Stochastic Dominance: A Tool for Evaluating Reinsurance Alternatives

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Abstract

During the course of reinsurance coverage negotiations, the prospective reinsured is often presented with a myriad of coverage options. In this situation, two questions naturally arise:

- *From the reinsured's perspective, which coverage option will yield the optimal long-run economic outcome?*
- *From the reinsurer's perspective, are the options consistently priced? In other words, do the risk loads ensure that each option places the reinsurer in the same long-run economic position?*

Stochastic dominance is an intuitive, easily implemented, analytical tool used by financial analysts to evaluate these types of questions. Furthermore, this tool is uniquely suited to the empirical output generated by DFA and other simulation models.

Stochastic dominance is a generalization of utility theory that eliminates the need to explicitly specify a firm's utility function. Rather, general mathematical statements about wealth preference, risk aversion, etc. are used to develop optimal decision rules for selecting between investment alternatives. This paper introduces stochastic dominance in a reinsurance context and explores its application to reinsurance pricing and risk loading.

The opinions expressed in this article are those of the author, not American Re-Insurance Company.

Introduction

During the course of reinsurance coverage negotiations, the prospective reinsured is often presented with a myriad of coverage options. In this situation, two questions naturally arise:

- From the reinsured's perspective, which coverage option will yield the optimal long-run economic outcome.
- From the reinsurer's perspective, have the options been consistently priced to ensure that each option places the reinsurer in the same long-run economic position.

Correctly answering these questions requires a means for consistently evaluating the balance between risk and reward under various investment scenarios[1]. The actuary has a variety of statistics for performing this evaluation: loss ratio, various volatility measures, expected policyholder deficit, return on equity, internal rate of return, etc. All of these statistics have significant shortcomings that have been discussed at length in the actuarial literature. For multi-period reinsurance contracts, however, several issues become particularly relevant:

[1] Throughout this paper the premium-reinsurance transaction will be treated as an investment where uncertain future cash flow is traded for certain cash flow today. This is consistent with the actuarial view of the insurance transaction except that terminal wealth rather than ultimate loss will be considered.

- Profit and loss are both dispersed over time.
- Ultimate premium and ultimate loss are frequently interdependent. This interdependency can be far more significant than that introduced by typical loss-sensitive contract features. For example, in some financial guaranty products large losses may actually preclude future premium collection.
- Overall volatility frequently isn't as important as the source of that volatility. From a reinsurance standpoint, we need to be able to differentiate between operating risk and catastrophe risk, loss volatility and profit volatility, etc.

DFA and other stochastic simulation techniques have become popular means for modeling these issues. These techniques, however, stop short of directly answering the essential question: do each of the firm's investment alternatives equally balance risk and reward? This paper introduces stochastic dominance as one tool for answering this question.

Stochastic dominance is an intuitive, easily implemented, analytical tool that is uniquely suited to the empirical output generated by DFA and other simulation models. Stochastic dominance is a generalization of utility theory that eliminates the need to explicitly specify a firm's utility function. Rather, general mathematical statements about wealth preference, risk aversion, etc. are used to develop optimal decision rules for selecting between investment alternatives.

Properties of Utility Functions

As stochastic dominance is a generalization of utility theory, we will begin with a discussion of utility functions. Simply stated, a utility function measures the relative value that a firm places on a business outcome. Within this definition, however, lies a significant limitation of utility theory: we can compare competing options, but we cannot assess the overall acceptability of any of those options. In other words, there is not objective, absolute scale for utility.

To specify a utility function we must have a measure that uniquely identifies each business outcome, typically some measure of profitability or terminal wealth, and a function that maps each business outcome to its corresponding utility. By convention utility is purely an ordinal measure. In other words, utility can be used to establish the rank ordering of outcomes, but cannot be used to determine the degree to which one is preferred over the other. For example, consider two outcomes A and B with corresponding utilities of 100 and 25. We can say that A is preferred over B, but we cannot say that A is four times more preferred than B. As a consequence of this ordinality, utility functions are not unique. Any positive, linear transformation of a utility function will still yield the same rank ordering of investment alternatives.[2]

Unfortunately, we rarely know *a priori* what outcomes will result from various investment alternatives. Instead, forecasted terminal wealth has some distribution which varies depending upon the investment alternative selected. Classical utility theory assumes that rational firms seek to maximize their expected utility and choose among their investment alternatives accordingly.[3] Mathematically, this is expressed as:

A is preferred to B if and only if terminal wealth satisfies $E_w[U(w_A)] - E_w[U(w_B)] \geq 0$
with at least one strict inequality $U(w_A) - U(w_B) \geq 0$. *(1)*

The mathematical features of the utility function *U* reflect the risk/reward motivations of the firm: several common risk/reward features are discussed below. These same features also determine what stochastic characteristics

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[2] For proof of this proposition, the interested reader should see Leigh Halliwell, ROE, Utility and the Pricing of Risk, 1999 CAS Spring Forum, Reinsurance Call Papers.

[3] For an axiomatic treatment of Maximum Expected Utility see Haim Levy, Stochastic Dominance, Investment Decision Making Under Uncertainty, Section 2.3.

the terminal wealth distribution must possess if one alternative is to be preferred over another. Evaluation of these stochastic characteristics is the basis of stochastic dominance analysis.

Increasing Wealth Preference
This feature captures the "more wealth is better" philosophy of firm behavior and is generally considered a universal feature of utility functions. For greater wealth to be preferred, the utility function must be monotonically increasing. Mathematically this is expressed as:

> *A utility function possesses increasing wealth preference if and only if $U'(w) \geq 0$ for all w with at least one strict inequality.* (2.1)

Risk Aversion
This feature captures the willingness of a firm to purchase insurance (*i.e.* to pay more than the expected loss to transfer an insurable loss). This is a subset of increasing wealth preference, a firm may have increasing wealth preference with or without exhibiting risk aversion, and is also generally considered a universal feature of utility functions. Mathematically this is expressed as:

> *A utility function possesses risk aversion if and only if it satisfies the conditions for increasing wealth preference and $U''(w) \leq 0$ for all w with at least one strict inequality.* (2.2)

It is not intuitively clear, however, that this mathematical definition of risk aversion is equivalent to the behavioral definition given above. To make this relationship clearer we must recognize that Equation 2.2 defines a concave function and apply Jensen's inequality.[4] This yields:

$$E_w[U(w)] \leq U(E_w[w])$$

Under risk aversion, then, the expected utility of a risky investment is less than the utility of the expected outcome. Why should this be the case? By proposition the firm has penalized the utility of the investment for the possibility of unfavorable outcomes. If we rewrite Jensen's inequality with a strict inequality we can show that:

$$E_w[U(w)] = U(E_w[w] - \pi)$$

This shows that the firm is indifferent between the return on a risky investment or a lower, risk-free wealth equal to $E_w[w] - \pi$ where π is the premium that the firm is willing to pay to eliminate risk.

