THE INTERPRETATION OF LIABILITY INCREASED LIMITS STATISTICS

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Several papers in the *Proceedings* deal with established ratemaking procedures for various lines of insurance and two such papers discuss in detail the methodology for liability insurance lines. In both papers attention is restricted to ratemaking techniques for basic limits coverage. The papers mention that statistics are collected for coverage above the basic limits, but they do not describe analysis of these statistics.[1] This limitation of the papers is understandable since their objective is to describe established ratemaking techniques and since there are no widely accepted methods in the increased limits area. Almost without exception both individual companies and rating bureaus express the premium for increased limits coverage as a function of the premium for basic limits coverage. Usually, the rates for increased limits of coverage are obtained simply by applying a factor to the appropriate basic limits rate.

Increased limits of liability are widely sold and the premiums involved can be substantial. Approximately two-thirds of automobile insureds purchase some increased limits coverage, and for some general liability sublines (e.g. professional malpractice) almost all insureds carry increased limits coverage. For automobile bodily injury liability, the charge for limits of $100,000 per person and $300,000 per accident is 41% of the basic limits charge, while comparable charges in the general liability line are as high as 159% of basic limits premium. In total, the premiums for increased limits of liability for automobile and general liability lines exceed the premiums for many other lines of insurance.

The actuary is faced not only with the problem of setting the increased limits charges, but also that of setting marketing strategy. Increased limits coverage is not only voluntary, but also evanescent. To a great extent it is sold, not bought. This is especially true with regard to the precise limit

[1] Stern, P. K., "Ratemaking Procedures for Automobile Liability Insurance," *PCAS* Vol. LII, p. 155.
Lange, J. T., "General Liability Insurance Ratemaking," *PCAS* Vol. LIV, p. 30.

selected. Even if the insured recognizes the need for increased limits coverage, it is difficult for him to decide objectively how much coverage to purchase. Should the insurer encourage him to buy high limits? While it would seem that profitability would be easily ascertainable, it can be an evasive question. Large liability losses may not be settled for many years. Inflationary trends have a substantially different impact in the increased limits area and, additionally, influence outstanding cases. Increased limits losses are in the tail of the distribution of losses by size and there is considerable statistical variation. Increased limits premiums are generally lumped in with basic limits premiums along with other unrelated charges. The situation is further complicated by reinsurance treaties and the fact that beyond some limit the insurer's expected losses are zero and it is charging only for bearing the potential risk. This paper discusses the analysis of increased limits statistics, indicating several major problem areas, in the hope that other actuaries will offer suggestions as to how research in this area might be carried forward.

Loss Ratios or Pure Premiums

In ratemaking, either a loss ratio or a pure premium approach is usually used; however, neither is well suited to the increased limits area. For the loss ratio approach to yield satisfactory results it is necessary that premiums be known with some degree of accuracy. Increased limits premium charges are determined by applying a factor to basic limits rates and are included with basic limits premiums and other charges (e.g. medical payments) in one lump sum premium. To reconstruct separate increased limits premiums would require that estimates concerning increased limits and other charges be made so that the total premium can be subdivided into components. Even if this were done and loss ratios were constructed, their utility would not be great. Since increased limits charges are a function of basic limits rates, the isolated fact that increased limits experience is good or bad does not tell the ratemaker whether or not the relationship between increased limits and basic limits rates is correct.

The use of pure premiums presents a slightly different problem. Increased limits losses are influenced by many of the same factors that influence basic limits losses. For example, one would logically expect that increased limits charges should depend on rating territory and classification in addition to limit purchased. This would involve subdividing the data into a great many categories. But since increased limits data is of lesser volume than basic limits and is subject to much greater statistical variation, it would appear that the resulting pure premiums for each category would have little credibility. Finally, the pure premium approach is not particularly convenient for testing the present rating procedure in which increased limits charges are expressed as a function of basic limits rates.

