Aggregate Loss Reserve Analysis by Accounting Date

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Abstract
This paper introduces new systematic procedures to estimate aggregate unpaid claims as of the current accounting date. Through the use of examples that introduce concepts in a natural progression, emphasis is placed on the reasonability and practicality of an accounting date reserving framework and its appeal to loss reserving practitioners. The accounting date framework provides a fresh perspective which differs from traditional actuarial reserving methods that typically derive unpaid claim estimates using individual accident year experience. Current accounting date aggregate unpaid claims are directly estimated from the emergence of aggregate claim experience which had been unpaid as of prior accounting dates. Exploration of this accounting date framework leads to techniques that may be understood as accounting date analogues of commonly used accident year reserving methods including the incurred development and Bornhuetter-Ferguson methods.

In addition to revealing visibly apparent aggregate unpaid claim estimates, the structure of appropriate accounting date reserving applications suggests improved accuracy over corresponding accident year development methods.

Keywords: loss reserve; reserving; unpaid claim estimate; IBNR; Bornhuetter-Ferguson; accounting date

1. INTRODUCTION

Basic loss reserving methods typically begin with individual accident year claim experience and develop each accident year to an estimated ultimate value. These estimated ultimate values are reduced by cumulative claim payments as of the current accounting date resulting in an unpaid claim estimate for each accident year as of that date. In applying this procedure, the sum of the individual accident year unpaid claim estimates is understood to be an aggregate unpaid claim estimate as of the current accounting date.

Traditional accident year development methods have several important potential drawbacks:

- They are indirect. Indirectly solving for unpaid claims by estimating ultimate costs and then reducing this estimate by cumulative claim payments to date provides no immediate visible sense of the order-of-magnitude of a reasonable aggregate unpaid claim estimate.

- The aggregate unpaid claim estimate may be unduly volatile. The focus is to obtain unpaid claim estimates by individual accident year rather than directly target an aggregate unpaid claim estimate.

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1 Accident year claim (or loss) is used throughout this paper since it is the most common organization of historical data. Techniques described in this paper are also applicable to data organized in other time intervals including policy year, underwriting year, report year and fiscal year. Similarly, the techniques are applicable to monthly, quarterly and biannual data. Finally, the techniques presented are applicable to dollars, claim counts, ALAE (DCCE), and loss & ALAE combined.
• They are often highly leveraged, especially for long-tailed lines of business. Small changes in historical experience or development factor selection may lead to large changes in unpaid claim estimates. Even when exposures are directly incorporated into accident year development methods (e.g., Cape Cod), the focus remains on estimating individual accident year unpaid claims rather than an aggregate unpaid claim estimate.

This paper approaches reserving techniques from a different perspective by asking the direct question:

How might we estimate aggregate unpaid claims as of the current accounting date from the historical aggregate emergence of claims that were unpaid as of prior accounting dates?

This is addressed by examining properties of the emergence of aggregate unpaid claims under certain common and reasonable actuarial assumptions. We then endeavor to capitalize on these properties to derive estimates of aggregate unpaid claims as of the current accounting date. Exploration of the accounting date framework leads to techniques that may be understood as accounting date analogues of commonly used accident year reserving methods including the incurred development and Bornhuetter-Ferguson methods.

The accounting date techniques presented are relatively easy to apply and allow for direct estimation of aggregate unpaid claims. Since historical loss data is recast such that certain experience for all accident years is combined and the aggregate unpaid claims are estimated from this combined data, statistical volatility is expected to decrease while credibility is expected to increase as compared with traditional accident year development methods. The accounting date representation further provides an observable order-of-magnitude indication of reasonable unpaid claim estimates. Recent research suggests that certain accounting date reserving techniques are particularly consistent with the type of actuarial methodologies that tend to produce relatively accurate unpaid claim estimates in comparison with reserving methods in common use. Section 11 discusses these concepts further.

1.1 Research Context

Other than by separating historical experience into individual accident year components, surprisingly little actuarial literature exists on the subject of directly estimating aggregate unpaid claims as of an accounting date. Saltzmann [16] sought to find an appropriate “yardstick” to measure aggregate loss reserve adequacy. Khury [12] introduces the idea of using “reserve ratios” (i.e., IBNR to premium, IBNR to reported loss, IBNR to paid loss, total reserve to premium, and total reserve to paid loss) as
tools for testing the reasonableness of loss reserves. The current NAIC IRIS Ratio 13 Estimated Current Reserve Deficiency to Policyholder’s Surplus [15 p. 204] includes an estimate of current accounting date aggregate unpaid loss & DCCE based upon the average of developed loss & DCCE reserves to earned premium for the two prior accounting years which is then applied to current accounting date earned premium. However, all these measures are only benchmark tests and are not intended for use in actually setting loss reserves.

1.2 Objective

The purpose of this paper is to set forth a framework and systematic procedures to estimate aggregate unpaid claims as of the current accounting date. Through the use of examples that introduce concepts in a natural progression, emphasis is placed on the reasonability and practicality of this accounting date reserving paradigm and its appeal to loss reserving practitioners. Appropriate use of these accounting date concepts may increase the accuracy of aggregate unpaid claim estimates as well as bring visual clarity to the unpaid claim estimation process.

1.3 Outline

The remainder of this paper presents a framework and describes techniques by which aggregate unpaid claims may be estimated as of the current accounting date:

- Section 2 discusses actuarial assumptions relied upon to apply accounting date techniques.
- Section 3 introduces payment development by accounting date.
- Section 4 discusses incurred development by accounting date.
- Section 5 describes expected unpaid losses.
- Section 6 presents a Bornhuetter-Ferguson method by accounting date.
- Section 7 describes a Cape Cod method by accounting date.
- Section 8 explores the use of alternative exposure measures.
- Section 9 explains the broad applicability of the accounting date framework.
- Section 10 addresses certain implementation challenges.
- Section 11 discusses the major results of this paper.
- Section 12 summarizes the main conclusions of the paper.
2. ACTUARIAL ASSUMPTIONS

A loss reserve analysis usually commences with information gathering and exploration of any trends and changes that may affect the historical database. This guides the loss reserve practitioner in: consideration of the predictive power of applicable actuarial methods; choice of appropriate loss reserving techniques, and; interpretation of results.

As indicated by Berquist and Sherman [2], [10 p. 81], unpaid claim estimation cannot be reduced to a “cookbook” of rules and methods; actuarial judgment is required at many critical junctures to assure that unpaid claim estimates are neither distorted nor biased. Berquist and Sherman identify certain areas where actuarial judgment is required:

— Determining the optimal combination of the kinds of claims data to be used in the estimation of unpaid claims
— Assessing the effect of changes in an insurer’s operations on the claims data that is used in estimating unpaid claims
— Adjusting the claims data for the influences of known and quantifiable events
— Evaluating the strengths and weaknesses of various estimation techniques
— Making the final selection of the unpaid claim estimate(s)

Mindful of the above, accounting date reserving techniques rely upon the following actuarial assumptions:

A1: The requisite claim and exposure experience is available. Techniques presented herein reorganize traditional accident year loss reserving claim and exposure experience into a new framework. Under certain conditions, less common exposure measures may be incorporated into the accounting date reserving paradigm.

A2: Except for noise (i.e., randomness in historical experience), accident year payments subsequent to the first year of development follow the same payment pattern.

A3: When case reserves are used as loss experience then, except for noise, there has been no change in the adequacy of case reserves.

A4: The exposure metric as of each stage of development provides a reasonable measure of the relative accident year exposure to remaining development. The exposure metric should reflect exposure volume including trend. Measurement of absolute exposure is not necessary.
A5: The historical experience is statistically credible.
A6: The historical experience is homogeneous.
A7: The presence or absence of large claims does not distort the historical experience.

While the valuation date and accounting date may not necessarily be equal, the current valuation date is assumed to equal the current accounting date for the purposes of this paper. Actuarial assumptions are denoted throughout this text by the shorthand references (e.g., A4) above.

When actual historical experience does not substantially satisfy certain actuarial assumptions relied upon by a particular technique (e.g., there has been a change in the claims environment), it is often possible to: restate historical experience on another basis; use alternative or supplementary data; or redefine the data to more completely satisfy actuarial assumptions. This is discussed further in Section 10.

The actuary should consider the use of multiple methods or models appropriate to the purpose, nature and scope of the assignment and the characteristics of the claims unless, in the actuary’s professional judgment, reliance upon a single method or model is reasonable given the circumstances. The relative strengths and weaknesses of appropriate actuarial techniques are evaluated in consideration of assignment objectives, the degree to which relevant actuarial assumptions are satisfied and the reasonableness of results.

As with all basic actuarial reserving methods, the methods presented herein provide deterministic single point estimates. Except in the most trivial situations and despite best efforts to satisfy actuarial assumptions, the actual future emergence of current accounting date unpaid claims is inherently uncertain.

3. PAYMENT DEVELOPMENT BY ACCOUNTING DATE

We introduce two payment development examples satisfying A1-A2 and A4-A7. A3 is not relevant since case reserves are not used as loss experience in payment development methods.

3.1 Static Example: No Noise

This first example contains no noise in the historical experience.

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2 Actuarial Standard of Practice No. 43 “Property/Casualty Unpaid Claim Estimates”, Section 3.6.1
3.1.1 Traditional actuarial triangle chain-ladder accident year representation

Exhibit 1, Table 1 displays payment development data in the familiar CL format. Typically, selected age-to-age LDFs are derived as some average of historical LDFs. For each stage of development, the appropriate product of selected LDFs is the selected CDF. In this static example, since LDFs are identical within each age-to-age interval, simple average LDFs and volume weighted LDFs are identical within each development interval. Similarly, simple average CDFs and volume weighted CDFs are equal as of each stage of development.

Exhibit 1, Table 2 displays case reserves by accident year. Since there is no noise in this example, the ratio of case reserves to cumulative loss payments is the same for all accident years as of each stage of development.

3.1.2 Traditional payment development approach

Exhibit 1, Table 3 displays the traditional payment development method used to derive unpaid loss estimates from cumulative loss payments. The product of cumulative loss payments as of the current accounting date and their corresponding CDFs produce Column (4) estimated ultimate losses by accident year. These estimated ultimate losses are then reduced by cumulative loss payments as of the current accounting date resulting in an unpaid loss estimate for each accident year as of the current accounting date. Estimated unpaid losses by accident year are added to produce a total estimate of unpaid losses as of the current accounting date. The sum of individual accident year Column (5) unpaid loss estimates equals the total unpaid loss estimate of $434,721 as of 12/31/12.\(^3\)

3.1.3 Accounting date representation

This paper presents an alternative approach that organizes the historical experience into an accounting date representation. Exhibit 1, Table 4 displays cumulative loss payment emergence by year-end accounting date and may be derived by the appropriate accumulation of cumulative loss payments from Exhibit 1, Table 1.

For example, year-end accounting date 2009 cumulative loss payments as of 12/31/12 (i.e., as of 3 years of emerged loss payments) of $205,714 are defined as loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior or, equivalently, as payments during calendar years 2010 through 2012 on accident years 2009 & prior. This may be derived from Exhibit 1, Table 1 as the

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\(^3\) Unless otherwise specified, tables in the text are displayed in rounded thousands of dollars (i.e., $000 Omitted).
The developed payments of Exhibit 1, Table 4 represent the historical emergence of aggregate losses that were incurred and unpaid as of each year-end accounting date. This representation provides useful information as it tracks the historical loss payment emergence of accounting date unpaid losses as opposed to tracking individual accident year loss payment development from accident year inception. Hence, the goal is to estimate the ultimate value of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 1, Table 4). How might we estimate aggregate unpaid claims as of the current accounting date from the historical aggregate emergence of claims that were unpaid as of prior accounting dates? Despite the absence of noise in this first example, the non-constant LDFs between each development interval resulting from different accident year exposure levels signifies that that an estimate of the bold rectangle value is not readily apparent directly from Exhibit 1, Table 4.

