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PERSONAL LINES PRICING: FROM JUDGEMENT TO FACT

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INTRODUCTION

The event of an insurer undertaking to write a new line of business is not, under the most optimistic circumstances, a common occurrence. Such comparative rarity precludes the uniform accumulation of statistical observations, and, therefore, analysis of the common attributes of such ventures is not feasible.

The most recent past has witnessed a vigorous new interest in the marketing of personal lines business by various insurers and especially by those whose traditional expertise has been in the life and accident and health lines. The fact that life insurers dominate this renewed interest is not critical to the concept of this paper. The intensity of this interest, however, has served to highlight certain problems with the pricing of the insurance product which, not only the new insurer, but any insurer expanding its portfolio must face.

The opening statement regarding the difficulty of obtaining data on such new ventures is further complicated by the diversity of approaches individually attempting to translate this interest into the market place. There are direct writing life insurance companies establishing fully self-reliant operations, agency companies branching out into mail order merchandising of personal lines, direct writing life insurance companies utilizing a property/casualty "partner" for the placement of its agents' personal lines production, and the list goes on.

While the diversity of style grows, one basic fact remains constant: an initial base price must be established. This initial pricing process by

necessity has to consider numerous marketing aspects, underwriting criteria, industry experience trends, etc. Ultimately, however, a price emerges which has to be demonstrated to the regulator to be sound and to the consumer to be competitive; all of this alongside a full anticipation by the insurer of a reasonable underwriting profit. One must hasten to add that there is comparatively little room for statistically based argument over the initial price as it admittedly represents the consensus judgement of actuarial, underwriting, and marketing expertise.

After experience evolves for a period of time, say one year, the first rate level review becomes due. Whatever pricing problems initially confronted the insurer are now further complicated by the presence of a smattering of experience. This is the point at which this paper is intended to attach.

PROBLEM

The problem will first be illustrated and then stated.

The initial base rate is predicated on a projected pure premium and assumed expense, profit, and contingency loadings. Attention will be focused on the pure premium segment, inasmuch as the expense assumptions problem is different and is subject to different considerations altogether.

The determination of the initial base pure premium, at least in the classical sense, makes use of the following information:

> The target market segment(s).
> The projected underwriting selection criteria.
> Pertinent experience[1] data.
> Pertinent collateral data.

The principal thrust, of course, is to achieve a correct actuarial balance among the multiplicity of factors which [will] interact throughout the insurance transaction.

Accordingly, the collectives identified above generate a model market segment, a model underwriting policy, and a model pure premium realized by some (group of) insurer(s). In short, a model prospective competitor[2] emerges whose pure premium represents the new writer's "kick-off" point

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[1] Policy contract considerations are not addressed in this presentation. That is, an already existent policy is contemplated by the new writer.

[2] This model "competitor" may be a rating organization.

in constructing its initial base pure premium. Let the new writer's initial pure premium be denoted by P_u, and the model competitor's pure premium be denoted by P_c. Initially, P_u is a function of P_c, and this relationship is intended to measure the extent to which the judgement of the new writer would anticipate P_u to differ[3] from P_c.

A period of time goes by during which both P_u and P_c are "tested" as actual experience figures are accumulated. Let the raw experience pure premiums be $\overline{P_u}$ and $\overline{P_c}$, respectively. Current standard credibility procedures would yield [new] experience adjusted pure premiums, $\overline{\overline{P_c}}$ and $\overline{\overline{P_u}}$, as follows[4]:

$$\overline{\overline{P_u}} = Z_u \cdot \overline{P_u} + (1\text{-}Z_u) \cdot P_u \qquad (I)$$

$$\overline{\overline{P_c}} = Z_c \cdot \overline{P_c} + (1\text{-}Z_c) \cdot P_c \qquad (II)$$

Now, P_u as a linear function of P_c $(P_u = K \cdot P_c)$ may be assumed (for illustration purposes) to be less than P_c (i.e., $K < 1$). Also, suppose (for illustration purposes) that:

$$\overline{P_c} > P_c \text{ and } \overline{P_u} < P_u \qquad (III)$$

which, when combined with (I) and (II) would produce:

$$\overline{\overline{P_c}} \geq P_c \text{ and } \overline{\overline{P_u}} \leq P_u \qquad (IV)$$

and altogether yielding:

$$\overline{\overline{P_u}} / \overline{\overline{P_c}} \leq K \qquad (V)$$

thus raising immediate question about the initial judgement regarding the magnitude of K whenever strict inequality holds for (V).

