DETERMINING THE PROPER INTEREST RATE FOR LOSS RESERVE DISCOUNTING: AN ECONOMIC APPROACH

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Abstract

By examining the underlying economic principles of insurance and finance, this paper shows how the proper interest rate for reserve discounting is a function of the degree of risk present in the outstanding reserve. When loss reserves are certain, the discounting interest rate is shown to be the market interest rate for a riskless security having a duration matching that of the loss payment. The unpaid loss is then allowed to be uncertain, and the risk adjustment to the discounting interest rate is derived.

An analysis of empirical property-liability data over a 15-year period is performed using a pricing model which incorporates the risk-adjusted interest rate. Results indicate that the risk adjustment for aggregate industry loss reserves is about three points of yield rate.

The effect of income taxes on the discounting interest rate is then explored. The appropriate rate in this case is shown to be based upon the risk-adjusted rate in the absence of taxes, and is identical to the pre-tax value when the tax reserve evaluation uses the risk adjustment. Finally, several other applications of the risk-adjustment method are given, including asset valuation, reinsurance and pricing of products.

INTRODUCTION

Historically, property-liability loss reserves have been maintained, at least in principle, at full, undiscounted value. The life insurance industry, on the other hand, has traditionally valued its liabilities on a discounted basis. Although there are some major differences between the two industries regarding the nature of their respective liabilities (life reserves are generally more certain and longer-term), proponents of reserve discounting have argued that these two industries should share the same accounting treatment.

The argument over the appropriateness of reserve discounting for the P/L industry may now be academic, since the 1986 Tax Reform Act has brought loss reserve discounting to the property-liability industry rather abruptly. Currently, the accounting community is generally in agreement (for example, see Reference [3]) that loss reserves should be carried at a discounted value. At this point, few would disagree that discounted reserves provide a more economically sound measure than undiscounted, or full-value reserves. Instead, debate centers over <u>which</u> accounting methods, such as GAAP or statutory, should use discounted reserves, and <u>how</u> reserves should be discounted.

The focus of this paper is a major element of the "how" of reserve discounting: how should the <u>interest rate</u>, through which the reserve is brought to present value, be determined? By examining underlying economic principles of insurance and finance, we will show how the appropriate interest rate is a function of the degree of <u>risk</u> present in the outstanding reserve. The risk-adjusted interest rate, being lower than yield rates available in the market, falls between the two extremes of not discounting (a zero interest rate) and discounting with a market rate.

To develop the concept of the risk-adjusted interest rate, we establish basic assumptions, including riskless loss reserves, and show how these imply a riskless discounting rate. This interest rate is a market rate for a security with a duration the same as the expected loss payment. Next, we allow the unpaid loss to be uncertain, and derive the risk adjustment using the notion of a reinsurer who assumes the loss reserve.

To assist in practical applications, we then analyze historical property-liability results to obtain empirical values of the risk adjustment. Then the effect of income taxes on our results is discussed in relation to the reserve valuation used for taxation. Finally, we explore some applications of the risk-adjustment procedure, including asset valuation, reinsurance and pricing of new products.

CERTAINTY MODEL FOR RESERVE VALUATION

Basic Assumptions

We begin with some basic assumptions, which later will be relaxed to provide more general, and more useful results. For brevity, we will use the term "loss" to include loss adjustment expense.

1. The amount and timing of loss payments are known with certainty. Of course this is unrealistic, but it serves to emphasize what happens when we later examine reserve valuation under uncertainty. An obvious implication of the certainty assumption is that the undiscounted reserve is fairly stated (at its unbiased, or "best" estimate).

2. Results are before income taxes. This assumption simplifies the analysis, and as shown later, the pretax interest rate is the basis for the reserve evaluation after taxes.

Separation of Asset and Liability Valuation

The economic rationale behind reserve discounting is that, since losses are paid in the future, less than a dollar of current invested assets is needed to pay a dollar of future loss. This principle has been historically construed to mean that the insurer's <u>own assets</u> should determine the interest, or yield rate by which loss reserves are discounted. However, there are severe problems with this approach. The insurer's actual assets should <u>not</u> determine the interest rate for several reasons, both economic and logical:

First, two insurers having identical full-value loss reserves but different assets would show different discounted reserves, since the yield rates on those assets would differ. The same liabilities should have the same value.

Second, the amount of invested assets for an insurer does not necessarily equal the amount of loss reserves, discounted or otherwise. Which assets, among those in the portfolio, all having differing yield rates, does one set aside to pay the future losses? The answer is unclear.

Third, two insurers could have identical liabilities and identical assets, say bonds, but purchased at different prices. The yields to maturity would differ, using the amortized cost basis of GAAP and statutory accounting. Here the accounting system gets in the way, since using the market value of the bonds would produce the same yield rates and therefore, same discounted reserves.

The above examples illustrate why the valuation of liabilities should be independent of the valuation of assets. Further, the insurer's actual assets should not influence the value of liabilities. This is particularly important for determining the discounting interest rate. Since the insurer's own assets are unsuitable as a basis for the discounting rate, we need to select a hypothetical asset for this purpose. Let's examine the criteria for choosing this asset.

Eliminating Default and Interest Rate Risk

Since we have assumed that the losses are certain, it is reasonable to expect that the value of the hypothetical asset will accumulate with interest to a certain amount sufficient to pay the loss. In other words, there must be a zero probability that the asset will default or otherwise drop in value. This criterion eliminates corporate and municipal bonds, as well as all categories of stocks and real estate. The only remaining candidates are those assets issued or backed by the U.S. Government.

The list of candidates for our hypothetical asset can be narrowed further when we consider the duration to maturity of the ideal asset. (Technically, the duration measurement includes the coupon payments as well as the face value of a bond, but for clarity we will assume no coupons are paid in our numerical examples.) The duration of the asset should match that of the loss payment. Otherwise, the underlying asset's value would be insufficient to pay the loss if either interest rates dropped and the term to maturity were less than the loss duration, or if interest rates increased and the term were greater than the loss duration.