### 3.1.4 Accounting date representation recast at current accounting date exposure level

Exhibit 1, Table 4 year-end accounting date emergence may be recast into a form that is especially useful for estimating unpaid claims as of the current accounting date. The emerged loss payments of Exhibit 1, Table 4 are recast on Exhibit 1, Table 5 at the year-end accounting date 2012 exposure level where the case reserves of Exhibit 1, Table 2 are used as an A4 measure of the relative accident year exposure to remaining payments as of each stage of development.\(^5\) Accordingly, Exhibit 1, Table 5

\[ \begin{align*}
85,700 - 82,993 &= 2,707 \quad \text{accident year 2001 contribution} \\
+ 88,350 - 81,375 &= 6,975 \quad \text{accident year 2002 contribution} \\
+ 95,000 - 82,000 &= 13,000 \quad \text{accident year 2003 contribution} \\
+ 93,840 - 76,500 &= 17,340 \quad \text{accident year 2004 contribution} \\
+ 95,000 - 82,000 &= 13,000 \quad \text{accident year 2003 contribution} \\
+ 93,840 - 76,500 &= 17,340 \quad \text{accident year 2004 contribution} \\
+ 93,840 - 76,500 &= 17,340 \quad \text{accident year 2004 contribution} \\
+ 86,573 - 66,290 &= 20,283 \quad \text{accident year 2005 contribution} \\
+ 85,999 - 59,780 &= 26,219 \quad \text{accident year 2006 contribution} \\
+ 79,444 - 46,607 &= 32,837 \quad \text{accident year 2007 contribution} \\
+ 63,163 - 28,282 &= 34,881 \quad \text{accident year 2008 contribution} \\
+69,857 - 18,383 &= 51,474 \quad \text{accident year 2009 contribution} \\
&= 205,714 \quad \text{Total}\(^4\)
\end{align*} \]

\(^4\) Totals may not add precisely due to rounding

\(^5\) While case reserves may not be a commonly used exposure base for traditional reserving methods that estimate individual accident year ultimate losses, case reserves can be a reasonable A4 accounting date reserving exposure metric. Exceptions would include (a) where zero case reserves at later stages of development do not signify negligible remaining exposure and (b) very long-tailed lines where few claims are reported in the early stages of development. Otherwise, when A1-A7 are satisfied, case reserves would be expected to be a reliable A4 measure of relative accident year exposure to remaining payments at each stage of development. Such case reserves would reflect the relative volume of remaining development exposure between accident years including trend. While A3 should be satisfied to accept case reserves as an
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displays the Exhibit 1, Table 4 emergence recast as if each year-end accounting date had emerged at the current year-end 2012 accounting date exposure level. For example, using the loss payments of Exhibit 1, Table 1, the Exhibit 1, Table 4 year-end 2009 accounting date cumulative emerged loss payments as of 12/31/12 (i.e., after 3 years) of $205,714 is recast as:

\[
\begin{align*}
\text{Year-End Accounting Date} & \quad \text{Year-End Accounting Date} \\
2012 & \quad 2009 \\
\text{Exposure} \quad \text{Exposure} & \\
(2,040/ 1,804) \times (85,700 - 82,993) & = 3,061 \quad \text{accident year 2001 contribution}
+ (3,958/ 3,720) \times (88,350 - 81,375) & = 7,421 \quad \text{accident year 2002 contribution}
+ (6,293/ 6,000) \times (95,000 - 82,000) & = 13,635 \quad \text{accident year 2003 contribution}
+ (9,533/ 9,180) \times (93,840 - 76,500) & = 18,007 \quad \text{accident year 2004 contribution}
+ (10,370/10,883) \times (86,573 - 66,290) & = 19,327 \quad \text{accident year 2005 contribution}
+ (15,932/13,634) \times (85,999 - 59,780) & = 30,638 \quad \text{accident year 2006 contribution}
+ (25,418/18,007) \times (79,444 - 46,607) & = 46,351 \quad \text{accident year 2007 contribution}
+ (31,399/18,855) \times (63,163 - 28,282) & = 58,087 \quad \text{accident year 2008 contribution}
+ (43,173/30,639) \times (69,857 - 18,383) & = 72,531 \quad \text{accident year 2009 contribution}
\end{align*}
\]

269,056 Total

This year-end 2009 accounting date emerged loss payments as of 3 years, recast at the year-end 2012 accounting date exposure level total of $269,056, is displayed in its corresponding position on Exhibit 1, Table 5. Appendix A provides a formula to recast accounting date cumulative loss payment emergence at the current accounting date exposure level.

In order for recast year-end accounting date experience to be useful, we must be able to consistently recast each year-end accounting date through the same stage of development. Ideally, this would be though ultimate development (10 years of accident year development in this example). Section 10 discusses approaches under less than ideal circumstances.

The recast Exhibit 1, Table 5 loss payments emerged by year-end accounting date at the year-end 2012 accounting date exposure level visibly clarifies an appropriate aggregate year-end 2012 accounting date unpaid loss estimate. The recast unpaid claims for each year-end accounting date are seen to inevitably emerge towards an ultimate of $434,721. This is the same figure derived from the traditional payment development method on Exhibit 1, Table 3.

\[
\begin{align*}
\text{A4 exposure metric, A3 is unnecessary to perform payment development accounting date reserving. It is important to recognize that A4 exposure metrics other than case reserves may be appropriate as discussed in Sections 8 and 9.}
\end{align*}
\]
We now make several important observations:

- In contrast to traditional estimates which require an estimated ultimate for each accident year, the central goal under an accounting date representation is to directly target only one quantity, i.e., estimated aggregate unpaid claims incurred as of the current accounting date.
- Where there is no noise in the data and despite variable accident year exposure, development factors remain constant within development interval under the recast accounting date representation.
- In contrast to traditional indirect accident year estimated ultimate approaches, a reasonable unpaid claim estimate is visibly apparent under a year-end accounting date representation appropriately recast at the current accounting date exposure level.
- Where there is no noise, the recast accounting date representation results in the same unpaid claim estimate as traditional development methods.
- Tail factors converge to unity faster under accounting date representations than for corresponding traditional accident year representations.
- Accident year payments during the first calendar year are not reflected in accounting date representations.
- The final diagonal of accounting date representations contains all calendar year activity through the current accounting date on losses incurred as of each prior year-end accounting date that remained unpaid as of each year-end accounting date.
- Especially for longer tailed lines of business, the data volume for accounting date representations tends to grow faster than under corresponding traditional accident year representations.  

3.1.5 Estimation of aggregate unpaid loss

While we may visually observe $434,721 as an obvious unpaid claim estimate as of 12/31/12 for our ‘no noise’ example, this may be formalized mathematically. We can apply development procedures to the emergence of loss payments by accounting year recast at the current accounting date exposure level. The lower portion of Exhibit 1, Table 5 displays LDFs and corresponding CDFs for the recast accounting

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6 Long-tailed lines of business may exhibit little activity for recent accident years as of the current accounting date (e.g., accident year 2011 cumulative loss activity as of 12/31/12 equals 0), but would be expected to exhibit considerably more activity for recent year-end accounting dates as of the current accounting date. Accordingly, especially for long-tailed lines of business, statistical reliability and credibility (A5) would be expected to be enhanced under the recast accounting date representation since accident year activity is aggregated.
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date loss payments. In this static example, since LDFs are identical within each development interval, simple average LDFs and volume weighted LDFs are identical within each development interval. As a result, simple average CDFs and volume weighted CDFs are identical at each development stage.

Exhibit 1, Table 6, Column (4) displays the indicated total emergence of unpaid year-end accounting date losses at year-end 2012 exposure levels using the recast accounting date payment development technique. As expected in this example without noise, the indicated unpaid loss for each prior year-end accounting date at the year-end 2012 accounting date exposure level equals $434,721.

3.1.6 Allocation of aggregate unpaid loss estimate to accident year

Rather than explicitly computing individual accident year unpaid claims as in the traditional payment development method, the accounting date reserving paradigm may be used to allocate the aggregate unpaid loss estimate to accident year by use of a top-down iterative approach that unwinds the exposure adjustment.

Exhibit 1, Table 6, Column (5) displays the indicated unpaid loss as of 12/31/12 at the 2012 year-end accounting date exposure level for each year-end accounting date. Beginning with accident year 2004, the oldest accident year with any remaining unpaid claim liability as of 12/31/12, we know that accident year 2004 is expected to have only one more year of loss payments beyond 12/31/12 (i.e., payments to be made during calendar year 2013). Recasting loss payments emerged at the 2012 year-end accounting date exposure level implies the following equation for accident year 2004:

\[
5,181 = \left(\frac{43,173}{25,500}\right)x(acc.\ yr.\ 2004\ estimated\ payments\ during\ yr.\ 10)
\]

Solving this equation yields:

\[
acc.\ yr.\ 2004\ estimated\ payments\ during\ yr.\ 10 = \left(\frac{25,500}{43,173}\right)x5,181 = \\
acc.\ yr.\ 2004\ est.\ unpaid\ loss\ as\ of\ 12/31/12 = 3,060
\]

Similarly, we have the following equation for accident year 2005:

\[
17,662 = \left(\frac{31,399}{20,400}\right)x(acc.\ yr.\ 2004\ estimated\ payments\ during\ yr.\ 10)
+\left(\frac{43,173}{24,735}\right)x(acc.\ yr.\ 2005\ estimated\ payments\ during\ yrs.\ 9,10)
\]

Using $3,060 as the acc. yr. 2004 estimated payments during yr. 10 and solving this equation results in:

\[
acc.\ yr.\ 2005\ est.\ payments\ during\ years\ 9,10 = \left(\frac{24,735}{43,173}\right)x[17,662- (31,399/20,400)x(3,060)]
\]

\[
acc.\ yr.\ 2005\ est.\ unpaid\ loss\ as\ of\ 12/31/12 = 7,421
\]
This process is continued iteratively to derive unpaid losses as of 12/31/12 for each accident year as displayed on Exhibit 1, Table 6, Column (7). Appendix C provides a formula to allocate the current accounting date aggregate unpaid loss estimate to accident year.

The total of all accident year unpaid claim estimates of the current year end accounting date equals the aggregate unpaid claims estimate. As expected in this ‘no noise’ example, the individual accident year unpaid losses derived in this manner equal the accident year unpaid loss estimates derived on Exhibit 1, Table 3 by using the traditional payment development method.

3.2 Payment Development with Noise

While the previous example without noise is illustrative of concepts, actual historical experience typically presents with significant noise in the historical experience. This section adds noise to the example introduced in Section 3.1.

3.2.1 Traditional actuarial triangle accident year representation

Exhibit 2, Table 1 displays loss payment experience in CL format. Since noise has been introduced, LDFs no longer remain constant within each development interval. Since interval LDFs are not constant, volume weighted average CDFs are not necessarily equal to unweighted simple average CDFs. Exhibit 2, Table 2 displays case reserves by accident year with noise added.

3.2.2 Accounting date representation

Exhibit 2, Table 3 displays the cumulative emergence of loss payments by year-end accounting date and may be derived by the appropriate accumulation of cumulative loss payments from Exhibit 2, Table 1 as described in Section 3.1.3. This tracks the historical emergence of accounting date unpaid losses and the goal is, once again, to estimate the ultimate value of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 2, Table 3).

3.2.3 Accounting date representation recast at current accounting date exposure level

Following procedures described in Section 3.1.4, the emerged loss payments of Exhibit 2, Table 3 are recast on Exhibit 2, Table 4 at the year-end accounting date 2012 exposure level where case reserves of Exhibit 2, Table 2 are used as an A4 measure of the relative accident year exposure to remaining payments as of each stage of development. By recasting all loss payment emergence at the 2012 year-end accounting date exposure level, LDFs within each development interval are now on a comparable basis. Weighted LDFs are weighted on the pre-recast actual loss experience of Exhibit 2, Table 3 to preserve
the weighting of actual experience.\(^7\)

Recasting the loss payments emerged as displayed on Exhibit 2, Table 4 provides an observable order-of-magnitude aggregate year-end 2012 current accounting date unpaid claim estimate. The recast unpaid claims for each recast year-end accounting date are observed to be emerging towards an ultimate somewhere in the low-to-mid four-hundred million dollar range.

