Building a Public Access PC-Based DFA Model

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Abstract

This paper describes the initial version of a DFA model for U.S. property-liability insurers that will be made available to all interested parties. Those wanting to test the model will be able to obtain a copy of the program on disk or access the model over the internet. The long-term goal of this research is to develop a usable, understandable model that meets the basic needs of the industry. The model is very much a work-in-progress, and comments and suggestions are encouraged.

Introduction

In developing a Dynamic Financial Analysis (DFA) model, there are two primary problems facing the property-liability actuary. First, insurance operations are affected by an almost overwhelming number of factors, many of which deserve considerable attention. Second, the proprietary nature of most existing models limits the amount of information that has been shared and made publicly available regarding the modeling process. The project described in this paper has the long-term objective of addressing both of these problems.

In recognition of the fact that no model can successfully consider every potential source of risk, our model focuses on the *key variables* that affect the financial results and condition of a typical property-liability insurer. While the key factors affecting the underwriting and investment operations of a company are included, the number of variables actually incorporated in the model is held to a minimum. Addressing only the more important quantifiable financial risks to which property-liability insurers are exposed facilitates model comprehension and communication. Many of these same factors have been considered in the development of other models. However, certain factors, important in analyzing property-liability insurers, have been overlooked in some models that have been derived from life insurance DFA models. Differences in the essential natures of life and property-liability businesses need to be taken into account in any property-liability DFA model.

Our response to the second problem mentioned above is *public access*: our model is being made available for use by anyone interested in the process. All assumptions, techniques and calculations are explained in enough detail that other researchers and practitioners, with an appropriate understanding of the basic concepts and issues, will be able to use the model. The publication of the process should foster peer review of the model and can lead to improvements in the overall methodology. This approach should provide a valuable learning tool for individuals wanting to understand DFA for property-liability insurers. It should also help the profession deal with the issue of developing standards of practice in this emerging and important area.

Dynamic Financial Analysis is a natural outgrowth of the increasing recognition of the interdependence among all underwriting and investment facets of the insurance business. Technological advances now allow the widespread use of much more sophisticated financial models than were previously possible. While these developments represent advances for the industry, they also present challenges. In addition, the importation of DFA models previously utilized for life insurers and in other countries has resulted, sometimes, in very complex models that lack appropriate focus on risks specific to U.S. property-liability insurers. All of these factors have combined to create a significant hurdle for individuals seeking to work in this area. Thus, one goal of this research is to simplify the process by generating a usable, understandable model that can serve as a reasonable approach to Dynamic Financial Analysis for property-liability insurers.

What is DFA?

Dynamic Financial Analysis is a new term for a standard task of casualty actuaries: planning for the future. In earlier days such an analysis might have been labeled simply an "actuarial" analysis. The new term reflects, in part, the latest terminology fad, with "dynamic" representing the recognition of the stochastic, or variable, nature of insurance assets, liabilities, and operations, and 'financial' reflecting the long overdue approach of integrating assets and liabilities of insurers. In addition, the advent and dissemination of personal computers (programmed first-hand by actuaries, rather than second-hand by data processing personnel), spreadsheet programs, and other (often sophisticated) computer software allows for the performance of more timely and involved calculations. Finally, new research that provides sophisticated mathematical models of factors such as interest rates, catastrophic losses, and investment and underwriting performance allows for better modeling of complex systems of interrelationships, well beyond the precision and detail attained in earlier actuarial planning models.

The casualty actuarial profession in the United States has been a relative latecomer to the area of dynamic financial analysis. European insurers, both life and casualty, have long recognized the importance of investment risk, and incorporated financial models in their approach to actuarial work. Life insurers in the United States and Canada, after experiencing the traumatic effects of interest rate shocks during the 1980s, began to focus on interest rate risk and utilized dynamic financial models to help manage asset-liability risk. Canadian casualty actuaries, following the developments of their life industry more closely, in part as a result of regulatory requirements, also developed models in this area.

This latecomer status of U.S. casualty actuaries to DFA modeling has both benefits and disadvantages. The major benefit is that the general structure of dynamic financial analysis is already in place: there are many functioning models, used primarily in life insurance and/or in other countries, that can be used as guides and points of reference. Much of the trial-and-error and initial testing stages of model development have been accomplished, and actuaries can now benefit from that documented experience.

The potential disadvantages, however, are that the unique characteristics of U.S. property-liability insurers can be overlooked, or trivialized, when creating DFA models for this industry, simply because prior models failed to consider and incorporate them. Adopting and using inappropriate models can, in certain situations, be even worse than having no model: confidence based on improper information can sometimes be more problematic than cautiousness based on uncertainty. We explore some of the unique characteristics of our industry in the following section.

How are U.S. Property-Liability Insurers Unique?

The typical U.S. property-liability insurer writes personal and/or commercial lines, with a mixture of property and liability exposures, in a regulated environment. Property lines are subject to a range of catastrophic risks -- e.g., windstorm, earthquake, freezing, and fire -- depending on the geographic distribution of its business. (While property insurance in any country is exposed to catastrophic risks, each country – and each geographic subdivision – has different specific risk characteristics.) Liability lines are exposed to the unique vagaries of the U.S. civil justice system, which is vastly different from the European and Canadian systems. Both personal and commercial lines are subject to state insurance regulation, which can (and does) impact rates, residual market size and subsidies, policy writing requirements, entry and exit conditions, and even retroactive premium rebates. For any company, the impact of insurance jurisdictional issues, and any associated volatility in operating results, depends on both the geographical and line of business distribution of the insurer's writings.

The investment portfolio of a typical U.S. property-liability insurer is primarily bonds – of varying types and maturities -- with a smaller investment in stocks. (See Exhibits 1 through 3.) The major component of liabilities for property-liability insurers is loss reserves; typically, the second most significant liability item is the unearned premium reserve, a portion of which will, in turn, develop into reserves for losses. (See Exhibit 4.) Both assets and liabilities are subject to statutory valuation requirements that, it can be argued in some cases, defy logic and consistency. Many regulatory requirements – theoretically, for the purpose of enhancing potential solvency -- impose conservative valuation measures, such as not allowing loss reserves to reflect the time value of money. Others, such as stating bonds at an amortized value based on the initial interest rate, can be either conservative or excessive, depending on the direction of subsequent interest rate changes. These statutory valuations, despite their recognized deficiencies, do impact insurer performance, both in terms of internal decision-making and external perception. Regulatory intervention is frequently imposed as a result of indications based on statutory valuation.

U.S. property-liability insurers write short-term policies that are generally offered for renewal at the policyholder's option. While those insurers selling through independent agents are dependent on the decision of the agent as to whether the policyholder has this choice, other insurers can contact the policyholder directly with an invitation to renew the contract under the current policy terms and rates. Although the reasons for this behavior are not fully understood, policyholders do generally accept this offer, as the retention rates of most U.S. property-liability insurers are in the 90 percent level. Additionally, also for reasons not fully understood, the loss experience of renewal business is generally lower than that of new business, with experience continuing to improve as the policy is renewed a greater number of times.

Because of the improving loss ratio on renewal business, a property-liability insurer's book of business represents a significant asset to the company. However, despite the fact that renewal rates approximate, or even exceed, those for some renewable life insurance policies which are valued as assets, this asset is not reflected on the balance sheets of property-liability insurers. Nevertheless, this characteristic does need to be considered when projecting operating results for property-liability insurers.

DFA Approaches

There are two primary techniques for modeling risk factors: scenario testing (a deterministic approach), and stochastic simulation. Each has its advantages and disadvantages. Scenario testing is used to project results under specific situations. One highly visible example of this approach is the Social Security System, where several sets of economic and demographic assumptions are used to develop three different forecasts of possible funding conditions. Another example is a set of scenarios used in New York to test the ability of life insurers to withstand specific changes in interest rates and other economic conditions. However, a more common framework for DFA models is stochastic simulation, in which a series of randomly generated events, or "trials," produces a large number of different outcomes. The distribution of these outcomes then forms the basis for various indications -- for example, the proportion of simulated outcomes that are considered "unacceptable" is used as a measure of insurer risk. "Unacceptable" outcomes can be based on failing a regulatory test, incurring a ratings decline from one of the financial ratings agencies, becoming insolvent, or any other established benchmark.