If condition (III) is revised so as to reflect a movement "in formation" such as:

$$\overline{P_c} > P_c \text{ and } \overline{P_u} > P_u$$
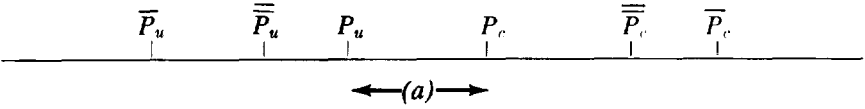$$\text{with } \overline{P_c} / P_c = \overline{P_u} / P_u \qquad (VI)$$

then the simple likelihood of Z_u being less than Z_c would still generate the same relationship as in (V) as well as the same questions arising therefrom.

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[3] A static hazard exposure (against a background of a static economy) is assumed so as to isolate the particular issue with which this paper is concerned.
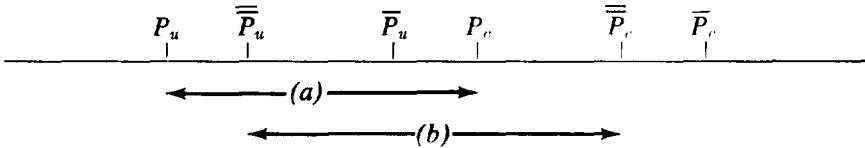
[4] Although trend is not specifically reflected in (I) or (II), footnote 3 assumed certain static conditions which provide license for this omission. Furthermore, the incorporation of trend is a relatively simple extension of the proposed modification procedures.

The following real number lines depict both situations described above as leading to (V):

(1) $K < 1; \overline{P}_u < P_u$ and $\overline{P}_c > P_c$

$$\overline{P}_u \qquad \overline{\overline{P}}_u \qquad P_u \qquad\qquad P_c \qquad\qquad \overline{\overline{P}}_c \qquad \overline{P}_c$$

$$\longleftarrow(a)\longrightarrow$$

$$\longleftarrow\!\!\text{——————}(b)\text{——————}\!\!\longrightarrow$$

(2) $K < 1; \overline{P}_u > P_u$ and $\overline{P}_c > P_c; Z_u < Z_c$

$$P_u \qquad \overline{\overline{P}}_u \qquad\qquad \overline{P}_u \qquad P_c \qquad\qquad \overline{\overline{P}}_c \qquad \overline{P}_c$$

$$\longleftarrow\!\!\text{————}(a)\text{————}\!\!\longrightarrow$$

$$\longleftarrow\!\!\text{——————}(b)\text{——————}\!\!\longrightarrow$$

(a) and (b) identify $|\,P_u\text{-}P_c\,|$ and $|\,\overline{\overline{P}}_u\text{-}\overline{\overline{P}}_c\,|$, respectively. For the sake of completeness, it should be noted that similar situations can occur for $K \geq 1$.

The interpretation problems attendant to the illustrated movements in the pure premium are numerous and do not lend themselves to ready explanation. Although not always as starkly present as suggested here, these movements need to be properly reconciled prior to "closing the book" on the revision of P_u. The examples provided herein demonstrate the potential inadequacy of present credibility procedures in effecting a proper transition from the initial pure premium through a series of experience modifications to a fully "seasoned" pure premium. This paper provides a means whereby standard credibility procedures, as presently utilized, may be augmented to produce a systematic transition from judgement to fact.

DISCUSSION

The motivation for the manner of construction of the solution would probably be aided by a brief discussion of some of the more prominent aspects of the judgement underlying the selection of the initial pure premium.

In considering the marketing aspects, suppose one is given an ongoing product distribution apparatus with varying degrees of penetration of certain population strata. The new writer has to carefully gauge the potential of this marketing force in several respects. Among them:

Will the already penetrated strata be the target strata for the new product?
Will the degree of penetration of each stratum contract or expand?

Will production quotas be established?
Will there be advertising support?
Will there be an inter-product (new & old) coordination?

This list is by no means exhaustive. It does illustrate, however, the many judgements which must be made en route to a full visualization of the typical expected risk. Each of these items is subject to a full range of judgements in its own right. Each must be gauged for the present and projected into the future. The combined judgements relative to all the pertinent marketing questions [if the questions are distilled to a single appropriate inquiry] will reduce to the marketing input in the process of arriving at the initial pure premium.