### 3.2.4 Estimation of aggregate unpaid loss

While we may observe an order-of-magnitude unpaid claim estimate as of 12/31/12, we can apply our formal development procedure to the emergence of loss payments by accounting year recast at the current accounting date exposure level.\(^8\)

Exhibit 2, Table 5, Column (4) displays the indicated total emergence of unpaid year-end accounting date losses at the current 2012 year-end accounting date exposure level. While each figure in Column (4) provides an estimate of unpaid losses as of 12/31/12,\(^9\) the most recent estimate of $433,929 is accepted as the payment development accounting date unpaid loss estimate as of 12/31/12.

### 3.2.5 Allocation of aggregate unpaid loss estimate to accident year

Exhibit 2, Table 5, Column (7) allocates the $433,929 aggregate estimated unpaid loss as of 12/31/12 to accident year using the iterative procedure described in Section 3.1.6.

### 4. INCURRED (REPORTED) DEVELOPMENT BY ACCOUNTING DATE

This section presents the incurred (reported\(^10\)) loss counterpart to the payment development discussion presented in the Section 3. We introduce two incurred development examples satisfying A1-A7.

#### 4.1 Static Example: No Noise

##### 4.1.1 Traditional actuarial triangle chain-ladder accident year representation

Exhibit 3, Table 1 displays reported losses in the familiar CL format. In this static example, since

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\(^7\) Friedland’s [10] Chapter 7 – Development Technique “Mechanics of the Development Technique” discussion beginning p. 85 is written in a traditional accident year development context. Her discussion may be adapted to accounting date development techniques.

\(^8\) Friedland’s [10] Chapter 7 – Development Technique ‘When the Development Technique Works and When it Does Not’ discussion beginning p. 95 is written in a traditional accident year development context. Her discussion may be adapted to accounting date development techniques.

\(^9\) Section 7 revisits this important point.

\(^10\) Reported losses equal cumulative loss payments plus case reserves.
LDFs are identical within each age-to-age interval, volume weighted LDFs are identical to simple average LDFs within each development interval and volume weighted CDFs are identical to simple average CDFs as of each stage of development.

4.1.2 Traditional incurred development approach

Exhibit 3, Table 2 displays the traditional accident year incurred development method used to derive unpaid loss estimates from reported losses. The product of reported losses as of the current accounting date and their corresponding CDFs produce Column (4) estimated ultimate losses by accident year. Estimated ultimate losses are then reduced by reported losses as of the current accounting date resulting in Column (5) IBNR estimates for each accident year as of the current accounting date. These accident year IBNR estimates are added to Column (6) current accounting date case reserves resulting in a Column (7) unpaid loss estimate for each accident year.\(^{11}\) Estimated unpaid losses by accident year are added to produce a total estimate of unpaid losses as of the current accounting date. The sum of individual accident year Column (7) unpaid loss estimates equals the total unpaid loss estimate of $434,721 as of 12/31/12. Since there is no noise, this total unpaid loss estimate is identical to the traditional payment development estimate derived in Section 3.1.2.

4.1.3 Accounting date representation

As with cumulative payments, our alternative approach organizes reported loss experience into an accounting date representation. Exhibit 3, Table 3 displays the cumulative reported losses emerged by year-end accounting date and may be derived as the sum of cumulative loss payments emerged by year-end accounting date of Exhibit 1, Table 4 and the appropriate accumulation of case reserves from Exhibit 1, Table 2.

For example, the year-end accounting date 2009 reported losses as of 12/31/12 (i.e., as of 3 years of reported loss emergence) of $253,840 are defined as loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior plus case reserves as of 12/31/12 on accident years 2009 & prior. Equivalently, this may be defined as loss payments during calendar years 2010 through 2012 on accident years 2009 & prior plus case reserves as of 12/31/12 on accident years 2009 & prior. The loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior equal $205,714 from Exhibit 1, Table 4. Case reserves as of 12/31/12 on accident years 2009 & prior of $48,126 equal

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\(^{11}\) This derivation of unpaid loss estimates by accident year is equivalent to solving for unpaid loss estimates as Column (4) accident year estimated ultimate losses less cumulative loss payments as of the current accounting date.
the sum of appropriate accident year contributions from Exhibit 1, Table 2:

\[
\begin{align*}
2,040 & \text{ accident year 2004 contribution} \\
+ & 3,958 \text{ accident year 2005 contribution} \\
+ & 6,293 \text{ accident year 2006 contribution} \\
+ & 9,533 \text{ accident year 2007 contribution} \\
+ & 10,370 \text{ accident year 2008 contribution} \\
+ & 15,932 \text{ accident year 2009 contribution} \\
+ 48,126 & \text{ Total}
\end{align*}
\]

The sum of these two components, $205,714 + $48,126, equals the $253,840 year-end accounting date 2009 reported losses emerged as of 12/31/12.

Exhibit 3, Table 3 tracks historical reported loss emergence of accounting date unpaid losses as opposed to tracking individual accident year reported loss development from accident year inception. It is important to observe that exhibits displaying cumulative reported losses emerged by accounting date display one additional diagonal (as of 0 years) for each accounting date compared with exhibits that display the corresponding cumulative loss payments emerged. In particular, the 2012 current accounting date contains an entry as of 0 years (i.e., as of 12/31/12) that equals the aggregate case reserves as of the current year-end accounting date. Our goal is to estimate the ultimate value of unpaid losses as of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 3, Table 3). As with Exhibit 1, Table 4, an estimate of the bold rectangle value is not readily apparent directly from Exhibit 3, Table 3.

4.1.4 Accounting date representation recast at current accounting date exposure level

Exhibit 3, Table 3 accounting year reported loss emergence may be recast into a form that is especially useful for unpaid claim estimation. Exhibit 3, Table 4 displays the recast cumulative reported losses emerged by year-end accounting date at the current accounting date exposure level and may be derived as the sum of recast cumulative loss payments emerged by year-end accounting date of Exhibit 1, Table 5 and the appropriate accumulation of recast case reserves of Exhibit 1, Table 2.

The emerged reported losses of Exhibit 3, Table 3 are recast on Exhibit 3, Table 4 at the 2012 year-end accounting date exposure level where the case reserves of Exhibit 1, Table 2 are used as an A4 measure of the relative accident year exposure to remaining reported losses (IBNR) as of each stage of development.

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12 Since there can be no emerged payments as of 0 years, reported emerged as of 0 years = case reserves as of 0 years.
For example, the emerged reported losses of accounting year-end 2009 as of 12/31/12 (i.e., after 3 years) from Exhibit 3, Table 3 of $253,840 is recast on Exhibit 3, Table 4 as $335,474. This is derived as the recast loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior equal to $269,056 from Exhibit 1, Table 5 plus recast case reserves as of 12/31/12 on accident years 2009 & prior of $66,418. The $66,418 of recast case reserves equals the sum of appropriate recast accident year contributions from Exhibit 1, Table 2, computed as:

\[
\begin{array}{lll}
\text{(Year-End Accounting Date)} & \text{(Year-End Accounting Date)} & \text{Amount} \\
2012 & 2009 & \\
\frac{9,533}{9,180} \times 2,040 & = & 2,118 \quad \text{accident year 2004 contribution} \\
+ \frac{10,370}{10,883} \times 3,958 & = & 3,771 \quad \text{accident year 2005 contribution} \\
+ \frac{15,932}{13,634} \times 6,293 & = & 7,354 \quad \text{accident year 2006 contribution} \\
+ \frac{25,418}{18,007} \times 9,533 & = & 13,456 \quad \text{accident year 2007 contribution} \\
+ \frac{31,399}{18,855} \times 10,370 & = & 17,269 \quad \text{accident year 2008 contribution} \\
+ \frac{43,173}{30,639} \times 15,932 & = & 22,450 \quad \text{accident year 2009 contribution} \\
\hline
\end{array}
\]

The year-end 2009 accounting date emerged reported losses as of 3 years, recast at the year-end 2012 accounting date exposure level of $335,474, is displayed in its corresponding position on Exhibit 3, Table 4. Appendix B provides a formula to recast accounting date reported loss emergence at the current accounting date exposure level.

It is important to observe that the recast year-end accounting date 2012 emerged reported losses of $148,116 displayed on Exhibit 3, Table 4 equals the pre-recast amount displayed on Exhibit 3, Table 3. This must always be true because the aggregate year-end accounting date 2012 case reserves recast at the 2012 year-end exposure level, by definition, equals the pre-recast aggregate year-end 2012 case reserves.

The recast Exhibit 3, Table 4 reported losses emerged by year-end accounting date at the year-end 2012 accounting date exposure level visibly clarifies an appropriate aggregate year-end 2012 accounting date unpaid loss estimate. The recast unpaid claims for each year-end accounting date are seen to inevitably emerge towards an ultimate of $434,721. This is the same figure derived from the traditional incurred development method of Exhibit 3, Table 2 as well as the ‘no noise’ payment development indication of Exhibit 1, Table 3 and the recast accounting date payment indication of Exhibit 1, Table 5.
The bullet point observations at the conclusion of Section 3.1.4 also apply to accounting date emerged reported loss representations. There are two additional observations for emerged reported losses under an accounting date representation:

- Accounting date reported emergence of unpaid claims converges to ultimate faster than accounting date payment emergence.
- Exhibits displaying cumulative reported losses emerged by year-end accounting date display one additional diagonal (as of 0 years) for each accounting date as compared with exhibits displaying the corresponding cumulative emerged loss payments. In particular, the current recast year-end accounting date contains an entry as of 0 years that equals total current year-end accounting date case reserves.

### 4.1.5 Estimation of aggregate unpaid loss

While we may observe $434,721 as an obvious unpaid claim estimate as of 12/31/12 for our ‘no noise’ example, this can be formalized using development factors.

Exhibit 3, Table 5, Column (4) displays the indicated total reported emergence of unpaid year-end accounting date losses at the 2012 year-end accounting date exposure level. As expected in this example with no noise, the indicated unpaid loss for each year-end accounting date at the 2012 year-end accounting date exposure level equals $434,721.

### 4.1.6 Allocation of aggregate unpaid loss estimate to accident year

As with loss payments, the emerged reported loss accounting date paradigm may be used to allocate the aggregate unpaid loss estimate to accident year by use of a top-down iterative approach that unwinds the exposure adjustment.

Exhibit 3, Table 5, Column (5) displays the indicated IBNR as of 12/31/12 at the 2012 year-end accounting date exposure level for each year-end accounting date. Beginning with accident year 2004, the oldest accident year with any remaining unreported losses as of 12/31/12, we know that accident year 2004 is expected to have only one more year of loss reportings beyond 12/31/12 (i.e., reportings to be made during calendar year 2013). Recasting reported losses emerged at the 2012 year-end accounting date exposure level implies the following equation for accident year 2004:

\[
1,727 = \frac{43,173}{25,500} \times \text{est. acc. yr. 2004 estimated reportings during yr. 10}
\]
Solving this equation yields:
acc. yr. 2004 estimated reportings during yr. 10 = (25,500/43,173)x$1,727 =
acc. yr. 2004 estimated IBNR as of 12/31/12 = $1,020

Similarly, we have the following equation for accident year 2005:

$7,614 = (31,399/20,400)x(acc. yr. 2004 estimated reportings during yr. 10)
+ (43,173/24,735)x(acc. yr. 2005 estimated reportings during yrs. 9,10)

Using $1,020 as the acc. yr. 2004 estimated reportings during yr. 10 and solving this equation results in:
acc. yr. 2005 est. reportings during years 9,10 = (24,735/43,173)x[$7,614 –(31,399/20,400)x($1,020)]
acc. yr. 2005 estimated IBNR as of 12/31/12 = $3,463

This process is continued iteratively to derive IBNR estimates as of 12/31/12 for each accident year as displayed on Exhibit 3, Table 5, Column (7). These IBNR estimates are added to the Column (8) case reserves as of 12/31/12 resulting in the Column (9) accident year unpaid loss estimates as of 12/31/12. Appendix C provides a formula to allocate the current accounting date aggregate IBNR estimate to accident year.