Classifying Risk

The risks facing insurers can be classified into two major categories: one for items listed on the balance sheet, and the other based on continuing operations (which would appear in the operating statement). Furthermore, each of these categories can be subdivided into two further categories. Balance sheet risk consists of asset risk and liability risk. Operating risk consists of underwriting risk and investment risk.

Asset risk involves the change in value of an existing asset. For a bond, this could result from a change in interest rates, a change in the debt rating, or default on interest or principal. For an equity, asset risk involves a change in the market price, which could be caused by some of the same factors affecting bond values, or by other changes affecting company profitability or operations. Other assets, such as agents balances, are exposed to default risk.

Liability risk is primarily related to the adequacy of the loss reserves. As statutory valuation requires loss reserves to be carried as the nominal value of all future payments, this risk involves the possibility that total payments will ultimately differ from the indicated estimate. Based on market valuation of loss reserves, however, the risk also includes timing and discount rate components as well as the total payment amount. In addition, liability risk includes the adequacy of the unearned premium reserve to cover losses that will emerge on existing policies.

Underwriting risk is the risk associated with business that the insurer will write in the future, either as new business or renewals of existing policies. This risk includes pricing risk -- the ability to obtain adequate premium levels on this business -- as well as the risk associated with stochastic losses and expenses.

Investment risk relates to investment income and capital gains to be earned on existing assets and new assets resulting from continuing operations. This is dependent on interest rates and other economic conditions.

The four risk components are complexly interrelated. An increase in interest rates, for example, would lead to a decline in the value of existing assets (especially bonds), but higher investment income on new investments. Adverse development on loss reserves would generate the need for premium increases, and impact future underwriting experience. The advantage of a DFA model is that it can allow for this type of interaction. However, a drawback is that these relationships are difficult to quantify. This leads to the need to develop answers to some basic modeling questions before proceeding.

Practical Questions

Getting started building a model to forecast the future financial results of a property-liability insurance company is difficult, because so many fundamental issues must be addressed before any applications can be constructed. Three of these basic issues are:

1 **What risk exposures will be modeled?** In addition to traditional insurance risk exposures (such as pricing, reserving, reinsurance, jurisdictional, and catastrophes), property-liability insurers are also exposed to financial risk (interest rates, default, market fluctuations) and general business risk (such as fraud, mismanagement, lawsuits and off-balance sheet items). Some can be quantified (interest rate risk, pricing risk) while others defy quantification (management fraud, novel interpretation of insurance contracts).

2. **How can the exposure be quantified?** Historical data may or may not be available, either internal to the company, or from external sources. What data is available may not be of sufficient volume to be reliable for modeling purposes.

3. **How should the risk factors be modeled?** Scenario testing and stochastic simulation are alternative approaches, but the latter is generally preferred by the authors since it accounts for the stochastic nature of insurance operations. It is likely that certain risk factors may be incorporated as stochastic variables, while others are treated in a deterministic fashion. In addition to the general approach of the DFA model (i.e., deterministic vs. stochastic), the interactions between various risk exposures must be considered.

After addressing these three fundamental issues, there are a host of other fundamental questions that must be addressed as well.

First, how complex (or simple) should the model be? Given the complexity of the risk exposures facing property/casualty insurance companies, the natural tendency is to construct a rather intricate model, attempting to quantify as many risk factors as possible. This tendency should be counterbalanced by the need for a workable model that can be adequately understood and communicated. Presumably, the model will be a work in process for many years to come as additional research is conducted addressing the risk exposures and their potential treatment in DFA models.

Second, should the model incorporate any level of management intervention in which certain decisions are pre-programmed into the model, such as curtailing growth in new business when the premium-to-surplus ratio attains a particular level? Other possible management interventions include the realization of capital gains or losses depending on the profitability in a particular year, specifying certain tax elections to minimize income tax liability (such as carry-forwards and carry-backs, and the portfolio mix of taxable vs. tax-exempt securities), or withdrawing from states that are persistently unprofitable. Although many of these decisions appear reasonable, they need to be viewed in the context of the basic premise of a DFA model. The purpose of a DFA model is to provide a tool which management can use to assess the future financial condition of their business, and make better informed decisions accordingly. In this context, all management decisions should be "off the table" in the development of a DFA model. We recognize that a prudent manager would make certain decisions given the anticipated financial results in a given year. However, circumventing the outcomes of the DFA model by programming automatic decisions into the process hinders the effectiveness of the model. That is, if management will be using the model to make better informed decisions, it does not seem appropriate to incorporate any part of the decision making process into the model itself.

Third, should the modeling be done on a direct or net basis? In order to address a variety of reinsurance issues, it seems plausible to consider direct results, netted down for the impact of reinsurance. If the reinsurance program is fairly straightforward (i.e., excess of loss, quota share, aggregate excess and catastrophe type coverage), the model should be able to accommodate a direct/ceded/net approach. If, however, the reinsurance program includes financial reinsurance, clash covers, multiple line aggregates, and so on, the modeling could be extremely cumbersome. Our model uses a direct/ceded/net approach, and assumes the current reinsurance program of a company will remain in place for the projection period.

Fourth, how should data be incorporated that is external to the individual company being modeled? In particular, should the concept of credibility be factored into the modeling? If so, how should that be accomplished? We submit that in a DFA exercise, the most relevant source of information is the historical financial results of the company being modeled. However, in forecasting future results for a relatively new company, or one that has new management or some other fundamental change in its recent operations, some external data will have to be used (be it financial results of peer companies or industry aggregates). This is a question we will defer to later versions of the model. In the meantime, the model assumes the data for the company being examined are fully credible.

Key Risks

In this section, we discuss some of the key risks that need to be recognized by a property-liability DFA model.

Pricing

Property-liability insurers have the opportunity to change the premium level prior to writing new or renewal business. Thus, as expenses or expected losses change, insurers can reflect these changes in the new rate levels. However, two problems can affect the ability of insurers to charge the correct price. First, since most insurance premiums are set prior to the policy being written, the insurer may incorrectly estimate future experience, causing the price to be either inadequate or excessive. Second, the freedom of insurers to set premium levels varies by state, with some states allowing relatively unrestricted pricing and other states having extensive restrictions. Thus, there are two components to pricing risk. The first component is handled in this model by having the loss ratio (exclusive of catastrophes – see next subsection) be a random variable with the mean value and standard deviation based on company experience. Loss ratios are simulated by line, with appropriate consideration given in the simulations to correlations of contemporaneous loss experience between lines. The second component of pricing risk is handled by a factor imposing a restriction on the ability of a company to make rate changes which are indicated by changes in loss frequency or severity. In our model, a factor of 1 would represent complete freedom to adjust rates in accordance with indications, while lower values are used when companies write in states with restrictive jurisdictional forces.

Catastrophes

In addition to normal pricing risk and the inherently stochastic nature of the loss process, property-liability insurers face the risk of a catastrophic loss. Hurricanes, earthquakes, winter storms, and fires all have the potential to significantly affect the financial condition of an insurer. This risk is separated out from the normal pricing risk described above. In this model, catastrophes are handled as follows, for each simulated year:

1) The number of catastrophes (by our definition, events of any type causing industry-wide losses in excess of $25 million) during the year is determined based on a Poisson distribution, with the parameter based on historical experience.
2) Each catastrophe is assigned to a specific geographical area, or "focal point," again based on historical tendencies.
3) Once assigned to a focal point, the aggregate-industry size of each catastrophe is determined, based on a lognormal distribution. The size of the event is affected by the location, as both the type of loss and the amount of insured property exposed to a loss is a function of where the catastrophe occurred. The parameters of the lognormal distribution are based on historical industry experience, appropriately adjusted to future cost levels.

4) The geographical distribution of the event by state is determined, based on a state-by-state frequency correlation matrix determined from historical patterns.
5) The loss is allocated to the company based on market share in the lines exposed to catastrophic risk.

Loss Reserving and Development

This is the major component of liability risk, and one that distinguishes, and complicates, dynamic financial analysis for property-liability insurers. The starting value used for the loss reserve in this model should be the value indicated by an analysis of the company's historical experience, not just the loss reserve stated in the latest financial report. However, even though the loss reserve is based on an actuarial analysis, it cannot be assumed to be exact – there is likely to be some random deficiency or redundancy. In addition to the stochastic nature of the loss reserve and payout processes, a complication is the correlation between loss reserve development and interest rates, since both are correlated with inflation. However, whereas the relationship between inflation and interest rates is well recognized and has been extensively documented, the relationship between inflation and loss development is much harder to quantify. Loss reserving techniques traditionally assume that past inflation rates will continue. If inflation increases over historical (or other forecasted) levels, then future loss payments are likely to exceed the amount reserved. The relationship between inflation and loss development is one area that needs additional research.