Skewness Preference (Ruin Aversion)
This feature is classically presented as an individual's willingness to play the lottery: to accept a small, almost certain loss in exchange for the remote possibility of huge returns. A firm's concern, however, is with the opposite situation, unwillingness to accept small, almost certain gain in exchange for the remote possibility of ruin. This is a subset of risk aversion, a firm may have risk aversion with or without exhibiting ruin aversion. Mathematically this is expressed as:

> *A utility function possesses ruin aversion if and only if it satisfies the conditions for risk aversion and $U'''(w) \geq 0$ for all w with at least one strict inequality.* (2.3)

As with risk aversion, it is not intuitively clear that the mathematical and behavioral definitions of ruin aversion are consistent. If we take a Taylor series expansion of the utility function about $E_w[w]$, and take the expectation with respect to *w*, we obtain:

[4] For a more complete discussion of risk aversion see Daykin, Pentikäinen, and Pesonen, Practical Risk Theory for Actuaries, Section 6.6.

$$U(w)=U(E_w[w])+U'(E_w[w])\cdot(w-E_w[w])+\frac{U''(E_w[w])}{2!}\cdot(w-E_w[w])^2+\frac{U'''(E_w[w])}{3!}\cdot(w-E_w[w])^3$$
$$E_w[U(w)]=U(E_w[w])+\frac{U''(E_w[w])}{2!}\cdot\sigma_w^2+\frac{U'''(E_w[w])}{3!}\cdot\mu_3$$

From this expression we can see that any investment feature that increases positive skewness μ_3 (or reduces negative skewness) acts to increase expected utility.

Stochastic Dominance

Utility theory and the features embedded in utility functions are elegant but practically ineffective constructs. An economist once asserted[5]:

> *"A man who seeks advice about his actions will not be grateful for the suggestion that he maximize expected utility."*

Few firms have the willingness or means to select and parameterize their own utility function.[6] The problem becomes, then, how can we use features such as increasing wealth preference, risk aversion, ruin aversion, etc. to select among investment alternatives without actually selecting a specific utility function?

First-Order Stochastic Dominance

Let us begin with the definition of preference given in Equation 1 and the most general constraint on a utility function given in Equation 2.1, increasing wealth preference. We can integrate Equation 1 by parts to yield:

$$E_w[U(w_A)]-E_w[U(w_B)]\geq 0$$
$$\int_{-\infty}^{\infty}U(t)f_A(t)dt-\int_{-\infty}^{\infty}U(t)f_B(t)dt\geq 0$$
$$\int_{-\infty}^{\infty}U(t)\cdot[f_A(t)-f_B(t)]dt\geq 0$$
$$U(t)\cdot[F_A(t)-F_B(t)]_{-\infty}^{\infty}-\int_{-\infty}^{\infty}[F_A(t)-F_B(t)]\cdot U'(t)dt\geq 0$$
$$\int_{-\infty}^{\infty}[F_B(t)-F_A(t)]\cdot U'(t)dt\geq 0 \qquad (3.1)$$

By Equation 2.1 we know that $U'(w)\geq 0$ so for Equation 3.1 to be true for all utility functions with increasing wealth preference we must have:

> *A is uniformly preferred to B under increasing wealth preference (A dominates B by* ***first-order stochastic dominance****) if and only if* $[F_B(w)-F_A(w)]\geq 0$ *for all w with at least one strict inequality.* *(3.2)*

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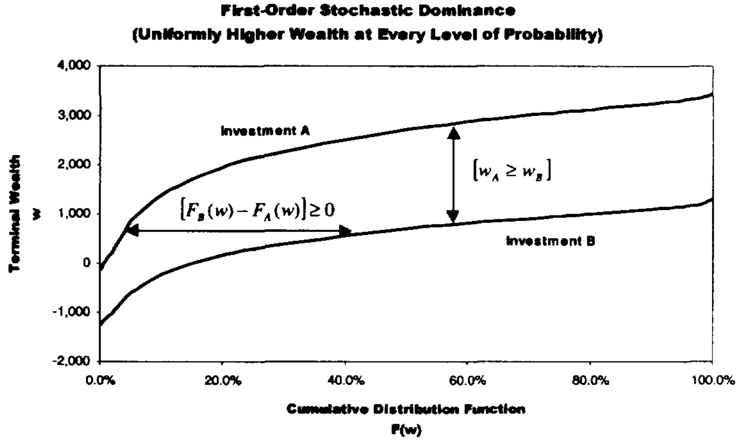
[5] A.D. Roy, Safety First and the Holding of Assets, Econometrica, July 1952

[6] Curiously, though, companies do show a willingness to tackle equally intractable investment selection measures such as ROE, allocated capital, expected policyholder deficit, etc.

Practical understanding of this constraint is straightforward if we place it on a Lee-graph.[7] This is shown in Figure 1 below. Note that this graph depicts ultimate wealth rather than ultimate loss as is commonly shown in actuarial applications.

Figure 1



This figure depicts the cumulative distribution functions for two investments A and B that satisfy Equation 3.2. From this graph we can see that first-order stochastic dominance is equivalent to uniformly higher terminal wealth at every level of probability. Accordingly, first-order stochastic dominance is a weak result; rarely will a firm be faced with such an obvious investment choice. The weakness of this result arises from the fact that first-order stochastic dominance results from a weak utility function constraint, increasing wealth preference.

Second-Order Stochastic Dominance

Let us now use a stronger utility function constraint, risk aversion, to develop investment selection criteria. We begin with the definition of preference given in Equation 1 and the risk aversion definition given in Equation 2.2. We can twice integrate Equation 1 by parts to yield[8]:

$$U'(\infty) \cdot \int_{-\infty}^{\infty} [F_B(t) - F_A(t)] dt - \int_{-\infty}^{\infty} U''(t) \int_{-\infty}^{t} [F_B(u) - F_A(u)] du dt \geq 0 \qquad (4.1)$$

Since risk aversion is a subset of increasing wealth preference we know that $U'(\infty) \cdot \int_{-\infty}^{\infty} [F_B(t) - F_A(t)] dt$ is positive. By Equation 2.2 we know that $U''(w) \leq 0$ so for Equation 4.1 to be true for all utility functions with risk aversion we must have:

A is uniformly preferred to B under risk aversion (A dominates B by ***second-order***