The total of all accident year unpaid claim estimates of the current year end accounting date equals the aggregate unpaid claims estimate. As expected in this ‘no noise’ example, the individual accident year unpaid losses derived in this manner equal the accident year unpaid loss estimates derived on Exhibit 3, Table 2 by using the traditional incurred development method.

4.2 Incurred Development with Noise

This section adds noise to the example introduced in Section 4.1.

4.2.1 Traditional actuarial triangle accident year representation

Exhibit 4, Table 1 displays reported losses in the traditional CL format derived as the sum of Exhibit 2, Table 1 and Exhibit 2, Table 2. Since noise has been introduced, LDFs no longer remain constant within each development interval. Since interval LDFs are not constant, volume weighted average CDFs do not necessarily equal the unweighted simple average CDFs.

4.2.2 Accounting date representation

Exhibit 4, Table 2 displays the cumulative reported losses emerged by year-end accounting date and may be derived as the sum of cumulative loss payments emerged by year-end accounting date of Exhibit
2, Table 3 and the appropriate accumulation of case reserves from Exhibit 2, Table 2 as described in Section 4.1.3. This tracks the historical reported emergence of accounting date unpaid losses and our goal is, once again, to estimate the ultimate value of unpaid losses as of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 4, Table 2).

4.2.3 Accounting date representation recast at current accounting date exposure level

Following procedures described in Section 4.1.4, reported losses emerged of Exhibit 4, Table 2 are recast on Exhibit 4, Table 3 at the year-end 2012 accounting date exposure level where case reserves of Exhibit 2, Table 2 are used as an A4 measure of the relative accident year exposure to remaining reported losses (IBNR) as of each stage of development. By recasting all reported loss emergence at the 2012 year-end accounting date exposure level, LDFs within each development interval are now on a comparable basis. Weighted LDFs are weighted on the pre-recast actual loss experience of Exhibit 4, Table 2 to preserve the weighting of actual experience.13

It is again important to observe that recast year-end accounting date 2012 emerged reported losses of $148,006 displayed on Exhibit 4, Table 3 equals the pre-recast amount displayed on Exhibit 4, Table 2. While this relationship must be true, the fact that each prior recast year-end accounting date emerged reported loss at 0 years also equals $148,006 is only true, in this instance, because accident year case reserves are used as the A4 exposure metric. Examples using different exposure metrics, presented in subsequent sections, help clarify this point.

Recasting the reported losses emerged as on Exhibit 4, Table 3 provides an observable order-of-magnitude aggregate year-end 2012 current accounting date unpaid claim estimate. It is visually apparent that the recast unpaid claims for each year-end accounting date are emerging towards an ultimate somewhere in the low-to-mid four-hundred million dollar range.

4.2.4 Estimation of aggregate unpaid loss

While we may observe an order-of-magnitude unpaid claim estimate as of 12/31/12, we can apply our formal development treatment to the emergence of reported losses by accounting year recast at the current accounting date exposure level.14

The recast accounting date representation results in a CDF which is appropriate to develop the

13 Footnote 7 applies.
14 Footnote 8 applies.
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current accounting date total case reserves to ultimate. Exhibit 4, Table 4, Column (4) displays the indicated total emergence of unpaid year-end accounting date losses at the current 2012 year-end accounting date exposure level. While each figure in Column (4) provides an estimate of unpaid losses as of 12/31/12, the most recent estimate is the only one that incorporates the entire actual available 2012 year-end accounting date experience (i.e., the aggregate case reserves as of 12/31/12). As such, the most recent estimate of $437,699 (= 148,006 x 2.957307) is accepted as the incurred development accounting date unpaid claim estimate.

While accident year case outstanding reserving methods appear in the actuarial literature [1], [8], [13], [20], the procedure described above is seen to reduce the current accounting date incurred development unpaid claim estimate to a particularly parsimonious formulation:

\[
\text{Aggregate Unpaid Claim Estimate} = \text{Aggregate Case Reserves} \times CDF
\]

4.2.5 Allocation of aggregate unpaid loss estimate to accident year

Exhibit 4, Table 4, Column (9) allocates the $437,699 aggregate estimated unpaid loss as of 12/31/12 to accident year using the iterative Column (7) IBNR procedure described in Section 4.1.6.

5. EXPECTED UNPAID LOSSES

The key assumption of the traditional accident year expected loss technique is that the actuary can better estimate total unpaid claims based on an *a priori* (or initial) estimate than from claims experience observed to date. In certain circumstances, claims experience reported to date may provide little information about ultimate claims (e.g., assumptions A1-A7 are not generally well satisfied) especially when compared to the *a priori* estimate.\(^{15}\)

To be compatible with our accounting date paradigm, expected loss by accident year is reframed as aggregate expected unpaid loss as of the current accounting date.\(^{16}\) Continuing with our Section 4.2 example, comparable industry experience\(^{17}\) is used to derive expected unpaid losses as of the current year-end accounting date. The critical assumption in this calculation is that the industry loss reserve to earned premium ratio by accident year as of the current accounting date is appropriate for the particular

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\(^{15}\) Adapted from Friedland’s [10] Chapter 8 – Expected Claims Technique p. 131.

\(^{16}\) Friedland’s [10] Chapter 8 – Expected Claims Technique is written in a traditional accident year ultimate context. Her Chapter 8 discussion of expected claims may be generally adapted to accounting date expected unpaid claims.

\(^{17}\) This ‘comparable industry experience’ is artificially constructed for illustrative purposes only and does not represent actual industry experience.
insurer under review. Exhibit 5 displays an example of this calculation which results in a Column (6) expected unpaid loss of $432,407 as of 12/31/12.

6. BORNHUETTER-FERGUSON BY ACCOUNTING DATE

The traditional Bornhuetter-Ferguson method is essentially a blend of development and expected loss techniques by accident year. The Exhibit 5, Column (9) aggregate unpaid loss estimate of $434,197 \[= 148,006 + (1 - 1/2.957307) \times (432,407)\] as of accounting date 12/31/12 is the result of an accounting date analogue to the traditional Bornhuetter-Ferguson method. As a hybrid of development and expected unpaid losses, the Bornhuetter-Ferguson by accounting date technique may be particularly suitable when assumptions A1-A7 are partially satisfied.

The accounting date analogue of the traditional Bornhuetter-Ferguson method is seen to reduce to a concise formulation:

\[
\text{Aggregate Unpaid Claim Estimate} = \text{Aggregate Case Reserves} + (1 - 1/\text{CDF}) \times \text{Aggregate Expected Unpaid Losses}
\]

Column (12) displays an accident year allocation of the aggregate $434,197 unpaid claim estimate.

Application of the Bornhuetter-Ferguson method by accounting date is ill-advised where the Column (8) CDF is below unity. Caution is advised if any Column (10) implied IBNR is negative.

7. CAPE COD BY ACCOUNTING DATE

The traditional Cape Cod method is a Bornhuetter-Ferguson accident year ultimate calculation where expected losses are obtained from reported loss experience instead of an independent, and often judgmental, selection. While we have previously observed relative consistency in the emergence of each recast accounting date at the current accounting date exposure level, the Cape Cod by accounting date technique explicitly reflects this important feature. Exhibit 6 displays a Cape Cod by accounting date technique applied to our example resulting in a Column (7) aggregate unpaid loss estimate of $437,867 as of accounting date 12/31/12. Column (12) displays an accident year allocation of the aggregate $437,867 unpaid loss estimate.

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18 Friedland’s [10] Chapter 9 – Bornhuetter-Ferguson Technique is written in a traditional accident year ultimate context. Her Chapter 9 discussion may be adapted to the Bornhuetter-Ferguson method by accounting date.

19 Adapted from Friedland [10] Chapter 10 – Cape Cod Technique p. 174.

20 Friedland’s [10] Chapter 10 – Cape Cod Technique is written in a traditional accident year ultimate context. Her Chapter 10 discussion may be adapted to the Cape Cod method by accounting date.
Application of the Cape Cod method by accounting date is ill-advised when the Column (3) CDF for the current year-end accounting date is below unity. Caution is advised if any Column (10) IBNR is negative.

8. EXPOSURE MEASURES

As indicated in Section 2, the exposure metric as of each stage of development is intended to provide a reasonable measure of the relative accident year exposure to remaining development. In order to properly apply the accounting date paradigm, it is important that the exposure metric reflects volume and total frequency and severity trend or, if necessary, be adjusted to reflect volume and total trend. Several alternative exposure metrics may be reasonable, as follows:

8.1 Case Reserves

Case reserves have been used as the exposure metric for examples presented in previous sections. Footnote 5 outlines situations under which case reserves may serve as a reasonable exposure measure.

8.2. Earned Premium

Earned premium is a commonly used exposure metric. Ideally, earned premium (or more precisely, the pure premium portion of earned premium) would be brought to the same premium adequacy level\(^\text{21}\) to more accurately measure relative exposure. Exhibit 7, Table 1 displays an example of (independently derived) earned premium at the same adequacy level for each accident year. As indicated by this exhibit, earned premium is insensitive to actual emerged experience since it is remains unchanged at each stage of development.

Using earned premium at the same adequacy level as the A4 exposure metric, Exhibit 7, Table 2 and Exhibit 7, Table 3 display techniques described in Sections 3.2.3-3.2.5 to derive unpaid claim estimates based upon loss payments emerged by year-end accounting date.

Using earned premium at the same adequacy level as the A4 exposure measure, Exhibit 8, Table 1 and Exhibit 8, Table 2 display techniques described in Sections 4.2.3-4.2.5 to derive unpaid claim estimates based upon reported losses emerged by year-end accounting date. Note that, unlike Exhibit 4, Table 3 where case reserves are used as the A4 exposure measure, only the Exhibit 8, Table 1 recast year-end accounting date 2012 reported losses as of 0 years equals actual aggregate case reserves as of

\(^{21}\) An example of the 'same premium adequacy level' would be where all earned premium is 7% inadequate. Under the assumption that all earned premium is at the same premium adequacy level, it would be appropriate to use actual (unadjusted) earned premium as the exposure measure. Used here, 'same premium adequacy level' is not to be interpreted as actual earned premium for each accident year should be brought to a common (e.g., current) rate level.
8.3 Claim Counts; Averages and Counts (Frequency/Severity)

Claim counts are a rich source of exposure metrics. Use of claim counts as an exposure metric allows the practitioner to incorporate and estimate average cost per claim. Claim count exposures provide a means to derive an accounting date analogue to traditional averages and counts (frequency/severity) methods. While claim counts already reflect frequency trend, they need to be adjusted to additionally reflect any severity trend. As an example, Exhibit 9, Table 1 displays (independently derived) projected remaining claim counts to be closed with payment where we are confident these are reasonable estimates. These exposures are sensitive to actual emerged experience but need to be adjusted to reflect severity trend. Although a suitable severity trend index would be appropriate, Exhibit 9, Table 2 restates the Table 1 claim count exposure assuming a constant 5% annual severity trend.

Using the trend adjusted claim count exposure metric, Exhibit 9, Table 3 and Exhibit 9, Table 4 display techniques described in Sections 3.2.3-3.2.5 to derive unpaid claim estimates based upon loss payments emerged by year-end accounting date. Exhibit 9, Table 4, Column (9) displays estimated unpaid average cost per claim projected to be closed with payment.

