ESTIMATING PURE PREMIUMS BY LAYER —AN APPROACH

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This paper presents an approach to the estimation of loss costs by layer of insurance coverage. This method uses the log-normal probability distribution as a model for claim sizes. Although the approach has been successfully applied to several different lines of liability insurance, it may not be applicable to property insurance.

The motivation for using a probabilistic model for claim sizes arises largely from the "long-tail" nature of liability insurance. The long tail derives from both the delayed reporting of claims as well as from the lengthy settlement period involved. The long tail makes it difficult to accurately price some liability insurance lines. Since it takes many years to settle claims, the latest year for which a vast majority of claims are settled may well be quite old. Conditions may have changed significantly since that latest mature year. Indeed, average claim costs have increased significantly in most liability lines over the past several years.

In choosing experience data, the ratemaker is thus forced to make a tradeoff between using less mature experience and using more mature (and older) experience with a larger trend factor, to estimate current costs. A method often used to produce more consistent, stable, and mature experience data is to limit individual claims to a certain size, often called "basic limits." A difficulty with this approach is that the value of the basic limits is changing over time. For example, $25,000 in 1963 claim costs was probably quite different than $25,000 in 1973 claim costs. Almost assuredly, the percentage of total limits losses below $25,000 per claim in 1963 was more than the respective percentage below $25,000 in 1973.

When the ratemaker's attention is focused on higher layers of liability, the problems caused by delayed settlements are more significant. The persistent inflation of recent years has pushed both jury verdicts and claim settlements to higher levels. Not only do more claims find their way into higher layers over time, but there is a leverage effect on their amounts; that is, the increase in amount applies only to the highest layer. This paper presents an aid to estimating pure premiums for the higher layers of liability.

The method described in this paper will be applied to two specific problems:

> *Problem No. 1:* A new company, formed to write medical malpractice insurance, wants to purchase excess of loss reinsurance to cover a layer of $900,000 excess of $100,000 per claim. How might the premium for this coverage be determined?

> *Problem No. 2:* Experience data is available for medical malpractice claims for policy years 1963 to 1974. The loss data is limited to $25,000 per claim and premiums are needed for $100,000 limits. What increased limits factors should be applied to the data to calculate the $100,000 pure premiums?

THE APPROACH

The approach assumes that the distribution of incurred claim sizes follows a log-normal probability distribution. Knowing two parameters of this distribution, such as the mean and coefficient of variation (CV)[1], one can calculate the percentage of incurred losses by layer. Rather than talking about the losses for a specific layer, it is simpler to talk in terms of the *excess loss distribution.* This distribution is the percentage of total limits losses which are above a certain amount, called the *attachment point,* per claim[2]. Assuming, for example, that the mean of the total limits claim size distribution is $50,000 and the CV is 3.0, the excess pure premium for an attachment point of $100,000 is about 40% of the total limits pure premium. For an attachment point of $250,000 it is about 21%, and for an attachment point of $1,000,000 it is about 5%. (See Table I.)

[1] The coefficient of variation is the ratio of the standard deviation to the mean.

[2] Excess losses above a given attachment point are defined as the sum of all claim values larger than the attachment point, less the number of claims above the attachment point multiplied by the value of the attachment point.

TABLE I

EXCESS LOSS DISTRIBUTION
(AS A PERCENTAGE OF TOTAL LIMITS LOSSES)

LOG-NORMAL ASSUMPTION

Attachment Point (Times The Mean)	Coefficient Of Variation 1.0	1.5	2.0	3.0	4.0	5.0
.5	56%	61%	65%	70%	73%	75%
1.0	32	41	47	55	60	63
1.5	20	30	37	46	52	56
2.0	13	22	30	40	46	50
2.5	9	18	25	35	41	46
3	6	14	21	31	37	42
4	3	9	16	25	32	36
5	2	7	12	21	27	32
10	—	2	5	11	16	21
15	—	1	3	7	12	15
20	—	—	2	5	9	12
25	—	—	1	4	7	10
50	—	—	—	1	3	5
100	—	—	—	—	1	2

The log-normal distribution has appeared previously in the *Proceedings* and other actuarial literature[3]. It is assumed that the natural logarithms of the claim sizes are distributed according to the normal (or Gaussian) probability law. The appendix gives a precise mathematical definition of the log-normal distribution. Exhibit I illustrates the case where the mean is 60 and the CV is 3. The main virtues of the log-normal distribution, from a modelling point of view, are that: (1) it can be a highly skewed distribution[4] and (2) it can be justified on a intuitive basis.

