant. In light of the role of informed judgment in selecting the full credibility standard, however, an explicit calculation of this type may not be required for moderately skewed distributions.

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ADDRESS TO NEW MEMBERS—MAY 12, 1986

CHARLES C. HEWITT, JR.

INTRODUCTION

It is a genuine pleasure for me to address the new members of the Casualty Actuarial Society today. While my remarks are primarily intended for the new Fellows and Associates, I hasten to recognize the role that your partners, spouses or otherwise, have played in the achievement which has just been recognized. So, my remarks are also addressed to those, here today, who have sacrificed in order that the person sitting next to you may now append the letters FCAS or ACAS to her name or his name.

Incidentally, for you spouses and partners of new Fellows, the old alibi, "But I have to study for exams" is no longer valid. Back to changing diapers and all the other joys of conjugal life.

When Phil Ben-Zvi called me to ask if I would accept this assignment I was particularly delighted for a very personal reason. One of the new Fellows is a young lady whom I recruited for the profession. Next to being a parent and enjoying the achievements of one's children, there is no greater satisfaction than participating in the success of someone for whom you have opened the door. The young lady knows who she is and I'm not going to embarrass her by identifying her publicly. But I do want to say to her, "Rhonda, you did it on your own and I'm very proud to be here today to participate in this important moment in your life!"

My remarks will be brief. I intend to cover three general areas which I will label COMMUNICATION, ACTUARY, and SPAN. The reason for this rather awkward choice of labels will become apparent later, although some of you may see through this selection of titles.

COMMUNICATION

Actuaries, as a class, are literal-minded people. They, too often, assume that words speak for themselves, thus ignoring the importance that tone of voice plays in oral communication, or that it is necessary to lay groundwork and provide emphasis in all forms of communication. They should be aware of "body language" as a means of understanding what others are trying to communicate. It took me thirty years of marriage to realize that my wife listened more closely to my tone of voice than to my words. It took me somewhat less time to understand that when she took out her emery boards and began working on her finger nails with a vigor that would have cut through solid oak, she was upset about something, but was not yet ready to discuss it.

In thinking about methods of communication, I am reminded of the story of the Irish priest whose sermons were constantly filled with vilification of the English. Word of this reached his bishop and the latter decided to attend mass on Palm Sunday and listen to the priest's sermon. As usual, the Irish priest managed to lambaste the English to a fare-thee-well. After mass the bishop took the priest aside. "My son," he said, "You do preach a good sermon, but do you really think it's proper to bring your political feelings into discussions of the Lord's work? Now, I would suggest that in the future you omit any reference to the English even though it's obvious your feelings on this subject are very strong." The following Sunday, Easter, the priest declared that his topic was to be the Last Supper. He described how Christ announced that one of his disciples had betrayed him, and how Christ proceeded to go around the supper table, one disciple at a time, asking who it was. "And each disciple answered firmly 'Not I, Lord' until Jesus came to Judas Iscariot. And, Jesus asked, 'Was it you, Judas, that betrayed me?' And, Judas replied, 'Blimey, guv'ner, it wasn't me!'"

Actuaries have a terrible time making themselves understood by people who are not actuaries! That bald statement of self-criticism is worth repeating. Yes, ACTUARIES HAVE A TERRIBLE TIME MAKING THEMSELVES UNDERSTOOD BY PEOPLE WHO ARE NOT ACTUARIES! Ask any senior member of our organization who is currently in an administrative position which factor most influences him or her when employing or promoting an otherwise qualified actuary, and he or she will readily identify the ability to explain ideas and results to others as paramount. Check our own *Proceedings* and find the number of times that presidential addresses take up this same issue.

To me, communication begins with putting oneself into the position of the person with whom you are communicating. What does that person expect to hear? Does the person have a lot of time or are they in a hurry? What analogies would be most readily appreciated? If you have an idea or set of facts that are worthy of being passed on, then, for heaven's sake, make the additional effort to pass them on properly. We all know the old philosophers' question, "If a tree falls in the forest and there is no one to hear it, is there any sound?" If you wish to pass on a thought or report something important, it may be lost if no one can understand you or if you can't make them want to listen.

ACTUARY

What does it mean to be an actuary? It means that you, and sometimes only you, will take the long-range point of view. Managers are here today and gone tomorrow, and most frequently look only for the short-term advantage that will further their own careers. Being an actuary means integrity; it means standing firm when you're in the right, not thinking any less of those who disagree with you, but trying to use facts and reason to overcome objections wrongfully come by.

Many of you will ultimately find yourselves in positions where very little of your day-to-day work is actuarial. Thumb through the CAS *Yearbook* and see how many of our members are in non-actuarial assignments. But as long as you bear the designation which you have studied so hard to achieve, and have received today, remember that other people think of you and respect you as an actuary. Others will continue to come to you for your actuarial advice or opinion.

Being an actuary is not unlike being a weather man. People will make snide remarks about you and your profession. Nevertheless, you will find that you have earned their respect for objectivity and honesty. Avoid the trap of telling people only what they want to hear. Learn to recognize the pros and cons in a decision; think them through and then be prepared to discuss both sides of an issue.

Speaking about being an actuary at all times, I came across the following statement which appeared recently in the *New York Times Sunday Magazine* in an article seriously questioning the need for liability insurance rate increases:

> *Actually liability awards are remarkably consistent. In constant dollars, the median award has hovered around $20,000 over the last 25 years, according to the Rand Corporation's Institute for Civil Justice, although the average award has risen appreciably, reflecting the impact of a few huge settlements.*

The context in which this statement is contained is clearly intended to make the layman feel that there is something called the 'median' which has a significant bearing on whether or not liability insurance rates should go up or down.

