Premium Earning Patterns for Multi-year Policies with Aggregate Deductibles

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Abstract

Multi-year policies with large aggregate deductibles or multiple triggers raise some interesting issues with respect to the correct amount of the unearned premium reserve. Examples in this paper illustrate some of the difficulties that arise when trying to establish such reserves. The theoretical approach taken here is that the pure premium portion of the unearned premium reserve should always be exactly adequate to cover the remaining risk. This can lead to some unusual and controversial earning patterns; there are even situations where negative premium is earned. In addition, the earning pattern for a particular loss scenario can differ materially from the earning pattern that is expected when the contract is written.

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Section 1: Introduction

Statutory accounting requires that reserves be established for covered losses that have occurred but are unpaid (loss reserves) and – effectively – for losses that have not yet occurred, but will be covered by policies already on the books (unearned premium reserves). Furthermore, these reserves need to be separate.

A problem can arise when a multi-year contract has a large aggregate deductible. If losses depleting the deductible occur faster than expected, the premium reserve at some point in time may be inadequate. Of course, it is also possible that those losses occur slower than anticipated in which case the premium reserve may be redundant. The approach taken in this paper is that at each point in time (or at the end of each accounting period) the pure premium portion of the unearned premium reserve should be exactly adequate. This, in turn, implies a certain earning pattern for the premium that, in some cases, requires that negative premium be earned.

This paper was inspired by a discussion in my workplace on capital allocation for second-event covers and similar types of transactions.[1] Shortly after this conversation, a very interesting discussion thread

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[1] Do you need more capital to write a second-event cover than to write a first-event cover, because its results are more volatile? Do you need less because the probability of a loss to the cover is more remote? (If so, once the first event occurs, you are now effectively on a first-event cover. Do you at this point allocate more capital? What if you don't have it?)

appeared on CASNET. Ruy Cardoso of Ernst & Young asked a hypothetical question which I will paraphrase here: *Losses are certain at $10 per month. You cover $20 excess $100 in aggregate. The contract begins 7/1/xx. What is the loss reserve at 12/31/xx (ignore investment income)?*

After a short section defining unearned premium, the bulk of this paper consists of several examples that illustrate some of the consequences of taking the "adequate pure premium reserve" approach to establishing the unearned premium reserve (UEPR). The examples in this paper have been designed to illustrate how the experience early in a multi-year contract affects the expected losses (to the contract, not ground-up) that occur later in the contract, and how this in turn should affect premium reserving and earning patterns. While the examples could be made more "realistic", it was felt that this would introduce complications not relevant to the central issue. For example, in our simplification of Ruy Cardoso's question above, we have assumed that there are certain losses of $20 per month. If the losses are certain, there are questions of risk-transfer. Similarly, in Section 4, the single premium policy has an indefinite term – even though such a policy would be highly unusual. Despite the simplifications, the examples and the technical considerations they illustrate are relevant.

After these examples, Section 7 provides some comments on Practical Considerations, including remarks relevant to the new requirement that an actuary opine on the adequacy of the unearned premium reserve under certain circumstances.

The author would like to thank the reviewers and colleagues who read and commented on early versions of this paper. The views and examples contained in this paper are those of the author. In some cases, the approach contained herein might result, for example, in earning premium faster than some state's regulations would allow. Naturally, one should consult with qualified accounting professionals to decide how to properly record the financials of complex or difficult contracts.

1.1 What is unearned premium?

According to the glossary of the IASA Property-Casualty Insurance Accounting text [5], "Unearned Premium [is] the portion of the premium applicable to the unexpired period of the policy". What is the Unearned Premium Reserve (UEPR)? Again from the glossary, "The sum of all premiums representing the unexpired portions of the policies or contracts which the insurer or reinsurer has on its books as of a certain date…." So, the UEPR is a liability that represents the premium for the unexpired risks on the insurer's books.

The Statement of Principles Regarding P&C Insurance Ratemaking [4] states that ratemaking is prospective, and that a rate is an estimate of the expected value of future costs. Also, a rate provides for *all* costs associated with the transfer of risk. This paper is concerned primarily with the pure premium portion of the rate – i.e. the expected loss and loss adjustment expense, not including other expenses.

Combining these two concepts, we see that the UEPR consists of the pure premiums and the other expenses for the unexpired portion of the risks that are currently on the insurer's books. From one valuation date to another, the amount of unexpired risk on an insurer's books changes: new risks may be written, and the unexpired portion of those risks that were on the books at the beginning of the period generally decreases. This is captured in the familiar accounting identity:

$$EP = WP + UEPR_{begin} - UEPR_{end}$$

where: EP is the premium earned during the period,

WP is the premium written during the period, and

$UEPR_{begin}$ and $UEPR_{end}$ are the UEPR at the beginning and end of the period, respectively

One can see that, *ceteris paribus*, if the $UEPR_{end}$ is made smaller, then the amount of premium earned is larger; and, conversely, if the $UEPR_{end}$ is made larger, then the amount of premium earned is smaller. Should it happen that the $UEPR_{end}$ for a certain policy is larger than the $UEPR_{begin}$ without any new premium being written (we shall see below how this might happen), then the above identity forces us to conclude that the premium earned on this policy during this period was negative.