As mentioned, loss development is subject to further variability unrelated to inflation. This variability is factored into the model by a normal random variable that allows for either favorable or adverse development. The volatility parameter is selected based on the company's size and past development patterns, as well as industry considerations (however, any tendency on the part of management – or the industry -- to consistently over- or under-reserve is considered separately, i.e., in the analysis of the appropriate beginning loss reserve level). In years in which the uncertainty regarding court decisions affecting loss payments is higher than usual or when other economic conditions generate greater volatility, this additional uncertainty would be reflected by an increase in the loss development parameters. Loss reserve development may also affect rate adequacy. Significant under-reserving, in addition to impacting surplus directly, generates the need for additional rate increases that may, depending on the jurisdictional environment (as discussed below), be difficult to obtain. Also, rate increases can affect the renewal rates on business, causing an additional effect on a company's operations.

Jurisdiction

In addition to having the potential to affect the responsiveness of rates to changes in economic conditions, the jurisdictions in which a company operates impose additional risks on insurers. Residual market subsidies, retroactive premium rebates, and benefit changes on workers compensation policies already written, are all examples of jurisdictional burdens on insurers that increase the financial risk of the company. Thus, an additional, jurisdictional, risk component, dependent upon the geographical distribution of writings, is added to the model. This risk is assumed to only have the potential for a negative impact on an insurer (an insurer is not likely to be the beneficiary of a retroactive premium surcharge on former policyholders). The number of jurisdictional "events" is simulated by a Poisson distribution, with the parameter based on the characteristics of the jurisdictional environment in which the insurer operates. The size of each simulated event is determined based on a lognormal distribution.

Interest Rates

Interest rate volatility has led to a major focus on modeling interest rates by many financial institutions, including life insurers. Extremely complex models, using multifactor stochastic variables and time series relationships, have been developed. Despite the complexity of these models, and their relative accuracy in particular situations, no single model is accepted as being correct. Each model has its shortcomings and recognized deficiencies.

Interest rates are an important factor for property-liability DFA models, as they affect asset values and investment returns, and, less directly, other economic parameters. However, the ability of property-liability insurers to re-price contracts, their lower leverage, and the generally shorter maturities of fixed income securities, make it less critical that interest rates be modeled to as high a degree of accuracy as is necessary for life insurers, banks and other financial institutions.

Duration is a measure of the sensitivity of a financial instrument to interest rate changes. For instruments whose cash flows are not affected by the interest rate change, the duration can be measured as the weighted average time to receipt of the cash flow. The sensitivity of an insurer's surplus to interest rates is determined as follows:

$$D_s S = D_A A - D_L L$$

or

$$D_s = (D_A - D_L)(A/S) + D_L$$

where

D = Duration (of the subscripted quantity),
S = Surplus,
A = Assets, and
L = Liabilities.

Property-liability insurers are much less highly leveraged than life insurers. A typical asset/surplus ratio for property-liability insurers is 3/1, whereas the typical ratio for life insurers is approximately 20/1. Assuming that property-liability insurers have a duration of liabilities of 4 and life insurers have a duration of liabilities of 10, then based on the above relationship, a two year asset-liability mismatch would lead to the following: For property-liability insurers:

$$D_s = (2)(3/1) + 4 = 10$$

For life insurers:

$$D_s = (2)(20/1) + 10 = 50$$

This means that a 1% increase in interest rates would reduce the surplus of a property-liability insurer by 10%, but the surplus of a life insurer would decline by 50%. Based on this relationship, interest rate risk, while important for property-liability insurers, is not as critical as it is for life insurers. Thus, in the trade-off between simplicity and realism, the interest rate model is selected to be one more easy to work with and explain.

The interest rate process used in this model is based on the work by Cox, Ingersoll and Ross (CIR)[1] and takes the following framework:

$$dr = a(b - r)dt + s(r^{1/2})dZ$$

where:

dr = the (instantaneous) change in the interest rate level,
a = a constant that represents the speed of adjustment in interest rates,

[1] J. C. Cox, J. E. Ingersoll and S. A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53 (1985) 385-407.

b = the long term mean interest rate level,
r = the current short-term interest rate level,
dt = an (instantaneous) unit of time,
s = the standard deviation of the random volatility measure, and
dZ = the standard normal distribution.

This model proposes that the short-term interest rate has two components, one a deterministic factor and the other a random factor. The deterministic factor, represented by the first term, is the movement from the current interest rate level toward the long term mean, with the amount of this movement set by the speed-of-adjustment factor (if this value were 1.0, then the deterministic component would cause the interest rate level to move all the way back to the long term mean). Thus, the CIR model is a "mean-reverting" model of interest rates. The other component, represented by the second term, is the random factor, which is the product of the volatility factor, the square root of the current interest rate level (to scale the moves to the current level of interest rates and prevent negative interest rates from occurring), and the standard normal variate.

The initial values for the model, based on historical data[2], are:

$a = .2339$
$b = .0808$
$r_0 = .05$
$s = .0854$

These values reflect a discretized (specifically, annual periods) version of the continuous-time CIR model. The values resulting from this approach represent our model's simulated short-term (or T-bill, or "risk-free") interest rate for each trial year. This rate, in addition to impacting bond values and investment returns, also impacts several other simulated model values, for example inflation and equity returns. In addition, interest rates appropriate for valuing longer-term government and corporate fixed income securities can be generated by allowing for a stochastic term or default premium to be added to the basic risk-free rate. (For example, historical term yield spreads on U.S. Government instruments are displayed graphically in Exhibit 5.)

Inflation

The inflation rate for each year is a random variable that is determined after the interest rate has been simulated. In our initial version of the model, the "expected" inflation rate for a given trial year is calculated by reducing the simulated annual interest rate by a constant 2 percentage points; this

[2] K. C. Chan, G. A. Karolyi, F. A. Longstaff, and A. B. Sanders, "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate," *Journal of Finance*, 47 (1992) 1209-1227.

expected value, along with a volatility parameter, then act as inputs into a normal distribution from which the "actual" inflation rate for the trial year is simulated. This approach recognizes the correlation between interest rates and inflation (see, Exhibit 6), but still allows for variability around the standard inflation-interest rate differential. Once chosen, the inflation rate affects loss experience on the current book of business, on policies to be written or renewed in the future, and the loss development patterns for current reserves. It also affects the indicated rate level changes for future years.

Future versions of the model will include an enhanced module relating the inflation rate to the contemporaneous interest rate, as well as possibly past rates. In addition, there is some empirical evidence that the level of future inflation is related to current government bond term spreads (e.g., see Exhibits 7 and 8). This and other projection techniques are currently being investigated.

Market

Equities represent risky assets whose values change over time in a largely random fashion. In our model, determining the change in equity values for each insurer is a two step process. In the first step, the change in the value of the overall equity market is simulated for each trial year. This change is a function of both historical equity risk premium patterns and contemporaneous changes in interest rates. (The latter relationship exists to the extent that equities can be priced as the present value of future dividends or free cash flow. The relationship between changes in interest rates and equity values thus tends to be negative - see Exhibit 9.) Then, once the market change is selected, the insurer's equity holdings are assumed to change in line with the Capital Asset Pricing Model, based on the beta, or systematic risk, of the insurer's equity portfolio.

Default

In addition to interest rate risk, fixed income securities pose the risk of default on interest or principal. Default rates are a function of both the underlying security (in line with the ratings assigned to the debt) and economic conditions (more volatile interest rates engender a higher level of defaults). The risk of default is included in the model.

Deterministic Values

At this point, a number of other values are assumed to be deterministic, although it is recognized that they do indeed vary in practice. However, the risk imposed by these aspects of insurance operations are considered secondary to the other elements. Specifically, the following factors are assumed not to vary from expected values: expenses, reinsurance collectible, agents balances, premium growth rates, new (and first renewal) business loss ratio penalty, and the asset allocation between stocks and bonds.

