Accident Year / Development Year Interactions

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Abstract

While traditional actuarial reserving methods assume that development patterns are stable over time, changes are often observed in practice. This paper explores the reasons for these changes and surveys the most relevant literature on methods that address the changes in development patterns. Finally, the paper suggests possible research for further improvements in reserving techniques.

Keywords. Loss Reserving, Interaction Terms

1. INTRODUCTION

1.1 Research Context

Common reserving methods, such as Chain-Ladder and Bornhuetter-Ferguson, rely upon an assumption that loss development patterns are stable over time. That is, loss development patterns do not change from one accident year[1] to the next. In practice, however, reserving actuaries observe changes in these patterns and make adjustments in the use of their methods to account for the changes.

When the loss data is summarized in a triangular format, it can be analyzed from three directions: accident year (AY), development year (DY), and payment/calendar year (CY). Most reserving methodologies assume that the AY and DY directions are independent. However, many factors can create dependencies between the three directions and violate this assumption. In the actuarial literature, these dependencies are sometimes referred to as "CY effects", reflecting the fact that frequently they are caused by CY trends or shocks. A more general way to describe these effects is to say that there is some interaction between the AY and DY variables, or that there is some other confounding variable that we have not accounted for. The main danger from ignoring these changes is "omitted variable bias" in our estimated reserves.

Recently, this danger has been highlighted empirically through the test of common reserving methods on a sample of actual triangles. The Casualty Actuarial Society (CAS) has made available a database of loss triangles from Schedule P to test common reserving methods. These methods were applied mechanically, generally using all-year averages to select patterns. The results showed some

[1] The discussions and techniques presented in this paper can be easily applied to a policy/underwriting year triangle.

systematic biases in the estimates, confirming in many cases, that patterns were not unchanged over even a ten year period.

Practicing reserving actuaries have always been aware of this phenomenon, and would not naively apply an all-year weighted average without looking for changes in the pattern. Various practical approaches are used when changing patterns are observed. The simplest approach is to base the selected pattern only on the latest diagonals, ignoring the upper left portion of the triangle. This method is clearly not the optimal option, but it is an easy solution. Other practical techniques have been created, which generally try to adjust the historical data such that the triangle of adjusted losses will have consistent patterns by year and therefore allow the analyst to use more diagonals or even the whole triangle.

This call paper will survey the actuarial literature for the methods that address AY/DY interactions and will give a brief description of each of these techniques, including some numerical examples. The purpose, however, is to communicate only the major concepts. The relevant papers will be referenced so that the interested reader can find the specific calculations needed to implement the techniques. There are also more advanced statistical models which will be described in much less depth.

All of the methods presented have some limitations that will be discussed in this survey. A common theme is that the methods generally assume that there is a single cause for the changing development pattern, and that an adjustment to the triangle can be made that will make the patterns consistent over time "all else being equal." The difficulty is that often multiple types of changes have taken place over the experience history, and the practical methods may not satisfactorily handle changes from multiple causes.

1.2 Objective

The purpose of the present paper is to explore the reasons for the changes in development patterns, survey some of the relevant literature on methods that address the changes in development patterns, and suggest future research.

1.3 Outline

The remainder of the paper proceeds as follows.

Section 2 will discuss the basic reasons as to why loss development patterns are different from one year to the next.

Section 3 will provide some diagnostics for evaluating whether or not a development pattern is changing over time.

Section 4 will survey the actuarial literature for the most common methods to account for changing patterns.

Section 5 will present opportunities for future research for practical and advanced methods.

Section 6 will present our conclusions.

2. BACKGROUND

Many factors can cause the loss development patterns to change from one accident year to the next. They can be internal (e.g., shift in the mix of business, change in claim settlements procedures) or external (e.g., law changes, inflation) to the company. They can also occur alone or simultaneously, making the identification of the real cause of the change more challenging.

2.1 Internal Changes Impacting the Patterns

Internal changes impacting the loss development patterns often relate to changes in the company's business and processes that directly or indirectly impact the loss data.

The change in the mix of business, for example, can manifest itself as a change in the geographical distribution, frequency or severity level of the claims, the retention limits, the deductible levels and others. For reinsurance companies, a change in the mix of business can come from a change in virtually every clause of the reinsurance contract: program type (excess vs. quota share), quota share percentage, attachment points, excess retention and limits, special features (corridors, caps), coverage of expenses, statute of limitation, and others. The type of marketing (direct vs. broker) may cause a shift from regional accounts, that are dominant when direct marketing is used, to national accounts, which rely more on brokers. Consequently, the actuary may observe a change in attachment points, limits and reporting lags. Additionally, changes in underwriting guidance can shift the focus from a profit seeking portfolio to a growth strategy, from small to large risks, or simply to a new type of risk with different development characteristics.

Changes in a company's procedures are also a major source for pattern distortion. The change can be related to the way the initial case reserves are established or the way claims are settled. For example, the settlement of claims can be impacted by a desire to fight claims, a change in guideline on whether to prioritize large claims or small claims, or other factors that cause a speed up or a delay in claim payments or reserve re-evaluation. A period of time with an understaffed claim department may create artificial changes in paid and reported loss development patterns. Expense related changes impacting the patterns can come from a simple change in the definition of allocated loss adjustment expenses, a shift from internal handling of claims to a Third Party Administrator (TPA), or a change in the TPA. This also creates opportunities for errors and delays in the claim processing.

Commutations can create one of the most significant pattern distortions for Schedule P loss triangles. In a typical commutation, the reinsurer transfers its current and future liability from particular ceded contracts back to the original insurer, along with an agreed upon payment. The reinsurer's loss triangles will no longer show any development for losses related to these commuted contracts. Any related reserves will be taken down and the final lump sum (or periodic payments) of the commutation "price" will be recorded as a paid loss. As a result, the ceding company will now start recording the reporting, payment, and development of these losses. Actuaries usually restate the historical loss triangles so that such transactions do not affect the development patterns. However, many industry studies and comparisons are done using Schedule P data, which is not restated for commutations. Thus, extra care must be used when applying reserving methods to Schedule P data.

Missing or incomplete loss data is a common issue for insurance and reinsurance companies. Whether due to a switch in data processing systems, a desire to start organizing the data differently (example: distinguish the medical and indemnity piece of a workers' compensation claim), or a significant delay in claim reporting, the missing or incomplete loss data compromise the reliance on historical patterns. In that case, actuaries usually exclude parts of the triangle from the analysis or try to find alternative methods to overcome this problem.