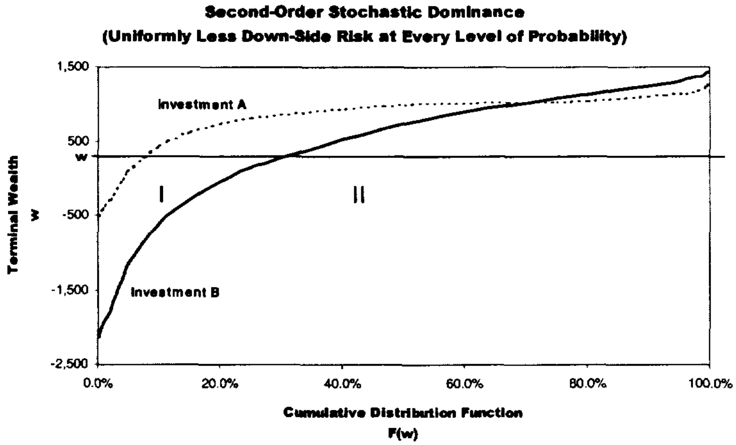
[7] Lee, Y.S., "The Mathematics of Excess of Loss Coverages and Retrospective Rating—A Graphical Approach," *PCAS* LXXV, 1988

[8] For details of this and other stochastic dominance derivations see Haim Levy, Stochastic dominance, Investment Decision Making Under Uncertainty.

***stochastic dominance)** if and only if* $\int_{-\infty}^{w}[F_B(u) - F_A(u)]du \geq 0$ *for all w with at least one strict inequality.* *(4.2)*

Again, practical understanding of this constraint is straightforward if we place it on a Lee-graph. This is shown in Figure 2 below.

Figure 2



This figure depicts the cumulative distribution functions for two investments A and B that satisfy Equation 4.2. From this graph, it is obvious that first-order stochastic dominance does not apply in this case. The two cumulative distribution functions intersect and, consequently, neither investment option results in uniformly higher wealth at every level of probability. How, then, can we recognize second-order stochastic dominance? On a Lee-graph, the limited expected value of investment A (limited to wealth *w*) is depicted by areas I and II combined. Similarly, the limited expected value of investment B is depicted by area II. Area I, then, may be interpreted as the difference between the limited expected values of investments A and B. Area I is also the constraint integral in Equation 4.2 for a specific wealth *w*. *Accordingly, second-order stochastic dominance is equivalent to a uniformly higher limited expected value at every wealth limit.*

By changing the variable of integration, it can also be shown that second-order stochastic dominance implies that area I is positive for every level of probability. This may be interpreted as "uniformly less down-side risk at every level of probability".

Third-Order Stochastic Dominance

Finally, let us use the definition of preference given in Equation 1 and the ruin aversion definition given in Equation 2.3. We can thrice integrate Equation 1 by parts to yield:

$$U'(\infty)\cdot\int_{-\infty}^{\infty}[F_B(t)-F_A(t)]dt-U''(x)\int_{-\infty}^{x}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\Bigg|_{-\infty}^{\infty}\ldots$$

$$\ldots+\int_{-\infty}^{\infty}U'''(w)\int_{-\infty}^{w}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudtdw\geq 0 \qquad (5.1)$$

Since risk aversion is a subset of ruin aversion we know that:

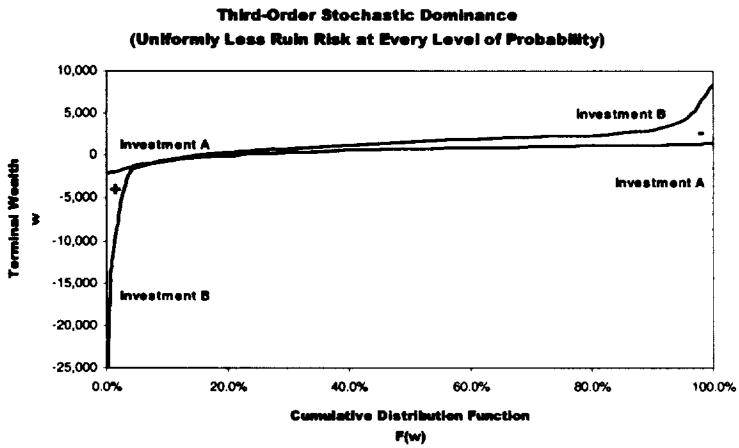
$$U'(\infty)\cdot\int_{-\infty}^{\infty}[F_B(t)-F_A(t)]dt-U''(x)\int_{-\infty}^{x}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\Bigg|_{-\infty}^{\infty}$$

:is positive. By Equation 2.3 we know that $U'''(w)\geq 0$, so for Equation 5.1 to be true for all utility functions with ruin aversion we must have:

> *A is uniformly preferred to B under ruin aversion (A dominates B by* ***third-order stochastic dominance****) if and only if* $\int_{-\infty}^{w}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\geq 0$ *for all w with at least one strict inequality.* (5.2)

This is shown graphically in Figure 3 below.

Figure 3



This figure depicts the cumulative distribution functions for two investments A and B that satisfy Equation 5.2. From this graph, it is obvious that the cumulative distribution functions intersect and first-order stochastic dominance does not apply in this case. Similarly, although not readily apparent from the graph, the negative area between the cumulative distribution functions is 50% larger than the positive area so second order dominance does not apply in this case. Investment A, however, has significantly less negative skewness (remote, but possible ruin). Unfortunately, there is no simple graphical means to explicitly test whether investment A and B satisfy the conditions of Equation 5.2.

Empirical Application

Although the dominance conditions defined in Equations 3.2, 4.2, and 5.2 appear mathematically formidable, their practical application in a stochastic simulation setting is relatively straightforward.

Consider the situation where a DFA model has been used to simulate *n* terminal wealth outcomes under two proposed reinsurance structures. Sorting those outcomes in ascending order for each structure creates estimates (albeit crude estimates) of each structure's cumulative distribution function in probability increments of *1/n*. We can then test for dominance of structure A over structure B as follows:

- Compute the difference between each percentile of structure A and the corresponding percentile of structure B; place these differences in a vector S_1. If every element of S_1 is positive then first, second, and third-order dominance apply.
- Compute the running sums of S_1 (*i.e.* for each element of S_1, compute the sum of that element and every prior element of S_1) and place in a vector S_2. If every element of S_2 is positive then second and third-order dominance apply.
- Compute the running sums of S_2 and place in a vector S_3. Then from every element of S_3 subtract the corresponding element $\frac{1}{2}$ S_2. If every element of S_3 is positive and the expected terminal wealth of each option satisfies $E_w[A]>E_w[B]$ then third-order dominance applies.