The Model

The model is set up to run in an Excel spreadsheet in conjunction with @Risk, two widely used computer software packages. The program can be run with a minimal number of mandatory inputs. Standard values are used for most variables, but these values can be replaced by alternatives if the user desires. The information that is required to run the model includes:

1. Premium written (direct and ceded) by line of business, by state
   a) Total new business by line
   b) First renewal by line (policies first written in the prior year)
   c) Second and subsequent renewals by line (policies first written two or more years prior)
2. Initial loss ratio by line
3. New business loss ratio penalty by line
4. First renewal loss ratio penalty by line
5. Growth rate by line
6. Renewal rate by line (percent of policies renewed from one year to the next)
7. Expenses by line
   a) Commissions
   b) General expenses
   c) Other acquisition expenses
   d) Premium taxes
   e) Policyholder dividends
   f) Fixed expenses
8. Assets
   a) Bonds
      i) By type of bond as listed in Schedule D - Part 1A
      ii) Par, book, cost, and market value for each maturity class (under 1 year, 1-5 years,...)
   b) Preferred stock - market value - affiliated and unaffiliated
   c) Common stock - market value - affiliated and unaffiliated

d) Other assets

9. Liabilities
   a) Loss and LAE reserves by line, by accident year
   b) Unearned premium reserves by line
   c) Other liabilities

10. Surplus

Information that may be input to override the standard values:

- i) Loss payment pattern by line
- ii) Loss ratio volatility measure
- iii) Beta of equity holdings
- iv) Interest rate parameters
- v) Catastrophe parameters
- vi) Inflation parameters
- vii) Jurisdictional parameters
- viii) Tax parameters
- ix) Interest rate sensitivity of assets

Output

The model is set up to generate 1000 runs of the next five years of experience, although the number of runs can be adjusted. The five year period was selected as a compromise, on the one hand to allow the effect of changes in operations to be apparent, but not so long that the underlying forecasts become completely unreliable. The results of each run are stored to facilitate the analysis of individual runs, but the sheer number of values available requires focusing on key factors. One summary statistic is a histogram showing the final surplus value (after five years) of all the runs. An example is shown as Exhibit 10 (which is a simplistic, single-period simulation of a fictitious company whose period-ending expected surplus was $ 350 (million)). This display facilitates an overview of the risk an insurer faces, especially if viewed in the context of the proportion of times the surplus is below a selected value. Alternatively, other financial measures can be determined. For example, the premium-to-surplus ratio in the last year of the period could be determined and displayed. Another approach would be to indicate the number of times the financial ratings from one of the insurance rating agencies is reduced by one or more levels.

An additional output of the model is a distribution of the number of Insurance Regulatory Information System (IRIS) tests that the company fails in each of the five years simulated. These twelve tests are calculated from the projected balance sheets and operating statements. Companies that fail four or more tests receive a priority classification and are subject to additional regulatory scrutiny. Thus, avoiding test failures can be a reasonable management objective.

The first three tests, Gross Premium Written to Policyholders' Surplus, Net Premium Written to Policyholders' Surplus, and Change in Net Written Premium, can all be calculated directly from values obtained from the model. The fourth test, Surplus Aid to Surplus, measures the degree to which surplus is enhanced by reinsurance transactions. Calculation of this value requires the amount of reinsurance commissions and would be based on the assumption that the reinsurance program does not change over time, which may not be the case. Thus, this estimate would be more of an approximation.

The fifth test determines if the Two Year Operating Ratio is below 100 percent, which can be calculated directly from the model values. The sixth test is based on whether the Investment Yield falls within typical guidelines, currently 4.5 to 10.0 percent. Since these guidelines change with market conditions, the model incorporates a variable guideline which is set at 2.5 percent above or below the current mid-maturity U.S. Treasury bond yield. Thus, if interest rates rose significantly, an insurer that had locked in low yielding debt would be classified as failing this test.

The next test measures Change in Policyholders' Surplus, which is determined directly from the model results. The eighth test calculates Liabilities to Liquid Assets and has as a failing value of 105 percent. Both of these quantities are determined by the model, and so the ratio can be calculated directly within the model. The ninth test is Agents' Balances to Policyholders' Surplus, which is calculated from the model output.

The last three tests are based on loss reserve adequacy: measures of One Year Loss Development, Two Year Loss Development, and the Current Estimated Deficiency, all as a percentage of Policyholders' Surplus. These calculations come out of the model results, but also require information about the carried loss and LAE reserves of the insurer, since the loss reserves input into the model are those indicated based on an actuarial analysis. Some insurers consistently over- or under-reserve, and these policies would impact the results of these tests.

The output of the IRIS tests would be a histogram indicating the proportion of the runs versus the number of failed tests, and this is provided for the current year and each of the five forecasted years.

Problem Areas

There are several areas involved with this model that require additional work and consideration. First, current tax provisions relating to the property-liability insurance industry are extremely complex. Investment income is generally taxed at a lower rate than underwriting income, reflecting capital gains, dividends, and investments in municipal bonds that are taxed at a lower rate, and that rate depends on when the securities were purchased. In addition, insurers are subject to the alternative minimum tax provisions and can have tax loss carry-forward and carry-back positions.

Our model uses the standard graduated tax rate schedule applied to total operating income to determine the tax liability. The user can adjust this calculation to apply different tax rates for underwriting and investment income. This would allow the taxation of investments in tax-favored instruments, such as municipal bonds or dividend-paying stocks, to be reflected more accurately. In order to keep the required input to the program to a manageable level, however, this model does not attempt to perform an exact tax calculation. There is proprietary software available that can be used to calculate taxes accurately, but this software must be purchased and is not available to be included in the public access model.

Second, each reinsurance program is unique, and reflecting the full effect of reinsurance would require tailoring the model specifically to each insurer. For certain types of reinsurance – e.g., catastrophe covers and quota shares – our projections of direct losses can be brought to a net basis in a straightforward manner. However, for other types of reinsurance – e.g., working excess covers – the adjustment from direct to net aggregate annual losses is much more difficult. In our model, the "net loss ratio risk" of the insurer with regard to such "problematic" reinsurance covers is selected to be one of three levels, depending on the combined effect of the size and stability of the direct business and the general characteristics of the reinsurance program. To run the model, the user need only specify whether the low, standard or high values should be applied. For a user that has a better concept of the underlying risk parameters for net losses, these values can be changed.

Standard fixed income securities, such as noncallable bonds, involve a cash flow stream that is not dependent on the level of interest rates. This type of investment represents a significant portion of the assets of most insurance companies. The change in value of these securities in relation to interest rate changes can be calculated straightforwardly. However, there are other types of assets, such as callable bonds and mortgage-related securities (including collateralized mortgage obligations), that are much more complicated. Our initial model is set up to deal with standard bonds by calculating an approximate duration from the input data. Applications involving more complex assets require the user to separately input values for interest rate sensitivity.

Future versions of our model will attempt to more precisely value the callability options of corporate bonds, as well as mortgage prepayment considerations underlying mortgage-related securities. The latter type of asset is becoming important to the property-liability insurance industry (see Exhibits 11 and 12). The prepayment rate is theoretically a function of one or more of the interest rate-mortgage coupon rate differential, the age distribution of the underlying mortgages, the characteristics of the underlying mortgage holders, the season, and the geographical distribution of the mortgaged real estate. A common industry model of the mortgage prepayment rate is the Public Securities Association (PSA) model; we are considering incorporating this approach to valuing mortgage-related securities into our model.

Using the Model

One of the most important questions relating to dynamic financial analysis is, "How do you use it?" Although relegated to a final section in work describing DFA, this question really needs to be the first thing decided before beginning a DFA project. The intended use of the model dictates the structure and content of the model.

A variety of different uses of DFA exist. One is to measure the likelihood of a company's insolvency given current operations. Fortunately, for most insurers, this value is very low, and it is hard to judge the significance of the difference between a situation that indicates the company will be insolvent 1 or 2 times in 1000 runs. However, the model allows management to examine the parameter values in the cases where the company did have financial problems in order to see if steps should be taken to reduce this risk even further.

A more widespread use of a DFA model is to examine the financial effect of different management strategies. Looking at both the range and probable outcomes from specific management decisions, such as expansion into a new line of business, withdrawing from a state or hedging investments, can provide valuable information about the potential impact of these strategies. Thus, DFA can be a useful planning tool. To facilitate this type of analysis, the model allows the user to set the volatility parameters to zero, so the model produces expected values of different strategies. The advantage of this ability is that it removes the stochastic features of the model so the outcome is not influenced by random fluctuations, and it allows the user to focus directly on the impact of the specific strategy in question.