2.2 External Changes Impacting the Patterns

There are several external changes affecting the loss development patterns. One of them is related to changes in law and more specifically tort reforms. As discussed by Kerin and Israel (1998), most often, tort reforms limit the amount of damages that can be paid in total, restrict the conditions under which a damage is paid, modify the rule of evidence and change the litigation behavior. Their impact on loss payments and reserves is not easily predicted and it is also difficult to restate the historical data when significant changes occur. Examples of such reforms include no fault repeal in auto liability, caps in damage awards in medical malpractice, revised interpretation of coverage provisions and changes in workers compensation benefit laws.

Another major external factor impacting the development pattern is the change in inflation. Payments are impacted by an increase in the cost of goods and services, medical costs, attorneys' fees and jury awards. Other economic and social influences may also distort the patterns. Examples include the increased workers compensation claim frequency after the 2007-2009 recession period and the reduced delays in claim reporting due to new technology.

Knowing the variety of factors that can create pattern distortions, the actuary's goal is 1) to explore the loss triangle data and identify if such distortions exist; 2) to identify what caused them; and 3) to find the appropriate reserving method to overcome these distortions. The following section provides a discussion of practical techniques that can help the actuary detect and analyze changes in loss development patterns.

3. LOSS DEVELOPMENT PATTERN DIAGNOSTICS

3.1 Examples of Practical Diagnostic Techniques

The first step in the identification of any change in the AY development pattern consists of an analysis of the triangular data. The analysis can start with a review of ratios of available loss data.

The changes in patterns can be detected directly in the loss development factor (LDF) triangle. A review of the incremental paid loss, reported loss or claim count triangles may also be helpful in identifying the effects of changes in business mix, missing data and others forces. The actuary can also look at ratio diagnostics. Cicci, Banerjee, and Jha (2011) and Friedland (2010) list the following examples of diagnostic tests:

- Paid loss to reported loss ratios
- Paid loss to on-level earned premium (or other on-leveled exposure measure)
- Reported loss to on-level earned premium (or other on-leveled exposure measure)
- Reported loss to reported counts (reported severity)
- Paid loss to closed with payment counts (paid severity)
- Case reserve to open counts (average case outstanding)
- Closed with payment counts to reported counts ratios
- Closed without payment counts to reported counts ratios

- Open counts to reported counts ratios

The ratio diagnostics are useful in identifying any of the pattern shifts discussed earlier. For example, the average case outstanding and paid loss to reported loss ratios could indicate changes in case reserve adequacy; changes in settlement rate could be revealed by any ratio involving paid losses and claim counts or the paid loss to reported loss ratio; other changes could be indicated by the closed to reported claim counts. As noted by Friedland (2010), when the diagnostic is a ratio, a signal for a change in the pattern can come from the numerator or from the denominator and it may not always be clear what is causing it. Also, a lack of a signal could be due to offsetting changes in the numerator and the denominator.

Here is an example of paid loss to reported loss ratios indicating a change in the most recent diagonals:

Table 1: Example for Ratio Diagnostics

Paid Loss to Reported Loss Ratios

AY	12	24	36	48	60
1	0.33	0.67	0.91	0.98	1.00
2	0.33	0.67	0.91	0.98	1.00
3	0.33	0.67	0.91	0.98	1.00
4	0.33	0.67	0.91	0.98	0.95
5	0.33	0.67	0.91	0.94	0.95
6	0.33	0.67	0.80	0.94	
7	0.33	0.60	0.80		
8	0.27	0.60			
9	0.27				

Paid Loss Triangle

AY	12	24	36	48	60
1	159	413	677	775	791
2	154	401	656	778	793
3	145	389	615	769	785
4	151	394	644	755	788
5	146	399	620	762	770
6	161	411	626	739	
7	158	412	556		
8	150	367			
9	113				

Reported Loss Triangle

AY	12	24	36	48	60
1	477	620	744	791	791
2	462	601	721	793	793
3	434	584	677	785	785
4	454	591	709	770	830
5	437	598	682	811	811
6	482	617	783	786	
7	473	687	695		
8	556	612			
9	420				

This example was constructed so that the two most recent diagonals show lower paid loss to reported loss ratios compared to prior diagonals. However, the reason for the shift is different for each diagonal. CY 8 was impacted by an increase in reported loss (i.e. increase in the denominator of the ratio diagnostic) that could be an indication of a case reserve strengthening. CY 9 experienced a decrease in payments (i.e. decrease in the numerator of the ratio diagnostic) that could be an indication of a slowdown in payments. In cases like this, a review of several ratio diagnostics can help isolate the effect of simultaneous changes and will provide more direction in identifying the real cause for the pattern instability.

3.2 Heatmaps

A practical tool for identifying patterns in any type of data is the heatmap, which is just a visual representation of the data, where the values are emphasized with colors. Its purpose is to reveal patterns or clusters that may not be visible without additional analysis. For example, a heatmap may be very helpful in the analysis of a large triangle with more than 20 accident and development periods, where changes in patterns may be difficult to spot through visual inspection. Heatmaps are convenient because they are easily created in an excel spreadsheet using conditional formatting.

The tables below provide examples of heatmaps. Let's take a look again at the paid and reported loss triangles from Table 1. Even without calculating the paid loss to reported loss ratio diagnostic, it is clear that both loss triangles experienced some changes. The paid triangle has a very light colored last diagonal indicating lower payments and the reported triangle has a bright colored diagonal for CY 8.

Table 2: Heatmaps of Paid and Reported Triangles

Paid Loss to Reported Loss Ratios

AY	12	24	36	48	60
1	0.33	0.67	0.91	0.98	1.00
2	0.33	0.67	0.91	0.98	1.00
3	0.33	0.67	0.91	0.98	1.00
4	0.33	0.67	0.91	0.98	0.95
5	0.33	0.67	0.91	0.94	0.95
6	0.33	0.67	0.80	0.94	
7	0.33	0.60	0.80		
8	0.27	0.60			
9	0.27				

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Paid Loss Triangle

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7	473	687	695		
8	556	612			
9	420				

When starting an analysis, the actuary may not know in advance if there will be any data distortions. A heatmap can save time and effort by immediately focusing the actuary's attention to the problem area. Table 3 first shows a paid loss development factors triangle with changing patterns and then shows the heatmap of the same triangle. The heatmap immediately identifies that in CY 8, all AYs have larger payments when compared to other calendar years. This could be due to a speed up of payments or payments on larger number of claims that were reported with a delay. Also, the heatmap shows that the latest diagonal exhibits a much lower loss development.