[3] For example, the log-normal distribution is mentioned in: Bickerstaff, D. R. "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model" PCAS LIX (1972); Hewitt, C. C. "Credibility for Severity" PCAS LVII (1970); Mayerson, A. L. "A Bayesian View of Credibility" PCAS LI (1964). It is also discussed in Harding, V. "Treatment of IBNR Claims," *IBNR*, Amsterdam: Netherlands Reinsurance Group (1972). A thorough discussion of the log-normal distribution can be found in Aitchison, J. and J. A. C. Brown, *The Lognormal Distribution*, Cambridge University Press (1957).

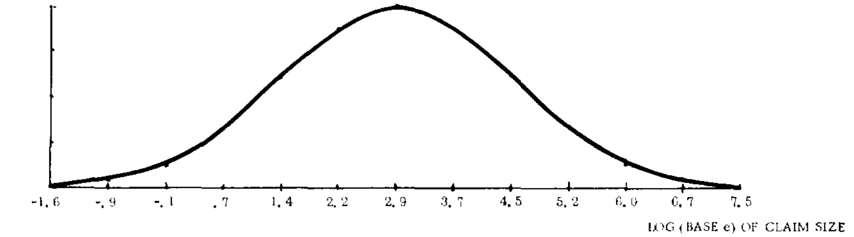
[4] The higher the CV, the more skewed the distribution. This can be seen in Table I.

EXHIBIT I

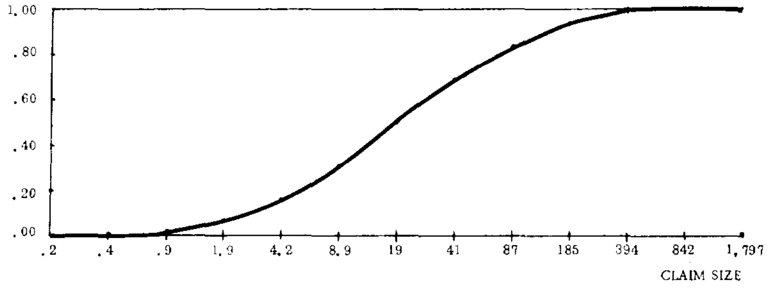
LOG-NORMAL PROBABILITY DISTRIBUTION

MEAN = 60 CV = 3.0

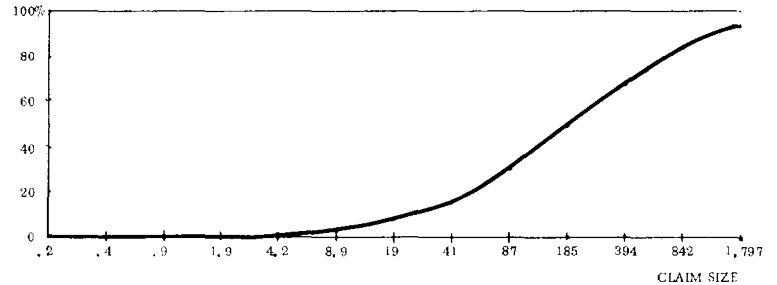
1. PROBABILITY DENSITY FUNCTION



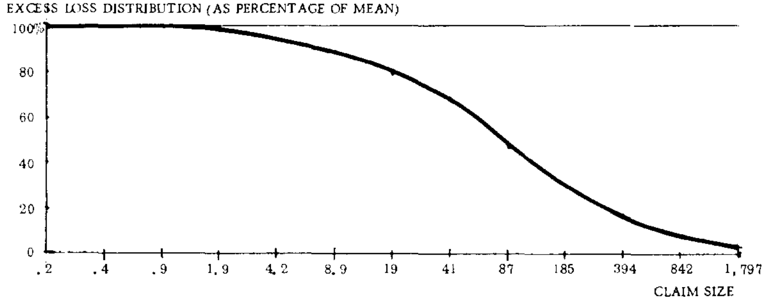
2. CUMULATIVE DISTRIBUTION FUNCTION



3. FIRST MOMENT DISTRIBUTION (AS PERCENTAGE OF MEAN)



4. EXCESS LOSS DISTRIBUTION (AS PERCENTAGE OF MEAN)



Intuitively, the log-normal distribution can be considered appropriate as an analog of the central limit theorem. The central limit theorem states that the average (or sum) of independent random variables will converge to the normal probability distribution. The normal distribution can thus be used as an approximation for the distribution of the sum of a number of independent random variables. If the individual random variables were logarithms, the sum of the logarithms would be approximately normally distributed. The sum of logarithms is analogous to the product of the anti-logarithms.