In terms of practical issues regarding its use, our model will be publicly available via computer disk, or by accessing web pages affiliated with the authors. As mentioned before, the model utilizes Excel and @Risk spreadsheet software. Data, as specified in the section "The Model" above, is input by the user on a general input sheet. The user will also have the capability of changing the default values for a variety of variables, allowing for flexibility in modeling many different corporate or economic environments.

Conclusion

Dynamic financial analysis is becoming one of the skills casualty actuaries will need to possess in the near future. By developing a basic DFA model that can be used to understand this technique, and making the model widely available to facilitate discussion and improvement, we hope to help practitioners enhance this skill and researchers develop better models. Dynamic financial analysis has the potential to provide insurers with an opportunity to assess their risk, examine alternative strategies, and develop effective risk management approaches. Hopefully, this work will foster improvements in this field.

Appendix

Summary of DFA Model Stochastic Variables

This Appendix summarizes some of the key variables appropriate for a DFA model. The summaries of each variable below give the following information:

a) Description of the variable
b) Reason for inclusion in a DFA model
c) Some possible sources for data regarding the variable
d) Analytical approach to the variable

More extensive descriptions of several of these variables are included in the "Key Risks" section of this paper.

Asset and Investment Risks

1. Short-Term Interest Rate
   a) Rate on U.S. treasury bills (e.g., one month maturity)
   b) Short-term rates are correlated with many other financial variables (e.g., inflation, returns on other assets, insurance pricing); in our model, many other stochastic variables are simulated partly as a function of this process. In addition, T-bill market values are a function of the short-term rate
   c) Sources: Ibbotson Associates[3], Federal Reserve Bank of St. Louis FRED Database (e.g., via CAS DFA web page)
   d) Cox, Ingersoll, Ross mean-reverting short-term interest rate model

2. Term Premium
   a) Premium added onto short-term rate to longer-term U.S. government instrument rates
   b) Affects longer-maturity government bond market values
   c) Sources: Ibbotson Associates, Federal Reserve Bank of St. Louis FRED Database (e.g., via CAS DFA web page)
   d) Modeled via a statistical distribution with mean and volatility parameters derived from the time series of historical term premium values, and their relationship to the short-term rate

3. Default Premium

[3] Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation: 1996 Yearbook*, 1996, Ibbotson Associates, Inc.

a) Premium added onto Treasury rates to get corporate bond rates
b) Affects corporate bond market values
c) Source: Ibbotson Associates
d) Modeled via a statistical distribution with mean and volatility parameters derived from the time series of historical default premium values, and their relationship to the short-term rate

4. Default Risk
    a) Likelihood of default by issuer of fixed-income securities
    b) Value of corporate bonds is reduced by default on interest or principal
    c) Sources: Academic studies[4]
    d) A function of the underlying securities (e.g., as reflected by debt ratings) and general economic conditions (e.g., interest rates)

5. Equity (Market) Premium
    a) Premium added onto short-term rate to get return on equity market
    b) Affects market value of equity portfolio
    c) Sources: Ibbotson Associates, Center for Research in Security Prices (CRSP), stock market links on CAS DFA web page
    d) Change in the equity market is a function of historical equity risk premium patterns and contemporaneous changes in interest rates

6. Prepayment Risk
    a) Risk of prepayment by mortgage holders
    b) Affects the market value of mortgaged-backed securities
    c) Sources: Industry and academic studies
    d) Modeled by Public Securities Association (PSA) model

Liability Risks

1. Loss Payout Pattern
    a) Percentage of ultimate losses paid in each calendar quarter
    b) The loss payout process is inherently stochastic, and impacts operating results each year through its effect on the payout of claims
    c) Sources: Individual company data, supplemented by industry statistics

[4] For example: Edward I. Altman, "Measuring Corporate Bond Mortality and Performance," *Journal of Finance*, 44 (1989) 909-922

d) A combination of two stochastic variables. The first variable is the expected historical claim payment pattern, and is modeled using a Beta distribution with parameters developed from company and industry historical experience. This variable is then combined with claims inflation (as described below) to develop calendar year payments and, in turn, estimates of reserves.

2. Loss Reserve Development (Redundancy / Deficiency)
    a) Redundancy or deficiency in the beginning loss reserve
    b) The loss process is inherently stochastic, and thus impacts operating results each year through the level of claim liabilities incurred
    c) Sources: Individual company and industry statistics
    d) Modeled via a statistical distribution with mean and volatility parameters based on company size and historical company and/or industry experience

3. Inflation
    a) Both general economic (CPI) and insurance claims inflation
    b) Affects future values of liabilities (e.g., claims payouts)
    c) Sources: Ibbotson Associates, Masterson indices (published in *Best's Review*), Federal Reserve Bank of St. Louis FRED Database (e.g., via CAS DFA web page)
    d) Future inflation a function of contemporaneous interest rates and current yield spreads; also some autoregressive properties

Underwriting Risks

1. Pricing - Rate per Exposure
    a) Premium charged per unit of exposure
    b) Affects level of premium income received
    c) Sources: Historical company and industry experience
    d) Modeled by taking into consideration the company's current average rate and the effect of interest rates, inflation rates, and the underwriting cycle. The process is similar to a prospective rate level indication.

2. Pricing - Exposures
    a) Underlying quantity of insurance sold
    b) Affects level of premium income received
    c) Sources: Various insurance industry statistics; general economic (e.g., interest rate, inflation, output, and employment)

conditions also affect exposure levels (data available, for example, via CAS DFA web page)
   d) A function of rate level and general economic conditions

3. Catastrophes
   a) Risk of large losses (i.e., greater than $ 25 million on an aggregate industry basis)
   b) Impacts claims costs, and thus operating results
   c) Sources: Property Claims Services division of American Insurance Services Group, Inc. (e.g., via Insurance Information Institute[5])
   d) Five-step simulation process – see "Key Risks" section

4. Jurisdiction
   a) Insurance risk unique to geographical location – e.g., residual market subsidies, legislative / judicial / regulatory environment, level of competition
   b) Impacts operating results
   c) Sources: Aggregate insurance industry data by line by state
   d) Simulate potential variability in loss ratios that are a function of a company's distribution of business by geographic location and line of business

5. *a priori* (or Underlying) Loss Ratio
   a) Ratio of losses and allocated loss adjustment expenses to premium
   b) The loss process is inherently stochastic, and thus impacts operating results each year through the level of claim liabilities incurred
   c) Sources: Company experience adjusted for changes in rate levels, new business premium writings, and catastrophes
   d) Modeled as a function of prior years' loss ratios and inherent variability due to internal and external influences

[5] Insurance Information Institute, *1997 Fact Book: Property/Casualty Insurance Facts*, 1996, Insurance Information Institute

DISTRIBUTION OF BONDS BY TYPE

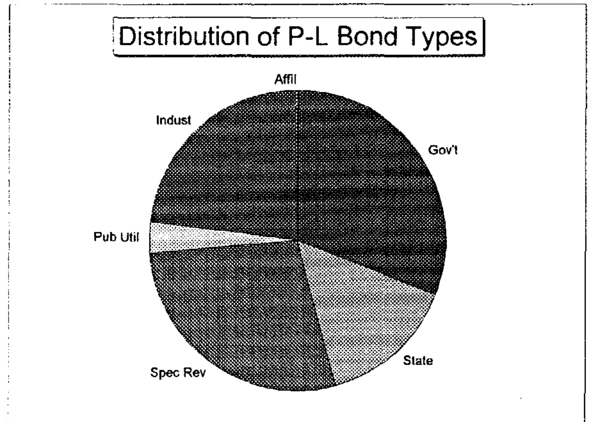
Exhibit 2

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

In Billions of Dollars

Bond Type	*Statement* Value	% of Total
Governments	154.6	30.9%
States, Territories, and Possessions	74.9	15.0%
Special Revenue	139.0	27.8%
Public Utilities	16.9	3.4%
Industrial and Miscellaneous	114.2	22.8%
Parents, Subsidiaries, and Affiliates	1.0	0.2%
	=======	=======
Total Bond Holdings	500.6	100.0%