Losses to Losses

While it would be possible to adapt either the pure premium or loss ratio approach for use in increased limits ratemaking, there is another ratio which might be employed. In rating increased limits coverage, the increased limits premium charge is expressed as a ratio of the basic limits premium, which in turn is a function of basic limits losses. This suggests relating increased limits losses for a particular policy limit to the corresponding basic limits losses for that same policy limit. The resulting ratio expresses the increased limits losses as a proportion of basic limits losses. If both sets of losses are estimates of expected losses (reflecting loss adjustment expense, loss development, adjustments for changes in cost and frequency), then the ratio of increased limits premiums to basic limits premium for that policy limit should be the same as the ratio of losses to losses in order to produce comparable results. In other words, the ratio of increased limits losses to basic limits losses corresponds to an increased limits factor.

For policies carrying limits of m/n where m/n is greater than basic limits

$$r_{m/n} = \frac{\text{increased limits losses}}{\text{basic limits losses}}$$

where increased limits losses equal total losses less basic limits losses, and basic limits losses reflect the application of the basic limits of liability to each loss.

$f_{m/n}$ = increased limits factor = $1.00 + r_{m/n}$

$p_{m/n}$ = premium for limits of $m/n = f_{m/n} \times$ (basic limits rate)

One disadvantage of the approach, as stated, is that in reviewing the charge for 15/30 limits one would be restricted to the use of the experience of 15/30 policies. To avoid this limitation, one could use the experience of all policies having at least 15/30 limits (e.g. 15/30, 20/20, 25/50, etc.) and limit the increased limits losses for all such policies to 15/30.

To carry out this approach in general, losses must be subdivided in two ways: first by policy limit purchased and then by layer of loss (less than

10/20, from 10/20 to 15/30, from 15/30 to 25/50, etc.). In the latter subdivision, a \$30,000 loss would have \$10,000 assigned to the first layer, \$5,000 to the second, \$10,000 to the third and \$5,000 to the fourth, assuming 10/20 basic limits. Following this procedure, the data could be arranged in an array where L denotes the losses in the cell, the first subscript denotes the upper limit of the layer of loss and the second subscript denotes the policy limit purchased by the insured. For example, $L_{15,25}$ denotes the amount of losses in the layer between limits of 10/20 and 15/30 (e.g. \$5000 for a \$15,000 or more claim) for policies with a 25/50 limit.

Layer of Loss	Policy Limit Purchased				
	10/20	*15/30*	*20/40*	*25/50*	. . .
Portion of losses less than 10/20	$L_{10,10}$	$L_{10,15}$	$L_{10,20}$	$L_{10,25}$	
Loss amounts between 10/20 and 15/30		$L_{15,15}$	$L_{15,20}$	$L_{15,25}$	
Loss amounts between 15/30 and 25/50			$L_{25,20}$	$L_{25,25}$	
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To evaluate the 15/30 limit, increased and basic limits losses for all policies with at least 15/30 limits would be compared

$$r_{15/30} = \frac{L_{15,15} + L_{15,20} + L_{15,25} + \ldots}{L_{10,15} + L_{10,20} + L_{10,25} + \ldots}$$

$$r_{15/30} = \sum_{i \geqslant 15} L_{15,i} \Big/ \sum_{i \geqslant 15} L_{10,i}$$

Similarly, to evaluate the 25/50 limit one would compute

$$r_{25/50} = \left[\sum_{i \geqslant 25} L_{25,i} \Big/ \sum_{i \geqslant 25} L_{10,i} \right] + r_{15/30}$$

It is probably not possible to have a layer of losses corresponding to every limit purchased, since the construction of the layers requires the subdivision of each excess loss. In the above example, the treatment of a policy limit not corresponding to a layer of losses is illustrated with the 20/40 limit. The limit factor for this limit could be set by interpolation. For limits corresponding to layers of loss, the following general formula is appropriate.

$$r_{m/n} = \left[\sum_{i \geqslant m} L_{m,i} \Big/ \sum_{i \geqslant m} L_{10,i} \right] + r_{m'/n'}$$

where $r_{m'/n'}$ denotes the r for next lowest layer of losses.

This formula assumes a $10,000/$20,000 basic limit and also assumes the application of the appropriate accident limit to the individual losses in each case.

