IMPLICATIONS OF SALES AS AN EXPOSURE BASE FOR PRODUCTS LIABILITY

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In Dorweiler's classic article, "Notes On Exposure and Premium Bases," he defines the term exposure as follows:

> When *critical conditions* and *injurable objects* exist in such relationship that accidents may result there is said to be *exposure*. The term *critical conditions* is intended to cover, rather broadly, the presence of or the absence of anything, objective or subjective, generally external to the injurable object, which contributes to the accident frequency and/or the accident severity.[1]

This somewhat intangible concept will be referred to in the remainder of the paper as the "true exposure." It is obviously important to select an exposure medium which will accurately measure the true exposure. The selected medium is called "the premium basis"[2] by Dorweiler, and will be called the exposure base or exposure units in this paper. Dorweiler suggests two criteria for the determination of a good choice of an exposure medium:

1. Magnitude of Medium should vary with hazard.
2. The Medium should be practical and preferably already in use.[3]

Thus, payroll is a good measure of exposure for workers' compensation insurance since, for a given classification, higher payroll tends to indicate higher expected losses, and since payroll information is relatively easy to obtain.

Finally, Dorweiler states that "the hazard varies directly with the product of the three variables: *critical conditions*, *injurable objects*, and *period of time*."[4] The differing premium rates for different classifications within a line of insurance are recognition of the critical conditions variable while the other two variables are reflected in the exposure base. Thus, beds, doctors, cars and units are the oft-used but short-hand versions of the more technically correct bed-years, doctor-years, car-years, or unit-years. The partition of payroll into a quantity and a temporal variable is less obvious, but payroll can be viewed

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[1] P. Dorweiler, "Notes on Exposure and Premium Bases," *PCAS* LVIII (1971), p. 59.

[2] Ibid., p. 60.

[3] Ibid., p. 61.

[4] Ibid., p. 59.

either as a wage rate multiplied by length of time worked, or as a surrogate for person-hours or person-years. While the existence of a time component is necessary for most exposure bases, there are exceptions. Generally, these exceptions involve a single use or consumption, so that for fillings (propane tanks) or blood donations or food products, there is not really a time component.

The dominant exposure base for products liability insurance, dollars of sales, cannot easily be decomposed into the injurable objects and time components. Yet many products classifications which use sales as an exposure base do not fall into the above exception classes. This paper will explore the implications of using sales as an exposure base. Sales can be thought of as the product of the number of units sold and the average price per unit; in order to simplify the discussion, we will assume the price per unit is fixed, so that total sales and number of units sold can be used interchangeably.

The products liability policy as considered in this paper covers occurrences during the policy period. Occurrences are *not* limited to those resulting from products manufactured during the current policy period but rather could result from any products still in existence. Hence, the true exposure is more accurately a function of total sales to date, less "expired" products, where expired means consumed, destroyed, or otherwise disposed of. Although this measure would be a preferable exposure base from the standpoint of the first of Dorweiler's two criteria, since it more closely varies with the hazard, it has not been considered a practical medium since the information is not generally readily available. Sales data for the current year are more easily obtainable and are thus the preferred exposure base.

It will be helpful to examine the relationship between current year sales and products in use. For a product with a very short lifetime (e.g. batteries), the total number of units in use is small relative to the number of units sold during the year. At the other extreme, the number of drill presses in use is far in excess of the number sold in the current year. It should be clear that the ratio of true exposure to current year sales depends on the distribution of the lifetime of the product, and generally increases with the length of the expected lifetime. It also depends on the length of time that the company has been in business, as will be seen later.

The following crude example with overly restrictive assumptions shows the main implications of the use of sales as an exposure base; a slightly more refined model will be used to draw conclusions.