DISTRIBUTION OF COMPANY ASSETS

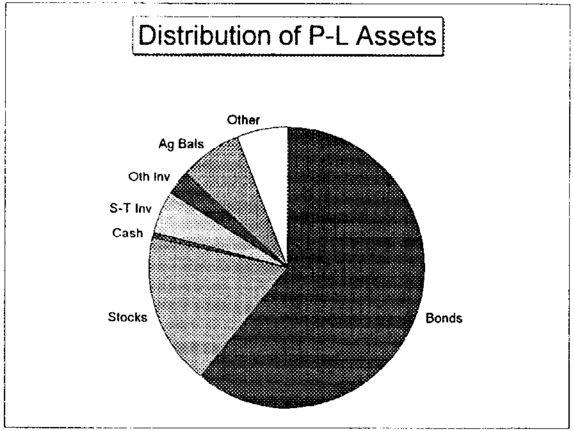
Exhibit 1

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

In Billions of Dollars

Asset Item	Statement Value	% of Total
Bonds	465.5	60.8%
Stocks	134.1	17.5%
Cash	4.9	0.6%
Short-Term Investments	37.5	4.9%
Other Invested Assets	22.0	2.9%
	----------	----------
Total Invested Assets	664.0	86.8%
Agents' Balances or Uncollected Premiums	55.4	7.2%
Other Assets	45.8	6.0%
	=======	=======
Total Assets	765.2	100.0%



DISTRIBUTION OF BONDS BY MATURITY

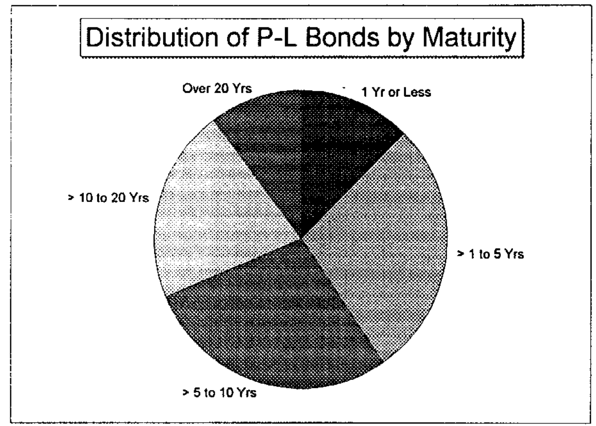
Exhibit 3

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

In Billions of Dollars

Bond Maturity	Statement Value	% of Total
1 Year or Less	62.2	12.4%
Over 1 Year, Through 5 Years	140.5	28.1%
Over 5 Years, Through 10 Years	140.9	28.1%
Over 10 Years, Through 20 Years	105.8	21.1%
Over 20 Years	51.2	10.2%
	=======	=======
Total Bond Holdings	500.6	100.0%



DISTRIBUTION OF COMPANY LIABILITIES AND SURPLUS

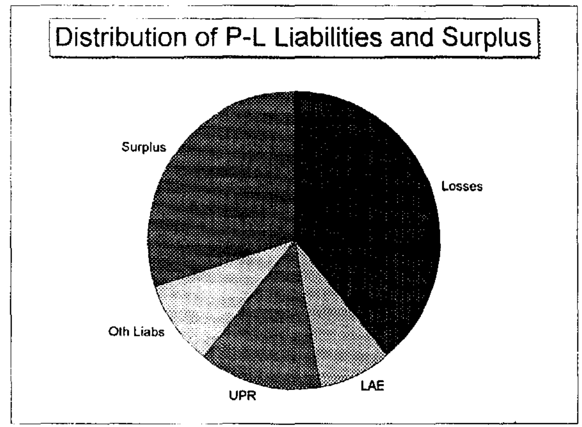
Exhibit 4

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

Statutory Values In Billions of Dollars

Liability or Surplus Item	Statement Value	% of Total L&S
Losses	298.9	39.1%
Loss Adjustment Expenses	62.0	8.1%
Unearned Premiums	103.9	13.6%
Other Liabilities	70.4	9.2%
	----------	----------
Total Liabilities	535.2	69.9%
Policyholders' Surplus	230.0	30.1%
	=======	=======
Total Liabilities and Surplus	765.2	100.0%



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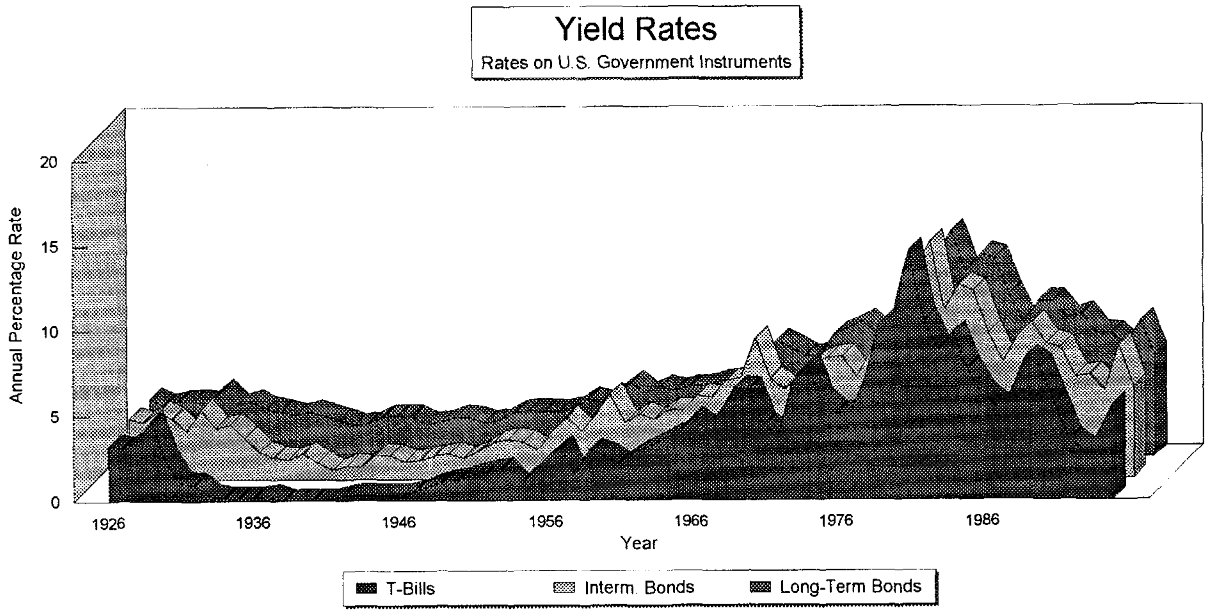
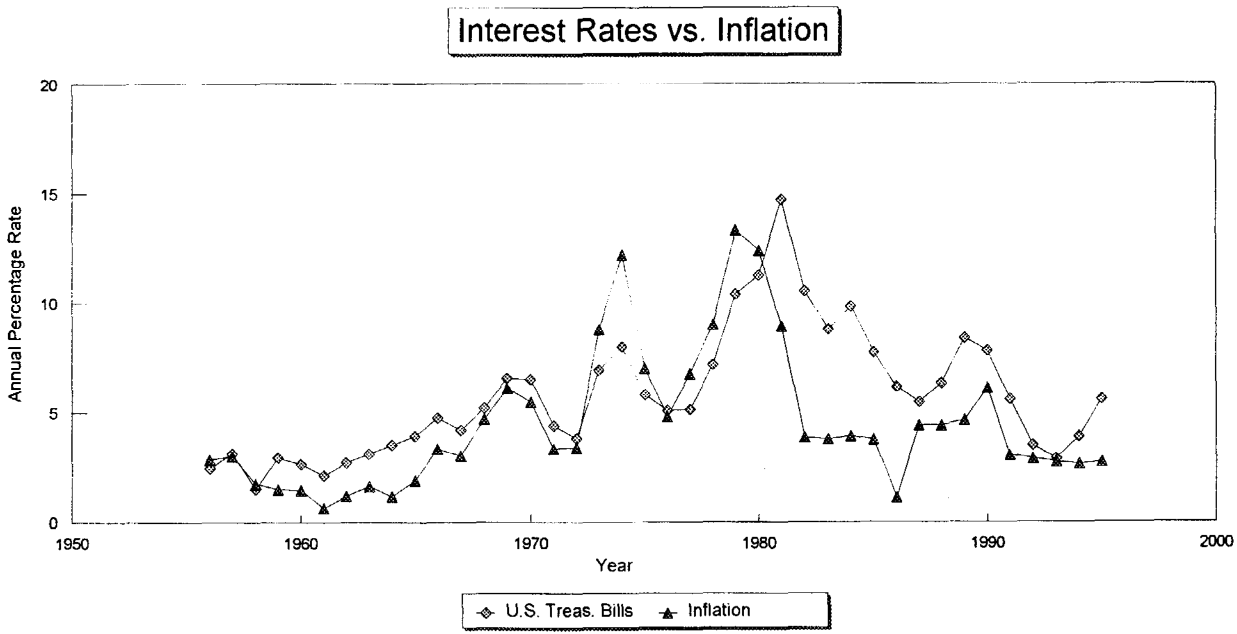