Much of the history and a survey of traditional estimation techniques for the UEPR can be found in James L. Morgan's Chapter 5 of the IASA text. Morgan reports that early in the 19th century the usual practice was that written premium was fully earned at policy inception, and that in 1848 the State of New York required insurers to carry a liability equal to an amount needed to reinsure all outstanding risks safely. Ignoring frictional costs and risk loads (as in the "Frictionless World" described below), this amount is the pure premium portion of the UEPR. A modern codification of this statutory requirement is section 1305 of the New York State Insurance Code.

1.2 A First Example

Now we turn to the question of the indicated UEPR for multiyear policies. For ease of exposition, let's first examine a simplified version of the problem. We will assume that there is a maximum of one loss in each year, each loss is exactly \$1000, and there is no investment income (i.e. all flows are discounted at 0%). We further assume that the probability that a loss occurs in any given year is 10%, and that different years are independent. For this simplified setup we want to compute the pure premium for the k^{th} loss during the next n years; we will denote this pure premium by PP(k,n). It will be convenient to have a name for a policy covering such a loss; for it we shall write Policy(k,n).

To illustrate:

PP(1,1) is the pure premium for a policy that pays \$1000 if there is at least[2] one loss during year 1, so PP(1,1) = \$1000 * 0.1 = \$100.

PP(1,2) is the pure premium for a policy that pays \$1000 if there is a loss during year 1 or year 2 (since we discount flows at 0% it does not matter which). The probability that there is no loss in two years is 0.9^2 = 81%, so the probability of at least one loss is 19% and PP(1,2) = \$190.

PP(2,2) is the pure premium for a policy that pays \$1000 if there are at least two losses during years 1 and 2. Since we are assuming at most one loss per year, this can only happen if there is exactly one loss in each of years 1 and 2. The probability of this is 0.10 * 0.10 = 1% and the pure premium is \$10.

Suppose that you purchased both Policy(1,2) and Policy(2,2). You would have full coverage for two years. In fact, your coverage would be identical to first purchasing Policy(1,1) and then one year later purchasing a second Policy(1,1). Your pure premium for the first set of policies would be \$190 + \$10 = \$200, and for the second your pure premium would be \$100 + \$100 = \$200 once more. This is no coincidence. Identical coverages must have identical pure premiums.

In a frictionless world with no transaction costs, where risk carriers are willing to cede or assume risks for their pure premiums, the following principle holds: If two sets of policies give identical coverage, they must have the same premium charge. (Below this simplified environment will be referred to as "The Frictionless World".) If this were not so, a portfolio consisting of a long position (assumed risk) and a short position (ceded risk) could be assembled which has positive net (pure) premium, but no net risk. This would violate the principle of no risk-free arbitrage.[3] (Below this will be referred to as "The No Arbitrage Principle".)

1.3 A Definition

The pure premium for a policy is equal to the expected losses at contract inception. However, as time passes the pure premium for the remaining expected losses will change. We will call the remaining expected losses the required pure premium reserve (RPR). This quantity will vary over time; when we need to be more specific, we will call it the required pure premium reserve at time *t* (RPR_t). This value RPR_t, by the way, is exactly the amount that one of the hypothetical risk carriers from The Frictionless World would require to assume the risk at time *t*.

So, at policy inception, the required premium reserve equals the pure premium for the policy. At policy termination, when no more losses can occur, the required premium reserve is zero. (Here and throughout the paper we assume that losses are paid as they are incurred and that there is no reporting

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[2] In this first example, there can be only one loss per year so for the first year "at least one" implies "exactly one".
[3] Such opportunities are also referred to as "free lunches", but, alas, we all know that there is no such thing.

lag). The RPR is very similar to the unearned premium reserve (UEPR), but it has the following difference: the UEPR contains premium elements other than pure premium (such as expense loads and risk loads). In The Frictionless World, an exactly adequate UEPR is equal to the RPR, so in the following discussion the terms are used interchangeably.

The RPR may depend on loss experience, as the following continuation of the earlier example illustrates:

The RPR for Policy(1,2) at time $t = 0$ is the pure premium, which we computed above as \$190.

After one year, we are in one of two states:

State	**Probability**	**RPR_1**
Loss Occurred	10%	No more cover remains, so $RPR_1 = 0$
No Loss Occurred	90%	Remaining cover is Policy(1,1), so $RPR_1 = 100$.

The decrease in the RPR during the first year is analogous to the (pure) premium earned during that period. The decrease in the RPR in the loss case is 190 and in the no-loss case is 90. The probability of the loss case is 10%, so the expected change in the RPR is $(0.1)(190) + (0.9)(90) = 100$. This is equal to the pure premium for a one-year cover (which is the coverage that you got during the first year of Policy(1,2)). Again, this is no coincidence.

Lemma: The (*a priori*) expected value of the change in the RPR during a period is equal to the (*a priori*) expected value of the losses occurring during that period.

Sketch of Proof: Consider a time period during the term; call this period D. Let B and A be the time periods (during the contract term) before and after period D, respectively.

|-------B(efore)---------**|--D(uring)--**|------------------A(fter)--------------------|

<-------------------------------------Policy Term---------------------------------->

At contract inception, the RPR is equal to the expected losses occurring during the whole policy period: B, D, and A combined. And at contract inception, we *expect* the RPR at the start of period D to be equal to the losses expected to occur during periods D and A. Similarly, at contract inception we *expect* the RPR at the start of period A to be equal to the losses expected to occur during period A.