Loss Development

In accident year ratemaking, it is generally necessary to measure subsequent "loss development" as reserves translated into paid claims with the passage of time. The customary techniques for the determination of loss development factors[2] can be applied in the increased limits area. Increased limits losses are, by definition, large liability cases and generally take a long time to be settled. Their magnitude cannot always be adequately estimated since there are few large cases and since these cases are frequently of an exceptional nature. As a result, it is necessary to measure loss development over a long period of time; significant changes in accident year losses may occur even at 60 and 72 months evaluations. A further consequence is that the factors to be applied to the first and second reportings of the losses are quite substantial. In the former case, the factors may exceed two. This implies that, even after giving careful attention to loss development factors, the actuary can give little credence to the latest year of experience alone and several years of data must be used in any analysis. If the results of the analysis must be explained to non-actuaries who might be disturbed by the magnitude of the factors, it might be well to recast the study on a calendar year basis, which although less accurate, avoids the use of loss development factors.

If loss development factors for increased limits are greater than basic limits, one might expect this same phenomenon might be observed if layers of increased limits coverage are compared. Studies have shown that increased limits loss development factors do increase for each successively higher layer of coverage. If this fact is neglected, it will distort any analysis of the factors for the higher limits (e.g. 50/100, 100/300).

[2] Stern, *op. cit.*, p. 162.

Loss Trends

In basic limits ratemaking, it is common practice to adjust the reported loss experience for prospective changes in claim cost and, for auto liability, in frequency. As basic limits rate levels are set using the latest available years of experience, and as basic limits rates are frequently revised, these adjustments are often of a routine nature. For increased limits of liability, the situation is quite different: a number of years of experience are used in ratemaking; increased limits tables are not frequently changed; increased limits trends are substantially greater than basic limits trends.

In a period of rising claim costs, the cost for excess or deductible insurance will rise more rapidly. One can grasp the general idea by considering two claims, one of $1,500, one of $15,000. If the $1,500 claim increases 10 percent, its basic limits portion increases 10 percent, while if the $15,000 claim increases 10 percent, its basic portion is unchanged at $10,000 while its excess portion increases 30 percent from $5,000 to $6,500, assuming $10,000 basic limits. Thus, if claim costs are increasing slightly each year due to inflationary pressures, the impact of this increase will be much greater on increased limits experience than on basic limits experience.

Increased limits cost trends increase more rapidly than basic limits for two reasons. First, the whole effect of the trend is in the excess portion of the increased limits claim while the effect on the basic limits portion is zero. Second, although uniform frequency trends affect equally basic and increased limits, a rising cost trend causes a rise in increased limits claim frequency since additional claims (previously only basic limits losses) break through the lower boundary of the increased limits layer of losses becoming new excess claims. If x represents the dollar amount of a loss, N the total number of claims, and $p(x)$ equals the probability that the value of a loss is x, then losses above basic limits of k, the increased limits losses, are:

$$N\left[\int_k^\infty x\,p(x)\,dx - k\int_k^\infty p(x)\,dx\right]$$

If losses increase by "a" percent, not only is the first term multiplied by $(1 + a)$ with no increase in the negative second term, but basic limits losses in the range $k/(1 + a)$ to k now become increased limits losses contributing the following amount to the total increased limits losses:

$$N\left[(1+a)\int_{\frac{k}{1+a}}^{k} x\,p(x)\,dx - k\int_{\frac{k}{1+a}}^{k} p(x)\,dx\right]$$

Combining this expression with the previous one yields the increased limits losses after the application of the trend. Dividing the new increased limits losses by the old gives the increased limits trend factor, which is equal to the basic limits trend factor plus a quantity which is always strictly greater than zero.

Thus it can be established mathematically that increased limits loss trends are greater than basic limits trends. How much greater? One can estimate the magnitude either directly from claim cost data or by the application of the above equations to a distribution. The lognormal distribution is a plausible model[3] for the distribution of claims by size and is relatively easy to work with. The parameters of the distribution may be estimated[4] from a sample of claims and the theoretical distribution of claims may be adjusted by a uniform trend factor. (The model may be further refined by injecting assumptions concerning the policy limits purchased because some claims are not increased fully by the trend factor since they would then exceed the insured's limit.)