To illustrate, assume that a loss payment of $121 is to be paid two years from now, and we buy, for $100, a Treasury note (having no interim interest payments) due one year from now. Its yield is 10%, so we get $110 a year from now. As long as we can reinvest the $110 at a 10% yield or greater a year from now to produce at least $121 two years from now, this arrangement will work. Suppose, however, that the yield drops to 8% a year from now; the $110 when reinvested will produce only $118.80 when it is time to pay the $121 claim.

Conversely, if we buy a three-year note yielding 10% for $100 (a value at maturity of $133.10), we can sell it to pay the claim for $121 in two years as long as interest rates hold at the 10% level. But if the market rate after two years increases to say 12%, then the value of the note at that point drops to 133.10/1.12, or $118.84, also insufficient to pay the loss.

Clearly, the duration of the hypothetical asset must match the duration of the loss in order to guarantee sufficient funds to pay the loss. This matching process is called immunization, and is well-known in the actuarial and financial literature (see [6], [9], [10] and [12]).

Imbedded vs. Market Interest Rate: the Exchange Principle

In our preceding numerical example, we assumed a 10% interest rate when we initially discounted the loss by buying a two-year riskless note for $100. In other words, the present value of the $121 loss payable two years hence, was $100. One year later, what is the present value? Some would argue that it is $110, and this amount is independent of the prevailing, or market interest rate at that time.

Their reasoning says that, as long as the original asset is held, its value will exactly match the $121 loss payment. Therefore, the value of the asset at one year is the amortized value, or $110 at the original purchase price and 10% yield. Hence the present value of the loss is also $110, or a 10% discount. In other words, the interest rate for discounting remains fixed through time for reserves previously recognized. Since GAAP and Statutory accounting methods record asset values at amortized cost, the interest yield from assets backing the unpaid losses would be the imbedded or portfolio yield, based on the purchase price, and not the market yield on the same securities.

The alternative argument is based upon the economic concept of value being worth in exchange. Suppose we wanted to transfer the $121 liability to a reinsurer at one year. If the yield rate at that time for the hypothetical asset were 12%, then the reinsurer would only need $108.04, and not $110, to purchase an asset to match the $121 loss due in one year. Thus, the underlying economic value of the loss is obtained by discounting at 12%, not 10%. On this basis, the appropriate interest rate is the market rate, not the imbedded yield based upon amortized value.

Using the imbedded yield for reserve valuation has economic meaning only if the underlying asset is in fact riskless with matched duration and it and the loss are valued together as a package. Since this condition is rarely met in practice, and we have already demonstrated why assets and liabilities should be valued independently, it is clear that market yield rates should determine the discounting interest rate.

Summary

The preceding analysis has shown that, for a certain unpaid loss amount, the appropriate discounting interest rate is a market rate, based on an asset which has no default risk, and has the same duration as the loss.

UNCERTAINTY MODEL FOR RESERVE VALUATION

We now bring the certainty model a step closer to the real world by allowing the unpaid loss to fluctuate randomly, in some sense, around its expected value. In other words, although we don't know the true value of the loss we have a "best guess" regarding that value, and that estimate equals the mathematical average of all possible values which the unpaid loss could take.

Loss Transfer to a Reinsurer: The Risk Adjustment

As previously discussed, one way to conceptualize the value of a liability is to consider what it would be worth if exchanged for cash. We shall call this the economic value. If paid enough, there will always be a reinsurer willing to assume the loss reserve. In the certainty model, the fair value for this exchange was the present value of the unpaid loss, discounted at the riskless yield rate. Is this the proper rate for a risky loss reserve?

To find the answer, we shall introduce the concept of a reinsurer whose only business is to assume loss reserves from other insurers. Obviously, this reinsurer would require some equity (known also as net worth, capital or surplus) to prevent insolvency in the event that the loss reserves develop adversely. We shall assume that the amount of equity is proportional to the amount of the reserve.

In fact, the required equity should be proportional to the discounted reserve. This is because the equity is itself invested, and will grow in proportion to the prevailing interest rate. The greater the interest rate, the more equity will be available to offset potential unfavorable unpaid loss development.

Now we can show the balance sheet of the hypothetical assuming reinsurer at the time the loss reserve is ceded (time = 0):

Assets	Liabilities	
Investments: $V_o(1+e)$	Discounted Reserve:	V_o
	Equity:	$E_o = eV_o$
	Total:	$V_o(1+e)$

Here, the ceding company paid an amount V_o in cash to the reinsurer, which is required to hold a fraction e of the discounted reserve as supporting equity. The reinsurer thus puts up an amount eV_o in cash, and has a total amount of $V_o(1+e)$ to invest. The balance sheet is in fact in balance, with assets equal to liabilities of $V_o(1+e)$.

Assume that the reinsurer invests the cash in a riskless asset (matched to the proper loss duration) which has an interest rate i. Although the reinsurer invests its <u>assets</u> at i, it must expect to produce a return on <u>equity</u> greater than i, because it is now liable for an uncertain, risky future loss payment. Let us denote the expected return on equity by R. Also, for convenience assume that the losses will still be unpaid after one year. Now examine the reinsurer's expected balance sheet one year later (time = 1):

Assets		Liabilities	
Investments:	$(1+e)V_0(1+i)$	Discounted Reserve:	$V_1 = V_0(1+i_A)$
		Equity:	$E_1 = E_0(1+R)$
		Total:	$(1+e)V_0(1+i)$

The total assets, which must equal total liabilities, have increased by a factor of $1+i$. Since we require a return on equity of R, the equity must increase by a factor of $1+R$. To keep the balance sheet in order, the reserve must grow by some factor $1+i_A$. The value of i_A which fits these constraints can be found directly, since

(1) $E_1 = E_0(1+R) = eV_0(1+R)$. This amount also equals the difference between total liabilities and the discounted reserve V_1:

(2) $(1+R)eV_0 = (1+e)V_0(1+i) - V_0(1+i_A)$.