On the underwriting side, the considerations are of necessity quite different. Although the average expected risk should have been largely identified, the underwriting policy must also accommodate several environmental conditions. For example:

The extent of new staff acquisition and training.
The existing (and probably) statutory underwriting constrictions.

Also, the underwriting policy proper has to be defined in the appropriate fine detail within the projected environment. Without detailing the many facets which have to be defined in order to produce such policy, the aggregate of all judgements may be reduced to certain key projections, such as:

Prospective rejection ratios.
Prospective non-renewal ratios.
Prospective frequency of loss.

These projections would naturally utilize the marketing input already provided. Also, as new business is processed, these ratios are subjected to the initial acid test. And, as the first experience pure premium is reviewed, the fact that this is the initial test must be acknowledged and injected into the interpretation.

The claim aspects face yet a different set of complications. The severity element must be projected with largely no historical performance yardsticks of the projected claim apparatus, at least for the new line of business. The first experience pure premium, on the other hand, is heavily impacted by judgement. To put it another way, case reserves, incurred but not reported reserves, and all other bulk loss reserves enter the first experience pure premium at full face. And, for third party coverages, these

reserves easily constitute the greater portion of the corresponding pure premium. Finally, questions regarding the claim environment must be correctly formulated and resolved.

These illustrations highlight the many, many facets which must not only be recognized but also be projected forward. The first experience pure premium, presumably, provides the first glimpse of how the composite of these judgements fared. As the section describing the problem noted, if one subjects this first glimpse merely to standard credibility procedures, potential pricing problems are likely to arise. The next section demonstrates how this potential may be eliminated.

SOLUTION

Once the projected target has been numerically established for an underlying component, the progress towards the full achievement of this target may be considered to be a function of both actual observations and time. For example, if one considers the claim operation to be fully "mature" once at least 10,000 claims have been paid and closed and the operation has been in existence for at least five years, then a simple progress function may be defined[5] as follows:

$$f(N, t) = [(N/10{,}000)\ (t/60)]^{1/2}$$

with:

$N =$ Number of claims paid and closed since inception of operations, and

$t =$ Number of months elapsed since inception of operations.

Actually in order to assure $f(N, t)$ will always be ≤ 1, the following construction is needed:

$$f(N, t) = \begin{cases} f(N, t) & \text{if} \quad f(N, t) \leq 1 \\ 1 & \text{if} \quad f(N, t) > 1 \end{cases}$$

Table I illustrates sample values of f for the indicated sample combinations of N and t.

To construct this piece of the solution formally, denote the progress functions corresponding to the measurable attributes for which numerical

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[5] This example is deliberately oversimplified. No attempt is made here to identify all the components which would contribute to the full maturation of a claim operation.

targets have been established by:

$$C_1, C_2, \ldots C_n$$

TABLE I
NUMBER OF CLAIMS PAID AND CLOSED

Time	...	374	...	2,805	...	8,022	...	12,422	...
.		.		.		.		.	
.		.		.		.		.	
.		.		.		.		.	
6 mos	...	0.06	...	0.17	...	0.28	...	0.35	...
.		.		.		.		.	
.		.		.		.		.	
.		.		.		.		.	
22 mos	...	0.12	...	0.32	...	0.54	...	0.67	...
.		.		.		.		.	
.		.		.		.		.	
.		.		.		.		.	
49 mos	...	0.17	...	0.48	...	0.81	...	1.00	...
.		.		.		.		.	
.		.		.		.		.	
.		.		.		.		.	
86 mos	...	0.23	...	0.63	...	1.00	...	1.00	...
.		.		.		.		.	
.		.		.		.		.	
.		.		.		.		.	

where n is the number of distinct subject attributes.