Table 3: Heatmap of a Loss Development Triangle

Paid Age-to-Age Factors

AY	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1	2.209	1.416	1.140	1.090	1.049	1.038	1.021	1.056	1.011
2	2.050	1.313	1.180	1.095	1.058	1.040	1.061	1.014	
3	2.553	1.338	1.146	1.087	1.048	1.088	1.014		
4	2.159	1.326	1.158	1.084	1.101	1.019			
5	2.247	1.270	1.165	1.161	1.033				
6	2.395	1.311	1.375	1.025					
7	2.295	1.895	1.028						
8	4.517	1.031							
9	1.054								

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Heatmap of Paid Age-to-Age Factors

AY	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
1	2.209	1.416	1.140	1.090	1.049	1.038	1.021	1.056	1.011
2	2.050	1.313	1.180	1.095	1.058	1.040	1.061	1.014	
3	2.553	1.338	1.146	1.087	1.048	1.088	1.014		
4	2.159	1.326	1.158	1.084	1.101	1.019			
5	2.247	1.270	1.165	1.161	1.033				
6	2.395	1.311	1.375	1.025					
7	2.295	1.895	1.028						
8	4.517	1.031							
9	1.054								

3.3 Limitations of the Diagnostics

These diagnostic tests are useful in identifying whether a problem exists in the triangle, either from changing patterns or due to missing data. Often the diagnostics cannot identify exactly what the problem is (as seen in the example with the paid loss to reported loss ratio). Some of the changes listed in Section 2 do not create sharp changes in the triangle, but rather gradual shifts over time. This makes it difficult for the analyst to hone in on the problem, or even to determine which dimension (e.g., accident year versus payment year) is involved.

For example, if our triangle is actually a combination of two types of risks – one with quick development and a second with slow development – and the mix is changing over time, then a changing development pattern will be observed. Our diagnostic tests will be unable to distinguish this mix problem from other possible causes such as, say, calendar year trend or changes in claim settlement practices.

Table 4: Example of Diagnostic Limitations

Slow Developing Policies with Growing Volume
Loss Triangle

AY	12	24	36	48	60
1	100	200	250	275	290
2	300	600	750	825	
3	500	1000	1250		
4	700	1400			
5	900				

Age-to-Age Factors

AY	12-24	24-36	36-48	48-60
1	2.000	1.250	1.100	1.055
2	2.000	1.250	1.100	
3	2.000	1.250		
4	2.000			

Quick Developing Policies with Shrinking Volume
Loss Triangle

AY	12	24	36	48	60
1	900	1350	1395	1395	1395
2	700	1050	1085	1085	
3	500	750	775		
4	300	450			
5	100				

Age-to-Age Factors

AY	12-24	24-36	36-48	48-60
1	1.500	1.033	1.000	1.000
2	1.500	1.033	1.000	
3	1.500	1.033		
4	1.500			

All Policies Combined
Loss Triangle

AY	12	24	36	48	60
1	1000	1550	1645	1670	1685
2	1000	1650	1835	1910	
3	1000	1750	2025		
4	1000	1850			
5	1000				

Age-to-Age Factors

AY	12-24	24-36	36-48	48-60
1	1.550	1.061	1.015	1.009
2	1.650	1.112	1.041	
3	1.750	1.157		
4	1.850			

This is an example of Simpson's Paradox, as described in more detail in Stenmark and Wu (2004). The "paradox" is that the sub-portfolios each have patterns that are unchanging and perfectly stable over time, but the changing mix gives an appearance of a changing pattern for the combined business. This phenomenon occurs frequently in insurance applications because data are aggregated to produce more credible volumes, and that aggregation means that the data are no longer truly homogeneous; conversely, when data is broken out into smaller homogeneous pieces, it is no longer easy to see the signal hidden in the noise.

A practical example is US Workers' Compensation loss development. The development patterns are different for medical and indemnity coverages, with medical coverage generally having a longer development tail. Over time, the portion of losses in the medical coverage has been growing. Even if the patterns for medical and indemnity were each stable on their own, the combined triangle would, all else being equal, show a slowing development pattern.

The triangle may therefore show that something is changing, but at an aggregated level the actuary will be unable to identify the nature of that change. This is sometimes referred to as the problem of "lurking" or a "confounding" variable. The unidentified confounding variable is not explicit in the model and manifests as an AY/DY interaction.

3.4 Communication

Once the actuary has detected a change in the pattern, he or she needs to investigate what caused it. Knowing the source of the problem is important because it provides a better insight into what pattern to expect in the future. It tells us what data we can trust and what data we need to adjust (example: if the paid loss to reported loss ratio is distorted it is necessary to know whether it is the paid or the reported data that experienced a change). Finally, it helps the actuary decide which reserving method to use.

As we have seen the diagnostics may be misleading. Even in the most obvious case of distortion, the actuary needs to confirm his or her findings with other parties involved in the data processing who may be closer to the source of change. Berquist and Sherman (1977) and Friedland (2010) provide questions that can help the actuary investigate and confirm the change in data through communication with other departments of the (re)insurance firm. For example, the actuary can ask a claim executive if there have been any recent significant changes in the guidelines for setting and reviewing the unpaid case reserves. A question for the underwriters could focus on the shift of business by territory or by type of distribution (direct vs. brokerage distribution). These types of conversations can provide insights into the causes of the pattern distortion. More importantly, they can lead to additional information that can help quantify the total impact of the pattern change.

The important note to keep in mind is that, even with the best intention to collaborate, the other party may not have noticed the change or may not be willing to recognize an event as the source for pattern distortion (e.g., a case reserve weakening may not be easy to admit to the auditing actuary). In a presentation at the CAS 2007 Casualty Loss Reserving Seminar, Richard Sherman cautioned the audience to "beware of quick, slick answers" that are designed to bias the investigation of the cause of pattern changes. He also raised awareness of the importance of carefully selecting parties who will be able to provide the actuary with the most valuable information. Benefits could be found in a conversation with the most knowledgeable party (for example the department executive) or with the less biased party (for example a middle level staff).

4. CURRENT METHODS TO ACCOUNT FOR CHANGING PATTERNS

The fact that patterns can change over time due to a variety of reasons is well-known. A number of practical approaches are used by reserving actuaries to account for these changes.