This equation reduces easily to

(3) $i_A = i - e(R-i)$.

The above relationship is simple and yet has profound implications. Since the capital requirement e and the excess return $R-i$ are both positive, it is clear that i_A, the interest rate for discounting risky reserves, must be <u>less than the riskless yield rate</u>.

If the discounting interest rate instead were the riskless rate, then the cash V_0 when invested in the riskless asset, would accumulate to exactly match the expected loss. Equity, however, would grow only at the rate i. Thus the reinsurer would get an expected return on equity equal to the riskless rate even though bearing risk. The inherent unfairness of this proposition would, of course, force the price of the reserve transfer to move upwards; i.e. the loss would be discounted at the lower rate $i_A < i$ in order to make the deal fair to both ceding and assuming parties.

The difference between the "market value" (V_o) of the reserve and the reserve discounted at the riskless rate, is the reinsurer's expected profit. This would be zero if there were no risk in the transaction.

Notice that Equation (3) is similar to the well-known Capital Asset Pricing Model, or CAPM (see Sharp [11] for background) which is

$$(4) \quad r = i + b(R-i).$$

Here r is the required return on a particular asset, R is the average return for all risky assets, and b is the "beta," or systematic risk. The price for risk, b(R-i) is a positive increment to the riskless yield. The greater the beta, the higher the required return. Since the CAPM applies to assets, and has a positive risk adjustment, it seems reasonable that in a pricing model for liabilities, the risk adjustment should be negative. In fact, Myers and Cohn [8] use such a negative liability beta for discounting losses in a pricing application.

An Illustrative Example

More insight into the economic basis for the risk-adjusted interest rate can be gained by using a simplified numerical example. Assume that we have an uncertain loss with an expected value of $110.25 to be paid in two years. The riskless yield rate is 8%, required equity is 25% of the discounted reserve, and the required return on equity is 20%.

Equation (3) produces a risk-adjusted discount rate of <u>5%</u> = .08 - .25(.20-.08), which is three points less than the riskless rate. Discounting the $110.25 for two years at this rate gives an economic value of $100, an amount which our hypothetical reinsurer is willing to accept in exchange for the reserve. The following exhibit shows how the reinsurer's expected balance sheet will appear at various stages:

					2	
Time	0		1		Before Loss Payment	After Loss Payment
Reserve	100	→(5%)→	105	→(5%)→	110.25	0
Equity (Pre-Dividend)	25		30		31.50	31.50
Assets (Pre-Dividend)	125	(8%) ↗	135	(8%) ↗	141.75	31.50
Dividend	-		3.75		-	31.50
Assets (Post-Dividend)	125		131.25		-	0
Required Equity	25		26.25		-	0
ROE			20%		-	20%
Equity Flow	-25		+3.75		-	+31.50

The reinsurer starts with $100 of cash from the ceding insurer, to which it adds $25 of equity funds to offset the risk of the uncertain reserve. The $25 of the reinsurer's own funds is a negative flow of equity.

The $125 of total assets is invested risk-free at 8%, accumulating to $135 a year later. Meanwhile, the economic value of the loss reserve has increased to \$105, since it is only a year from payment. Equity is therefore 135 - 105 = \$30, which is 20% greater than the \$25 initial capital. However, the needed equity is only .25 (105) = \$26.25, so the extra \$3.75 is returned as a dividend (positive equity flow). Alternatively, the \$3.75 could be used to back an additional assumption of 3.75/.25 = \$15 of discounted reserves.

The dividend reduces assets to \$131.25, which grow at 8% to \$141.75 at the end of the second year. This amount is reduced to \$31.50 after the loss is paid. With no remaining reserve, there is no need to hold equity against the reserve, and so the \$31.50 is returned as a final dividend. Again the equity value (\$31.50) represents a 20% return on the needed equity at the start of the period (\$26.25).

In this example, the risk premium, or reinsurer's expected profit, is the difference between the reserve discounted at the risk-adjusted rate (\$100) and the reserve discounted at the riskless rate (\$110.25÷(1.08x1.08) = \$94.52), or \$5.48. If the price for the reserve transfer were \$94.52, then the reinsurer would expect to earn only an 8% return on equity.

Besides earning a 20% return on the required equity during each year, the reinsurer also realizes a 20% <u>internal rate of return</u> (IRR) on the equity flows. The IRR is the interest rate at which the present value of the cash flows, positive and negative, must equal zero. For our example, we get

$$0 = -25 + 3.75(1.2)^{-1} + 31.5(1.2)^{-2}.$$

Appendix I demonstrates algebraically that, when the risk-adjusted interest rate is used for discounting, the internal rate of return on equity flows equals the return on required equity in each calendar period.

Effect of Interest Rate Movements on the Risk Adjustment

Since the required return on equity, R, is itself an interest rate (the investor in an insurance company demands a yield on funds invested), it will tend to move in parallel with market interest rates. Assume that the required return is always a constant spread c above the market riskless rate i. Then if the market rate changes to i', the required return becomes $R' = i'+c$. The risk adjustment is $e(R'-i') = ec = e(R-i)$.

Thus, the value of the risk adjustment will be independent of the level of interest rates. This property will simplify applications to reserve discounting, because we can assume that the risk adjustment for a particular type of reserve should remain stable over time.

With the risk adjustment being stable over time, it is possible that if interest rates become low enough, the risk-adjusted yield rate (for particularly risky types of reserves) would become negative. In other words, the proper "discounted" reserve would be greater than the best-estimate undiscounted reserve!

Loss Reserve Transfers: the Settlement Rate

The idea of determining the discounted reserve through a marketplace transfer is shared by the AICPA Task Force on Discounting Applications, in their September 9, 1987 Issues Paper [3, p.55]. Their advisory conclusion is that a settlement rate should be used to determine the present value of future cash flows in recognizing "monetary liabilities with uncertain terms." They further state that the discount rate should be such that

> "If the liability is capable of being settled currently, the rate inherent in the price at which an independent third party (that is, other than the creditor) would assume the liability currently."