If we have a number of independent variables, whose product is the observed claim size, we can expect the sum of the logs of these variables to be approximately normally distributed; the claim size would then be approximately log-normally distributed. We might thus expect that any line of business where several independent factors can be multiplied together to determine the claim size will have a log-normal claim size distribution.

Considering an automobile accident, we may theorize that a number of independent factors interact multiplicatively to determine the liability claim size, such as:

- the speed of the vehicles before impact
- the health of the injured party
- the protection (e.g., with seat belts, interior padding), of the victim
- the income of the victim
- the skill of the plaintiff's attorney, and
- the skill of the defendant's claims adjusters.

Regardless of the intuitive justification, the choice of claim size distribution must be sustained in practice. As will be pointed out later, the log-normal distribution seems to provide a good fit for medical malpractice insurance claims.

The log-normal assumption applies to the individual claim sizes (i.e., the claim count). A related distribution is the (first) moment distribution. The moment distribution gives the total amount of losses on claims which are smaller than a given size. Exhibit I, Section 2, illustrates the cumulative distribution function of the claim count distribution. Exhibit I, Section 3, illustrates the cumulative moment distribution, as a percentage of the mean.

The excess loss distribution gives the total amount of losses above a given attachment point per claim. It differs from the complement of the first moment distribution in that the amount of the attachment point is subtracted from every claim greater than the attachment point. Exhibit 1, Section 4, illustrates the excess loss distribution, as a percentage of the mean.

The log-normal distribution has two parameters. In practical usage, the mean and CV can be used as the two parameters. The log-normal distribution has the desirable property that, for a given CV, the distribution can be completely described by a function of a factor times the mean. This means, for example, that the distribution for an attachment point of $100,000 and a mean of $50,000 is the same as for a $200,000 attachment point and $100,000 mean. In both cases the attachment point is 2.0 times the mean. Tables of the log-normal distribution can thus be prepared (see Table I) as a function of the CV and a factor times the mean. Exhibit II depicts the excess loss distribution graphically as a function of the CV and attachment point, which is defined as a ratio to the mean.

We now tackle the two problems posed earlier:

Problem No. 1: For simplicity we may assume that each primary policy is sold for $1,000,000 limits. We have concluded from other analysis that $6,300 is an appropriate pure premium for the coverage. This pure premium is made up of a gross frequency of 26.5%; 50% of the claims are closed without a payment; and the total limits average closed-paid claim will be $50,000. Based on other evidence, we assume that claim sizes are log-normally distributed with a CV of 3.0. Pure premium by layer can thus be calculated as in Table II. From this table we see that coverage from $100,000 to $1,000,000 would cost about $2,650 —$330 = $2,320 in claims per exposure unit. (Pure premium for coverage up to $1,000,000 is $6,630 — $330 = $6,300.)

TABLE II

EXCESS PURE PREMIUMS BY LAYER

Attachment Point		Excess Losses	
(1000's)	Times Mean	% Of Total	Per Unit
0	0.	100	6,630
25	0.5	70	4,640
50	1.0	55	3,640
100	2.0	40	2,650
250	5.0	21	1,390
500	10.0	11	730
1,000	20.0	5	330
2,500	50.0	1	66

Problem No. 2: For simplicity we may assume that the total limits mean claim size in 1964 is $10,000; that total limits claim sizes are increasing at 15% annually; and that claim sizes are log-normally distributed with a CV of 3.0. We can then calculate the excess losses for each attachment point for each year. Increased limits factors can be calculated directly from the excess loss distribution. Table III illustrates this problem. It should be noted that the increased limits factors are increasing.

TABLE III

INCREASED LIMITS FACTORS FOR $100,000 OVER $25,000

Policy Year	Ratios To Total Limits Mean[a]		Percent Excess Losses[b]		Indicated Increased Limits Factor[c]
	$25,000	$100,000	$25,000	$100,000	
1963	2.9	11.5	31%	10%	1.32
1964	2.5	10.0	35	11	1.35
1965	2.2	8.7	37	13	1.39
1966	1.9	7.6	41	15	1.44
1967	1.6	6.6	44	17	1.49
1968	1.4	5.7	48	19	1.54
1969	1.2	5.0	50	21	1.59
1970	1.1	4.3	53	24	1.64
1971	.9	3.8	56	26	1.70
1972	.8	3.3	60	29	1.76
1973	.7	2.8	63	32	1.82
1974	.6	2.5	65	35	1.88

Notes: a. Adjusted for 15% annual inflation.

b. Based on log-normal distribution with CV 3.0.