It follows that the *a priori* expected change in RPR is equal to the *a priori* expected value of losses occurring during period D, which is what the lemma says. QED.

In the above example, expected losses were $100 and the expected change in the RPR was also $100. Notice that while the expected change was $100, an actual change of $100 is not possible in this example (it is either $90 or $190).

Section 2: The "Adequate Pure Premium Reserve" Approach

In my opinion, the change in RPR is a correct measure for pure premium earned during the period, and that the pure premium portion of the UEPR should be the RPR. Applying this approach to the example of the previous section: in the no-loss case we would earn premium of $90 during the first period, and in the loss case we would earn premium of $190.

Under current accounting rules: in the loss case, since there is no more cover, all future premiums would be accrued and earned in the current period[4], so earned (pure) premium would be $190, just as the "adequate pure premium reserve" approach indicates. In the no-loss case, I believe that most companies would simply earn half of the pure premium ($95) during the first year (and some might recognize that they have a $5 premium deficiency, since the pure premium for year two is $100).

My view is that at policy inception we expected to earn $100, but that in fact we earned either $190 or $90 depending on our experience. Before you agree with me too quickly, let's look at another example:

Consider the expected change in the RPR for Policy(2,2) during year 1. This policy, you will recall, pays $1000 for the second loss in two years. The pure premium for this policy is $10, so this is the RPR at time 0.

After one year we are again in one of two states:

[4] Under US-GAAP, at least for reinsurers, this is the content of EITF93-6, Issue 3 "How should the ceding and assuming companies account for changes in future coverage resulting from experience under the reinsurance contract?"

State	**Probability**	**RPR_1**
Loss Occurred	10%	Remaining cover is Policy(1,1), so $RPR_1 = 100$
No Loss Occurred	90%	Since there can be only one loss per year, there can now be no second loss: $RPR_1 = 0$.

In the no-loss case, which occurs 90% of the time, the decrease in RPR is \$10. In the loss case, the decrease in RPR is -\$90. The expected decrease in RPR is (0.9)(10) + (0.1)(-90) = 0.

The lemma tells us that this must be the expected value of the losses occurring during the first year. Does this make sense?

Yes! This policy pays only on the second loss, and since we assume there can be only one loss per year, the second loss cannot occur during year 1. That is why the expected losses during year 1 are zero.

2.1 Standard Premium-Accrual Methodology Considerations

I am not certain how companies would account for the above cover today. Some would argue that since the second loss cannot occur in year 1, no premium should be earned in year 1 on this cover; they would earn all 10 in year 2. Others might earn 5 in the first year and 5 in the second year.

I would argue that in the no-loss case all 10 should be earned in the first year, but that in the loss case *negative* 90 should be earned during the first year. The "adequate pure premium reserve approach" implies that the amount of pure premium earned during a period must be that amount such that the remaining RPR contains exactly the expected pure premium required for the remaining policy period.

At inception, the company's expectation was to earn nothing during year 1 on this policy because the insured event could not occur during this period. But in fact one of two things happened: they had either an underwriting gain of 10 or an underwriting loss of 90.

The standard premium accrual procedure referred to before (i.e. accruing all future premium when no more cover remains) together with the No Arbitrage Condition (described earlier) leads to the same conclusion as the "adequate pure premium reserve approach", as we will now illustrate.

Recall that the portfolio consisting of Policy(1,2) and Policy(2,2) together gave identical coverage to the portfolio consisting of Policy(1,1) along with a one year deferred Policy(1,1). So, by the No Arbitrage Principle, the premiums and how they are earned should be the same. During year 1, the premium earned on Policy(1,1) is equal to 100. The premium earned during year 1 on each of Policy(1,2) and Policy(2,2) depends on the results of year 1:

Loss case: probability = 10%

Policy(1,2) earned premium = 190 implies$\Rightarrow$ Policy(2,2) earned premium = -90

or

No-loss case: probability = 90%

Policy(2,2) earned premium = 10 implies$\Rightarrow$ Policy(1,2) earned premium = 90.

In the loss case, the premium earned on Policy(1,2) is 190 by the standard premium accrual procedure. Using the No Arbitrage Principle, since the total premium earned on the two policies during year 1 must be 100, the premium earned on Policy(2,2) must be -90.

Similarly, in the no-loss case, the premium earned on Policy(2,2) should be all 10, because no coverage remains. No Arbitrage forces the premium earned on Policy(1,2) to be 90, because the sum must be 100.

If you are uncomfortable with earning all of the premium for Policy(2,2) in the no-loss case in year 1, consider what happens to the pair of policies in year 2 given that there was no loss in year 1. The coverage is identical to the coverage afforded by a one year deferred Policy(1,1), so the earned premium in year 2 must be the same: 100. In fact, the coverage during year 2 for Policy(1,2) alone is the same as for a Policy(1,1) because we are given that there was no loss in year 1. So the premium earned on Policy(1,2) during year 2 must be 100. Since the total premium earned is also 100, no premium can have been earned on Policy(2,2). Over the life of the Policy(2,2) $10 must be earned; if none is earned in year 2, all of it must have been earned in year 1.

2.2 Reconciling Total Earnings

The total amount of pure premium earned during the life of the policy is always equal to the initial pure premium. If some "negative premium" is earned during one period, it is recovered in later periods (or is balanced by some "over-earning" in prior periods). The total change in the RPR from contract inception to contract termination is the *a priori* pure premium. This is an important point. The negative premium earned is not "new" premium, the written premium stays the same, it is just earned in a different pattern.