Consider the Widget Manufacturing Company, on which we impose the following assumptions:

1. The company's first year of operation is year 0.
2. The company's final year of operation is year 6.
3. All widgets are produced at the beginning of the year and have a lifetime of exactly 4 years.
4. The potential for a claim-producing occurrence is constant over the lifetime of the widget.
5. No inflation occurs (either in the cost of the widgets or in the size of the claims).
6. There are other companies producing widgets; the total number of widgets sold by the industry each year is constant and has been constant since at least four years prior to the first year of data used in the ratemaking calculations.
7. The products liability insurance rate (based on the experience of the entire widget industry and traditional insurance ratemaking procedures) is $1 per widget sold. Although this rate includes only the loss cost portion of the premium, it will be referred to as "the premium."
8. The company sells 100 widgets each year.

Based on these assumptions, Table 1 shows the life cycle of the widgets produced by this company.

TABLE 1

WIDGET MANUFACTURING COMPANY
PRODUCTS LIABILITY EXPOSURES AND PREMIUM

		Calendar Year									
		0	1	2	3	4	5	6	7	8	9
Year of Manufacture	0	100	100	100	100						
	1		100	100	100	100					
	2			100	100	100	100				
	3				100	100	100	100			
	4					100	100	100	100		
	5						100	100	100	100	
	6							100	100	100	100
True Exposure*		100	200	300	400	400	400	400	300	200	100
Calendar Year Widgets Sold		100	100	100	100	100	100	100	0	0	0
Premium Charged**		100	100	100	100	100	100	100	0	0	0
"Correct" Premium**		25	50	75	100	100	100	100	75	50	25

*Total number of widgets in existence.
**In dollars; see text for discussion.

The rows on Table 1 show how many of the widgets produced in a year were in use at various points in time; the columns display the total number of widgets in use during each calendar year. For each of the calendar years 3, 4, and 5, the number of true exposure units is 400, equal to the life of the product multiplied by annual number sold. This formula is valid (under the given assumptions) for all years in a steady-state situation (i.e., not including "start-up" years or "tail" years). The exposures used in the premium calculation (number sold) will then be one-fourth of the true exposures in all steady-state years. However, since the calculation of the insurance rate from the steady-state industry experience also uses the sales exposure base, the published rate applied to sales produces the appropriate premium in all steady-state years. (Note that this depends on the assumption regarding the level industry exposures.) In the years following cessation of production, there should be a premium charge equal to the steady state premium times .75, .50 and .25, for the first, second and third subsequent years, respectively. Equally important, it is clear that the start-up years should receive a premium *reduction*. The "correct" premiums are shown in Table 1 as the product of the true exposures and the true rate of $.25 per true exposure. This has several implications for current rating methodologies. It is apparent that it is appropriate to charge a premium following cessation of production, but under the given assumptions this premium is exactly equal to the premium credit that *should* have been given when the company began production. Hence, an insuring company covering the Widget Manufacturing Company's entire lifetime of production would receive the same total dollars under either rating system, but would receive them earlier under the present system. Under the given assumptions the only difference between the two methods, albeit a significant one, is investment income.

The situation is analogous to that of claims-made professional liability coverage. Some of the similarities are:

1. A claims-made policy covers a report year, claims reported during the current year based on the present and all prior occurrence years; a products policy covers an occurrence year, which consists of present year occurrences arising from the present and all prior "manufacturing" years.
2. A claims-made policy is incomplete in the sense that additional coverage is necessary beyond the expiration of the policy, even if the insured ceases practice. This is the so-called "occurrence tail," consisting of incurred claims that have not yet been reported ("made"). Occurrence coverage for products liability is incomplete in the sense that additional

coverage is necessary beyond the expiration of the policy, even if the insured ceases production. This "tail" consists of the occurrences arising in the future from products that have not yet expired.

3. The early claims-made years have a lower premium because there are fewer insured incidents from prior years than there are in a mature claims-made year. The early years of a products exposure *should* have a lower premium because there are fewer existing products in use than in a "mature" products year.

Now that the basic concepts have been covered, a more refined model will be constructed to examine the implications further. The discussion will make use of the following definitions:

Firm: A single insured, producing a single product. When a company makes products falling into differing classifications, the company will be considered as the sum of various firms.