The accounting issue presented here is how to determine the settlement rate for a loss reserve when there is no "market" for the particular type of reserve being considered. Although portfolio transfers (ceding reserves to another insurer) are increasingly common, they are done by a minority of insurers. The AICPA task force recommends that a risk-free rate be used in discounting reserves for which a settlement rate cannot be objectively determined.

We argue, however, that in practice all loss reserves are risky and to discount them properly, a risk adjustment is necessary, even if the magnitude of the adjustment must be approximated.

The following section develops a method for estimating the risk adjustment, and provides an approximate value for its use in discounting the typical property-liability loss reserve.

EMPIRICAL RESULTS

Determining the Riskless Yield Rate

Determining the risk-free interest rate is relatively straightforward. It should be the yield on the U.S. Government treasury note or bond whose duration corresponds to the anticipated duration of loss payment. Since a loss reserve normally consists of a series of expected payments, ideally each payment at each different duration should determine a distinct yield rate. However, for most applications, using a single yield rate based on the average duration, should be sufficient. The applicable yield rates should be the market rates available (found daily in the Wall Street Journal, for example,) at the reserve evaluation date.

The following table shows average yield rates on treasury securities over the last eleven years:

TABLE 1
U.S. Treasury Securities: 1976-1986 Average Yield Rates

Year	90-Day	3-5 Year	10-Year
1976	4.97%	6.98%	7.61%
1977	5.27	6.84	7.42
1978	7.19	8.30	8.41
1979	10.07	9.62	9.44
1980	11.43	11.51	11.46
1981	14.03	14.34	13.91
1982	10.61	12.96	13.00
1983	8.61	10.62	11.10
1984	9.52	12.07	12.44
1985	7.48	9.89	10.62
1986	5.98	7.19	7.68

Source: Data Resources Inc. [5]

Determining the Risk Adjustment: Industry Pricing Model

Having found the riskless yield rate (i) without much difficulty, the risk adjustment, e(R-i), can also be found, but not quite so readily.

To facilitate estimating the risk adjustment, we will use a basic pricing model developed from our earlier results. The model uses average industry cash flows, expense and loss ratios to produce implicit ROE and equity requirements. The following simplifying assumptions are used:

1) The average policy has a one-year term.

2) At the time a policy is written, the loss reserve is defined to equal the expected loss (normal accounting methods instead set up an unearned premium reserve, which translates into a loss reserve as the premium is earned and losses are incurred). The reason for this definition is that a significant portion of the risk of writing insurance is related to the uncertainty of losses which will arise from unearned exposure. This is known as pricing risk, and is at least as great as that from adverse loss development (reserve risk), given that the losses have already occurred.

3) The amount of equity needed to support the pricing risk from expected loss in the unearned premium reserve equals that for the same magnitude of loss reserves. Some would argue that the pricing risk is greater than the reserve risk (for example, see Aldin and Jones [1]). We will later test the sensitivity of our results to the allocation of equity between pricing and reserve risk.

4) Premiums are collected, on average, .25 years after the policy effective date.

5) Commissions and taxes, licenses and fees are paid as premiums are collected. Other underwriting expenses are paid 50% when the policy is written and 50% evenly over the policy term (giving an average payment date of .25 years).

6) Policyholder dividends are paid 2.25 years after the policy effective date.

7) All non-loss cash flows are discounted at the Treasury security rate which matches the average loss duration. As discussed later, premiums should actually be discounted at a rate slightly greater than the riskless rate (since there is credit risk present), but the results are not sensitive to this assumption.

8) There is no investment expense. Since the explicit investment strategy is passive, using only Treasury securities, the cost should be negligible.

9) All results are before federal income taxes. Their effect on our findings will be explored later.

The preceding assumptions are admittedly crude and the interested reader is encouraged to fine-tune them. Nevertheless, we are now nearly armed with enough tools to produce a result. Next we define some additional notation:

Symbol	Meaning
P	Premium minus underwriting expenses; premium is 1 unit
D	Expected dividends
L	Expected losses and loss expenses
Z	Risk adjustment; $e(R-i)$
V_m	Mean discounted reserve over the policy term; includes expected loss portion of unearned premium
f	Fraction of loss unpaid at the end of the policy year
u	Average duration of P cash flows
w	Average duration of dividends paid
t	Average duration of loss & loss expense payments
C	Present value of cash flows at policy inception

The present value of the net cash flows from the policy is

(5) $C = P(1+i)^{-u} - D(1+i)^{-w} - L(1+i_A)^{-t}$.

The present value of the policy cash flows at the effective date is the cash-equivalent worth of the policy, since the risky expected loss payments have been discounted to adjust for the price of the risk. However, to hold this policy, equity is required since the actual loss payments will be risky. Just as in the earlier hypothetical reinsurer case, the value of the contract can be determined by investing the cash equivalent C in a riskless security with yield i. The required equity (averaging eV_m over the policy term) is also invested to produce riskless yield i. The return on equity, measured at the end of the policy term, is therefore

(6) $R = [(1+i)(eV_m + C)/eV_m] - 1$. This expression reduces to

(7) $Z = e(R-i) = (1+i)C/V_m$.

The average reserve over the policy term is $.5L(1+f)$ and the average discounted reserve is approximately $V_m = .5L(1+f)(1+i-Z)^{.5-t}$. Now we let $A = (1+i)/.5(1+f)L$ and $B = P(1+i)^{-u} - D(1+i)^{-w}$, which are known quantities, and finally we get

(8) $Z = A[B - L(1+i-Z)^{-t}]/(1+i-Z)^{.5-t}$

which can be solved for Z. Notice that we do not need to determine either e or R. For a given level of risk, if the required equity is reduced/increased, the expected return (in excess of the riskless return) is increased/reduced to compensate. Consequently, e and R-i are inversely related, and their product is constant.