It is should be noted that this process is nothing more than a "mark to market" of the outstanding UEPR.

The UEPR for a given policy is amortized over the policy's term. This amortization occurs according to some schedule. Commonly, for most lines of business this amortization is done linearly over the term; this produces the familiar pro-rata earning pattern. This pattern is theoretically correct for a policy with no aggregate deductible, no aggregate limit, and an underlying loss process that is compound with Poisson frequency. For a further discussion of compound distributions see for example Ross's text [6]. For certain lines of business (e.g. extended warranty, ocean marine cargo cover, credit insurance on a declining balance) other amortization patterns and, hence, earning patterns are used. The "adequate unearned premium reserve" process described above can be thought of as adjusting this amortization schedule to include the latest data.

Traditionally, one thinks of unearned premium reserves flowing into loss reserves and surplus as the policy term progresses. Sometimes the losses occur slower than expected, and an unexpectedly large portion of this flow goes to surplus; other times losses occur faster than expected, and (unfortunately) in these cases surplus may flow into loss reserves. In the example we worked through above, it is the unearned premium reserve, not the loss reserve, that has become inadequate and requires supplementation from surplus. This is discussed further in *Section 7.4: Is It Loss or Is It Premium?*

Section 3: A Less Simplified Example

Now let's start to relax the conditions that we imposed in Sections 1 and 2 for the first example. We now allow more than one loss in each year. For simplicity, we will assume that in each year there are 0, 1, or 2 losses with probabilities 1/2, 1/3, and 1/6 respectively. We will continue to ignore investment income and will again assume a constant loss amount, but this time, to make the arithmetic simple, the constant loss amount will be 216 instead of 1000.

The pure premiums for Policy(k,n) may be computed as follows. First compute the probability of having exactly k losses by the end of year n; the result of this calculation[5] is displayed in Table 1.

TABLE 1 (probability of exactly k losses in n years)

No. of losses	One Year	Two Years	Three Years
0	50.00%	25.00%	12.50%
1	33.33%	33.33%	25.00%
2	16.67%	27.78%	29.17%
3	0.00%	11.11%	20.37%
4	0.00%	2.78%	9.72%
5	0.00%	0.00%	2.78%
6	0.00%	0.00%	0.46%

Then sum these to produce the probability of having at least k losses in n years; see Table 2 for these values.

[5] The probabilities are most easily computed recursively. For example: Pr(2,2) = 1/2*Pr(2,1) + 1/3*Pr(1,1) + 1/6*Pr(0,1). That is, the only way to have exactly two losses at the end of year two is to have had no loss in year 2 AND exactly two losses in year 1, OR exactly one loss in year 2 AND one loss in year 1, OR two losses in year 2 AND no loss in year 1. (Here the events joined by AND are independent and the events joined by OR are mutually exclusive.)

TABLE 2 (probability of at least *k* losses in *n* years)

No. of losses	One Year	Two Years	Three Years
0	100.00%	100.00%	100.00%
1	50.00%	75.00%	87.50%
2	16.67%	41.67%	62.50%
3	0.00%	13.89%	33.33%
4	0.00%	2.78%	12.96%
5	0.00%	0.00%	3.24%
6	0.00%	0.00%	0.46%

Finally, multiply by the constant loss amount of 216 to compute the pure premiums shown in Table 3.

TABLE 3 (pure premiums for Policy(*k*,*n*))

Loss *k*	$n = 1$	$n = 2$	$n = 3$
1	108	162	189
2	36	90	135
3	0	30	72
4	0	6	28
5	0	0	7
6	0	0	1

Consider Policy(2,3), which covers the second loss in three years. The pure premium for this coverage is \$135. How much of this premium do we expect to earn during the first year?

Half of the time there will be no loss the first year, and the RPR for the last two years of the policy must be \$90 – the pure premium for Policy(2,2). So in this case \$135 - \$90 = \$45 would be earned in the first year.

Similarly, one-third of the time there will be one loss during the first year; then the RPR for the last two years must be $162 (the pure premium for Policy(1,2), which is equivalent to the remaining coverage) and $135 - $162 = -$27 would be earned during the first year.

Finally, one-sixth of the time there are two losses in year 1. In this case there is no more coverage available. The RPR for the last two years is zero, and the full $135 would be earned during year 1.

Combining the above calculations we find that *at policy inception* the expected earned premium for year 1 is

$$1/2\ (\$45) + 1/3\ (-\$27) + 1/6\ (\$135) = \$36.$$

Year 3's expected earnings are similarly easy to calculate: during the first two years of the cover there is a 1/2 * 1/2 = 1/4 chance that there have been no losses and a 1/2 * 1/3 + 1/3 * 1/2 = 1/3 chance of exactly one loss. From Table 2, we see that the pure premium for Policy (2,1) is 36 and for Policy (1,1) is 108. From this we see that *at policy inception* we expect to earn 1/4 ($36) + 1/3 ($108) = $45 during year 3.

During the life of the policy we will earn exactly $135. If *at policy inception* we expect to earn $36 in year 1 and $45 in year 3, it follows that we must expect *at policy inception* to earn $135 - $36 - $45 = $54 during year 2.

Does this mean that we should earn the premium over the three years in this pattern: $36, $54, $45?