Algorithmically, this might be simply structured as follows[9]:

```
INPUT   n       number of simulations n
        A,B     vectors of terminal wealth under each respective structure
OUTPUT  order   a TRUE/FALSE triple indicating the presence of first,
                second, and third order stochastic dominance.

SORT(A, ascending)
SORT(B, ascending)

EA = p(1) * x(1)
EB = p(1) * y(1)

S1(1) = A(1) - B(1)
S2(1) = S1(1)
S3(1) = 1/2 S2(1)

FOR i = 2 TO n
   EA = (p(i) - p(i - 1)) * x(i)
   EB = (p(i) - p(i - 1)) * y(i)

   S1(i) = A(i) - B(i)
   S2 (i) = S2(i-1) + S1(i)
   S3 (i) = S3(i-1) + 1/2 [S2(i-1) + S2(i)]
NEXT i

order= {FFF}
IF MIN(S3)≥0 AND EA>EB THEN order= {FFT}
IF MIN(S2)≥0 THEN order= {FTT}
IF MIN(S1)≥0 THEN order= {TTT}

RETURN order
```

9 This algorithm is based upon a change of variable transformation of equations 3.2, 4.2, and 5.2. In the algorithm, integration occurs over the probability measure rather than over the quantiles.

Evaluating Reinsurance Alternatives

We will now apply these tools to a simple, financial guaranty reinsurance contract and examine the impact of contract features such as risk loads, profit sharing, loss layering, and co-participation. Although the details of this contract and its modeling do not impact the use of stochastic dominance analysis, a brief description is appropriate to the place the results in context.

- The underlying insurance covers default losses on consumer loans written during the policy period. Default loss rates are heavily influenced by economic variables such as unemployment.
- Premium is received periodically in proportion to outstanding principal.
- Losses are paid as incurred.
- Profit sharing is a percentage of the favorable deviation from expected underwriting performance.
- Terminal wealth of each simulated outcome is measured as the net present value (NPV) of all cash flows. Cash flows are consistent with the Feldblum internal rate of return model and include premium, loss, recovery, expense, capital, investment income, taxes, etc.[10]
- Both the primary insurer and reinsurer are assumed to allocate risk capital equal to a multiple of the expected, outstanding, undiscounted loss. Although this assumption is unrealistic, it serves to highlight the fundamentally different timing risk faced by the primary insurer and reinsurer.

The results of the simulation model are a vector of NPVs that capture the stochastic risk/reward characteristics of the modeled contract. Initially, four coverage options were considered. The simulation results for these options are summarized in Exhibits 1A-1D and in Figure 4 below.

Figure 4
Stochastic Dominance from Primary Insurer's Position

	Gross Position	$2.5M XS $5.0M and 95% Participation	$2.5M XS $5.0M and 75% Participation	$2.5M XS $7.5M and 95% Participation	$5.0M XS $7.5M and 95% Participation
Gross Position		{FFF}	{FFF}	**{FTT}**	**{FTT}**
$2.5M XS $5.0M and 95% Participation	{FFF}		{FFF}	{FFF}	{FFF}
$2.5M XS $5.0M and 75% Participation	{FFF}	{FFF}		{FFF}	{FFF}
$2.5M XS $7.5M and 95% Participation	{FFF}	{FFF}	{FFF}		**{TTT}**
$5.0M XS $7.5M and 95% Participation	{FFF}	{FFF}	{FFF}	{FFF}	

Each triple indicates first, second, and third order dominance of the column option over the corresponding row option.

This figure presents the stochastic dominance of each option over every other option.[11] For example, we can see that the $2.5M XS $7.5M option dominates the gross position by second-order stochastic dominance. Second-order stochastic dominance means that the expected utility of the $2.5M XS $7.5M option exceeds the expected utility of the gross position for all utility functions that incorporate risk aversion. In several cases, however, we see

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[10] Feldblum, S., "Pricing Insurance Policies: The Internal Rate of Return Model," CAS Study Note, May 1992.

[11] Note that, in general it is unnecessary to test the stochastic dominance of every option pairing. Dominance is transitive: if A dominates B, and B dominates C, then A dominates C. This property can be used to limit the number of comparisons required.

that neither option dominates the other. This means that the dominance results obtained using a specific utility function can always be reversed by appropriately selecting a different utility function.

Figure 4 also highlights the impact of attachment point and layer size; on an NPV basis large, high layers place the primary insurer in a better economic position. Exhibit 2A shows why this is so. These options greatly reduce the primary insurers negative NPV potential with only a modest decrease in positive NPV potential.

The reinsurer's position is shown in Figure 5 below.

Figure 5
Stochastic Dominance from Reinsurer's Position

	$2.5M XS $5.0M and 95% Participation	$2.5M XS $5.0M and 75% Participation	$2.5M XS $7.5M and 95% Participation	$5.0M XS $7.5M and 95% Participation
$2.5M XS $5.0M and 95% Participation		{FFF}	{FFF}	{FFF}
$2.5M XS $5.0M and 75% Participation	{FFF}		{FFF}	{FFF}
$2.5M XS $7.5M and 95% Participation	{FFF}	{FFF}		{FFF}
$5.0M XS $7.5M and 95% Participation	{FFF}	**{FFT}**	**{TTT}**	

Each triple indicates first, second, and third order dominance of the column option over the corresponding row option.

As might be expected, the $5.0M XS $7.5M coverage option that is most desirable from the primary insurer's position is the least desirable from the reinsurer's position. This arises from weak pricing, and the transfer of ruin risk from the primary insurer to the reinsurer. These features are shown in Exhibit 2B. This reversal in desirability will generally hold unless the primary insurer and reinsurer have vastly different capital structures.

In reality, of course, there are other considerations that motivate the selection between reinsurance alternatives. For example, an otherwise desirable option may result in significantly lower retained premium volume. This reduction in premium volume may, in turn, create accounting difficulties for the insurer. The natural response to this situation is to incorporate a profit-sharing provision. This is shown for the $5.0M XS $7.5M coverage option in Exhibits 3A-B and in Figure 6 below.

Figure 6
Stochastic Dominance from Primary Insurer's Position
Impact of 15% Profit-Sharing Provision

	$5.0M XS $7.5M and 95% Participation	$5.0M XS $7.5M, 95% Participation, and 15% Profit Share
Gross Position	**{FTT}**	**{FTT}**
$2.5M XS $5.0M and 95% Participation	{FFF}	{FFF}
$2.5M XS $5.0M and 75% Participation	{FFF}	{FFF}
$2.5M XS $7.5M and 95% Participation	**{TTT}**	**{FTT}**
$5.0M XS $7.5M and 95% Participation		{FFF}
$5.0M XS $7.5M, 95% Participation, and 15% Profit Share	**{FTT}**	

Each triple indicates first, second, and third order dominance of the column option over the corresponding row option.

The profit-sharing option increases the primary insurer's retained premium, has dominance characteristics similar to the original $5.0 XS $7.5M option, and reduces the reinsurer loss ratio by 10 points. In Exhibit 3A, note that the expected loss ratio is not the ratio of expected loss to expected premium. In this example, the profit sharing provision makes the premium and loss amounts highly interdependent. The reinsurer's expected premium is premium is lower with the profit sharing provision, but the highest premiums occur when the losses are highest. This characteristic reduces the reinsurer's expected loss ratio.