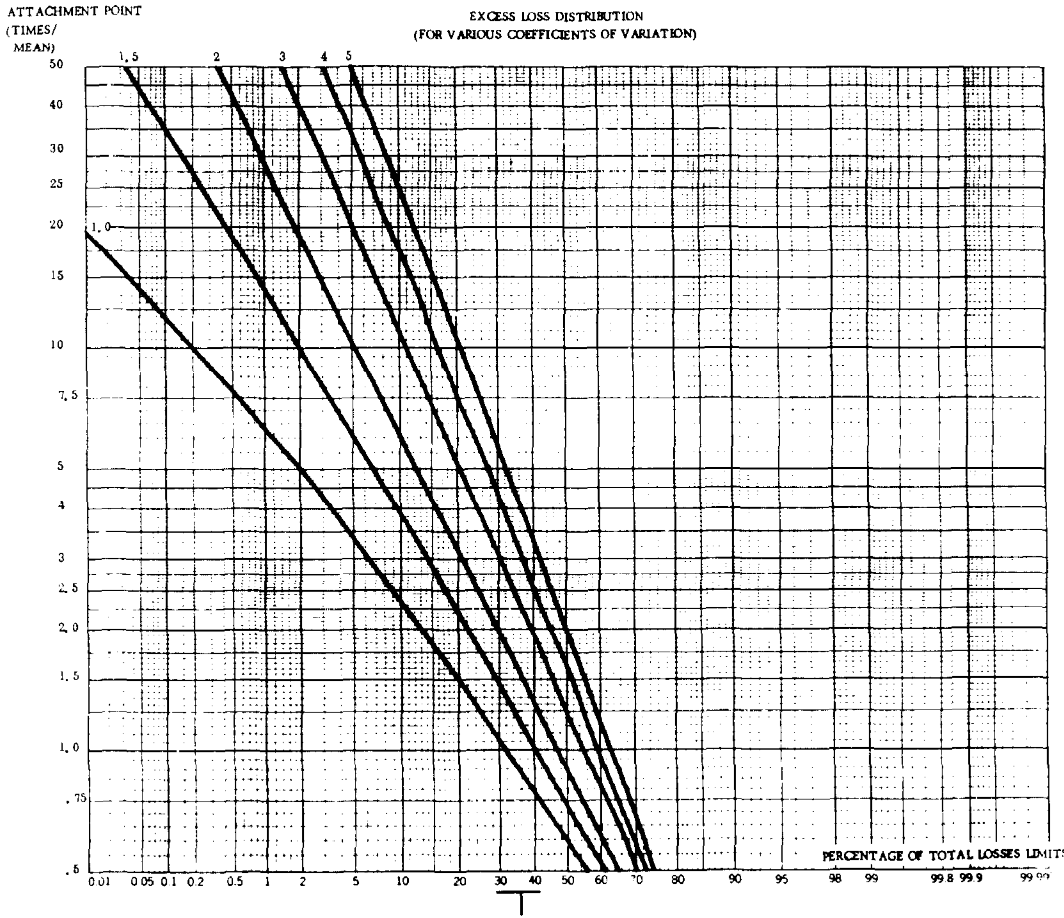
c. Other columns have been rounded. This is calculated as:

$$\frac{100 - E_{100,000}}{100 - E_{25,000}}$$ where E_x is the percentage of total limits losses above x per claim.

PARAMETER ESTIMATION

To use the approach of this paper, one needs to make assumptions about the total limits mean and CV of the claim size distribution. The basic limits mean is often available from other actuarial analysis. For a given choice of basic limits mean and CV, there is total limits mean. The more difficult parameter to estimate is the CV.

EXHIBIT II


EXCESS LOSS DISTRIBUTION
(FOR VARIOUS COEFFICIENTS OF VARIATION)
ATTACHMENT POINT (TIMES/ MEAN)
1.0
1.5
2
3
4
5
50
40
30
25
20
15
10
7.5
5
4
3
2.5
2.0
1.5
1.0
.75
.5
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
PERCENTAGE OF TOTAL LOSSES LIMITS

A number of practical problems arise in estimating the CV; these include:

- individual claim values are not always known
- claim values tend to cluster at target values, such as $2,500, $5,000 or $10,000
- a large number of nuisance claims are often settled for small amounts, such as $250, $500 or $1,000[5]
- many claims are closed without a payment.