Some of these approaches consist solely of data rearrangements and no method changes. They rely on additional data that can eliminate or explain the changes in the patterns. Berquist and Sherman (1977) discuss two means of obtaining data that is relatively unaffected by a given problem:

1) Data substitution – for example: the use of earned exposure in place of claim count when count data is disrupted. Earlier, it was noted that net data patterns can be easily distorted by changes in the reinsurance structure. In this case, the actuary may use the data substitution technique and rely on gross data. This approach relies on the assumption that the substitute data is available.

2) Subdivision of data into homogeneous groups of exposures – for example: when there have been changes in the mix of business. The actuary must be careful however of the decrease in credibility associated with the data split.

The most common approach currently used by actuaries is to eliminate distorted data. For example, when the actuary observes changing age-to-age factors down the columns of a triangle, he or she will make use of the latest few diagonals and ignore the earlier factors in the upper left corner of the triangle. This may be considered a default "only the latest diagonals" (OLD) method. This approach not only diminishes the statistical accuracy of LDF averages (they will be based on only a few points) but it will also affect the credibility of any reserve variability estimates. In other words, the actuary should be looking to use more data, not less.

The methods described in the five sections below improve on this in several ways. This survey of the actuarial literature will briefly describe methods for handling CY trends, changes in case reserve adequacy, changes in settlement rates, and missing data problems. Finally, statistical models will be discussed at a high level.

4.1 Calendar Year Trends

As noted above, a basic assumption of the Chain-Ladder method is that the columns of a development triangle are proportional to each other. Taylor (1977) notes that this assumption holds when "exogenous influences" such as monetary inflation and mix of business are relatively stable. But he also notes that:

> "It is crucial to the logic underlying the Chain-Ladder method that the 'exogenous influences' should not be too great. If this assumption does not hold, then the conclusion, that the columns of the run-off triangle are proportional, goes away too, and the Chain-Ladder method can give misleading results."

Taylor provides a "separation" method to isolate the calendar year effects from the development year effects. In order to apply this method, we need a development triangle of paid losses and an exposure base of claim counts by accident year. As Taylor states, getting a good estimate of ultimate counts by year can be "problematic" but we will assume here that it is available.

The separation method as outlined by Taylor requires that we distinguish the frequency and severity components within each accident year, so that changes along the diagonal can be assumed to be due to severity effects only. This requires that the triangle be adjusted such that each row represents the average severity rather than the aggregate loss dollars. To make this adjustment, we divide each row by an estimate of its ultimate frequency.

The accuracy of the separation method depends upon getting quality claim count information. For the example below, we will assume that all numbers have been adjusted to a common level, but in practice this assumption needs care.

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Table 5: Separation Model – Example

Cumulative Payments

AY	12	24	36	48	60
1	500	1,106	1,530	1,764	1,903
2	505	1,141	1,608	1,888	
3	530	1,230	1,790		
4	583	1,423			
5	700				

Age-to-Age Factors

AY	12-24	24-36	36-48	48-60
1	2.212	1.384	1.152	1.079
2	2.260	1.409	1.174	
3	2.320	1.455		
4	2.440			

Taylor gives a direct algebraic method for calculating a CY or payment year trend factor from this data. The method does not require any iterative optimization routines or special software, so it can be performed in a simple spreadsheet.

The implied trends by payment year are 1.0%, 5.0%, 10.0%, and 20.0%, which apply to incremental payments in the triangle. In this example, we have deliberately made sharply increasing trends so that the resulting increasing age-to-age factors in each column are obvious. If the payment year trend is constant, then no change in age-to-age factors would be observed.

Table 6: Separation Model – Example (cont.)

AY	Trend	CY Index
1		1.000
2	1%	1.010
3	5%	1.061
4	10%	1.167
5	20%	1.400

Index by Calendar Year

AY	Year 1	Year 2	Year 3	Year 4	Year 5
1	1.000	1.010	1.061	1.167	1.400
2	1.010	1.061	1.167	1.400	
3	1.061	1.167	1.400		
4	1.167	1.400			
5	1.400				

These CY Index factors are used to de-trend the incremental losses in the nominal triangle. The de-trended incremental losses are then accumulated by accident year to produce an inflation-free triangle. In the idealized example, this inflation-free triangle produces age-to-age factors that are constant down each column.

Table 7: Separation Model – Example (cont.)

Cumulative Payments

AY	12	24	36	48	60
1	500	1100	1500	1700	1800
2	500	1100	1500	1700	
3	500	1100	1500		
4	500	1100			
5	500				

Age-to-Age Factors

AY	12-24	24-36	36-48	48-60
1	2.200	1.364	1.133	1.059
2	2.200	1.364	1.133	
3	2.200	1.364		
4	2.200			

Future losses, estimated by completing the lower right portion of the data, then need to be put onto a nominal basis using an assumption about the future inflation.

Taylor notes that this method gives a good estimate so long as the change in patterns is due to a payment year effect, which is "particularly appropriate when claim costs are dominated by high rates of inflation." He goes on to caution that there may be other causes of changing patterns that would not be appropriately addressed by this method: "It is not so appropriate in respect of influences such as changing mix of business within a risk group, which is related rather to policy year." As we noted earlier, it is not easy to diagnose from the data what is causing the patterns we see, so investigation beyond the triangle is needed.

This method does have limitations though. We need a reliable measurement of counts as well as dollars, and the reserve estimate is dependent upon our ability to forecast the CY trend index into the future. In addition, this method applies only to paid loss data, and is not directly applicable to case incurred losses. Even with these limitations, however, it is an improvement over the OLD method, because it uses the entire triangle and not only the latest diagonals.

The use of calendar year trends has been advanced in several papers in the actuarial literature. Butsic (1981) produced a similar model to that of Taylor, adding interest rate discounting in the reserve. Barnet and Zehnwirth (2000) show how a calendar year trend can be estimated in a loglinear regression model. Gluck and Venter (2009) give a survey of the literature to 2009, especially with regard to more advanced statistical models.

4.2 Case Reserve Adequacy

When an actuary sees changes in reported losses, it is important to investigate what the real cause for these changes is. Given that the consistency of the reported incurred loss data depends not only on stable average case reserve, but also on stable claim reporting, and stable average payments, we can easily see that a pattern distortion may be due to changes in any (possibly multiple) of these three elements. Depending on what the real source of the disruption is, different data adjustments may be appropriate.