No, because these are *a priori* expectations. As we have seen in earlier sections, the premium earned during year 1 need not equal the *a priori* expected earned premium. Also, at the end of year 1 our expectations for the earnings in years 2 and 3 will probably be different than they were at inception.

The first two rows of Table 3 contain all of the information needed to compute the actual amount of premium earned to date at the end of each year. For example, suppose there is exactly one loss and it occurs in year 2. Then we should earn \$45 in the first year, because when we start year two, the remaining coverage is the second loss in two years – a Policy(2,2). During year three we are in a first-loss position, so we need to earn \$108 because at the start of year 3, the remaining coverage is the first loss in one year – a Policy(1,1). Since the total amount earned over the three years must be \$135, we find that the year two (actual) earnings must be *negative* 18. So the actual earning pattern observed in this case would be (\$45, - \$18, \$108), which differs markedly from the *a priori* expectation.

Section 4: An Indefinite-Term Example

In this example we will assume a 1/10 chance of loss each year and go back to the simplified model of at most one loss per year. Loss severity is assumed constant at \$3000. We will continue to ignore investment income. The policy that we consider in this example covers one loss, but has no time limit. The policy will stay in effect until there is a loss, at which time it will pay \$3000.

4.1 Pure Premium and Earning Patterns

What is the pure premium for this coverage? Let P be this premium. Then P must pay for two things. One-tenth of the time there is a loss during year 1 of \$3000 and $RPR_1 = 0$. The other nine-tenths of the time, there is no loss in the first year and RPR_1 is the pure premium for a policy that pays \$3000 whenever the loss occurs – but this is exactly what *P* is. We have:

$$P = 1/10\ (\$3{,}000 + 0) + 9/10\ (0 + P)$$

Solving for P, one finds P = $3000.

Upon reflection this is not very surprising, since $3000 will be paid out eventually (recall that we are still ignoring investment income). So the pure premium equals the expected loss, which is $3000.
How does one earn the premium for such a policy? In the loss case, the premium earned in year 1 is $3000; in the no-loss case the premium earned in year 1 is $0 (since RPR_1 remains at $3000). So, at policy inception the expected earned premium for the first year is $300.

What about later years? The answer depends on when you ask the question.

At the start of the first year, we expect to earn $270 during the second year and $243 during the third. But these are the *a priori* expectations at the start of the first year; after one year has passed there has been either one loss or no loss, and with this additional information the expected values for earned premium change.

At the start of the second year there are two possibilities: either there was a loss in year 1 (in which case no coverage remains) or there was no loss in year 1 (in which case there is coverage for year 2). Also, since we are assuming no late reporting, you will know which case applies. The conditional expectation (given no loss in year 1) for the premium earned in year 2 is $300. Similarly, the conditional expectation (given no loss is year 1) for the premium earned in year 3 is \$270. On the other hand, the conditional expectation (given no loss in years 1 *and* 2) for the earned premium in year 3 is \$300.

The expected earning pattern at the start of any year, for that and subsequent years, is: (\$300, \$270, \$243, …) with each term being 9/10 of the previous term. When a year passes without loss, each of these terms shifts forwards. It should come as no surprise that this infinite geometric series sums to \$3000.

Why is no premium earned during no-loss years? Because the RPR at the start of the no-loss year is \$3000, and it is also \$3000 at the end of the year. The change in the RPR, in this case 0, is the earned premium. During a loss year, the RPR is \$3000 at the start of the year and it is \$0 at the end of the year (because no more coverage remains), so the amount earned during the year is \$3000.

Note that the company shows no underwriting gain or loss, no matter what the outcome. In the no-loss case there is no movement in the reserves; in the loss case the RPR becomes the loss reserve. This is a consequence of the indefinite policy term. Since the cover continues until there is a loss, having a no-loss year only delays the inevitable payment; and without investment income the delay does not benefit us. We relax this restriction below.

4.2 Investment Income

Now let's modify the last example to take into account investment income. Assume that all losses are paid at the end of the year, and that invested funds earn interest at a rate of 5%.

Now the equation for the present value of the pure premium reads:

$$P = 1/10 * (\$3000)/1.05 + 9/10\ (P/1.05)$$

One tenth of the time we pay a loss of $3000 (discounted one year) and nine tenths of the time the present value of RPR_1 is P (discounted one year). Solving for P, we find that P = $2000.

How should this premium be earned? Should the fact that we now consider investment income affect how we earn the premium?

Suppose that we have a loss in year 1. Then, as before, the $RPR_1 = 0$, so we earn the full $2000 during year 1. We also have investment income of $100. On the other hand, suppose that we have no loss in the first year. Then RPR_1 = $2000, and again we have investment income of $100. What should be done with the investment income?

To investigate that question, we examine an alternative way to construct this same coverage. Consider an annual policy that pays $1000 at the end of the year if there is a loss, for a premium payable at the end of the year[6] of $100 (the pure premium for the policy). In effect, this policy provides similar coverage to the first year of the original policy, subject to a $2000 self-insured retention. Imagine that the insured sets aside this $2000 in a special account. During the year, $100 in investment income is earned on the $2000 (this is paid to the insurer as premium) and, if there is a loss, the $2000 set aside and the $1000 from the insurer combine to provide the $3000.