Depending upon the specific situation, the entire claim size distribution may not be log-normally distributed. It is often possible to eliminate some claims from consideration, such as very small claims or claims closed without a payment. The remaining distribution may then closely approximate a log-normal distribution.

This author has found it most convenient to estimate the CV from the observed excess loss distribution. To accomplish this, claims are grouped by interval and the percentage of the total limits losses in excess of a given interval is calculated. There is a unique CV for a given combination of excess percentage and ratio of the attachment point to the mean of the total limits distribution. For example, if excess losses above an attachment point of 2.0 times the mean are 40%, this implies a CV of 3.0. The uniqueness property is illustrated by Exhibit II.

Following the procedure above, the CV is estimated for a number of attachment points. If the estimated CV is the same for each attachment point tested, the distribution can safely be assumed to be log-normally distributed with the observed mean and given CV. If the estimated CV's are randomly distributed about a given value, that value is an appropriate estimate of the CV. If the estimated CV's form a progression (such as 6, 5, 4, 3), the observed data is not log-normally distributed. In the latter case, the data can be truncated, and the remaining data fitted to a log-normal distribution.

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[5] Considering that the average allocated expense payment in medical malpractice is over $2,000, there is an incentive to pay a token settlement even when there is no negligence.

This estimation procedure is highly empirical. This may not be a serious drawback since the observed distribution of claim sizes may not be log-normally distributed[6]; one or two large claims, by presence or omission, may distort the observed data. The practical difference resulting from the use of 3.0 versus 2.9, for example, will be small.

An example of the estimation procedure will now be given. The data is shown in Table IV. As might be expected, most claims are relatively small, but a significant amount of the loss dollars are on higher intervals. Estimating the CV from the claim count distribution can be misleading because a majority of the claims are small and the majority of the claim dollars are on a small number of large claims. In the given example, the largest 2.5% of the claims account for 50.9% of the claim dollars. Estimating the CV from the moment distribution can be misleading because of the targeting problem. For example, there may be twenty-five claims for exactly $100,000. Should these claims be considered larger than $100,000 or smaller than $100,000; or are 50% larger and 50% smaller? Using the excess loss distribution largely avoids the targeting problem and it puts the emphasis on the layers where losses have occurred.

If there are a large number of claims closed without a payment, the distribution which includes them is not likely to be log-normally distributed. Table V illustrates this. The basic difference between estimating the CV with and without claims closed without a payment is the indicated mean of the distribution. The data is log-normally distributed if the estimated CV's for different attachment points are the same. If the estimated CV's for higher attachment points exhibit a downward trend, this indicates that the observed mean is too small. In other words, it shows that claims are concentrated too close to the mean. One can raise the observed mean by eliminating claims closed without a payment or by eliminating some of the smaller claims.

Table VI illustrates the estimation process when claims below a given amount (such as $10,000) are excluded from the analysis. The basic difficulty involved in this procedure is in estimating the number and amount of claims which should have appeared below the truncation point. The trun-

[6] The above estimation procedure clearly indicates when the log-normal distribution does not provide a good fit for the data. This occurs when successive estimated CV's form a progression.

TABLE IV

SAMPLE DATA

Interval	Attachment Point	Number Of Claims	Claim Count Distribution: All Claims	Claim Count Distribution: Paid Claims	Indemnity On Interval ($1000's)	Moment Distribution	Excess Losses[a] ($1000's)	Excess Losses[a] Percent
0	0	2,370	51.4%	0%	$ 0	0%	$50,615	100.0%
1-10,000	10,000	1,496	83.8	66.7	4,500	8.9	38,645	76.4
10,001-25,000	25,000	365	91.7	83.0	6,437	21.6	30,128	59.5
25,001-100,000	100,000	267	97.5	94.9	13,933	49.1	14,245	28.1
100,001-300,000	300,000	99	99.7	99.3	16,488	81.7	4,457	8.8
300,001-1,000,000	1,000,000	15	100.0	100.0	7,207	95.9	1,050	2.1
Over 1,000,000	—	1	100.0	100.0	2,050	100.0	—	—
Total		4,613	(4,613)	(2,243)	$50,615			

Source: AIA (See Table VII)

Notes: a. Excess losses above a given attachment point are the sum of all claims values larger than the attachment point less the number of claims above the attachment point times the value of the attachment point. For example, there are 16 claims larger than $300,000 with an aggregate value of $9,257,000. The excess losses above $300,000 are thus 9,257,000 − 16(300,000) = 4,457,000 or 8.8% of the total limits losses.