Product: The output of a firm. Classifications for ratemaking will be assumed to consist of a single product, insofar as loss-producing potential and useful lifetimes are concerned.

Industry: All firms producing a given product. The industry can be viewed as the sum over all firms producing that product, or as the sum over all insurance companies for that particular classification.

Let $f(t)$ be the probability that a product expires exactly t years after being produced.

$$\text{Define } F(t) = \int_0^t f(s)ds \tag{1}$$

The function $F(t)$ represents the proportion of products that expire within t years after production.

$$\text{Define } G(t) = 1 - F(t) \tag{2}$$

The function $G(t)$ represents the proportion of products that are still in use t years after production. This function defines the distribution of the lifetime of the product.

Define $H(a, b)$ as the true exposure in the time interval (a, b) arising from production of a single unit at time $t = 0$. The true exposure is equal to the product of the length of time and the average number of products in use. The interval (a, b) has length $b - a$, and the average number of products in use can be easily calculated:

$$\text{Average number of products} = \frac{\int_a^b G(t)dt}{(b - a)}$$

Hence, the true exposure is:

$$H(a, b) = (b - a) \frac{\int_a^b G(t)dt}{(b - a)}$$

$$= \int_a^b G(t)dt \tag{3}$$

Appendix A provides further discussion of these functions and an alternate derivation of true exposure.

Since practical applications will generally be dealing with one-year units of time beginning at integral values of t, the following definition will simplify notation without sacrificing generality. Define

$$J(n) = H(n, n + 1) \tag{4}$$

$J(n)$ represents the true exposure in year n arising from a unit of production at time $t = 0$, where n will generally be assumed to be integer-valued. Assuming the distributions of the useful lifetimes of products manufactured at time $t = 0$ apply equally well for all manufacturing years, $J(n)$ can also be viewed as the exposure in year $m + n$ arising from unit production in year m.[5]

Define A_m as the number of products sold in year m. For this model, all production and sales are assumed to occur at the beginning of the period.

Consider policy year m. The true exposure is the sum of the exposure contributions of each of the manufacturing years. The current year's sales, A_m, multiplied by the exposure per unit, $J(0)$, yields $A_mJ(0)$; the previous year's output is A_{m-1} and the exposure per unit is $J(1)$. Hence, the total true exposure for policy year m is:

$$\text{True Exposure}_m = \sum_{k=0}^{\infty} A_{m-k}J(k) \tag{5}$$

[5] In actuality, the distributions may change over time due to improvements in the product. This could be included in the model by defining $J(m, n)$, but the less general model is presented for the sake of clarity.

(The upper limit of infinity is used for notational convenience only. The summation should be thought of as extending over the lifetime of the product, which is finite for virtually all products.)

Assume that production is increasing at the constant rate of $(1 + g)$ per year. Again, a more general model could be constructed by defining $g(m)$ as the growth rate in year m.

$$\text{Define } v = \frac{1}{1+g}\,; \tag{6}$$

$$\text{then} \quad A_{m-1} = vA_m$$

$$\text{and} \quad A_{m-k} = v^k A_m \tag{7}$$

Substituting (7) in (5) yields:

$$\text{True Exposure}_m = \sum_{k=0}^{\infty} v^k A_m J(k) \tag{8}$$

$$= A_m \sum_{k=0}^{\infty} v^k J(k) \tag{9}$$

Hence, the true exposure for year m is proportional to the traditional exposure, A_m, where the factor of proportionality is independent of m. Assuming that the total industry experience in the ratemaking base has the same distribution of lifetimes and the same growth factor as the particular firm examined above, it should be clear that rates determined by comparing past losses (adjusted for development and trend) to past sales, should be applicable to current sales, since the corrective factor of proportionality, $\Sigma_{k=0}^{\infty} v^k J(k)$, is the same for current sales as for past sales. Conversely, whenever growth patterns of a firm differ from those of the total industry, sales may *not* be a good measure of exposure.