Industry Data and Model Results

By examining industry data over the recent past, we can infer the risk adjustment, assuming that the actual performance over this period is commensurate with the risk borne by the industry. Unfortunately, we will never know if the industry was as profitable as it should have been.

From Best's Aggregates and Averages [2] we have complied average results over the period 1976 through 1984. This is a complete underwriting cycle, from the year after the peak of the previous cycle (high combined ratio) to the latest peak year. This approach was used by the Insurance Services Office in the 1987 Insurer Profitability Study [7]. Using a complete cycle reduces the impact of loss reserving bias on underwriting results, since reserves are relatively weak when underwriting results are poor and conversely when results are good (see Conning [4] for an analysis of this phenomenon).

The industry results are summarized below:

TABLE 2
1976-1984 Average Combined Ratio Components

	Nominal Value	Duration	Present Value
Loss and Loss Expense	.767 = L	2.30	Depends on Z
Underwriting Expense	.268	0.25	.262
Policyholder Dividends	.016	2.25	.013
Combined Ratio	1.051		
Premium	1.000	0.25	.977
Premium - U/W Exp. - Div.	.716	0.25	.702 = B

The present values above are computed using $i = .0972$, which is the average of the 90-day and 3-5 year Treasury security yields from Table 1. This composite has an average duration of about two years, which is close to the mean loss duration of 2.3 years. This value was calculated from 1985 industry Schedule P and Schedule O data, using the method employed in obtaining the loss payment patterns for reserve discounting under the 1986 Tax Reform Act. The 2.3-year duration is consistent with the result (1.72 years from occurrence date, plus .5 years from policy effective date to occurrence date) obtained by Woll [13] using a more sophisticated method.

The 1985 Schedules P and O data also indicate that 40.9% of losses are paid after one year; therefore $f = .591$. The average undiscounted reserve over the policy term is therefore 79.6% of incurred loss, or .610. We now have all the variables needed in Equation (8): $A = 1.799$, $B = .702$, and $L = .767$. Finally, we get $Z = .043$. Thus, with equal pricing and reserve risk, the implicit risk adjustment to the riskless yield is 4.3 points over this period.

This analysis can be applied to industry data over the preceding (1970-1975) underwriting cycle as well. The average combined ratio was 100.9%, corresponding to a lower riskless yield of 6.4%. Assuming that the mean loss duration was slightly less, at 2.2 years (with $f = .58$), the implicit risk adjustment is 3.7 points for this period. Given the approximate nature of these estimates, the risk adjustments for the two cycles are in fair agreement. Presuming that the two values should theoretically be equal, these results might suggest that the 1970-75 cycle was less profitable than the recent 1976-84 cycle.

Incorporating Pricing Risk

Our results so far have assumed that pricing risk (in the expected loss portion of the unearned premium reserve) requires the same equity as loss reserve risk. Now assume that pricing risk (in proportion to expected loss) requires k times as much equity as reserve risk. The risk adjustment for discounting the loss reserve is now

$$(9)\quad Z = A'[B - L(1+i-kZ)^{-.5}\,(1+i-Z)^{.5-t}]/(1+i-Z)^{.5-t}.$$

where $A' = A(1+f)/(k+f)$. The following are the implied risk adjustments from the historical data, as a function of k:

	Risk Adjustment				
Cycle	k=1.0	k=1.5	k=2.0	k=2.5	k=3.0
1970-75	.0370	.0317	.0277	.0246	.0220
1976-84	.0435	.0374	.0327	.0291	.0262

The true value of k is probably in the range of 1.5 to 2.5, giving roughly a 3-point value to the risk adjustment. A more precise value could be obtained by applying the model to lines of business having widely different loss payment durations, such as property and general liability. Nevertheless, the preceding results show that the risk adjustment is materially large, and should not be ignored.

THE EFFECT OF INCOME TAXES

The presence of federal income taxation poses an additional complication to the development of the discounting interest rate.

Certainty Model

We can appreciate how income taxes affect the discounting interest rate by again considering a hypothetical reinsurer in a numerical example.

Assume that a certain loss payment of $108 is due one year from now, and the riskless yield is 8%; the income tax rate is 30% and taxes are recognized and paid at annual intervals. A premium of $100 is paid to the reinsurer assuming the loss reserve. The value now of the future loss payment will depend upon the basis of reserve valuation for tax purposes:

	Tax-Basis Reserve Valuation			
	Discounted at 8%		Undiscounted	
	t = 0	t = 1	t = 0	t = 1
Premium	$100	$0	$100	$0
Reserve Value	100	0	108	0
Paid Loss		108		108
Underwriting Income	0	-8	-8	0
Investment Income	0	8	0	8.19
Taxable Income	0	0	-8	8.19
Tax: U/W Income	0	-2.4	-2.4	0
Inv. Income	0	2.4	0	2.46
Total	0	0	-2.4	2.46
Assets	100	0	102.4	0.13

In the case of reserve discounting at the riskless rate for tax purposes, the premium equals the reserve initially, so there is a zero underwriting gain and no tax liability. One year later, the loss is paid for \$108 and the reserve changes from \$100 to 0, giving on \$8 underwriting loss. This amount exactly offsets the investment income generated, creating zero taxable income and zero taxes. Assets have accumulated to \$108 with investment income, and are reduced to zero when the loss is paid. Thus no profit is returned to the reinsurer in this transaction -- which must be the case, because the reinsurer has borne no risk.

If the tax basis is undiscounted reserves, then the same transaction will produce an initial underwriting loss, having a cash value (the tax loss can be offset against taxable income from other sources) of \$2.40. This amount is invested at 8%, producing an additional \$0.19 of pretax investment income, or \$0.13 after-tax. The \$0.13 equals the after-tax investment income on the underwriting loss tax benefit of \$2.40, or .056 x 2.4. In order for the value of this transaction to equal zero, the premium must be reduced to \$99.82. The effective discounting interest rate is therefore 8.195%, which is greater than the riskless rate of 8%.