TABLE V

ESTIMATING THE CV:
EXCLUSION OF CLAIMS CLOSED WITHOUT A PAYMENT

Case I. Includes all claims. Mean $11,000				Case II. Excludes claims closed without a payment. Mean $22,600		
Attachment Point				Attachment Point		
(1000's)	Times Mean	Excess Percent	Estimated CV[a]	Times Mean	Excess Percent	Estimated CV[a]
10	.9	76 %	[b]	.4	76 %	4.5
25	2.3	60	[b]	1.1	60	4.2
100	9.1	28	6.8	4.3	28	3.6
300	27	8.8	5.4	13	8.8	3.1
1,000	91	2.1	4.8	44	2.1	3.2

[a] Estimated from tables.
[b] More than 10.0.

cation point and the mean of the remaining claims are known. Unfortunately, the relationship between these two items does not specify a unique CV (see Exhibit III). We must therefore pick a provisional CV, calculate the number and amount of claims below the truncation point and then see if the CV estimated from the excess load distribution is the same as our provisional value. Table VI shows that the CV is about 2.4. This result implies that about 38% of the claims should have been truncated or that there should have been about 1,205 claims. Instead the data shows 2,243 claims. We might conclude that there were over 1,000 nuisance claims which cost an average of about $2,300 each.

TABLE VI
ESTIMATING THE CV: TRUNCATION

Case Study:	Truncation Point	$ 10,000
	Remaining Amount	$46,100,000
	Remaining Count	747
	Remaining Mean	$ 61,700
Ratio:	Truncation Point to Remaining Mean	.162

Assumption:	CV 2.0	CV 2.5	CV 3.0
Ratio: Remaining Mean to Complete Mean[a]	1.35	1.58	1.80
Estimated Complete Mean (1000's)	45.7	39.1	34.3
Ratio: Truncation Point to Complete Mean	.22	.26	.29
Percent of Total Amount Truncated[b]	3.5	4.8	5.8
Percent of Total Count Truncated[b]	28.6	39.6	47.7
Estimated Total Amount (1000's)	47,800	48,500	48,900

		CV 2.0			CV 2.5			CV 3.0		
Attachment Point (1000's)	Excess Losses (1000's)	Attachment (Times/ Mean)	Percent Excess	Estimated CV[b]	Attachment (Times/ Mean)	Percent Excess	Estimated CV[b]	Attachment (Times/ Mean)	Percent Excess	Estimated CV[b]
$ 10	$38,600	.22	81	2.0	.26	80	2.5	.29	79	3.0
25	30,100	.55	63	2.0	.64	62	2.5	.73	62	3.0
100	14,200	2.2	30	2.2	2.6	29	2.5	2.9	29	2.8
300	4,500	6.6	9	2.0	7.7	9	2.3	8.7	9	2.5
1,000	1,000	22	2.1	2.2	26	2.1	2.4	29	2.0	2.5