Using the trend adjusted claim count exposure metric, Exhibit 10, Table 1 and Exhibit 10, Table 2 display techniques described in Sections 4.2.3-4.2.5 to derive unpaid claim estimates based upon reported losses emerged by year-end accounting date. Exhibit 10, Table 2, Column (11) displays estimated unpaid average cost per claim projected to be closed with payment.22

8.4 Other Exposure Measures

Freidland [10 p. 35, 132] extends the list of potential exposure measures to include: payroll, number of vehicles, etc. for particular coverages. The Struzzieri and Hussain [19] ‘Best Exposure Base’ section adds base class equivalent exposures and contains other valuable exposure discussion. Several of these other exposure measures may require trend adjustments.

Section 9 expands the meaning of “exposures” in different contexts to include exposure metrics beyond those discussed in this section.

22 Friedland’s [10] Chapter 11 – Frequency-Severity Techniques is written in a traditional accident year ultimate context. Her Chapter 11 discussion of frequency/severity techniques may be generally adapted to accounting date averages and counts methods.
9. BROAD APPLICABILITY

We have narrowly referred to the quantity being estimated by development methods as “losses” (or “claims”) and the exposure base as “exposures”. However, the accounting date paradigm has much broader application. Accounting date techniques described herein are useful any time we make a development-based projection where the ratio of remaining accident year “losses” to “exposures” is expected to be equal at each stage of development. For example, if we are estimating unpaid DCCE where we expect a constant ratio of accident year unpaid DCCE to unpaid loss at each stage of development, then unpaid “losses” are unpaid DCCE and “exposures” could be estimated unpaid losses when we are confident we have reasonable estimates of unpaid losses.23

10. ACCOUNTING DATE IMPLEMENTATION CHALLENGES

As previously indicated, factors to consider in an unpaid claim analysis require professional actuarial judgment.24 This section briefly addresses several accounting date implementation challenges requiring actuarial judgment.

10.1 Data Availability

For all but relatively fast developing lines of business, it is optimal to have accident year experience available for older accident years as well as several years of calendar year activity (e.g., Exhibit 1, Table 1 upper right corner experience and 10 calendar year diagonals). If this experience were not readily available, one could: (1) obtain compatible supplementary (e.g., industry, prior insurer, competitor) experience where the exposure measure is consistent with available experience; (2) perform the accounting date representation though a common (though incomplete) stage of development and estimate tail development factors; and/or (3) create pseudo-data based upon available experience. These three approaches may also be useful in situations where some available experience is relatively old and deemed unrepresentative of future development.

10.2 Supplementary Experience

As indicated in Section 10.1, supplementary experience may permit completion of accounting date representations through a further stage of development than would otherwise be possible.

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23 This entire Section is derived from Gluck [11] p. 505-6 who also provides additional examples where we may apply this general principle.
24 These factors are outlined in Actuarial Standard of Practice No. 43 “Property/Casualty Unpaid Claim Estimates”, especially Section 3.6.
Supplementary data may also be used to increase the A5 credibility of experience. The use of supplementary experience should be carefully weighed and balanced with the consideration of the use of tail development factors and pseudo-data.

10.3 Tail Development Factors

At comparable late stages of development, recast accounting date CDFs typically converge to unity more quickly than for traditional accident year reserving methods. However, additional historical data is often necessary to attain this quicker convergence. The actuary should consider the trade-offs and interplay between faster convergence, reliance on supplementary experience and the use of pseudo-data. When we perform accounting date representations through a late (but incomplete) common stage of development, we may capitalize on faster convergence and estimate tail development by adapting accident year tail factor procedures discussed in the actuarial literature.\(^{25}\) When A1-A7 are satisfied, all other things being equal, faster CDF convergence implies accounting date tail development factors with less leverage and less uncertainty than for traditional accident year reserving methods.

10.4 Pseudo-Data

In addition to increasing A5 credibility, pseudo-data may also permit completion of accounting date representations through a further stage of development than would otherwise be possible. For example, if accident year 2002 & prior experience were unavailable on Exhibit 2, Table 1, then we would be unable to create Exhibit 2, Table 3 with as many year-end accounting dates and through 9 years of development. However, we could create pseudo-data to substitute for the missing experience. On the theory that accident year 2003 is the most recent fully developed accident year, a simple approach would be to use accident year 2003 experience to serve as the missing experience. A more nuanced approach would consider all accident year 2003 & subsequent experience in the creation of pseudo-data. As with previously discussed data availability tools, the actuary should consider the impact of pseudo-data and its interaction with supplementary data and tail development factors.

10.5 Actuarial Consistency Assumptions Initially Unsatisfied

Assumptions A1-A7 should be satisfied to make optimal use of accounting date reserving methods. When assumptions A1-A7 are satisfied, the noise that remains is expected to be reduced and credibility

\(^{25}\) Friedland’s [10] Chapter 7 - Development Technique “Step 5 - Select Tail Factor” is written in a traditional accident year context. Her discussion may be adapted to an accounting date framework.
increased by aggregating all accident years.\textsuperscript{26} When assumptions A1-A7 are not initially satisfied, it may be appropriate to pre-process the data using approaches described by Berquist and Sherman \textsuperscript{2} that address situations where an insurer’s historical experience has been inconsistent as a result of changes in operations and procedures.\textsuperscript{27}

\section*{11. SUMMARY RESULTS AND DISCUSSION}

This paper introduces the accounting date reserving paradigm. The general principle is always the same: recast the aggregate emergence of unpaid claims of prior year-end accounting dates at the current accounting date exposure level; use this recast emergence as basis to estimate the current accounting date aggregate unpaid claims; and, if necessary, allocate the aggregate unpaid claim estimate to accident year using an iterative top-down procedure.

\subsection*{11.1 Accounting Date Analogues to Basic Reserving Methods}

The new reserving techniques presented are seen to be accounting date analogues to basic reserving methods including:

- Payment Development
- Incurred Development
- Bornhuetter-Ferguson
- Cape Cod
- Averages & Counts (Frequency/Severity)

\subsection*{11.2 Characteristics of Accounting Date Reserving Paradigm}

As discussed, highlights of the accounting date paradigm are:

- In contrast to traditional estimates which require an estimated ultimate for each accident year, the central goal under the accounting date representation is to directly target only one quantity, i.e., the aggregate estimate of unpaid claims incurred as of the current accounting date.
- In contrast to traditional indirect accident year estimated ultimate approaches, a reasonable unpaid claim estimate is visibly apparent under a year-end accounting date representation appropriately recast at the current accounting date exposure level.

\textsuperscript{26} As a consequence of The Law of Large Numbers
\textsuperscript{27} Friedland’s \textsuperscript{10} Chapter 13 - Berquist-Sherman Techniques provides a summary. Fleming and Mayer \textsuperscript{7} also address an aspect of this issue.
• Tail factors converge to unity faster in the accounting date representation than in the traditional accident year representation.

• Accident year payments during first year calendar year are not reflected in the accounting date representation.

• The final diagonal of the accounting date representation contains all calendar year activity through the current accounting date on losses incurred as of the year-end accounting date that had remained unpaid as of that accounting date.

• Especially for longer tailed lines of business, the data volume for the accounting date representation tends to grow faster than under the traditional accident year representation.

• Accounting date reported emergence of unpaid claims converges to ultimate faster than accounting date payment emergence.

• Exhibits displaying cumulative reported losses emerged by year-end accounting date display one additional diagonal (as of 0 years) for each accounting date as compared with exhibits displaying the corresponding cumulative emerged loss payments. In particular, the current recast year-end accounting date contains an entry as of 0 years that equals total current year-end accounting date case reserves.

• When appropriate assumptions are satisfied, the accounting date reserving paradigm is associated with improved accuracy over traditional accident year reserving methods as further discussed below.

11.3 Accounting Date Paradigm Consistent with Improved Accuracy

When assumptions A1-A7 are satisfied, two powerful forces imply improved accuracy of the accounting date reserving paradigm over traditional accident year reserving methods: forward-looking and aggregation. 28

11.3.1 Forward-looking

The recent Forray [8], [9] empirical studies “…suggest that there are many more valuable methods for reserve analysis beyond the [accident year] incurred- and paid-chain-ladder methods and that the paid chain ladder, in particular, should not receive the weight it often does.” 29 Forray’s analysis found

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28 When assumptions are insufficiently satisfied and absent appropriate adjustments, these forces may serve to leverage distinctive individual accident year attributes and distort the resulting unpaid claim estimate.

29 Forray goes on to note: “Of course, this is a general observation, and a particular company’s circumstances always
Aggregate Loss Reserve Analysis by Accounting Date

that the best-performing reserving methods “…were observed to satisfy the following two criteria: 1. each relies at least in part on case reserves (‘Criteria 1’); and “2. amounts paid to date do not directly influence the indicated unpaid loss (‘Criteria 2’).” Despite the inclination to place more reliance on paid loss triangle experience (“real money changing hands, less vulnerable to changes in case reserving practices, etc.”), Meyers [14] has also recently observed instances of superior empirical results using reported loss experience.

While all accounting date reserving methods incorporate forward-looking A4 exposure measures, accounting date incurred methods also rely upon forward-looking A3 case reserves.

11.3.2 Aggregation

When assumptions A1-A7 are satisfied, the noise that remains is expected to be reduced and credibility increased as a result of aggregating accident years.

11.3.3 Excellent candidates for improved accuracy – accounting date incurred methods

The Section 4 accounting date incurred development method (i.e., aggregate case reserves x CDF): essentially relies on forward-looking case reserves (Criteria 1) in conjunction with a forward-looking exposure adjusted CDF; and uses limited amounts of paid to date (to estimate CDF) which do not directly influence the indicated unpaid loss (Criteria 2). Furthermore, when assumptions A1-A7 are satisfied, the accounting date incurred development method capitalizes on the aggregation of accident years which would be expected to result in reduced volatility and commensurate increased credibility. As such, all accounting date incurred methods³⁰ are excellent candidates to be relatively more accurate performing methods as compared with reserving methods in common use.

11.4 Areas for Future Research

Future areas of research include:

1. Compare accounting date reserving methods with traditional actuarial reserving methods using relative “method skill” measures [8], [9] as well as other performance analytics. Empirically test the hypothesis that incurred development accounting date methods produce relatively more accurate aggregate unpaid claim estimates than analogous accident year methods.

³⁰ This includes: incurred development; (incurred) Bornhuetter-Ferguson; (incurred) Cape Cod; and (reported) averages & counts.
2. Explore the impact of changing environments (e.g., changes in payment pattern, changes in case reserve adequacy, changes in calendar year inflation trend) on accounting date reserving methods. As described by Boles and Staudt [3], compare the performance of accounting date reserving techniques to other reserving methods under changing environments.

3. Investigate techniques to organize or modify historical experience such that actuarial assumptions A1-A7 are well satisfied for application to accounting date reserving methods.

4. Consider optimal weighting scheme(s) to credibly represent historical experience and recast it at the current accounting date exposure level.

5. Adapt tail development factor and expected unpaid loss procedures to apply to the accounting date paradigm.

6. Analyze impacts, trade-offs, interactions and sensitivities associated with the use of various combinations of supplementary data, tail factors and pseudo-data. Consider the appropriate balance of stability and responsiveness.

7. Generalize Appendix A, B and C formulas to incorporate all situations including where no actual accident year experience has reached maturity as well as for run-off business.

8. Experiment with the most effective exposure measures to use under different circumstances. Is it advisable to use different exposures for payments versus case reserves? Would a hybrid exposure metric be more effective than any one particular exposure measure?