The most extreme examples of the inappropriateness of a sales exposure base occur when a firm begins or ceases production. In the latter case, production in year m is zero, so the usual exposure measure will also be zero. The true exposure may, however, be significant. In policy year m, the first year following the end of production, the true exposure is the sum of the previous years' sales still in use:

$$\text{True Exposure}_m = \sum_{k=1}^{\infty} A_{m-k} J(k) \tag{10}$$

Again, assuming annual growth rate g,

$$A_{m-2} = vA_{m-1},$$

$$A_{m-k} = v^{k-1}A_{m-1},$$

and (10) can be written as:

$$\text{True Exposure}_m = A_{m-1} \sum_{k=1}^{\infty} v^{k-1}J(k) \tag{11}$$

Changing convention slightly, let A_m represent the normal production for year m under growth rate g, and the true exposure will be manipulated through the use of the index of summation. This allows (11) to be written as:

$$\text{True Exposure}_m = A_m \sum_{k=1}^{\infty} v^k J(k) \tag{12}$$

If the firm desires coverage in the first year following cessation of production, the appropriate factor to be applied to the current rates is the ratio of the true actual exposures to the true exposures contemplated in the rate:

$$\sum_{k=1}^{\infty} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{13}$$

Similarly, the factor to be applied in the nth year after the end of production is:

$$\sum_{k=n}^{\infty} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{14}$$

Of course, a problem occurs when one considers to what this factor should apply. In theory, it is applicable to the sales that *would* have occurred had production not been ceased. Since this is a subjective estimate, in practice, the most recent year's sales would probably be the best exposure to which this factor could be applied.

A similar type of analysis is required when a firm begins to produce a new product. (New product means new to the firm where an established products liability rate for the product already exists, as opposed to a completely new product requiring the calculation of an appropriate rate. The latter case is beyond the scope of this paper, although the ideas presented here should be valuable in the process of determining the new rate.) As above, A_m will refer to expected production in year m under growth assumption g, and actual production will be manipulated via the index. Suppose a firm begins production in year m. Production (sales) in year m will be A_m and the exposure will be $A_m \sum_{k=0}^{0} v^k J(k)$.

Hence, the appropriate rate in the first year of production is the normal rate multiplied by the factor:

$$\sum_{k=0}^{0} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{15}$$

Similarly, the factor applicable to the nth year of production will be:

$$\sum_{k=0}^{n} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{16}$$

Note that this factor becomes equal to one when n is equal to or greater than the lifetime of the longest lived product.

An example falling in between the two extremes of production startup or cessation is a firm with a growth rate g' differing from the industry growth g. The factor applicable to the industry rate will be the ratio of the actual true exposures to the true exposures implicit in the rates:

$$\sum_{k=0}^{\infty} (v')^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \qquad \text{where } v' = \frac{1}{1+g'} \tag{17}$$

For a growth rate g' less than g, v' will be greater than v, hence the rate applicable to sales will be *greater* than the industry rate. For a firm growing faster than the industry, the correct premium rate will be less than the industry rate.

We now consider a less simplified but more realistic numerical example. The following assumptions are imposed on the Widget Manufacturing Company:

1. The company's first year of operation is year 0.
2. The company's final year of operation is year 7.
3. The company and industry growth rates are 10%.
4. The lifetimes of widgets are distributed as illustrated in Table 2 for widgets produced at the beginning of year 0. All failures are assumed to occur at the beginning of a year, so that column 1 contains the discrete counterparts of the $f(t)$, column 2 contains the $F(t)$, and column 3 contains the $G(t)$. Since failures all occur at the beginning of the year, $G(t)$ is constant throughout each year, and column 3 also represents the $J(t)$.
5. All other earlier assumptions hold.

TABLE 2

Discrete Distribution of Widget Lifetimes

Year	(1) Proportion Failing in Year	(2) Cumulative Failures	(3) Proportion still in use 1.00 − (2)
0	.00	.00	1.00
1	.05	.05	.95
2	.10	.15	.85
3	.20	.35	.65
4	.30	.65	.35
5	.20	.85	.15
6	.10	.95	.05
7	.05	1.00	.00

Based upon these assumptions, Table 3, which displays Widget Manufacturing Company exposures by calendar year and production year, can be constructed.