Notes: a. See Exhibit III
b. From Tables

EXHIBIT III

ESTIMATING THE COMPLETE MEAN FROM A TRUNCATED LOG-NORMAL DISTRIBUTION

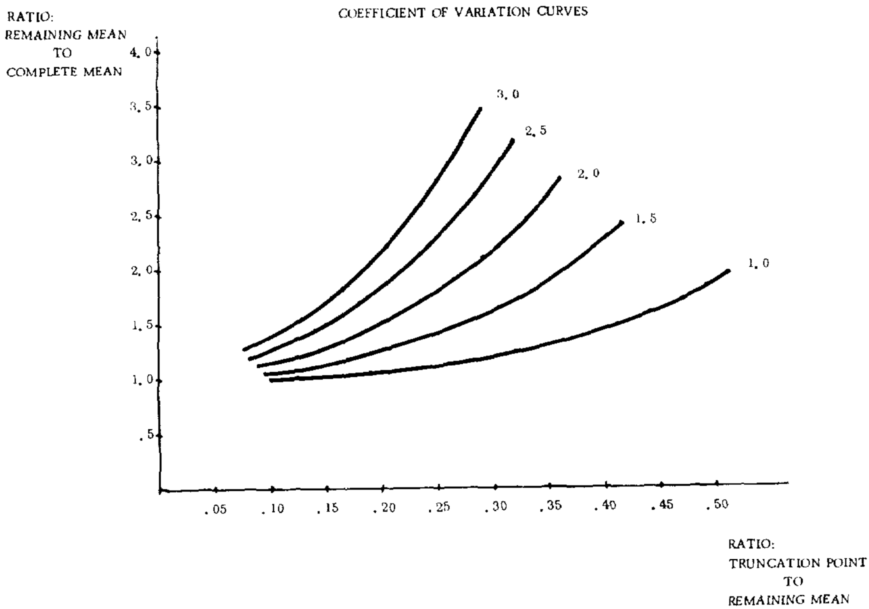

Table VII shows the estimation of the CV for two large groups of countrywide medical malpractice claims. The first group (the AIA study) has already been used in the previous analysis. Calculating the mean from all claims closed with a payment, indicates a CV of about 3.1 to 3.6 for attachment points in excess of $100,000. As previously shown, eliminating nuisance claims indicates a CV of about 2.4 for all attachment points. The second group (NAIC) indicates a higher estimated CV. This is partially due to one more claim in excess of $1,000,000. The higher CV may also be due to the broader group of companies which were included in the study.

SENSITIVITY ANALYSIS

Nuisance claims and other problems tend to distort the estimation process. Nuisance claims may be removed by estimating parameters for a truncated distribution. Because it is somewhat more cumbersome to estimate the CV from a truncated distribution, this section briefly analyzes the magnitude of the errors involved in estimating the CV, while excluding only claims closed without a payment.

Case I: Assume that we fit a log-normal distribution with an actual CV = 2.5 (without nuisance claims) to a distribution with a CV = 3.5. What is the actual error in the postulated excess distribution? Table VIII, Section I, demonstrates that this error is within 2% if the total limits costs. Smaller errors could be obtained by reducing the CV at higher attachments or estimating the CV from a truncated distribution.

Case II: Assume the actual distribution has a CV = 2.5. If about 38% of the claims are nuisance claims, what do we expect the estimated CV's to be for various attachment points? Table VIII, Section II shows that the estimated CV at about 3 times the mean would be 3.9; the estimates are expected to decline to 3.1 at an attachment of 65 times the mean. This is a typical pattern of estimated CV's, which may occur when there are a large number of nuisance claims.

TABLE VII
ESTIMATING THE COEFFICIENT OF VARIATION

I. AIA CLOSED CLAIM STUDY (1974)*

Interval	Number Of Claims	Indemnity On Interval ($1,000's)	Excess Losses ($1,000's)	Excess Losses Percent	Estimated C.V.
1-10,000	1,496	$ 4,500	$38,645	76.4%	4.5
10,001-25,000	365	6,437	30,128	59.5	4.2
25,001-100,000	267	13,933	14,245	28.1	3.6
100,001-300,000	99	16,488	4,457	8.8	3.1
300,001-1,000,000	15	7,207	1,050	2.1	3.2
Over 1,000,000	1	2,050	—	—	—
Total	2,243	$50,615			
Closed No Payment	2,370				

II. NAIC CLOSED CLAIM STUDY (DECEMBER, 1975)**

Interval	Number Of Claims	Indemnity On Interval ($1,000's)	Excess Losses ($1,000's)	Excess Losses Percent	Estimated C.V.
1-10,000	1,124	$ 3,082	$20,800	71.7%	4.0
10,001-50,000	372	7,851	11,029	38.0	3.9
50,001-100,000	83	5,422	6,857	23.6	3.7
100,001-300,000	51	7,607	2,950	10.2	4.0
300,001-1,000,000	5	2,050	1,000	3.4	4.5
Over 1,000,000	2	3,000	—	—	—
Total	1,637	$29,012			
Closed No Payment	2,711				

*Report to the All-Industry Committee Special Malpractice Review: 1974 Closed Claim Survey, Preliminary Analysis of Survey Results. December 1, 1975. Report 9.
**Volume 1, Number 1, December, 1975. Summary 22.

Case III: What if the observed distribution is actually a mixture of two or more distributions which have different means, but the same CV? There might, for example, be different means for different insurers, geographical areas, specialties, or accident years. Table VIII, Section III illustrates the case where half the claims have a mean of 10 and half have a mean of 30; both groups have observed estimated CV = 3.5. The estimated CV for the combined distribution is very close to 4.0. We thus observe what we would have expected, that a mixture of means will increase the coefficient of variation. It should thus be expected that CV's for individual insurers and states should be somewhat below those previously shown in the previous countrywide studies.[7]

[7] This may also explain why the NAIC study, which was based on a broader group of insurers, shows a higher CV than the AIA study.