Similarly, from the reinsurer's position, we can determine the risk load required to make the $5.0 XS $7.5M option stochastically equivalent to the other options. This is shown in Exhibits 4A-B and in Figure 7 below.

Figure 7
Stochastic Dominance from Reinsurer's Position
Impact of $175K Up-Front Risk Load

	$2.5M XS $5.0M and 95% Participation	$2.5M XS $5.0M and 75% Participation	$2.5M XS $7.5M and 95% Participation	$5.0M XS $7.5M and 95% Participation	$5.0M XS $7.5M, 95% Participation and Risk Load
$2.5M XS $5.0M and 95% Participation		{FFF}	{FFF}	{FFF}	{FFF}
$2.5M XS $5.0M and 75% Participation	{FFF}		{FFF}	{FFF}	{FFF}
$2.5M XS $7.5M and 95% Participation	{FFF}	{FFF}		{FFF}	{FFF}
$5.0M XS $7.5M and 95% Participation	{FFF}	**{FFT}**	**{TTT}**		**{TTT}**
$5.0M XS $7.5M, 95% Participation and Risk Load	{FFF}	{FFF}	{FFF}	{FFF}	

Each triple indicates first, second, and third order dominance of the column option over the corresponding row option.

The risk loaded $5.0 XS $7.5M option dominates the unloaded option and is stochastically equivalent to the other options.

Summary

This paper has introduced stochastic dominance as a means for using general mathematical statements about wealth preference, risk aversion, etc. to optimally select between reinsurance alternatives. There are, however, many other utility criteria that can be treated similarly. These criteria include decreasing absolute risk aversion, dominance when investment in a risky alternative may be combined with risk-free investment, etc. [12]

The dominance computational approach described above is relatively crude but highlights the practical simplicity of these measures. In a DFA system equipped to handle large vectors, simple quadrature techniques could yield faster and more precise dominance measures.

A key factor to the success of any selection scheme is ensuring that all relevant factors enter the selection process. For a financial transaction such a reinsurance, these factors include timing and magnitude of premium, loss, expense, capital flows, etc. Net present value is an obvious measure that incorporates all of these factors, but the analyst should be careful to consider any other factors unique to the transaction.

[12] The interested reader should see Haim Levy, Stochastic dominance, Investment Decision Making Under Uncertainty.

Exhibit 1A

Discounted Cash Flow Simulation Summary

Reinsurance for: $2.5M XS $5.0M and 95% Participation

	Gross Position	Reinsurer Position	Net Position
Expected Gross Loss	6,434,702	1,261,194	5,173,507
99 %-ile Gross Loss	10,716,402	2,375,000	8,341,405
Expected Present Value Loss	4,045,420	688,741	3,356,679
Present Value Loss StDev	1,143,902	548,581	659,560
Present Value Loss CV	28.3%	79.6%	19.6%
Expected Present Value Premium	5,848,264	927,041	4,921,222
Present Value Premium StDev	35,384	5,609	29,775
Expected Loss Ratio	69.3%	74.6%	68.3%
5% EDPR Asset Requirement	7,455,214	2,160,739	5,323,744
Expected NPV	1,400,306	97,313	1,259,281
NPV StDev	1,656,289	755,786	966,253
NPV Percentile			
0.5%	(3,067,400)	(1,084,449)	(2,032,880)
1.0%	(2,604,222)	(1,052,792)	(1,632,946)
2.0%	(2,146,669)	(1,014,437)	(1,169,684)
3.0%	(1,828,486)	(993,927)	(877,782)
4.0%	(1,556,306)	(977,793)	(640,470)
5.0%	(1,400,140)	(963,954)	(485,084)
10.0%	(739,608)	(902,050)	103,688
15.0%	(339,485)	(851,936)	467,931
20.0%	16,458	(800,466)	780,819
25.0%	295,500	(739,069)	967,700
50.0%	1,425,523	98,675	1,277,937
75.0%	2,572,617	961,498	1,608,944
80.0%	2,831,681	1,071,370	1,755,052
85.0%	3,157,097	1,073,982	2,039,723
90.0%	3,543,737	1,076,043	2,423,725
91.0%	3,627,151	1,076,390	2,505,893
92.0%	3,708,663	1,076,807	2,589,458
93.0%	3,817,528	1,077,388	2,697,378
94.0%	3,944,447	1,077,824	2,823,127
95.0%	4,078,382	1,078,375	2,957,338
96.0%	4,207,987	1,079,074	3,086,430
97.0%	4,349,506	1,079,770	3,225,969
98.0%	4,572,637	1,080,653	3,448,245
99.0%	4,899,414	1,082,331	3,774,678
99.5%	5,168,612	1,083,422	4,042,043

Exhibit 1B

Discounted Cash Flow Simulation Summary

Reinsurance for: $2.5M XS $5.0M and 75% Participation

	Gross Position	Reinsurer Position	Net Position
Expected Gross Loss	6,451,096	999,204	5,451,892
99 %-ile Gross Loss	10,653,112	1,875,000	8,778,116
Expected Present Value Loss	4,056,666	546,371	3,510,296
Present Value Loss StDev	1,141,512	433,595	750,786
Present Value Loss CV	28.1%	79.4%	21.4%
Expected Present Value Premium	5,847,883	724,881	5,123,002
Present Value Premium StDev	35,333	4,380	30,953
Expected Loss Ratio	69.5%	75.7%	68.6%
5% EDPR Asset Requirement	7,473,958	1,707,329	5,788,591
Expected NPV	1,383,970	61,624	1,285,386
NPV StDev	1,652,840	650,947	1,096,849
NPV Percentile			
0.5%	(3,056,340)	(864,552)	(2,239,867)
1.0%	(2,569,597)	(839,523)	(1,765,352)
2.0%	(2,088,553)	(808,864)	(1,318,973)
3.0%	(1,819,643)	(794,607)	(1,067,332)
4.0%	(1,597,769)	(782,485)	(854,391)
5.0%	(1,421,366)	(774,091)	(698,810)
10.0%	(770,619)	(728,936)	(83,918)
15.0%	(368,627)	(689,928)	284,452
20.0%	(44,663)	(650,254)	579,899
25.0%	265,211	(602,295)	823,869
50.0%	1,438,970	77,101	1,314,519
75.0%	2,548,065	739,087	1,794,697
80.0%	2,828,032	834,105	1,971,850
85.0%	3,118,079	836,253	2,244,900
90.0%	3,492,526	837,724	2,617,799
91.0%	3,568,557	838,057	2,693,880
92.0%	3,679,274	838,443	2,804,875
93.0%	3,778,461	838,820	2,902,609
94.0%	3,901,795	839,244	3,026,551
95.0%	4,059,046	839,657	3,184,435
96.0%	4,206,809	840,246	3,330,522
97.0%	4,366,401	840,857	3,487,938
98.0%	4,605,362	841,702	3,726,486
99.0%	4,966,033	842,777	4,085,449
99.5%	5,149,202	843,568	4,268,836