TABLE 3

Widget Manufacturing Company Products Liability Exposure

		Calendar Year													
		0	1	2	3	4	5	6	7	8	9	10	11	12	13
Year of	0	100.0	95.0	85.0	65.0	35.0	15.0	5.0	0	0	0	0	0	0	0
Manufacture	1		110.0	104.5	93.5	71.5	38.5	16.5	5.5	0	0	0	0	0	0
	2			121.0	115.0	102.9	78.7	42.4	18.2	6.1	0	0	0	0	0
	3				133.1	126.4	113.1	86.5	46.6	20.0	6.7	0	0	0	0
	4					146.4	139.1	124.4	95.2	51.2	22.0	7.3	0	0	0
	5						161.1	153.0	136.9	104.7	56.4	24.2	8.1	0	0
	6							177.2	168.3	150.6	115.2	62.0	26.6	8.9	0
	7								194.9	185.1	165.6	126.7	68.2	29.2	9.7
True Exposure		100.0	205.0	310.5	406.6	482.2	545.5	605.0	665.5	517.6	365.9	220.2	102.9	38.1	9.7

In this example, only calendar years 6 and 7 are in an equilibrium state, where the traditional rate is the correct rate. For all years in an equilibrium state, the ratio of true exposures to calendar year sales will be a constant; in this case:

$$605.0/177.2 = 665.5/194.9 = 3.41$$

(See Appendix B for more discussion of the ratio of true exposures to sales.)

In the prior Widget Manufacturing Company example, we assumed a premium rate of \$1.00 per \$100 of sales; the corresponding rate per true exposure was \$.25. In the current example, this \$.25 rate per true exposure implies a premium rate of \$.853 per \$100 of sales. Since losses are a function of true exposures rather than sales, the rate per unit of sales can be calculated by multiplying the rate per unit of true exposure (\$.25 per \$100) by the ratio of true exposures to sales. In the prior example, this factor was 4; the rate per \$100 of sales was thus $4 \times \$.25 = \$1.00$. In this example, the factor is 3.41 so the rate per \$100 of sales is $3.41 \times \$.25 = \$.853$. If the industry growth rate has been constant during the period between the first year used in ratemaking and the present, this calculation does not have to be made explicitly but will work out automatically. Note that a further implication of this discussion is that a material change in the industry growth rate will make the results of the present ratemaking methodology somewhat inappropriate.

Table 4 compares the premium that would normally be charged using sales as an exposure base and the "correct" premium using the true exposure.

Several observations can be made from Table 4. In the equilibrium years (6 and 7), the premium charged is the same under both measures of exposure. In the start-up years (0 through 5), the premium based on sales exceeds the correct premium, the difference being significant in the early years and decreasing over time. The correct premium exceeds the sales-based premium in the tail years, since there are no sales in those years. Note that the total premium over all years is *not* the same for the two exposure bases. The premiums are identical in all equilibrium years, but the excesses in the early years do not make up for the deficiencies of the later years. However, it cannot be concluded that traditional ratemaking does not provide enough premium over the life cycle of an insured. To answer this question, it is necessary to examine the nature of the

TABLE 4

WIDGET MANUFACTURING COMPANY

PRODUCTS LIABILITY PREMIUMS ON DIFFERENT EXPOSURE BASES

(1) Year	(2) Sales	(3) True Exposure	(4) Premium Based on Sales (2) × .854	(5) "Correct" Premium (3) × .25
0	100.0	100.0	85.4	25.0
1	110.0	205.0	93.9	51.3
2	121.0	310.5	103.3	77.6
3	133.1	406.6	113.7	101.6
4	146.4	482.2	125.0	120.6
5	161.1	545.5	137.6	136.4
6	177.2	605.0	151.3	151.3
7	194.9	665.5	166.4	166.4
Subtotal*	1143.6	3320.2	976.6	800.6
8	0	517.6	0	129.4
9	0	365.9	0	91.5
10	0	220.2	0	55.1
11	0	102.9	0	25.7
12	0	38.1	0	9.5
13	0	9.7	0	2.4
Total*	1143.6	4574.5	976.6	1143.6