TABLE VIII

SENSITIVITY ANALYSIS TO PARAMETER ESTIMATION

I. ERROR IN ASSUMING A LARGER COEFFICIENT OF VARIATION

Attachment Point	Theoretical Distribution Mean = 50 CV = 2.5	Assumption Of CV = 3.5 (Fitted At 100) Mean = 35	(Fitted At 250) Mean = 31
	Excess Loss Distribution Percentages		
100	35%	35%	33%
250	17	18	17
500	8	10	9
1,000	3	4	4
2,000	1	2	2

II. ERROR IN FITTING COEFFICIENT OF VARIATION

Attachment Point	Theoretical Distribution True Mean = 50 CV = 2.5 Excess Losses	Observed Mean = 31 Estimated Coefficient Of Variation
100	35%	3.9
250	17	3.5
500	8	3.4
1,000	3	3.2
2,000	1	3.1

III. MIXTURE OF DIFFERENT MEANS

Attachment Point	Component Distributions Mean = 10 CV = 3.5	Mean = 30 CV = 3.5	Composite Distribution (Observed)	Fitted Distribution Mean = 20 CV = 4.0
	Excess Loss Distribution Percentages			
5	72%	87%	83%	83%
10	58	78	73	73
20	43	66	60	60
50	24	47	42	41
100	14	32	28	27
200	7	20	16	16
500	2	8	7	7
1,000	1	4	3	3
2,000	—	1	1	1

Case IV: An unanswered and, in many situations, a crucial question is whether or not the coefficient of variation is changing over time. If not, one can estimate the total limits mean at a future date from a trending procedure. This mean and the CV will then completely determine the claim size distribution at the future date.

OTHER APPLICATIONS

Although the data in this paper comes from the medical malpractice line, claim sizes in many other lines appear to be log-normally distributed. Allocated expense payments also seem to be log-normally distributed. It is expected that the log-normal distribution may be appropriate whenever a large number of independent factors contribute multiplicatively to the claim size. Property lines may not provide a proper fit due to: (1) a tangible fixed upper limit on most property claims and (2) widely varying values at risk.

Examples in this paper have stressed excess losses. In many cases the log-normal distribution also yields suitable approximations for deductibles. A potential problem which may call for special handling, however, is nuisance claims.

CONCLUSION

This paper has presented an approach to estimating pure premiums by layer of insurance. It should be helpful to primary carriers for: (1) evaluating the basic limits experience of long-tail lines and (2) evaluating the cost of excess of loss reinsurance. It should be useful to reinsurers, if they have the basic limits experience of their reinsureds; in this case the approach is beneficial because the primary market tends to be more stable and its claims develop more quickly.

The method assumes that claim sizes, except for some nuisance claims, follow the log-normal distribution. In order to apply the method, the actuary needs to know the mean and coefficient of variation of the total limits claim size distribution. The mean is often estimated in the ratemaking process, leaving the coefficient of variation as an unknown. Countrywide data has been presented to estimate the CV for medical malpractice insurance. One sample showed a CV of 2.4, when nuisance claims have been excluded. If nuisance claims are included in the mean, the countrywide CV appears to full in the range from 3.0 to 4.0. For individual carriers or states, the CV should be lower.

APPENDIX

I. THE LOG-NORMAL DISTRIBUTION

The log-normal distribution (with parameters μ and σ^2) is defined as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma x}\, e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2} \qquad X > 0$$

The mean is $M = e^{\mu + \frac{1}{2}\sigma^2}$

The variance is $e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$

The coefficient of variation is $\beta = (e^{\sigma^2} - 1)^{\frac{1}{2}}$

Let the cumulative distribution function be

$$X1(\alpha \mid \beta) = \int_0^{\alpha M} f(u \mid \beta)\, du$$

where α is a ratio to the mean.

The (first) moment distribution is also log-normally distributed with parameters $\mu + \sigma^2$ and σ^2. This distribution is defined as:

$$X2(\alpha \mid \beta) = \frac{1}{M}\int_0^{\alpha M} u f(u \mid \beta)\, du$$

II. THE EXCESS LOSS DISTRIBUTION

Define $X3(\alpha)$ to be the percentage of total limits losses to be excess of α times the mean of the claim size distribution.

$$X3(\alpha \mid \beta) = (1 - X2(\alpha \mid \beta)) - \alpha(1 - X1(\alpha \mid \beta))$$

One property of the log-normal distribution is:

$$X2(\alpha \mid \beta) = X1\left(\frac{\alpha}{1+\beta^2} \mid \beta\right)$$