With a one-time premium of $2000 and a limit of $3000, the insurer has only $1000 at risk. So in this second setup, the insurer is entitled to only $100 (= $1000 * 10%) in annual pure premium. This, as we have seen, is the investment income generated by the one-time premium payment of $2000.

We see that the insured can obtain identical coverage in two ways: by setting aside the $2000 and paying an annual premium of $100, or by paying a one-time premium of $2000. The No Arbitrage Principle says that since the two coverages are identical, their pure premiums must be equal. In order for this to work out, we need to view the investment income on (discounted) premium as premium – in fact, this is implicit in the pricing equation.

Now we can determine the earning pattern for the original multiyear policy, and answer the question about what to do with the investment income. In a year with no loss, premium of $100 is earned. In a loss year, premium of $2100 (the original premium plus one year's investment income) is earned.

This result is related to the "Paid-up Insurance Formula for Life Reserves". (see for example, [3])

Section 5: The Continuous-Time Case

We now will quickly look at an example in continuous time. For simplicity we shall go back to discounting at 0% (no investment income). For this example, we consider a constant loss amount of $1000 per occurrence and we assume that the frequency of losses on an annual basis is Poisson with

[6] The premium is made payable at the end of the year to remove timing effects.

parameter 4/3. Our cover, Policy(2,3), will have a term of three years and will pay for the second loss during the period. Let's look at the probabilities of paying off in each of years 1, 2, and 3.

Since losses are Poisson, we have the following probabilities for year 1:

Losses during Year 1	Probability
0	$e^{-4/3}$= 26.360%
1	4/3 $e^{-4/3}$ = 35.146%
2 or more	1 – 7/3 $e^{-4/3}$ = 38.494%

Notice that of these three possible outcomes for year 1, the most likely is that the second loss occurs during the first year – even though we expect only one and one third losses per year.

For the first two years we have:

Losses during Years 1 and 2	Probability
0	$e^{-8/3}$= 6.948%
1	8/3 $e^{-8/3}$= 18.529%
2 or more	1 – (11/3) $e^{-8/3}$ = 74.523%

During years 1 and 2 we will pay 74.523% of the time. During year 1 we paid 38.494% of the time, so it follows that during year 2 we will pay 74.523 – 38.494 = 36.029% of the time.

To find the probability of paying in year 3, we observe:

Losses during Year 1, 2, and 3	Probability
0	e^{-4} = 1.832%
1	4 e^{-4} = 7.326%
2 or more	1 – 5 e^{-4} = 90.842%

So, the probability of paying in year 3 is 90.842% - 74.523% = 16.319%.

With these probabilities we see that at contract inception, we expect to earn the $908.42 = $1000 * 90.842% of pure premium over three years in the following yearly pattern: $384.94, $360.29, and finally $163.19.

But as in the discrete time case, this expectation is only valid at contract inception. As soon as any time has passed (or rather, once some period has passed and you know how many losses there were during that period) the expected future pattern changes. Below is a graph showing the earning pattern expected at contract inception:

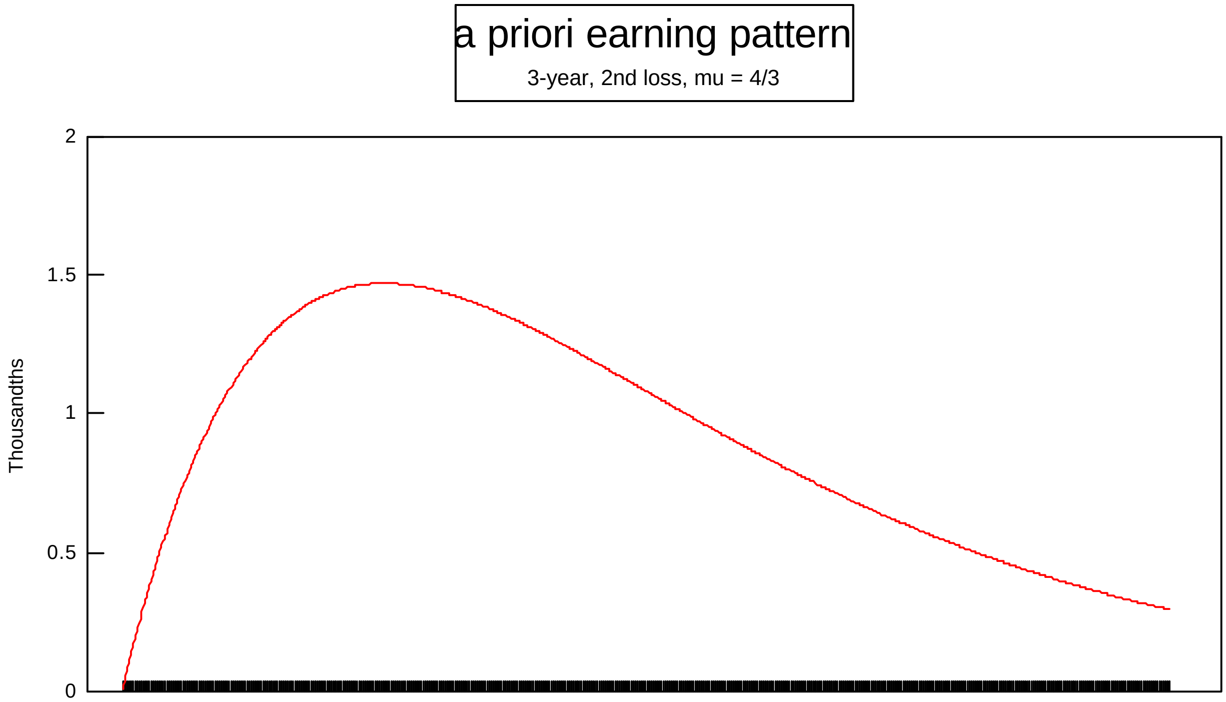

Figure 1

Note that the earning is initially slow (in fact it is instantaneously zero at contract inception) and then rapidly increases towards the end of the first year. Also notice that the earnings in the last year are expected to be small. The initial slow earning is due to the fact that it is very unlikely for two losses to occur in a short period of time. The earnings expected later in the policy term are small because, *a priori*, we expect to be off risk by then (by virtue of having already paid the loss!).