(For our guests, here today, who are not actuaries, let me say that the 'median' is simply the middle number in a string of numbers which have been arranged in order from smallest to largest or vice versa.)

To illustrate, come with me on a shopping trip to the supermarket. I'll try to be semi-realistic and yet keep my example simple. We buy a loaf of bread for 80 cents, a quart of milk for 70 cents, and a pound of coffee for $3.50. The median price of 70 cents, 80 cents and $3.50 is the middle amount—80 cents. Let's add up the bill. Ignoring sales tax, we'll expect to pay 70 cents plus 80 cents plus $3.50. That's $5.

Next week we'll go to the store and buy three loaves of bread at 80 cents per loaf, three quarts of milk at 70 cents, and three pounds of coffee at $3.50 per pound. The amounts, by item, are 70 cents, 70 cents, 70 cents, 80 cents, 80 cents, etc. Clearly the median price is still 80 cents. Would we expect the supermarket to charge $5, the same as last week?

Alternatively, suppose that bread and milk had stayed the same price but coffee had jumped to $4.50 a pound because of a freeze in Brazil. Then, if, next week, we bought the same items, but only one of each, would we expect the same total at the register, since the median price is still only 80 cents?

If the median liability award 'hovers' around $20,000, but the number of awards, or claims, doubles or triples, should we expect our bill for liability insurance to remain the same? If this median remains relatively constant but the larger awards get bigger and bigger, should we expect the cost of liability insurance to stay constant? Finally, if it's the large awards that are the major problem, should the cost of excess liability covers ignore this fact and not change?

The author of the *New York Times* article from which I quoted is a member of this Society. Unfortunately, this person would seem to have forgotten what it means to be an actuary. Now, I don't want my remarks to be misinterpreted as saying that you should never espouse a position that is unpopular with the majority of your actuarial brethren. Far from it; some of the older members will remember that Charlie Hewitt has been on the unpopular side of more than one issue. What I am saying is: get your facts straight and then interpret them objectively, i.e., actuarially.

SPAN

When congratulating new Fellows, my stock inquiry has always been, "Now what are you going to do with all your free time?" Recently, I received the reply, "Well, I'm certainly not going to read any more papers by Valerius!" Naively I responded, "You know I knew Valerius." Now, it should be explained that Nels Valerius is a fine old gentleman, and at last report was still living in Cheshire, Connecticut. He received his Fellowship in 1928.

The group of younger members with whom I was conversing looked aghast. One of them said, in disbelief, "You knew Valerius!?" When I nodded assent, the young member blurted out, "Boy, you're a real link with the past!" Now I must confess that I even knew Dorweiler—and he was the man who hired Valerius.

The point I'd like to make with you is that our careers as actuaries will span a considerable period of time; our lives will span an even longer period of time. Most of us focus, with the greatest intensity, on the present, and pay decreasing amounts of attention to either the past or the future. Picture a Normal curve with no beginning and no end, and with time as the x-axis. The present moment in time is the mode (median and mean, also). The height of this curve at any point in time can represent the effect that other times in our careers (or our lives) have upon our present actions and decisions. What we did or thought yesterday, or expect to do or think tomorrow will usually affect today's thought and actions far more than those things did one year ago or will do one year from now.

As you grow older you will appreciate that looking upon the full span of a career (or a life) will give a better perspective as to the importance of what's happening right now, or what happened yesterday, or what might happen tomorrow. Try to live your life and your careers without the perspective that today's deeds are all-important. Realize that what took place in the past has some importance, but with an ever-lessening intensity as we go backward in time. Similarly, although tomorrow seems awfully important and will be even more important when it becomes today, the other tomorrows further off must be acknowledged as having bearing on our actions and thoughts today.

CONCLUSION

COMMUNICATION, ACTUARY, SPAN—by now some of you may have perceived that the initial letters of these awkwardly chosen titles for my subjects spell *C A S*—for Casualty Actuarial Society. Once again, I'm reminded of a story; this time about the late Herman Hickman of whom I suspect most of you have never heard.

Herman was a 300-pound college and professional football player and sometime professional wrestler and during a brief period a notably unsuccessful football coach at Yale University. During this tenure as Yale's football coach, he told the story that during a halftime intermission he gave his team a pep talk in which he chose to use the letters of Yale—*Y A L E*—as the theme around which he would inspire his players to better deeds in the second half of the game.

"*Y*," said Herman, "is for You. You must get out there and fight, fight, fight. *A* is for All. All of us must give every ounce of our ability to win this football game. *L* is for Loyalty. It's our loyalty to dear old Yale that will enable us to go on to victory. *E* is for Each and Every one of us who must give his all to insure that we walk off the field today triumphant. *Y A L E*; those letters spell victory." Newly inspired, the Yale team charged out of the locker room. Trailing behind the rest of the team were two substitutes who had not played in the first half and had little prospect of playing at all. Unaware that the coach was immediately behind them, one sub turned to the other and said, "What did you think of the coach's pep talk?" The other replied, "All I can say is thank heavens we don't go to California Polytechnic University at San Luis Obispo."

It has been a pleasure and a privilege to address the new members of the Casualty Actuarial Society. You have my congratulations and my best wishes for both long and successful careers in whatever line of endeavor you may choose. Thank you.