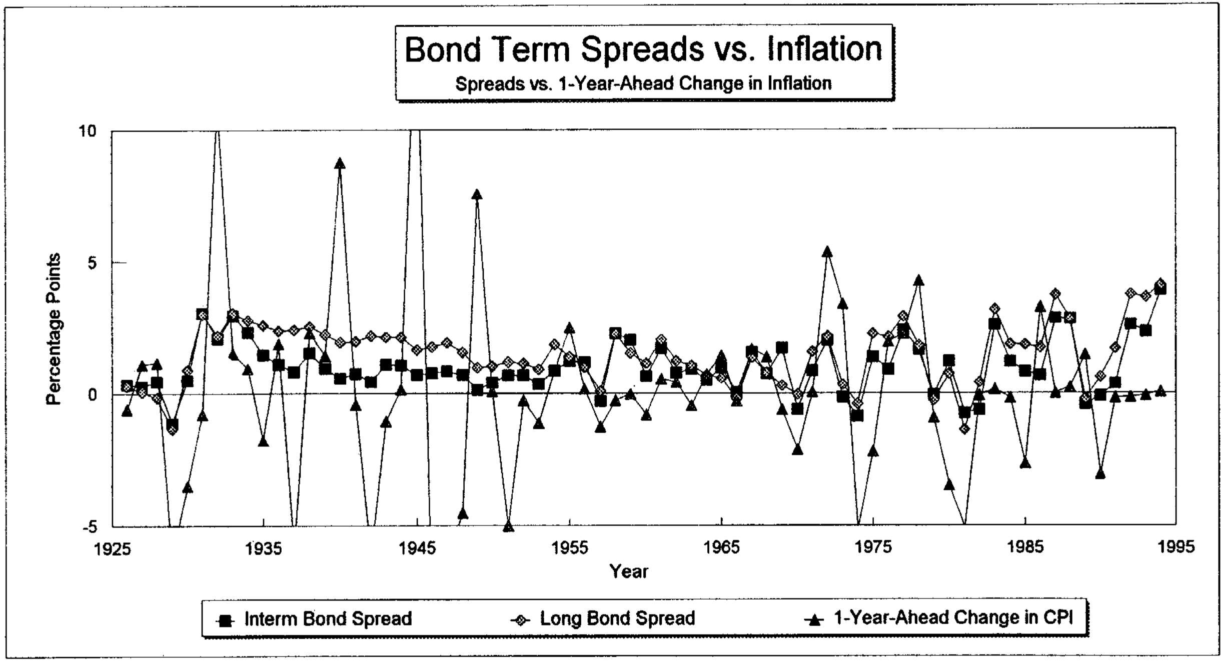
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 6



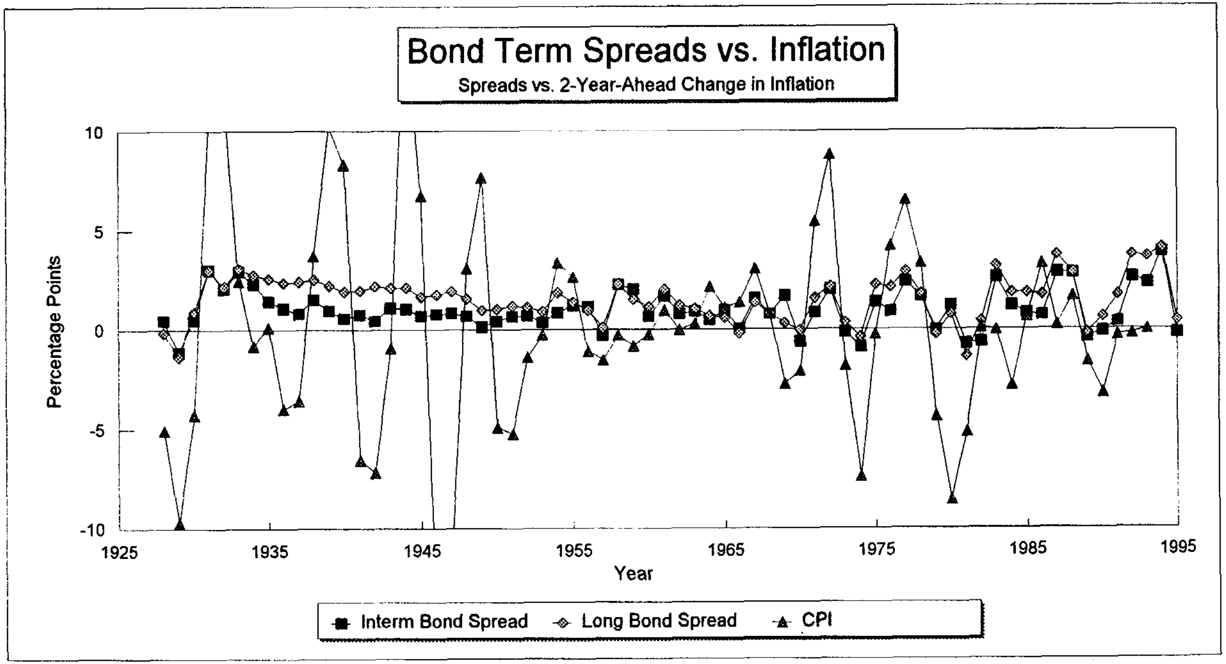
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 7



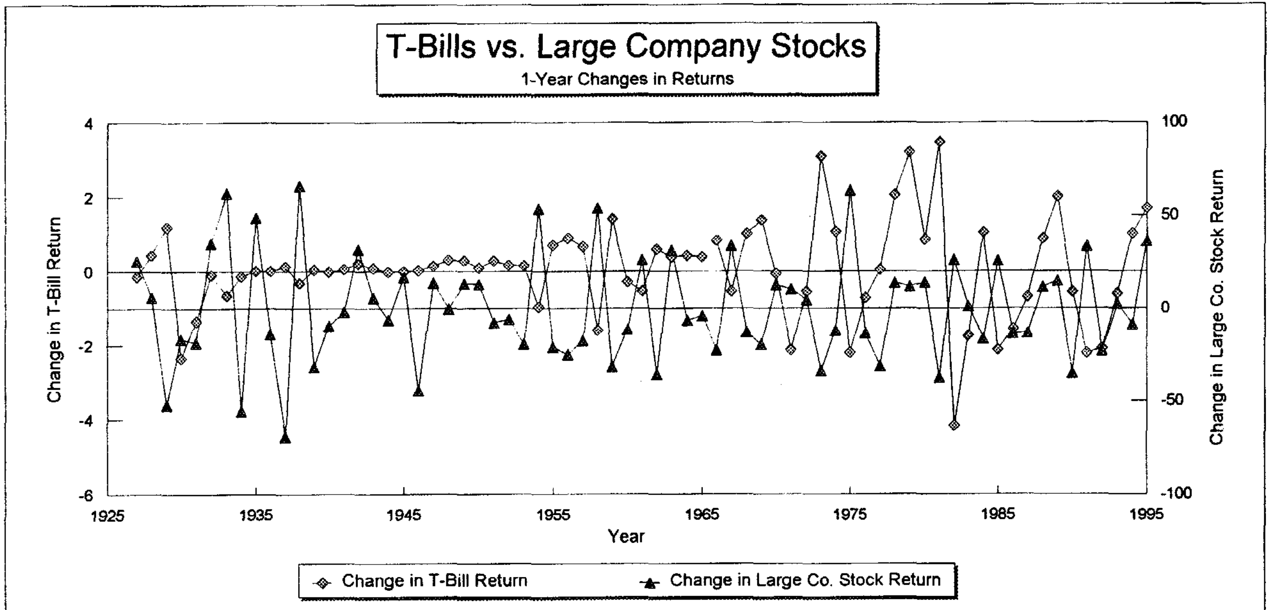
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 8