In addition to obtaining basic and increased limits trend factors, the data may be grouped by layer of loss so that separate trend factors by increment of coverage may be calculated. Generally, this action will result in a basic limits trend factor less than the total limits trend factor, and in increasing trend factors for each layer of loss with the highest trend factor obtaining for the highest layer of loss. This last trend factor may be as much as twice the total limit factor. This result parallels that discussed for loss development. If these two points are neglected, one could easily be misled in an analysis of data for the higher limits of liability.

An alternative approach, which unfortunately does not lead to a gradation of trend factors by layer of loss, is separately to fit total limits and basic limits claim costs (from the same population) to a line.[5] One may subtract the basic limits average cost from the total limits average cost and also subtract the basic limits average annual change (from the fitted line) from the corresponding total limits figure. The resulting average cost over

[3] Bailey, R. A., "Experience Rating Reassessed," *PCAS* Vol. XLVIII, p. 60.
Benchert, L. G., "The Lognormal Model for the Distribution of One Claim," *ASTIN Bulletin* Vol. II, p. 9.

[4] Aitchison, J. and Brown, J., *The Lognormal Distribution* (Cambridge University Press, Cambridge, 1957), p. 39.
Gjeddeback, N. F., "Contributions to the Study of Grouped Observations," *Skandinavisk Aktuarietidskrift* Vol. 32, p. 135 ff.

[5] Stern, *op. cit.*, p. 172.

basic limits and average annual change over basic limits may be used to compute the increased limits loss trend corresponding to the basic and total limits trends. (It should be noted that both of the average dollar amounts over basic limits are expressed as in terms of all claims — both basic and excess — and thus this trend factor reflects the added frequency discussed above.) This may be illustrated with the following example:

	Average Claim Cost	Average Annual Change in Claim Cost From Fitted Line
Total limits	\$1100	\$100
Basic limits	1000	80
Difference	\$ 100	\$ 20

Total limits trend: $\frac{100}{1100} = 9\%$

Basic limits trend: $\frac{80}{1000} = 8\%$

Increased limits trend: $\frac{20}{100} = 20\%$

While this approach is not perfect it can be easily applied to readily available data, is relatively simple to explain, and does demonstrate the magnitude of the problem.

Credibility

It is well known that a way of increasing the relative credibility of a body of data is to exclude or limit the larger losses.[6] It follows that these large losses by themselves have much less credibility than do the basic losses. The amount of variation (the standard deviation or coefficient of variation) in each increased limits increment (or layer of loss) may be compared to the amount of variation in basic limits data in order to determine the degree of increased variation. This approach leads to the conclusion that increased limits experience requires higher credibility factors, but such approach does

[6] Roberts, L. H., "Credibility of 10/20 Experience as Compared with 5/10 Experience," *PCAS* Vol. XLVI, p. 235.

not lead to a determination of exact factors. Perhaps the Mayerson-Jones-Bowers formula[7] could be adapted to deal with a portion of the loss distribution and thus yield a credibility standard.

Reinsurance Costs

When increased limits coverage is written two insurance carriers are often involved: the primary (direct) insurer and the reinsurer. Some allowance must be made for the fact that both of these carriers incur administrative (operating) expenses. The basic limits rates include a provision only for the expenses of the direct or primary insurer. Increased limits premiums are determined from basic limits premiums; unless some provision is made in the analysis for the expenses of the reinsurer, the increased limits charges would be inadequate in that they would fail to contain the necessary allowance for the expenses incurred by the reinsurer and paid by the primary insurer. It is recognized that this increased expense to the primary insurer results in lower risk; therefore, the element of reinsurance expense might be combined with that of risk. However, reinsurance is an important consideration in determining profitability and the adequacy of increased limits charges.

Risk

While it is obvious that the risk assumed in insuring increased limits coverage is greater than the risk assumed in insuring basic limits coverage, it is difficult to measure this difference in assumed risk quantitatively.

Letting x denote the losses of a policy, p its expected losses, and $f(x)$ the probability that losses for an individual policy do not exceed x, risk has traditionally been defined by actuaries as follows:[8]

$$\text{Risk} = \left[\int_0^\infty (x-p)^2 \, df(x) \right]^{1/2}$$

$$\text{where } p = \int_0^\infty x \, df(x)$$

[7] Mayerson, A., Jones, D., and Bowers, N., "The Credibility of the Pure Premium," *PCAS* Vol. LV, p. 175.