9. Conceive of the recast accounting date representation as sample emergence from the aggregate distribution of unpaid future payments which have been incurred and unpaid as of the current accounting date. From this perspective, consider use of the recast accounting date representation as a basis to address the stochastic analysis and estimation of loss variability [4].
12. CONCLUSION

As actuarial science has evolved, the continued widespread practice of estimating unpaid claims on an individual accident year basis may have been motivated by several considerations including: conception of the total unpaid claim estimate as the sum of individual accident year ultimate estimates reduced by cumulative payments to date; the link to ratemaking, which requires cost estimates for an individual future policy year and is often derived by trending forward individual accident year estimated ultimate loss costs; statutory annual statement Schedule P reporting requirements by individual accident year; and the natural tendency to apply familiar methods. Actuarial reserving methods that develop individual accident years to estimated ultimate values have become ingrained into common actuarial practice. However, as we have seen, this familiar paradigm may not take full advantage of reasonable actuarial assumptions.

This paper introduces a new accounting date paradigm that provides practical and powerful additions to the loss reserving methodologies available to actuaries. In addition to revealing visibly apparent aggregate unpaid claim estimates, the structure of appropriate accounting date reserving applications suggests improved accuracy over corresponding accident year development methods.

Acknowledgement

The author gratefully acknowledges the very valuable suggestions and feedback of reviewers Jon Michelson, Dana Joseph, Karl Goring and Deborah Rosenberg as well as the editorial advice of Steven Katten.
Appendix A, B and C formulas pertain to specific exhibits presented in this paper and may not necessarily be more generally applicable.

Appendix A

Where required data for appropriate application is available, compute cumulative emerged loss payments $a_{i,j}$ as of year-end accounting date $i$, at year-end valuation date $j$, recast at current year-end accounting date $c$ exposure level as:

$$a_{i,j} = \sum_{k=0}^{k=n-2} \left( e_{c-k}^i \cdot e_{i-k}^j \cdot (p_{i-k}^j - p_{i-k}^j) \right)$$

where, $i < j$

- $n$ = number of years until accident year payments reach ultimate
- $e_m^s$ = exposure to remaining payments for accident year $m$ as of year-end $s$
- $p_m^s$ = cumulative loss payment for accident year $m$ through year-end $s$

Appendix B

Where required data for appropriate application is available, compute cumulative emerged reported losses $b_{i,j}$ for year-end accounting date $i$, at year-end valuation date $j$, recast at the current year-end accounting date $c$ exposure level as:

$$b_{i,j} = a_{i,j} + \sum_{k=0}^{k=n-2} \left( e_{c-k}^i \cdot e_{i-k}^j \cdot (r_{i-k}^j - p_{i-k}^j) \right)$$

where, $i \leq j$

- $a_{i,j}$ = computed via Appendix A and equals 0 when $i=j$
- $n$ = number of years until accident year payments reach ultimate
- $e_m^s$ = exposure to unreported loss (IBNR) for accident year $m$ as of year-end $s$
- $r_m^s$ = case reserves of accident year $m$ as of year-end $s$

Appendix C

Where required data for appropriate application is available, compute the unpaid claim [or IBNR] estimate $u_i$ iteratively for accident year $i$ associated with the aggregate unpaid claim [or IBNR] estimate $d_c$ at current year-end accounting date $c$ as:

$$u_i = (e_i^d / e_i^c) \left[ d_i - \sum_{k=c-n+1}^{k=i-1} (e_{c+k-1}^c \cdot e_k^i) \cdot u_k \right]$$

where, $i \leq c$

- $n$ = number of years until accident year payments reach ultimate
- $e_m^s$ = remaining exposure for accident year $m$ as of year-end $s$
- $d_i$ = estimated aggregate remaining unpaid [or IBNR] at year-end accounting date $i$ at year-end accounting date $c$ exposure level
13. REFERENCES

Abbreviations and notations

ALAE, allocated loss adjustment expenses
BF, Bornhuetter-Ferguson
CAS, Casualty Actuarial Society
CDF, cumulative age-ultimate development factor
CL, chain-ladder
DCCE, defense and cost containment expenses
IBNR, incurred but not reported loss (i.e., all unreported development beyond case reserves)
IRIS, Insurance Regulatory Information System
LDF, age-to-age loss development factor
NAIC, National Association of Insurance Commissioners
PCAS, Proceedings of the Casualty Actuarial Society

Biography of Author

Bertram A. Horowitz is President of Bertram Horowitz, Inc. Actuarial and Risk Consultants which provides property/casualty and title insurance actuarial and risk assessment services. He has a B.S. degree in Applied Mathematics from the State University of New York at Stony Brook and a M.S. in Mathematics from Brown University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. Mr. Horowitz is the former Special Deputy Superintendent and Financial Actuary of the New York State Insurance Department (now the New York State Department of Financial Services). He has served on the CAS Committee on Reserves and has been an active participant in the development of actuarial research, principles and standards.

Mr. Horowitz may be contacted at:
bert@bertramhorowitz.com

Bertram Horowitz, Inc.
www.bertramhorowitz.com
### CUMULATIVE LOSS PAYMENTS BY ACCIDENT YEAR

($000 Omitted)

<table>
<thead>
<tr>
<th>Accident Year</th>
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<th>As of 4 Years</th>
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### Aggregate Loss Reserve Analysis by Accounting Date

- **Average LDF**: 2.000000
- **Average CDF**: 6.333333
- **Weighted LDF**: 2.000000
- **Weighted CDF**: 6.333333
## NO NOISE IN CASE RESERVES

### CASE RESERVES BY ACCIDENT YEAR

($000 Omitted)

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*Aggregate Loss Reserve Analysis by Accounting Date*

Casualty Actuarial Society E-Forum, Fall 2013
### NO NOISE IN PAYMENT PATTERN

#### TRADITIONAL PAYMENT DEVELOPMENT METHOD BY ACCIDENT YEAR

($000 Omitted)

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<th>Accident Year</th>
<th>Cumulative Loss Payments as of 12/31/12</th>
<th>Cumulative Loss Development Factor to Ultimate</th>
<th>Payment Development Method Estimated Ultimate Losses</th>
<th>Ultimate Losses as of 12/31/12</th>
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(2) Exhibit 1, Table 1 final diagonal

(3) Exhibit 1, Table 1 corresponding CDF; payments completed as of 10 years
### CUMULATIVE LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE

($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date

Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 1, Table 1

<table>
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<tr>
<th>Year-End Accounting Date</th>
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<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
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### Exhibit 1

**Table 5**

**NO NOISE IN PAYMENT PATTERN OR CASE RESERVES**

**LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL USING CASE RESERVES AS EXPOSURE MEASURE**

($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date

Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 1, Table 1 Exposure Adjusted to 2012 Year-End Accounting Date Exposure Level

<table>
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<th>Year-End Accounting Date</th>
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<td>197,268</td>
<td>1363909</td>
<td>269,056</td>
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<tr>
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<td>197,268</td>
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<td>2010</td>
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<td>197,268</td>
<td>1363909</td>
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<td>197,268</td>
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<td>197,268</td>
<td>1363909</td>
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<td>1202395</td>
<td>323,511</td>
<td>1127883</td>
<td>364,883</td>
<td>1085310</td>
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Average LDF: 1.829724, 1.363909, 1.202395, 1.127883, 1.085310, 1.053151, 1.029926, 1.012061, 1.000000

Average CDF: 4.032178, 2.203708, 1.615729, 1.343759, 1.191399, 1.097750, 1.042348, 1.012061

Weighted LDF: 1.829724, 1.363909, 1.202395, 1.127883, 1.085310, 1.053151, 1.029926, 1.012061

Weighted CDF: 4.032178, 2.203708, 1.615729, 1.343759, 1.191399, 1.097750, 1.042348, 1.012061
### Aggregate Loss Reserve Analysis by Accounting Date

#### NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

**ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12; ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

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<tr>
<th>Year-End</th>
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<th>Factor</th>
<th>Recast Cumulative Loss Payments As of 12/31/12</th>
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<th>(5)= (4)-(2)</th>
<th>(7)</th>
<th>(6)</th>
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<td>Total</td>
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</table>

(2) Exhibit 1, Table 5 final diagonal

(3) Exhibit 1, Table 5 corresponding CDF

(7) Iterative Formula

* Accept most recent indication
### Exhibit 2

#### Table 1

**NOISE IN PAYMENT PATTERN**

#### CUMULATIVE LOSS PAYMENTS BY ACCIDENT YEAR

($000 Omitted)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
<th>As of 9 Years</th>
<th>As of 10 Years</th>
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Average LDF: 1.994686 1.468456 1.303729 1.172235 1.122012 1.094118 1.068156 1.052044 1.033033

Average CDF: 6.379437 3.198216 2.177945 1.670550 1.425099 1.270128 1.160869 1.086797 1.033033

Weighted LDF: 1.993299 1.467589 1.308666 1.172358 1.122104 1.093854 1.068275 1.052074 1.032894

Weighted CDF: 6.370319 3.195867 2.177631 1.670390 1.424813 1.269789 1.160874 1.086880 1.032894

---

_Casualty Actuarial Society E-Forum, Fall 2013_
## Exhibit 2
### Table 2

### NOISE IN CASE RESERVES

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<tr>
<th>Accident Year</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
<th>As of 9 Years</th>
<th>As of 10 Years</th>
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</table>
### Exhibit 2

**Table 3**

**NOISE IN PAYMENT PATTERN**

**CUMULATIVE LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE**

($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date

Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1

<table>
<thead>
<tr>
<th>Year-End Accounting Date</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
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<th>As of 7 Years</th>
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<td>1,125,961</td>
<td>223,732</td>
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</table>

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**Casualty Actuarial Society E-Forum, Fall 2013**

41
**Exhibit 2**

**Table 4**

NOISE IN PAYMENT PATTERN AND CASE RESERVES

LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL

USING CASE RESERVES AS EXPOSURE MEASURE

($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date

Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1 Exposure Adjusted to 2012 Year-End Accounting Date Exposure Level

<table>
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<tr>
<th>Year-End Accounting Date</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
<th>As of 9 Years</th>
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<td>1.396273</td>
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Average LDF: 1.829681 1.367451 1.203560 1.129112 1.085062 1.053198 1.028405 1.011532

Average CDF: 4.042031 2.209145 1.615521 1.342285 1.188798 1.095604 1.040264 1.011532

Weighted LDF: 1.829531 1.366944 1.203286 1.128899 1.084878 1.053156 1.028416 1.011532

Weighted CDF: 4.037726 2.206973 1.614531 1.341768 1.188563 1.095573 1.040275 1.011532
### NOISE IN PAYMENT PATTERN AND CASE RESERVES

**ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12; ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

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<tr>
<th>Year-End</th>
<th>Recast Cumulative Loss Payments As of 12/31/12</th>
<th>Weighted Exposure Level</th>
<th>Indicated Total Emergence at 2012 Year-End</th>
<th>Payment Development Indicated Unpaid Loss as of 12/31/12</th>
<th>Accident Year Allocation of Aggregate Accounting Date Payment Development</th>
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<td>Total 433,929</td>
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(2) Exhibit 2, Table 4 final diagonal

(3) Exhibit 2, Table 4 corresponding Weighted CDF

(7) Iterative Formula

* Accept most recent indication
### Reported Losses by Accident Year: Exhibit 1, Table 1 + Exhibit 1, Table 2

($000 Omitted)

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<th>As of 3 Years</th>
<th>As of 4 Years</th>
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Average LDF: 1.250000 1.220000 1.147541 1.114286 1.076923 1.047619 1.039773 1.027322 1.010638 1.000000

Average CDF: 2.375000 1.900000 1.557377 1.357143 1.217949 1.103952 1.079645 1.050825 1.000000
### NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

#### TRADITIONAL INCURRED DEVELOPMENT METHOD BY ACCIDENT YEAR

($000 Omitted)

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<tr>
<th>Accident Year</th>
<th>Reported Losses as of 12/31/12</th>
<th>Cumulative Factor to Ultimate</th>
<th>Lost Development Method Estimated Ultimate Losses as of 12/31/12</th>
<th>Incurred Development</th>
<th>BNR Estimate as of 12/31/12</th>
<th>Case Reserve as of 12/31/12</th>
<th>Loss Estimate as of 12/31/12</th>
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<td><strong>148,116</strong></td>
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(2) Exhibit 3, Table 1 final diagonal
(3) Exhibit 3, Table 1 corresponding CDF; reportings completed as of 10 years
(6) Exhibit 1, Table 2 final diagonal
NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 1, Table 4 plus appropriate accumulation of Case Reserves of Exhibit 1, Table 2