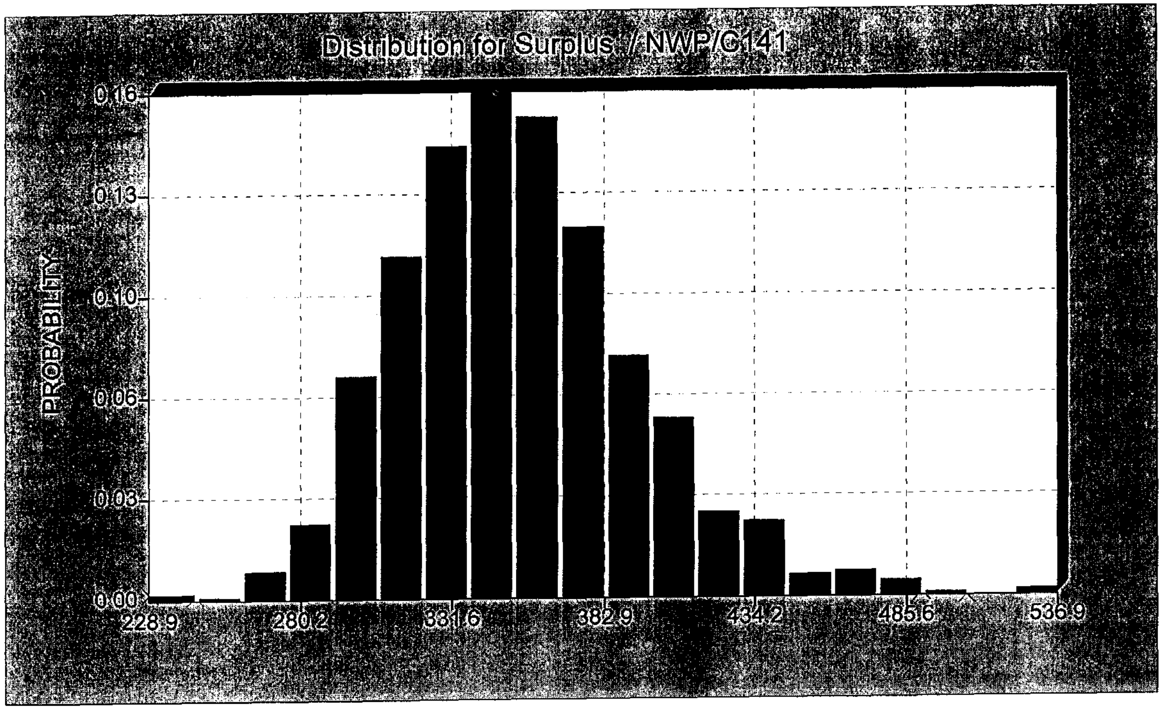
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 9



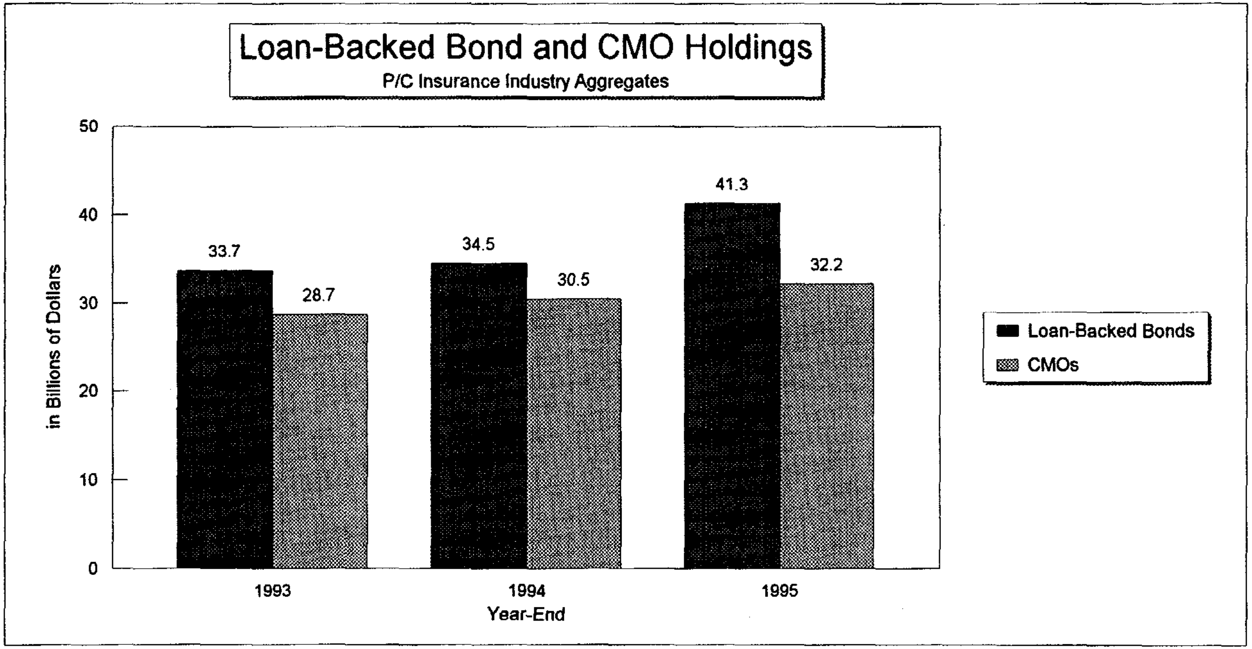
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 10

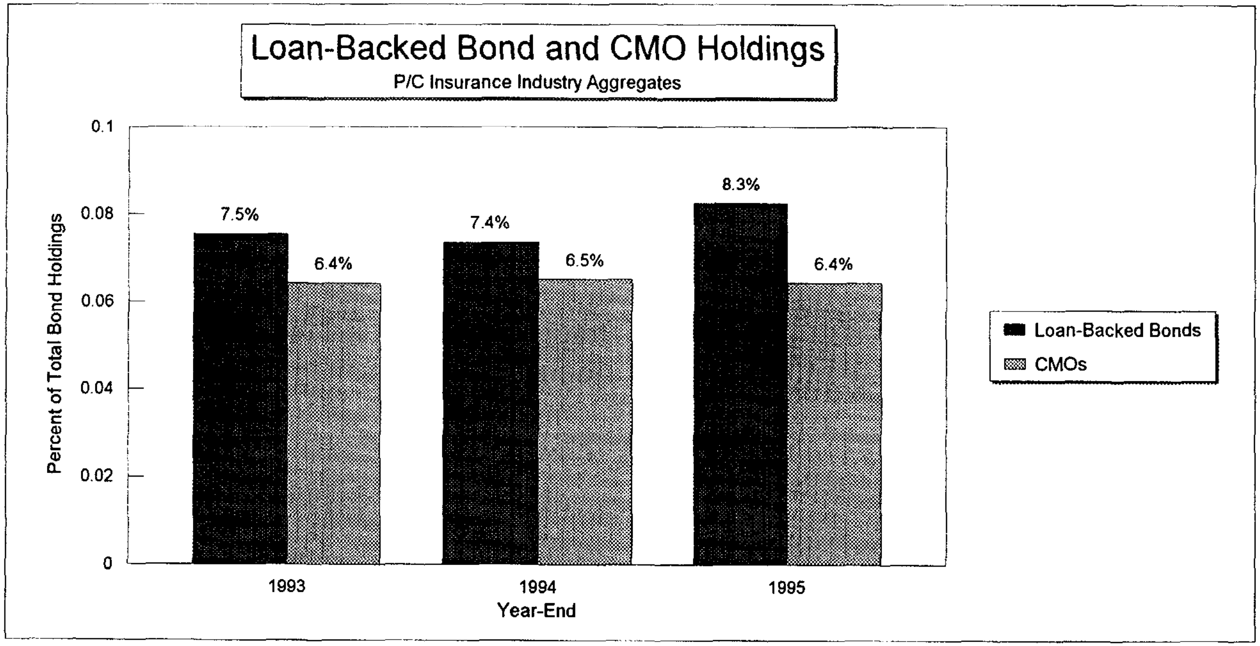


Distribution for Surplus / NWP/C141
PROBABILITY
0.16
0.13
0.10
0.06
0.03
0.00
228.9
280.2
331.6
382.9
434.2
485.6
536.9

Exhibit 11



Data per Best's Aggregates & Averages (1996)



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