When faced with changes in case reserve adequacy, the actuary may be able to perform exact adjustments to the case reserves if they are set by formula (e.g., workers' compensation indemnity tabular reserves). In these cases, the system can re-evaluate the case reserves using current assumptions on mortality or interest rates and produce an "as if" triangle using the more recent assumptions.

In situations for which we are uncertain of the reasons for changes in case reserve adequacy, Berquist and Sherman (1977) provide a method for making an appropriate adjustment. That approach is nicely described by Duvall (1993), as follows:

> "Given a shift in reserving practices, the Berquist-Sherman adjustment for the shift begins by obtaining the rate of inflation in average closed claims. Next, the average reserve at the most recent valuation date is calculated for each year. These average reserves are trended back to earlier valuation dates at the estimated trend rate to obtain the average reserve at each age for each year in the experience period. The computed average reserves are then multiplied by the number of open claims at each age to get the estimated cost of open claims. Cumulative claim payments are then added to get an estimate of incurred losses on a basis that is consistent with current reserving practice."

Thorne's (1978) discussion of the Berquist and Sherman method points out the difficulty and actuarial judgment involved in the selection of the severity trend used to trend back the most recent average reserves.

One way to determine, or at least confirm, the severity trend selection is to use Duvall's (1992) regression technique. Duvall's model has two purposes: 1) to detect shifts and trends in the loss development factor parameters and, if a change is observed, 2) to provide an objective way to restate the reported incurred losses for early valuations on a basis that is consistent with recent valuations. The first step in his model is to present the reported incurred loss as a function of the number of claims, the average claim cost and the loss development factor at each valuation date. Next, for each of these factors, Duvall specifies a regression function and estimates the parameters using the triangular data. He states:

> "The LDF function is central to the objective of this paper. Changes in reserving practices must be manifest in changes in the parameters of this function if they are to be detected. Therefore, it is important that the function be capable of providing an excellent fit to the observed development patterns."

The estimates from this regression model can be used as an objective way to determine a severity trend and restate the recent reported incurred losses to earlier valuations on a basis that is consistent with the current valuations. This approach can also be applied in cases where we have a change in settlement rates.

4.3 Changing Settlement Rates

Berquist and Sherman (1977) also present a method for reducing the impact of changes in settlement rates by adjusting the cumulative closed claim and paid loss triangles.

The method starts with a review of disposal rates. The disposal rate can be seen as a type of ratio diagnostic. It is defined as the cumulative closed claim counts for each accident year and maturity, divided by the ultimate claim counts. A change in the disposal rate pattern is an indication of a change in the rate of claim settlement. Next, a representative disposal rate pattern is selected (for example the most recent diagonal) and it is assumed to be valid for all accident years. The adjusted closed claim counts are obtained by multiplying the selected disposal rate by the ultimate claim counts. The method approximates the relationship between the paid losses and the closed claim counts, before any adjustments, with a function. It then uses this relationship to obtain the adjusted paid losses based on the adjusted closed claim triangle.

Thorne's (1978) comments on this technique are that "lack of recognition of the settlement patterns by size of loss can be an important source of error" and "it may be necessary to modify the technique to apply to size of loss categories adjusted for 'inflation' ". Exhibit I of his discussion paper provides an example of how a shift in claim settlement (from small to large claims) increases the error in the reserves estimates.

Fleming and Mayer (1988) propose a variation of the Berquist-Sherman method where the adjustment is made not only to the paid losses but also to the outstanding losses. The procedure is described in pages 196 -199 and an example is given in Exhibit 5 of their paper.

As with the other methods described, this method can only be applied if reliable count data is available. This can be a challenge because counts can be compiled differently over time (e.g., the treatment of closed-without-pay claims). Counts can also be distorted by accident year changes. For example, a small increase in deductibles can greatly reduce claim counts, giving the appearance of a slow-down in settlements. Similarly in Workers' Compensation, a change in the states or industries covered can alter the mix of "medical only" versus "lost time" claim counts, giving a misleading impression of claims handling practices. These types of "confounding variable" need to be investigated before the methods are applied.

The change in settlement rates can be also addressed with a Bayesian model. More details of this technique will be provided in Section 4.5.4.

4.4 Incremental Development

Often, when data are missing for older accident years or when changes in definitions or mix of business have made it inappropriate to combine the data in a cumulative triangle for the purpose of the reserve estimation, the general practice is to "cut" the triangle and work only with accident years that are not distorted but contain data for all maturities. Throwing away the data is not an optimal solution. Instead, the actuary can make use of any non-distorted incremental data from old accident years.

The Sherman-Diss (2004) paper describes the Mueller Incremental Tail (MIT) method that can help achieve this goal. This method works for triangles that are missing values in the upper left corner, but have incremental amounts for the more mature years. The method consists of three steps:

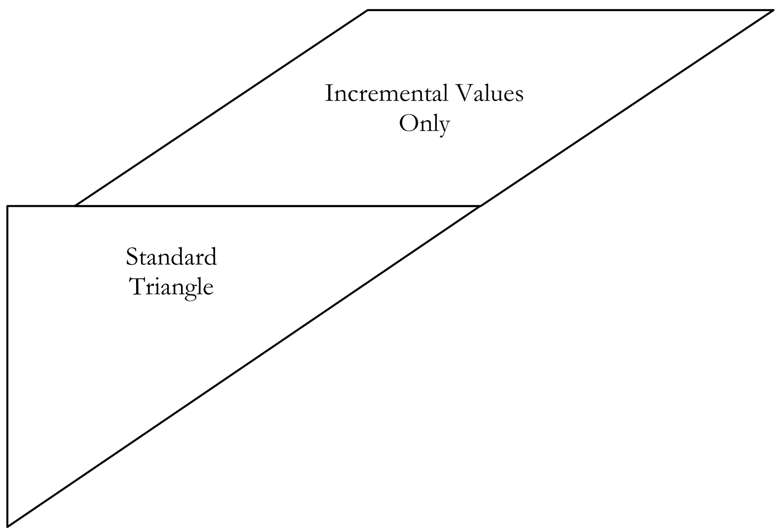
1) Calculation of incremental age-to-age factors for all available data. This is done by taking the ratio of incremental paid at age n+1 to incremental paid at age n

2) Calculation of an anchored decay factor representing the incremental payments made in year n relative to payments made in an anchor year. For example, it calculates the payments for years 16 to 37 relative to the incremental payments in year 15. The sum of the decay factors for years 16 to 37 can be viewed as a "cumulative decay factor" relative to year 15;

3) Calculation of a tail factor: The cumulative decay factor is then combined with a traditional

age-to-age factor for year 14 to 15, based on more recent data, to create a full cumulative loss tail factor.