In general, let d be the duration until a loss of 1 unit is paid, P_0 the fair premium or after-tax discounted reserve (\$99.82 above), and let the tax-basis reserve be discounted at interest rate h. The preceding logic allows us to determine (the derivation is shown in Appendix II; for the certainty case, $i_A = i$):

(11) $P_0 = [(i-h)(1+j)^{-d} - (i-j)(1+h)^{-d}]/(j-h)$,

where $j = (1-T)i$ is the after-tax riskless interest rate. When the discounting rate h equals the riskless rate i, we get $P_o = (1+i)^{-d}$; i.e., the fair value of the transaction is the reserve discounted at the pretax riskless rate. This relationship underlies the rationale for the discounted reserve provision of the 1986 Tax Reform Act, where the Federal Midterm rate (which is close to the Treasury rate) is used to discount reserves on a pre-tax basis.

If h is less than i, then we see how the tax benefit created by using undiscounted reserves could be enormous if the duration to payment is long and/or the tax rate is high. In particular, if T = 46% (the tax basis prior to 1987) and i = 8%, we get the following results for a future loss payment of 1 unit.

	P_o		
d (Years)	h=.08	h=0	Effective Discount Rate (h=0)
0	1.000	1.000	8.00%
1	.926	.923	8.31
2	.857	.850	8.48
5	.681	.647	9.10
10	.463	.361	10.72
18.4	.243	0	Infinite

This unintended "subsidy" of insurers by the U.S. government did not go unnoticed. It was removed by the 1986 Tax Reform Act, which requires reserve discounting.

Uncertainty Model

In the case of certain unpaid losses, using the pretax riskless yield rate for reserve evaluation (both as a tax basis and for income accounting) makes economic sense, since the fair exchange value for the reserve does not depend upon the tax rate. Stated differently, if an insurer sold a policy making a zero profit in the absence of the tax law, the same policy should also generate zero income under the provisions of the tax law.

However, when the loss reserve is <u>uncertain</u>, the riskless rate is not appropriate. Appendix II derives the uncertainty version of Equation (11):

$$(12) \quad P_o = [(i-i_A)(j-h)V_o + (i-j)(h-i_A)J_o - (i-j)(j-i_A)U_o]/Y,$$

where $Y = (j-h)(j-i_A)$, $V_o = (1+i_A)^{-d}$ is the pretax economic (risk-adjusted) value of the reserve, $J_o = (1+j)^{-d}$ is the reserve discounted at the after-tax riskless yield, and $U_o = (1+h)^{-d}$ is the tax-basis discounted reserve. Notice that the after-tax economic reserve P_o is the weighted average (the coefficients sum to 1) of the three types of discounted reserves: V_o, U_o and J_o.

For the case where there are no taxes, then $j = i$ *and we get* $P_o = V_o$, *which* matches our earlier result. Also, when the tax discounting rate h, equals the *risk-adjusted discount rate* i_A, *we get* $U_o = V_o$ *and thus Equation (12) reduces* to $P_o = V_o$. In other words, the tax basis which preserves the economic value of the reserve liability is that which discounts using the <u>risk-adjusted pretax yield rate</u>.

The following example illustrates this point. Here we assume that $i = 8\%$, $T = 30\%$, $R = 20\%$ and $e = .25$. Thus, $i_A = 5\%$. An expected loss of \$105 is paid one year from now: a reinsurer takes on the liability in exchange for the risk-adjusted fair value, or premium, of \$100:

	Discounted at 8%		Discounted at 5%	
	t = 0	t = 1	t = 0	t = 1
Premium	100.00		100	
Reserve (Tax Basis)	97.22	0	100	0
Paid Loss		105.00		105.00
Required Equity	25.00	0	25	0
Underwriting Income	2.78	-7.78	0	-5.00
Investment Income	0	9.93	0	10.00
Tax: U/W Income	0.83	-2.33	0	-1.50
Inv. Income	0	2.98	0	3.00
Total	0.83	0.65	0	1.50
Assets	124.17	28.45	125	28.50
Return on Equity (After-Tax)		13.80%		14.00%

Since the pretax required ROE is 20% and the tax rate is 30%, the after-tax required ROE is 14%. This value is achieved only if the pretax risk-adjusted interest rate of 5% is used in valuing the reserve. Using the pure riskless rate creates a tax penalty which is not commensurate with the economic value of the liability. Because the 1986 Tax Reform Act mandates a yield rate (currently 7.2%) which is not risk-adjusted, the industry is perhaps being burdened with a tax liability which is larger than intended.

Effect of Taxes on Empirical Results

Because there had been an economic benefit (now removed) to holding undiscounted reserves for tax purposes, our earlier estimates of the risk adjustment can be modified to remove the effect of the tax benefit. Over the 1975-84 underwriting cycle, the corporate tax rate was 46%, but due to the ability to hold tax-exempt bonds having a yield higher than the after-tax taxable bonds, the *effective* tax rate was much lower. (Essentially, an insurer with an underwriting loss, prior to 1987, never had to pay income taxes, since it could generate just enough taxable investment income to offset the underwriting loss and invest the balance of its portfolio in tax-exempt bonds; the cost of this strategy, or the effective tax, is the lost investment income due to the difference in yields between taxable and tax-exempt bonds.)

Prior to 1987, the effective tax rate can be determined by the ratio of tax-exempt to taxable bond yields of equivalent risk. The average ratio for AAA Municipal to Corporate bond yields over 1976-84 (see Data Resources, Inc. [5]) was .702, giving an effective tax rate of 29.8%, which rounds to 30%.

Using Equation (12) with the 1976-84 cycle values of $i = .0972$, $T = .3$ and $d = 2.3$, we arbitrarily set $R-i = .15$ and solve for e. This produces the pretax risk adjustment Z corresponding to the effective risk adjustment Z', which is understated in our earlier analysis due to the tax benefit of using undiscounted reserves:

Z'	i'_A	e	$Z = .15e$
.040	.0572	.284	.0426
.030	.0672	.221	.0331
.020	.0772	.158	.0237

The above results show that the tax impact on the estimated risk adjustment is about 0.3 percentage points of yield, and therefore our earlier conclusion that the risk adjustment is roughly 3 points remains unchanged.