But again, this graph shows only the *a priori* expectation at contract inception. When the first loss occurs, the RPR for the second-loss cover immediately jumps. In effect the policy then converts from a second-loss cover to a first-loss cover; this is a manifestation of the "memory-less" feature of the Poisson distribution. The first loss to occur is not a loss event for the cover, and no loss reserves are required. But suddenly the RPR is inadequate and an underwriting loss has been incurred (because some “negative premium” has been earned – this premium will be earned back over the remainder of the contract).

Section 6: An Example with Expenses

In the real world, the UEPR contains many components in addition to the RPR’s pure premium. There may be, for example, on-going contract maintenance expense[7]. Effectively such expense forms an annuity that runs until contract termination. One quick example will give a flavor of the complications.

Recall the earlier example of an indefinite-term policy that pays $3000 when the loss occurs, has annual loss probability of 10%, and no investment income. Assume that on-going contract maintenance expense is $150 per year. Letting *G* stand for the expense-loaded premium, the premium equation now reads:

$$G = 1/10\ (\$3000 + \$150) + 9/10\ (G + \$150)$$

That is, one-tenth of the time we have expenses of $150 and a loss of $3000, and the other nine-tenths of the time we have expenses of $150 and $RPR_1 = G$ (because of the indefinite term).

Solving for *G*, we find that *G* = $4500.

The company with this risk on its books suffers an underwriting loss (after expenses) of $150 each year that there is no loss, but has an underwriting gain of $1350 the year that the loss occurs![8]

The interested reader may find it amusing to work out the effect on this example of including 5% investment income as before.

Section 7: Some Practical Ramifications of the Methodology

The preceding examples illustrate some theoretical issues, but the practicing actuary must consider the broader practical effects of any change to common practice. Questions of materiality and practicality also should be addressed.

7.1 Actuarial Reserve Opinions

The NAIC SAO Instructions for Property-Casualty [2] specify that the SCOPE paragraph include the Reserve for Direct, Ceded, and Net Unearned Premiums. Also, these three items must be covered in the OPINION and RELEVANT COMMENTS paragraphs. This applies to all insurers that write direct and/or assumed contracts or policies (excluding financial guaranty, mortgage guaranty, and surety contracts) with terms of thirteen months or more, which the insurer cannot cancel, and for which the insurer cannot increase premiums during the term.

The insurer is required to establish an adequate unearned premium reserve. For each of the three most recent policy years, the gross unearned premium reserve must be no less than the largest result of three

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[7] Had these expenses have been deferred policy acquisition expenses, there would be additional accounting complications.
[8] What's happening here is that we have an annuity with an expected life of ten years funding the expenses. When we have a no-loss year, the expected life of the annuity stays at ten (instead of decreasing to nine) and we show an underwriting loss of the difference. When we have a loss year, the expense annuity is no longer needed (its expected life drops from ten to zero). The release of the reserve supporting this annuity yields the underwriting gain.

tests. The three tests (in slightly simplified form) are:

1) The best estimate of the amounts refundable to the contract holders at the reporting date.

2) The gross premium multiplied by the ratio of (a) over (b) where:

   (a) equals the projected future gross losses and expenses to be incurred during the unexpired term of the contracts; and

   (b) equals the projected total gross losses and expenses under the contracts.

3) The amount of the projected future gross losses and expense to be incurred during the unexpired term of the contracts (as adjusted), reduced by the present value of the future guaranteed gross premiums.

The examples in this paper are intended to be non-cancelable and to have fixed premiums (generally payable at contract inception, to avoid irrelevant complications). The example contract terms are more than thirteen months, so except for the proscribed lines of business the rule applies. How do our examples fare under these tests?

For simplicity, we shall assume that there are no refund provisions in the policy, so the Test 1 lower bound on the unearned premium reserve is zero.

The second test requires that we estimate gross losses and expense. The examples in this paper for the most part have been concerned with pure premiums (i.e., only the expected losses, with no provision for expenses). Under the simplifying assumption that expenses are zero, Test 2 tells us to estimate the projected future gross loss to be incurred, and to divide this by the projected total gross loss. This ratio is then multiplied by the gross premium to obtain the second lower bound on the unearned premium reserve.

The third test requires that the unearned premium reserve be at least as large as the expected future losses and expenses to be incurred during the contract (as adjusted). The amount of the projected future gross losses to be incurred is exactly the RPR at the statement date. The “adjustments” in question are for future premiums (our examples have none) and for investment income up until the loss is incurred but not beyond (our losses are immediately payable, so the example with investment income complies). [The test also specifies a company-specific maximum interest rate. We will assume that 5% meets this test.]