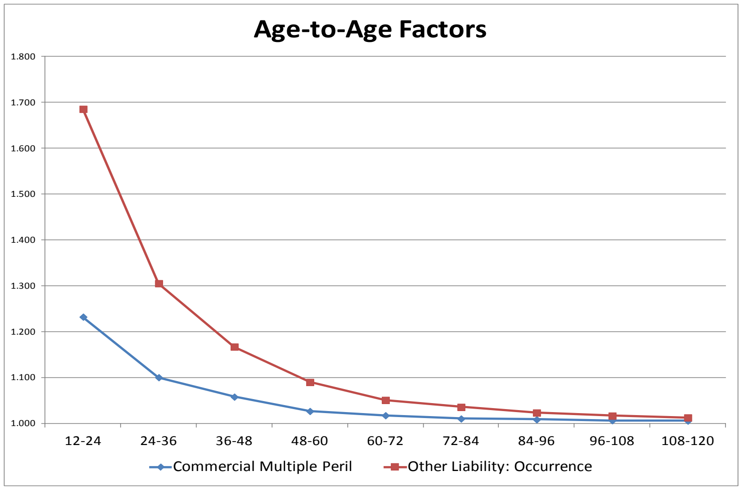
Figure 1: Graphical representation of the MIT method



The MIT method was originally created for incremental payments on long-tailed Workers' Compensation losses in a database that did not include payments for early periods (the upper left triangle). In this case, the data was missing because it was not available to the analysts. However, the technique could also be applied if the early payments were missing because they were below a self-insured retention. This suggests that the incremental method may be useful when there is a changing mix of primary and excess business in a portfolio or when commutations are not excluded from the data.

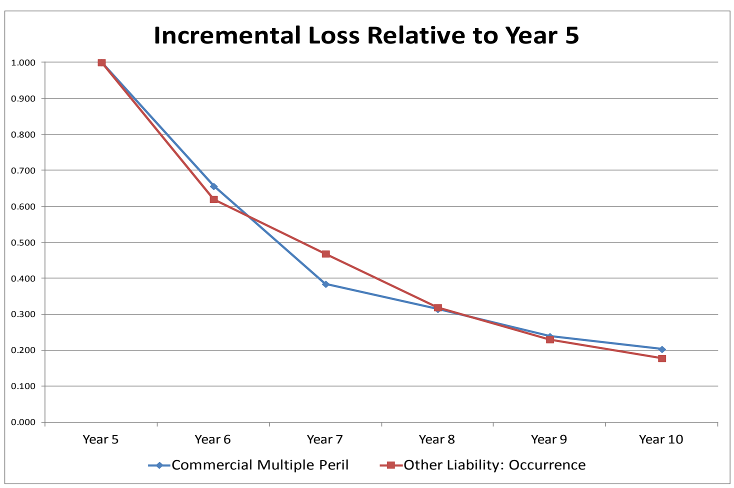
An interesting application of this method could be made in situations where the mix of business is different by accident year. To illustrate this, we can look at two triangles from consolidated industry Schedule P. The loss triangles and the calculation of the "normal" and anchored loss development patterns are provided in Appendix I. The development patterns for Other Liability (occurrence) and Commercial Multiple Peril (CMP) are quite different. Other Liability includes monoline, ground-up losses along with some losses from excess and umbrella policies. CMP includes losses from property as well as from liability. As we would expect, a much larger percentage of total losses are reported within the first few years for CMP. This early loss reporting acts as a ballast to reduce the age-to-age factors.

Figure 2: CMP vs Other Liability Loss Development



However, if we use the Mueller Incremental Tail (MIT) concept, then instead of age-to-age factors on cumulative losses, we anchor the factors at a later age. Figure 3 shows the incremental reported amounts relative to the amount incurred in the fifth year. These patterns look much more similar, implying that the losses contributing to the "tail" may be similar in both triangles.

Figure 3: CMP vs Other Liability Relative Incremental Loss Development



This approach is not recommended as an alternative to segregating the data into homogeneous groups. However, it does suggest that the incremental method may be useful on triangles where the changing mix of business cannot be fully identified in the historical triangle, or where the credibility of the data will be significantly impaired by the split of the data.

4.5 Statistical Models

The methods listed above have been designed so that the actuary can adjust the development triangle and make selections from the data without the need for special software. The transparency of assumptions and ease of calculation are clear advantages.

The methods do not make explicit assumptions about the variances or shape of the random variables that give rise to the observed data. For this reason, it is difficult to evaluate the results in terms of whether the final estimates are "best" (unbiased, minimum variance) estimates, or whether the adjustments are based on significant signals or could have been produced by random noise. Assumptions that are never made explicit are, by definition, untestable.

The main hurdle to implementation of a statistical model is the learning curve required to master the concepts and software. The statistical models listed below are roughly in order of difficulty in the learning curve required.

4.5.1 Generalized Linear Models (GLM)

Generalized Linear Models (GLM) are a generalization of linear regression models that allow for much greater flexibility in the relationship between the explanatory variables and the response variable being forecast, and in the variance structure of that response variable. The recent text "Predictive Modeling Applications in Actuarial Science" includes a good description of GLM (Dean 2014), along with the connection to reserving (Taylor 2014).

The use of GLMs for reserving was first suggested by Wright (1990), but was given very clear exposition by Renshaw and Verrall (1998). The initial observation was that the traditional Chain-Ladder method was actually a GLM model, and therefore making that explicit allowed for statistical tests and variance calculations to be performed easily. The GLM framework also allows for the introduction of exposure measures, market cycles, and calendar year effects to be included. Taylor's "separation method" discussed above, as well as the MIT incremental method, are special cases in the same GLM.

Perhaps the greatest benefit of a GLM perspective is that interactions between the accident years and development years can be explicitly recognized and included in the model. Taylor (2014) gives a good introduction to the use of GLMs in reserving, including the introduction of interaction terms.

4.5.2 Hierarchical or "Mixed" Models

Generalized Linear Models can also be helpful to account for changes in the mix of business because they allow us to look at multiple triangles simultaneously. For example, two triangles may represent different businesses that both include general liability exposures; they may have different development patterns but share the same sensitivity to inflation changes. A GLM can estimate some parameters separately for each data set and some parameters which are common across data sets.