Exhibit 1C

Discounted Cash Flow Simulation Summary

Reinsurance for: $2.5M XS $7.5M and 95% Participation

	Gross Position	Reinsurer Position	Net Position
Expected Gross Loss	6,462,796	286,341	6,176,455
99 %-ile Gross Loss	10,653,564	2,375,000	8,278,568
Expected Present Value Loss	4,063,954	145,729	3,918,224
Present Value Loss StDev	1,145,733	317,789	933,895
Present Value Loss CV	28.2%	218.1%	23.8%
Expected Present Value Premium	5,847,670	194,730	5,652,940
Present Value Premium StDev	35,397	1,179	34,218
Expected Loss Ratio	69.6%	75.6%	69.4%
5% EDPR Asset Requirement	7,497,643	1,975,802	6,725,875
Expected NPV	1,373,430	(87,069)	1,401,946
NPV StDev	1,658,887	416,986	1,358,544
NPV Percentile			
0.5%	(2,924,299)	(1,784,287)	(1,203,682)
1.0%	(2,559,135)	(1,747,818)	(867,906)
2.0%	(2,172,291)	(1,679,688)	(550,375)
3.0%	(1,865,060)	(1,583,820)	(410,617)
4.0%	(1,631,226)	(1,403,701)	(341,995)
5.0%	(1,457,331)	(1,252,183)	(294,399)
10.0%	(803,962)	(735,199)	(142,629)
15.0%	(362,131)	(409,520)	(21,645)
20.0%	(30,618)	(175,692)	86,754
25.0%	251,385	19,002	205,940
50.0%	1,418,219	117,984	1,241,750
75.0%	2,538,568	118,943	2,361,333
80.0%	2,818,583	119,164	2,640,833
85.0%	3,123,231	119,429	2,945,515
90.0%	3,501,341	119,742	3,322,936
91.0%	3,591,830	119,819	3,413,701
92.0%	3,688,810	119,900	3,510,436
93.0%	3,788,884	119,984	3,610,227
94.0%	3,890,887	120,072	3,712,491
95.0%	4,039,946	120,189	3,861,064
96.0%	4,207,436	120,300	4,028,644
97.0%	4,367,960	120,439	4,188,823
98.0%	4,609,218	120,639	4,430,311
99.0%	4,911,281	120,903	4,732,051
99.5%	5,149,562	121,069	4,970,000

Exhibit 1D

Discounted Cash Flow Simulation Summary

Reinsurance for: $5.0M XS $7.5M and 95% Participation

	Gross Position	Reinsurer Position	Net Position
Expected Gross Loss	6,447,604	307,354	6,140,250
99 %-ile Gross Loss	10,821,753	3,155,662	7,666,088
Expected Present Value Loss	4,055,644	156,183	3,899,462
Present Value Loss StDev	1,151,154	359,812	919,754
Present Value Loss CV	28.4%	230.4%	23.6%
Expected Present Value Premium	5,847,818	181,518	5,666,300
Present Value Premium StDev	35,559	1,104	34,455
Expected Loss Ratio	69.5%	86.9%	68.9%
5% EDPR Asset Requirement	7,497,643	2,160,739	5,369,422
Expected NPV	1,385,326	(221,598)	1,497,510
NPV StDev	1,666,722	483,755	1,338,691
NPV Percentile			
0.5%	(3,111,259)	(2,717,962)	(526,894)
1.0%	(2,687,016)	(2,395,528)	(447,661)
2.0%	(2,185,265)	(1,965,001)	(354,267)
3.0%	(1,850,594)	(1,729,253)	(296,284)
4.0%	(1,644,310)	(1,573,346)	(251,133)
5.0%	(1,436,436)	(1,390,406)	(216,695)
10.0%	(815,536)	(870,691)	(66,469)
15.0%	(382,078)	(532,050)	43,789
20.0%	(49,652)	(301,338)	154,262
25.0%	251,955	(94,298)	275,404
50.0%	1,463,452	(1,865)	1,355,963
75.0%	2,571,284	(1,408)	2,463,557
80.0%	2,849,078	(1,293)	2,740,528
85.0%	3,142,407	(1,178)	3,033,545
90.0%	3,496,940	(1,023)	3,387,437
91.0%	3,596,807	(988)	3,487,895
92.0%	3,688,003	(949)	3,578,764
93.0%	3,777,264	(910)	3,667,856
94.0%	3,903,084	(870)	3,794,067
95.0%	4,027,268	(811)	3,917,714
96.0%	4,169,303	(753)	4,059,890
97.0%	4,354,107	(696)	4,244,906
98.0%	4,572,765	(595)	4,463,199
99.0%	4,886,248	(481)	4,776,443
99.5%	5,138,597	(386)	5,028,484