Each C_i is a function of time, t, as well as some raw observation, u_i, as follows:

$$C_1 = f_1(u_1, t), C_2 = f_2(u_2, t), \ldots, C_n = f_n(u_n, t)$$

Since each C_i is subject to a maximum of 1 (objective accomplished), a new set of truncated functions, $\bar{f}_i$ will be needed. For example, for every i, re-define C_i as follows:

$$C_i = \bar{f}_i(u_i, t)$$

with:

$$\bar{f}_i(u_i, t) = \begin{cases} f_i(u_i, t) & \text{if} & f_i(u_i, t) \leq 1 \\ 1 & \text{if} & f_i(u_i, t) > 1 \end{cases}$$

Having established $\overline{f}_i\,(u_i,\,t)$, a system of time-dependent weights is needed:

$$w_1(t),\,w_2(t),\,\ldots,\,w_n(t)$$

subject to the condition:

$$\sum_i w_i(t) = 1 \quad \text{for every } t.$$

$w_i(t)$ attaches to $\overline{f}_i(u_i,\,t)$ in the process of aggregating the combined progress of the new writer as the raw data are accumulated.

Accordingly, the experience modified pure premium, $\overline{\overline{\overline{P}}}_u$, is subject[6] to the weight function W, defined as follows:

$$W\,(u_1,\,u_2,\,\ldots,\,u_n,\,t) = \sum_i w_i(t)\,\overline{f}(u_i,\,t) \qquad \text{(VII)}$$

Therefore, the revised experience modified pure premium, P_u, can now be constructed as:

$$\overline{\overline{\overline{P}}}_u = W \cdot \overline{\overline{\overline{P}}}_u + (1\text{-}W)\,(K \cdot \overline{\overline{P}}_c)$$

$$= W \cdot [Z_u \cdot \overline{P}_u + (1\text{-}Z_u) \cdot P_u] +$$

$$[(1\text{-}W) \cdot K]\,[Z_c \cdot \overline{P}_c + (1\text{-}Z_c) \cdot P_c] \qquad \text{(VIII)}$$

Some observations should be made regarding the nature of the underlying components and their collective impact on $\overline{\overline{\overline{P}}}_u$:

(a) $u_i \, \varepsilon \, [a_i,\,b_i]$, where a_i is the value of u_i at $t=0$ and b_i is the target for u_i.

(b) $t \, \varepsilon \, [0,\,T]$, where T is the maximum period needed for all $\overline{f}_i\,(u_i,\,t)$ to achieve a value of 1 regardless of the behavior of u_i.

(c) $\overline{f}_i(u_i,\,t) \to 1$ as $t \to T$ or $u_i \to b_i$.

(d) $W(u_1,\,u_2,\,\ldots,\,u_n,\,t) \to 1$ as $t \to T$ or $u_i \to b_i$ for every i.

(e) $(1\text{-}W) \cdot K \to 0$, as $W \to 1$ [directly from (d)].

(f) $\overline{\overline{\overline{P}}}_u \to \overline{\overline{P}}_u$, as $W \to 1$ [directly from (e)].

While it may be obvious, it is probably worth noting that $\overline{\overline{\overline{P}}}_u$, as stated in (VIII), essentially reconstructs the value of K as given by $\overline{K} = \overline{\overline{\overline{P}}}_u/\overline{\overline{P}}_c$. Thus

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[6] This modification in fact assures that the actual rate revision does not reflect [temporary] operational conditions at full face, such as by way of the construction suggested in (VIII).

when the second experience review becomes due, the process of reconstructing $\overline{K}$ becomes iterative with $\overline{K}$ as the basis judgement of the now not-so-new writer. With this in mind, statement (e) is true regardless of what stage of development K derives from. Also, statement (f) depicts how the original judgement as reflected by K evolves through a succession of $K's$ and ultimately [judgement] reduces to zero as the experience pure premium, $\overline{\overline{P}}_u$, [usually] eventually displaces $\overline{\overline{P}}_u$ until it $[\overline{\overline{P}}_u]$ becomes the operating basis pure premium without qualification.

DEMONSTRATION

Given a situation producing the following circumstances subject only to normal credibility procedures at the conclusion of the first twelve months of operation:

$$K = .80$$

$$P_c = \$50.00, \overline{P}_c = \$54.00, \overline{\overline{P}}_c = \$52.80 \quad (Z_c = .70)$$

$$P_u = \$40.00, \overline{P}_u = \$36.00, \overline{\overline{P}}_u = \$39.20 \quad (Z_u = .20)$$

Also, suppose that overall operational progress is dependent on precisely two attributes which have been identified as follows:

Total sales, S, as measured by the total direct premiums written since the inception of operations.

Number of claims closed and paid, N, since the inception of operations.