* Totals may not add correctly due to rounding.

coverage for the "tail" years, in this case, years 8 through 13. Among the possibilities are:

1. If the firm has ceased coverage because it has gone out of business, it will probably not purchase insurance in years 8 through 13. In any event, the insurance company will not be liable for occurrences in the "tail"

years. In this case, the insurance company will have received \$976.60 but will only have to pay out \$800.60 (Table 4 subtotals through year 7).

2. If this product represents only a minor portion of the insured company's total sales, then the insurance company is likely to continue coverage on the discontinued product, even though there is no premium collected for this product during the tail years. In this case, the insurance company will receive \$976.60 but will have \$1143.60 in expected losses (but will also receive significant investment income).

3. If the product represents a major portion of the company's sales but the company remains in business after discontinuing this product, the insurer may refuse to provide coverage for claims arising from this product occurring after year 7, unless additional premium payments are made. Assuming that the insurer can estimate the appropriate premium for the tail years as \$343.00 (\$1143.60 − \$800.60), the insurer will receive \$1319.60 (976.60 + \$343.00) for \$1143.60 in expected losses. Although an informed insured might realize that there were overcharges in the early years, the arguments will be useless if there has been a change in insurers. However, the insured may be in a position to demand premium credits for its newer products, since the reasoning used by the insurer to charge premium for the "tail" years is identical to the reasons for expecting a credit in the early years.

One other relationship on this table should be pointed out. The "excess" premium in year 0 is $85.4 - 25.0 = 60.4$. The "deficiency" in year 8 is $129.4 - 0 = 129.4$. The difference between the two is attributable to growth in production: $60.4 \times (1.1)^8 = 129.4$. A similar relationship holds true for years 1 and 9, 2 and 10, etc. The excess premium arises from an overstating of exposures in the early years, while the deficiency arises from an understating of exposures. But the understating of exposures occurs eight years later than the overstating, by which time the exposures have grown by the factor $(1.1)^8$. It should be clear that, with an assumption of no growth (as in the first example given), the excesses and deficiencies cancel out. In either case, investment income and changing carriers complicate the analysis.

In actual practice, the distortions caused by the use of sales will be less than indicated in this example. Normally, a company produces a number of products and the elimination of a single product would have a relatively small effect. It is also likely that an eliminated product will be replaced by another, new product. To the extent that the two products have the same distribution of lifetimes and the same potential for loss, the errors will cancel each other. In the case where a company completely ceases production (if, for example, it goes out of business), it may not even purchase insurance.

Despite the significant potential distortions which may result from the use of sales as an exposure base, it is not the intention of this paper to suggest that a change in exposure bases is necessary. The problems involved in attempting to objectively measure the true exposure would outweigh the benefits in most cases. Rather, it is the intention of this article to outline the implications of sales exposure bases so that the effects will be understood and the appropriate actions can be undertaken in the extreme cases. A few examples may help explain typical problems that may be encountered.