Cost of Discounted Reserves Provision of 1986 Tax Reform Act

The handy Equation (12) can be used once more to estimate the economic cost of the reserve discounting feature in the 1986 Tax Reform Act. Here we compute P_o, the economic value of the expected loss, on a newly written policy with reserve discounting for taxes at the current h = .072 and with the undiscounted h = 0; we also assume that the riskless yield is 8%, with an effective tax rate of 30%:

Average Duration of Unpaid Loss (Years)	P_o (h = 0)	P_o' (h = .072)	$(P_o-P_o')/P_o$
1	.95283	.95130	0.16%
2	.90829	.90389	0.48
2.3	.89541	.88992	0.61
3	.86621	.85777	0.97
4	.82644	.81293	1.64
5	.78882	.76935	2.47

The cost of the reserve discounting provision increases with the average loss duration and with the market interest rate. For the industry-average loss duration of 2.3 years, the cost is about 0.6% of incurred loss and loss expense or about 0.5% of premium. This result is not surprising when one considers that over the life of an average policy (from inception to when the final loss payment is made) the tax deduction for losses is the same whether or not they are discounted. Reserve discounting merely defers the deduction, so that the economic cost is the lost investment income on the cash used to pay the accelerated taxes, which are subsequently recovered.

Summary

We have demonstrated that the appropriate interest rate for reserve discounting under income taxation should be the same as that without taxes:

the <u>pretax riskless yield minus the risk adjustment</u>. If the tax basis uses a different rate, however, there should either be a correction to the risk-adjusted yield or the difference between the after-tax economic reserve value (P_0) and the pretax value (V_0) should be treated as a separate asset or liability.

APPLICATIONS

General Loss Reserve Discounting

It is important to know the economic value of a company, not only for mergers and acquisitions, but also to provide sound internal management decisions. A major element of economic valuation is the worth of loss reserves, which should be determined using a risk-adjusted discount rate.

The empirical results we have derived are crude and serve only to indicate the approximate magnitude of the risk adjustment. Since combined P/L industry data for all lines of business were used, the results will apply to a typical line of busines. The analysis presented here could be extended to individual Annual Statement lines to deduce implied risk adjustments by line.

In the absence of data, one could also apply judgement to determine line differences in the risk adjustment. For example, Workers' Compensation reserves should have a lower risk than Other Liability reserves, even though the average payment durations are about the same, because Workers' Compensation loss reserves consist partly of fixed, more predictable, life pension benefits.

A major distinction of risk by reserve type would be case vs. IBNR reserves. Case reserves apply to known claims, and should be less subject to adverse development (after being adjusted for bias) than claims which have not even been reported. Assuming a 3-point risk adjustment for the composite of case and IBNR reserves, the risk adjustment would probably be about 1.5 to 2.5 points for case reserves and about 3.5 to 5 points for IBNR reserves.

Reinsurance

Our results can be readily adapted to portfolio transfers of loss reserves. Once both parties to the transaction agree on the undiscounted reserve, applying the risk adjustment will provide the assuming reinsurer's profit margin.

Discounting with the risk-adjusted yield rate would also apply to the commutation of individual claims subject to a reinsurance treaty.

Valuing Assets and Other Balance Sheet Items

The method employed to evaluate loss reserves will also apply to other balance sheet items. On the liability side, we have already discussed the valuation of the expected loss portion of the unearned premium reserve. The risk adjustment is probably greater than that for the equivalent loss reserve once the losses have occurred (the riskiness of the expected loss, prior to occurrence, is quite likely even greater than that of IBNR for the same type of business).

Assets can also be valued using the risk-adjustment. Premiums receivable are analogous to reserves, since there is a time element: cash is collected in the future. Generally, the collection of the premium is not certain, because the insured or agent may not pay. Therefore the credit risk requires equity (denoted by e_c) in proportion to the discounted receivable.

Again we use the concept of an external party (perhaps a bank), who will pay an amount S to accept the receivable of 1 unit one year from now. The bank will need to put up e_cS in equity to support the receivable worth S now and

$S(1+i_A) = 1$ a year from now. Total cash (now) is $e_cS - S$, a negative amount, and grows to $(e_cS-S)(1+i)$ with riskless interest in one year (the bank must borrow to finance the receivable). This amount is increased by the expected premium collection of $S(1+i_A)$; the total amount equals the initial equity plus required return on equity: $(e_cS-S)(1+i) + S(1+i_A) = e_cS(1+R)$, which reduces to

(13) $i_A = i + e_c(R-i)$.

Therefore, the proper interest rate for evaluating a risky asset is greater than the riskless rate. Not surprisingly, this relationship looks like the Capital Asset Pricing Model, having a positive risk adjustment, shown in Equation (4). In general, the yield adjustment for credit risk will be much smaller than the reserve risk adjustment. Banks tend to have a much lower equity-to-loan ratios than insurers have equity-to-reserve ratios.

Product Pricing

The pricing model used to determine empirical estimates of the risk adjustment is similar in concept to the Myers-Cohn model and with the same inputs will give results identical to the internal rate of return model used by Aldin and Jones, the National Council on Compensation Insurance and others. (Recall that Appendix I shows the equivalence of the risk-adjustment method and the IRR model.)

More importantly, however, we have demonstrated how the risk and return elements necessary for a pricing application are also incorporated in the appropriate method for economic evaluation of loss reserves and other balance sheet items.

Accounting For Income

The risk-adjustment process allocates equity to cover risk inherent in the various balance sheet components, mainly loss and unearned premium reserves. This method implies an accounting model wherein profits are earned over the life of the policy (until all losses are paid) according to the reduction in risk relating to the change in balance sheet items. This has intuitive appeal, and reflects the economic evaluation of the insurance transactions.