Furthermore, suppose that the corresponding $\overline{f}'s$ have been constructed (very simply) as follows:

Sales:

$$\overline{f}_1(u_1, t) = \overline{f}_1(S, t)$$

$$= \begin{cases} (S/10^7)\,(t/36)^{1/2} & \text{if} \quad (S/10^7)\,(t/36)^{1/2} \leq 1 \\ 1 & \text{if} \quad (S/10^7)\,(t/36)^{1/2} > 1 \end{cases}$$

Claims:

$$\overline{f}_2(u_2, t) = \overline{f}_2(N, t)$$

$$= \begin{cases} [(N/10{,}000)\,(t/60)]^{1/2} & \text{if} \quad [(N/10{,}000)\,(t/60)]^{1/2} \leq 1 \\ 1 & \text{if} \quad [(N/10{,}000)\,(t/60)]^{1/2} > 1 \end{cases}$$

Given that:

$$S = \$2,825,000$$
$$N = 8,022$$

$\bar{f}_1$ and $\bar{f}_2$ derive immediately:

$$\bar{f}_1 (2,825,000 \; ; 12) = 0.16$$
$$\bar{f}_2 (8,022 \; ; 12) = 0.40$$

Finally, suppose a system of weights $w_i(t)$ has been defined as follows:

$$w_1(t) = (2t^2 - 5t + 1) / [2t(t + 1)]$$
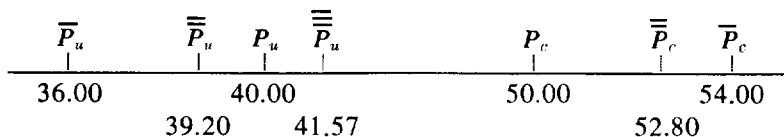$$w_2(t) = (7t - 1) / [2t(t + 1)]$$

And, for $t = 12$,

$$w_1(t) = .73 \quad \text{and} \quad w_2(t) = .27$$

Substituting in (VII) and (VIII) respectively yields:

$$W(S, N, t) = \Sigma_i \, w_i(t) \, \bar{f}_i \, (u_i, t)$$
$$= w_1(t) \, \bar{f}_1 \, (S, t) + w_2(t) \, \bar{f}_2 \, (N, t)$$
$$= .22$$

$$\text{and } \overline{\overline{\overline{P}}}_u = \$41.57$$

Once again, on a real number line, the following depicts the entire demonstration:

$\overline{P}_u$	$\overline{\overline{P}}_u$	P_u	$\overline{\overline{\overline{P}}}_u$	P_c	$\overline{\overline{P}}_c$	$\overline{P}_c$
36.00		40.00		50.00		54.00
	39.20		41.57		52.80	

To illustrate the iterative properties of the process as described herein, the new value of K (at 12 mos) reduces to

$$\overline{K} = \overline{\overline{\overline{P}}}_u / \overline{\overline{P}}_c = \$41.57/\$52.80 = 0.787$$

Thus the initial judgement setting K at .800 has been augmented and is now reset at $\overline{K} = 0.787$. When the second review becomes due, equation (VIII) will utilize $\overline{K}$, ultimately producing $\overline{\overline{K}}$, and so on. It should be noted that the "evolution" of K, through its various updates, does not impact the con-

tinued application of (VIII) with the same originally selected $\overline{f_i}$'s and w_i's.

CONCLUSION

Although the discussion has been couched in terms of the pure premium, the principles espoused here are equally applicable to any measurable aspect of the insurance transaction with, of course, appropriate modifications to the construction. The use of the pure premium is primarily in deference to the genesis of the idea within a personal lines framework.

One important application presents itself when an insurer elects to expand its operation to another jurisdiction. The "seasoning" of this new book of business will be an important aspect in assessing operational results during the early stages.

A word about the matter of selecting the functions $\overline{f_i}$ and w_i. Although each function can take innumerable forms—just which form(s) is the most responsive to the particular prospective modus operandi of the new writer is a matter of great import. These selections truly represent a new writer's insight, experience, and planning. This issue is not of as much moment for the already operational writer simply expanding its operations geographically as there probably exists a great store of knowledge about likely performance standards.

Finally, while the role of judgement in the ratemaking process could not be denied, this effort hopes to have established a framework for the systematic and consistent application of judgement by the new writer as the character of its operations evolves into an ongoing posture.