So in our examples, the RPR is the lower bound on the unearned premium reserve specified by Test 3.

7.2 Perspectives on Aggregate Deductible Business

In a multi-year contract with an aggregate deductible, the experience of the first few years can influence the required premium reserve in two ways. First, the aggregate deductible may be depleted faster or slower than planned; second, adverse or favorable experience during the initial period may influence one’s view of the future ground-up experience. This paper addresses only the former.

There is an additional way to view such policies. The later years of a multi-year policy with an aggregate deductible can be thought of as excess layers, each year/layer having a retention that depends on the earlier years’ experience. If the total losses to date have been small, little of the aggregate deductible has been eroded and the retention (the remaining aggregate deductible) for the later years is higher. Since higher layers have lower premiums, the RPR is small. Similarly, if early experience has been unfavorable, much of the aggregate deductible will have been eroded. The retention will be lower and the RPR will be large. In essence, early experience determines which layers the later years' coverage corresponds to.

7.3: What to Do about Negative Premium

In chapter 14 of the IASA text, David L. Holman and Chris C. Stroup discuss US-GAAP accounting for P&C insurers. Under US-GAAP there is a notion of a premium deficiency reserve (PDR). Holman and Stroup write: "Projections, therefore, are periodically updated, based on new information about expected cash flows. GAAP requires that a premium deficiency be recognized if the sum of expected loss and loss adjustment expenses, expected dividends to policyholders, maintenance costs, and unamortized (or deferred) policy acquisition costs, exceed the related unearned premiums related thereto." If this is the case, the unamortized policy acquisition costs are reduced to make up the shortfall. If that alone is not sufficient, a liability is reported for the remaining deficiency. Interestingly, Canadian statutory accounting provides a line item (Line 15) for Premium Deficiency (see chapter 18 of the IASA text). European actuaries speak of the "reserve for unexpired risks", which is similar in concept to a premium deficiency reserve.

So, under US-GAAP one might establish a PDR to handle "negative premium" earnings. Effectively, negative premium is earned by the reduction of an asset (the unamortized policy acquisition cost) and/or the establishment of an additional liability

Statutory accounting does not have the notion of a premium deficiency, although in principle one could include one by using the write-in lines. However, due to US income tax regulation, there may be a material difference between treating the shortfall as premium or as some other type of liability. The interested reader should see chapter 13 of the IASA text or the Almagro and Ghezzi paper from the *Proceedings* [1].

7.4: Is It Loss or Is It Premium?

The argument can be made that instead of altering the premium earning methodology, we should put up loss reserves corresponding to the losses that are eroding the aggregate deductible. That is, there is an increase in expected losses to the cover caused by events that have occurred prior to the statement date. The amounts to be put up are not in dispute; they would be exactly the amount needed to make the booked unearned premium reserve match the RPR. The difference is that these reserves would be characterized as loss instead of premium.

But these reserves behave more like premium than loss in two important ways. First, they amortize over the remaining policy period. To see the second reason, consider a two-trigger two-year policy. In order for the policy to pay, two events, A and B, must occur during a two-year period. Say event A occurs in year 1, and as a result some additional reserve (either a loss reserve or a premium deficiency reserve) is needed. Suppose now you wanted to completely reinsure this risk. You could do this by purchasing cover for event B. Observe that this reinsurance is completely prospective. Being prospective, it should be funded from premium reserves, not loss reserves.[9]

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[9]Claims-made policies and "sunset clauses" in reinsurance agreements can further blur the line between premium reserves and loss reserves. Suppose that an event has occurred, but that it has not been reported yet. Assuming that a reserve is appropriate should it be premium or loss? This reserve amortizes over the remaining reporting period (acts like premium). On the other hand, the underlying loss event has already occurred. Is the reporting a second trigger?

Section 8: Conclusions

We could use the "adequate pure premium reserve" approach to answer Ruy Cardoso's question, which was mentioned at the start of this article: *Losses are certain at $10 per month. You cover $20 excess $100 in aggregate. The contract begins 7/1/xx. What is the loss reserve at 12/31/xx?*

Assuming no expenses or investment income, the UEPR would be $20 (because that is the RPR remaining), and the loss reserve would be $0 (because no covered loss has occurred). No premium (positive or negative) would have been earned to date. This question sparked a very lengthy and enlightening discussion, and I urge the interested reader to look over the thread (link: http://www.casact.org/RESEARCH/casnet.htm)

The "adequate pure premium reserve" approach outlined in this paper is internally consistent, even though it leads to some controversial implications such as negative earned premium. But the idea of negative earned (and written) premium already is used in some instances, such as the treatment of ceded proportional reinsurance. US GAAP and Canadian accounting have a notion of a premium deficiency reserve (PDR), and additionally in some European jurisdictions there is a notion of an "unexpired risk reserve". These entries could be used to record unexpected changes in the required premium reserve.

However, there are some operational problems with using what might be called "the negative premium approach": it might distort loss and expense ratios; it can make budgeting difficult; and for US taxpayers, the treatment of the UEPR for US taxation is different than for other reserves, which could lead to complications.

The good news is that "on average" the standard methodology should give the same results as this method for a large book of uncorrelated risks, written evenly throughout the year. However, the type of analysis outlined in this paper is probably justified for those risk carriers with a few large risks or for single risks that are large enough to distort the book.

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