<table>
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<tr>
<th>Year-End Accounting Date</th>
<th>After 0 Years</th>
<th>After 1 Year</th>
<th>After 2 Years</th>
<th>After 3 Years</th>
<th>After 4 Years</th>
<th>After 5 Years</th>
<th>After 6 Years</th>
<th>After 7 Years</th>
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</table>

Casualty Actuarial Society E-Forum, Fall 2013
### Exhibit 3
Table 4

**NO NOISE IN PAYMENT PATTERN OR CASE RESERVES**

**CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL USING CASE RESERVES AS EXPOSURE MEASURE**

($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date

Derived as Exhibit 1, Table 5 plus Case Reserves of Exhibit 1, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

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<th>Year-End Accounting Date</th>
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<th>After 2 Years</th>
<th>After 3 Years</th>
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<th>After 5 Years</th>
<th>After 6 Years</th>
<th>After 7 Years</th>
<th>After 8 Years</th>
<th>After 9 Years</th>
</tr>
</thead>
</table>

Average LDF: 1.505397, 1.284923, 1.170929, 1.110057, 1.069499, 1.043603, 1.027582, 1.017827, 1.003988
Average CDF: 2.935012, 1.949660, 1.517336, 1.295840, 1.167364, 1.091505, 1.045901, 1.017827, 1.003988
Weighted LDF: 1.505397, 1.284923, 1.170929, 1.110057, 1.069499, 1.043603, 1.027582, 1.017827, 1.003988
Weighted CDF: 2.935012, 1.949660, 1.517336, 1.295840, 1.167364, 1.091505, 1.045901, 1.017827, 1.003988
## NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

**ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12; ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)= (2)x(3)</th>
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<td><strong>Accident Year</strong></td>
<td><strong>Accident Year</strong></td>
<td><strong>Indicated IBNR</strong></td>
<td><strong>Incurred Development</strong></td>
<td><strong>Case Incurred Development</strong></td>
<td><strong>Aggregate Unpaid Loss</strong></td>
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<td><strong>Losses As of 12/31/12</strong></td>
<td><strong>Weighted Exposure Level Factor</strong></td>
<td><strong>Exposure Level</strong></td>
<td><strong>Exposure Level</strong></td>
<td><strong>Year-End Case Reserves</strong></td>
<td><strong>Accounting Date Incurred Development</strong></td>
<td><strong>Aggregate Development</strong></td>
<td><strong>Aggregate Unpaid Loss</strong></td>
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(2) Exhibit 3, Table 4 final diagonal  
(3) Exhibit 3, Table 4 corresponding CDF  
(7) Iterative Formula  
(8) Exhibit 1, Table 2 final diagonal
## Aggregate Loss Reserve Analysis by Accounting Date

### NOISE IN PAYMENT PATTERN AND CASE RESERVES

#### REPORTED LOSSES BY ACCIDENT YEAR: Exhibit 2, Table 1 + Exhibit 2, Table 2 ($000 Omitted)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
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### Average LDF

- Average LDF: 1.247022
- Average CDF: 2.391364
- Weighted LDF: 1.244711
- Weighted CDF: 2.398138

### Aggregate Loss Reserve Analysis by Accounting Date

- Casualty Actuarial Society E-Forum, Fall 2013
### Table 2

**NOISE IN PAYMENT PATTERN AND CASE RESERVES**

**CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE**

($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date

Derived as Exhibit 2, Table 3 plus appropriate accumulation of Case Reserves of Exhibit 2, Table 2

<table>
<thead>
<tr>
<th>Year-End Accounting Date</th>
<th>After 0 Years</th>
<th>After 1 Year</th>
<th>After 2 Years</th>
<th>After 3 Years</th>
<th>After 4 Years</th>
<th>After 5 Years</th>
<th>After 6 Years</th>
<th>After 7 Years</th>
<th>After 8 Years</th>
<th>After 9 Years</th>
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<tbody>
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</tr>
</tbody>
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*Casualty Actuarial Society E-Forum, Fall 2013*
### Aggregate Loss Reserve Analysis by Accounting Date

#### NOISE IN PAYMENT PATTERN AND CASE RESERVES

**CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL USING CASE RESERVES AS EXPOSURE MEASURE**

($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date

Derived as Exhibit 2, Table 4 plus Case Reserves of Exhibit 2, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

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<th>Year-End Accounting Date</th>
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<th>3 Years</th>
<th>4 Years</th>
<th>5 Years</th>
<th>6 Years</th>
<th>7 Years</th>
<th>8 Years</th>
<th>9 Years</th>
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</table>

Average LDF: 1.510170, 1.286589, 1.173305, 1.110088, 1.072055, 1.043836, 1.027962, 1.012432, 1.003791

Average CDF: 2.958485, 1.959041, 1.522663, 1.297755, 1.168056, 1.090482, 1.044687, 1.016270, 1.003791

Weighted LDF: 1.509636, 1.286796, 1.173275, 1.110150, 1.071929, 1.043710, 1.027948, 1.012448, 1.003791

Weighted CDF: 2.957307, 1.958953, 1.522349, 1.297521, 1.168780, 1.090352, 1.044689, 1.016286, 1.003791
## NOISE IN PAYMENT PATTERN AND CASE RESERVES

**Exhibit 4**

**Table 4**

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<th>(4)=(2)x(3)</th>
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<th>(7)</th>
<th>(8)</th>
<th>(9)=(7) - (8)</th>
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<tr>
<td><strong>Allocation of Total Unpaid Claim Estimate to Accident Year</strong></td>
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<td><strong>($000 Omitted)</strong></td>
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<td>442,078</td>
<td>99,230</td>
<td>2008</td>
<td>30,801</td>
<td>16,315</td>
<td>47,116</td>
</tr>
<tr>
<td><strong>2009</strong></td>
<td>333,522</td>
<td>1.297521</td>
<td>432,752</td>
<td>147,747</td>
<td>2009</td>
<td>50,893</td>
<td>24,910</td>
<td>75,803</td>
</tr>
<tr>
<td><strong>2010</strong></td>
<td>282,850</td>
<td>1.522349</td>
<td>430,597</td>
<td>214,794</td>
<td>2010</td>
<td>71,103</td>
<td>31,618</td>
<td>102,721</td>
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<td><strong>2011</strong></td>
<td>223,988</td>
<td>1.958953</td>
<td>438,782</td>
<td>289,693</td>
<td>2011</td>
<td>96,931</td>
<td>43,001</td>
<td>139,932</td>
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<tr>
<td><strong>2012</strong></td>
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<td>2.957307</td>
<td><strong>437,699</strong></td>
<td><strong>437,699</strong></td>
<td>2012</td>
<td><strong>437,699</strong></td>
<td><strong>437,699</strong></td>
<td><strong>437,699</strong></td>
</tr>
</tbody>
</table>

**Total** | 289,693 | 148,006 | **437,699** | **437,699** |

(2) Exhibit 4, Table 3 final diagonal
(3) Exhibit 4, Table 3 corresponding Weighted CDF
(7) Iterative Formula
(8) Exhibit 2, Table 2 final diagonal
### Aggregate Loss Reserve Analysis by Accounting Date

**Exhibit 5**

**NOISE IN PAYMENT PATTERN AND CASE RESERVES**

**ACCOUNTING DATE EXPECTED UNPAID LOSSES AS OF 12/31/12:**

**ACCOUNTING DATE BORNUETTER-FERGUSSON UNPAID LOSSES AS OF 12/31/12:**

**ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Industry Earned Premium</th>
<th>Industry Loss Reserve</th>
<th>Industry Earned Premium as of 12/31/12 Ratio as of 12/31/12</th>
<th>Industry Expected Unpaid Loss Reserves</th>
<th>Industry Cumulative Development Factor as of 12/31/12</th>
<th>Industry BF Indicated</th>
<th>Implied BF Indicated</th>
<th>Industry Allocation of IBNR Aggregate Loss</th>
<th>Aggregate Unpaid Loss as of 12/31/12</th>
</tr>
</thead>
<tbody>
<tr>
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<td>3,703,297</td>
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<td>43.001</td>
<td>95.433</td>
<td>96.034</td>
<td>139.034</td>
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**Year-End Accounting Date 2012 Total**

<p>| | | | | | | | | | | |</p>
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</thead>
<tbody>
<tr>
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<td>2.957307</td>
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<td>286,191</td>
<td>434,197</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(3), (4) figures are used here to illustrate methodology and do not represent actual industry figures

(7) Exhibit 2, Table 2 final diagonal

(8) Exhibit 4, Table 4, Column (3) Year-End Accounting Date 2012

(11) Total = Total (9) - Total (7); otherwise (10)x[Total (11)/Total (10)]
### Aggregate Loss Reserve Analysis by Accounting Date

**NOISE IN PAYMENT PATTERN AND CASE RESERVES**

**ACCOUNTING DATE CAPE COD AGGREGATE UNPAID LOSS ESTIMATE AS OF 12/31/12; ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

<table>
<thead>
<tr>
<th>Year-End</th>
<th>Indicated Through 12/31/12</th>
<th>Recast Reported Losses</th>
<th>Indicated Accident Year</th>
<th>BNR Allocation of Aggregate Accident Year</th>
<th>Cape Cod Case IBNR Reserves</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>Exposure Level</td>
<td>Weight Exposure Level</td>
<td>Exposure Level</td>
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<td>432,240</td>
<td>1.000000</td>
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<td>1.003791</td>
<td>436,851</td>
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<td>0.643548</td>
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<tr>
<td>2005</td>
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<td>0.957222</td>
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<td>1.168750</td>
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<td>0.712001</td>
</tr>
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<td>1.297521</td>
<td>432,752</td>
<td>0.770760</td>
<td>0.763947</td>
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<td>437,699</td>
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<td>1.000000</td>
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</table>

\[
\text{437,953} = \text{Expected Unpaid Loss as of 12/31/12} \]

(2) Exhibit 4, Table 3 final diagonal

(3) Exhibit 4, Table 3 corresponding Weighted CDF

(4) Expected Unpaid Loss at 12/31/12 equals weighted average of Column (4), weighted on Columns (5) and (6)

(6) [Exhibit 4, Table 2 final diagonal][corresponding Exhibit 4, Table 3 final diagonal]

(7) (2)=1-1/(3)(5)(Expected Unpaid Loss as of 12/31/12)

(10) Iterative Formula

(11) Exhibit 2, Table 2 final diagonal

---

*Casualty Actuarial Society E-Forum, Fall 2013*
### NOISE IN EARNED PREMIUM

<table>
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<tr>
<th>Accident Year</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
<th>As of 9 Years</th>
<th>As of 10 Years</th>
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</table>
## NOISE IN PAYMENT PATTERN AND CASE RESERVES

**LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE**

($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date  
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1 Exposure Adjusted to 2012 Accounting Date Exposure Level

<table>
<thead>
<tr>
<th>Year-End Accounting Date</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
<th>As of 9 Years</th>
</tr>
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<tbody>
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<td>198,872</td>
<td>318,237</td>
<td>361,992</td>
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<td>370,324</td>
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<tr>
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<td>203,482</td>
<td>331,284</td>
<td>371,587</td>
<td>407,434</td>
<td>422,556</td>
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<tr>
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<td>200,154</td>
<td>324,141</td>
<td>371,587</td>
<td>407,434</td>
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</tr>
</tbody>
</table>

Average LDF: 1.827998  
Average CDF: 4.043237  
Weighted LDF: 1.827809  
Weighted CDF: 4.038622  

Casualty Actuarial Society E-Forum, Fall 2013  
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### Table 3