This accounting model lies between the two extremes of statutory accounting (undiscounted reserves, with profits being delayed until future investment income is realized) and current tax accounting (discounted reserves using a full unadjusted yield rate, where all income is recognized over the policy term).

CONCLUSION

We have studied the problem of the proper interest rate for reserve discounting from an economic perspective -- what is the worth of a loss reserve in exchange to another party? The problem is easily solved when the reserve is certain, and becomes more difficult when risky reserves are considered, with income taxes being an additional complication.

To restate our major results, we have determined that:

1. The interest rate is a market rate, for a U.S. Government security, or portfolio of securities, with a duration equal to that of the expected loss payments.

2. The above rate must be reduced to adjust for the risk of adverse loss development. The adjustment equals the product of the required equity to support the risky reserve and the excess of the required return on equity over the riskless yield.

3. The discounting interest rate must be a pretax value. If the tax basis is not the risk-adjusted pretax yield rate, the difference between the pretax economic (risk-adjusted) present value and the after-tax economic value of the loss reserve should be explicitly recognized.

4. The approximate value of the risk adjustment is 3 points of interest for a typical reserve. This number would vary depending

on the nature of the reserve and should be higher for IBNR than for case reserves. Further research into determining the risk adjustment by reserve category will be welcome.

5. By examining the relationship between risk, required equity and required return on equity, we have demonstrated how the risk-adjusted yield rate is appropriate for both evaluation of loss reserves (and other balance sheet items) and pricing of products.

6. The accounting model implied by the risk-adjustment procedure earns profits over the life of the policy (not just over the policy term) according to the reduction of risk associated with the change in balance sheet items. This consequence of economic evaluation earns income slower than with reserves discounted at market interest rates, but faster than with undiscounted reserves.

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APPENDIX I

<u>Equivalence of Internal Rate of Return on Equity Flows and Calendar-Period Return on Equity</u>

Using the notation in the text, the equity, before dividend, at time t+1 is $eV_t(1+R)$. The required equity is $eV_t(1+i_A) = eV_{t+1}$. Therefore, the dividend (equity flow) at time t+1 is $d_{t+1} = eV_t\ (R-i_A)$. Assume that the loss is paid at t = n. At that time the dividend d_n will be returned as well as the accumulated needed equity eV_n. The present value of the dividends discounted at the required return on equity R is

$$\text{(I-1)} \qquad PV = eV_n(1+R)^{-n} + \sum_{t=0}^{n-1} eV_t(R-i_A)(1+R)^{1-t}$$

Since $V_t = V_o(1+i_A)^t$ and $\sum_{t=0}^{n-1} a^t = (1-a^n)/(1-a)$, the result becomes

$$\begin{aligned}\text{(I-2)} \quad PV &= eV_n(1+R)^{-n} + e(R-i_A)V_o(1+R)[1-(1+i_A)^n(1+R)^{-n}]/[1-(1+i_A)(1+R)^{-1}] \\ &= eV_n(1+R)^{-n} + eV_o[1-(1+i_A)^n(1+R)^{-n}] \\ &= eV_o\end{aligned}$$

Since the initial equity contribution is a $-eV_o$ equity flow, the present value of the total equity flows is <u>zero</u>, and the internal rate of return on the equity flows is R.

APPENDIX II

Risk-Adjusted Yield Rate for Loss Reserves With Income Taxes

Using the notation in the text, we get the following balance sheet items and cash transactions for a hypothetical reinsurer assuming an expected loss of 1 unit to be paid in d years:

	t=0	t=1
Fair Premium	P_o	P_1
Loss Reserve (Tax)	U_o	$U_1 = U_o(1+h)$
Loss Reserve (Economic)	V_o	$V_1 = V_o(1+i_A)$
Tax Payment	$T(P_o-U_o)$	$T(U_o-U_1)$
Tax Liability	$T(P_o-U_o)$	$T(U_1-P_1)$
Invested Assets	$I_o = P_o+eV_o-T(P_o-U_o)$	$I_1 = I_o(1+j)-T(U_o-U_1)$
Needed Equity	$E_o = eV_o$	eV_1
Actual Equity	$E_o = eV_o$	$E_1 = I_1-P_1-T(U_1-P_1)$

There is an immediate tax liability of $T(P_o-U_o)$, since the taxable underwriting profit is P_o-U_o. This tax must be paid at t = 0, and thus reduces (increases, if $U_o > P_o$) the initial assets of the premium P_o plus the required equity E_o.

The invested assets grow at the riskless rate i; but the investment income is taxed at the tax rate T one year later, so the assets actually grow at the after-tax riskless rate j. Invested assets at t=1 are further reduced (increased) by the tax on the underwriting gain (loss) U_o-U_1 due to the change in the tax-basis discounted reserve. Equity at t=1 equals the invested assets minus the value of the reserve and its accompanying tax liability (if the reinsurer were to cede the reserve at this point for its fair value P_1, it would have to pay a tax on the U_1-P_1 underwriting gain of the transaction). Thus, the equity after one year, E_1, should equal $I_1-P_1-T(U_1-P_1)$, which also should equal the required after-tax return on equity or $R' = (1-T)R$. Therefore,

(II-1) $$eV_o(1+R') = [P_o + eV_o - T(P_o-U_o)](1+j) - T(U_o-U_1) - T(U_1-P_1) - P_1.$$

This reduces to

(II-2) $$(1+j)P_o = P_1 + V_o(i-i_A) - (i-j)U_o.$$ In general,

(II-3) $$(1+j)P_t = P_{t+1} + V_t(i-i_A) - (i-j)U_t.$$

This relationship can be solved recursively, starting backwards from $P_d = 1$ to P_{d-1} and so forth, to P_o. After some manipulation, we finally get

(II-4) $$P_o = [(i-i_A)(j-h)V_o + (i-j)(h-i_A)J_o - (i-j)(j-i_A)U_o]/Y$$

where $Y = (j-h)(j-i_A)$, with $J_o = (1+j)^{-d}$, $U_o = (1+h)^{-d}$ and $V_o = (1+i_A)^{-d}$.