1. If a manufacturer has *recently* discontinued a hazardous product, losses will still continue to occur even though there are no sales. If the firm is being experience-rated, the indicated modification will be too low and a schedule debit may be appropriate. A few years later, the situation is reversed. The loss experience, but not the exposures, of the discontinued product will be included in the experience rating calculations. Since the future losses arising from that discontinued product should decrease, the experience modification is now too high and a schedule credit may be appropriate. Similarly, a firm adding a major product should get a schedule credit from the manual rate. At the very least, this paper will be helpful for understanding and explaining to the insured why changes in the experience modification are occurring.
2. A manufacturer recently requested advice as to whether it should join a captive insurer, since its loss experience was significantly better than that contemplated in the existing premium rates. (The premium rates were developed for that particular industry by a specialty company.) However, the firm had been in existence for only three years, and was producing a product with an expected lifetime of fifteen to twenty years. This paper makes it clear that the experience during these early years of production *should* be significantly better than that of established manufacturers of the same product. For the purpose of illustration, assume that the lifetime of each product is exactly fifteen years, and there has been no growth in sales for the industry. The established manufacturers will have fifteen units of true exposure for every one sold in the current year, while the firm in question only has three units of true exposure for each one sold in this year. The true exposure of the new firm (in this year) is only 20% of the exposure of the established firms: the firm

deserves an 80% credit from the manual rate. The amount of credit should decrease over time as the true exposures increase. It was recommended that the firm should not join the association captive, where it would share disproportionately in the other members' losses, but rather the firm should remain with its insurer and try to negotiate a premium credit.

3. Projections of losses for a manufacturer typically involve a regression line fit to the history of annual loss pure premiums (losses divided by exposures). In the start-up years of a product, such an analysis may indicate a significantly increasing pure premium. However, if the firm is nearing the equilibrium stage, the pure premiums should begin to plateau (ignoring inflation), rather than continue their steep rise. In the absence of an understanding of the implications of this paper, artificially high losses may be projected. Similarly, an understanding of these concepts will aid in projecting future losses arising from a discontinued product.
4. Another manufacturer is producing a product with a lifetime in excess of one hundred years, but has been producing the product for only thirty years. It should be clear that the total true exposure is increasing each year, even if sales are constant. If the nature of true exposure were not considered, it would be difficult to understand why losses are growing each year, even after adjusting for inflation and sales growth.
5. If products produced now have longer lifetimes than products produced in the past, the true exposure will increase even though sales (inflation adjusted) are not increasing.
6. For a product such as an elevator, it is likely that the exposure to loss is not constant over the elevator's lifetime, but more concentrated in the later years. A firm may not have increasing sales now, but may find it has an increasing "inventory" of older elevators which are more likely to produce losses.

Summary

The use of sales as an exposure base for products liability insurance can have a distorting effect under certain circumstances. However, it is neither necessary nor feasible to change the exposure base. As long as the effects of this distortion are understood, the impact can be estimated and corrected. Certainly the effects are not trivial to calculate, since this calculation requires an estimate of the distribution of the lifetime of a product, but even crude estimates will result in more accurate premiums in some of the extreme cases.

APPENDIX A

Derivation of True Exposure Arising from a Product

We have defined $f(t)$ to be the probability density function of the lifetime of a product manufactured at time 0. It can be viewed either as the probability of expiration at time t or as the portion of products that have a lifetime of length t. Since the true exposure is the product of the number of objects and the length of their lifetimes, the exposure arising from a single unit of production can be calculated by multiplying the lifetime t by the portion of products with lifetime t, and summing over all values of t. Hence,

$$\begin{array}{c}\text{True exposure from}\\ \text{unit of production}\end{array} = \int_0^\infty tf(t)dt$$

For the purposes of this paper, it is necessary to calculate the exposure during the time interval $(0, t)$. The expression

$$\int_0^t sf(s)ds$$

does *not* represent the total exposure in the interval $(0, t)$; it represents only the exposure arising from products which expire at or before time t. The exposure from the products still unexpired must be added. Since the portion of products still unexpired is $G(t)$, the total exposure during the time interval $(0, t)$, is

$$tG(t) + \int_0^t sf(s)ds$$

The following derivation will show that $\int_0^t G(s)ds$ is equal to the above expression, and hence, is equivalent to the true exposure during the time period $(0, t)$ from a product manufactured at time 0.

From equation (2),

$$G(s) = 1 - F(s).$$

From equation (1),

$$G(s) = 1 - \int_0^s f(r)dr.$$

Differentiating both sides,

$$G'(s) = -f(s).$$

Multiply by s,

$sG'(s) = -sf(s)$.