Exhibit 2A

Primary Insurer's NPV Distribution

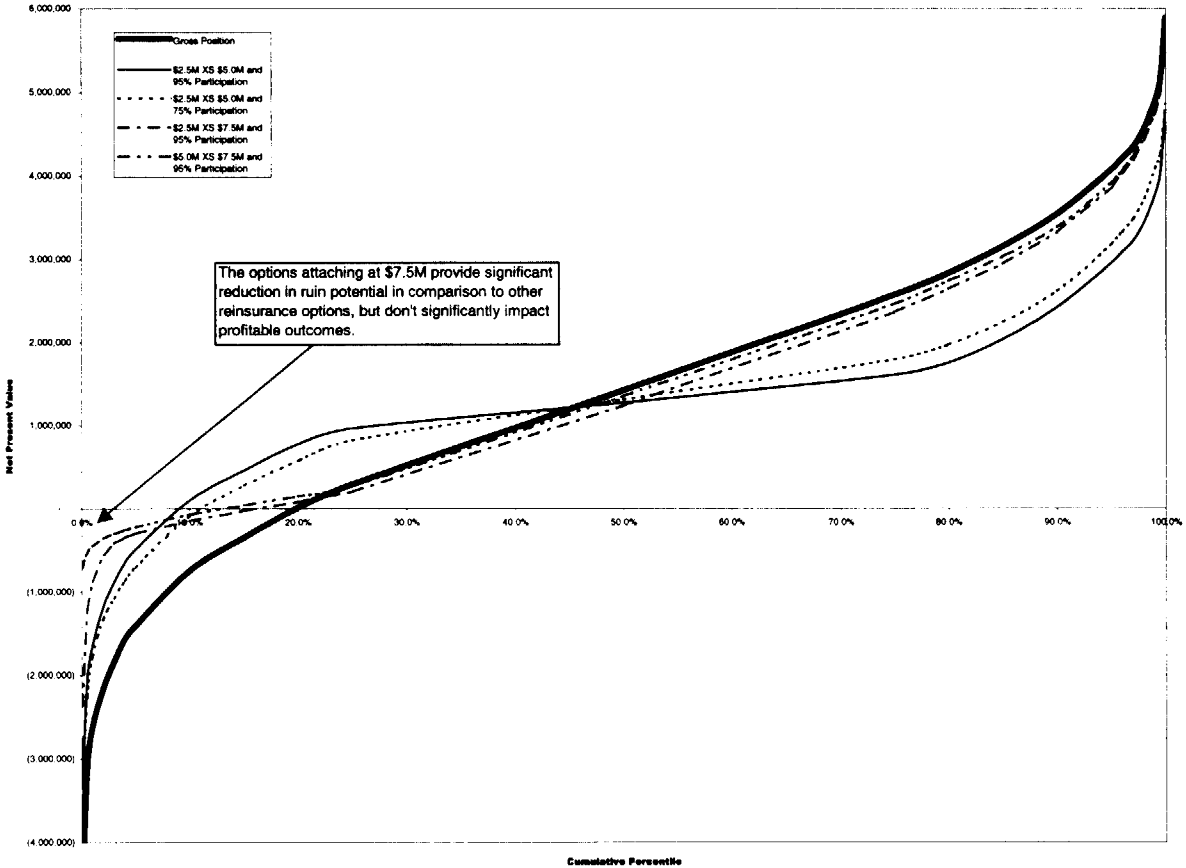

Exhibit 2B

Reinsurer's NPV Distribution

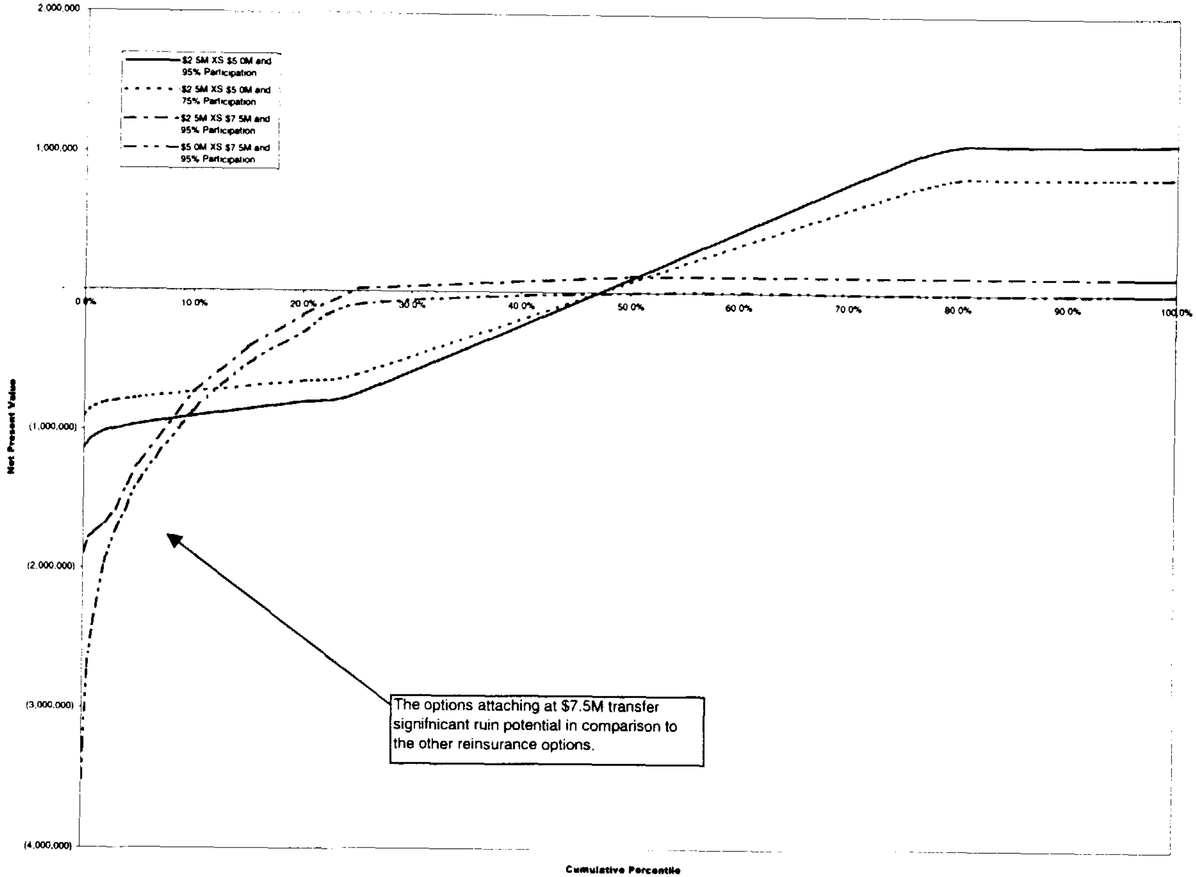

Exhibit 3A

Discounted Cash Flow Simulation Summary

Reinsurance for: $5.0M XS $7.5M, 95% Participation, and 15% Profit Share

	Gross Position	Reinsurer Position	Net Position
Expected Gross Loss	6,435,347	300,569	6,134,778
99 %-ile Gross Loss	10,869,245	3,200,782	7,668,462
Expected Present Value Loss	4,046,159	152,767	3,893,392
Present Value Loss StDev	1,149,512	359,667	920,149
Present Value Loss CV	28.4%	235.4%	23.6%
Expected Present Value Premium	5,848,224	110,172	5,738,052
Present Value Premium StDev	35,439	51,075	79,291
Expected Loss Ratio	69.3%	76.2%	68.0%
5% EDPR Asset Requirement	7,482,872	2,569,961	6,675,197
Expected NPV	1,399,213	(203,819)	1,493,624
NPV StDev	1,664,260	477,953	1,364,992
NPV Percentile			
0.5%	(3,147,559)	(2,493,850)	(793,473)
1.0%	(2,688,566)	(2,164,245)	(667,358)
2.0%	(2,142,620)	(1,742,806)	(529,959)
3.0%	(1,838,877)	(1,534,710)	(437,263)
4.0%	(1,565,092)	(1,341,805)	(365,421)
5.0%	(1,394,500)	(1,204,678)	(316,855)
10.0%	(765,971)	(775,272)	(120,783)
15.0%	(355,475)	(481,009)	17,975
20.0%	(11,471)	(256,635)	148,831
25.0%	277,677	(84,014)	283,945
50.0%	1,453,188	(11,706)	1,355,251
75.0%	2,570,952	(8,891)	2,469,628
80.0%	2,841,058	(8,212)	2,739,284
85.0%	3,153,680	(7,436)	3,051,641
90.0%	3,528,588	(6,534)	3,425,008
91.0%	3,614,095	(6,295)	3,510,601
92.0%	3,703,841	(6,097)	3,600,562
93.0%	3,818,423	(5,837)	3,714,394
94.0%	3,941,536	(5,556)	3,836,358
95.0%	4,067,541	(5,245)	3,963,807
96.0%	4,197,025	(4,856)	4,091,900
97.0%	4,364,462	(4,493)	4,258,946
98.0%	4,561,318	(4,025)	4,454,600
99.0%	4,897,503	(3,284)	4,790,311
99.5%	5,190,787	(2,482)	5,082,836