**NOISE IN PAYMENT PATTERN**

**ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12**

**USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE;**

**ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

<table>
<thead>
<tr>
<th>(1) Year-End Accounting Date</th>
<th>(2) Recast Cumulative Loss Payments As of 12/31/12</th>
<th>(3) Weighted Exposure Level Factor</th>
<th>(4)= (2)x(3)</th>
<th>(5)= (4)- (2)</th>
<th>(6) Accident Year Indicated Unpaid Loss as of 12/31/12</th>
<th>(7) Accounting Date Allocation of Aggregate Payment Development</th>
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<td>26,927</td>
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<td>322,072</td>
<td>2011</td>
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<td></td>
<td></td>
<td>428,065</td>
<td></td>
<td>2012</td>
<td>134,962</td>
</tr>
</tbody>
</table>

**Total**

|        | 428,065 |

---

(2) Exhibit 7, Table 2 final diagonal

(3) Exhibit 7, Table 2 corresponding Weighted CDF

(7) Iterative Formula

* Accept most recent indication
Aggregate Loss Reserve Analysis by Accounting Date

Exhibit 8
Table 1

NOISE IN PAYMENT PATTERN AND CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 7, Table 2 plus Case Reserves of Exhibit 2, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

<table>
<thead>
<tr>
<th>Year-End Accounting Date</th>
<th>As of 0 Years</th>
<th>As of 1 Year</th>
<th>As of 2 Years</th>
<th>As of 3 Years</th>
<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
<th>As of 7 Years</th>
<th>As of 8 Years</th>
<th>As of 9 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>146,256</td>
<td>222,131</td>
<td>283,486</td>
<td>331,470</td>
<td>364,626</td>
<td>391,045</td>
<td>409,674</td>
<td>420,719</td>
<td>425,714</td>
<td>427,355</td>
</tr>
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<td>2004</td>
<td>146,320</td>
<td>222,437</td>
<td>284,152</td>
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Average CDF: 2.957138  1.959045  1.522834  1.297783  1.169155  1.090479  1.044739  1.016359  1.003856
Weighted LDF: 1.508888  1.286646  1.173365  1.110062  1.072019  1.043645  1.027907  1.012471  1.003856
Weighted CDF: 2.955693  1.958855  1.522451  1.297508  1.168861  1.090337  1.044739  1.016375  1.003856
### Aggregate Loss Reserve Analysis by Accounting Date

#### NOISE IN PAYMENT PATTERN AND CASE RESERVES

**ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12**

**USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE;**

**ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

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<th>Indicated Accounting Date</th>
<th>Allocation of Aggregate Unpaid Loss as of 12/31/12</th>
<th>Total</th>
<th>Recast Reported Indicator Accident Year Losses</th>
<th>Indicated Accident Year Losses</th>
<th>IBNR Allocation of Total Total</th>
<th>IBNR as of 12/31/12</th>
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<th>Recast Reported Indicator Accident Year Losses</th>
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Total 289,454 148,006 437,460

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(2) Exhibit 8, Table 1 final diagonal

(3) Exhibit 8, Table 1 corresponding Weighted CDF

(7) Iterative Formula

(8) Exhibit 2, Table 2 final diagonal
### NOISE IN CLAIM COUNTS

**PROJECTED REMAINING CLAIM COUNTS TO BE CLOSED WITH PAYMENT**

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<th>As of 2 Years</th>
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<th>As of 4 Years</th>
<th>As of 5 Years</th>
<th>As of 6 Years</th>
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### SEVERITY ADJUSTED PROJECTED REMAINING CLAIM COUNTS TO BE CLOSED WITH PAYMENT

Exhibit 9, Table 1 
**Accident Year 2003 inflated/deflated annually by 5%**

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NOISE IN CLAIM COUNTS AND PAYMENT PATTERN

**LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL USING SEVERITY ADJUSTED REMAINING CLAIM COUNTS AS EXPOSURE MEASURE**
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1 Exposure Adjusted to 2012 Accounting Date Exposure Level

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<th>As of 8 Years</th>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Average LDF | 1.794628 | 1.349382 | 1.209110 | 1.127755 | 1.089701 | 0.953049 | 0.853230 | 0.789265 | 0.740032 |
Average CDF | 1.829471 | 1.396832 | 1.204772 | 1.137252 | 1.085128 | 0.956770 | 0.884714 | 0.826814 | 0.783931 |

Weighted LDF | 1.858437 | 1.381993 | 1.208387 | 1.129128 | 1.085128 | 0.956770 | 0.884714 | 0.826814 | 0.783931 |
Weighted CDF | 1.844729 | 1.358151 | 1.200068 | 1.121431 | 0.978502 | 0.914831 | 0.856770 | 0.796814 | 0.753931 |
**NOISE IN CLAIM COUNTS AND PAYMENT PATTERN**

**ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12**

Using severity adjusted remaining claim counts as exposure measure;
Allocation of total unpaid claim estimate to accident year

($000 Omitted)

<table>
<thead>
<tr>
<th>Year-End Accounting Date</th>
<th>Exposure Level Factor</th>
<th>Recast Cumulative Loss Payments As of 12/31/12</th>
<th>Weighted Total Emergence at 2012 Year-End</th>
<th>Projected Average Payment as of 2012 Year-End</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>1.000000</td>
<td>434,486</td>
<td>434,486</td>
<td>307,285</td>
</tr>
<tr>
<td>2004</td>
<td>1.011866</td>
<td>432,973</td>
<td>438,111</td>
<td>322,212</td>
</tr>
<tr>
<td>2005</td>
<td>1.040639</td>
<td>423,068</td>
<td>440,261</td>
<td>324,331</td>
</tr>
<tr>
<td>2006</td>
<td>1.095816</td>
<td>398,503</td>
<td>436,686</td>
<td>324,331</td>
</tr>
<tr>
<td>2007</td>
<td>1.188459</td>
<td>365,362</td>
<td>434,218</td>
<td>282,494</td>
</tr>
<tr>
<td>2008</td>
<td>1.340801</td>
<td>322,702</td>
<td>432,679</td>
<td>238,396</td>
</tr>
<tr>
<td>2009</td>
<td>1.612368</td>
<td>272,052</td>
<td>438,648</td>
<td>205,042</td>
</tr>
<tr>
<td>2010</td>
<td>2.202511</td>
<td>200,071</td>
<td>440,659</td>
<td>177,141</td>
</tr>
<tr>
<td>2011</td>
<td>4.020162</td>
<td>107,313</td>
<td>431,414</td>
<td>144,036</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
<td>431,414</td>
<td>136,853</td>
</tr>
</tbody>
</table>

* Accept most recent indication

---

**(2) Exhibit 9, Table 3 final diagonal**

**(3) Exhibit 9, Table 3 corresponding Weighted CDF**

**(7) Iterative Formula**

**(8) Exhibit 9, Table 1 final diagonal**

---

**Casualty Actuarial Society E-Forum, Fall 2013**

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### Exhibit 10

#### Table 1

**NOISE IN CLAIM COUNTS, PAYMENT PATTERN AND CASE RESERVES**

**REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL**

*Using Severity Adjusted Remaining Claim Counts as Exposure Measure*

($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date

Derived as Exhibit 9, Table 3 plus Case Reserves of Exhibit 2, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

<table>
<thead>
<tr>
<th>Year-End Accounting Date</th>
<th>After 0 Years</th>
<th>After 1 Year</th>
<th>After 2 Years</th>
<th>After 3 Years</th>
<th>After 4 Years</th>
<th>After 5 Years</th>
<th>After 6 Years</th>
<th>After 7 Years</th>
<th>After 8 Years</th>
<th>After 9 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>148,440</td>
<td>221,082</td>
<td>285,071</td>
<td>333,847</td>
<td>372,259</td>
<td>397,742</td>
<td>400,827</td>
<td>417,883</td>
<td>1,030,671</td>
<td>430,700</td>
</tr>
<tr>
<td>2007</td>
<td>144,843</td>
<td>222,175</td>
<td>287,447</td>
<td>336,843</td>
<td>372,492</td>
<td>1,068,238</td>
<td>397,910</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>145,399</td>
<td>223,877</td>
<td>287,319</td>
<td>334,238</td>
<td>1,097,058</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2009</td>
<td>150,814</td>
<td>227,059</td>
<td>289,156</td>
<td>339,296</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>152,204</td>
<td>224,643</td>
<td>290,355</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>148,060</td>
<td>223,707</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>148,006</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Average LDF**

1.516970

**Average CDF**

1.999999

**Weighted LDF**

1.50851

**Weighted CDF**

1.947347

---

**Aggregate Loss Reserve Analysis by Accounting Date**

Casualty Actuarial Society E-Forum, Fall 2013

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### Exhibit 10
Table 2

**NOISE IN CLAIM COUNTS, PAYMENT PATTERN AND CASE RESERVES**

**ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12**

**ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR**

($000 Omitted)

<table>
<thead>
<tr>
<th>(1) Year-End</th>
<th>(2) Recast Reported Losses As of 12/31/12</th>
<th>(3) Weighted Exposure Level</th>
<th>(4) Indicated IBNR as of 12/31/12</th>
<th>(5)= (4)-(2)</th>
<th>(6) Accident Year Allocation of Aggregate Indicated IBNR</th>
<th>(7) Accident Year Allocation of Case Incurred Development as of 12/31/12</th>
<th>(8) Reserves Aggregate Unpaid Loss as of 12/31/12</th>
<th>(9)= (7)+ (8)</th>
<th>(10) Number of Remaining Claims to be Closed</th>
<th>(11)= (9)x1,000/(10) Projected Average per Remaining Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>434,486</td>
<td>1.000000</td>
<td>434,486</td>
<td>1,701</td>
<td>1,018</td>
<td>1,973</td>
<td>2,990</td>
<td>10</td>
<td>299,035</td>
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</tr>
<tr>
<td>2004</td>
<td>436,271</td>
<td>1.003900</td>
<td>437,973</td>
<td>7,096</td>
<td>3,279</td>
<td>4,068</td>
<td>7,348</td>
<td>23</td>
<td>319,472</td>
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</tr>
<tr>
<td>2005</td>
<td>432,935</td>
<td>1.016391</td>
<td>440,031</td>
<td>18,620</td>
<td>7,519</td>
<td>6,255</td>
<td>13,774</td>
<td>43</td>
<td>320,330</td>
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<tr>
<td>2006</td>
<td>417,579</td>
<td>1.044590</td>
<td>436,199</td>
<td>35,827</td>
<td>11,683</td>
<td>9,476</td>
<td>21,159</td>
<td>75</td>
<td>282,124</td>
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<tr>
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<td>370,905</td>
<td>1.168014</td>
<td>433,223</td>
<td>100,445</td>
<td>30,633</td>
<td>16,315</td>
<td>46,948</td>
<td>227</td>
<td>206,818</td>
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</tr>
<tr>
<td>2009</td>
<td>339,296</td>
<td>1.296040</td>
<td>439,741</td>
<td>150,934</td>
<td>51,271</td>
<td>24,910</td>
<td>76,180</td>
<td>432</td>
<td>176,344</td>
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</tr>
<tr>
<td>2010</td>
<td>290,355</td>
<td>1.519826</td>
<td>441,289</td>
<td>213,420</td>
<td>69,664</td>
<td>31,618</td>
<td>101,282</td>
<td>670</td>
<td>151,167</td>
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</tr>
<tr>
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<td>223,707</td>
<td>1.954019</td>
<td>437,127</td>
<td>288,219</td>
<td>96,781</td>
<td>43,001</td>
<td>139,782</td>
<td>1,023</td>
<td>136,639</td>
<td></td>
</tr>
</tbody>
</table>

**Total**

| 288,219 | 148,006 | **436,225** | 2,613 | 166,944 |

---

(2) Exhibit 10, Table 1 final diagonal
(3) Exhibit 10, Table 1 corresponding Weighted CDF
(7) Iterative Formula
(8) Exhibit 2, Table 2 final diagonal
(10) Exhibit 9, Table 1 final diagonal