[8] Borch, K., "The Theory of Risk," *Journal of the Royal Statistical Society, Series B*, Vol. 29, p. 432, attributes this definition to Hansdorf, F., "Das Risico bei Zufallsspielin," *Leipziger Berichte* Vol. 49 (1897), p. 497.

Plotkin has employed the variance in his calculation of risk, while Houston has suggested that the standard error of mean pure premium be used as measure of the risk assumed by the insurer.[9] He argues that the insurer's risk includes not only the variation inherent in the pure premium distribution, which would be measured by the standard deviation and variance, but also includes the expected variation of the average pure premium. All of these suggested measures illustrate that risk is essentially a variance, not a mean, concept.

In actuarial literature, the usual way of meeting risk is through the use of a safety loading (proportional to the risk) in the premium.[10] This is not inconsistent with economic theory which links level of profit to degree of uncertainty. Each insurer is of finite capacity and need not assume every possible risk. If the profit were the same on a 10,000/20,000 policy as on a 1,000,000/2,000,000 policy, why should a prudent insurer assume the added risk of a 1,000,000/2,000,000 or even a 100,000/200,000 policy. The argument of reinsurance does not blunt this point since the insurer must pay a greater reinsurance premium if he writes 1,000,000/2,000,000 policies than if he limited himself to 10,000/20,000. Some element in the formula, either a safety loading, a larger profit margin, or an increment for reinsurance expense, would seem necessary in the analysis of increased limits statistics.

It would seem that this element should increase as the risk increases at higher limits. For limits above $100,000 (e.g. $1,000,000), risk is more important than the pure premium, since the frequency of $1,000,000 liability claims is miniscule. While the element for risk is obviously necessary, and easily justified intuitively, it is difficult to calculate analytically. If a pure premium distribution could be obtained, the measures described in previous paragraphs might be applied.

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[9] Conrad, G. and Plotkin, I., "Risk/Return: U.S. Industry Pattern," *Harvard Business Review,* March-April 1968, p.. 90, and *Prices and Profits in the Property and Liability Insurance Industry* (American Insurance Association, New York, 1967).
Houston, D. B., "Risk, Insurance and Sampling," *Journal of Risk and Insurance* Vol. XXI, p. 511.

[10] Borch, K., *op. cit.*
Cahill, J. M., "Deductible and Excess Coverages, Liability and Property Damage Lines Other Than Automobile," *PCAS* Vol. XXIII, p. 18.
Cramér, H., "Collective Risk Theory, a Survey from the Point of View of the Theory of Stochastic Processes," *Skandia Jubilee Volume,* Stockholm.
Lange, J. T., "Application of a Mathematical Concept of Risk to Property-Liability Insurance Ratemaking," *Journal of Risk and Insurance,* Vol. XXXVI, p. 383.

Conclusion

The subjects discussed in the paper could (and have been) brought together into a complete analysis of a set of increased limits statistics. Such an analysis has not been presented in the paper since it would imply both a level of development and a degree of acceptance of the idea which is not warranted. Numerical exhibits might detract from the philosophical discussion which is necessary at this stage in the development of ratemaking procedures for increased limits coverage. On the other hand, it is interesting to note that the application of the procedures outlined in this paper to actual numerical data has led to conclusions contrary to those based simply upon overall, approximate increased limits loss ratios.

Increased limits coverage has usually been thought of as profitable to insurers and one may question whether refined calculations are necessary. Yet a paradox appears if one reviews the experience of the reinsurers, many of whom write on a "manual excess basis" receiving the manual increased limits premium (less direct expenses) as their premium. Despite their freedom from regulation, the reinsurers have not found this area profitable in recent years. Perhaps our conventional wisdom about increased limits profitability is more faith than fact and is based upon a superficial analysis which neglects the long term nature of these claims, the additional expense of reinsurance, the large risks assumed by the company, and the greatly magnified impact of trend and development on higher limits of liability.