Exhibit 3B

Primary Insurer's NPV Distribution with Profit Share

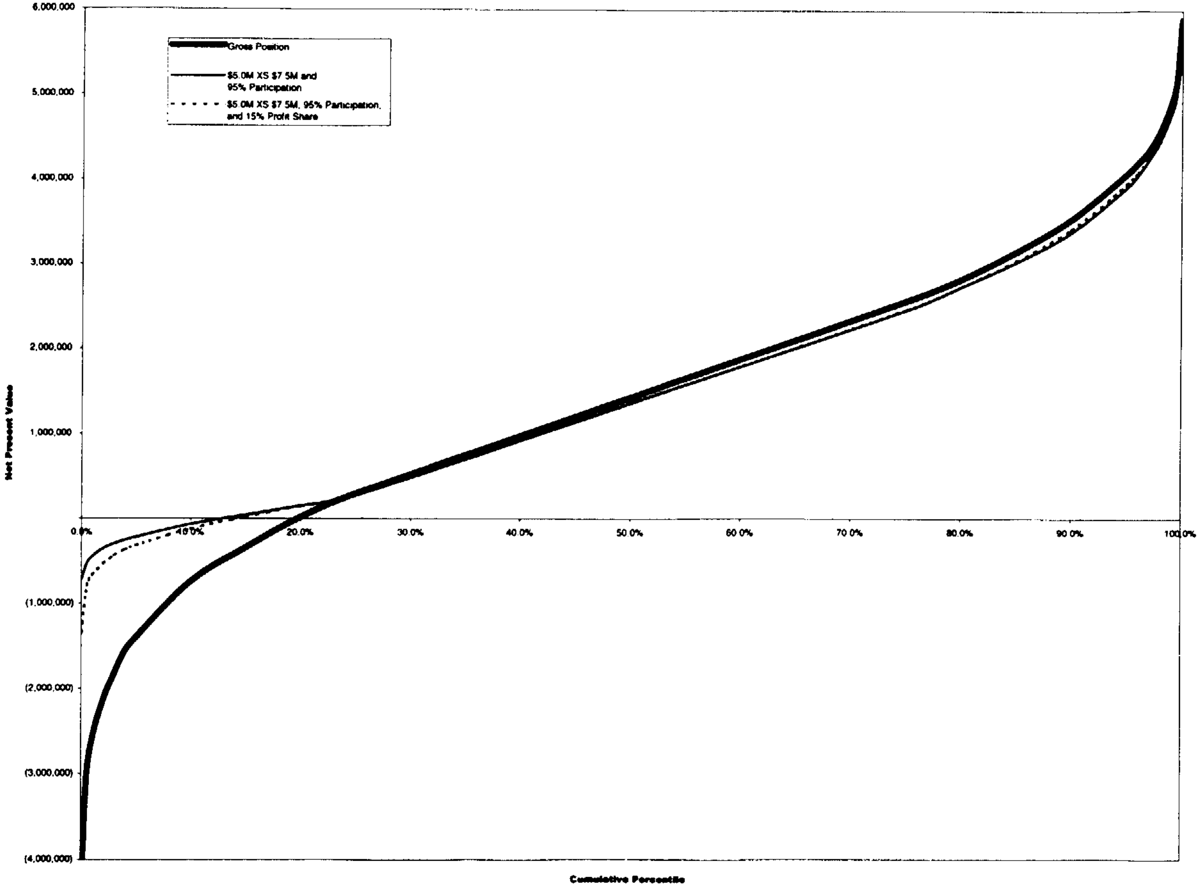

Exhibit 4A

Discounted Cash Flow Simulation Summary

Reinsurance for: $5.0M XS $7.5M and 95% Participation

$175,000 Reinsurer Up-Front Risk Load

	Gross Position	Reinsurer Position	Net Position
Expected Gross Loss	6,447,604	307,354	6,140,250
99 %-ile Gross Loss	10,821,753	3,155,662	7,666,088
Expected Present Value Loss	4,055,644	156,183	3,899,462
Present Value Loss StDev	1,151,154	359,812	919,754
Present Value Loss CV	28.4%	230.4%	23.6%
Expected Present Value Premium	5,847,818	356,518	5,666,300
Present Value Premium StDev	35,559	1,104	34,455
Expected Loss Ratio	69.4%	43.8%	68.8%
5% EDPR Asset Requirement	7,497,643	3,488,047	4,819,579
Expected NPV	1,385,326	(46,598)	1,497,510
NPV StDev	1,666,722	483,755	1,338,691
NPV Percentile			
0.5%	(3,112,871)	(2,542,962)	(701,894)
1.0%	(2,643,474)	(2,220,528)	(622,661)
2.0%	(2,147,691)	(1,790,001)	(529,267)
3.0%	(1,829,527)	(1,554,253)	(471,284)
4.0%	(1,587,342)	(1,398,346)	(426,133)
5.0%	(1,426,016)	(1,215,406)	(391,695)
10.0%	(802,720)	(695,691)	(241,469)
15.0%	(362,892)	(357,050)	(131,211)
20.0%	(42,692)	(126,338)	(20,738)
25.0%	253,091	80,702	100,404
50.0%	1,456,500	173,135	1,180,963
75.0%	2,587,327	173,592	2,288,557
80.0%	2,860,000	173,707	2,565,528
85.0%	3,145,366	173,822	2,858,545
90.0%	3,490,693	173,977	3,212,437
91.0%	3,587,072	174,012	3,312,895
92.0%	3,688,946	174,051	3,403,764
93.0%	3,792,679	174,090	3,492,856
94.0%	3,903,975	174,130	3,619,067
95.0%	4,026,003	174,189	3,742,714
96.0%	4,171,133	174,247	3,884,890
97.0%	4,344,469	174,304	4,069,906
98.0%	4,548,910	174,405	4,288,199
99.0%	4,867,692	174,519	4,601,443
99.5%	5,175,732	174,614	4,853,484

Exhibit 4B

Reinsurer's NPV Distribution with Risk Load