Integrate over $(0, t)$,

$$\int_0^t sG'(s) = \int_0^t -sf(s)$$

Integrating by parts yields

$$sG(s)\Big|_0^t - \int_0^t G(s)ds = -\int_0^t sf(s)ds$$

or

$$\int_0^t G(s)ds = tG(t) + \int_0^t sf(s)ds.$$

The right-hand side of this equation has a useful verbal interpretation. The true exposure between zero and t is the sum of two pieces:

1. The portion of products still in use at time t, multiplied by the length of exposure: $tG(t)$.
2. The exposure arising from products which expire during the period: $\int_0^t sf(s)ds$.

APPENDIX B

True Exposure and Sales

The relationship between the true exposure in a year, $\Sigma_{k=0}^{\infty} A_{m-k}J(k)$, and the sales in the year, A_m, is affected by two factors:

1. The distribution of the lifetime of the product. This factor is reflected in the function $J(k)$.
2. The growth pattern of the calendar year sales, reflected in the A_{m-k}.

If the number of products sold does not change from year to year (as in the first example given in the text), then $A_{m-k} = A_m$ for all k. Then the ratio of true exposure to sales is given by

$$\frac{\sum_{k=0}^{\infty} A_{m-k}J(k)}{A_m} = \frac{\sum_{k=0}^{\infty} A_m J(k)}{A_m} = A_m \frac{\sum_{k=0}^{\infty} J(k)}{A_m} = \sum_{k=0}^{\infty} J(k)$$

In this circumstance, the ratio is solely dependent on the function $J(k)$. Rewriting,

$$\begin{aligned}\sum_{k=0}^{\infty} J(k) &= \sum_{k=0}^{\infty} H(k,\, k + 1)\\ &= H(0,\, 1) + H(1,\, 2) + \ldots\\ &= \int_0^1 G(s)ds + \int_1^2 G(s)ds + \ldots\\ &= \int_0^{\infty} G(s)ds\end{aligned}$$

As shown in Appendix A, this expression represents the expected or average lifetime of the product. Hence, under the condition of constant sales, the ratio of true exposure to sales will be equal to the expected lifetime of the product.

If sales are not constant, the relationship becomes more complicated. However, it can be said that if sales are growing at a constant rate, then the ratio of true exposure to sales will be less than the expected lifetime of the product. This can easily be seen using (9). The ratio of true exposure to sales will be

$$A_m \frac{\sum_{k=0}^{\infty} v^k J(k)}{A_m} = \sum_{k=0}^{\infty} v^k J(k);$$

when the rate of increase $(1 + g)$ is greater than one, v is less than one and $\Sigma_{k=0}^{\infty} v^k J(k)$ will be strictly less than $\Sigma_{k=0}^{\infty} J(k)$.

As a specific example, consider the equilibrium state of Widget Manufacturing Company in the main text. The ratio of true exposures to sales was $665.5/194.9 = 3.41$.

The numerator can be broken into its components (refer to Table 3 in the text): $665.5 = 5.5 + 18.2 + 46.6 + 95.2 + 136.9 + 168.3 + 194.9$.

This can be further decomposed as:

$$\frac{(.05)(194.9)}{(1.1)^6} + \frac{(.15)(194.9)}{(1.1)^5} + \frac{(.35)(194.9)}{(1.1)^4} + \frac{(.65)(194.9)}{(1.1)^3}$$

$$+ \frac{(.85)(194.9)}{(1.1)^2} + \frac{(.95)(194.9)}{(1.1)^1} + \frac{(1.00)(194.9)}{(1.1)^0}$$

which should be recognizable as $\Sigma_{k=0}^{\infty} v^k J(k) A_m$.

If there were no growth in sales, the denominators in the above expression would be unity, and the sum would reduce to $(4.0) \times (194.9) = 779.6$; the effect of growth in sales is to reduce the contribution of prior years sales to true exposures (i.e. reduce the numerator), without affecting the denominator.