However, if the data is split into many triangles, then it may be impractical to estimate parameters for all of the components separately. This is where hierarchical models (also known as mixed or multi-level models) can be introduced.

A good example of the application of mixed models is given by Schmid (2012). He was looking at residual market triangles for Workers' Compensation. These pools are segregated by state and can have very different volumes by policy year as business shifts between voluntary and involuntary placement, resulting in several triangles with similar – but not identical – patterns. The hierarchical approach allows for separate parameters to be estimated for each state pool, but also controlled such that the parameters for any one state could not be too far apart from some overall average. This "total credibility" approach allows reserving for each pool, while also borrowing strength from the larger sample of triangles.

Guszcza (2008) introduces the use of hierarchical models for reserving, allowing individual accident year patterns to deviate from some overall average pattern.

4.5.3 Models Using Detailed Data

A more extreme case of segregating the reserving data is to go down to the individual claim level detail. Guszcza and Lommele (2006) describe the advantage of this approach because it would automatically capture the mix of coverages, types of losses, and changes in policy limits. They note that "A danger of using summarized loss triangles is that they can mask heterogeneous loss development patterns."

The danger that Guszcza and Lommele describe is another form of Simpson's Paradox, and is a result of using the highly aggregated data provided in the traditional development triangle.

This point has been made by England and Verrall (2002):

> "…it has to be borne in mind that traditional techniques were developed before the advent of desktop computers, using methods which could be evaluated using pencil and paper. With the continuing increase in computer power, it has to be questioned whether it

would not be better to examine individual claims rather than use aggregate data."

In many cases, however, the individual claim data may not be readily available. Reinsurers, for example, would not have access to the individual claim-level data from ceding companies. A compromise between complete aggregation and micro-level reserving might be a model that uses treaty-level data.

The use of individual claim level data also introduces the problem that late reported or "incurred but not yet reported" (IBNYR) claims must be modeled separately.

4.5.4 Bayesian Models

Some of the recent literature on statistical modeling in loss reserving has proposed the use of Bayesian models.

Bayesian models allow (in fact, require) the user to specify prior knowledge of development factors and variables influencing the development patterns. The prior knowledge takes the form of a distribution of model parameters that is revised as actual loss data is observed. Increased computer speed and the availability of Markov Chain Monte Carlo (MCMC) simulation techniques have made the models more accessible to actuaries.

The key advantage of these models is that even very complex non-linear interactions between accident year and development year dimensions can be evaluated. If the prior distributions are set meaningfully, the models can also work with relatively sparse data sets and still produce useful information. Meyers (2015) shows that a non-linear "growth" function can include a non-linear interaction term, which he termed the "Changing Settlement Rate" (CSR) model, and found that it was able to correct bias in some of the data sets he reviewed.

There are still challenges to making Bayesian models fully accessible to reserving actuaries. First, they require "prior knowledge" about patterns to be explicitly incorporated as multivariate distributions of parameters. These prior distributions are not trivial to create. Second, the MCMC algorithm needs to be calibrated and monitored by the user to ensure that the results have truly converged to approximate the posterior distribution.

5. OPPORTUNITIES FOR FUTURE RESEARCH

This call paper has been intended as a brief survey of existing literature on methods addressing changes in development patterns over time.

We have seen that most of the methods are limited in that they assume "all else being equal" from other effects. In other words, the techniques may not be reliable if more than one type of change is taking place simultaneously. If policy limits written are changing, or business mix is shifting from manufacturing to service industry risks, and at the same time case reserve adequacy is changing, then we have no available methods for correctly adjusting the data. In technical language, this is an example of a misspecified model and can lead to biased results.

The way forward is to recognize that all of the factors that cause patterns to change can be viewed as different types of interaction terms. Viewed in terms of a regression model, our explanatory variables are the accident year and development year indices; the traditional Chain-Ladder model assumes that these two variables act independently. Instead, we need to include models that allow for interactions between these two explanatory variables. The exact form of this interaction may be different based on the cause of the change (mix of business, CY inflation, reserving practices, etc.), but they all fall under this concept.

We have also seen that identification of the cause of changing patterns is problematic when only highly aggregated triangles are available. More data such as information about the mix of business may be needed to help understand how and why the accident year and development year dimensions are not independent. In most cases this is done judgmentally with few practical suggestions in the literature as to how it can be quantified objectively.

While "technical" models such as GLM or Bayesian MCMC have begun to move in this direction, they have yet to allow for full flexibility in the types of interactions or – more importantly – to provide a friendly user interface for the average reserving analyst.

Some concrete suggestions on moving this forward:

1) Identify the types of additional data needed for evaluating pattern changes, such as

    a) Historical policy limit and risk profiles

    b) Historical rate change indices

    c) Inflation and benefit change indices

2) Advance research on models that can look at multiple triangles, potentially down to the individual claims level; focus on practical implementation.

3) Create a library of the form of interactions appropriate for different factors, such as

   a) Calendar year trend is the simplest interaction term as in GLM

   b) Glenn Myers monograph on the speed-up is a good start on settlement patterns.

6. CONCLUSIONS

In this call paper we have seen that there are a number of reasons that development patterns can be different from one accident year to the next. These include calendar year trends, changing settlement patterns, changing case reserve adequacy, changing mixes of business, and others. All of these produce triangles in which the AY and DY dimensions are not independent, but instead show interactions. This violates a basic assumption of the Chain-Ladder method.

We have surveyed several practical methods for addressing these interactions. The methods can be as simple as ignoring portions of the triangle, or adjusting the historical data for known changes. These methods have proven useful to reserving actuaries because they are easy to implement, but also because they are tied to the reasons that patterns are changing, and therefore, help to give a more complete story for the reserve estimate.

However, most of these tools depend upon knowing *a priori* what adjustments need to be made to the data, and then restating the development triangles to current cost levels (or case reserve adequacy, or settlement rate) using a reliable measure of claim counts. Having reliable counts is necessary, but it is the assumption that we know the cause of the changing patterns that is most critical. If multiple changes are happening simultaneously – for example, a change in policy limits as well as a change in case reserve adequacy – then the methods will fail.

The long-term improvement in reserving models points us to the use of more data: including more detailed loss statistics, policy limit profiles, measures of exposure, and external indices such as cost inflation. Statistical modeling is the recommended framework for bringing in additional information.

For those building statistical models, the challenge is to make the models more accessible to the practicing actuary, including the flexibility to allow clear intervention points where the knowledgeable actuary can adjust the intermediate results when needed. Statistical models may be better absorbed by practicing actuaries if they can easily incorporate adjustments such as changes in case reserve adequacy or claim closure rates.

Abbreviations and notations:

AY, accident year (row dimension of triangle)
BF, Bornhuetter-Ferguson method
CL, Chain-Ladder method
CY, calendar year
DY, development year (column dimension of triangle)
GLM, generalized linear models
GLMM, generalized linear mixed models
MIT, Mueller Incremental Tail method
OLD, only the latest diagonal(s) method
TPA, Third Party Administrator

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Appendix I-A

Commercial Multiple Peril

Cumulative Incurred Loss+ALAE

	12	24	36	48	60	72	84	96	108	120
1974–2003					…					
2004	10,062,877	12,019,810	13,120,330	13,800,717	14,048,563	14,228,194	14,343,555	14,483,141	14,548,973	14,631,706
2005	10,807,279	13,426,225	14,338,169	15,034,751	15,342,112	15,537,527	15,597,310	15,688,265	15,790,959	
2006	9,497,881	11,602,993	12,651,289	13,302,004	13,668,841	13,806,668	13,938,202	14,051,374		
2007	10,595,875	12,728,205	13,930,135	14,601,754	14,888,062	15,094,464	15,193,434			
2008	14,050,047	16,969,555	18,106,007	18,766,853	19,210,347	19,408,707				
2009	11,339,648	13,883,191	14,991,666	16,232,739	16,558,188					
2010	12,302,948	14,937,969	16,189,574	16,944,556						
2011	15,602,014	18,354,523	19,759,192							
2012	13,342,603	16,316,017								
2013	11,939,724									

Age-to-Age Loss Development Factors[1]

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
All Year Weighted Avg	1.232	1.100	1.058	1.026	1.017	1.010	1.008	1.006	1.005

Incremental Triangle

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1974–2003					…					
2004	10,062,877	1,956,933	1,100,520	680,387	247,846	179,631	115,361	139,586	65,832	82,733
2005	10,807,279	2,618,946	911,944	696,582	307,361	195,415	59,783	90,955	102,694	
2006	9,497,881	2,105,112	1,048,296	650,715	366,837	137,827	131,534	113,172		
2007	10,595,875	2,132,330	1,201,930	671,619	286,308	206,402	98,970			
2008	14,050,047	2,919,508	1,136,452	660,846	443,494	198,360				
2009	11,339,648	2,543,543	1,108,475	1,241,073	325,449					
2010	12,302,948	2,635,021	1,251,605	754,982						
2011	15,602,014	2,752,509	1,404,669							
2012	13,342,603	2,973,414								
2013	11,939,724									

Age-to-Age Loss Development Factors Anchored to Year 5

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1974–2003					…					
2004					1.000	0.725	0.465	0.563	0.266	0.334
2005					1.000	0.636	0.195	0.296	0.334	
2006					1.000	0.376	0.359	0.309		
2007					1.000	0.721	0.346			
2008					1.000	0.447				
2009					1.000					

0.465 = Incremental Loss in Year 7 (115,361) / Incremental Loss in Year 5 (247,846)

Anchored Loss Development Factors[2]

	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
All Year Weighted Avg	1.000	0.656	0.384	0.314	0.239	0.203

1. The average age-to-age loss development factors were used in Figure 2
2. The anchored age-to-age loss development factors were used in Figure 3

Appendix I-B

Other Liability

Cumulative Incurred Loss+ALAE

	12	24	36	48	60	72	84	96	108	120
1974–2003					…					
2004	4,490,851	6,542,233	8,412,758	9,851,474	10,929,582	11,596,339	11,830,132	12,126,474	12,415,106	12,510,225
2005	4,314,808	6,802,700	8,537,640	10,029,125	10,959,584	11,312,724	11,735,144	12,152,153	12,429,060	
2006	4,442,035	7,156,625	9,360,538	10,885,689	11,757,797	12,400,293	13,002,514	13,236,483		
2007	4,555,135	7,646,174	9,975,072	11,537,284	12,772,506	13,788,485	14,272,563			
2008	4,068,513	7,126,421	9,490,827	11,206,561	12,664,704	13,367,708				
2009	4,096,903	7,034,898	9,119,711	10,963,616	12,109,594					
2010	3,752,463	6,649,357	9,213,646	10,933,500						
2011	3,670,262	6,637,029	9,098,742							
2012	3,571,801	6,543,045								
2013	3,584,497									

Age-to-Age Loss Development Factors[1]

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
All Year Weighted Avg	1.686	1.305	1.167	1.090	1.051	1.035	1.023	1.016	1.012

Incremental Triangle

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1974–2003					…					
2004	4,490,851	2,051,382	1,870,525	1,438,716	1,078,108	666,757	233,793	296,342	288,632	95,119
2005	4,314,808	2,487,892	1,734,940	1,491,485	930,459	353,140	422,420	417,009	276,907	
2006	4,442,035	2,714,590	2,203,913	1,525,151	872,108	642,496	602,221	233,969		
2007	4,555,135	3,091,039	2,328,898	1,562,212	1,235,222	1,015,979	484,078			
2008	4,068,513	3,057,908	2,364,406	1,715,734	1,458,143	703,004				
2009	4,096,903	2,937,995	2,084,813	1,843,905	1,145,978					
2010	3,752,463	2,896,894	2,564,289	1,719,854						
2011	3,670,262	2,966,767	2,461,713							
2012	3,571,801	2,971,244								
2013	3,584,497									

Age-to-Age Loss Development Factors Anchored to Year 5

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1974–2003					…					
2004					1.000	0.618	0.217	0.275	0.268	0.088
2005					1.000	0.380	0.454	0.448	0.298	
2006					1.000	0.737	0.691	0.268		
2007					1.000	0.823	0.392			
2008					1.000	0.482				
2009					1.000					

0.217 = Incremental Loss in Year 7 (233,793) / Incremental Loss in Year 5 (1,078,108)

Anchored Loss Development Factors[2]

	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
All Year Weighted Avg	1.000	0.620	0.468	0.319	0.230	0.178

1. The average age-to-age loss development factors were used in Figure 2
2. The anchored age-to-age loss development factors were used in Figure 3

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